

Decision Trees & Random Forests

BUGS Meeting

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Decision Trees

Goal: Predict

- Will I get El Cáncer?
- Will I develop Diabetes?
- Is my boyfriend/girlfriend cheating on me?
- Will my Bacteria develop Antibiotic Resistance?

	A	B	C	D	E	F	G	H
1	0.166390734	0.0739022542	0.9380422919	0.4197909476	0.199372394	0.8773471711	0.0832315471	0.9400430233
2	0.8615271223	0.7757295209	0.56219381	0.4953289942	0.7901157516	0.3168325007	0.0901134394	0.5927609941
3	0.9898183863	0.1606356235	0.6343072962	0.6000025657	0.2041446567	0.8752995632	0.9870366976	0.9585429376
4	0.4160293543	0.6541336076	0.533131226	0.2588331876	0.5838729884	0.5999851158	0.5517696308	0.6071065792
5	0.015721427	0.3214682985	0.78565533648	0.1555887529	0.724996398	0.3202909518	0.0761947159	0.1158882901
6	0.6832840093	0.1885837403	0.4081572187	0.2237776744	0.3958876091	0.7385271892	0.0843965665	0.3581091606
7	0.2270948424	0.0753671026	0.0494126878	0.1149176771	0.5402233903	0.9147158584	0.8683215391	0.4956815843
8	0.559706779	0.3547628012	0.6737842499	0.4400304139	0.3052613081	0.9529025748	0.908458523	0.7750171428
9	0.0639803573	0.8327802755	0.754443808	0.673489338	0.8825760537	0.5428943438	0.0870754241	0.725444772
10	0.8509917245	0.1453037842	0.969216953	0.9897150856	0.9354477939	0.9258715529	0.1733232688	0.6480473382
11	0.2713187919	0.2857065005	0.903035834	0.9298100988	0.1260013632	0.0767446451	0.4399159383	0.2829078324
12	0.3704964726	0.4851600723	0.443642485	0.9373102742	0.7847810986	0.3864274416	0.7541396178	0.3867283773
13	0.9759758923	0.7371094345	0.0455157282	0.8346701767	0.4383846298	0.6298020934	0.0993899375	0.5817343793
14	0.6351593339	0.3977721159	0.5993215961	0.2587002467	0.1375043502	0.2660007051	0.1666408479	0.8403079875
15	0.0716924155	0.6319312733	0.3823427393	0.6420220807	0.2364369316	0.3835056194	0.5611884128	0.3820433638
16	0.8422852685	0.542874712	0.5883257678	0.760320741	0.8315882224	0.6276306058	0.5803535685	0.8411028145
17	0.9408779606	0.5220144195	0.4812654063	0.5331868522	0.1047852791	0.7975936416	0.8026251938	0.9925388684
18	0.1095553513	0.2625067541	0.9741186467	0.9848365255	0.2483271887	0.0985997624	0.2721076927	0.1593648221
19	0.4997927954	0.096071915	0.2369617911	0.8118898307	0.6460952105	0.8707811364	0.4498663761	0.254943708
20	0.0429012946	0.0803355075	0.5060224588	0.9520570131	0.1408375208	0.1400302523	0.9902096502	0.0871451208
21	0.7057224081	0.4630195114	0.9144776522	0.3820045677	0.6039446075	0.8557347907	0.9241318835	0.4712547897
22	0.2484984729	0.2377738538	0.3104894005	0.3790386592	0.5252200689	0.1912480153	0.8910088711	0.0505195535
23	0.5463155243	0.5151090947	0.085890304	0.506719203	0.0661523314	0.0243307806	0.0956105	0.685405273
24	0.1472119791	0.4978118786	0.3711289708	0.8731593913	0.1267880907	0.1878780541	0.9781146096	0.5520577445
25	0.790510121	0.4327639828	0.7940384434	0.0472936828	0.4442862018	0.2649206629	0.6887534886	0.1463450387
26	0.238609707	0.3432042131	0.0508109827	0.053046589	0.8862833569	0.4834029363	0.7091768233	0.9173125281
27	0.2727451844	0.7989924805	0.8074715317	0.8630560155	0.520064408	0.9395466181	0.7433063779	0.7300025792
28	0.0233547923	0.892362818	0.8213618146	0.0381375295	0.7760785248	0.9095269332	0.4853258794	0.3151255539
29	0.431277638	0.8813389968	0.6464634317	0.1806829796	0.0149378479	0.1696688208	0.0184137947	0.4957891628
30	0.9908212749	0.394212218	0.7033778366	0.671705693	0.3933258315	0.6299235066	0.3324489791	0.7547388354
31	0.3143209	0.6560834548	0.4499266401	0.4371305082	0.0300377	0.4826313867	0.608E405084	0.3349117569
32	0.8232727759	0.9862914453	0.2903483235	0.4740527456	0.5629339886	0.5561574434	0.9774761845	0.778319096

Here is my Data
 How do I know? Entropy

Other Variables (age, gender, etc.)

Samples →

Variable of Interest (cancer, diabetes, cheater)

Here is my Data

How do I know? Entropy

THE FOLLOWING **PREVIEW** HAS BEEN APPROVED FOR
ALL AUDIENCES
BY THE MOTION PICTURE ASSOCIATION OF AMERICA INC.

THE FILM ADVERTISED HAS BEEN RATED



www.filmratings.com

www.mpaa.org

Horse	1	2	3	4	5	6	7	8	Length
P(winning)	1/2	1/4	1/8	1/16	1/32	1/32	1/32	1/32	
Message	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1	3 bits



Message



Info Theory & Entropy

A Quick Detour

Horse	1	2	3	4	5	6	7	8	E[Length]
P(winning)	1/2	1/4	1/8	1/16	1/32	1/32	1/32	1/32	
Message	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1	3 bits
Optimal	0	10	110	1110	1111 00	1111 01	1111 10	1111 11	2 bits



Message →



Info Theory & Entropy

A Quick Detour

This is what you need to remember:

$$H(x)$$

Amount of **information** encoded in variable x

How frequent is this outcome?

$$= \sum_x p(x) \log \frac{1}{p(x)}$$

Average over all outcomes?

