Here, we define the state variables and equations that simulate bacterial growth and life strategies in our three-species synthetic community. The model components are assigned according to the description of [the generic model](https://github.com/danielriosgarza/hungerGamesModel/wiki/Generic-model) and informed by our investigation of the [growth kinetics](https://github.com/danielriosgarza/hungerGamesModel/wiki/Experiments) of each species. We assigned parameters that represent the best fit to three independent monoculture experiments, each with two or more biological replicates.

Some notable features found in [experiments](https://github.com/danielriosgarza/hungerGamesModel/wiki/Experiments), which we incorporated into the model are summarized below (also see how we [fit models to experiments](https://github.com/danielriosgarza/hungerGamesModel/wiki/Fitting-models-to-experimental-data)):

##### *Blautia hydrogenotrophica* DSM 10507 (referred to as "Bh").

1. Bh shows preference for trehalose over glucose:
   * When both trehalose () and glucose () are present in the medium, Bh first consumes trehalose and neglects glucose until trehalose is depleted.
   * Bh's genome contains a [trehalose-specific PTS gene](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/strainSummaries/bh/blastAnalysis/trehaloseSpecific.tsv), which is found to be overexpressed when Bh grows on trehalose. Interestingly, the genome does not contain the [glucose-specific IIA component of the PTS system](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/strainSummaries/bh/blastAnalysis/glucoseSpecific.tsv) gene that is found in closely-related *Blautia* and *Ruminococcus* strains. In the presence of trehalose, the non-PTS glucose transporter is inhibited while the trehalose-specific PTS gene is actively expressed. To capture this behavior in our model, we used the transition function (), which triggers a switch from a subpopulation () that does not uptake glucose to a subpopulation that uptakes glucose () based on the concentration of trehalose.
   * We found that adding a higher concentration of trehalose to the media () completely prevented Bh from shifting to glucose consumption. This behavior was not observed when increasing the concentration of pyruvate.
   * We also found that increasing trehalose leads to an increase in lactate production, while increasing pyruvate results in an increase in acetate, but not lactate production.
   * In standard WC medium, pyruvate was depleted before the glucose was consumed. However, when we supplemented the medium with pyruvate, we observed co-consumption of glucose and pyruvate. In contrast, the presence of trehalose inhibits the uptake of glucose.
2. Bh exhibits higher growth rates on glucose compared to trehalose and pyruvate:
   * We observed higher growth-rates during the glucose-consuming stage compared to the trehalose-consuming stage.
3. Bh exhibits glucose co-limitation:
   * Not all of the glucose is consumed from the media before the culture enters the stationary and death phases, suggesting co-limitation with another substrate. The [core-metabolic pathway](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/Figures/bhExperiments/modelGeneExp.png) suggests that growth on glucose would be favored by glutamate fermentation, which would provide an additional mol of CO2 and a reduced ferredoxin that could be used to pump protons through the RNF system. This favors ATP production through the membrane ATPase (ATPS4), which is driven by a proton gradient. We modeled this behavior as a co-consumption of glucose and glutamate as the genes for glutamate fermentation are significantly overexpressed during growth on glucose. We confirmed glutamate depletion from the spent media by measuring the levels of amino acids before and after fermentation.

##### *Bacteroides thetaiotaomicron* VPI-5482 (referred to as "Bt")

1. Bt produces a range of fermentation acids that significantly decrease the medium pH:
   * In our experiments, Bt was observed to rapidly consume glucose and pyruvate, while producing a variety of fermentation acids. This resulted in a swift drop in the medium's pH.
2. Bt is inhibited by low pH values:
   * Bt is known to be sensitive to low pH levels, a fact we corroborated by incubating the cells across different pH ranges. These experiments revealed that while the population did not grow at pH levels < 5, most cells remained viable. As such, we modelled pH's impact as a growth inhibitor.
   * But we also found that when carbon sources are exhausted most Bt cells lose their viability. This was confirmed through assessing cell permeability with PI staining. To represent this in our model, we introduced transition functions from active to inactive subpopulations that are triggered by nutrient depletion at low pH.
3. Bt fixes CO2, producing succinate:
   * Bt fixes CO2 by converting phosphoenolpyruvate (a C3 molecule) into oxaloacetate (a C4 molecule) in a process that mirrors carbon fixation in plants. Via the [reductive carboxylation of phosphoenolpyruvate](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/Figures/btExperiments/logos/modelGeneExp.png), Bt generates ATP and produces succinate.
4. Bt shows a second growth peak in WC medium:
   * We consistently observed a [second growth peak](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/Figures/btExperiments/live.png) before the majority of the cell population transitioned to an inactive state. We attributed this second peak to the consumption of mannose, which is present in low concentrations in the complex medium. Mannose depletion was confirmed through single-point measurements and suggested by the [gene expression data](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/Figures/btExperiments/logos/modelGeneExp.png).

