

# SCHEDULING

FLORIAN DAHMS\*, ANNA-LENA FRANK†, SEBASTIAN KÜHN†, AND DANIEL PÖHLE†

**Abstract.** In this article, we discuss the optimization of a linearized traffic flow network model based on conservation laws. We present two solution approaches. One relies on the classical Lagrangian formalism (or adjoint calculus), whereas another one uses a discrete mixed-integer framework. We show how both approaches are related to each other theoretically. Numerical experiments are accompanied to show the quality of solutions.

**Key words.** traffic networks, network optimization

**AMS subject classifications.** 90B20, 90C11

## 1. Extended Abstract.

### 1.1. problem and objectives.

- assemble to order for capacity planning
- 3%, 5%, 3 min C&R-App

### 1.2. literature.

- TAKT
- OpSysTra

### 1.3. innovation.

- first infrastructure manager to use optimization models for freight train timetable planning
- customer value: shorter travel time, higher infrastructure capacity and faster response time
- first fully automated process in short-term capacity planning from ordering to the actual departure of the train

### 1.4. data.

- digital infrastructure model
- timetable of passenger trains

### 1.5. practical relevance.

- release of C&R-App in 2019

As explained earlier the C&R-App gives the cargo customer a valid timetable after an average of three minutes. As for now the app is working as design. However, the app is not used by the customer yet since it is still in a evaluation phase where bugs can be find and be fixed.

Nevertheless this situation changes next year as the release of the C&R-App is planned next year. This has an extremely impact of the timetable procedure since then the customer gets his timetable within three minutes. Today a short-term creation of a timetable takes up to 72 hours, since we have to take all the details of the construction of one train into account and the construction is done manually. Therefore customer order there timetable a long time ahead even if they don't exactly know whether they need the timetable at the requested time or not. For example when goods are relaoded from a

---

\*???, ???, Frankfurt, Germany (dahms@???.de)

†I.NMF 32, DB Netz AG, Frankfurt am Main, Germany (Anna-Lena.Frank@deutschebahn.com, Sebastian.Se.Kuehn@deutschebahn.com, Daniel.Poehle@deutschebahn.com)

ship to a train. Here, it is really hard to find out when the goods are exactly on the train and therefore the ordered timetable does not fit to the actual time anymore. But since the customer ordered a timetable we have to dispose the train through our network which makes the network unstable.

As a conclusion the new App gives not only the customer a chance to actually stick on their timetable it also gives us better data for the construction of a timetable.

As you can see our solution is not only academic it also solves real world problems since  $x\%$  of the customer requests can be answered automatically.

What we want to do in the future.

We want to give the customer the possibility to select within three different timetables to give him more options what kind of timetable he wants.

Furthermore after going live with the C&R-App we also want to enable the automatic planning of the "Netzfahrplan" which does the timetable of the cargo trains automatically for one year and the planning of the "Baufahrplan" which reschedule the cargo trains due to constructions.

This next steps do not come all at one time. Since we want to learn how the customer react to the new procedure and we want to have the chance to change the things the customer complain about. With this way we can come together with our customer to a better quality of the timetable.

This new way of the construction of the cargo train timetable prepares DB Netz for the future with an increased efficiency since we have less effort for staff and customer. We are not only more efficient with the planning of timetable we also have a higher capacity utilization of the infrastructure and we are able to offer our cargo customer a better timetable since the customer reaches their destination earlier.

**2. Introduction.** Modeling, simulation and optimization of traffic flow networks based on partial differential equations have been investigated intensively during the last years, see for instance for an overview [4, 11, 15, 16, 17, 19, 21].

For optimization purposes, different applications such as optimal routing of traffic at intersections [11, 15, 16], traffic light control [12], evacuation planning [13, 14] and air traffic control [1] are of interest. Since in all problems the underlying optimization problem is restricted by nonlinear partial differential equations, relaxed models with simplified dynamics have been investigated. In this context, two different solution approaches emerge. On the one hand, nonlinear continuous optimization techniques have been successfully applied. To compute the optimum, the first order optimality system is derived and solved by a descent type method, see [18, 22, 25]. On the other hand, suitable discretization of the dynamics leads to linear network flow models that have been widely considered in the field of combinatorial optimization [2, 8, 9, 20].

