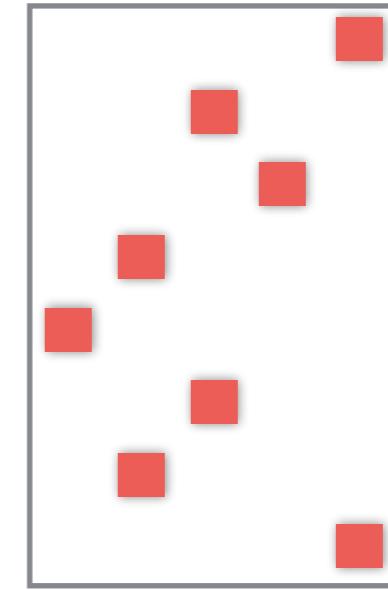
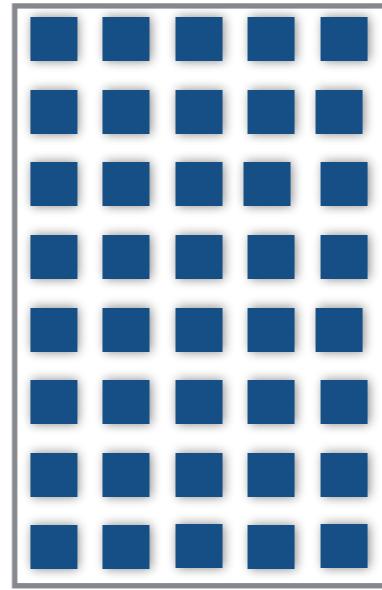
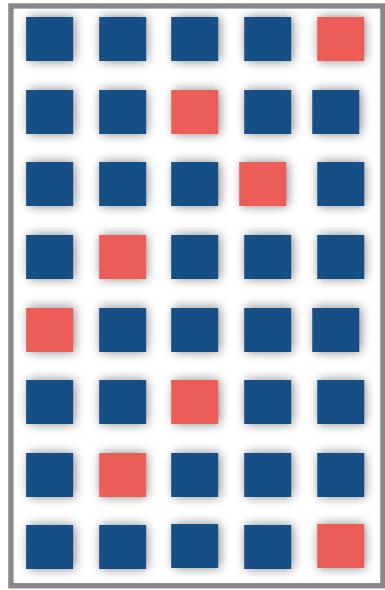


Random Consensus Robust PCA

Daniel Pimentel-Alarcón & Robert Nowak

Wisconsin Institute for Discovery
UNIVERSITY *of* WISCONSIN-MADISON
Department of Electrical and Computer Engineering

AISTATS 2017



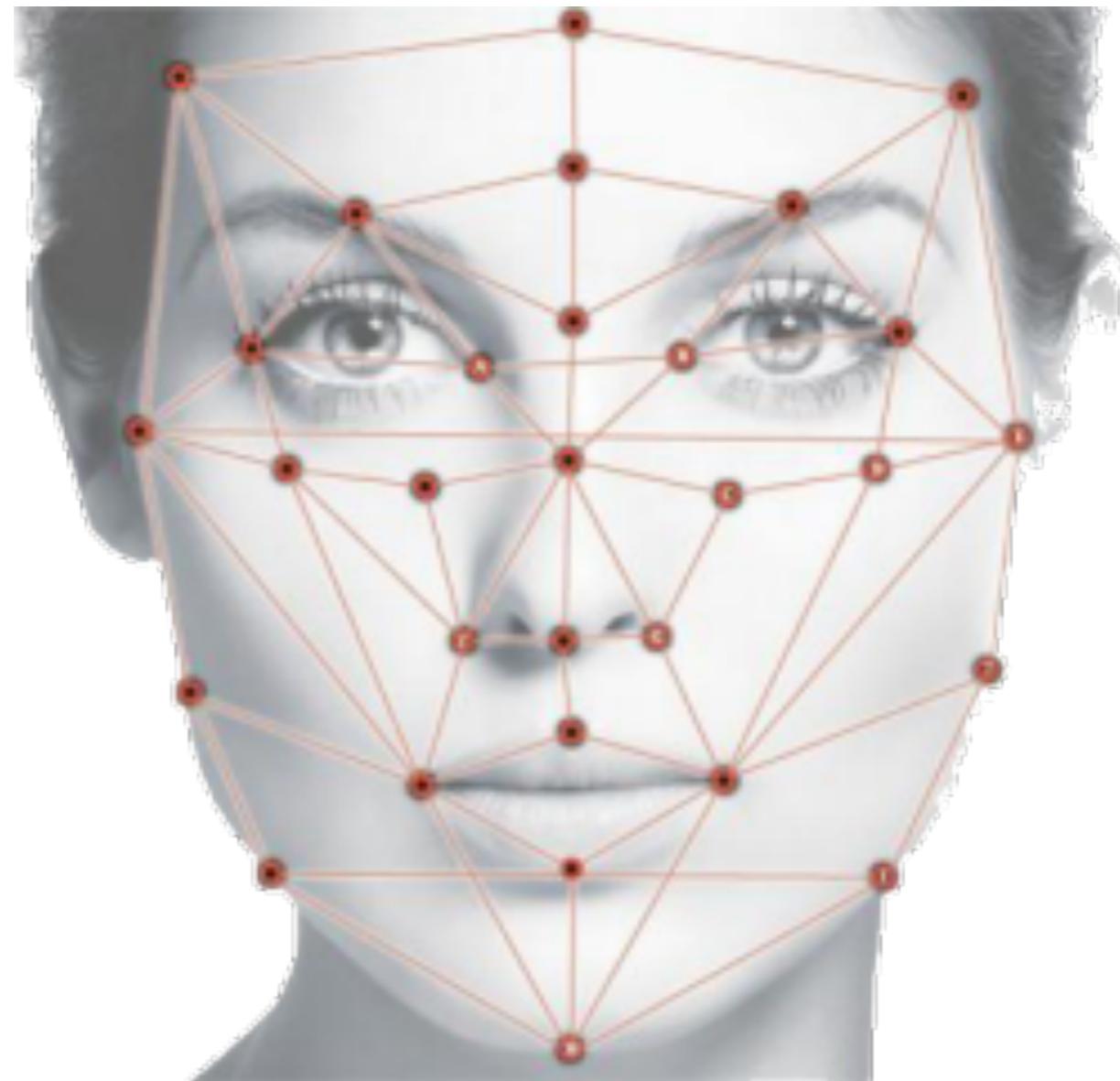
$$X = L + S$$

Low-Rank Sparse

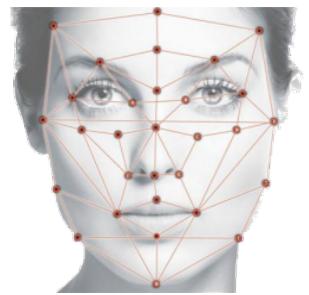
Robust PCA



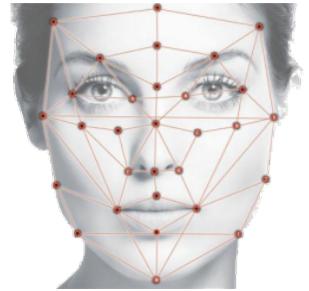
What is
this good
for?



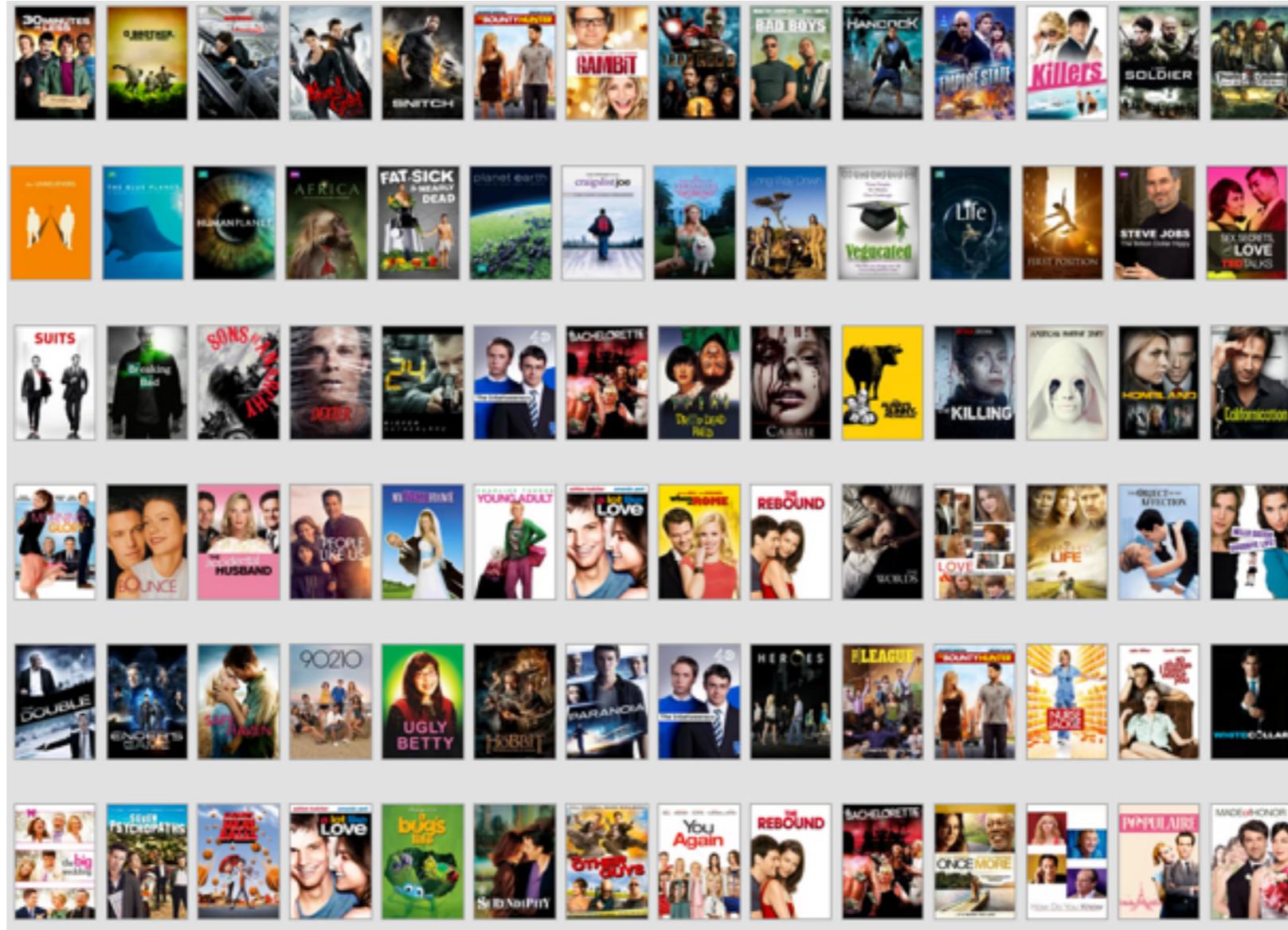
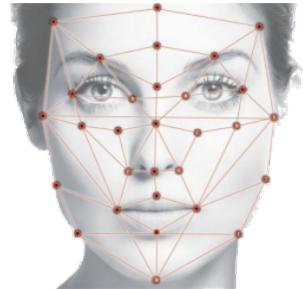
Lots of Applications



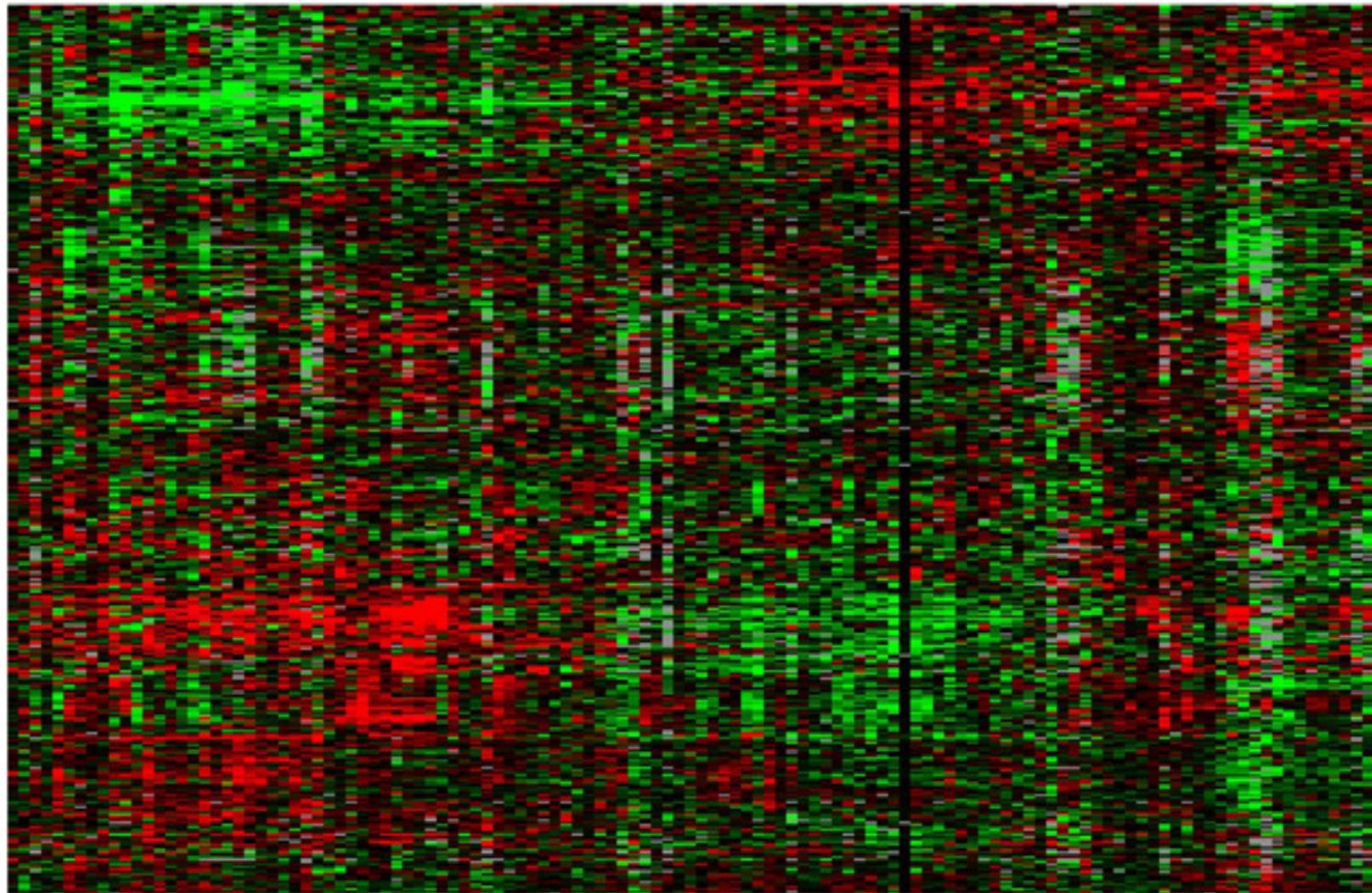
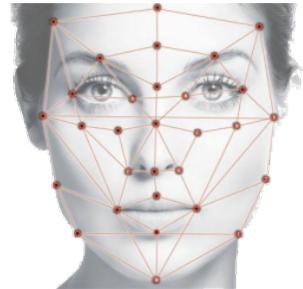
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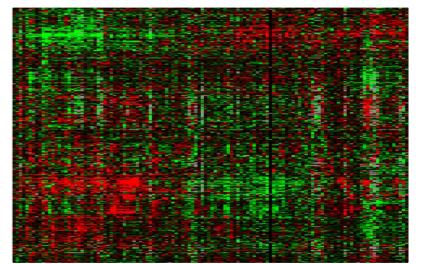
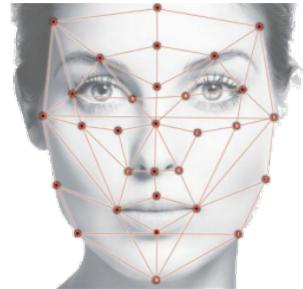
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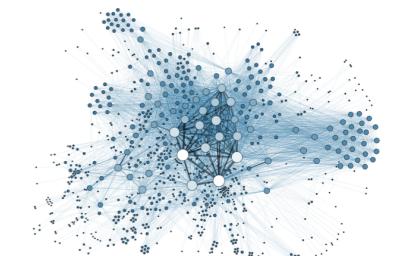
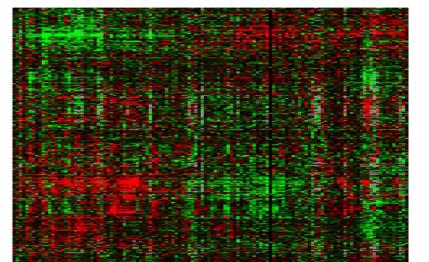
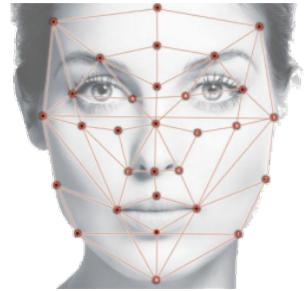
Lots of Applications



Lots of Applications



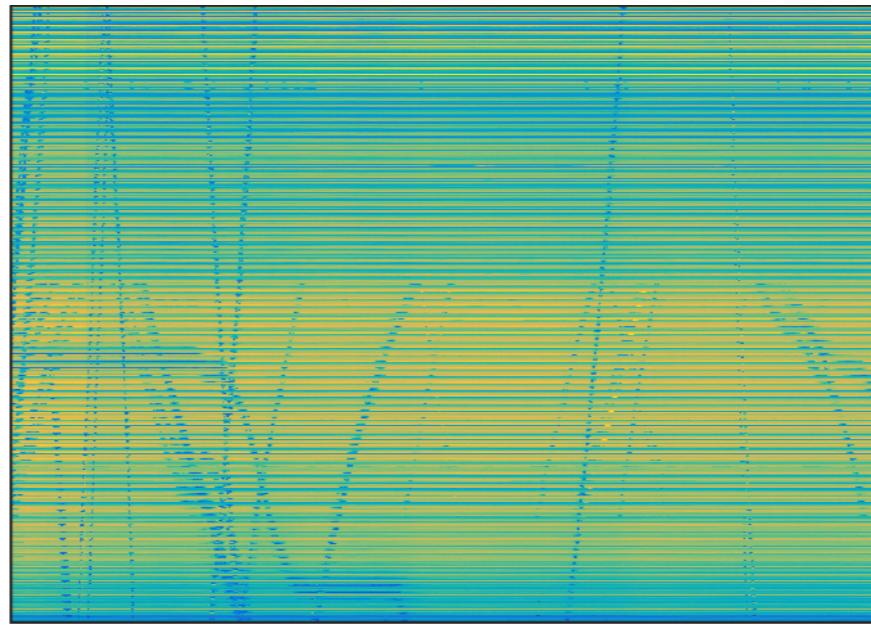
Lots of Applications



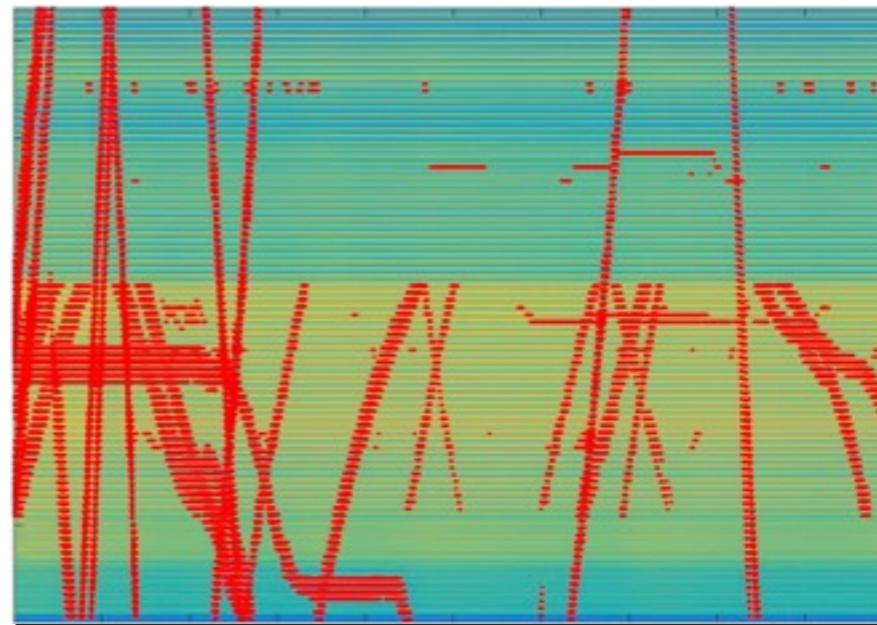
Lots of Applications



Background segmentation



Background segmentation



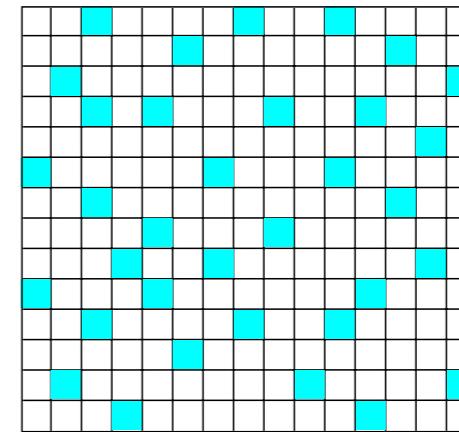
Background segmentation

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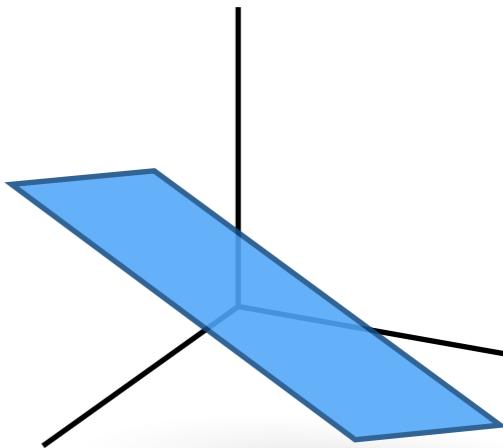
$$\begin{aligned} & \text{minimize} && \|\mathbf{L}\|_* + \lambda \|\mathbf{S}\|_1 \\ & \text{subject to} && \mathbf{X} = \mathbf{L} + \mathbf{S} \end{aligned}$$

Existing theory

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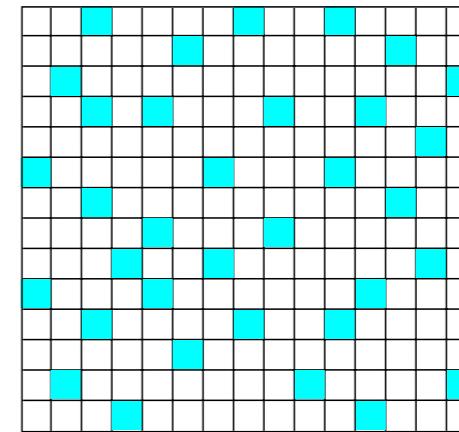


Uniform Sampling + Incoherence

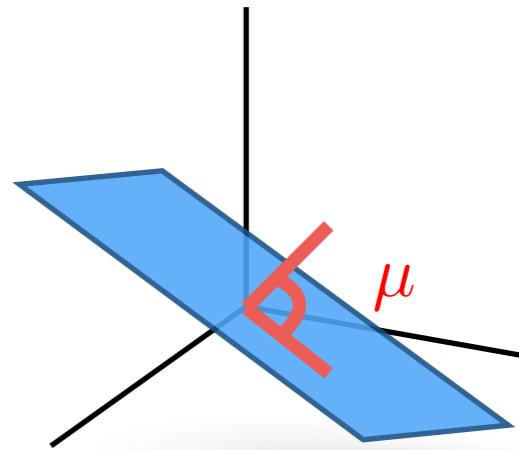


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- [1] F. De La Torre and M. Black, *A framework for robust subspace learning*, International Journal of Computer Vision, 2003.
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Uniform Sampling + Incoherence

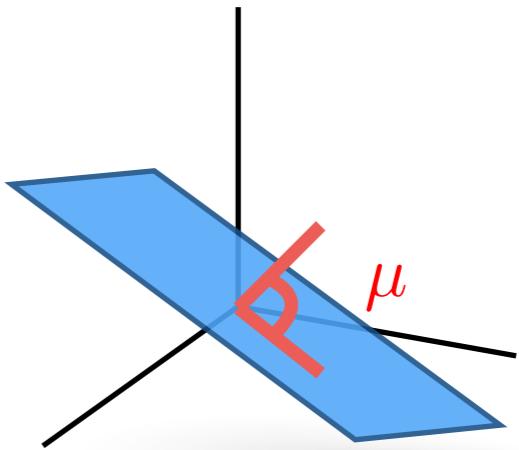


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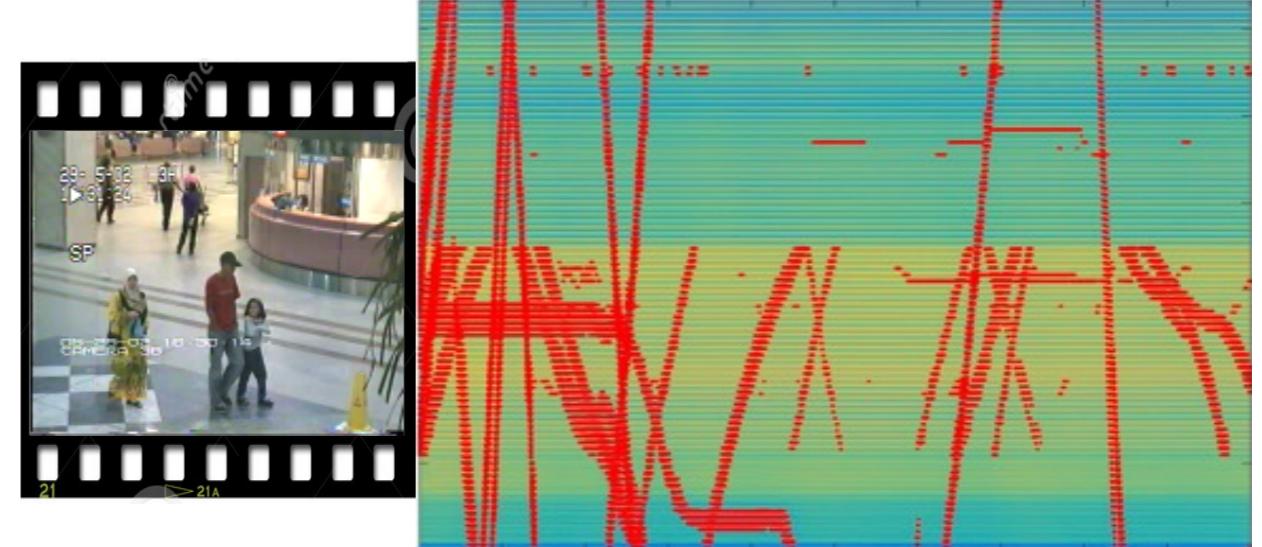


Uniform Sampling + Incoherence

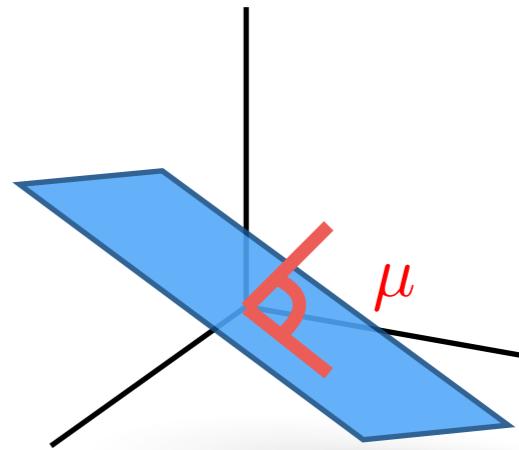


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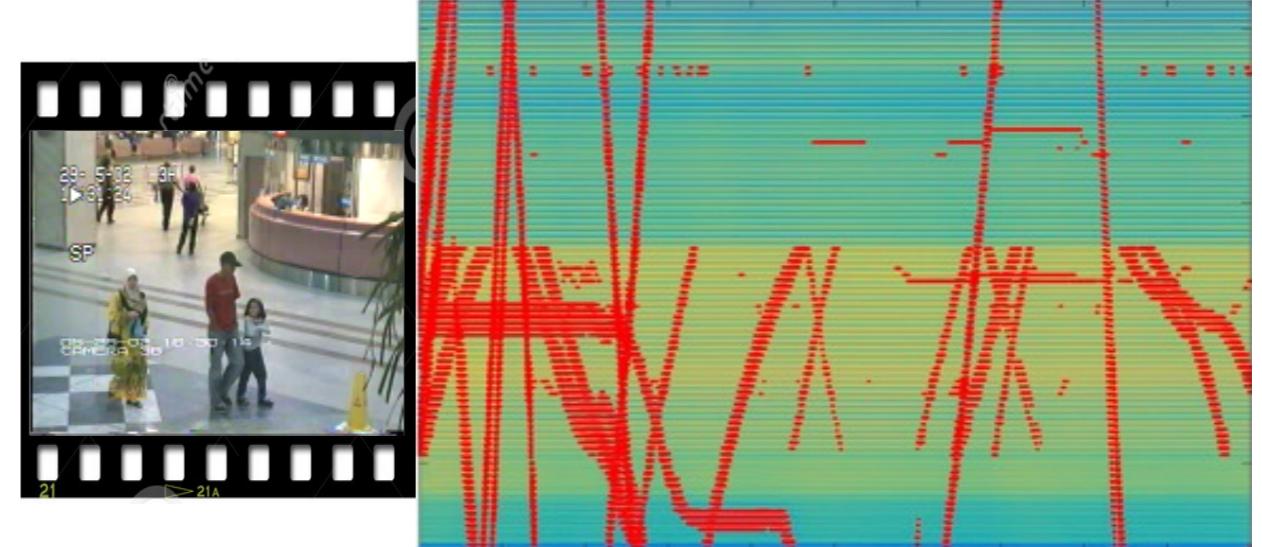


Uniform Sampling + Incoherence

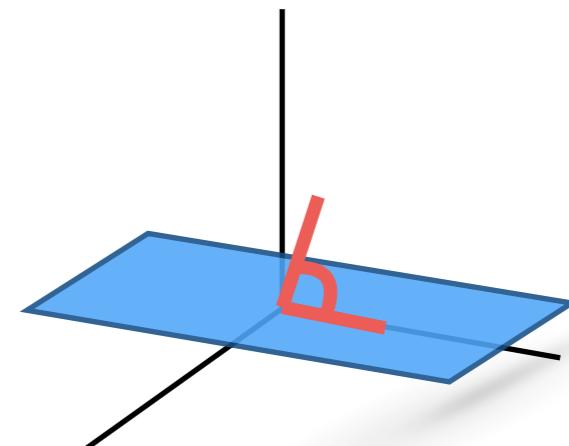


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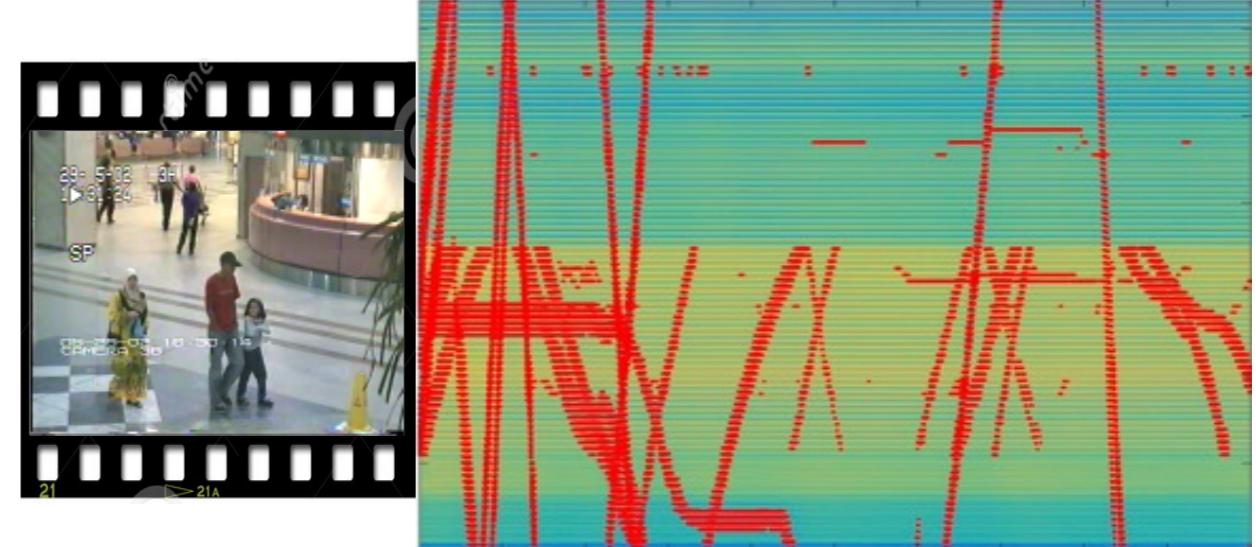


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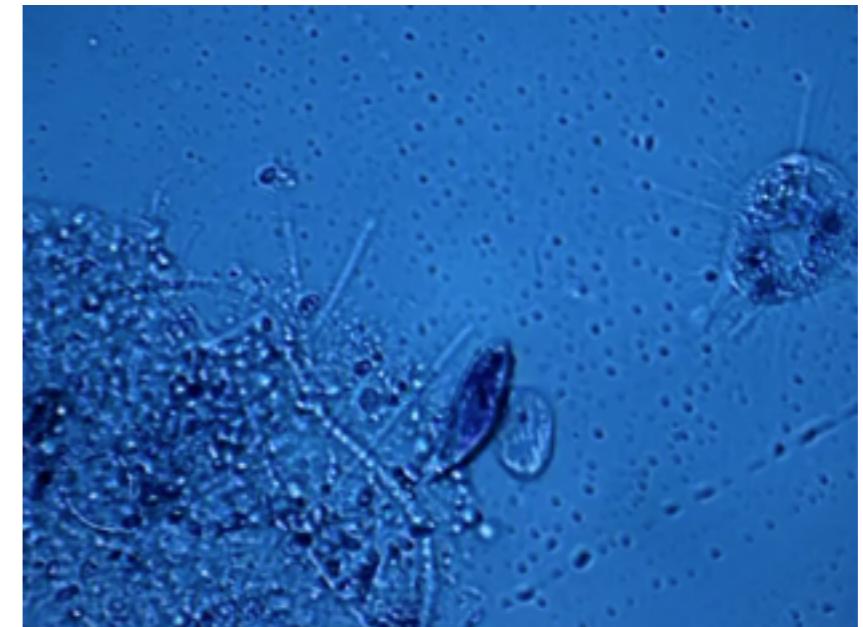


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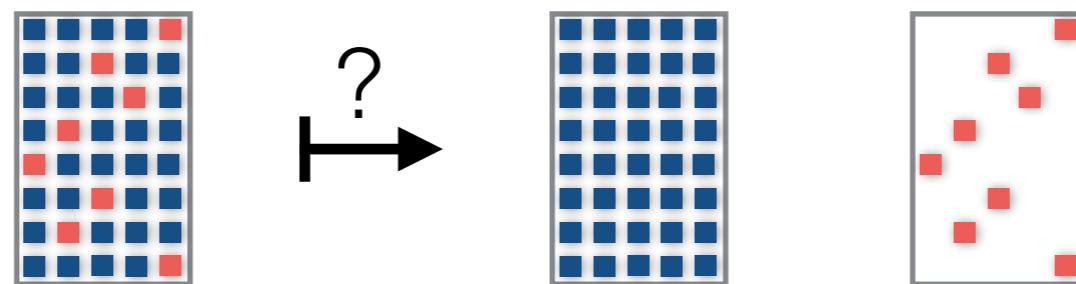


Uniform Sampling + Incoherence



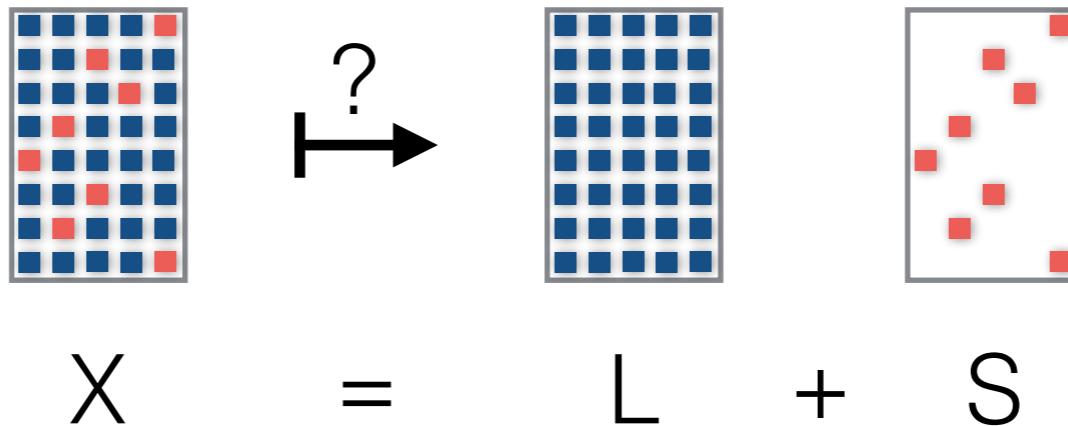
Existing theory

In general



$$X = L + S$$

In general



To answer this:

Totally different way to think about the problem

- ~~Incoherence~~
- ~~Uniform~~
- ~~With high probability~~
- ~~Optimization~~
- Arbitrary
- Deterministic
- With probability 1
- Algebraic/Geometric



THE FOLLOWING **PREVIEW** HAS BEEN APPROVED FOR
ALL AUDIENCES
BY THE MOTION PICTURE ASSOCIATION OF AMERICA INC.