How many bits should I spend encoding this outcome?

(more likely outcomes get fewer bits)

Info Theory & Entropy

A Quick Detour

	Gender	PhD?
1	0	0
2	1	0
3	1	0
4	0	0
5	0	0
6	0	0
7	1	0
8	1	0
9	1	0
10	1	1
11	0	0
12	0	0
13	0	0
14	1	0
15	0	0
16	1	0

$$H(x) = \sum_x p(x) \log \frac{1}{p(x)}$$

Info Theory & Entropy

Example

	Gender	PhD?
1	0	0
2	1	0
3	1	0
4	0	0
5	0	0
6	0	0
7	1	0
8	1	0
9	1	0
10	1	1
11	0	0
12	0	0
13	0	0
14	1	0
15	0	0
16	1	0
$p(0)$	1/2	15/16
$p(1)$	1/2	1/16

$$\begin{aligned}
 H(x) &= \sum_x p(x) \log \frac{1}{p(x)} \\
 &= p(0) \log \frac{1}{p(0)} + p(1) \log \frac{1}{p(1)} \\
 &= \frac{1}{2} \log(2) + \frac{1}{2} \log(2) \\
 &= \frac{1}{2} + \frac{1}{2} \\
 &= 1
 \end{aligned}$$

Info Theory & Entropy

Example

	Gender	PhD?
1	0	0
2	1	0
3	1	0
4	0	0
5	0	0
6	0	0
7	1	0
8	1	0
9	1	0
10	1	1
11	0	0
12	0	0
13	0	0
14	1	0
15	0	0
16	1	0
$p(0)$	1/2	15/16
$p(1)$	1/2	1/16
$H(x)$	1	

$$\begin{aligned}
 H(x) &= \sum_x p(x) \log \frac{1}{p(x)} \\
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 &= 1
 \end{aligned}$$

Info Theory & Entropy

Example

	Gender	PhD?
1	0	0
2	1	0
3	1	0
4	0	0
5	0	0
6	0	0
7	1	0
8	1	0
9	1	0
10	1	1
11	0	0
12	0	0
13	0	0
14	1	0
15	0	0
16	1	0
$p(0)$	1/2	15/16
$p(1)$	1/2	1/16
$H(x)$	1	

$$\begin{aligned}
 H(x) &= \sum_x p(x) \log \frac{1}{p(x)} \\
 &= p(0) \log \frac{1}{p(0)} + p(1) \log \frac{1}{p(1)} \\
 &= \frac{15}{16} \log\left(\frac{16}{15}\right) + \frac{1}{16} \log(16) \\
 &= \frac{15}{16}(0.093) + \frac{1}{16}(4) \\
 &= 0.337
 \end{aligned}$$

Info Theory & Entropy

Example

Most informative!



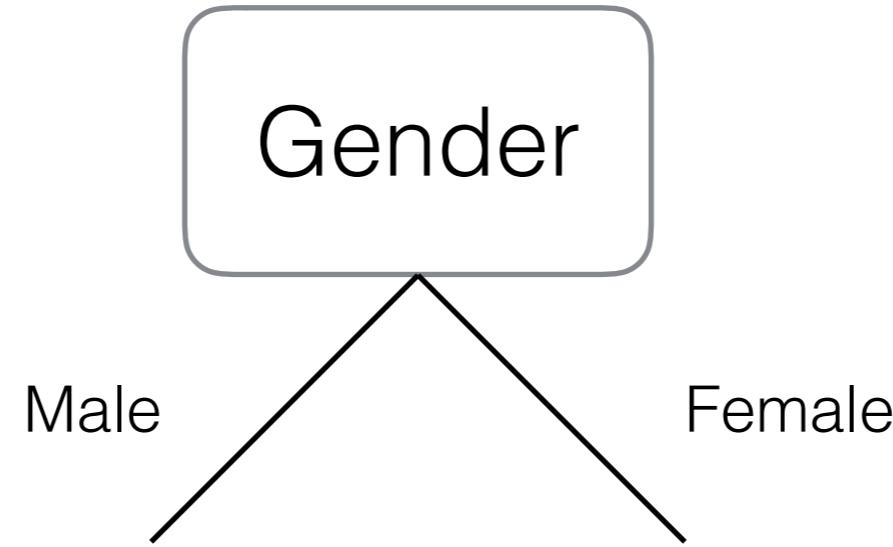
	Gender	PhD?
1	0	0
2	1	0
3	1	0
4	0	0
5	0	0
6	0	0
7	1	0
8	1	0
9	1	0
10	1	1
11	0	0
12	0	0
13	0	0
14	1	0
15	0	0
16	1	0
$p(0)$	1/2	15/16
$p(1)$	1/2	1/16
$H(x)$	1	0.337

$$\begin{aligned}H(x) &= \sum_x p(x) \log \frac{1}{p(x)} \\&= p(0) \log \frac{1}{p(0)} + p(1) \log \frac{1}{p(1)} \\&= \frac{15}{16} \log\left(\frac{16}{15}\right) + \frac{1}{16} \log(16) \\&= \frac{15}{16}(0.093) + \frac{1}{16}(4) \\&= 0.337\end{aligned}$$

