WORK LOG OF JUNE 13 2025

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1. General

- In `splitForest`, the typed value `(lrdr,krdr)` is matched against the typed value `plusAssoc j1 j2' j2''`.

2. Type signatures and function evaluations

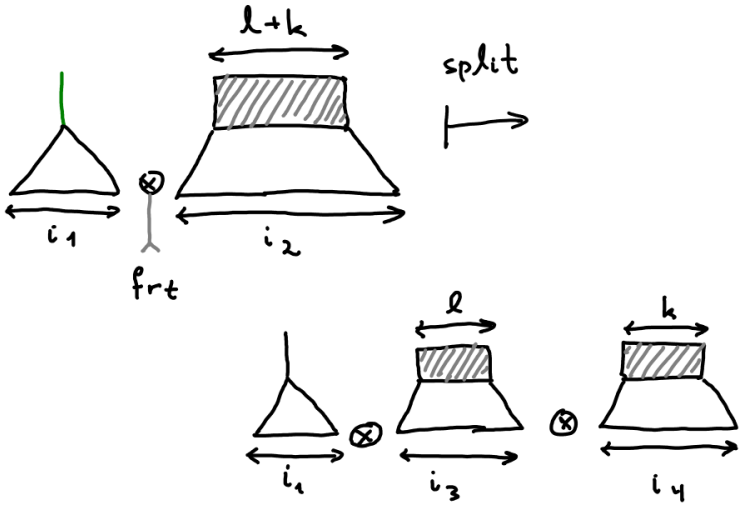
2.1. The type signature of the `splitForest` function is

```
splitForest :: forall f a b z q. SNat a -> SNat b ->
  Forest f q (a+b) ->
  (
   forall j j'. (j+j') ~ q =>
   (Forest f j a, Forest f j' b) -> z
  ) -> z
```

And the evaluation of its recursive case is given by

```
splitForest (SS (sl :: SNat l))
            (sk :: SNat k)
            (Cons (t :: f j1) (frt :: Forest f j2 (l+k)))
            c =
  splitForest sl sk frt $
    (
     \((lrdr :: Forest f j2' l),(krdr :: Forest f j2'' k)) ->
        case plusAssoc (j1 :: Proxy j1)
                       (j2' :: Proxy j2')
                       (j2'' :: Proxy j2''') of
        Dict -> c (Cons t lrdr , krdr)
    )
```

this can be visualized in the following picture:



2.2. The signature of the `Operad` typeclass is

```
class (Graded f) Operad where
  ident :: f One
  compose :: f n -> Forest f m n -> fm
```

And our instance of interest is the type `MoveTree`, which is parametrized by the `Nat` kind. Its signature is coupled with that of `Trees`, also parametrized by `Nat`. These signatures are

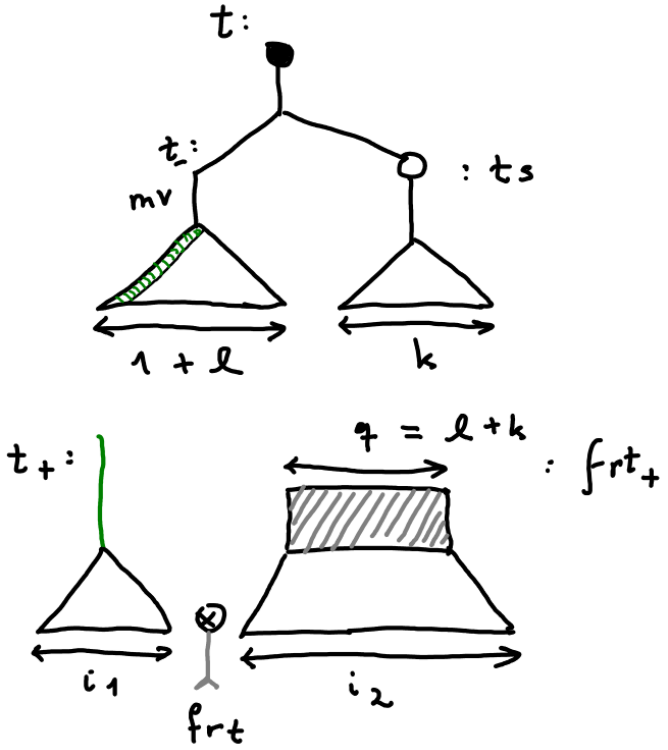
```
data MoveTree n where
  Leaf :: MoveTree One
  Fan :: Trees n -> MoveTree n

data Trees n where
  NilT :: Trees Z
  (:+) :: (Move, MoveTree a) -> Trees b -> Trees (a+b)
```

Therefore, the main challenge in making move trees an operad instance is defining the `compose` function, especially in the recursive case.

Here's an overview of the instantiation:

```
instance Operad MoveTree where
  ident = Leaf
  compose Leaf (Cons (t :: MoveTree m) Nil) =
    case plusZ :: Dict (m ~ (m + Z)) of Dict -> t
  compose (Fan ((mv, t) :+ ts)) frt =
    let ans = splitForest (grade t) (grade ts) frt lambda
        lambda = \(mst1, mst2) -> Fan ((mv,tree) :+ trees)
        tree = compose t mst1
        (Fan trees) = compose (Fan ts) mst2
    in ans
  compose _ _ = error "Composition␣undefined!"
```

This depicts the composition arguments:



And this is a scheme of the composition's result: