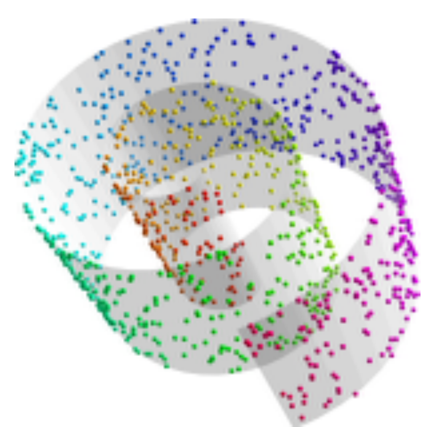
Matrix Completion goes Rogue (nonlinear)!



Daniel L. Pimentel-Alarcón
Georgia State University


Nigel Boston


Rob Nowak


Steve Wright

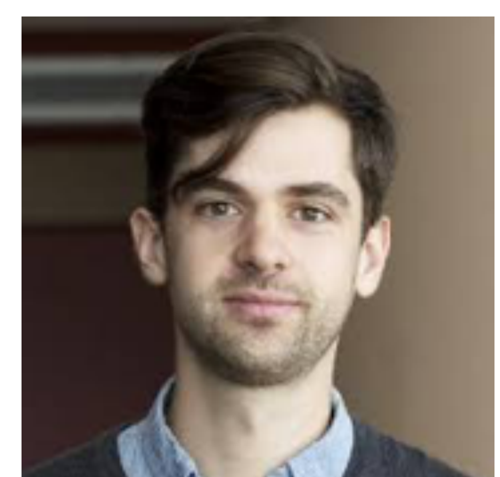
Joint work with:


Becca Willett

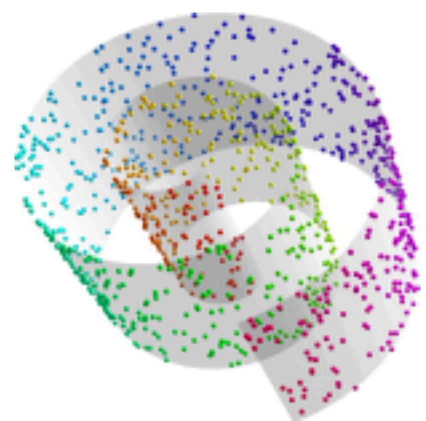

Laura Balzano


Roummel Marcia

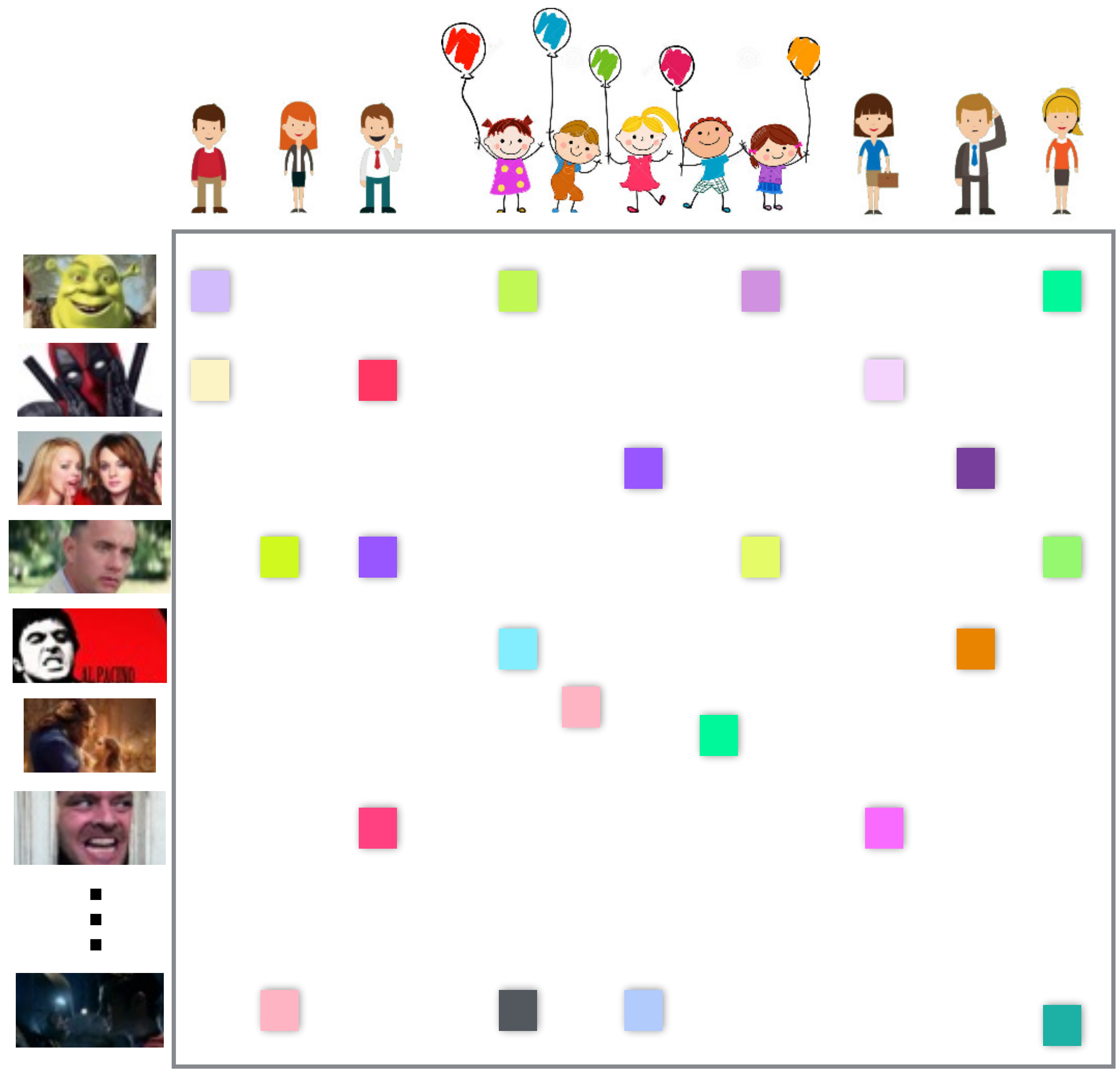

Claudia Solís


Greg Ongie

Matrix Completion goes Rogue (nonlinear)!

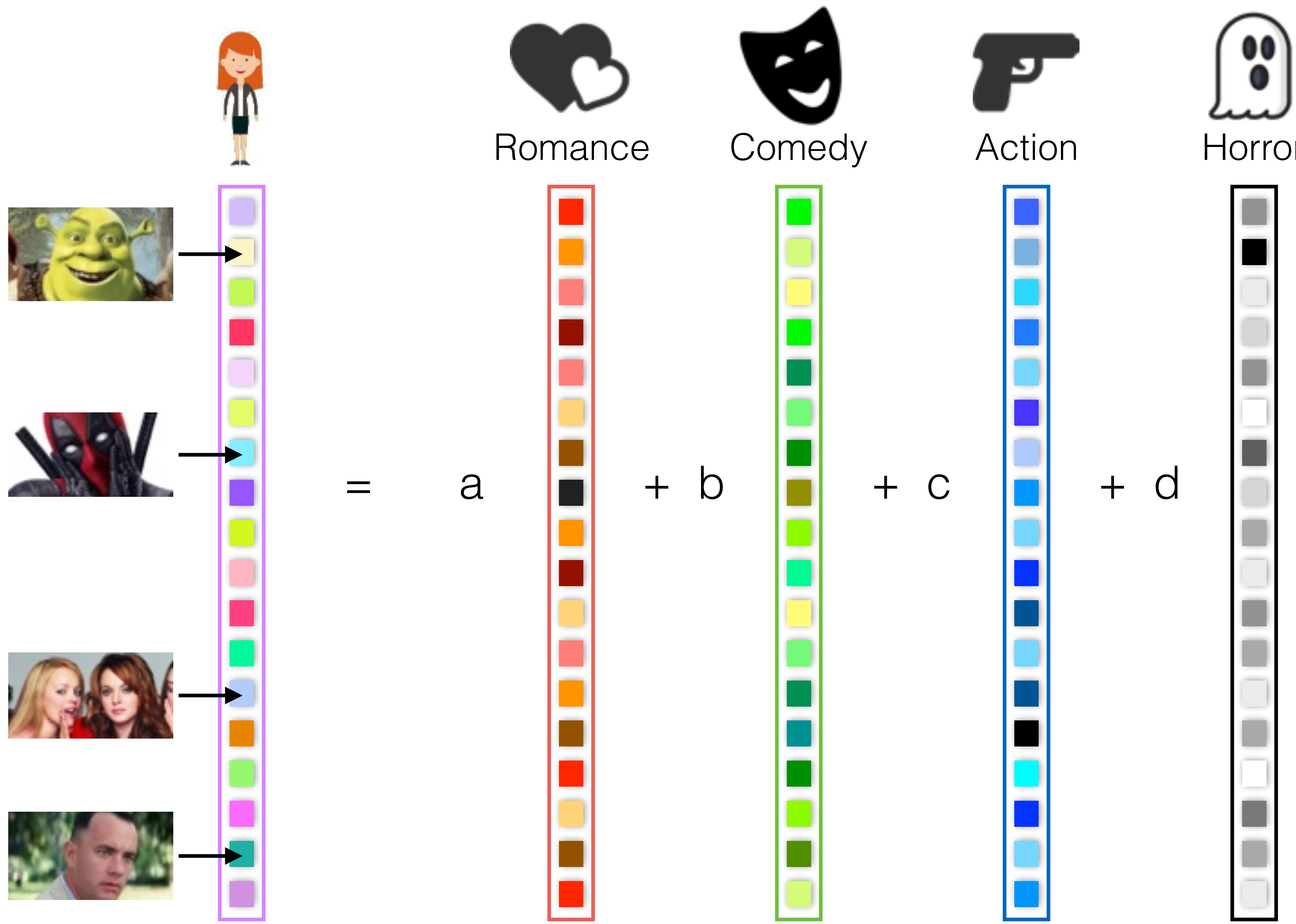


Daniel L. Pimentel-Alarcón
Georgia State University

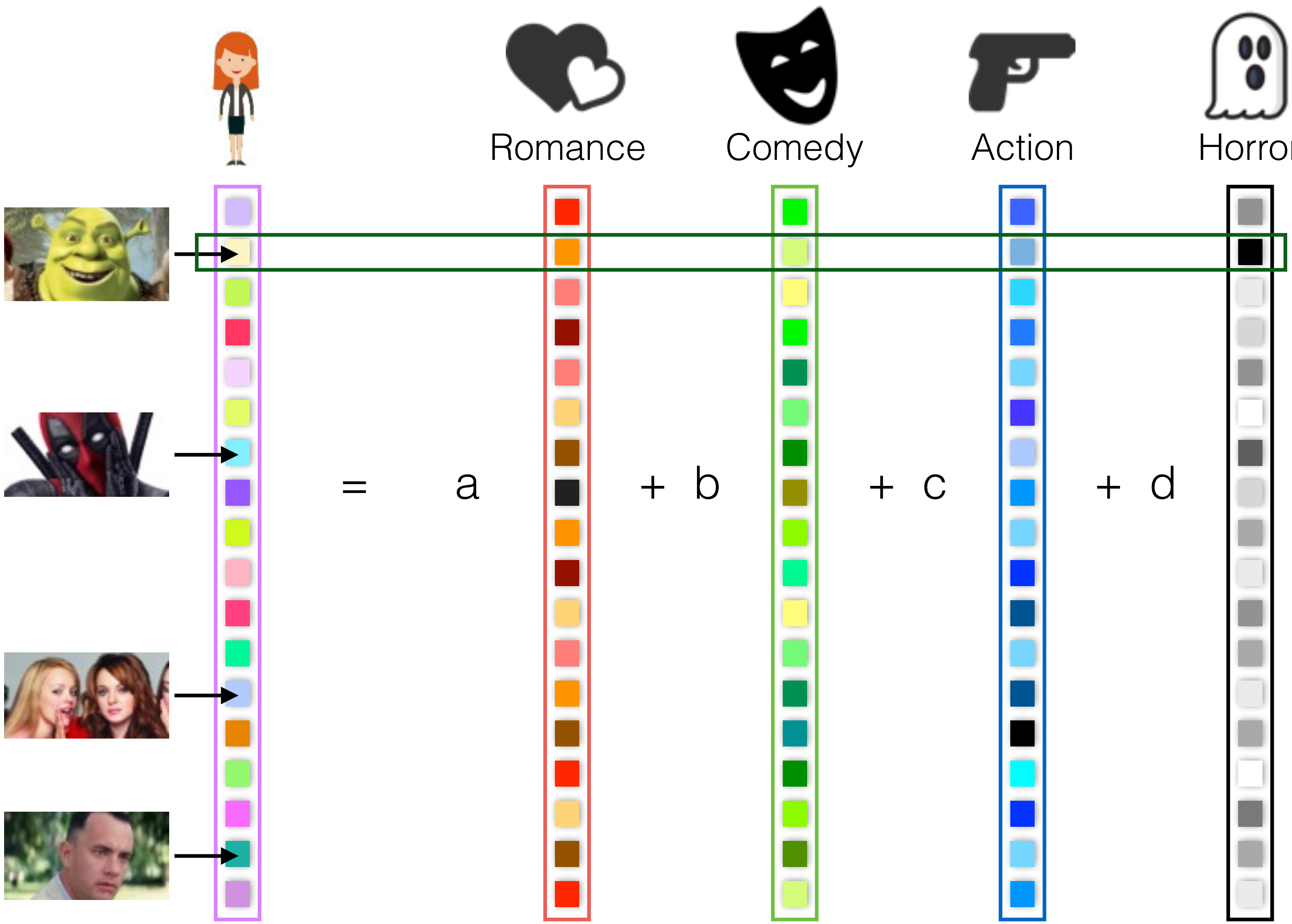


Textbook Example

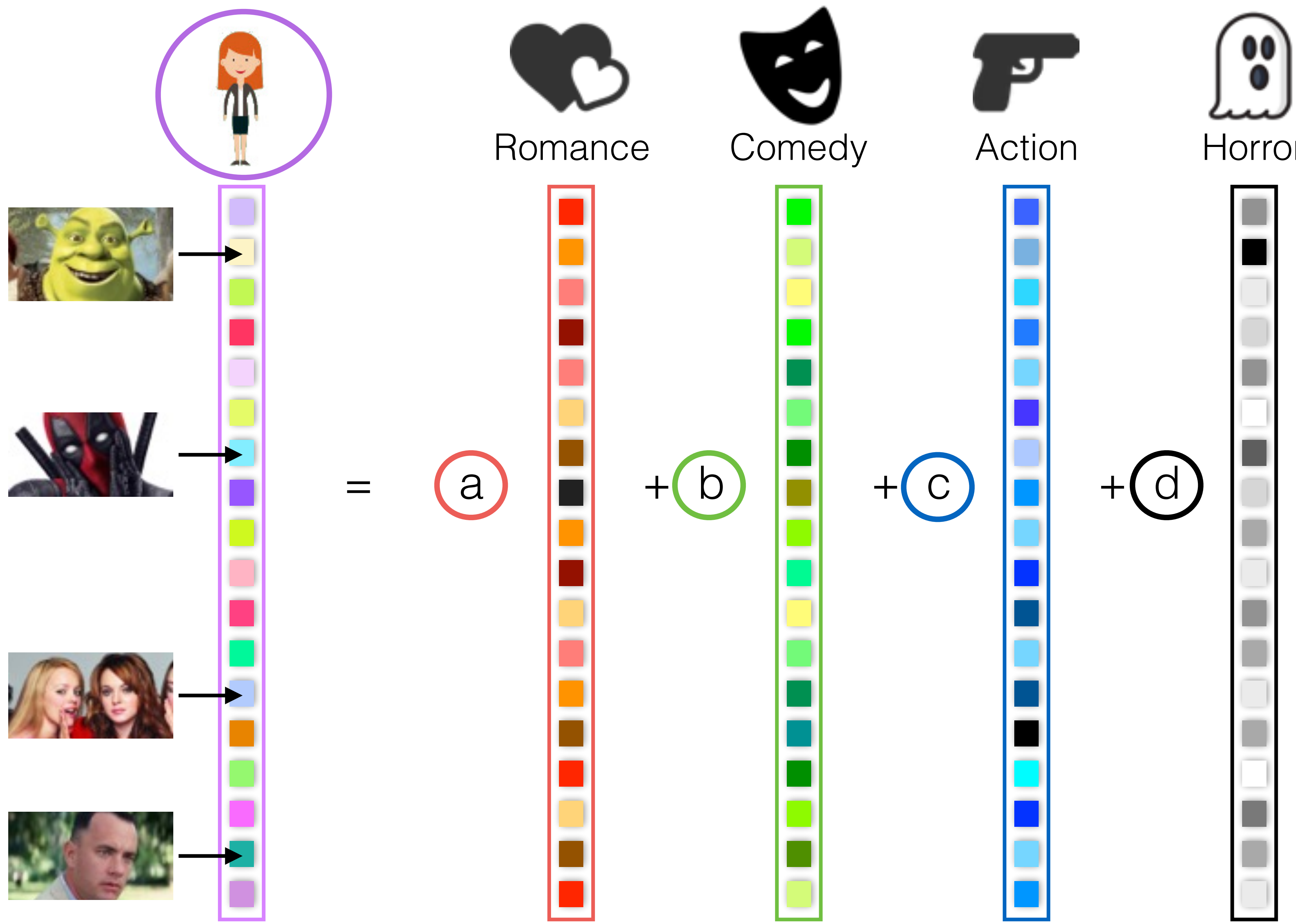
Recommender Systems



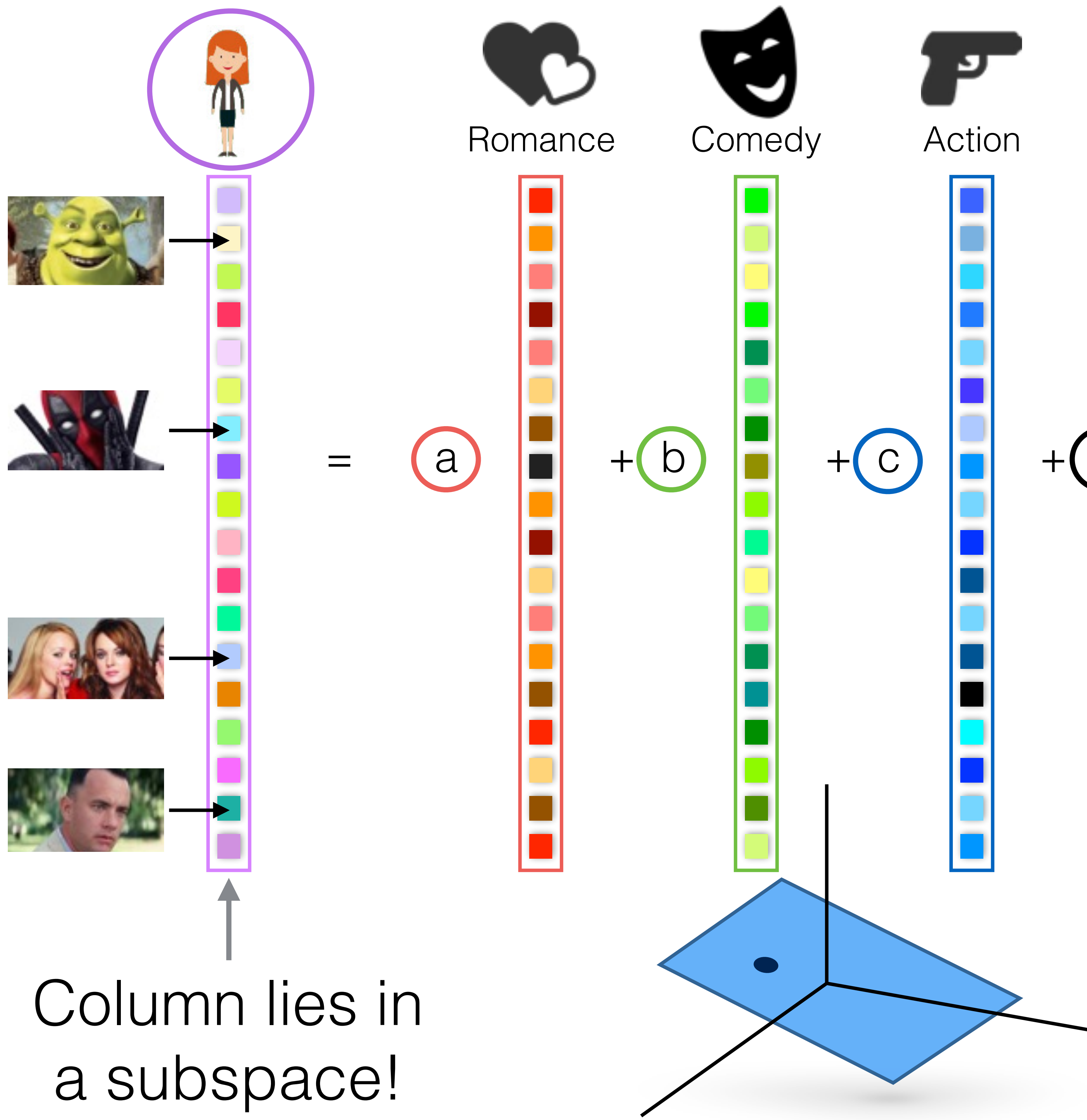
Romance
Comedy
Action
Horror
= a
+ b
+ c
+ d

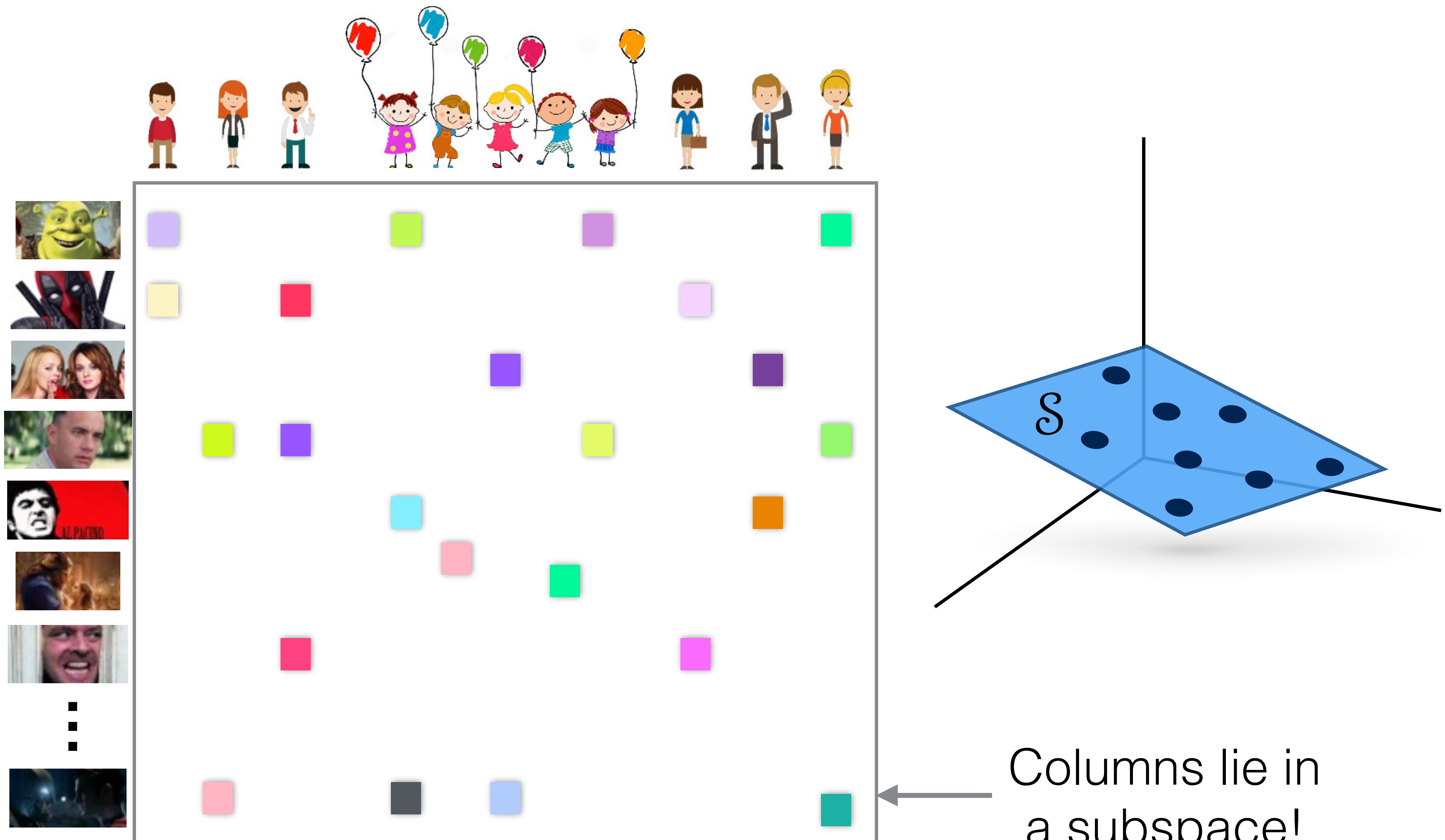


Romance
Comedy
Action
Horror
= a
+ b
+ c
+ d

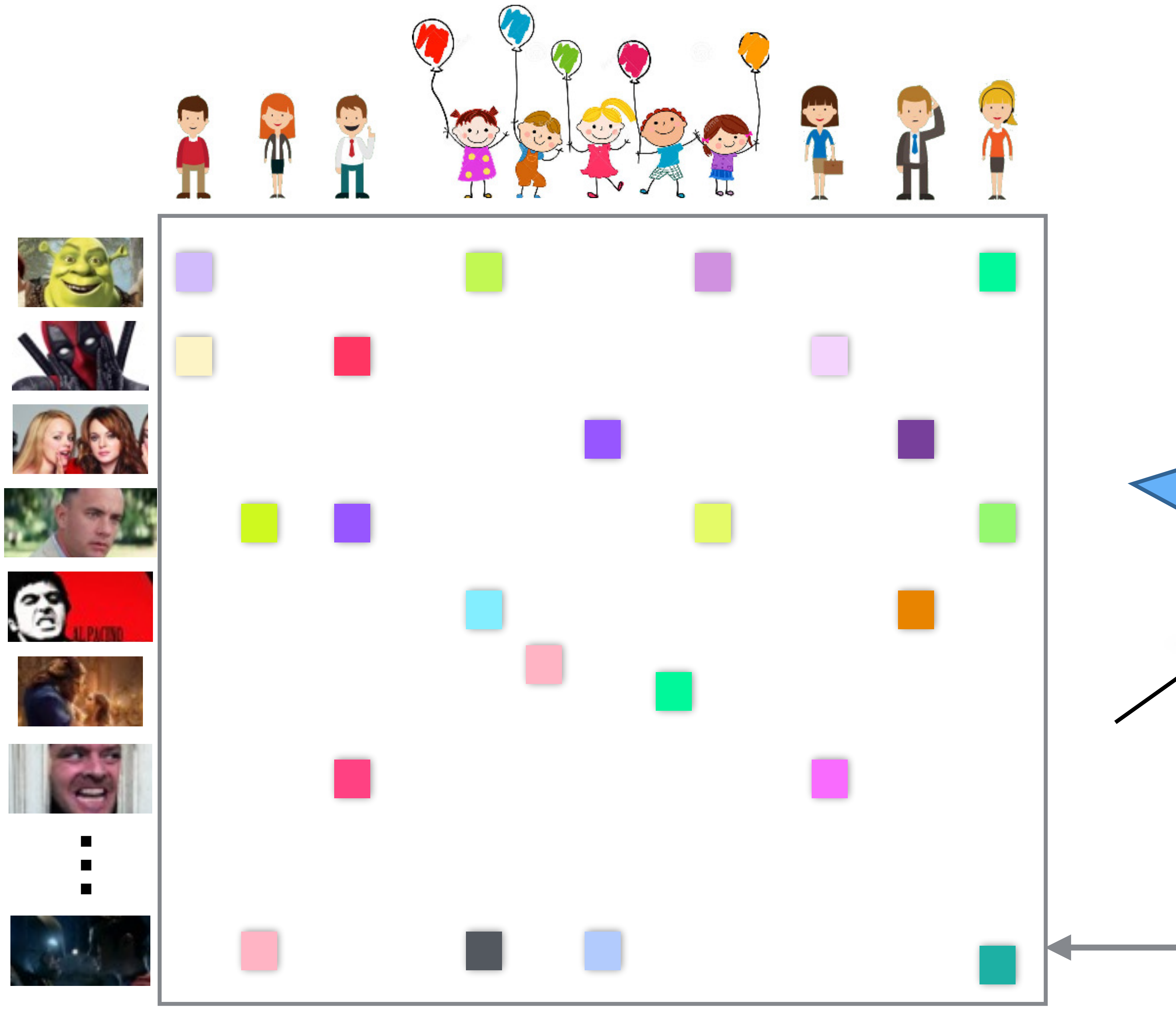


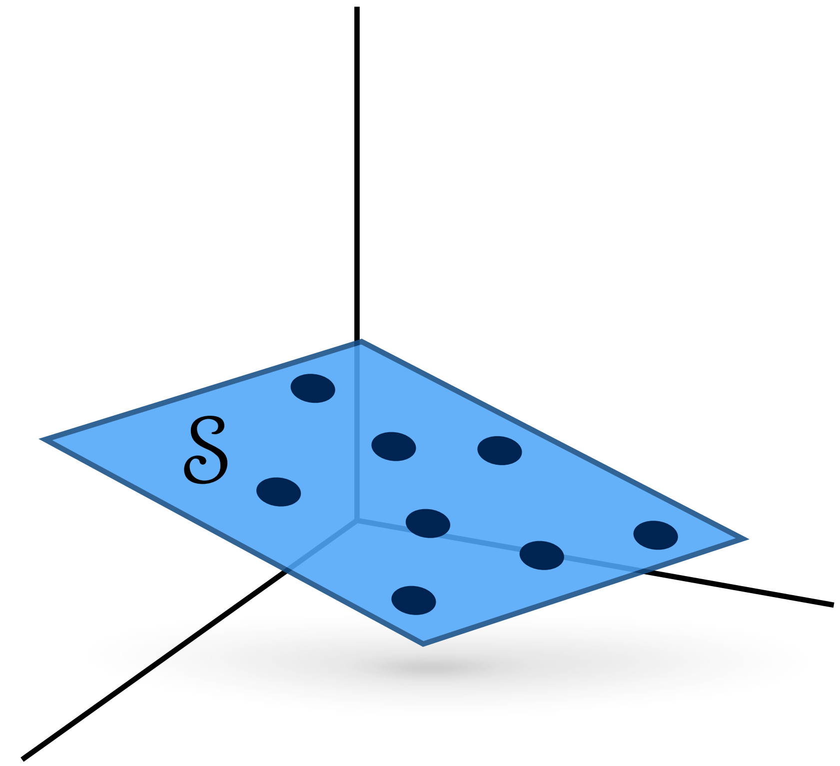
Romance
Comedy
Action
Horror
=
a
+
b
+
c
+
d


Romance
Comedy
Action
Horror
= a + b + c + d
Column lies in
a subspace!



We want to find this Subspace!

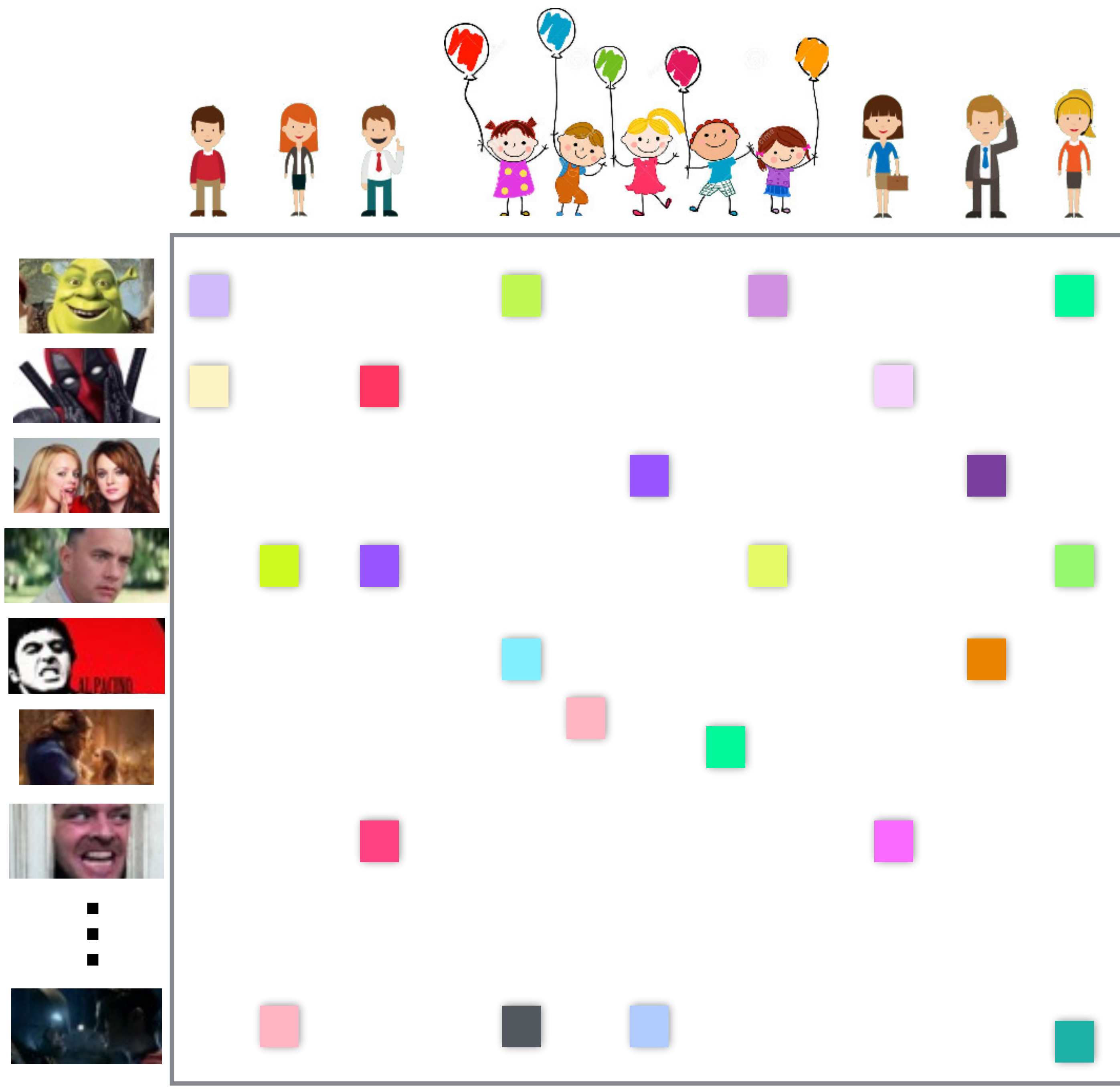


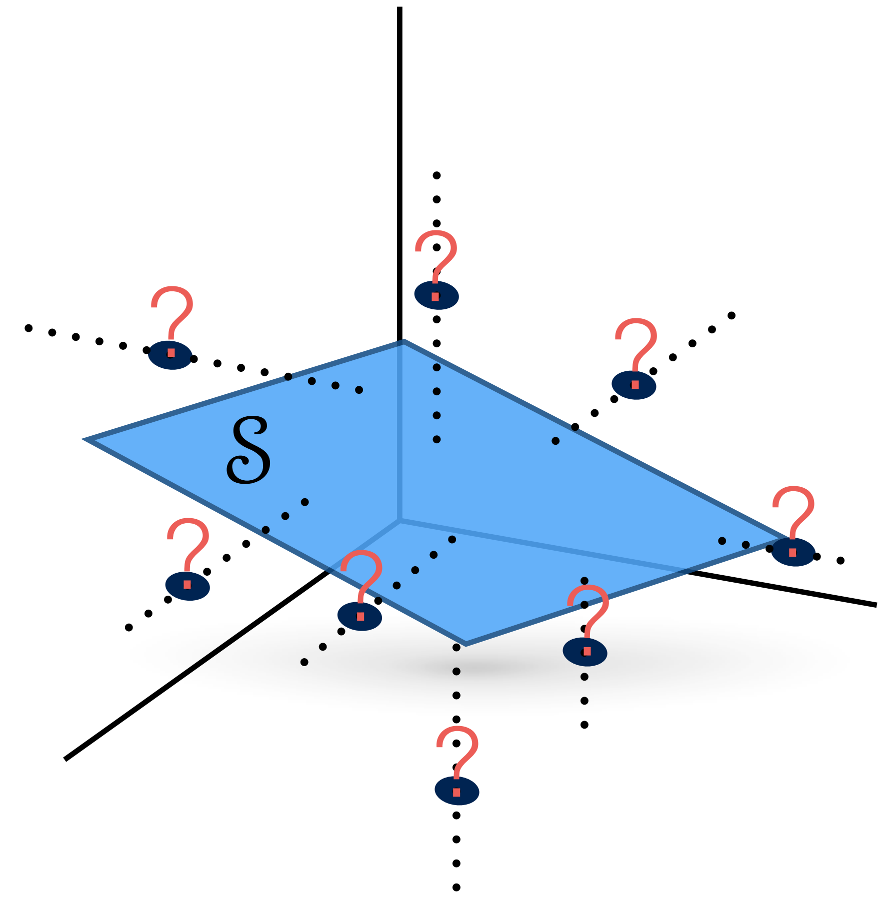


Columns lie in
a subspace!

We want to find this Subspace!

Problem is: data is incomplete!





Columns lie in
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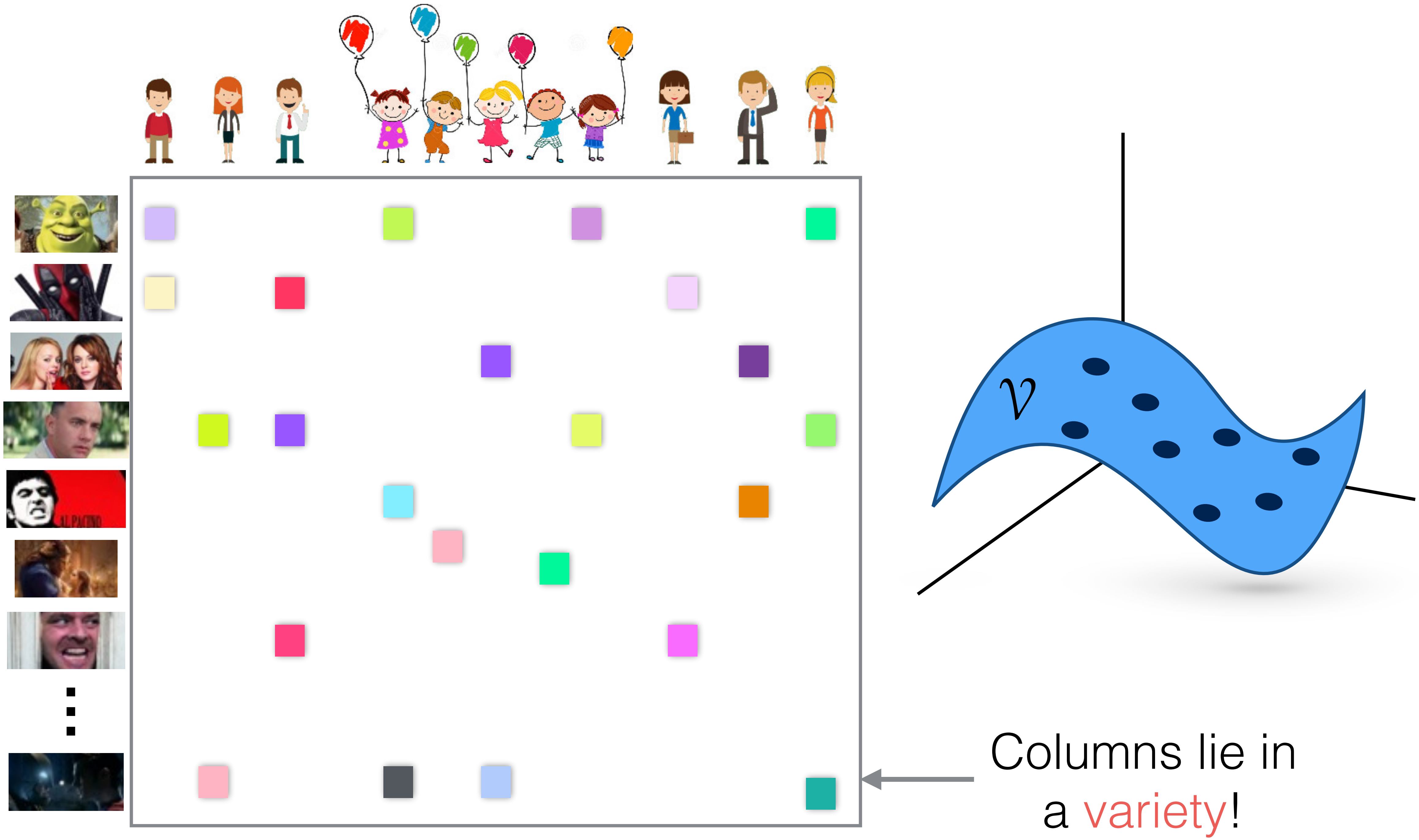
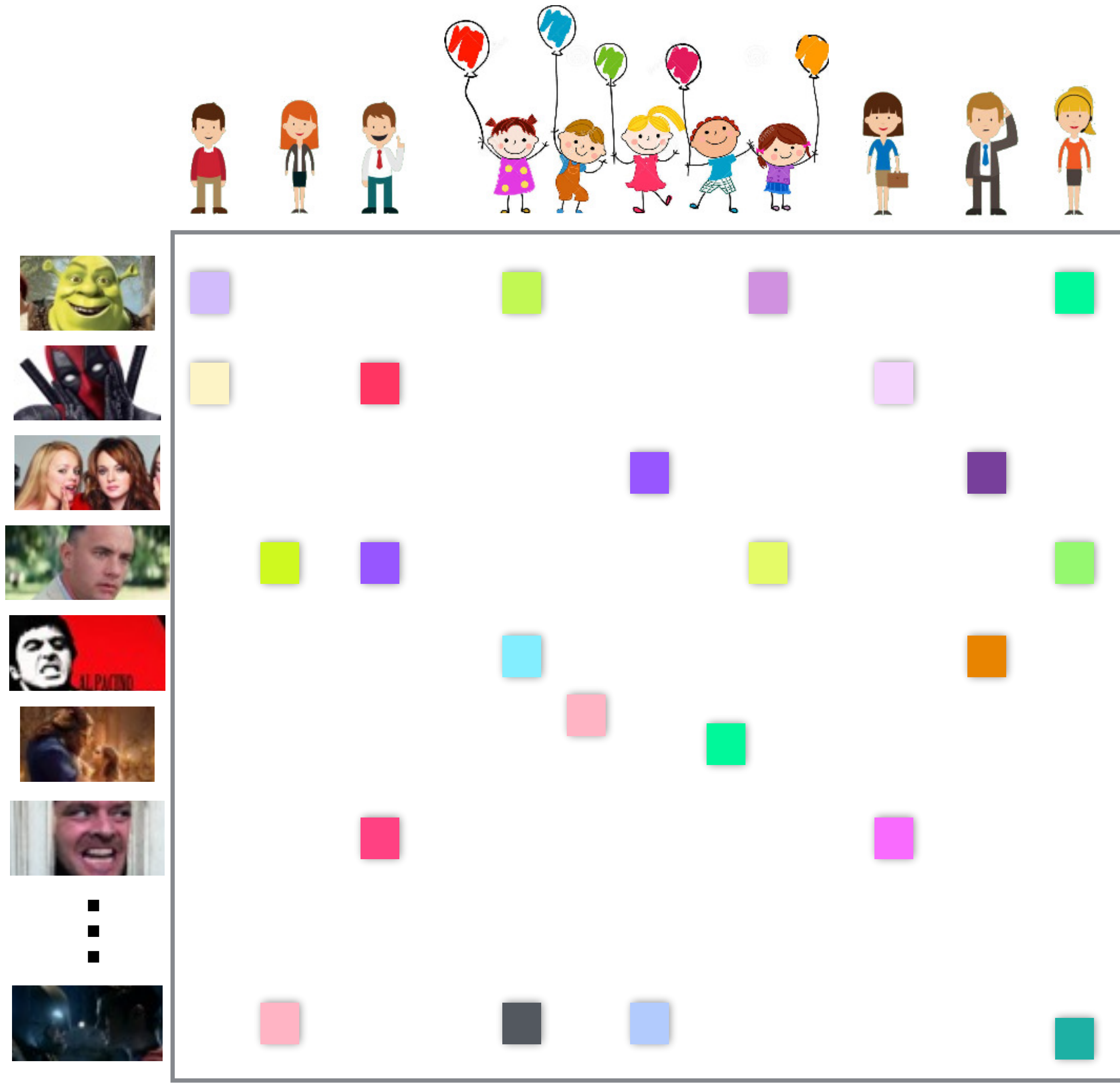
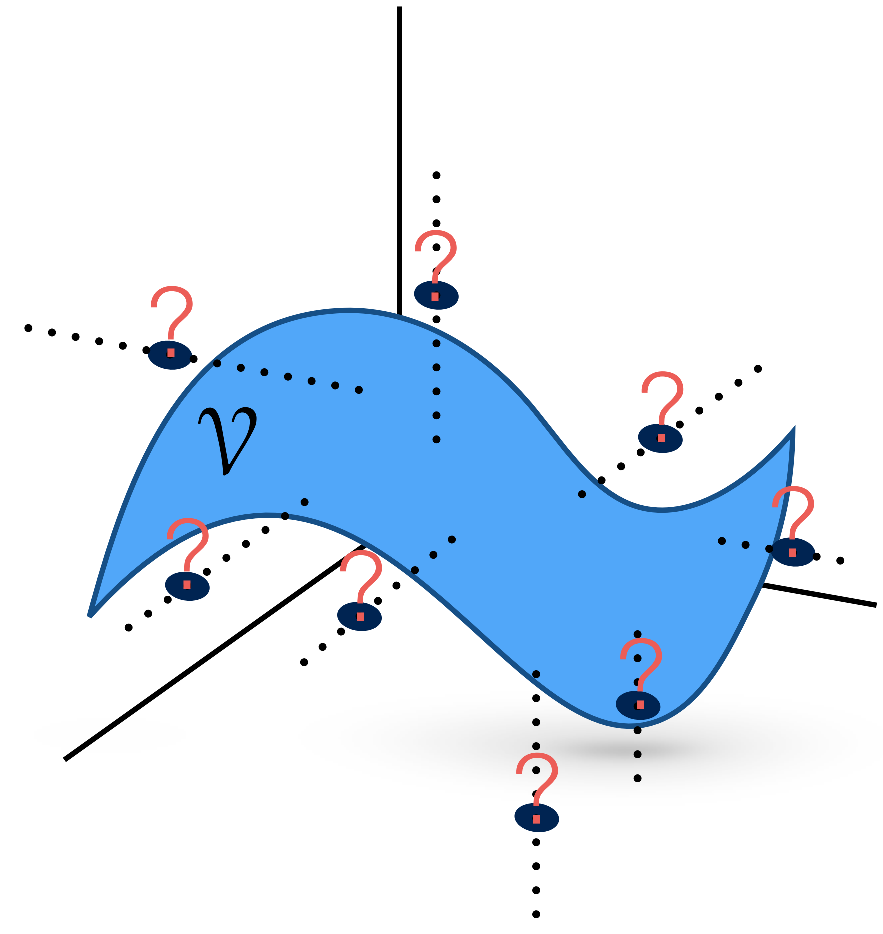


What if dependencies are nonlinear?!

Problem is: data is incomplete!





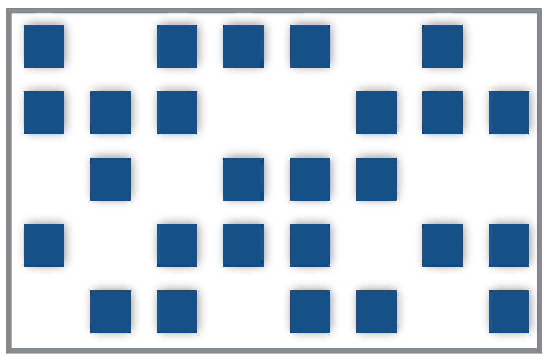
Columns lie in
a variety!

What if dependencies are nonlinear?!

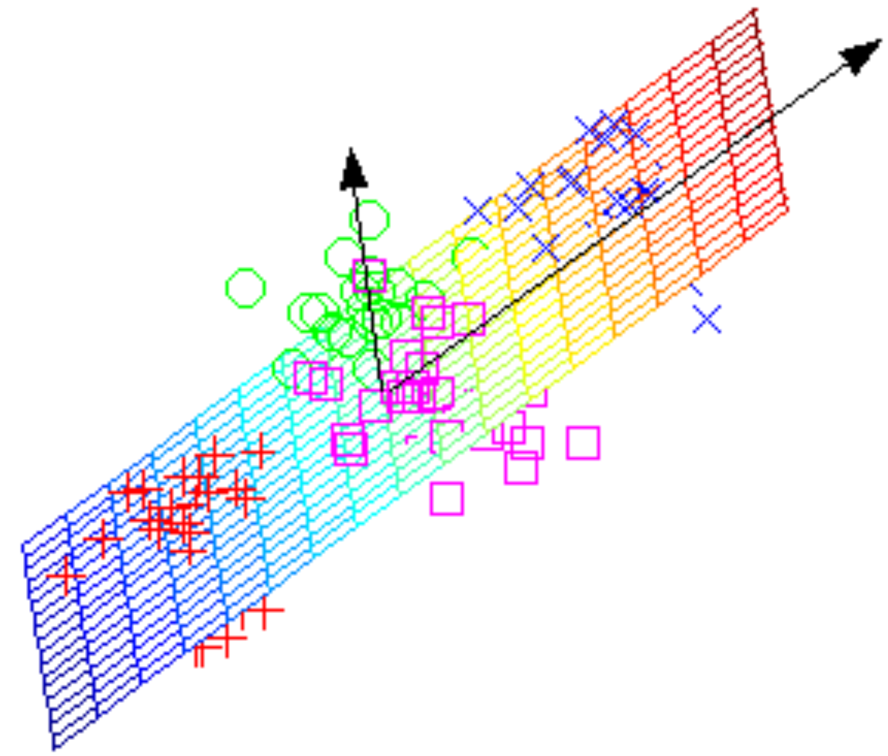
Problem is: data is incomplete!

What am I telling you?

$\mathbf{X}$



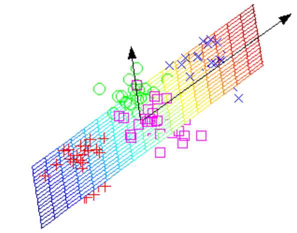
$\in$



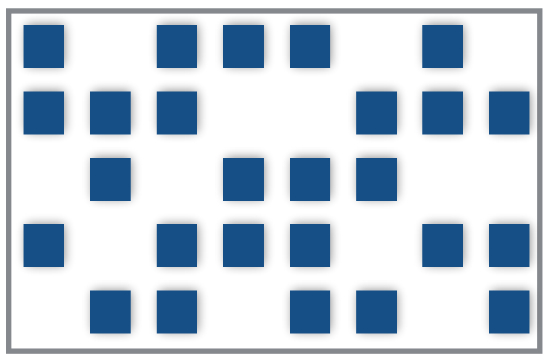
Given: Incomplete
data matrix

Goal: Find linear subspace
that explains data

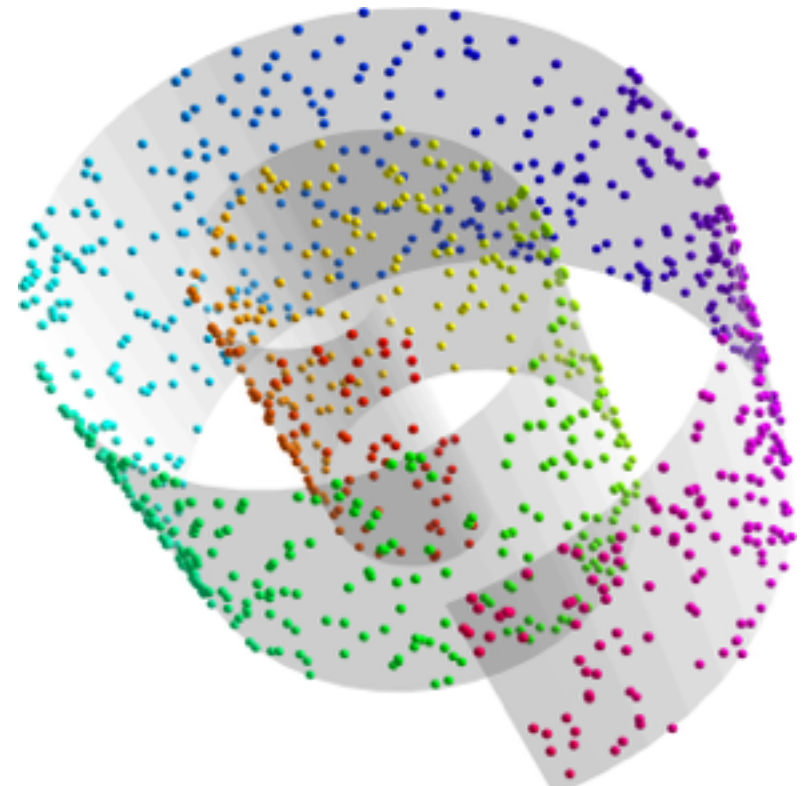
Low-Rank Matrix Completion



$\mathbf{X}$



$\in$



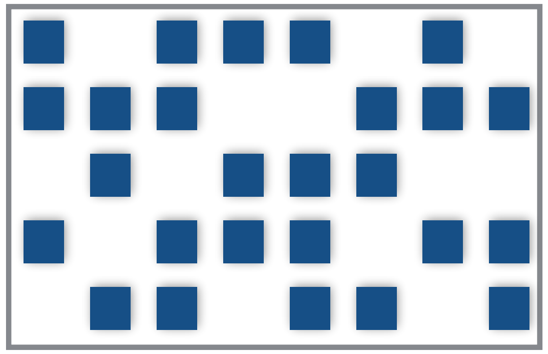
Given: Incomplete data matrix

Goal: Find ~~linear subspace~~ that explains data

Algebraic Variety!

~~Low-Rank~~ Matrix Completion

$\mathbf{X}$



$\in$

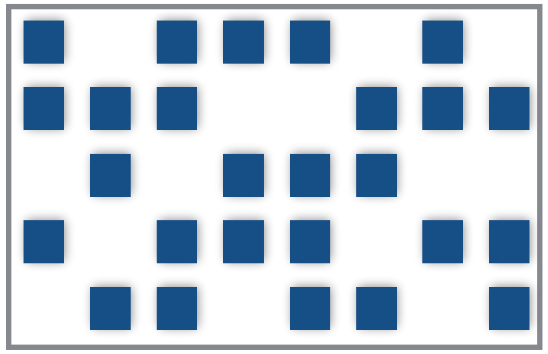
Given: Incomplete data matrix

Goal: Find ~~linear subspace~~ that explains data

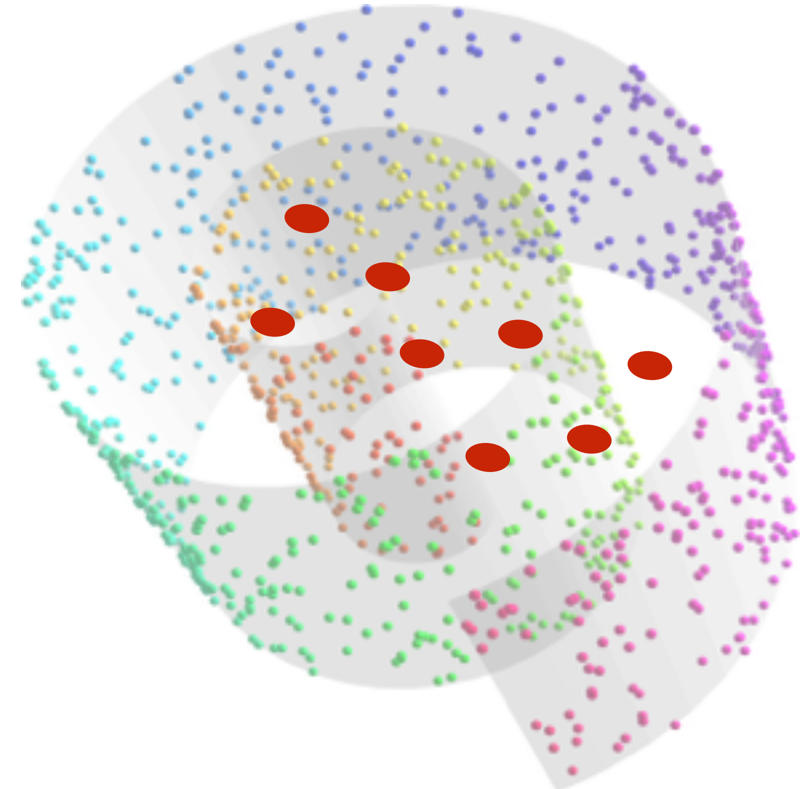
Algebraic Variety!

Is it possible?
When?
How?

$\mathbf{X}$



$\in$



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Goal: Find ~~linear subspace~~ that explains data

Algebraic Variety!

Is it possible?
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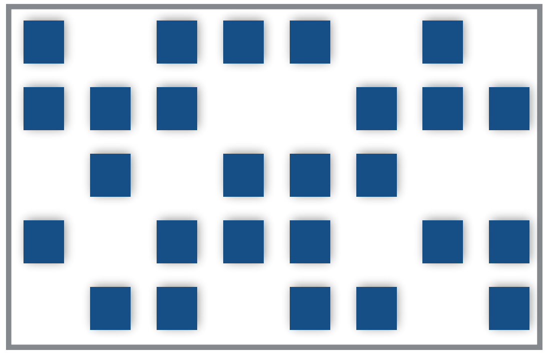
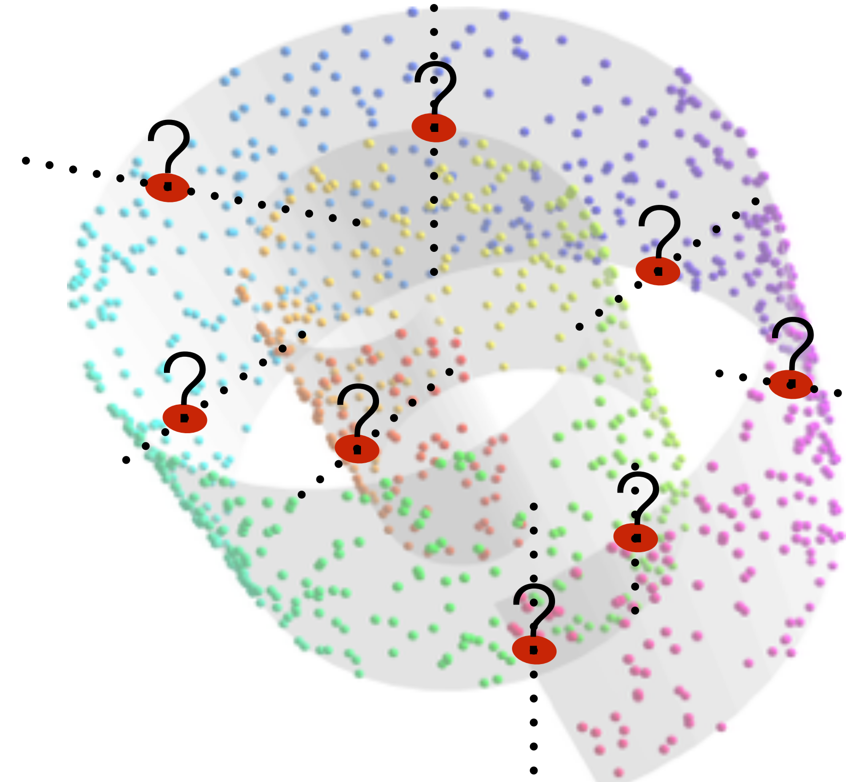
$\mathbf{X}$



$\in$



Given: Incomplete data matrix

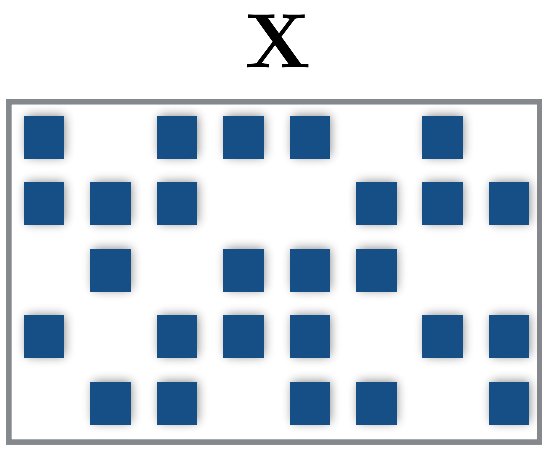
Goal: Find ~~linear subspace~~ that explains data

Algebraic Variety!

