

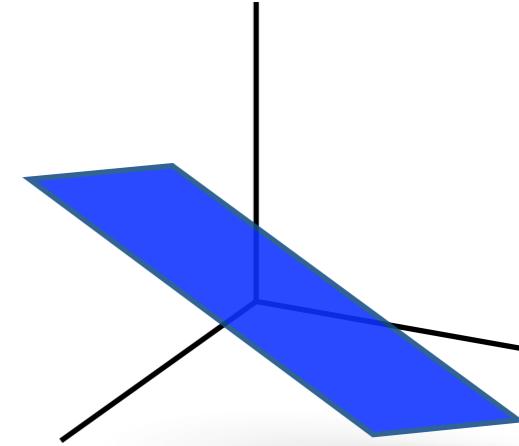
Selective Erasures for High-Dimensional Robust Subspace Tracking

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Georgia State University
Computer Science

SPIE 2018

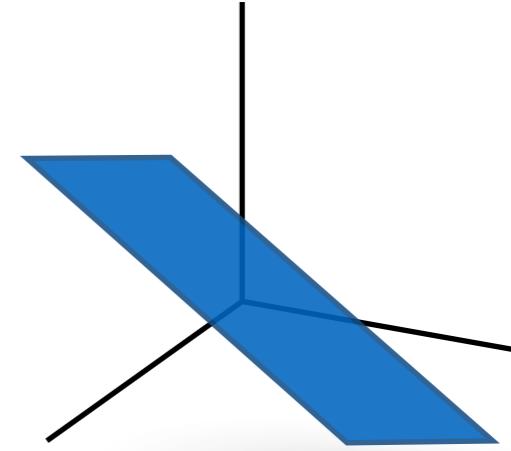
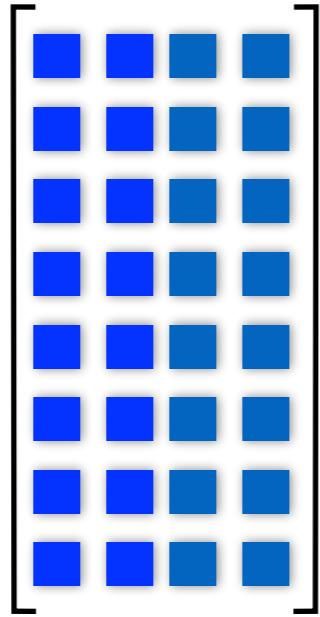
$$\begin{bmatrix} \text{blue square} & \text{blue square} \\ \text{blue square} & \text{blue square} \end{bmatrix}$$



L

Low-Rank

Subspace Tracking

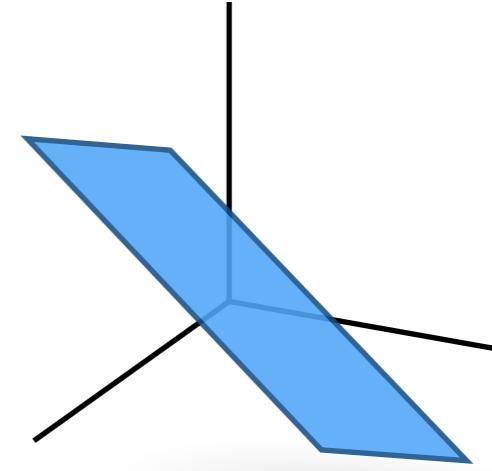


L

Low-Rank

Subspace Tracking

$$\begin{bmatrix} \text{blue} & \text{blue} & \text{dark blue} & \text{dark blue} & \text{light blue} & \text{light blue} \\ \text{blue} & \text{blue} & \text{dark blue} & \text{dark blue} & \text{light blue} & \text{light blue} \\ \text{blue} & \text{blue} & \text{dark blue} & \text{dark blue} & \text{light blue} & \text{light blue} \\ \text{blue} & \text{blue} & \text{dark blue} & \text{dark blue} & \text{light blue} & \text{light blue} \\ \text{blue} & \text{blue} & \text{dark blue} & \text{dark blue} & \text{light blue} & \text{light blue} \\ \text{blue} & \text{blue} & \text{dark blue} & \text{dark blue} & \text{light blue} & \text{light blue} \end{bmatrix}$$

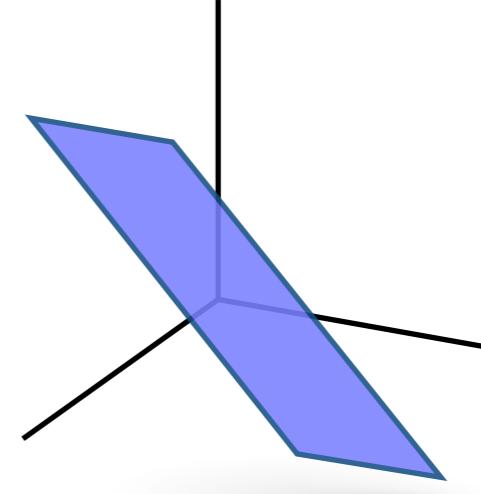


L

Low-Rank

Subspace Tracking

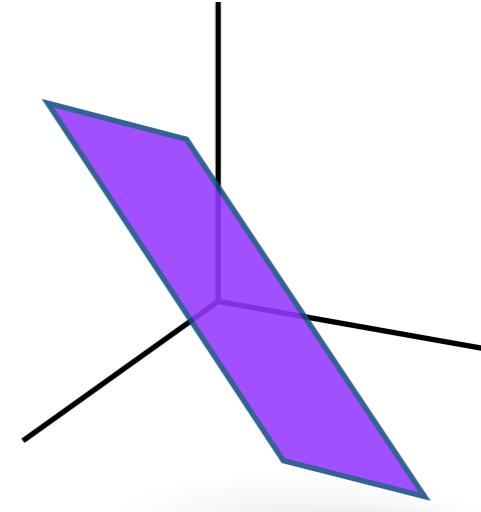
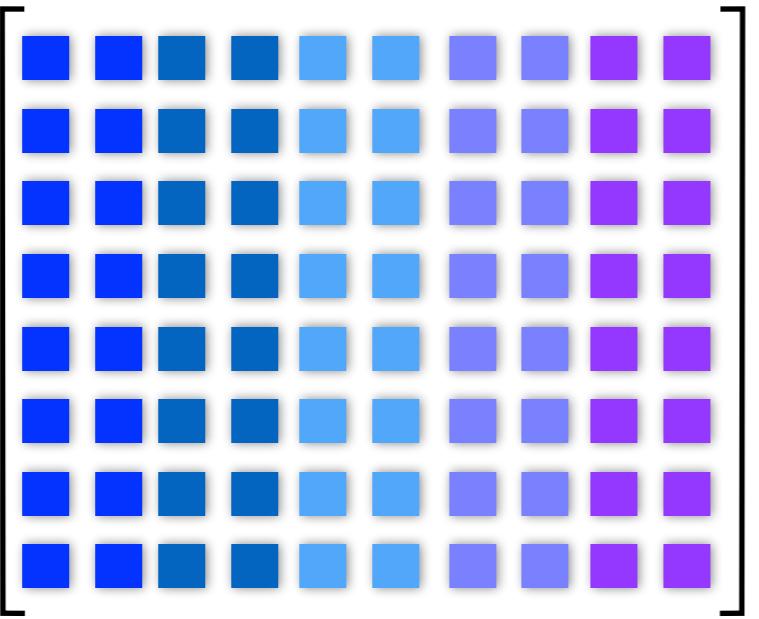
$$\begin{bmatrix} \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \\ \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \\ \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \\ \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \\ \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \\ \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \\ \text{Blue} & \text{Blue} & \text{Dark Blue} & \text{Blue} & \text{Light Blue} & \text{Purple} & \text{Light Purple} \end{bmatrix}$$



L

Low-Rank

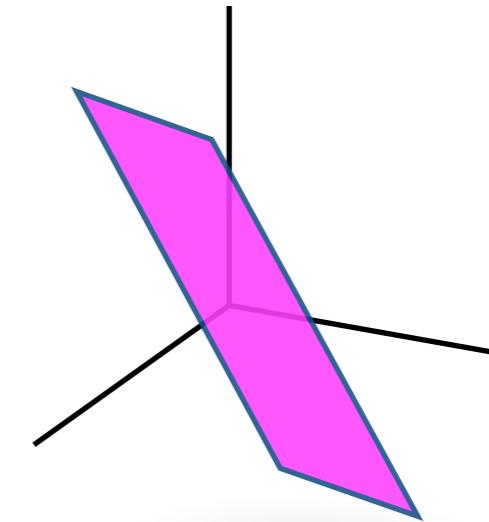
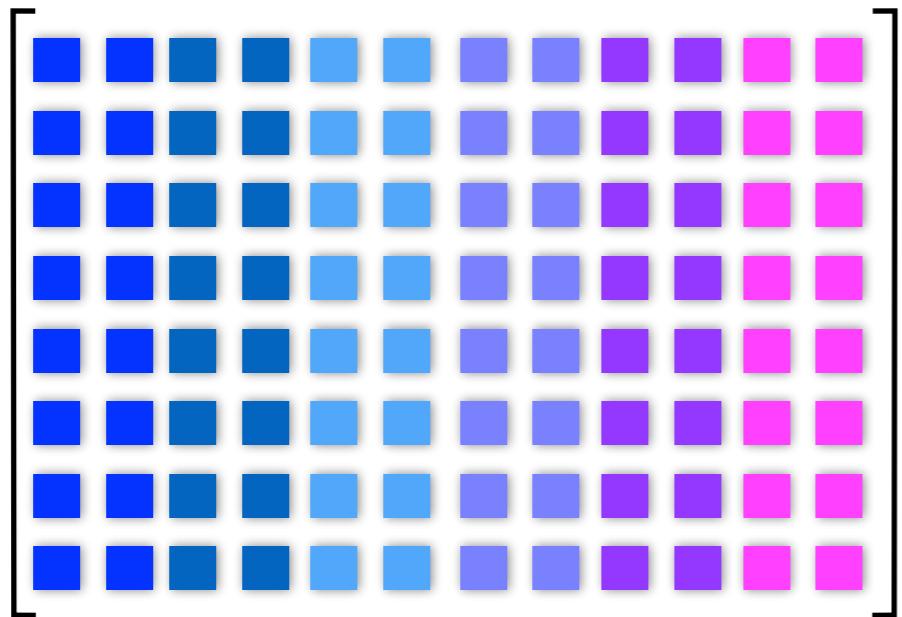
Subspace Tracking



L

Low-Rank

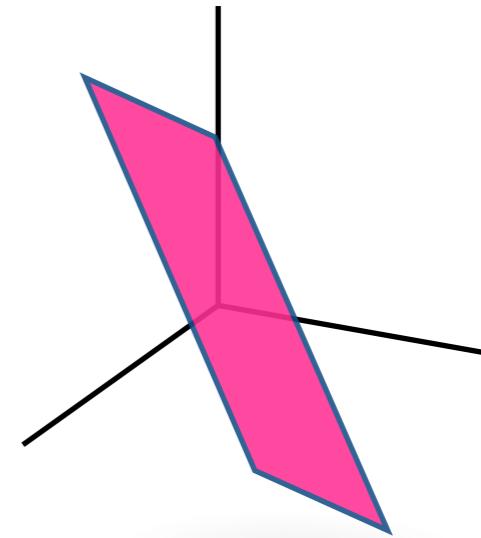
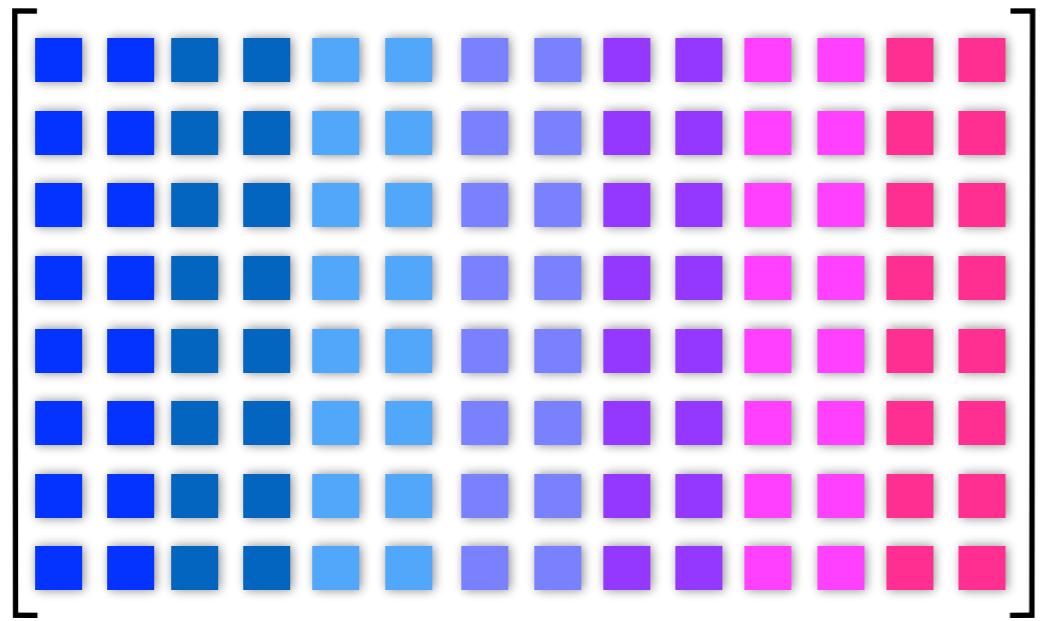
Subspace Tracking