THE FILM ADVERTISED HAS BEEN RATED

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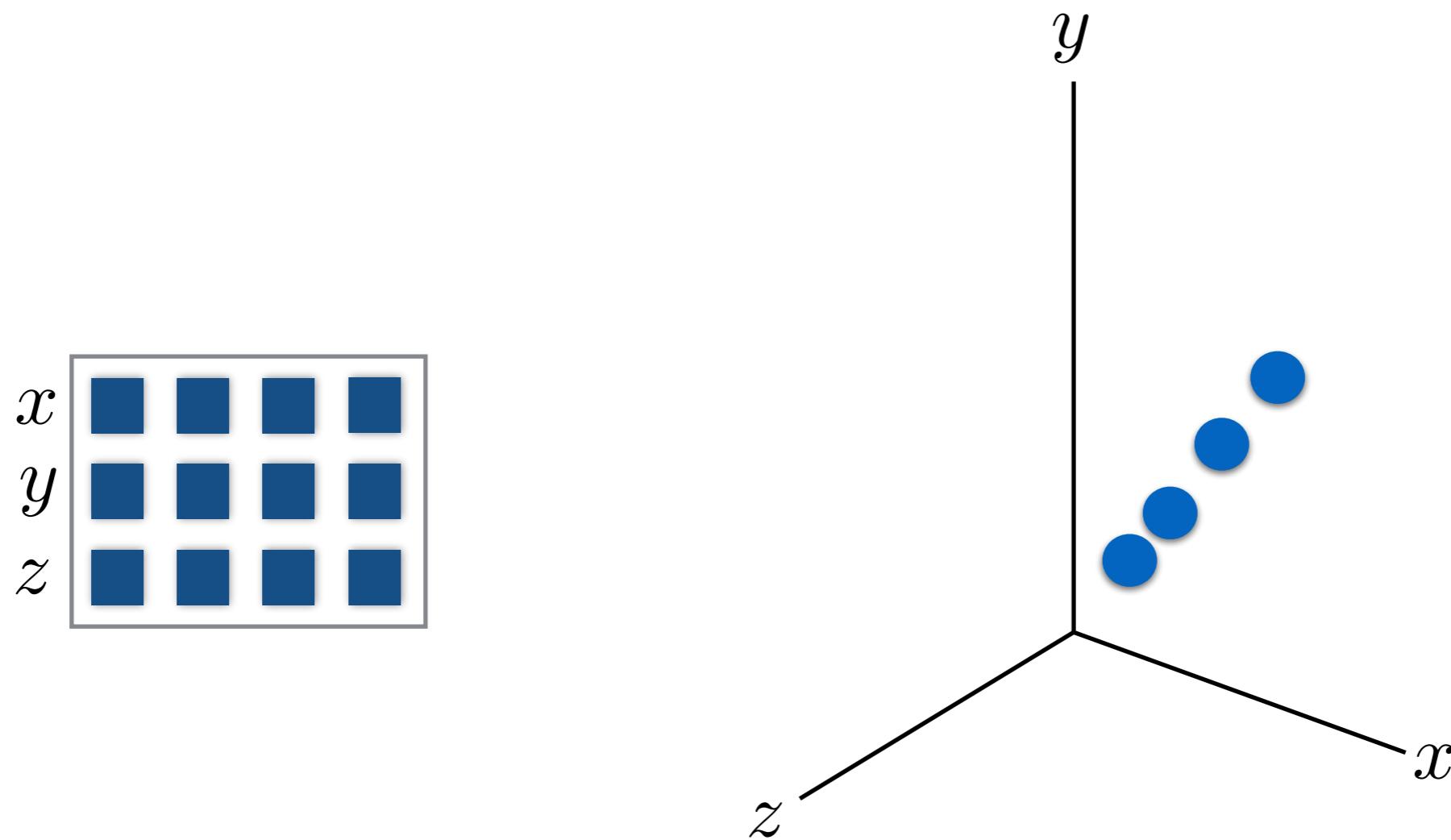
RESTRICTED

UNDER 17 REQUIRES ACCOMPANYING PARENT OR GUARDIAN

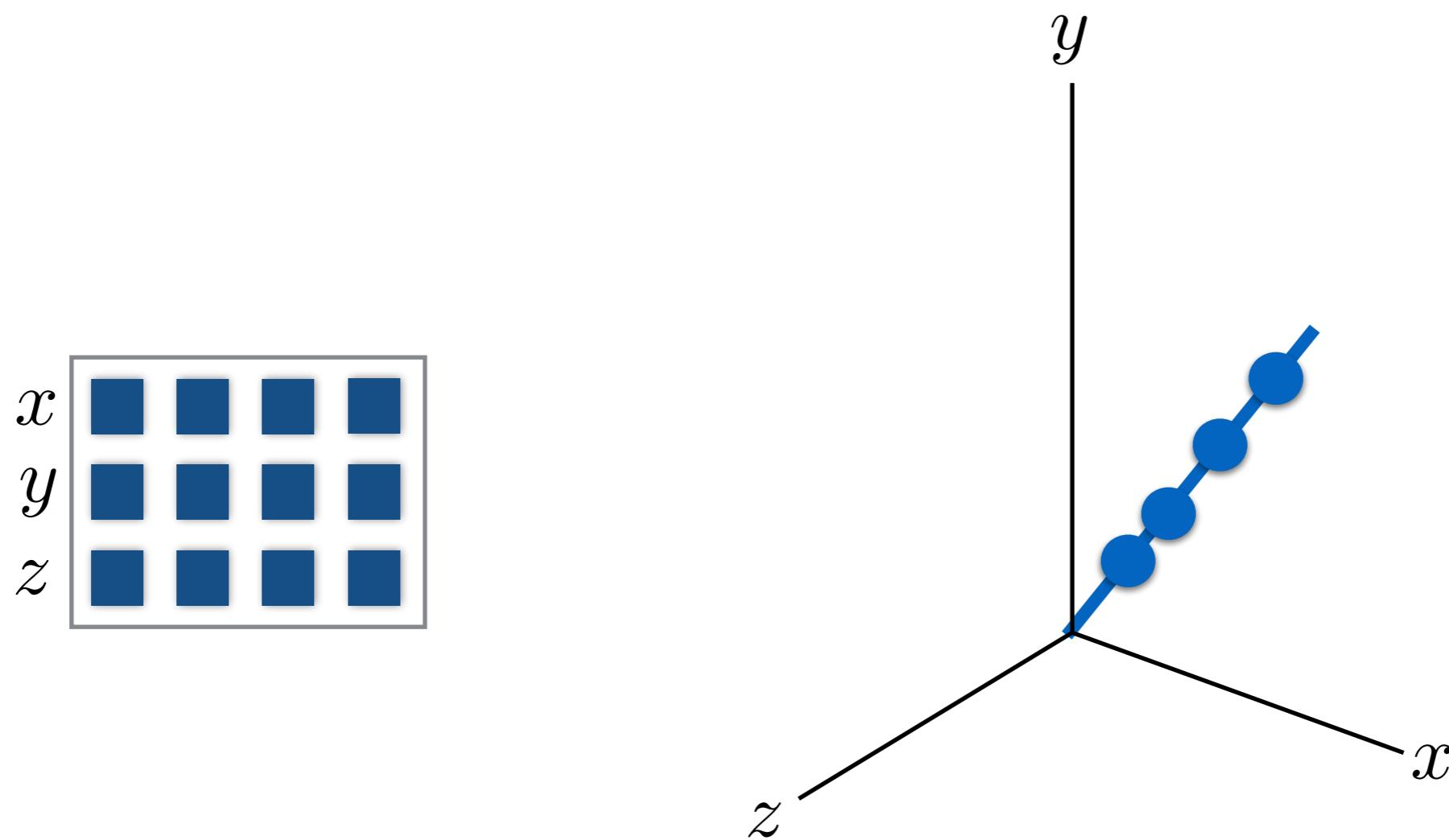
GEOMETRY

www.filmratings.com

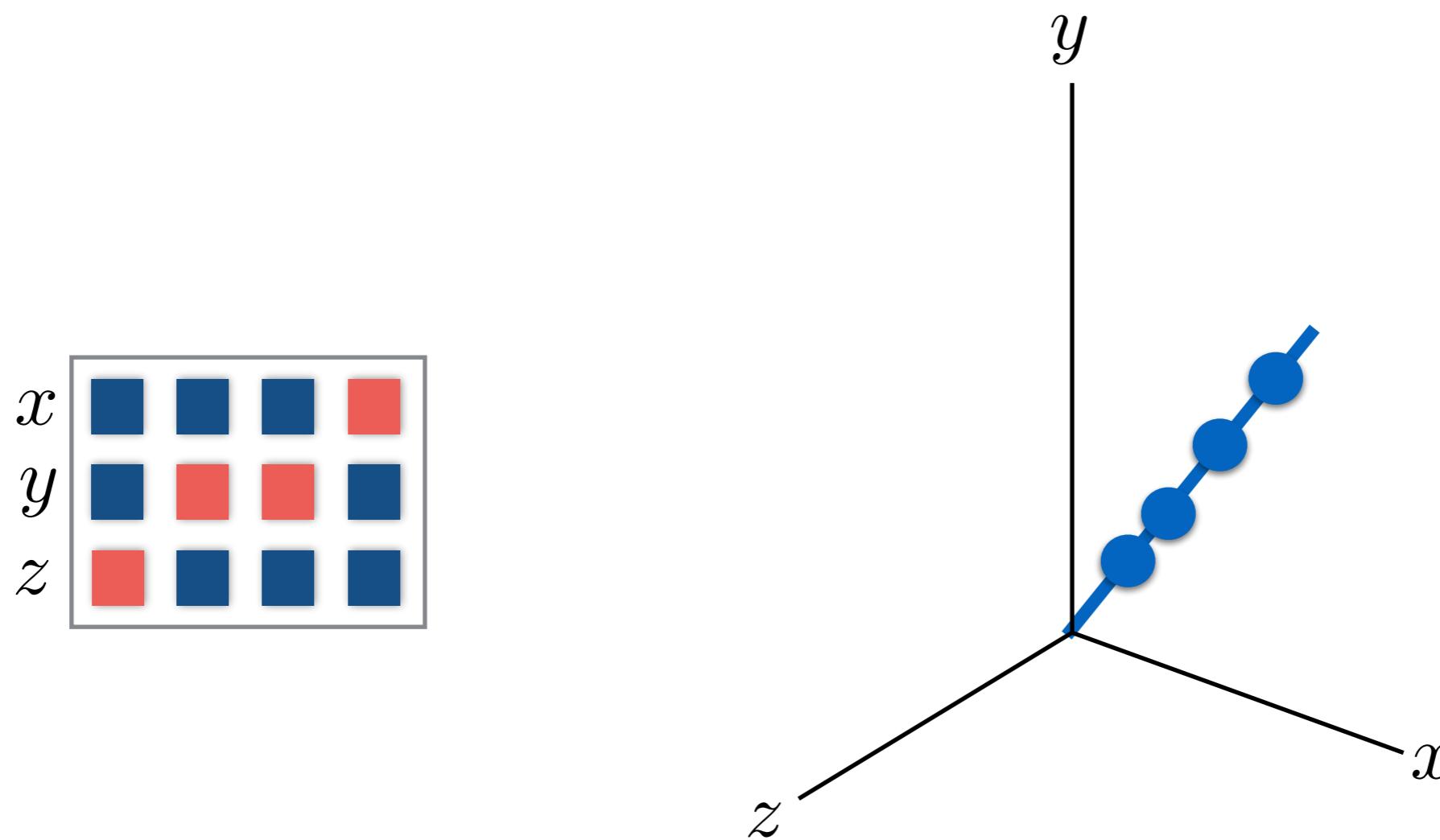
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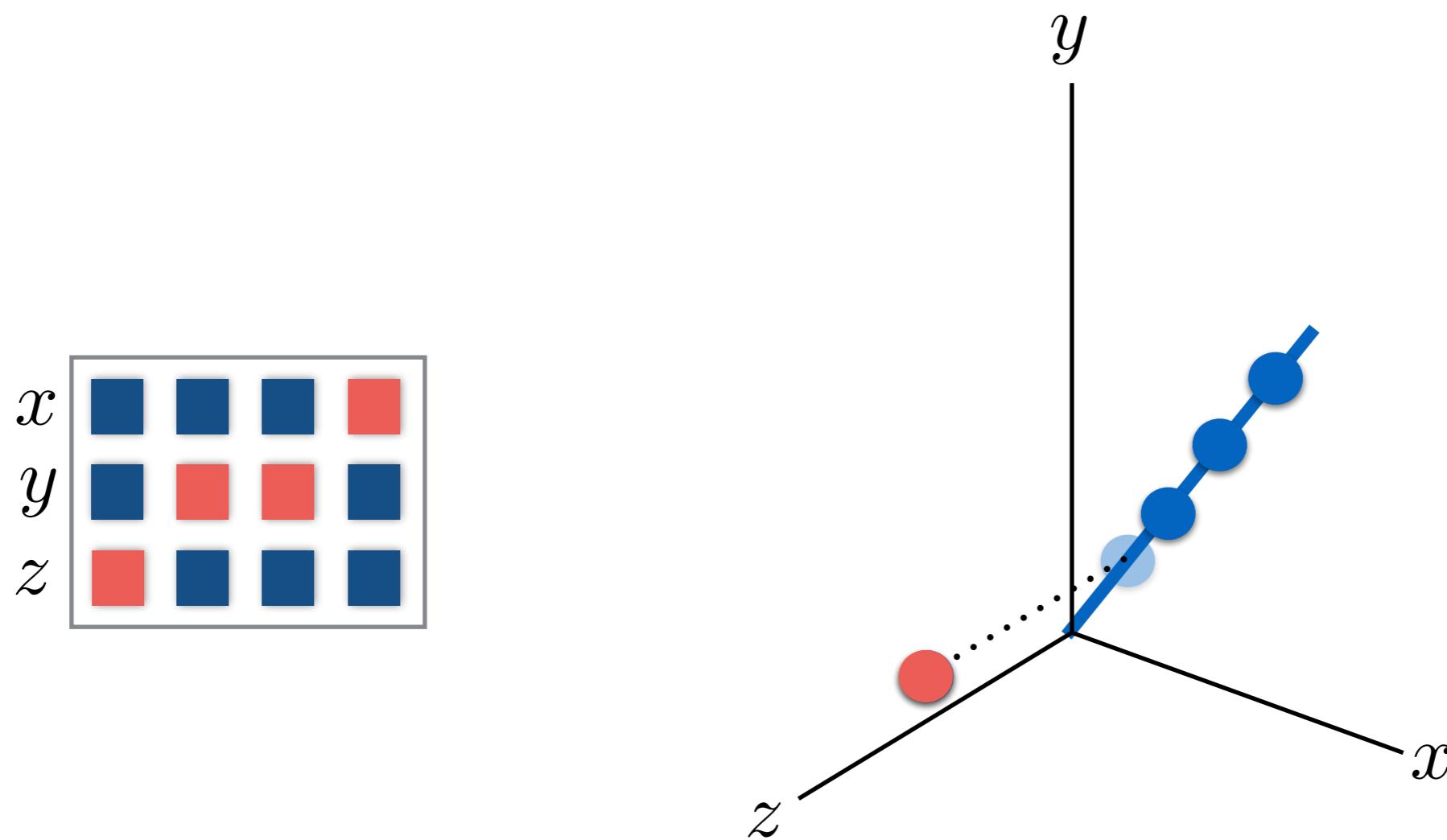
- **PCA:** Finds Subspace that Explains Data.



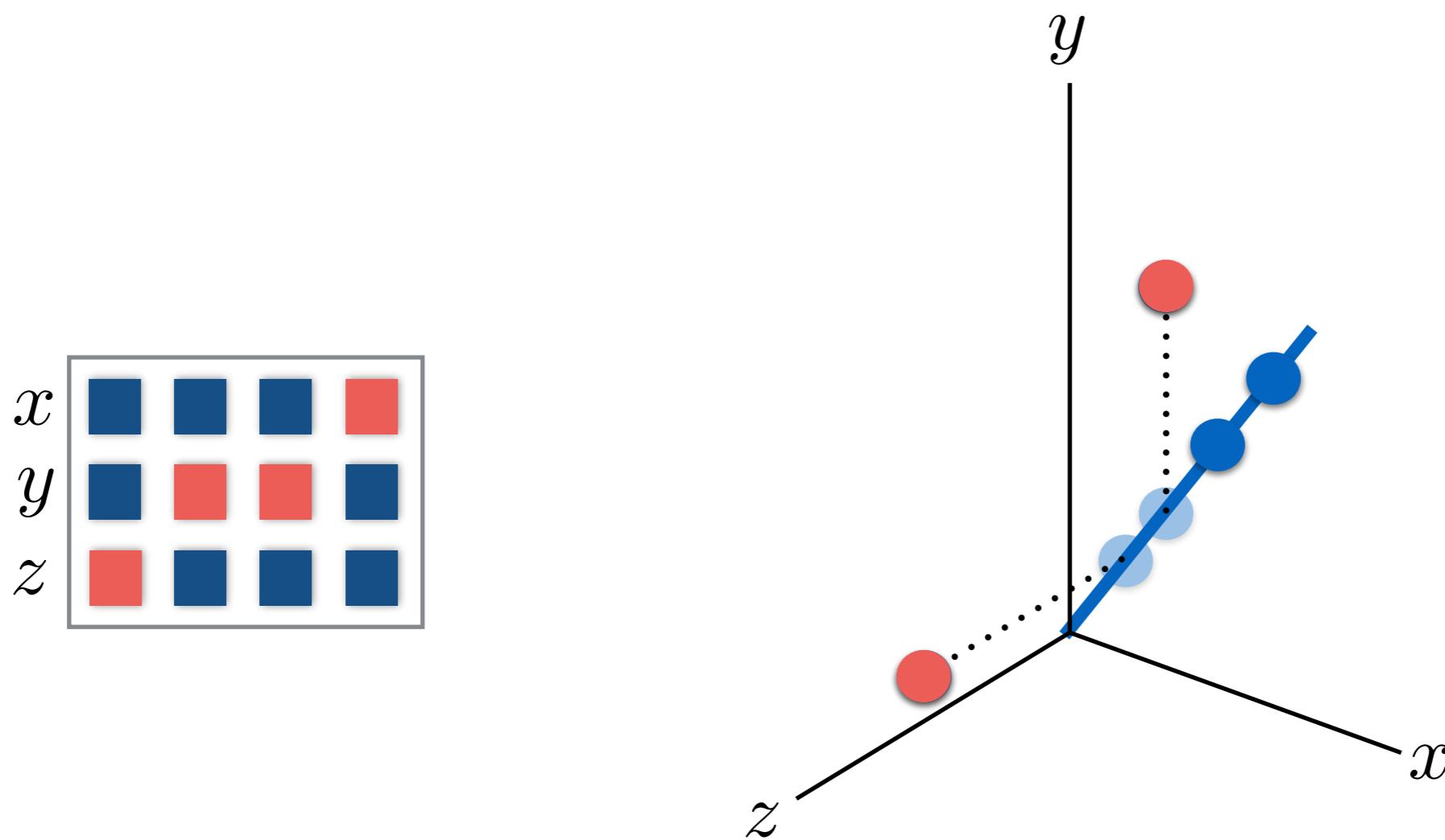
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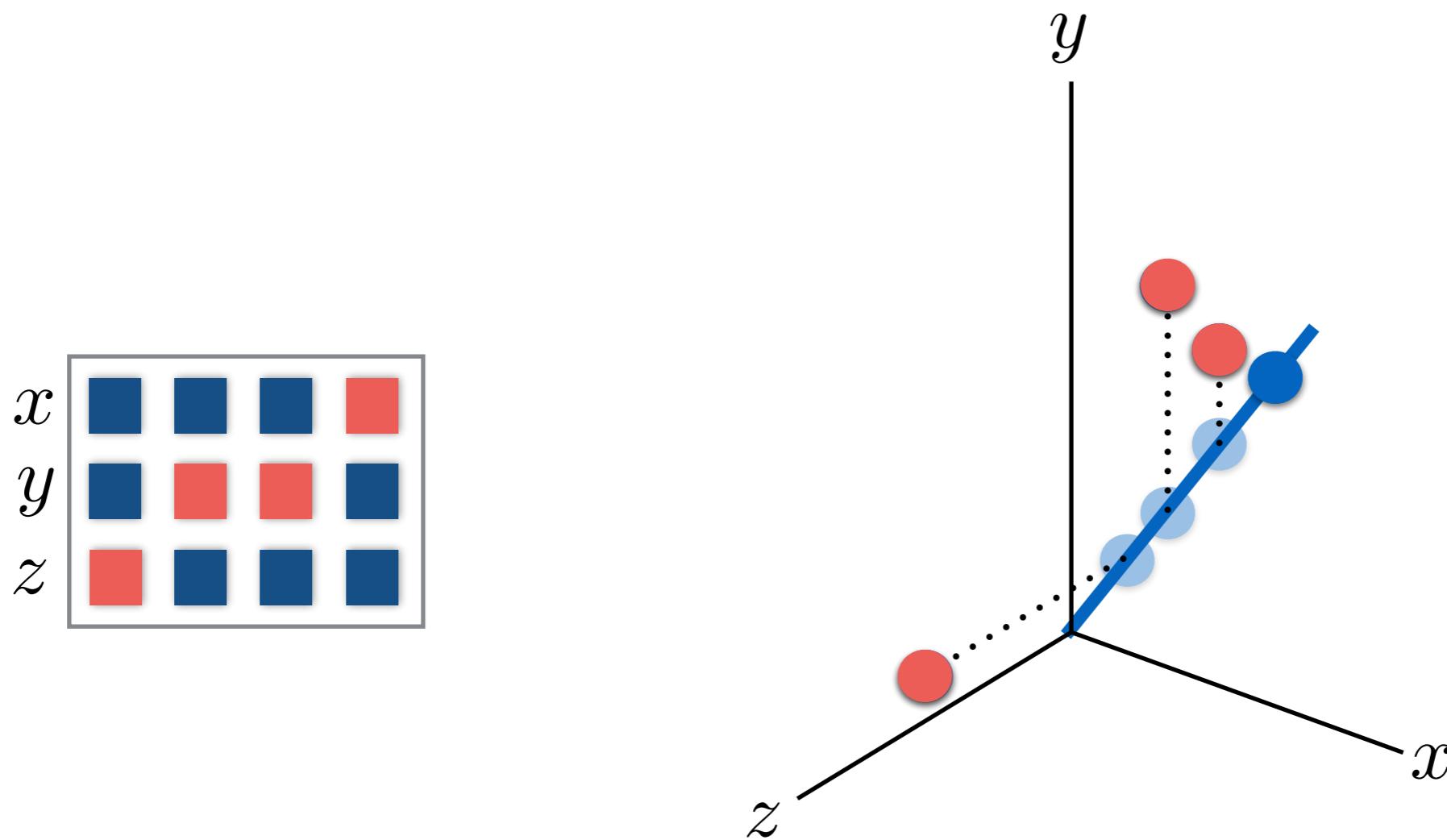
- **PCA:** Finds Subspace that Explains Data.
- **Complication:** corrupted entries in **EVERY** column!