##### *Roseburia intestinalis* L1-82 (referred to as "Ri")

1. Ri is a butyrate producer:
   * By studying its core [metabolic pathway](https://github.com/danielriosgarza/hungerGamesModel/blob/main/files/Figures/riExperiments/logos/modelGeneExp.png), we found that Ri produces butyrate through the reverse beta oxidation pathway. In our experiments, Ri quickly consumed glucose and pyruvate and produced butyrate, acetate, and lactate.
2. Ri has lesser impact on the medium pH
   * Unlike Bt, Ri exerts a weaker effect on the pH of the medium despite its high growth rate.
3. Ri enters a slow growth mode characterized by the consumption of lactate and acetate:
   * We observed that some of the lactate and acetate that are produced during growth in glucose and pyruvate, later get consumed, leading to a gradual increase in butyrate.
   * Following the consumption of glucose and pyruvate and production of butyrate, lactate, and acetate, most cells burst and are no longer detected by flow cytometry. However, a subset of cells can persist for several days, possibly entering a slow growth mode. We modeled this behavior by having cells quickly die in the absence of glucose and transition to a slow growth mode when triggered by lactate.

### States

| symbol | type | state | units |
| --- | --- | --- | --- |
|  | subpopulation | Bh subpopulation that consumes trehalose |  |
|  | subpopulation | Bh subpopulation that consumes glucose |  |
|  | subpopulation | Bh inactive subpopulation |  |
|  | subpopulation | Bh dead subpopulation |  |
|  | subpopulation | Bt subpopulation that consumes glucose |  |
|  | subpopulation | Bt subpopulation that consumes mannose |  |
|  | subpopulation | Bt inactive subpopulation |  |
|  | subpopulation | Bt dead subpopulation |  |
|  | subpopulation | Ri subpopulation that consumes glucose |  |
|  | subpopulation | Ri subpopulation that consumes lactate and acetate |  |
|  | subpopulation | Ri inactive subpopulation |  |
|  | subpopulation | Ri dead subpopulation |  |
|  | metabolite | trehalose |  |
|  | metabolite | pyruvate |  |
|  | metabolite | glucose |  |
|  | metabolite | glutamate |  |
|  | metabolite | lactate |  |
|  | metabolite | acetate |  |
|  | metabolite | mannose |  |
|  | metabolite | succinate |  |
|  | metabolite | formate |  |
|  | metabolite | butyrate |  |
| pH | pH | potential of hydrogen |  |

### Equations

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | live Bh subpopulation that grows on trehalose () or pyruvate (), and produces acetate () and lactate () |

$\frac{d x\_a}{dt} = x\_a \left[\space \xi\_{x\_a} \space \mu max\_{x\_a} ( \space \frac{s\_1}{s\_1 + K\_{x\_a, s\_1}} \space + \frac{s\_2}{s\_2 + K\_{x\_a, s\_2}}) - (Z\_1 + Z\_3 )\right] + x\_b Z\_2$ (1)

$\xi\_{x\_a}(pH) = \frac{1}{\varphi(p\_{x\_a}; \alpha\_{x\_a}, \frac{\alpha\_{x\_a} - 1}{p\_{x\_a}})} \space \space \varphi(pH; \alpha\_{x\_a}, \frac{\alpha\_{x\_a} - 1}{p\_{x\_a}})$ (2)

The function: [wiki/Generic-model#varphi](https://github.com/danielriosgarza/hungerGamesModel/wiki/Generic-model#varphi)

transition from to , triggered by low trehalose concentration ():

