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PERMISSIVE UPDATES

ABSTRACT

David Lewis asked in "A problem about permission" about the effects on context, specifically on what he called the "sphere of permissibility," of successively forbidding and allowing various sorts of behavior. The framework of truthmaker semantics sheds useful light on this problem. Update procedures are definable in the truthmaker framework that capture more than Lewis was able to just with worlds. Connections are drawn with epistemic modals, belief revision and the semantics of exceptives. We consider how a truthmaker account of permissive update might be integrated into a larger semantic/pragmatic account of deontic language. Bridge principles are articulated associating types of deontic act with distinct update rules. Distinct forms of permission trigger distinct procedures because they aim at spheres more or less strongly verifying $\diamond\varphi$, the claim that φ is permissible.

1 INTRODUCTION

This paper concerns a problem on the border of semantics and pragmatics:[2] how to make sense of permissions to do what was previously forbidden [**?**]? The problem is semantic in that it turns on the meanings of the sentences used to convey permissions. It is pragmatic in that an account is needed of how the speech acts performed with these sentences alter the conversational context, in ways that may or may not be semantically encoded.

Our focus will be on PERMISSIVE UPDATE: we are interested in how conversational participants update their understanding of what they are permitted to do as new permissions are granted. **?** showed that the problem is non-trivial; it cannot be handled by any simple variation on the standard account of the conversational updates associated with assertion [e.g., **?**].

Sections 2 to 4 review the standard account and lay out some of its problems, both predictive and structural. Sections 5 to 8 argue that the truthmaker framework (**?**, **?**) can avoid these

problems, and more generally that it provides better resources than the world framework for tracking permissive updates.[3] Consider, for instance, inference patterns such as the following:

(1)   Starting context: You must work Monday through Friday.
        New permission: You can take Friday off.
        Resulting context: You must work Monday through Thursday.[4]

(2)   Starting context: You must interview all the students in Grade 5 or 6.
        New permission: You do not need to interview boys.
        Resulting context: You must interview all the non-boys in Grade 5 or 6.

Standard stories, such as those based on AGM [**?**], or (the nearly equivalent) closest possible worlds approach, do not capture these inferences.[5]

Section 9 shows that truthmaker-based update operations cannot be relied on to satisfy the AGM axioms for contraction. Section 10 looks briefly at the relation with exceptive constructions. The remainder of the paper, sections 11 to 19, tries to connect truthmaker-based updates up with the semantics of permission (with digressions in 14 and 17 on weak permission vs strong, duality, and free choice). We consider four options.

The first option, in section 12, is to build our proposed update functions for permission into the semantics of permission statements. This option is unworkable, we argue, due in large part to its failure to respect duality. Next we try a standard Kratzerian possible world semantics for permissive modals, treating the updates as pragmatics-driven (section 13). A third option (section 15) is to give a (static) truthmaker semantics for permission statements, without attempting to link the update mechanism to the semantics.

Finally we attempt to integrate the semantics and update rules more closely by positing two grades of verification: strict and loose (section 16). Speech acts with content $\Diamond\varphi$ are graded by the kind of verification they aim at (section 18). "Allowing" φ aims at a loosely verifying sphere, while "inviting" φ aims at a sphere that strictly verifies $\Diamond\varphi$. Since a loosely verifying sphere is delivered by one sort of update procedure ("requirement-reduction"), and a strictly verifying sphere by another ("possibility-adding"), the speech acts of allowing φ and inviting it trigger different updates. More generally permissive acts are distinguished teleologically, in terms of the kind of sphere they aim at and the sphere's normative implications (section 19).

2 THE PROBLEM OF PERMISSIVE UPDATE

Start with the framework laid out in **?**. Conversation is viewed as a game. Speech acts are moves that change the game's governing parameters: the *score*. One of the parameters Lewis considers

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[3] The truthmaker framework has affinities with inquisitive semantics [e.g., **?**] as well as the older literature on alternatives [**?**, e.g.].

[4] A referee was skeptical of this inference, arguing that permission to take Friday off could, in the right context, extend back to the rest of the week. Suppose, e.g., it is understood that the boss's plans are completely thwarted if a single work day is missed. Better to stick with a standard issue premise semantics (like Kratzer's), which explains why our intuitions would fluctuate, than treat inference (1) as a data point. But it is not clear that our intuitions do fluctuate, if the question is whether you were *permitted* to take the week off. Suppose the boss says, when you fail to show up on Monday, that permission had only been given to take *Friday* off. This is surely correct. You are to be congratulated, perhaps, for realizing that you were not needed at all this week; but not for "realizing" that she was permitting you, in saying *You may take Friday off*, to take Monday-Thursday off too. (See also note 54.)

[5] That is, standard AGM does not validate (1) and (2) (which is not to say it *in*validates them). It is hard to be definitive here, since AGM-style operations are quite various. For the connection between AGM and the closest world approach, see **?**.
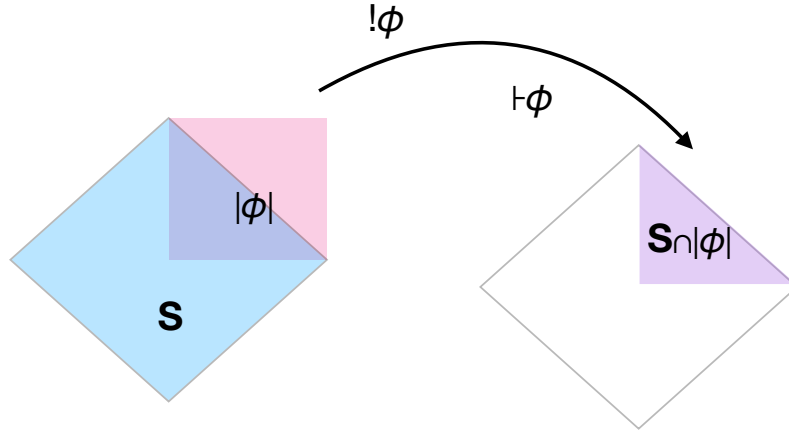
Figure 1: Intersecting with φ-worlds

is the "sphere of permissibility" discussed below. But it will be easier to begin with another sphere, employed by Stalnaker in his theory of assertion (**??**).

Conversation for Stalnaker has speakers making statements in order to build up the *common ground*, the assumptions shared by all involved. Assertive contents and common grounds are modeled as sets of worlds; such sets in Stalnaker-ese are propositions.[6] An assertion is associated with the set of worlds in which the sentence uttered is true, while the common ground is associated with the set of worlds (the "context set") satisfying all of the commonly held assumptions of participants in the conversation.

This model, highly idealized, assigns a certain functional role of assertion. Helping ourselves here to Frege's speech act marker ⊢ for assertion (the "assertion stroke"), the role is this: ⊢φ aims to induce a new context set $\mathbf{S}'$ obtained from the old one $\mathbf{S}$ by *intersecting* $\mathbf{S}$ with the proposition $|\varphi|$ asserted.

$$\vdash\varphi \colon \mathbf{S} \to \mathbf{S}' = \mathbf{S} \cap |\varphi|.$$

As Stalnaker puts it, the "essential effect" of an assertion ⊢φ is to remove all those worlds from the context set $\mathbf{S}$ that are incompatible with its propositional content $|\varphi|$. Lewis suggests that commands work similarly to assertions. The only difference is that this time $\mathbf{S}$ is the set of initially *permissible* worlds, the sphere of permissibility rather than believability.

$$!\varphi \colon \mathbf{S} \to \mathbf{S}' = \mathbf{S} \cap |\varphi|.$$

The essential effect of a command !φ is to remove from $\mathbf{S}$ all worlds incompatible with the proposition $|\varphi|$ that's commanded. $\mathbf{S}'$ is calculated the same way in both cases; it is $\mathbf{S} \cap |\varphi|$.

No doubt there is something right about the Stalnaker/Lewis model of assertions/commands; it remains at the core even of many revisionary accounts of conversational update today.[7]

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[6]Officially the *common ground* in Stalnaker is a set of propositions, while the *context set* is the set of worlds where all the propositions are true. A set of propositions is informationally richer, clearly, than a set of worlds, but the extra information is mostly not exploited by Stalnaker. A different sort of second-order information will be drawn on by the update procedures sketched below.

[7]See **??** for the model's relation to contemporary dynamic theories in the spirit of Heim and Veltman.