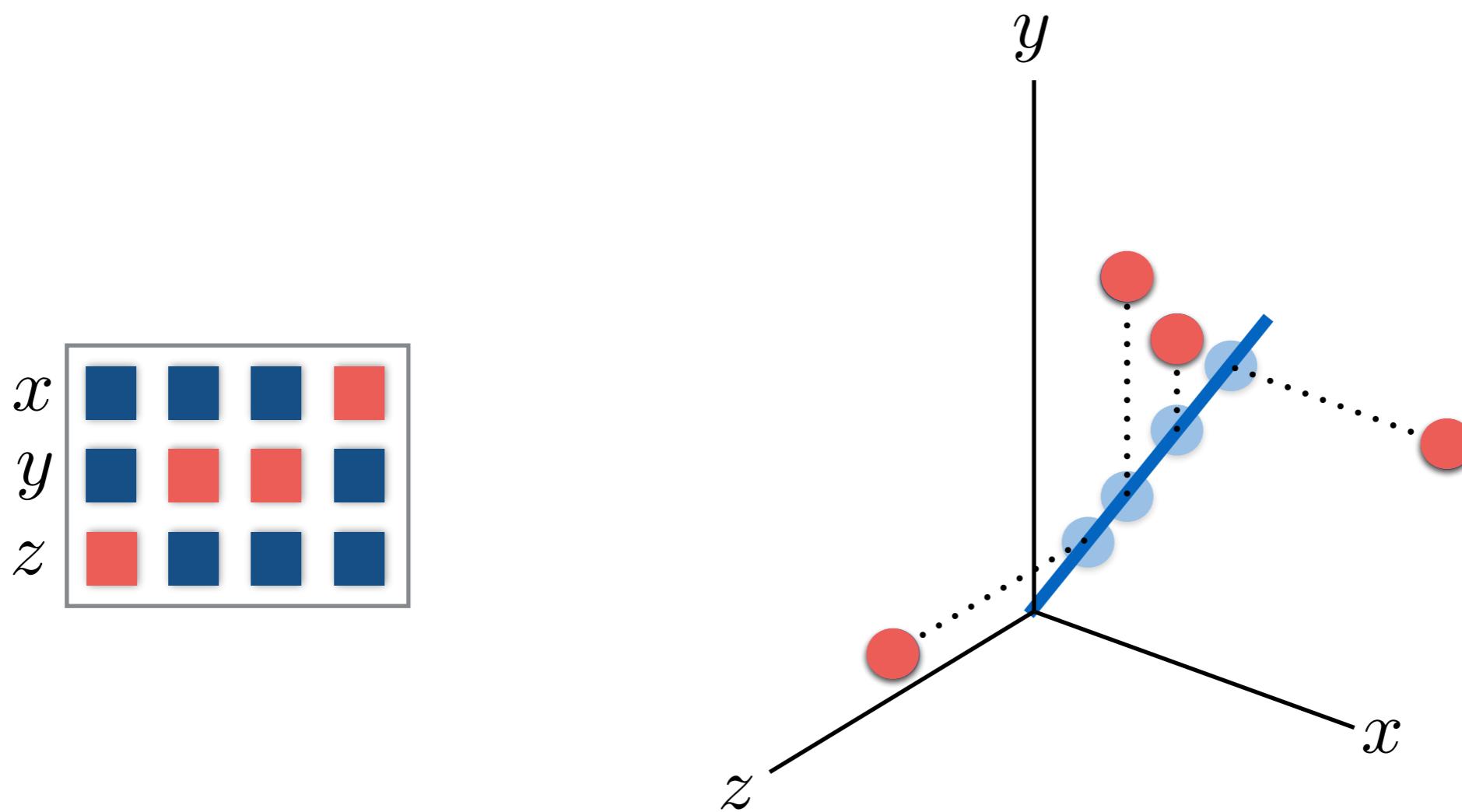
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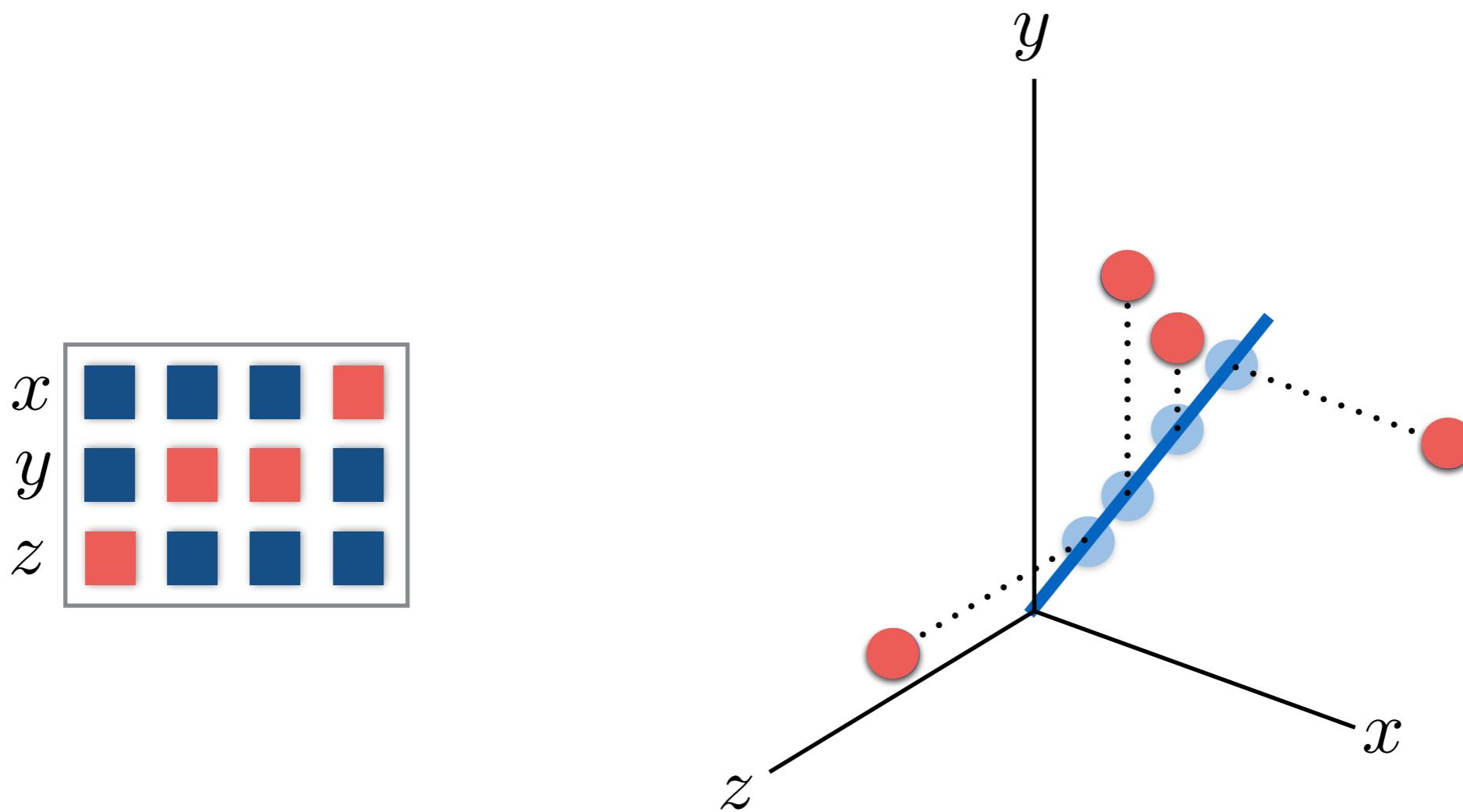
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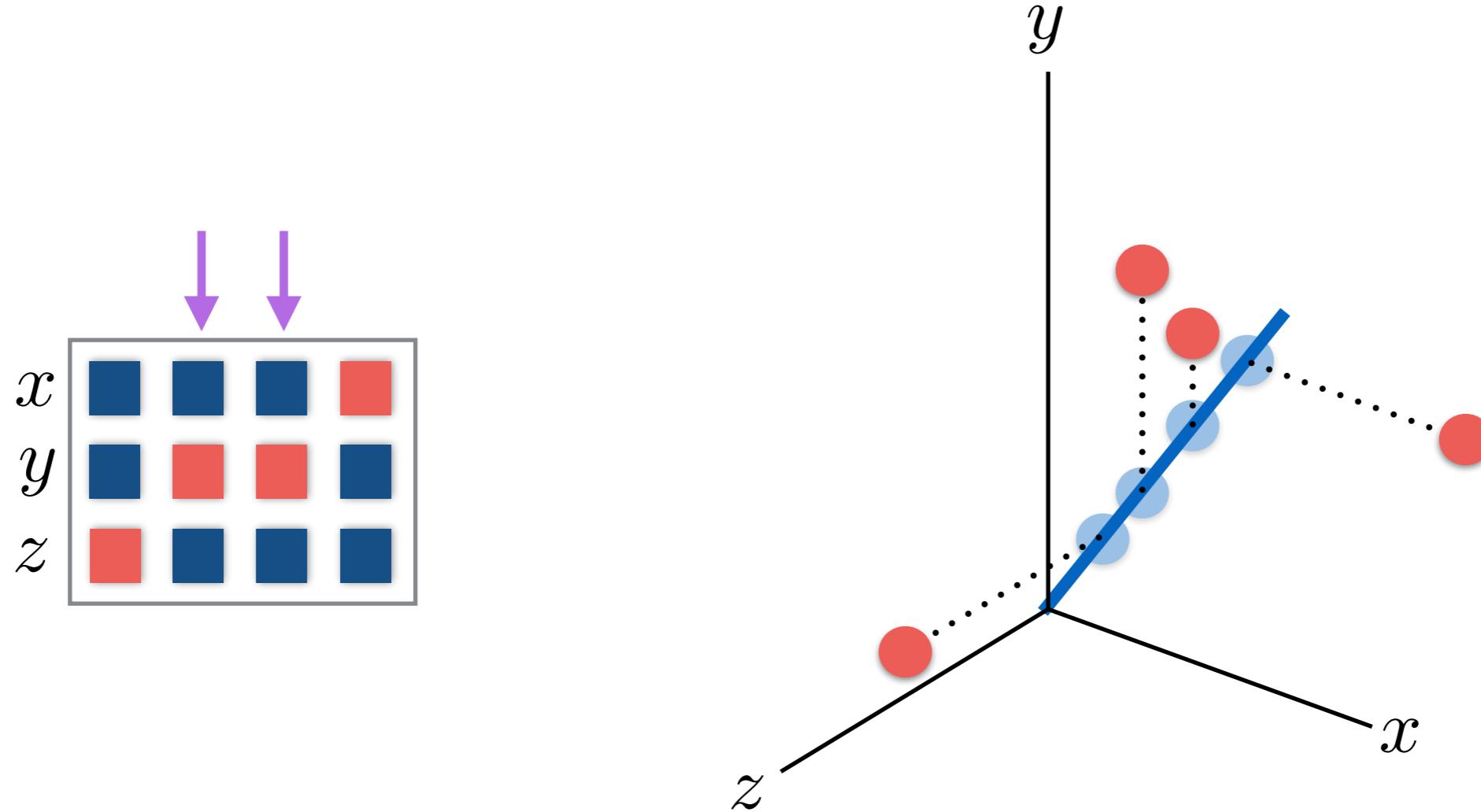
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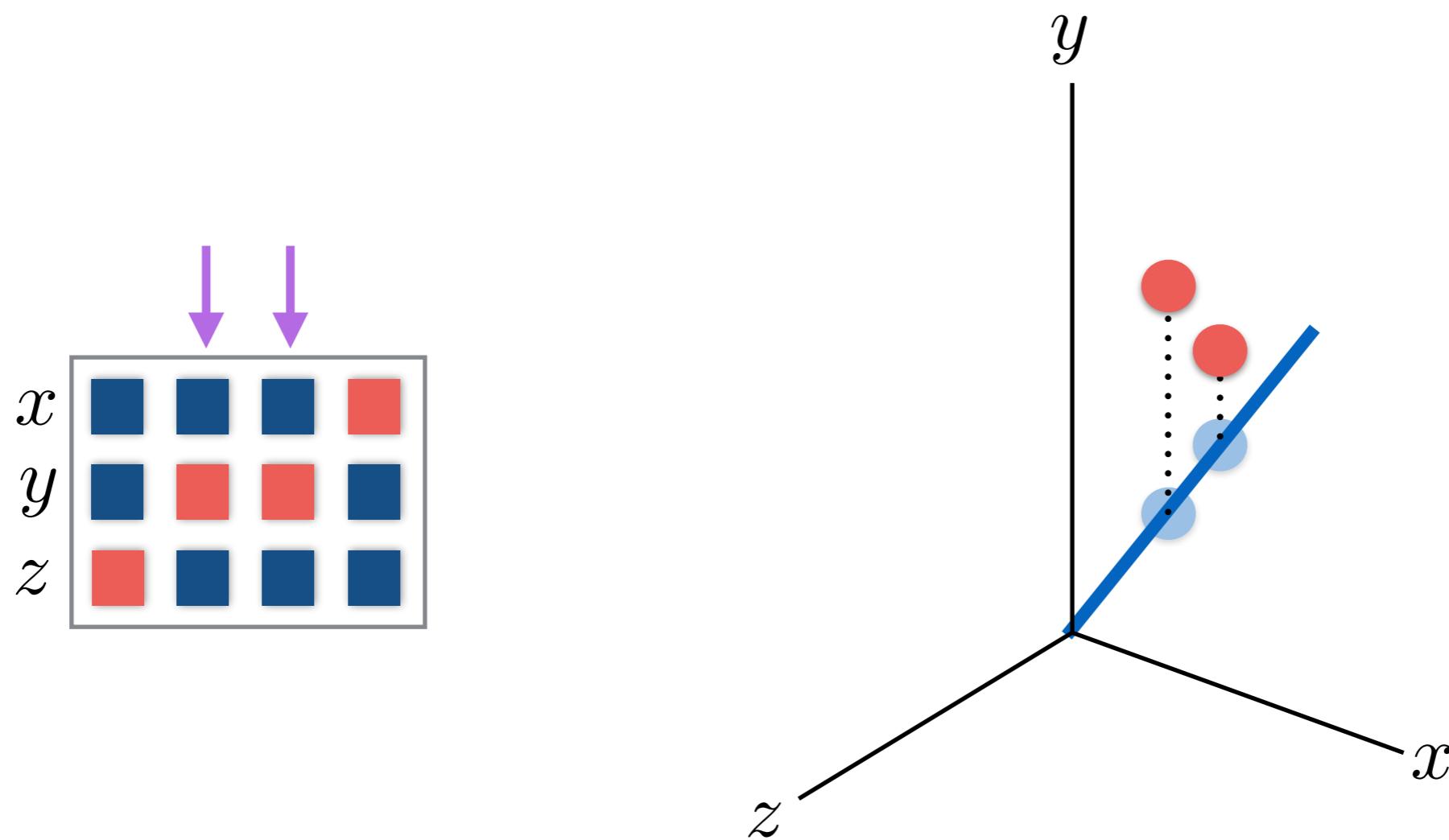


- **PCA**: Finds Subspace that Explains Data.
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- ALL columns are **outliers**!



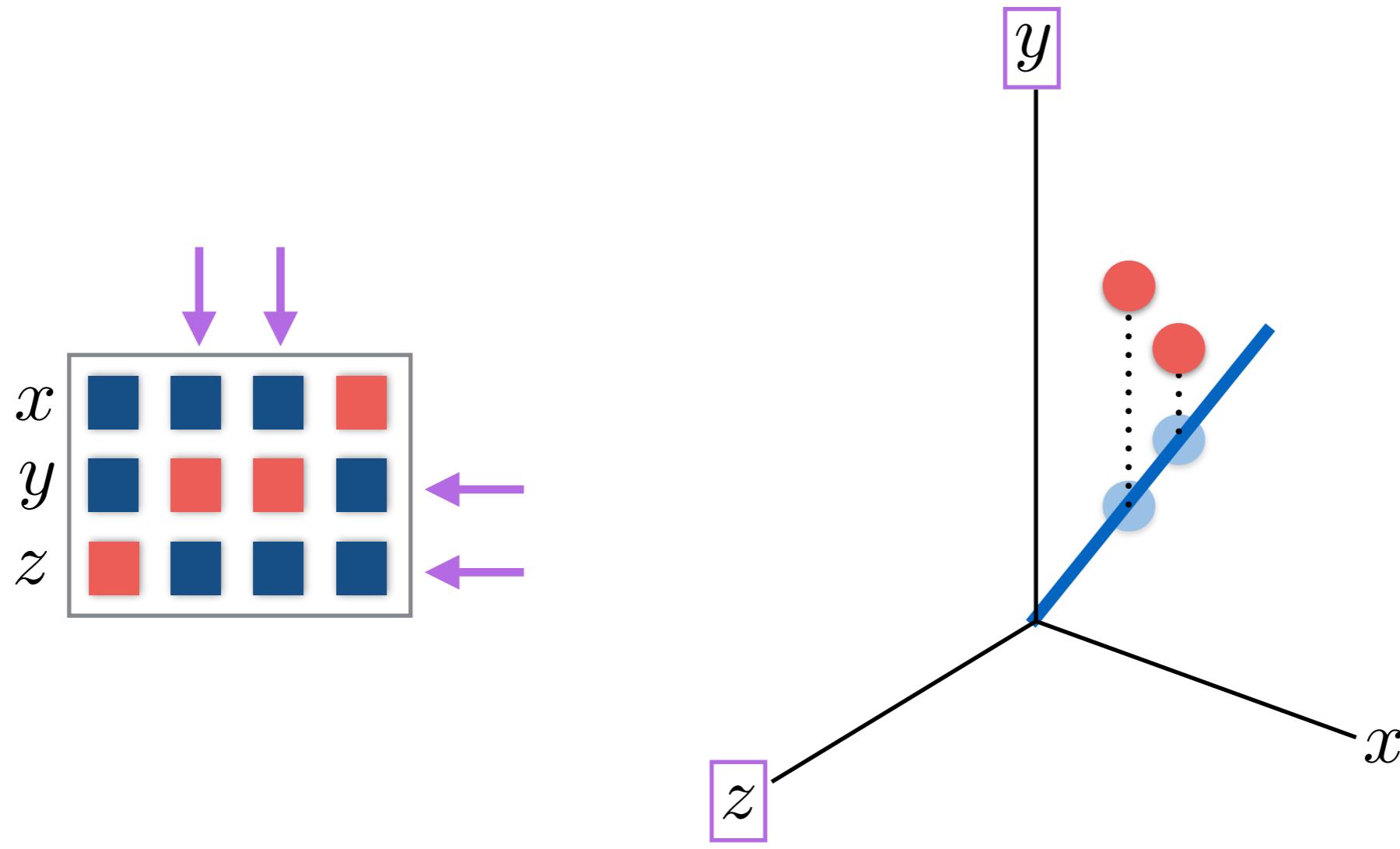
Our main idea

- Take a few columns at a time (as RANSAC)



