CS 8850: Advanced Machine Learning **Fall 2017**

Homework 5: Parameter Estimation

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Problem 5.1 (Subspace with gaussian noise)**.** Suppose you observe a random vector $\boldsymbol{x} \in \mathbb{R}^{\mathrm{D}}$ given by:

$$\boldsymbol{x} \;=\; \mathbf{U}\mathbf{c}^{\star} + \boldsymbol{\epsilon},$$

where $\mathbf{U} \in \mathbb{R}^{\mathrm{D}\times\mathrm{R}}$ is known, $\boldsymbol{\epsilon} \in \mathbb{R}^{\mathrm{D}} \sim \mathcal{N}(0, \mathbf{I})$. Find the MLE of $\mathbf{c}^{\star}$.

Problem 5.2 (Gaussian MLE)**.** Let $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{\mathrm{N}}$ be D-dimensional random vectors distributed i.i.d. $\mathcal{N}(\boldsymbol{\mu}^{\star}, \boldsymbol{\Sigma}^{\star})$.

(a) Find the MLEs $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$ of $\boldsymbol{\mu}^{\star}$ and $\boldsymbol{\Sigma}^{\star}$.

(b) Are these unbiased estimators? If not, can you propose modified estimators that are unbiased?

(c) What are the MLEs of the eigenvalues and eigenvectors of $\boldsymbol{\Sigma}^{\star}$? Hint: consider the invariance property of the MLE.