However, certain conversational moves are not naturally explicable in world-deletion terms. An example is implicit in Stalnaker's account of

> a nondefective conversation as a game where the common context set is the playing
> field and the moves are either attempts to reduce the size of the set in certain ways
> *or rejections of such moves by others* (**?**, italics added)

This assumes we can see *when the move is made* that φ cannot be taken for granted—that it ought to be left open whether φ. What if we realize it only later, after the $\overline{\varphi}$-worlds are gone? Or maybe we run later into evidence against φ; surely this cannot be ruled out. It is clear enough what to *say* in such cases (*As it turns out, it might be that* $\neg\varphi$) but not what to do with this statement. Presumably we will need to *expand* **S** to include at least some $\neg\varphi$-worlds. Stalnaker doesn't tell us which such worlds should be added, and it is far from obvious. What is the update effect supposed to be of *It might be that* ψ?

Lewis's "problem about permission" is the deontic analogue of this. Which worlds are supposed to be added to **S** by, *Actually, come to think of it, it's OK if* ψ? Lewis conceives this statement in speech act terms as an act $¡\psi$ of permitting ψ. One might similarly posit an act $¬!\psi$ of *reopening the question* of whether ψ, standing to assertion ($\vdash\psi$) as permission stands to command ($!\psi$). The problem in both cases is the same. The set of permissible worlds *grows* when permission is granted to do something previously forbidden, just as the context set—the set of believable worlds—grows when a question is reopened that was earlier closed. You can't grow a set by deleting some of its members. Yet world-deleting updates (intersective updates) are the only ones we've got at this point.

Lewis asks us to imagine a situation in which one person, whom he calls the Master and we call Madge, issues commands to another, whom he calls the Slave and we call Simon. If Simon is obedient, he will do whatever Madge tells him to, unless she later relents. At any stage in the process, we model Simon's options by the set of allowable worlds, the aforementioned sphere of permissibility.[8] The problem of permission is the problem of modeling how this sphere evolves as deontic directives are given.[9]

A command, according to Lewis, behaves like a Stalnakerian assertion. He works with the following simple example. Madge tells Simon that he is to work every day (Monday-Friday). The propositional content of the command is the set of worlds where Simon does work every day. The command's effect on the permissibility sphere is to remove all those worlds in which Simon defies the command. This is the same, of course, as intersecting the old sphere with the worlds where Simon works every day.

What about permission? Lewis is not much concerned with permission to do something φ which was not in any case forbidden.[10] The case that matters to him, and us, is permission

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[8]The contents of permissions and commands are "agent-centered" on some views. We will mostly be ignoring this issue. Nothing much changes if we switch to sets of centered worlds, where the agent is assumed to be at the center. More radical departures, like switching to sets of action-types, can make a difference [see **???**, for one proposal]; this has less to do, however, with agent-centeredness than granularity. φ *is permitted* will be run together in this paper with *You may see to it that* φ and even *You may* φ.

[9]"Directives" because we want to put aside here purely *reportative* acts, where φ is represented as "already" mandatory or permissible—mandatory/permissible as matters stand. One way to tell the difference is that reporters need not possess any sort of normative authority. They are charting the sphere of permissibility, not attempting to change that sphere.

[10]Though this might still serve a purpose, such as assuring Simon the sphere was not about to shrink, or singling out
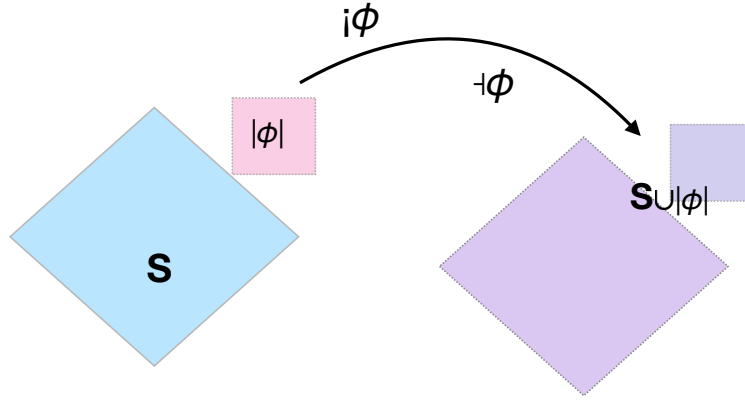
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Figure 2: Adding in all φ-worlds

granting, where something previously impermissible is brought in out of the cold. Suppose that Madge, having first required Simon to work Monday-Friday each week, relents and permits him to take Fridays off (φ). How is the new sphere to be generated, when φ is permitted? Intersection is not an option, for φ-worlds are wholly absent from the sphere as it existed previously: $\mathbf{s} \cap |\varphi|$ $= \varnothing$. Madge's intent was not to add on a new requirement, but suspend an old one. To cash out this metaphor with an explicit suspension rule proves surprisingly difficult.

3 WHY UPDATE IS DIFFICULT TO EXPLAIN JUST WITH WORLDS

3.1 PREDICTIVE PROBLEMS

A natural first thought is to just union in all the worlds where the permitted action occurs, as in figure 3.1. That this approach fails so badly is what makes the problem interesting. Among the worlds where Simon does not work on Fridays are those where he does not work *any* day. But, surely, granting permission to take Fridays off is does not make it permissible to stop coming in entirely.[11]

A second thought is that we should add in the *closest* worlds in which the newly permitted action occurs, as in figure 3.1. This too faces severe challenges. One is that of figuring out what kind of closeness relation is needed. Structurally speaking, what we need is not a relation between worlds, but a relation that, given a set of worlds, tells us what the closest worlds to that set are.[12] As the sphere changes, which worlds are closest can change in any number of ways. It would be nice to have some sort of handle on what determines proximity to a sphere. 'Closeness' in the (?) sense used in the counterfactuals literature will not work, Lewis shows. The closest worlds in *that* sense where Simon takes Friday off may be ones where he stays at his place of work, but doesn't conduct any business there (or does equivalent work elsewhere on a volunteer

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for approval an act which was previously just not verboten. A representation of the deontic order that distinguishes what is explicitly permitted from what has merely not been forbidden is sketched in section 14. For other ideas on this, see **????**.

[11] There may be an *inference* to this effect in some cases (perhaps the best explanation of the Friday-off permission is that Madge is abandoning her project). But we are speaking here of what Madge has permitted, not what she has given evidence of no longer caring about. See Lewis and notes 4 and 54 for more on this distinction.
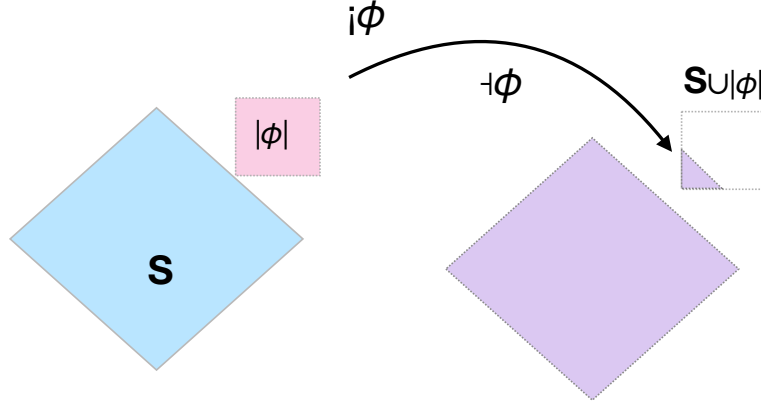
[12] See, for instance, **?** and **?**.

Figure 3: Adding in the closest φ-worlds

basis).

Following a suggestion of Stalnaker's, we might appeal here to a notion of 'comparative permissibility.' Given a sphere of permissibility, we can compare worlds outside the sphere in terms of how outrageous they are. Some such worlds will flout the norms more egregiously than others. When a new permission is given, we let in the *least* φ-worlds that are least impermissible by the standards defining the old sphere.[13]

But this gives the wrong results in many cases (**?**). Having been forbidden to eat meat on moral grounds, the least impermissible world where you eat it anyway is one where you eat only a tiny amount of the dimmest animal. But permission to eat meat intuitively extends to worlds in which you eat, say, an entire hamburger.[14] Similarly if you are permitted for the first time to drive the car, this does not mean you can only drive for one second, even if this would be the least impermissible option from the perspective of a prohibition on car driving.