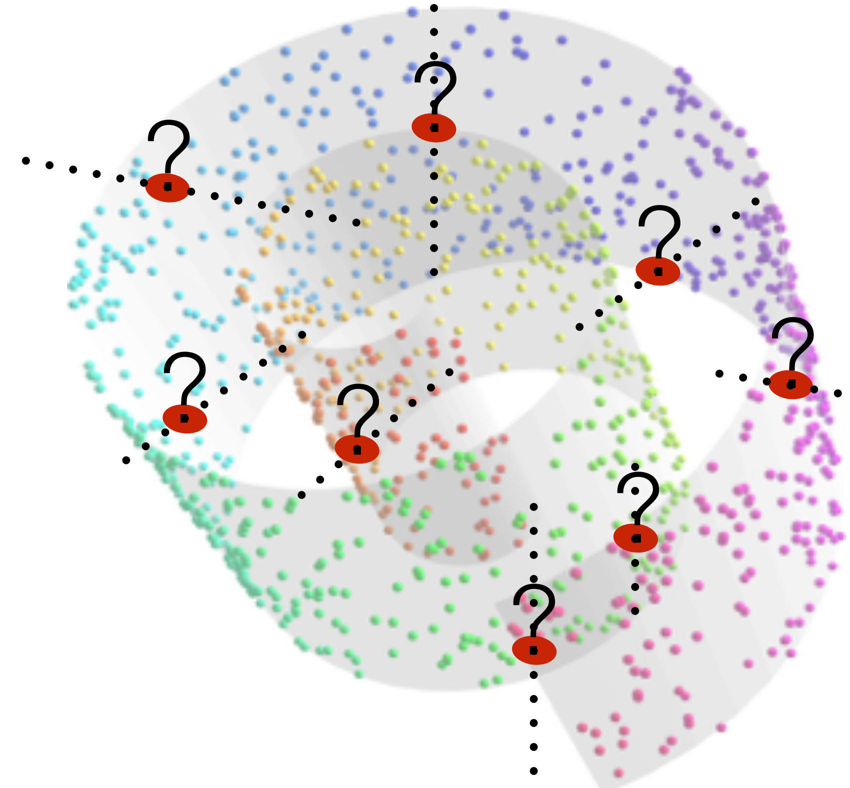
Is it possible?

When?

How?



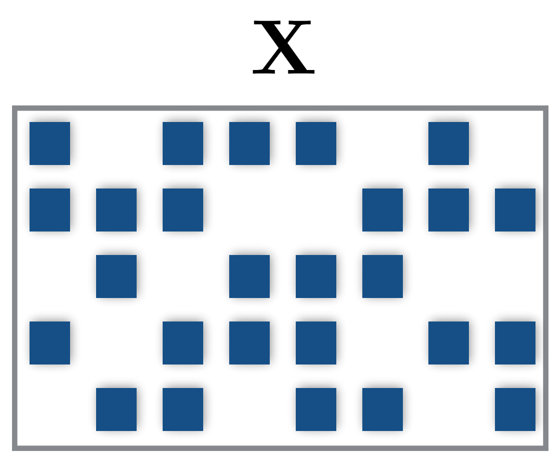
$\in$



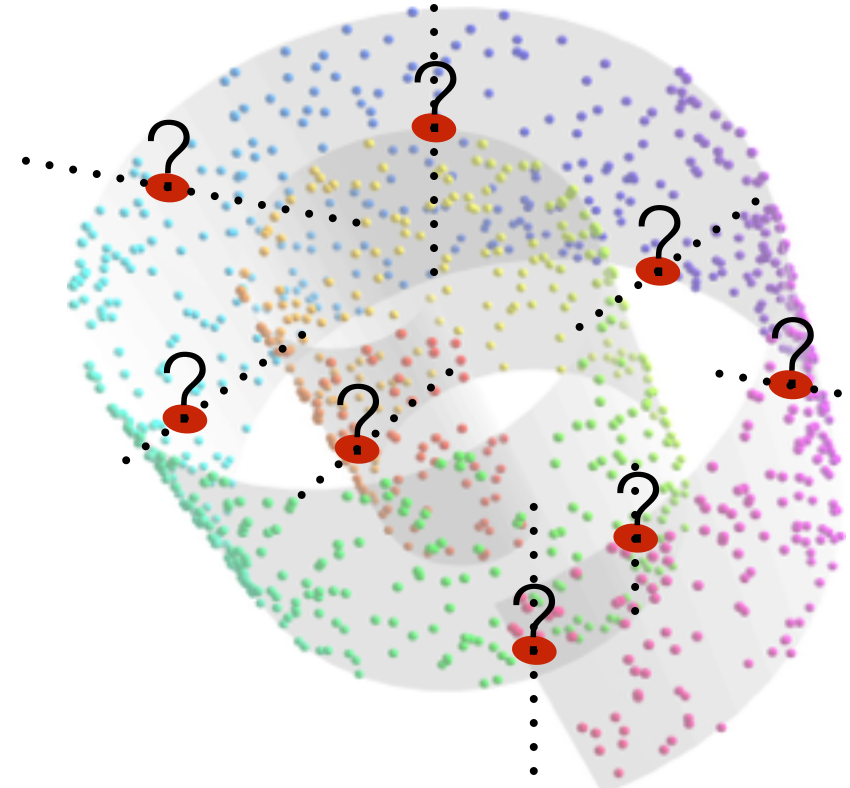
Given: Incomplete
data matrix

Goal: Find ~~linear subspace~~
that explains data
Algebraic Variety!

Is it possible? Yes
When?
How?



$\in$



Given: Incomplete data matrix

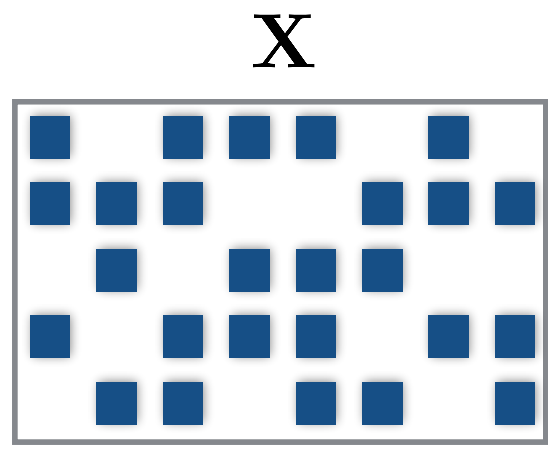
Goal: Find ~~linear subspace~~ that explains data

Algebraic Variety!

Is it possible? Yes

When? When you observe *the right entries*

How?



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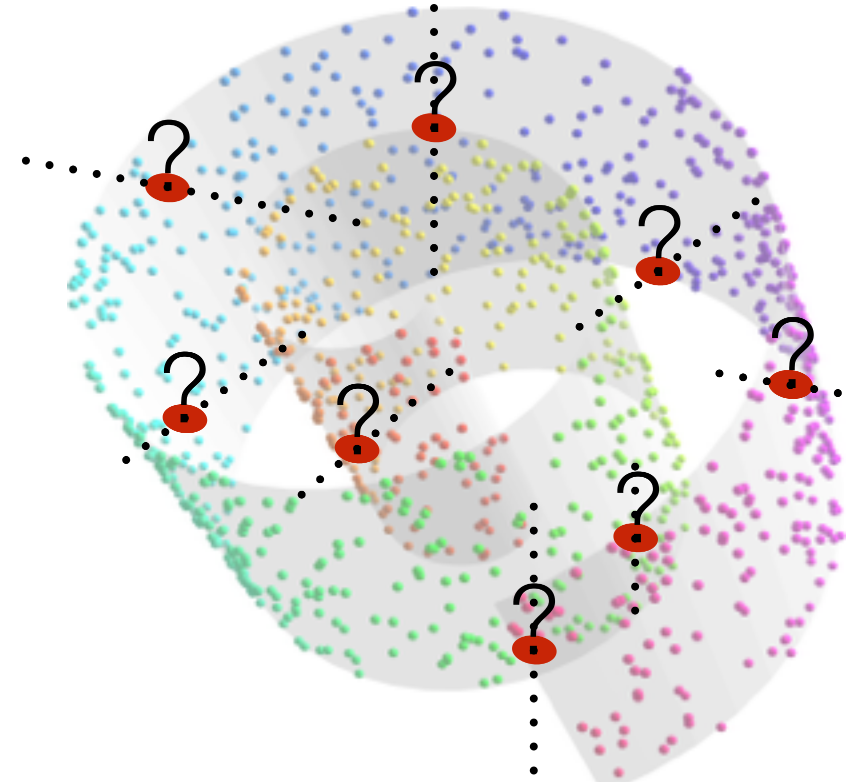


Given: Incomplete data matrix

Goal: Find ~~linear subspace~~ that explains data

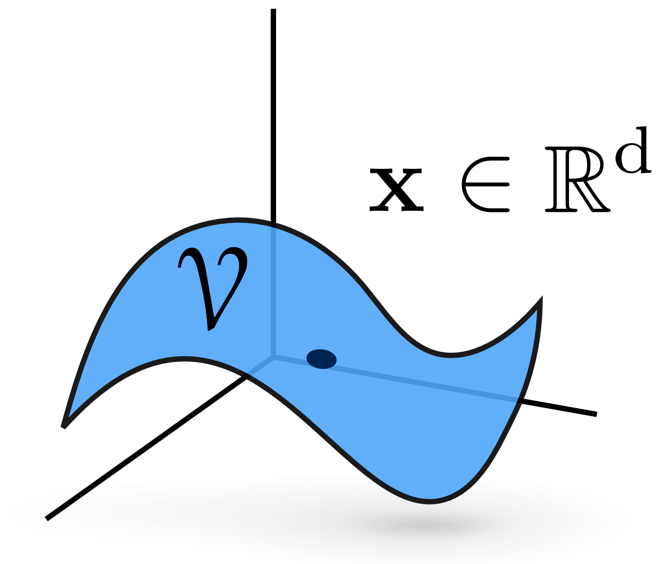
Algebraic Variety!

Is it possible? Yes

When? When you observe *the right entries*

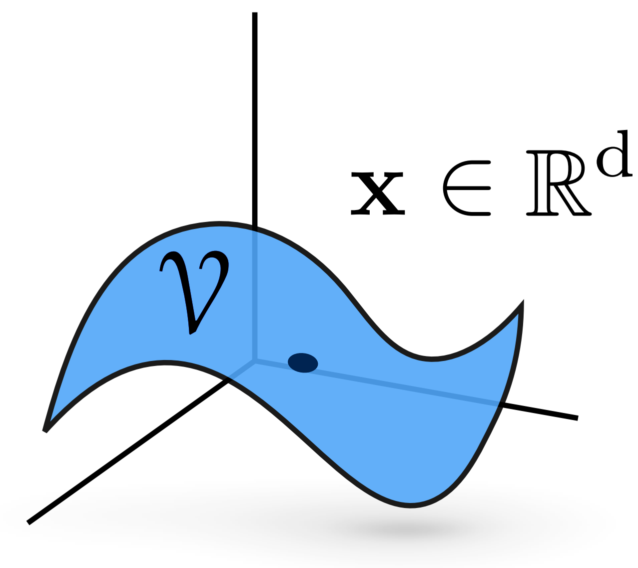
How? Using tensors


ROLL
UP
YOUR
SLEEVES!



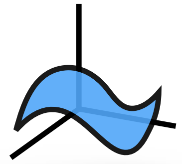
$$\begin{cases} f_1(\mathbf{x}) = 0 \\ f_2(\mathbf{x}) = 0 \\ \vdots \\ f_{\mathrm{N}}(\mathbf{x}) = 0 \end{cases}$$

Consider a point in a variety



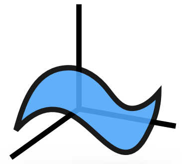
$$\begin{cases} v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 = 0 \\ v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 = 0 \\ \vdots \\ v_{N1}x_1^2 + v_{N2}x_1x_2 + v_{N3}x_1x_3 + \cdots + v_{ND}x_d^2 = 0 \end{cases}$$

Consider a point in a variety



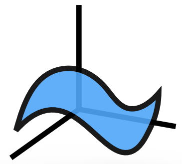
$$\mathbf{x} \in \mathcal{V} \qquad v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 = 0$$

$$\begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1D} \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_1x_2 \\ x_1x_3 \\ \vdots \\ x_d^2 \end{bmatrix} = 0$$



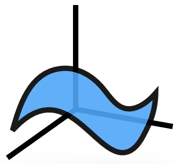
$$\mathbf{x} \in \mathcal{V} \quad \left\{ \begin{array}{lcl} v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 & = & 0 \\ v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 & = & 0 \end{array} \right.$$

$$\begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1D} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2D} \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_1x_2 \\ x_1x_3 \\ \vdots \\ x_d^2 \end{bmatrix} = \mathbf{0}$$



$$\mathbf{x} \in \mathcal{V} \begin{cases} v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 = 0 \\ v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 = 0 \\ \vdots \\ v_{N1}x_1^2 + v_{N2}x_1x_2 + v_{N3}x_1x_3 + \cdots + v_{ND}x_d^2 = 0 \end{cases}$$

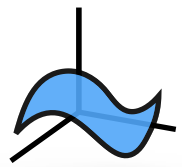
$$\begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1D} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2D} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{N1} & v_{N2} & v_{N3} & \cdots & v_{ND} \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_1x_2 \\ x_1x_3 \\ \vdots \\ x_d^2 \end{bmatrix} = \mathbf{0}$$



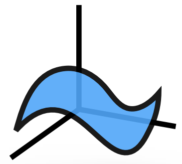
$$\mathbf{x} \in \mathcal{V} \begin{cases} v_{11}x_1^2 + v_{12}x_1x_2 + v_{13}x_1x_3 + \cdots + v_{1D}x_d^2 = 0 \\ v_{21}x_1^2 + v_{22}x_1x_2 + v_{23}x_1x_3 + \cdots + v_{2D}x_d^2 = 0 \\ \vdots \\ v_{N1}x_1^2 + v_{N2}x_1x_2 + v_{N3}x_1x_3 + \cdots + v_{ND}x_d^2 = 0 \end{cases}$$

$$\begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1D} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2D} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{N1} & v_{N2} & v_{N3} & \cdots & v_{ND} \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_1x_2 \\ x_1x_3 \\ \vdots \\ x_d^2 \end{bmatrix} = \mathbf{0}$$

$$V^\top \mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$



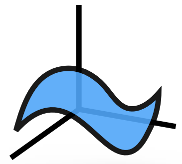
$$\mathbf{x} \in \mathcal{V} \iff V^{\mathsf{T}} \mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$



$$\mathbf{x} \in \mathcal{V} \iff \mathrm{V}^{\mathsf{T}} \mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$

$$\Updownarrow$$

$$\mathbf{x}^{\otimes \mathbf{2}} \in \ker \mathrm{V}^{\mathsf{T}}$$



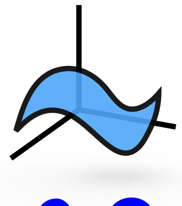
$$\mathbf{x} \in \mathcal{V} \quad \iff \quad \mathrm{V}^{\mathsf{T}}\mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$

$$\Updownarrow$$

$$\mathbf{x}^{\otimes \mathbf{2}} \in \ker \mathrm{V}^{\mathsf{T}}$$

$$\|$$

$$\mathcal{S}$$



$$\mathbf{x} \in \mathcal{V} \iff \mathsf{V}^\mathsf{T}\mathbf{x}^{\otimes\mathbf{2}} = \mathbf{0}$$

$$\Updownarrow$$

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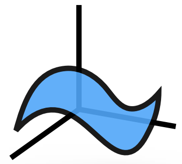
$$\|$$

$$\mathbf{x}^{\otimes\mathbf{2}} \in \mathcal{S}$$



$$\mathbf{x} \in \mathcal{V} \iff V^{\mathsf{T}} \mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$

$$\Updownarrow$$

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$$\|$$

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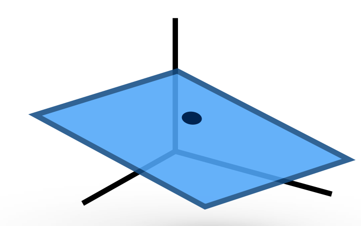
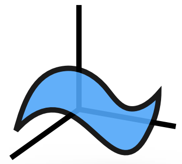




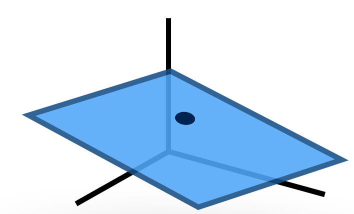
$$\mathbf{x} \in \mathcal{V} \iff V^{\mathsf{T}}\mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$

$$\Updownarrow$$

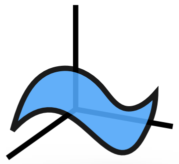
$$\mathbf{x}^{\otimes \mathbf{2}} \in \ker V^{\mathsf{T}}$$

$$\|$$

$$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$$



Variety = Tensorized Subspace!



$$\mathbf{x} \in \mathcal{V} \iff V^{\top}\mathbf{x}^{\otimes \mathbf{2}} = \mathbf{0}$$

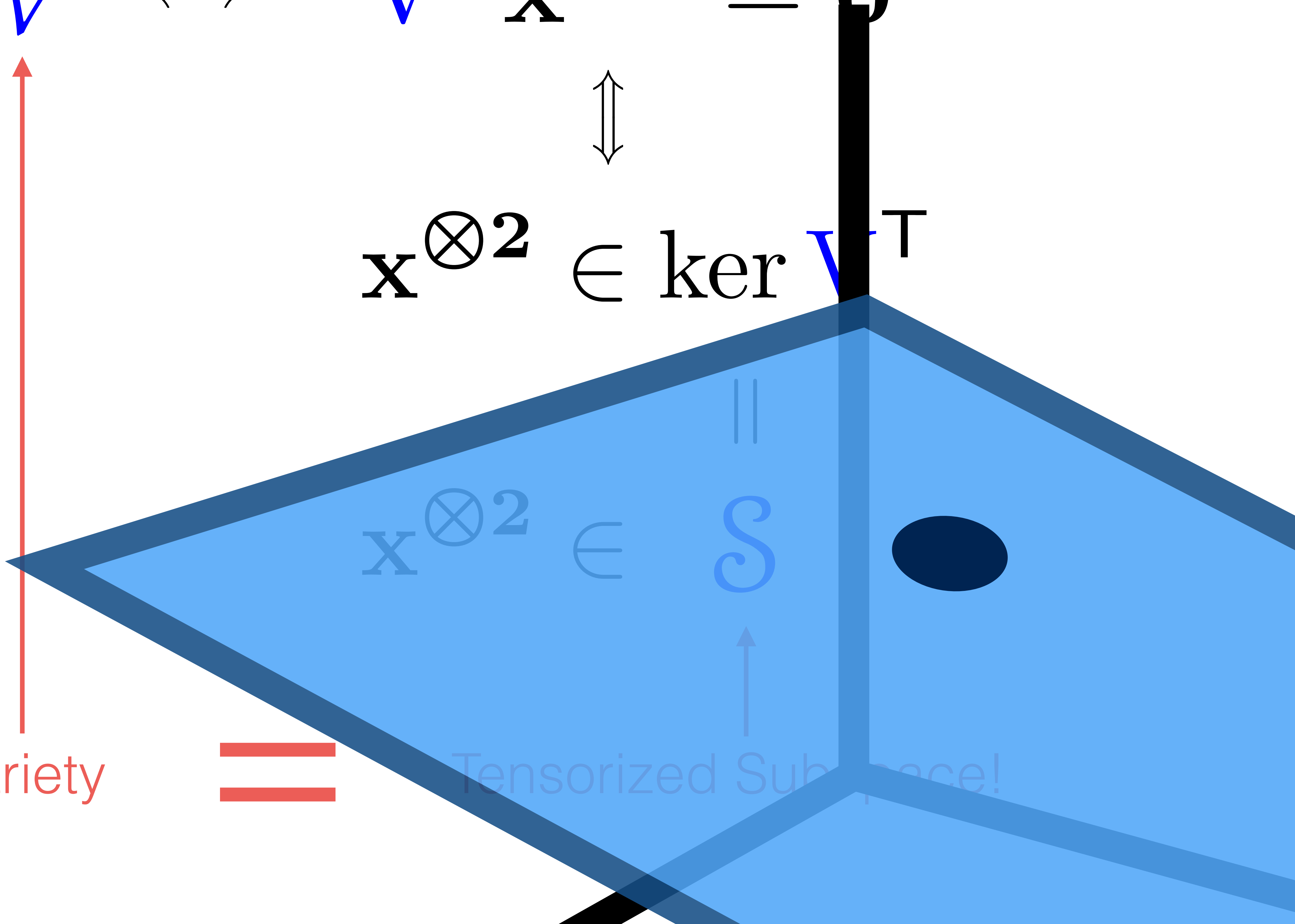
$$\Updownarrow$$

$$\mathbf{x}^{\otimes \mathbf{2}} \in \ker V^{\top}$$

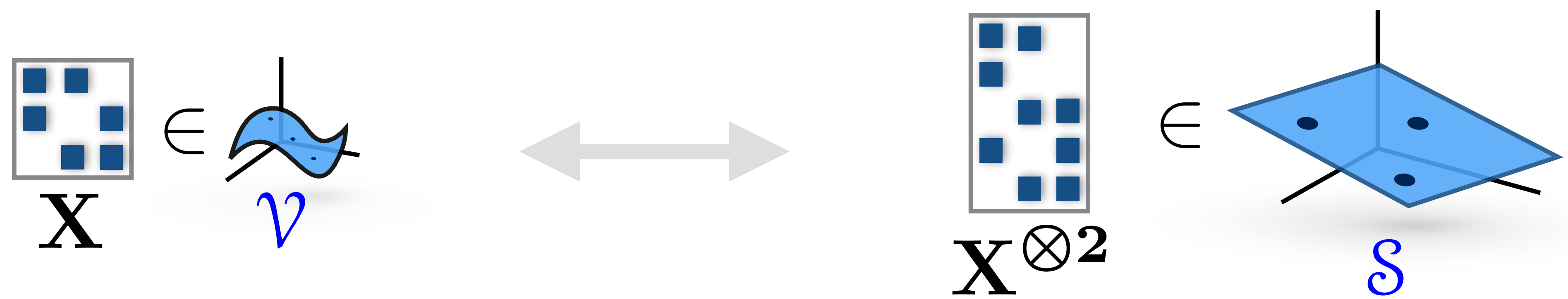
$$\|$$

$$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$$

Variety = Tensorized Subspace!



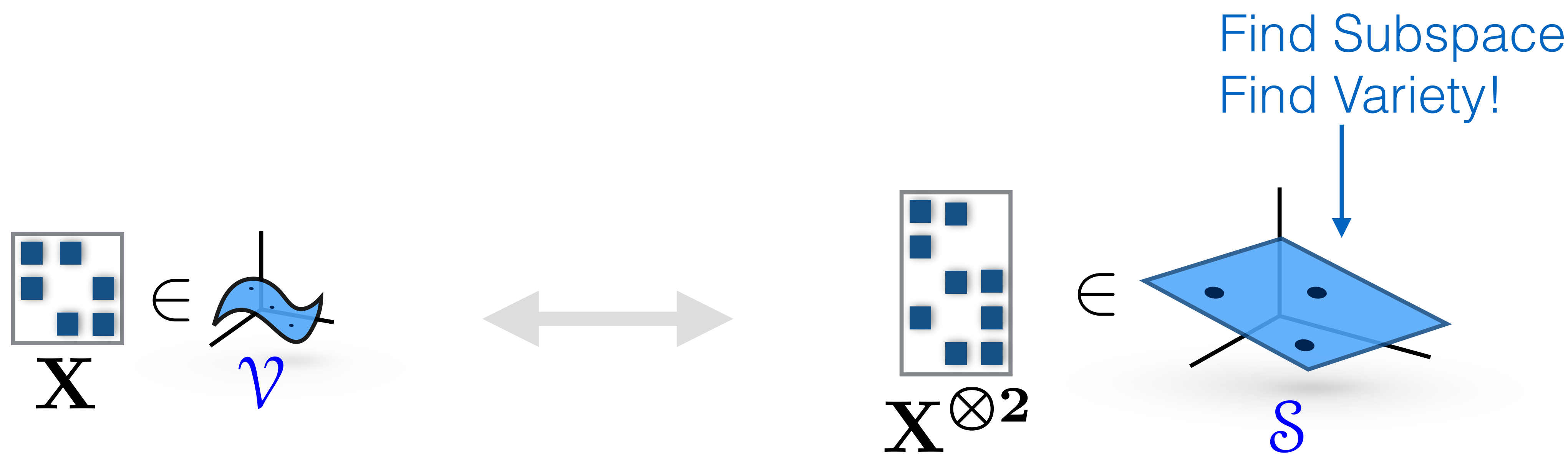
What does this mean?



Algebraic Variety
(nonlinear)

Tensorized Subspace
(linear)

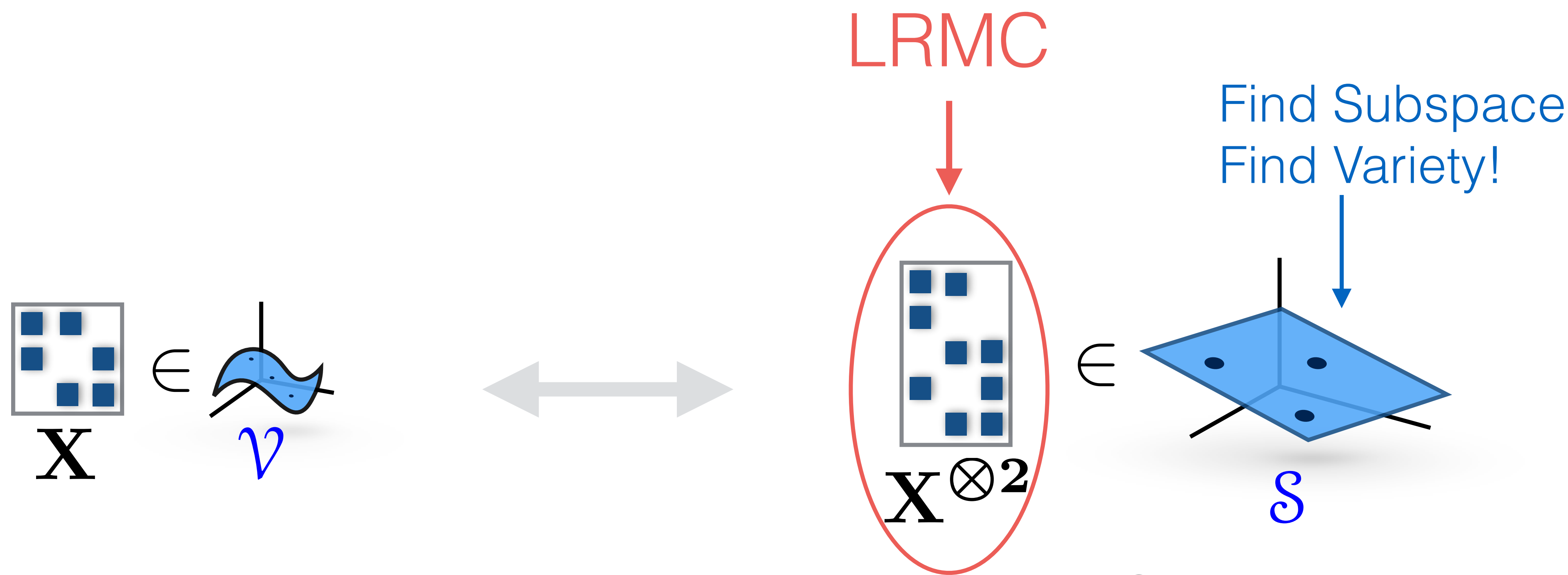
What does this mean?



Algebraic Variety
(nonlinear)

Tensorized Subspace
(linear)

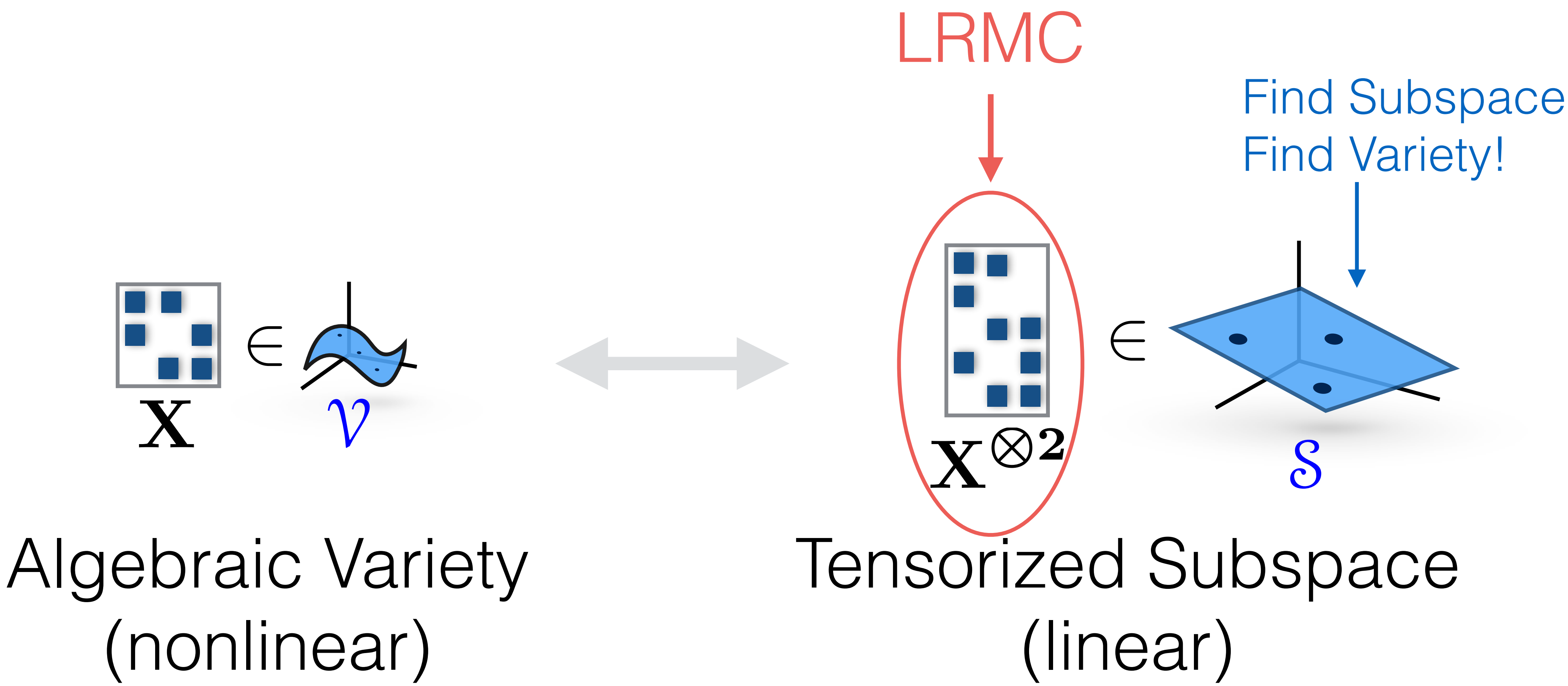
What does this mean?



Algebraic Variety
(nonlinear)

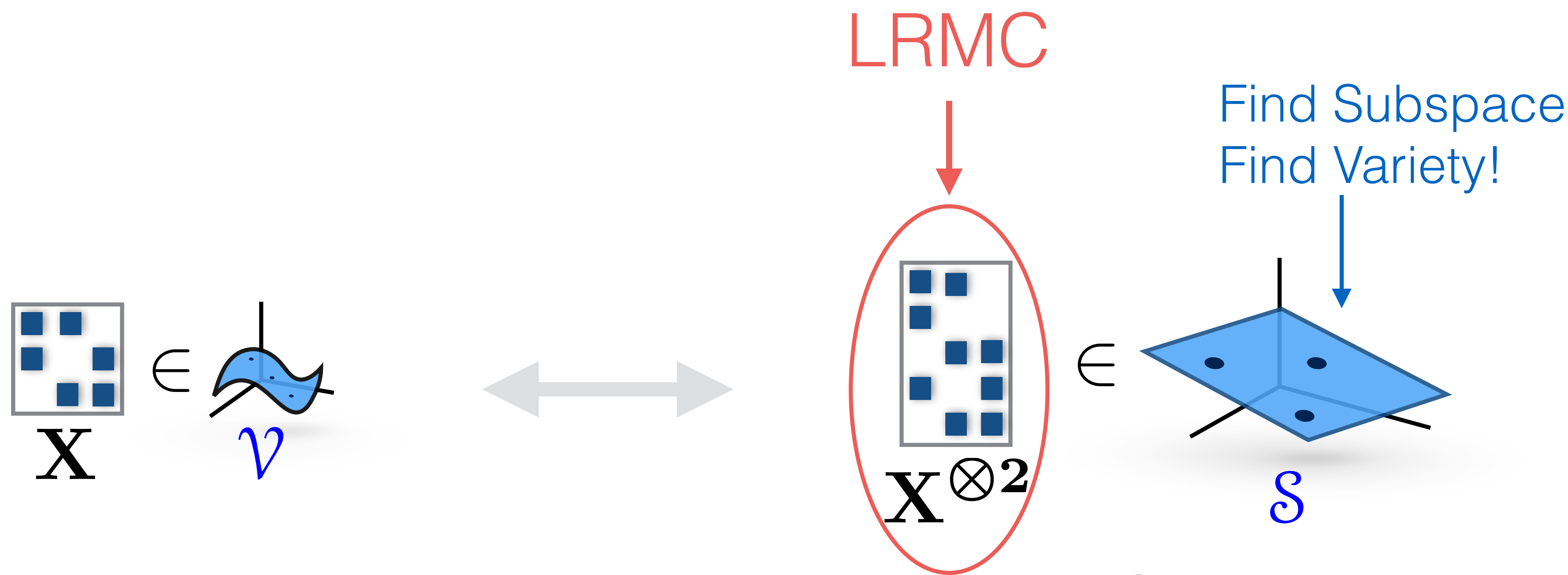
Tensorized Subspace
(linear)

What does this mean?



What does this mean?

Is this just standard Low-Rank Matrix Completion?



Algebraic Variety
(nonlinear)

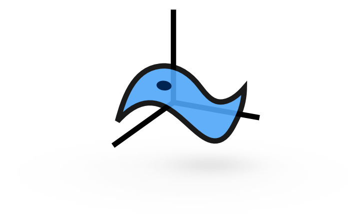
Tensorized Subspace
(linear)

What does this mean?

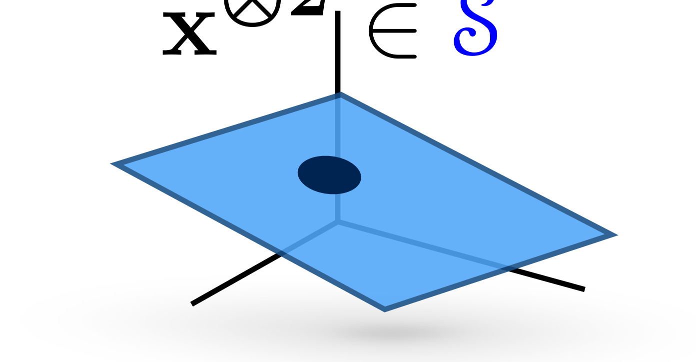
Is this just standard Low-Rank Matrix Completion?

More or less…

$\mathbf{x} \in \mathcal{V}$



$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$

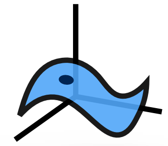


$$\mathbf{x}$$

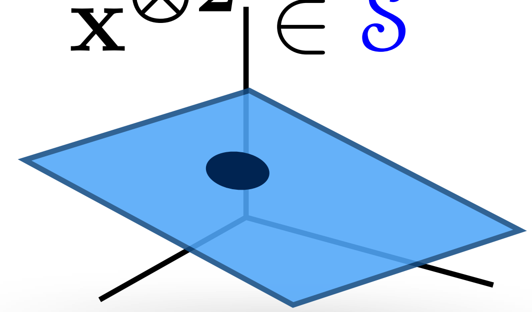
$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix}$$

Recall…

$\mathbf{x} \in \mathcal{V}$



$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$

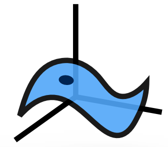


$$\mathbf{x} \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

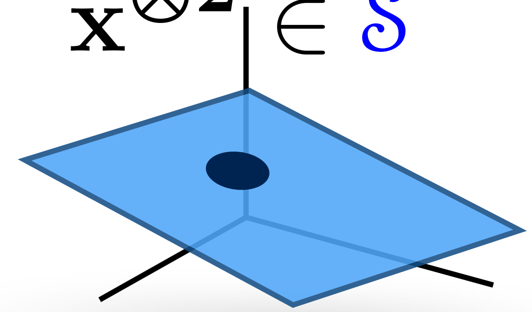
$$\mathbf{x} \otimes \mathbf{x} \qquad \begin{bmatrix} x_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ x_2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ x_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{bmatrix}$$

Recall…

$\mathbf{x} \in \mathcal{V}$



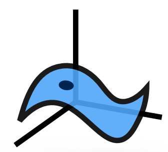
$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$



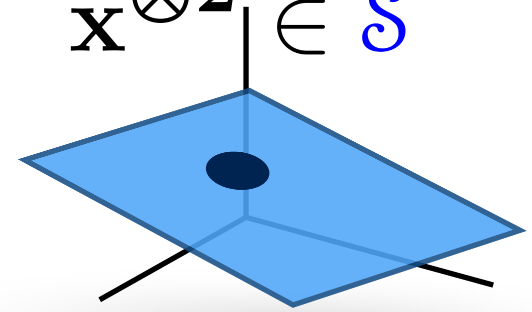
$$\begin{array}{ccc} \mathbf{x} & & \mathbf{x} \otimes \mathbf{x} \\ \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} & \longrightarrow & \begin{bmatrix} \mathrm{x}_1\mathrm{x}_1 \\ \mathrm{x}_1\mathrm{x}_2 \\ \mathrm{x}_1\mathrm{x}_3 \\ \\ \mathrm{x}_2\mathrm{x}_1 \\ \mathrm{x}_2\mathrm{x}_2 \\ \mathrm{x}_2\mathrm{x}_3 \\ \\ \mathrm{x}_3\mathrm{x}_1 \\ \mathrm{x}_3\mathrm{x}_2 \\ \mathrm{x}_3\mathrm{x}_3 \end{bmatrix} \end{array}$$

Recall…

$\mathbf{x} \in \mathcal{V}$



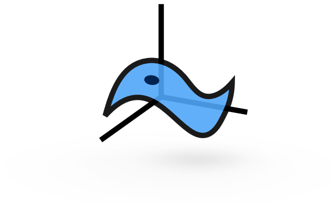
$\mathbf{x}^{\otimes 2} \in \mathcal{S}$



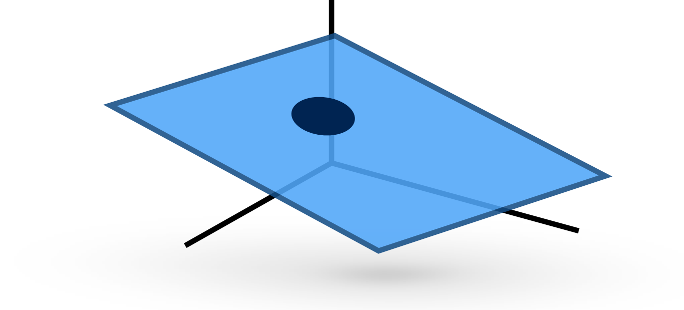
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \mathbf{x} \otimes \mathbf{x} = \begin{bmatrix} x_1x_1 \\ x_1x_2 \\ x_1x_3 \\ \\ x_2x_1 \\ x_2x_2 \\ x_2x_3 \\ \\ x_3x_1 \\ x_3x_2 \\ x_3x_3 \end{bmatrix}$$

Recall…

$\mathbf{x} \in \mathcal{V}$



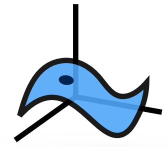
$\mathbf{x}^{\otimes 2} \in \mathcal{S}$



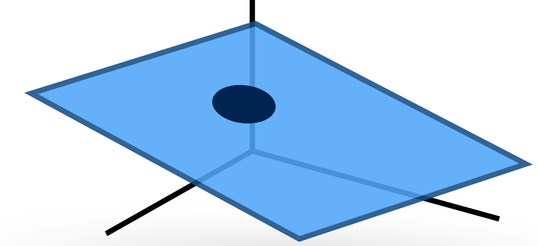
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \mathbf{x} \otimes \mathbf{x} = \begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_1 x_3 \\ x_2 x_1 \\ x_2 x_2 \\ x_2 x_3 \\ x_3 x_1 \\ x_3 x_2 \\ x_3 x_3 \end{bmatrix}$$

Recall…

$\mathbf{x} \in \mathcal{V}$



$\mathbf{x}^{\otimes 2} \in \mathcal{S}$



$$\mathbf{x}$$

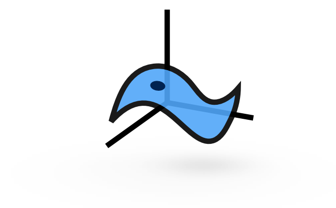
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\mathbf{x} \otimes \mathbf{x}$$

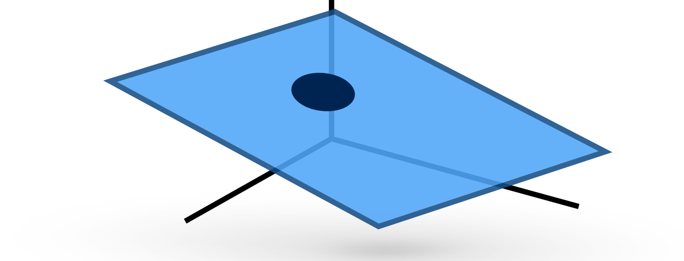
$$\begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_1 x_3 \\ \\ x_2 x_1 \\ x_2 x_2 \\ x_2 x_3 \\ \\ x_3 x_1 \\ x_3 x_2 \\ x_3 x_3 \end{bmatrix}$$

Recall...

$\mathbf{x} \in \mathcal{V}$



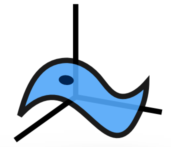
$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$



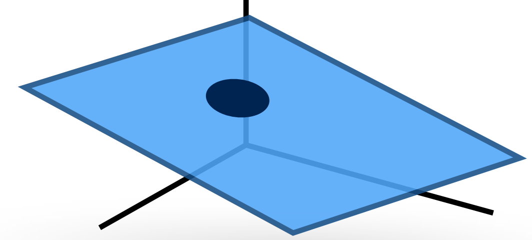
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \longrightarrow \mathbf{x} \otimes \mathbf{x} = \begin{bmatrix} x_1x_1 \\ x_1x_2 \\ x_1x_3 \\ x_2x_1 \\ x_2x_2 \\ x_2x_3 \\ x_3x_1 \\ x_3x_2 \\ x_3x_3 \end{bmatrix} \longrightarrow \mathbf{x}^{\otimes \mathbf{2}} = \begin{bmatrix} x_1x_1 \\ x_1x_2 \\ x_1x_3 \\ x_2x_2 \\ x_2x_3 \\ x_3x_3 \end{bmatrix}$$

Recall...

$\mathbf{x} \in \mathcal{V}$



$\mathbf{x}^{\otimes \mathbf{2}} \in \mathcal{S}$

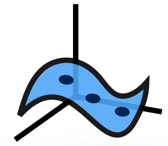


$$\mathbf{x} \quad \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \cdot \end{bmatrix}$$

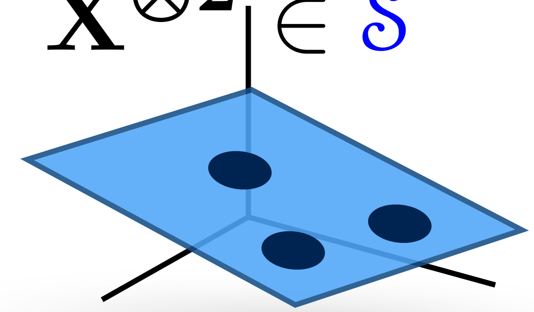
$$\mathbf{x}^{\otimes \mathbf{2}} \quad \begin{bmatrix} \mathrm{x}_1\mathrm{x}_1 \\ \mathrm{x}_1\mathrm{x}_2 \\ \cdot \\ \mathrm{x}_2\mathrm{x}_2 \\ \cdot \\ \cdot \end{bmatrix}$$

The sampling is highly restricted!

$\mathbf{X} \in \mathcal{V}$



$\mathbf{X}^{\otimes 2} \in \mathcal{S}$



$$\mathbf{X}$$

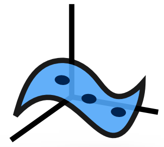
$$\begin{bmatrix} x_1 & x_1 & \cdot \\ x_2 & \cdot & x_2 \\ \cdot & x_3 & x_3 \end{bmatrix}$$

$$\mathbf{X}^{\otimes 2}$$

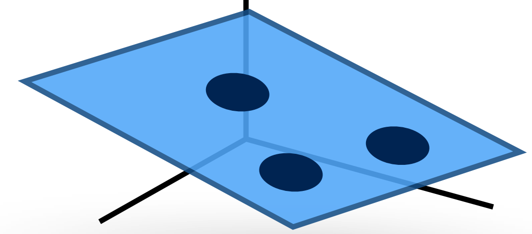
$$\begin{bmatrix} x_1^2 & x_1^2 & \cdot \\ x_1 x_2 & \cdot & \cdot \\ \cdot & x_1 x_3 & \cdot \\ x_2^2 & \cdot & x_2^2 \\ \cdot & \cdot & x_2 x_3 \\ \cdot & x_3^2 & x_3^2 \end{bmatrix}$$

The sampling is highly restricted!

$\mathbf{X} \in \mathcal{V}$



$\mathbf{X}^{\otimes 2} \in \mathcal{S}$



$$\mathbf{X}$$

$$\begin{bmatrix} x_1 & x_1 & \cdot \\ x_2 & \cdot & x_2 \\ \cdot & x_3 & x_3 \end{bmatrix}$$

$$\mathbf{X}^{\otimes 2}$$

$$\underbrace{\begin{bmatrix} x_1^2 & x_1^2 & \cdot \\ x_1 x_2 & \cdot & \cdot \\ \cdot & x_1 x_3 & \cdot \\ x_2^2 & \cdot & x_2^2 \\ \cdot & \cdot & x_2 x_3 \\ \cdot & x_3^2 & x_3^2 \end{bmatrix}}_{3/20}$$

Impossible!

$$\underbrace{\begin{bmatrix} x_1^2 & x_1^2 & x_1^2 & & \cdot \\ \cdot & x_1 x_2 & x_1 x_2 & & \cdot \\ \cdot & x_1 x_3 & \cdot & \cdots & \cdot \\ x_2^2 & \cdot & \cdot & & x_2^2 \\ \cdot & \cdot & x_2 x_3 & & x_2 x_3 \\ x_3^2 & \cdot & \cdot & & x_3^2 \end{bmatrix}}_{17/20}$$

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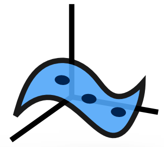
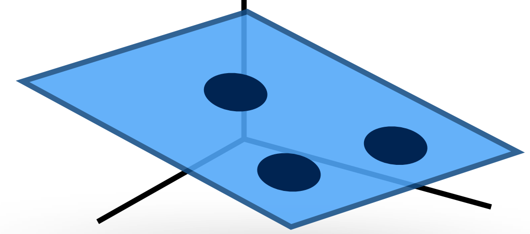
$\mathbf{X} \in \mathcal{V}$



$\mathbf{X}^{\otimes 2} \in \mathcal{S}$



$$\mathbf{X} \qquad \begin{bmatrix} x_1 & x_1 & \cdot \\ x_2 & \cdot & x_2 \\ \cdot & x_3 & x_3 \end{bmatrix}$$

$$\mathbf{X}^{\otimes 2} \qquad \underbrace{\begin{bmatrix} x_1^2 & x_1^2 & \cdot \\ x_1x_2 & \cdot & \cdot \\ \cdot & x_1x_3 & \cdot \\ x_2^2 & \cdot & x_2^2 \\ \cdot & \cdot & x_2x_3 \\ \cdot & x_3^2 & x_3^2 \end{bmatrix}}_{3/20}$$

Impossible!

$$\underbrace{\begin{bmatrix} x_1^2 & x_1^2 & x_1^2 & & \cdot \\ \cdot & x_1x_2 & x_1x_2 & & \cdot \\ \cdot & x_1x_3 & \cdot & & \cdot \\ x_2^2 & \cdot & \cdot & \cdots & x_2^2 \\ \cdot & \cdot & x_2x_3 & & x_2x_3 \\ x_3^2 & \cdot & \cdot & & x_3^2 \end{bmatrix}}_{17/20}$$

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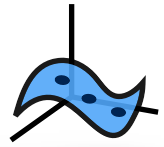
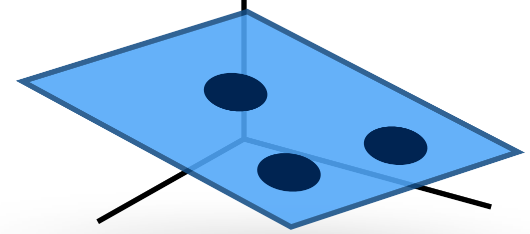
$\mathbf{X} \in \mathcal{V}$



$\mathbf{X}^{\otimes 2} \in \mathcal{S}$



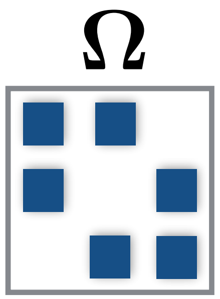
Impossible!

$$\begin{array}{ccc}
\mathbf{X} & \mathbf{X}^{\otimes 2} & \\
\begin{bmatrix} x_1 & x_1 & \cdot \\ x_2 & \cdot & x_2 \\ \cdot & x_3 & x_3 \end{bmatrix}
&
\underbrace{\begin{bmatrix} x_1^2 & x_1^2 & \cdot \\ x_1x_2 & \cdot & \cdot \\ \cdot & x_1x_3 & \cdot \\ x_2^2 & \cdot & x_2^2 \\ \cdot & \cdot & x_2x_3 \\ \cdot & x_3^2 & x_3^2 \end{bmatrix}}_{3/20}
&
\underbrace{\begin{bmatrix} x_1^2 & x_1^2 & x_1^2 & & \cdot \\ \cdot & x_1x_2 & x_1x_2 & & \cdot \\ \cdot & x_1x_3 & \cdot & & \cdot \\ x_2^2 & \cdot & \cdot & \cdots & x_2^2 \\ \cdot & \cdot & x_2x_3 & & x_2x_3 \\ x_3^2 & \cdot & \cdot & & x_3^2 \end{bmatrix}}_{17/20}
\end{array}$$

Small letters in LRMC:

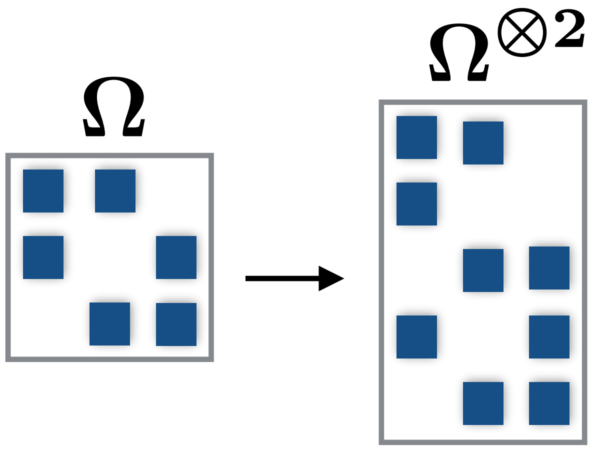
Incoherence and Uniform Sampling

In general



Given: available samplings

In general



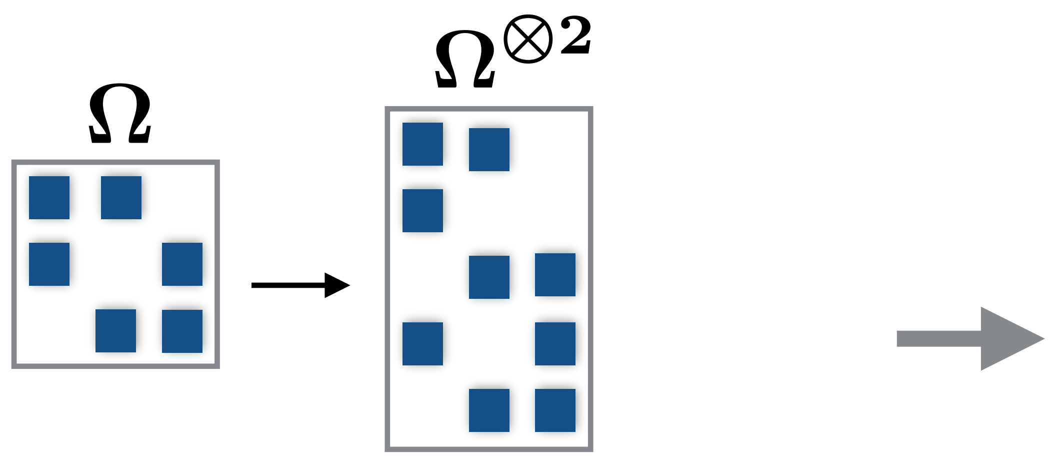
Given: available samplings

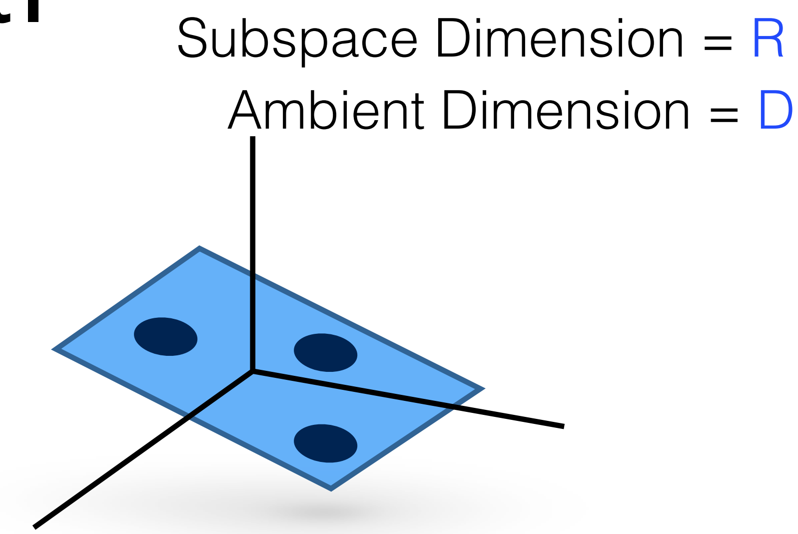
In general

Given: available samplings

Can we find $\mathcal{S}$?

In general





Given: available samplings

Can we find $\mathcal{S}$?

Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathcal{V}$ is in general position. With probability 1, $\mathcal{S}$ can be uniquely recovered if and only if there is a matrix $\boldsymbol{\Omega}_\star^{\otimes 2}$ formed with D-R columns of $\boldsymbol{\Omega}^{\otimes 2}$ such that every $\boldsymbol{\Omega}_\ell^{\otimes 2}$ formed with a subset of columns in $\boldsymbol{\Omega}_\star^{\otimes 2}$ satisfies:

$$\#\text{rows\_with\_observations}(\boldsymbol{\Omega}_\ell^{\otimes 2}) \geq \#\text{columns}(\boldsymbol{\Omega}_\ell^{\otimes 2}) + \text{R}.$$

Furthermore, this condition is true if and only if $\dim\ker \mathbf{A}^\mathsf{T} = \text{R}$, whence $\mathcal{S} = \ker \mathbf{A}^\mathsf{T}$.

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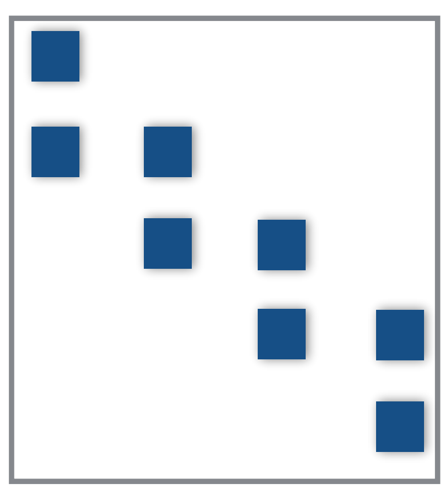
D=5, R=1

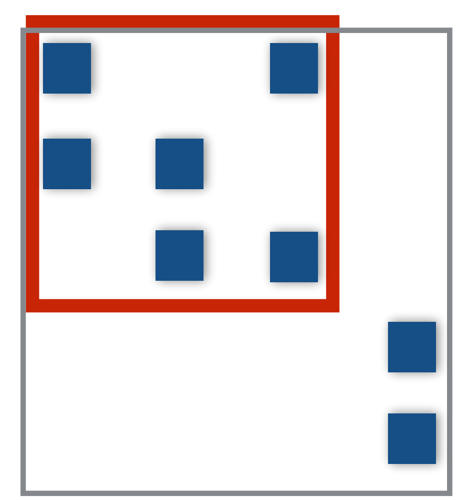
$\mathbf{\Omega}^{\otimes 2} =$



Good

3<3+1

$\mathbf{\Omega}^{\otimes 2} =$



Bad

Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

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Furthermore, this condition is true if and only if $\dim\ker \mathbf{A}^\mathsf{T} = \mathrm{R}$, whence $\mathcal{S} = \ker \mathbf{A}^\mathsf{T}$.

In words:

- Yes, it is possible to find the subspace $\mathcal{S}$.
- Iff you observe *the right* entries (rows vs cols condition).
- There is an easy way to check this rows vs cols condition.
- If the condition is satisfied, there is an easy way to find $\mathcal{S}$.