Info Theory & Entropy
Example

Gender

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ			
1	0.13630734	0.07302542	0.058420513	0.419509476	0.10633204	0.873474111	0.063231547	0.54043233	0.77754913	0.54791554	0.206252101	0.0302497031	0.530728796	0.053051506	0.106493486	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0													
2	0.8917222	0.7152959	0.61951981	0.49548942	0.910517516	0.21882607	0.060135294	0.59260641	0.48745017	0.57597	0.23468935	0.602625275	0.5024484816	0.5024484816	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0													
3	0.41023535	0.51832078	0.53372109	0.28851578	0.505872084	0.09893581	0.517106320	0.870719572	0.55580	0.7589	0.09157509	0.063224605	0.710719572	0.60482770	0.710719572	0.09157509	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0												
4	0.16517217	0.32168298	0.78581364	0.155898759	0.72496348	0.36239016	0.07947157	0.151882409	0.312958253	0.479774162	0.063224605	0.492718404	0.059051506	0.185567155	0.059051506	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0												
5	0.22709484	0.047587202	0.549128072	0.211047177	0.540223308	0.04718884	0.588215391	0.666881943	0.706598941	0.053271619	0.605232151	0.548454783	0.587970553	0.088992020	0.286324605	0.362390120	0.610804588	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0										
6	0.59570579	0.347452861	0.21751784	0.440021419	0.352502748	0.004559533	0.7795	0.96612731	0.578645667	0.950887	0.38	0.484524	0.286324605	0.502539137	0.232205208	0.593359443	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0											
7	0.27711976	0.28705405	0.50134784	0.32981096	0.70771165	0.0771165	0.12611641	0.419614	0.953475274	0.297504512	0.629045121	0.549273282	0.587970553	0.088992020	0.286324605	0.362390120	0.610804588	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0										
8	0.85399172	0.1940160	0.14520374	0.38971508	0.38481716	0.28851705	0.17287110	0.17277110	0.91348834	0.849273282	0.582724368	0.584927328	0.088992020	0.286324605	0.362390120	0.610804588	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0											
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11	0.27711976	0.28705405	0.50134784	0.32981096	0.70771165	0.0771165	0.12611641	0.419614	0.953475274	0.297504512	0.629045121	0.549273282	0.587970553	0.088992020	0.286324605	0.362390120	0.610804588	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0										
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14	0.77131976	0.28705405	0.50134784	0.32981096	0.70771165	0.0771165	0.12611641	0.419614	0.953475274	0.297504512	0.629045121	0.549273282	0.587970553	0.088992020	0.286324605	0.362390120	0.610804588	0.063224605	0.464687203	0.562503217	0.490273983	0.063224605	0.40671622	0.986245063	0.0302497031	0.226022988	0.286325458	0.379461646	0										
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17	0.77131976	0.28705405	0.50134784	0.32981096	0.70771165	0.0771165	0.12611641	0.419614	0.953475274	0.297504512	0.629045121	0.549273282	0.587970553	0.																									



Most Informative Variable:

First Decision in my Tree

Then What?

Males Females

Gender

2) Split According to Most Informative Variable

3) Find Most Informative Variable in Each Subset

Gender Age Income

Males

Females

- 2) Split According to Most Informative Variable
- 3) Find Most Informative Variable in Each Subset