The point that there is no clear way of homing in on the relevant notion of closeness is not devastating. After all, the notion used in the Lewis/Stalnaker account of counterfactuals is elusive too.[15] But closeness gives us in this case *much less* than we wanted, and less than can be obtained by other means. The inferences in (1) and (2) are simple: if you have been required to work every day and you are then given Friday off, then you are a) not required to work on Friday, and b) still required to work every other day of the week. The closeness account captures a), but says nothing useful about b).[16]

3.2 STRUCTURAL PROBLEMS

World-based rules *yielding the right predictions* are not easy to find. This might be written off just to a lack of imagination. But there are structural problems as well, suggesting that the worldly approach is misguided in principle.

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[13] See **?**, chapter 6, for this kind of proposal.

[14] Thanks here to Caspar Hare.

[15] **?** was keen to defend a reductive account of closeness, but he was not successful, due to Morgenbesser cases among others. **?** prefers to treat closeness as a context-dependent primitive.

[16] The standard theory of belief revision, AGM [**?**], which is essentially equivalent to the closeness account [**?**], has a similar problem. It is also overly opinionated, for reasons considered in section 9.

For one thing, world-based rules make permissive update *intensional*. Suppose that $\diamond\varphi$ is the sentence one typically utters to permit φ,[17] and that $\oplus$ is the operation on sets of worlds that governs permissive update: $\mathbf{S}' = \mathbf{S} + \diamond\varphi = \mathbf{S} \oplus |\varphi|$. Then if φ holds in the same worlds as φ', permitting the one yields the same sphere as permitting the other:

$\mathbf{S} + \diamond\varphi$
$= \mathbf{S} \oplus |\varphi|$
$= \mathbf{S} \oplus |\varphi'|$ (since $|\varphi| = |\varphi'|$)
$= \mathbf{S} + \diamond\varphi'$.

This seems quite wrong in many cases. Marie Antoinette invited the poor to eat cake *C*). She did not invite them to *either* eat cake *or* eat cake and behead Louis (*D*), her not long for the world husband. And yet *C* holds in precisely the same worlds as $C \vee C \wedge D$.

A second structural issue turns on the distinction between two kinds of consequence: those that are "part" of what is permitted/commanded (as posting the letter is part of posting and burning it), and those that merely "follow" from what is permitted/commanded (as posting *or* burning the letter follows from posting it, without being part of posting it). The contrast here is deontically significant, since to command posting is not to command posting-or-burning, while to command posting-*and*-burning *is* to command inter alia that the letter be burned. But the distinction between parts and mere implications requires a notion of content ("thick" content) finer-grained than worlds can provide. The set of ψ-worlds is a subset of the set of φ-worlds, whether ψ is a part of φ, or φ has ψ merely as a downstream, after the fact, implication.

The final issue is this. *Must* and *may* are widely supposed to be duals. Permitting $\neg\varphi$ has at least *something* to do with not demanding that φ. It is hard to see how this is going to be capturable with the usual sort of world-based update rule.[18] For $\mathbf{S} + \diamond\neg\varphi$ (using $\square$ and $\diamond$ for *must* and *may*) is a superset of $\mathbf{S}$, while $\mathbf{S} + \neg\square\varphi$ is— on the standard dynamic clause for negation— a subset of $\mathbf{S}$, obtained by removing from $\mathbf{S}$ all the worlds in $\mathbf{S} + \square\varphi$. If updating with $\diamond\neg\varphi$ adds worlds without deleting any, and updating with $\neg\square\varphi$ deletes worlds without adding any, then clearly the update effects of $\diamond\neg$ and $\neg\square$ are not giving us much of a handle on how not insisting on φ lines up with permitting $\neg\varphi$.

4 DIGRESSION: REQUIREMENT-LISTS

All these problems, especially the structural ones, become less daunting if update rules are understood to be defined on sets of truthmakers rather than sets of worlds. First though let's consider an alternative approach, also mentioned by Lewis, that offers to address them on a more conservative basis.

Suppose we took propositions as the unit of analysis. What evolves over time is a list of them: those that Simon is required to see to the truth of. When a permission is granted, we strike from the going list of requirements all those incompatible with the newly permitted φ. The permissibility sphere at any given stage of the conversation is the set of worlds verifying

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[17]As φ unadorned is the sentence used to assert that φ.

[18]Though the problem here may be as much to do with the *update* aspect as the *world* aspect. See section 12 for more on duality.

each proposition then on the list.[19] Call this the *requirement-list* approach. Figure 4 applies it to the work-all-week example.
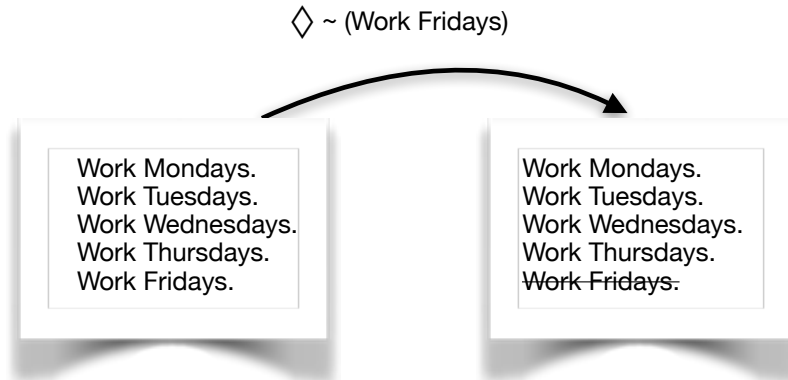
$$\lozenge \sim (\text{Work Fridays})$$

Figure 4: Removing requirements

Requirement-lists help with duality insofar as not mandating φ—keeping φ off the list— is like permitting $\neg\varphi$— keeping the list free of requirements incompatible with $\neg\varphi$. They help with hyperintensionality insofar as removing $\overline{F}$-incompatible requirements from the list has different results, depending on whether M, T, and so on were separately required, as in Figure 4, or there was instead a single requirement of working Monday-Friday, as in Figure 5.
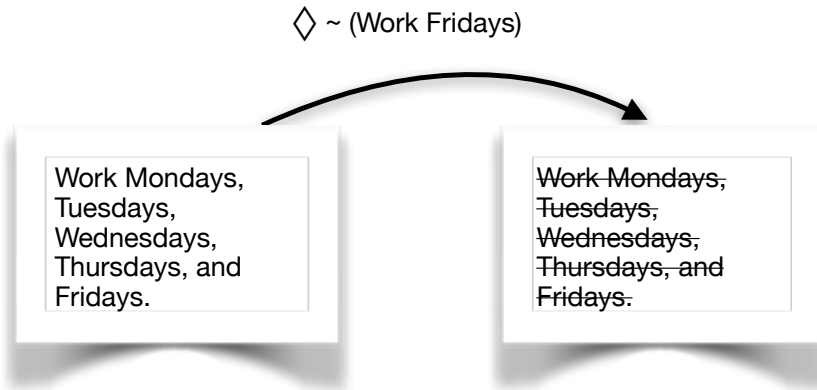
$$\lozenge \sim (\text{Work Fridays})$$

Figure 5: Requirements reframed

But requirement-lists do not seem to help with the problem of parts vs downstream consequences. And while they help with *an* intensionality problem—how can necessarily equivalent requirements $\mathscr{R}$ and $\mathscr{R}'$ lead, when φ is permitted, to distinct spheres?— the problem actually raised was different— how can necessarily equivalent permissions φ and φ' lead to distinct spheres? (Bearing in mind that φ is compatible with a requirement just if φ' is compatible with that requirement.)

A further difficulty is noted by Lewis. One effect of permitting φ ought to be that the permissible worlds come to include some where φ is true (unless of course φ is impossible). This

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[19]Compare Stalnaker's official notion of context set as the set of worlds satisfying every proposition with common ground status.

effect is not guaranteed on the present approach. It may happen that φ is compatible with each requirement taken individually, yet incompatible with their conjunction. If so, then φ comes out still impermissible, on the present approach, even after being permitted.

How is this to be addressed? A rule seems needed that tells us how to cut φ-inconsistent sets back to sets whose members are jointly consistent with φ. Presumably the requirements will have to be ranked in some way, and a subset will be preferred if it sacrifices low-ranking requirements to those that are higher-ranked. (Here work in the AGM tradition [**?**], **?** is relevant.)

Not that there is nothing intrinsically objectionable about rankings, their use in update rules causes a famous problem, obsessed over in the literature of iterated belief revision.[20] Update rules have got above all to apply to the results of earlier updates. Their whole point and purpose is to be brought repeatedly to bear over the course of a conversation. But then, the information needed to update rankings will have to be available at every stage. And it isn't, on the present approach. Nothing has been said about how new requirements are to be slotted into the order when a new command is issued. Nothing has been said either about how newly issued permissions change the ranking of the requirements that remain. So far, then, we do not have anything worthy of the name "update rule."[21] Whether truthmaker-based update procedures do any better in this respect remains to be seen, of course, but the news when it comes will be good.

