Homework 3 Spectral Leakage and Tapering

1.

(i) Create a vector of 300 time samples with a sample interval of 0.01 seconds and use this to calculate a sine wave with a period of 1 second. Calculate the FFT (*see notes below for useful links*) of the sine wave and plot its amplitude against frequency. Use the `xlim([0 5])` command to zoom in on frequencies between 0 and 5 Hz. Use this as a "base map" onto which to plot other spectra you calculate.

(ii) Calculate and plot the amplitude of the FFT of the first 275 samples of the sine wave. Explain why it differs from the amplitude spectrum of the 300-sample sine wave, perhaps with the help of a sketch in the time domain. (Note: Be sure to look at the width as well as the height of the 1 Hz peak).

(iii) Pad the 300-sample sine wave with 1748 zeros and plot the amplitude of the FFT against frequency on the plot. Can you explain what you see? – It may help to look at the amplitude spectrum of a boxcar function with 2048 samples of which the first 300 are set to one and the remainder to zero.

(iv) A Hanning taper is 1 cycle of a cosine function to which 1 has been added before dividing by two. Calculate a 300-sample Hanning taper with the command `scipy.signal.hanning(300)`. In the time domain, plot the sine wave, the Hanning taper and their product.

(v) Plot the amplitude spectra of the tapered 300-sample sine wave. How does it compare with the spectrum of the untapered sine wave?

(vi) Plot the amplitude spectra of the Hanning window and use it to explain what you see in part 5. It is also interesting to use the `semilogy` command to make the y-axis logarithmic.

(vii) Plot the amplitude spectra of first 275 samples of sine wave after tapering with a 275-sample Hanning window. How does it compare with the spectrum of the untapered 275-sample sine wave?

(viii) Finally, taper the 300-sample sine wave with the 300-sample Hanning window, pad with 1748 zeros and plot the amplitude spectra. What is the effect of padding with zeros?

2.

(i) Download the KATNP acceleration file from Canvas. This is the east-west component of an accelerogram inside the city of Kathmandu during the 2015 M7.8 Gorkha, Nepal earthquake. Plot the waveform and plot the amplitude spectrum using `semilogx()`. Are there any obvious spectral peaks? What do you think they mean?

(ii) Apply a taper of your choosing and plot the spectra with and without the taper. What, if any, are the differences?

(iii) Recall that integration in the time domain is equal to division by f in the frequency domain. Verify this property of the Fourier transform using the `scipy.integrate.cumtrapz()` function.

Notes:

There are lots of mini tutorials on how to obtain an amplitude or power spectrum using the FFT functions in Matlab or Python, just google "Amplitude spectrum with FFT in Matlab/Python". For example:

In Python: https://pythonnumericalmethods.berkeley.edu/notebooks/chapter24.04-FFT-in-Python.html

In Matlab: https://www.mathworks.com/help/matlab/ref/fft.html