Repeat

Gender Age

Males

Oldenburg

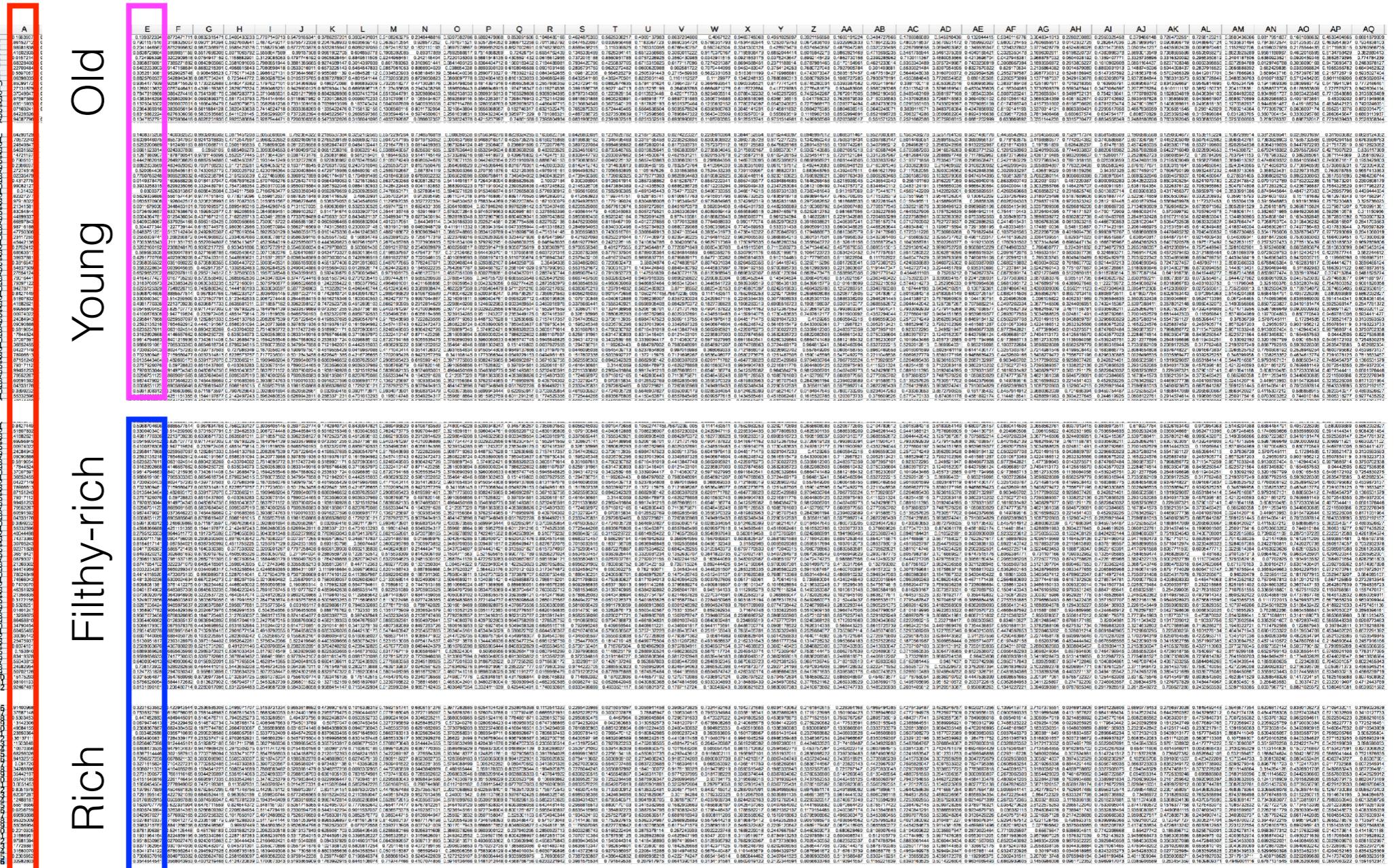
Females

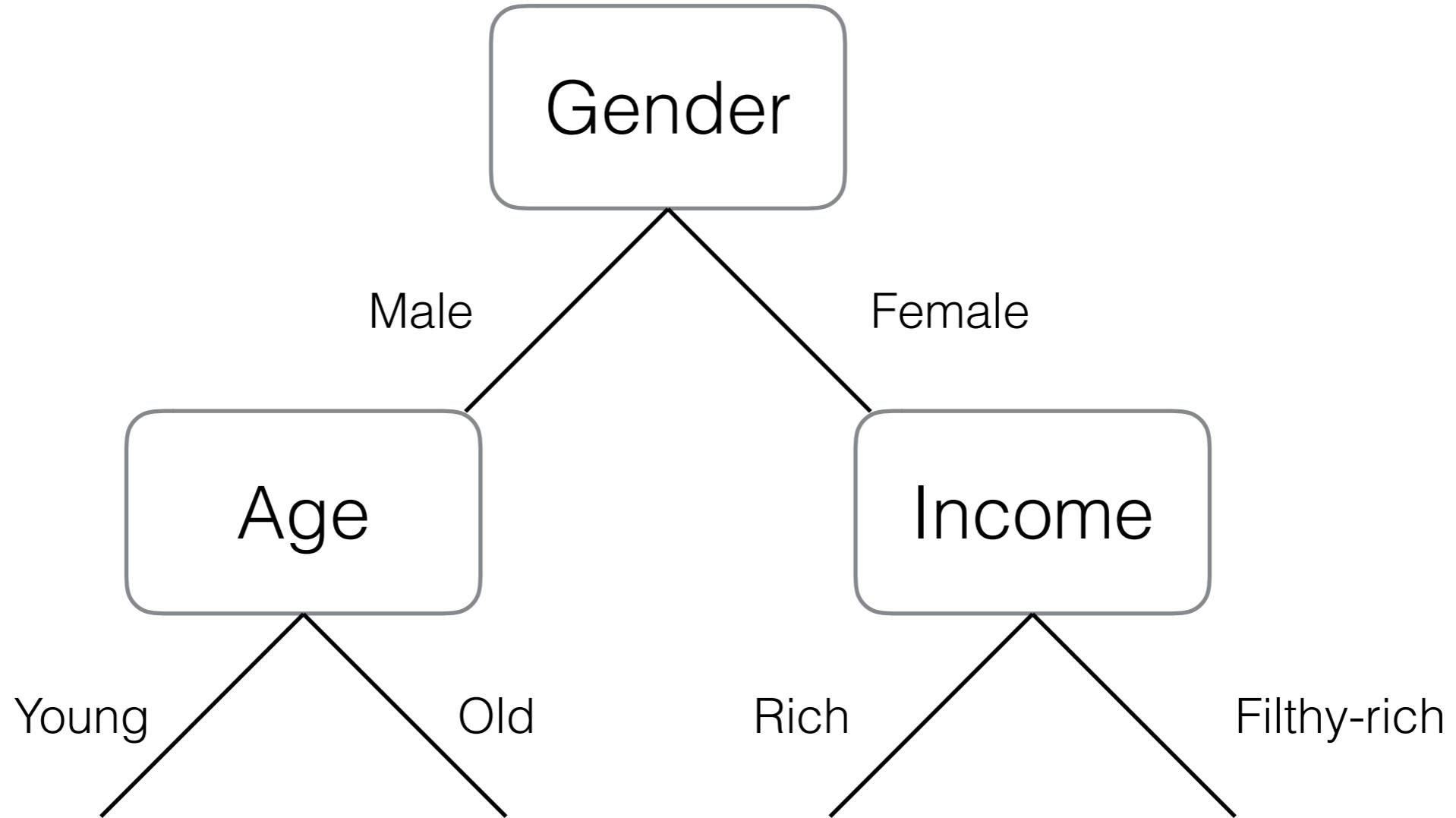
Filthy-rich
Rich

Income

- 2) Split According to Most Informative Variable
- 3) Find Most Informative Variable in Each Subset

Repeat





Each Informative Variable:

One Decision in my Tree

Then What?

Gender Age

Males

10

ung

6

Filthy

Rich

In

- 2) Split According to Most Informative Variable
- 3) Find Most Informative Variable in Each Subset

Repeat

When do we stop?