L

Low-Rank

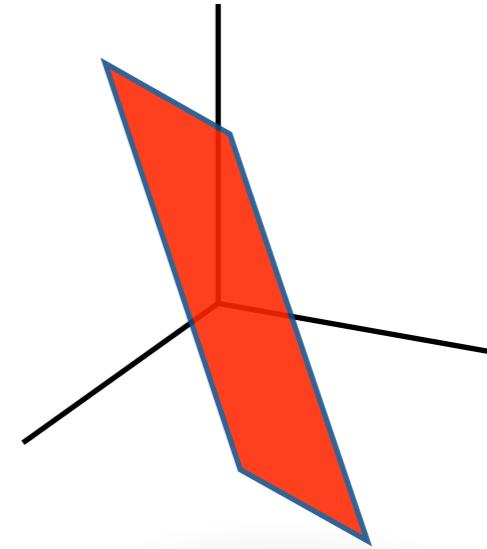
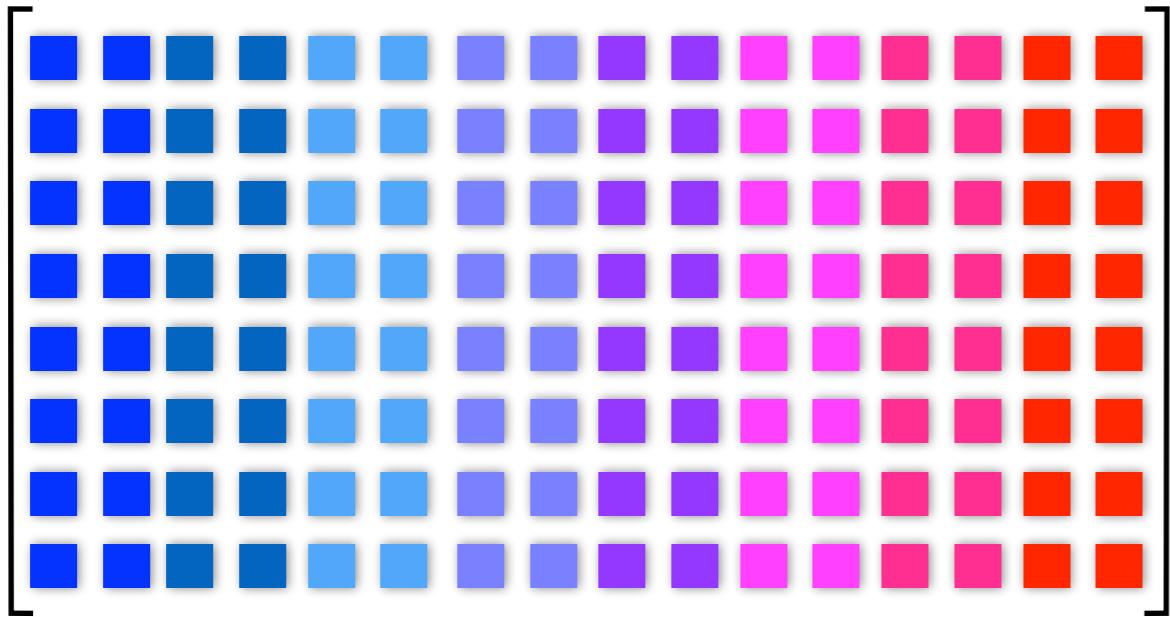
Subspace Tracking



L

Low-Rank

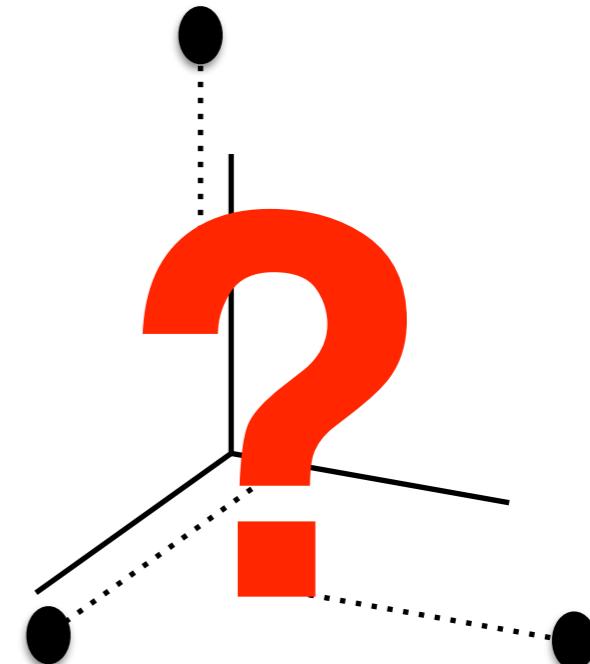
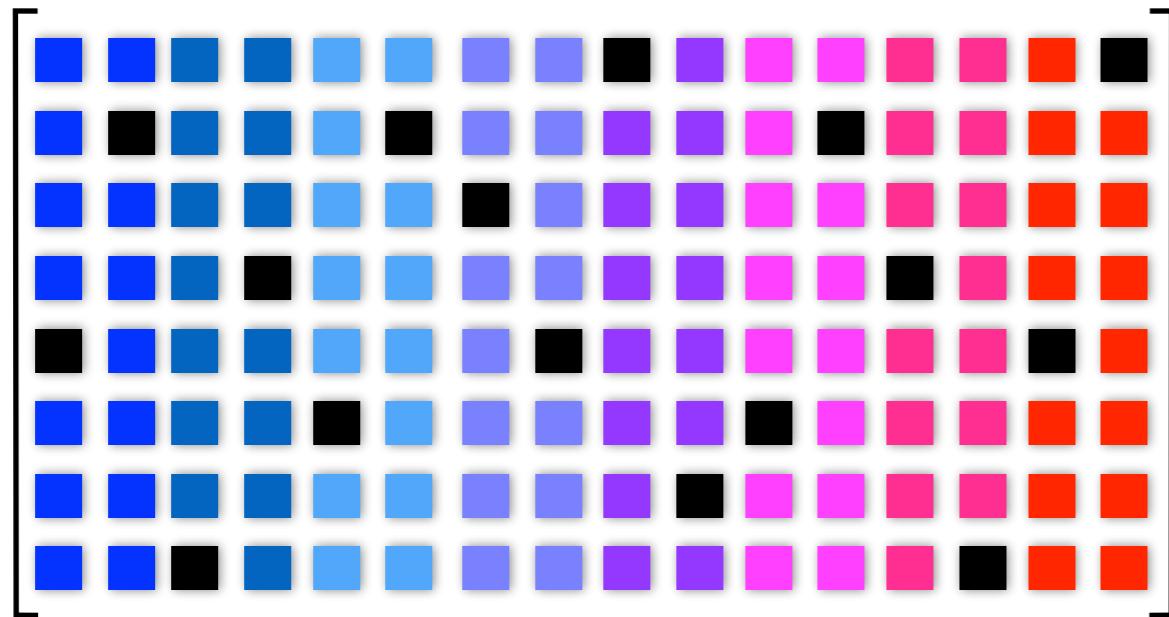
Subspace Tracking



L

Low-Rank

Subspace Tracking



L + S

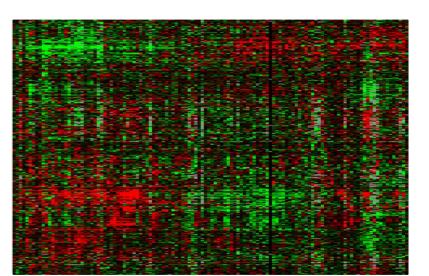
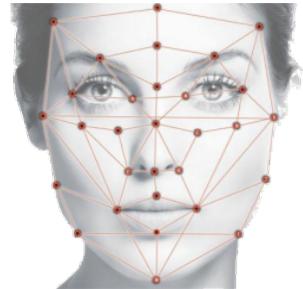
Low-Rank

Sparse

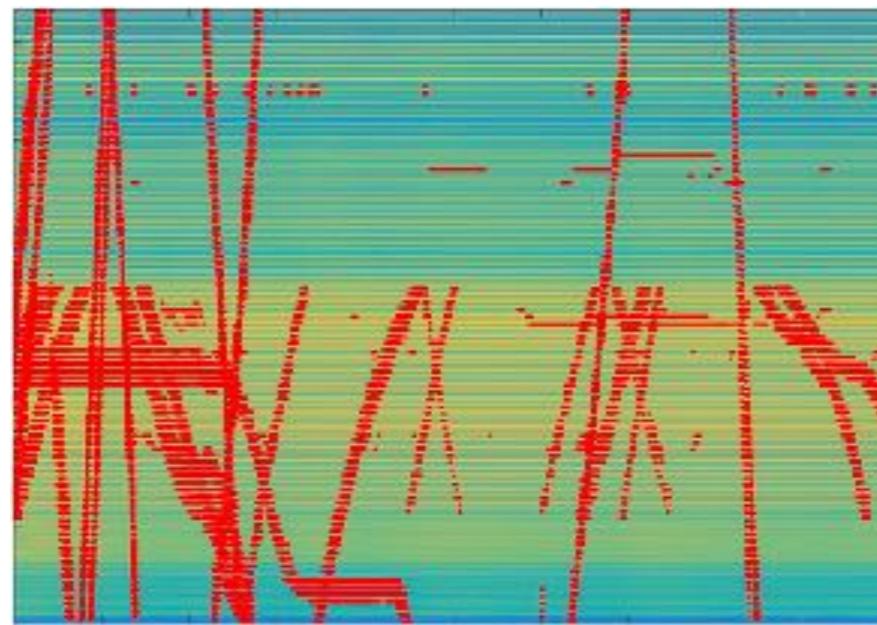
Robust Subspace Tracking



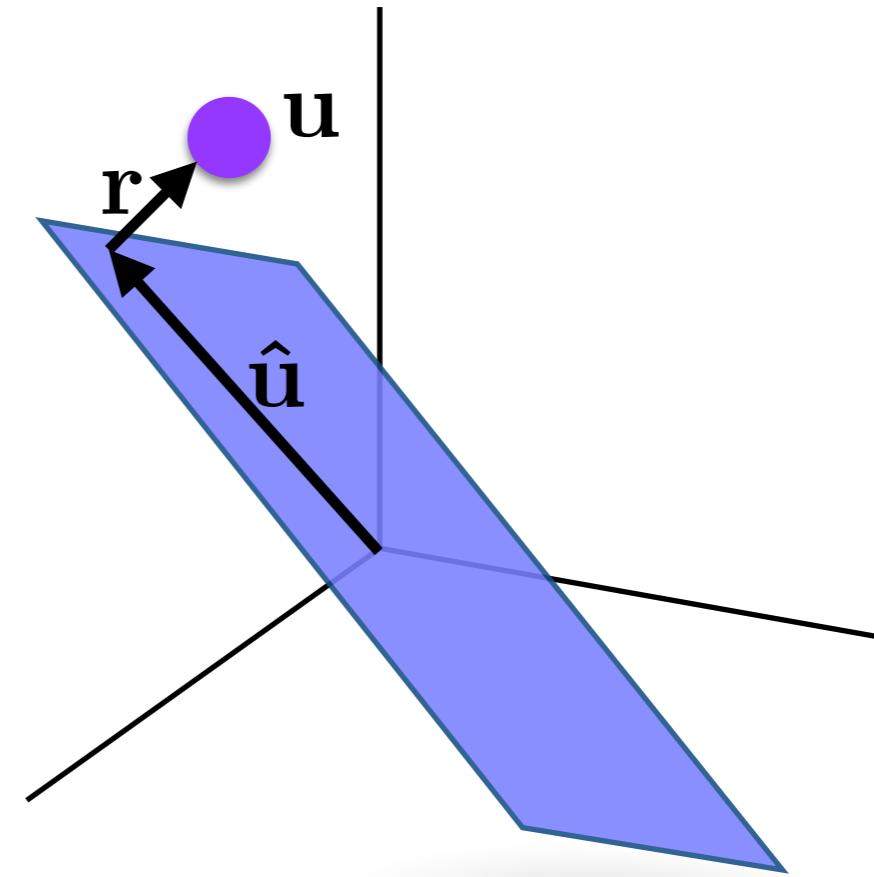
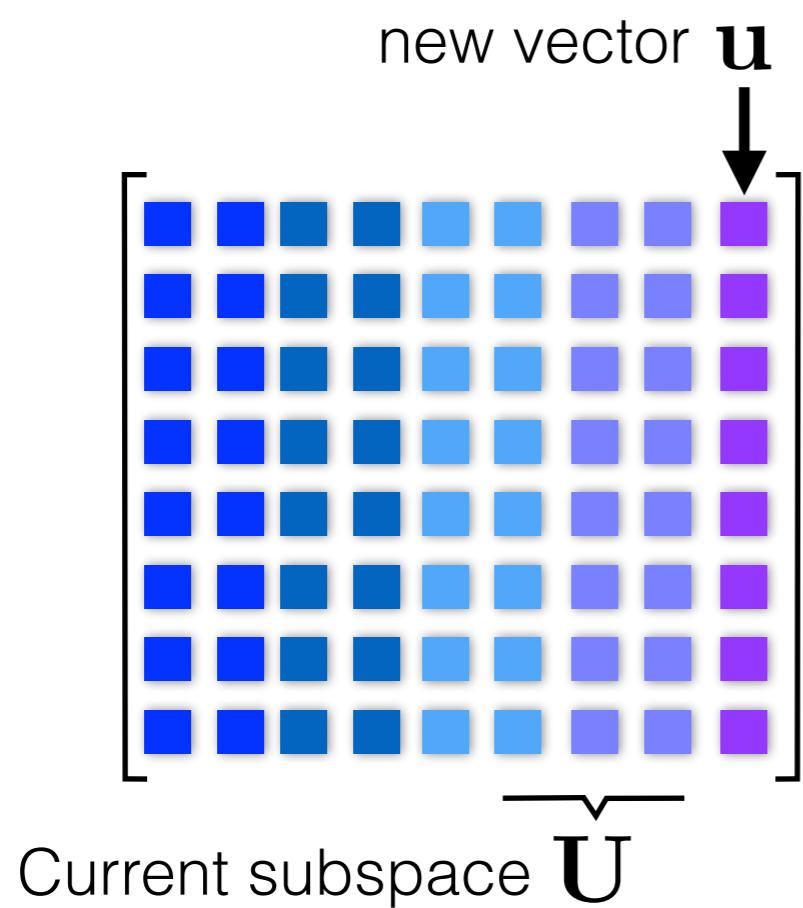
What is
this good
for?