(3)

transition from to triggered by high trehalose concentration ():

(4)

transition from to :

(5)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | live Bh subpopulation that grows on glucose () and glutamate (), and produces acetate () |

$\frac{d x\_b}{dt} = x\_b \left[\space \xi\_{x\_b} \space \mu max\_{x\_b} ( \frac{s\_3}{s\_3 + K\_{x\_b, s\_3}}\space \frac{s\_4}{s\_4 + K\_{x\_b, s\_4}} + \frac{s\_2}{s\_2+K\_{x\_b,s\_2}}) - (Z2 + Z\_4)\right] + x\_a Z\_1$ (6)

$\xi\_{x\_b}(pH) = \frac{1}{\varphi(p\_{x\_b}; \alpha\_{x\_b}, \frac{\alpha\_{x\_b} - 1}{p\_{x\_b}})} \space \space \varphi(pH; \alpha\_{x\_b}, \frac{\alpha\_{x\_b} - 1}{p\_{x\_b}})$ (7)

transition from to triggered by low concentrations of glucose () or glutamate():

(8)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | Inactive Bh subpopulation that is PI-positive and eventually bursts ([eq. 10](#equation10)) |

(9)

(9)

fixed burst rate:

(10)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | live Bt subpopulation that grows on glucose () or pyruvate (), and produces acetate (), lactate (), formate (), and succinate () |

(11)

$\xi\_{x\_e}(pH) = \frac{1}{\varphi(p\_{x\_e}; \alpha\_{x\_e}, \frac{\alpha\_{x\_e} - 1}{p\_{x\_e}})} \space \space \varphi(pH; \alpha\_{x\_e}, \frac{\alpha\_{x\_e} - 1}{p\_{x\_e}})$ (12)

transition from to triggered by low glucose () and high mannose () concentrations:

(13)

transition from to triggered by low glucose concentration and low pH:

(14)

transition from to triggered by high glucose:

(15)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | live Bt subpopulation that grows on mannose (), and produces acetate () and succinate () |

(16)

$\xi\_{x\_f}(pH) = \frac{1}{\varphi(p\_{x\_f}; \alpha\_{x\_f}, \frac{\alpha\_{x\_f} - 1}{p\_{x\_f}})} \space \space \varphi(pH; \alpha\_{x\_f}, \frac{\alpha\_{x\_f} - 1}{p\_{x\_f}})$ (17)

transition from to triggered by low mannose concentration () and low pH:

(18)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | inactive Bt subpopulation that is PI-positive and eventually bursts ([eq. 20](#equation20)) |

(19)

(20)

Fixed burst rate:

(20)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | live Ri subpopulation that grows on glucose () or pyruvate (), and produces acetate (), lactate (), and butyrate () |

(21)

$\xi\_{x\_i} (pH) = \frac{1}{\varphi(p\_{x\_i}; \alpha\_{x\_i}, \frac{\alpha\_{x\_i} - 1}{p\_{x\_i}})} \space \space \varphi(pH; \alpha\_{x\_i}, \frac{\alpha\_{x\_i} - 1}{p\_{x\_i}})$ (22)

transition from to triggered by high lactate concentration ():

(23)

transition from to triggered by low glucose () concentration and pyruvate ():

(24)

transition from to triggered by high concentration of glucose () or pyruvate ():

(25)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | live Ri subpopulation with slow growth that consumes acetate () and/or lactate (), and produces small amounts of butyrate () |

(26)

$\xi\_{x\_j}(pH) = \frac{1}{\varphi(p\_{x\_j}; \alpha\_{x\_j}, \frac{\alpha\_{x\_j} - 1}{p\_{x\_j}})} \space \space \varphi(pH; \alpha\_{x\_j}, \frac{\alpha\_{x\_j} - 1}{p\_{x\_j}})$ (27)

Fixed death rate:

(28)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | inactibe Ri subpopulation that is (PI-positive) and eventually bursts ([eq. 30](#equation30)) |

(29)

(30)

Fixed burst rate:

(31)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | Trehalose concentration |