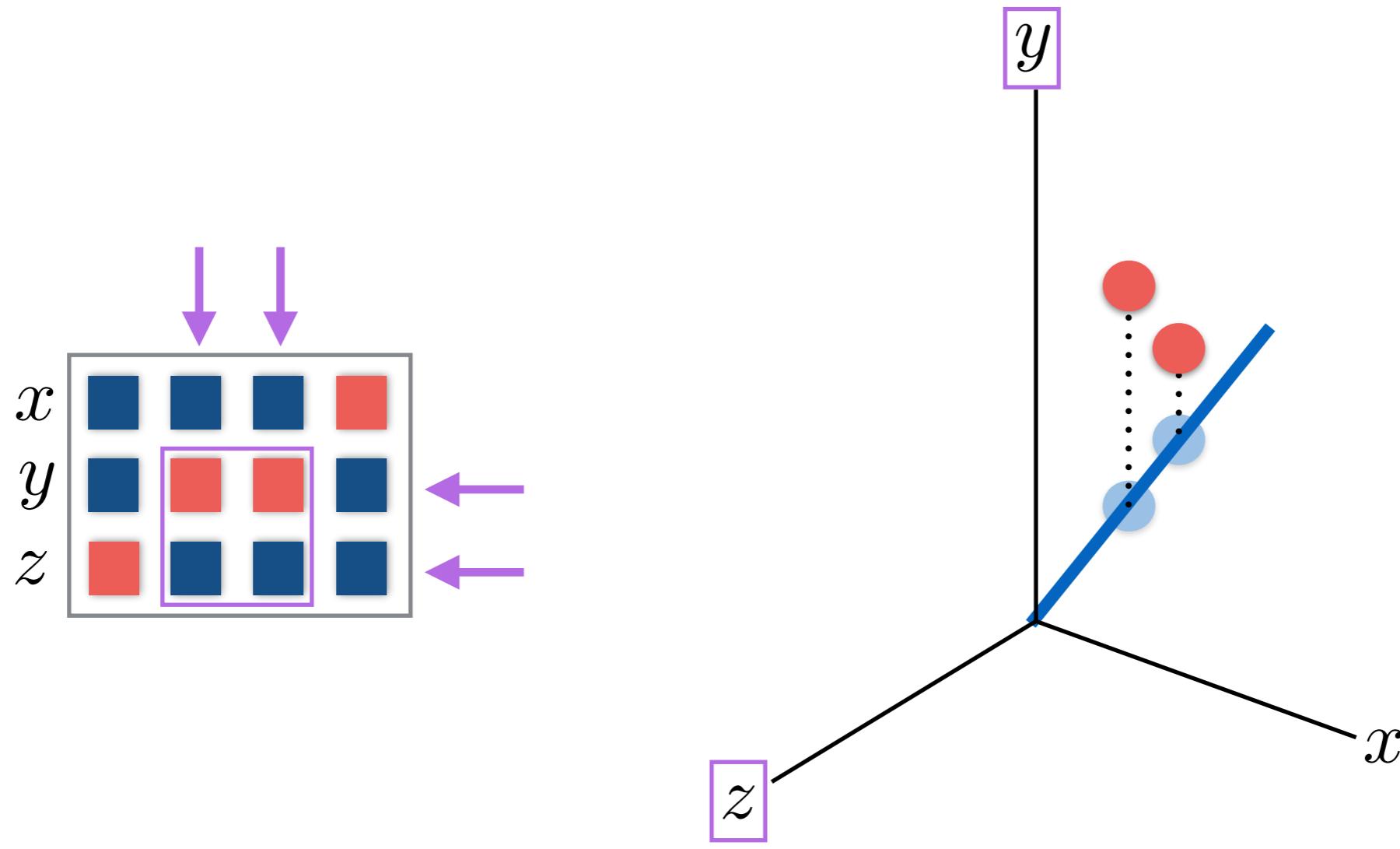
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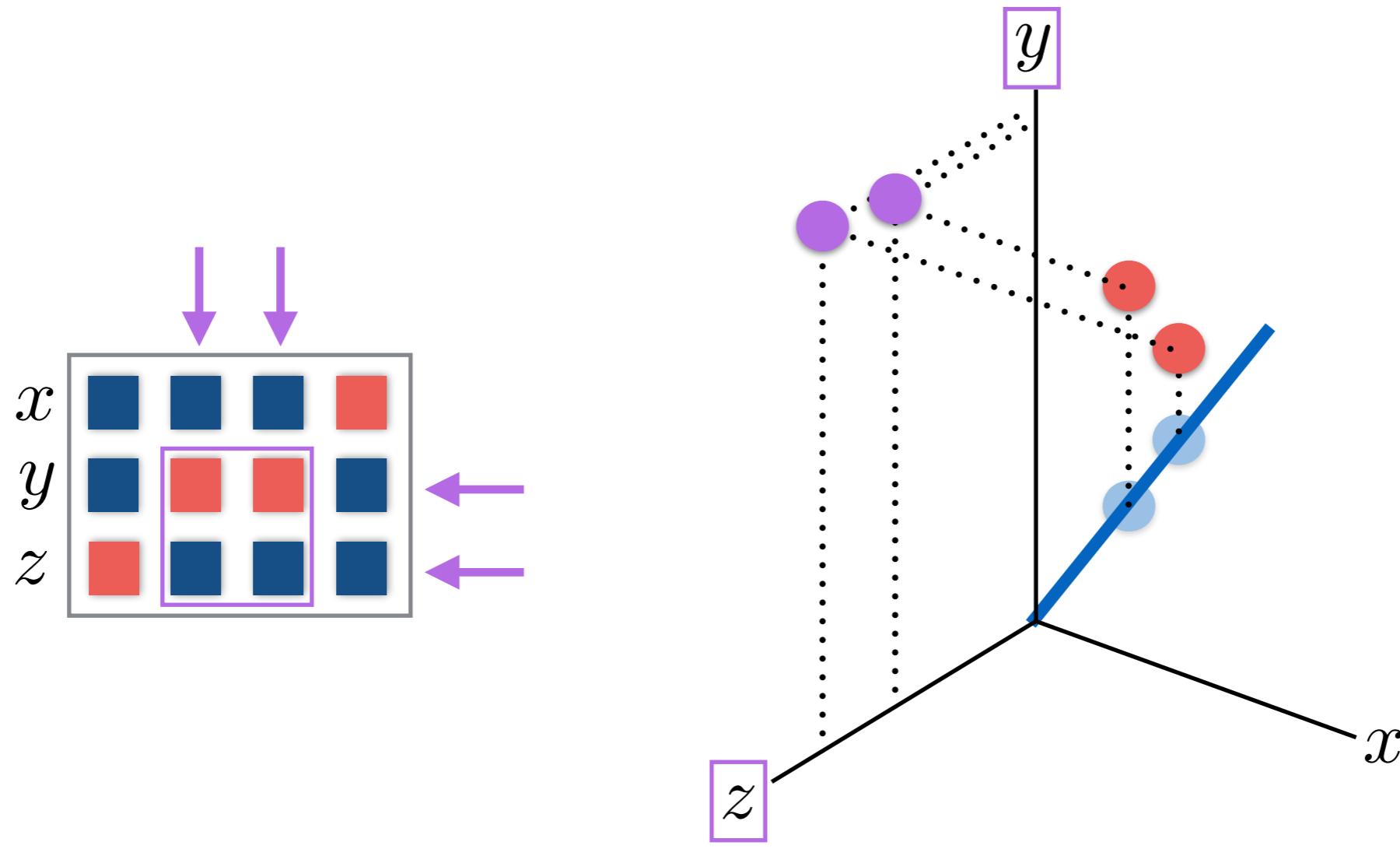
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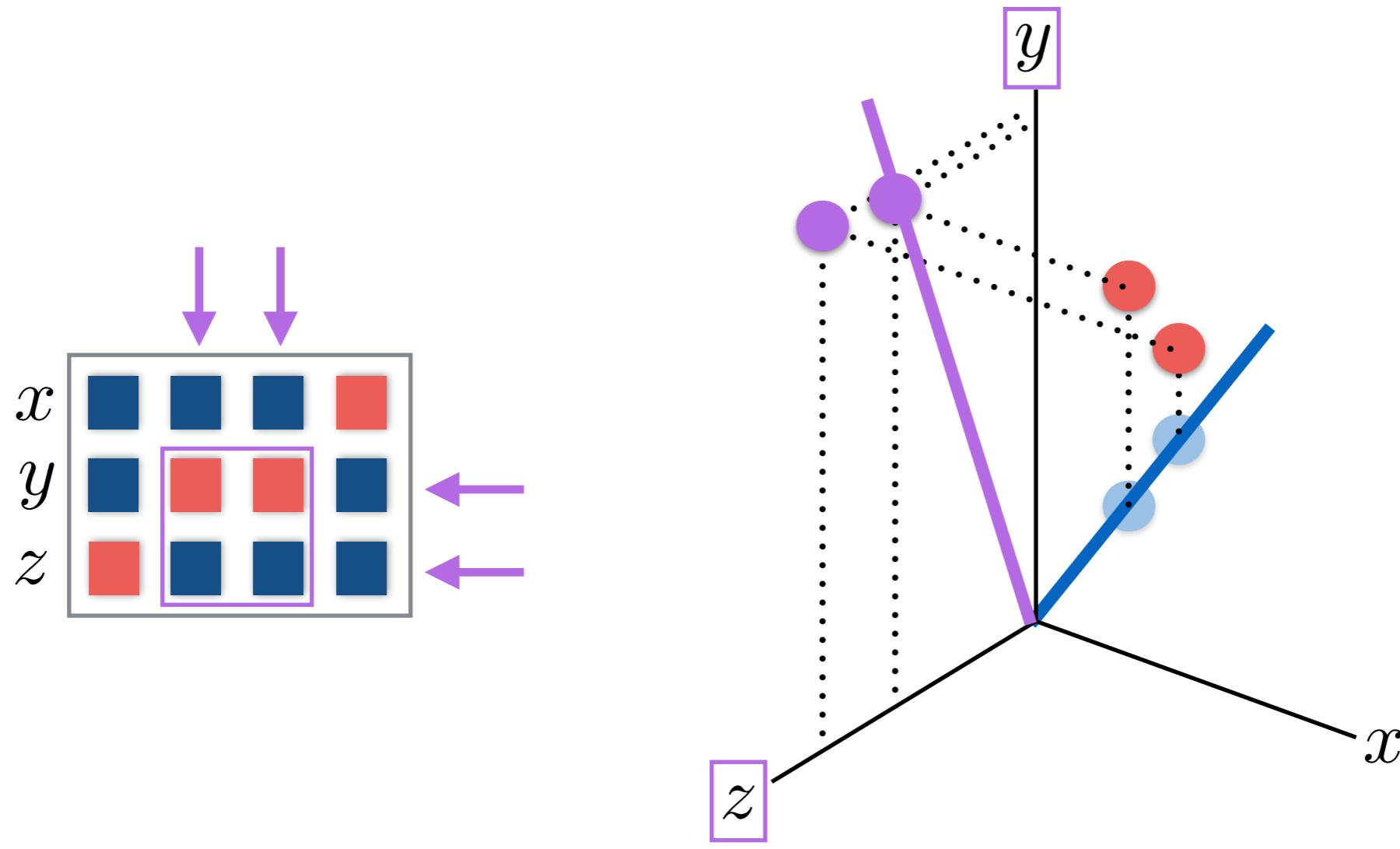
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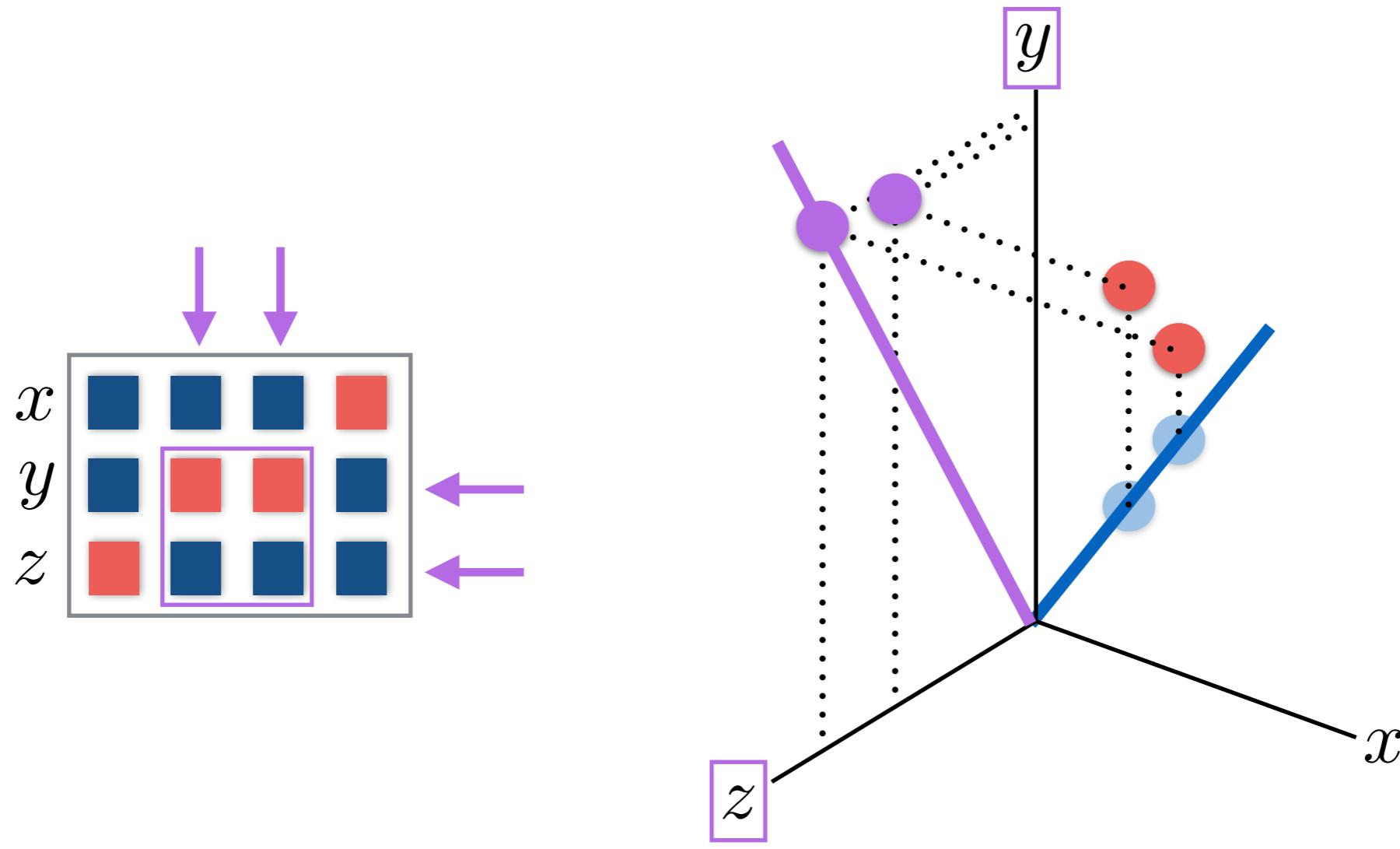
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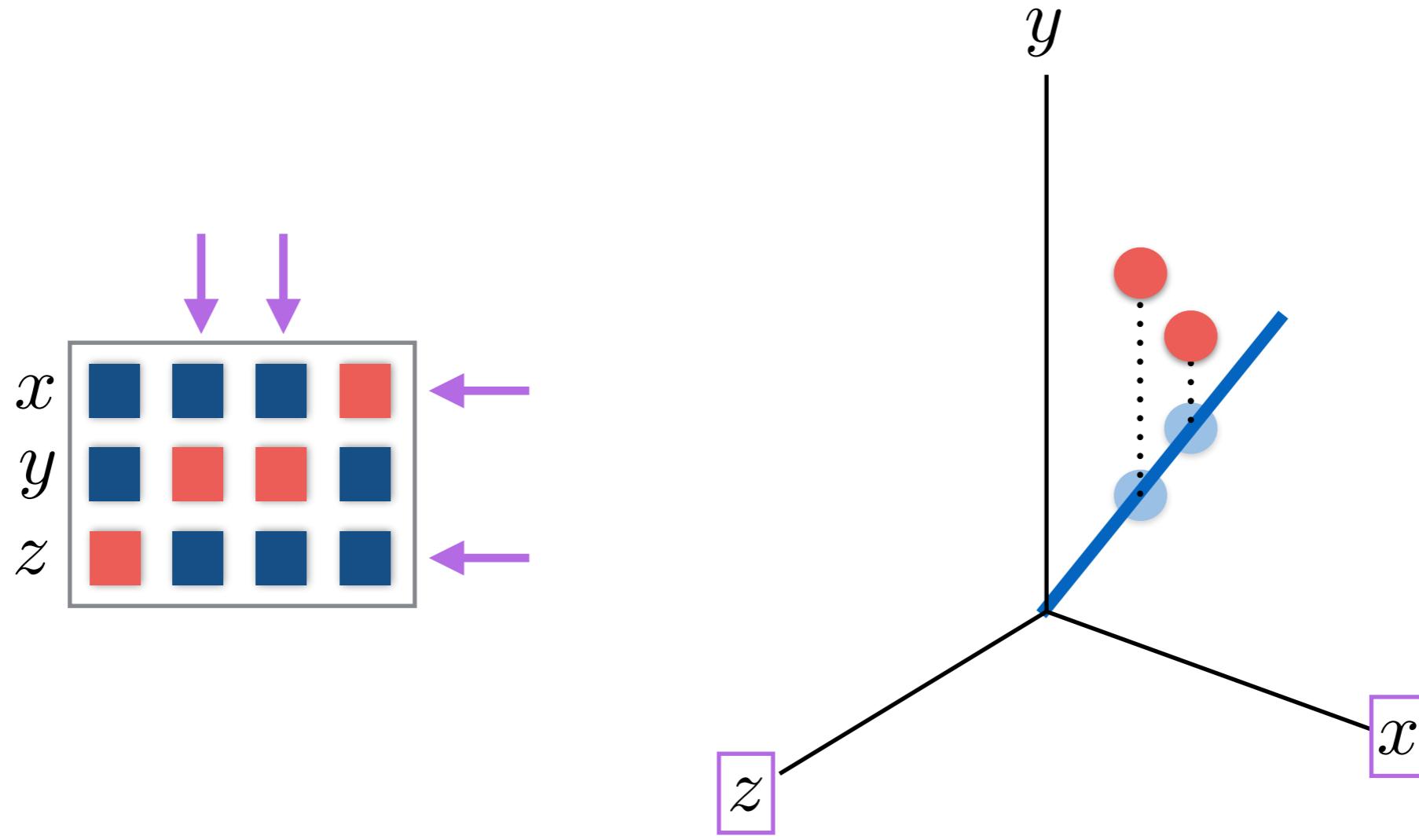
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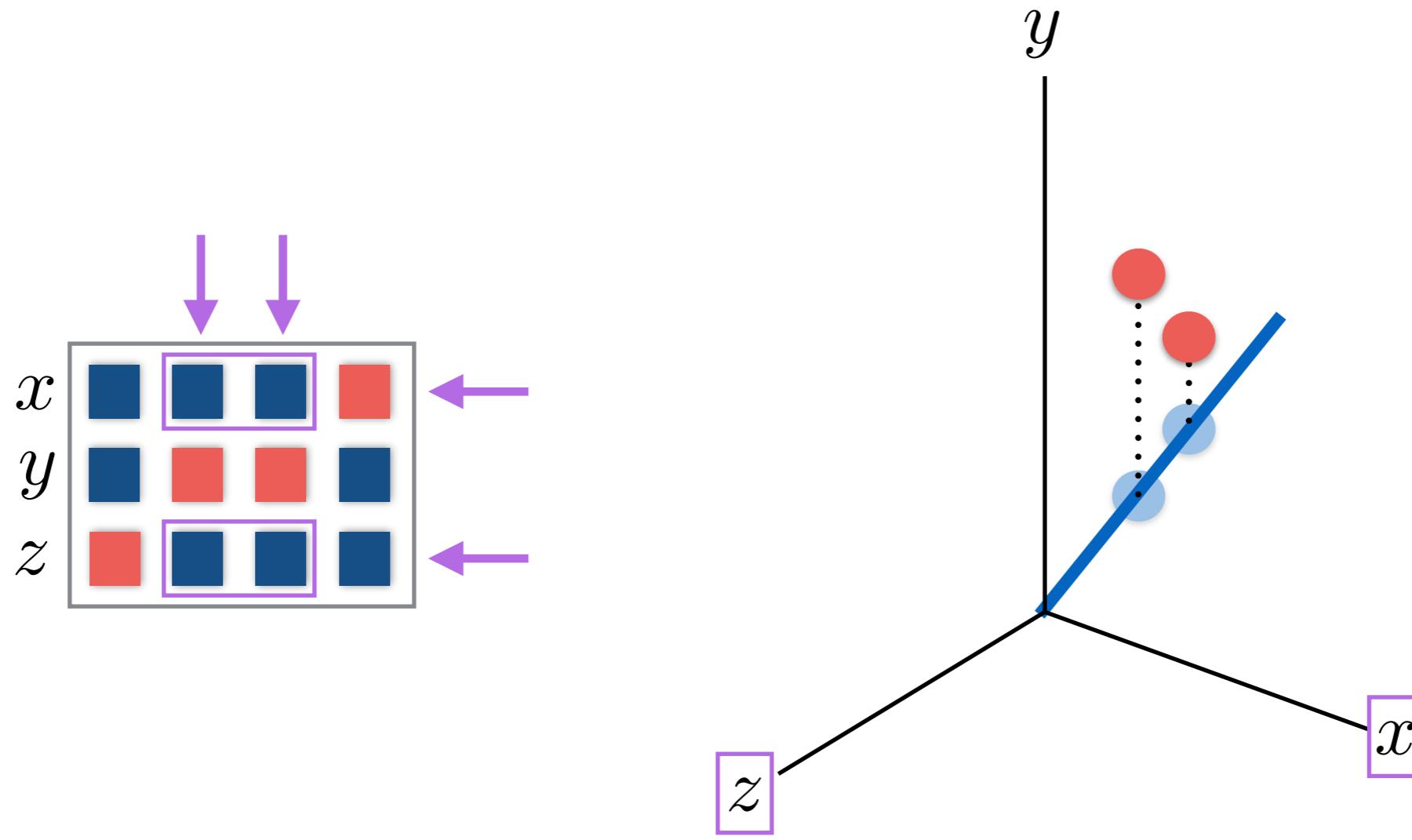
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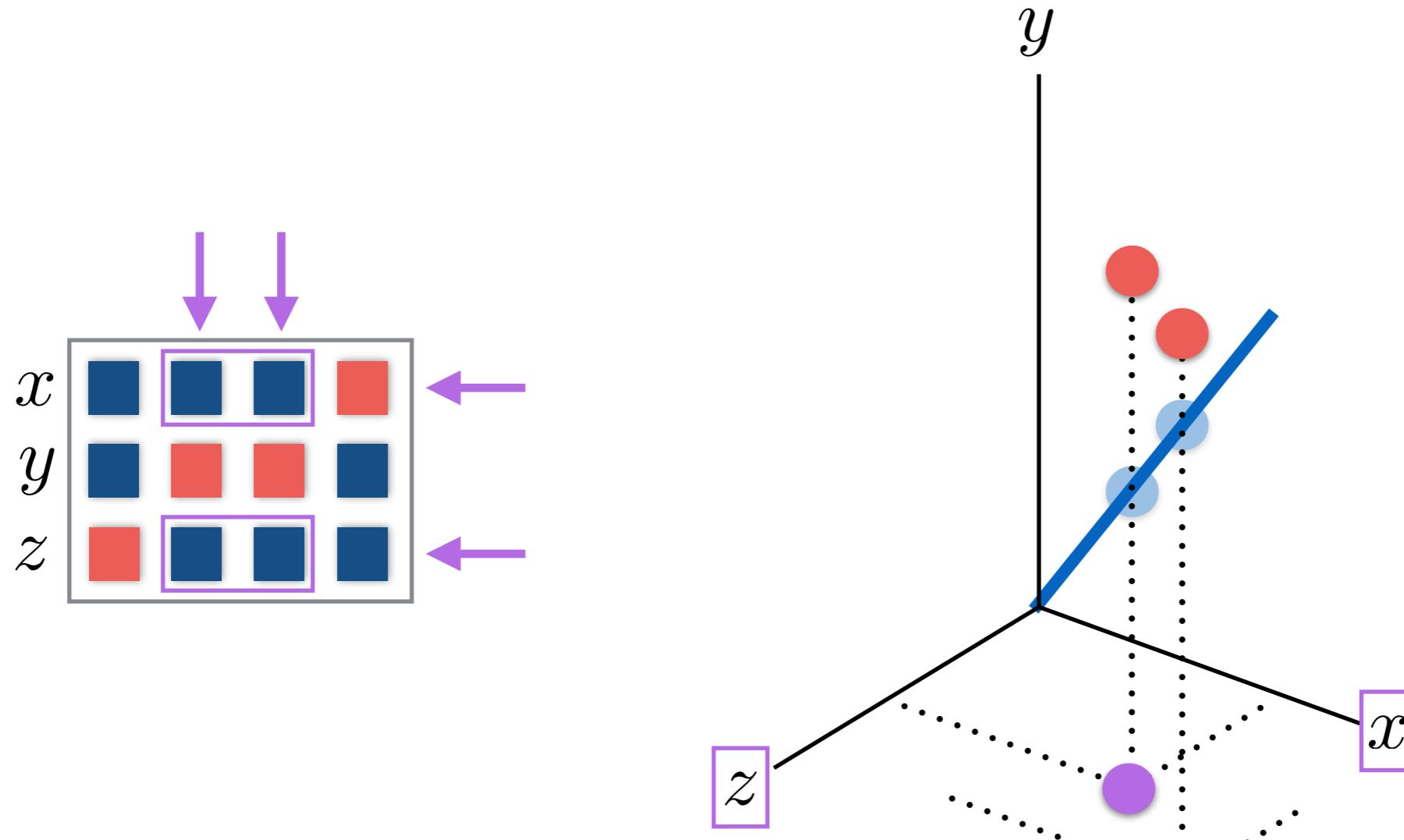
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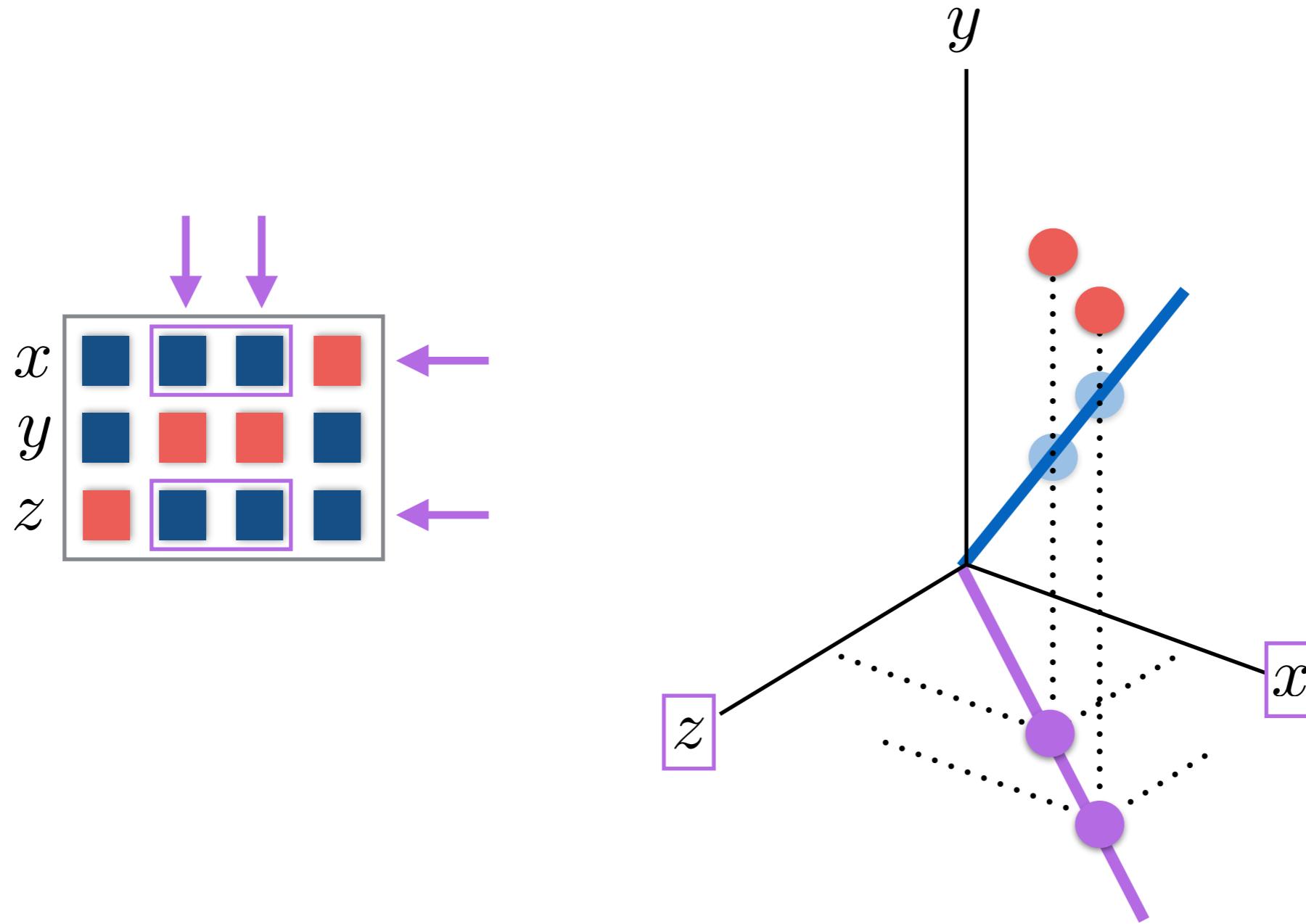
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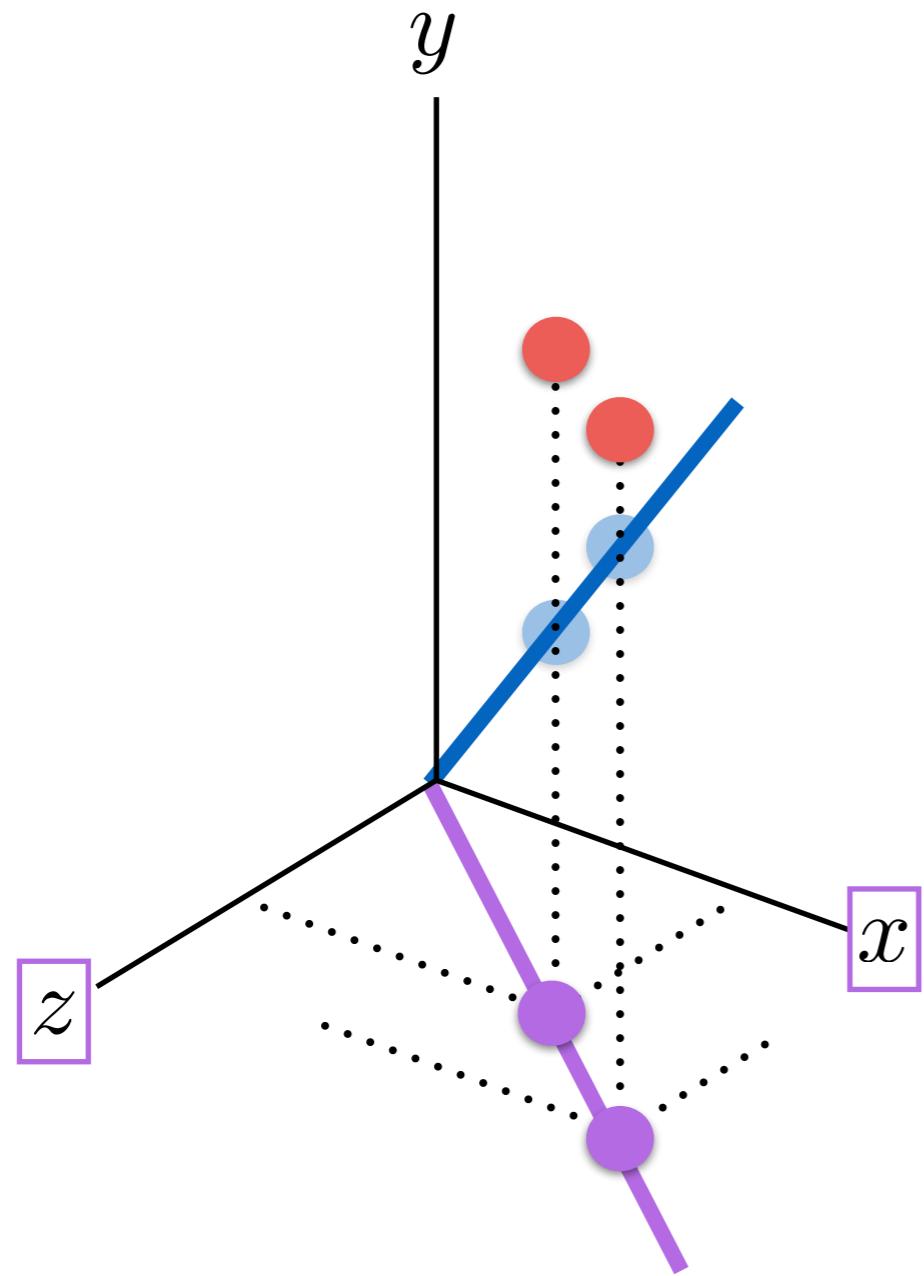
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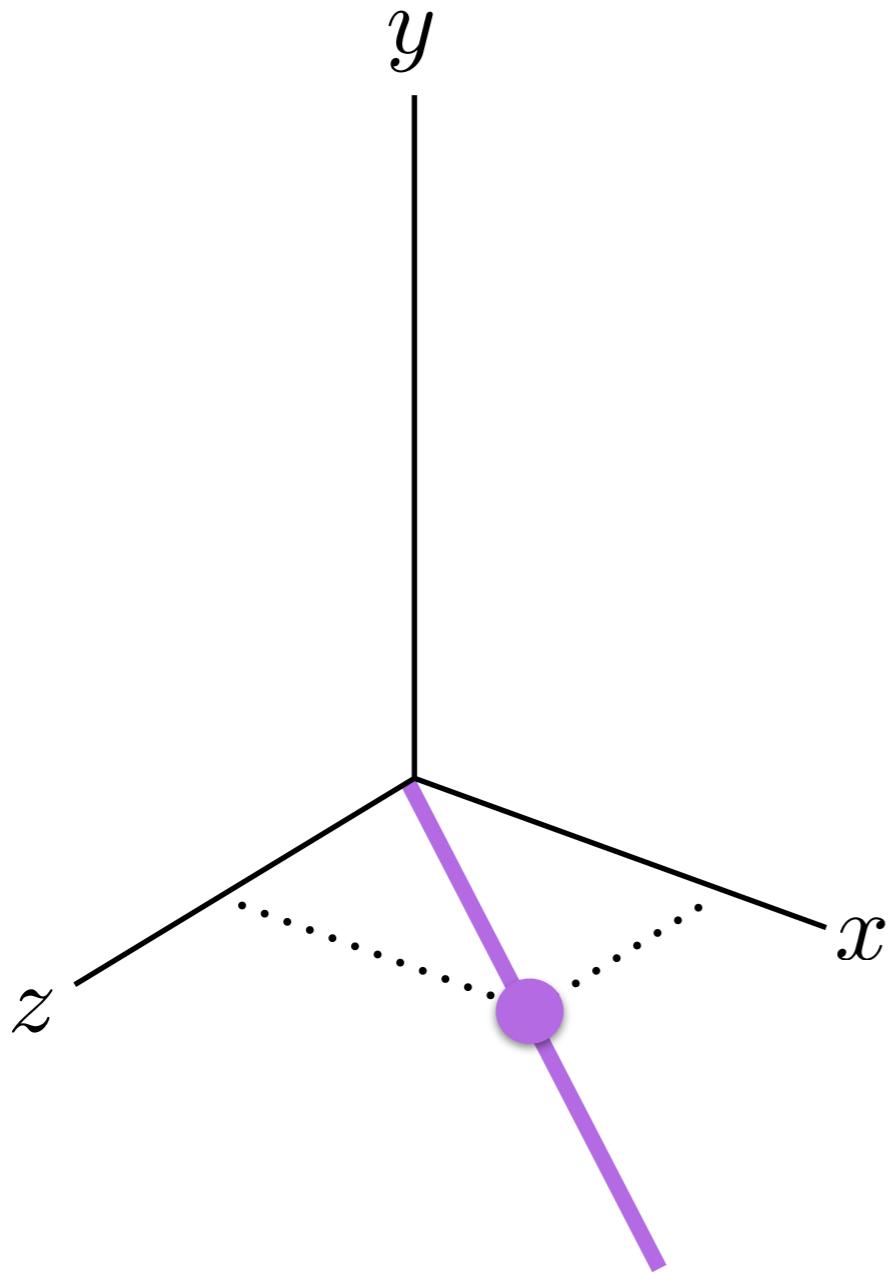
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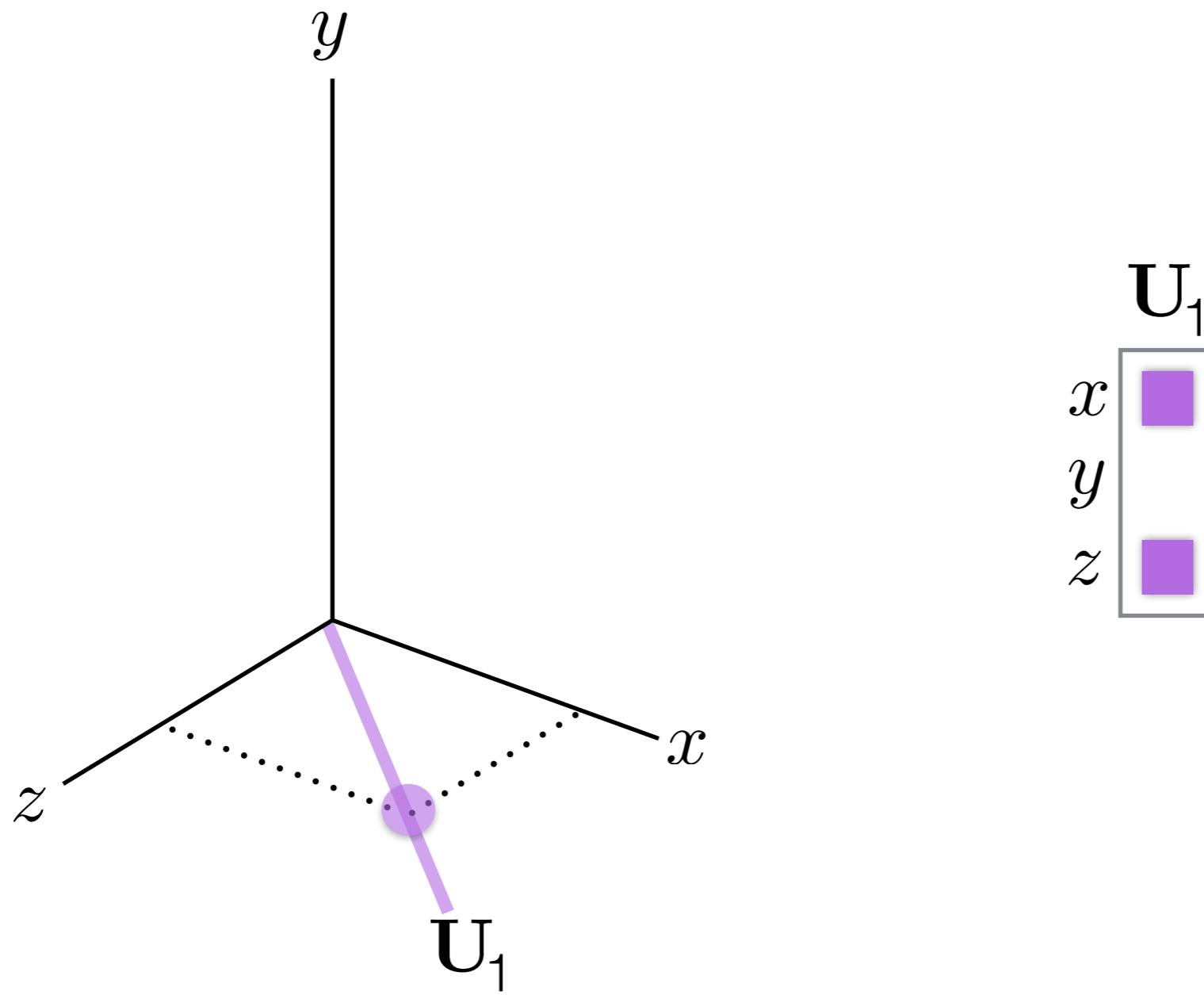
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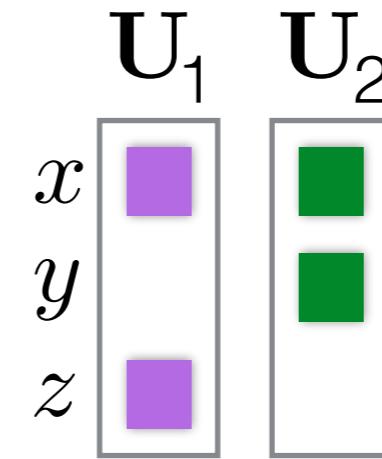
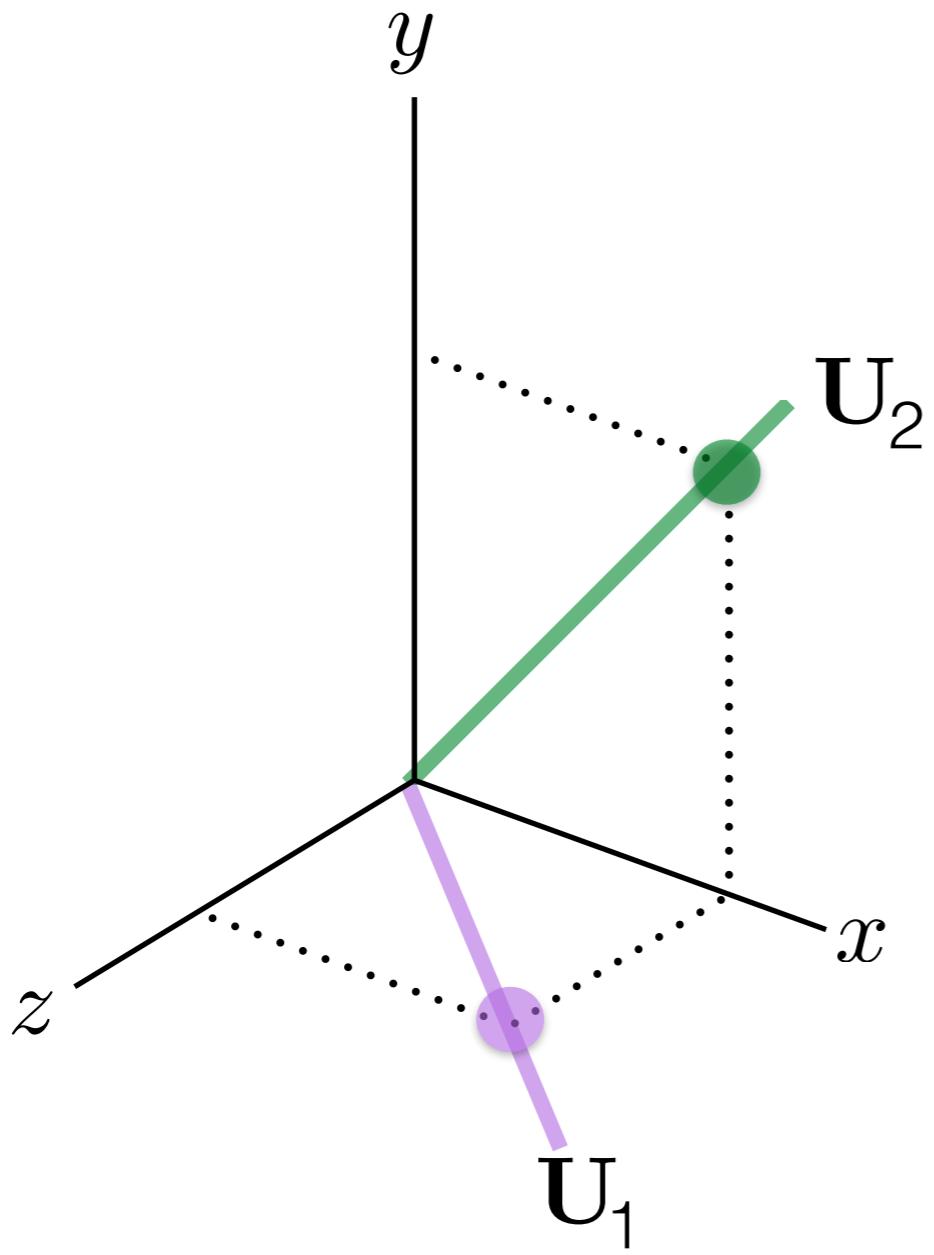
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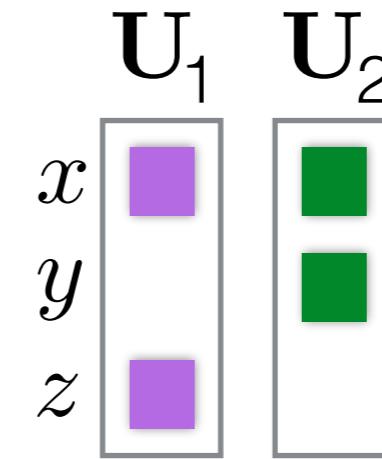
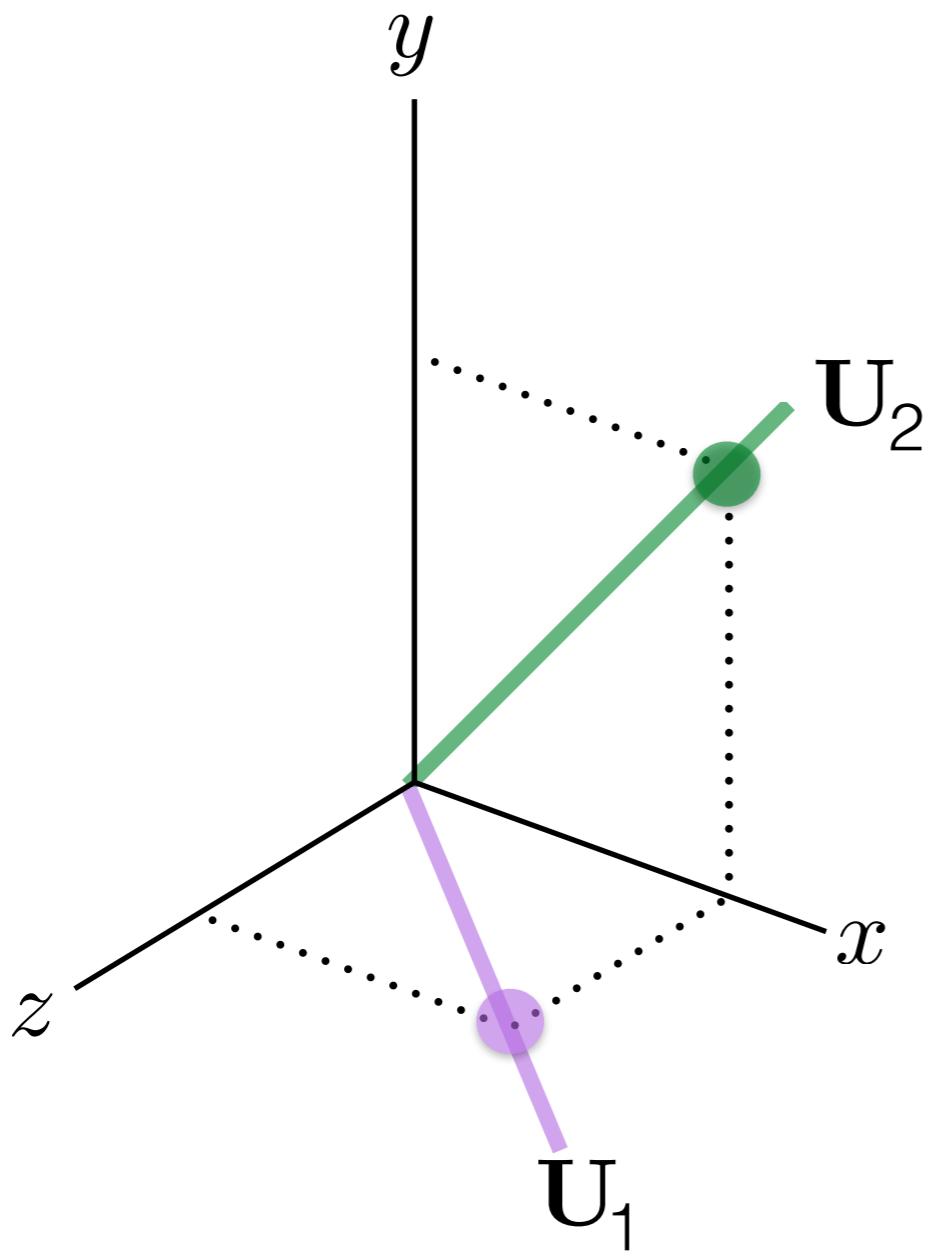
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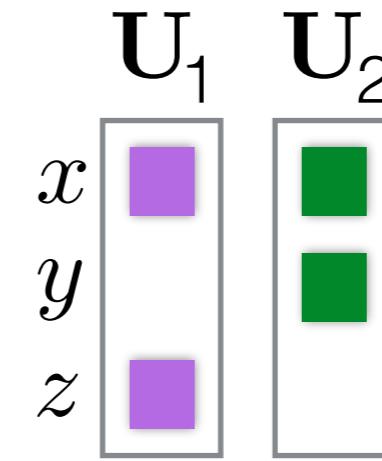
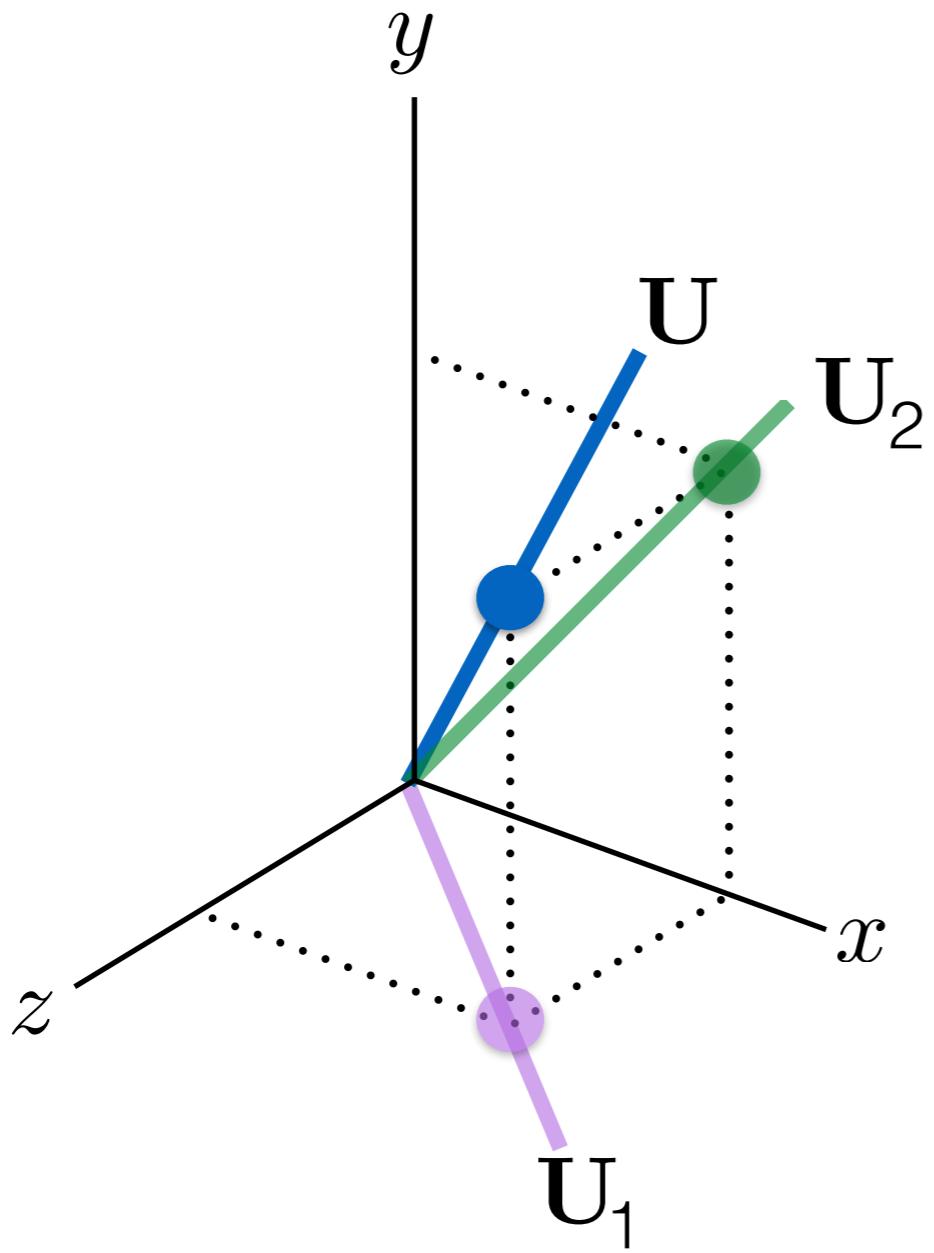
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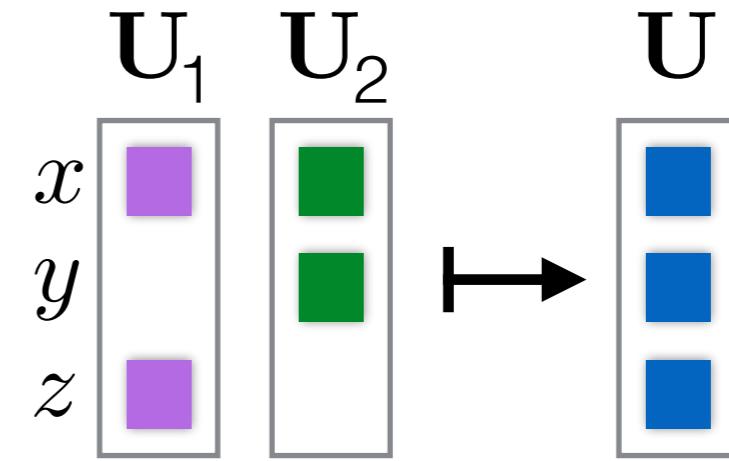
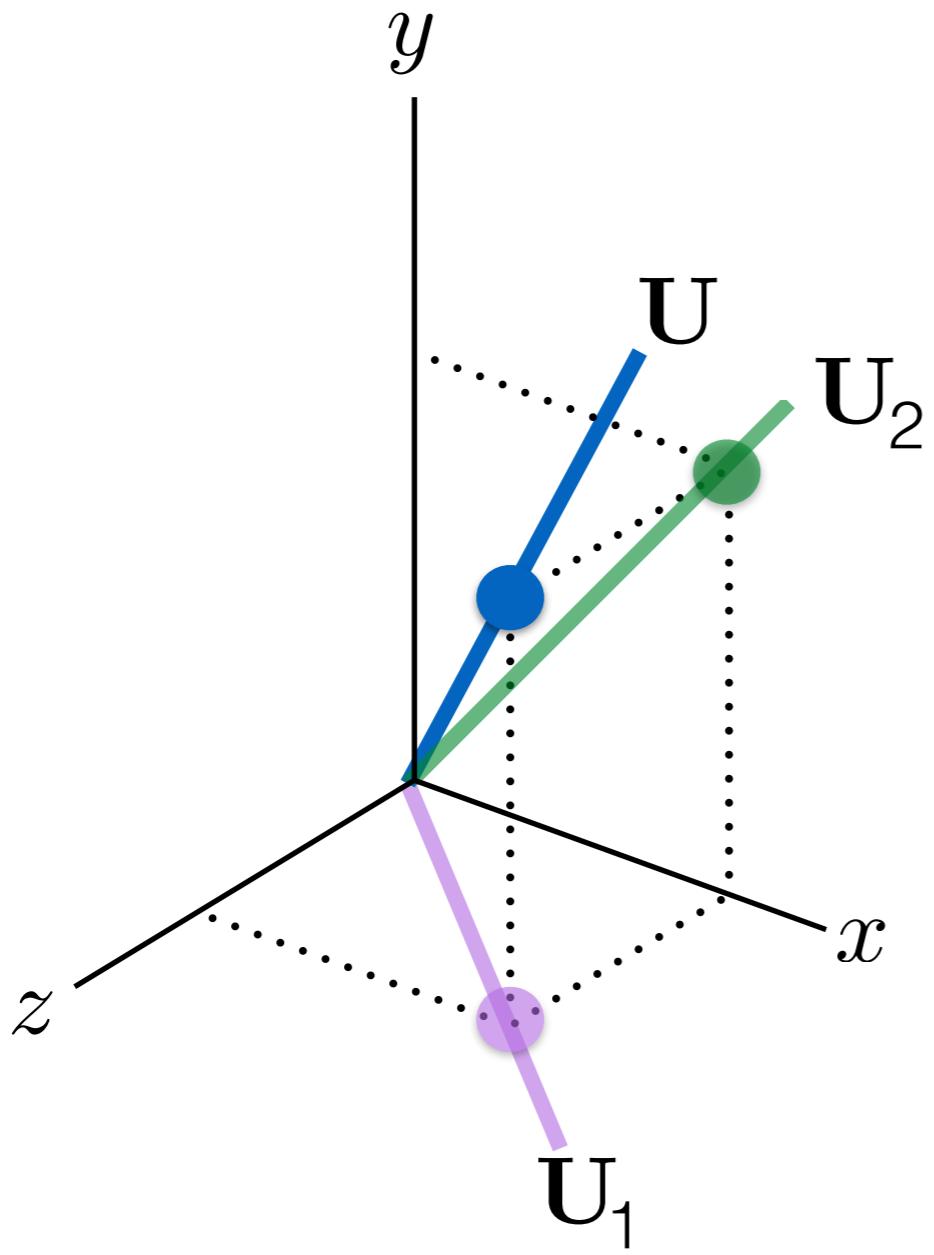
Now the question is:

Given canonical projections, can I find the subspace?