Lots of Applications



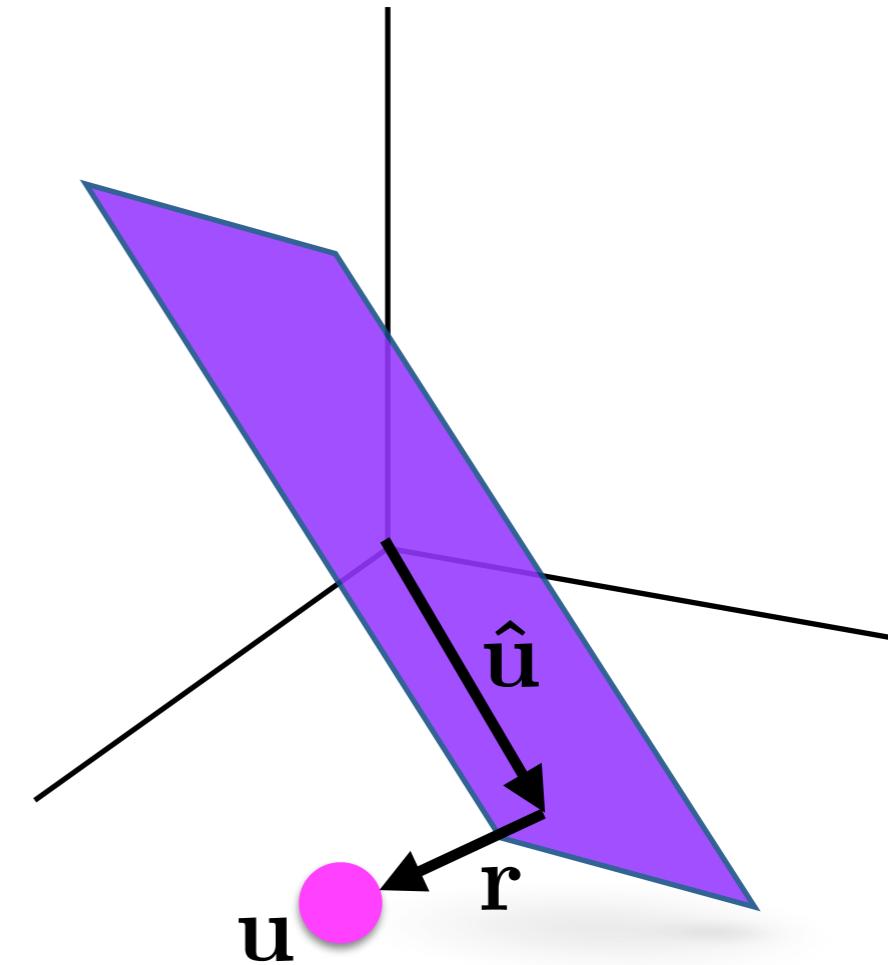
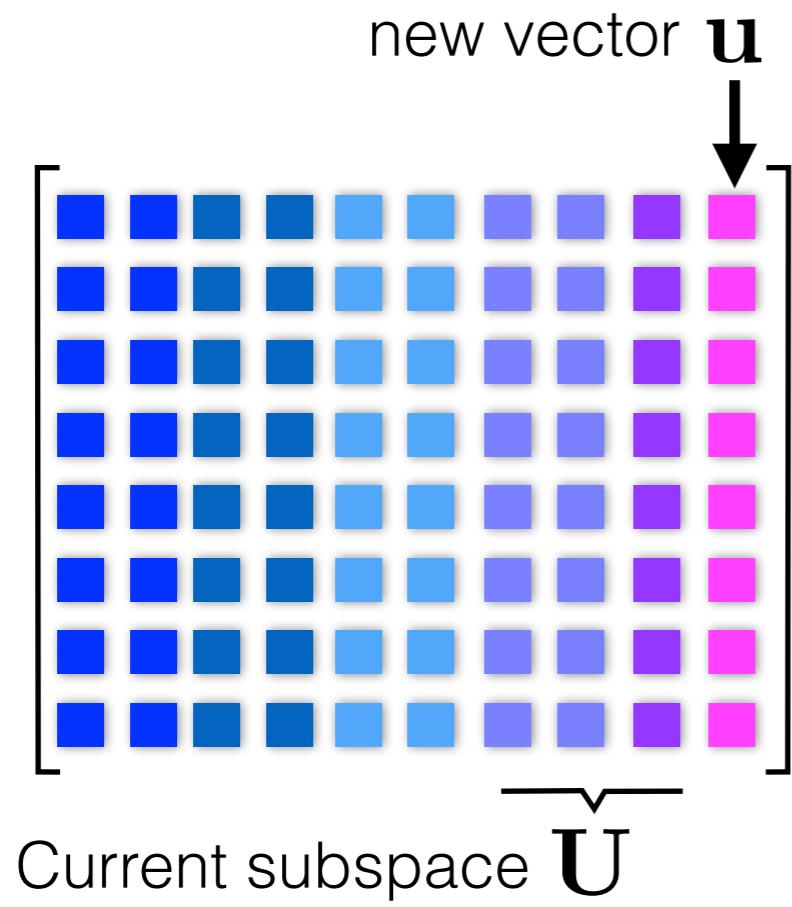
Background segmentation



$$\text{Update} = \mathbf{U} + \left((\cos(\sigma\eta) - 1) \frac{\hat{\mathbf{u}}}{\|\hat{\mathbf{u}}\|} + \sin(\sigma\eta) \frac{\mathbf{r}}{\|\mathbf{r}\|} \right) \frac{\boldsymbol{\theta}^T}{\|\boldsymbol{\theta}\|}$$

Existing theory

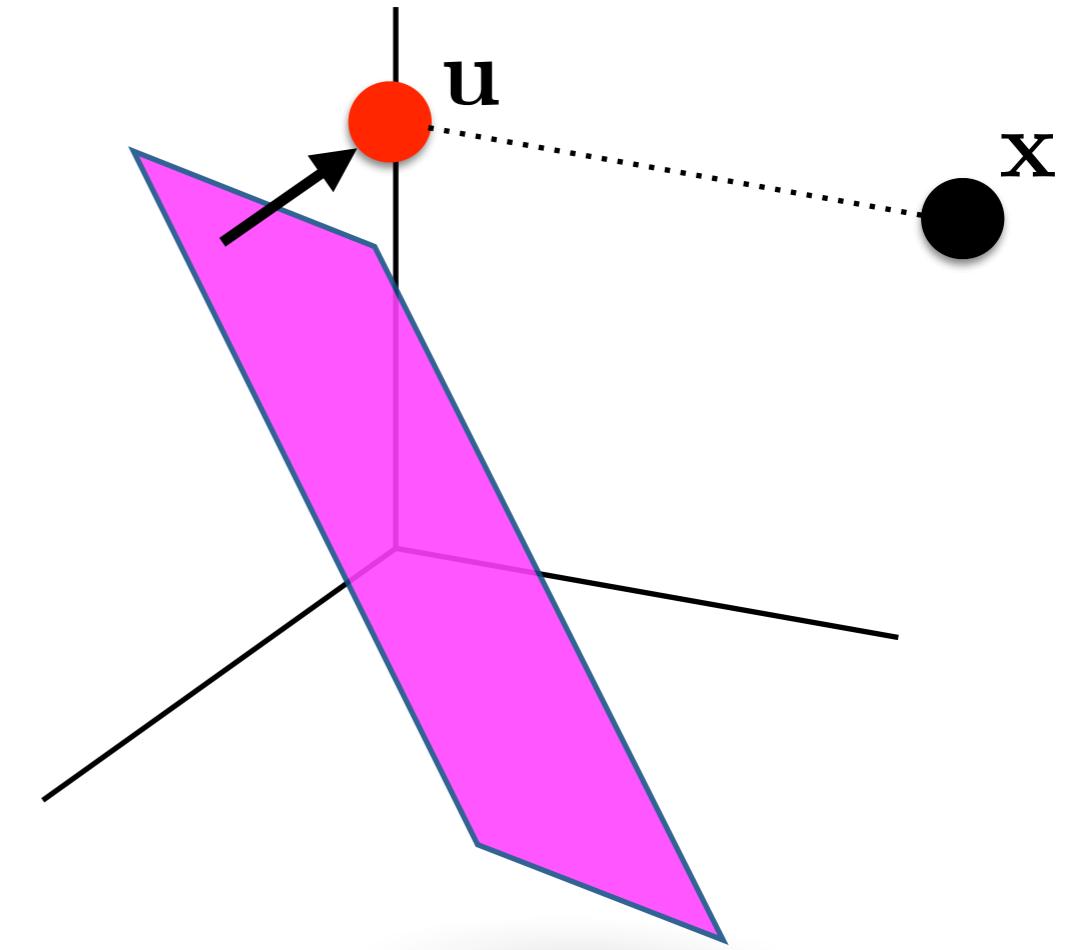
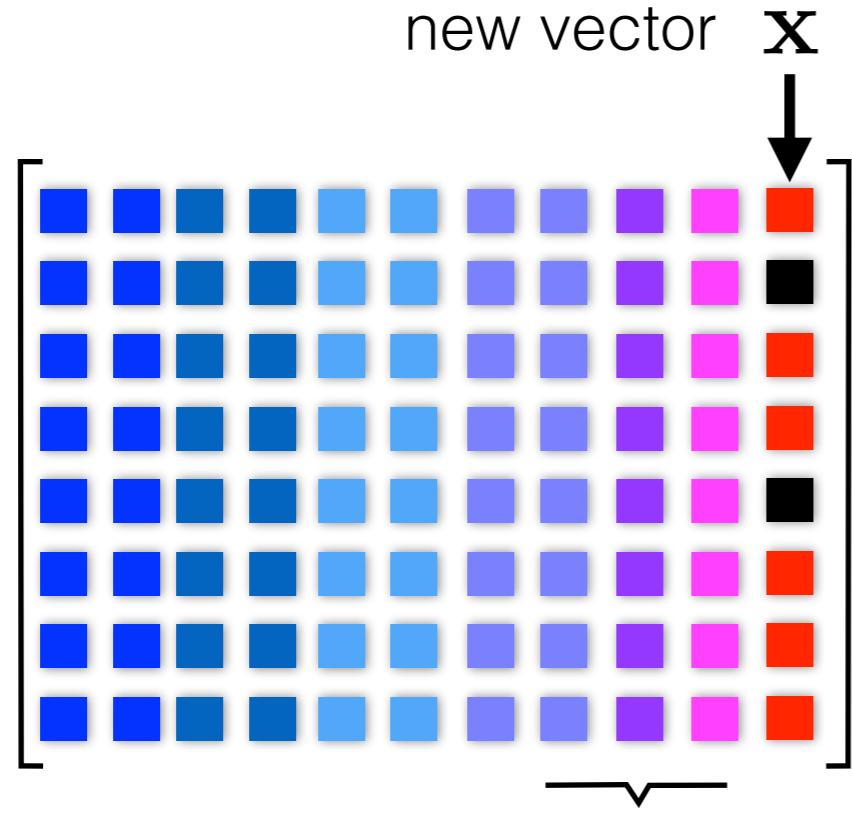
Forget about errors for a second...