	Gender	PhD?	Will Die?
1	0	0	1
2	1	0	1
3	1	0	1
4	0	0	1
5	0	0	1
6	0	0	1
7	1	0	1
8	1	0	1
9	1	0	1
10	1	1	1
11	0	0	1
12	0	0	1
13	0	0	1
14	1	0	1
15	0	0	1
16	1	0	1
$p(0)$	1/2	15/16	0
$p(1)$	1/2	1/16	1
$H(x)$	1	0.337	

$$\begin{aligned}
 H(x) &= \sum_x p(x) \log \frac{1}{p(x)} \\
 &= p(0) \log \frac{1}{p(0)} + p(1) \log \frac{1}{p(1)} \\
 &= 0 \log\left(\frac{1}{0}\right) + 1 \log(1) \\
 &= 0
 \end{aligned}$$

Info Theory & Entropy

Extreme Case



	Gender	PhD?	Will Die?
1	0	0	1
2	1	0	1
3	1	0	1
4	0	0	1
5	0	0	1
6	0	0	1
7	1	0	1
8	1	0	1
9	1	0	1
10	1	1	1
11	0	0	1
12	0	0	1
13	0	0	1
14	1	0	1
15	0	0	1
16	1	0	1
$p(0)$	1/2	15/16	0
$p(1)$	1/2	1/16	1
$H(x)$	1	0.337	0

$$\begin{aligned}
 H(x) &= \sum_x p(x) \log \frac{1}{p(x)} \\
 &= p(0) \log \frac{1}{p(0)} + p(1) \log \frac{1}{p(1)} \\
 &= 0 \log\left(\frac{1}{0}\right) + 1 \log(1) \\
 &= 0
 \end{aligned}$$

This Variable Provides
No Information!

Info Theory & Entropy

Extreme Case

Wales

Females

Gender Age

Income

When do we stop? **Interest** is **Uninfo** (~zero entropy)

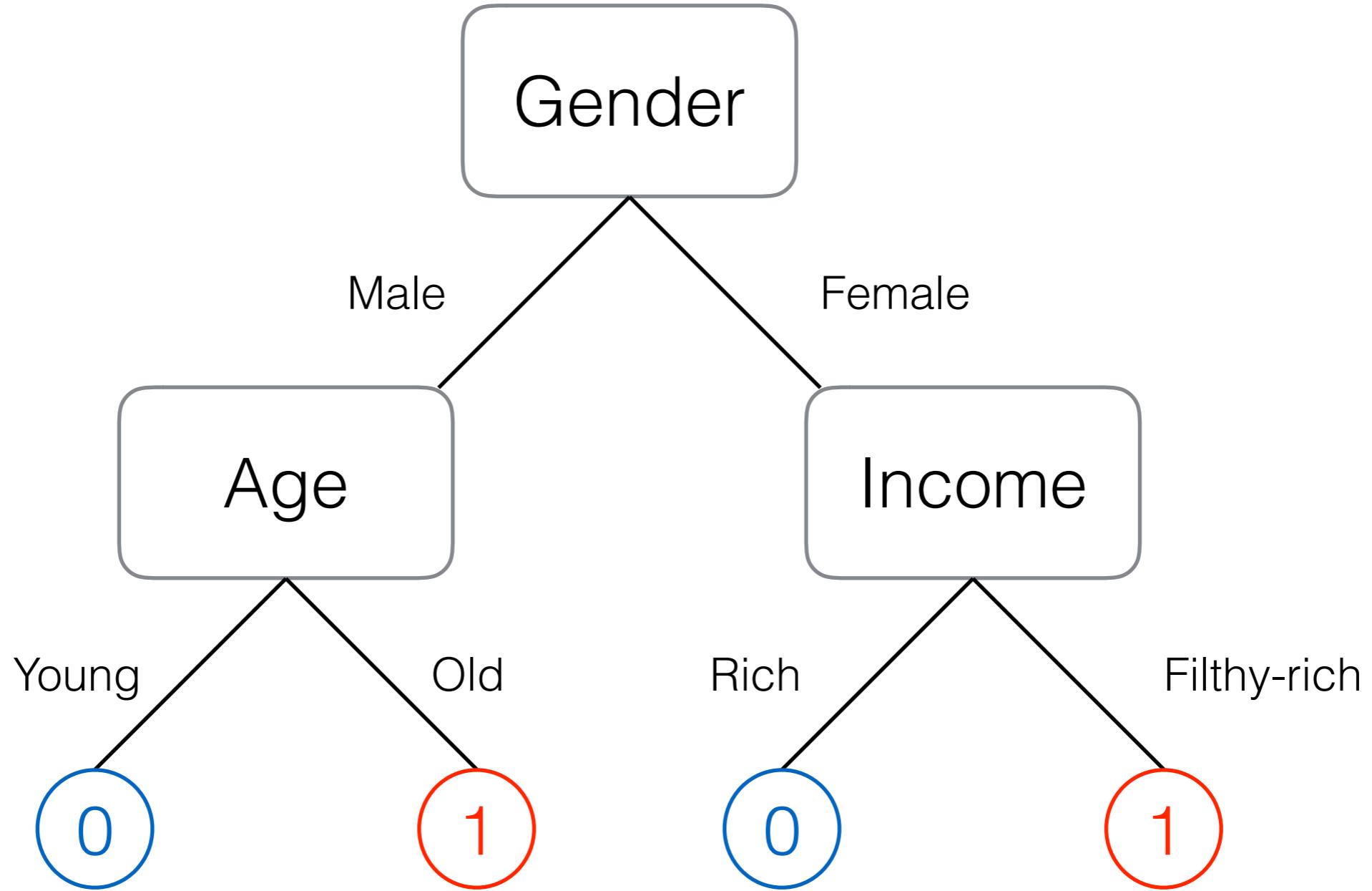
When **Variable of Interest** is **Uninformative** in Subset

1

6

4

8



Finally...

