CS 4780/6780: Fundamentals of Data Science © Spring 2019

Homework 3: Review of Probability

Instructor: Daniel L. Pimentel-Alarcón Due 03/07/2019

Problem 3.1. Consider a 6-faced fair die.

(a) What is the probability of obtaining an even number?

(b) Given that the die rolled an odd number, what is the probability that it was a 3?

Problem 3.2. Let x be a random variable with the following distribution:

$$\mathbb{P}(x = \mathrm{k}) \;=\; \begin{cases} 1/2^{\mathrm{k}} & \text{if } 1 \le \mathrm{k} \le 3 \\ 1/8 & \text{if } \mathrm{k} = 4 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $\mathbb{E}[x]$.

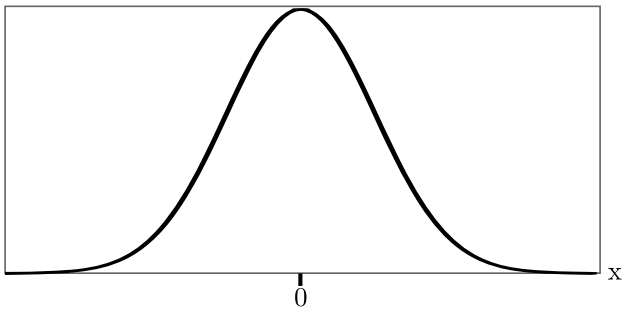
(b) Compute $\mathbb{E}[x^2]$.

(c) Compute $\mathsf{var}(x)$.

Problem 3.3. Recall that the Normal distribution (also known as gaussian) with mean μ and variance σ^2 is given by:

$$\mathsf{p}(\mathrm{x}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\mathrm{x}-\mu}{\sigma}\right)^2}$$

The following figure shows this density when $\mu = 0$ and $\sigma^2 = 1$. In this same figure, sketch the gaussian distribution with mean $\mu = 1$ and $\sigma^2 = 4$.



Problem 3.4. Consider the following samples from an i.i.d. random variable:

Variable	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
x	-0.5	-1.6	-1.5	-2.4	2.1
y	0.8	-0.6	-2.1	-0.5	2.1

Compute the sample covariance of x and y.