$\frac{ds\_1}{dt} = -(\gamma\_{x\_a, s\_1})(\frac{s\_1}{s\_1 + K\_{x\_a,s\_1}})(\xi\_{x\_a} \space \mu max\_{x\_a}x\_a)$ (32)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | pyruvate concentration |

$\frac{ds\_2}{dt} = - \left[(\gamma\_{x\_a, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_a, s\_2}})(\xi\_{x\_a} \space \mu max\_{x\_a} x\_a) + (\gamma\_{x\_b, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_b, s\_2}})(\xi\_{x\_b} \space \mu max\_{x\_b} x\_b) + (\gamma\_{x\_e, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_e, s\_2}})(\xi\_{x\_e} \space \mu max\_{x\_e} x\_e) + (\gamma\_{x\_i, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_i, s\_2}})(\xi\_{x\_i} \space \mu max\_{x\_i} x\_i) \right]$ (33)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | glucose concentration |

$\frac{ds\_3}{dt} =- \left[ (\gamma\_{x\_b, s\_3})(\frac{s\_3}{s\_3 + K\_{x\_b, s\_3}}\frac{s\_4}{s\_4 + K\_{x\_b, s\_4}})(\xi\_{x\_b} \space \mu max\_{x\_b} x\_b) + (\gamma\_{x\_e, s3})(\frac{s\_3}{s\_3 + K\_{x\_e, s\_3}})(\xi\_{x\_e} \space \mu max\_{x\_e} x\_e) + (\gamma\_{x\_i,s\_3})(\frac{s\_3}{s\_3 + K\_{x\_i, s\_3}})(\xi\_{x\_i} \space \mu max\_{x\_i} x\_i)\right]$ (34)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | glutamate concentration |

$\frac{ds\_4}{dt} = - (\gamma\_{x\_b, s\_4}) (\frac{s\_3}{s\_3 + K\_{x\_b, s\_3}}\frac{s\_4}{s\_4 + K\_{x\_b, s\_4}})(\xi\_{x\_b} \space \mu max\_{x\_b} x\_b)$ (35)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | lactate concentration |

$\frac{ds\_5}{dt} = -(\gamma\_{x\_j, s\_5})(\frac{s\_5}{s\_5 + k\_{x\_j, s\_5}})(\xi\_{x\_j} \space \mu max\_{x\_j} x\_j) + (\gamma\_{x\_a, s\_5, s\_1})(\frac{s\_1}{s\_1 + k\_{x\_a,s1}})(\xi\_{x\_a} \space \mu max\_{x\_a} x\_a) + (\gamma\_{x\_e, s\_5, s\_2})\frac{s\_2}{s\_2+k\_{x\_e, s\_2}}(\xi\_{x\_e} \space \mu max\_{x\_e} x\_e) + (\gamma\_{x\_i, s\_5, s\_3})\frac{s\_3}{s\_3 + K\_{x\_i, s\_3}}(\xi\_{x\_i} \space \mu max\_{x\_i} x\_i)$ (36)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | acetate concentration |

$\frac{ds\_6}{dt} = -(\gamma\_{x\_j, s\_6})(\frac{s\_6}{s\_6 + K\_{x\_j, s\_6}})(\xi\_{x\_j} \space \mu max\_{x\_j} x\_j) + \left[(\gamma\_{x\_a, s\_6, s\_1})(\frac{s\_1}{s\_1 + K\_{x\_a, s\_1}}) + (\gamma\_{x\_a, s\_6, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_a, s\_2}}) \right] (\xi\_{x\_a} \space \mu max\_{x\_a} x\_a) + (\gamma\_{x\_b, s\_6, s\_3, s\_4})(\frac{s\_3}{s\_3 + K\_{x\_b, s\_3}}\frac{s\_4}{s\_4 + K\_{x\_b, s\_4}}) (\xi\_{x\_b} \space \mu max\_{x\_b} x\_b) + \left[(\gamma\_{x\_e, s\_6, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_e, s\_2}}) + (\gamma\_{x\_e, s\_6, s\_3})(\frac{s\_3}{s\_3 + K\_{x\_e, s\_3}})\right](\xi\_{x\_e} \space \mu max\_{x\_e} x\_e) + (\gamma\_{x\_f, s\_6, s\_7})(\frac{s\_7}{s\_7 + K\_{x\_f, s\_7}})(\xi\_{x\_f} \space \mu max\_{x\_f} x\_f) + \left[(\gamma\_{x\_i, s\_6, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_i, s\_2}}) + (\gamma\_{x\_i, s\_6, s\_3})(\frac{s\_3}{s\_3 + K\_{x\_i, s\_3}})\right](\xi\_{x\_i} \space \mu max\_{x\_i} x\_i)$ (37)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | mannose concentration |