Put Result on my Decision Tree

And enjoy! (start predicting)



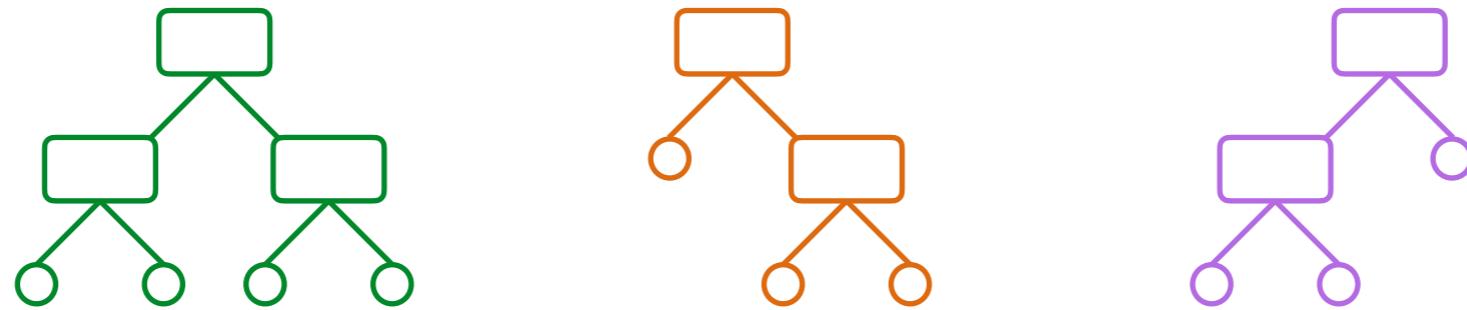
(Didn't I promise
partial nudity?)

What could possibly go wrong?
Overfitting & Bias

- **Overfitting.** My tree is accurate, but only for my given data (for which I already know the answer)
