Explaining Presupposition Projection with Dynamic Semantics[*]

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1 Introduction

The pattern by which complex sentences inherit the presuppositions of their parts (*presupposition projection*) has been a major topic in formal pragmatics since the 1970s. Heim's classic paper "On the Projection Problem for Presuppositions" (1983) proposed a replacement of truth-conditional semantics with a dynamic semantics that treats meanings as instructions to update the common ground. Heim's system predicts the basic pattern of presupposition projection quite accurately. The classic objection to this program, recently pressed by Schlenker (2008a, 2009), is that the treatment of binary connectives is stipulative, and other, equally natural treatments fail to make the right predictions about presupposition projection. I give a variation on Heim's system that is designed to escape this objection.

2 Explaining Presupposition Projection

Consider these three sentences:

(1) a. John stopped smoking.
b. If John used to smoke, then John stopped smoking.
c. Either John didn't use to smoke, or he stopped smoking.

Sentence (1-a) presupposes that John used to smoke but neither sentence (1-b) nor sentence (1-c) does.[1] This is an instance of the pattern of presupposition projection, the way complex expressions inherit or, as in this case, fail to inherit the presuppositions of their parts. Ideally, there would be something relatively simple we could say about *why* (1-b) and (1-c) don't give rise to presuppositions. Saying it has turned out to be surprisingly difficult.

The matter is a bit confused by the fact that there is one thing we can say that sounds quite nice and covers this small set of data, but that doesn't generalize. According to Gazdar (1979) the reason (1-b) and (1-c) don't presuppose that John used to smoke, is that these presuppositions are not consistent with the conversational assumptions necessary for (1-b) and (1-c) to be appropriate utterances. Indeed, it does seem that (1-b) and (1-c) would be rather odd things to say in contexts in which we already took for granted that John used to smoke. The problem with this story is that when we slightly modify (1-b) and (1-c) so that they no longer have this feature, the presuppositions do not magically reappear:[2]

∗ This paper is a substantial rewrite of Rothschild (2008a), it is different enough that I decided to give it a new title to avoid confusion. I am grateful to Be Birchall, Emmanuel Chemla, Haim Gaifman, Nathan Klinedinst, James Shaw, Benjamin Spector, Philippe Schlenker and workshop audiences at the ENS and the University of Chicago for their helpful comments. I also benefitted from many suggestions and comments from two anonymous referees at *S&P*.

1 I'm assuming a basic familiarity with the notion of presupposition as currently used within the linguistic semantics community in this paper. See e.g. Soames (1989) or Beaver (2001).

2 This observation is due to Soames (1982).

(2) a. If John used to smoke heavily, then John stopped smoking.
b. Either John didn't use to smoke heavily, or he stopped smoking.

So the simple conversational condition strategy of explaining presupposition projection fails.

Gazdar's account, though inadequate, does have the virtue that it gives a clear explanation for the pattern in (1-a) to (1-c) without appeal to any special notions about the meanings of the connectives appearing in those examples (i.e. "if ... then ..." and "or"). Standard accounts, as we shall see, mostly lack this feature: they generally make *stipulations* about how sentential connectives and quantifiers treat presuppositions that do the work of predicting the pattern of presupposition projection.

This paper is an extended discussion of how dynamic semantics can account for presupposition projection. I argue that Heim's (1982) account, like other leading accounts, relies on stipulations to capture the pattern of presupposition suggestion. However, I suggest a way of modifying her account to make the same predictions without recourse to stipulations peculiar to individual connectives or quantifiers. My modification of Heim's account goes roughly as follows: Heim defined the meaning of connectives and quantifiers by means of *rewrite rules* that allow one to state the truth conditions for complex dynamic formulas in a language using only set theory and simple dynamic formulas. Heim assigned a separate rewrite rule for each binary connective and her stipulations are located in the details of each of these rules. I suggest that we can use a semantics where sentences are defined iff there exists *some* rewrite rule for the connective that both gets the truth-conditions correct and does not lead to a presupposition failure. This account, once a simple linear order constraint is added, yields exactly the predictions about presupposition projection of Heim's dynamic semantics (which themselves were modeled on earlier work by Karttunen and others). Thus, I do not think there is any inherent problem with the lack of explanatory power in dynamic semantics, and, the choice between dynamic semantics and other approaches to presupposition projection needs to rest on other criteria.[3]

3 Common Grounds and Projection Rules

One of the marks of linguistic presuppositions is that when a sentence presupposes a proposition an assertion of the sentence seems to take the proposition for granted. We might describe presuppositions by saying that a sentence, S, presupposes a proposition, p, when an assertion of S is only felicitous in a context in which the mutual assumptions of the conversational participants include p. This definition, due to Stalnaker (1974) and Karttunen (1974), takes linguistic presupposition to give rise to acceptability conditions on the *common ground*, the collection of mutually accepted assumptions among conversational participants.[4]

Here is a more careful description of the framework: in a conversation any utterance is made against the common ground, which we model as the set of worlds not ruled about by the mutual assumptions of the conversational participants. When one asserts a proposition, a, the normal effect, if the audience accepts the assertion, is the removal of the worlds where a is false from the common ground. One way of working presuppositions into this framework is to assume that

3 This paper is thus largely in response to the recent criticism of dynamic semantics put forward by Schlenker (2006, 2008a). Schlenker (2009) also tries to rehabilitate dynamic semantics in a way that answers the explanatory worry, but his theory is, I think, much further from Heim's original program than the one presented here.

4 My use of this framework is mostly for convenience: you might think presuppositions are instead something like default inferences, in which case you can rephrase everything in another way. Examples of this are Karttunen and Peters (1979), Chemla (2008).

certain sentences are such that they are only felicitously asserted in certain common grounds. In particular, we say that if a sentence A presupposes $\underline{a}$, then A is only felicitously uttered in common grounds that entail $\underline{a}$ (i.e. $\underline{a}$ is true in every world in the common ground). When it is felicitous, the effect of an assertion of A is to remove certain worlds from the common ground.

In this framework we can see the projection problem as the problem of defining what conditions complex sentences put on the common ground in terms of what conditions their parts do. Here are some such rules that are widely accepted:

i. $\neg A$ is acceptable in c iff $c \models \underline{a}$

ii. $A \wedge b$ is acceptable in c iff $c \models \underline{a}$

iii. $A \vee b$ is acceptable in c iff $c \models \underline{a}$

iv. $A \rightarrow b$ is acceptable in c iff $c \models \underline{a}$

v. $b \wedge A$ is acceptable in c iff $c \models b \rightarrow \underline{a}$

vi. $b \vee A$ is acceptable in c iff $c \models \neg b \rightarrow \underline{a}$

vii. $b \rightarrow A$ is acceptable in c iff $c \models b \rightarrow \underline{a}$

For example, rules 5, 6, and 7 explain why the following sentences fail to presuppose anything:

(3) John used to smoke and he's stopped.

(4) John didn't use to smoke, or he's stopped

(5) If John used to smoke, then he's stopped.

According to these rules the presuppositions of sentences (3) to (5) are trivial. For instance, the presupposition of (3) by rule 5. is *if John used to smoke, then he used to smoke*. So the entire sentence is correctly predicted not to presuppose anything. I'll leave it to the reader to check whether all the rules seem to work. These rules can easily be elaborated into totally general rules that predict the full presupposition of any complex sentence based on the presuppositions of its parts. Such a set of rules would essentially be the *filtering* rules developed by Karttunen (1973). There is some debate over the empirical merits of these rules, but I want to put this aside here.[5]

Suppose rules along the lines of i. to vii. suffice to describe the pattern of presupposition projection. Merely stating these rules fails entirely to *explain* why the pattern of presuppositions project can be so described. Heim (1983b) was a landmark paper partly because it gave a semantics of presuppositional expressions (and complexes formed out of these) from which these rules of presupposition projection follow. I will outline her account and discuss a major criticism of it, due to Scott Soames and Mats Rooth. They argued that Heim's semantics has features which effectively amount to stipulations of presupposition projection properties (Soames 1982, Heim 1990, Schlenker 2008a).

4 Dynamic Semantics

I'm going to present propositional dynamics semantics in a non-standard way, as this will facilitate some of the later technical results. While the basic ideas may be familiar to some readers, it is worth skimming through this section to get a sense of the notation.

5 There is a long tradition that argues against these conditional presuppositions (most notably Geurts 1996). I will briefly discuss this issue in section 8.

The major change in Heim's dynamic semantics, from the Stalnakerian framework discussed above, is that the meanings of sentences are no longer propositions, sets of possible worlds, but instead ways of changing the common ground. Thus, a sentence has as its semantic value a function from sets of possible worlds to sets of possible worlds (i.e. a function with domain $P(W)$ and range $P(W)$).[6]

Using this kind of semantic value we can reproduce the Stalnakerian treatment of assertion. Instead of having a sentence S denote a set of possible worlds p we can have the sentence denote the function that goes from a common ground c to the intersection of p and c. In other words, a sentence does not denote a proposition, it denotes a function that captures what it is to update any common ground with the sentence. Heim named this sort of function a *context change potential (CCP)*.

Much of the allure of dynamic semantics, in particular the treatment of donkey anaphora, comes from its treatment of variables which a propositional fragment cannot capture. However, most of Heim's treatment of presupposition projection can be expressed in a propositional fragment. For now I will only discuss the propositional case, and introduce variables and quantifiers later in §9.

Presuppositional meanings, in Heim's semantics, are encoded by partial functions from context to contexts. Consider a sentence like "John stopped smoking". In a classical semantics we would assign this as its meaning the set of possible worlds where John used to smoke and doesn't any more. However, in a partial, dynamic semantics we treat this as a function that takes as its argument any set of worlds c and is defined if and only if c includes only worlds in which John used to smoke. If the function is defined, it returns the intersection of c with the proposition that John doesn't smoke now. However, since "John stopped smoking" is not defined when the context does not entail that John used to smoked, it is infelicitous in such a context. Thus, the partiality of the CCPs captures their presuppositional behavior.