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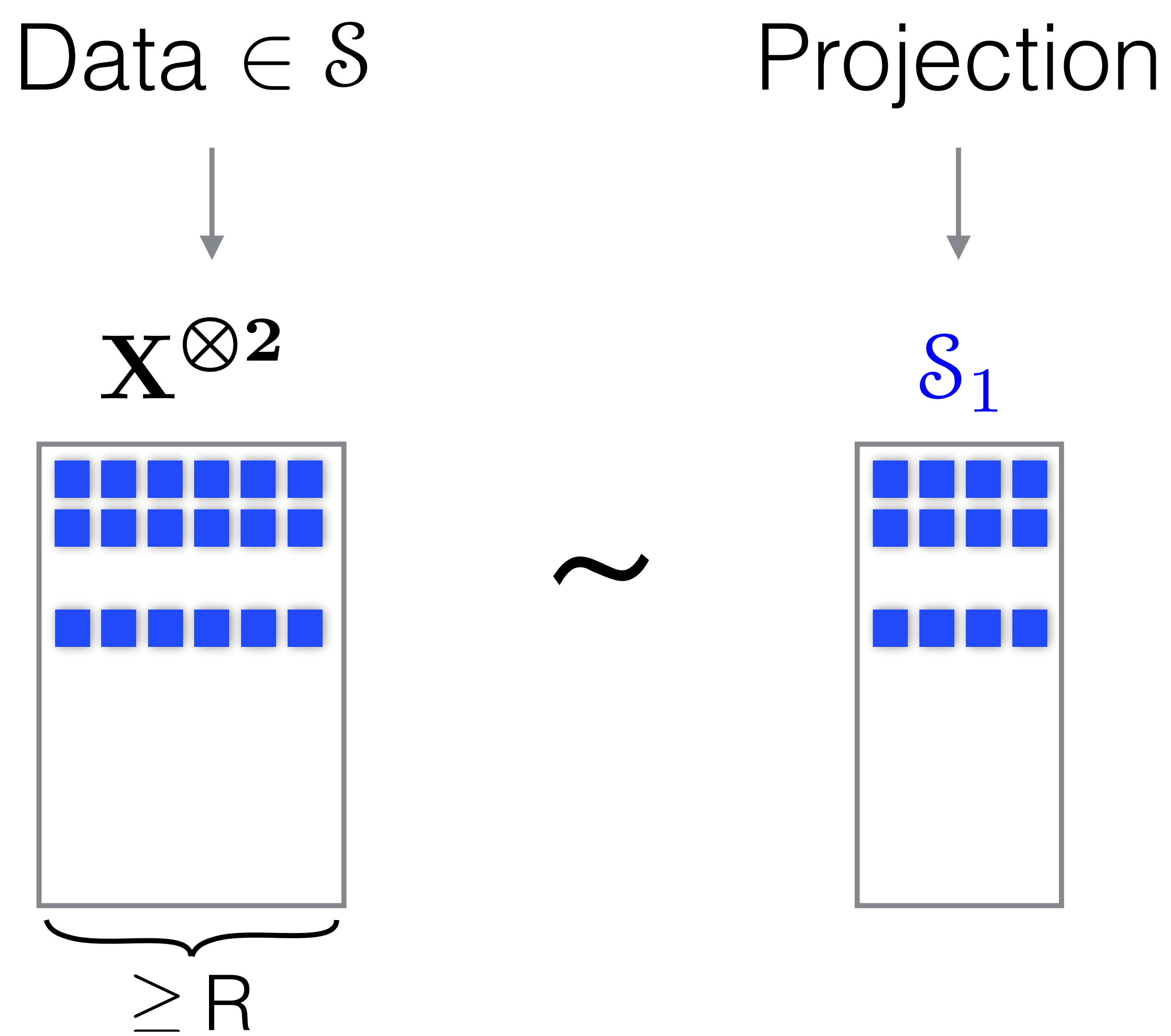
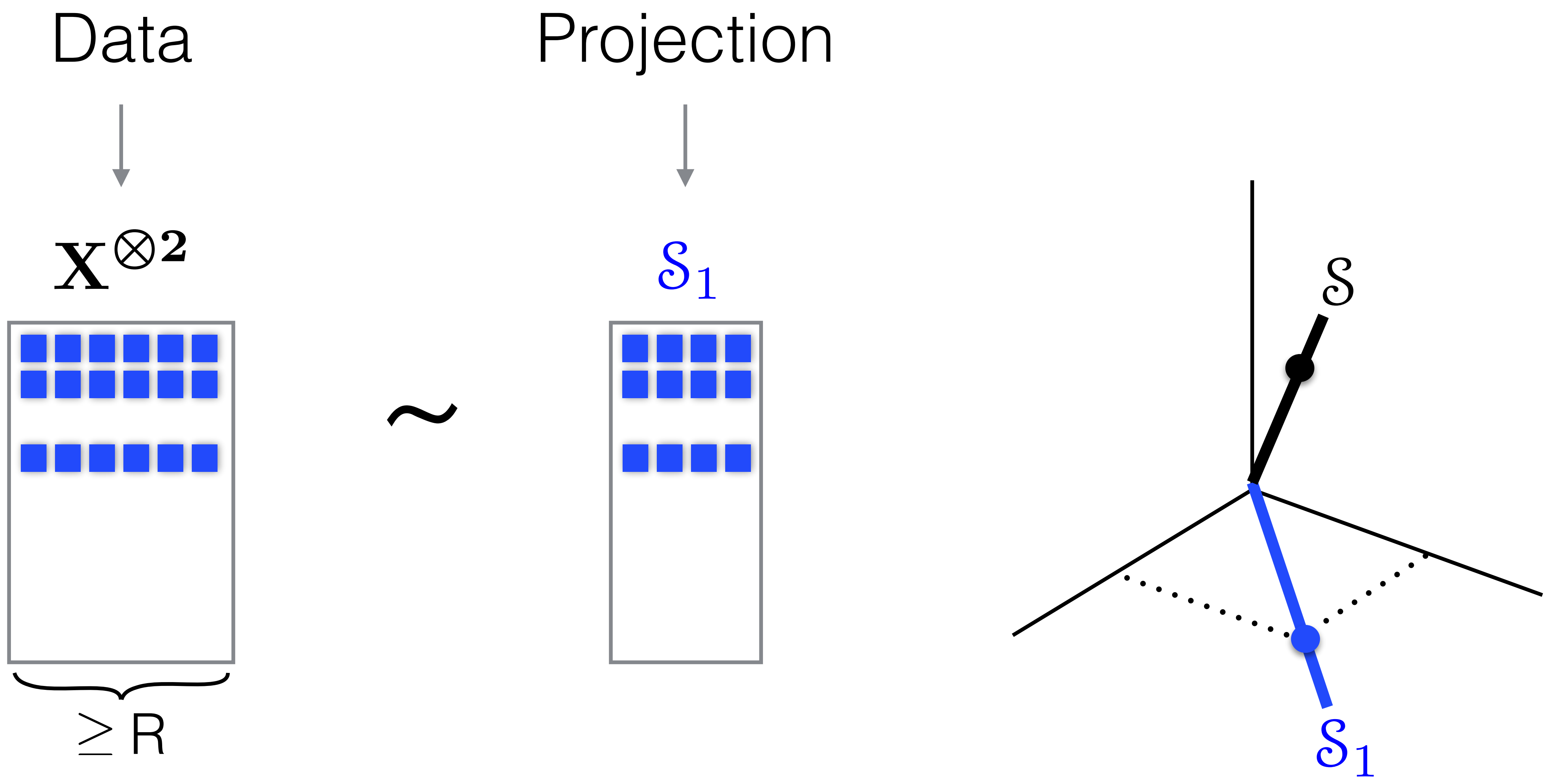


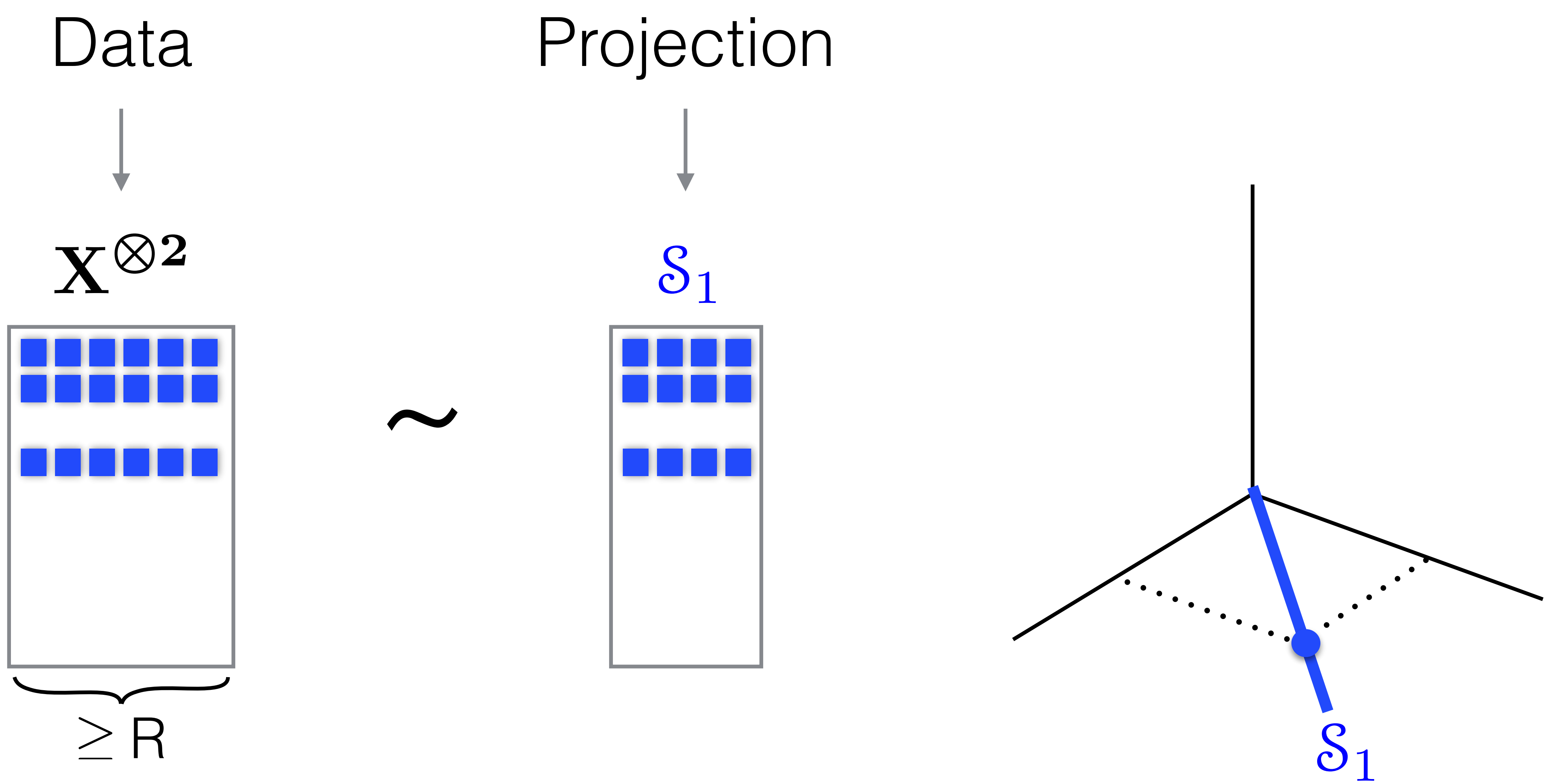
A Flavor of our Ideas

Data ~ Projection

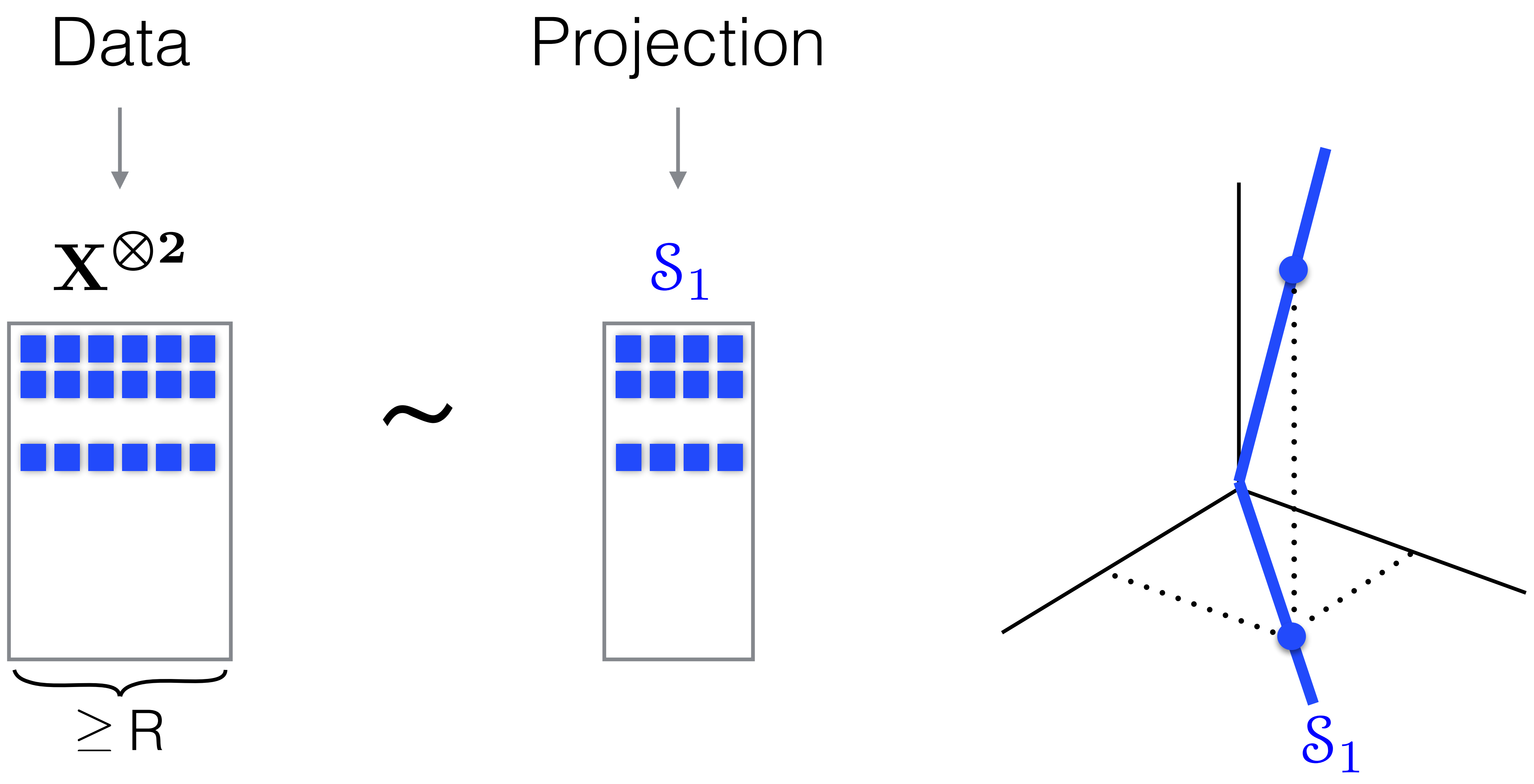


A Flavor of our Ideas

Data ~ Projection



How is this information useful?



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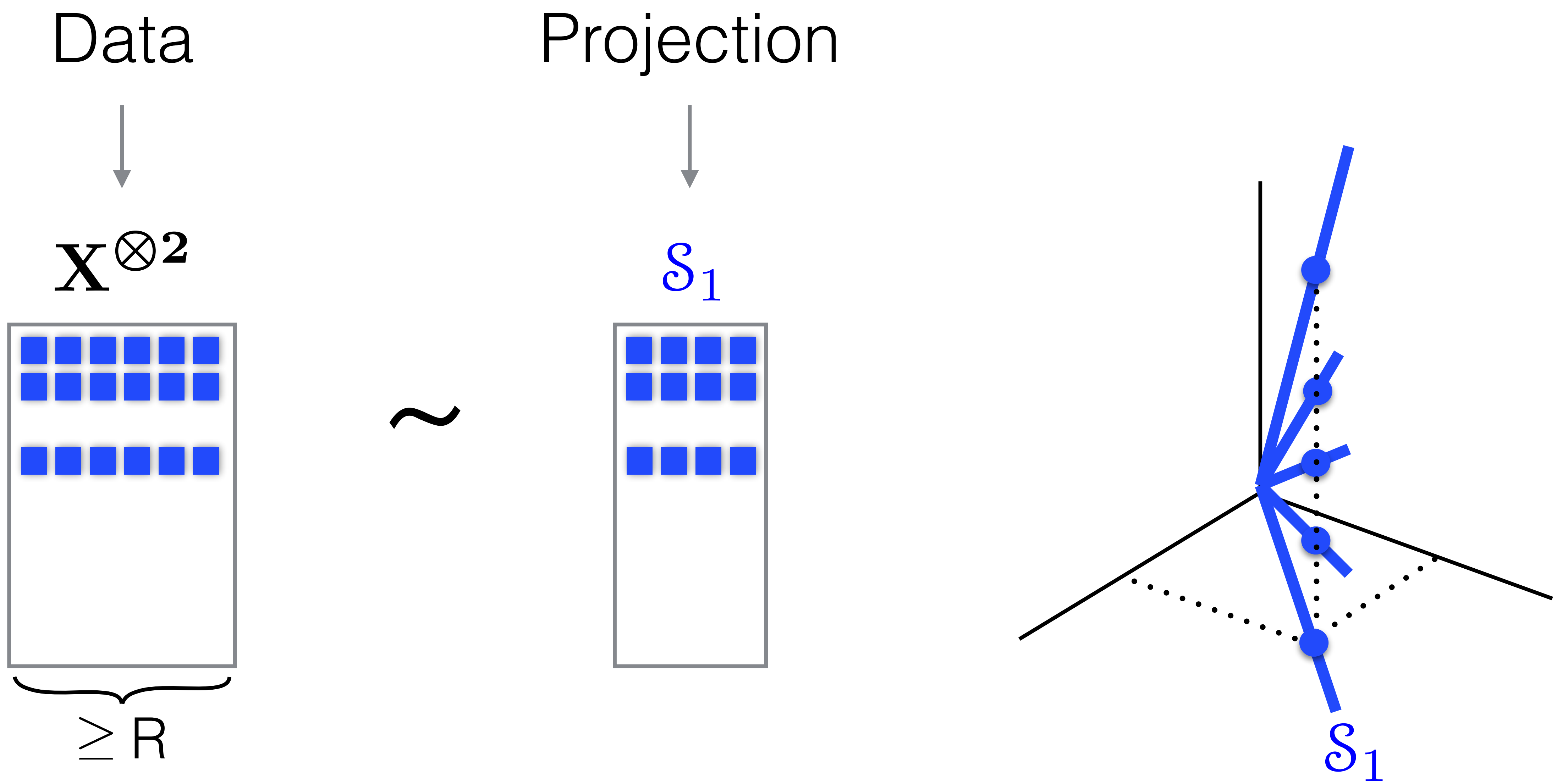

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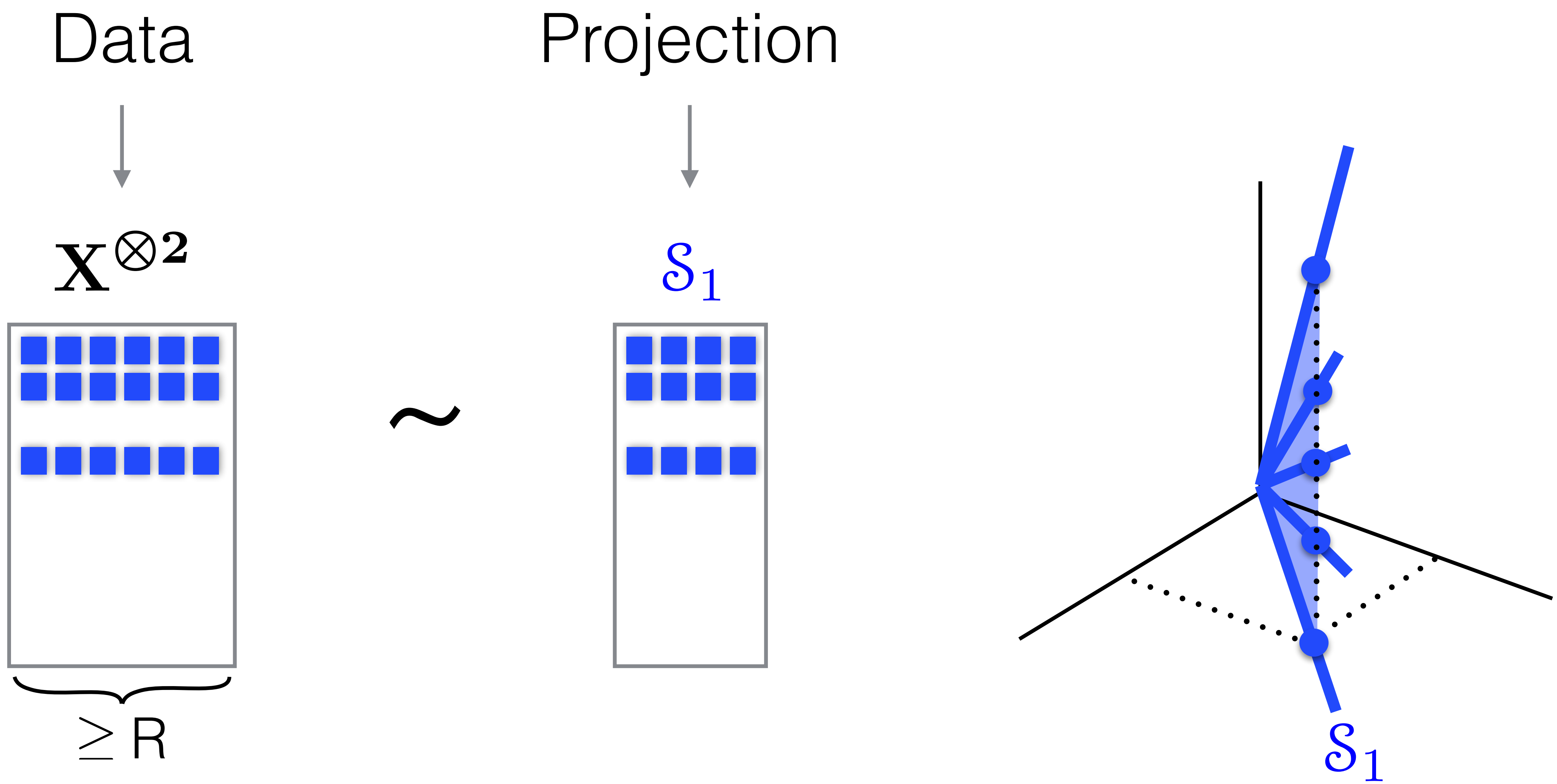
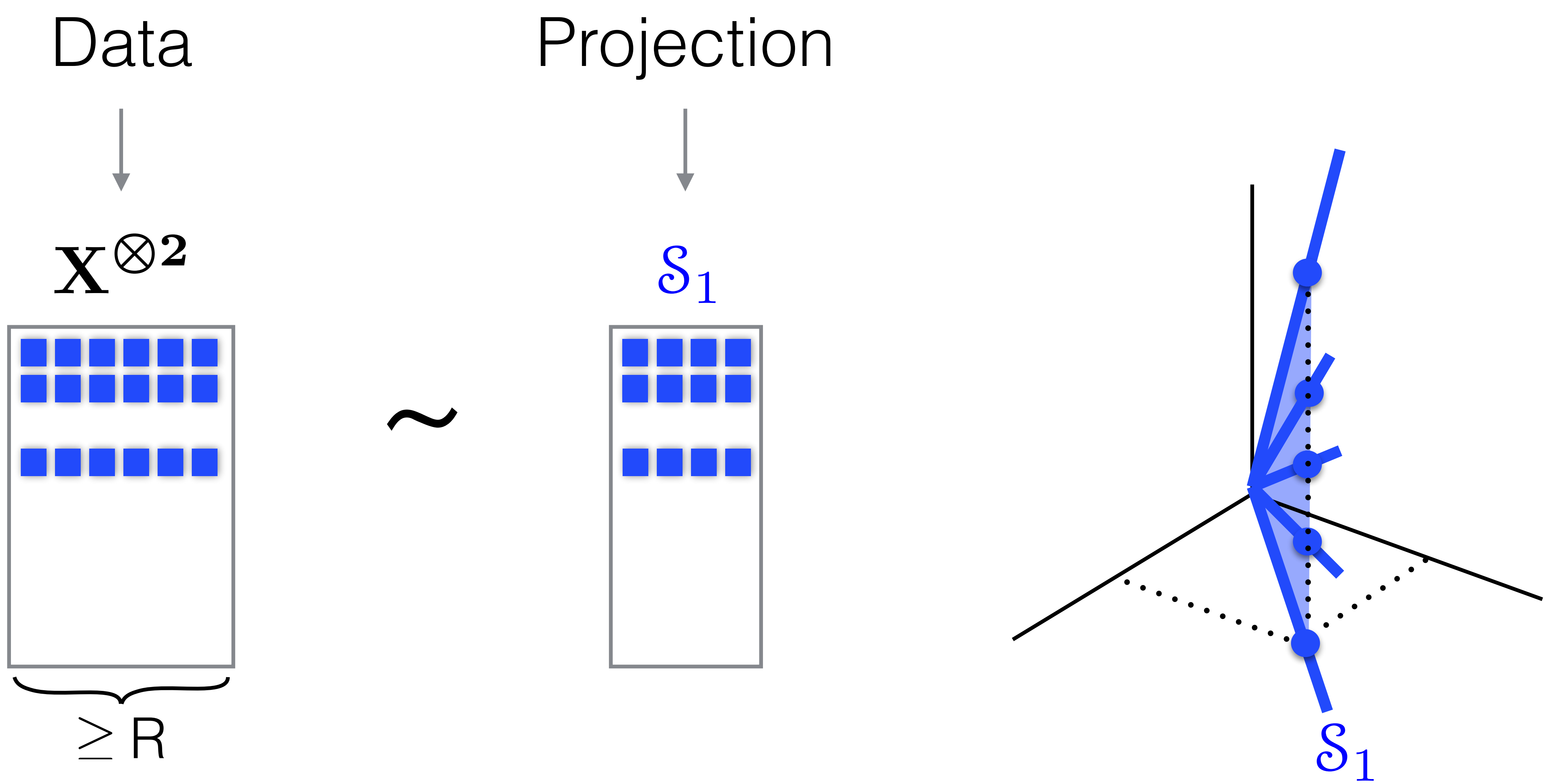


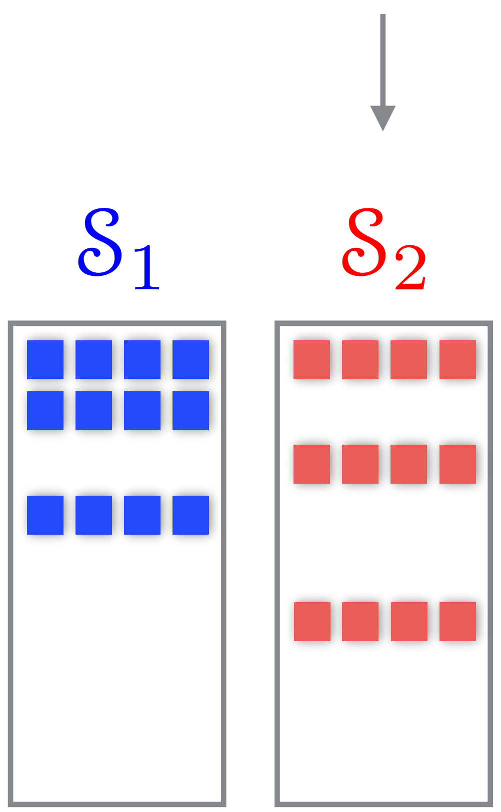
How is this information useful?

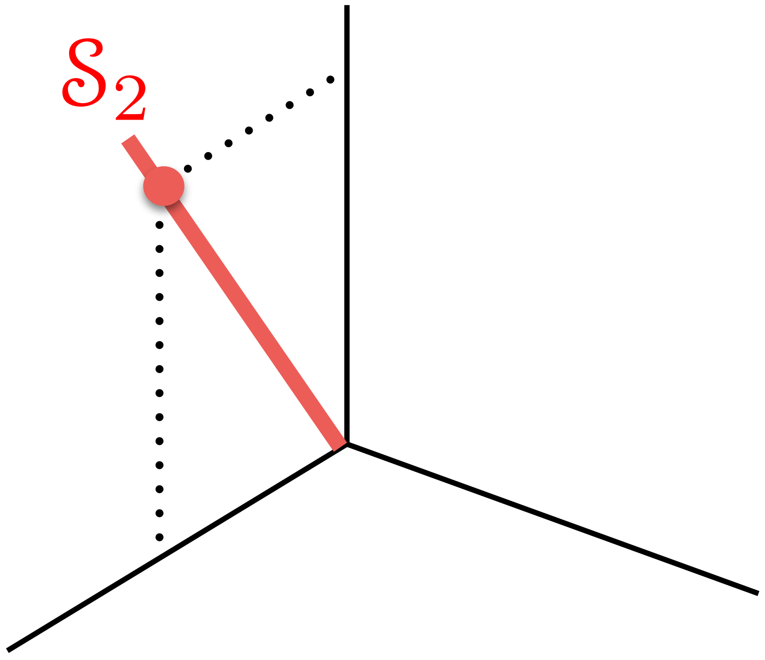


The projection $\mathcal{S}_1$ imposes one restriction on what $\mathcal{S}$ may be.

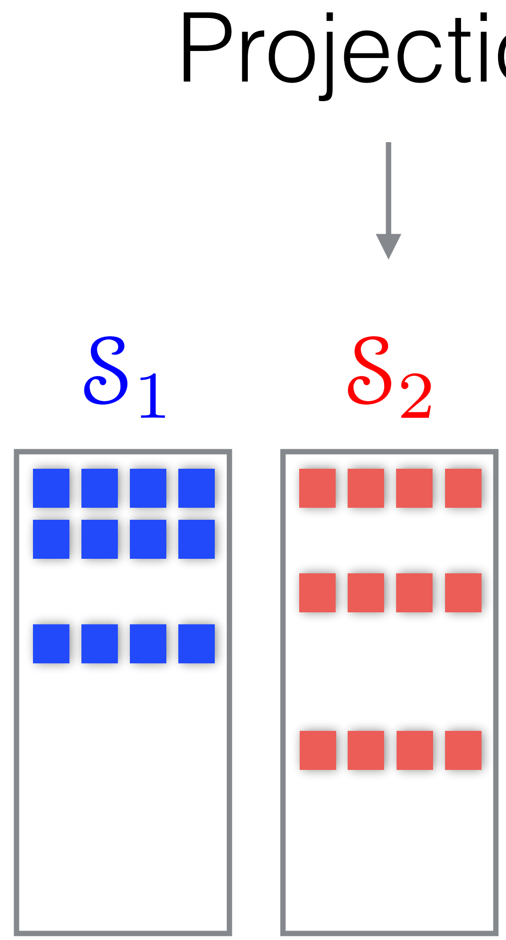
How is this information useful?

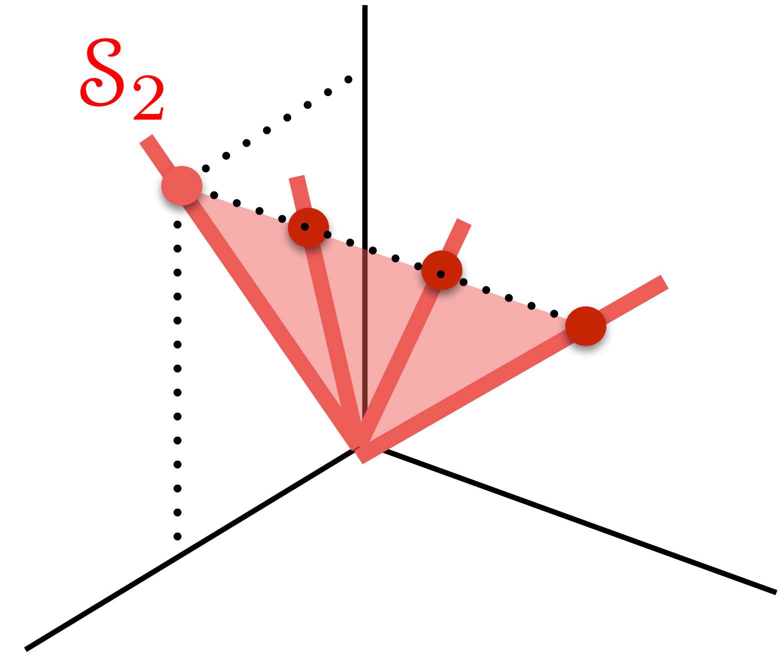
Projection





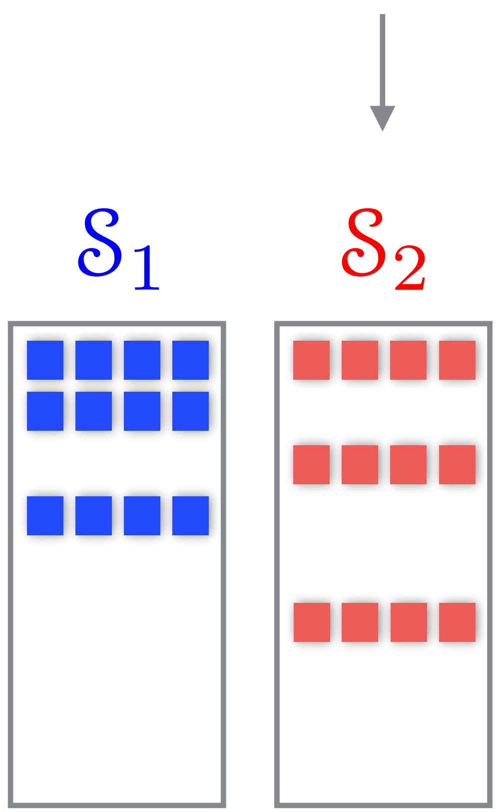
If we observe more projections…

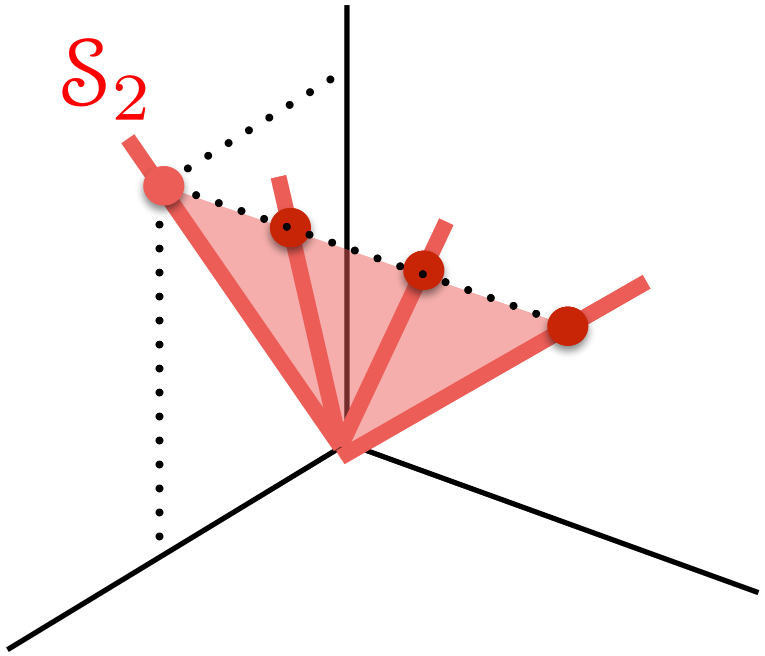




If we observe more projections…

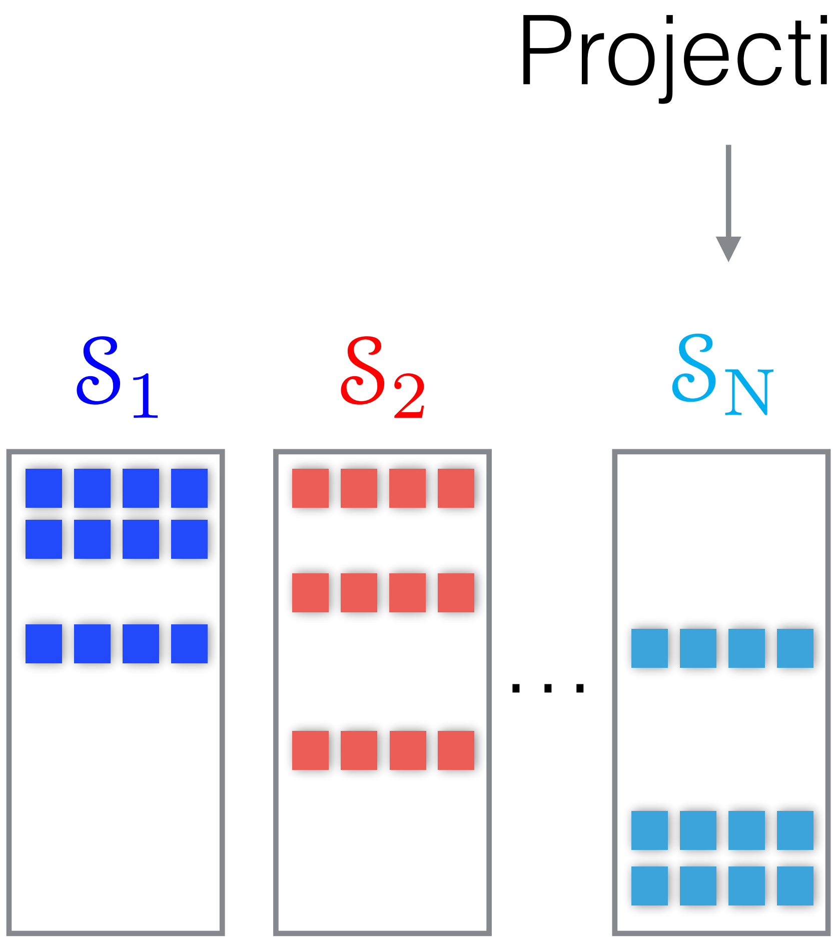
Projection

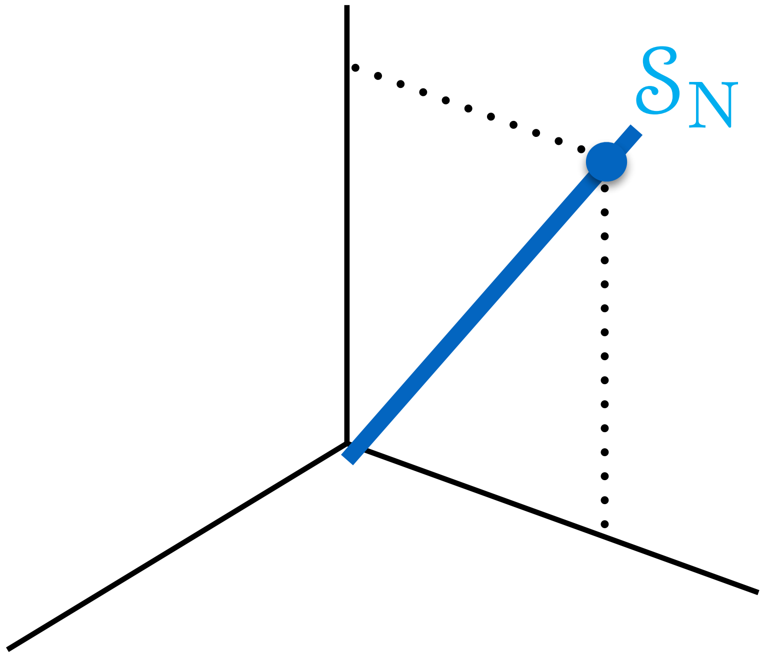




If we observe more projections…

We get more restrictions…





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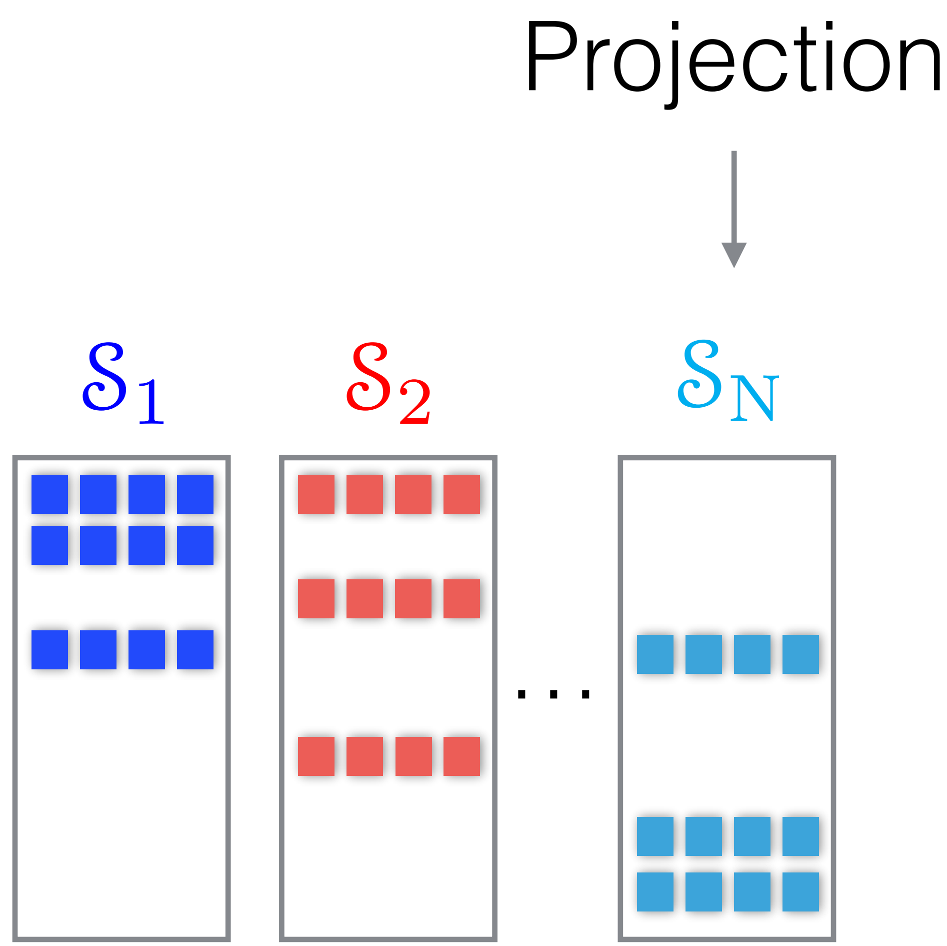

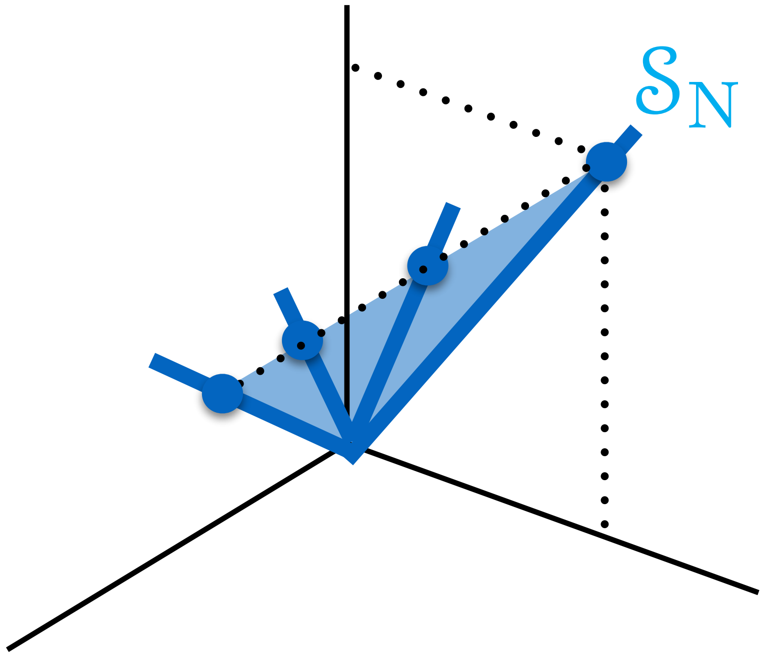

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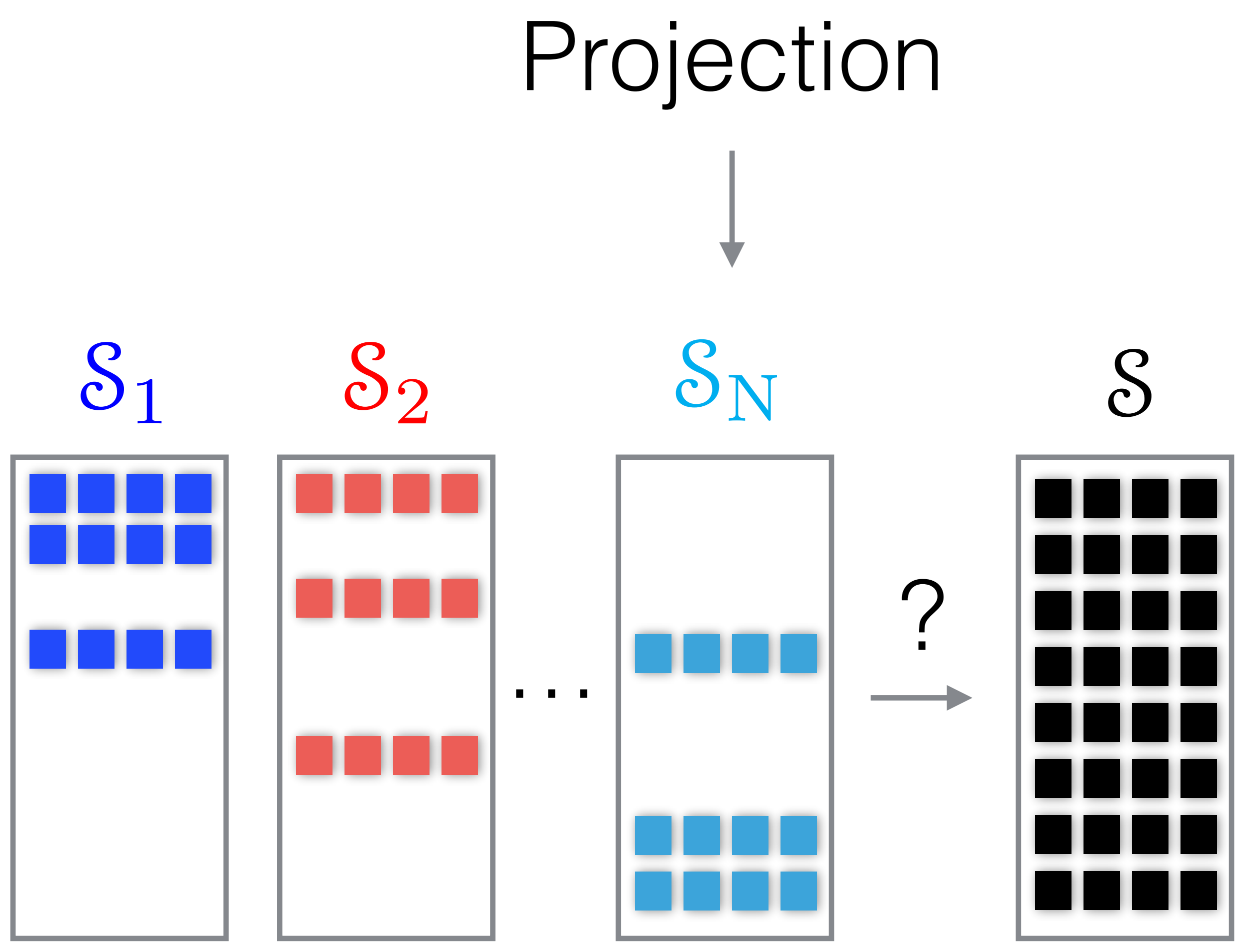
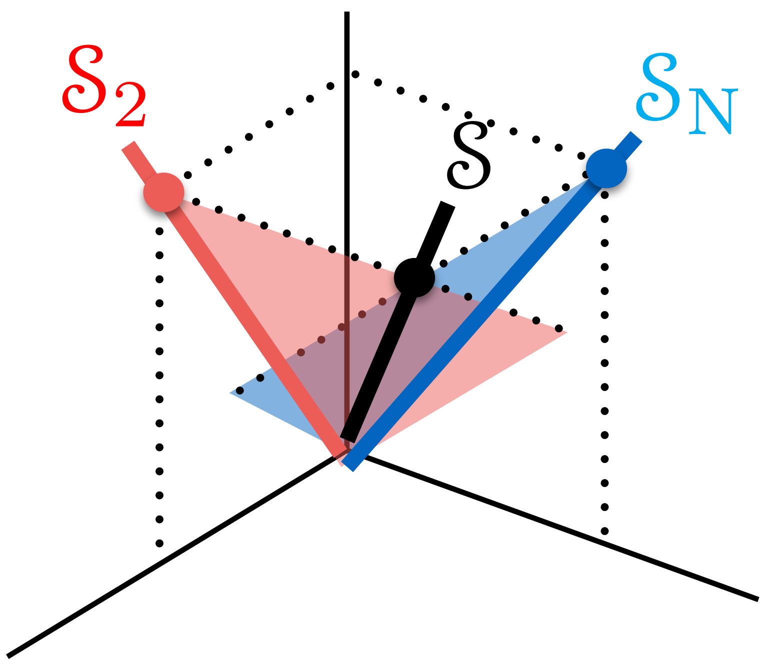
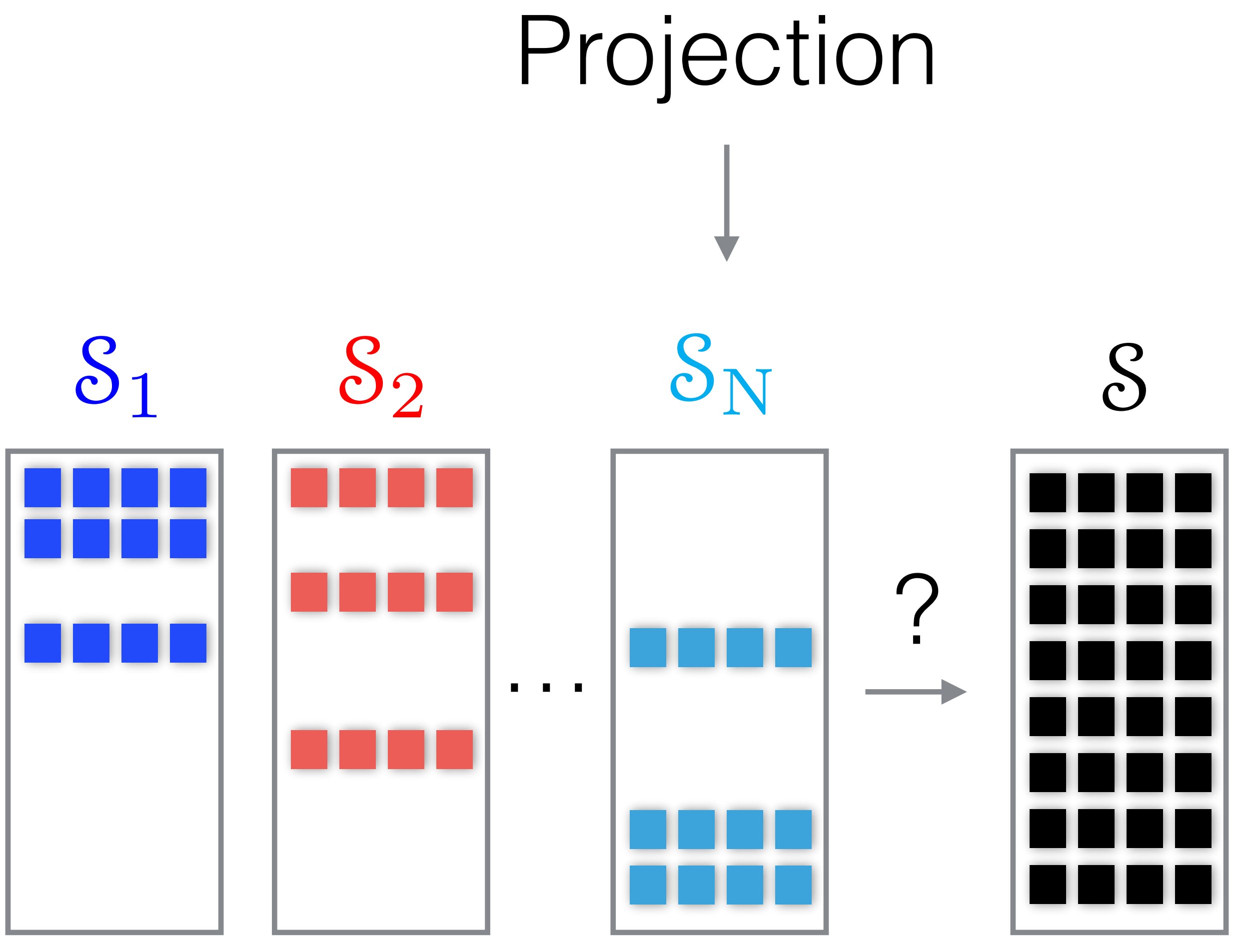
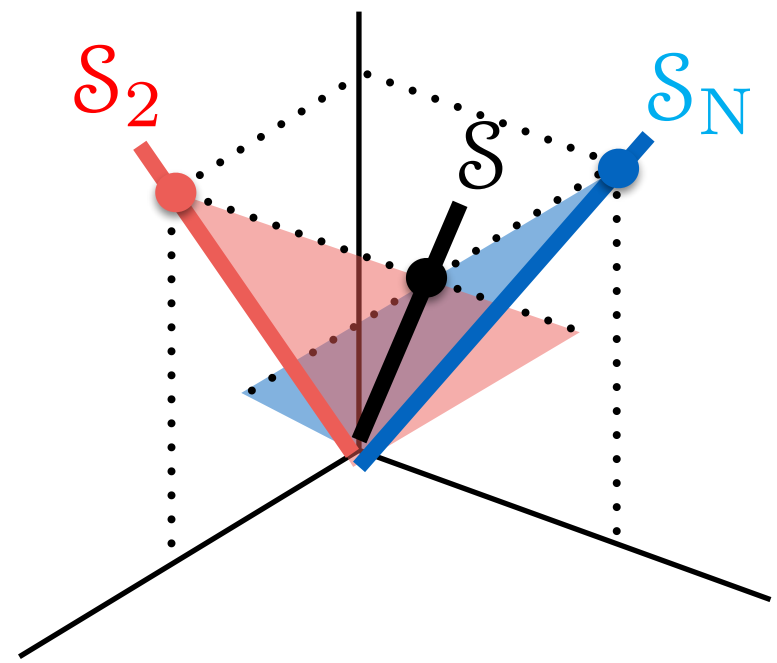




If we observe more projections…

Can we find the whole subspace?

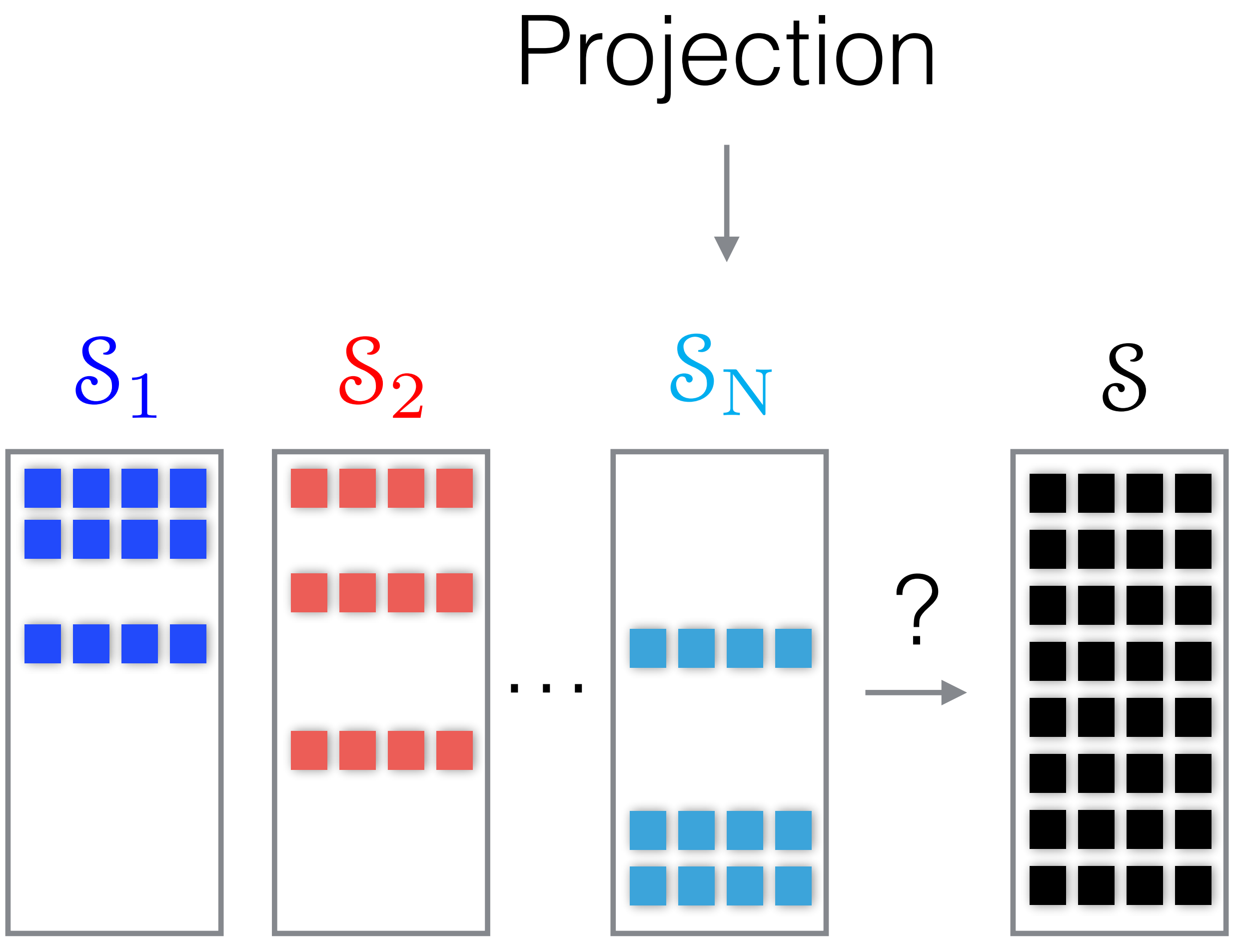


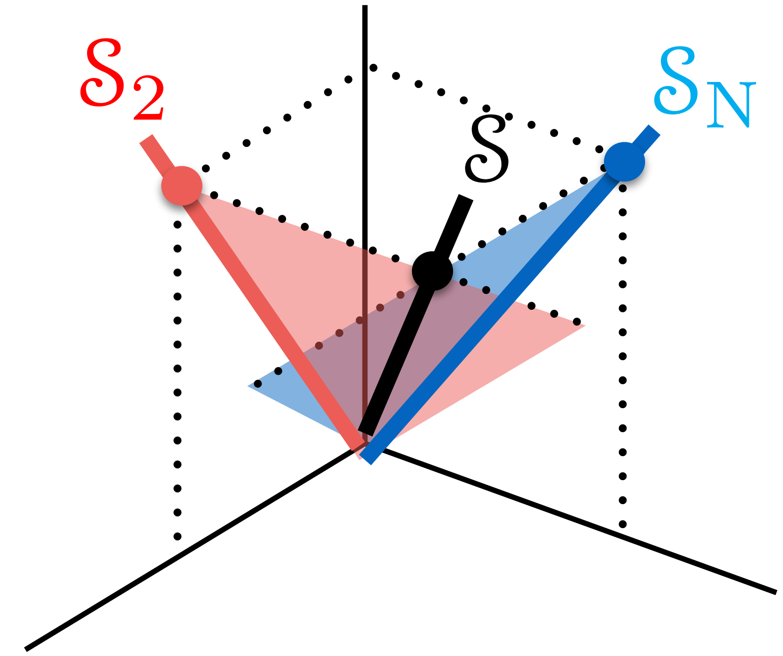


If we observe more projections…

Can we find the whole subspace?

Sometimes you can,
sometimes you can't.





If we observe more projections…

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Sometimes you can,
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Depends on which
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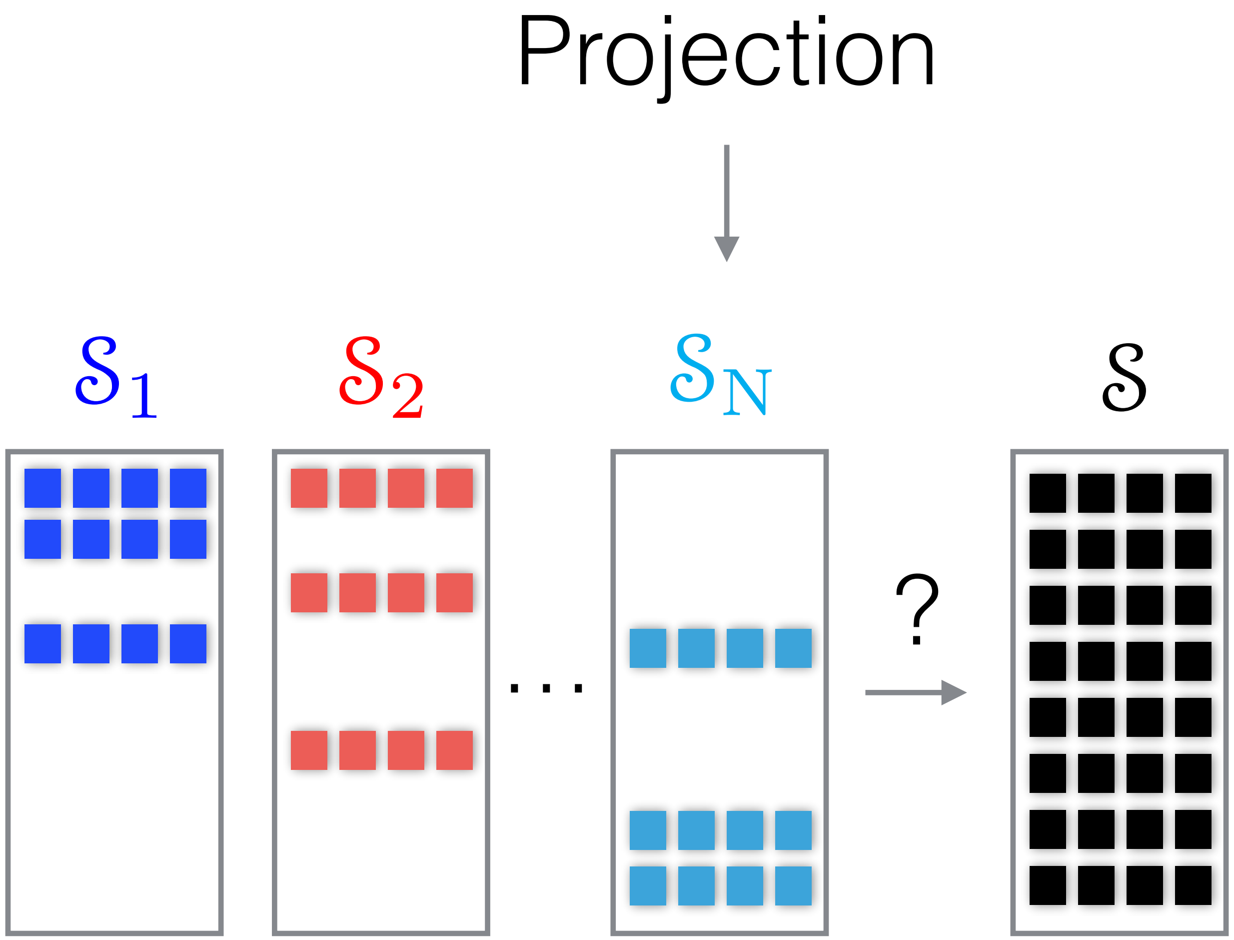

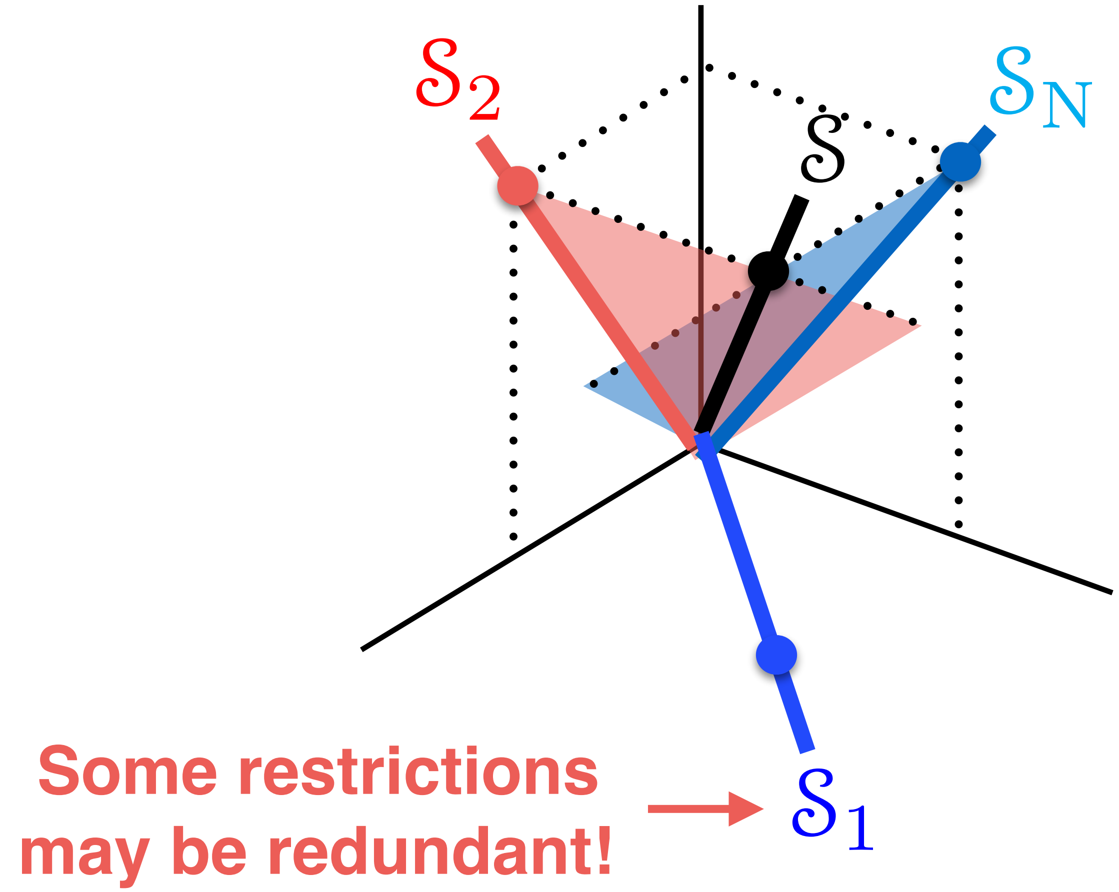

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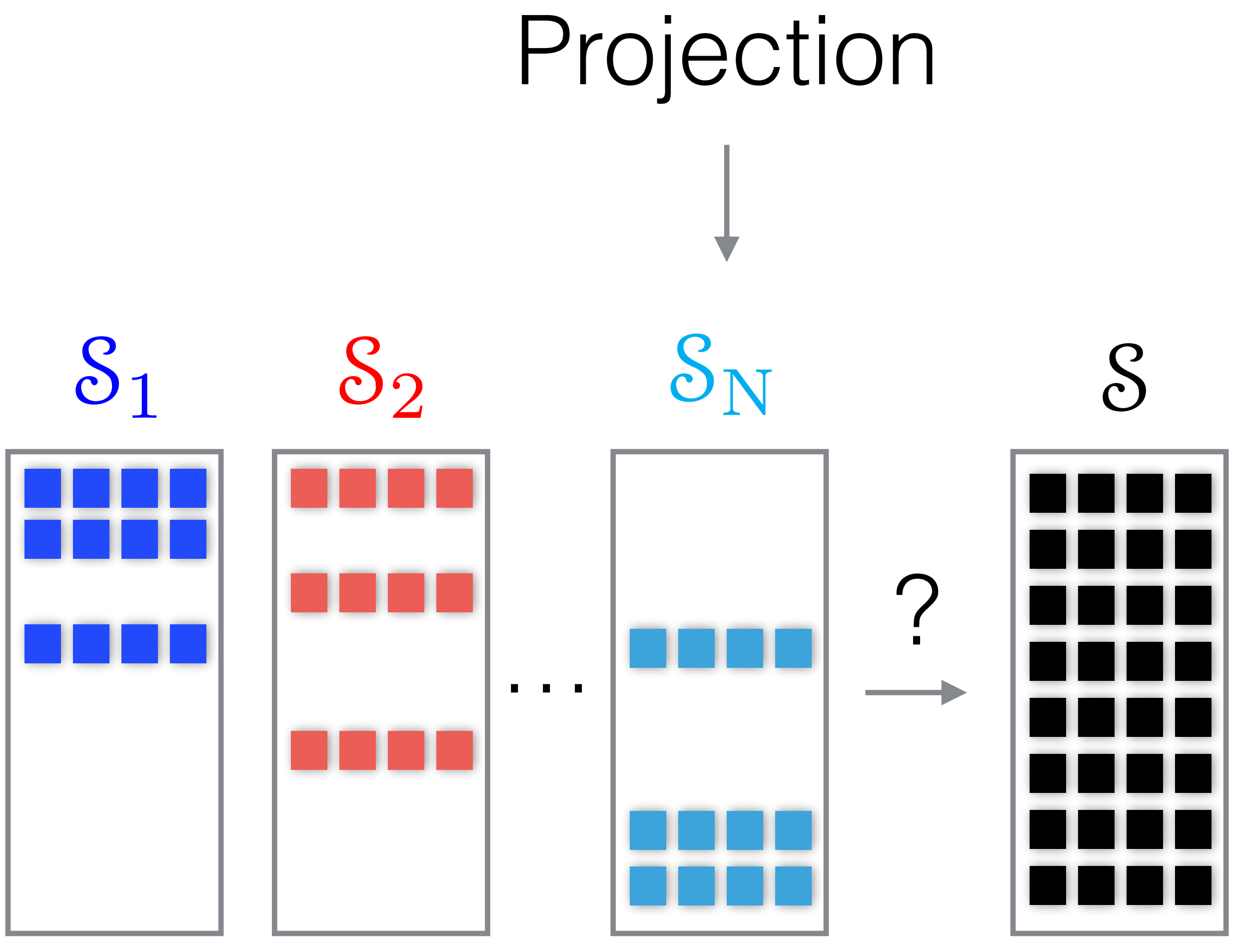
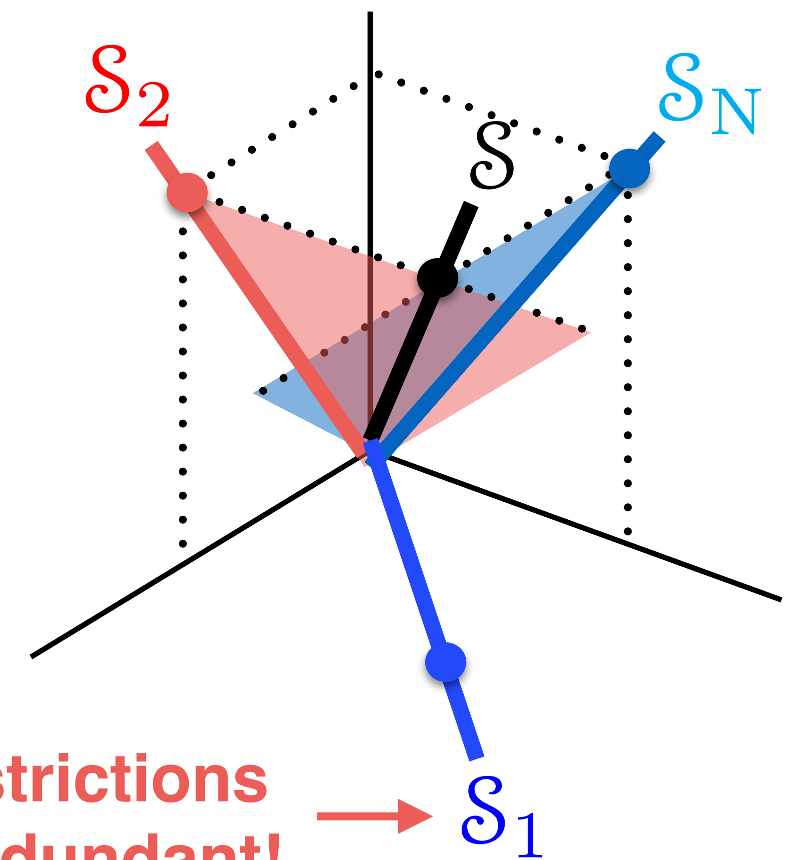




Some restrictions may be redundant!

