Quantization Noise Analysis of Rounding and Truncation Methods for Sinusoidal Signals

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Abstract : Quantization noise is an problem in converting an analog signal to digital and there are two methods called as Rounding and Truncation to minimize the error during the digitization process. The behavior of quantization noise depends on the components of the sinusoidal signal and the conditions under which the conversion process takes place. This paper presents an analysis of the behavior of quantization noise for a sinusoidal signal. Computer simulation to quantize on the sinusoidal input signal is done using the Rounding and Truncation methods. Signal to quantization noise ratio (SQNR) comparisons for the Rounding and Truncation methods of quantization error is done for a sinusoidal signal. For selected quantization bit rates from 3-bit to 16-bit, the SQNR is calculated using the formula and then simulation is performed to determine the SQNR using the Truncation method and Rounding method. According to simulation results, it has been observed that there is difference of about 5 dB between SQNR error using the formula and the SQNR using the Rounding for quantization error. Similarly, it has been observed that there is difference of about 6 dB between the SQNR error using the formula and the SQNR using the Truncation for quantization error. It has also been observed that there is difference of about 1 dB between the SQNR quantization errors using the Truncation and using the Rounding.

Keywords: quantization, signal to quantization noise ratio, rounding and truncation

1. INTRODUCTION:

Analog-to-digital converters (ADCs) play essential role in novel technology as they bridge the gap between real-world and digital domains, converting analog signals into digital format for various applications like telecommunications, medical devices, multimedia, and industrial automation. Quantization is an integral process within the ADC framework, entails the discretization of a continuous analog signal into a finite set of discrete digital levels. This process introduces a trade-off between accuracy and data size, as the number of quantization levels affects the precision of the digital representation. Applying the right quantization method is crucial to ensure optimal performance and minimize information loss.

In recent years, in the fields of communication and control systems and signal processing, the application of uniform quantization has played an important role in reducing the sensitivity of analog signals and representing them with discrete values. Using this technique, which has a wide range of applications in digital signal processing, innovative solutions to challenging problems in quantization have been introduced in various engineering disciplines. The difficulties of signal-to-noise ratio limiting output feedback control subject to channel input quantization were also addressed by Rojas & Lotero (2015), who offered helpful solutions for control systems using constrained communication resources. Uniform quantization has been the focus of extensive study in communication systems. Researchers have studied the impact of uniform quantization in dealing with packet outages and stochastic systems, for example in the disciplines of communication and control systems. Sheng et al. (2017) highlights the importance of output-feedback control for nonlinear stochastic systems with sequential packet outages and uniform quantization effects, and shed light on maintaining system stability and performance in challenging communication situations. Bashar et al. (2019) made significant contributions to the field by investigating the max-min ratio and energy efficiency of cell-free massive MIMO uplink with uniform quantization and showed how quantization techniques can improve the performance of modern communication systems. Goyal's (2011) work on scalar quantization with random thresholds has also proven to be effective in improving the performance of quantization processes in communication scenarios.

Nowadays, the integration of uniform quantization into image compression has attracted a great deal of interest from researchers. Tsubota & Aizawa (2023) conducted a comprehensive study on uniform quantization in deep image compression and provided valuable insights into the trade-off between image quality and file size. As multimedia data storage and transmission becomes increasingly critical in the digital age, efficient compression of images and videos through uniform quantization has significant implications for data management and distribution. Moreover, the application of uniform quantization in multi-agent systems has opened new avenues for effective coordination and decision making among agents. Li et al. (2022) contributed to the development of robust and scalable multi-agent systems by investigating distributed semi-agreement control under round-robin protocol and uniform quantization for stochastic multi-agent systems. Furthermore, the research by Na & Neuhoff (2018, 2019) investigated the convexity and monotonicity of the mean square error (MSE) decay of symmetric scalar uniform quantization and provided valuable insights into the fundamental properties of uniform quantization processes.

The impact of uniform quantization in signal processing and communication systems has been extensively studied in different contexts, as shown in the papers by Zou et al. (2017). In their studies the authors investigate the effects of uniform quantization in networked control systems using the try-once-discard protocol. They investigate how quantization affects the stability and boundedness of the system, important considerations in the design and implementation of networked control systems. On the other hand, Chen et al. (2019) focus on the application of non-uniform quantization in millimeter-wave MIMO systems. The authors propose a non-uniform quantization codebook-based hybrid coding technique to minimize the feedback overhead required for efficient communication in MIMO systems operating at millimeter wave frequencies. Both papers shed light on the effects of quantization in different systems and provide valuable insights for future developments and optimizations in signal processing and communications.