$$\text{Update} = \overline{\mathbf{U}} + \left((\cos(\sigma\eta) - 1) \frac{\hat{\mathbf{u}}}{\|\hat{\mathbf{u}}\|} + \sin(\sigma\eta) \frac{\mathbf{r}}{\|\mathbf{r}\|} \right) \frac{\boldsymbol{\theta}^T}{\|\boldsymbol{\theta}\|}$$

Existing theory

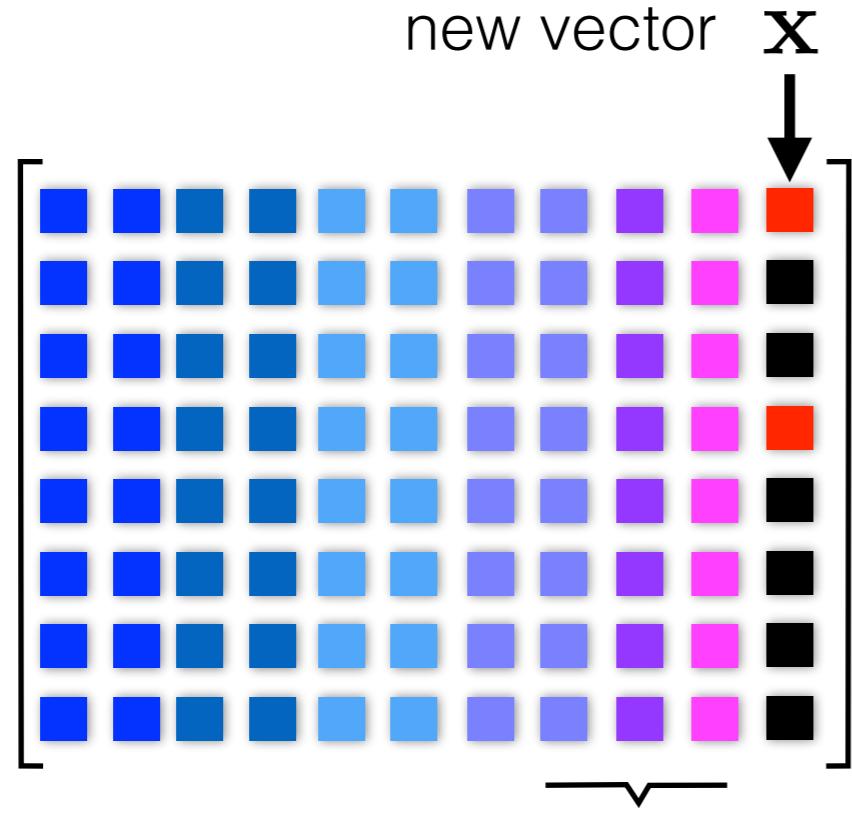
Forget about errors for a second...



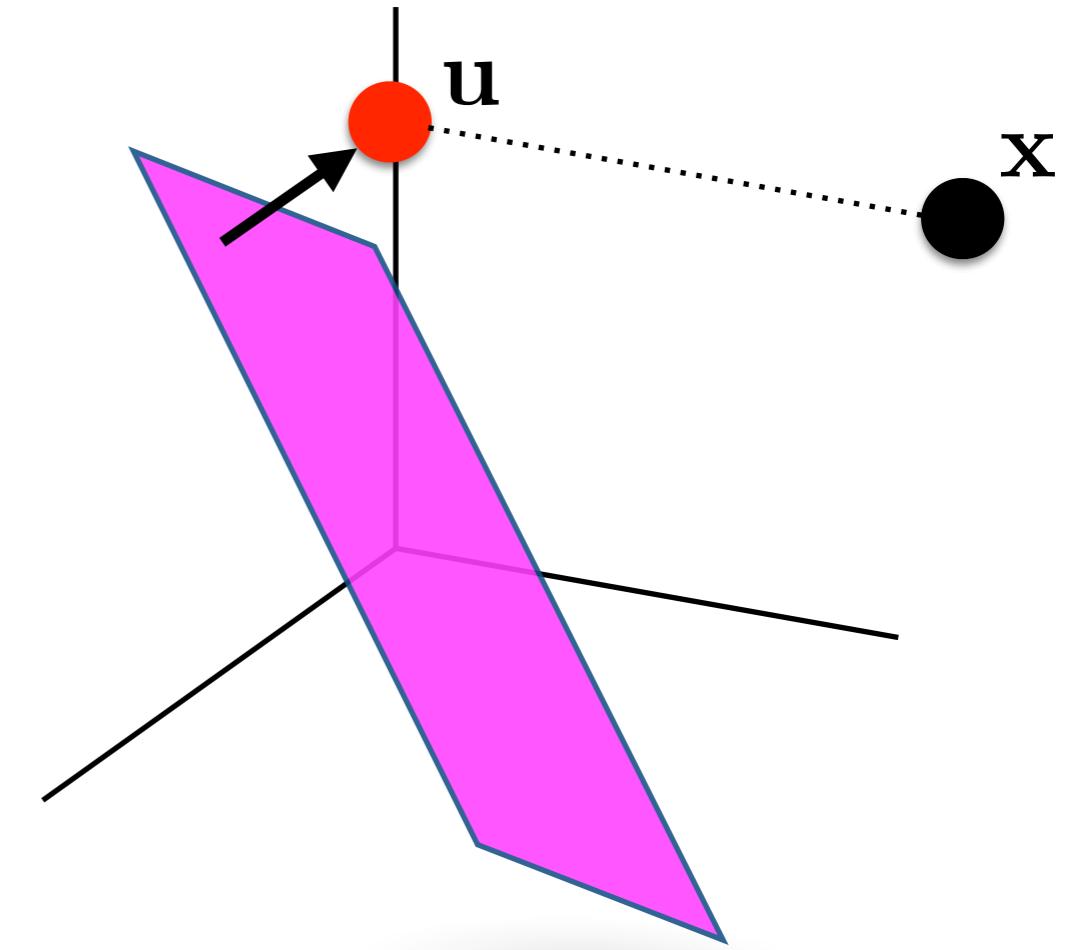
$$\min_{\theta \in \mathbb{R}^r} \|\overbrace{\mathbf{U}}^{\mathbf{u}} \theta - \mathbf{x}\|_1 \leftarrow \text{Solution with few nonzero entries}$$

Existing theory

Now with sparse errors...



Current subspace $\overline{\mathbf{U}}$



$$\min_{\theta \in \mathbb{R}^r} \|\overline{\mathbf{U}}\theta - \mathbf{x}\|_1 \leftarrow \text{Solution with few nonzero entries}$$

↑
Won't work!!

Existing theory

Too many errors?

Selective Erasures

Our solution

Current
subspace

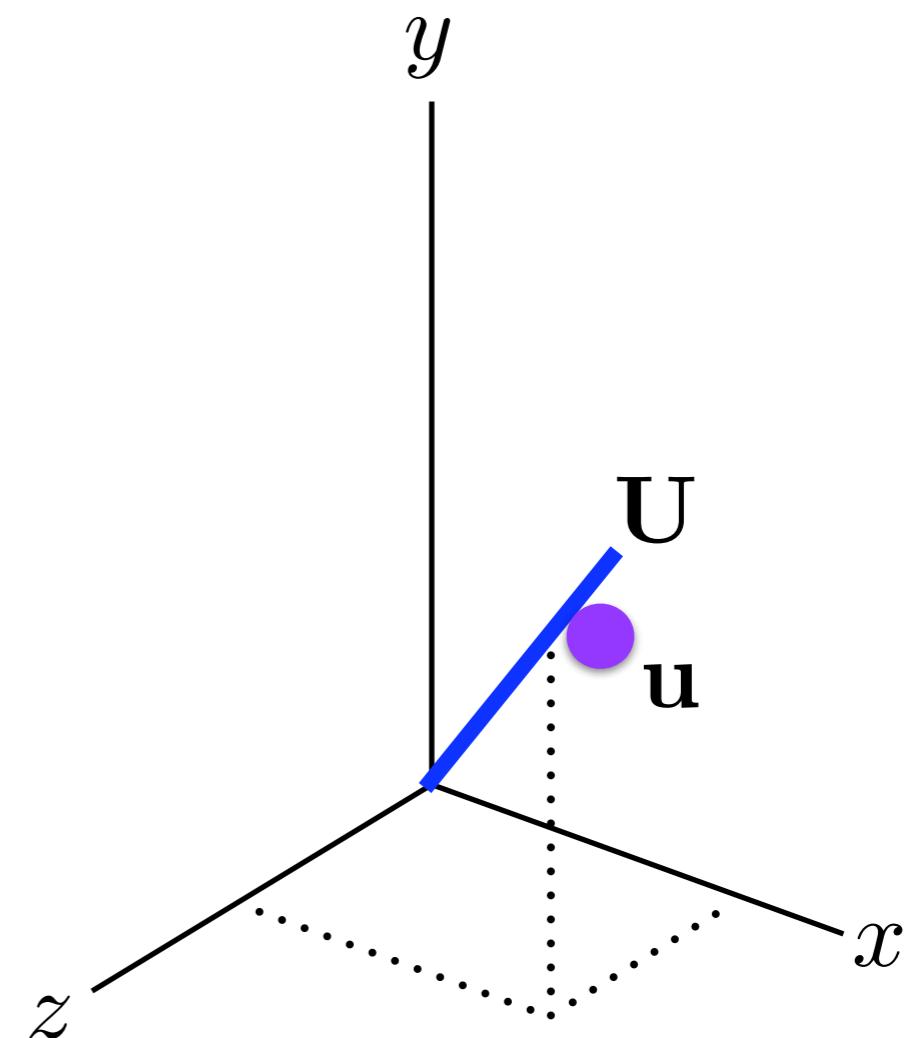
\mathbf{U}

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

\mathbf{u}

$$\begin{bmatrix} \text{purple} \\ \text{purple} \end{bmatrix}$$



Main Idea

Remove coordinates that don't help

Current
subspace

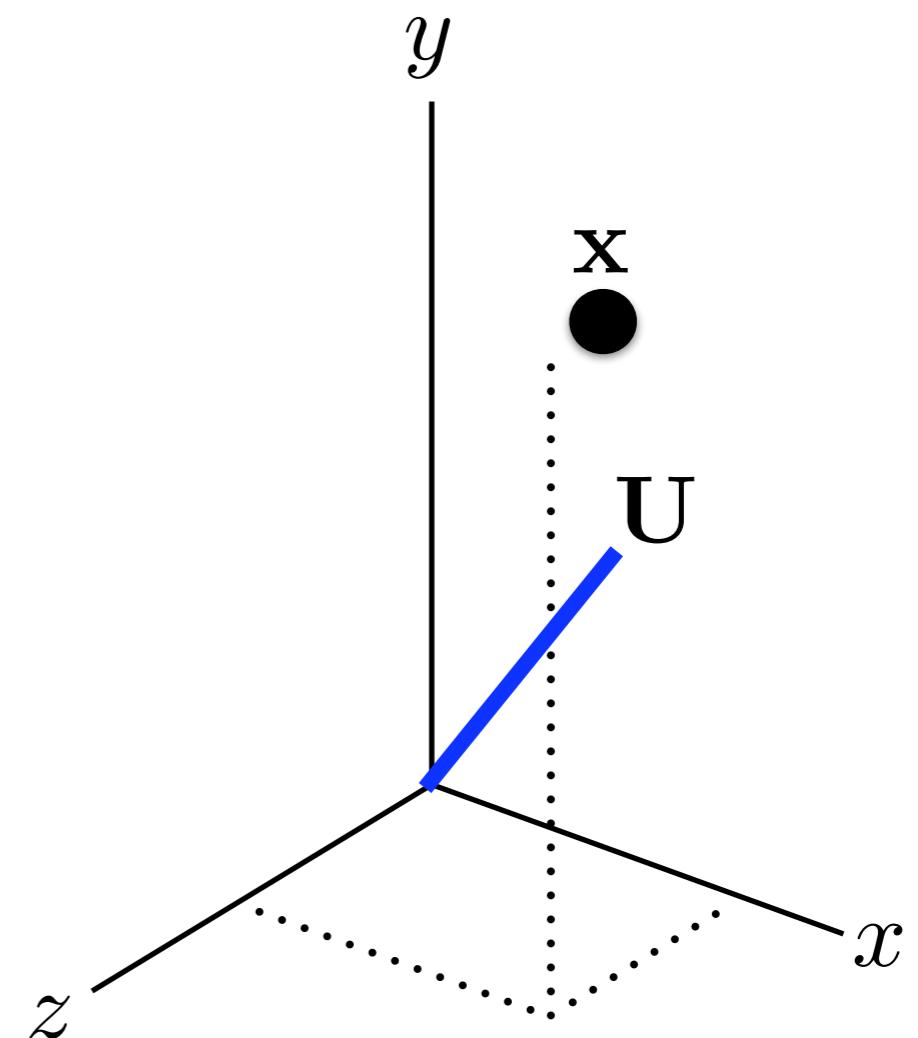
U

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

x

$$\begin{bmatrix} \text{black} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$