If we observe more projections…

Can we find the whole subspace?

Sometimes you can, sometimes you can't.

Depends on which projections you get.

You need to observe *the right* projections

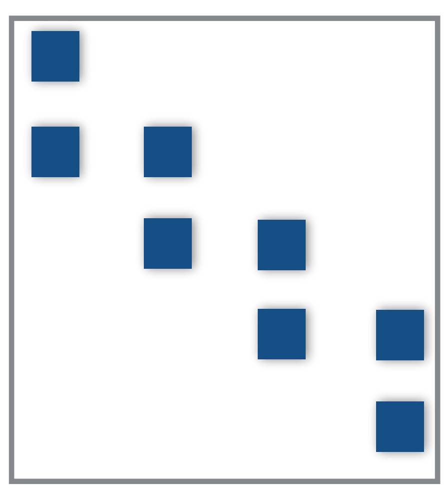
Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

Suppose $\mathcal{V}$ is in general position. With probability 1, $\mathcal{S}$ can be uniquely recovered if and only if there is a matrix $\boldsymbol{\Omega}_{\star}^{\otimes 2}$ formed with D-R columns of $\boldsymbol{\Omega}^{\otimes 2}$ such that every $\boldsymbol{\Omega}_{\ell}^{\otimes 2}$ formed with a subset of columns in $\boldsymbol{\Omega}_{\star}^{\otimes 2}$ satisfies:

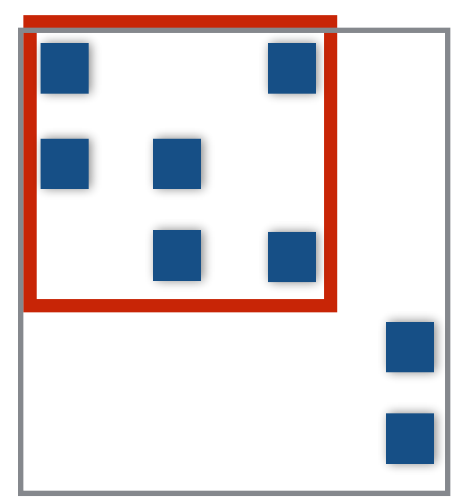
$$\#\text{rows\_with\_observations}(\boldsymbol{\Omega}_{\ell}^{\otimes 2}) \geq \#\text{columns}(\boldsymbol{\Omega}_{\ell}^{\otimes 2}) + \text{R}.$$

Furthermore, this condition is true if and only if $\dim\ker \mathbf{A}^{\mathsf{T}} = \text{R}$, whence $\mathcal{S} = \ker \mathbf{A}^{\mathsf{T}}$.

D=5, R=1

$\boldsymbol{\Omega}^{\otimes 2} =$ 

Good

$\boldsymbol{\Omega}^{\otimes 2} =$ 

Bad

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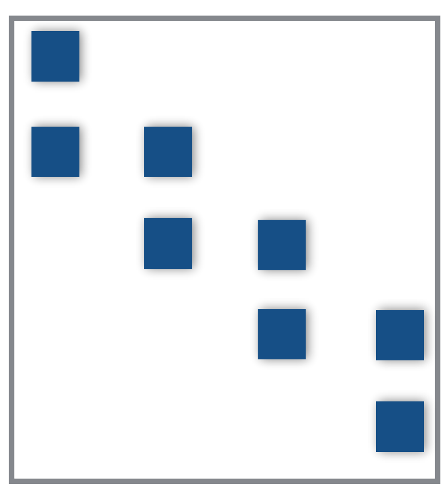
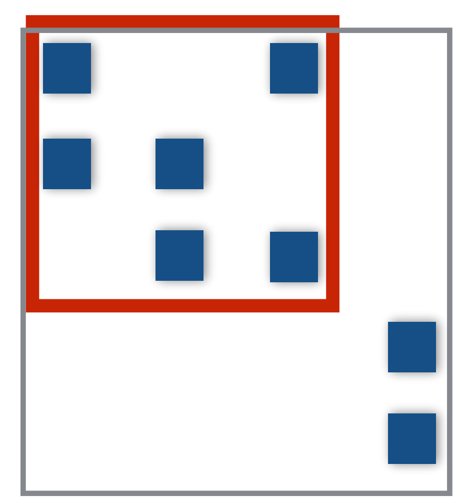
$\boldsymbol{\Omega}^{\otimes \mathbf{2}} =$ 

Indicates coordinates involved in given projections

Good

$\boldsymbol{\Omega}^{\otimes \mathbf{2}} =$ 

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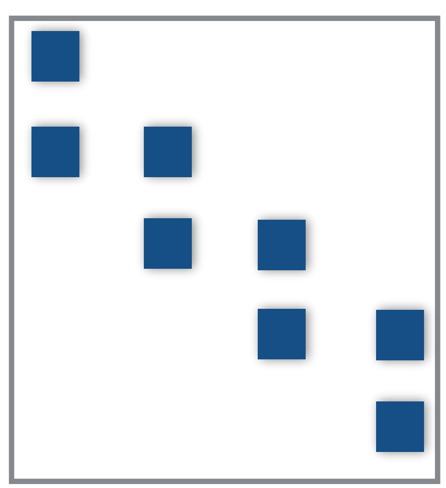
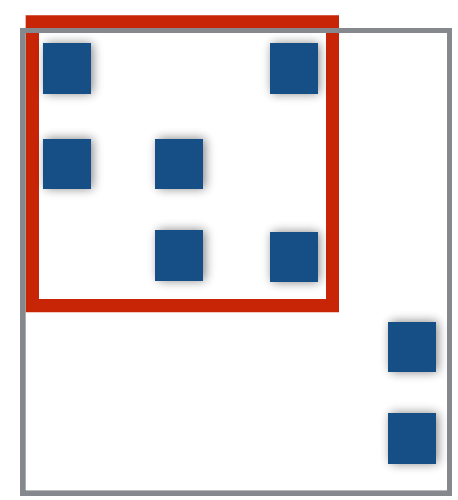
Encodes information of given projections

D=5, R=1

$\boldsymbol{\Omega}^{\otimes 2} =$ 

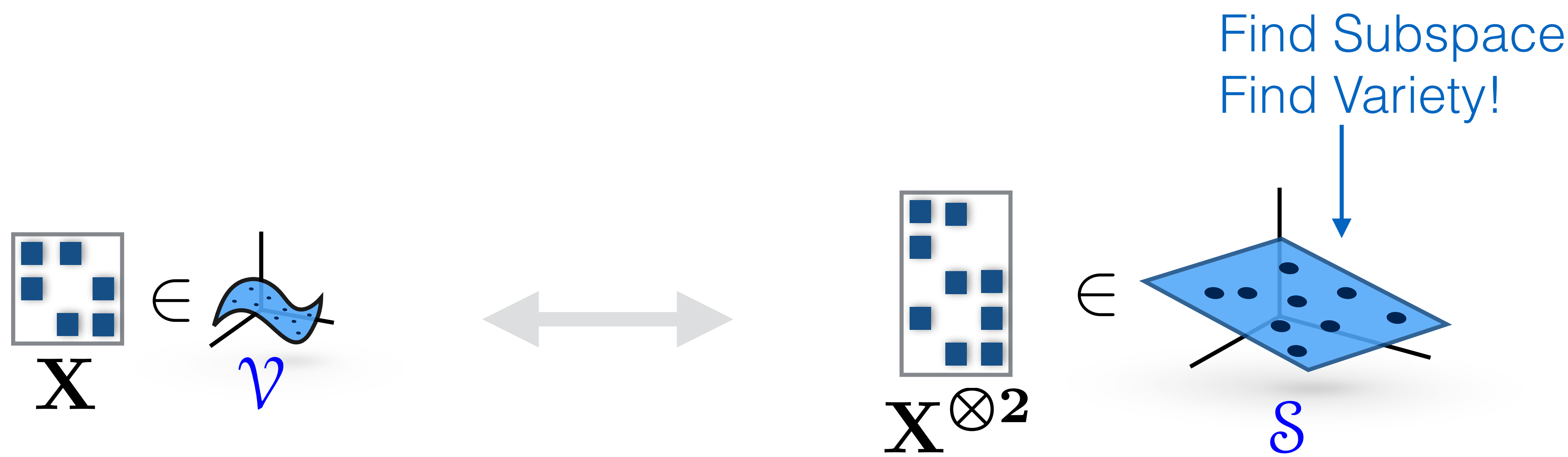
Indicates coordinates involved in given projections

Good

$\boldsymbol{\Omega}^{\otimes 2} =$ 

Bad

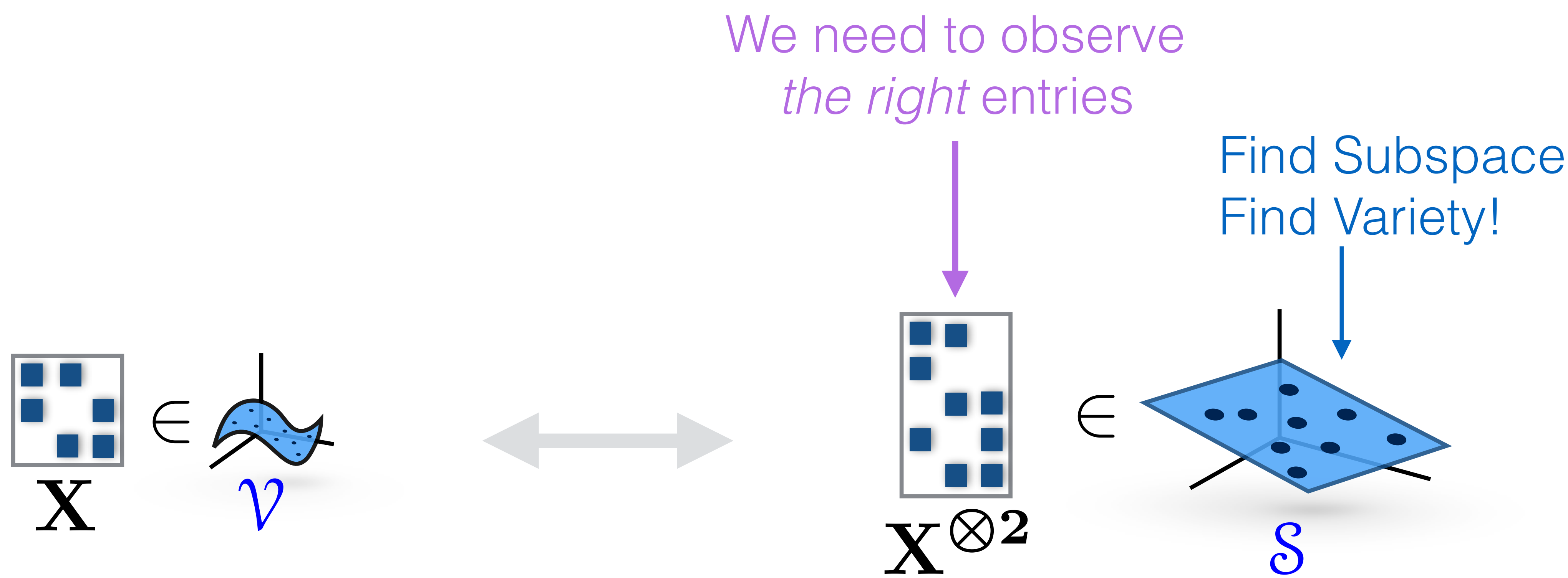
So, what do we know so far?



Algebraic Variety
(nonlinear)

Tensorized Subspace
(linear)

So, what do we know so far?



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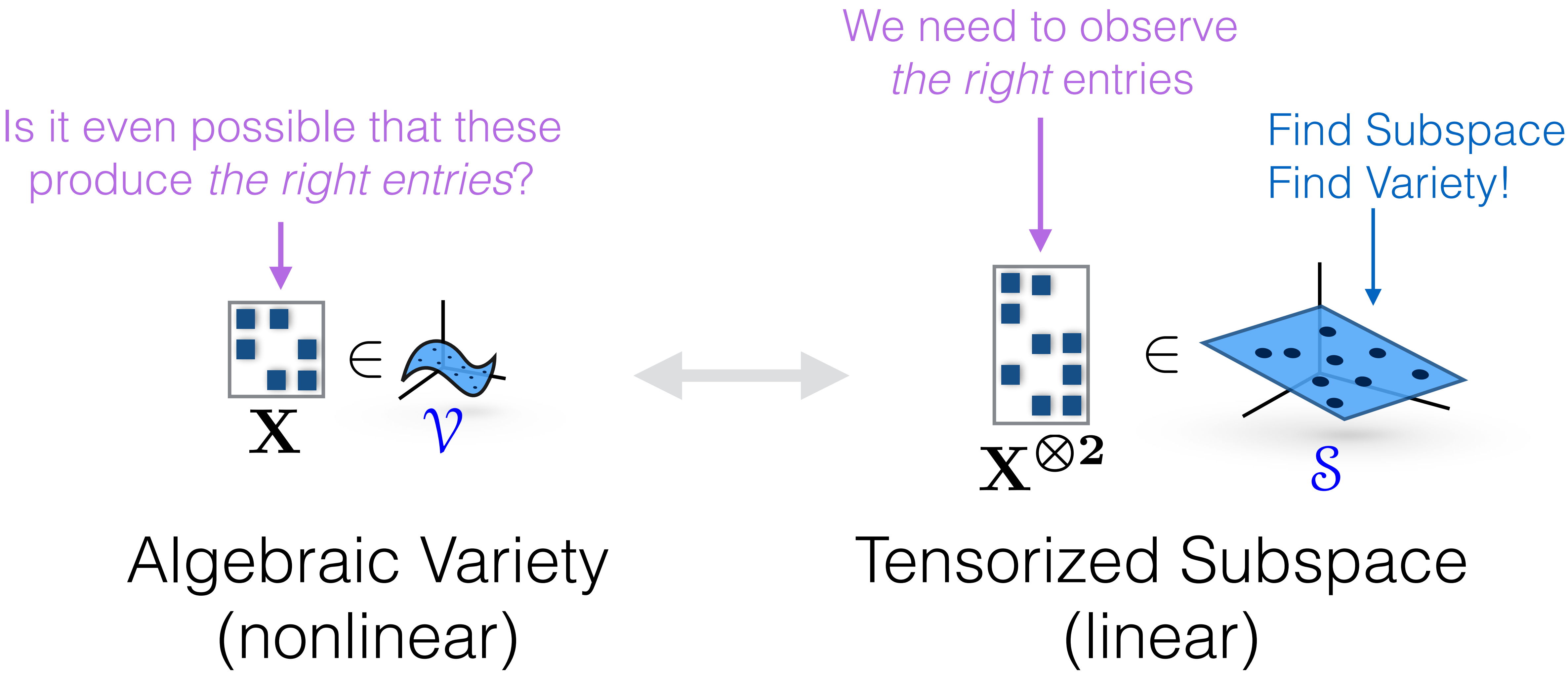


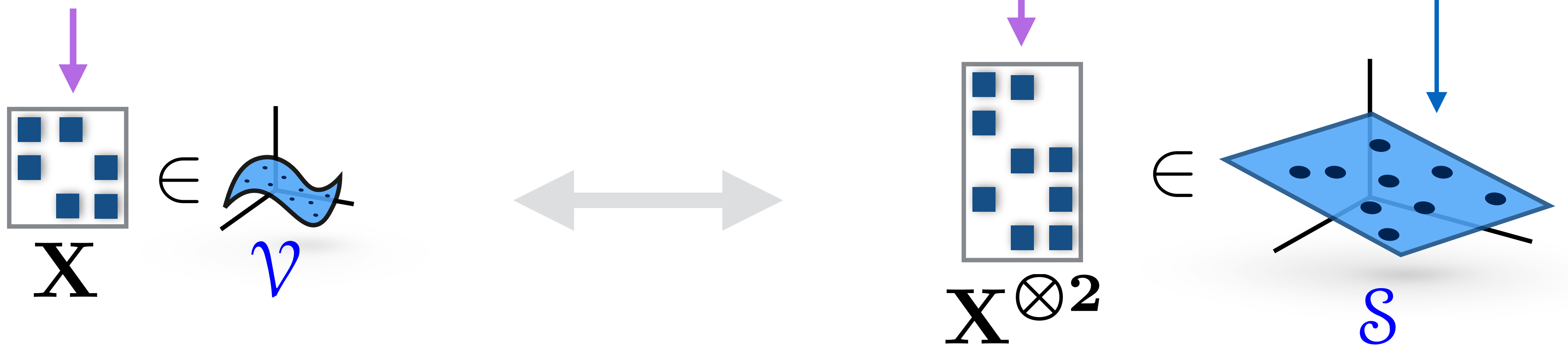
So, what do we know so far?

We need to observe *the right* entries

Is it even possible that these produce *the right entries*?

Yes :)

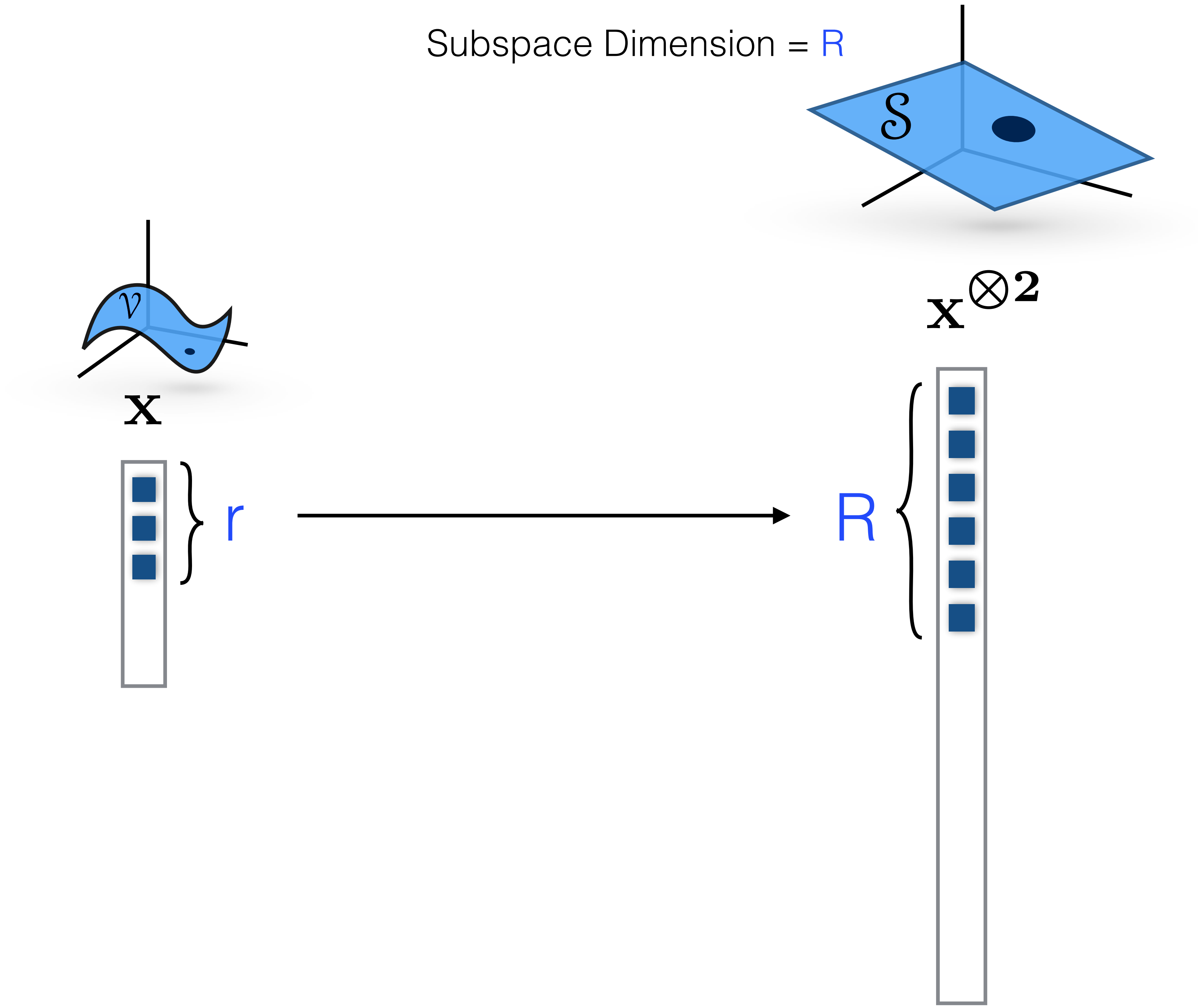
Find Subspace, Find Variety!



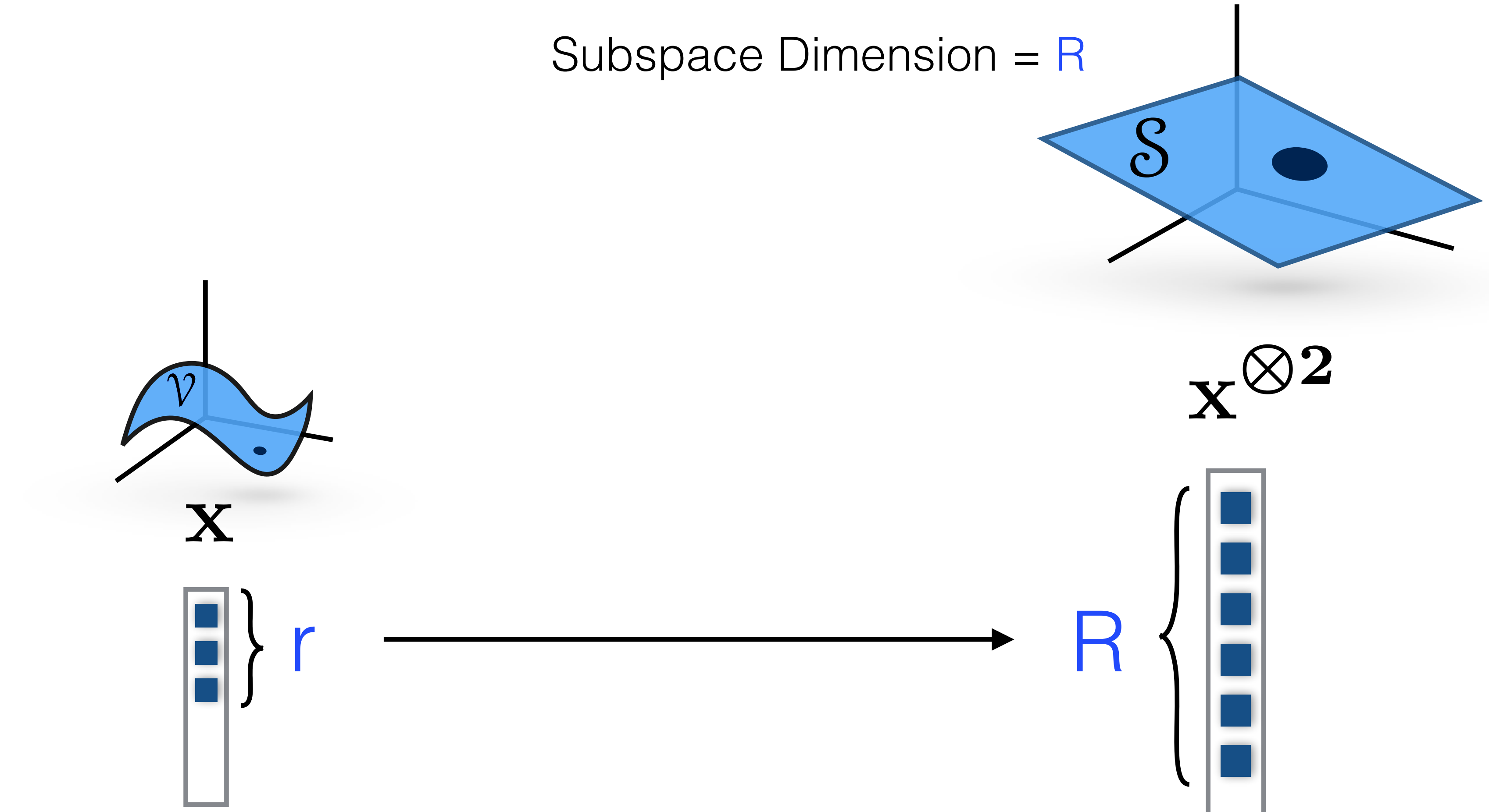
Algebraic Variety (nonlinear)

Tensorized Subspace (linear)

So, what do we know so far?



Subspace Dimension = R
$\mathcal{S}$
$\mathbf{x}^{\otimes 2}$
$\mathcal{V}$
$\mathbf{x}$
r
R



Theorem (P.-A., Ongie, Balzano, Willett, Nowak, 2017)

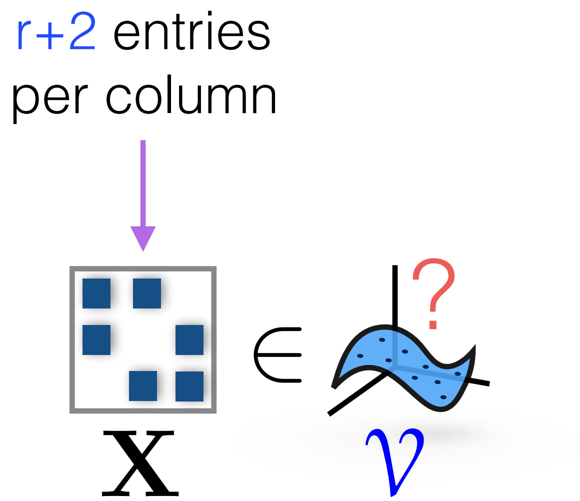
Suppose $\mathcal{V}$ is in general position. Suppose each column $\mathbf{x}$ has m samples.

(i) If m < r, then $\mathcal{S}$ cannot be uniquely determined.

(ii) There are cases with m = r and m = r+1 where $\mathcal{S}$ cannot be uniquely determined.

(iii) If m ≥ r+2, then $\mathcal{S}$ can be uniquely determined (if you observe *the right* entries).

So, what do we know so far?



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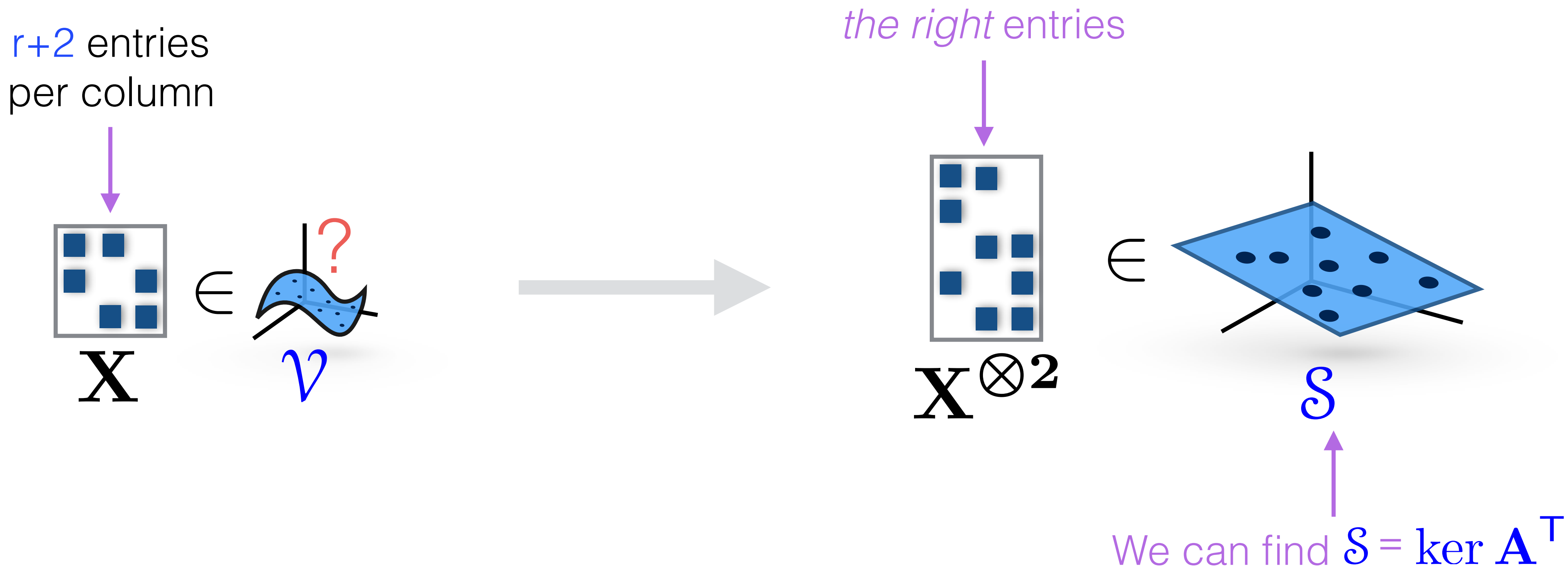

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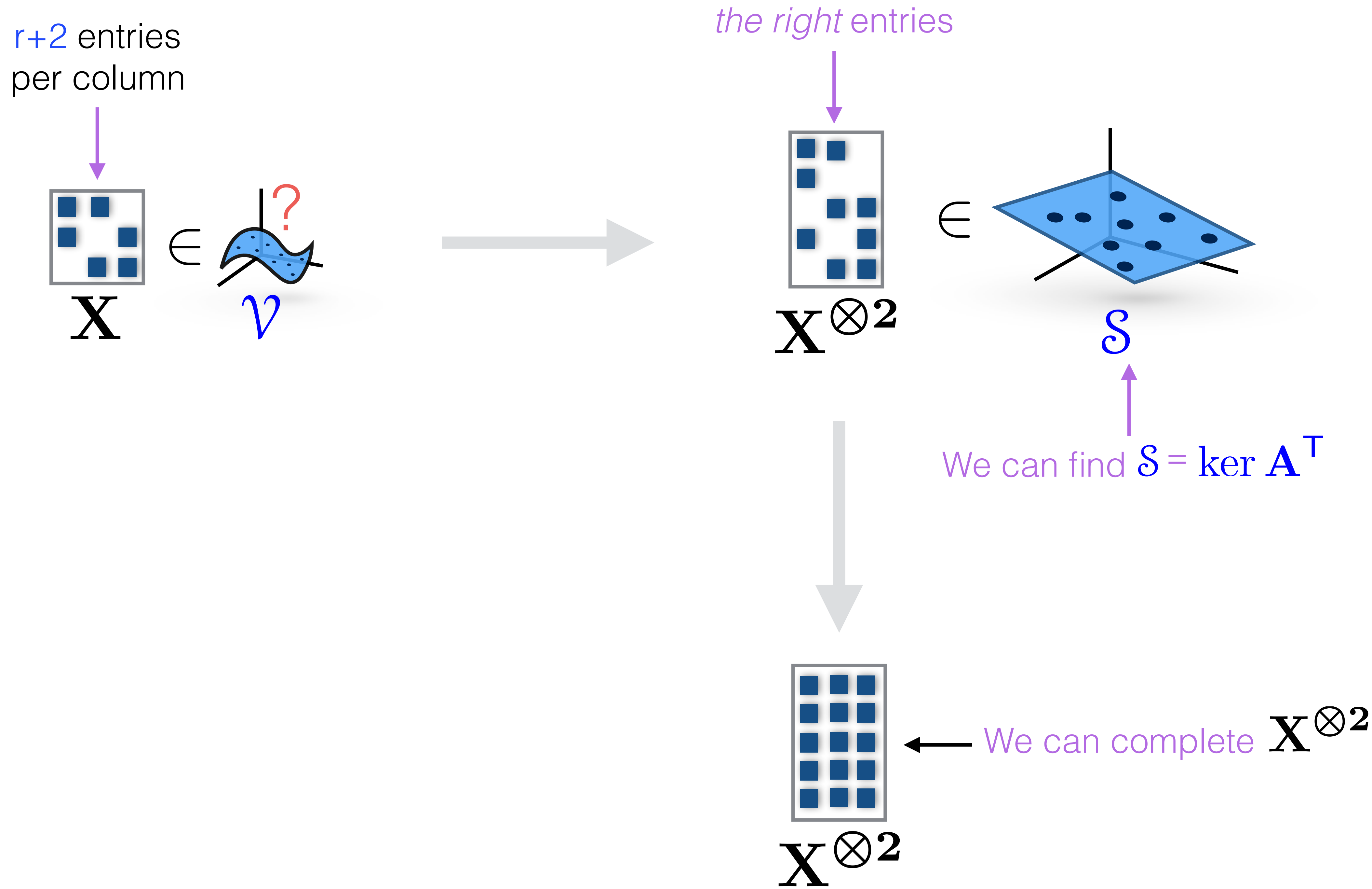

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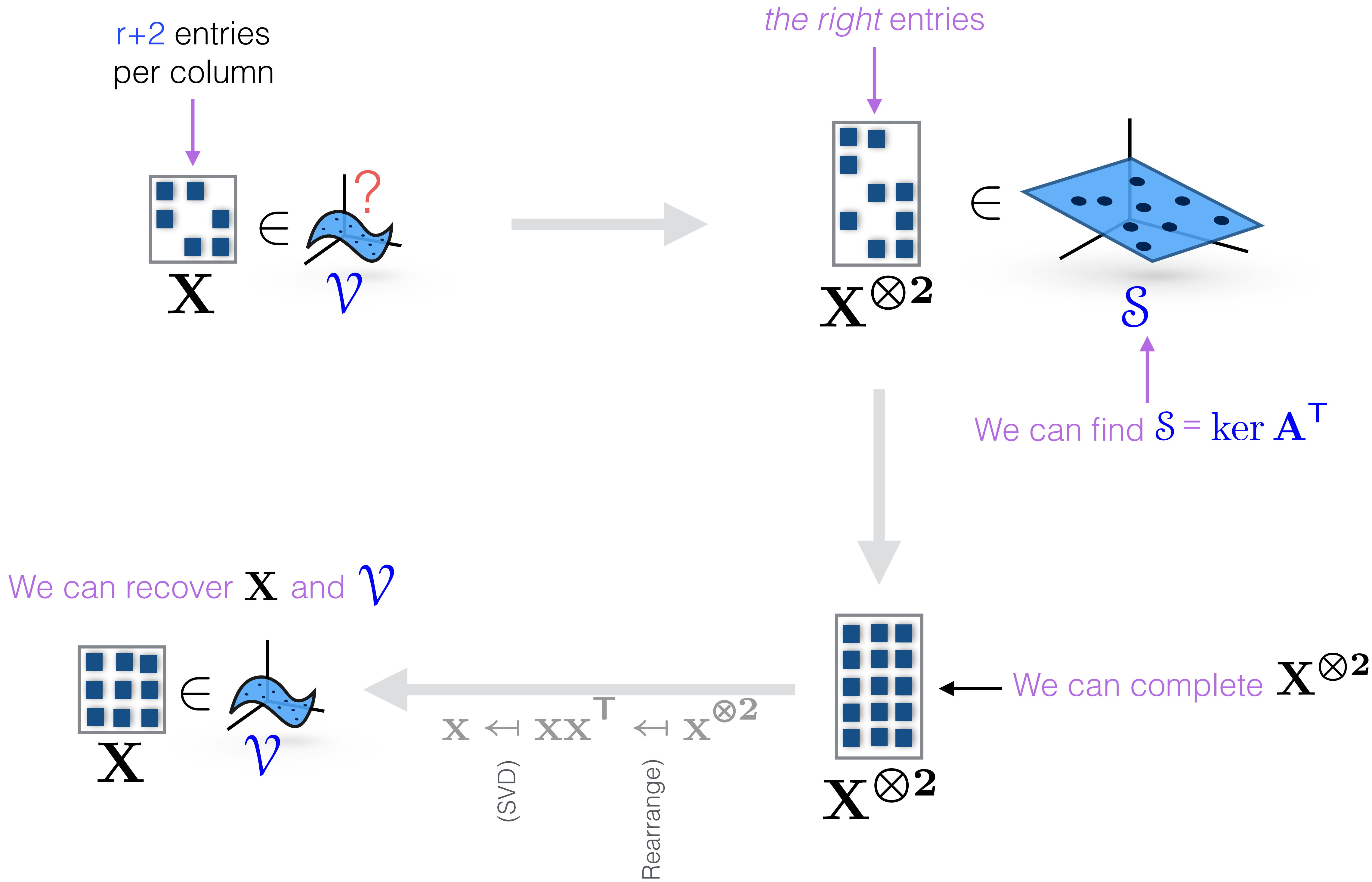

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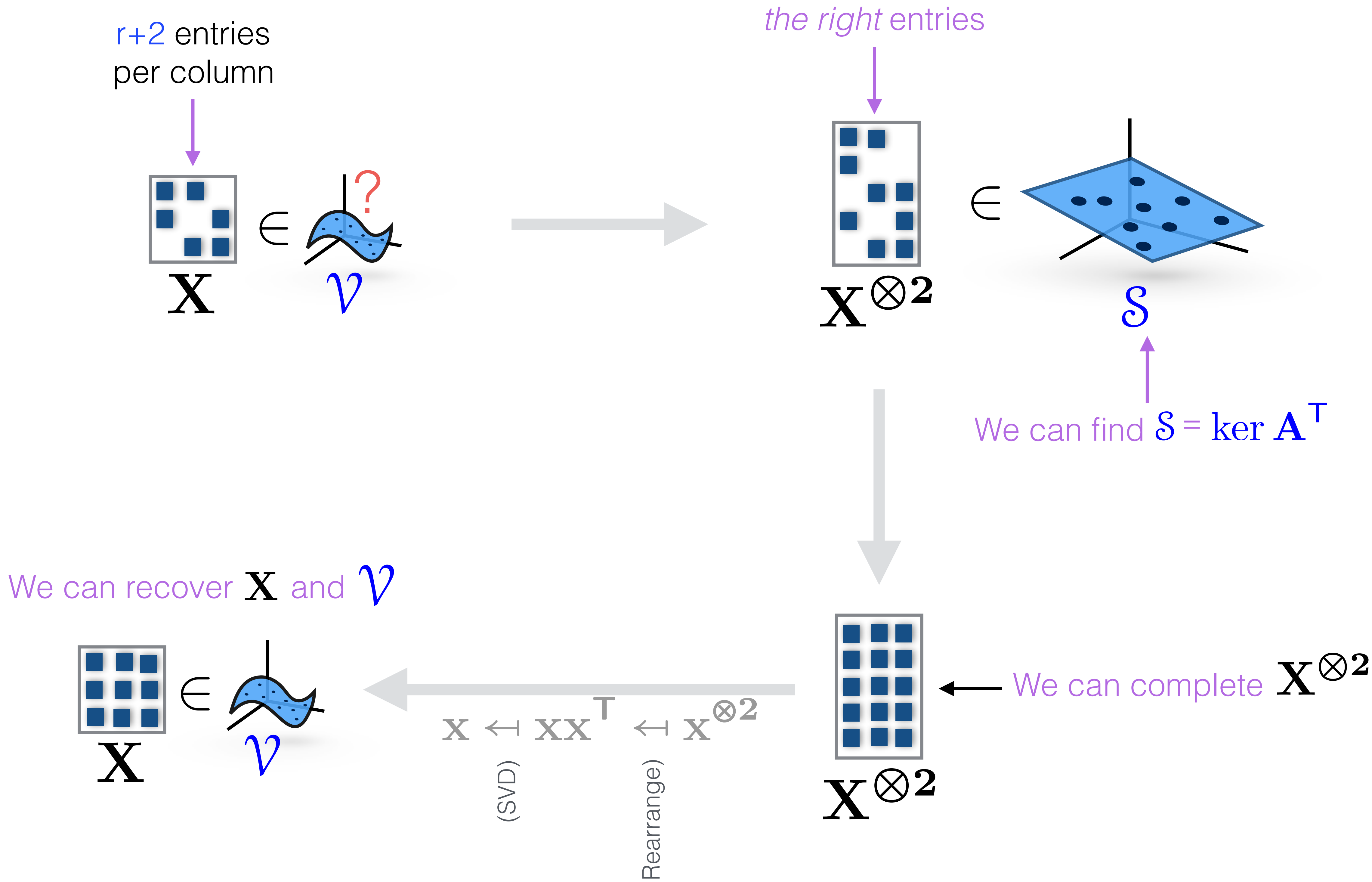


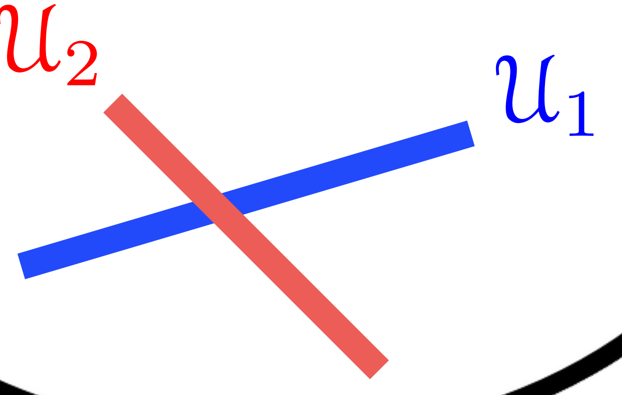
So, what do we know so far?

(Provable Algorithm)


What is this good for?

What is this good for?

Unions of Subspaces!




What is this good for?
Unions of Subspaces!

Multiple subspaces

We want to find them all!

Multiple subspaces

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Multiple subspaces

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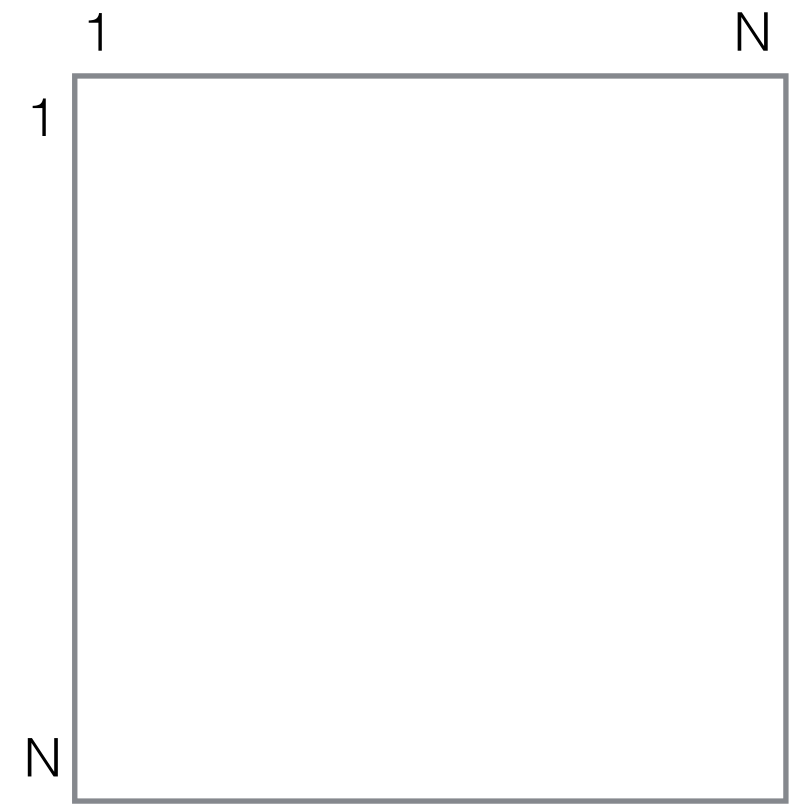

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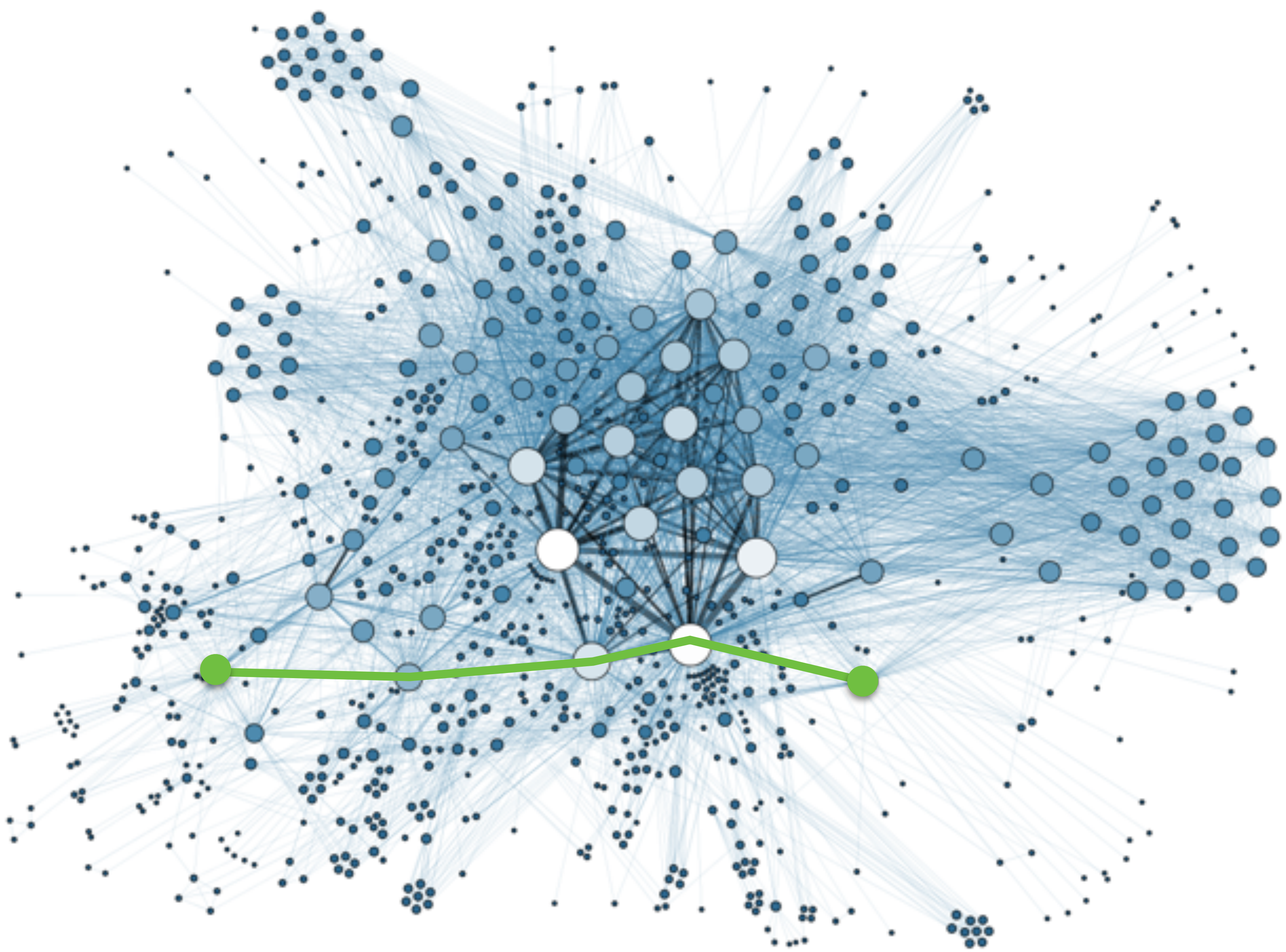

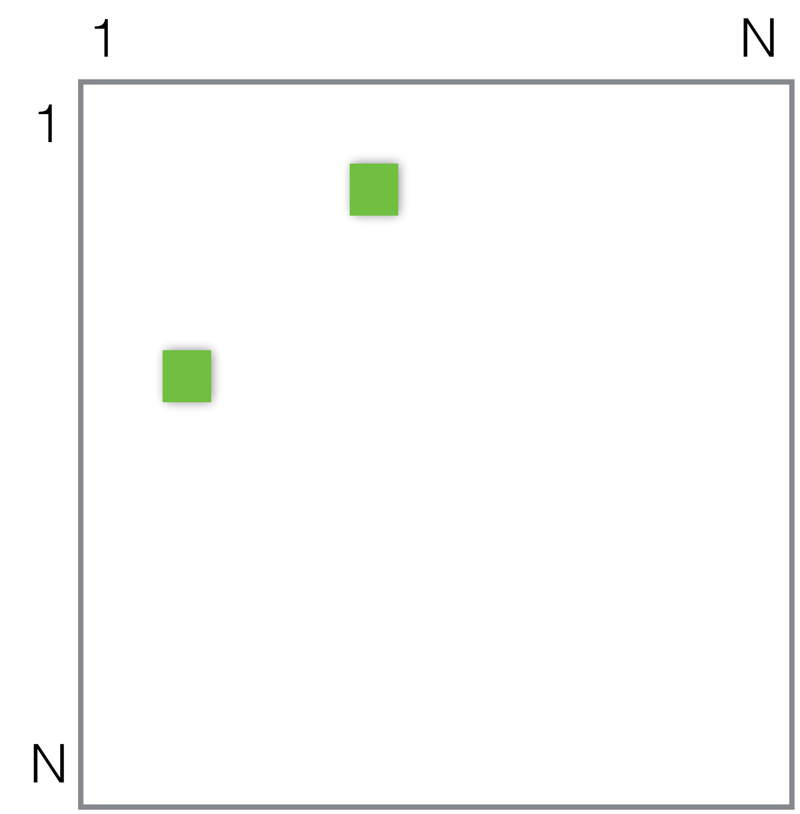

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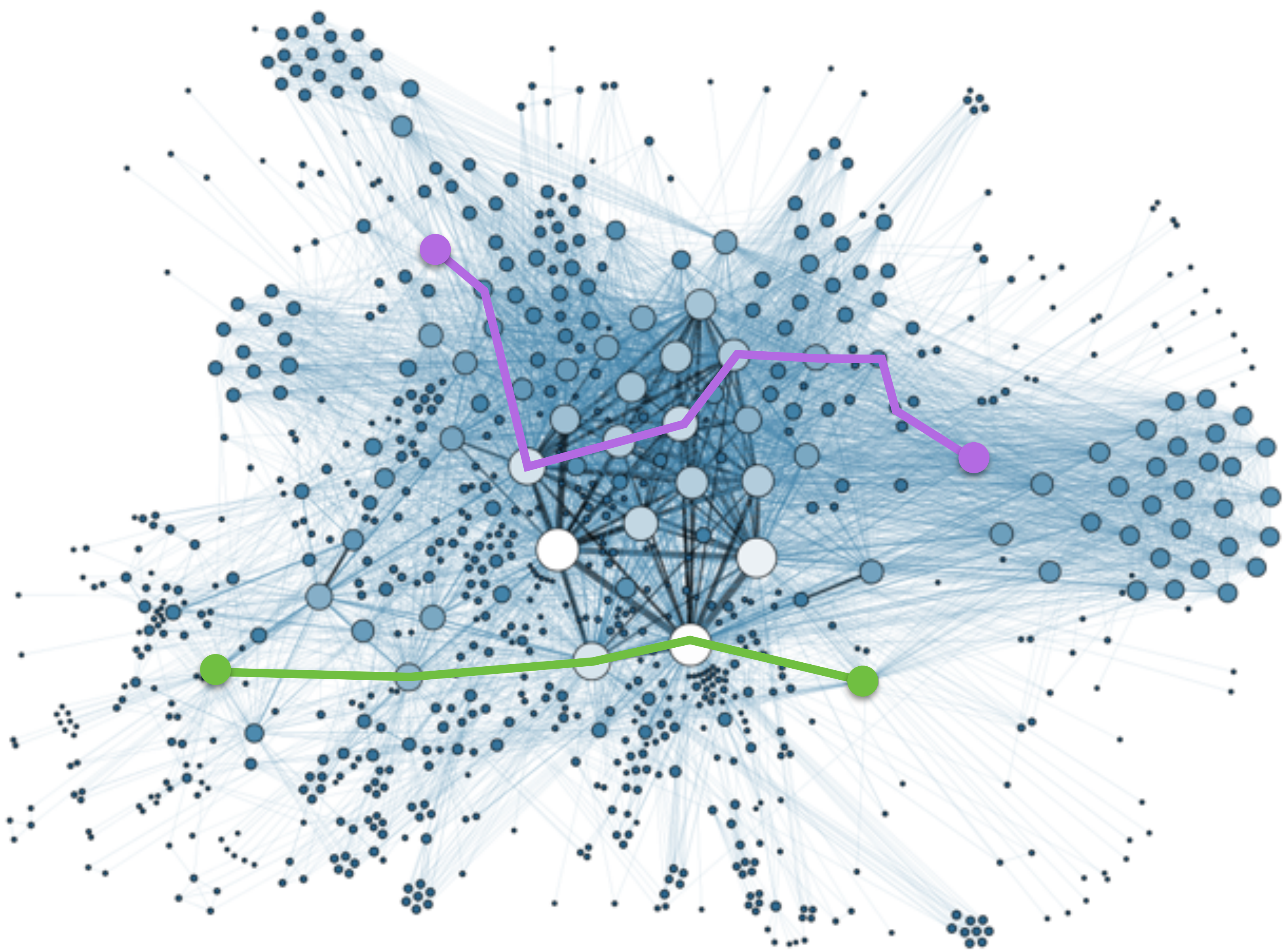

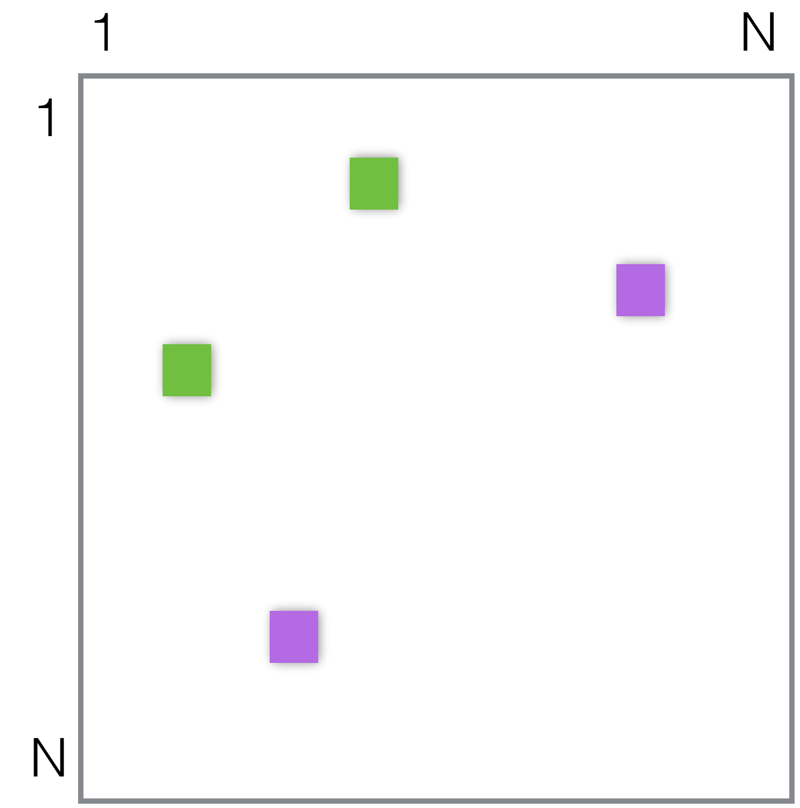

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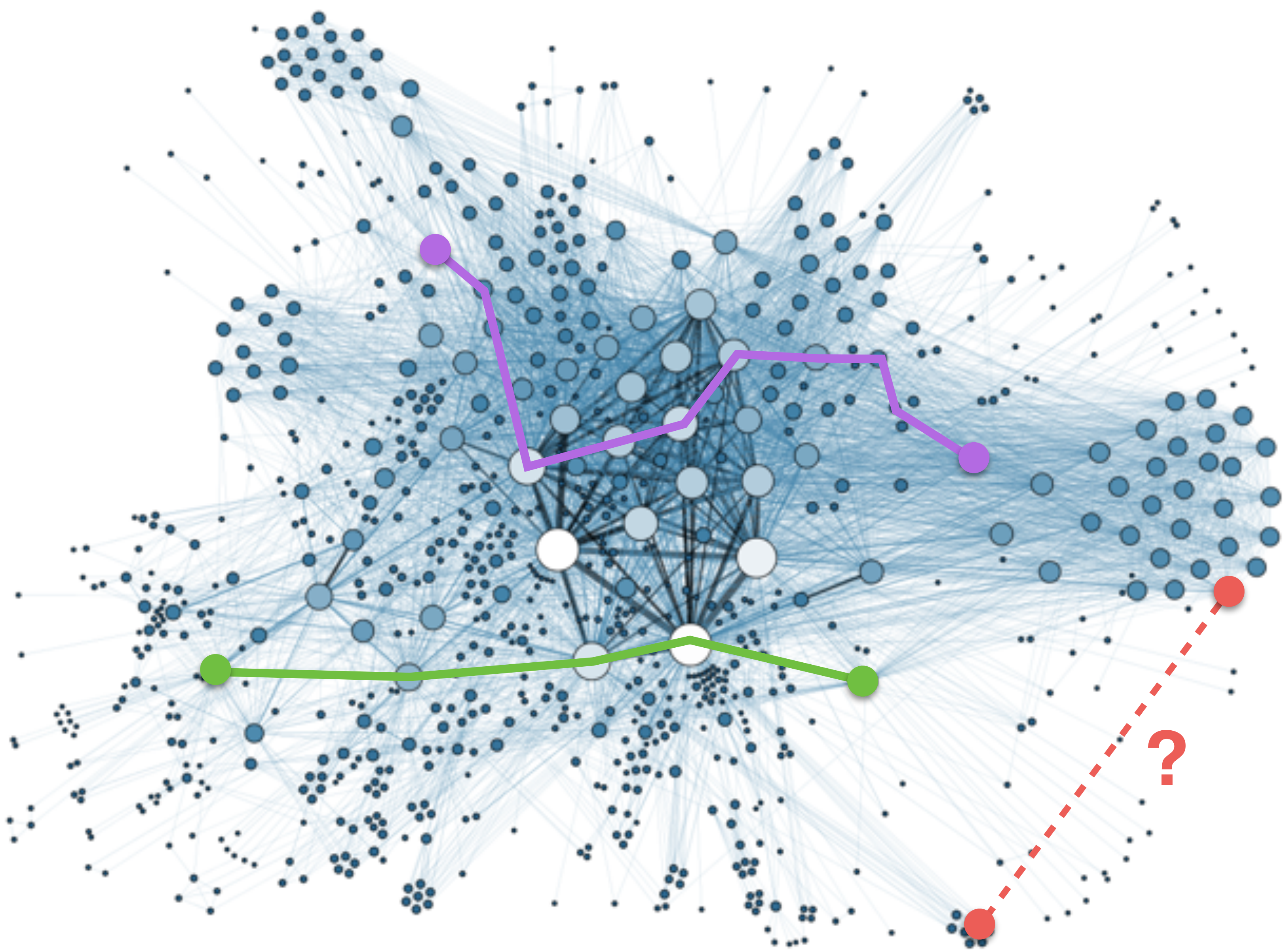