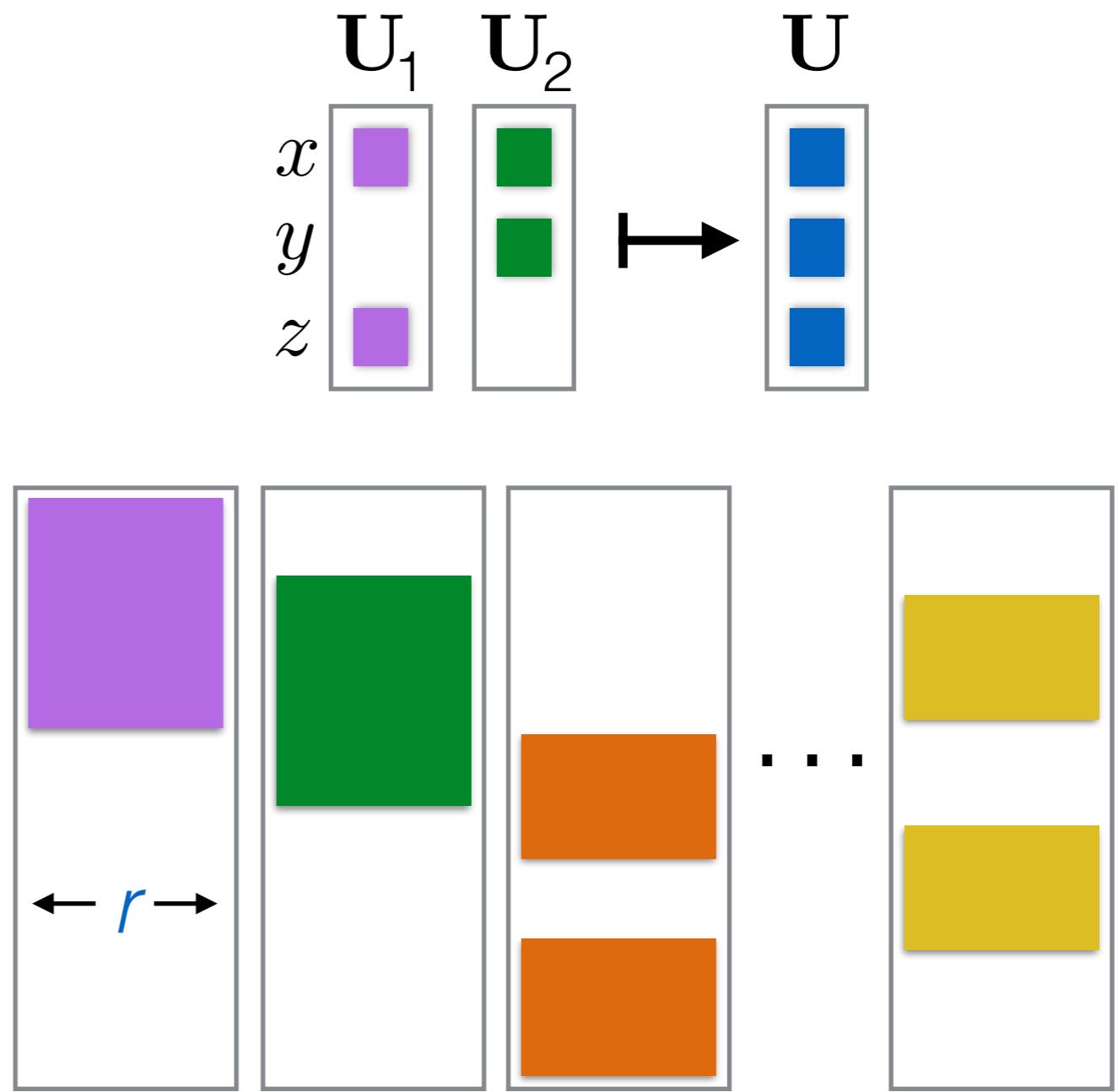
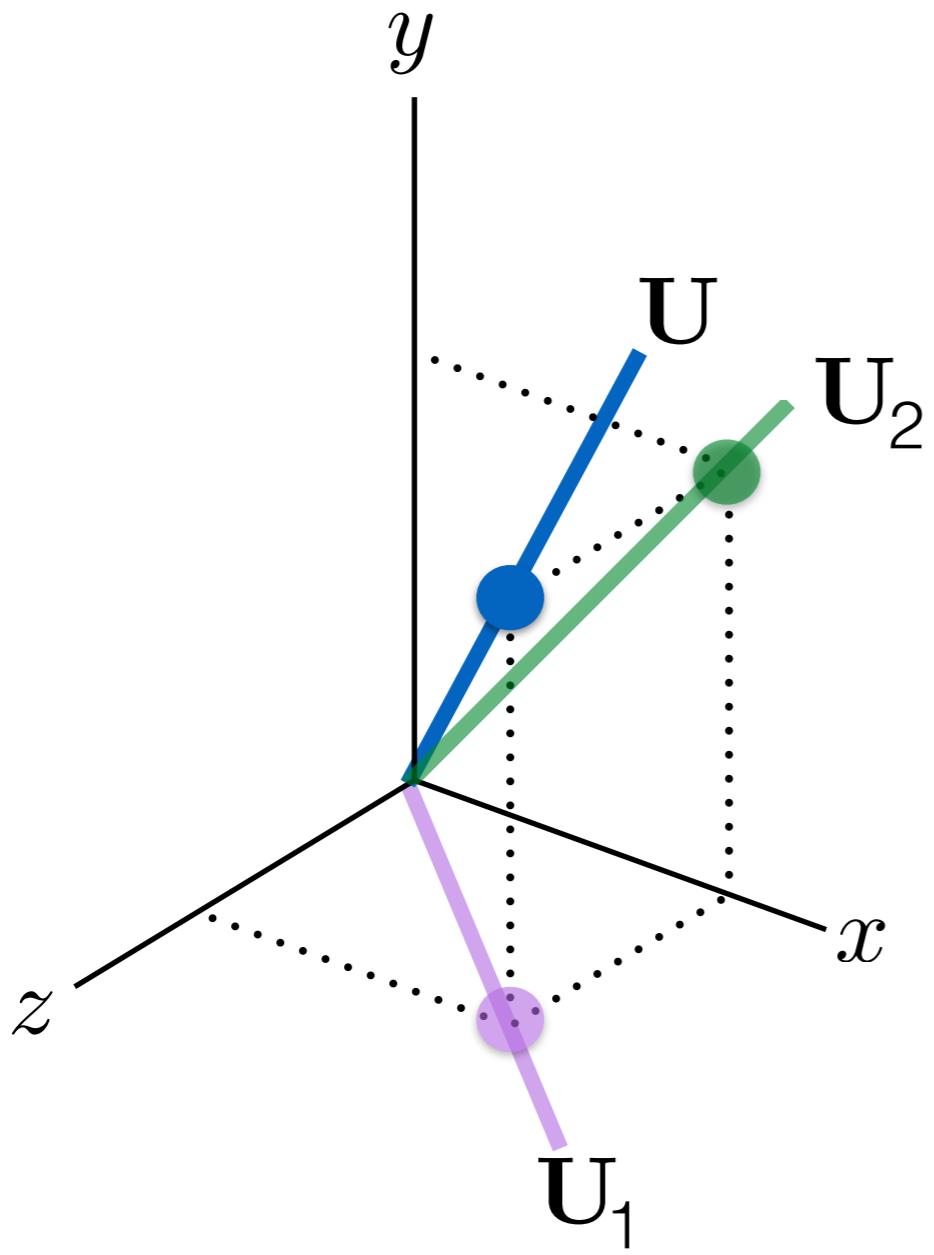
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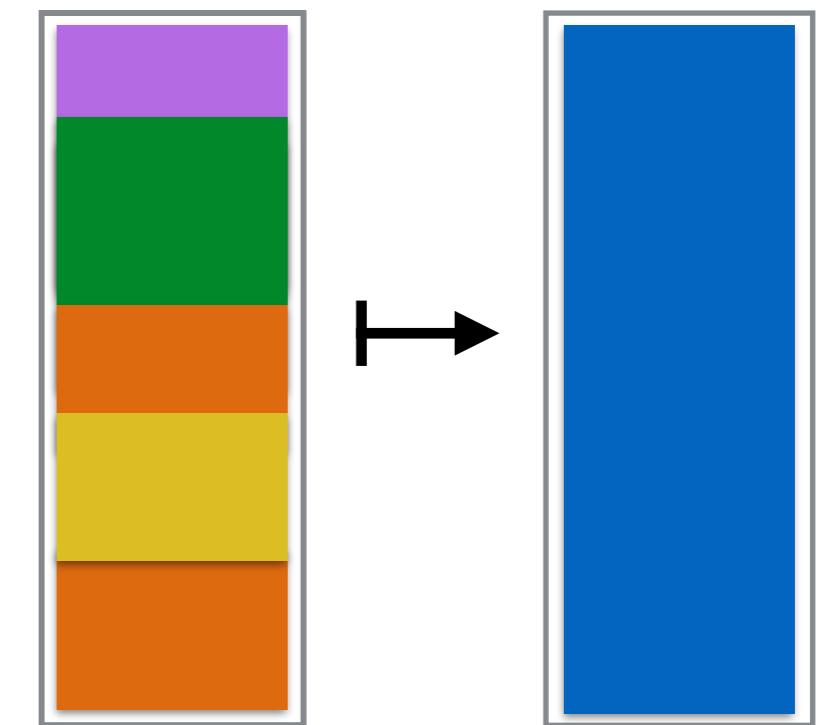
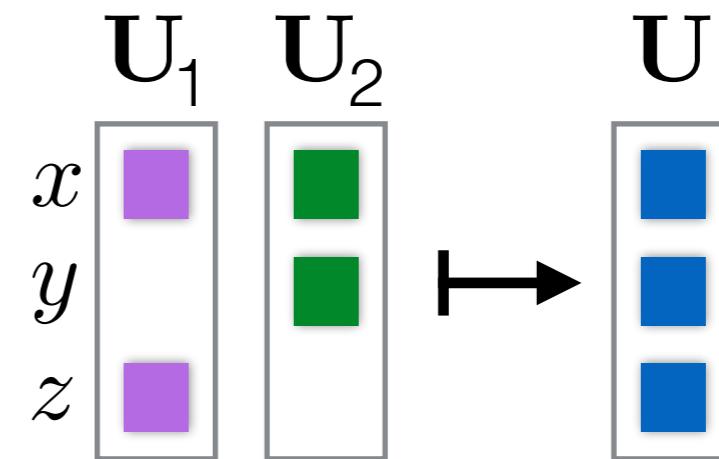
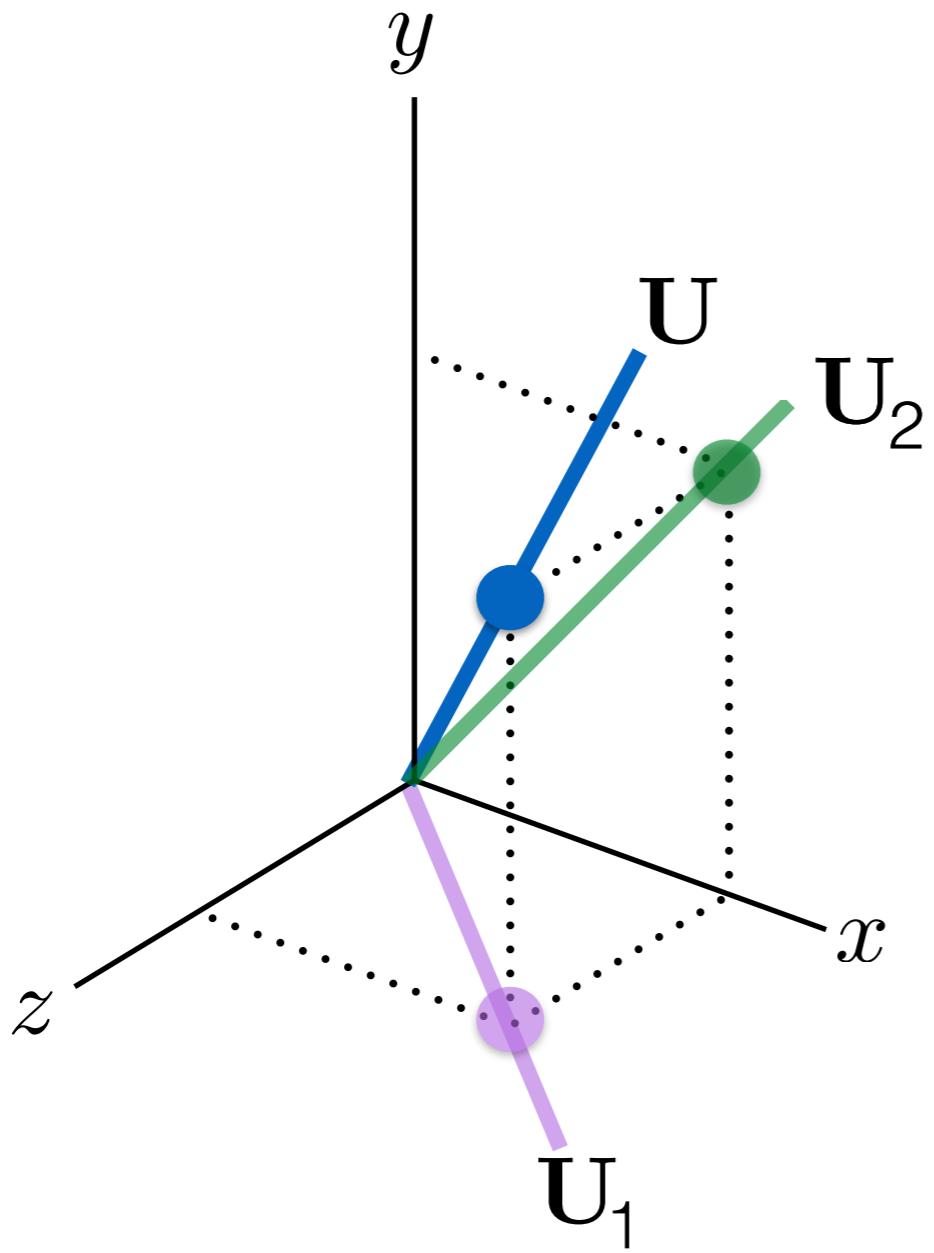
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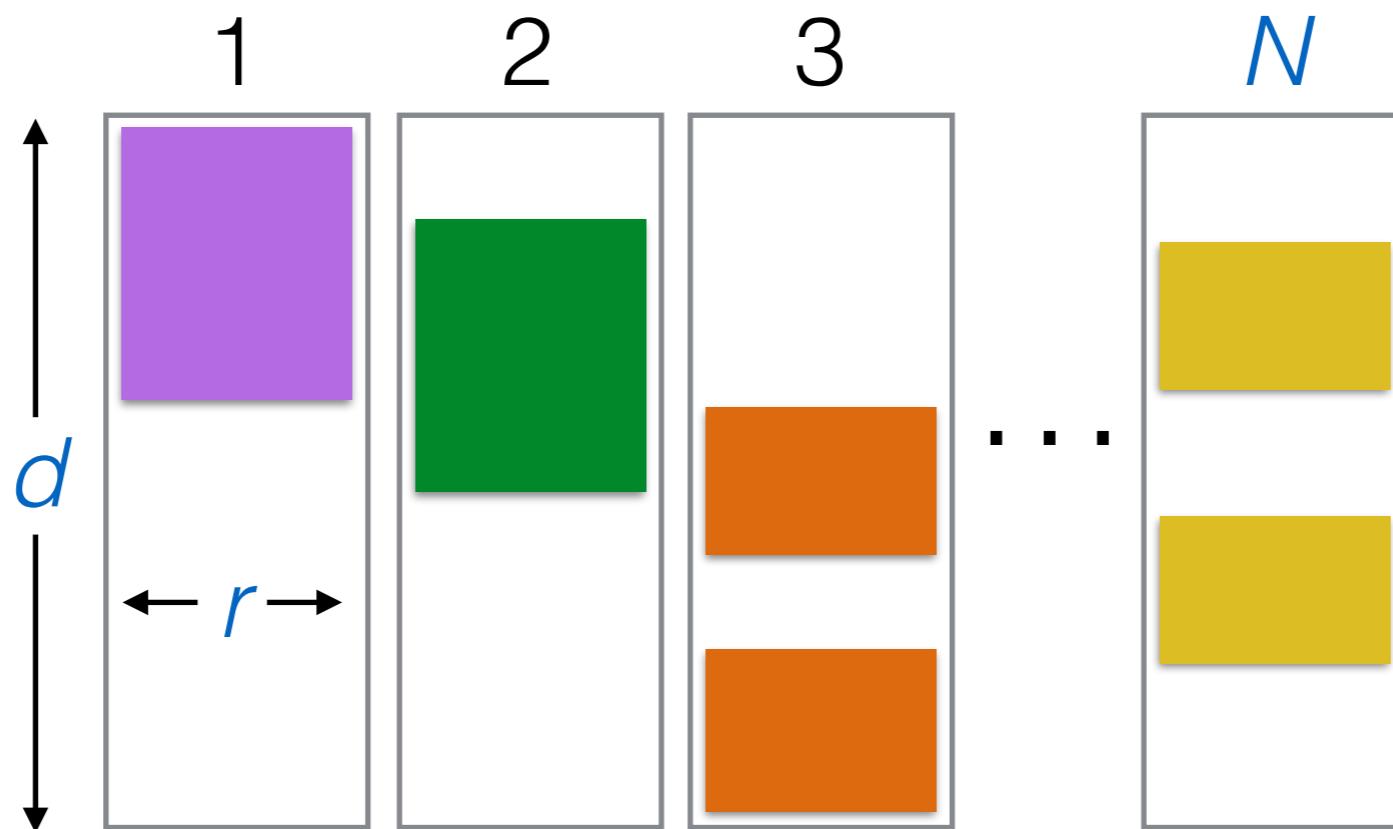
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Theorem

(Pimentel, Boston, Nowak, ISIT, 2015)

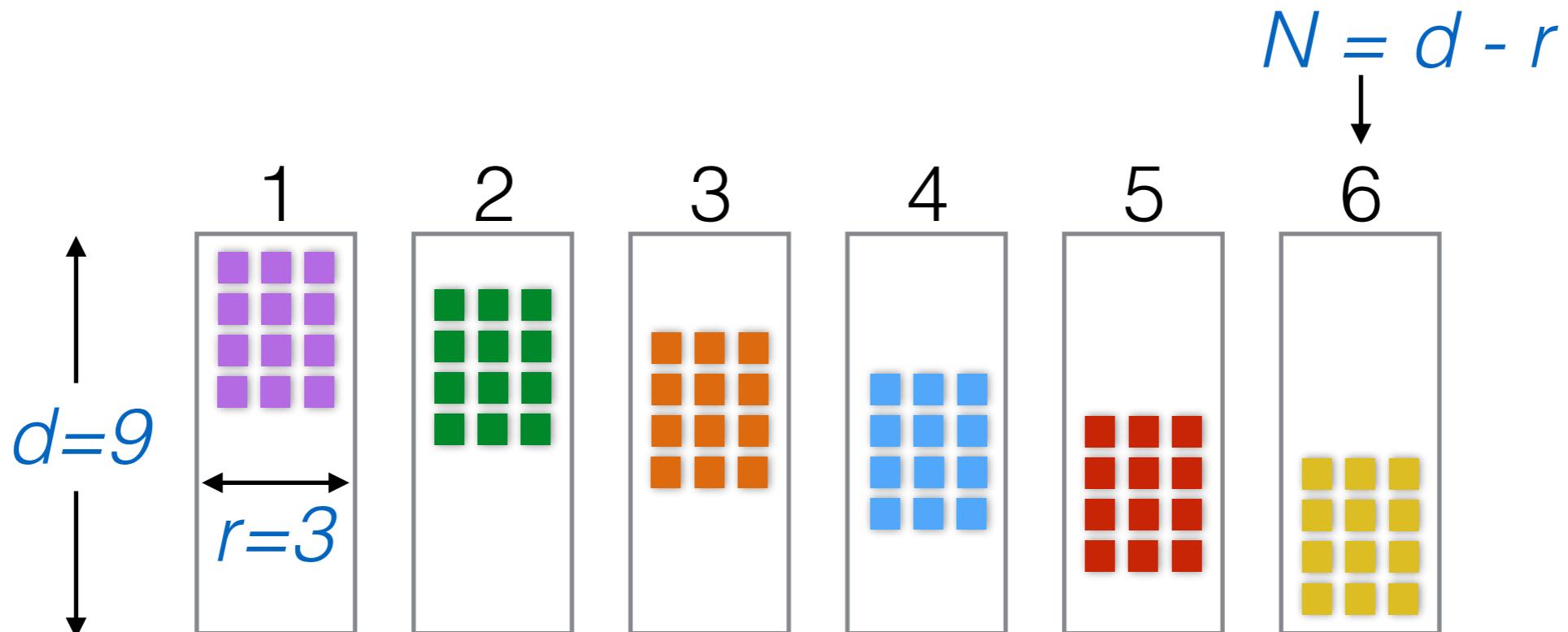
A subspace can be recovered from $N = d - r$ canonical projections if and only if every subset of n projections involves at least $n + r$ coordinates.

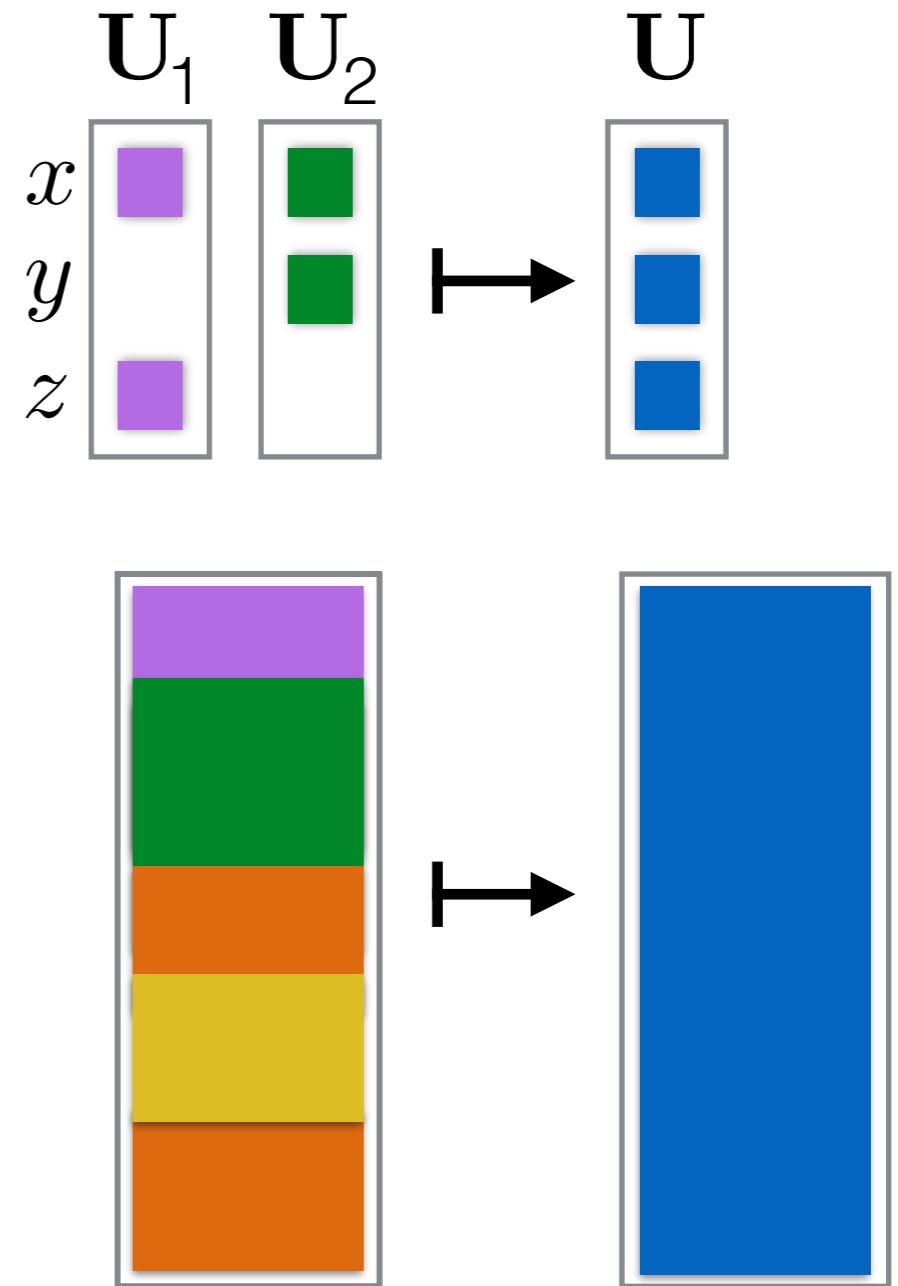
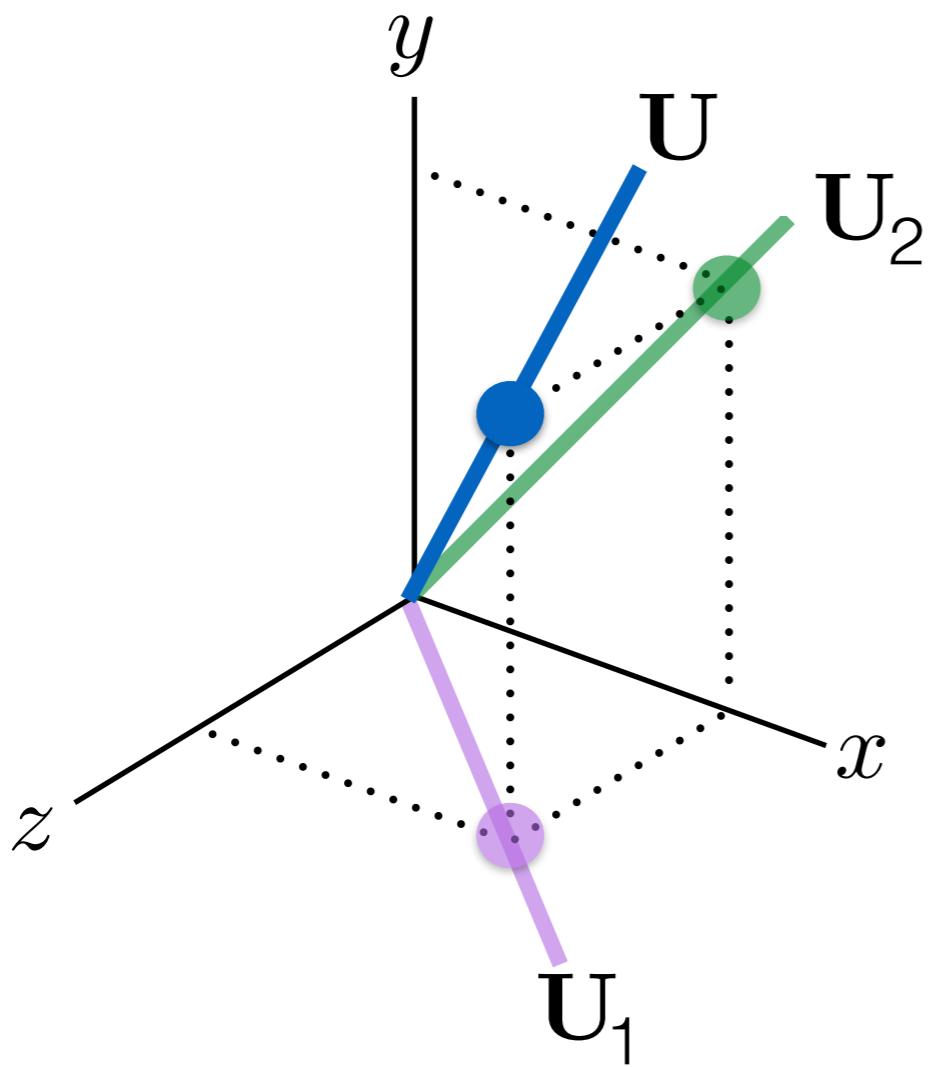


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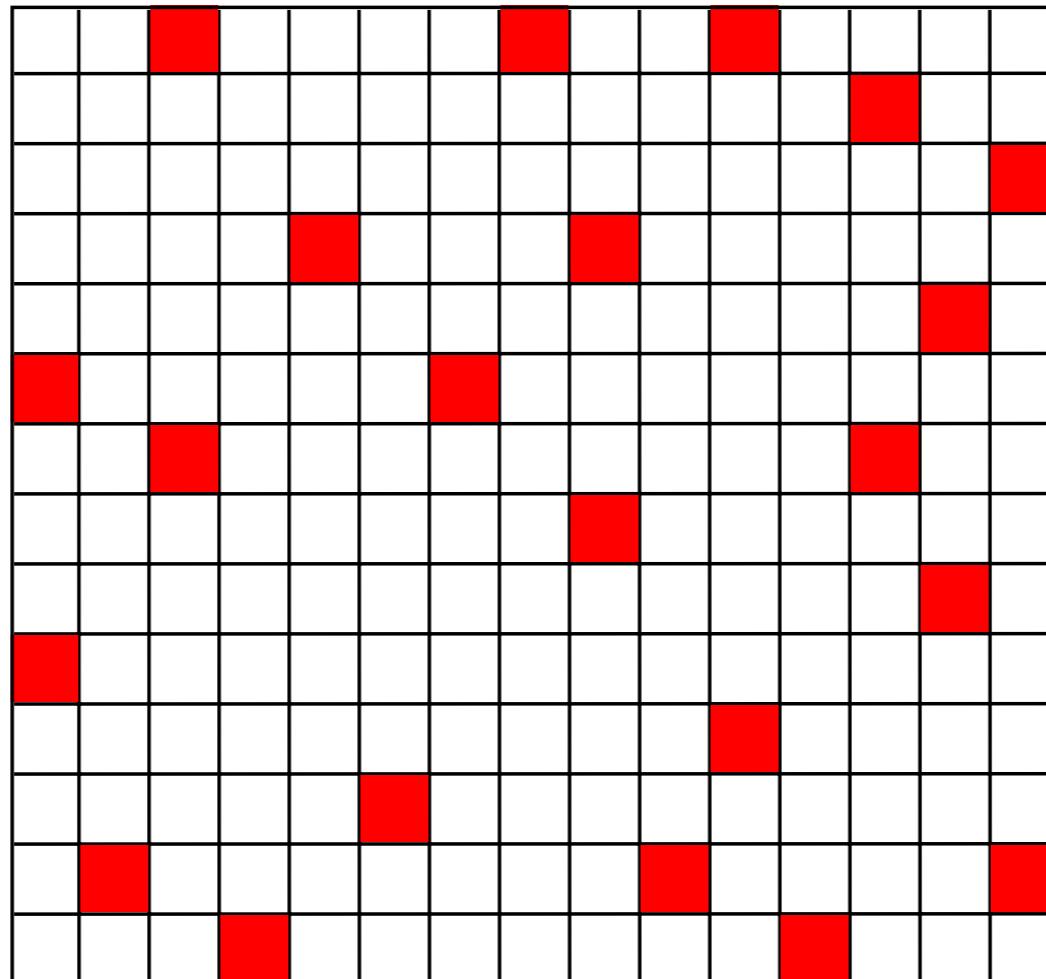


This tells me

Which projections I need to reconstruct the subspace.