 - Not a lot of “predictive power”.
- **Bias.** It may heavily depend on my particular sample.
 - If I add/remove a few people, the result may be very different!

What could possibly go wrong?
Overfitting & Bias

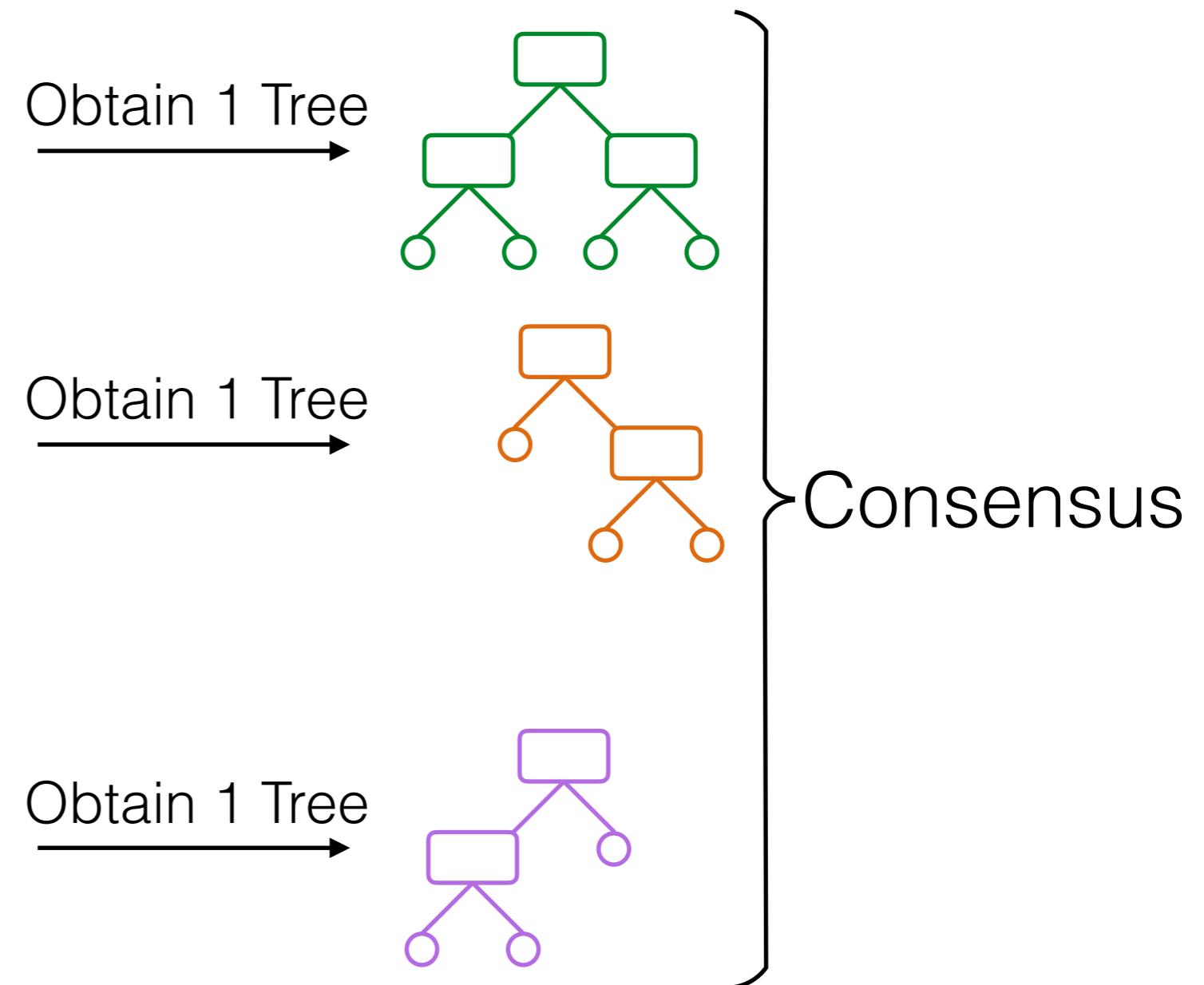
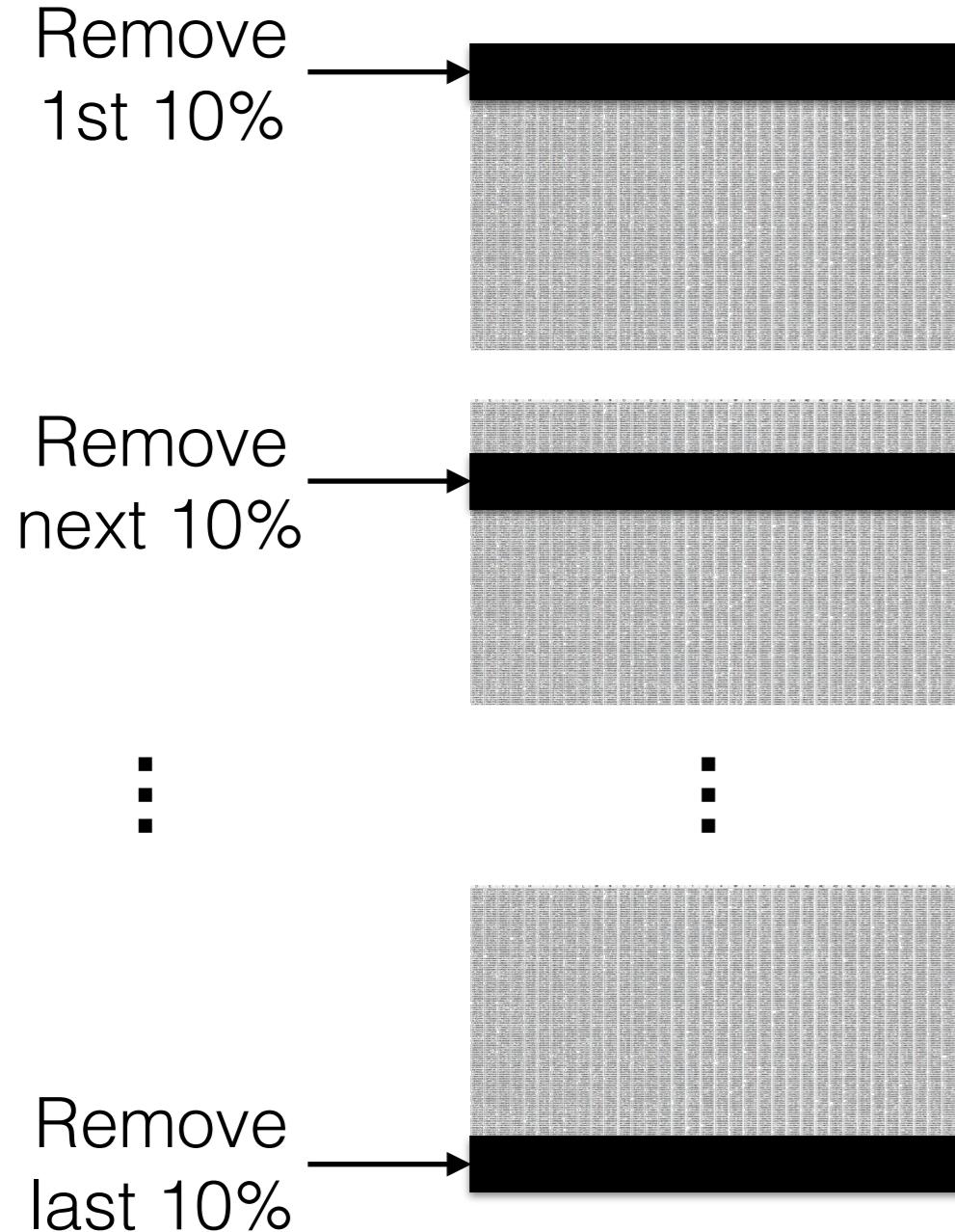
No worries:
Random Forests