5 WORLDS AND TRUTHMAKERS

A sentence φ will be associated henceforth not only with $|\varphi|^+$ and $|\varphi|^-$, the worlds w where it is true (false), but also $\|\varphi\|^+$ and $\|\varphi\|^-$, comprising its various *ways* of being true (or false) in those worlds. Truth-ways, or truthmakers, can be understood for many purposes as set-of-worlds propositions that guarantee φ's truth, that is, subsets **p** of $|\varphi|^+$. $|\varphi|^+$ can always be recovered from $\|\varphi\|^+$ (it's the set of worlds where a member of $\|\varphi\|^+$ obtains), so $\|\varphi\|$ ($= <\|\varphi\|^+, \|\varphi\|^->$) may as well be φ's sole semantic value.[22] Usually $\|\varphi\|$ is taken to be a bicameral proposition made up of $\|\varphi\|^+$ and $\|\varphi\|^-$, but we will sometimes, when no confusion arises, let $\|\varphi\|$ be $\|\varphi\|^+$.

Not all subsets of $|\varphi|^+$ count as truthmakers for φ—only those that explain *why* φ is, or would be, true. A simple model of how this works, in a language $\mathscr{L}$ with the syntactic and semantic structure of propositional logic, is as follows; it's essentially Fine's canonical state space model.[23]

Take first the monadic notion of being the kind of thing σ that can play the role of making a sentence true (or false)—states, Fine calls them. σ in our model will be a set or sum of literals — "literals" being negated ($\overline{A}$) or unnegated (A) atoms. States correspond in an obvious way to valuations (generally gappy, sometimes glutty as well) of the language. A crude notation will be used which renders σ as a *concatenation* of the relevant literals. Concatenation is understood

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[20] **?**, **?**

[21] The 'comparative permissibility' approach reviewed in the last section is open to a similar objection. We are told how a new command or permission changes what is absolutely permissible, but not what happens to comparative permissibility.

[22] Fine develops a world-free framework that defines propositions directly in terms of truthmakers (**?**). This approach is more powerful, since necessarily equivalent truthmakers need not be identical for Fine. The approach in the next paragraph via conjunctions of literals also does not identify necessarily equivalent truthmakers. So does the valuational approach if we allow atoms to be assigned both truth-values. Not a lot is going to turn on these issues and we are mostly going to ignore them.

[23] $\mathscr{L}$'s atomic sentences are $A, B, C.... \in \mathscr{A}$. A valuation $\mathscr{M}$ of $\mathscr{L}$ is a function from $\mathscr{A}$ to the truth values $0, 1$. (Gappy valuations are functions assigning in some cases neither value. Glutty valuations are functions assigning in some cases both values.) Valuations play the world role. Propositions are thus sets of valuations. The proposition expressed by φ is the set of valuations in which φ is true by the truth-table test.

here as a kind of commutative conjunction, so order is not relevant; AB is a different string than BA, but they are the same state σ. A state $A\bar{B}$ is said to be *composed* of A and $\bar{B}$, and A and $\bar{B}$ are its parts. The possible truthmakers composed of atomic sentences A and B are A, B, $\bar{A}$, $\bar{B}$ AB, $\bar{A}B$, $A\bar{B}$, $\bar{A}\bar{B}$.[24] We can speak also of the trivial state $\top$, a tautological σ with no proper parts. $\top$ will be treated as an ex officio element of every state.

Armed with this notion of a potentially truthmaking state σ, we move on to the relational notion of being a truthmaker *for* a particular sentence φ. A paper of van Fraassen's (**?**) gives a method for associating $A\bar{B}$-type truthmakers with sentences φ, that is, for determining which states in the monadic sense are truthmakers for which φs.[25] The plural here is important, for a sentence may have several truthmakers.[26] The disjunction $A \lor \neg B$ has two, A and $\bar{B}$. By contrast the conjunction $A \land \neg B$ has just the one truthmaker $A\bar{B}$. This reflects the intuitive thought that there are two ways for a disjunction to be true, whereas there is just one way for a conjunction to be true. Disjunctions have likewise one falsemaker, while conjunctions tend to have more than one.

There are two initial constraints on what the truthmakers for a sentence S can look like. **s** is the set of S's truthmakers only if

    a) every member of **s** necessitates S, and
    b) the truth of S guarantees that at least one member of **s** obtains.

A set of states **s** that satisfies a) and b) will be called EXTENSIONALLY ADEQUATE for S. There clearly exist extensionally adequate sets for any sentence in the language, because each PC sentence can be written in disjunctive normal form; the truthmakers are more or less the disjuncts. However, just as sentences have oftentimes more than one disjunctive normal form, we cannot expect for each S a unique extensionally adequate set. $\{AB, C\}$, $\{A B\bar{C}, C\}$, $\{AB, C, CE\}$, and so on, are all extensionally adequate for $(A \land B) \lor C$. One cannot expect even a unique *minimal* set of this kind. $\{AB, \bar{B}\bar{C}, \bar{A}C\}$ is extensionally adequate for $A{\equiv}B \lor B{\equiv}C$, but so is $\{BC, \bar{A}\bar{B}, A\bar{C}\}$. Neither of these two sets can be reduced further by dropping a state or shrinking one.[27]

This is where van Fraassen's rules come in. He associates with every sentence in $\mathscr{L}$ a *unique* set of truthmakers. The rules are as follows, writing $\|\varphi\|^+$ ($\|\varphi\|^-$) for the set of φ's truthmakers (falsemakers), and writing ss' for the union/fusion of truthmakers s and s' ($\approx$ the set of all value-assignments made either by s or s').[28]

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[24] Fine calls such composition *fusion*, emphasizing the mereological aspect of truthmakers.

[25] His paper differs in terminology and notation from ours.

[26] It can even have more than one in a world; but here we are talking about *potential* truthmakers.

[27] Since sets of states are understood 'disjunctively' we will sometimes speak of them as being true when one of their members is true and false otherwise. Thus, we can speak of sets of states, too, as necessarily equivalent.

[28] ss' can also be conceived as the set of all literals corresponding to assignments made by s or s'. This is sharply to be distinguished from a different set, one level up: the truthmaker set for $\varphi \lor \psi$ is the union of the truthmaker set for φ with the truthmaker set for ψ. Sets of literals (states) are conjunctive. Sets of truthmakers (sets of sets of literals) are disjunctive. The truthmakers are φ's various ways of being true; φ is equivalent to their disjunction.

<u>Van Fraassen's Recursive Rules</u>

$$\|A\|^+ = \{A\} \text{ (where } A \text{ is any atomic formula)}$$
$$\|A\|^- = \{\overline{A}\} \text{ (where } A \text{ is any atomic formula)}$$
$$\|\neg\varphi\|^+ = \|\varphi\|^-$$
$$\|\neg\psi\|^- = \|\psi\|^+$$
$$\|\varphi \wedge \psi\|^+ = \{ss' : s \in \|\varphi\|^+, \ s' \in \|\psi\|^+\}$$
$$\|\varphi \wedge \psi\|^- = \|\varphi\|^- \cup \|\psi\|^-$$
$$\|\varphi \vee \psi\|^+ = \|\varphi\|^+ \cup \|\psi\|^+$$
$$\|\varphi \vee \psi\|^- = \{ss' : s \in \|\varphi\|^-, \ s' \in \|\psi\|^-\}$$

These rules yield, for instance, that

$$\|A \wedge \neg B \wedge C\|^+ = \{A\overline{B}C\}$$
$$\|A \wedge (B \vee C)\|^+ = \{AB, AC\}$$
$$\|A \equiv B\|^+ = \{AB, \overline{A}\overline{B}\}$$
$$\|A \wedge \neg A\|^+ = \{A\overline{A}\}$$

The van Fraassen mapping of sentences to truthmakers (and his understanding of truthmakers as concatenations, or sets, of literals) is our starting point. It cannot be the final story about deontic truthmakers, for a number of reasons. To capture the full range of permitted φs, one needs additional devices not present in propositional logic (quantifiers, for instance). Second, deontic sentences $\Diamond\varphi$ will need a different *kind* of truthmaker.