It is helpful to note that Heim's treatment of presuppositions as partially defined CCPs is technically similar to the older tradition of modeling presuppositions with a trivalent semantics. On a trivalent semantics each sentence can be true in some worlds, false in others, and undefined in others. Stalnaker (1973) proposed that the following pragmatic rule should govern the assertion of such sentences:[7]

(6) Only assert a trivalent sentence S in a common ground c, if S is true or false in every world in c.

If we followed this rule, then S could only result in a felicitous update of c if c entails that S is either or true or false. So we could, in a dynamic system, model S by a CCP that is defined iff c entails that the trivalent S is true or false. When it is defined the result of updating c by S is to intersect c by all the worlds where s is true.

So far, Heim's treatment is simply a different technical framework for expressing a trivalent semantics. The interest comes when we introduce the compositional rules for complex sentences. Before we do that, however, we need to state the details of the semantics in a more precise way. We could simply give a semantics for sentences which assigns as values not propositions, but rather CCPs. For ease of exposition here, I will introduce a language that includes

6 Notation: W denotes the set of all possible worlds. And for any set X, $P(X)$ denotes the set of all subsets of W, i.e. the powerset of W.

7 See Soames (1989) for an interesting criticism of this pragmatic rule. Soames's most important point is that if we use trivalence to capture vagueness as well as presupposition failure, this rule predicts that a vague sentence has non-trivial presuppositions. This prediction seems false.

not just CCPs but also formulas representing contexts or common grounds. So this language will include two parts: 1) the context part for formulas representing common grounds and 2) the CCP part for sentences expressing context change potentials. Properly speaking then, the part of the formal language that corresponds to the actual spoken (or written) language is only the CCP-part. This formal language combines representations of conversational assumptions and representations of sentence-meanings. However, this is just a notational convenience, not itself a substantive assumption.

Syntax

- lower case letters a, b, c to represent atomic sentences (these will be used to model contexts)
- upper case letters A, B, C to represent atomic CCPs (these represent sentences in human language)
- the set of CCPs is defined as follows:
  - any atomic CCP is a CCP
  - if ϕ and ψ are CCPs then so are $\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$, and $\phi \rightarrow \psi$
- the set of complex sentences is defined as follows:
  - any atomic sentence is a sentence
  - if α and β are sentences then so are $\alpha \wedge \beta$, $\alpha \vee \beta$, and $\alpha \backslash \beta$[8]
  - if α is a sentence and ϕ is a CCP then $\alpha[\phi]$ is a sentence

CCPs, simple and complex, are the only thing that represent natural language sentences in this language. The actual complex sentences in the language represent contexts, sometimes ones which have been combined or updated by CCPs in various ways. So, for instance, $c[A \wedge B]$ represents the update of context c by the complex CCP $A \wedge B$. It may seem that the rules for combining contexts to get such formulas as $a \wedge b$ are pointless since contexts themselves are not syntactic objects, but as we will see they will come in handy for giving the semantics of complex CCPs.

Partial Semantics

Following standard practice, I'll use the "$[\![]\!]$" sign to designate the semantic value of a sentence or a CCP. An interpretation I sets the semantic values for both the atomic sentences and the atomic CCPs while the values of complex formulas is given by recursive semantic rules. For every atomic sentence α, $[\![\alpha]\!]_I$ is a set of possible worlds (i.e. a subset of w). For every atomic

8 Throughout, I assume and suppress when unnecessary standard parenthetical notation to mark order of operations.

CCP α, $[\![\alpha]\!]_I$ is a function from sets of possible worlds to sets of possible worlds (i.e. from $P(W)$ to $P(W)$). [9]

The value of all complex sentences, as we will see, are sets of possible worlds. A sentence ϕ entails ψ (which we write $\phi \models \psi$) iff on every interpretation, I $[\![\phi]\!]_I \subseteq [\![\psi]\!]_I$. The value of all complex CCPs, which we will give later, are functions from $P(W)$ to $P(W)$, and we will not need entailment relations for these.

Let us first discuss the semantic value of sentences of the form $\alpha[\psi]$ where α is a sentence and ψ is an *atomic* CCP. Then $[\![\alpha[\psi]]\!]_I = [\![\psi]\!]_I([\![\alpha]\!]_I)$—in other words the result of applying the function denoted by the CCP, ψ to the denotation of the sentence, α. $[\![\alpha[\psi]]\!]$ is undefined when $[\![\psi]\!]([\![\alpha]\!])$ is undefined. (Since presuppositions arise because of undefinedness, such assumptions matter.)

We will also give recursive semantic rules for the connectives. I'll simply do this for an arbitrary binary connective $*$. If α and β are sentences then $[\![\alpha * \beta]\!] = \{w : w \in [\![\alpha]\!] * w \in [\![\beta]\!]\}$. So for example: $[\![\alpha \wedge \beta]\!]_I$ is $\{w : w \in [\![\alpha]\!] \wedge w \in [\![\beta]\!]\}$.[10] This is the standard interpretation of the connectives: $[\![\alpha \wedge \beta]\!]$ is the set of all worlds in $[\![\alpha]\!]$ and $[\![\beta]\!]$. Throughout, I'll assume that $[\![\alpha * \beta]\!]$ is undefined if either $[\![\alpha]\!]$ or $[\![\beta]\!]$ is undefined. This semantics is partial since we have only given a semantics for sentences formed *without* use of the recursive syntax for CCPs. We have no way of handling any formula that includes a complex CCP such as $a[A \wedge B]$ or $a[\neg A]$.

Semantics of Complex CCPs

Heim's treatment of complex CCPs can be expressed a number of different ways. On my treatment here, which is very close to her original treatment, the meaning of complex CCPs is defined recursively in terms of the semantics of the language already given. We give the following rules:

Heim's Semantics for Complex CCPs If α is a sentence and ϕ and ψ are CCPs then.

- $\alpha[\neg\phi] = \alpha \backslash \alpha[\phi]$
- $\alpha[\phi \wedge \psi] = (\alpha[\phi])[\psi]$
- $(\alpha[\phi] \vee \psi) = \alpha[\phi] \vee (\alpha[\neg\phi])[\psi]$
- $(\alpha[\phi] \supset \psi) = \alpha[\neg\phi] \vee (\alpha[\phi])[\psi]$

(The equal sign is used here to designate equality of semantic value, not syntactic equality, semantic evaluation brackets here and later are suppressed for readability).

Assessment

These rules complete the semantics for the language as repeated applications of them will yield an interpretation of any formula.[11] Two main properties recommend this semantics for complex CCPs:

9 Hence I can be specified by a triplet, $\langle W,S,C\rangle$, where W is a set of possible worlds, S is a function from atomic sentences to subsets of W and, C is a function from atomic CCPs to functions from $P(W)$ to $P(W)$. I will often suppress mention of I.

10 We can define $\backslash$ as just an abbreviation of $\wedge\neg$, so $[\![\alpha \backslash \beta]\!]_I$ is $\{w : w \in [\![\alpha]\!] \wedge \neg w \in [\![\beta]\!]\}$.

11 One can quibble about whether this semantics is compositional, but if it is not in some sense, it should at least be clear that it can easily be reformulated into a compositional semantics.

i. It seems to get the truth conditions of complex sentences correct.

ii. The rules of presupposition projection fall out of it.

To see the way in which the proposal gets the truth-conditions right requires looking at an example of complex CCPs. Consider the CCPs we should assign to "It stopped raining", R, and "John is tall", J:

- $[\![R]\!]$ = function f s.t. $f(p)$ is defined if and only if p is a set of possible worlds in all of which it used to rain, and when defined $f(p) = p \cap \{w \in p : w$ it doesn't rain now in $w\}$
- $[\![J]\!]$ = function g that takes any set of possible worlds p and returns $\{w \in p :$ John is tall in $w\}$

Now we can ask what happens when we update a context c with the complex CCP $R \wedge J$. Applying the semantics above, we get: $c[R \wedge J] = (c[R])[J] = g(f(c))$ which is defined iff every world in c is one in which it used to rain, and when defined the value is $\{w \in c :$ it doesn't rain now in w and John is tall in $w\}$. When defined, this is exactly the context you would get when you update it with the propositions that it stopped raining and that John is tall. So the rule for conjunction allows complex CCPs to mimic the effect on the common ground of adding complex sentences in a classical semantics. Similar remarks apply to all the other definitions above (as long as we understand the conditional as the material conditional).