Random Forests

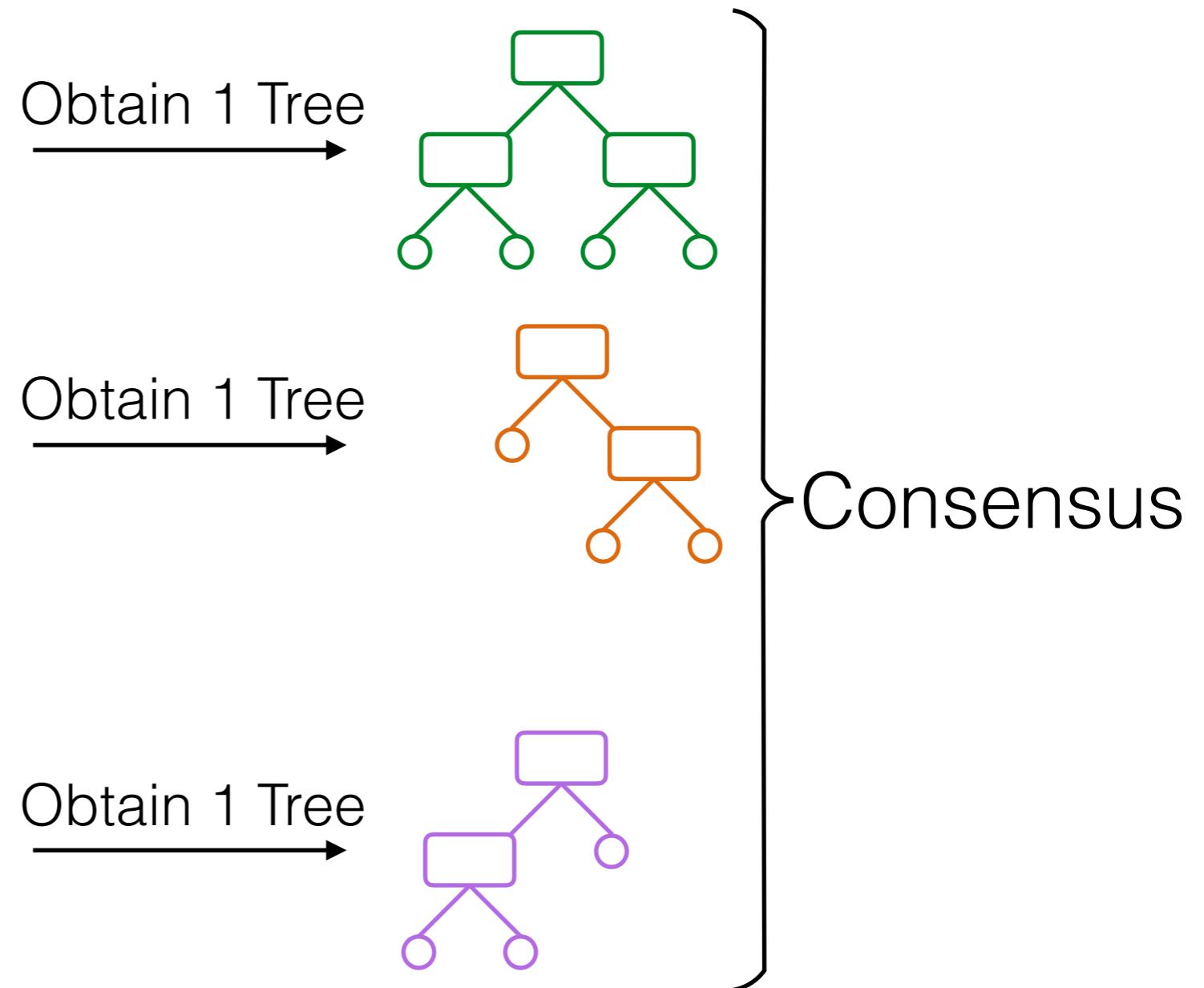
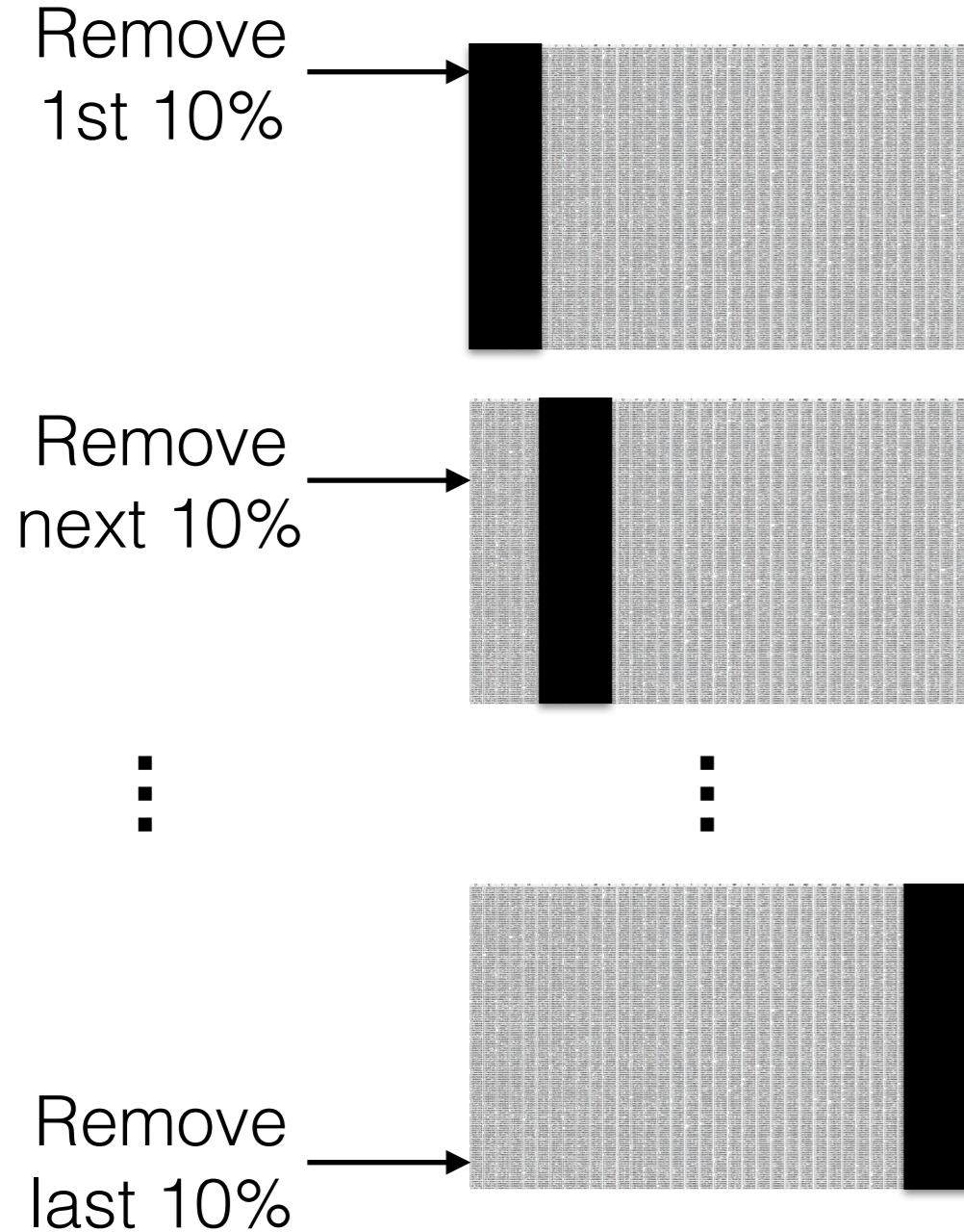
Main Idea: Do many decision trees,
each with a random subsample

aka
Bootstrap Bagging



Random Forests

Main Idea: Do many decision trees,
each with a random subsample



Random Forests

Main Idea: Do many decision trees,
each with a random subsample

- Good for **Prediction**, but bad for **Description**.
- Fast to train, but **slow** to predict.
- Poor performance on **unbalanced** data.

Advantages/Disadvantages

- Neural Networks
- Regression (Linear, Logistic, Polynomial)
- Other clustering methods (e.g., subspaces)

Alternatives

Questions?