Trivalent Presupposition Projection

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1 Introduction

It is traditional in philosophy and linguistics to model presuppositions using a trivalent logic. The usual idea is that a sentence is true or false only in situations in which its presuppositions are satisfied. So for example, "John stopped smoking," would be true (T) if John used to smoke and doesn't now, false (F) if John used to smoke and smokes now, and neither true nor false (U) otherwise.

Presupposition projection is the term used in linguistics to describe the way presuppositions of simple sentence (or predicates) get inherited by larger complexes they appear in. Consider, for example, this sentence with a presuppositional expression under a quantifier:

(1) No boy in the class stopped smoking.

(1) is usually taken to presuppose that every boy used to smoke. But clearly we don't want to just stipulate this by treating (1) as an atomic sentence that has this presupposition. Rather, on a trivalent view, we want to have a partially defined predicate "stopped smoking" and give a semantics for quantifiers like "no" such that when we put the partial predicate under the quantifier "no" with a restrictor "boy", we get a sentence that is only true or false when every boy used to smoke, and otherwise is truth-valueless.

There is, of course, a long tradition of giving semantics for logical operators (including quantifiers) that extend their truth-conditional meaning to a logic with undefinedness. Most famous, are the Weak Kleene and the Strong Kleene truth tables and the system of supervaluations. How these systems relate to the empirical facts of presuppositions projection is discussed in **?**Soames (1982); Schlenker (2008). Extending these systems to binary quantifiers, I will argue, does not give results that seem very empirically plausible.

The goal here is to give a trivalent logic with binary quantifiers that yields the predictions of presupposition projection of Heim (1983). Peters (1979) essentially does just this, proving that Kartunnen's projection rules can be stated as part of a compositional system. However, Peters simply stipulated a trivalent semantics for each connective and quantifier. Thus, from Peters we cannot discern a general principle that takes us from how a quantifier and connective works classically (i.e. in the bivalent case) and yields its presuppositions projection behavior.

Here we give a predictive account: For each binary quantifier Q we can associate a relation R that determines the truth conditions in the usual way familiar from generalized quantifier theory. We give a general semantics for quantifiers in terms of that relation R that extends the truth-conditions of quantifiers to cases where there is some undefinedness. The treatment naturally extends to the sentential connectives since they can be seen as instances of binary quantification where there are no bound variables.

My goal here is not to advocate trivalence as the best theory of the nature of presuppositions. How you interpret the third truth value, U, is something I have little to say about, except to criticize the most standard views according to which we think of it as simple lack of truth or falsity. I do tentatively suggest that it's reasonable to think of the third truth value as a second form of falsity, one specially marked in the grammar. In this sense, trivalence might just be a method for keeping track of presuppositions, not a theory of what presupposition are. If we take this tact then we can see the theory presented here as essentially giving a descriptive algorithm of presuppositions projection: it's explanatory advance over Heim's dynamic semantics is that the description is more general.

This system bears marked similarities to the supervaluationist frameworks discussed by George (2008) and **?**. However, as I will explain, standard supervaluationist-inspired treatments of presupposition projection make eccentric predictions for the presuppositions of quantified expressions. I find the attempt of George (2008) to add further principles to make these predictions more palatable implausible. The system here, like those systems presented in Schlenker (2006, to appear, 2008), closely matches the predictions of Heim (1983). In addition the method of doing so bears some similarity to Schlenker (2008). In comparison with that theory, the system presented here is much leaner as it is essentially just descriptive. Heim's predictions themselves face emprical problems Chemla (2007), but it is not certain that these cannot be overcome.[1]

2 Pragmatics of Trivalence

Although many consider the relation between a trivalent logic and the empirical phenomenon of presupposition projection to be a self-evident, this is due more to historical accident than any deep conceptual connection. Trivalent truth tables as well as supervaluationism were originally introduced as a means of explaining presuppositions. However, when we think more carefully we can see that the relationship between trivalence and linguistic presupposition is far from obvious.

Merely saying that a sentences S is neither true nor false in some circumstances does not explain why an utterance of S should be infelicitous unless the conversational background contains certain assumptions. What is needed is a bridging principle of this form:

Trivalent Assertion Rule (TAR) Only assert/question S if the set of mutual assumptions in the conversational situation (the Stalnakerian "common ground") ensures that S is either true or false."

While such a principle may seems perfectly obviously, it is not only not obvious, it is actually likely to be false. Suppose, for instance, that vague statements such as "John is bald" are neither true there false in some borderline cases. Then a straightforward application of the principle above would suggest that we can only ask the question, "Is John bald?" in cases where we know that John is not a borderline case. This is obviously a bad prediction.[2]

So if we treat the third-truth value as lack of truth-value, then we need to ensure that either a) there is only one form of lack of truth-value, that involving presuppositions, or b) that the form of lack of truth-value involving presuppositions is distinguished by the grammar/pragmatic processes

[1] See for instance, **?** for discussion.