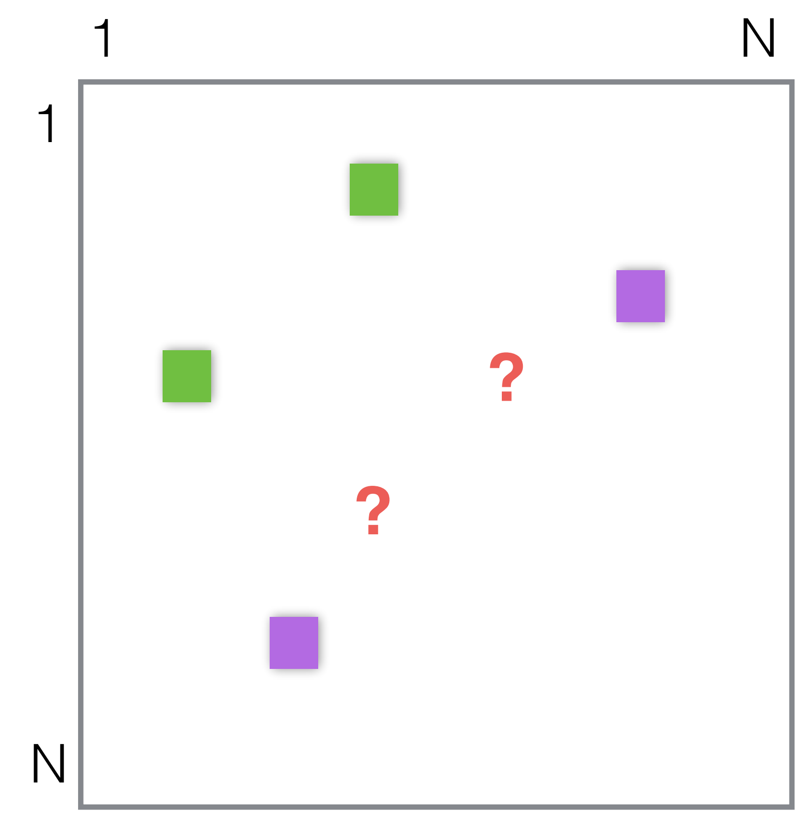

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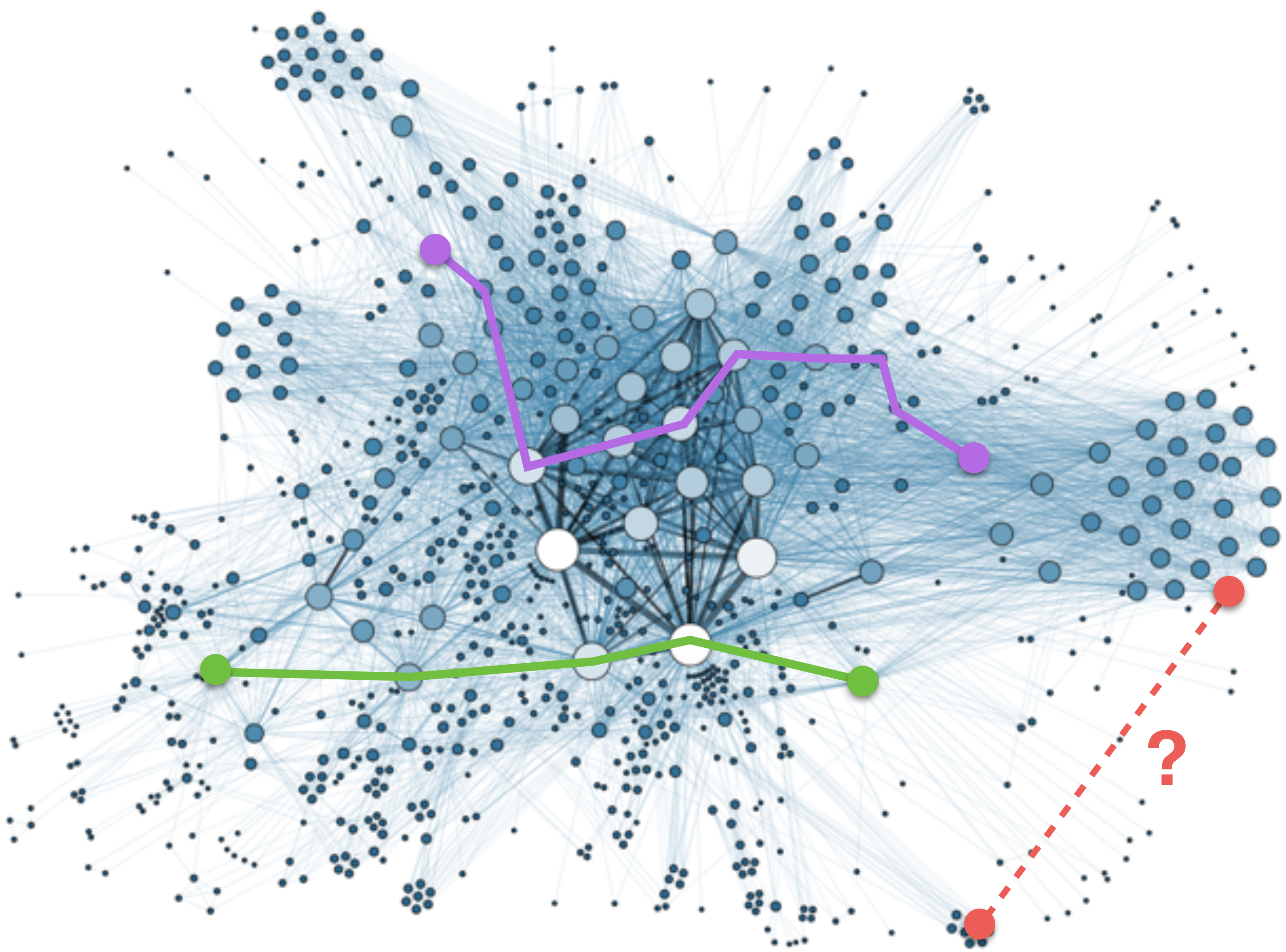
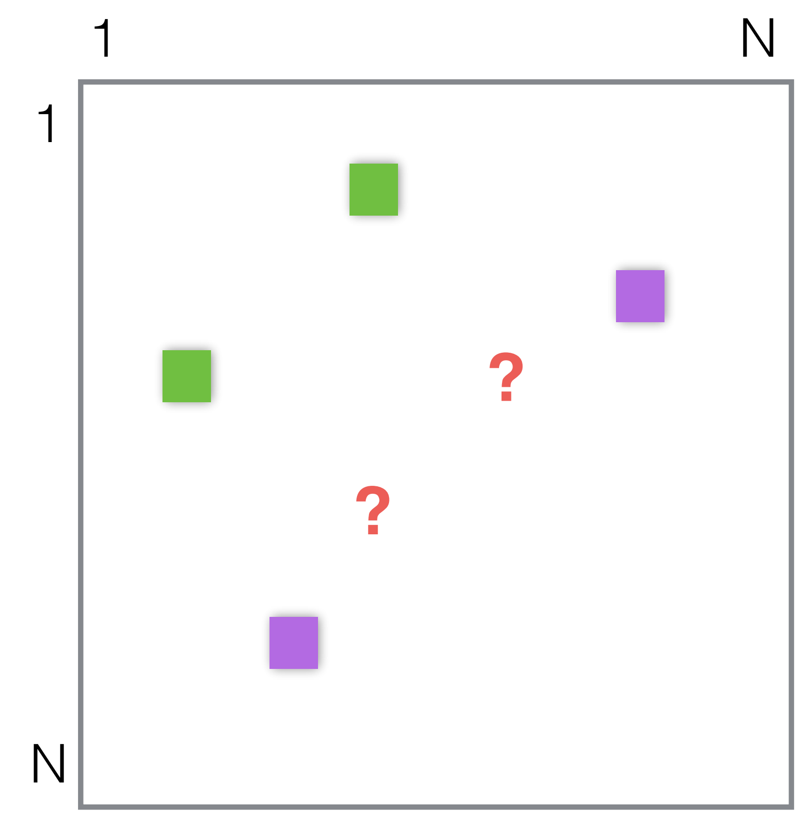
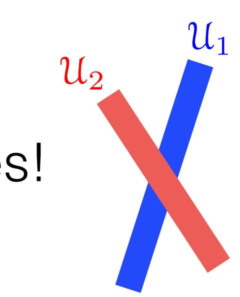




Columns lie in a union of subspaces!



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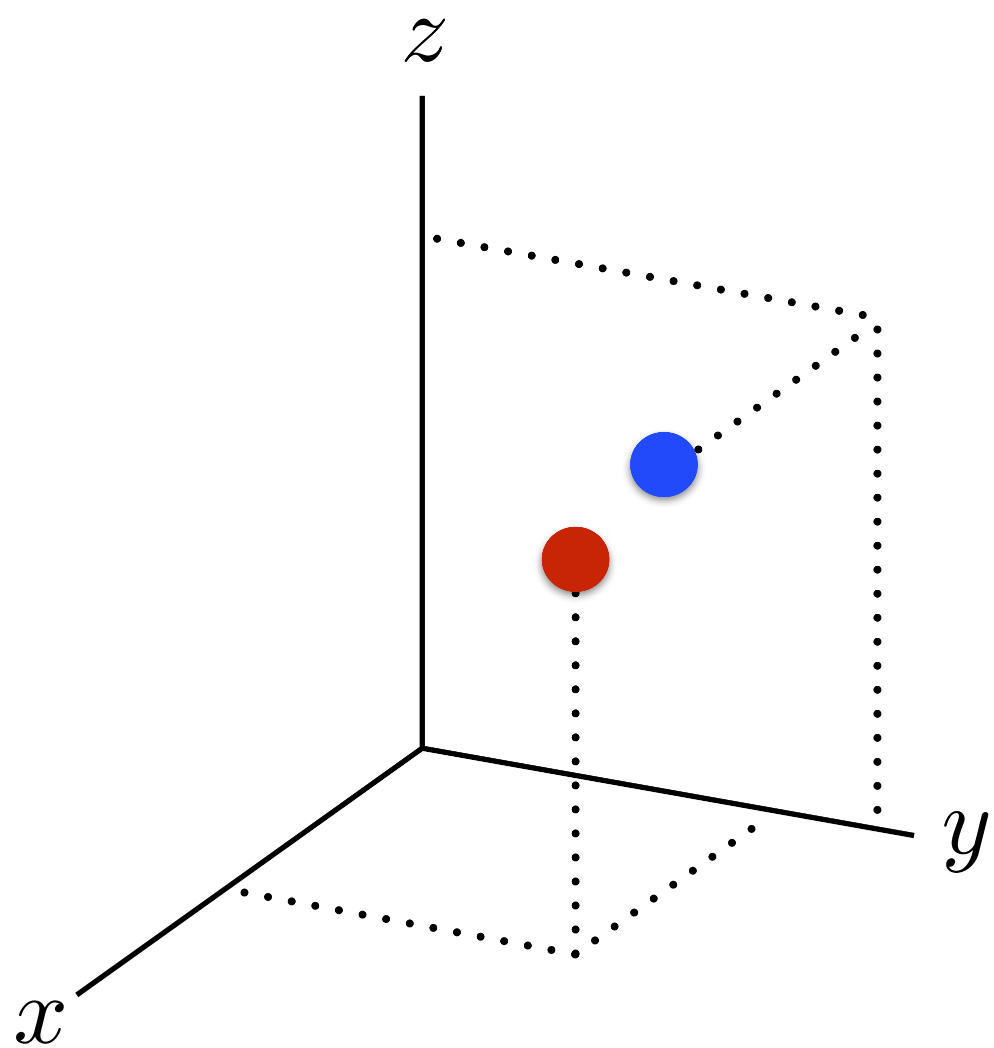
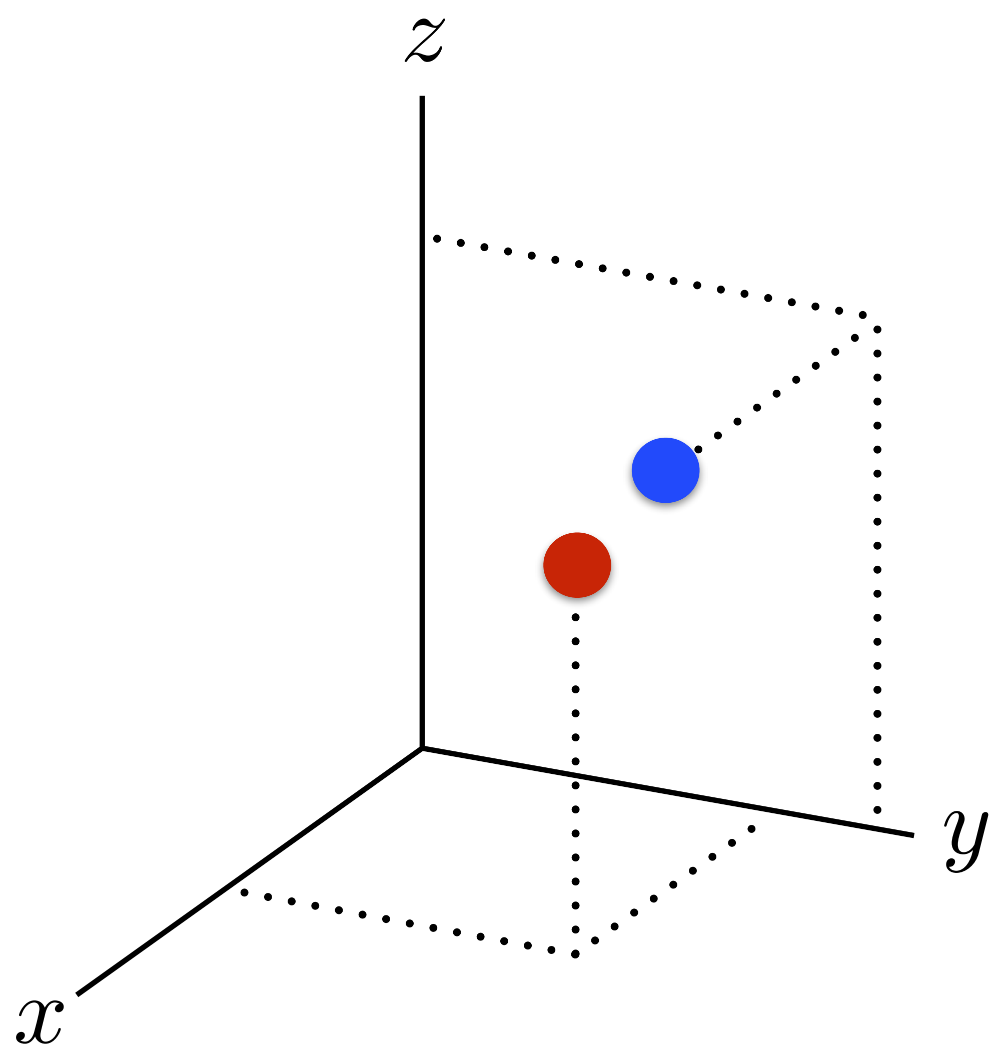


$$\begin{bmatrix} x_1 & \cdot \\ y_1 & y_2 \\ \cdot & z_2 \end{bmatrix}$$

Things get more complicated

- We don’t know where points are :(
- We don’t know which go together :(



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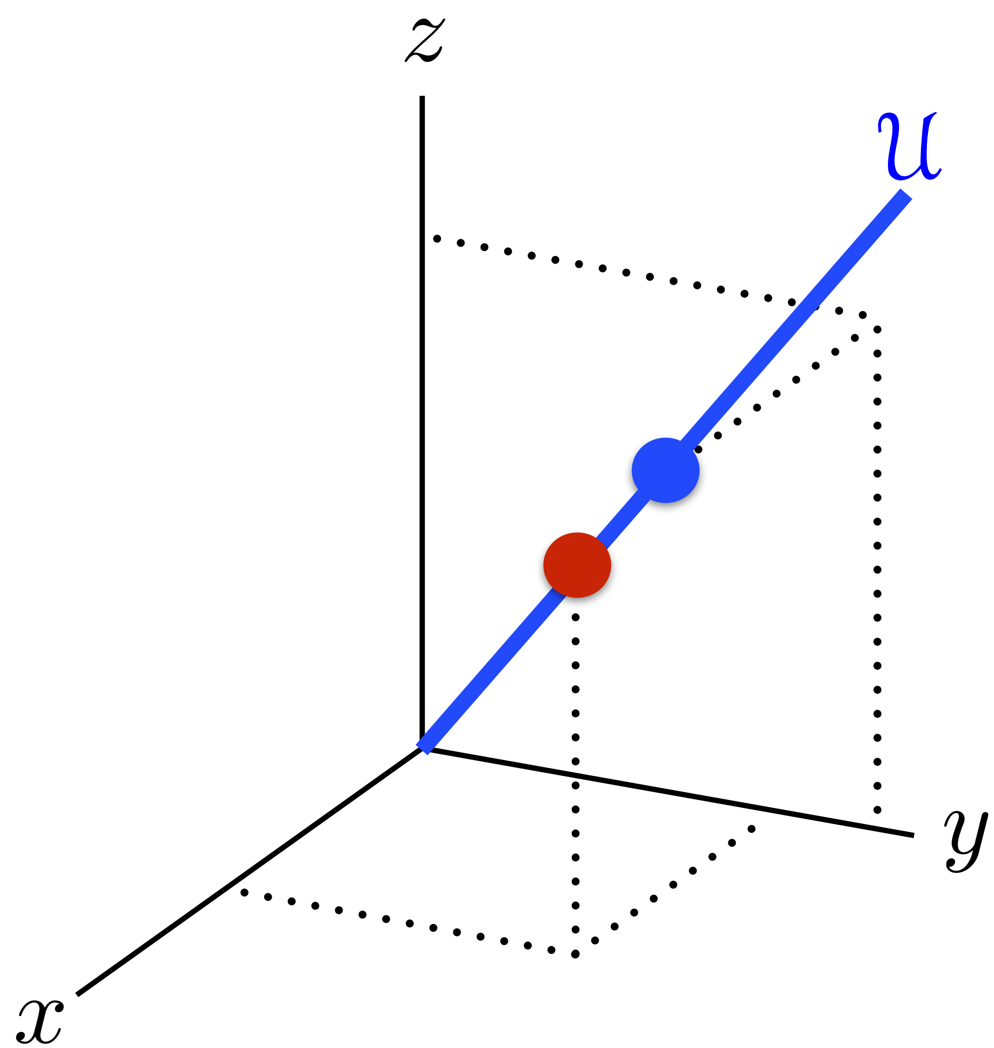

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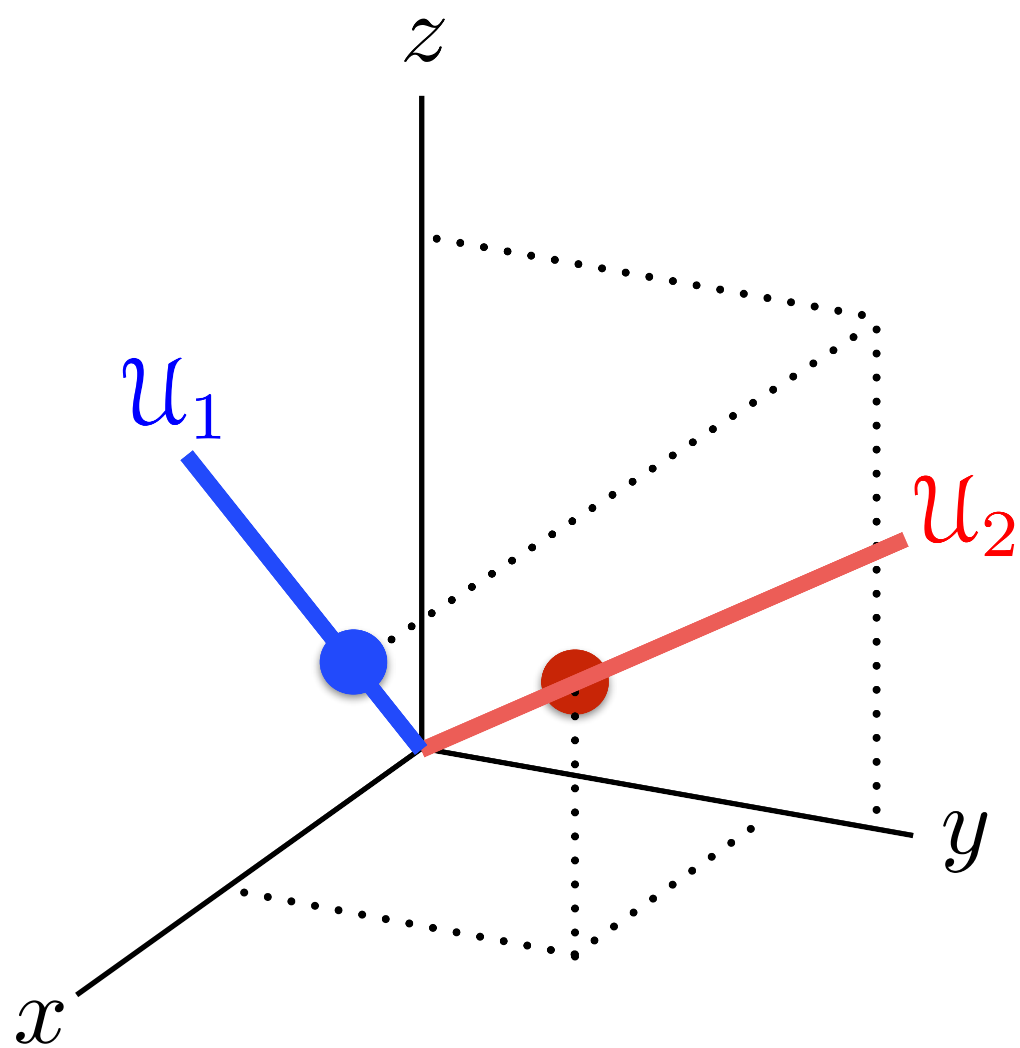

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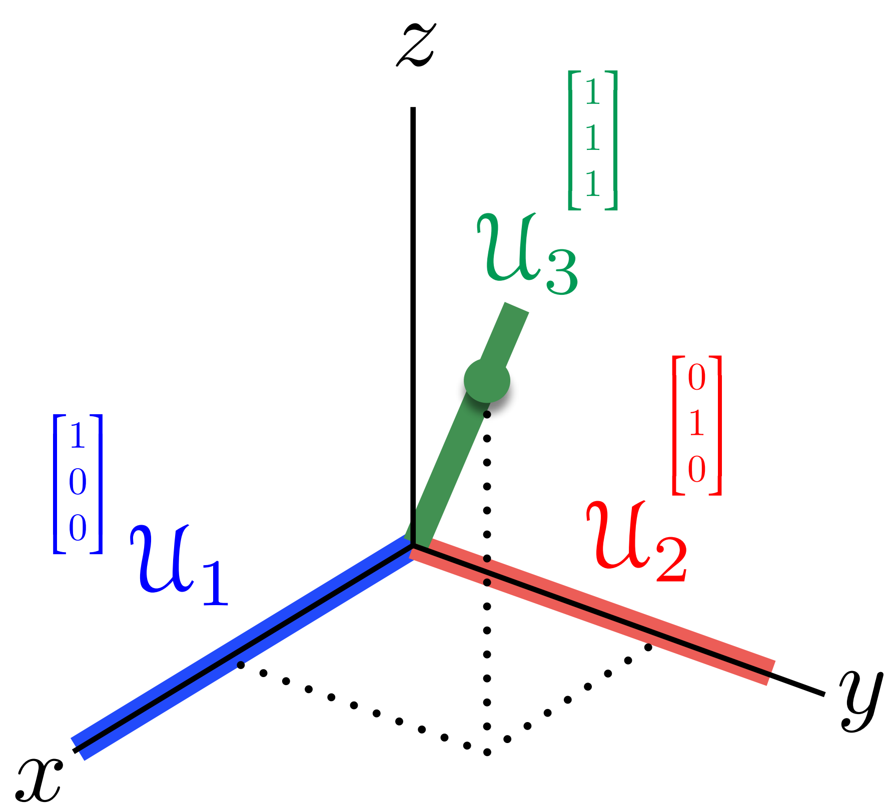
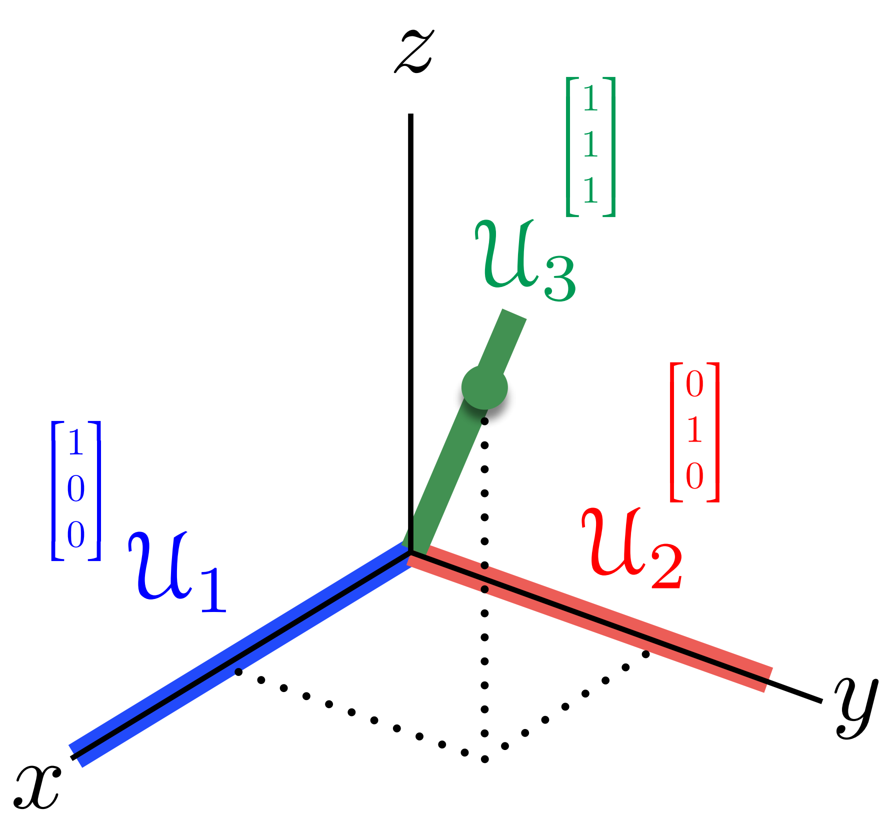


Unions of Subspaces are Varieties!



$$\mathcal{V} = \left\{ \mathbf{x} : \begin{array}{rcl} xy(x-y) & = & 0 \\ xy(x-z) & = & 0 \\ z(x-y) & = & 0 \\ z(x-z) & = & 0 \end{array} \right\}$$

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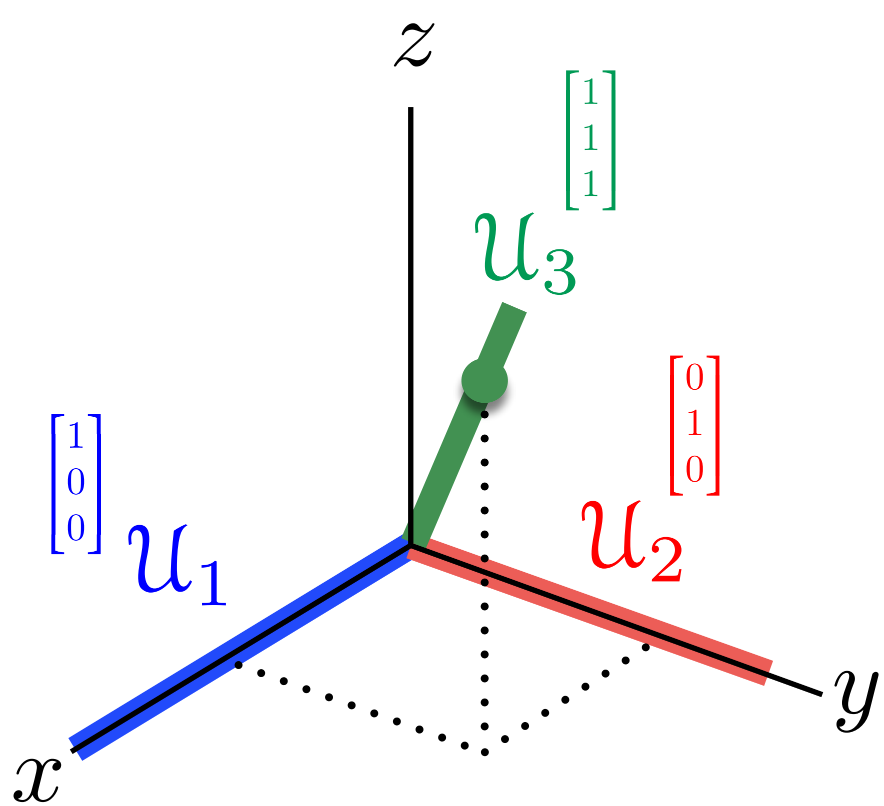

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$$= \left\{ \mathbf{x} : \begin{array}{rcrcrcrcc} -\frac{5}{6}xy & + & \frac{1}{2}xz & + & \frac{5}{30}yz & + & \frac{5}{30}z^2 & = & 0 \\ -\frac{5}{30}xy & - & \frac{1}{2}xz & + & \frac{5}{6}yz & - & \frac{5}{30}z^2 & = & 0 \\ -\frac{5}{30}xy & - & \frac{1}{2}xz & - & \frac{5}{30}yz & + & \frac{5}{6}z^2 & = & 0 \end{array} \right\}$$

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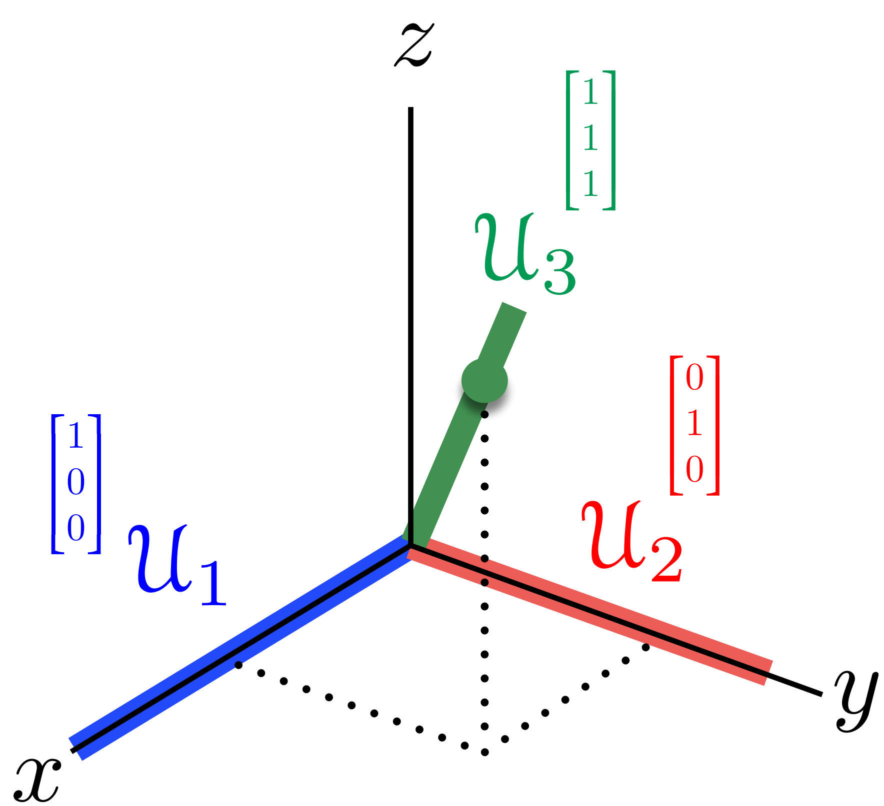

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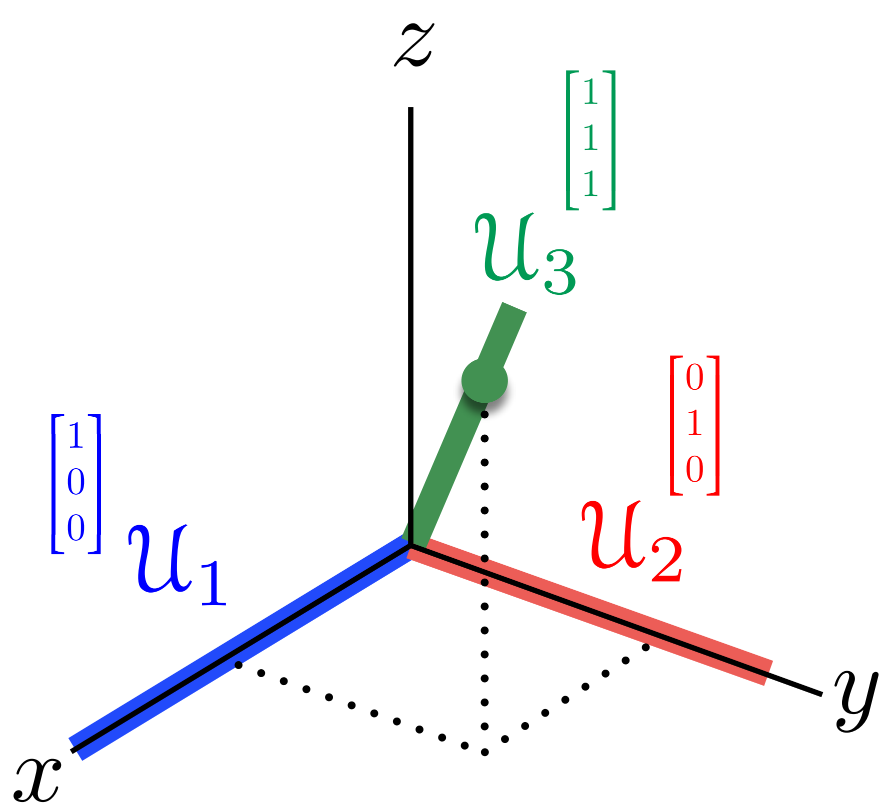
$$= \left\{ \mathbf{x} : \mathbf{x}^{\otimes \mathbf{2}} \in \ker \mathbf{V}^{\mathsf{T}} =: \mathcal{S} \right\}$$

Unions of Subspaces are Varieties!



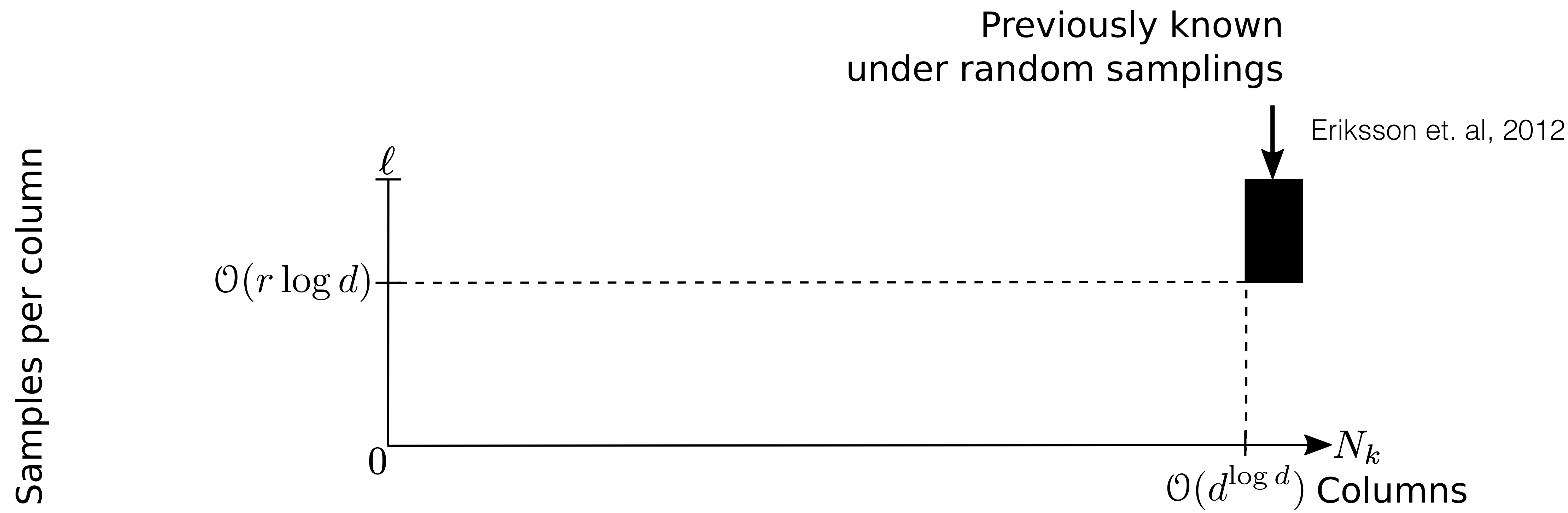
$$\mathcal{V} = \left\{ \mathbf{x} : \begin{array}{rcl} xy(x-y) & = & 0 \\ xy(x-z) & = & 0 \\ z(x-y) & = & 0 \\ z(x-z) & = & 0 \end{array} \right\}$$

$$= \left\{ \mathbf{x} : \begin{array}{rcrcrcrcl} -\frac{5}{6}xy & + & \frac{1}{2}xz & + & \frac{5}{30}yz & + & \frac{5}{30}z^2 & = & 0 \\ -\frac{5}{30}xy & - & \frac{1}{2}xz & + & \frac{5}{6}yz & - & \frac{5}{30}z^2 & = & 0 \\ -\frac{5}{30}xy & - & \frac{1}{2}xz & - & \frac{5}{30}yz & + & \frac{5}{6}z^2 & = & 0 \end{array} \right\}$$

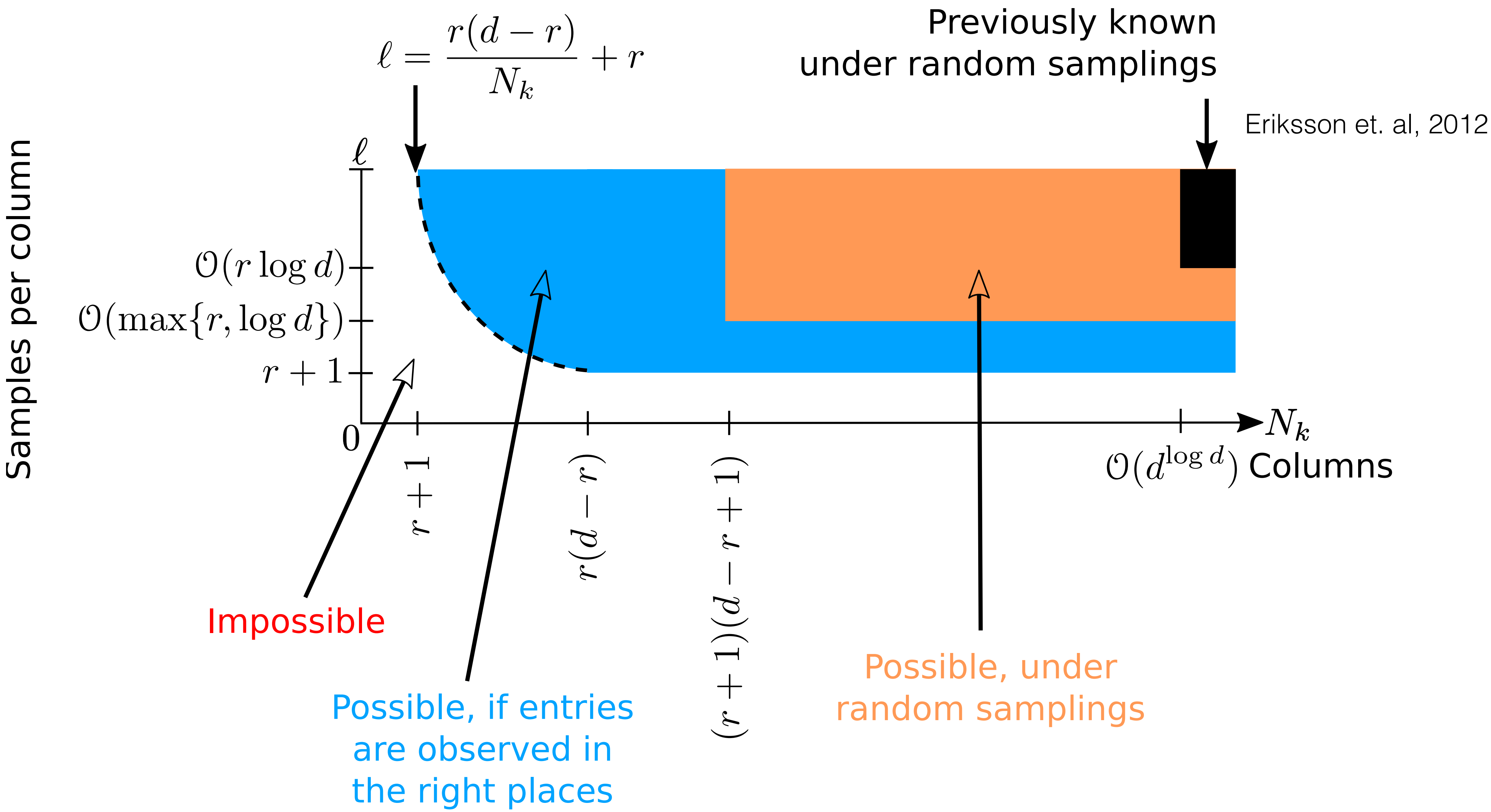
$$= \left\{ \mathbf{x} : \mathbf{x}^{\otimes 2} \in \ker \mathbf{V}^{\mathsf{T}} =: \mathcal{S} \right\}$$

Unions of Subspaces are Varieties!

So we can use our theory!

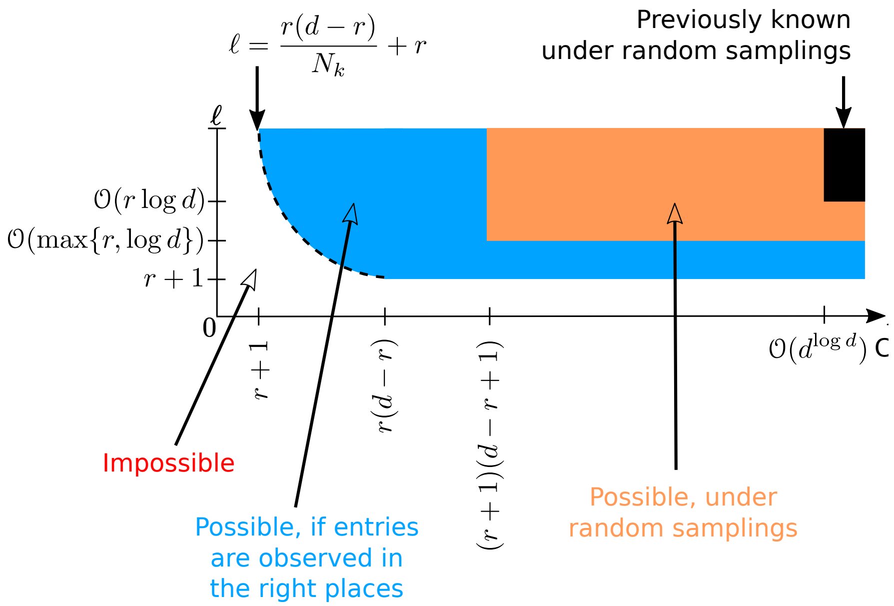


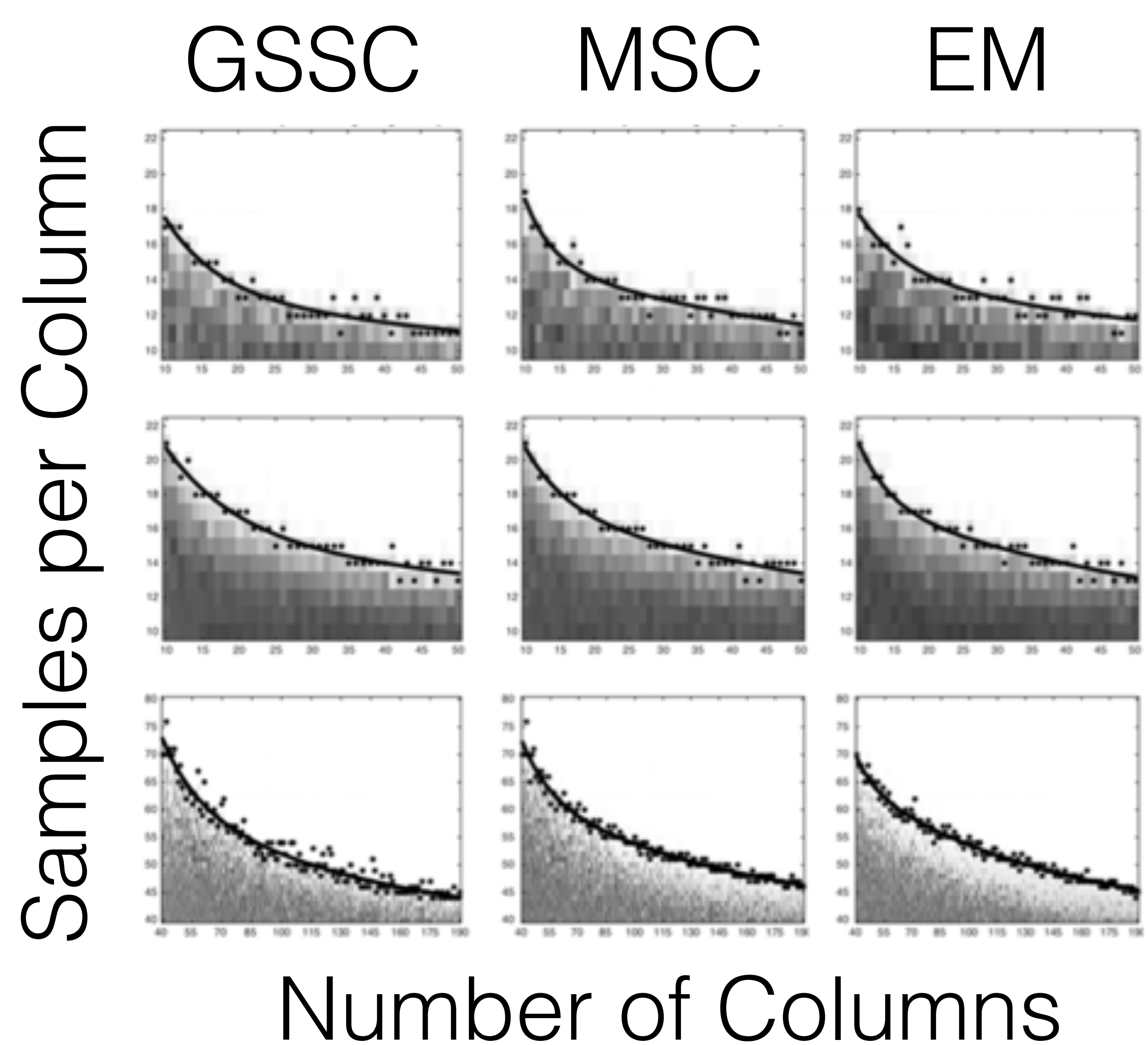
Information-theoretic requirements



Information-theoretic requirements

[4] Pimentel et. al, 2016





Theory matches Practice


Mmmh… You begin to convince me…

Mmmh... You begin to convince me...

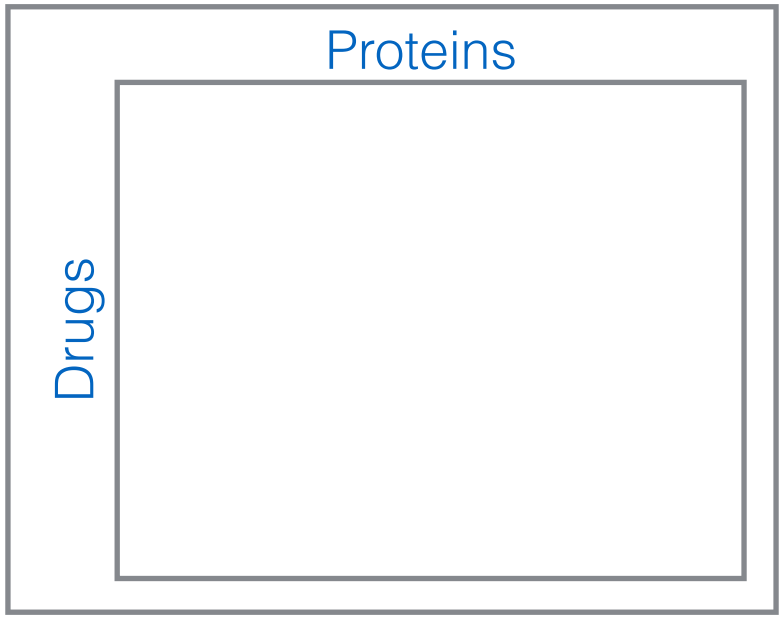
(Tough crowd!)

Mmmh…
You begin
to convince
me…

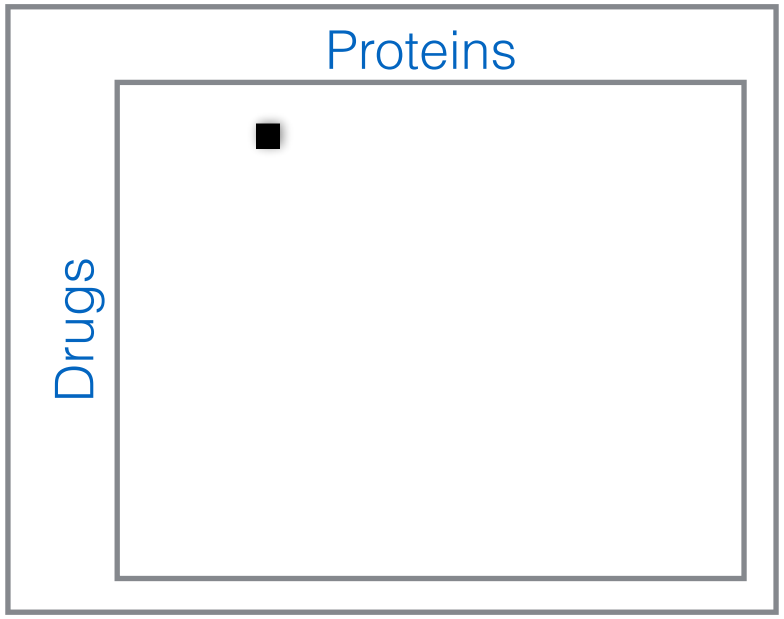
(Tough crowd!)