In conclusion, uniform quantization has proven to be a versatile and powerful tool in a variety of engineering applications, from improving the stability and performance of control systems to improving the efficiency and spectral utilization of communication systems. Applications of uniform quantization in the field of image compression have also shown promising results in enabling efficient data storage and transmission, while its integration into multi-agent systems has led to improved coordination and decision-making among agents. The research by Na & Neuhoff has contributed further to the fundamental understanding of MSE distortion in uniform quantization. As the technology continues to evolve, further research and development in this area is expected to pave the way for further applications and optimizations of uniform quantization, taking engineering disciplines to new heights of efficiency and performance.

This paper presents an analysis of the variation of quantization noise with increasing bit-rate for sinusoidal signals in time-domain. This paper is organized as follows. Section 2 provides a background of a uniform quantizers. Section 3 introduces modelling of the time-domain uniform quantization both in Simulink and MATLAB code. Section 4 presents an analysis of observations. Section 5 concludes the paper.

2. BACKGROUND:

Recent studies on quantization have been focused on deep neural networks for a better signal compression. Tsubota & Kiyoharu (Tsubota, 2021) compared the seven different approximation methods of uniform quantization to apply in deep image compression. They evaluated these methods on two datasets. In deep image compression, quantization is the critical part of image compression. They emphasized that uniform quantization is preferred because it is both a rate-distortion optimized quantization and a standard operation in deep image compression. They compared the performance of the approximations using the rate-distortion curve. They found better results with the combination of different approximation methods.

One of the latest summaries of the quantization methods is presented by Szyduczynski et al. (Szyduczyński, 2023). The study emphasizes the importance of quantizers in converting analog blocks to digital circuits especially in the design of semiconductor circuits operating in the time domain. The paper also highlights quantization challenges for high data rate wireless communication systems and leading-edge applications such as Digital Phase Locked Loops. In the paper, it is mentioned that technological developments and requirements in CMOS technology support the increase in time resolution in quantizers. The paper states that for high resolution digital applications and optimization, increasing the signal input range, reducing the quantization time and power consumption are important. It is also emphasized here that quantizers are critical components of many cutting-edge low-power wireless/wired communication systems such as phase detectors, digital filters, voltage-controlled-oscillator circuits (Zhong, 2022) digitization of phase locked loops (PLL) (Chandrasekaran, 2018), mobile networks, internet of things (IoT) (Yousefirad, 2022), LIDAR systems (Yoshioka, 2018). It is also emphasized that time domain quantization provides digitization of phase locked loop (PLL) by replacing analog phase detectors and loop filters, which improves noise performance meaning size, power, scalability, programmability and digital calibration.

The study by Dehner et al. (Dehner, 2016) analyzes the noise power spectrum of the quantization error at the output of a limited word-length digital multiplier in fixed point arithmetic and states that it is often based on simplifying assumptions. The study also states that the quantization error is modeled as a random variable independent of the input signal. The mathematical analysis is based on the extension of the Gaussian distribution to Hermite polynomials. The error model analysis results presented in the study provide superior results for word lengths in the eight-bit range. The results presented here will provide a basis for the analysis of finite word length applications of digital filters and digital controllers. Liu et al. (Liu, 2022) states that non-uniform quantization performs well on neural networks, but accepts the complex nature of non-uniform quantization compared to uniform quantization and the additional loads it imposes on the hardware in which it is located. In this scope, in their study, a non-uniform to uniform quantization (N2UQ) method is proposed and it is stated that the strengths of both quantization methods are combined. It is also stated that their proposed N2UQ method provides up to 1.7% improvement in the processing of image signals compared to unequal quantization.

West and Scheets (West, 2012) focus on the resolution increase of a uniform quantizer in their study. In this scope, the study analyses the dithering signal addition method used to increase the signal accuracy at the quantizer output in the literature, and it is stated that the resolution of the uniform quantizer is increased with the deterministic dither signals used.

In their study, Seifullaev et al. (2019) emphasize the effect of smooth quantization on parameter estimation of composite distributions in their work. Their work explores how the application of uniform quantification affects the accuracy of estimating the parameters of composite distributions. By exploring the challenges and limitations of this quantification method, the authors provide valuable insights into their impact on the overall accuracy and performance of parameter estimation techniques. The findings presented in this paper are important to understand the trade-offs associated with using uniform quantization, especially in scenarios where precise parameter estimation is needed for optimum system performance. If quantization error is well described, modeling the error as a noise source can be a good solution to remove it. In order to describe the behavior of a quantization noise, observing the amplitude distribution of the noise in uniform quantization levels can give us a quantization level graph to decide to a good quantization level and also good quantization method (Proakis J.G., 2013) (Haykin, 2009). Figure-1 shows a general view of input-output voltage relationship during quantization and Figure-2 shows a quantized outputs for a 3-bit uniform quantization. The quantization error is obtained by modeling the quantization process as a Rounding or Truncation approximations and then calculating the quantization error as the difference between the sampled original signal and the quantized signal. The theoretical SQNR can be found by using the Eq.1 where b is the quantization bit rate (Haykin, 2009).