Main Idea

Remove coordinates that don't help

Current
subspace

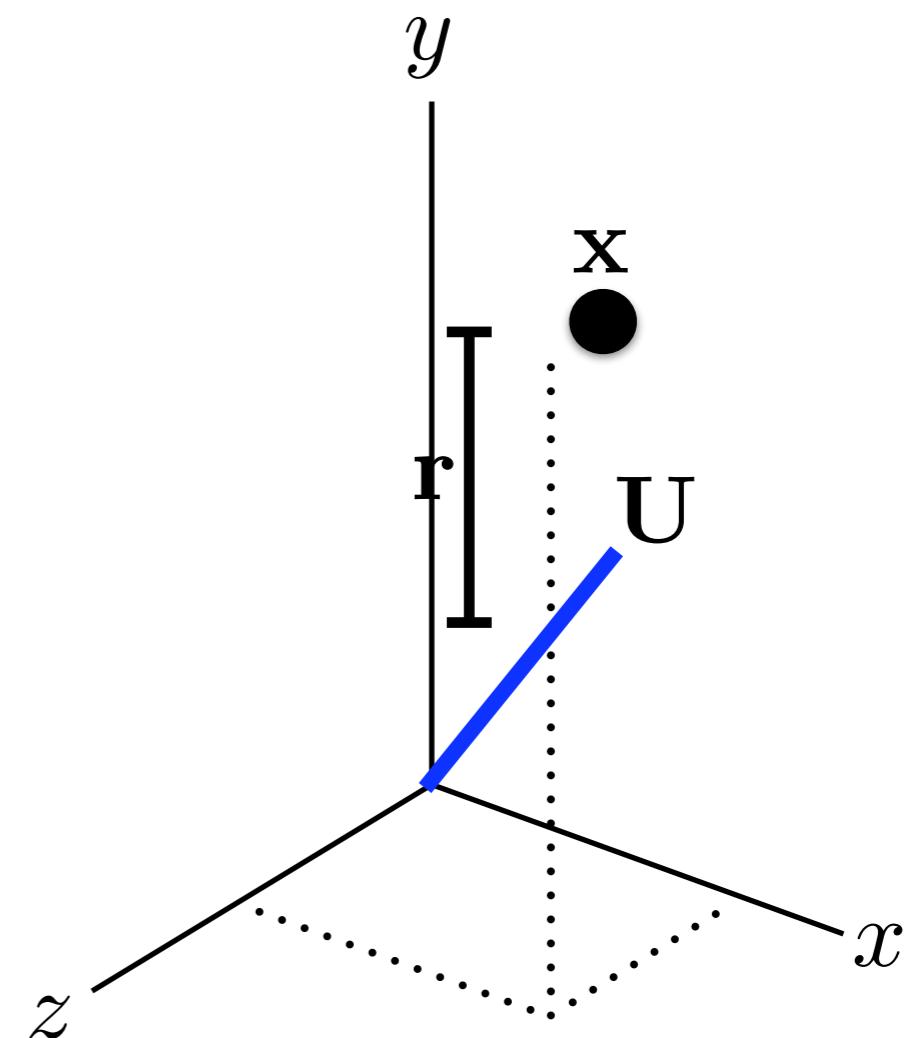
U

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

x

$$\begin{bmatrix} \text{black} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$



Main Idea

Remove coordinates that don't help

Current
subspace

\mathbf{U}

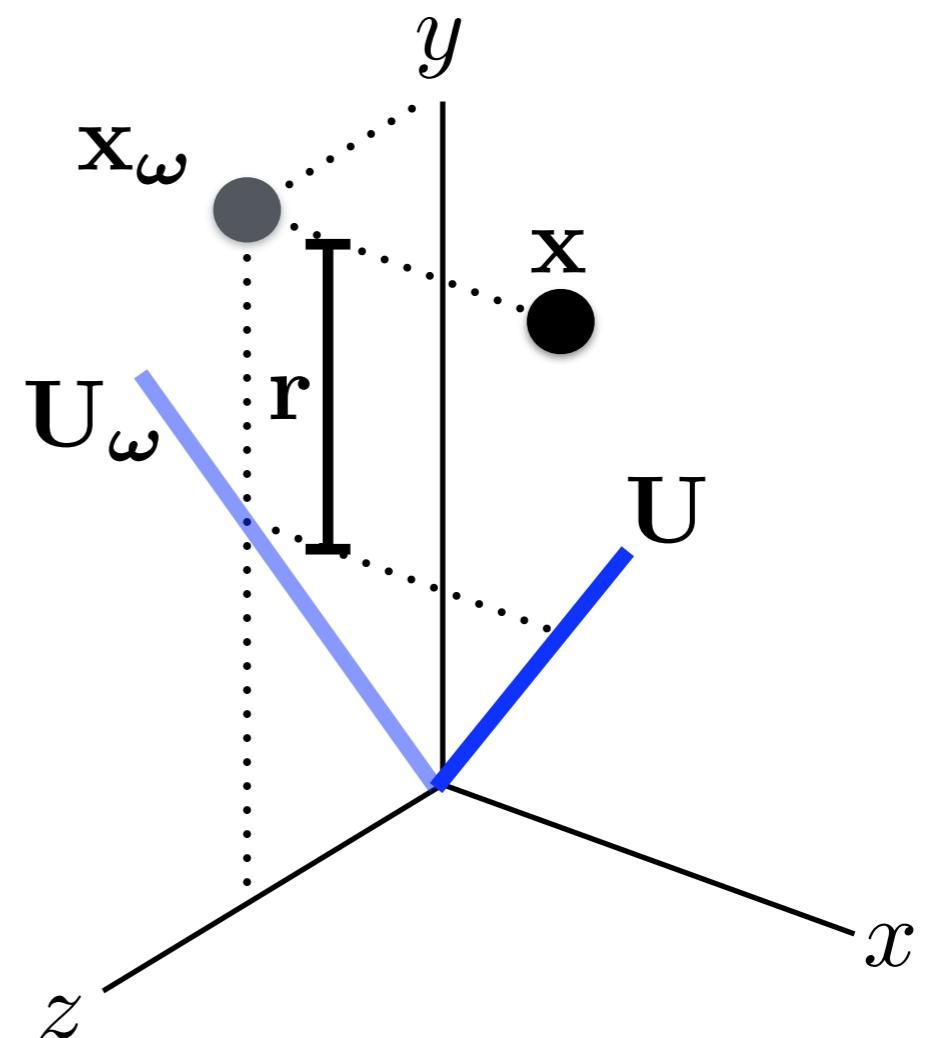
$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

\mathbf{x}

$$\begin{bmatrix} \text{black} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$

Removing
helps?



Main Idea

Remove coordinates that don't help

Current
subspace

\mathbf{U}

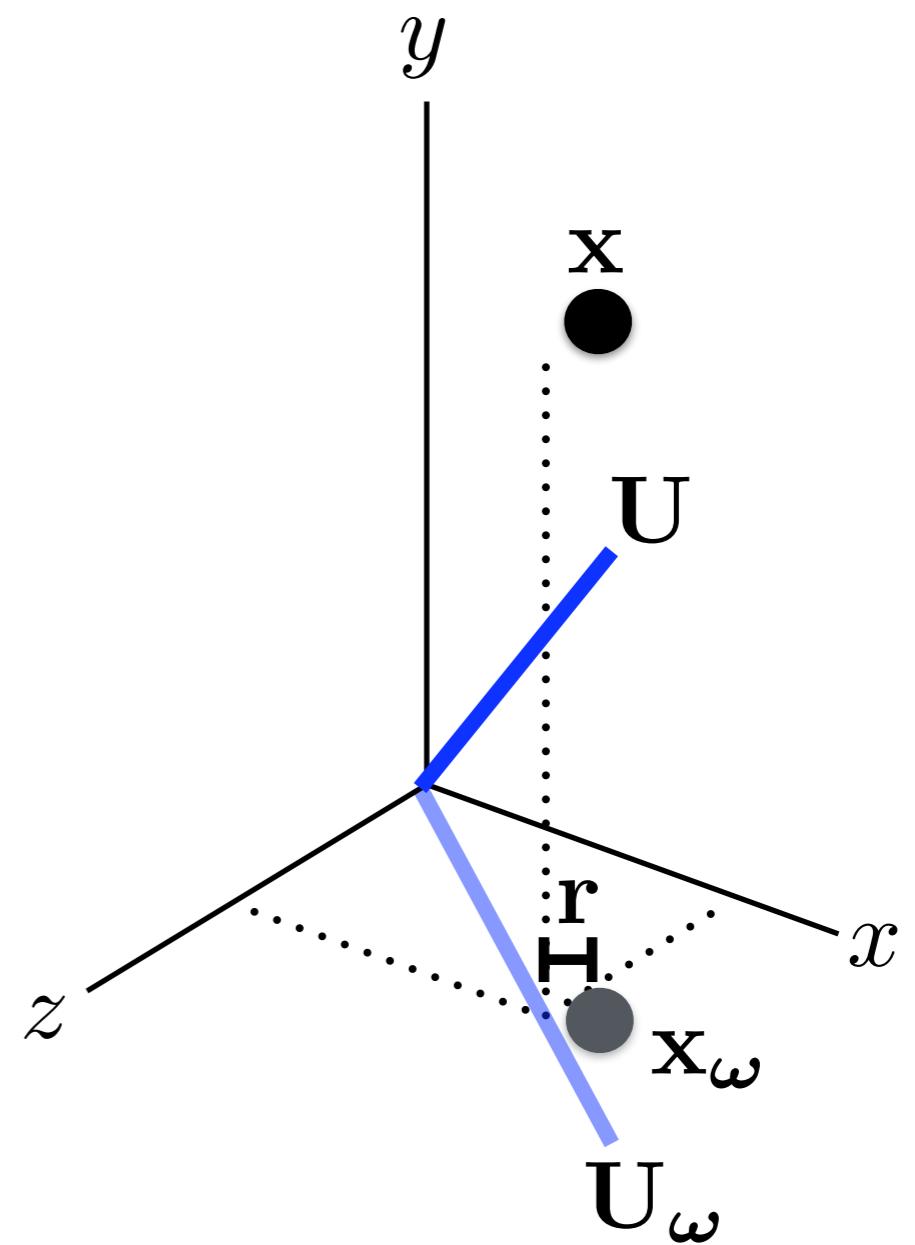
$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

\mathbf{x}

$$\begin{bmatrix} \text{black} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$

Removing
helps?



Main Idea

Remove coordinates that don't help

Current
subspace

\mathbf{U}

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

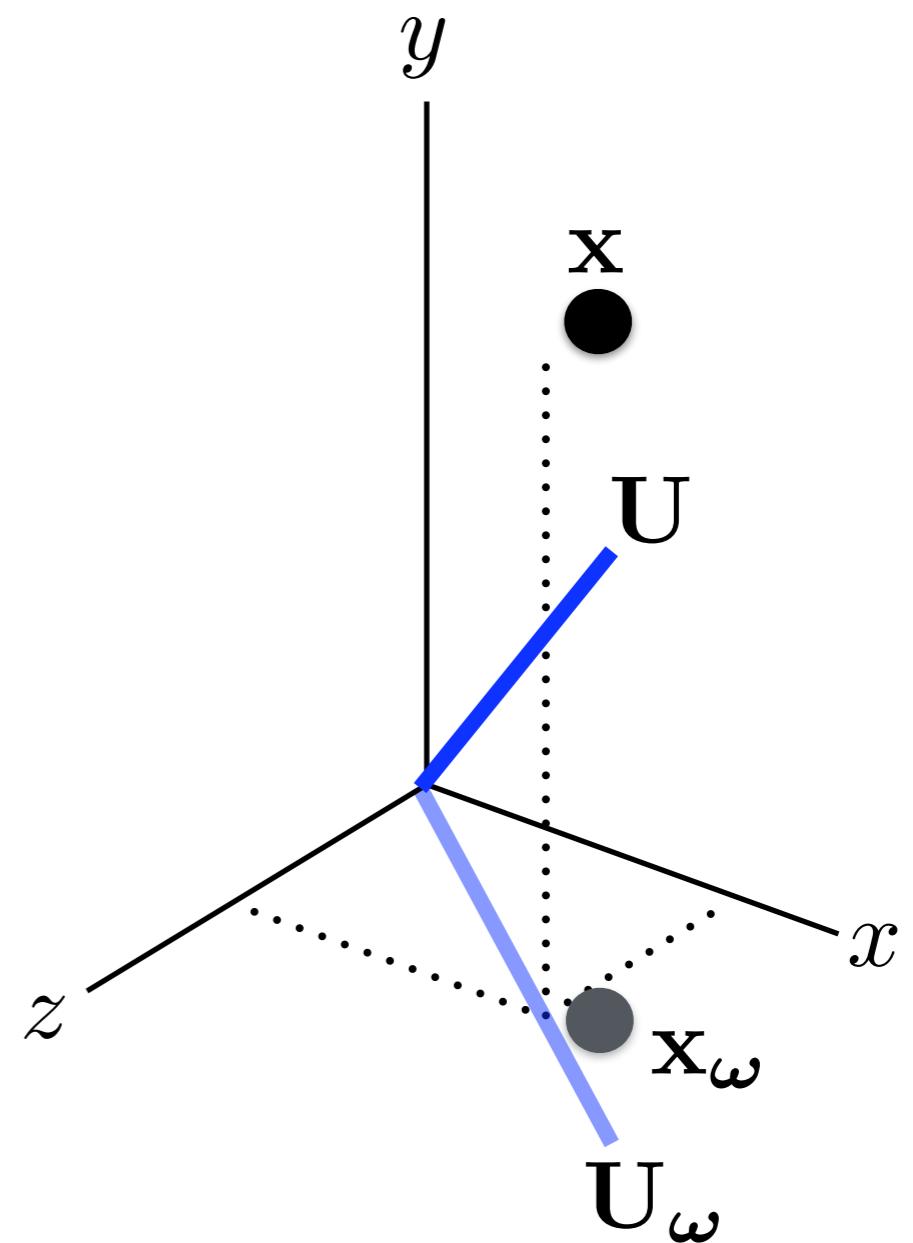
new
vector

\mathbf{x}

$$\begin{bmatrix} \text{black} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$

Removing
helps?