All right,
Here are other
Applications…

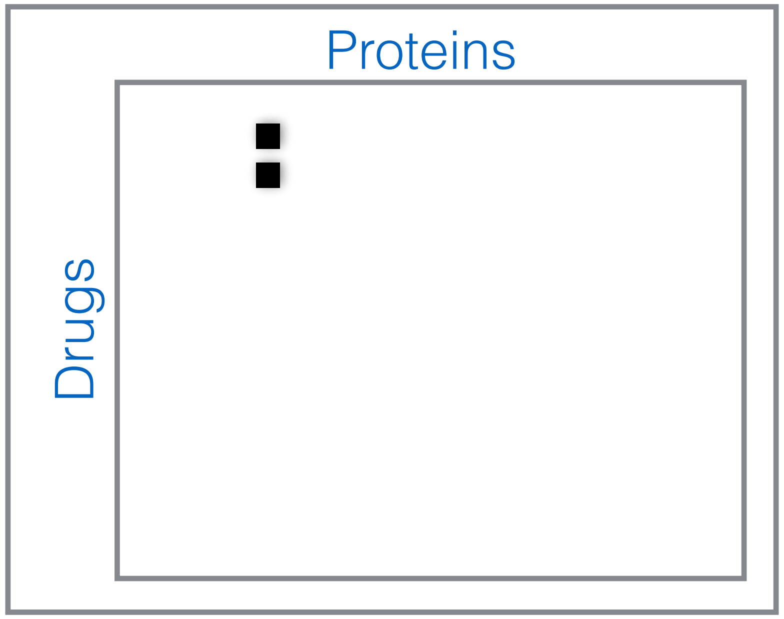
Drug Discovery



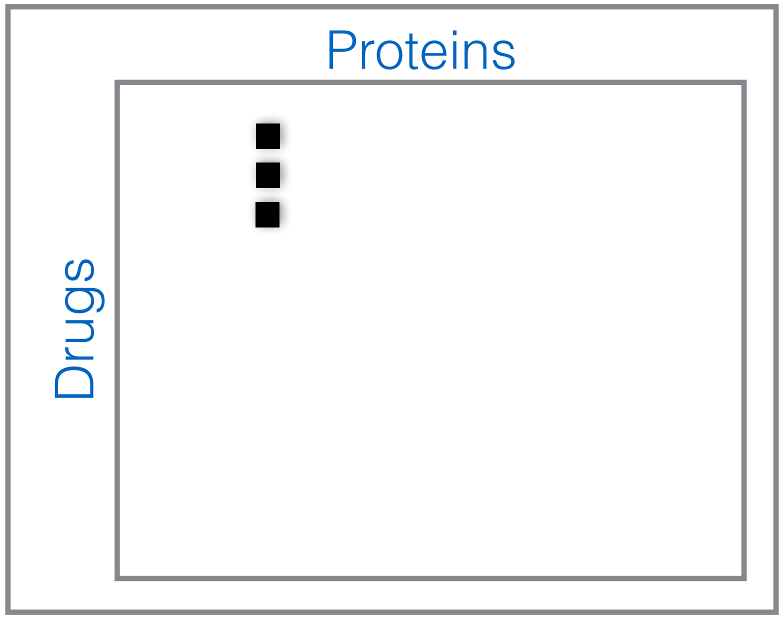
Drug Discovery



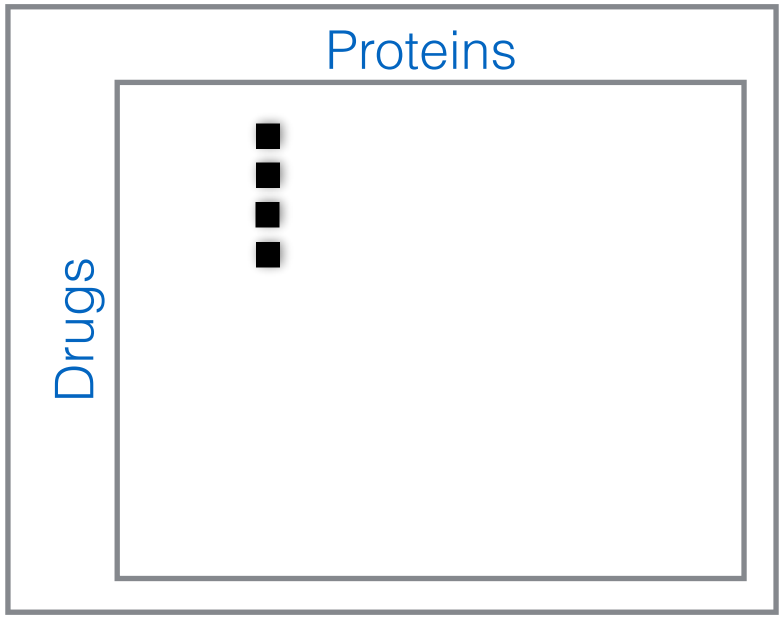
Drug Discovery



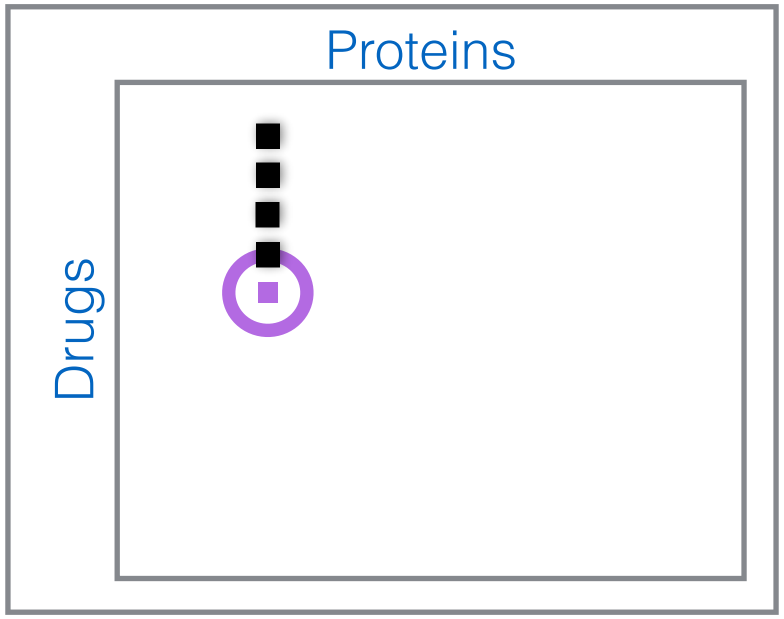
Drug Discovery



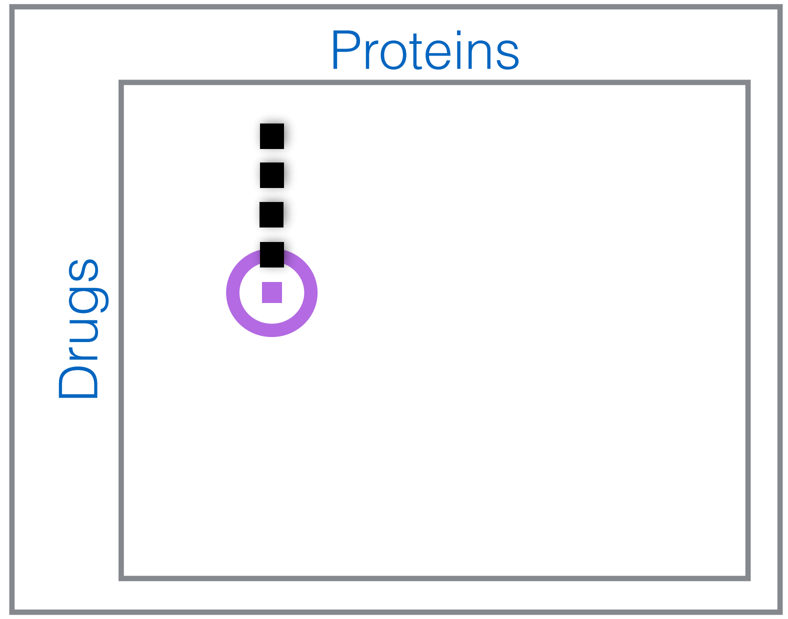
Drug Discovery



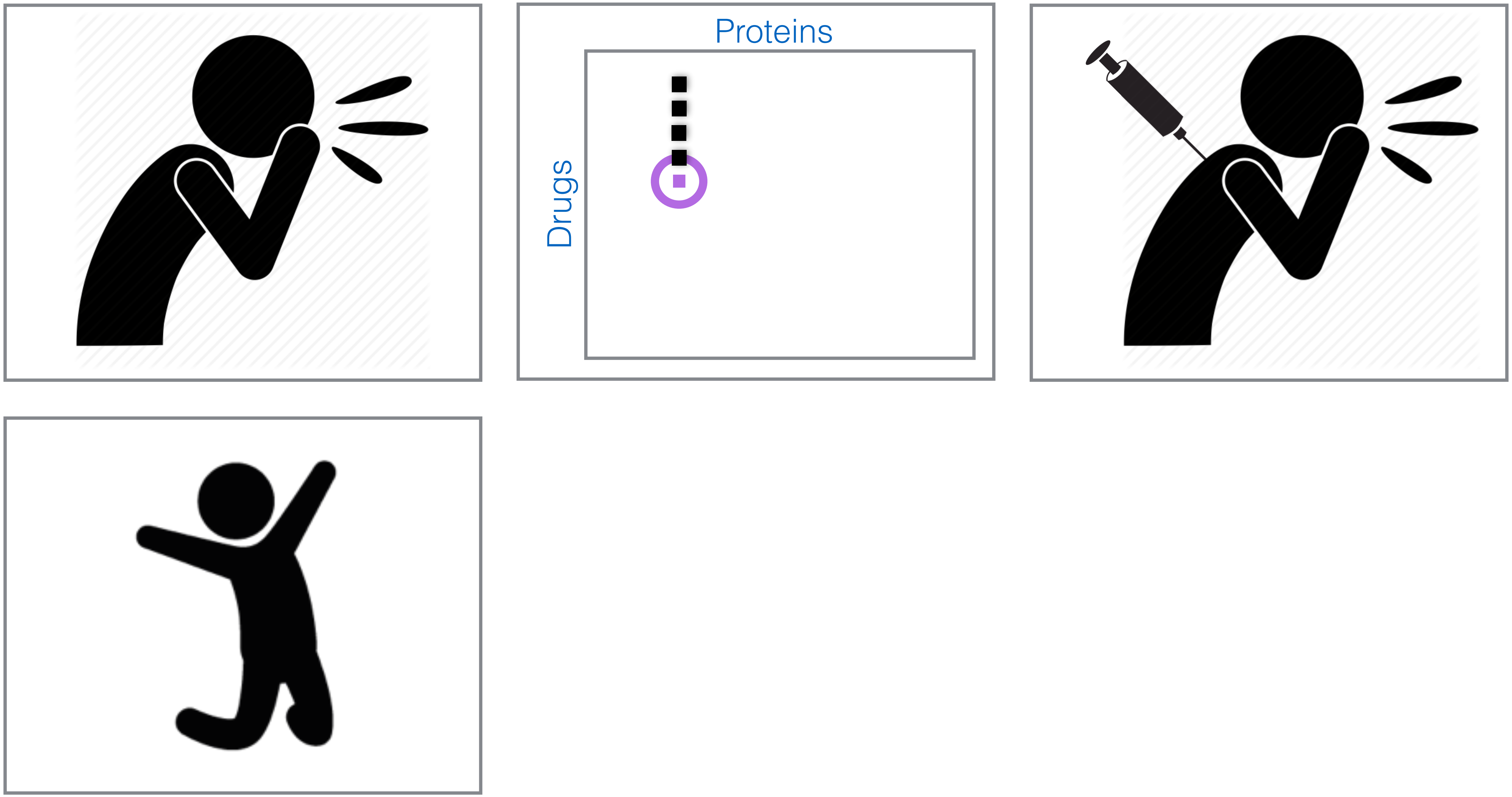
Drug Discovery



Drug Discovery



Drug Discovery



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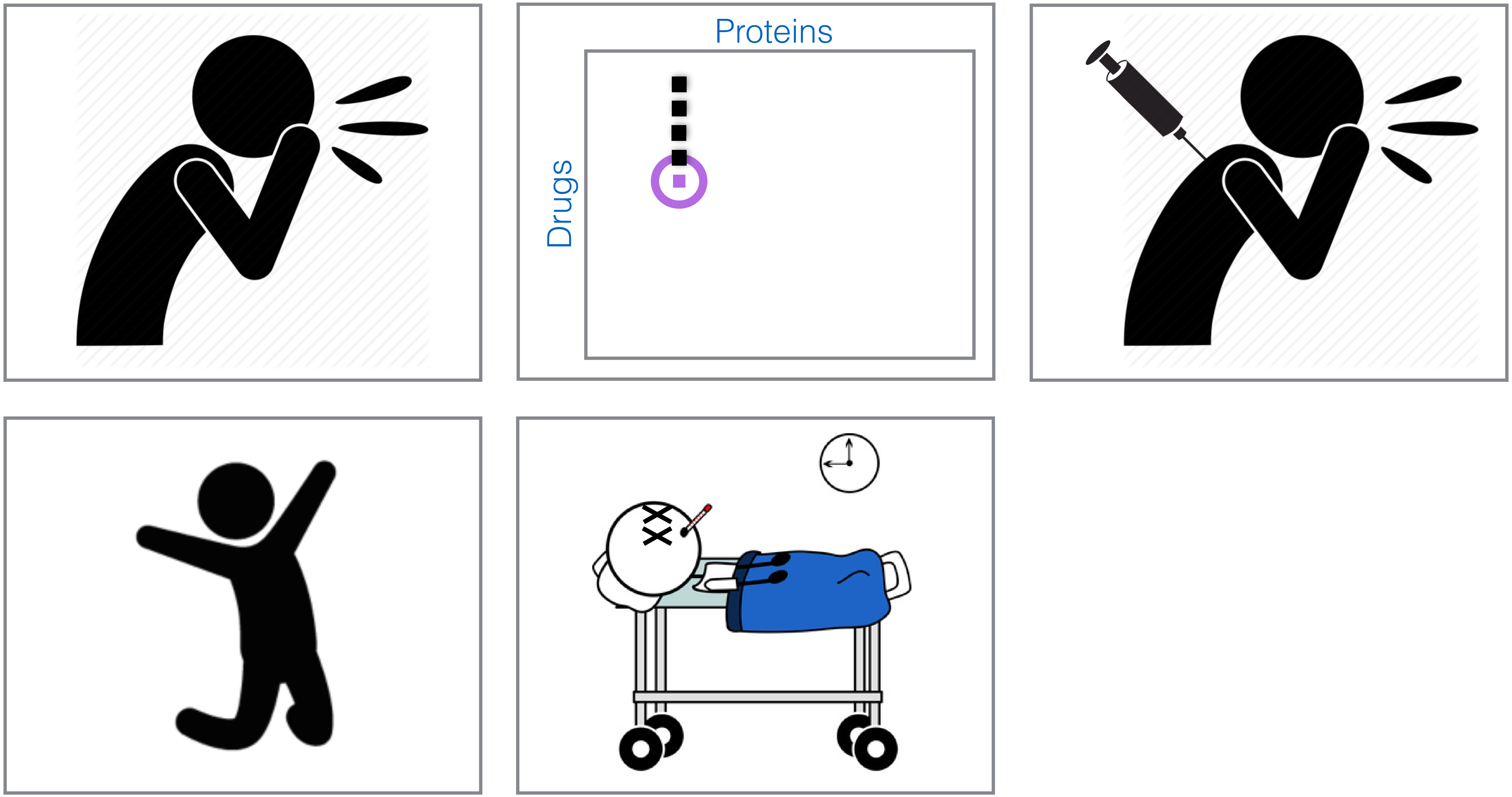

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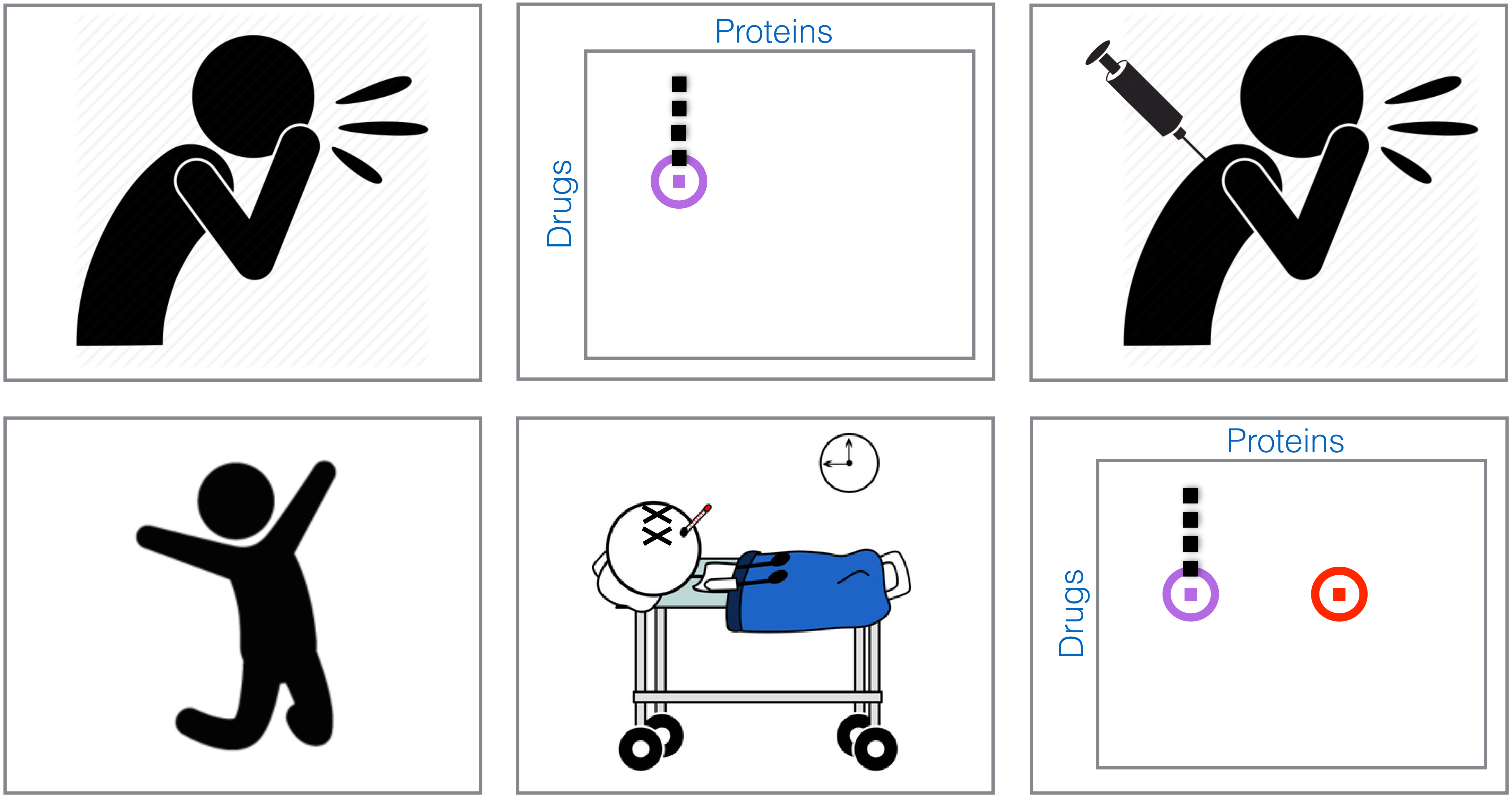

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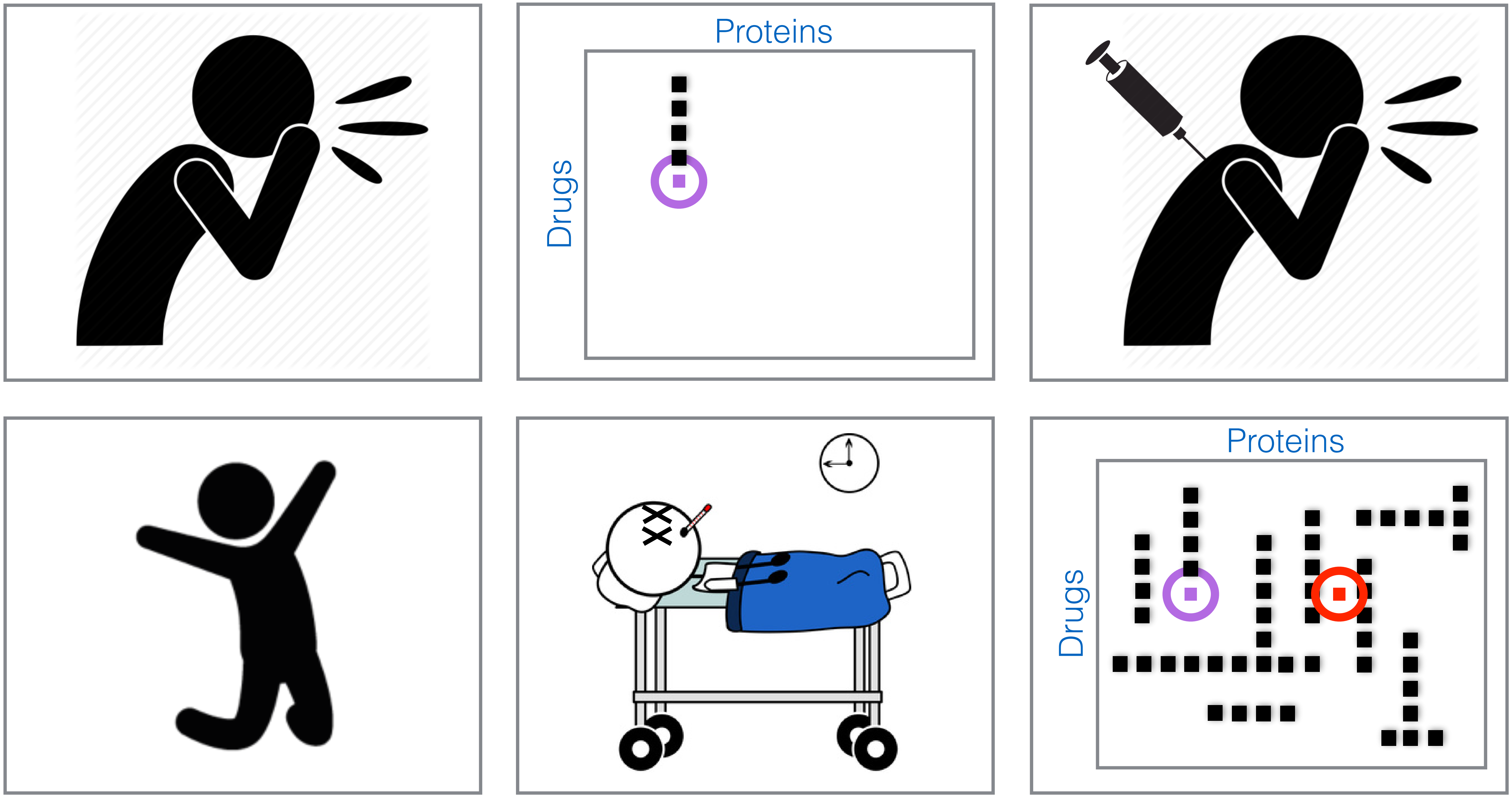
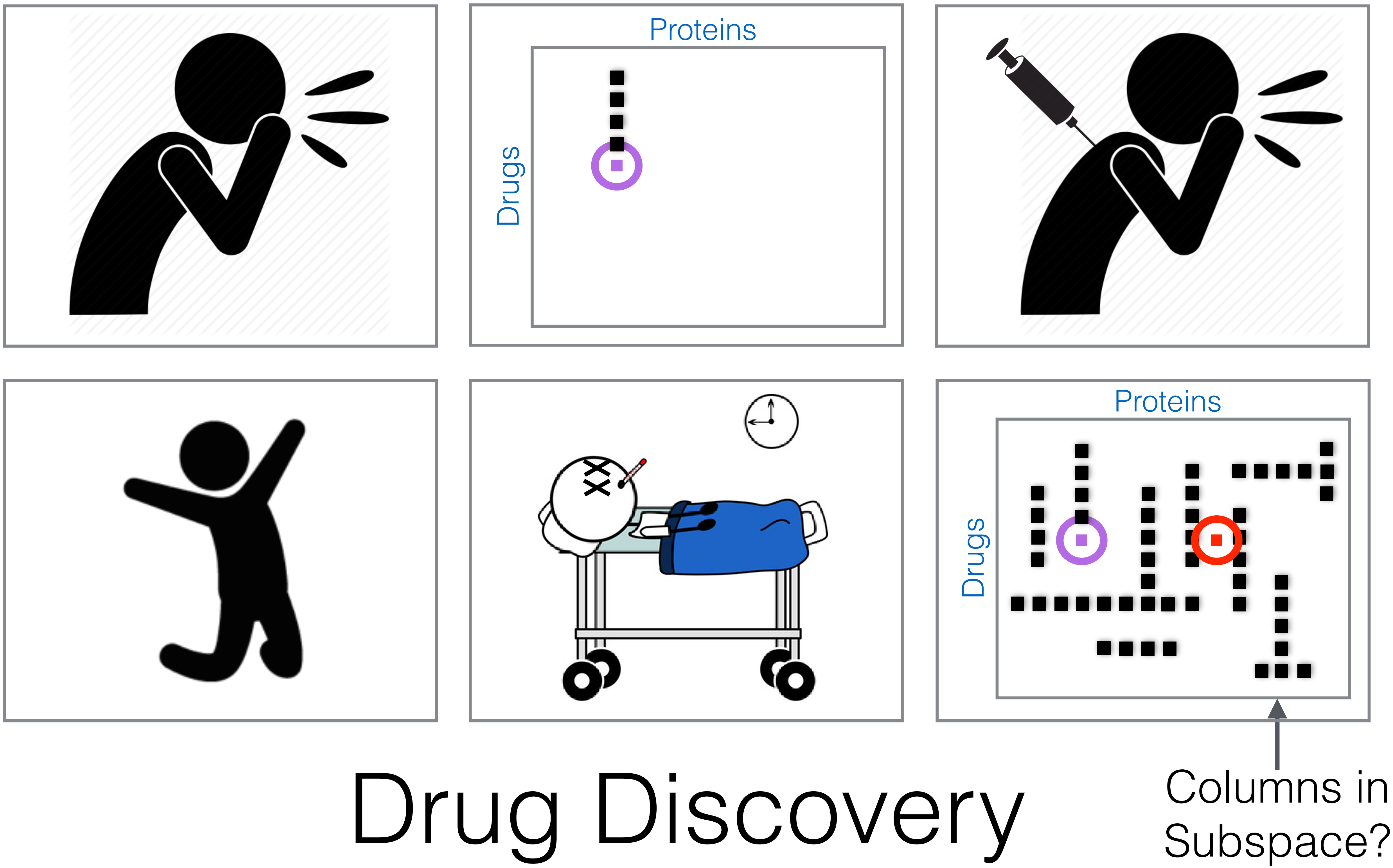
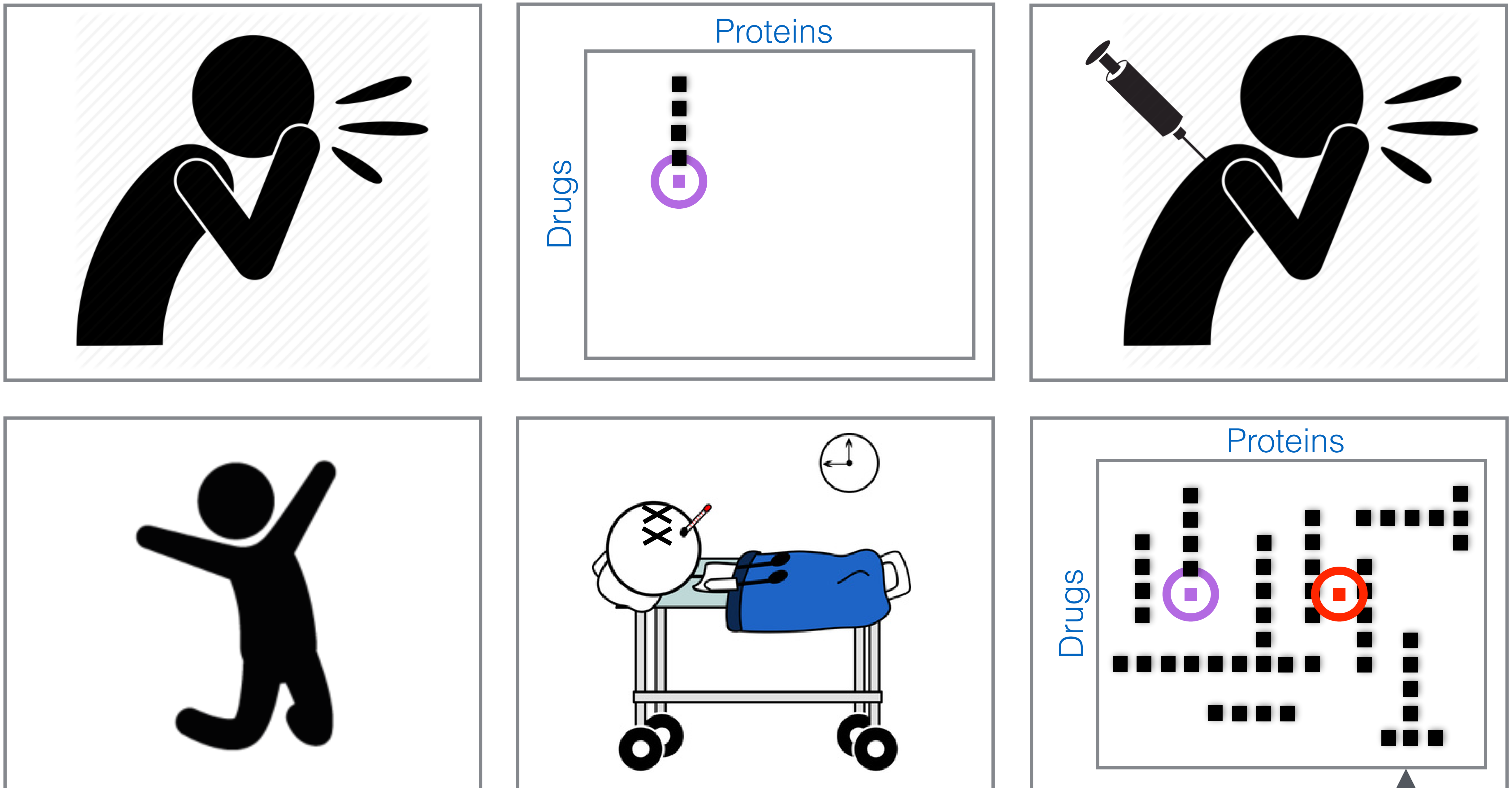


Drug Discovery

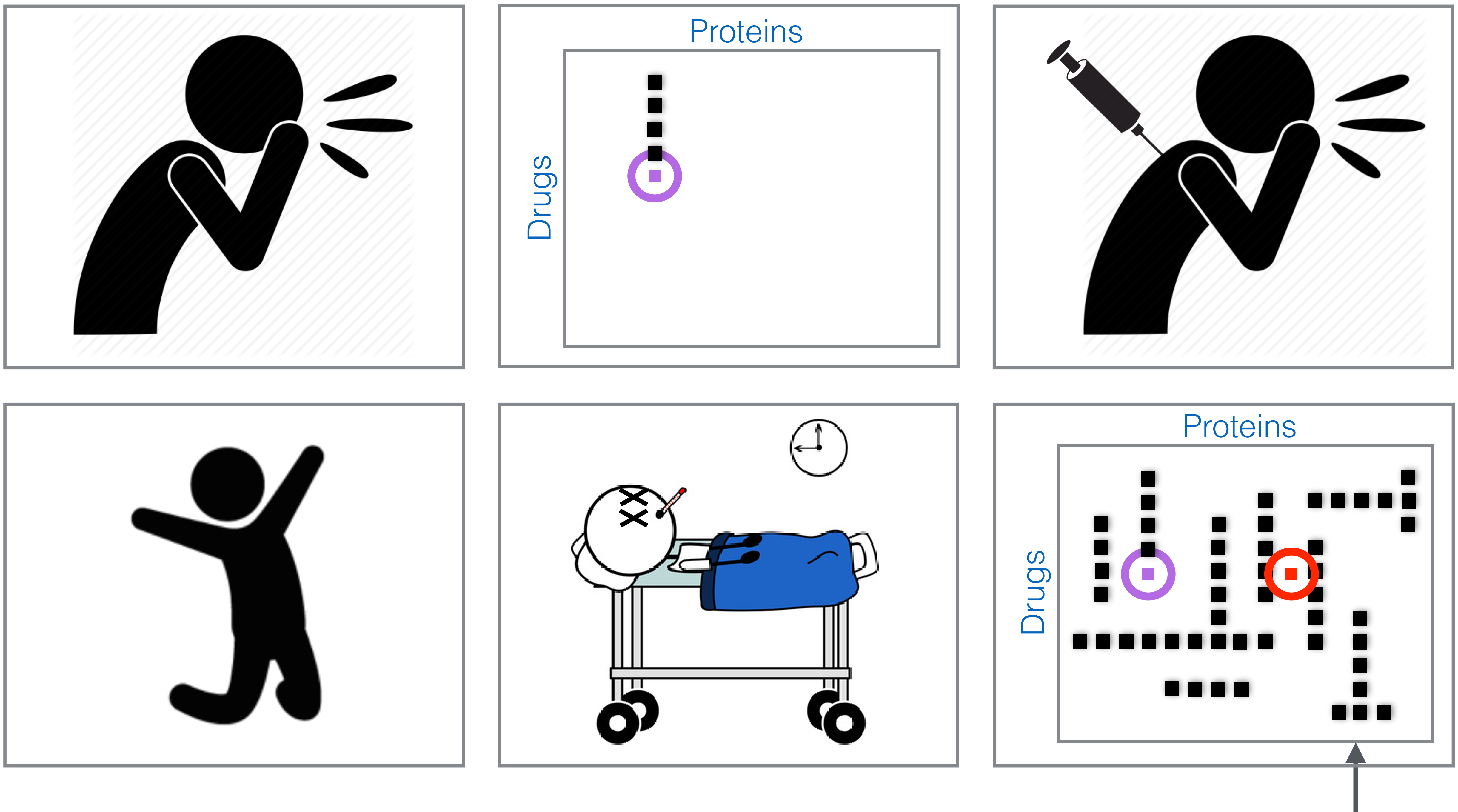


Drug Discovery



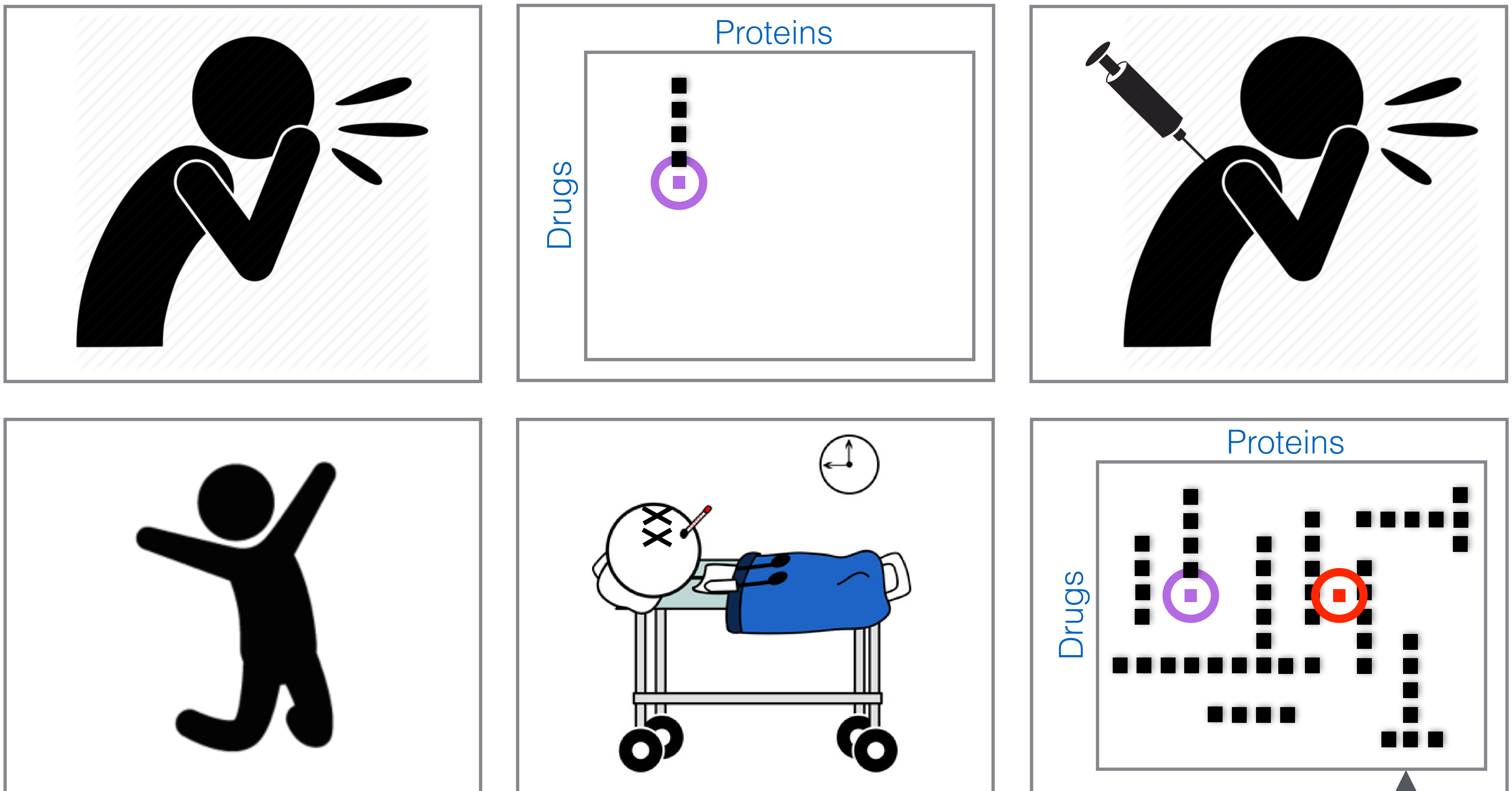
Drug Discovery

Columns in Subspace? Union?



Drug Discovery

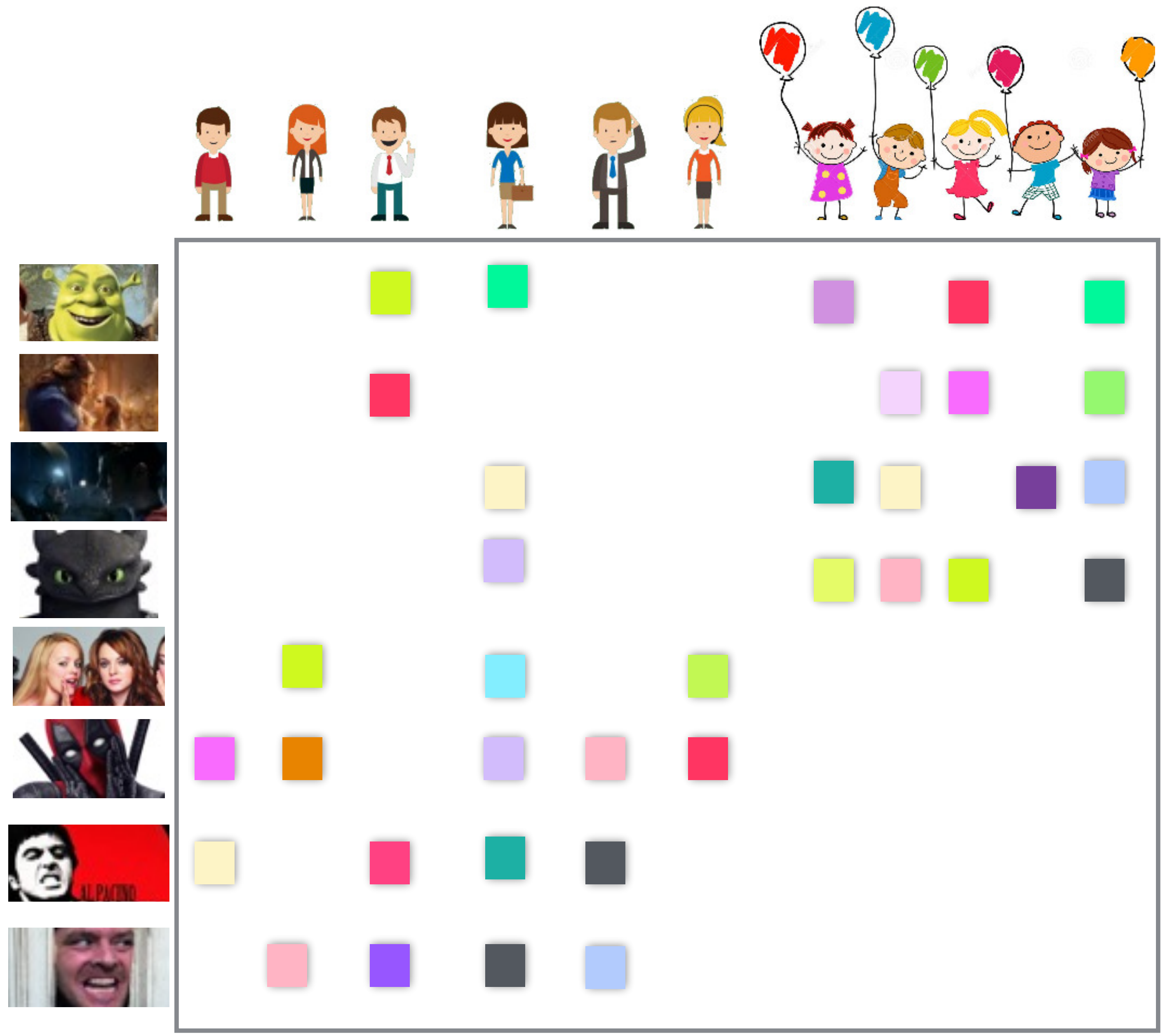
Columns in
Subspace?
Union?
Variety?



Columns in
Subspace?
Union?
Variety?

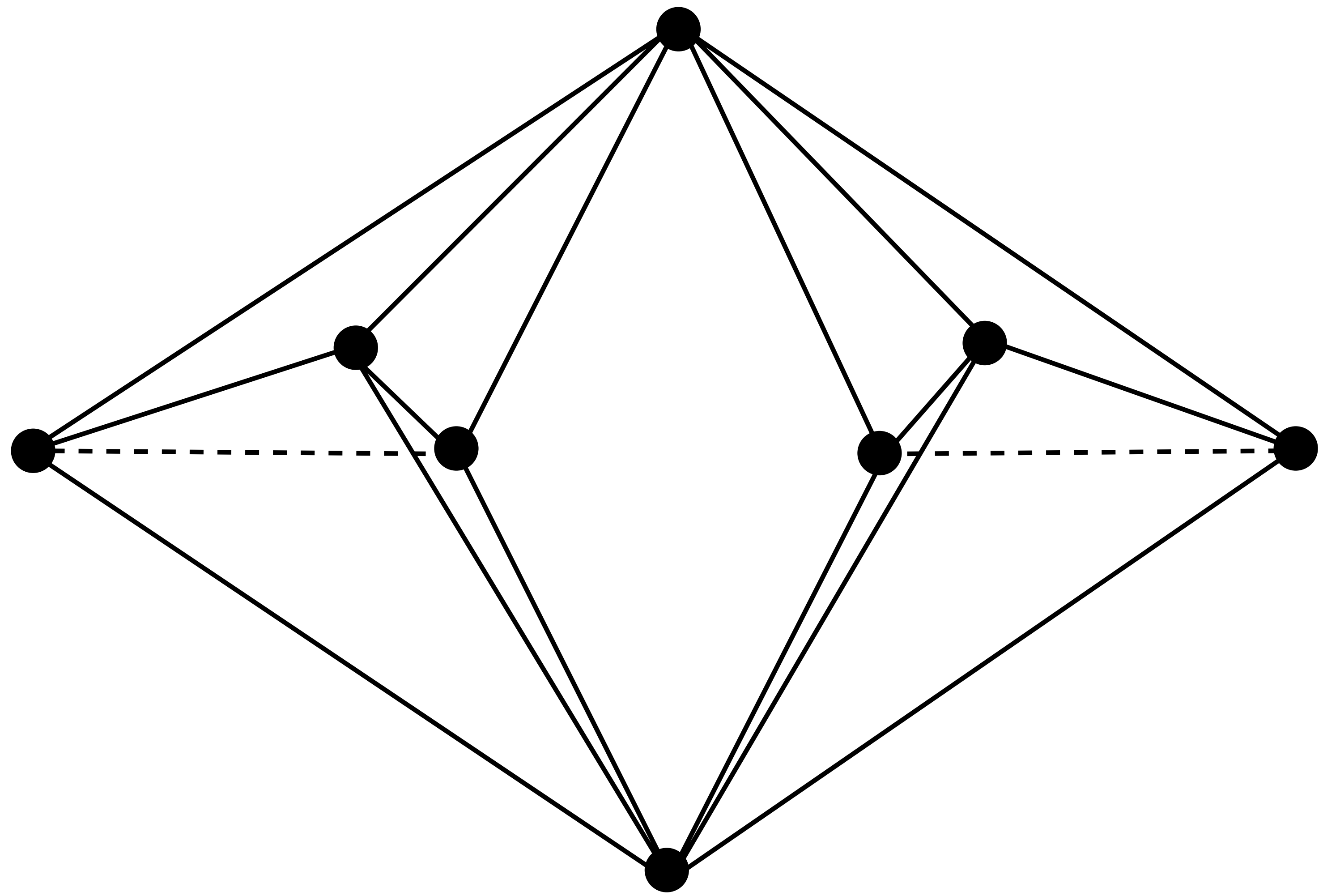
Drug Discovery

Adaptive Sampling



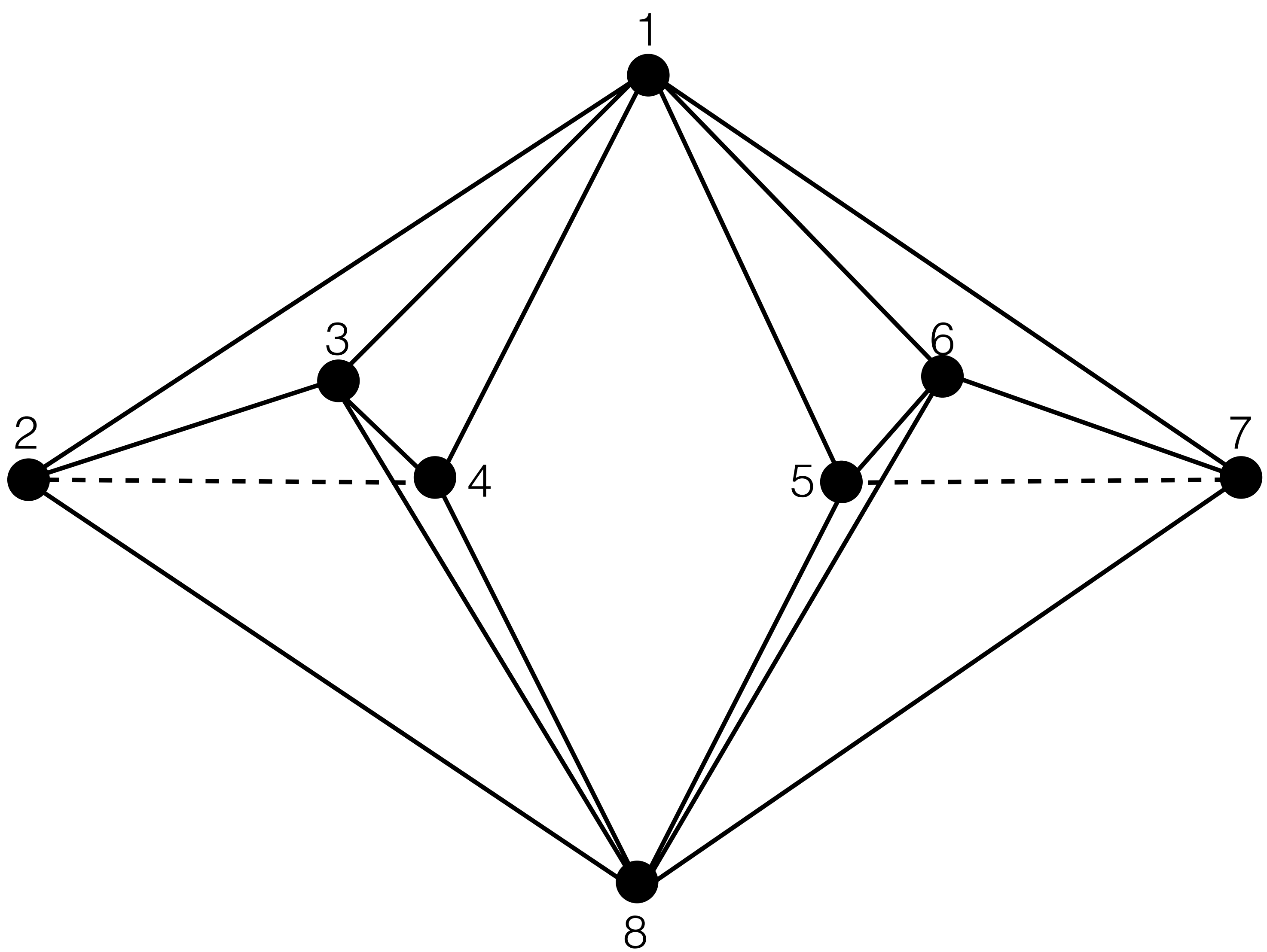
Recommender Systems

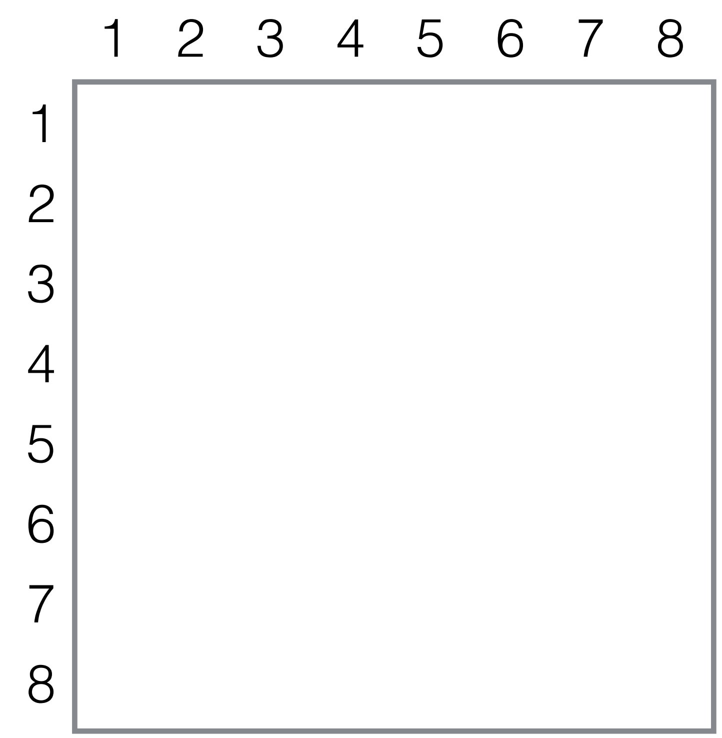
- **Non-uniform Sampling!**
- **Coherent Subspace?**



Rigidity and Graph Inference

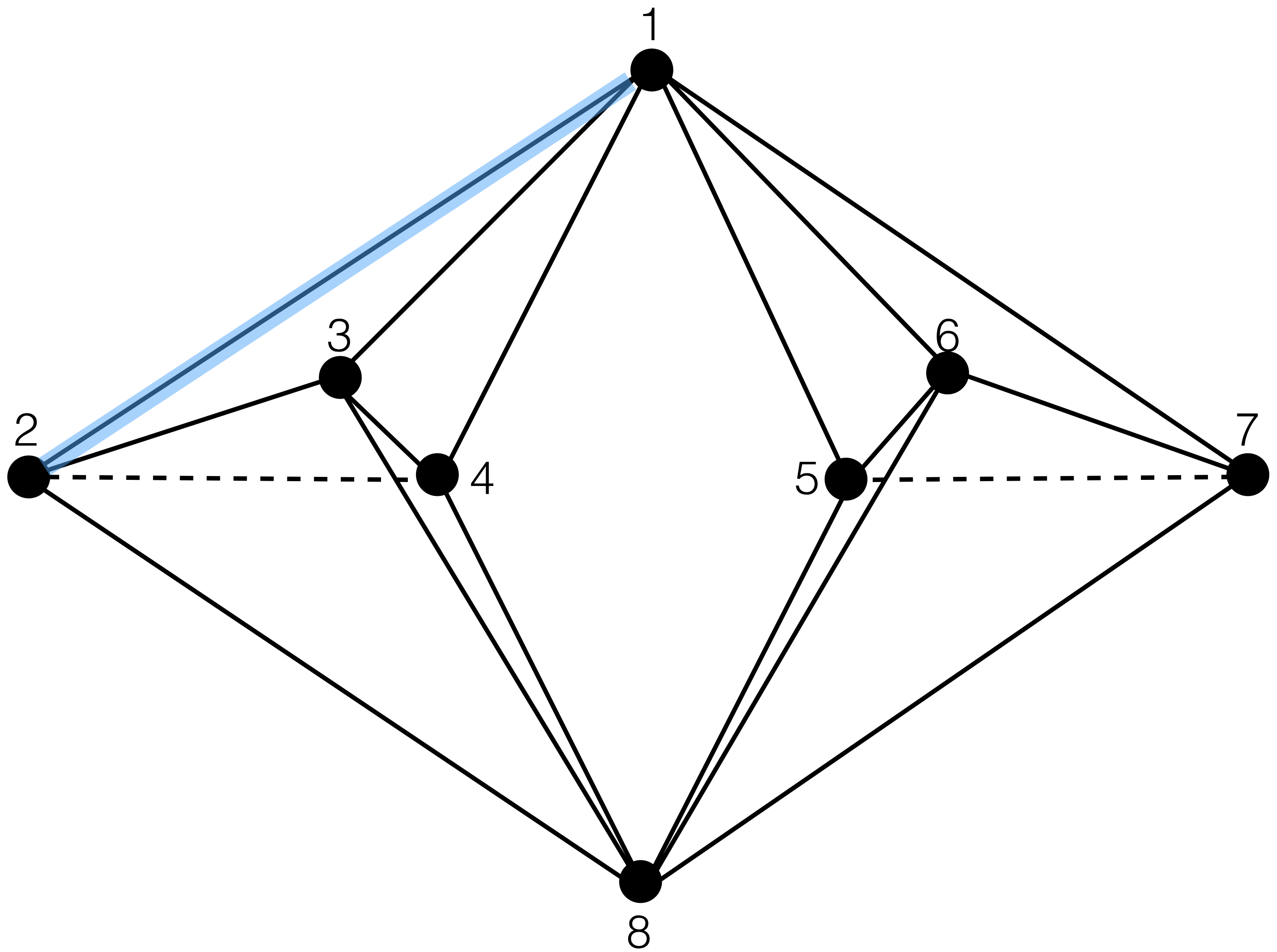
- **Non-uniform Sampling!**
- **Coherent Subspace!**

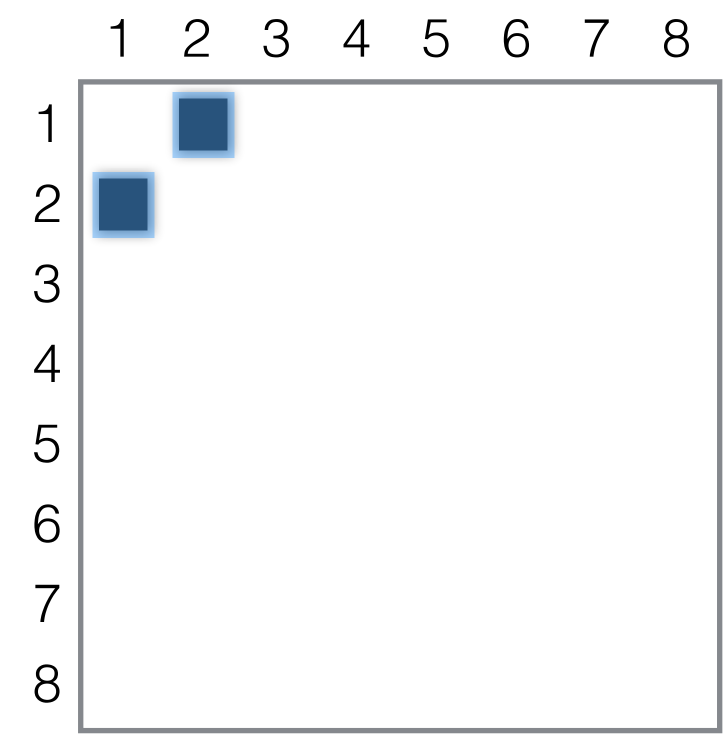




Rigidity and Graph Inference

- **Non-uniform Sampling!**
- **Coherent Subspace!**





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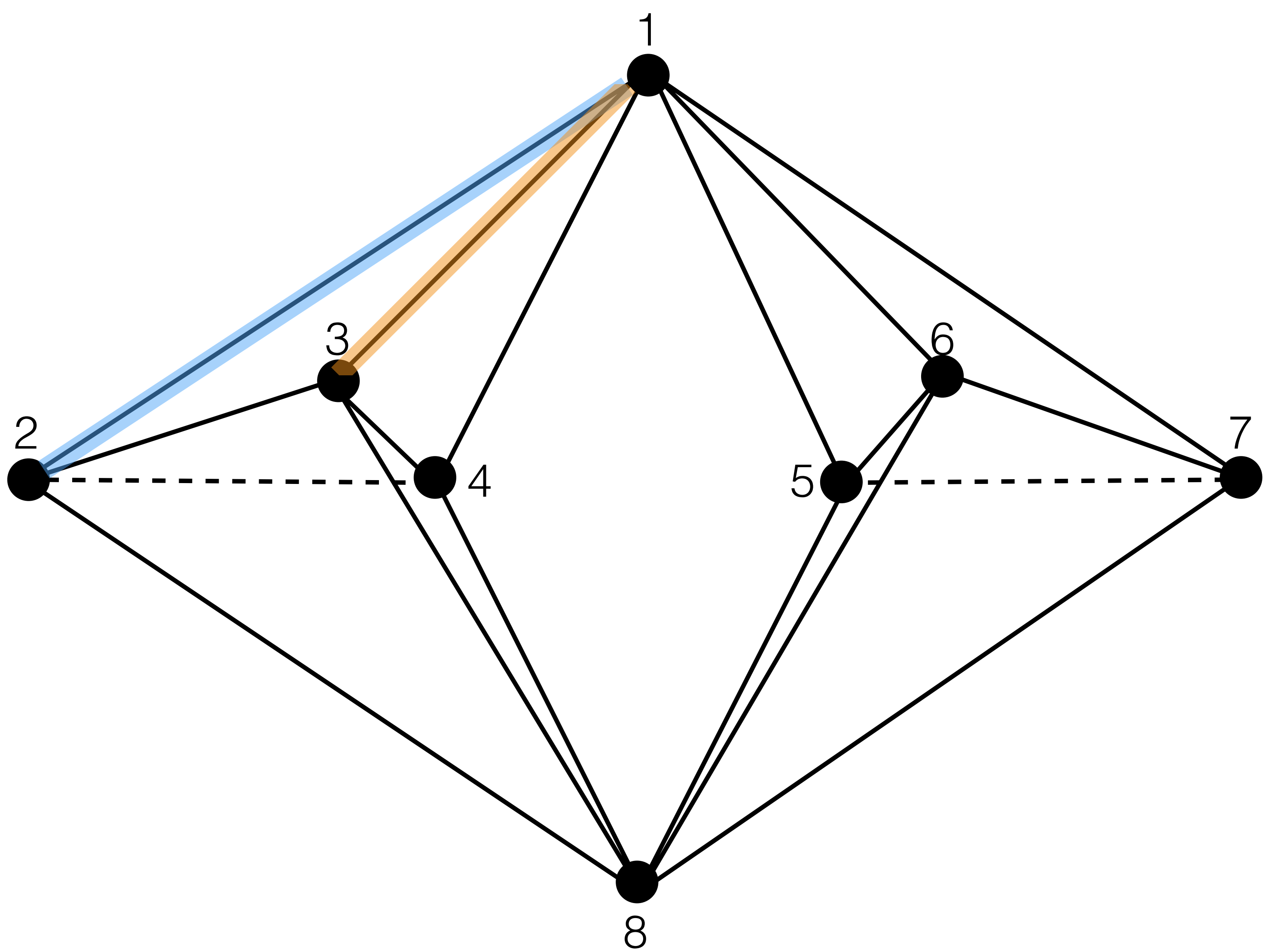

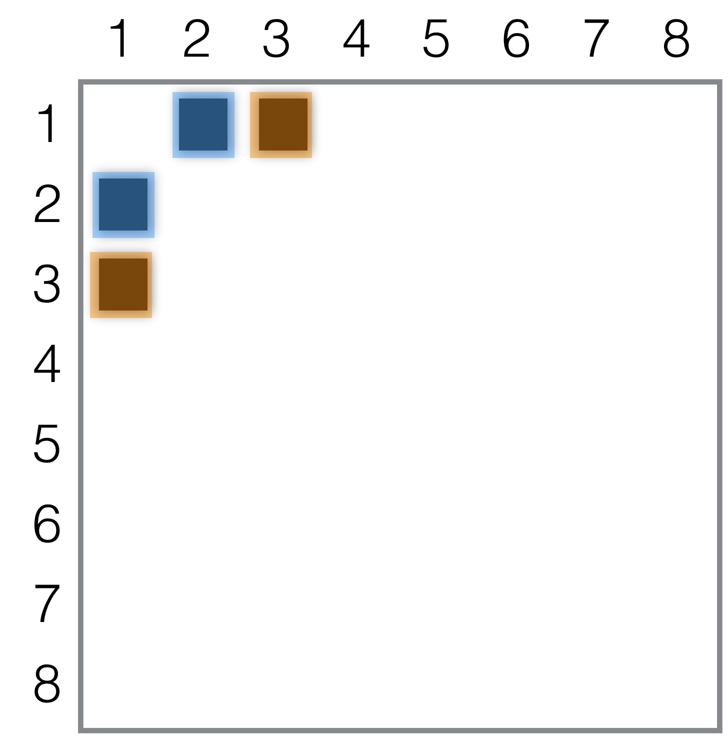

Rigidity and Graph Inference

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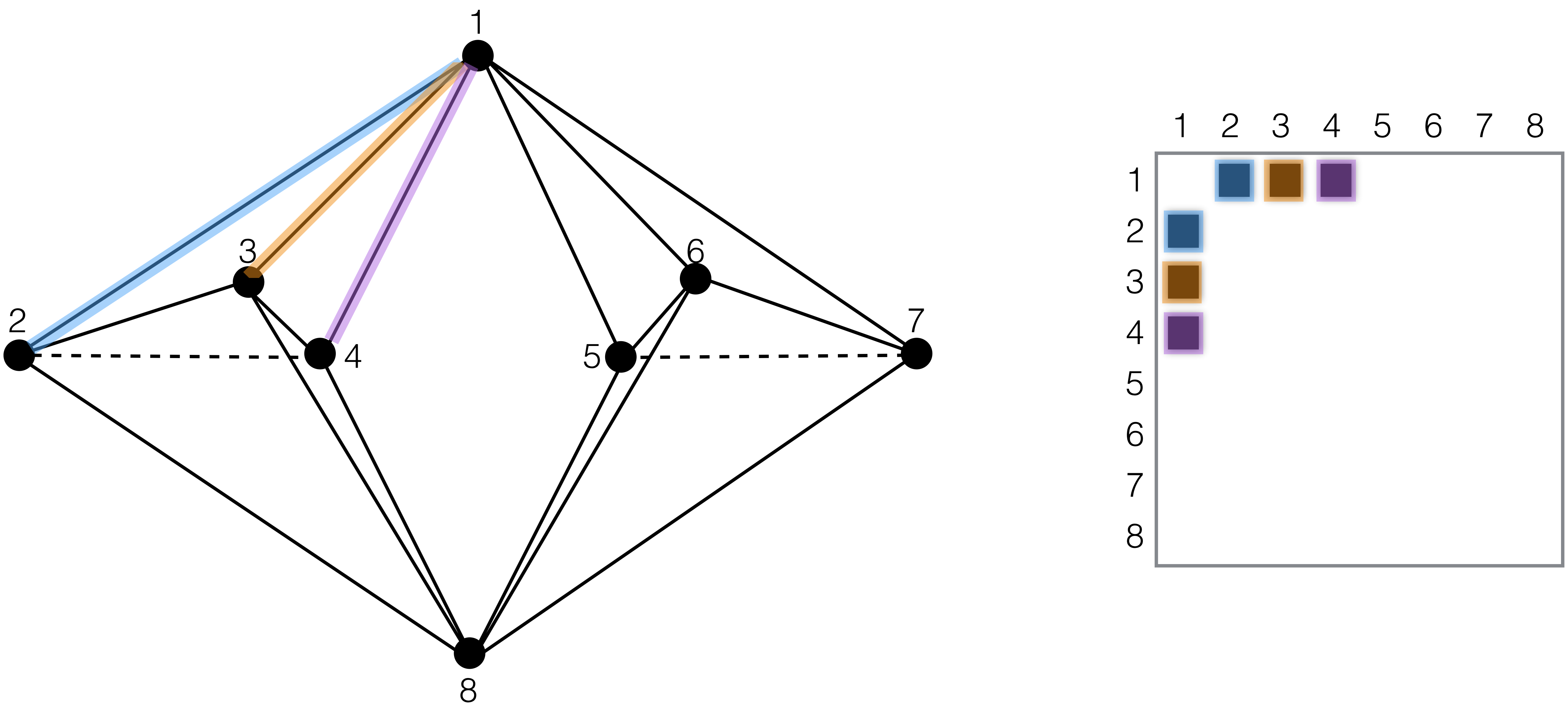

Rigidity and Graph Inference

- Non-uniform Sampling!
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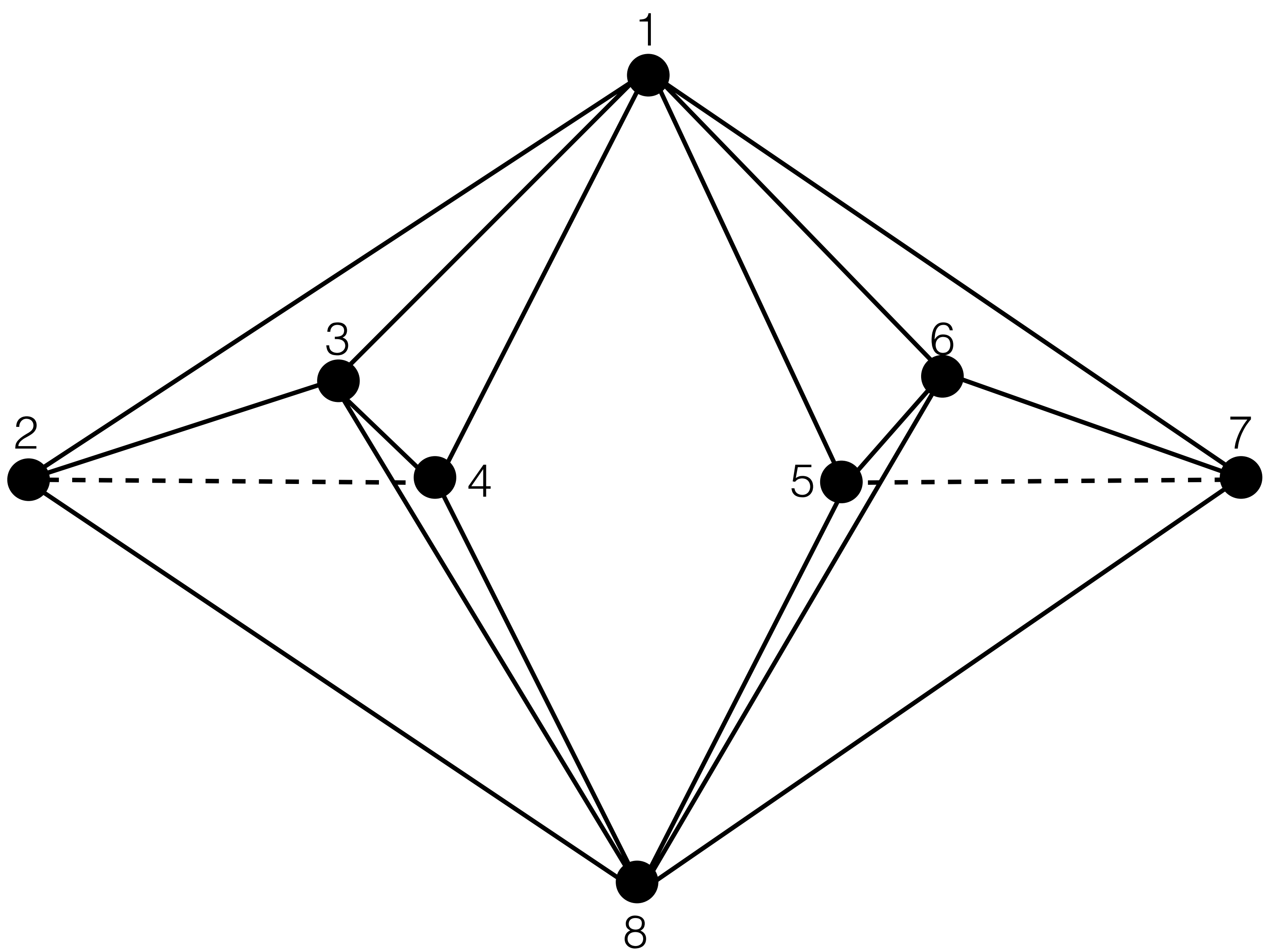
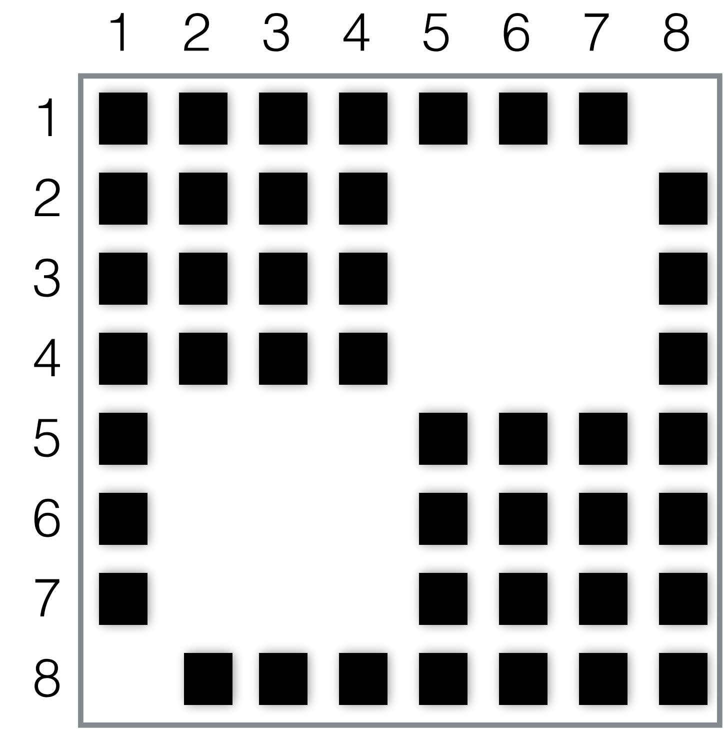
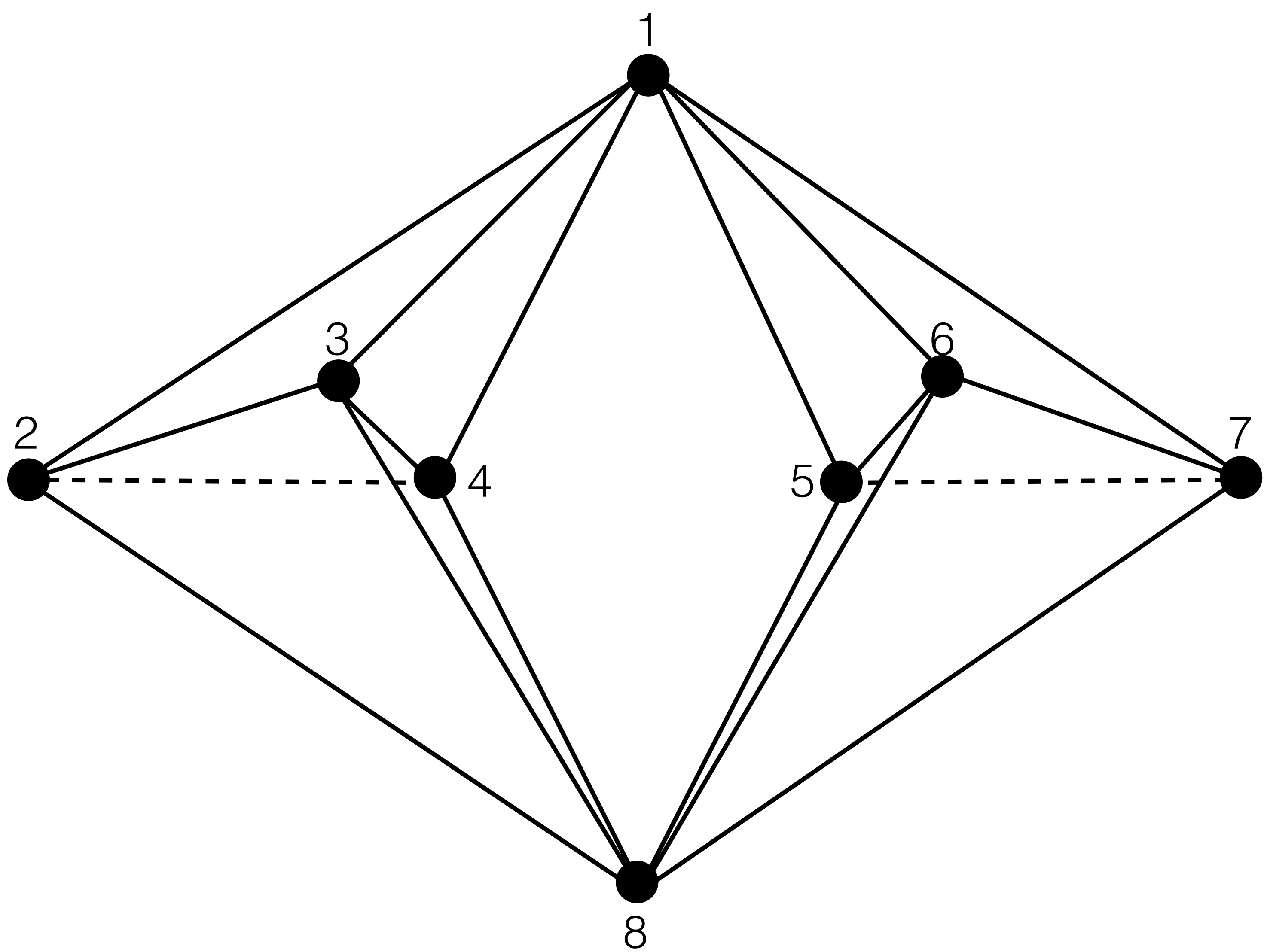
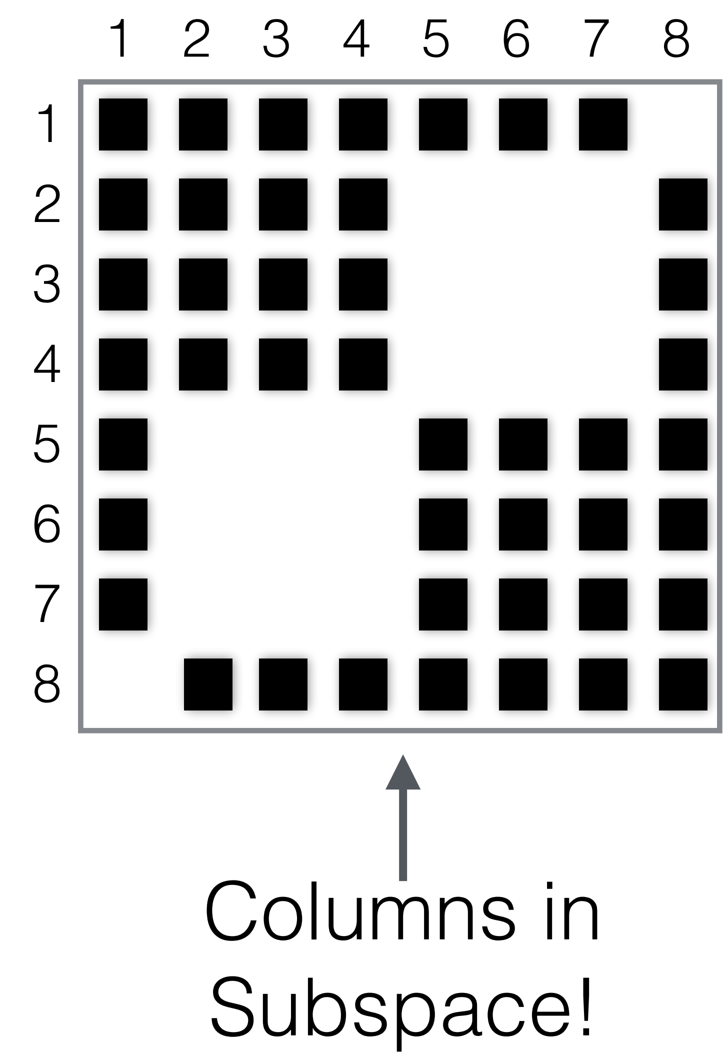




Rigidity and Graph Inference

- Non-uniform Sampling!
- Coherent Subspace!