[2] I owe most of these points to the excellent discussion of trivalence and presuppositions in Soames (1989). Soames differs from me in thinking that these considerations make trivalent thinking implausible, I think rather they require us simply to be clear about what we are doing when we are giving a trivalent account.

in some way from other cases of lack of truth value. In addition, we will then to stipulate **TAR** or a restriction of **TAR** to those forms of lack of truth-value of the presupposition type.

Given these facts, I think it is apparent that there is little motivation for insisting that U be understood as a third-truth-value rather than as some sort of distinguished form of falsity, which special pragmatic rules apply to. Indeed we need not even think that for every lexical expression it is stipulated in the lexicon that it sometimes has the value U rather than F, rather we may think that the attribution of U to certain expression in certain contexts is the result of pragmatic processes. All this is just to propose using trivalent theories not as deep explanation of what presuppositions are, but rather as a simple formal technique for keeping track of presuppositions.

3 Syntax

- Language includes: set of n-place predicates, F, G (n may be 0).
- Set of quantifiers Q associated with relations as usual: these include *every*, *no*, *and*, *or*.
- Set of variables x, y, etc.
- If F is an n-place predicate, $x_1, \ldots, x_n$ are variables, then $F(x_1, \ldots, x_n)$ is a formula.
- If α and β are formulas and x is a variable and Q a quantifier then $Q_x(\alpha, \beta)$ is a formula.
- We write $and_x(\alpha, \beta)$ as $\alpha \& \beta$ and as a convention assume that x is free in α and β (there are no official binary connectives beside the quantifiers). Similarly for *or*. (We can also add negation with a quantifier which we will assume to take the same formula twice.)

4 Semantics

- Interpretations I which map predicates and tuples of objects in domain D onto truth values $\{T, F, U\}$ (we suppress mention of I when convenient, and just refer directly to whether, e.g., F is true for a given object)
- Each quantifier Q has a relation R_Q associated with it (this is fixed, not determined by I).

- $S_1 R_{every} S_2$ iff $S_1 \subset S_2$
- $S_1 R_{no} S_2$ iff $S_1 \cap S_2$ is empty
- $S_1 R_{or} S_2$ iff either S_1 or S_2 is nonempty
- $S_1 R_{and} S_2$ iff both S_1 or S_2 are nonempty

- Notation: $\mathcal{P}(S)$ = powerset of S
- If α is of form $F(x_1, x_2, \ldots)$ and assignment function f assigns $x_1 \to o_1$, $x_2 \to o_2$, etc then $[\![\alpha]\!]^f$ is equal to whatever truth-value I assigns to combination of F and $(o_1, o_2$, etc.).
- Notation: For any formula α, value V in $\{T, F, U\}$, and variable x, $\alpha_{f,x,V} = \{o \in D : [\![\alpha]\!]^{f_{x \to o}} = V\}$
- For any two sentences α, β that are always defined, the semantics should yield the following result (to get the bivalent case right!): $[\![Q_x(\alpha, \beta)]\!]^f = T$ iff $\alpha_{f,x,T} R_Q \beta_{f,x,T}$

We will first give the standard Strong-Kleene/Supervaluationist semantics and then move on to the new system.

5 Strong-Kleene/Supervaluationist Semantics for Complex Expressions

Here, I give a semantics for these expressions based on the Strong-Kleene logic or the supervaluationist program. In a standard supervaluationist framework we have an infinite number of possible interpretation functions giving all possible truth values to those values that are not defined. This allows us, e.g., to make $A \& \neg A$ to be false even if A has no defined truth value. However, this feature is undesirable for presuppositions. For tautologies can yield presuppositions. For example, (2) seems to presuppose that John used to smoke.

(2) Either John stopped smoking or he didn't.

This actually makes the supervaluationist semantics easier to state. To get the truth value of a complex expression of the form $Q_x(\alpha, \beta)$ with respect to f we simply quantify over all possible combinations of T and F for $[\![\alpha]\!]^{f_{x->o}}$ (or $\beta^{f_{x->o}}$) for each o in D for which it is U. For the complex to be T (or F) is just for it to have the T (or F) on each of those combinations. Otherwise it is U.

5.1 Symmetric

The symmetric version is just a straightforward implementation of this framework.

- $[\![Q_x(\alpha, \beta)]\!]^f = T$ iff $\forall A' in \mathcal{P}(\alpha_{f,x,U}), \forall B' in \mathcal{P}(\beta_{f,x,U}) : (\alpha_{f,x,T} \cup A')R_Q(\beta f, x, T \cup A')$
- $[\![Q_x(\alpha, \beta)]\!]^f = F$ iff $\forall A' in \mathcal{P}(\alpha_{f,x,U}), \forall B' in \mathcal{P}(\beta_{f,x,U}) : \neg(\alpha_{f,x,T} \cup A')R_Q(\beta f, x, T \cup A')$
- If neither of the two conditions above holds then $[\![Q_x(\alpha, \beta)]\!]^f = U$

Fact 1. *$[\![Every_x(\alpha, \beta)]\!]^f \neq U$ iff either ($\exists o \in D$ s.t. $[\![\alpha]\!]^f = T$ and $[\![\beta]\!]^f = F$) or ($\forall o \in D [\![\alpha]\!]^f = F$ or $[\![\beta]\!] = T$)*

Obviously this presuppositions is not entirely standard and does not seem to match our observations of the presuppositions of sentences of this form such as this:

(3) Every student stopped smoking.