In fact, there exist a few research results comparing both optimization tools, see [5, 7, 10, 11, 26]. One might assume that a one-to-one relation between nonlinear continuous and discrete optimization techniques holds true. This is especially the case when the governing dynamics in the network are linear. Then, appropriate numerical discretizations can be chosen, such that the original optimization problem either leads to numerically solving the finite-dimensional optimality system, i.e. the so-called *discretize-then-optimize approach*, or the interpretation as a mixed-integer

programming model (MIP). However, it remains the question of detecting local or global optima. We know from the theory of linear programming, in particular the strong duality theorem [3, 23, 24], that under certain circumstances a global optimum can be reached. This is usually not the case for the adjoint calculus. The solution of the optimality system via gradient methods often gets stuck in local optima.

In this work we want to close the gap between the two solution procedures that first appear to be quite varied. To do so, we start with the traffic flow network model where the evolution of traffic density on roads is governed by the linearized Lighthill-Whitham-Richards (LWR) equations, see [6]. For the coupling conditions at the intersections, we stick to the ones presented by Coclite-Garavello-Piccoli [4] but do not use a predefined distribution matrix since these parameters will be obtained as solutions of the optimization problem. In a next step, we discretize the full network model in space and time and formally derive the adjoint equations and the mixed-integer model (MIP) as well. We show the equivalence of both approaches by comparing the dual variables of the MIP and the adjoint variables. A formal, detailed proof for a simple network is given. Finally, we compare the optimization results of the MIP and the adjoint equations for different networks and objective functions numerically.

The paper is organized as follows: Section 3 contains a description of the network and the models used for describing flow on roads. Section 4 introduces the optimization problem to be considered. Herein, we also present the main theorem of this article, which we prove in the following sections. In section ??, the adjoint equations for the former optimization problem are derived. Thereafter, the mixed-integer model is introduced in section 5. In section ??, we conclude the proof of our main theorem. Numerical examples are given in section 6.

### 3. Traffic Flow Network Model.

### 4. Optimization Problem.

### 5. Mixed Integer Program (MIP).

### 6. Numerics.

**Acknowledgements.** This work was financially supported by BMI, Project DigiKap, FKZ ???.

## REFERENCES

- [1] A. M. BAYEN, R. L. RAFFARD, AND C. J. TOMLIN, *Adjoint-based control of a new eulerian network model of air traffic flow*, IEEE Transactions on Control Systems Technology, 14 (2006), pp. 804–818.
- [2] M. CAREY AND E. SUBRAHMANIAN, *An approach to modelling time-varying flows on congested networks*, Transportation Research Part B: Methodological, 34 (2000), pp. 157–183.
- [3] V. CHVÁTAL, *Linear Programming*, W. H. Freeman and Company, New York, 1983.
- [4] G. M. COCLITE, M. GARAVELLO, AND B. PICCOLI, *Traffic flow on a road network*, SIAM Journal on Mathematical Analysis, 36 (2005), pp. 1862–1886.
- [5] C. D'APICE, S. GÖTTLICH, M. HERTY, AND B. PICCOLI, *Modeling, Simulation and Optimization of Supply Chains: A Continuous Approach*, SIAM book series on Mathematical Modeling and Computation, 2010.
- [6] C. D'APICE, R. MANZO, AND B. PICCOLI, *Packet flow on telecommunication networks*, SIAM Journal on Mathematical Analysis, 38 (2006), pp. 717–740.
- [7] P. DOMSCHKE, B. GEISSLER, O. KOLB, J. LANG, A. MARTIN, AND A. MORSI, *Combination of nonlinear and linear optimization of transient gas networks*, INFORMS Journal on Computing, 23 (Fall 2011), pp. 605–617.