With regard to point ii above, the question is what the definedness conditions of complex CCPs are in terms of the definedness conditions of their parts. Looking at conjunction we can see that $c[R \wedge J]$ is defined only if $g(f(c))$ is defined and the only way this can fail to be true is if $f(c)$ is not defined, which depends just on $f(c)$ being defined. By definition $f(c)$ is defined iff c only includes worlds where it used to rain. This matches the predictions of standard accounts: for the sentence to not have a presupposition failure c must include the information that it is raining. If we switch the order of R and J we get the standard Karttunen prediction that $c[J \wedge R]$ is defined if and only if in every world in c in which John is tall it used to rain. If we work through the predictions for all the connectives we get standard predictions, ones that capture the generalizations in §3. The general predictions, which I will call the *Karttunen/Heim projection rules*, are as follows:[12]

Proposition 1. *Suppose ϕ and ψ are CCPs s.t. for any sentence α iff there exists sets of possible worlds $\underline{\phi}$ and $\underline{\psi}$ s.t. $\alpha[\phi]$ is defined iff $\alpha \subseteq \underline{\phi}$ and $\alpha[\psi]$ is defined iff $\alpha \subseteq \underline{\psi}$. It follows on Heim's semantics for complex CCPs that:*

- $\alpha[\neg\phi]$ *is defined iff* $\alpha \subseteq \underline{\phi}$
- $\alpha[\phi \wedge \psi]$ *is defined iff* $\alpha \subseteq \underline{\phi}$ *and* $\alpha[\phi] \subseteq \underline{\psi}$
- $\alpha[\phi \vee \psi]$ *is defined iff* $\alpha \subseteq \underline{\phi}$ *and* $\alpha[\neg\phi] \subseteq \underline{\psi}$
- $\alpha[\phi \to \psi]$ *is defined iff* $\alpha \subseteq \underline{\phi}$ *and* $\alpha[\phi] \subseteq \underline{\psi}$

12 This label is slightly inaccurate as neither Karttunen nor Heim endorsed this rule for disjunction.

5 Explanatory Challenge

A persistent criticism of Heim's program concerns the relation between points 1. and 2. above. Although Heim does not directly say so, it is clear that she thought that her dynamic framework had the property that once you assign truth-conditionally adequate semantics for the complex expressions, the Karttunen rules for presupposition projection will follow.[13]

In its strongest interpretation this claim is trivially false. Given the partial semantics for CCPs in §4, there are numerous possible ways we could extend the the semantics to handle complex CCPs. Some of them will result in the disappearance of *all* standard presuppositions when the presuppositional trigger appears in a complex clause. Consider, for instance, this rule for handling conjunctive CCPs:

- $\alpha[\phi \wedge \psi]$ is defined iff there is largest subset a of $[\![\alpha]\!]$ such that $= [\![\psi]\!]([\![\phi]\!](a))$ is defined. If it is defined, $\alpha[\phi \wedge \psi] = [\![\psi]\!]([\![\phi]\!](a))$

The effect of this cumbersome CCP, in Heim's terminology, is to locally accommodate the presuppositions of ϕ and ψ. In other words, its effect is to alter the intermediate context representations used to calculate the meaning to make sure that the calculation does not fail. Heim herself introduces local accommodation as the way of understanding cases in which presuppositions fail to appear—what was previously called "cancellation" in the presupposition literature. So, it is obvious from her paper that there are ways of treating the semantics of complex expressions that do not capture Karttunen's projection rules. We can conclude that with no restrictions at all on how to determine the meaning of complex CCPs we do not predict anything about their projection properties just by using the partial dynamic semantics outlined in §4.

We can, however, restrict our attention to the class of semantics for CCPs made using *rewrite rules* like the one Heim gives in which the semantics for complex CCPs is given entirely in terms of its component CCPs and the other operators in the language. If we limit ourselves to such rewrite rules, we would eliminate the local accommodation rule above as a possible semantic rule for conjunction, as it is not expressible as a rewrite rule. On a natural reading of Heim's paper her implicit claim is that any rewrite rule which correctly captures the truth conditions for a sentence will yield Karttunen's rules for presupposition projection.[14]

However, both Scott Soames and Mats Rooth made observations that indicate that even a rewrite semantics need not get the facts for presupposition projection right. There are many "deviant" rewrite semantics for complex CCPs that match the truth-conditions of Heim's but have different definedness conditions. For example the following rule is often cited as a deviant rule for conjunction:

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13 She wrote about the quantifier 'no': "Here as elsewhere, the theory I am advocating gives me no choice: Once I have assigned 'no' a CCP that will take care of its truth-conditional content, it turns out that I have to side with Cooper [about the presupposition projection properties of 'no']" It is clear from her later writing that she thought this applied also to the binary connectives so that, in some sense, the projection properties follow from the truth-conditional meaning. Heim (1990) writes "In my 1983 paper, I was less cautious than Karttunen or even Stalnaker and claimed that if one only spelled out the precise connection between truthconditional meaning and rules of context change, one would be able to use evidence about truth conditions to determine the rules of context change, and in this way motivate those rules independently of the presupposition projection data that they are supposed to account for."

14 In fact it is difficult to know what Heim had in mind, since there is no precise claim in the paper along these lines, and whatever she thought, as she later acknowledged, was wrong. So any interpretation of her paper will, by necessity, seem somewhat uncharitable. I choose this interpretation since it makes sense of the Soames and Rooth objection, which Heim (1990) admitted as valid.

(7) $\alpha[\phi \wedge \psi] = (\alpha[\psi])[\phi]$

When defined, this rule will always do what normal conjunction does (in a sense to be made precise later). However, its definedness conditions are different from Heim's rule for conjunction. It is defined if and only if the *second* conjunct is defined in the starting context c and the *first* conjunct is defined when applied to the result of updating the c with the second conjunct. This is the opposite of the normal predictions. According to it, the following sentence should have no presuppositions:

(8) Mary knows John is tall and John is very tall.

The normal theory, however, would suggest that the use of (8) presupposes that John is tall.[15] So, in fact, merely limiting oneself to a dynamic semantics based on rewrite rules of the kind Heim uses does not determine the rules of presupposition projection.

It will be useful to be even more explicit about Heim's implicit claim which Soames and Rooth refuted. There are really two notions that need to be spelled out: 1) the class of rewrite rules that could give the semantics for complex CCPs and 2) what exactly it means for a rewrite rule to be "equivalent" to standard conjunction, disjunction etc.

REWRITE RULES. I will use $*$ to represent an arbitrary binary connective. A rewrite rule semantics for $*$ is a rule for rewriting a formula of the form $\alpha[\psi * \phi]$ as a formula formed out of the sentence α and the CCPs ϕ and ψ using all the standard rules of syntactic composition for L except in so far as no new atomic formulas or CCPS may be added and no rules for forming complex CCPs may be used. This gloss yields this formal definition:

- For any sentence α and CCPs ψ and ϕ:
  - α is a rewrite rule
  - if α is a rewrite rule then so are $\alpha[\psi]$ and $\alpha[\phi]$
  - if α and β are rewrite rules then so are $\alpha \wedge \beta$, $\alpha \vee \beta$, and $\alpha \backslash \beta$.

We need not only have rewrite rules for binary operators: we can also have them for unary operators such as $\neg$. A rewrite rule for a sentence of the form $\alpha[\neg\phi]$ is the same as above except we can only use the CCP ϕ. Our definition of rewrite rules rules such as these:

(9) a. $\alpha[\psi * \phi] = \alpha[\phi] \vee (\alpha[\psi])[\phi]$
b. $\alpha[\psi * \phi] = \alpha[\psi] \backslash (\alpha \wedge \alpha[\phi])$

However, the following are not rewrite rules:

(10) a. $\alpha[\psi * \phi] = \alpha[\phi \vee \psi] \vee (\alpha[\psi])[\phi]$ (uses a complex CCP formed out of ϕ or ψ)
b. $a[\psi * \phi] = a[\psi] \wedge (c[\psi])[\phi]$ (uses a sentence c not identical to a)

TRUTH-CONDITIONAL ADEQUACY. Intuitively, truth-conditional adequacy is the property, that Heim and Soames and Rooth's rewrite rules for conjunction share, of doing the same thing as a classical semantics would have done. We can characterize this property precisely: If the common ground is represented by a sentence α then the result of updating the common ground with a (classical/bivalent) sentence β is just $\alpha \wedge \beta$: updating is equivalent to conjunction. Recall that a rewrite rule is a way of rewriting a sentence of the form $\alpha[\phi * \psi]$ in a form where the CCPs

15 The data supporting this theory is actually less clear-cut than usually acknowledged, as I'll discuss later.

ϕ and ψ no longer are allowed to be part of complex CCPs (and no other sentences or CCPs are added). So, we have both the complex sentence that we start with $\alpha[\phi * \psi]$ and the sentence generated by the rewrite rule, γ. A rewrite rule is *adequate* iff for arbitrary sentences a, p, and q, $a \wedge (p * q)$ is logically equivalent to γ' where γ' is formed by taking γ and making the following syntactic changes:

- for any sentence β, and any CCP, τ, replace $\beta[\tau]$ with $\beta \wedge \tau$
- replace every instance of α with a, ϕ with p, ψ with q, and $\backslash$ with $\wedge\neg$.

This definition ensures that if the CCPs themselves behave classically then any adequate rewrite rules are exactly equivalent to the standard, Stalnakerian update.

Here are some examples of adequate rewrite rules for $\vee$ according to these rules:

(11) a. $\alpha[\psi \vee \phi] = \alpha[\phi] \vee \alpha[\psi]$
b. $\alpha[\psi \vee \phi] = \alpha\backslash((\alpha\backslash\alpha[\phi])\backslash(\alpha\backslash\alpha[\phi])[\psi])$

These are not adequate rewrite rules, however:

(12) a. $\alpha[\psi \vee \phi] = \alpha[\phi] \wedge \alpha[\psi]$
b. $\alpha[\psi \vee \phi] = \alpha\backslash(\alpha\backslash\alpha[\phi])[\psi]$

With this explicit understanding of rewrite rules and truth-conditionally adequacy we can now state a version of Heim's implicit claim about her dynamic semantics:

(13) For any given connective $*$, any two rewrite rules that are *truth-conditionally adequate* for $*$ will determine the same presupposition projection properties, which are the properties outlined by Karttunen (for, at least, conditionals, disjunction, negation).

Once it is stated baldly we can easily see that the conjecture is false by using the deviant rewrite rule for conjunction (7) suggested by Soames and Rooth. This rewrite rule is adequate for disjunction but has different projection properties.[16]

In this way, the basic framework of dynamic semantics combined with notion of a rewrite semantics fails itself to give a sufficiently constrained system to predict the rules of presupposition projection. This, of course, does not show that the framework is wrong. But it does leave the theory needing a separate stipulation for the semantics of each binary connective. If there were only three connectives this might not seem such a bad situation, but, of course, there are a host of other constructions with presupposition projections besides "and", "or" and "if" that a theory should make predictions about. These include quantifiers and other connectives like "unless" and "because". Ideally, we want to find generalizations that explain whatever pattern of presupposition projection is observed across all these different expressions.[17]

6 Explanatory Approaches

It is worth looking at, in general, what a semantics would need to do to overcome the particular form of explanatory inadequacy that Heim's dynamic semantic suffers from. Of course,

16 Note that the problem is not essentially about the order of the arguments: (11-a) also has different projection properties, but shares the same order of arguments as Heim's rule does.