$$\mathrm{SQNR(dB)} = 10\log_{10}\mathrm{SQNR} = 1.76 + 6.02\mathrm{b} \qquad (1)$$

Figure 1. Quantization input-output relationship

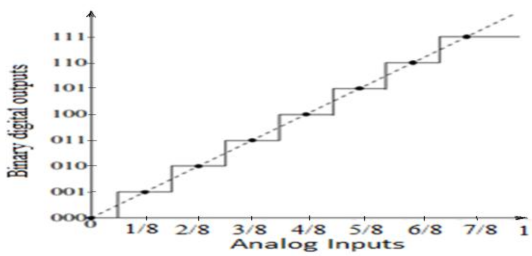

Figure 2. Quantization of normalized analog inputs for a 3-bit ADC

If analog input is a DC value, the quantization error will be constant. But, if input is a sinusoidal signal, quantization error changes according to quantization level of the quantizer (Proakis J.G., 2013). Quantization can be in time-domain or frequency-domain. frequency domain quantization provides more flexibility than time-based quantization in controlling noise/distortion of the components of the signal. On the other hand, frequency-based quantization causes non-linear distortion in distortion depending on the quantization bit sampling depth. Besides, because frequency-based procedures in signal processing requires complex calculations, computational complexity is much less in time-domain quantization. Furthermore, for a given N-sampled sequence of sine-wave, for a quantization in frequency domain, discrete Fourier transform (DFT) cycle (M) should be carefully selected by considering the cases in (Neeman, 2004).In this study, we present the behavior of an SQNR error of an input sinusoidal signal for a time domain quantization at different quantization levels by holding the sampling rate at a constant value and by increasing the quantization bit rate from 3-bit through 16-bit. We assume the quantization steps are uniform. In telephony lines non-uniform quantization levels can be used that have a logarithmic characteristic.

3. MODELLING OF THE TIME DOMAIN QUANTIZATION:

The Matlab Simulink model of the proposed time domain quantization for quantizing a 3-bit and 8-level Sine wave can be seen in the Figure-3. A sinusoidal waveform of frequency 1 Hz and sampling frequency 1000 Hz was configured at source, and pass to zero order hold. Output of zero order hold pass to quantizing encoder. A subtractor is used to get Quantization error. Quantizing encoder was configured for 8 levels and 3 bits initially, step size of 0.25 and further from 4th bit to 16 bits. Time scope is configured for three parameters display these are sine wave, quantized sine wave and quantization error. Running RMS used for input sine wave and quantized sine wave and were sent to workspace as array ‘out.yout’ and ‘out.yout1’ and SQNR was calculated. SQNR was calculated in this way from 3 bit to 16 bit using this Simulink model. Simulation time of one second is used and frequency of sinusoidal signal is 1 Hz. Using the Simulink model in Figure-3, the waveform for the SQNR result of 3 bits (SQNR is 13.5957) can be seen in Figure-4. Here, the black line represents the sine wave, the brown line the quantized sine wave, and the yellow line the sine wave, respectively. align quantization error.

Besides the Simulink results, the SQNR results for the Rounding and Truncation approximations of the quantization error are obtained by writing the Matlab code. All test results and theoretical results can be seen in Table-1. According to the results in Table-1, it is easily seen that the higher the sampling rate, the better the quantization. As a result, we can easily say that the accuracy increases with increasing bit rate. In addition, the Rounding error results are better or very close to the Truncation error results at every bit level. Furthermore, Simulink and Matlab code test results appear to be consistent at every bit level.

```
% SQNR calculation in a time-domain uniform quantizer
% Define digital sine-wave
t = [0:1/500:4]; % Sinus interval
N=length(t);
nbits=2   %start with number of bits =2
sig = sin(2*pi*t)      % Original signal
% Compute the maximum and minimum values of the signal
max_value = max(sig)
min_value = min(sig)
% loop for the calculation of all SQNRs between [3-16] bits
  for j=1:14
      nbits=nbits+1   %increase number of Bits
      stepsize=(max_value-min_value)/(2^nbits-1)
      %theoritical SQNR error for nbits signal
      SQNRNumerical(j) =1.76 + 6.02*nbits
      % Quantize the signal by Rounding each value to the nearest
quantization level
      rounded_signal =round(((sig) - (min_value)) / step_size) * step_size
+ (min_value);
      % Truncate the values based on quantization indices and step size
      truncated_signal =floor(((sig) - (min_value)) / step_size) * step_size
+ (min_value) ;
      powersig=mean(sig.^2);
      logsig=10*log10(powersig);
      %SQNR calc for rounding
      errorrounded=mean((sig-rounded_signal).^2);
      sqnr_rnd(j)=10*log10(powersig/errorrounded)
      %SQNR calc for truncation
      error_trun=mean((sig-truncated_signal).^2);
      sqnr_trun(j)=10*log10(powersig/errorfloor)
  end
```