It does not seem that (3) presupposes that either one student didn't stop smoking or every student used to smoke.

5.2 Asymmetric

To make the system asymmetric, we can make it that $[\![Q_x(\alpha, \beta)]\!]^f \neq U$ only in cases where one can tell just from looking at the first formula α that no undefinedness in α could possibly cause any undefinedness in the whole formula, regardless of our choice of β.

To do this we simply take the above definition and expand undefined cases to include all cases where: $\exists B \in D$ s.t. $\exists A' \in \mathcal{P}(\alpha_{f,x,U})$ s.t. $\alpha_{f,x,T} R_Q B \not\leftrightarrow \alpha_{f,x,T} \cup A' R_Q B$

Unfortunately this does not affect the prediction for sentences like (3) where the restrictor itself has no presuppositions (and hence is always defined).

So we can see that the supervaluationist framework even when made asymmetric, still fails to reproduce Heim's predictions for the presuppositions of quantified sentences. However, the system does reproduce Heim's results (as summarized in Schlenker (to appear, 2008) for the binary connectives.

6 New System

6.1 Asymmetric Version

In this theory, the asymmetric version is simpler than the symmetric theory. Consider a formula of the form $[\![Q_x(\alpha, \beta)]\!]^f = U$. What we do is essentially the following: make sure that for every possible set of truth conditions for β over the defined objects (i.e. the $o \in D$ s.t. $[\![\beta]\!]^{f_{x\to o}} = T$ or F), that every assignment of values to the undefined parts of both α and β yields the same results. If they do then whatever they yield is the value of $[\![Q_x(\alpha, \beta)]\!]^f$ otherwise it is undefined.

- $[\![Q_x(\alpha, \beta)]\!]^f = U$ iff $\exists B \in \mathcal{P}(\beta_{f,x,T} \cup \beta_{f,x,F}), \exists B' \in \mathcal{P}(\beta_{f,x,U}), \exists A' \in \mathcal{P}(\alpha_{f,x,U}) : \alpha_{f,x,T} R_Q B \neq (\alpha_{f,x,T} \cup A') R_Q (B \cup B')$
- If $[\![Q_x(\alpha, \beta)]\!]^f \neq U$ then $Q_x(\alpha, \beta) = T$ if $\alpha_{f,x,T} R_Q \beta_{f,x,T}$ otherwise $[\![Q_x(\alpha, \beta)]\!]^f = F$

Fact 2. $[\![Every_x(\alpha, \beta)]\!]^f \neq U$ *iff* $\forall o \in D, [\![\alpha]\!]^{f_{x\to o}} \neq U$ *&* $\forall o \in D : [\![\alpha]\!]^{f_{x\to o}} = T \to [\![\beta]\!]^{f_{x\to o}} \neq U$

Fact 3. $[\![No_x(\alpha, \beta)]\!]^f \neq U$ *iff* $\forall o \in D, [\![\alpha]\!]^{f_{x\to o}} \neq U$ *&* $\forall o \in D : [\![\alpha]\!]^{f_{x\to o}} = T \to [\![\beta]\!]^{f_{x\to o}} \neq U$

Fact 4. *Suppose x is not free in α or β, then $[\![And_x(\alpha, \beta)]\!]^f \neq U$ iff $[\![\alpha]\!]^f = F$ or $[\![\alpha]\!]^f = T$ and $[\![\beta]\!]^f \neq U$*

Fact 5. *Suppose x is not free in α or β, then $[\![Or_x(\alpha, \beta)]\!]^f \neq U$ iff $[\![\alpha]\!]^f = T$ or $[\![\alpha]\!]^f = F$ and $[\![\beta]\!]^f \neq U$*

7 Symmetric Systems

The symmetric version just is a disjunction of the asymmetric version done either direction.

- $[\![Q_x(\alpha,\beta)]\!]^f = U$ iff $\exists B \in \mathcal{P}(\beta_{f,x,T} \cup \beta_{f,x,F}), \exists B' \in \mathcal{P}(\beta_{f,x,U}), \exists A' \in \mathcal{P}(\alpha_{f,x,U}) : \alpha_{f,x,T} R_Q B \neq (\alpha_T \cup A') R_Q (B \cup B')$ and $\exists A \in \mathcal{P}(\alpha_{f,x,T} \cup \alpha_{f,x,F}), \exists A' \in \mathcal{P}(\alpha_{f,x,U}), \exists B' \in \mathcal{P}(\beta_{f,x,U}) : A R_Q \beta_{f,x,T} \neq (A \cup A') R_Q (\beta_{f,x,T} \cup B')$
- If $[\![Q_x(\alpha,\beta)]\!]^f \neq U$ then $[\![Q_x(\alpha,\beta)]\!]^f = T$ if $\alpha_{f,x,T} R_Q \beta_{f,x,T}$ otherwise $[\![Q_x(\alpha,\beta)]\!]^f = F$

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