17 Of course, we could make these generalizations at the level of the lexicon. That is they could be similar in status to Horn's generalizations about the the universal lack of "nand" across languages.

if one is giving a semantic account of presupposition projection, as Heim does, the semantics itself should yield the projection properties, and so there must be some degree of stipulation. However, what Heim wanted was a semantic system according to which *any connective with a given truth conditional meaning will have the same projection properties*. For any semantics to have this property, it will need to include some general principles that apply to the treatment of all logical connectives. I suggested that Heim's dynamic semantics seems to suggest only one such principle: the principle that semantics of compound CCPs be expressible as rewrite semantics. This cross-connective principle, however, proved to be inadequate. In order to overcome the explanatory problem, we need to formulate, within the dynamic system, a principle that does predict the facts about presupposition projection.

Explanatory theories have been developed *outside* the dynamic framework for deriving the basic pattern of presupposition projection. Heim (1990) gives the following suggestion for what resources one might use to explain the pattern of presupposition projection:

> If we wanted to deduce at least some aspects of [Karttunen's projection rules] from deeper principles or independent evidence, in what direction should we look? Two possibilities come to mind: explore to what extent these rules are predictable from the linear order of the constituent clauses, and to what extent they might follow from facts about each connective's truthconditional meaning.

The suggestion then is to have some sort of recipe that, takes as input a connective's truth-conditional meaning and the syntactic position (or the linear order) of its arguments, and outputs its presupposition projection properties. While at the time of writing her paper there were no systems that were able to derive Karttunen's rules of presupposition projection, since Schlenker's work, a number of such theories have been developed, which take up exactly this suggestion. Schlenker (2006, 2008a, 2009) details two related systems that yield recipes for *deriving* the Karttunen results using exactly the components Heim suggested, truth-conditional meaning and linear order. There are other strategies: George (2007) and Fox (2008) develop trivalent systems that derive the Karttunen rules for a propositional fragment, while Chemla (2008) provides a way of deriving Karttunen's rules as a form of scalar implicature.[18]

There are various different ways of making a dynamic semantic system that is explanatory in the sense above. In a sense, all we need to do is postulate a *constraint* on the space of possible semantics for compound CCPs, that has the effect that any semantics for a connective $*$ that satisfies the constraint and gets the truth-conditions right also captures the projection properties. Of course, the degree to which such a principle actually provides an *explanation* of the Karttunen rule will depend on how plausible (and simple) the principle itself is. For example, we gain no explanatory purchase if the constraint simply amounts, itself, to a stipulation of the Karttunen rules. This is because the constraint used should not only satisfy the Karttunen/Heim rules, but also tell us for *any* arbitrary connective what its presupposition projection properties are. For, if it does not do this, the principle is not at all *predictive*. This eliminates the mere stipulation of the Karttunen projection rules as a possibility since the rules, as we have so far understood them, do not generalize.[19]

It is possible to give a way of constraining dynamic semantics to do exactly this. Here is a constraint on the range of possible connectives in dynamic semantics that will do the trick for

18 See Schlenker (2008c) for a review of these theories, including an earlier version of the current one.

19 Of course, even generality alone is not sufficient. Any partially predictive system can be expanded to a fully general one in any number of arbitrary ways. The system must also show some theoretical virtues, such as simplicity or connections to other phenomena.

us:

(14) For any connective $*$ the semantic rule for $\alpha[\phi * \psi]$ satisfies the following:

i. It must be a rewrite rule adequate for $*$.

ii. Every instance of ϕ must appear before every instance of ψ in the righthand side of the rewrite rule.

iii. There must be no other rewrite rule adequate for $*$ that leads, in any case, to a weaker presupposition.

It is possible to show that any semantics for connectives that satisfies this constraint, will also yield the Karttunen projection rules.[20] So, in the propositional case, dynamic semantics can be constrained by a unified principle that will allow us to predict the pattern of presupposition projection.

Merely giving this constraint does not provide a very clear sense of where it comes from. This section is really devoting to motivating and justifying this constraint. What I will show, roughly, is that the loosest possible system, in which we use any rewrite rule that is defined, combined with an order constraint will yield Karttunen/Heim projection rules.

First, I will discuss the form of the loosest possible rewrite system, and discuss in fact, why it has some empirical advantages of the Karttunen/Heim rules. Then I will explain how to add an order constraint to it to match exactly the Karttunen/Heim rules. To summarize the next two sections: In §6.1, I introduce the loosest possible rewrite system and show that it makes *very* similar predictions to Heim's except that the rules do not take into account order, which Heim's system does. In §6.2, I show that when you add an order constraint for all connectives, you get a system that reproduces Heim's exactly.

6.1 Loosest Rewrite System

By the loosest possible rewrite system, I mean a semantics for connectives in which one is allowed to choose whatever rewrite rule works in order to satisfy the presuppositions of a given sentence.[21] One way of implementing this is to give an explicit semantics for any arbitrary binary connective which allows one, in effect, to choose whichever rewrite rule, in any instance, is defined. It goes as follows:

(15) $\alpha[\phi * \psi]$ is defined iff a) there exists a rewrite rule adequate for $*$ s.t. when applied to $\alpha[\phi * \psi]$ it gives a formula whose semantic value is defined and b) all such rewrite rules have the same semantic value. When it is defined, $\alpha[\phi * \psi]$ equals that unique semantic value.

This is a liberal, anything-goes rule, that allows updating with any rewrite rule. The presupposition projection behavior yielded by this rule depends on the exact property of the CCPs in the language. Here we define (very standard) properties we will need to get definite results out of this language.

Monotonic Definedness A CCP ϕ has monotonic definedness conditions, if for any sentence

20 This is a simple corollary of other results discussed in this chapter, it requires certain, very natural, assumptions about the meaning and definedness conditions of atomic CCPs which are given later.

21 In many ways, this seems to be a formal version of the suggestion Soames (1989) makes for handling presupposition projection in disjunction.

α, if $\alpha[\phi]$ is defined , then for any sentence α' where α' is stronger than α (i.e every world in $[\![\ \alpha']\!]$ is in $[\![\ \alpha]\!]$), $\alpha'[\phi]$ is defined.

Intersective Meaning A CCP ϕ has an intersective meaning, if there exists a set of possible worlds P such that for any α when $\alpha[\phi]$ is defined it denotes $[\![\alpha]\!] \cap P$.

Note that Heim (and others) understanding of standard CCPs have these features. For example, "John knows it's raining" is defined in a context c iff c entails that it's raining in c (a monotonic definedness condition) and when defined always has the effect of intersecting the context with the proposition that John knows it's raining (an intersective meaning).[22]

Given (15) and the two conditions above, we can prove exactly what the projection rules for the connectives are. These are given in the following result.[23]

Proposition 2. *Suppose ϕ and ψ are CCPs with monotonic definedness conditions and intersective meanings, it follows on the semantics of (15) that:*

- $\alpha[\neg\phi]$ *is defined iff* $\alpha[\phi]$ *is defined.*
- $\alpha[\phi \wedge \psi]$ *is defined iff* $(\alpha[\phi])[\psi]$ *or* $(\alpha[\psi])[\phi]$ *is defined*
- $\alpha[\phi \vee \psi]$ *is defined iff* $(\alpha[\neg\phi])[\psi]$ *or* $(\alpha[\neg\psi])[\phi]$ *is defined*
- $\alpha[\phi \rightarrow \psi]$ *is defined iff* $(\alpha[\phi])[\psi]$ *or* $(\alpha[\neg\psi])[\phi]$

This theory differs substantially from the Karttunen/Heim rules above. The major difference lies in the fact that all binary connectives have disjunctive definedness conditions in this system, whereas only one of the disjuncts serves as the definedness condition on the Karttunen/Heim rules. This disjunctive condition makes the order of the disjuncts and conjuncts irrelevant in this system. So, in this sense, the rules in Proposition 2 might be called *symmetric* while the Heim/Karttunen rules are clearly *asymmetric*.[24]

Despite a preference in the literature for asymmetric theories of presupposition projection, there are many cases which can only be handled by a symmetric theory. Usually the cases are slightly more complex than the very standard cases, but I think the judgments are relatively clear.[25]

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22 As Kai von Fintel pointed out these assumptions are not entirely innocuous. It is commonly claimed that the presupposition of an indicative conditional is that its antecedent is at least possible according to the context: this is a non-monotonic presupposition. So the results below, Facts 2 and 3, do not cover cases with indicative conditionals with such presuppositions. In addition, if we follow Veltman's (1996) account of epistemic modals, then epistemic modals do not have intersective meanings, so, Facts 2 and 3, does not cover sentences with such epistemic modals. My system does make predictions for such cases, they are just not covered by these results.

23 The proof of this result is lengthy and difficult so I have put it in the appendix.

24 Similar comments actually apply to the disjunctive rule for conditionals in Proposition 2, though the definition of symmetric needs to change for conditionals which are not symmetric themselves.

25 These observations build on work by Schlenker (2008a, 2009). The reason why we need to look at complex cases is there may be be independent pragmatic principles interfering with our judgments in many simple cases. For example, the reason $A \wedge \underline{a}$ is unacceptable may be that there is a prohibition against saying $A \wedge B$ if A entails B (but not vice versa). So, for example, as Schlenker, following Stalnaker, notes, the following sort of sentence is odd:

(a) John is a practicing, accredited doctor and he has a medical degree.