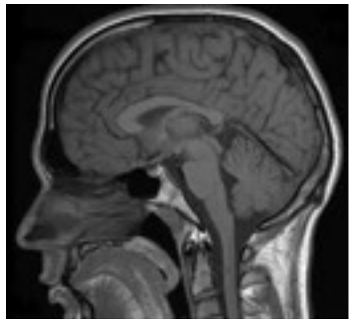
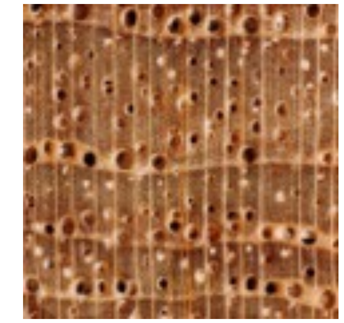
Rigidity and Graph Inference

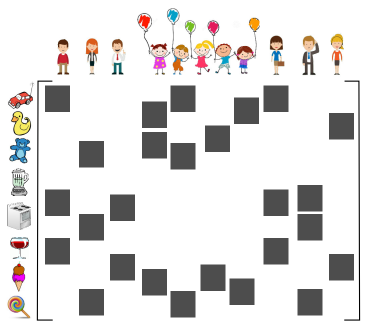
- Non-uniform Sampling!
- Coherent Subspace!

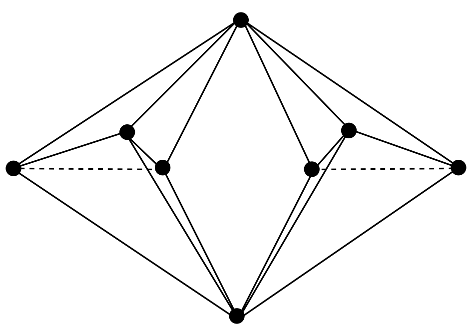
Countless Applications

- **Non-uniform Sampling**
- **Coherent Subspace**



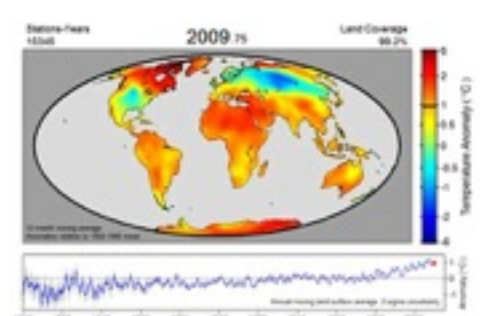




Countless Applications

- **Non-uniform Sampling**
- **Coherent Subspace**







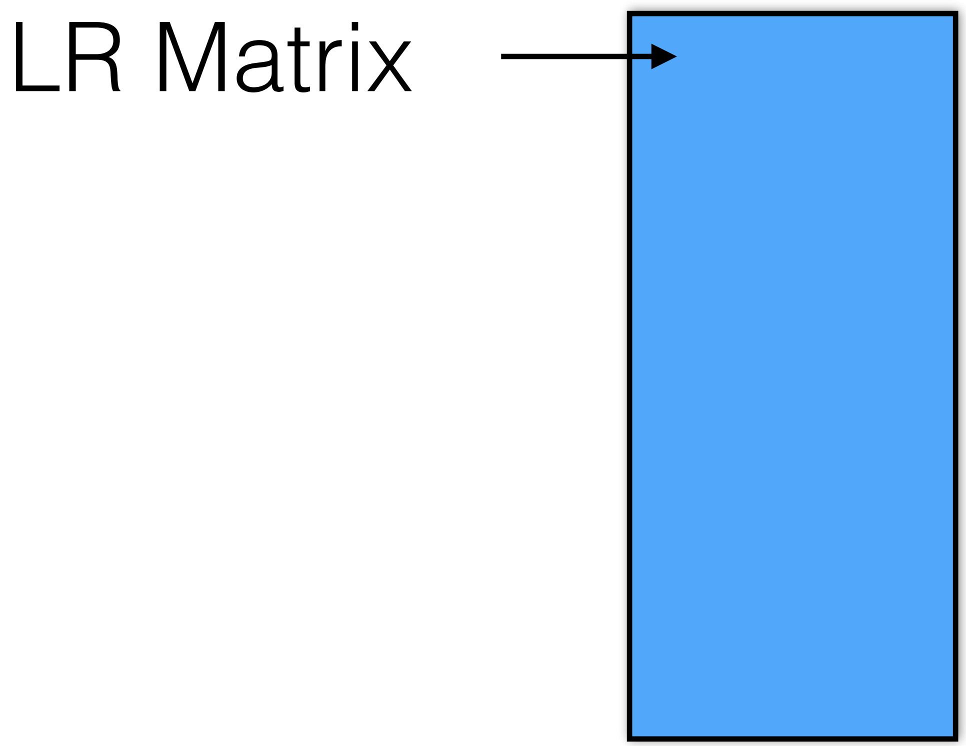
WOW, AMAZING
PLEASE TELL
ME MORE

How am I on time?

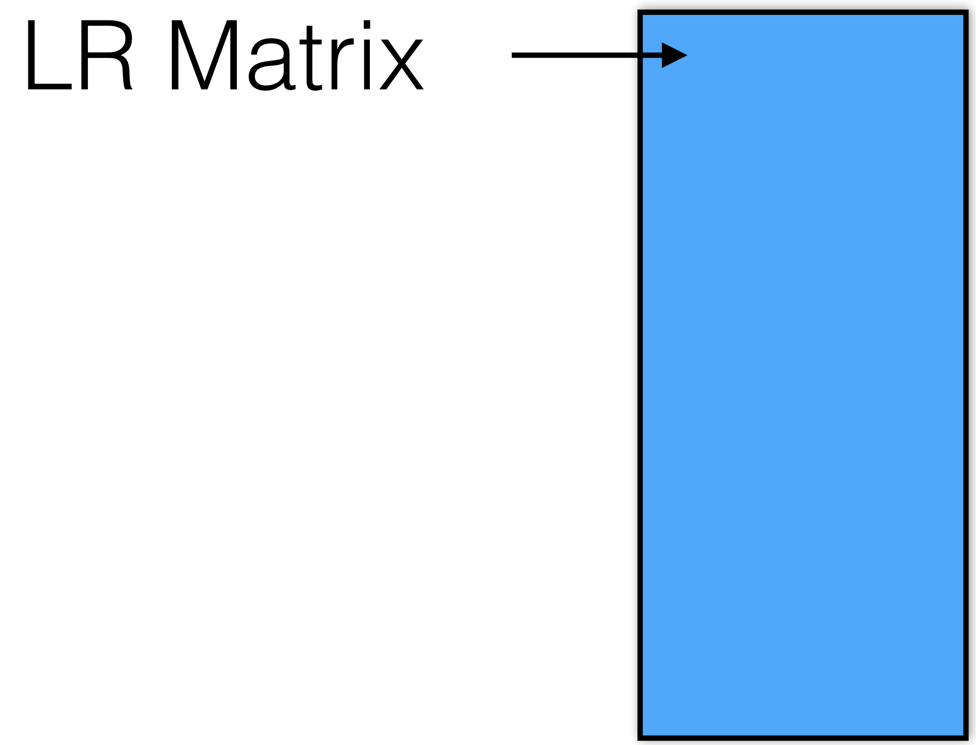
How am I on time?

Beyond Missing Data

LRMC
(Low-Rank Matrix Completion)



RPCA
(Robust Principal Component Analysis)

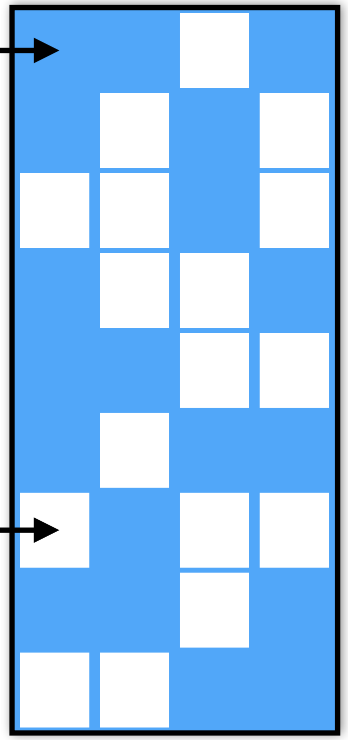


LRMC

(Low-Rank Matrix Completion)

LR Matrix

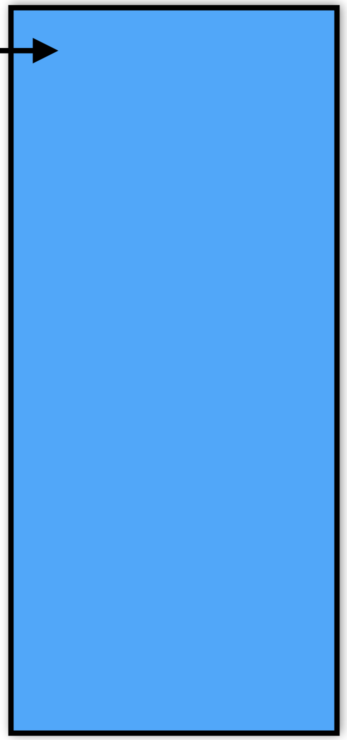
Tons of Missing Entries



RPCA

(Robust Principal Component Analysis)

LR Matrix



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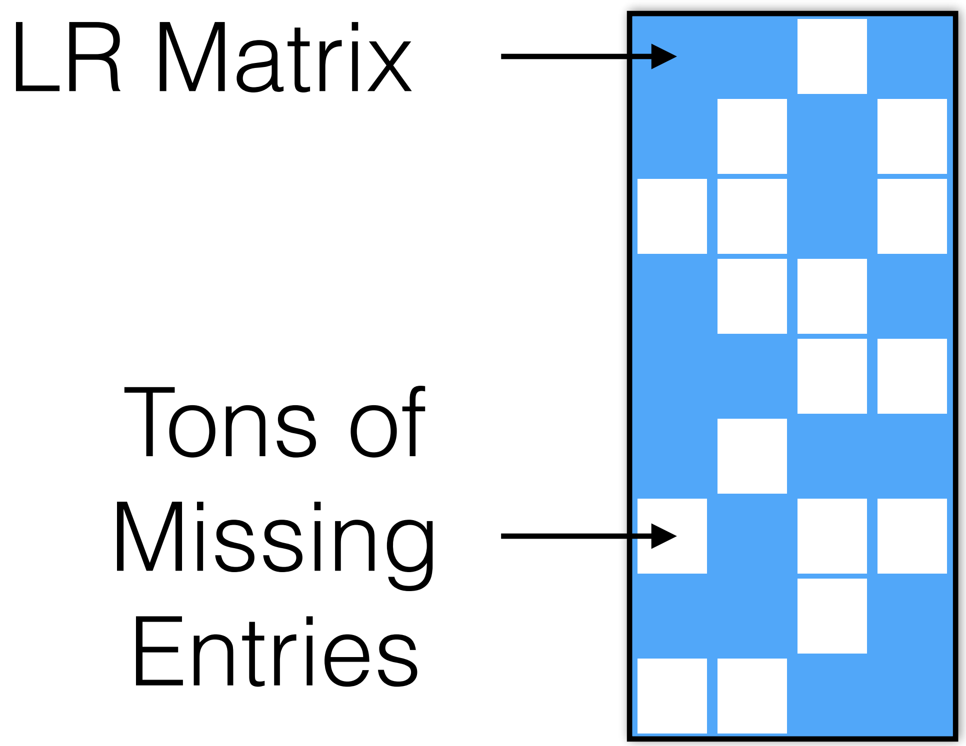

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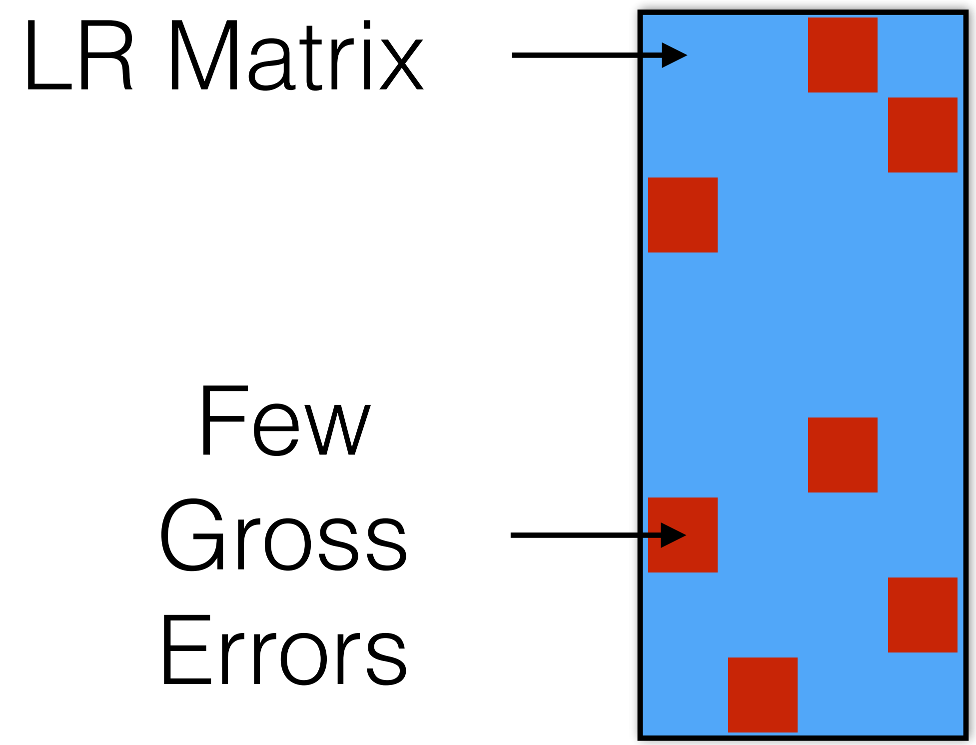
(Robust Principal Component Analysis)

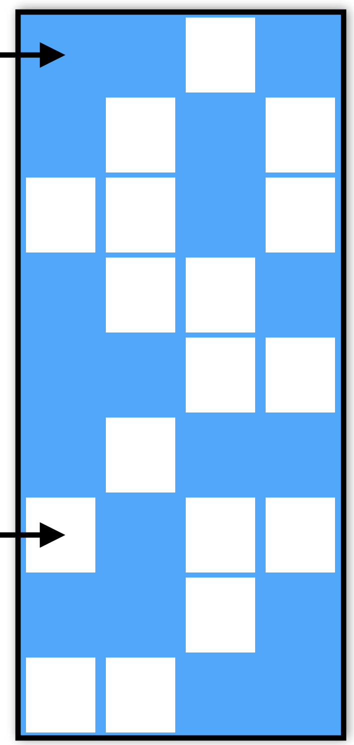


LRMC

(Low-Rank Matrix Completion)

LR Matrix

Tons of Missing Entries



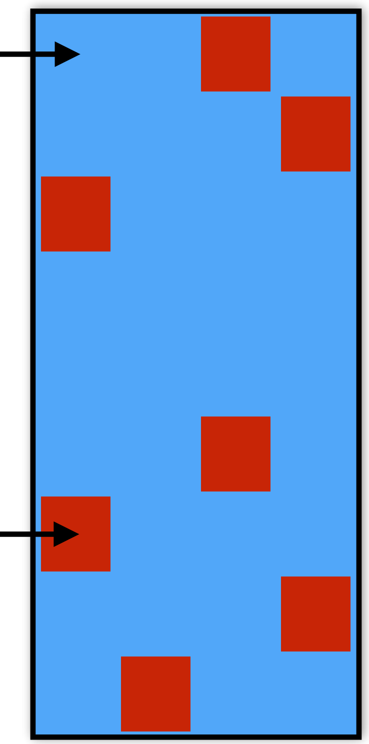
- [x] Know Locations
- [ ] Don’t know values

RPCA

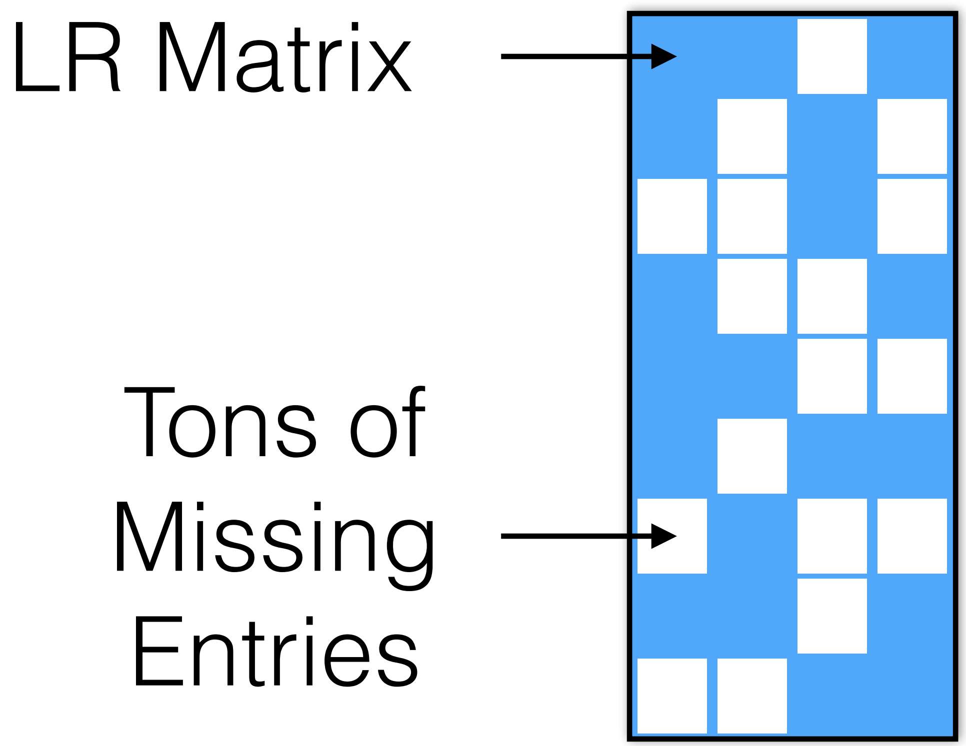
(Robust Principal Component Analysis)

LR Matrix

Few Gross Errors



LRMC
(Low-Rank Matrix Completion)



- [x] Know Locations
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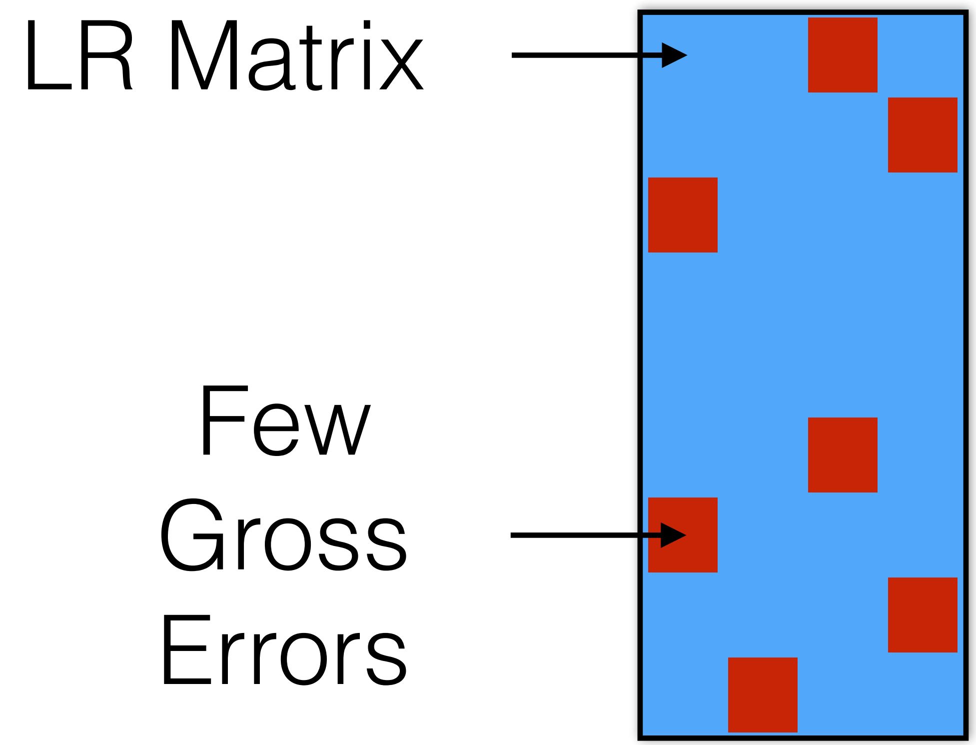
RPCA
(Robust Principal Component Analysis)



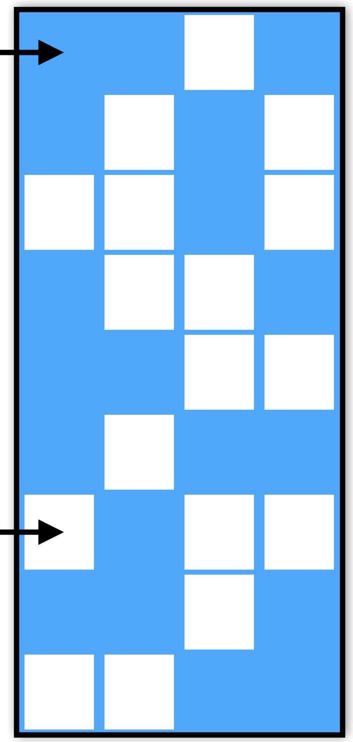
- [ ] Don’t know Locations
- [x] Know all values

LRMC

(Low-Rank Matrix Completion)

LR Matrix

Tons of Missing Entries



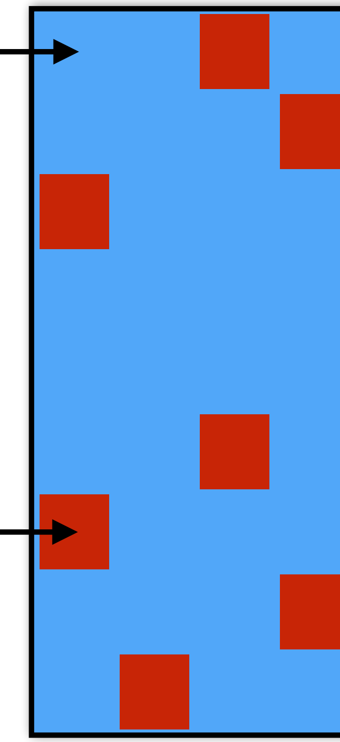
- [x] Know Locations
- [ ] Don’t know values

RPCA

(Robust Principal Component Analysis)

LR Matrix

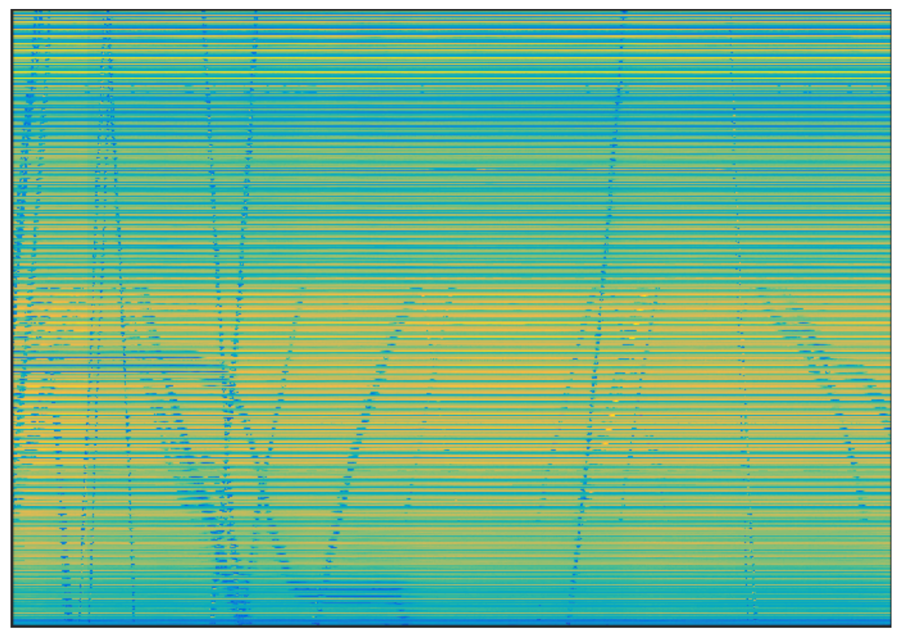
Few Gross Errors



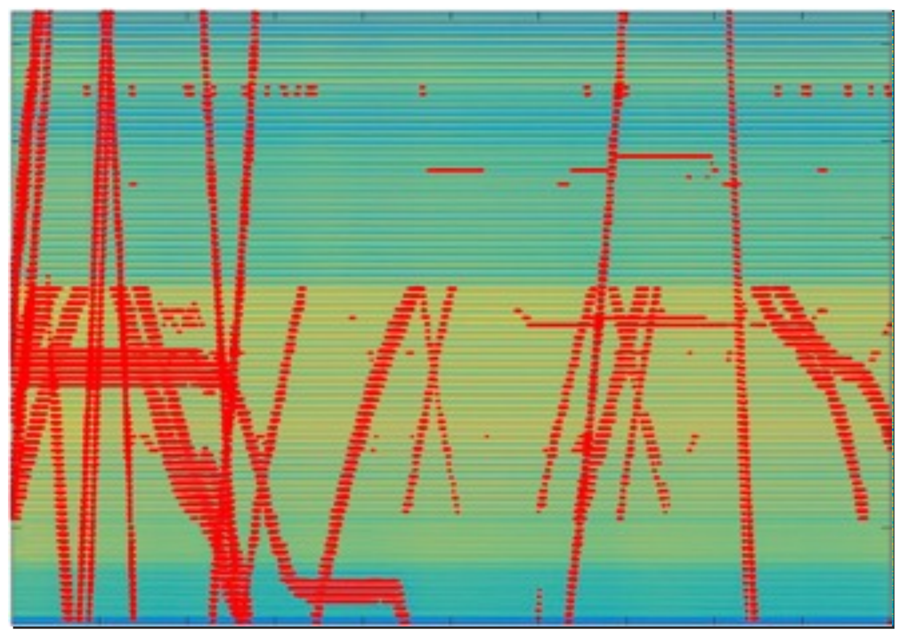
- [ ] Don’t know Locations
- [x] Know all values

Common goal: find the Subspace

Background segmentation



Background segmentation



Background segmentation

Existing Approaches

$$\text{minimize} \quad \|\mathbf{L}\|_* + \lambda\|\mathbf{S}\|_1$$
$$\text{subject to} \quad \mathbf{X} = \mathbf{L} + \mathbf{S}$$

[1] F. De La Torre and M. Black, *A framework for robust subspace learning*, International Journal of Computer Vision, 2003.

[2] Q. Ke and T. Kanade, *Robust L_1 norm factorization in the presence of outliers and missing data by alternative convex programming*, IEEE Conference on Computer Vision and Pattern Recognition, 2005.

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[6] V. Chandrasekaran, S. Sanghavi, P. Parrilo and A. Willsky, *Rank-sparsity incoherence for matrix decomposition*, SIAM Journal on Optimization, 2011.

[7] L. Mackey, A. Talwalkar and M. Jordan, *Divide-and-conquer matrix factorization*, Advances in Neural Information Processing Systems, 2011.

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[9] T. Bouwmans and E. Zahzah, *Robust PCA via principal component pursuit: a review for a comparative evaluation in video surveillance*, Computer Vision and Image Understanding, 2014.

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[11] Z. Lin, R. Liu and Z. Su, *Linearized alternating direction method with adaptive penalty for low rank representation*, Advances in Neural Information Processing Systems, 2011.

[12] X. Yuan and J. Yang, *Sparse and low-rank matrix decomposition via alternating direction methods*, 2009.

[13] Z. Lin, A. Ganesh, J. Wright, L. Wu, M. Chen and Y. Ma, *Fast convex optimization algorithms for exact recovery of a corrupted low-rank matrix*, Computational Advances in Multi-Sensor Adaptive Processing, 2009.

[14] Y. Shen, Z. Wen, and Y. Zhang. Augmented Lagrangian Alternating Direction Method for Matrix Separation based on Low-Rank Factorization, Optimization Methods and Software, 2011.

Existing Approaches

$$\begin{array}{ll} \text{minimize} & \|\mathbf{L}\|_* + \lambda \|\mathbf{S}\|_1 \\ \text{subject to} & \mathbf{X} = \mathbf{L} + \mathbf{S} \end{array}$$

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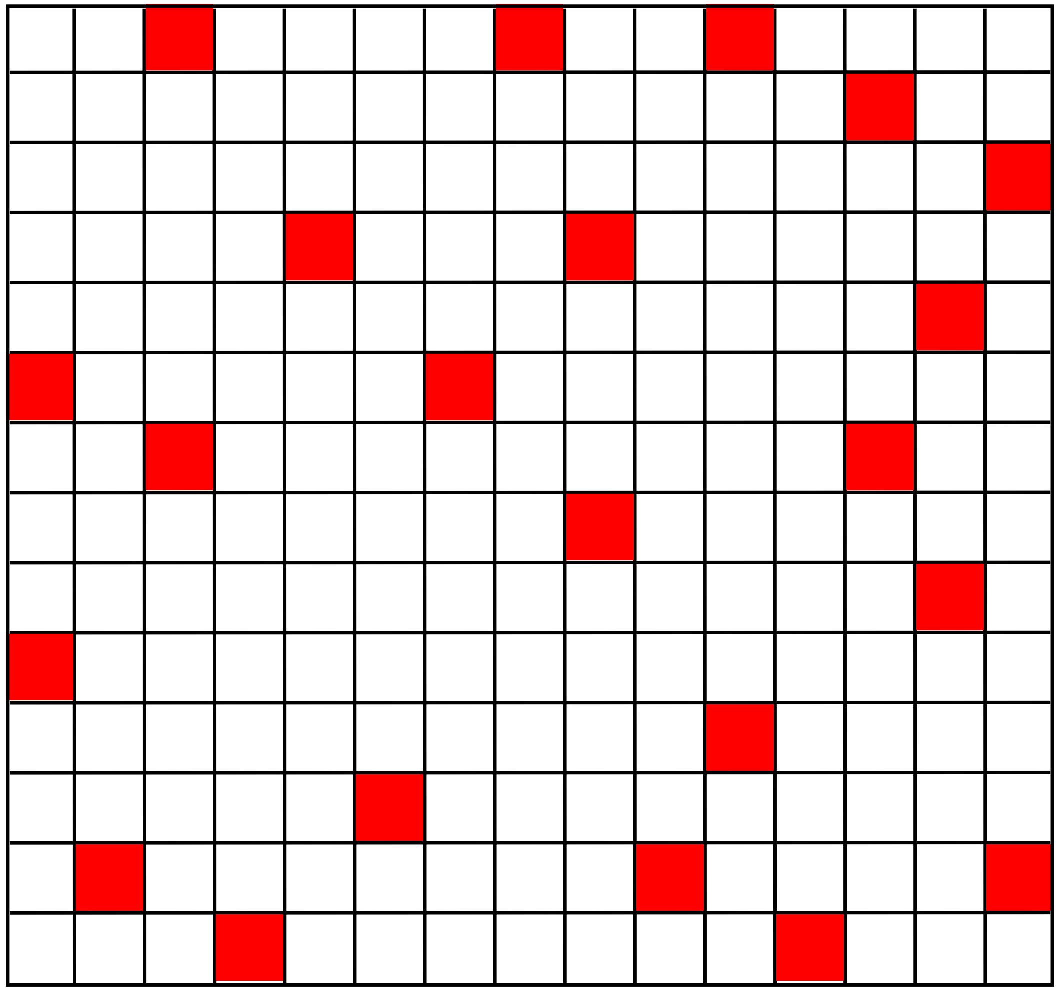
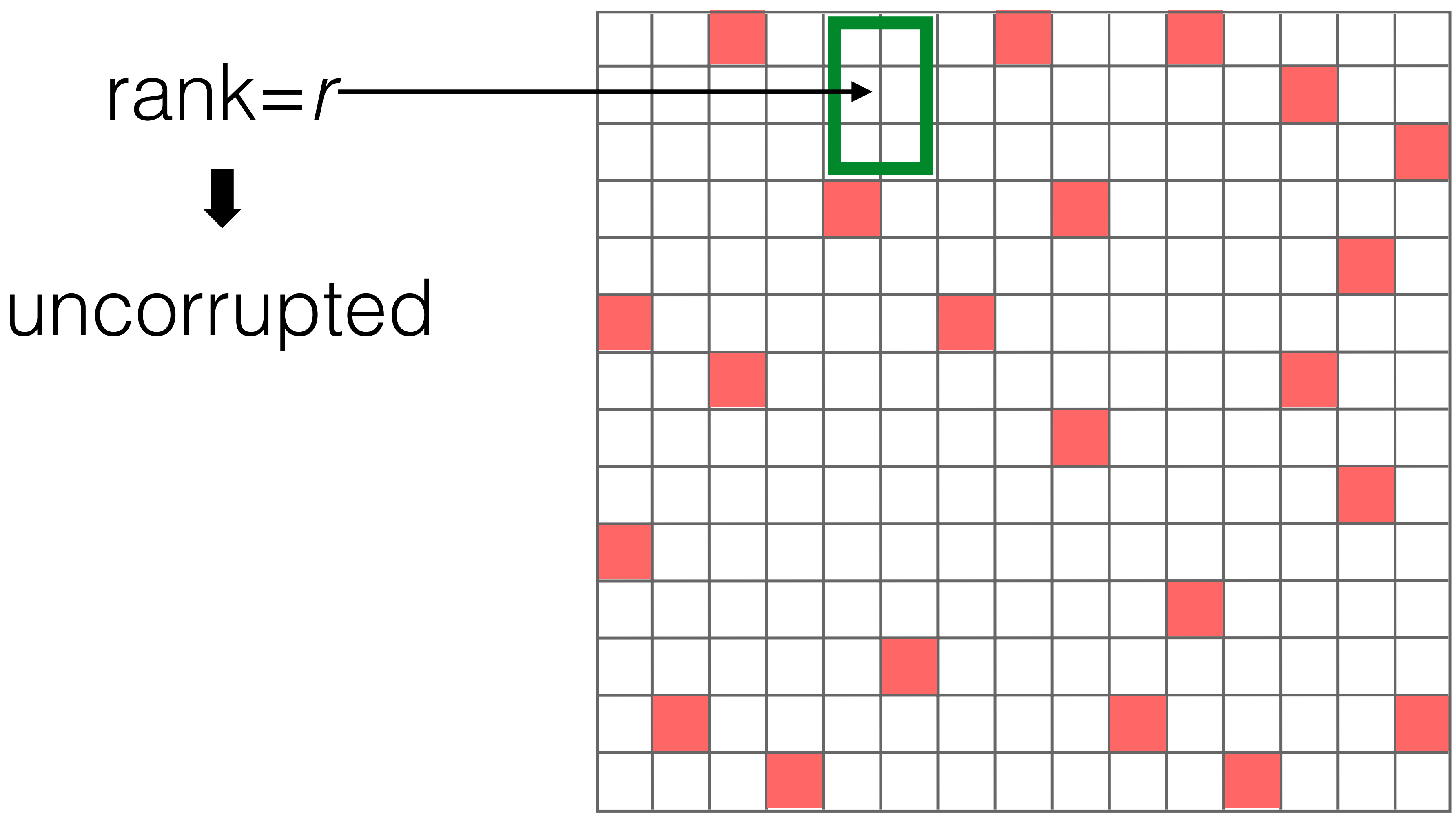
Using our Theory

Totally different way to think about the problem

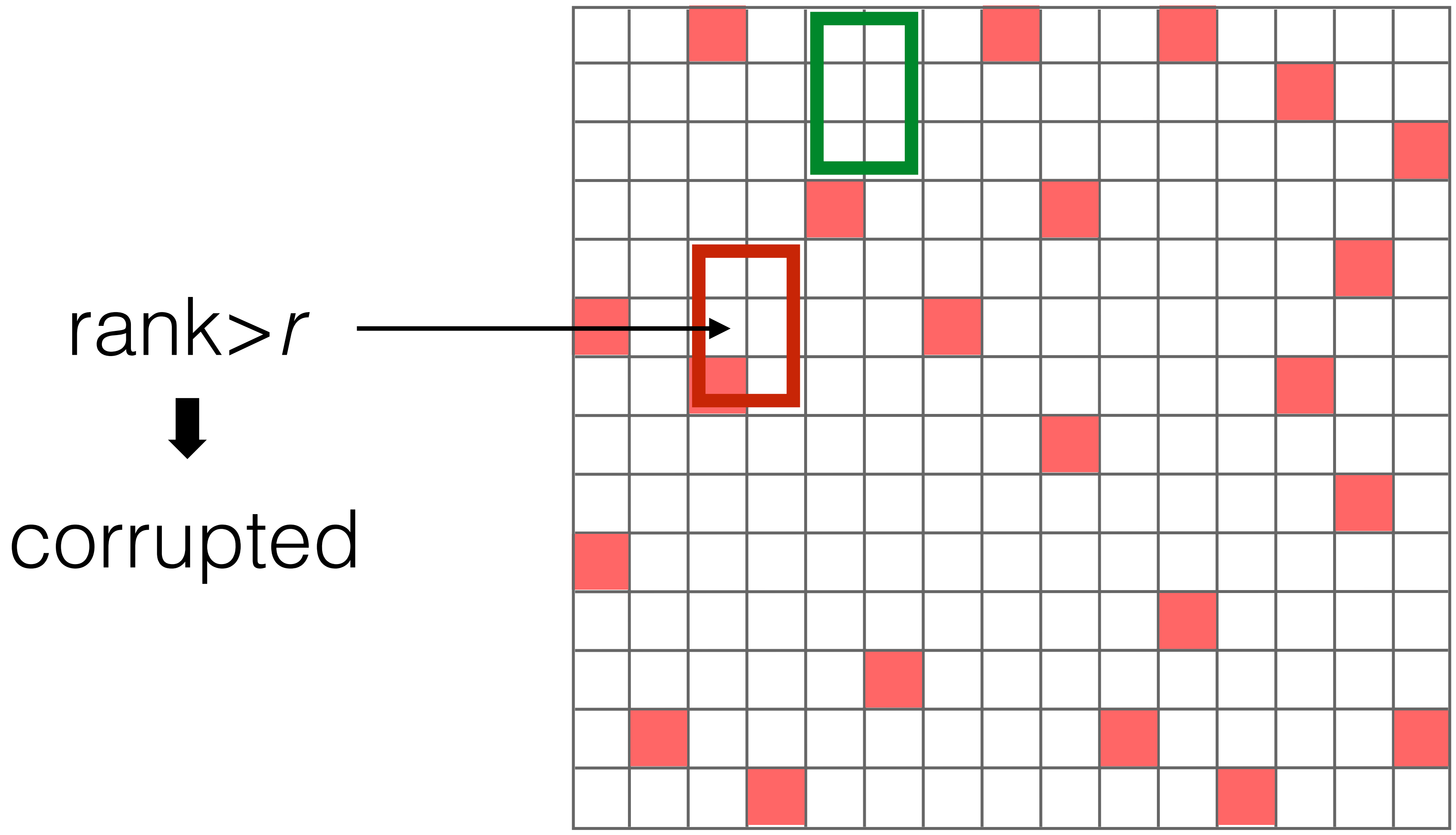
Use incomplete-data tricks



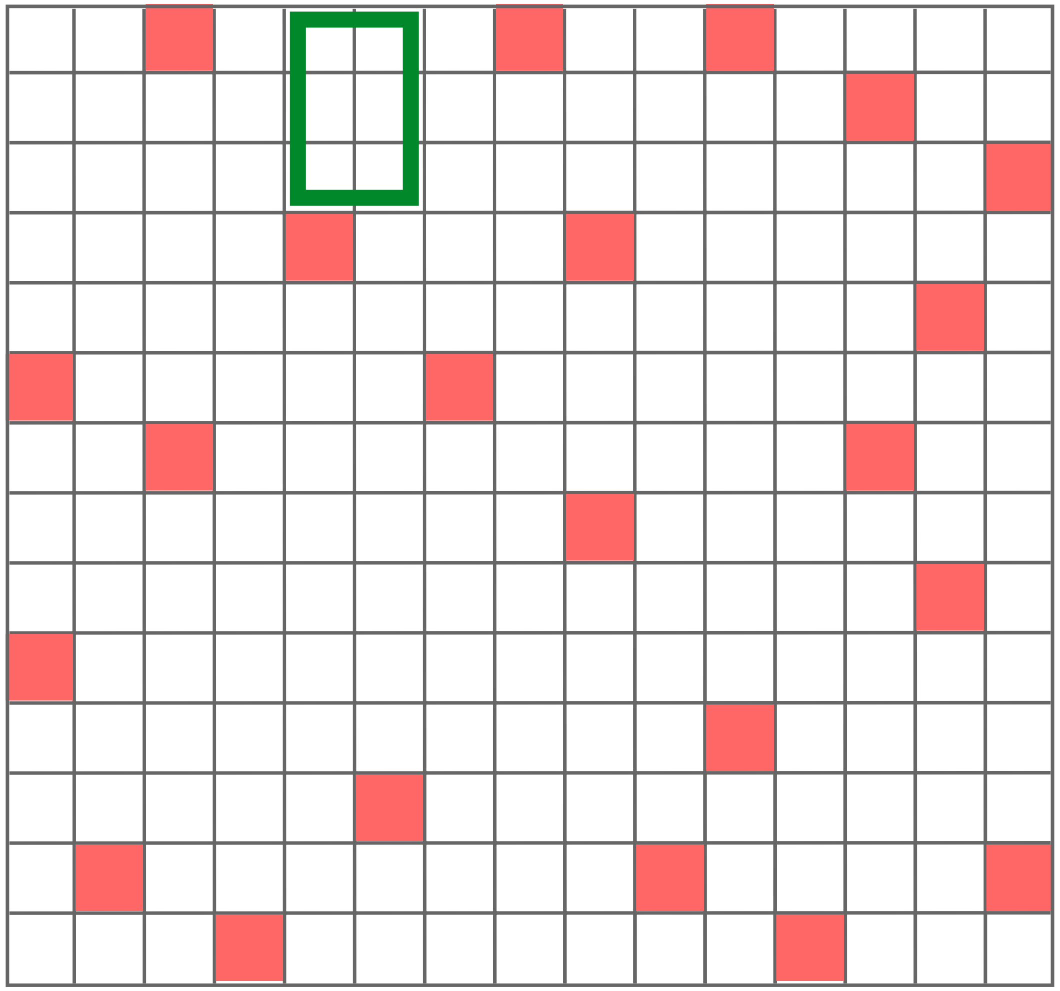
Use incomplete-data tricks



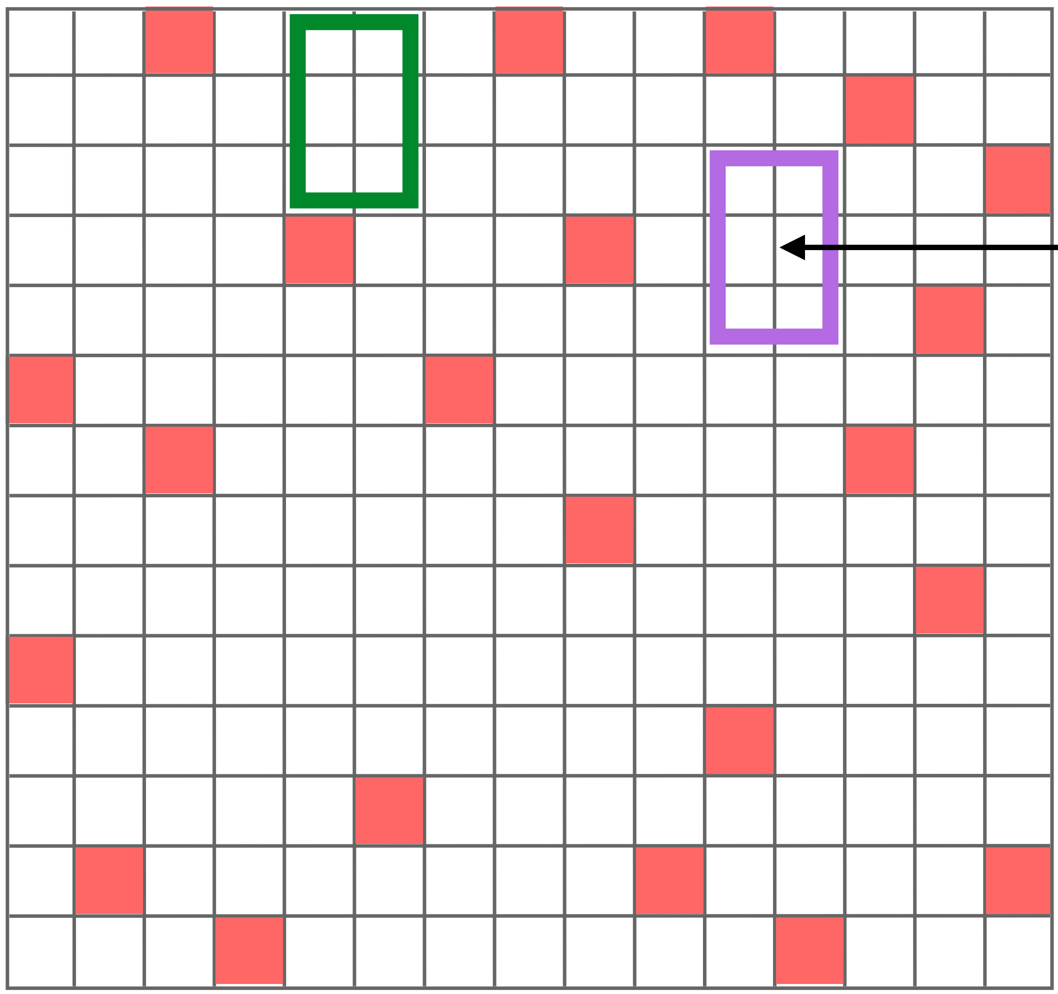
Use incomplete-data tricks



Use incomplete-data tricks

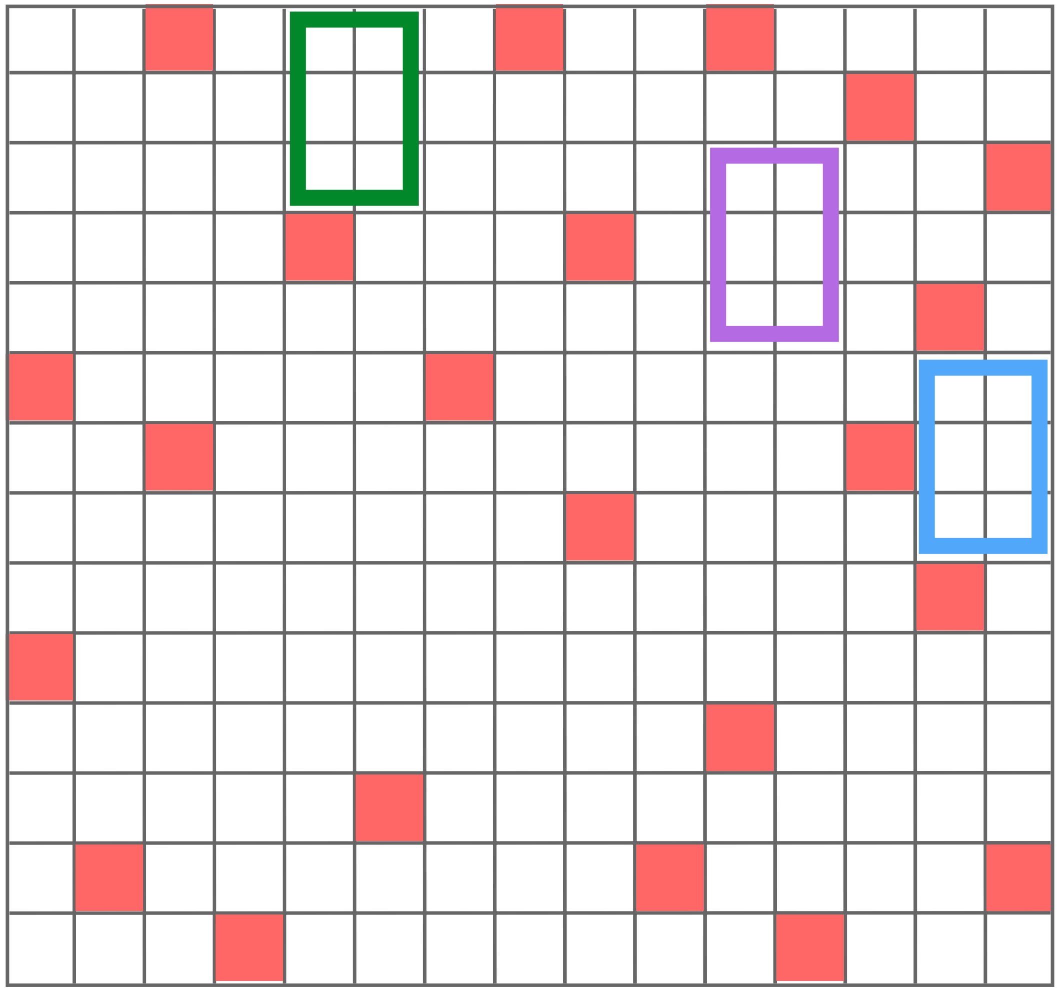


Use incomplete-data tricks



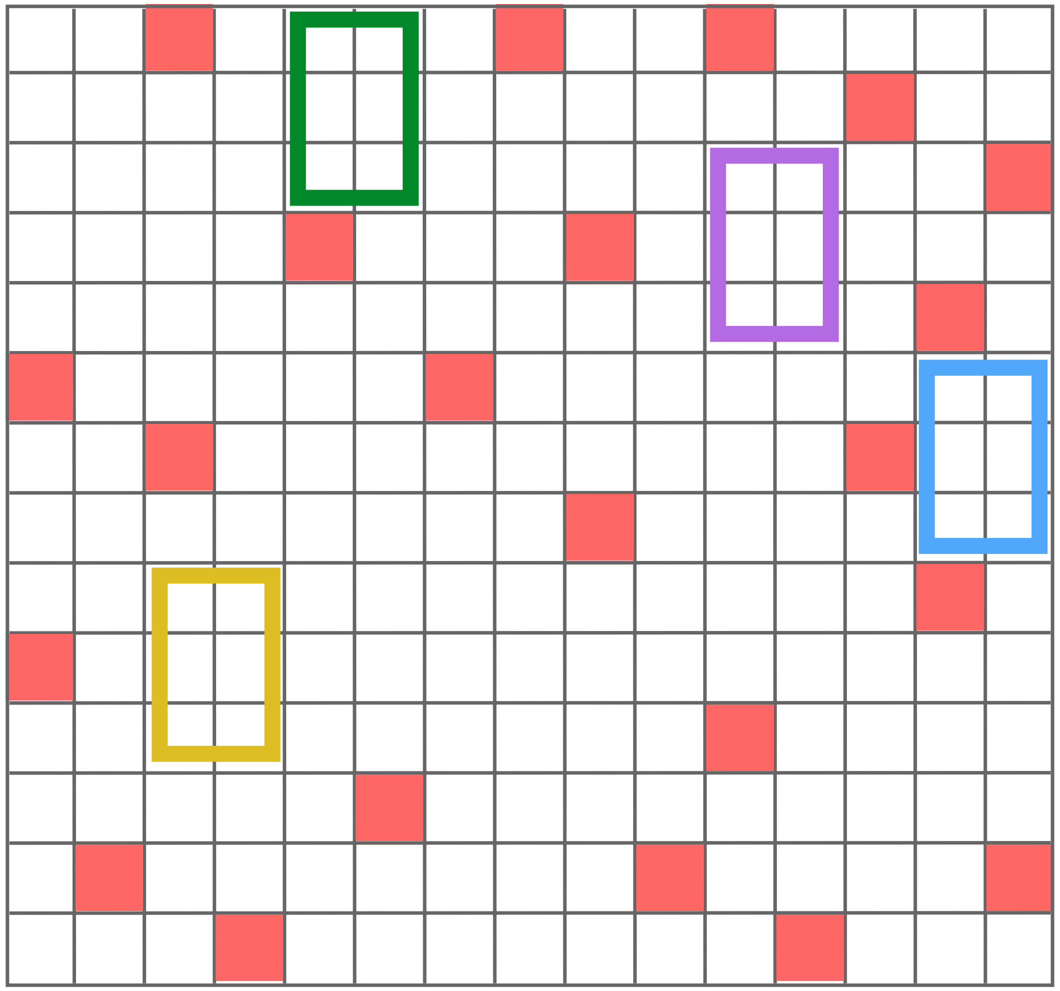
Keep finding uncorrupted pieces

Use incomplete-data tricks



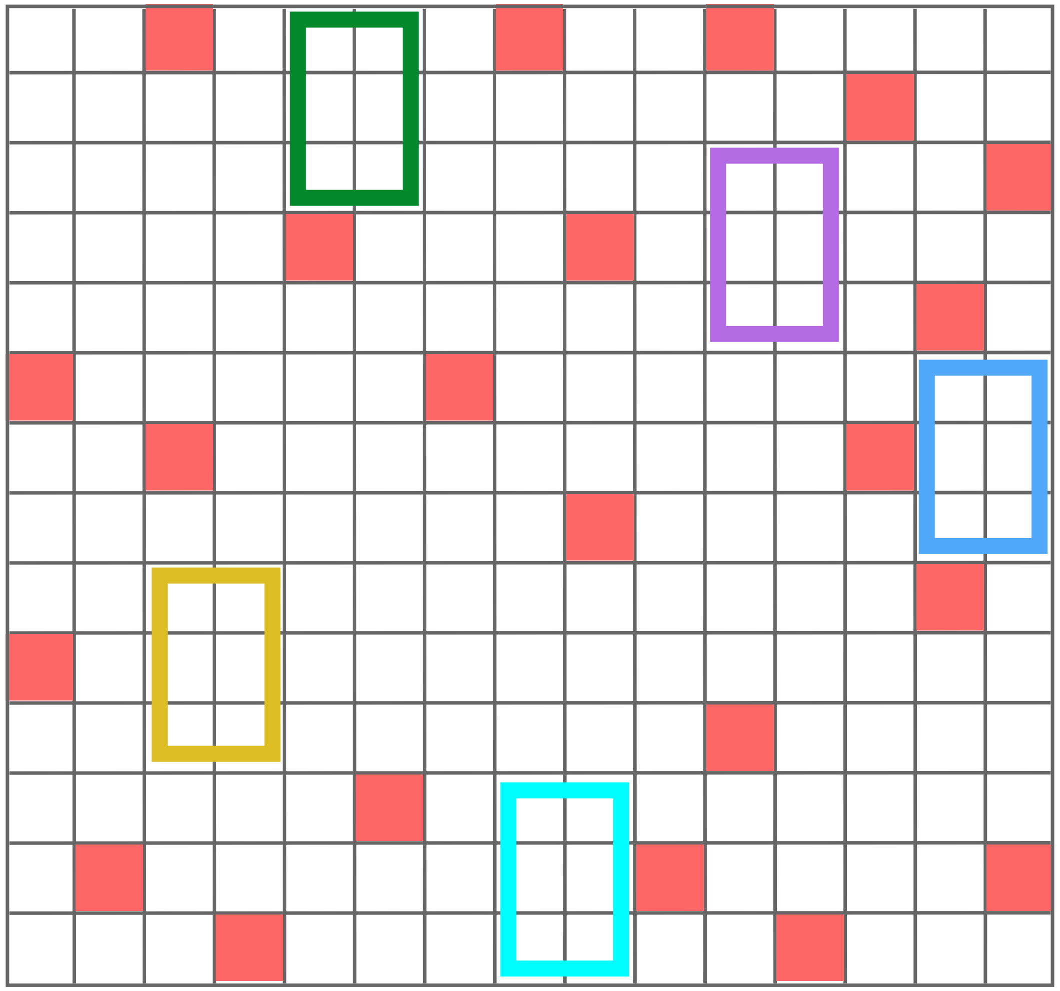
Keep finding uncorrupted pieces

Use incomplete-data tricks



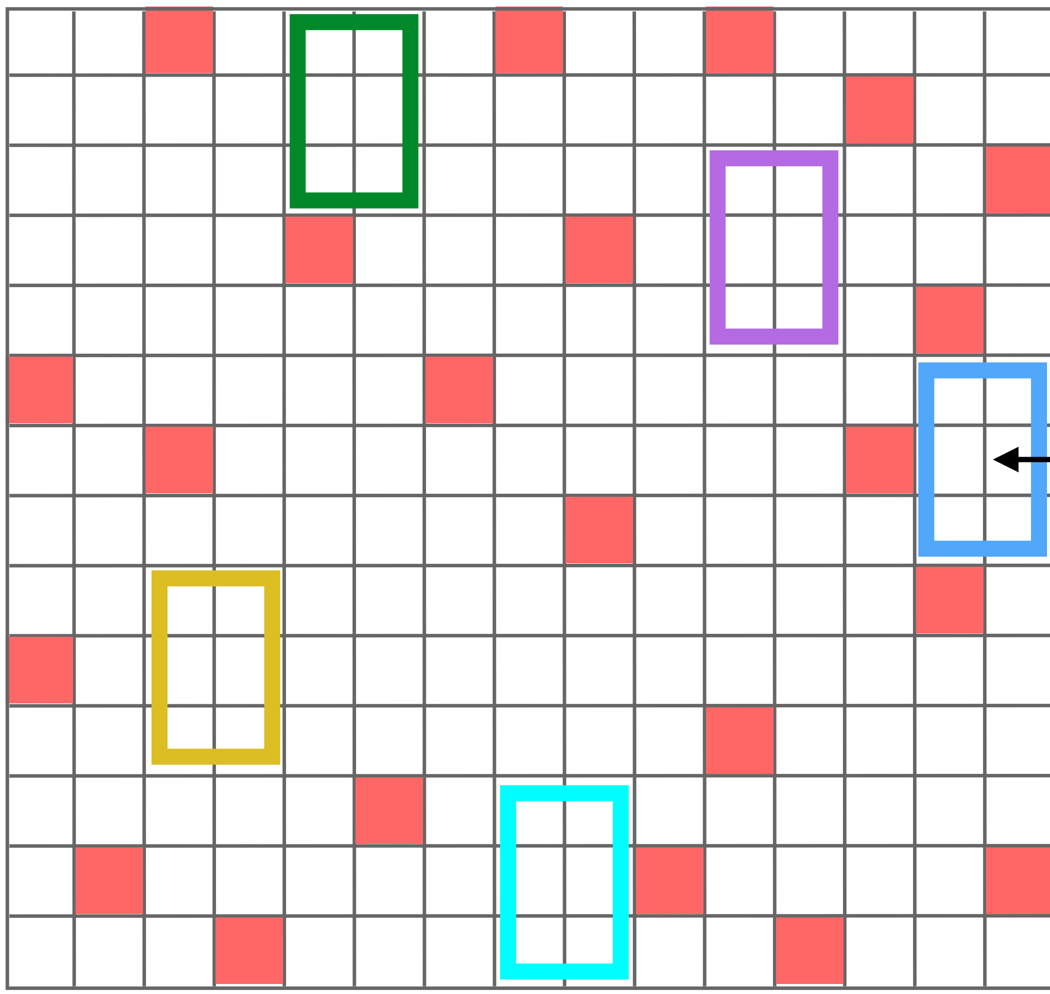
Keep finding uncorrupted pieces

Use incomplete-data tricks



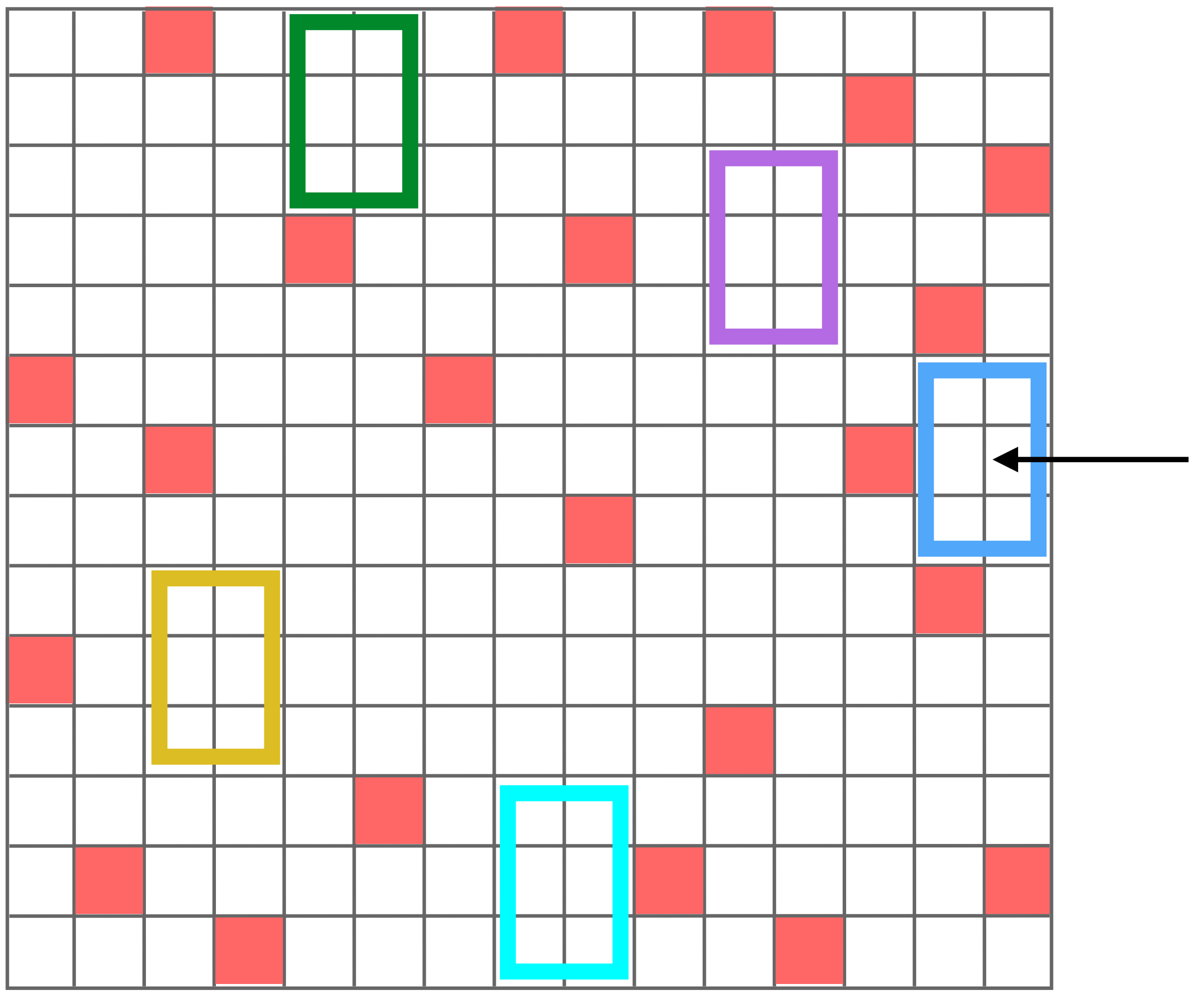
Keep finding uncorrupted pieces

Use incomplete-data tricks



Each piece gives you a Projection!