Erase!



Main Idea

Remove coordinates that don't help

Current
subspace

\mathbf{U}

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

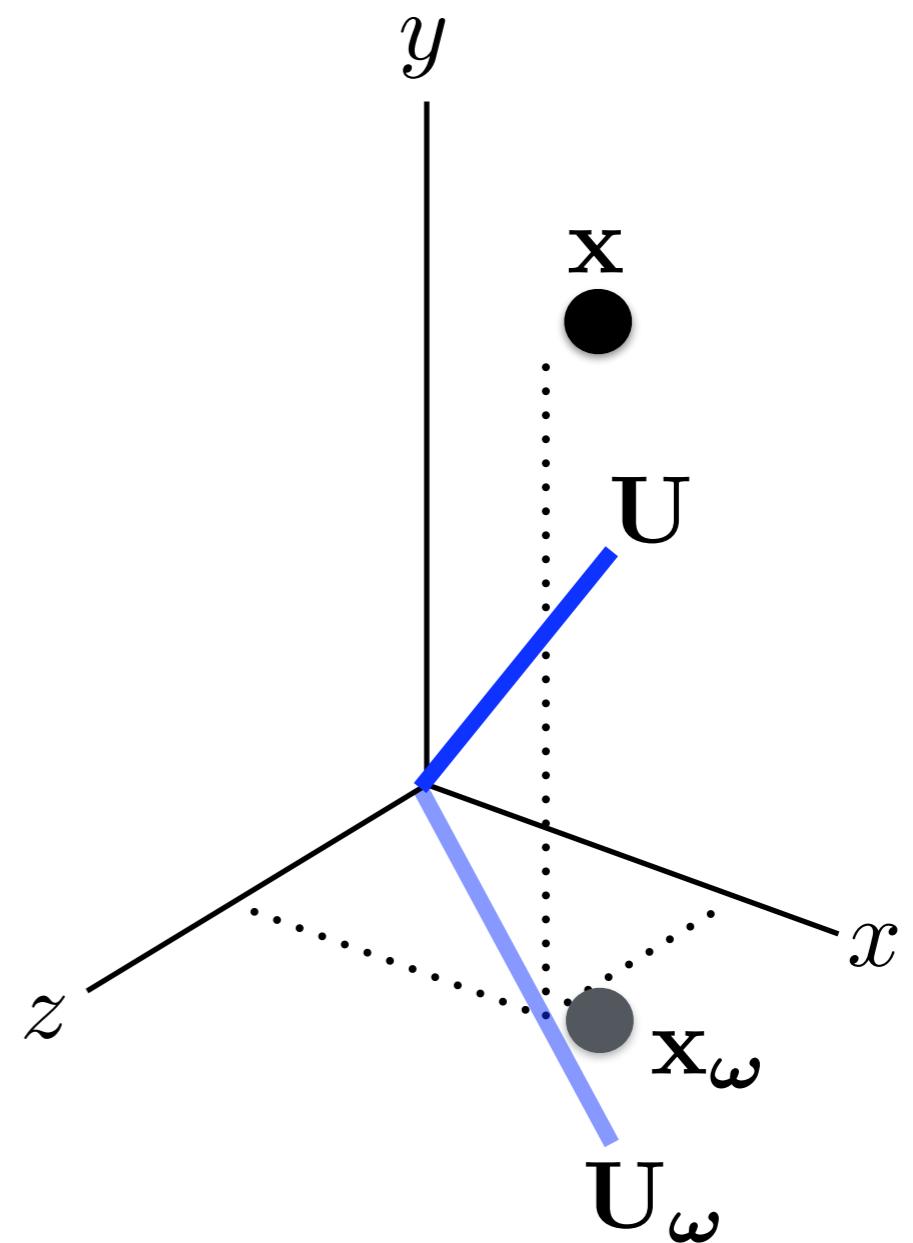
new
vector

\mathbf{x}

$$\begin{bmatrix} \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$

Removing
helps?

Erase!



Main Idea

Remove coordinates that don't help

Current
subspace

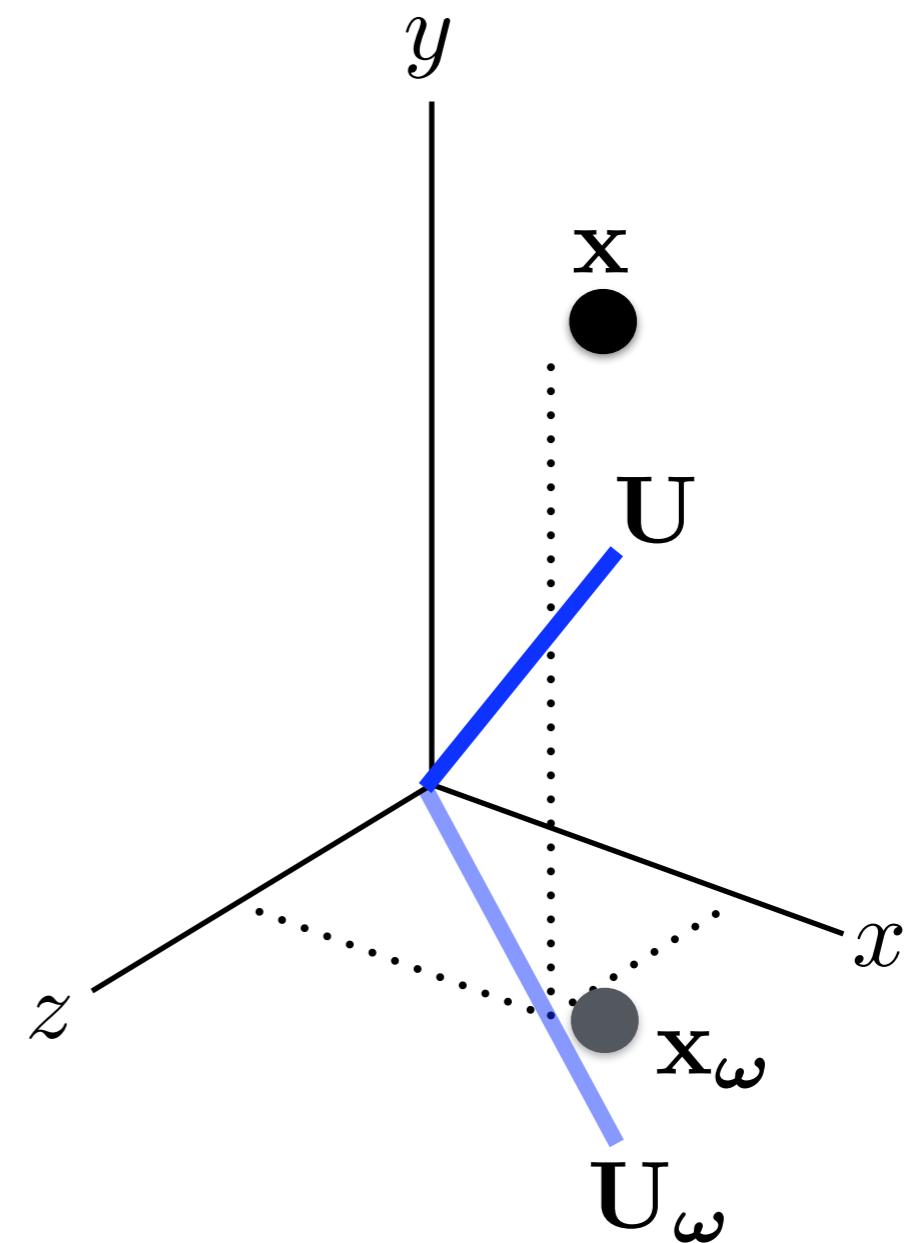
\mathbf{U}

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

\mathbf{x}

$$\begin{bmatrix} \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$



Main Idea

Remove coordinates that don't help

Current
subspace

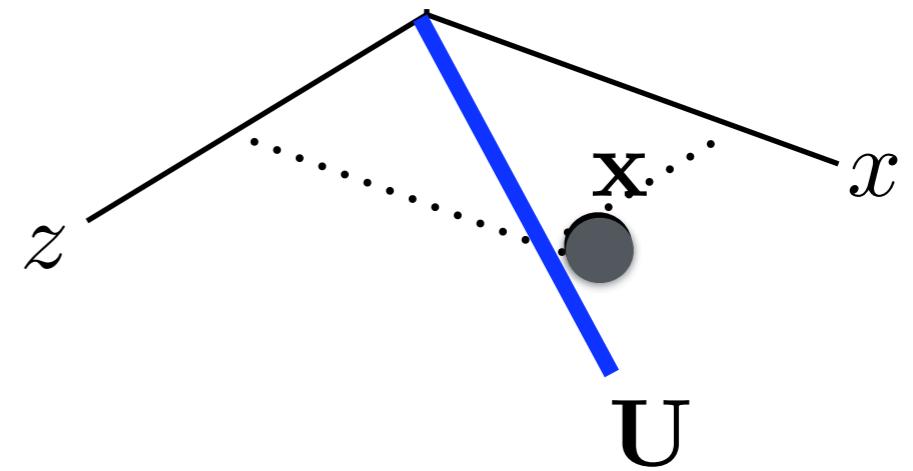
U

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

x

$$\begin{bmatrix} \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$



Main Idea

Remove coordinates that don't help

Current
subspace

U

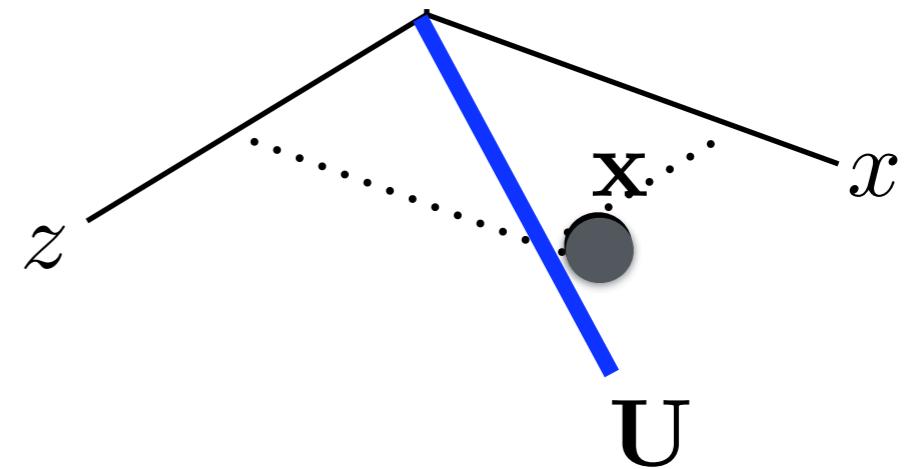
$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

x

$$\begin{bmatrix} \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$

Removing
helps?