Whereas the reverse order is more normal:

(b) John has a medical degree and he is a practicing, accredited doctor.

The following sentences are examples where the standard asymmetric theories predict that there are presuppositions, but the non-incremental version of the theory above predicts no presuppositions:

(16) If John doesn't know it's raining and it is in fact raining heavily, then John will be surprised when he walks outside.

(17) It's unlikely that John still smokes, but he used to smoke a lot.

(18) Either the bathroom is well hidden or there is no bathroom.

In all these cases, I find the standard judgement is that there is no presupposition perceived nor do the examples seem to be marked in a way that indicates cancellation.

6.2 Adding an Order Constraint

However, we may think that there is something right about the asymmetric rules. If we want to match the Heim/Karttunen rules exactly we need to add a constraint on what kind of rewrite rules we are allowed to use. The constraint required appears to be something like an order constraint, as Heim suggested in the quote above. Recent work by Schlenker implements such a constraint to give explanatory theories of presupposition projection that match Karttunen's.[26] Schlenker (2006, 2008a, 2009) builds in the order component by using a kind of incremental checking routine: one has to check at each stage of the derivation that all presuppositional expressions mentioned so far will be satisfied no matter what formulas follow.

Although Schlenker's incremental checking routine could be put on top of the loose system above,[27] a simpler way to replicate the Heim/Karttunen rules is to build an order constraint into the notion of rewrite rules used. An *order-constrained rewrite rule* for the formula $\alpha[\psi * \phi]$ is a rewrite rule that does not allow any instance of the CCP ψ to operate on a formula that contains ϕ. Corresponding to this more constrained notion of a rewrite rule is a refinement of our previous semantics for complex CCPs:

(19) $\alpha[\phi * \psi]$ is defined iff a) there exists an *order-sensitive* rewrite rule adequate for $*$ s.t. when applied to $\alpha[\phi * \psi]$ it gives a formula whose semantic value is defined and b) all such rewrite rules have the same semantic value. When it is defined $\alpha[\phi * \psi]$ has that unique semantic value.

It takes little effort to show that on this system we reproduce what I called the Heim/Karttunen projection, as the following proposition states.

Proposition 3. *Suppose ϕ and ψ are CCPs and with monotonic definedness conditions and intersective meanings and α is a sentence. It follows on the semantics of (19) that the projection properties match those of Heim's System, listed here:*

26 The order constraint may operate on the purely linear order of a sentence, but it is more likely that it works on some less superficial kind of order (i.e. something at a deeper syntactic or semantic level). For example, it's widely thought that conditionals have the same presupposition projection whether the antecedent appears before or after the consequent (see, e.g., Heim 1990):

(i) a. John will know that there's been a break-in, if there has been one.
b. If there has been a break-in, John will know it.

27 This is what I did in the earlier versions of this system (Rothschild 2008b,a).

- *$\alpha[\neg\phi]$ is defined iff $\alpha \backslash \alpha[\phi]$ is defined.*
- *$\alpha[\phi \wedge \psi]$ is defined iff $(\alpha[\phi])[\psi]$ is defined.*
- *$\alpha[\phi \vee \psi]$ is defined iff $\alpha[\phi] \vee (\alpha[\neg\phi])[\psi]$ is defined.*
- *$\alpha[\phi \rightarrow \psi]$ is defined iff $\alpha[\neg\phi] \vee (\alpha[\phi])[\psi]$ is defined.*

7 Individual Connectives: Theory and Data

The preceding discussion has been rather abstract. In this section, I will review the predictions the two semantics in the previous section make for different connectives and discuss how they compare both to other accounts and to what we seem to find empirically.

7.1 And

7.1.1 Background

Conjunction is somewhat special as there has always been a rather obvious "pragmatic" story of how presupposition projection works with projection, due to Stalnaker (1974). Stalnaker's trick is to view conjunction as a form of consecutive assertion. If all goes well, by the time we get to the second assertion (i.e. the second conjunct), the common ground has already been updated with the first assertion. In this case we should expect the Karttunen/Heim rule for conjunction.[28] The most basic problem that this pragmatic story faces is that it does not naturally extend to *embedded* uses of conjunction. For example, a conjunction inside the antecedent of a conditional yields the same presupposition as a conjunction outside of one. In the case of an embedded conjunction, neither conjunct is asserted so the consecutive assertion account of conjunction cannot apply.

The only way to deal with such cases and maintain the pragmatic story, and what I take Stalnaker (2010) to be doing in his response to Schlenker, is to view the antecedents of conditionals as being *suppositionally* asserted, and thus view the conjunction here as a set of two suppositional assertions. The presupposition triggers, then, respond to suppositional common grounds rather than real common grounds in conditionals.[29] For this theory to be genuinely predictive (for embedded conjunctions) we need an account of all the types of common ground relevant in any arbitrary embedded context. I do not know of any such account, so I will put aside this suggestion here and conclude that the pragmatic strategy is, if not unworkable, at least unworked-out. Pragmatic theories do not generalize compositionally, as Karttunen's and Heim's theory do.

7.1.2 Explanatory Dynamic Account

On the loosest version of my account, any adequate rewrite rule is acceptable for conjunction. As in all cases, there are only two relevant rewrite rules for $\alpha[\phi \wedge \psi]$, these are $(\alpha[\phi])[\psi]$ and $(\alpha[\psi])[\phi]$. The reason these are the only options worth considering is that one of these is defined iff some other rewrite rule is defined.[30] Thus, the loose semantics, without the order

28 Schlenker (2008a, 2009, 2010) has extensively criticized this account and all points I make here can be found in his work.

29 This style of explanation also seems to me what Soames (1982) has in mind.

30 For any adequate rewrite will need to include either $\alpha[\phi]$ or $\alpha[psi]$ so one must be defined.

constraints, predicts that conjunction can allow filtering of presuppositions in either direction. If we add the order constraint, then the loosest rule we can use is $(\alpha[\phi])[\psi]$, so we reproduce the Heim/Karttunen projection rule. It is not easy to test empirical evidence about which version is better. To get clear examples we first have to rule out the possibility that the relevant judgments are due to simply violating pragmatic maxims against redundancy. One way to do this is to negate the presuppositional expression and so consider pairs like this:

(20) a. Mary is pregnant, and John doesn't know it.
b. John doesn't know Mary is pregnant, and she is.

It seems me that (20-a) is distinctly better than (20-b). However if we replace "and" with "but" in (20-b), then the situation is less clear. The judgments here are subtle and controversial enough that the introspective judgments are not obviously going to decide this question.[31]

7.2 Or

7.2.1 Background

The treatment of the projection properties of "or" has attracted much less of a consensus then that of "and". Indeed, Heim's original paper does not even given a dynamic rule for "or". Proposed rewrite rules vary from rules that do not allow any presupposition filtering such as $\alpha[\phi \vee \psi] = \alpha[\phi] \vee \alpha[\psi]$ to the rule I used here that allows filtering of the second presupposition by the negation of the first: $\alpha[\phi \vee \psi] = \alpha \backslash (\alpha[\neg\phi])[\neg\psi]$.

What is interesting about disjunction, and most difficult for traditional dynamic theories, is that *no* rule for disjunction seems to capture the empirical facts about presupposition projection in disjunction. It has long been recognized that disjunction appears to have *symmetric* properties of presupposition projection. So the following two examples seem equally acceptable when there is no presupposition that John used to smoke:

(21) a. Either John didn't use to smoke or he stopped.
b. Either John stopped smoking, or he didn't use to.[32]

What has not been observed to my knowledge is that *there is no adequate rewrite rule that yields symmetric predictions for disjunction*. Thus, disjunction alone provides a strong empirical argument against any stipulation of rewrite rules within dynamic semantics along the lines of Heim (1983b). This, then, suggests that if we are going to have a dynamic system, we should have a looser one such as the one I propose here.

There are some explanatory approaches that yield symmetric projection rules of disjunction. However, as I pointed out in my reply to Schlenker (2008a), the predictions of Schlenker's system while they do capture both examples in (21) are, in fact, extremely liberal and so also predict that presuppositions can cancel each other across disjunction.[33] So, the following example would be acceptable on his system with no presupposition:

(22) Either he doesn't regret that he used to smoke, or he didn't stop smoking.

31 Chemla and Schlenker (2009) provide empirical results on this question—and for symmetric readings of other connectives—providing limited evidence for the symmetric readings being somewhat acceptable.

32 We might replace "used to smoke" with "used to smoke heavily" in both examples to eliminate the possibility of a Gazdar-style explanation of this symmetry.

33 See Rothschild (2008b). David Beaver (2008) makes a similar point in his reply in the same volume.

This seems to me like a bad prediction.[34] By contrast, the trivalent accounts of presupposition projection (George 2007, Fox 2008) give symmetric readings without this problem.

7.2.2 Explanatory Dynamic Account

For disjunction the two relevant rewrite rules are $\alpha[\phi \vee \psi] = \alpha\backslash(\alpha[\neg\phi])[\neg\psi]$ and $\alpha[\phi \vee \psi] = \alpha\backslash(\alpha[\neg\psi])[\neg\phi]$. Again, we can restrict our consideration to these two rules as for any defined adequate rewrite rule, *one* of these two rules will always be defined when it is. The availability of *both* these rules gives us the symmetric definedness conditions for conjunction. We do not, however, over-generate and allow presuppositions to cancel each other in examples like (22).

When we add in the order constraint the only relevant rule possible is $\alpha[\phi \vee \psi] = \alpha\backslash(\alpha[\neg\phi])[\neg\psi]$ which only allows filtering of presuppositions in the second disjunction.

7.3 If

7.3.1 Background

It is widely accepted that the material conditional account of "if" is inadequate for natural language conditionals.[35] Nonetheless, following Heim (1983b) and Schlenker (2006) I give a semantic analysis of conditionals that is equivalent to the material conditional account. This is of some use, as the material conditional is, in many instances, truth-conditionally equivalent to more sophisticated analyses.