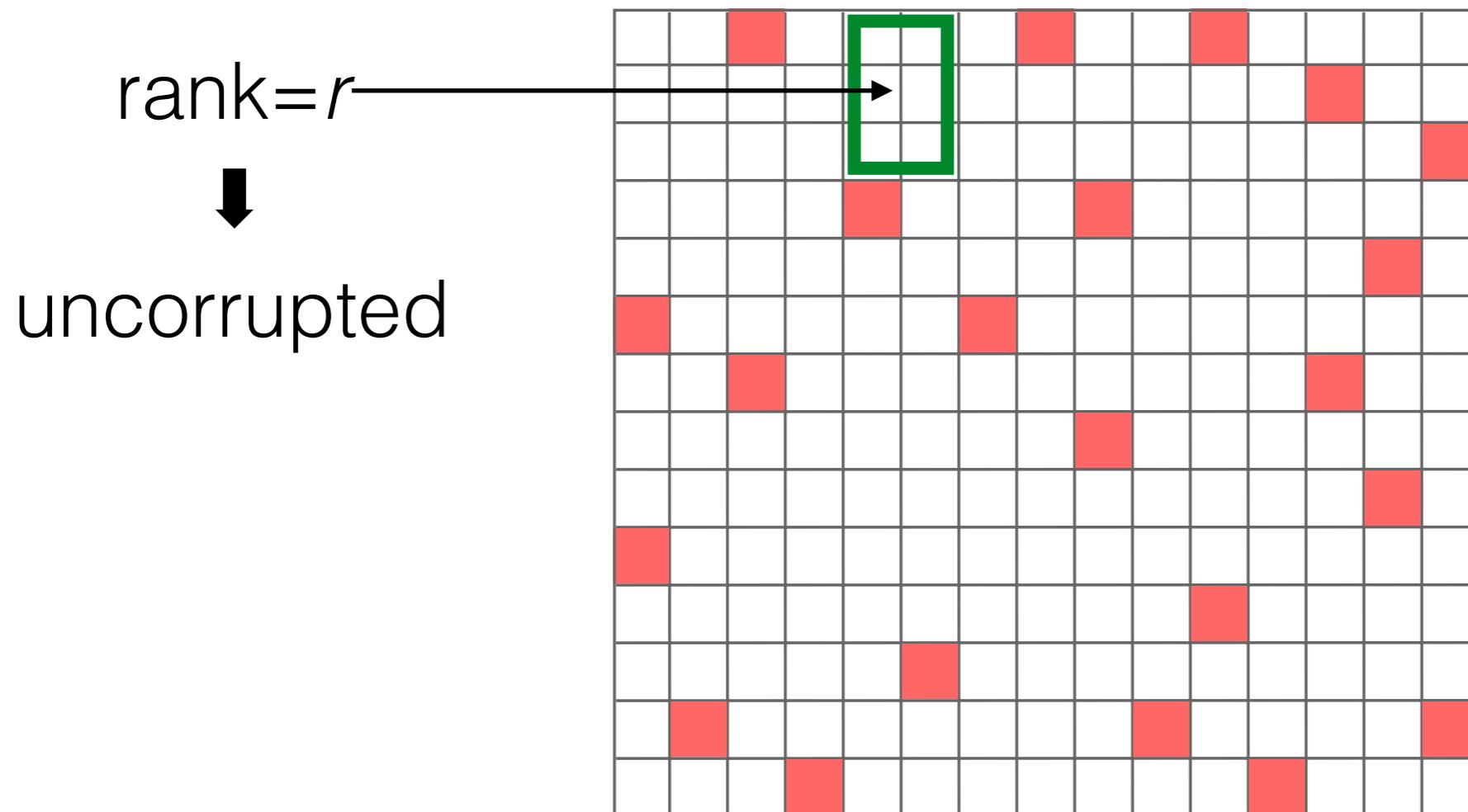
Our Algorithm: R2PCA

(Ransac Robust PCA)



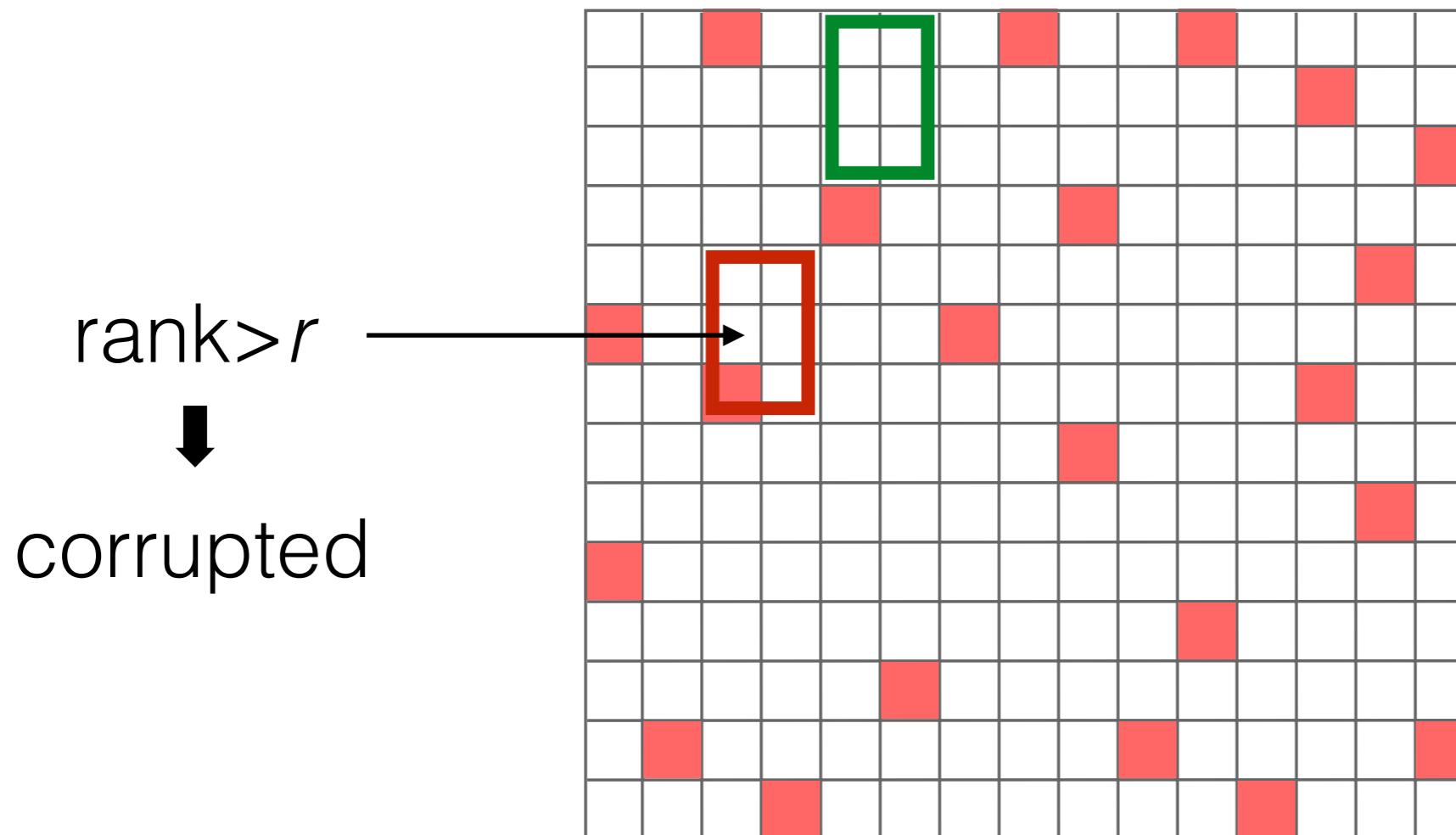
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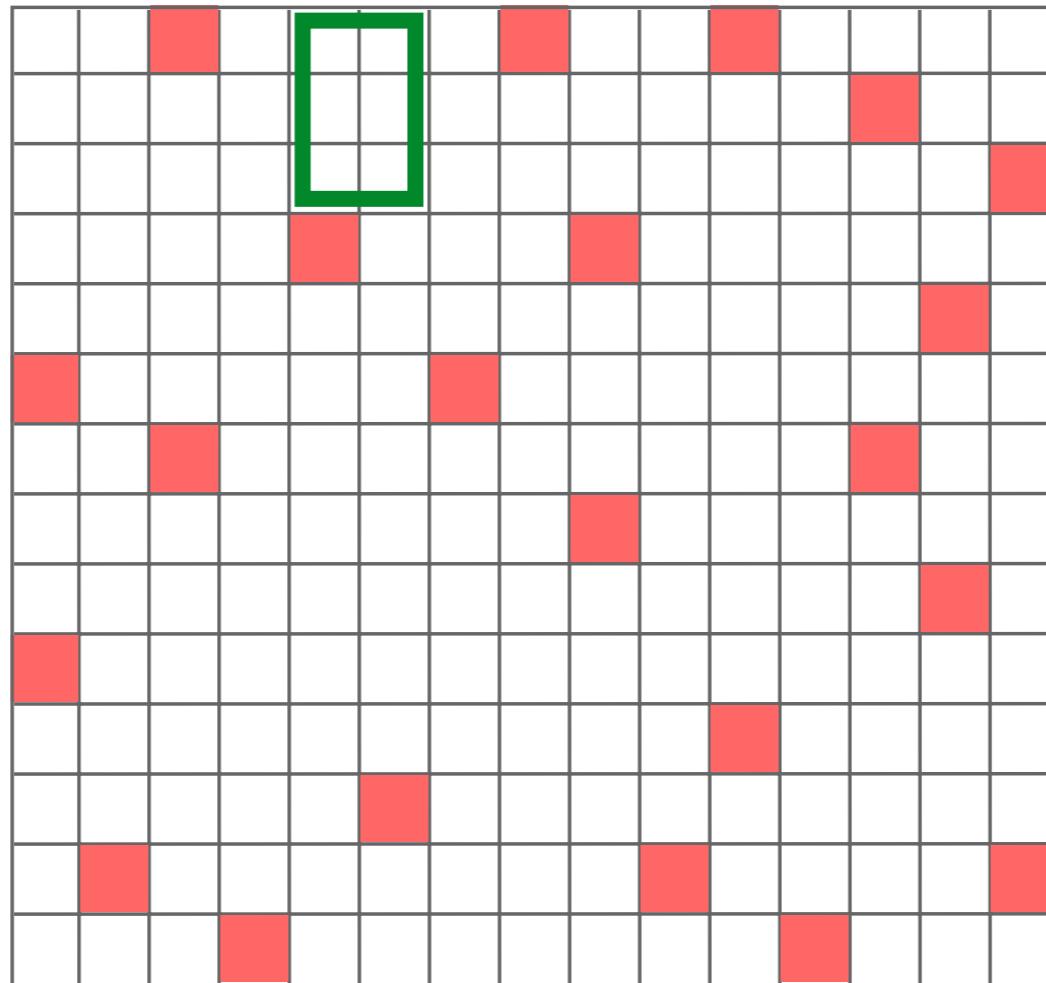
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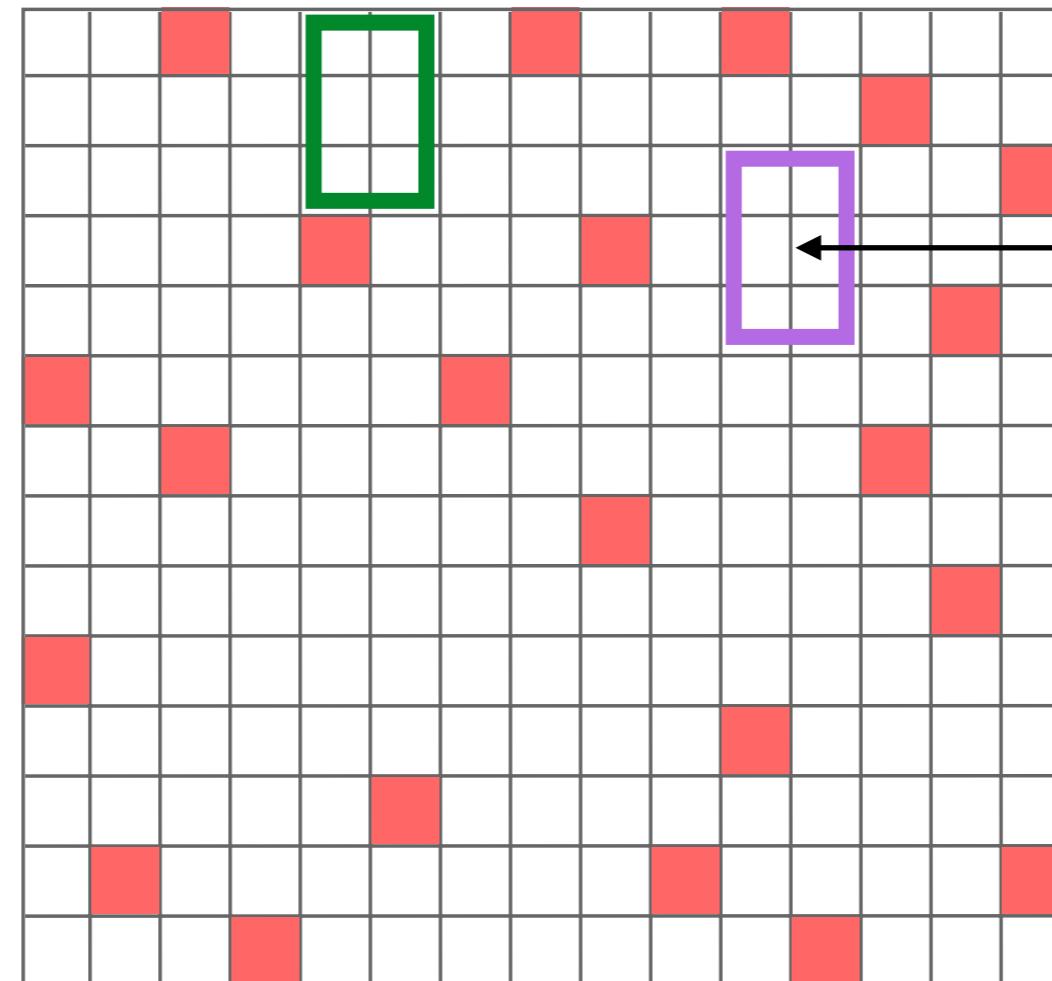
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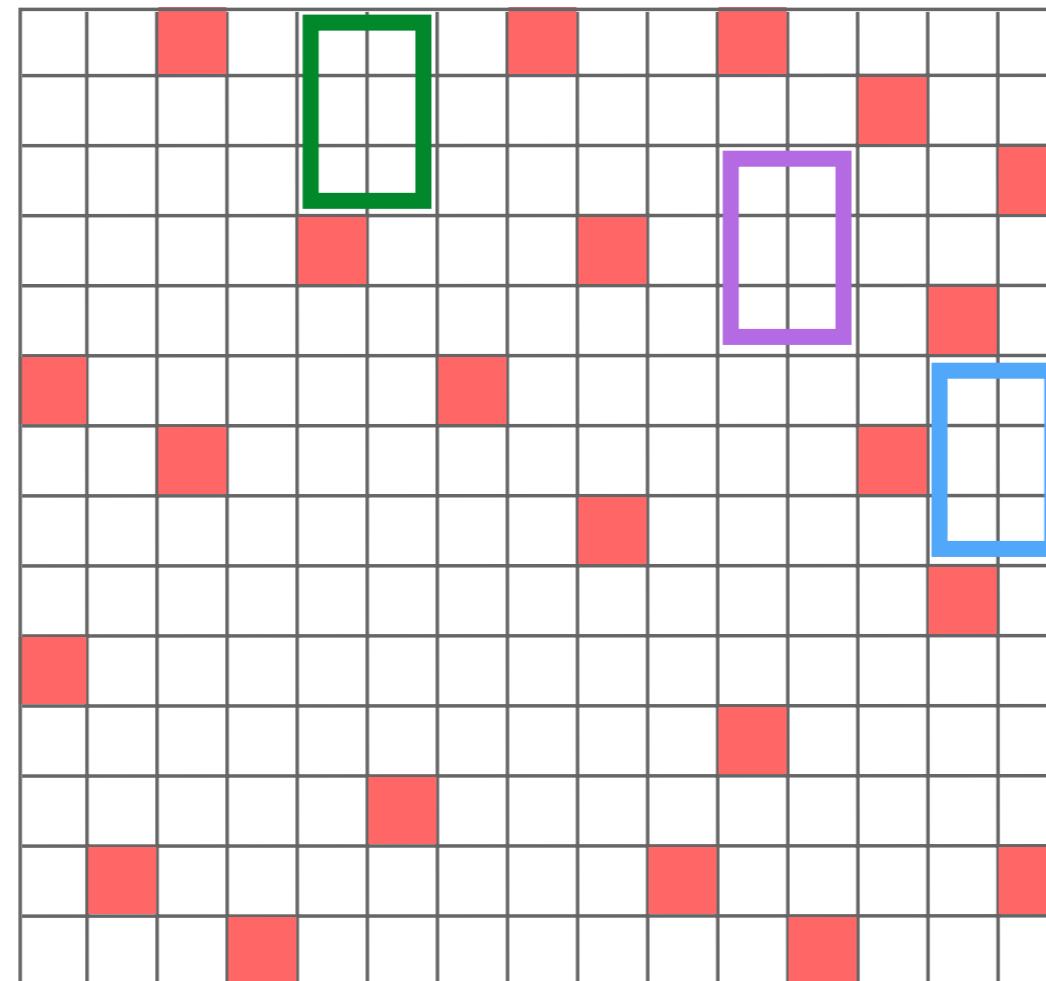
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Keep finding
uncorrupted
projections

Our Algorithm: R2PCA

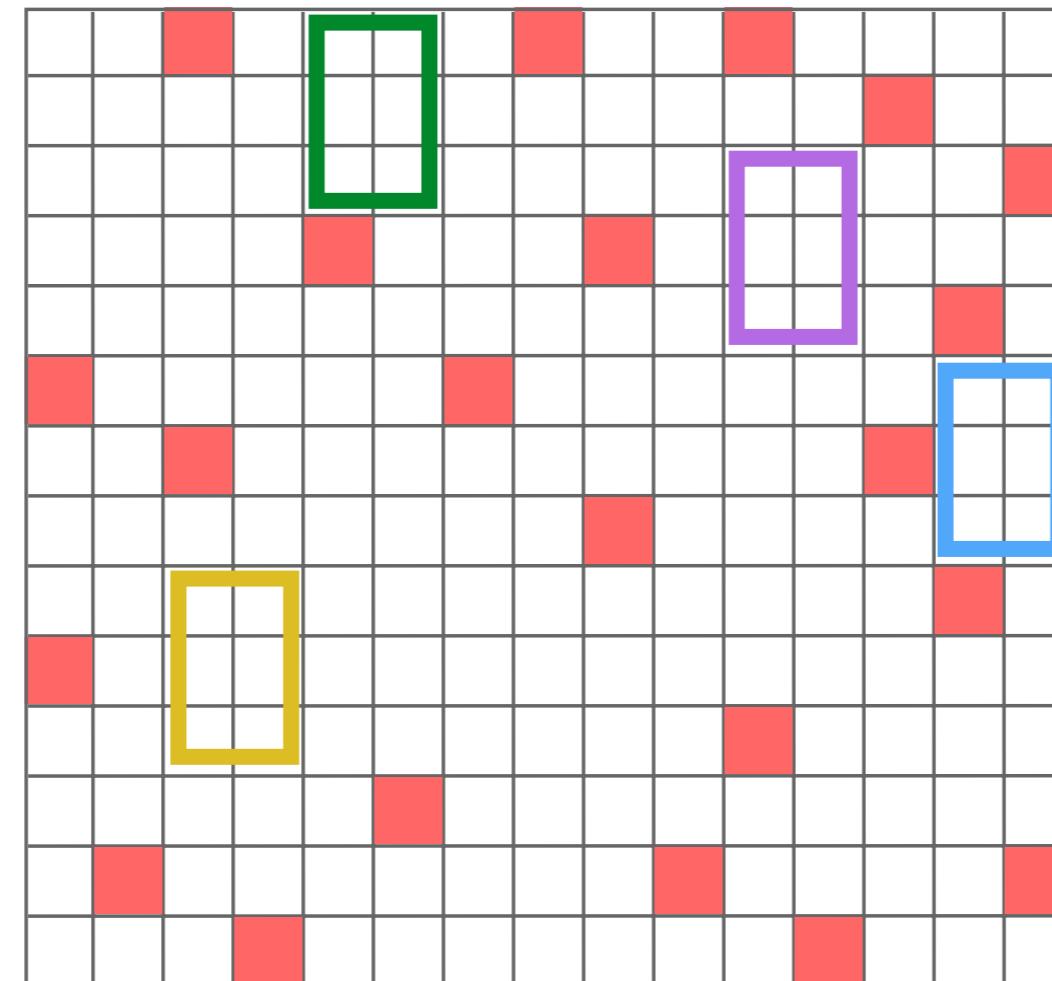
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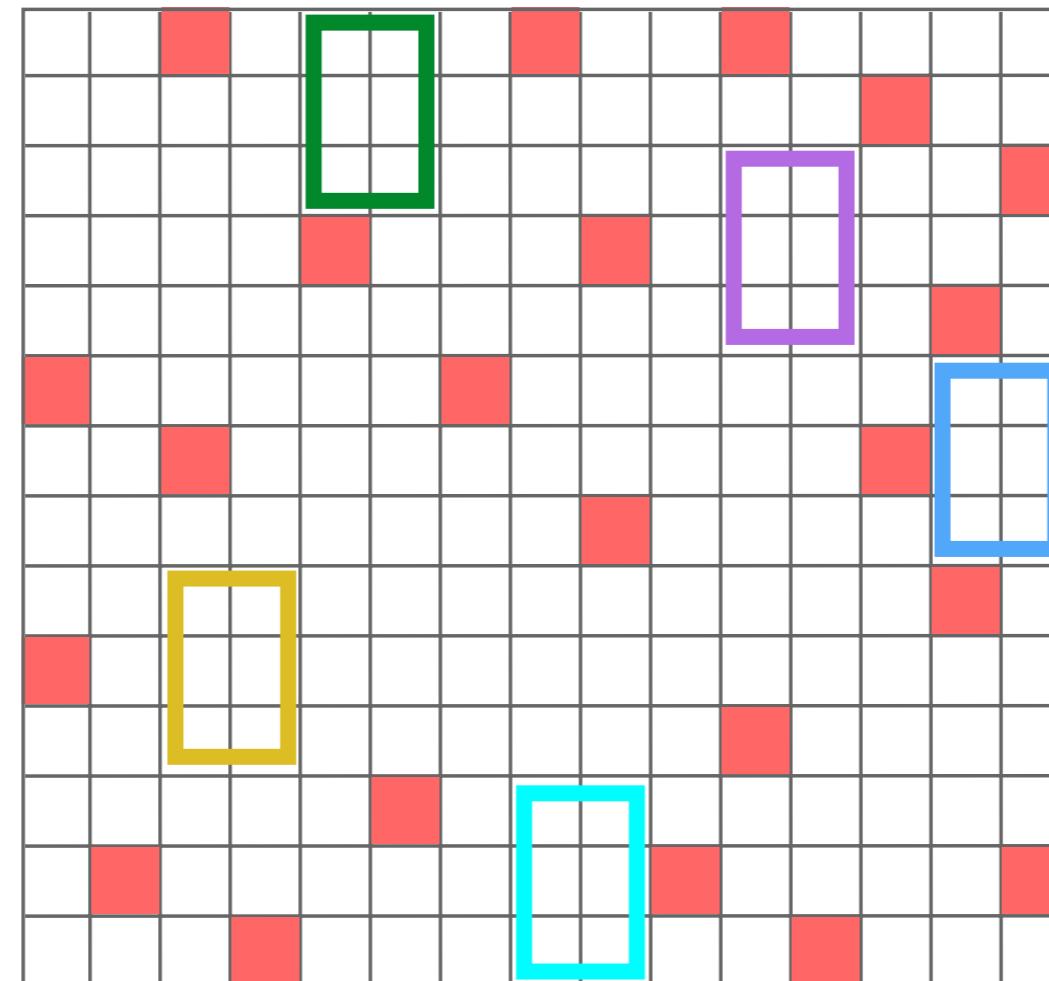
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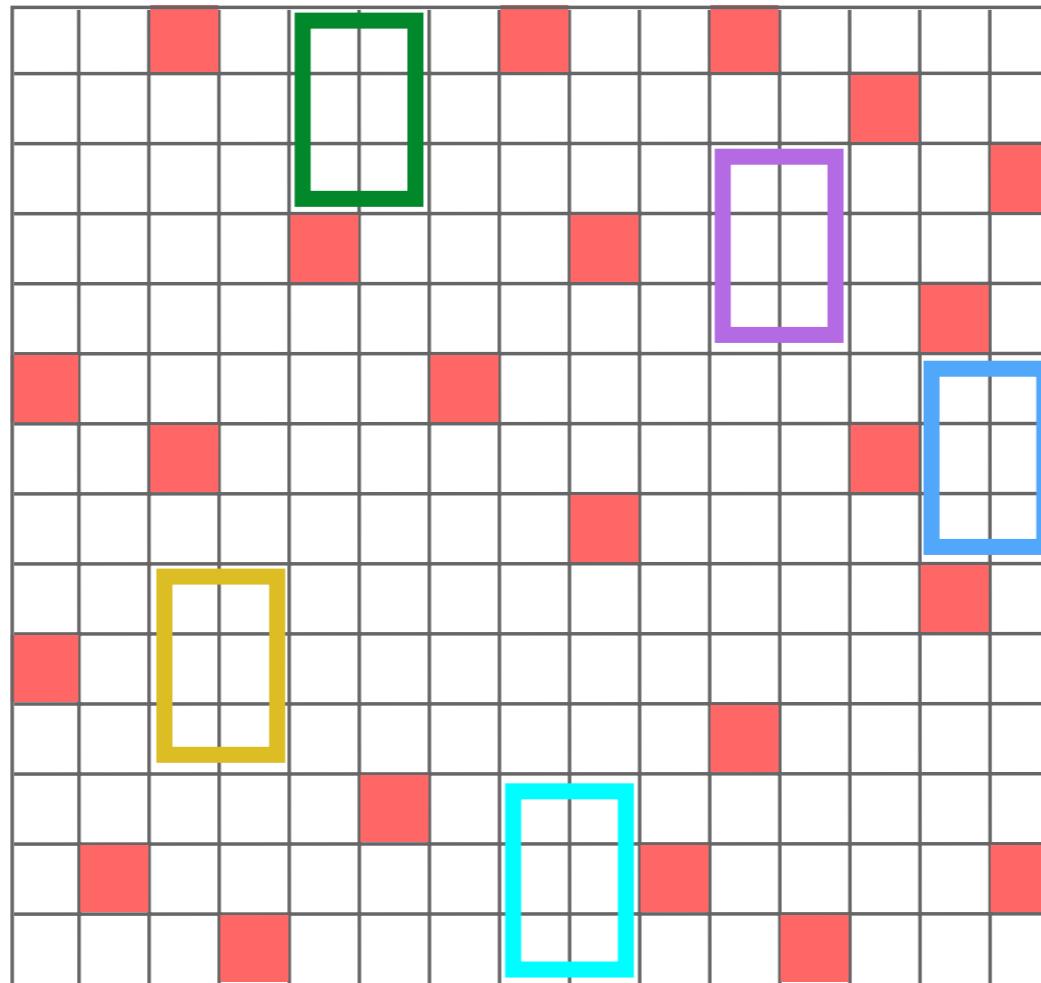
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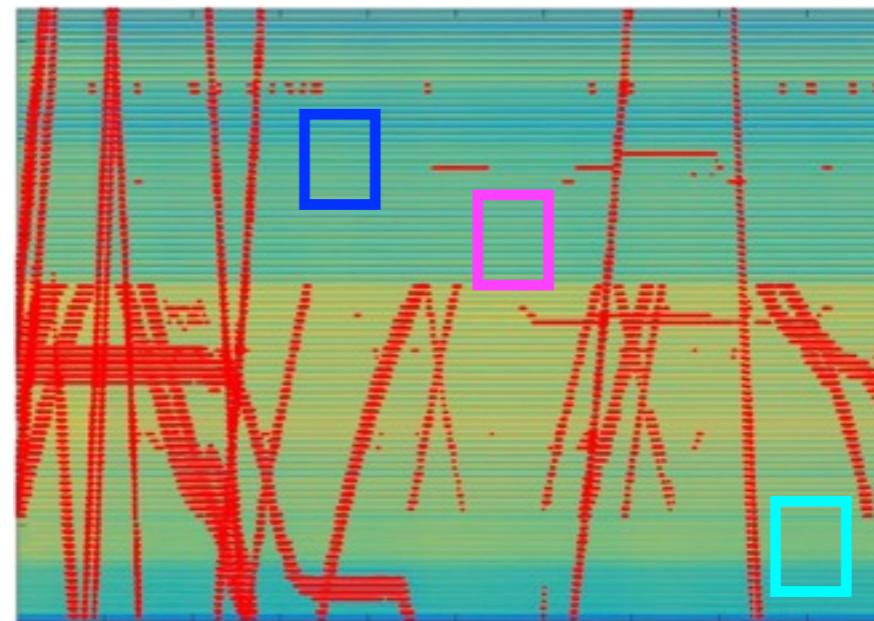
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Keep finding
uncorrupted
projections

If we find *the right projections*,
we can find the subspace



Background segmentation

Original Frame



This Work
(Pimentel, Nowak, 2017)



RPCA-ALM
RPCA-ALM (Lin et. al, 2011-2016)



In many cases, similar results

Original Frame



This Work
(Pimentel, Nowak, 2017)



RPCA-ALM
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In other cases, better

Original Frame



This Work
(Pimentel, Nowak, 2017)

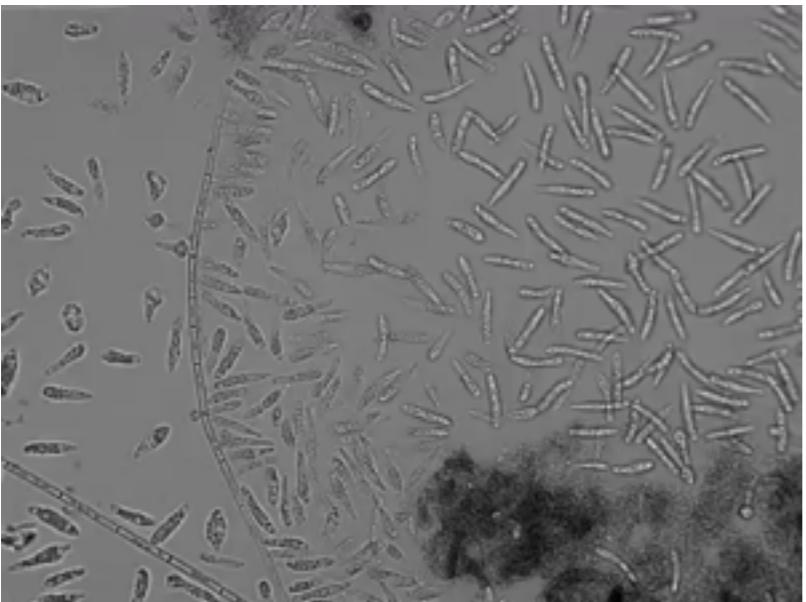


RPCA-ALM
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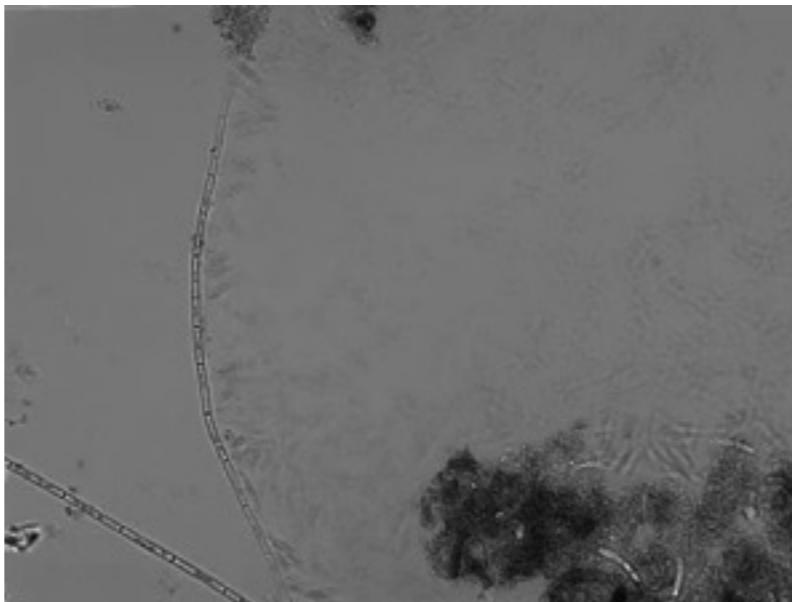
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Original Video



