Short Description

Chaos and pattern in the Hénon–Heiles potential

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1 Motivation

Michel Hénon and Carl Heiles worked on the non-linear motion of a star around a galactic center with the motion restricted to a plane. They found that the third integral of motion only existed for a limited number of initial conditions. By now we know these orbits are chaotic and can be described by the methodes of nonlinear dynamics. My goal is to analyze the system and learn these methodes. I will compare my results with the original [2] and the relevant literature.

2 Basic concept

The potential and the Hamiltonian are the following:

$$V(x,y) = \frac{1}{2}(x^2 + y^2 + \lambda \left(x^2 y - \frac{1}{3} y^3 \right), \quad H = \frac{1}{2}p_x^2 + \frac{1}{2}p_y^2 + V(x,y). \tag{1}$$

Usually when we analize the systems chaotic properties we set λ to unity. The equations of motion then follow from the Hamiltonian:

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = -x - 2xy, \quad \frac{\mathrm{d}p_y}{\mathrm{d}t} = -y - x^2 + y^2, \quad \frac{\mathrm{d}x}{\mathrm{d}t} = p_x, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = p_y. \tag{2}$$

The celestial object is binded near the origin until it moves further away, then it is released from the potential which can be seen in Figure 1.

These equations can be solved numerically and analyzed with the methods in the chapter Discrete and Continuous Nonlinear Dynamics [3].

3 Project layout

First I will solve the equations and plot the trajectories for different initial conditions. The result will only depend on the energy so I will use equipotential lines. I shall find multiple bounded orbits if $E < 1/6$. I will examine the emerging chaos for a small parameter change.

I will produce the two Poincaré sections and phase space diagrams to analyze the setup. In the phase space I will classify the paths showing exampes to the main types of structures. I will try to reproduce the same results by making a bifurcation diagram with the absolute value of the instantaneus velocity at the beginning of every period and the energy of the system. I shall find the fix points and the number of states should be the same.

For quantitative results I will use the Lyapunov coefficients for every dimension and try to adapt the Shannon entropy. If there is enough time I will try to find some characteristic time and implement the alternative phase space plot with τ lag.

I expect it will be a little harder to make sense of the data, baceuse the problem is in two dimensions.

4 Objectives

I will study a continuous nonlinear system using the techniques described in the book and compare my results with the literature. I will implement every methode and write the code in python or matlab within

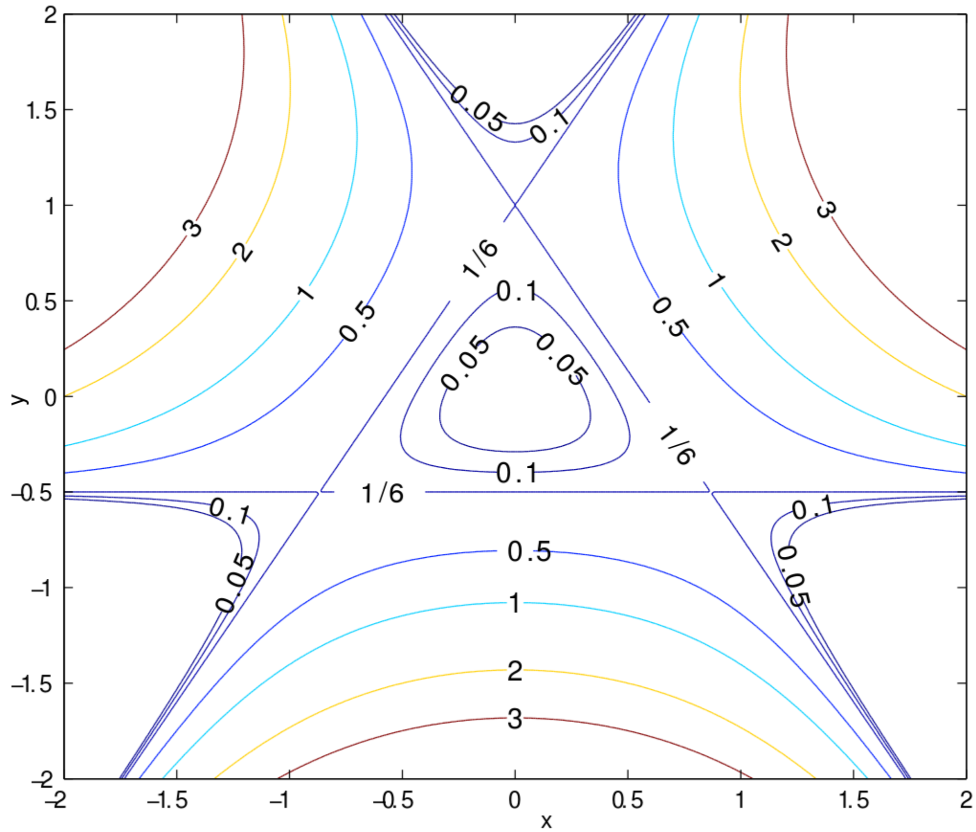

Figure 1: Hénon–Heiles potential [1]

the expected limits.

References

[1] URL: `https://en.wikipedia.org/wiki/H%5C%C3%5C%A9non%5C%E2%5C%80%5C%93Heiles_system`.

[2] Michel Henon and Carl Heiles. "The applicability of the third integral of motion: Some numerical experiments". In: 69 (Feb. 1964), p. 73. DOI: 10.1086/109234.

[3] Rubin H Landau, Jose Paez, and Cristian C Bordeianu. "A survey of computational physics". In: *A Survey of Computational Physics*. Princeton University Press, 2011.