Algorithm 1. Pseodocode of the Quantization error calculation in Matlab

From the obtained results in Table-1, SQNR results in each bit levels are plotted. Figure-5 shows the relationship between SQNR(dB) of number of bits and theoretical SQNR in Eq.1 Figure-6 is the graphical description of the relationship between number of bits and SQNR(dB) obtained from Simulink. Similarly, Figure-7 and Figure-8 are the graphical description of the relationship between number of bits and SQNR(dB) obtained using Truncation error and Rounding error, respectively. Figure-9 shows all the SQNR results in Table-1 in one graph.

Table 1. Simulation results for each quantization levels in Rounding and Truncation method

Level No	No. of bits (b)	No. of Quantization Levels ($L = 2^b$)	theoretical SQNR using the Equation (1)	experimental SQNR using Simulink model	SQNR with Truncation approximation	SQNR with Rounding approximation
1	3	8	19.8200	13.5957	13.4876	14.4233
2	4	16	25.8400	19.6715	19.2790	19.9716
3	5	32	31.8600	25.7171	25.4992	27.8401
4	6	64	37.8800	31.7598	31.3341	32.1807
5	7	128	43.9000	37.8033	37.3590	39.2021
6	8	256	49.9200	43.8447	43.7739	46.5270
7	9	512	55.9400	49.8572	50.3047	51.4120
8	10	1024	61.9600	55.9426	55.7833	54.4518
9	11	2048	67.9800	61.8030	61.4554	61.5649
10	12	4096	74.0000	67.9295	67.5237	69.3817
11	13	8192	80.0200	74.0070	74.0185	76.6486
12	14	16384	86.0400	80.1614	79.8202	79.5837
13	15	32768	92.0600	86.1506	86.2125	87.4188
14	16	65536	98.0800	92.0339	91.9306	91.2323