$\frac{ds\_7}{dt} = -(\gamma\_{x\_f, s\_7}) (\frac{s\_7}{s\_7 + K\_{x\_f, s7}})(\xi\_{x\_f} \space \mu max\_{x\_f} x\_f)$ (38)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | succinate concentration |

$\frac{ds\_8}{dt} = (\gamma\_{x\_e, s\_8, s\_3})(\frac{s\_3}{s\_3 + K\_{x\_e, s\_3}})(\xi\_{x\_e} \space \mu max\_{x\_e} x\_e) + (\gamma\_{x\_f, s\_8, s\_7})(\frac{s\_7}{s\_7 + K\_{x\_f, s\_7}})(\xi\_{x\_f} \space \mu max\_{x\_f} x\_f)$ (39)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | formate concentration |

$\frac{ds\_9}{dt} = (\gamma\_{x\_e, s\_9, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_e, s\_2}})(\xi\_{x\_e} \space \mu max\_{x\_e} x\_e)$ (40)

| diagram | state | unit | description |
| --- | --- | --- | --- |
| fig: |  |  | butyrate concentration |

$\frac{ds\_{10}}{dt} = \left[(\gamma\_{x\_i, s\_{10}, s\_2})(\frac{s\_2}{s\_2 + K\_{x\_i, s\_2}}) + (\gamma\_{x\_i, s\_{10}, s\_3})(\frac{s\_3}{s\_3 + K\_{x\_i, s\_3}})\right](\xi\_{x\_i} \space \mu max\_{x\_i} x\_i) + \left[(\gamma\_{x\_j, s\_{10}, s\_5}) (\frac{s\_5}{s\_5 + K\_{x\_j, s\_5}}) + (\gamma\_{x\_j, s\_{10}, s\_6}) (\frac{s\_6}{s\_6 + K\_{x\_j, s\_6}})\right](\xi\_{x\_j} \space \mu max\_{x\_j} x\_j)$ (40)

