ISIS1105
DEFINICIONES TAREA 4

SECCIÓN 3

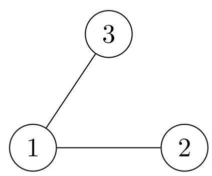
1. Basics

Definition 1. A *graph* is a pair $\langle N, A\rangle$ such that A consists of subsets of N of size 2. We call N the set of *nodes* and A the set of *edges*.

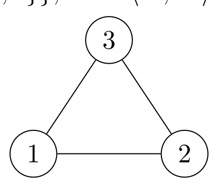
Definition 2. Let $G = \langle N, A\rangle$ be a graph.

(1) The *order* of G is $\#N$.
(2) The *degree* of $v \in N$ is $\#\{e \in A \mid v \in e\}$.

Example 1. Let $N = \{1, 2, 3\}$ and let $A = \{\{1, 2\}, \{1, 3\}\}$. Then $\langle N, A\rangle$ can be dawn as



Node 1 has degree 2, and the other nodes have degree 1.
If now $A' = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$, then $\langle N, A'\rangle$ can be drawn as



Now all three nodes have degree 2.

Definition 3.

(1) A $k-$*path* is a sequence of nodes $\mathbf{p} = v_0, ..., v_{k-1}, v_k$ Such that $v_i \neq v_j$ and $\{v_i, v_{i+1}\} \in A$ for all $i = 0, ..., k-1$. We also say that $\mathbf{p}$ is a v_0v_k-path of length k, since it involves that many edges.
(2) A $k-$*cycle* is a sequence of nodes $\mathbf{c} = v_0, ..., v_{k-2}, v_{k-1}$ Such that $v_i \neq v_j$ except for $v_0 = v_{k-1}$, and $\{v_i, v_{(i+1) \,\mathbf{mod}\, k}\} \in A$ for all $i = 0, ..., k-1$.

Definition 4.

(1) The nodes u, v are said to be *connected* if they are equal or there exists a $uv-$path, and we denote this by $u \sim v$. Notice that the resulting relation $\sim$ is an equivalence relation. Given the node u, its *connected component* is the subgraph induced by the edges connected to it. We use the notation C_u for the connected component of the node u.

(2) In general $G = C_{u_1} \sqcup C_{u_2} \sqcup ... \sqcup C_{u_t}$ with $1 \leq t \leq n$, where n is the order of G. Namely, a graph is the disjoint union of the connected components of its nodes. We also say that "C_v is a connected component of G".

(3) The graph G is said to be *connected* if there exists a path between any two different nodes. Equivalently, G has a single connected component. If G has n nodes called $0, 1, \dots, n-1$ and is connected, then $G = C_0 = C_1 = \cdots = C_{n-1}$.

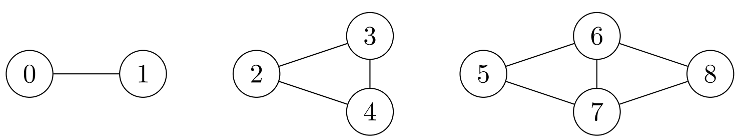
Example 2. Let $G = \langle N, A \rangle$ with

$$\begin{cases} N = \{0, 1, \dots, 8\}. \\ A = \{\{0,1\},\ \{2,3\},\ \{2,4\},\ \{3,4\},\ \{5,6\}, \\ \qquad \{5,7\},\ \{6,7\},\ \{6,8\},\ \{7,8\},\}. \end{cases}$$

Then G has three connected components:

$$\begin{aligned} C_0 &= C_1 \\ C_2 = C_3 &= C_4 \\ C_5 = C_6 = C_7 &= C_8 \end{aligned}$$

This can be visualized in the following picture of G:



The degrees of the nodes are

$$\begin{aligned} \deg(0) = \deg(1) &= 1 \\ \deg(2) = \deg(3) = \deg(4) = \deg(5) = \deg(8) &= 2 \\ \deg(6) = \deg(7) &= 3. \end{aligned}$$

Definition 5. A graph is *acyclic* if there is at most one path joining any two different nodes.