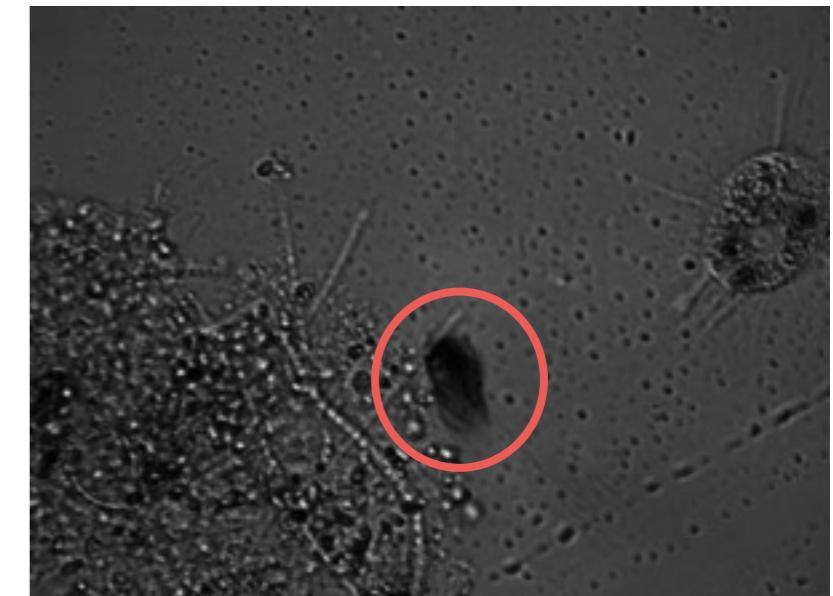
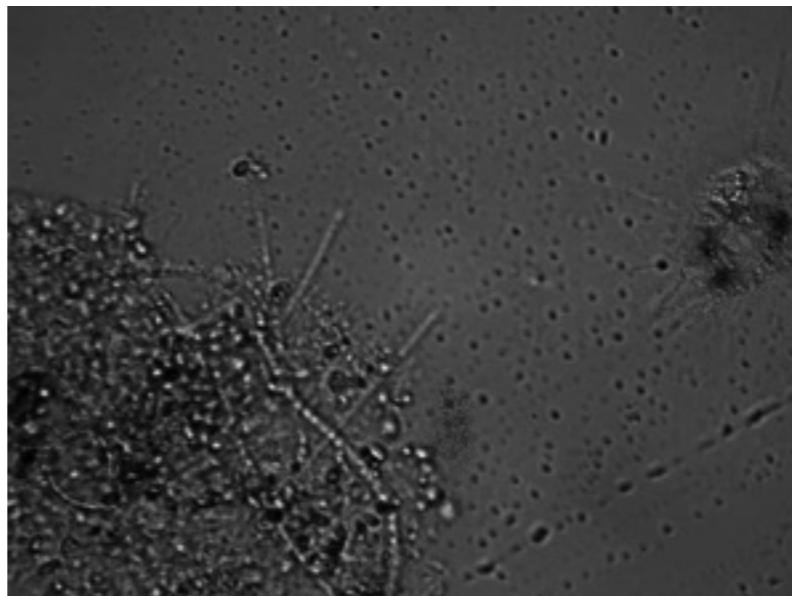
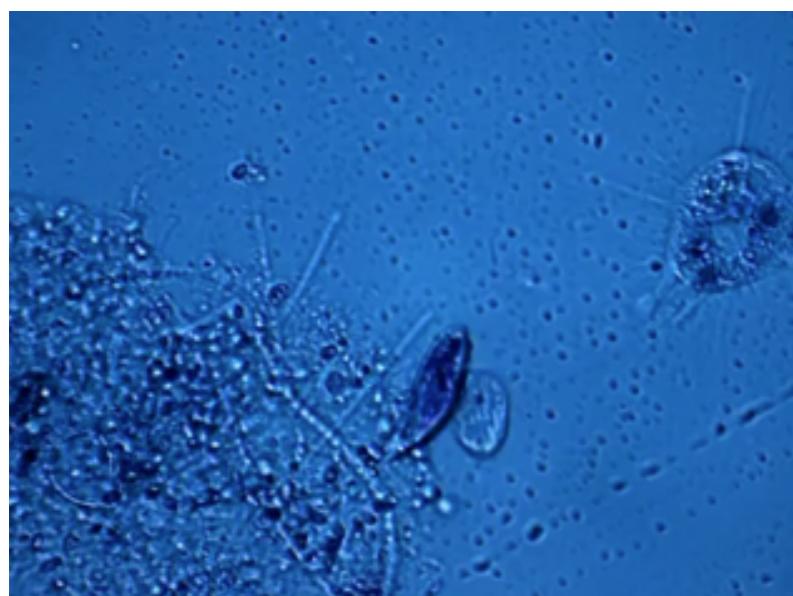
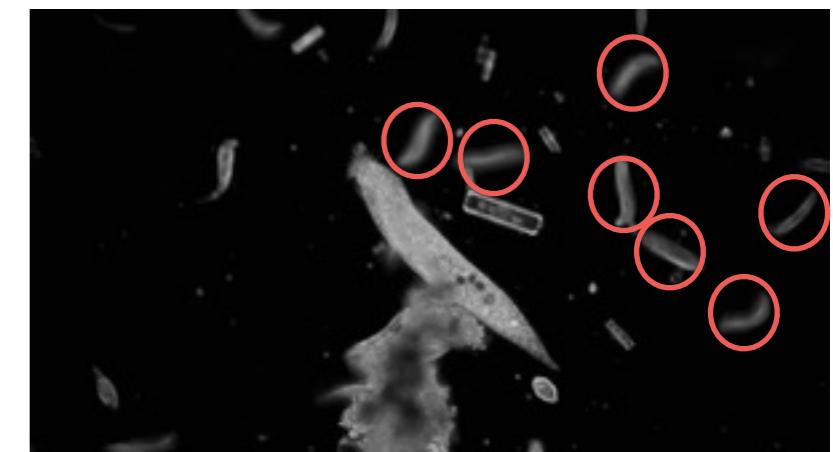
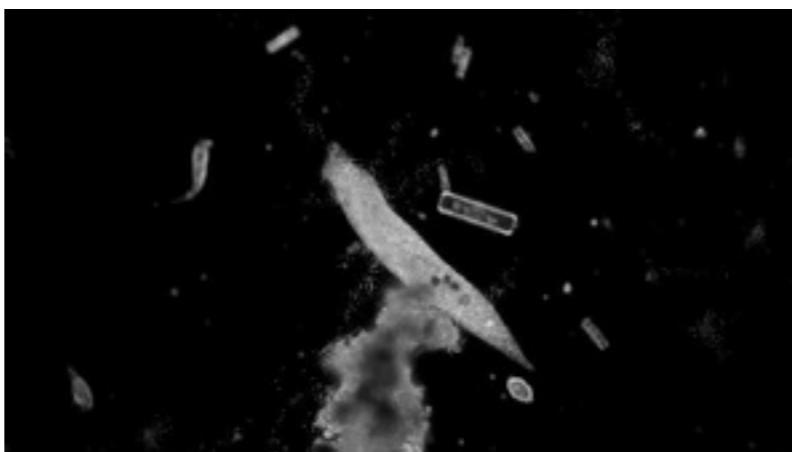
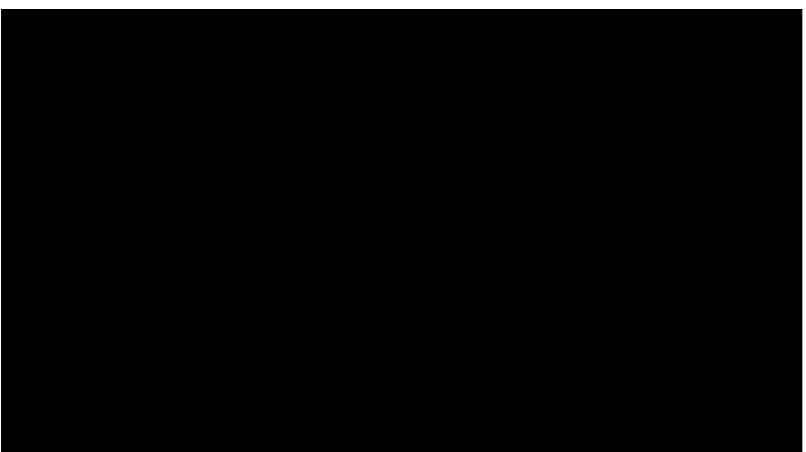
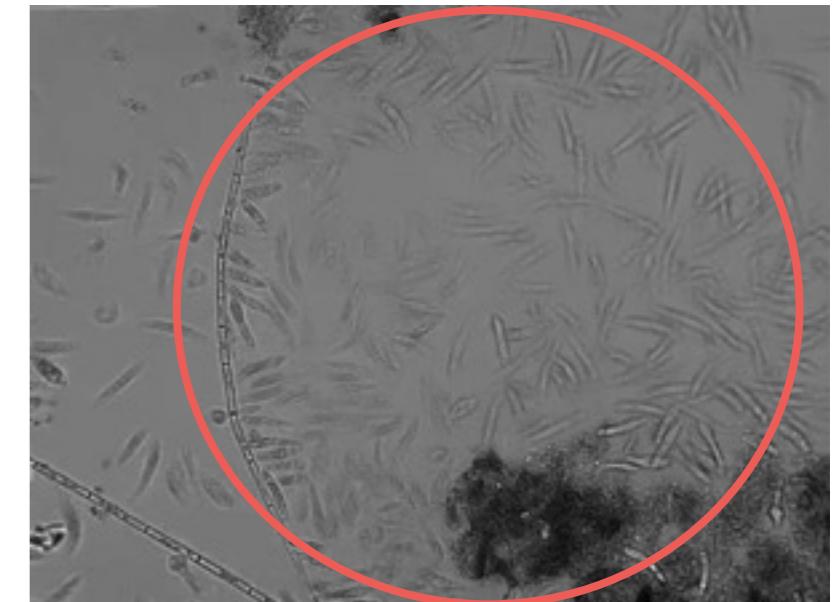
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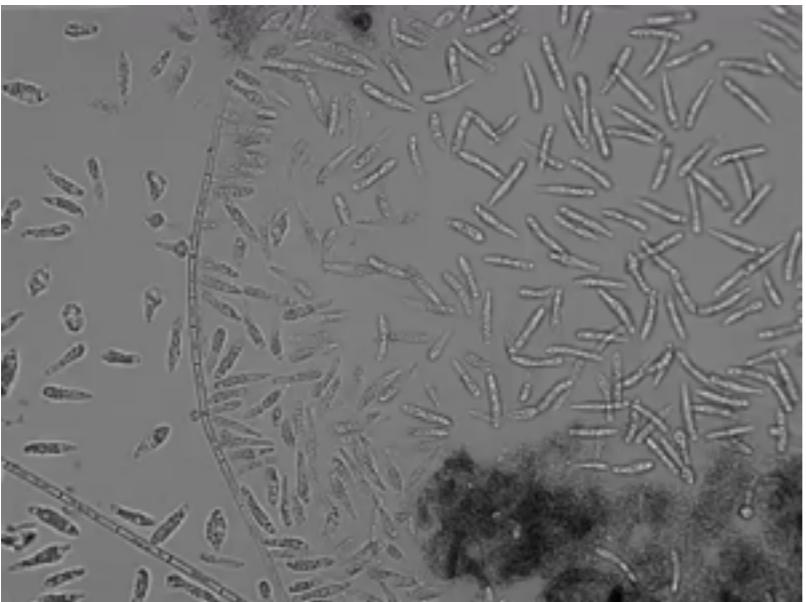


RPCA-ALM

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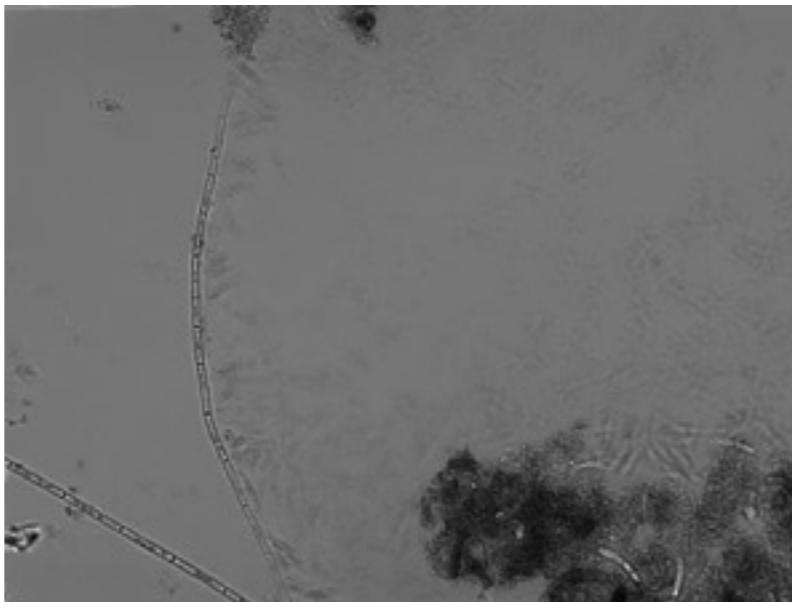


Original Video



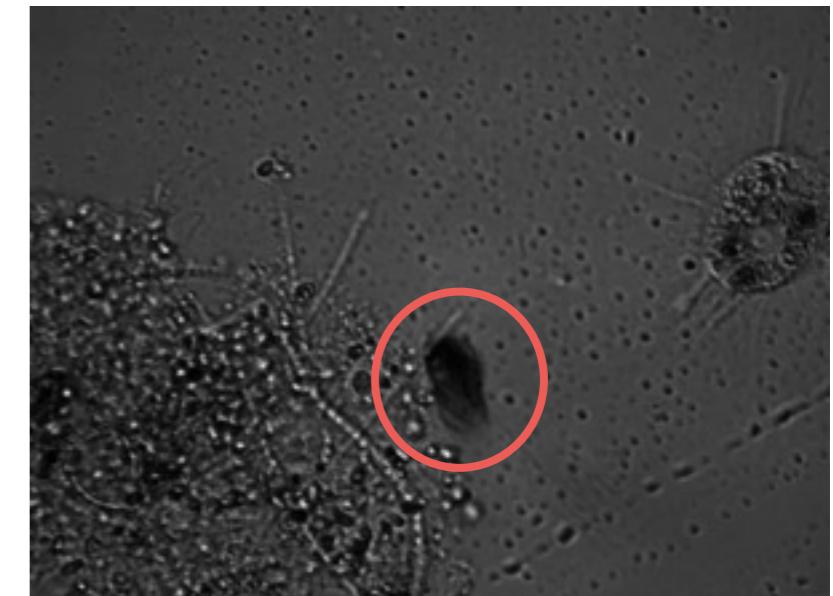
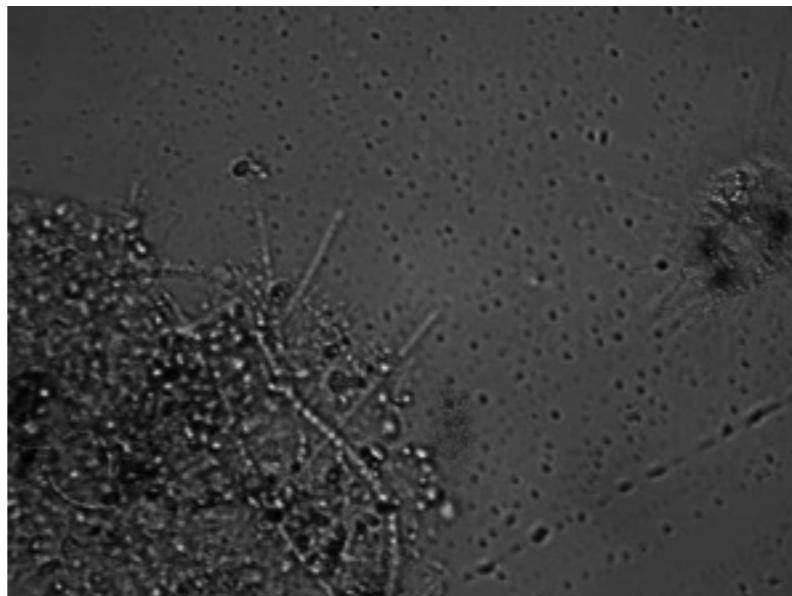
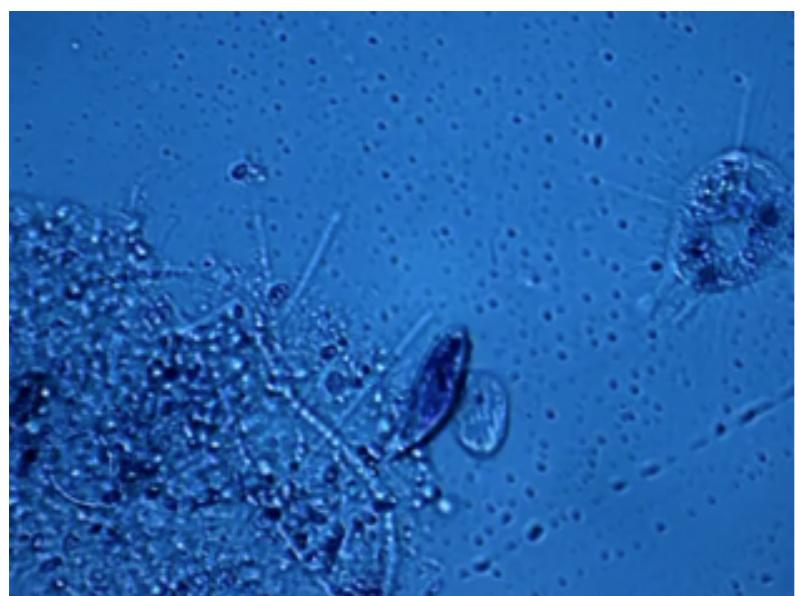
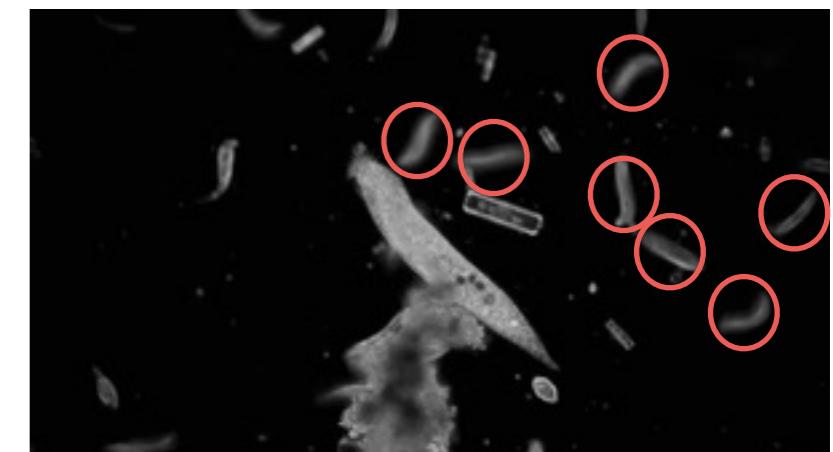
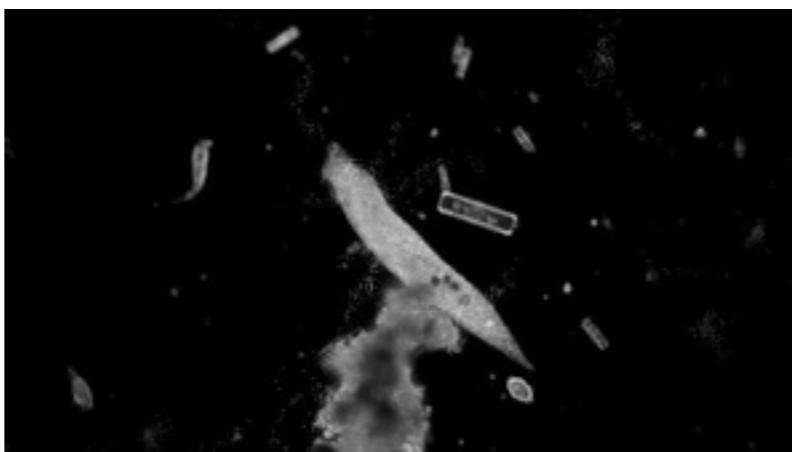
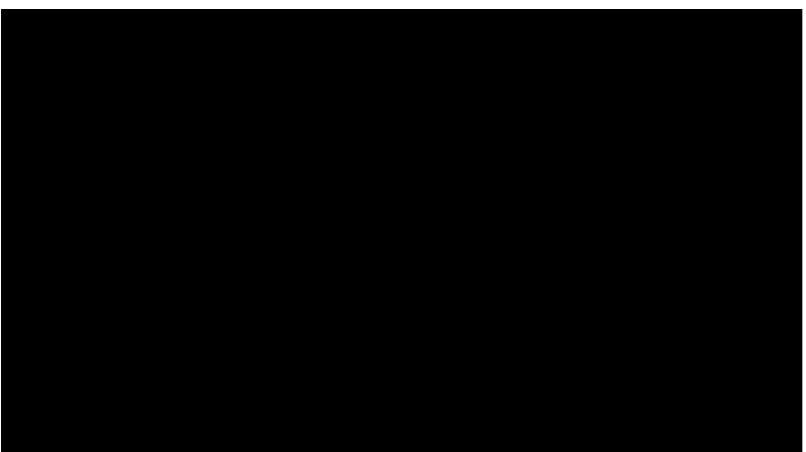
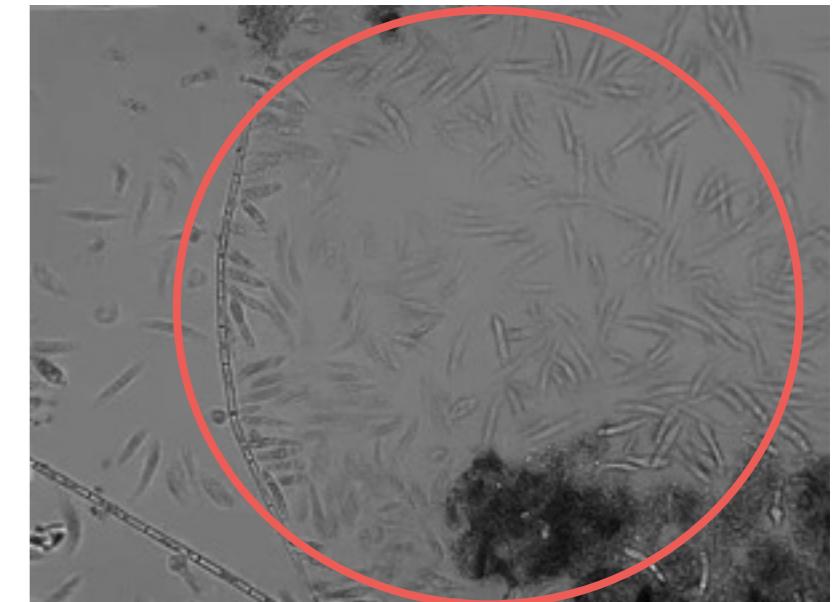
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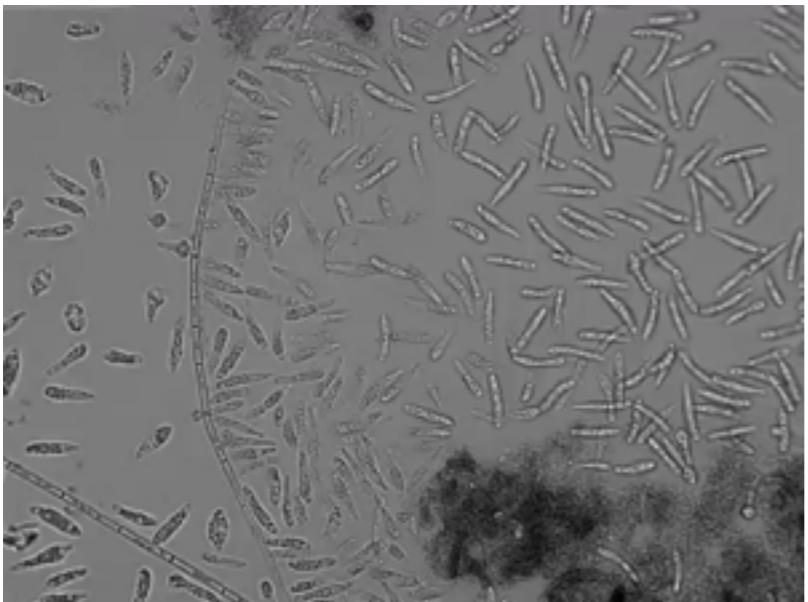


RPCA-ALM

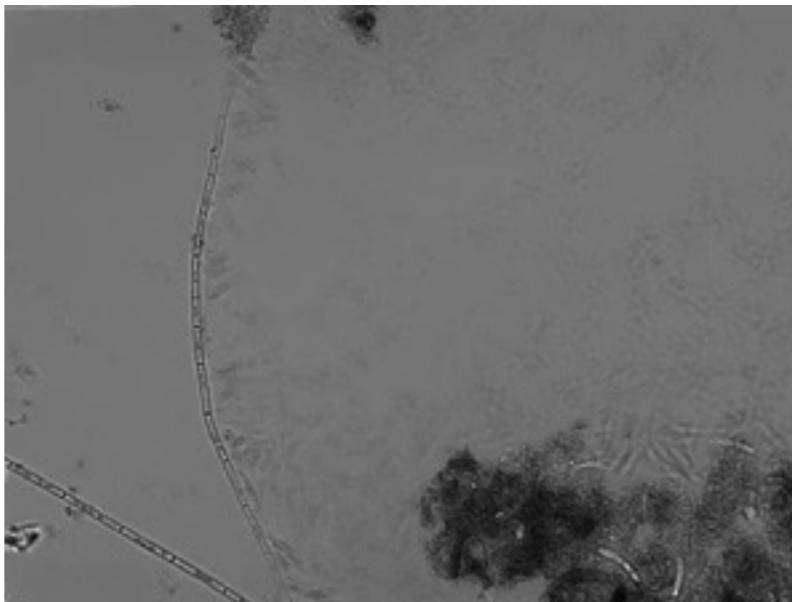
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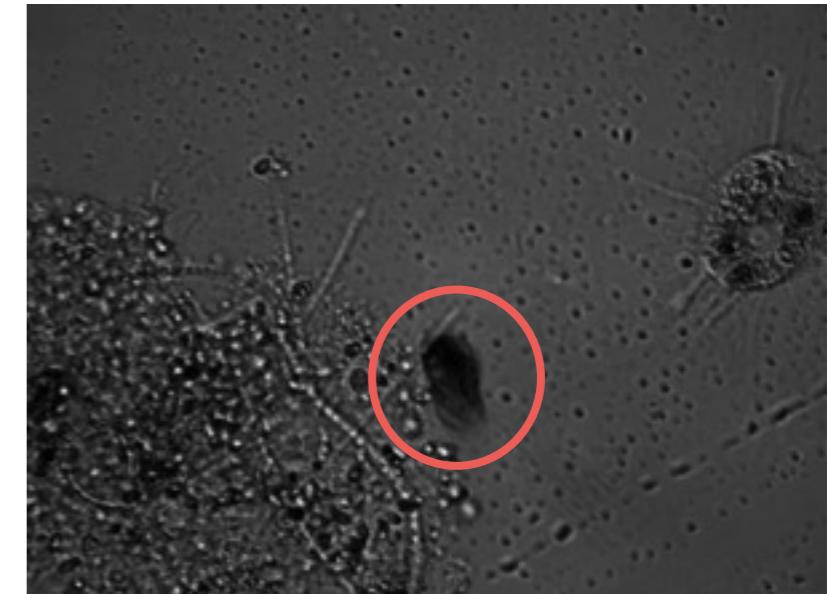
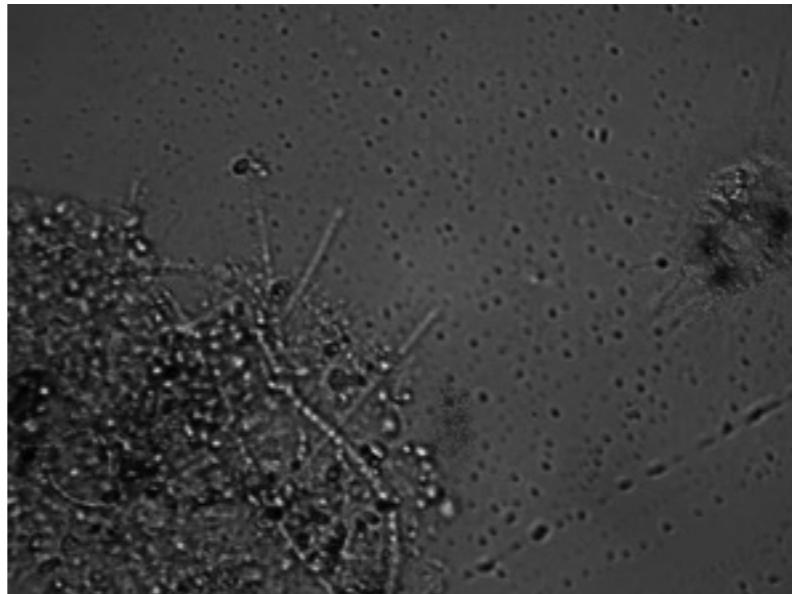
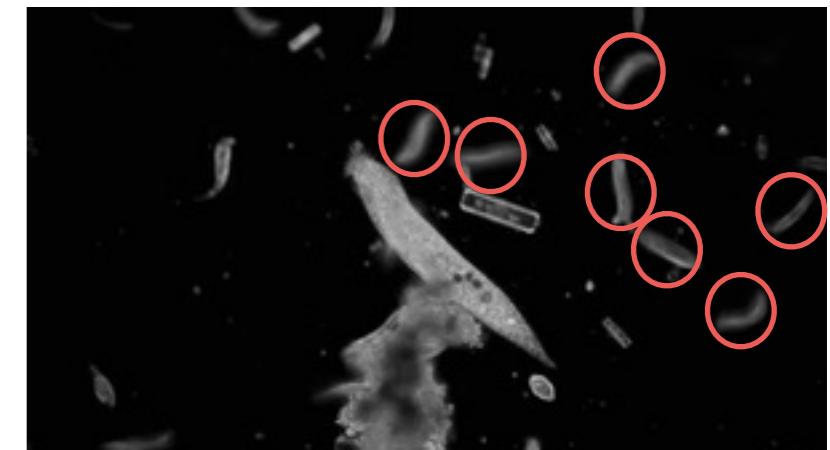
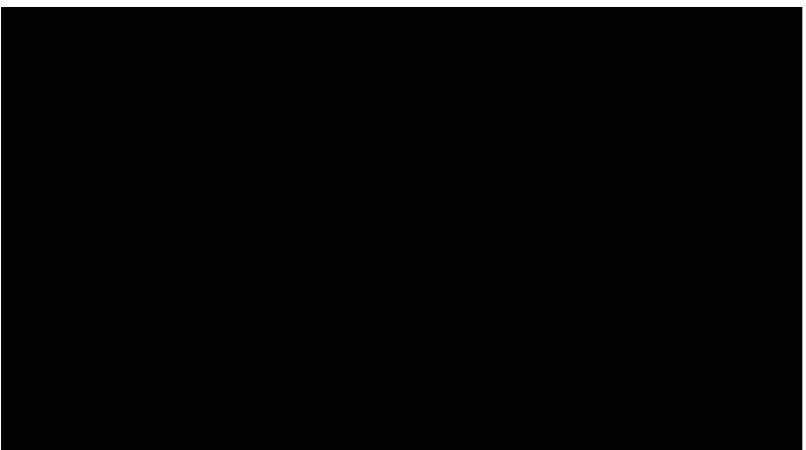
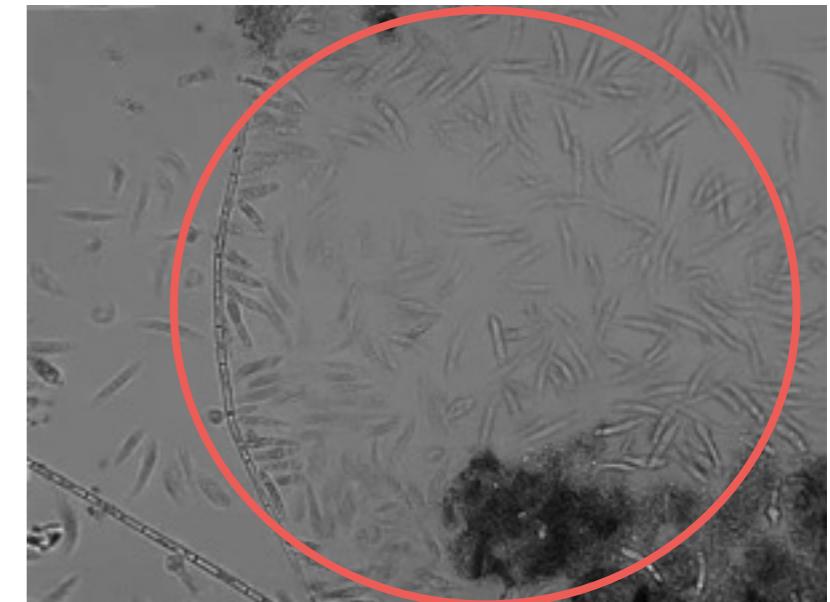
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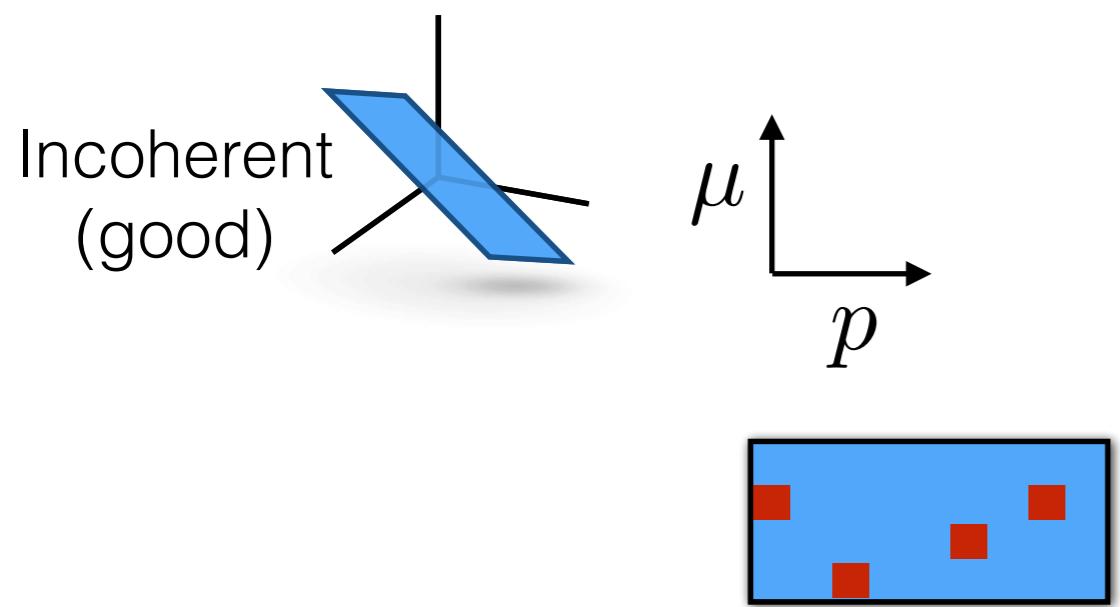