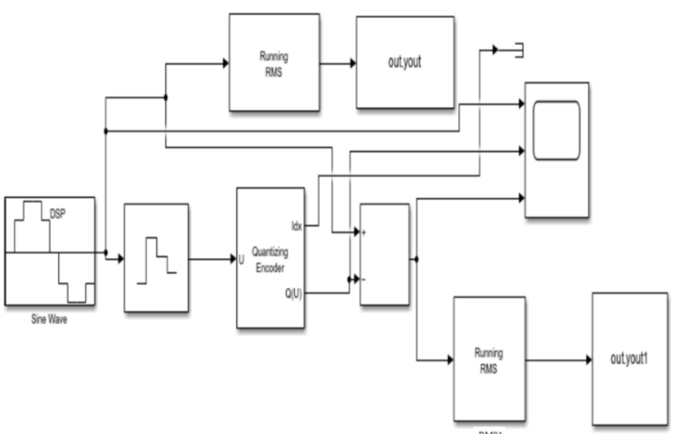

Figure 3. Simulink model of the time domain quantization

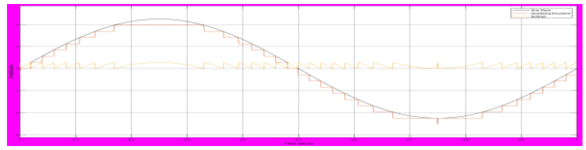

Figure 4. The Simulink result for the quantization of 3 bits

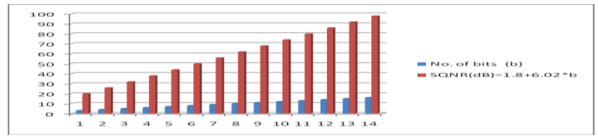

Figure 5. Plot between no. of bits and SQNR in Eq.1

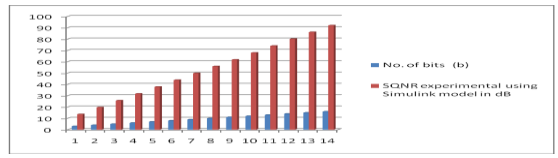

Figure 6. plot between no. of bits and SQNR obtained in Simulink

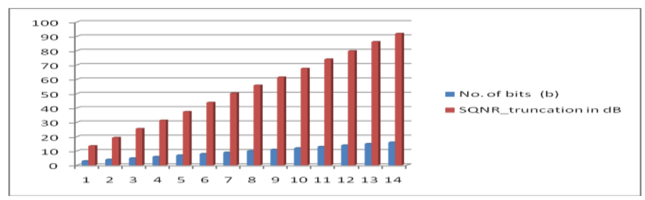

Figure 7. plot between no. of bits and SQNR(Truncation)

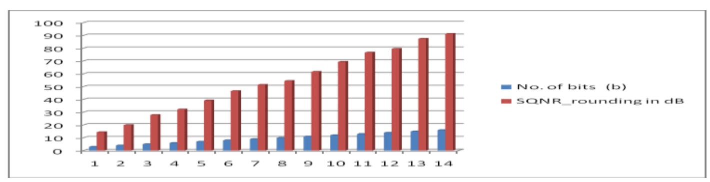

Figure 8. plot between no. of bits and SQNR(Rounding).

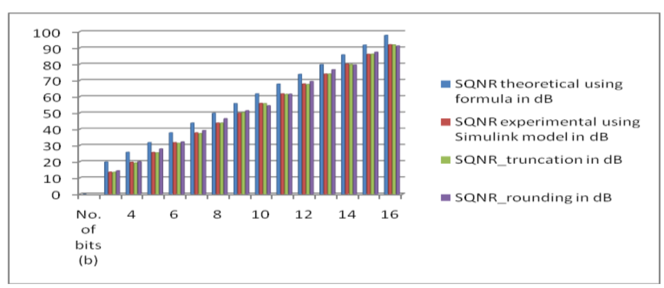

Figure 9. plot between no. of bits and SQNR (formula), SQNR (experimental), SQNR (Truncation) and SQNR(Rounding)

4. OBSERVATIONS:

Computer simulation to quantize on the sinusoidal input signal is done using the Rounding &,Truncation and through Simulink model methods. Signal to quantization noise ratio (SQNR) comparisons for using formula, Rounding & Truncation and through Simulink model of quantization error is done for a sinusoidal signal. For 1 KHz quantization bit rates from 3-bit to 16-bit, the SQNR is calculated using the formula and then simulation is performed to determine the

SQNR using the Truncation method, Rounding method and using Simulink model. According to simulation results, it has been observed that there is difference of about 5 dB between SQNR using the formula and the SQNR using the Rounding. Similarly, it has been observed that there is difference of about 6 dB between the SQNR using the formula and the SQNR using the Truncation. It has also been observed that there is difference of about 1 dB between the SQNR using the Truncation and using the Rounding and SQNR experimental using Simulink model is close to SQNR using Truncation. The precision of the model increases when bit-rate increases. From the test results, we generally say that the truncation method has less accurate quantization than rounding while quantizing a sine signal. However, the interesting thing is that the use of LSB in the truncation method gives near or better results at bit rates of 10 bits and above, although it gives poor results at low bit rates compared to the rounding method.

5. CONCLUSION:

Uniform quantization has emerged as a fundamental technique with diverse applications in many kind of digital signal processing systems. The research papers reviewed provided valuable information on the challenges and benefits of using uniform quantization in these applications. The fact that this technique has an important role in increasing efficiency in novel research projects such as deep image compression offers a potential for detailed investigation of efficiency and reliability of this technique. The results of the uniform quantization method investigated in this study will provide the basis for the non-uniform quantization methods that will be looked at later in the study and will be a crucial source of information for academic scholars that use uniform quantization. Research and development in the field of quantization, which is used in all applications involving analog and digital signal processing, is likely to lead to techniques where quantization error is further minimized.In this study, SQNR analysis of a uniform quantizer for sinusoidal signals was performed in different bit levels and best approximation method was searched. The SQNR values of the approximation methods were found both by designing the Matlab code and creating a simulink model. It can be seen from the test results that the SQNR increases nearly 6 dB for every doubling of the quantization levels. The theoritical SQNR value is approximately 6 dB greater than the SQNR calculated using the Truncation method and approximately 5 dB greater than the SQNR calculated using the Rounding method. In addition, it was observed that there is a difference of approximately 1 dB in favor of the Rounding method between the SQNR result using Truncation and the SQNR result using Rounding. Besides, experimental SQNR using Simulink model is closer to SQNR using Truncation

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