A third reason for not resting content with this model is that commands and permissions do not always come wrapped in a linguistic package. Which conduct counts as permissible in a given setting may be implicitly understood, not laid out in explicit verbal directives.[29] Even when there is a directive to analyze, the truthmaker structure behind it need not always track what is there syntactically. A command to water the plants while I am gone might be satisfied by the complex act of watering them Monday, Tuesday, and so on. But the days don't themselves seem to be reflected in the syntactic structure of the command. Or again, *Help yourself to anything in the fridge* may express one permission when the question is *What am I supposed to do with myself when you're away?* another when it's *What if I am too broke to order out?* We consider later, in section 13, the problem of how to find truthmakers without syntactic antecedents. For now we defer the issue and assume a perfect mapping (à la van Fraassen) between verbally permitted φs and their truthmakers.

6 STRUCTURAL ADVANTAGES OVER WORLDS

Putting these qualifications aside, let us begin to employ the apparatus of truthmakers to model permissive updates. The set of permissible worlds (the worlds we're allowed to realize) is given by a set of truthmakers. If one thinks of this set as the coarse-grained proposition *It's all OK*, then the truthmakers are what *make* it OK. Truthmakers so conceived are *licit-makers*, and that is a word we will sometimes use. The sphere on which updates operate is not the set of permitted

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[29]Recall Wittgenstein's example of telling the sitter to teach the children a game, and returning to find your five year old playing high stakes poker. "I didn't mean that kind of a game."

worlds, but the set of licit-makers. These determine in turn which worlds are permitted (call that the coarse-grained sphere, if you like).

Where do the licit-makers come from? One might identify them with the van Fraassen truthmakers of all φ such that φ has been permitted. Or, one might try to derive them from the van Fraassen *falsemakers* of all prior *proscriptions*. The set of permissible worlds in the second case is *Ain't misbehaving* (as opposed to *Am behaving*), and the licit-makers would be the different ways of staying out of trouble. (See the discussion of strong and weak permission in section 14; behavior that is not proscribed is *weakly* permitted.) Either way, each licit-maker lays out a permitted option for Simon.

Our model is analogous in one respect to the requirement-list approach canvassed in section 4; the coarse-grained sphere is an epiphenomenon of something finer-grained existing one level up, or down. The fine-grained item is a set of licitators (each in turn a combination of literals), though, rather than a set of requirements. This is important because where the requirement-propositions *conjunctively* represent what is required, the set of licitators does it *disjunctively*. The answer to "What am I required to do?" is: something permissible or other.

The sphere **s** of permissibility is a set of truthmakers (licit-makers). What about the permissions φ that expand the sphere? These too are associated with sets of truthmakers; $\|\varphi\| = \mathbf{p}$ comprises φ's various ways of being true. Each state in **p** represents one way of taking up the liberty that has been afforded.

Lewis's problem now takes a quite different form: how does one adjust the sphere of permissibility, with its truthmaker structure, to reflect the (similarly articulated) permission just granted? A method is needed to go from the set **s** of truthmakers comprising the initial sphere, and the set **p** representing the new permission, to a new set of truthmakers defining between them the updated sphere.

A simple case to start with: Suppose there are just four elementary states: *You eat a cookie* (C), *You eat an orange* (O), and their negations ($\bar{C}, \bar{O}$). Suppose that Simon is required to eat a cookie and to eat an orange. Then the coarse sphere—the set of permitted worlds— contains just those in which Simon eats a cookie and Simon eats an orange. The truthmaker defining this set is CO and so the set of currently operative licit-makers is the singleton set $\{CO\}$.

Now suppose that Madge allows Simon not to eat an orange. From a worldly perspective, it is clear that this permission adds worlds where Simon eats a cookie but not an orange (as well as keeping the worlds where he eats both). From the state perspective, we have choices: the new sphere of permissibility could be $\{C\}$, but it could also be something extensionally equivalent such as $\{CO, C\bar{O}\}$. So we go from the original sphere $\{CO\}$ and the new permission $\{\bar{O}\}$ either to $\{C\}$ or $\{C\bar{O}, CO\}$. One suspects that there are going to be quite a lot of update functions yielding a set of states extensionally equivalent to $\{C\}$. We will have to cut the range down somehow. Soon we will outline some specific ways of doing this— of updating permissibility spheres based on their licit-maker structure and that of the newly permitted φ.

First, though, let's look quickly back at the structural problems noted above for world-based update rules. The first was intensionality: if φ holds in the same worlds as φ', then $\mathbf{S} + \diamond\varphi = \mathbf{S} \oplus |\varphi| = \mathbf{S} \oplus |\varphi'|$ (since $|\varphi| = |\varphi'|$) $= \mathbf{S} + \diamond\varphi'$. But now, $|\varphi| = |\varphi'|$ does not at all mean that $\|\varphi\| = \|\varphi'\|$. If $\|C\|$ comes apart from $\|C \vee CD\|$, why should $\mathbf{s} \oplus \|C\|$ not come apart from $\mathbf{s} \oplus$

$\|C \vee CD\|$?

The second structural problem was to do with deontically distinctive forms of implication. Possible worlds semantics has essentially only one sort of consequence relation: implication, which relates φ to ψ just when every φ-world is a ψ-world. Implication has a deontically important sub-relation, we said, inclusive entailment.[30] An order to $\varphi \wedge \psi$ carries through to φ, but not $\varphi \wedge \psi$'s "mere consequence" $\varphi \vee \chi$. Truthmaker semantics lets us single out inclusive entailment as follows. φ inclusively entails ("entails") ψ if φ's truthmakers all imply truthmakers for ψ, and ψ's truthmakers are all implied by truthmakers for φ.

Duality will mostly be left for later, but one point can be made immediately. The worry was that more worlds ought to be permissible according to $\mathbf{s} + \diamond \neg \varphi$ than according to $\mathbf{s}$. But the standard dynamic clause for negation tells us that $\mathbf{s} + \neg \square \varphi = \mathbf{s} \setminus (\mathbf{s} + \square \varphi)$, which is a *subset* of $\mathbf{s}$. Of course we have no idea as yet what an operator $\boxplus$ standing to commands as $\oplus$ does to permissions (in the sense that $\mathbf{s} + \square \varphi = \mathbf{s} \boxplus \|\varphi\|$) might look like. But the objection does not require us to know: $\mathbf{s} \setminus (\mathbf{s} \boxplus \|\varphi\|)$ is bound to be a subset of $\mathbf{s}$ regardless of how $\boxplus$ is defined. That was the duality problem.

But, the standard dynamic clause for negation is not the only one possible. χ is associated In truthmaker semantics both with a set $\|\chi\|^+$ of truthmakers and a set $\|\chi\|^-$ of falsemakers. The update effect of $\neg \chi$ may well involve swapping $\|\chi\|^+$ out for $\|\chi\|^-$ in some way. Suppose it turned out (details below) that $\|\neg \square \varphi\|^+ = \|\square \varphi\|^- = \|\diamond \neg \varphi\|^+$. Then the following chain of identities seems not implausible, where $\oplus$ is a generic update operator taking spheres $\mathbf{s}$ and the contents of uttered sentences χ to revised spheres $\mathbf{s}'$:

$$\mathbf{s} + \diamond \neg \varphi$$
$$= \mathbf{s} \oplus \|\diamond \neg \varphi\|^+ \ (= \mathbf{s} \oplus \|\neg \varphi\|^+)$$
$$= \mathbf{s} \oplus \|\neg \square \varphi\|^+$$
$$= \mathbf{s} + \neg \square \varphi.$$

Similarly it seems not implausible that

$$\mathbf{s} + \neg \diamond \varphi$$
$$= \mathbf{s} \oplus \|\neg \diamond \varphi\|^+$$
$$= \mathbf{s} \oplus \|\square \neg \varphi\|^+ \ (= \mathbf{s} \boxplus \|\neg \varphi\|^+)$$
$$= \mathbf{s} + \square \neg \varphi.$$

This is speculative for now—we don't even know what $\|\diamond \neg \varphi\|^+$ is, much less that it's identical to $\|\neg \square \varphi\|^+$—but it at least shows that there are not the same structural obstacles to duality on the truthmaker approach as there were on the worldly approach.

An obstacle remains, you might think, if updating with $\neg \square \varphi$ is still a *pruning* operation— as it turns out to be, on the first proposal to be considered (REQUIREMENT-REDUCTION). But while it is true that no worlds are rendered permissible if $\mathbf{s}$ itself is cut back (fewer licit-makers s means fewer worlds are licitated), it turns out to be the individual licit-makers in $\mathbf{s}$ that get cut back. The result is an *expanded* set of permissible worlds. For a world is permissible if it satisfies a licit-maker; and to satisfy a pruned licit-maker is *easier* than satisfying the original, full-sized s from which it derives.