This Work
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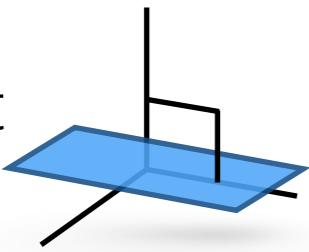
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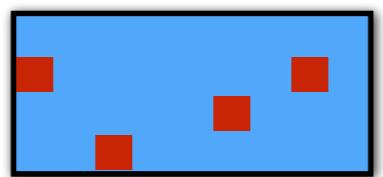
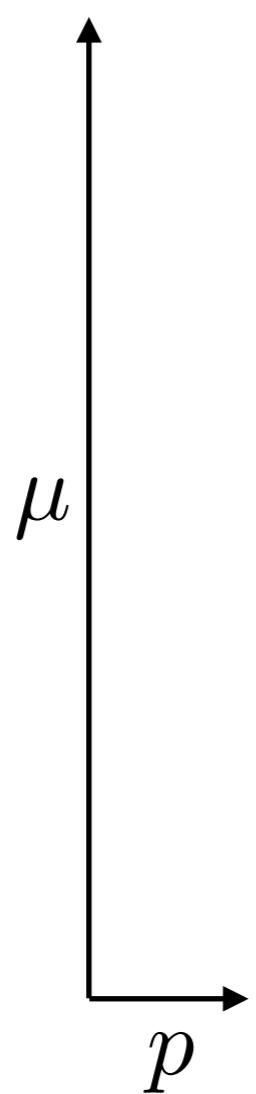
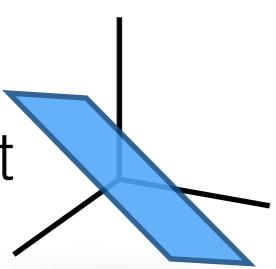


Performance Analysis

Coherent
(bad)

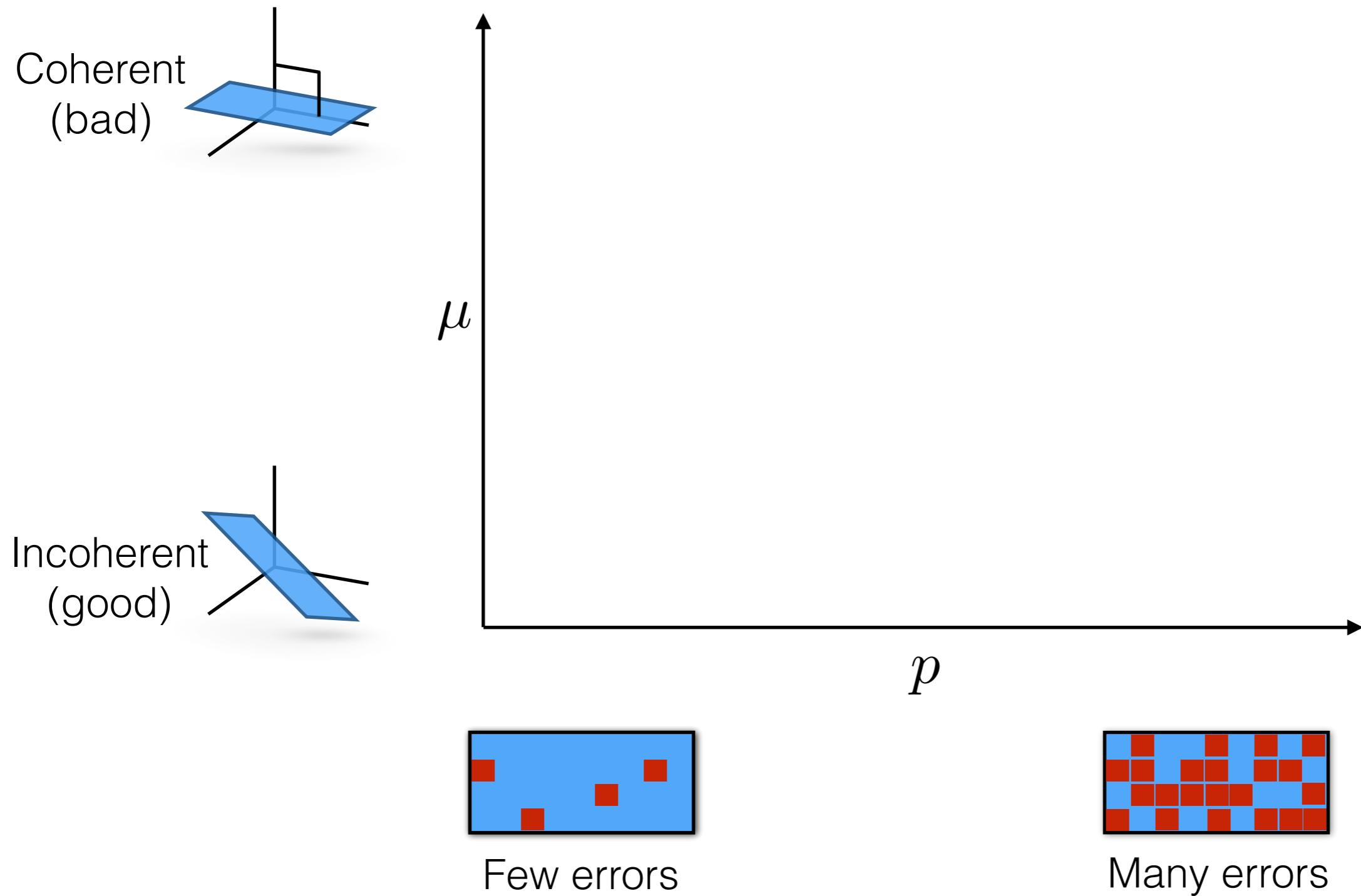


Incoherent
(good)



Few errors

Performance Analysis

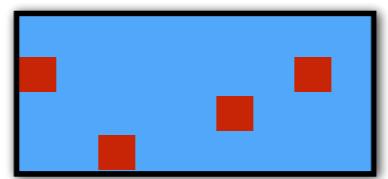
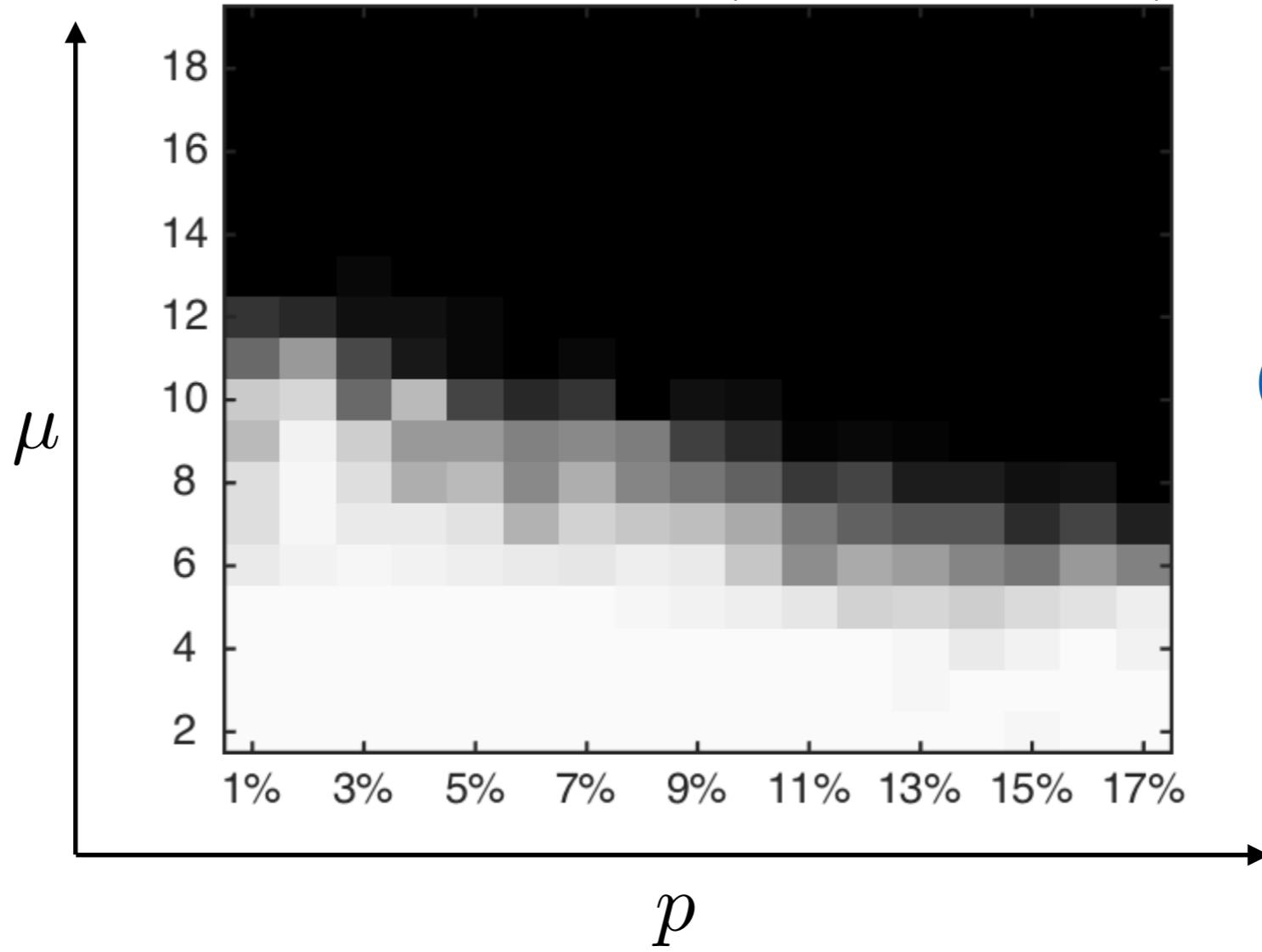


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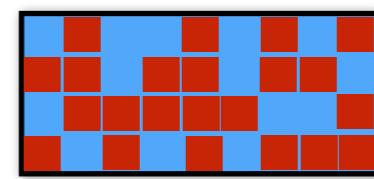
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(bad)

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RPCA-ALM (Lin et. al, 2011-2016)

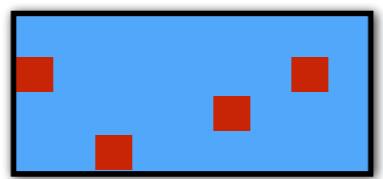
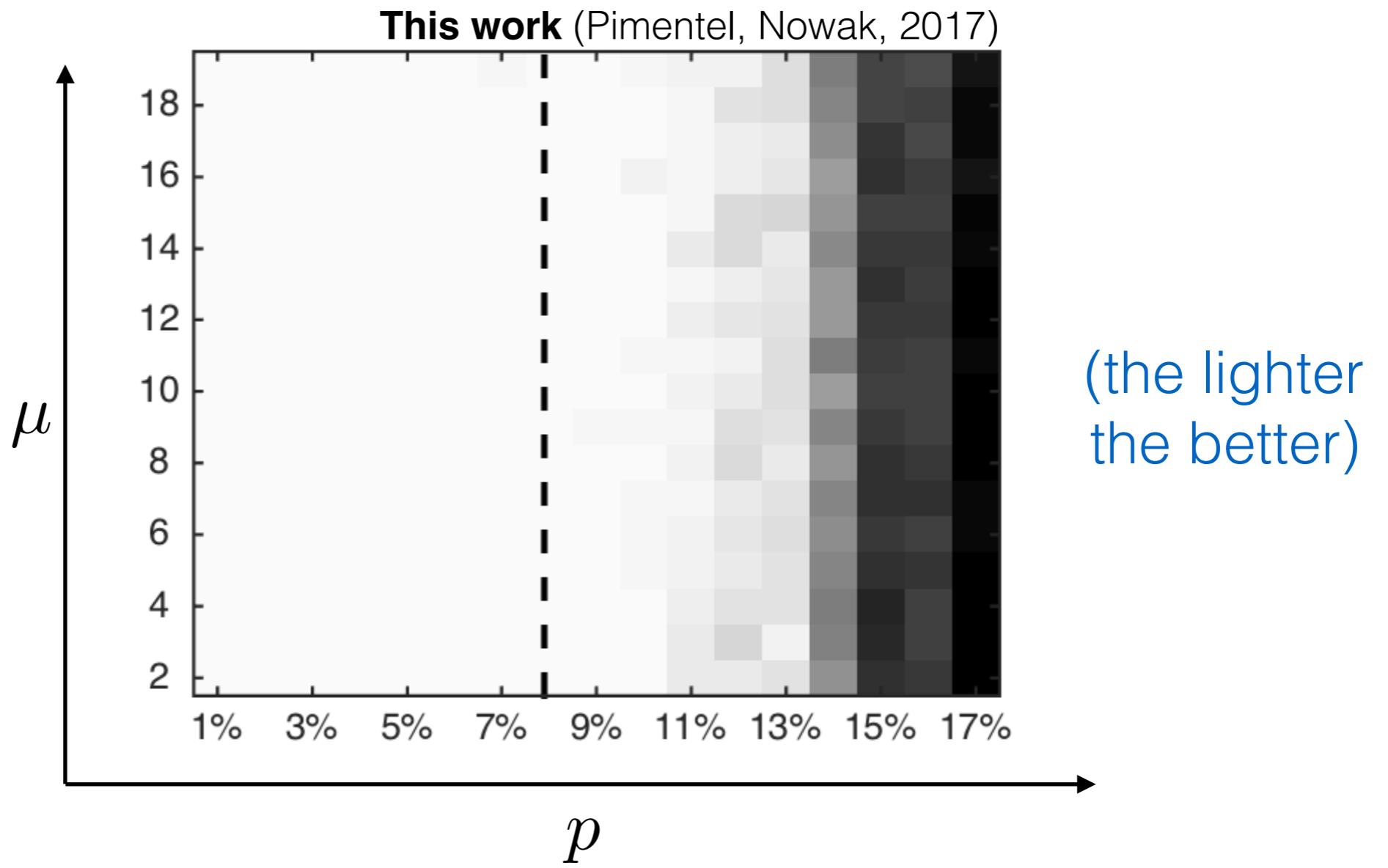
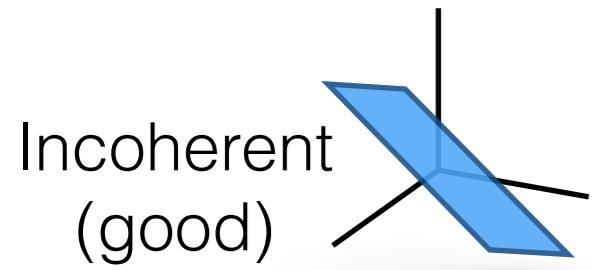
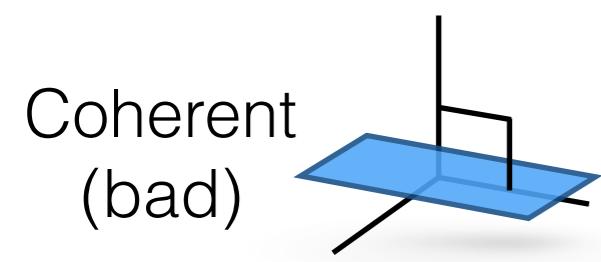


Few errors

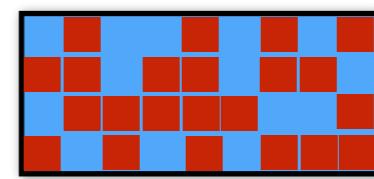


Many errors

Performance Analysis

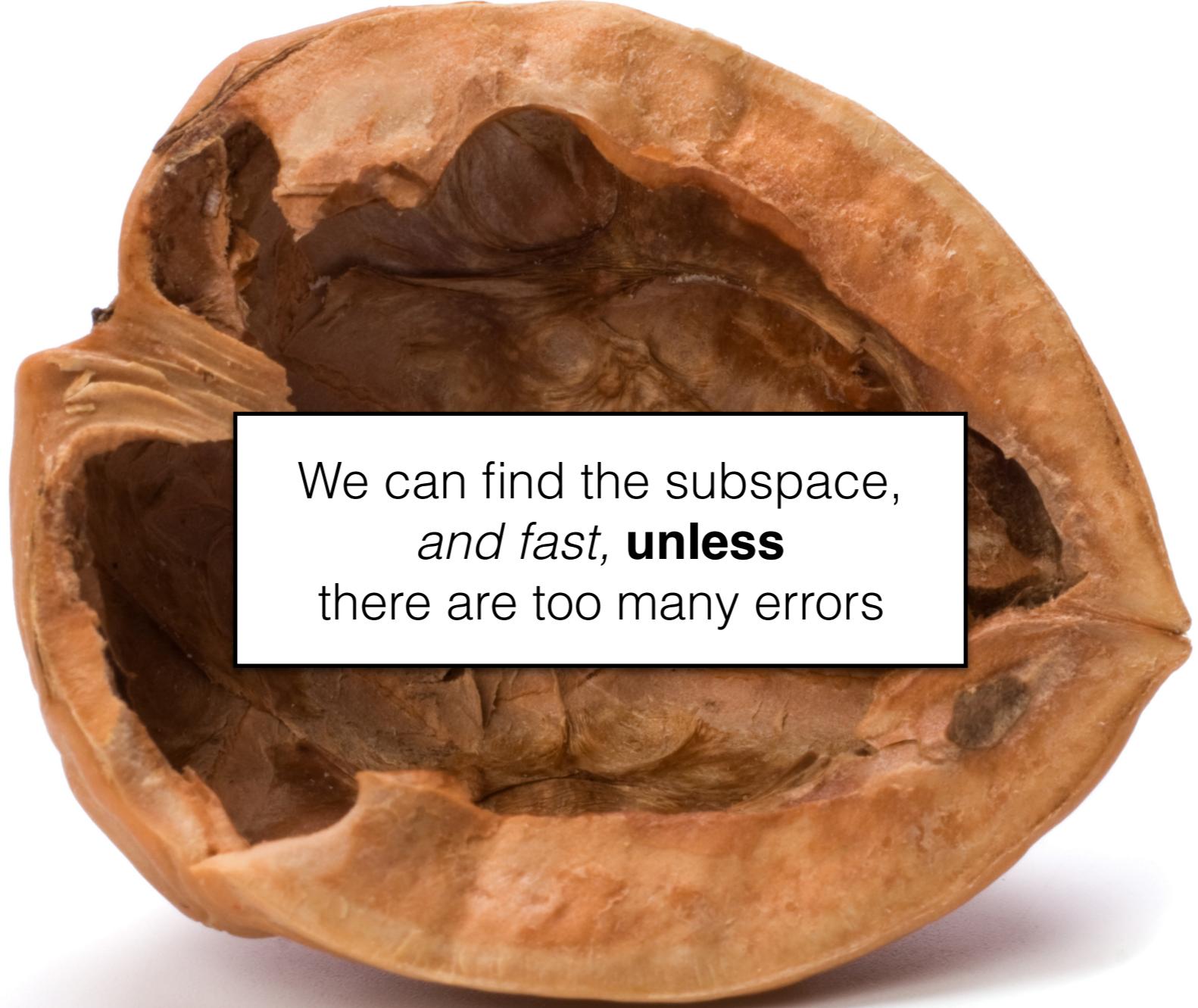


Few errors



Many errors

Performance Analysis



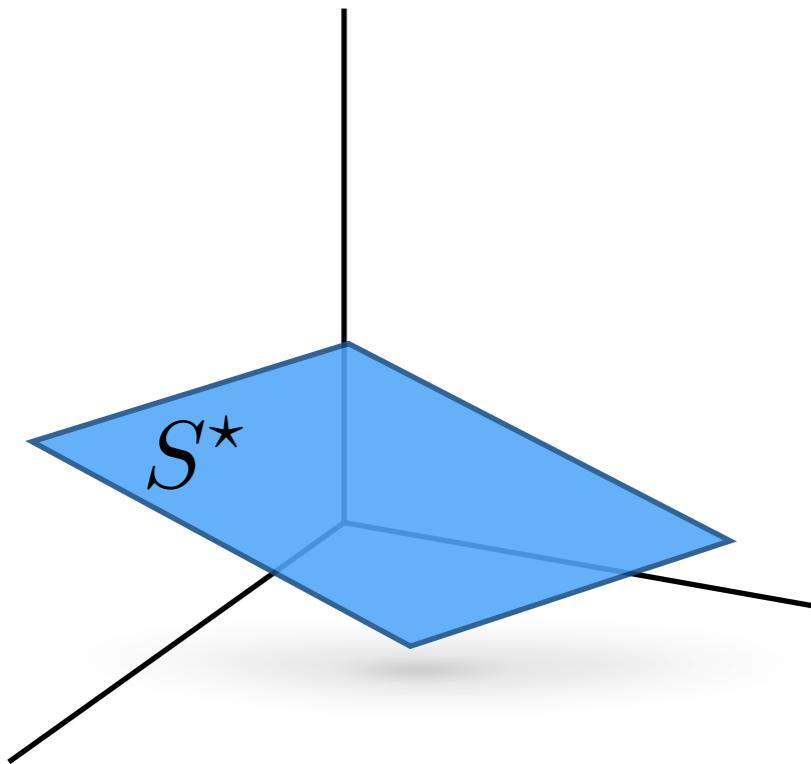
We can find the subspace,
and fast, unless
there are too many errors

Our main result in a nutshell

Pimentel, Nowak, AISTATS, 2017

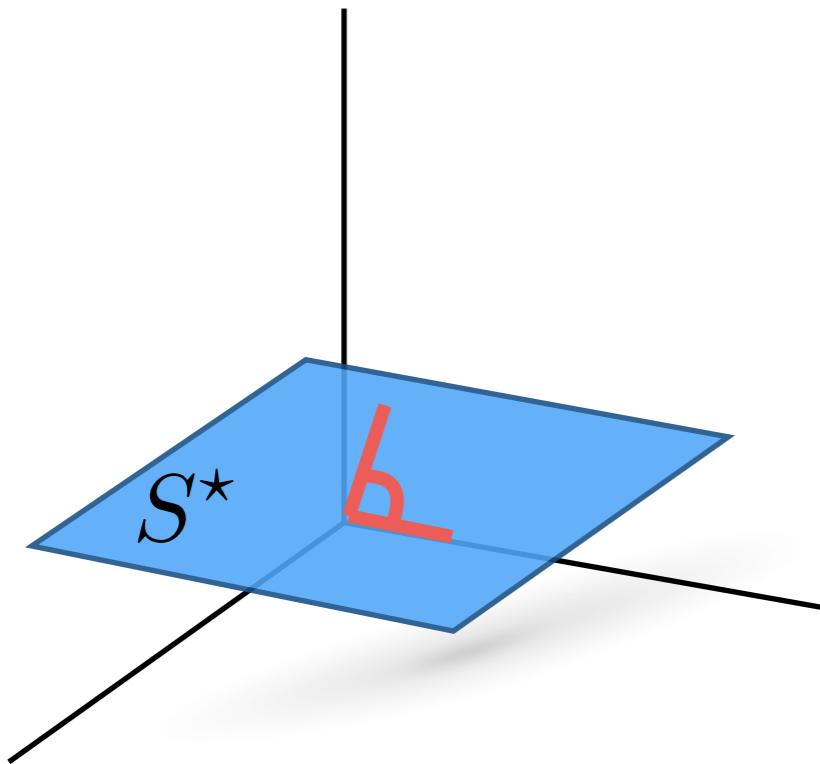


S^* = r -dimensional subspace in general position.



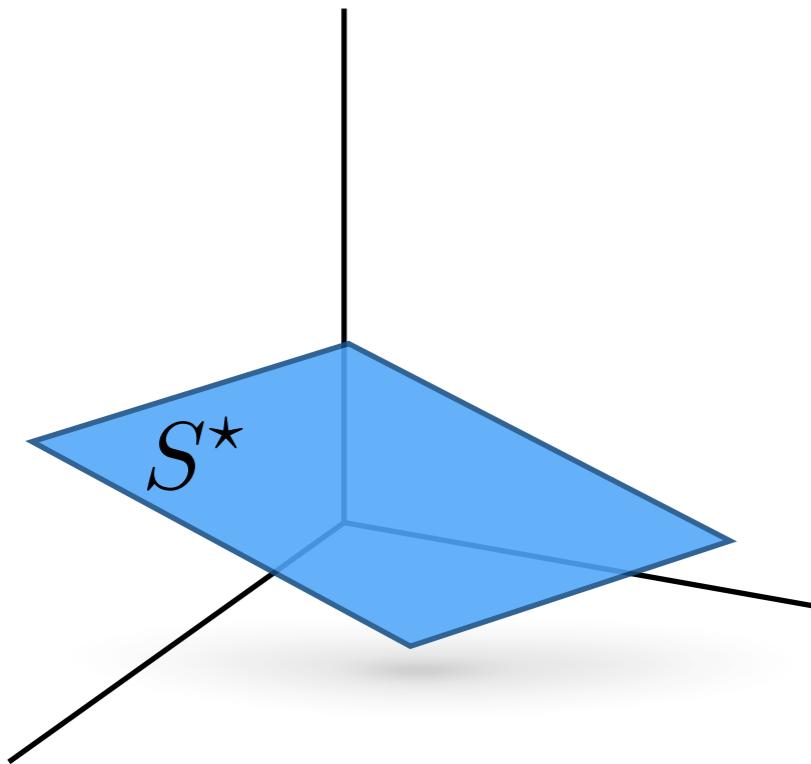
Assumptions

S^* = r -dimensional subspace in general position.



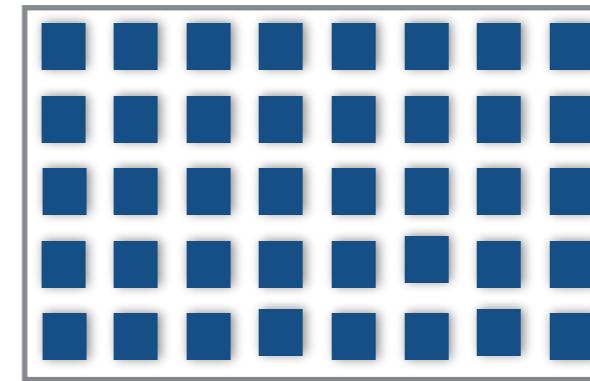
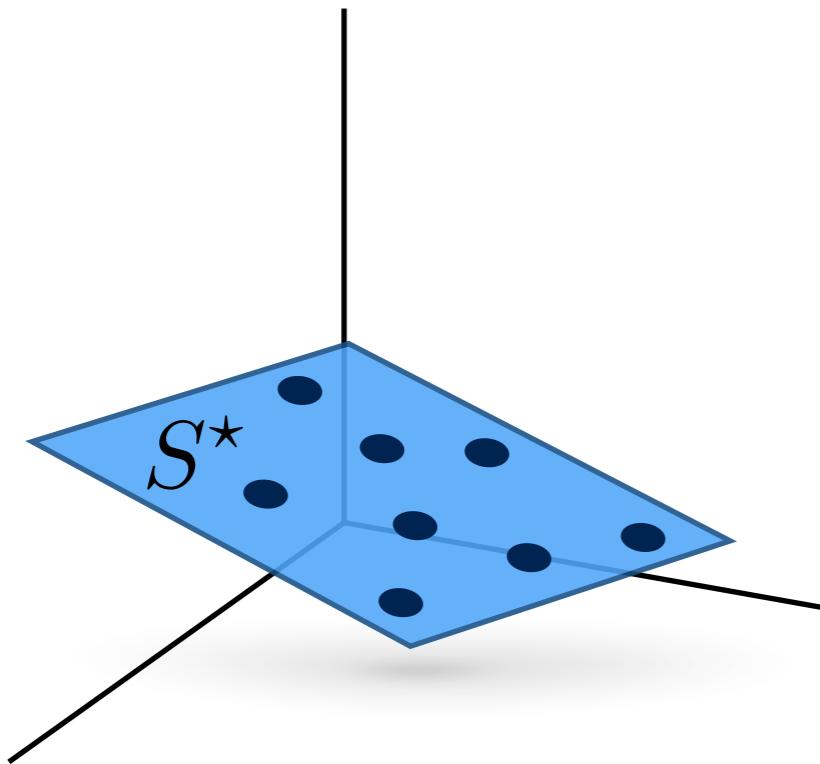
Assumptions

S^* = r -dimensional subspace in general position.



Assumptions

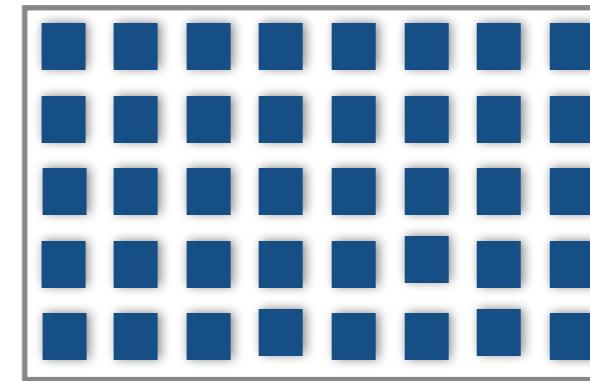
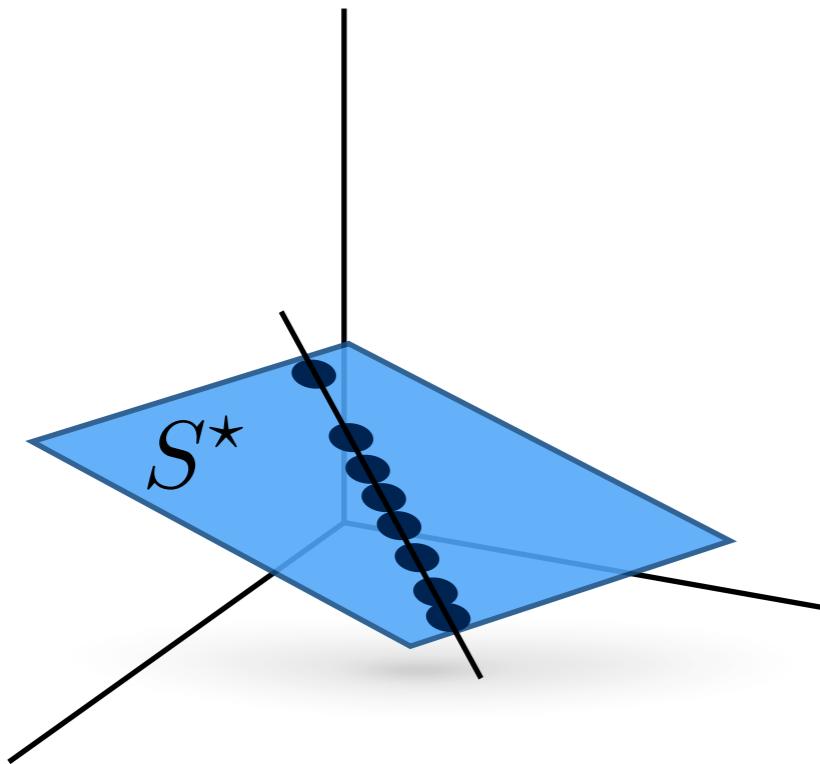
S^* = r -dimensional subspace in general position.



Columns lie in S^* generically.

Assumptions

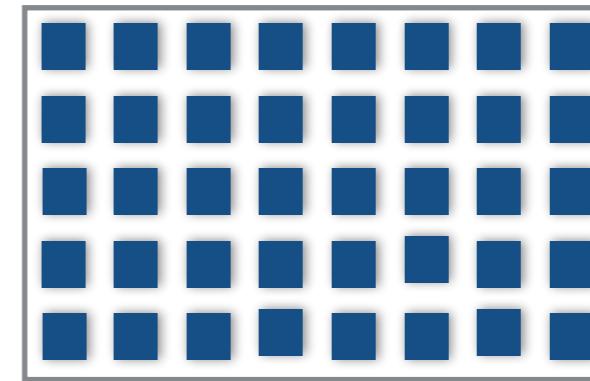
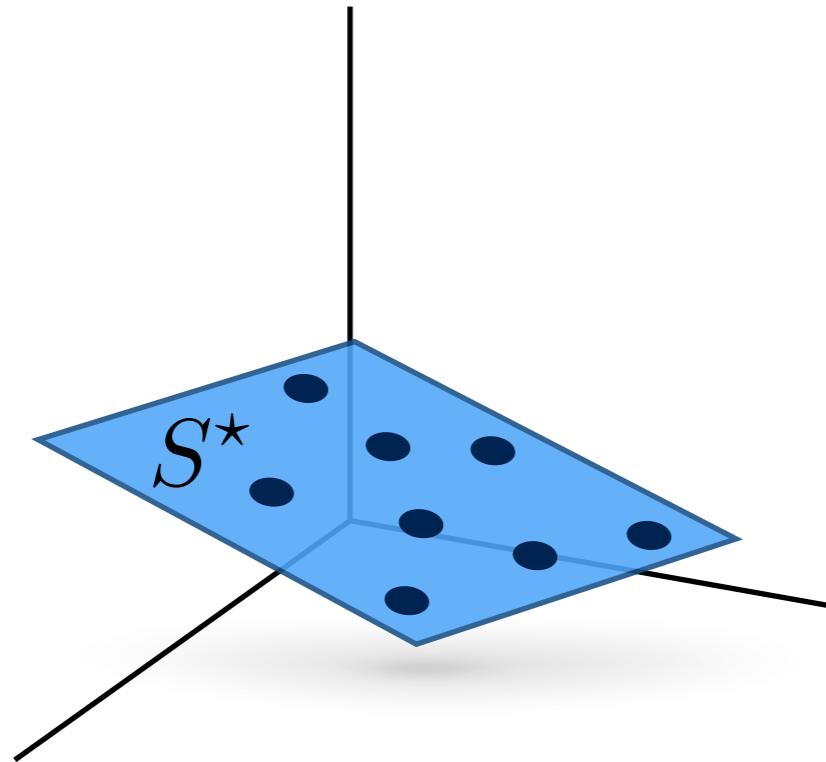
S^* = r -dimensional subspace in general position.



Columns lie in S^* generically.

Assumptions

S^* = r -dimensional subspace in general position.



Columns lie in S^* generically.

Assumptions

Take-home Message

- New (algebraic) method for Robust PCA.
- Arbitrary (non-uniform, even adversarial) sparsity patterns.
- No coherence assumptions.



Rob Nowak



Nigel Boston

Joint work with:

Thank you!