With conditionals the most standard generalization, and that given in the Heim/Karttunen rules above, is that the presupposition of the antecedent is projected out of the clause, but that the presupposition of the consequent is only projected to the extent that it is not entailed by the antecedent. So, the presupposition of a sentence of the form $A \rightarrow B$ is the same as that of a sentence of the form $A \wedge B$. This is captured by the rewrite rule $\alpha[\phi \rightarrow \psi] = \alpha\backslash(\alpha[\phi])[\neg\phi]$.

7.3.2 Explanatory Dynamic Account

On the loose system, the two relevant rewrite rules for the material conditional are $\alpha[\phi \rightarrow \psi] = \alpha\backslash(\alpha[\phi])[\neg\phi]$, and $\alpha[\phi \rightarrow \psi] = \alpha\backslash(\alpha[\neg\phi])[\psi]$. Once we add the order constraint the only rule that is applicable is the first, which gives the standard predictions.

But what about the reverse rule, $\alpha[\phi \rightarrow \psi] = \alpha\backslash(\alpha[\neg\phi])[\psi]$? It's hard to find any clear evidence that this rule has a role to play. Testing it requires evaluating a sentence where the presupposition of the antecedent is satisfied by the negation of the consequent. Some simple examples of this kind can be ruled out on the ground of having some other pragmatic infelicity:

(23) If John knows Mary is pregnant, then she isn't. (Conditional makes no sense).

If we complicate this by adding negation we can get a clearer example:

(24) If John doesn't know that Mary is pregnant, then she isn't.

This sentence is clearly acceptable without a presupposition, but it is not clear if this is because of a sort of backwards filtering allowed by the reverse rule, or just plain cancellation. After all,

34 Schlenker (2008b) suggests a repair strategy but gives little motivation for it.
35 Some of the many arguments are reviewed in Kratzer (1986) and Edgington (1995).

it's possible to say:

(25) John doesn't know that Mary is pregnant, because she isn't.

So we would need more complex, controlled examples to test the possibility (or lack thereof) of reverse filtering in conditionals. I cannot find any cases where I have clear judgments, so again, it is not clear how to decide between the symmetric and the asymmetric versions of the theory.

7.4 Other Connectives

7.4.1 Background

We criticized Heim's account for failing to make predictions about the presupposition projection properties of arbitrary truth-functional connectives. Schlenker (2008a) emphasizes that with respect for instance "unless", we would hope that a theory of presupposition projection would be able to tell us, given it's truth-conditions, what it's projection properties will be. More generally, we should expect that any two connectives with the same truth-conditional properties should have the same projection properties. A natural case of this is found with "and" and "but", which according to most are truth-conditionally equivalent, with "but" being distinct in virtue of having additional, non-truth-conditional force.

7.4.2 Explanatory Dynamic Account

If we are to treat "unless" as a connective with a truth table (which, as with conditionals, requires a simplification of its semantics) then the natural truth conditions are α *unless* β iff $\alpha \vee \beta$. In this case we should expect it to have the projection properties of disjunction. This seems to make good predictions as we often get disjunction-like patterns of presupposition projection. For instance, the negation of the first part of a sentence connected with "unless" can satisfy a presupposition in the second part:

(26) Unless he didn't talking to her yesterday, John will regret talking to Mary.

As with conditionals, it is difficult to tell whether the symmetric predictions from the looser systems are also found.

With "but" it seems clear that the projection properties are exactly those found with "and", a prediction that this explanatory version of dynamic semantics makes.

(27) John is sick, but Bill doesn't know it.

I will not prove the point here, but I am confident that across arbitrary truth-table definable propositional connectives, the order-sensitive account is equivalent to the order-sensitive predictive accounts proposed by Schlenker (2008a, 2009), Chemla (2008), Fox (2008), and George (2007).

8 Proviso Problem

One feature that is worth noting about the related semantics for complex CCPs in (14), (15), and (19) is that all of them ensure that the presuppositions of complex sentences are *as weak as possible* given the range of rewrite rules available. As some Philippe Schlenker and an anonymous reviewer have pointed out, this is in some sense the opposite of what is demanded by certain solutions to what is called the proviso problem (Geurts 1996). Consider a sentence like this:

(28) If Sue is here, then Mary stopped smoking.

On all of these semantics as well as Heim's semantics the presupposition of (28) is this material conditional, Sue is here sup Mary used to smoke. However, the normal judgment is that (28) just presupposes that Mary used to smoke.

This naturally leads to the question of whether we could modify the semantics here to account for these cases of *stronger* presuppositions. What we would need would be a principle that systematically disallows certain rewrite rules that lead to conditional presuppositions. If these rewrite rules are disallowed we will then be forced to get non-conditional presuppositions. I do not want to explore this question here in detail, but I will suggest one direction we could go.

Note that a conditional presupposition is only realized if we use a rewrite rule in which a CCP is applied to a sentence that has already has already had a CCP applied to it. Consider, for example, a context sentence c, a CCP Q, with intersective meaning q and with no presupposition and a CCP R with intersective meaning r and presupposition $\underline{r}$. In this case, the rewrite rule $(c[Q])[R]$ leads to the conditional presupposition $q \,\mathrm{sup}\, \underline{r}$ whereas the CCP $c[Q] \wedge c[R]$ leads to the simple presupposition $\underline{r}$.

If we wish to remove the possibility of certain conditional presuppositions, then we need to make unavailable some rewrite rules, and thus make the semantics more restrictive. I believe, this is possible, in a way similar to the proposal in Schlenker (forthcoming), but I will not explore this here.[36]

9 Adding Quantification

Heim's paper is also well-known for its systematic treatment of presupposition projection under quantifiers. In essence, adding quantification does nothing to change the conclusions that we made for binary quantifiers. As with the binary connective, the semantics that Heim gives would require stipulations for each quantifier in order to capture the presupposition projection facts (or rather, what she takes to be the facts). If we extend the loose system above to include quantifiers in a natural way, then we can capture, without stipulations, these same generalizations.

Expressions that trigger presuppositions can have variables in them. So, for instance, sentences of the form "x stopped smoking" give rise to the presupposition that x used to smoke. We can then bind such variables with quantifiers in examples like this:

(29) Every student stopped smoking.
= Every_x (student x, stopped smoking x)

Heim discusses sentences of this form, and gives general predictions for how they project presuppositions. She argues that the presupposition of a sentence of the form "Every_x (fx, gx)", where gx presupposes that x satisfies $\underline{g}$ and fx has no presupposition, is that every object satisfying f satisfies $\underline{g}$. Returning to (29) we can state Heim's prediction as follows:

(30) Every_x (student x, stopped smoking x) presupposes:

36 See Singh (2007) for another way of handling the proviso problem that could be integrated with this semantics.

Every$_x$ (student x, used to smoke x)

To summarize: the presuppositions in the matrix predicate of a universal quantifier are universal across all objects satisfying the restrictor predicate.

Heim goes beyond and "every" and argues that the dynamic semantics framework will make predictions of the same form for all quantifiers. As with binary connectives, this claim turns out to be overly optimistic (even if we restrict ourselves to rewrite semantics for quantifiers): in fact, separate stipulations are needed for each quantifier. In the rest of this section, I will show that, if we assume that quantifiers are conservative and both predicates are related to the truth conditions in a non-trivial way, we can replicate Heim's generalizations with the semantics given above in §§6.1–6.2, suitably expanded to include quantification. Some may wish to skip the rest of this section, where I demonstrate these points, as the technical details are rather involved without containing much of interest beyond what was already in the propositional case. In the next section, I will discuss the empirical adequacy of the rules given here in a more informal way.

Syntax

- a, b are atomic sentences
- F, G are n-place CCPs predicates
- x_1, x_2 are variables
- the set of CCPs is defined as follows:
  - if F is an n-place CCP predicate, and $x_1 \ldots x_n$ are variables, then $F(x_1 \ldots x_n)$ is a CCP
  - if ϕ and ψ are CCPs, then so are $\phi \wedge \psi$, $\neg\phi$, $\phi \vee \psi$, $\phi \rightarrow \psi$, and $Q_i(\phi, \psi)$
- the set of complex sentences is defined as follows:
  - any atomic sentence is a sentence
  - if α and β are sentences then so are $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \backslash \beta$ and $Q_i(\alpha, \beta)$
  - if α is a sentence and ϕ is a CCP then $\alpha[\phi]$ is a sentence

Partial Semantics with Quantifiers and Variables

The main difference in the semantics from the propositional case is that all sentences now denote sets of world assignment function pairs rather than just sets of worlds. Intuitively sentences denote those pairs that make the sentence true—so, the semantic value of a sentence α is a subset of the set $\{(f, w) : f \text{ is an assignment function and } w \text{ is a world}\}$. However, besides this change the non-quantificational operators work in exactly the same way as in the non-quantificational system above.

To be precise let us assume a standard predicative system: Assignment functions are functions from variables to individuals. For every atomic sentence α, $[\![\alpha]\!]$ is a set of pairs of worlds and assignment functions. For every n-place CCP predicate F, the interpretation I assigns F two different n-place relations (across possible worlds), the first we will call the *presuppositional relation* and the second the *assertive relation*.

For an atomic CCPs, $F(x_1 \ldots x_n)$, and a sentence α, $\alpha[F(x_1 \ldots x_n)]$ is defined iff for every pair (f, w) in α, $(f(x_1) \ldots f(x_n))$ is in the extension of the presuppositional relation associated with F. If $\alpha[F(x_1 \ldots x_n)]$ is defined then it its semantic value is $\{(f, w) \in \alpha : (f(x_1) \ldots f(x_n))$ is in the assertive relation of F at $w\}$.