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[30]Studied originally by Angell, it is best known (**?**) as analytic containment. "Inclusive entailment" is from **?**:59.

7   TRUTHMAKER-BASED UPDATE RULES

Truthmakers in our model are just sets of literals in $\mathscr{L}$. But we can generalize fairly easily to other models of truthmakers.[31] If the set all truthmakers is $\mathscr{T}$, an update rule is a function of the type $2^{\mathscr{T}} \times 2^{\mathscr{T}} \to 2^{\mathscr{T}}$.[32] Upward pointing arrows $\uparrow$ will be used for permissive update functions. Using infix notation, $\mathbf{s} \uparrow \mathbf{p} = \mathbf{s}'$ means that the permissibility sphere $\mathbf{s}$ updated by $\mathbf{p}$ is equal to $\mathbf{s}'$, where $\mathbf{s}$, $\mathbf{p}$ and $\mathbf{s}'$ are all sets of truthmakers.

Only one substantive desideratum for $\uparrow$ has been mentioned so far, namely that $\{CO\} \uparrow \{\bar{O}\}$ should be $\{C\}$ — or another set of truthmakers extensionally equivalent to $\{C\}$. Of course, a single observation puts few constraints on the overall shape of $\uparrow$. Now we look at a couple of structural desiderata that point the way from $\{CO\} \uparrow \{\bar{O}\} = \{C\}$ to some plausible ways of defining $\uparrow$ generally.

Initially it seems that updates should be non-trivial ($\mathbf{s}\uparrow\mathbf{p}$ is distinct from $\mathbf{s}$) only when the permitted φ was previously impermissible. This will take some unpacking, however, for while "previously impermissible" had only one reading in the possible worlds framework—φ holds in no $\mathbf{s}$-worlds— now we have two readings:

φ is *definitely* $\mathbf{s}$-impermissible iff, for *each* $s\epsilon\mathbf{s}$, φ holds in no s-worlds.

φ is *potentially* $\mathbf{s}$-impermissible iff, for *some* $s\epsilon\mathbf{s}$, φ holds in no s-worlds.

I take it we don't want to say that updates are trivial unless φ was *definitely* impermissible. The point of a permission may be to make φ definitely permissible (compatible with each s in the sphere), when it was formerly only potentially permissible (compatible with some s in the sphere). This is ruled out from the start, if φ has to be definitely impermissible before $\uparrow\mathbf{p}$ can get a grip.[33] The proper assumption therefore is this:

TRIVIALITY

($\mathbf{s}\uparrow\mathbf{p}$) $\neq \mathbf{s}$ only if some $p \in \mathbf{p}$ is incompatible with some $s \in \mathbf{s}$.[34]

Propositions $\mathbf{p}$ and $\mathbf{s}$ are said to be *orthogonal* when each $p \in \mathbf{p}$ is compatible with each $s \in \mathbf{s}$. TRIVIALITY thus says that permitting φ changes nothing when $\|\varphi\|$ is orthogonal to $\mathbf{s}$.[35]

How should $\uparrow$ operate in *non*-trivial cases, where the permitted φ was potentially impermissible? Our second structural assumption is that the update operation on sets of truthmakers (like $\mathbf{s}$ and $\mathbf{p}$) is induced by an analogous operation $\Uparrow$ on the truthmakers themselves.

FACTORIZABILITY

A function $\Uparrow$ from pairs of states to sets of states exists such that $\mathbf{s}\uparrow\mathbf{p} = \bigcup\{s\Uparrow p : s \in \mathbf{s}, p \in \mathbf{p}\}$.[36]

Now our account of the essential effect of a permissive act $\mathsf{i}\varphi$ is almost complete. Once we specify a permissive update function $\Uparrow$ on states, the essential effect of $\mathsf{i}\varphi$ will be to transform

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[31] Kit Fine has explored the options as thoroughly as anyone.

[32] $2^{\mathscr{T}}$ is the powerset of the set of truthmakers— we are talking then about functions taking a pair of sets of truthmakers to a new set of truthmakers.

[33] An example is given in the discussion of *Vacuity* in section 9.

[34] As should be clear, given that truthmakers are propositions, truthmakers are *compatible* if they can obtain together.

[35] Later we will be questioning even this weak form of TRIVIALITY, to allow for permissions making an act *expressly* permissible which was previously only unforbidden.

[36] A function in other words from $\mathscr{T} \times \mathscr{T}$ to $2^{\mathscr{T}}$. Really we should say that $\mathbf{s}\uparrow\mathbf{p}$ consists of $\bigcup\{s\Uparrow p : s \in \mathbf{s}, p \in \mathbf{p}\}$ *along possibly with the members of* $\mathbf{p}$, since $\mathbf{p}$ on some rules (such as PA below) is carried over automatically into $\mathbf{s} \uparrow \mathbf{p}$. Note that TRIVIALITY would follow if we added to FACTORIZABILITY that $s\Uparrow p = s$ when p is compatible with s. Looking ahead a bit, this extra condition holds for $\Uparrow_{RR}$ but not $\Uparrow_{PA}$.

sphere **s** to the sphere **s**⇑**p** obtained by unioning every $s ⇑ p$, s ranging over the members of **s** and p over the members of $\|\varphi\|$. One such ⇑ is implicit in the cookie-orange example:

REQUIREMENT REDUCTION
$s \Uparrow_{\text{RR}} p = \{s\backslash\bar{p}\}$, where $s\backslash\bar{p}$ is the largest part of s that's compatible with p.

Cookie-eating (C) is the largest part of eating both a cookie and an orange (CO) that is compatible with what was permitted: $\overline{O}$, not eating an orange. So REQUIREMENT REDUCTION tells us that the sphere after orange-avoidance is permitted should shrink from $\{CO\}$ to $\{C\}$

To make this explicit, we need to explain what is meant by "largest part of s". Following **?**, parthood should be a partial order ⊑ on truthmakers (or on whatever is playing the truthmaker role). If we're aiming for full generality, we should reach here for the notion of a residuated lattice. Sticking though to our simplified model, $q \sqsubseteq s$ if q is composed only of literals figuring also in s.[37] So the largest part of s compatible with p is naturally defined as the part of s obtained by deleting (i) each literal A such that $\bar{A}$ is part of p, and (ii) each literal $\bar{A}$ such that A is part of p. What RR does at an intuitive level is take the set of states defining the permissibility sphere, and the set of φ's truthmakers, as inputs, and deliver as output a pointwise reduction, or paring back, of the states in the first set by the licit-makers in the second set. For every way s in which the old prerogatives could be exercised, and every option p granted the new permission to be exercised, we prune s to allow for p.

Suppose for instance that you are required to work either Monday, Wednesday and Friday, or Tuesday, Wednesday and Thursday. Then the going sphere of permissibility is $\{MWF, TWR\}$ (M is a literal corresponding to working Monday, and so on with the other letters as expected). Now you are told you can take Wednesday off. This has only a single truthmaker, so the permission granted is $\{\bar{W}\}$. Note first that the permission is non-trivial, as it is incompatible with both elements of the going sphere of permissibility. Applying the REQUIREMENT REDUCTION rule, we get the following:

$$\{MWF, TWR\} \Uparrow_{\text{RR}} \{\bar{W}\} = \{MF, TR\}$$

This seems like a good result. But we will want to consider another possible rule as well. Here is a case where RR might seem inadequate. Suppose you are not allowed to eat either lemon drops or chocolate. Then permission is granted to run 50 kilometers and eat lemon drops. Representing this in the obvious way, we get

$$\{\bar{L}\bar{C}\} \Uparrow_{\text{RR}} \{KL\} = \{\bar{C}\}$$

Apparently (since nothing is forbidden on the right hand side but chocolate eating), you are permitted now to eat lemon drops *whether you run fifty kilometers or not*. This might seem too liberal an understanding of the new normative order. The new sphere should, one might think, be $\{\bar{L}\bar{C}, KL\bar{C}\}$. That is, you are allowed to eat lemon drops not outright, but only together with running that kilometer (and of course chocolate-avoidance). A different definition of ⇑ yields this result exactly:

POSSIBILITY-ADDING

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[37] If we think of truthmakers as sets of literals, then ⊑ is just the subset relation ⊆.