### Parameters

| # | species | Parameter | symbol | fitted values | description |
| --- | --- | --- | --- | --- | --- |
| 1 | Bh | xa\_mumax |  |  | maximum growth rate of |
| 2 | Bh | xb\_mumax |  |  | maximum growth rate of |
| 3 | Bh | xa\_pHopt |  |  | optimal pH |
| 4 | Bh | xb\_pHopt |  |  | optimal pH |
| 5 | Bh | xa\_pHalpha |  |  | pH sensitivity |
| 6 | Bh | xb\_pHalpha |  |  | pH sensitivity |
| 7 | Bh | xa\_k\_s1 |  |  | Monod constant for trehalose () consumption by |
| 8 | Bh | xa\_k\_s2 |  |  | Monod constant for pyruvate () consumption by |
| 9 | Bh | xb\_k\_s3 |  |  | Monod constant for glucose () consumption by |
| 10 | Bh | xb\_k\_s4 |  |  | Monod constant for glutamate () consumption by |
| 11 | Bh | xb\_k\_s2 |  |  | Monod constant for pyruvate () consumption by |
| 12 | Bh | xa\_g\_s1 |  |  | rate constant for trehalose () consumption by |
| 13 | Bh | xa\_g\_s2 |  |  | rate constant for pyruvate () consumption by |
| 14 | Bh | xa\_g\_s6\_s1 |  |  | rate constant for acetate () production from consuming trehalose () |
| 15 | Bh | xa\_g\_s5\_s1 |  |  | rate constant for lactate () production from consuming trehalose () |
| 16 | Bh | xa\_g\_s6\_s2 |  |  | rate constant for acetate () production from consuming pyruvate () |
| 17 | Bh | xb\_g\_s3 |  |  | rate constant for glucose () consumption by |
| 18 | Bh | xb\_g\_s4 |  |  | rate constant for glutamate () consumption by |
| 19 | Bh | xb\_g\_s2 |  |  | rate constant for pyruvate () consumption by |
| 20 | Bh | xb\_g\_s6\_s3\_s4 |  |  | rate constant for acetate () production from consuming glucose () and glutamate () |
| 21 | Bh | xb\_g\_s6\_s2 |  |  | rate constant for acetate () production from consuming pyruvate () |
| 22 | Bh | z1\_r |  |  | rate of transition from to |
| 23 | Bh | z2\_r |  |  | rate of transition from to |
| 24 | Bh | z3\_r |  |  | rate of transition from to |
| 25 | Bh | z4\_r |  |  | rate of transition from to |
| 26 | Bh | z5\_r |  |  | rate of transition from to |
| 27 | Bh | z1\_l\_s1 |  |  | half saturation constant for trehalose () |
| 28 | Bh | z2\_l\_s1 |  |  | half saturation constant for trehalose () |
| 29 | Bh | z4\_l\_s3\_s4 |  |  | half saturation constant for glucose () and glutamate () |
| 30 | Bh | z1\_h\_s1 |  |  | Hill coefficient for trehalose () |
| 31 | Bh | z2\_h\_s1 |  |  | Hill coefficient for trehalose () |
| 32 | Bh | z4\_h\_s3\_s4 |  |  | Hill coefficient for glucose () and glutamate () |
| 33 | Bt | xe\_mumax |  |  | maximum growth rate of |
| 34 | Bt | xf\_mumax |  |  | maximum growth rate of |
| 35 | Bt | xe\_pHopt |  |  | optimal pH |
| 36 | Bt | xf\_pHopt |  |  | optimal pH |
| 37 | Bt | xe\_pHalpha |  |  | pH sensitivity |
| 38 | Bt | xf\_pHalpha |  |  | pH sensitivity |
| 39 | Bt | xe\_k\_s3 |  |  | Monod constant for glucose () consumption by |
| 40 | Bt | xe\_k\_s2 |  |  | Monod constant for pyruvate () consumption by |
| 41 | Bt | xf\_k\_s7 |  |  | Monod constant for mannose () consumption by |
| 42 | Bt | xe\_g\_s2 |  |  | rate constant for pyruvate () consumption by |
| 43 | Bt | xe\_g\_s3 |  |  | rate constant for glucose () consumption by |
| 44 | Bt | xe\_g\_s5\_s2 |  |  | rate constant for lactate () production from consuming pyruvate () |
| 45 | Bt | xe\_g\_s6\_s2 |  |  | rate constant for acetate () production from consuming pyruvate () |
| 46 | Bt | xe\_g\_s6\_s3 |  |  | rate constant for acetate () production from consuming glucose () |
| 47 | Bt | xe\_g\_s8\_s3 |  |  | rate constant for succinate () production by consuming glucose () |
| 48 | Bt | xe\_g\_s9\_s2 |  |  | rate constant for formte () production by consuming pyruvate () |
| 49 | Bt | xf\_g\_s7 |  |  | rate constant for mannose () consumption by |