Main Idea

Remove coordinates that don't help

Current
subspace

U

$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

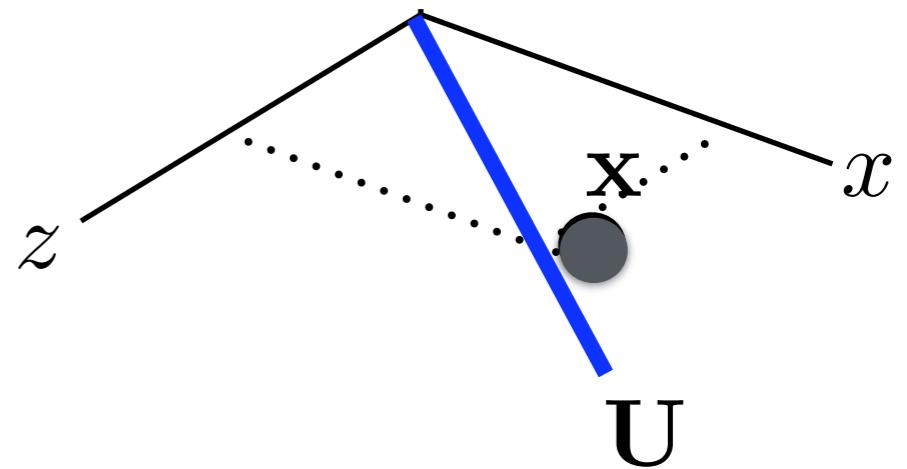
new
vector

x

$$\begin{bmatrix} \text{black} \\ \text{purple} \\ \text{black} \\ \text{purple} \\ \text{black} \\ \text{black} \\ \text{purple} \end{bmatrix}$$

Removing
helps?

Erase!



Main Idea

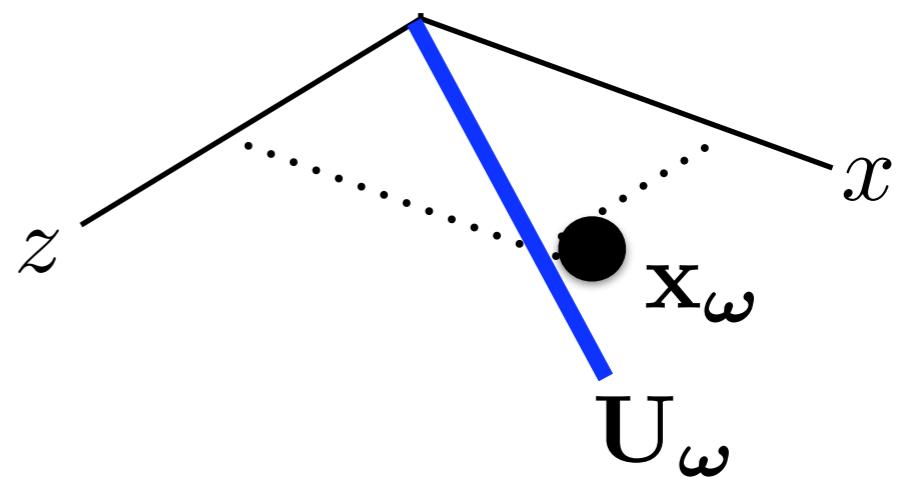
Remove coordinates that don't help

Current
subspace

$$\mathbf{U}_\omega = \begin{bmatrix} & \\ \text{blue} & \text{blue} \\ & \\ \text{blue} & \text{blue} \\ & \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

$$\mathbf{x}_\omega = \begin{bmatrix} & \\ \text{purple} & \\ & \\ \text{purple} & \\ & \\ \text{purple} & \end{bmatrix}$$



Main Idea

Recover full column

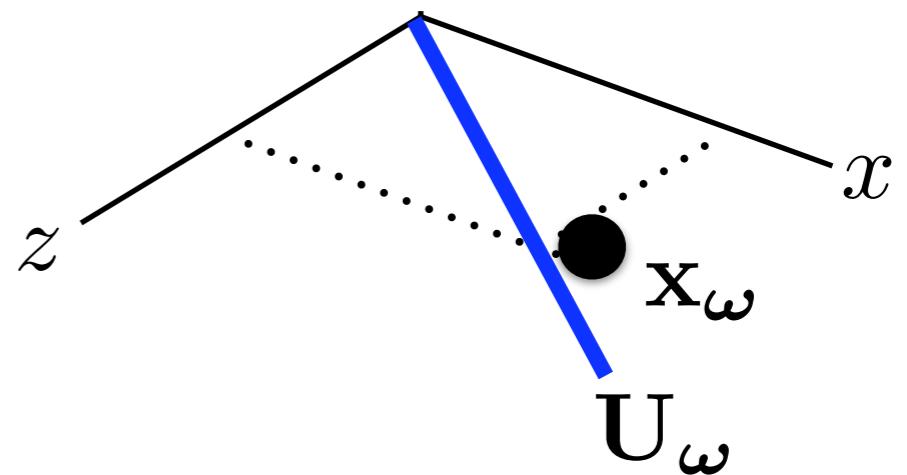
Current
subspace

$$\mathbf{U}_\omega = \begin{bmatrix} & \\ \text{blue squares} & \\ & \end{bmatrix}$$

new
vector

$$\mathbf{x}_\omega = \begin{bmatrix} & \\ \text{purple square} & \\ & \end{bmatrix}$$

- $\mathbf{U}_\omega \theta = \mathbf{x}_\omega$
- $\theta = (\mathbf{U}_\omega^\top \mathbf{U}_\omega)^{-1} \mathbf{U}_\omega^\top \mathbf{x}_\omega$
- $\mathbf{u} = \mathbf{U}\theta$



Main Idea

Recover full column

Current
subspace

\mathbf{U}

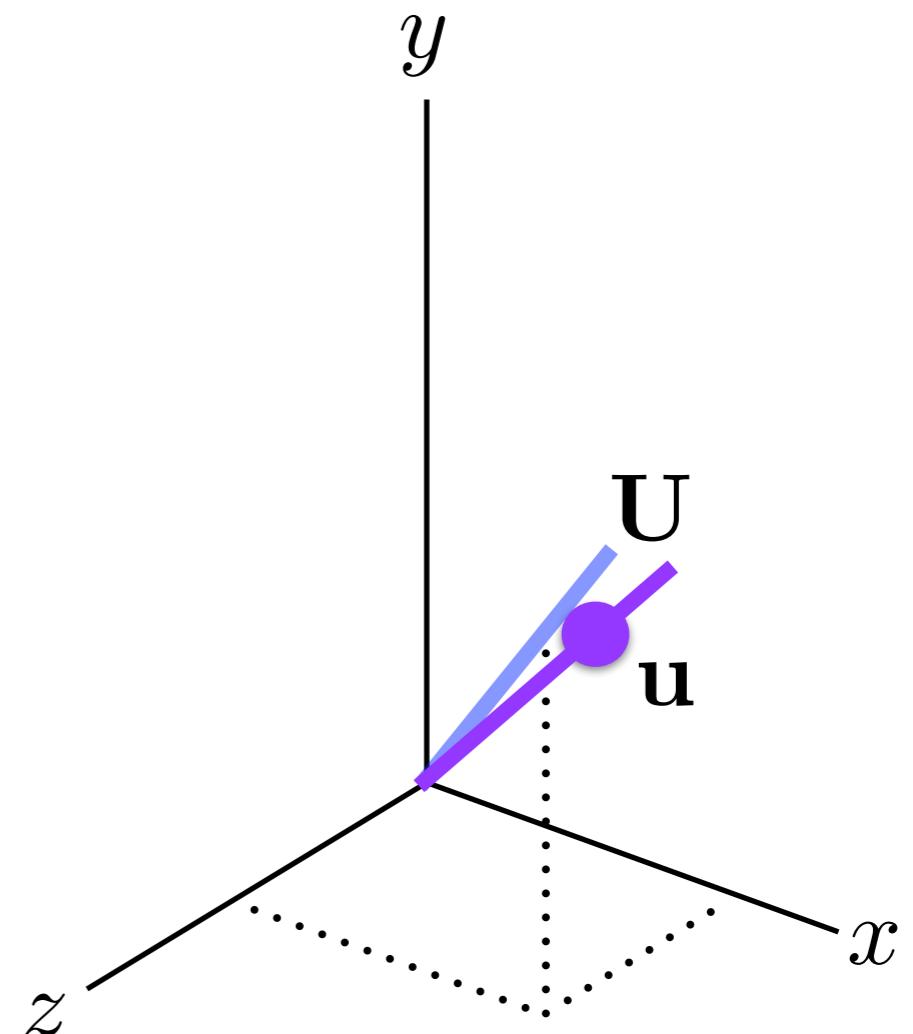
$$\begin{bmatrix} \text{blue} & \text{blue} \\ \text{blue} & \text{blue} \end{bmatrix}$$

new
vector

\mathbf{u}

$$\begin{bmatrix} \text{purple} \\ \text{purple} \\ \text{purple} \\ \text{purple} \\ \text{purple} \\ \text{purple} \\ \text{purple} \end{bmatrix}$$

- $\mathbf{U}_\omega \theta = \mathbf{x}_\omega$
- $\theta = (\mathbf{U}_\omega^\top \mathbf{U}_\omega)^{-1} \mathbf{U}_\omega^\top \mathbf{x}_\omega$
- $\mathbf{u} = \mathbf{U}\theta$



$$\mathbf{U} + \left((\cos(\sigma\eta) - 1) \frac{\hat{\mathbf{u}}}{\|\hat{\mathbf{u}}\|} + \sin(\sigma\eta) \frac{\mathbf{r}}{\|\mathbf{r}\|} \right) \frac{\theta^\top}{\|\theta\|}$$

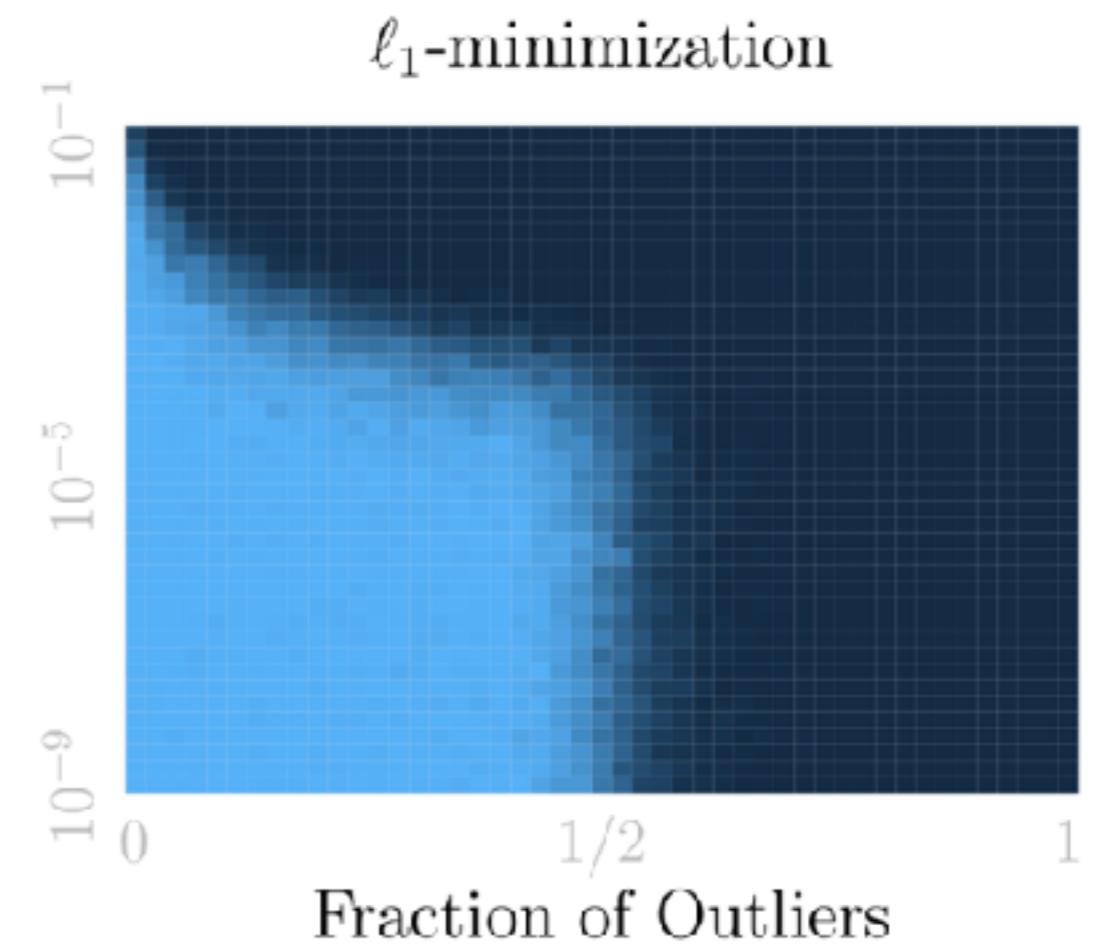
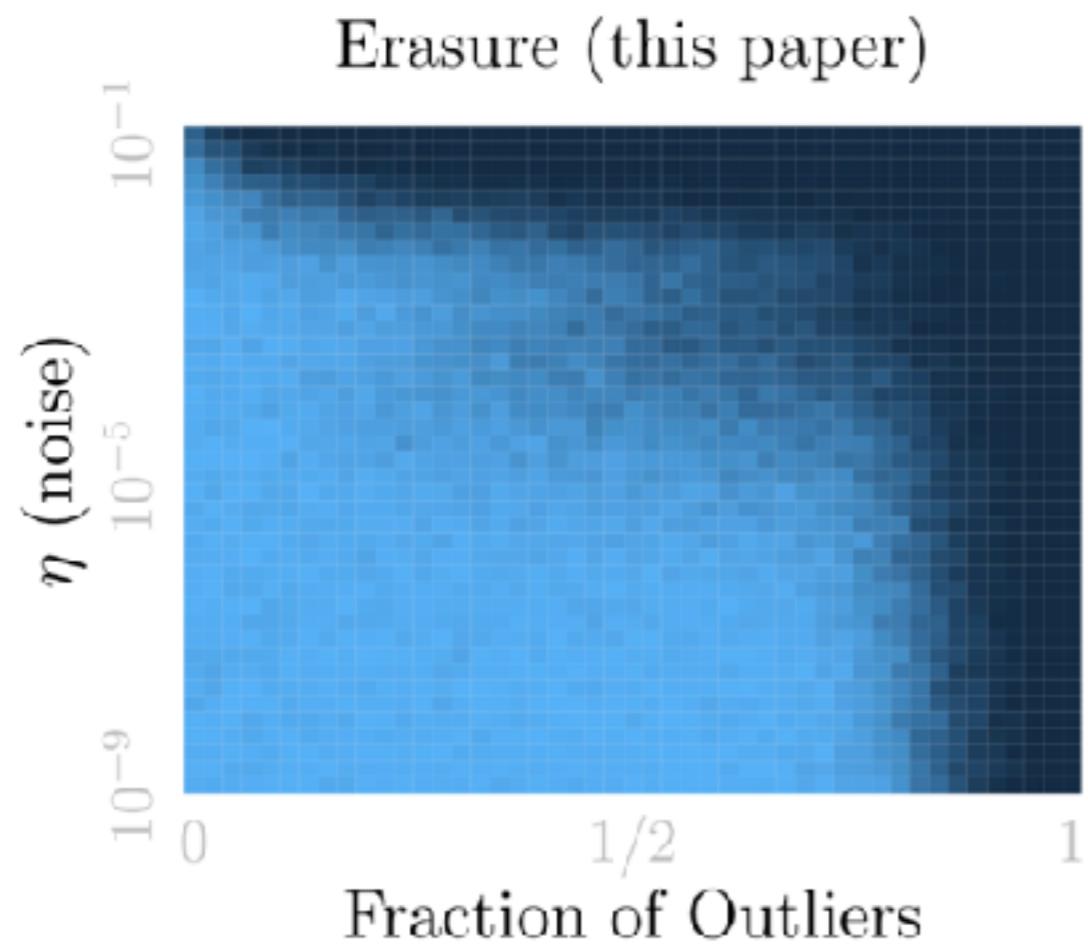
Main Idea