We will also give recursive semantic rules for the connectives (outside of CCPs). First, for connectives we can use the same rule as in the propositional system: if α and β are sentences then $[\![\alpha * \beta]\!] = \{(f, w) : (f, w) \in [\![\alpha]\!] * (f, w) \in [\![\beta]\!]\}$. Second, we need to treat quantifiers operating on sentences. To do this we associated with each quantifier Q a binary relation Q_R, as is standard in the theory of generalized quantifiers. Our semantics for quantifiers is then stated as follows:

- $[\![\ \mathbf{Q}_x(\alpha, \beta)]\!] = \{(f, w) : \{o : (f_{x \to o}, w) \in \alpha)\} R_Q \{o : (f_{x \to o}, w) \in \beta)\}\}$

This will only work (intuitively) if x is *free* in α and β. (By definition, x is free in α iff for all w, if (f, w) is in α then so is (f', w) if f' only differs from f in its assignment to x.) For an example of how this works, consider $\alpha = \{(f, w) : f(x)$ is a man in $w\}$, $\beta = \{(f, w) : f(x)$ is tall in $w\}$, $R_Q = \subset$. Then $Q_x(\alpha, \beta) = \{(f, w) :$ set of all men in w is a subset of the the set of all tall things in $w\}$. So, in this case, Q is the universal quantifier.

Heim's Semantics for Complex CCPs

Heim's rules for the standard connectives and negation are as they were described in §4. The only addition needed is the treatment of quantification CCPs. Her rewrite rule for "every" can be stated as follows:

(31) $\alpha[\text{Every}_x(\phi, \psi)] = \alpha \wedge \text{Every}_x(\alpha[\phi], (\alpha[\phi])[\psi])$

For this rule to work we need to assume that x is free in ϕ and ψ:. If we look at (31) we can see that $\alpha[\text{Every}_x(Fx, Gx)]$ is defined iff $\alpha[F(x)]$ and $(\alpha[Fx])[Gx])$ are both defined. Supposing x is free in α, the definedness condition is that for every world w that appears in the denotation of α every individual in w must satisfy the presupposition of F and every individual which satisfies F must also satisfy the presupposition of G for the formula to be defined.

As with binary connectives, Heim could have defined things differently and gotten the same basic truth conditions. Indeed, the simplest definition is as follows:

(32) $\alpha[\text{Every}_x(\phi, \psi)] = \alpha \wedge \text{Every}_x(\alpha[\phi], \alpha[\psi])$

This would have $\alpha[\text{Every}_x(F(x), G(x)]$ presuppose that for every world w that appears the denotation of α every individual in w must satisfy the presupposition of F and the presupposition of G. So this would give us much stronger presuppositions than Heim actually predicts.

In order to get general predictions of presupposition projection for each quantifiers, Heim needs to stipulate in the definition for all quantifiers a form like that of in (31) to ensure that the quantifier triggers the right presuppositions. So, we might want, again, to find a system that can produce these results without resort to such stipulations.

It is worth nothing in this context how facts about anaphora fit into the picture here. In Heim (1982, 1983a) the semantics for "every" in (31) is motivated by consideration of anaphora. In particular, Heim wanted to ensure that an indefinite description in the restrictor of a quantifier could be anaphorically picked out by a definite pronoun in the matrix.[37] Such anaphora,

37 A similar argument could be made with respect to the semantics for conjunction that Heim gives.

in the context of Heim's treatment of pronouns and indefinites (both of which are variables), would be allowed by (31) but not by (32). It may seem then, that in a language with variables, anaphora provides an *independent* motivation for Heim's semantics. It seems to me, however, that this is not the right way to think about things. Rather we should, like Heim, view anaphora and presupposition as two related problems which require a common solution. Thus, that the semantics for "every" in (31) provides a natural treatment of anaphora in quantification expressions, is not an *independent* consideration in favor of that semantics. Anaphora does not solve the explanatory problem.

Looser Semantics with Order Constraint

What we need is a *general* definition of the meaning of complex CCPs that can include quantifiers. We will maintain our previous definitions for CCPs whose top operator is a binary connective. But we need a new rule for CCPs whose top operator is a quantifier. To do this we will define again two concepts: a) being a rewrite rule for $\alpha[Q_x(\phi, \psi)]$ and b) the property of a rewrite rule being adequate for a given quantifier.

The definition of a rewrite rule is the same as before except we also allow adding quantifiers to connect two sentences. The definition of adequacy is again based on logical equivalence to what happens in the "normal" case. Suppose we have a complex sentence of the form $\alpha[Q_x(\phi, \psi)]$ and the sentence generated by the rewrite rule, γ. A rewrite rule is *adequate* iff for arbitrary sentences a, p, and q (where x is free in the denotation of a), $a \wedge (Q_x(p, q))$ is logically equivalent (on the assumption that x is free in a) to γ' where γ' is formed by taking γ and making the following syntactic changes:

- for any sentence β, and any CCP, τ, replace $\beta[\tau]$ with $\beta \wedge \tau$
- replace every instance of α with a, ψ with p, and ψ with q.

Using these definitions for rewrite rules and adequacy, we can, again, consider a system in which a complex CCP is defined just in case there is an adequate, order-sensitive, rewrite rule defined for it (order sensitivity is defined as before).

The resulting semantics makes different predictions for different logically possible quantifiers. However, luckily, it makes the same predictions for the presuppositions projection for quantifiers that have two properties shared by all natural language quantifiers. Here are the two properties:

Conservativity A quantifier Q_x is conservative just in case for any atomic formulas a, b: $Q_x(a, b)$ is logically equivalent to $Q_x(a, b \wedge a)$.

Non-triviality of matrix and restrictor The matrix and restrictor predicates are non-trivial iff there is no formula logically equivalent to $Q_x(a, b)$ that does not quantify over both a and b.

For quantifiers with these two properties we can now state the result giving us Heim's universal projection rules:

Proposition 4. *For a quantifier Q satisfying both the Conservativity and Non-Triviality, CCPs ϕ and ψ, variable x, and sentence α, $\alpha[Q_x(\phi, \psi)]$ is defined iff only if $\alpha[\phi]$ is defined and $\alpha[\phi][\psi]$ is defined.*

This reproduces exactly Heim's predictions for quantifiers: the presupposition of the matrix predicate is only projected for those individuals that satisfy the restrictor.

10 Quantifiers: Theory and Data

10.1 Existential Quantifiers

As this loose system shares the prediction of Heim, it also faces the same serious empirical challenges. The major one, which Heim discusses, is the very strong predictions for existential quantifiers. Consider, for instance:

(33) A man stopped playing the guitar.

If we think of "a man" as a regular existential quantifier, then we presuppose that every man was playing the guitar earlier. Heim discusses examples of this form and agrees that the predicted presupposition seems far too strong.

Her somewhat notorious solution is to posit a process of "local accommodation" where the presupposition is only accommodated for what, in effect, is an existential witness (or the intended referent) of the indefinite in (33) rather than the entire domain of men. In our looser semantics we could also help ourselves to local accommodation to deal with this problem.[38] A related, but perhaps more palatable, tact would be to follow the extensive tradition within discourse representation theory that attempts to explain facts like this. If we abandon a quantification treatment of indefinites (as Heim also did), we may treat the presupposition as only applying to the discourse referent rather than to the domain, in some way (see, e.g. Kamp and Reyle 1993).

As Schlenker (2008a) notes, another strategy is to view sentences like (33) as including covert domain restriction. This domain restriction, as Schwarzschild (2002) suggests, could be as narrow as a single object. If domain restrictions form part of the restrictor, then we would expect that the presupposition only affects the one individual in question.[39]

Neither of these solutions is ideal, and recent empirical data suggests things may well be more subtle than this style of account is able to account for (Chemla 2009). However, at the least, the explanatory account here is in the same position as standard dynamic accounts with respect to this problematic area.

10.2 Restrictors

Another interesting feature of this account is the prediction of universal projection of presuppositions in the restrictor. We predict, for instance, that if a quantificational restrictor contains a presuppositional predicate then the quantificational sentence will presuppose that every element in the domain satisfies the presupposition of the predicate. Thus, like Heim's, our semantics makes very strong predictions for the restrictors of quantifiers. We might think that common nouns come with *sortal* presuppositions of various sorts. So, for example, "bachelor"

38 Heim's system may seems superficially very different here, as she does not treat "a man" as a quantifier but rather as just a free variable. However, since it is the assumption that the variable is *free* that is doing all the work, the system is not greatly different than the approach here couched in terms of generalized quantifiers.

39 As an anonymous reviewer noted, cases like (33) generally seem to be ones in which a) the speaker can identify the referent of the indefinite and b) the presupposition only obtains for that referent. Singleton domain restrictions, with presuppositions only projecting over the restriction, is one way to formally model that intuition in this system.

might presuppose male and/or marriageable; after all, saying x is not a bachelor, seems to take for granted that x is male and not a Catholic priest. If this is correct, then, on the theory given here, a sentence like (34) presupposes that every element in the domain is marriageable.[40]

(34) Every bachelor is happy.

This is obviously an unacceptably strong prediction. The two ways we can avoid this prediction is a) to deny that common nouns can have presuppositions, or b) to suppose that some presupposition-less domain restriction prevents the presupposition from projecting onto the whole domain.

Similar, if less severe, problems arise for presupposition in relative clauses. Take for instance, the following example:

(35) Every student who knows he failed the exam will want to leave before the results are announced.

This example should presuppose that every student failed the exam, which seems too strong. However, this example might simply be a case of presupposition cancellation. If we try a non-cancelable ("hard") presupposition trigger like "regret" the judgment might seem right:

(36) Every student who regrets cheating should come forward.

So the prediction of universal projection in the restrictor, is problematic but not indefensible.

