Homework 4: Review of Linear Algebra

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Problem 4.1. Consider:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 4 & -1 \\ -2 & 0 & 3 \\ 0 & -4 & 4 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 3 & -1 & 2 & 4 \\ -2 & 4 & 2 & 1 \\ 1 & 2 & -4 & -3 \end{bmatrix}.$$

Compute $\mathbf{AB}$, $\mathbf{BA}$, $\mathbf{A}^\mathsf{T}\mathbf{B}^\mathsf{T}$, and $\mathbf{B}^\mathsf{T}\mathbf{A}^\mathsf{T}$, where $\cdot^\mathsf{T}$ denotes the *transpose* operator.

Problem 4.2. Consider:

$$\mathbf{x} = \begin{bmatrix} 8 \\ -3 \\ 0 \\ 1 \\ -7 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} -5 \\ 9 \\ -4 \\ 6 \\ -2 \end{bmatrix}.$$

Compute $\|\mathbf{x} - \mathbf{y}\|_1$ and $\|\mathbf{x} - \mathbf{y}\|_2$.

Problem 4.3. Let

$$\mathbf{U} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 5 \end{bmatrix}.$$

Compute the projection of $\mathbf{x}$ onto $\text{span}\{\mathbf{U}\}$.

Problem 4.4. Compute the minimizer and minimum of

$$g(\beta) \;=\; 3\beta^2 - 5\beta + 4.$$

Problem 4.5. Let $f(\mathbf{W}, \boldsymbol{\beta})$ be a function with parameters $\mathbf{W}$ and $\boldsymbol{\beta}$. Suppose you want to find

$$\underset{\mathbf{W},\boldsymbol{\beta}}{\arg\min}\; f(\mathbf{W}, \boldsymbol{\beta}).$$

To this end you will use gradient descent, with initial points given by:

$$\mathbf{W}_0 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad \boldsymbol{\beta}_0 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

If

$$\nabla \mathbf{W}\Big|_{t=0} = \begin{bmatrix} 0.2 & -0.3 \\ -0.1 & 0.2 \end{bmatrix}, \qquad \nabla \boldsymbol{\beta}\Big|_{t=0} = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix},$$

what are the values of $\mathbf{W}_1$ and $\boldsymbol{\beta}_1$ after the first iteration of gradient descent?

Problem 4.6. In your preferred language, code gradient descent, and test it on g from Problem 4.4.

(a) Deliver your code.

(b) What value of η did you choose?

(c) What minimizer did you obtain?

(d) What minimum did you obtain?

(e) Do these values agree with your answer from Problem 4.4? How would you fix any discrepancies?