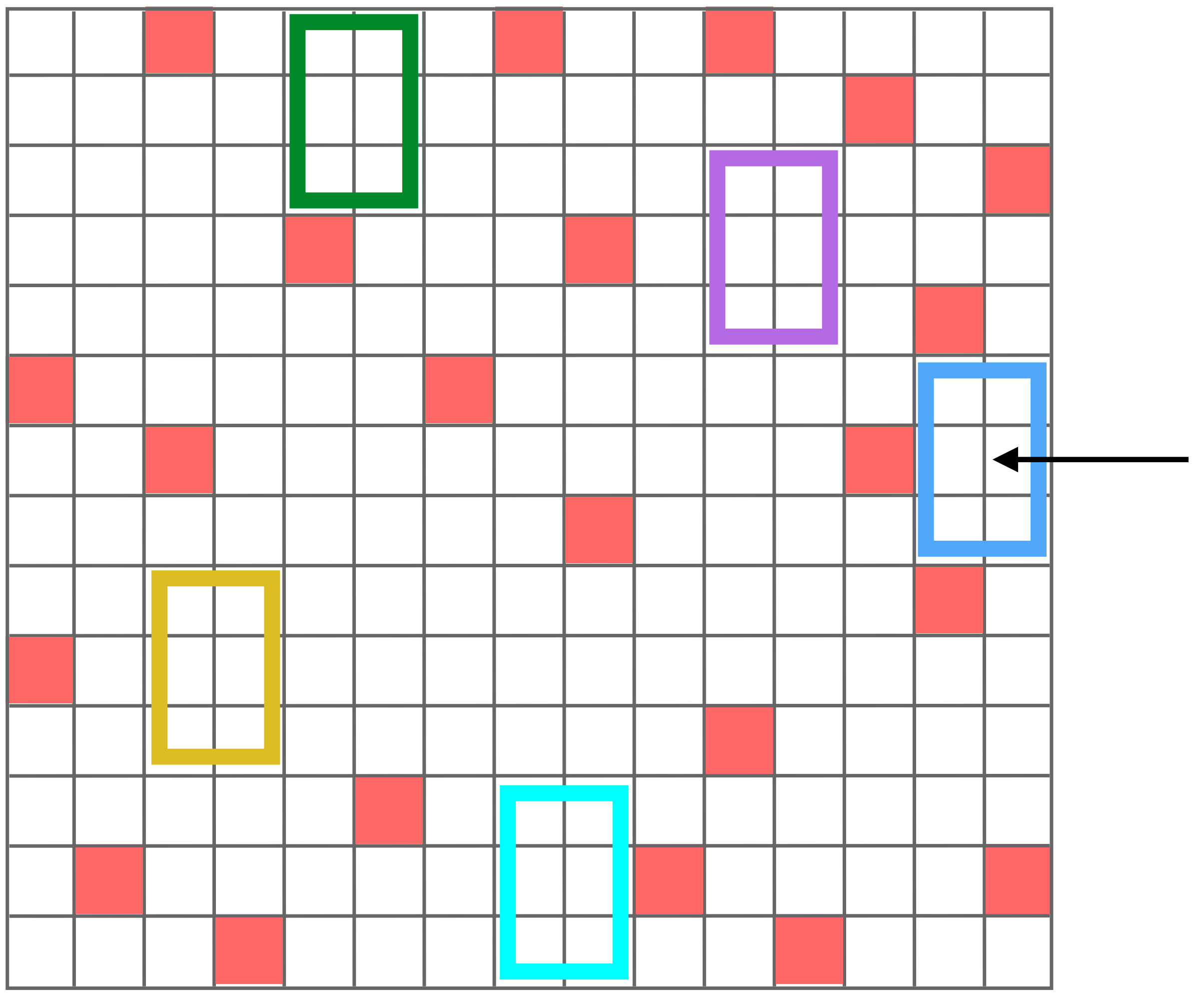
Use incomplete-data tricks



Each piece gives you a Projection!

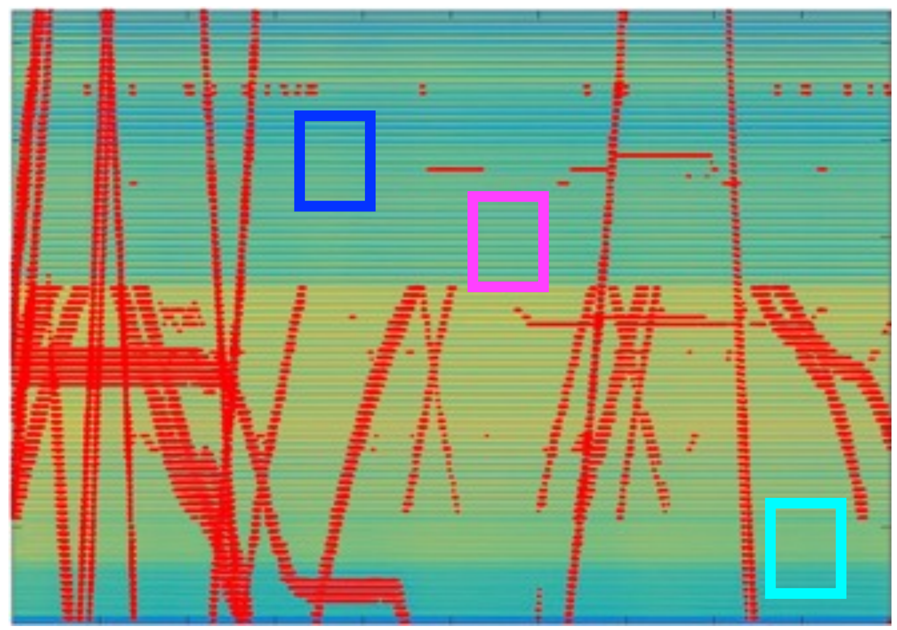
If pieces are *observed in the right places*, we can find the subspace

Use incomplete-data tricks



Each piece gives you a Projection!

If pieces are *observed in the right places*, we can find the subspace efficiently



Background segmentation

Original Frame | Our Work | RPCA-ALM

[5] Pimentel et. al (2017) | RPCA-ALM (Lin et. al, 2011-2016)

In many cases, similar results

Original Frame

Our Work

[5] Pimentel et. al (2017)

RPCA-ALM

RPCA-ALM (Lin et. al, 2011-2016)

In other cases, better

Original Frame

Our Work

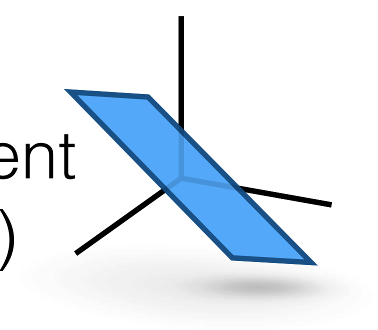
[5] Pimentel et. al (2017)

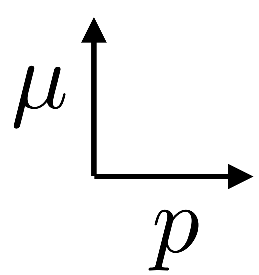
RPCA-ALM

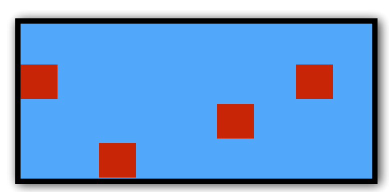
RPCA-ALM (Lin et. al, 2011-2016)

In other cases, better

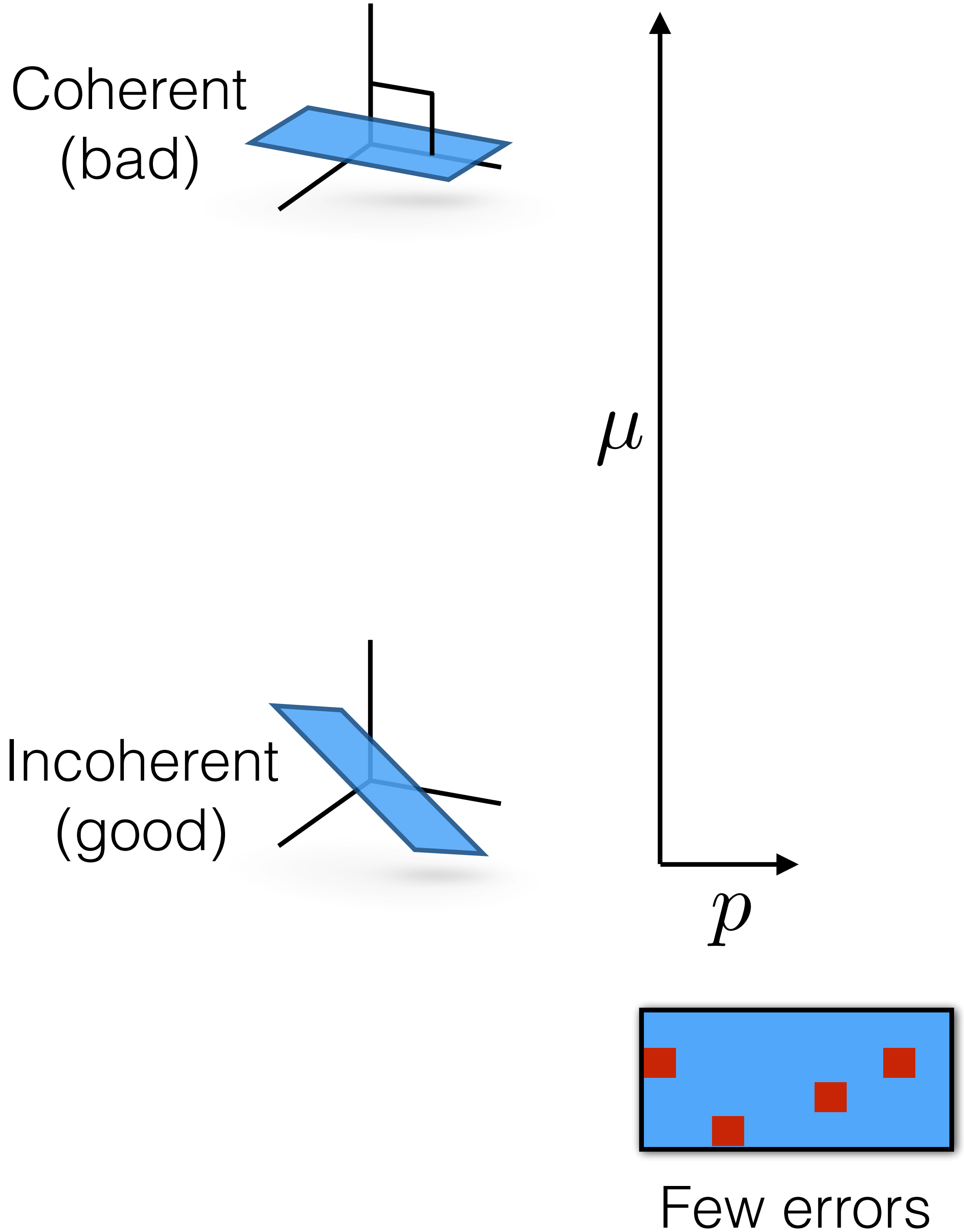
Incoherent
(good)



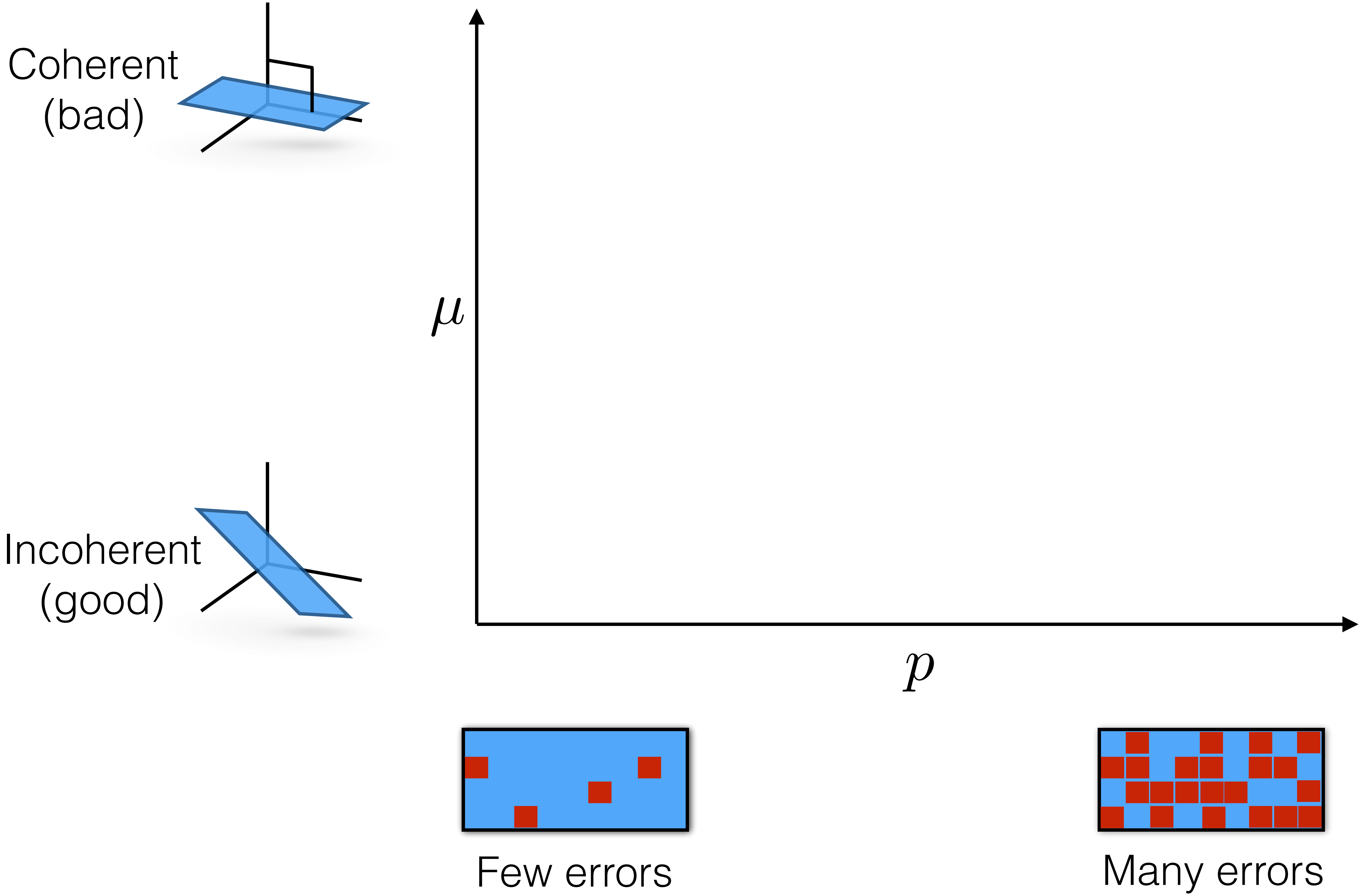




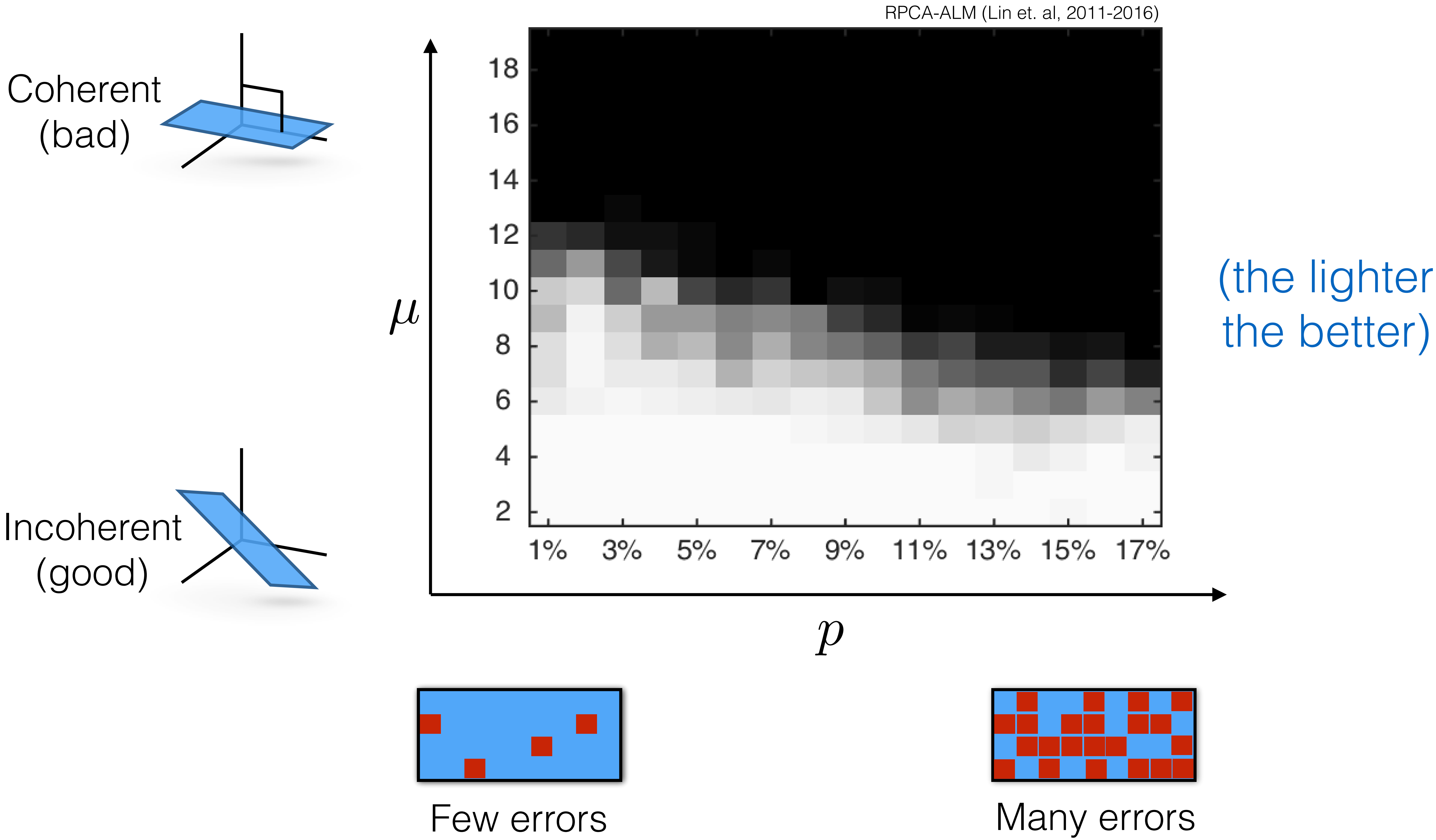
Performance Analysis



Performance Analysis

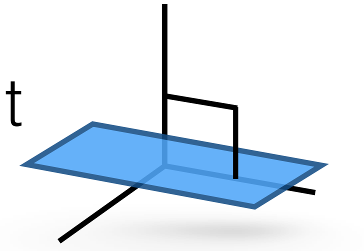


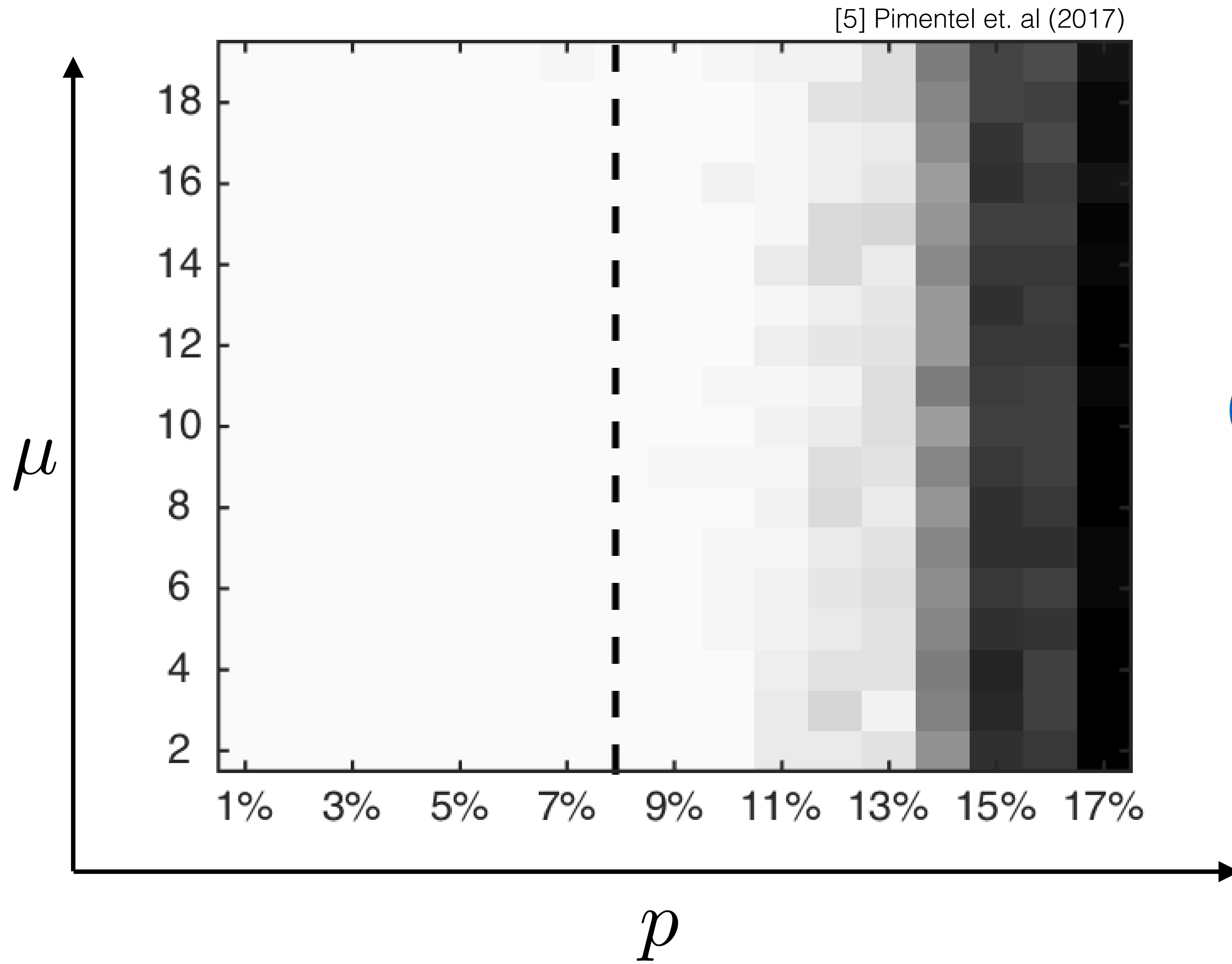
Performance Analysis



Performance Analysis

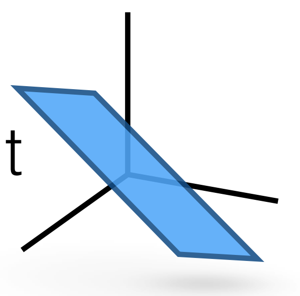
Coherent
(bad)

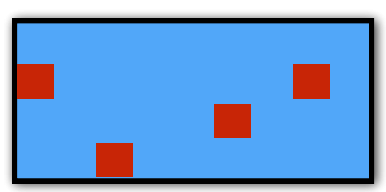




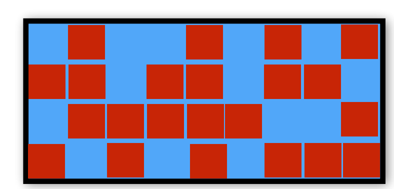
(the lighter
the better)

Incoherent
(good)





Few errors

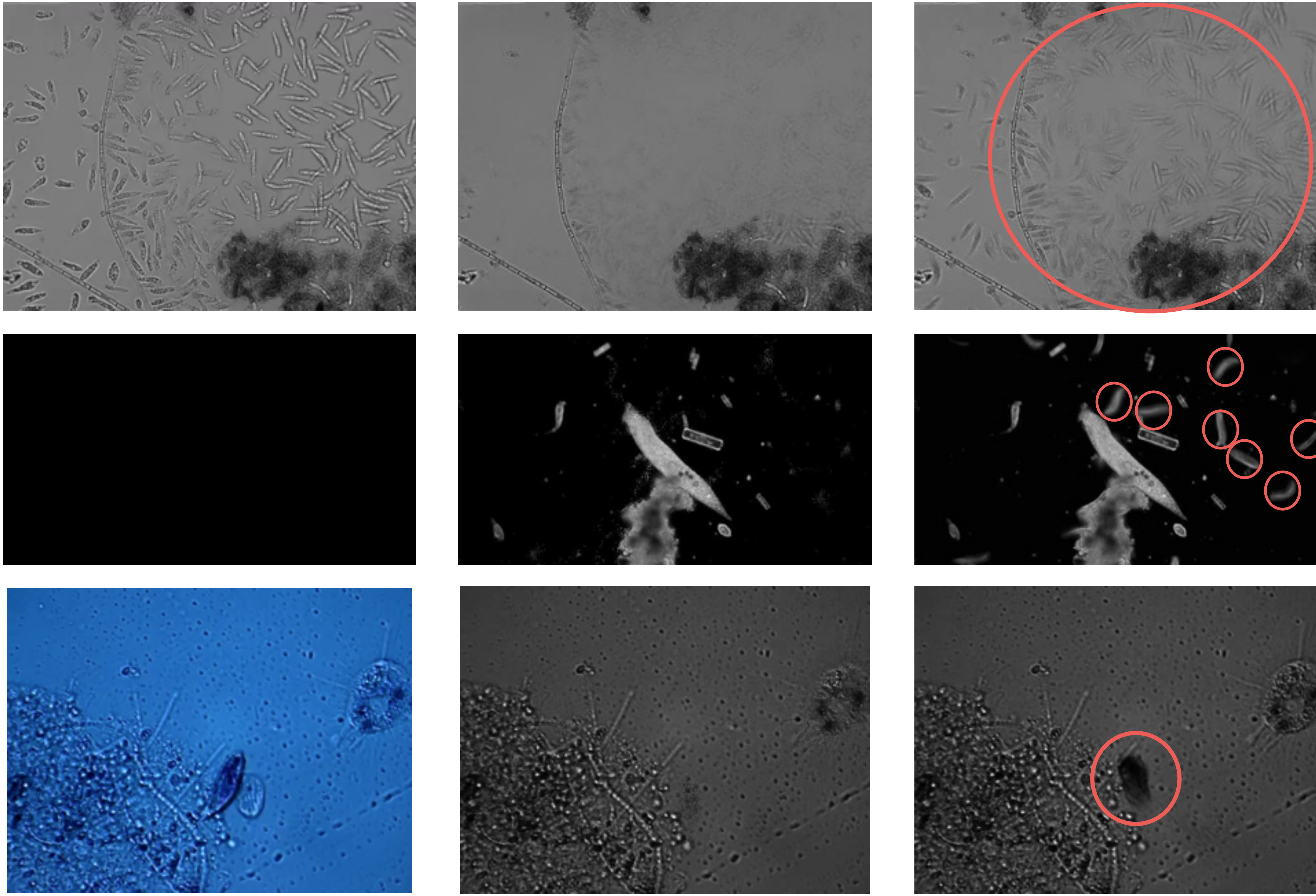


Many errors

Performance Analysis

Original Video | Our Work | RPCA-ALM

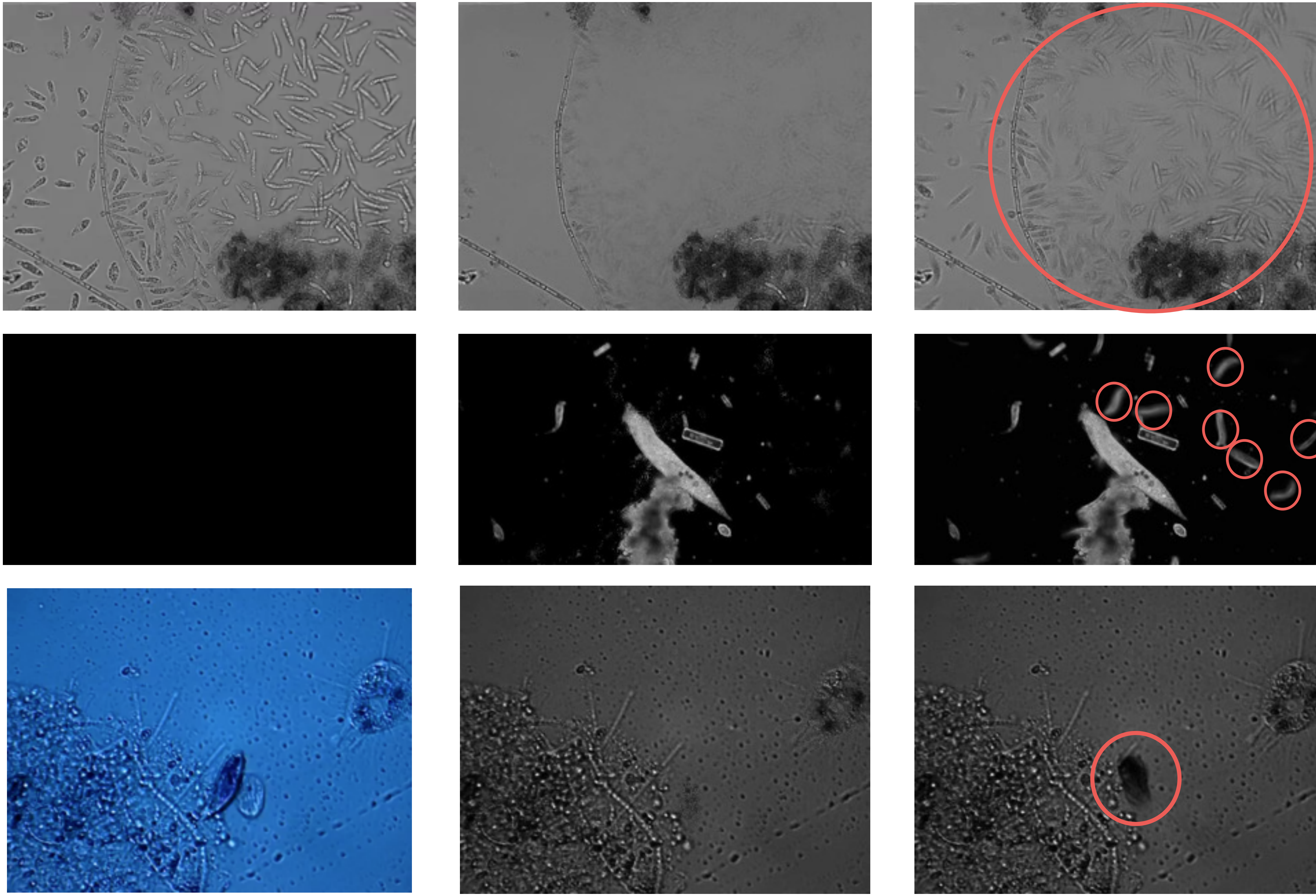
[5] Pimentel et. al (2017) | RPCA-ALM (Lin et. al, 2011-2016)

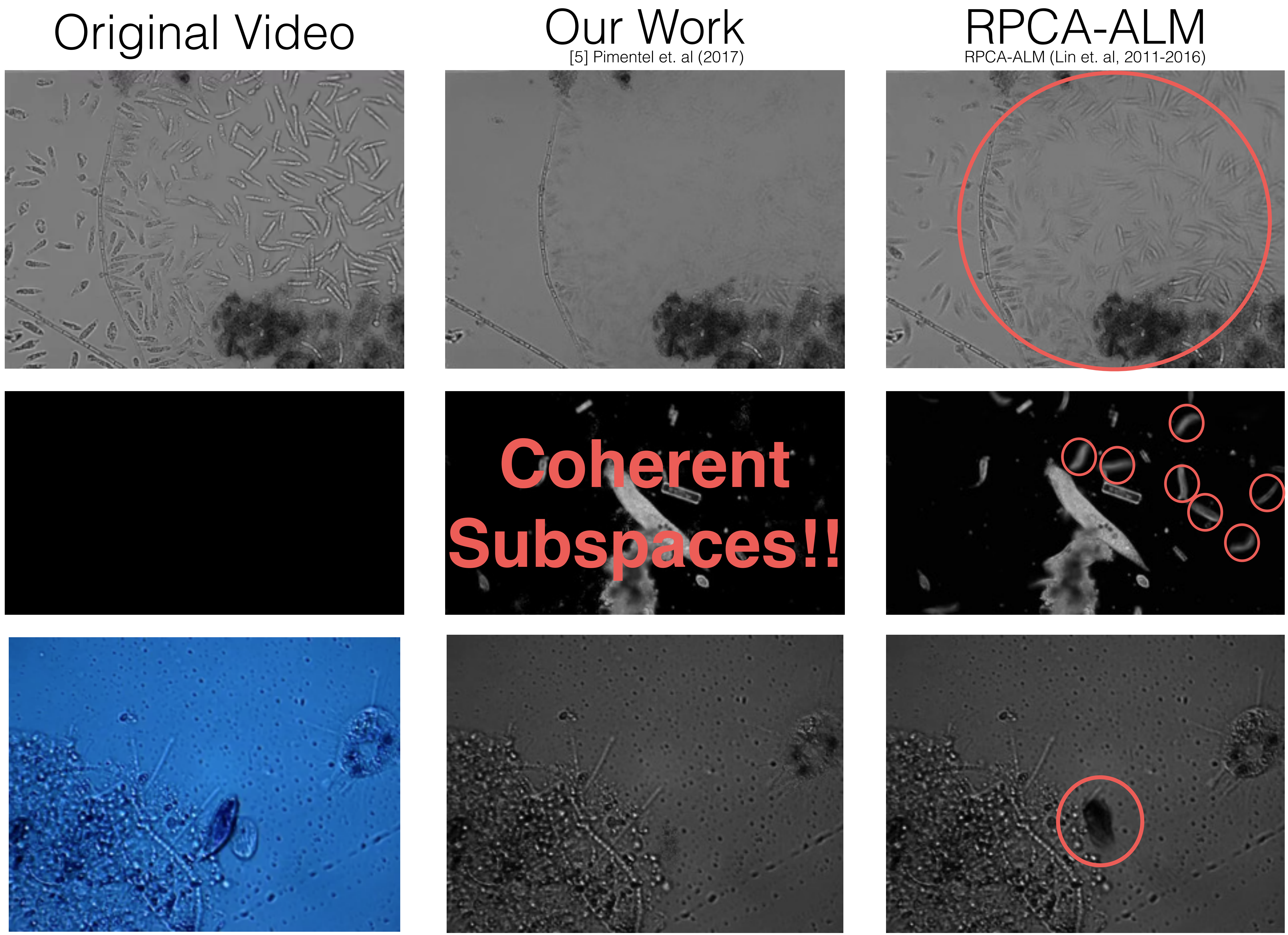


Original Video | Our Work | RPCA-ALM

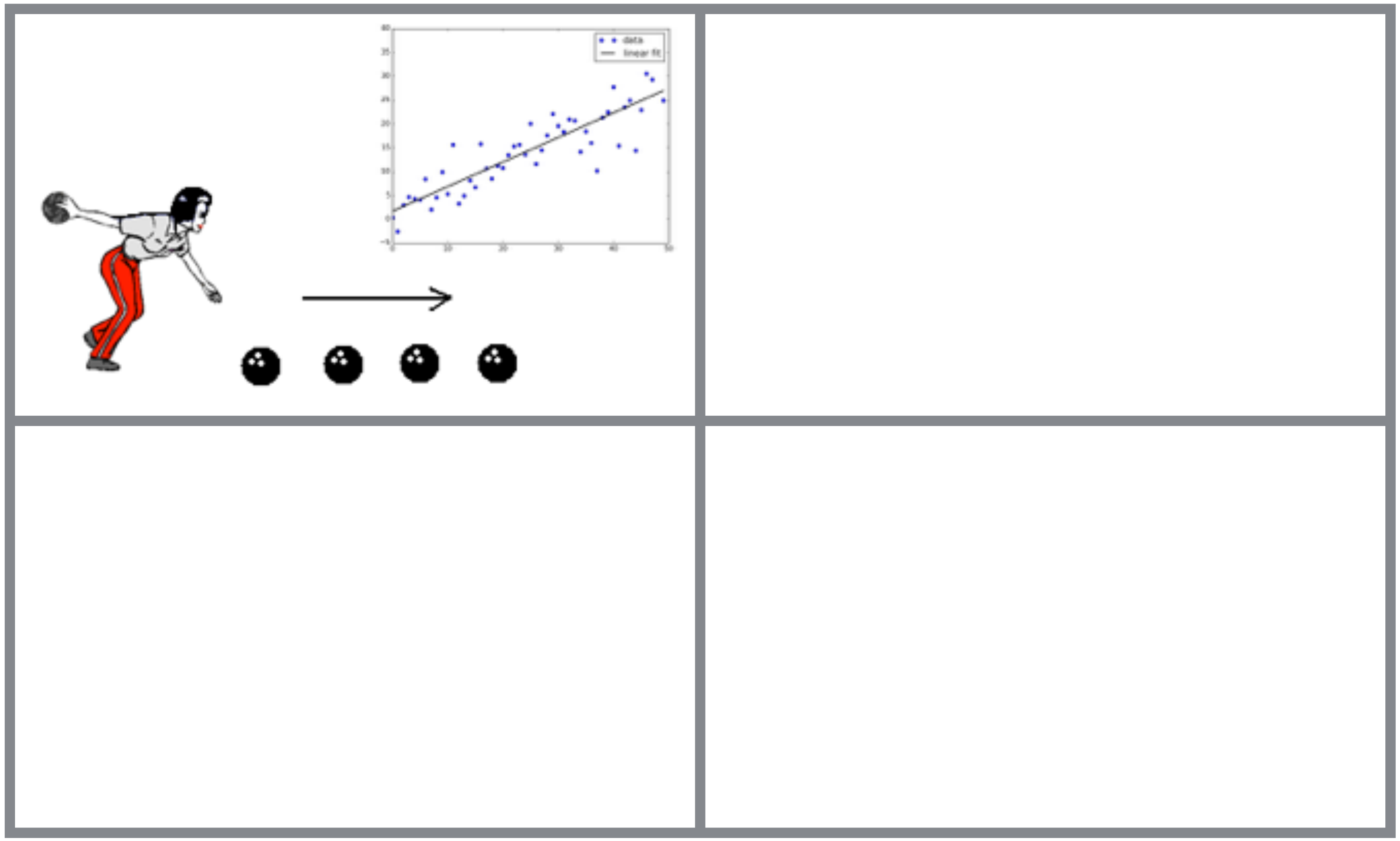
[5] Pimentel et. al (2017)

RPCA-ALM (Lin et. al, 2011-2016)

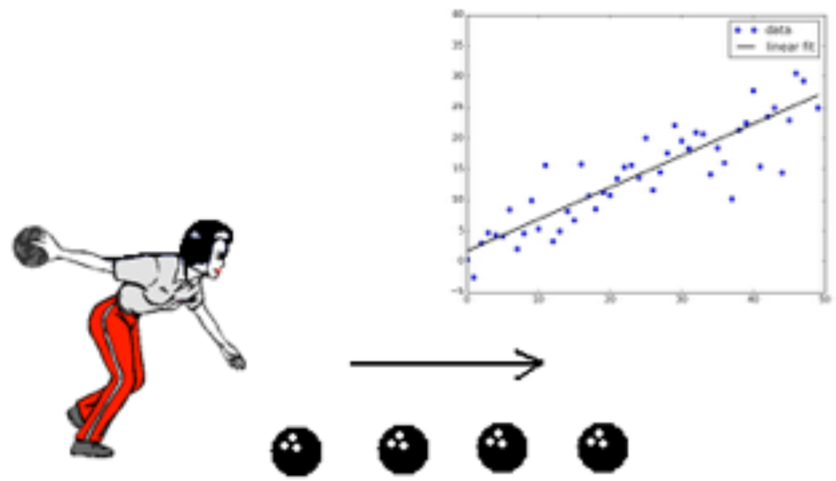
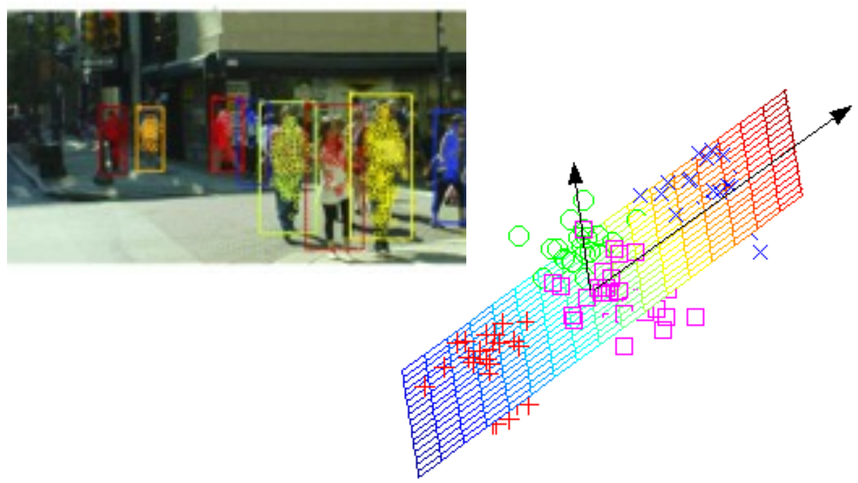



Original Video
Our Work
[5] Pimentel et. al (2017)
RPCA-ALM
RPCA-ALM (Lin et. al, 2011-2016)
Coherent
Subspaces!!

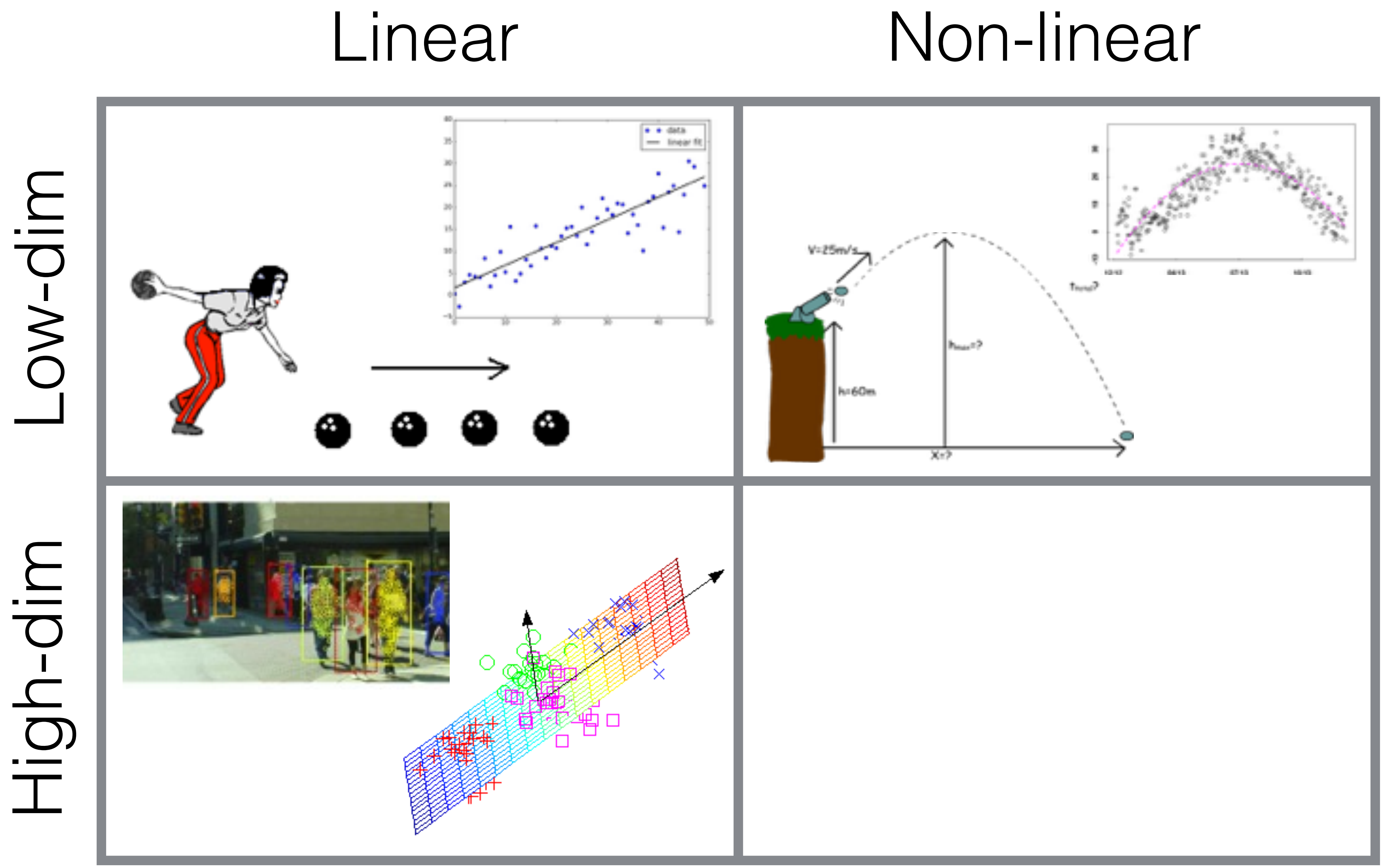
To Sum up…



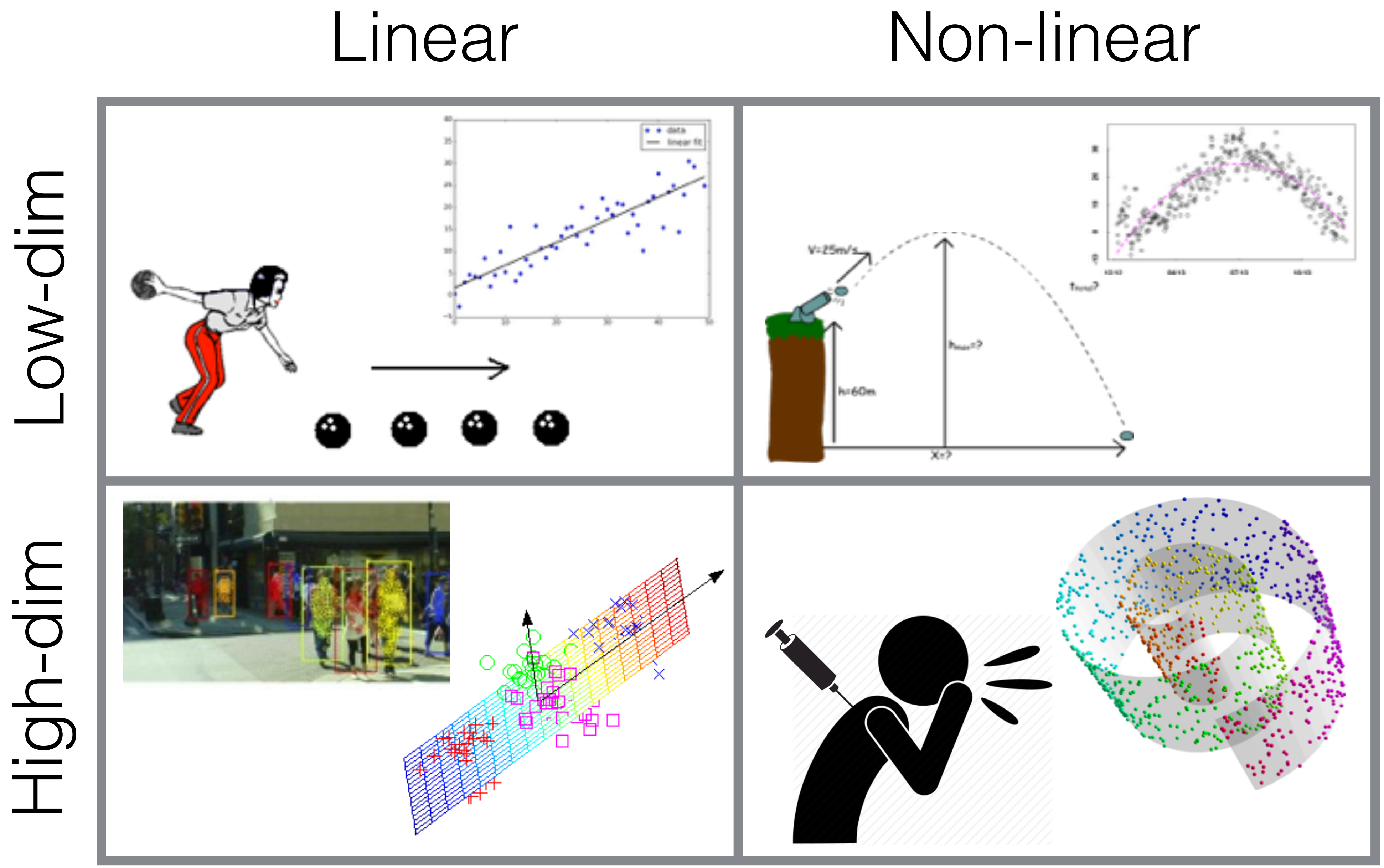
To Sum up…

Low-dim		
High-dim		

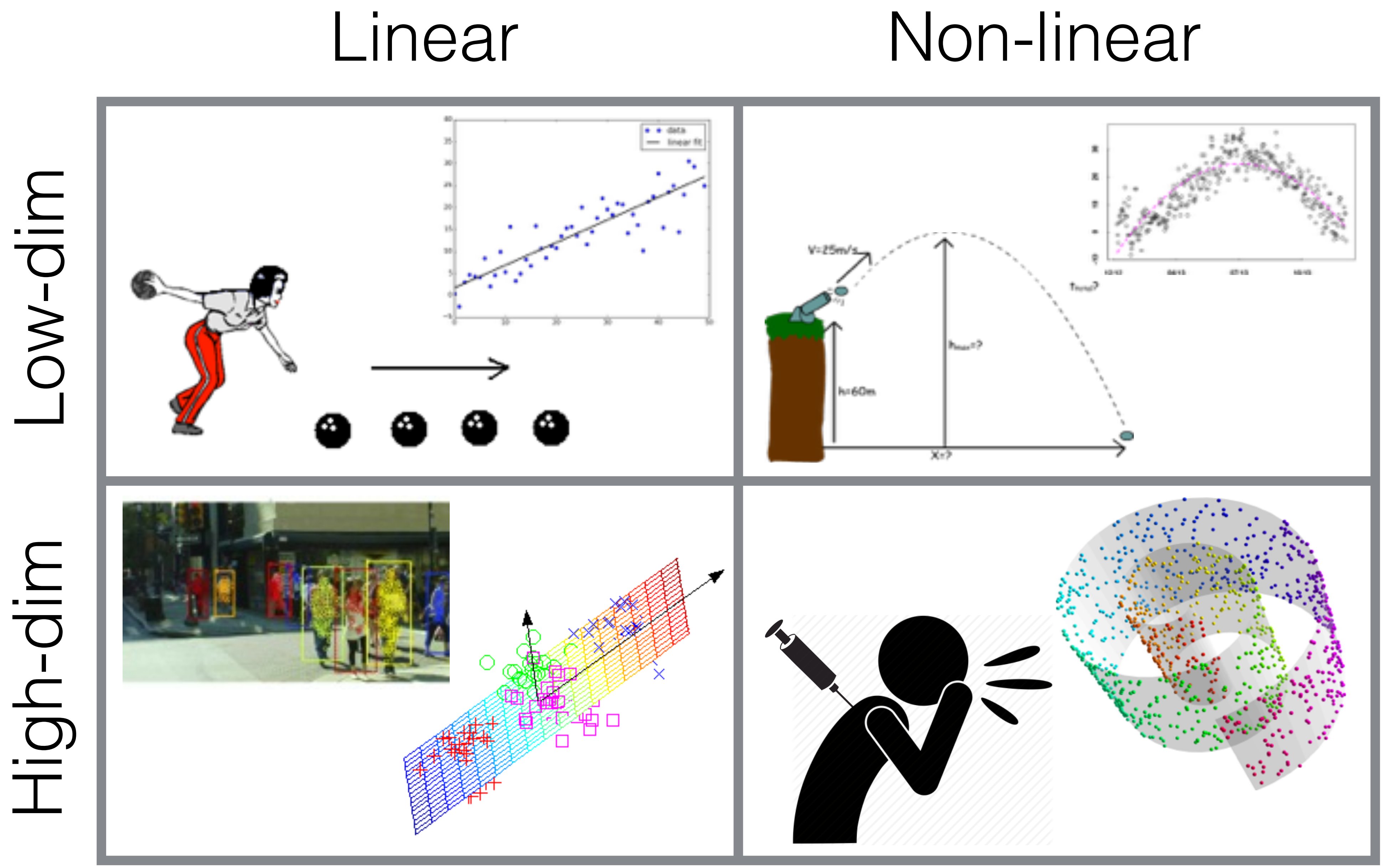
To Sum up…



To Sum up…



To Sum up…



- Tensors
- Projections
- Deterministic Samplings
- Algorithm
- Applications

To Sum up…

Nigel Boston

Rob Nowak

Steve Wright

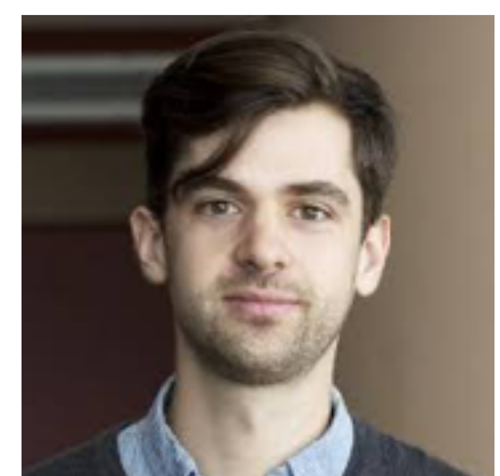
Joint work with:

Becca Willett

Laura Balzano

Roummel Marcia

Claudia Solís



Greg Ongie

Thank you

pimentel@gsu.edu

[1] **Low Algebraic Dimension Matrix Completion**

[2] **Deterministic Conditions for Subspace Identifiability from Incomplete Sampling**

[3] **A Characterization of Deterministic Sampling Patterns for Low-Rank Matrix Completion**

[4] **The Information-Theoretic Requirements of Subspace Clustering with Missing Data**

[5] **Random Consensus Robust PCA**

https://danielpimentel.github.io/publications