$$s \Uparrow_{\text{PA}} p = \{s, p \star s\}, \text{ where } p \star s \text{ is } pr, \text{ for } r \text{ the largest part of } s \text{ compatible with } p.^{38}$$

Let's check that "cheating"-worlds, with lemon drops unredeemed by exercise, are now blocked. The newly permissible worlds are those defined by *KL* (running fifty kilometers and eating lemon drops) conjoined with the largest *KL*-compatible part of $\overline{CL}$, viz. $KL\overline{C}$. The combination that results—-running and lemon drops without chocolate—involves no unredeemed lemon-drop-eating, so the definition succeeds. Our examples taken together come out as follows:[39]

$$\{MWF, TWR\} \uparrow_{\text{PA}} \{\bar{W}\} = \{MWF, TWR, M\bar{W}F, T\bar{W}R\}$$

$$\{CO\} \uparrow_{\text{PA}} \{\bar{O}\} = \{CO, C\bar{O}\}$$

$$\{\bar{L}\bar{C}\} \uparrow_{\text{PA}} \{MR\} = \{\bar{L}\bar{C}, KL\bar{C}\}$$

Permission to take Wednesday off, after being required to work either Monday, Wednesday, and Friday or Tuesday, Wednesday, and Thursday, leaves it open to us (in addition to what was permitted beforehand) to work Monday, Friday, and *not* Wednesday, or else Tuesday, Thursday, and not Wednesday. Permission not to eat an orange, following on an order to eat an orange and chocolate, leaves it open to us (in addition to what was formerly permitted) to eat just chocolate. Permission to run and eat lemon drops, after a ban both on lemon drops and chocolate, allows us to: run, eat lemon drops, and avoid chocolate.[40]

Which of the two update rules is to be preferred? Rather than pushing one rule over the other, we claim that either might be appropriate, depending on what the speaker means to be doing. There are different kinds of permission, in other words, which ought by their very nature to act differently on the normative common ground.

There will be opportunities later to consider how RR-triggering permissions might be expected to differ from PA-triggering permissions. Section 14 looks for instance at the distinction between "weakly" permitted acts, which are not forbidden, and "strongly" permitted acts, which are positively singled out as OK. One can see even now that the intended effect of a weak permission is better captured by RR than PA, while the intended effect of a strong permission is better captured by PA. When Simon, initially required to eat both a cookie and an orange, is permitted not to eat an orange, RR yields C—it removes *O* from the code entirely—while PA yields a sphere $\{CO, C\overline{O}\}$ smiling explicitly on an option involving orange-avoidance.

A second difference, already noted, is that RR-style permission look to be "trickle-down" in the sense that conjuncts of permitted acts are rendered permissible too. If $P \wedge Q$ is consistent with $s \epsilon \mathbf{s}$, then P and Q are going to be individually consistent with s as well. PA-style permission is package-deal.[41] Permitting $P \wedge Q$ adds pq to the pq-consistent part of $s \epsilon \mathbf{s}$, but it does not add p

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[38] In other words, $p \star s$ is the truthmaker composed of every literal in p and every literal in $s \upharpoonright \bar{p}$ — roughly, p with as much of s as can consistently be added to p.

[39] The leading candidates for $\{CO\} \uparrow \{\bar{O}\}$ were $\{C\}$ and $\{CO, C\bar{O}\}$. RR picked the first, PA the second.

[40] All of this assumes a principle of PRECISION: $\mathbf{s}$ makes a permissible (meaning, $\mathbf{s}$ verifies $\Diamond A$, where a is A's one truthmaker), only if $a \epsilon \mathbf{s}$; it is not enough that a be part of something in $\mathbf{s}$. PRECISION granted, POSSIBILITY-ADDING would have us either hew entirely to the old requirements, or defy them to the largest extent allowed. A possible advantage in that case of REQUIREMENT-REDUCTION is that it gives us *choices* about how much of our freedom to exercise. Alternatively we might have a rule of INCLUSIVE possibility-adding: $s \Uparrow_{\text{IP}} s = \{s\} \cup \{x : s \upharpoonright \bar{p} \sqsubseteq x \sqsubseteq s \star p\}$. You can see how the new rule is more liberal by comparing its response to "You may B and C," when A, B, and C had all been previously forbidden, to POSSIBILITY-ADDING's response: $\{\bar{A}\bar{B}\bar{C}\} \uparrow_{\text{PA}} \{BC\} = \{\bar{A}\bar{B}\bar{C}, \bar{A}BC\}$; $\{\bar{A}\bar{B}\bar{C}\} \uparrow_{\text{IP}} \{BC\} = \{\bar{A}\bar{B}\bar{C}, \bar{A}B, \bar{A}C, \bar{A}BC\}$. Inclusive possibility-adding is ignored in the main text, because we will not in the end be assuming PRECISION.

[41] Assuming SEN, anyway.

alone to the p-consistent part of s, or q alone to the q-consistent part.[42]

So far we are just pointing out possibilities. Skeptics about RR will insist that *all* permissions are package-deal. "Trickle down" permissions might be explained as conjunctions of simple permissions, one per conjunct. If Simon has the option of taking just one day off of the two permitted, that is because a pair of (separately exercisable) permissions were given, one for each day. None of this needs to be decided right now. The point for now is that the truthmaker framework allows us to draw a number of not unintuitive distinctions. Some of these distinctions will be looming larger as we proceed.

8 PREDICTIVE ADVANTAGES OVER WORLDS

Two main styles of permissive update have been distinguished: requirement reduction and possibility-adding. Table 1 illustrates their differences in the case where $\Box(A \wedge B)$ is followed by $\Diamond(\bar{B} \wedge C)$:[43]

Rule	Calculates $\{s\} \uparrow \{p\}$ like this:	Yielding
RR	$\{s \backslash \bar{p}\}$, where $s \backslash \bar{p}$ is the part of s compatible with p	$\{A\}$
PA	$\{s, p*s\}$, where $p*s$ is p fused, or conjoined, with $s \backslash \bar{p}$	$\{AB, A\bar{B}C\}$

Table 1: Two ways of updating $\{AB\}$ with $\Diamond(\bar{B} \wedge C)$

Both of these rules deal appropriately with the examples that started us off, the ones that challenged closest-world accounts. The first was

(1) Starting context: You must work every day of the week (Monday-Friday).

New permission: You can take Friday off.

Resulting context: You must work every day other than Friday (Monday-Thursday).

One can easily check that

$\{MTWRF\} \uparrow_{RR} \{\overline{F}\} = \{MTWR\}$.
$\{MTWRF\} \uparrow_{PA} \{\overline{F}\} = \{MTWRF, MTWR\overline{F}\}$.

The output spheres here agree that you must still, after permission is given to take Friday off, continue to work every day other than Friday. A sphere requires what all its component licit-makers imply in common, and that implication is *MTWR* whether $\mathbf{s}'$ is $\{MTWR\}$ or $\{MTWRF, MTWR\overline{F}\}$. The second example:

(2) Starting context: You must interview all the students in Grade 5 or 6.

New permission: You do not need to interview boys.

Resulting context: You must interview all the non-boys (henceforth, girls) in Grade 5 or 6.

A plausible representation is,

(i) $\Box(B_5 \wedge G_5 \vee B_6 \wedge G_6)$

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[42]**?** has an interesting discussion of the contrast here; we get the term "package-deal" from that paper. He maintains, and one of our referees agrees, that for most "conjunctive permission sentences, [the] package-deal prediction is empirically wrong" (p.133).

[43]The rule of *inclusive* possibility-adding (note 40) gives us $\{A, AB, A\bar{B}, AC, A\bar{B}C\}$.

(ii) On second thought, $\Diamond(\neg B_5 \wedge \neg B_6)$

(iii) Still, though: $\Box(G_5 \vee G_6)$

Again, what do the rules say?

$$\{B_5 G_5, B_6 G_6\} \uparrow_{RR} \{\overline{B}_1 \overline{B}_2\} = \{G_5, G_6\}$$
$$\{B_5 G_5, B_6 G_6\} \uparrow_{PA} \{\overline{B}_1 \overline{B}_2\} = \{B_5 G_5, B_6 G_6, G_5, G_6\}$$

Both output spheres $\mathbf{s}'$ makes $G_5 \vee G_6$ obligatory, assuming that $\mathbf{s}' \vDash \Box\varphi$ if φ is implied by each of its component licit-makers.[44] Whether $\mathbf{s}'$ is $\{G_5, G_6\}$ or $\{B_5 G_5, B_6 G_6, G_5, G_6\}$, all its members imply the disjunction of G_5 with G_6.