11 Conclusion

I have shown that Heim's treatment of presupposition projection can be extended to generate the same results without the stipulations. However, the loose semantics introduced here may seem to some to be closer in spirit to a trivalent semantic system, like the strong-Kleene truth tables, than it does to Heim's original semantics. A more sustained defense of dynamic semantics would need to show that the extra complexity of the system (the treatment of sentences as expressing CCPs rather than partial propositions) is doing real work for us.

Appendix: Proofs

Proposition 2. *Suppose ϕ and ψ are CCPs with monotonic definedness conditions and intersective meanings, it follows on the semantics of (15) that:*

- $\alpha[\neg\phi]$ *is defined iff* $\alpha[\phi]$ *is defined.*
- $\alpha[\phi \wedge \psi]$ *is defined iff* $(\alpha[\phi])[\psi]$ *or* $(\alpha[\psi])[\phi]$ *is defined.*
- $\alpha[\phi \vee \psi]$ *is defined iff* $(\alpha[\neg\phi])[\psi]$ *or* $(\alpha[\neg\psi])[\phi]$ *is defined.*
- $\alpha[\phi \rightarrow \psi]$ *is defined iff* $(\alpha[\phi])[\psi]$ *or* $(\alpha[\neg\psi])[\phi]$ *is defined.*

Proof. In each case, the right-to-left direction is easier than the left-to-right direction. For the right-to-left direction, all we need to show is that there are a set of rewrite rules acceptable for the given connective such that whenever the condition on the right is met, one of the rewrite

40 Emmanuel Chemla suggested this case to me.

rule is defined. (Given the intersective meaning assumption and the definition of adequacy, if two rewrite rules are defined they will always yield the equivalent meaning.) The following rules are sufficient:

Negation $\alpha\backslash\alpha[\phi]$

Conjunction $(\alpha[\phi])\psi$ and $(\alpha[\psi])\phi$

Disjunction $\alpha\backslash(\alpha[\neg\phi])[\neg\psi]$ and $\alpha\backslash(\alpha[\neg\psi])[\neg\phi]$[41]

Conditional $c\backslash(\alpha[\phi])[\neg\psi]$ and $c\backslash(\alpha[\phi])[\neg\psi]$

What remains, then, is the left-to-right direction. This requires proving for each complex CCP that if the complex CCP on the left is defined, then the condition on the right is satisfied. Given the semantics of (15), this is equivalent to showing that if the right-hand side condition is not met, then there is no adequate rewrite rule for the expression on the left-hand-side that is defined. For each connective, we will prove by induction on the complexity of rewrite rules (as defined in section 5) that if the condition on the right-hand side is not defined then no defined rewrite rules for the connective exists. I work it out in detail for the case of conjunction and sketch the proofs for the other cases (all of which are quite similar).

Conjunction Suppose neither $(\alpha[\phi])\psi$ nor $(\alpha[\psi])\phi$ is defined. Let's suppose now that $\alpha[\phi]$ is defined but $\alpha[\psi]$ is not (they cannot both be defined on this supposition, given the monotonic definedness conditions). On this assumption we will show there is no adequate rewrite rule for $\alpha[\phi \wedge \psi]$ that is defined. To do this, we will show by induction on complexity that every defined rewrite rule either entails $\alpha[\phi]$ or is entailed by $\alpha[\neg\phi] \vee \neg\alpha$. (We use the $\neg$ sign applying to sentences in the usual sense: $\neg\alpha$ denotes the complement of α.) If this holds, then no rewrite rule will be acceptable for conjunction, since $\alpha[\phi \wedge \psi]$ should have neither of these properties when defined by an adequate rewrite rule.

Base step: only formula is α, so trivial. Induction step: Suppose γ and γ' are rewrite rules for $\alpha[\phi \wedge \psi]$ satisfy the inductive property of either begin logically weaker than $\alpha[\phi]$ or stronger than $\neg\alpha[\phi]$. We have two ways of getting more complex results of rewrite rule for $\alpha[\phi \wedge \psi]$: adding a CCP to γ or γ' or using the connectives $\backslash$, $\wedge$, and $\vee$ to connect γ and γ'. I'll go through these in turn:

Adding ϕ: Suppose γ is logically weaker than $\alpha[\phi]$. Then adding $[\phi]$ will not change this property. Suppose γ is logically stronger than $\neg\alpha[\phi]$. Then, adding $[\phi]$, given it's intersective meaning, will not change this property.

Adding ψ: Suppose γ is logically weaker than $\alpha[\phi]$. $\gamma[\psi]$ if it were defined might not have the property, but it will not be defined since $(\alpha[\phi])[\psi]$ is not defined since 1) γ is weaker than $\alpha[\phi]$ by assumption and 2) ϕ has monotonic definedness conditions. If γ is stronger than $\neg\alpha[\phi]$, then $\gamma[\psi]$ will be as well.

Forming $\gamma \wedge \gamma'$ Suppose γ and γ' are weaker than $\alpha[\phi]$. Then so is their conjunction. And if one of the two is stronger than $\neg\alpha[\phi]$, then their conjunction is too.

Forming $\gamma \vee \gamma'$ Suppose γ and γ' are both stronger than $\neg\alpha[\phi]$. Then, so is their disjunction. And if one of the two is weaker than $\alpha[\phi]$ then their disjunction is too.

41 I use $\alpha[\neg\phi]$ as a shorthand for $\alpha\backslash\alpha[\phi]$. This saves a lot of space, and, as we are proving, they are, in fact, equivalent in both definedness and denotation when defined.

Forming $\gamma\backslash\gamma'$ If γ is stronger than $\neg\alpha[\phi]$ then so is $\gamma\backslash\gamma'$. If γ is weaker than $\alpha[\phi]$ and γ' is weaker than $\alpha[\phi]$ then $\gamma\backslash\gamma'$, then γ and γ' is stronger than $\neg\alpha[\phi]$.

On the other hand, suppose γ is weaker than $\alpha[\phi]$ and γ' is stronger than $\neg\alpha[\phi]$. Then $\neg\gamma'$ is stronger then $\neg\alpha[\phi]$, and, therefore, so is $\gamma \wedge \neg\gamma' = \gamma\backslash\gamma'$.

So it follows that if $\alpha[\phi]$ is defined but $\alpha[\psi]$ is not then there is no adequate rewrite rule for $\alpha \wedge \beta$ when the condition on the RHS is not met. By symmetry, the same follows if $\alpha[\psi]$ is defined but $\alpha[\phi]$ is not. That concludes the proof. (The case where $\alpha[\psi]$ *and* $\alpha[\phi]$ are undefined follows immediately from either of the symmetric cases.)

Disjunction We need to show that if neither $\alpha[\neg\phi][\psi]$ nor $\alpha[\neg\psi][\phi]$ is defined, then $\alpha[\phi \vee \psi]$ is not defined. Suppose $\alpha[\phi]$ is defined but $\alpha[\psi]$ is not. We do this again by induction on the results of rewrite rules. We show by induction that every rewrite rule defined for $\alpha[\phi \vee \psi]$ either is logically weaker than $\alpha[\neg\phi]$ or is logically stronger than $\neg\alpha[\neg\phi]$. From this it follows that no defined rule is adequate. This follows similarly to the proof for conjunction, and the proof is completed by the same symmetry argument.

Negation Suppose $\alpha[\phi]$ is not defined. Then we can show by induction that no rewrite rule will be both defined and adequate for negation. We can show this by showing that every rewrite rule for $\alpha[\neg\phi]$ that is defined is either equivalent in meaning to a contradiction or to α itself.

Conditional This follows from the discussions of disjunction and negation and the fact that the conditional is equivalent to $\alpha[\neg\phi \vee \psi]$.

□

Proposition 3. *Suppose ϕ and ψ are CCPs and with monotonic definedness conditions and intersective meanings and α is a sentence. It follows on the semantics of (19) that the projection properties match those of Heim's System, listed here:*

- $\alpha[\neg\phi]$ *is defined iff* $\alpha\backslash\alpha[\phi]$ *is defined*
- $\alpha[\phi \wedge \psi]$ *is defined iff* $(\alpha[\phi])[\psi]$ *is defined*
- $\alpha[\phi \vee \psi]$ *is defined iff* $\alpha[\phi] \vee (\alpha[\neg\phi])[\psi]$ *is defined*
- $\alpha[\phi \rightarrow \psi]$ *is defined iff* $\alpha[\neg\phi] \vee (\alpha[\phi])[\psi]$ *is defined*

Proof. Negation, of course, is unaffected by the order rule since it only takes one argument. For the rest of the connectives the proofs follow from minor modification of the proofs of Proposition 2 . For instance, for conjunction we can eliminate the possibility that $\alpha[\phi]$ is undefined, since if it were we could not introduce ϕ in the rewrite rule in any way (except to add it to an empty denotation, where it would have no effect), thus there will be no adequate rewrite rule. One we eliminate that possibility the definedness conditions follow. □

Proposition 4. *If a quantifier Q satisfies both the Conservativity and Non-Triviality principles above then and CCPs ϕ and ψ are well behaved atomic CCPs and x is free in sentence α, then $\alpha[Q_x(\phi, \psi)]$ has an adequate, order-constrained rewrite rule that is defined iff $\alpha[\phi]$ is defined and $\alpha[\phi][\psi]$ is defined.*

Proof. I give only a sketch here: The existence of the rewrite rule is easy to prove: We just use: $\alpha[Q_x(\phi,\psi)] = \alpha \wedge Q_x(\alpha[\phi], \alpha[\phi][\psi])$. Give the order constraint it's clear that if $\alpha[\phi]$ is undefined there is no way of producing a defined logically equivalent rewrite rule. So we can assume it must be defined. What about $\alpha[\phi][\psi]$? If it is undefined, then so is $\alpha[\psi]$. But there is going to be no adequate rewrite rule that does not make use of $\alpha[\psi]$ or $\alpha[\phi][\psi]$ or something equivalent to one of these because of the non-triviality assumption. $\square$

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