Update Subspace as Before



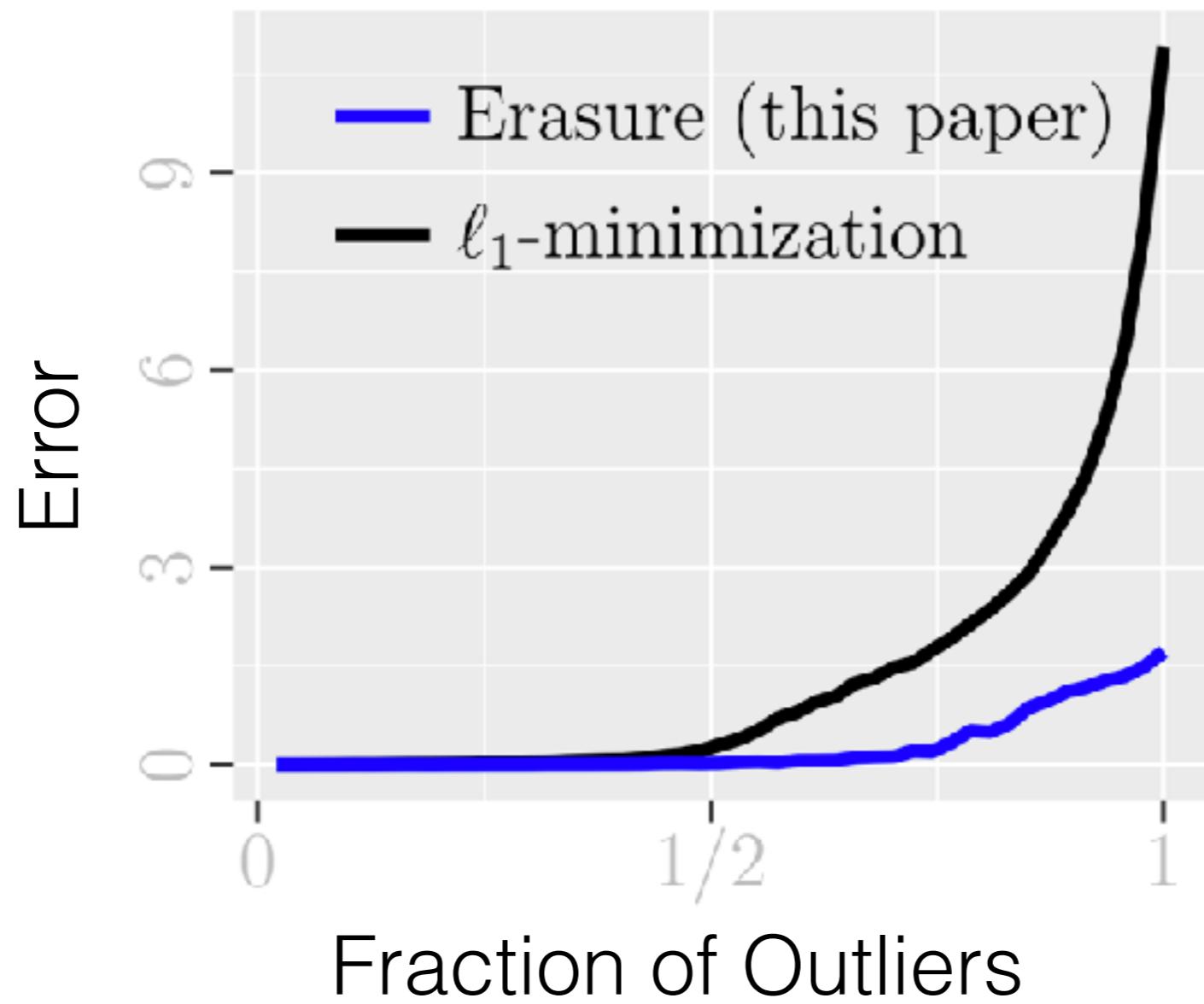
WOW, AMAZING

**PLEASE TELL
ME MORE**

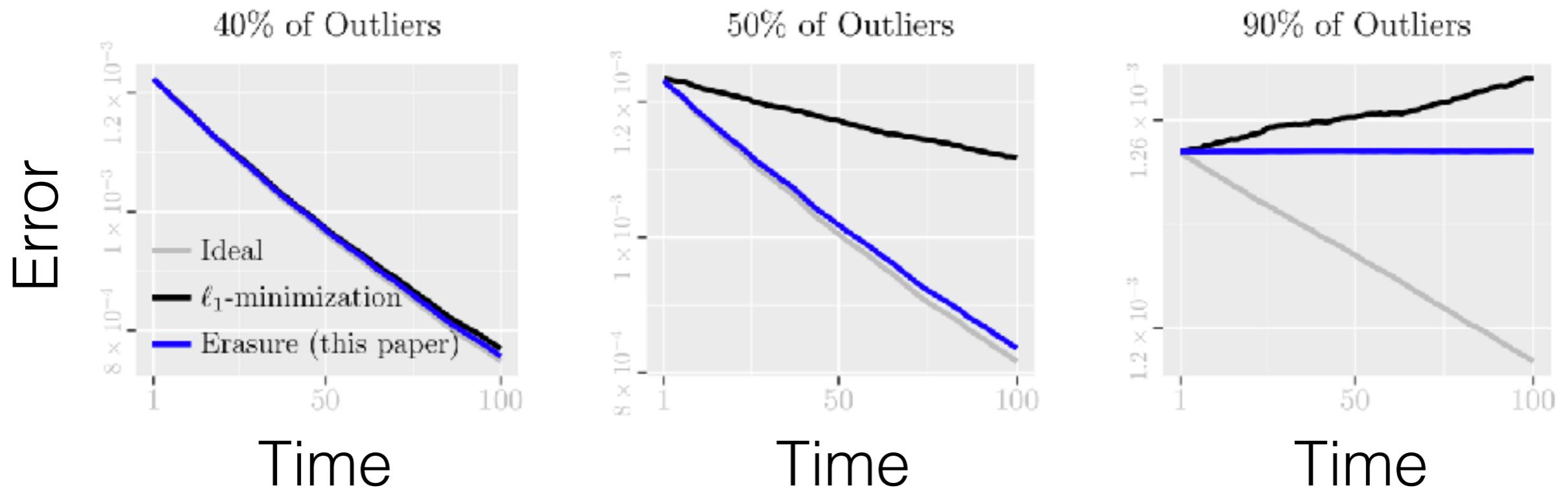


Detecting Outliers

(The lighter the better)



Detecting Outliers

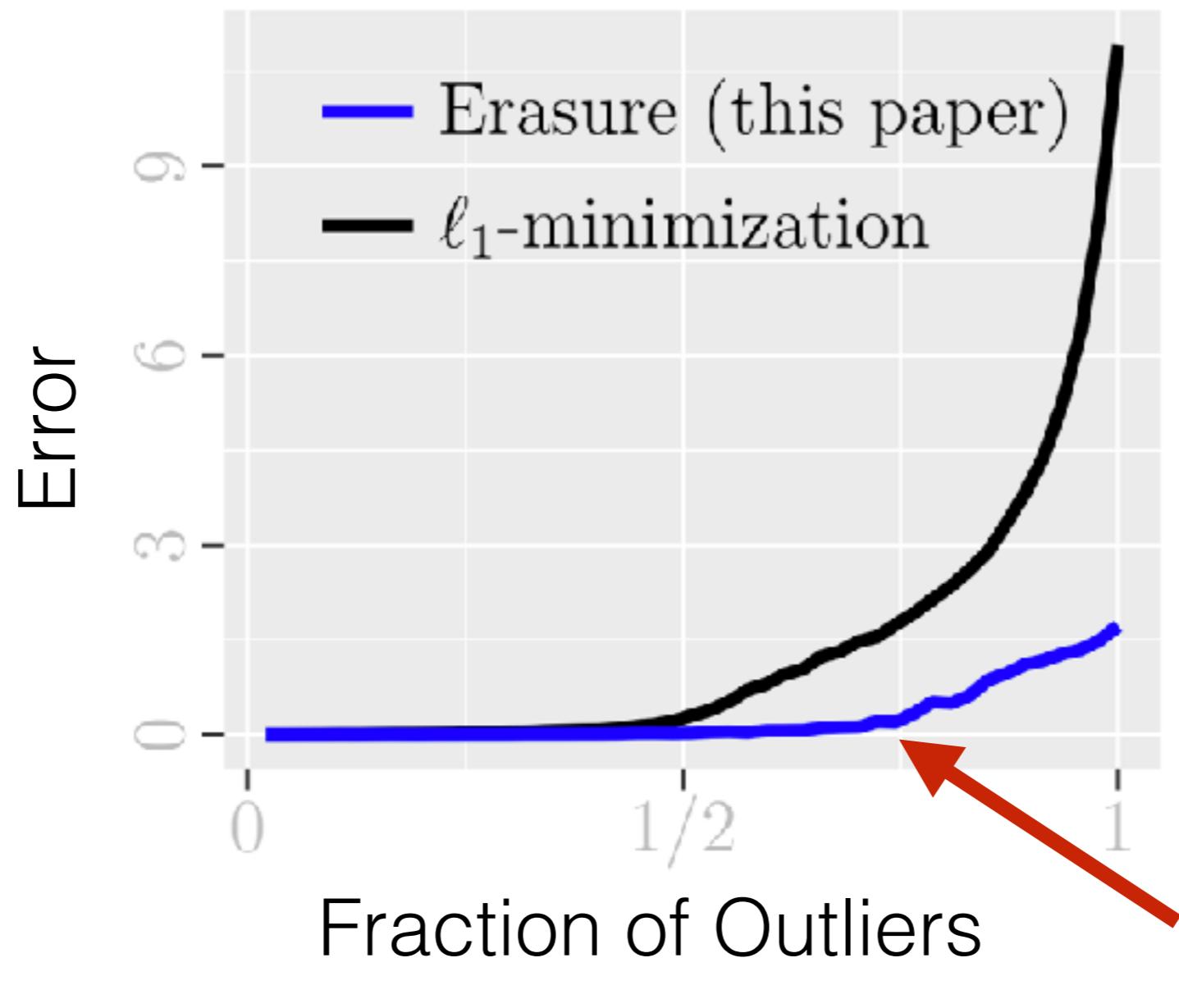


Subspace Tracking

Drawbacks?

No reason why it
should work!

Drawbacks?



Still room for improvement

Drawbacks?

Thank you!

pimentel@gsu.edu
<https://danielpimentel.github.io>