- [8] L. FLEISCHER AND E. TARDOS, *Efficient continuous-time dynamic network flow algorithms*, Operations Research Letters, 23 (1998), pp. 71–80.
- [9] L. R. FORD AND D. R. FULKERSON, *Constructing maximal dynamic flows from static flows*, Operations Research, 6 (1958), pp. 419–433.
- [10] A. FÜGENSCHUH, B. GEISLER, A. MARTIN, AND A. MORSI, *The transport pde and mixed-integer linear programming*, in Models and Algorithms for Optimization in Logistics, Cynthia Barnhart, Uwe Clausen, Ulrich Lauther, and Rolf H. Möhring, eds., no. 09261 in Dagstuhl Seminar Proceedings, Dagstuhl, Germany, 2009, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, Germany.
- [11] A. FÜGENSCHUH, M. HERTY, A. KLAR, AND A. MARTIN, *Combinatorial and continuous models for the optimization of traffic flows on networks*, SIAM Journal on Optimization, 16 (2006), pp. 1155–1176.
- [12] S. GÖTTLICH, M. HERTY, AND U. ZIEGLER, *Modeling and optimizing traffic light settings on road networks*, preprint, (2012).
- [13] S. GÖTTLICH, S. KÜHN, J.P. OHST, S. RUZKA, AND M. THIEMANN, *Evacuation dynamics influenced by spreading hazardous material*, Networks and Heterogeneous Media (NHM), 6 (2011), pp. 443–464.
- [14] H. W. HAMACHER AND S. A. TJANDRA, *Mathematical modelling of evacuation problems: A state of the art*, Berichte des Fraunhofer ITWM, 24 (2001), pp. 1–38.
- [15] M. HERTY AND A. KLAR, *Modeling, simulation, and optimization of traffic flow networks*, SIAM Journal on Scientific Computing, 25 (2003), pp. 1066–1087.
- [16] ———, *Simplified dynamics and optimization of large scale traffic networks*, Mathematical Models and Methods in Applied Sciences (M3AS), 14 (2004), pp. 579–601.
- [17] H. HOLDEN AND N. H. RISEBRO, *A mathematical model of traffic flow on a network of unidirectional roads*, SIAM Journal on Mathematical Analysis, 26 (1995), pp. 999–1017.
- [18] C. T. KELLEY, *Iterative methods for optimization*, Society for Industrial and Applied Mathematics, Philadelphia, 1999.
- [19] A. KLAR, R.D. KÜHNE, AND R. WEGENER, *Mathematical models for vehicular traffic*, Surveys on Mathematics for Industry, 6 (1996), pp. 215–239.
- [20] E. KÖHLER AND M. SKUTELLA, *Flows over Time with Load-Dependent Transit Times*, SIAM Journal on Optimization, 15 (2005), p. 1185.
- [21] M. J. LIGHTHILL AND G. B. WHITHAM, *On Kinematic Waves. II. A Theory of Traffic Flow on Long Crowded Roads*, Royal Society of London Proceedings Series A, 229 (1955), pp. 317–345.
- [22] S. G. NASH AND A. SOFER, *Linear and Nonlinear Programming*, McGraw-Hill, New York, 1996.
- [23] G. L. NEMHAUSER AND L. A. WOLSEY, *Integer and combinatorial optimization*, Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley & Sons, 1999.
- [24] A. SCHRIJVER, *Theory of Linear and Integer Programming*, John Wiley & Sons, Chichester, 1986.
- [25] P. SPELLUCCI, *Numerische Verfahren der Nichtlinearen Optimierung*, Birkhäuser-Verlag, Basel, 1993.
- [26] D. SUN, I. S. STRUB, AND A. M. BAYEN, *Comparison of the performance of four eulerian network flow models for strategic air traffic management*, Networks and Heterogeneous Media, 2 (2007), p. 569.