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1

The free operads of Baez are obtained via $C-$trees, where C is a *collection*.

Definition 1. A $C - n-$*tree* is a planar $n-$tree such that each vertex with k children is labeled by an element of C_k.

2

The identification of phylogentetic trees with the operations of $\mathrm{Com} + \mathbf{R}^{\geq 0}$ is thanks to the following theorem in [1]:

Theorem 1. *If O and O' are operads such that O' has only unary operations, then there is a bijection between $(O + O')_n$ and equivalence classes of $(O, O'_1) - n-$trees such that no unary vertex is labeled with 1_O and no internal edge is labeled with $1_{O'}$.*

2.1. The equivalence relation considered in theorem 1 is the one given by the *natural* way to permute labeled rooted trees, namely respecting "genealogy" or, more formally, its *partial order structure*.

3. Comments

3.1. The definition of *operad* in [1] coincides with that of May in *Geometry of iterated loop spaces*. Namely, it is a symmetric operad in the symmetric monoidal category of topological spaces. With the exception that more than one 0-ary operations are allowed.

3.2. The forgetful functor

$$U : \mathrm{Op} \to \mathrm{Top}^{\mathbf{N}}$$

is natural because operad maps are degree-preserving.

3.3. Operad maps are defined by preserving the commutative diagrams of operads, and being continuous[1].

References

[1] Baez, J. C., and Otter, N. Operads and phylogenetic trees, 2017.

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[1]Read May in case there are topological or otherwise considerations I'm ignoring.