| 50 | Bt | xf\_g\_s6\_s7 |  |  | rate constant for acetate () prouduction from consuming mannose () |
| 51 | Bt | xf\_g\_s8\_s7 |  |  | rate constant for succinate () production by consuming mannose () |
| 52 | Bt | z6\_r |  |  | rate of transition from to |
| 53 | Bt | z7\_r |  |  | rate of transition from to |
| 54 | Bt | z8\_r |  |  | rate of transition from to |
| 55 | Bt | z9\_r |  |  | rate of transition from to |
| 56 | Bt | z10\_r |  |  | rate of transition from to |
| 57 | Bt | z6\_l\_s3 |  |  | half-saturation constant for glucose () |
| 58 | Bt | z6\_l\_s7 |  |  | half-saturation constant for mannose () |
| 59 | Bt | z7\_l\_s3 |  |  | half-saturation constant for glucose () |
| 60 | Bt | z7\_l\_pH |  |  | half-saturation constant for the pH |
| 61 | Bt | z8\_l\_s7 |  |  | half-saturation constant for mannose () |
| 62 | Bt | z8\_l\_pH |  |  | half-saturation constant for the pH |
| 63 | Bt | z10\_l\_s3 |  |  | half-saturation constant for glucose () |
| 64 | Bt | z6\_h\_s3 |  |  | Hill coefficient for glucose () |
| 65 | Bt | z6\_h\_s7 |  |  | Hill coeffiecient for mannose () |
| 66 | Bt | z7\_h\_s3 |  |  | Hill coefficient for glucose () |
| 67 | Bt | z7\_h\_pH |  |  | Hill coefficient for the pH |
| 68 | Bt | z8\_h\_s7 |  |  | Hill coefficient for mannose () |
| 69 | Bt | z8\_h\_pH |  |  | Hill coefficient for the pH |
| 70 | Bt | z10\_h\_s7 |  |  | Hill coefficient for mannose () |
| 71 | Ri | xi\_mumax |  |  | maximum growth rate of |
| 72 | Ri | xj\_mumax |  |  | maximum growth rate of |
| 73 | Ri | xi\_pHopt |  |  | optimal pH |
| 74 | Ri | xj\_pHopt |  |  | optimal pH |
| 75 | Ri | xi\_pHalpha |  |  | pH sensitivity |
| 76 | Ri | xj\_pHalpha |  |  | pH sensitivity |
| 77 | Ri | xi\_k\_s2 |  |  | Monod constant for pyruvate () by |
| 78 | Ri | xi\_k\_s3 |  |  | Monod constant for glucose () by |
| 79 | Ri | xj\_k\_s5 |  |  | Monod constant for lactate () by |
| 80 | Ri | xj\_k\_s6 |  |  | Monod constant for acetate () by |
| 81 | Ri | xi\_g\_s2 |  |  | rate constant for pyruvate () consumption by |
| 82 | Ri | xi\_g\_s3 |  |  | rate constant for glucose () consumption by |
| 83 | Ri | xj\_g\_s5 |  |  | rate constant for lactate () consumption by |
| 84 | Ri | xj\_g\_s6 |  |  | rate constant for acetate () consumption by |
| 85 | Ri | xi\_g\_s5\_s3 |  |  | rate constant for lactate () production from consuming glucose () |
| 86 | Ri | xi\_g\_s6\_s2 |  |  | rate constant for acetate () production from consuming pyruvate () |
| 87 | Ri | xi\_g\_s6\_s3 |  |  | rate constant for acetate () production from consuming glucose () |
| 88 | Ri | xi\_g\_s10\_s2 |  |  | rate constant for butyrate () production by consuming pyruvate () |
| 89 | Ri | xi\_g\_s10\_s3 |  |  | rate constant for butyrate () production by consuming glucose () |
| 90 | Ri | xj\_g\_s10\_s5 |  |  | rate constant for butyrate () production by consuming lactate () |
| 91 | Ri | xj\_g\_s10\_s6 |  |  | rate constant for butyrate () production by consuming acetate () |
| 92 | Ri | z11\_r |  |  | rate of transition from to |
| 93 | Ri | z12\_r |  |  | rate of transition from to |
| 94 | Ri | z13\_r |  |  | rate of transition from to |
| 95 | Ri | z14\_r |  |  | rate of transition from to |
| 96 | Ri | Z15\_r |  |  |  |
| 97 | Ri | z11\_l\_s5 |  |  | half-saturation constant for lactate () |
| 98 | Ri | z12\_l\_s3\_s2 |  |  | half-saturation constant for glucose () and pyruvate () |
| 99 | Ri | z15\_l\_s3\_s2 |  |  | half-saturation constant for glucose () and pyruvate () |
| 100 | Ri | z11\_h\_s5 |  |  | Hill coefficient for lactate () |
| 101 | Ri | z12\_h\_s3\_s2 |  |  | Hill coefficient for glucose () and pyruvate () |
| 102 | Ri | z15\_h\_s3\_s2 |  |  | Hill coefficient for glucose () and pyruvate () |