One needs truthmaker structure for the rules to get a grip. This may give us pause, given that the requirement-list and closest-world approaches were faulted for making predictions only relative to a determination of what propositions make it onto the list, or what the closeness relation should be. Have we simply replaced stipulations about closeness and requirements with stipulations about truthmakers? Does truthmaker structure have to be reverse engineered from the results it is meant to generate?

This is a worry, but the cases are different; for one can get an independent grip on truthmaker structure, more anyway than on closeness or requirements. Truthmakers are obtainable in many cases compositionally, using the van Fraassen rules; they can be read off syntactic structure. The truthmaker approach fares at least as well here as alternative and inquisitive semantics ([**?**], [**?**]).

Second, there are other, less syntax-bound, ways of coming to grips with truthmaker structure. The states σ making a statement true determine its parts, and which larger statements it is part of, given the reduction in section 6 of part/whole to truthmaking: $\varphi \geq \psi$ if and only if each of φ's truthmakers implies a truthmaker for ψ, and each of ψ's truthmakers is implied by a truthmaker for φ.[45] Part/whole relations manifest themselves in judgments about partial truth—φ is partly true if it has a part ψ that is wholly true—and agreement—we agree to the extent that the content $\mathbf{s}$ of your statement overlaps the content $\mathbf{t}$ of ours, that is, s and t have a part $\mathbf{p}$ in common, that is, $\mathbf{p}$'s members are all implied by members of $\mathbf{s}$ ($\mathbf{t}$), and each truthmaker in $\mathbf{s}$ ($\mathbf{t}$) implies one in $\mathbf{p}$.[46] **?** and **???** point to other phenomena—to do with semantic explanation, inductive confirmation, knowledge attributions, scalar implicature, verisimilitude, and presupposition failure—bearing on a sentence's ways of being true.[47] Truthmaking is a load bearing notion throughout semantics; it is not just conjured up for the problem of permissive update.

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[44]This is not our last word on the verification of $\Box\varphi$; see section 15.

[45]A further condition worth considering is that each of ψ's falsemakers be implied by (better, identical to) a falsemaker for φ (**?**, chapter 3).

[46]Suppose you say that Mary is in Paris Monday to Wednesday, and we say she is there Tuesday to Thursday. We agree she is there on Tuesday and Wednesday. But we do not agree that she is in Paris on Monday or Thursday. This is predicted insofar as *She is in Paris Monday or Thursday*'s truthmakers are either not implied by a truthmaker for *She is in Paris Monday-Wednesday* or not implied by a truthmaker for *She is in Paris Tuesday-Thursday*.

[47]See also **?**, **?**, **?**, **?**, **?**, **?**, and a number of earlier papers by Gemes (**?**, **?**, **?**).

9 CONNECTION TO BELIEF REVISION

The problem of permissive updates is reminiscent of the problem of belief revision, at least as that problem is understood in AGM and related literature.[48] Suppose we model our beliefs by a set of possible worlds **B**. Then we learn a new proposition **P** that is incompatible with **B**. The question is how to update **B** to reflect the new information. The updated belief-set **B*** will imply **P**, since it is **P** that has just been learned. How does one get from **B** to **B***?

An apparent difference with permissive update should be mentioned; it turns out not to run very deep. When **P** is permitted against the backdrop of an existing sphere of permissibility **S**, the new sphere **S**′ should be *compatible* with **P**—there should be some permissible **P**-worlds—rather than implying it. Since at least **?**, however, it is common to divide belief update into two sub-operations: first, *contraction*, cutting **B** back so that it no longer entails ¬**P** (that is, adding **P**-worlds to **B**), and then *expansion*, removing all ¬**P** worlds from the result.[49] It is the first sub-operation, taking **B** to **B**~(¬**P**), that interests us here. **B**~(¬**P** is a bit like **B** + "**P** might be the case after all," which was compared earlier to **S** + "**P** may be done after all." Let's look then at the standard AGM axioms for contraction, writing **B**◇**P** for **B**~(¬**P**):

(1) *Success* **B**◇**P** has **P**-worlds in it, assuming **P**-worlds exist
(2) *Inclusion* **B** is a subset of **B**◇**P**.
(3) *Vacuity* If **B** is consistent with **P**, then **B**◇**P** = **B**.
(4) *Recovery* **B**◇**P** ∩ $\overline{\mathbf{P}}$ ⊆ **B**
(5) *Overlap* **B**◇(**P**∨**Q**) ⊆ **B**◇**P** ∪ **B**◇**Q**
(6) *Enclusion* **B**◇**P** ⊆ **B**◇(**P**∨**Q**), provided **B**◇(**P**∨**Q**) is consistent with **P**

This is not the place to go into the motivations for these principles. What we do want to ask is which of them govern permissive update as defined earlier. The question has to be framed carefully, since ↑ operates on sets of *states*, while ◇ operates on sets of worlds. Each set **x** of states, though, determines a set of worlds: the worlds [**x**] where some state in the set obtains ({*w* | some state in **x** holds in *w*}). Here is what we want to know:

   if [**s**]◇[**p**] is taken to be [**s**↑**p**], does ◇ satisfy the AGM axioms?

Of course, ◇ is not even well-defined if there are **s*** and **p*** such that [**s**] = [**s***] and [**p**] = [**p***] but [**s**↑**p**] ≠ [**s***↑**p***]. But let us put that issue aside. For it turns out that a contraction operator ◇ defined in terms of ↑ as above *would not satisfy the AGM axioms even if it existed*. Truthmaker-based update rules allow for trickier forms of sphere evolution than are allowed by AGM.[50]

Focussing for definiteness on ↑$_{PA}$, or POSSIBILITY ADDING (written for now without the subscript), we show that if truthmakers are obtained recursively by the van Fraassen rules, then two of the AGM axioms fail. (If truthmakers are understood non-recursively, as minimal truth-guarantors, then two more fail; this is left to the footnotes.)

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[48] **?** is among the many to draw this connection. See also **?**.
[49] In AGM one generally treats belief sets as sets of propositions or sentences, rather than worlds; hence what looks like expansion in worldly terms is called contraction in the AGM tradition.
[50] We are speaking here of regular AGM, not AGM with bases (**?**).

(1) *Success* ✓   Does permission to P always render at least some $[\mathbf{p}]$-worlds permissible? $\mathbf{s} \uparrow \mathbf{p}$ = the union over all s in $\mathbf{s}$ and p in $\mathbf{p}$ of $\{s, p*s\}$, where $p*s$ is p fused with s's largest p-compatible part. Each $p*s$ is p-compatible and so P-compatible (since p necessitates P). But then $[\mathbf{s} \uparrow \mathbf{p}]$, the set of worlds where at least one member of $\mathbf{s} \uparrow \mathbf{p}$ holds, is bound to contain P-worlds.

(2) *Inclusion* ✓   Does permission to P always expand the set of permissible worlds? Yes. Every licit-maker in the old sphere $\mathbf{s}$ is automatically carried over to $\mathbf{s} \uparrow \mathbf{p}$. Hence worlds licitated by a member s of $\mathbf{s}$ are still licitated by s when $\mathbf{s}$ is updated to $\mathbf{s} \uparrow \mathbf{p}$.

(3) *Vacuity* ✗   Does permitting a P that holds in some already permissible w leave everything unchanged? Not always! Suppose the initial sphere is $\mathbf{s} = \{AB\}$ and Madge then permits $B{\supset}C$ ($\mathbf{p} = \{\bar{B}, C\}$). $\mathbf{s} \uparrow \mathbf{p}$ is $\{AB, A\bar{B}, ABC\}$. Thus worlds where A holds and B fails, which were originally forbidden, are now OK, notwithstanding that the content of the permission P ($B{\supset}C$) already held in some permitted worlds (some AB-worlds). $B{\supset}C$ is consistent after all with $A{\wedge}B$.

(4) *Recovery* ✗   Suppose that Madge permits P and then immediately forbids it. Can a world be permissible when she's done which was not permissible at the outset? *Recovery* says no, but ↑ allows it. Let the starting sphere $\mathbf{s}$ be $\{AB, \bar{A}\bar{B}\}$ and let P be $B{\supset}A$ (($\mathbf{p} = \{\bar{B}, A\}$). When P is permitted, the sphere evolves from $\mathbf{s}$ to $\mathbf{s} \uparrow \mathbf{p} = \{AB, \bar{A}\bar{B}, A\bar{B}, \bar{A}\bar{B}\}$. The $\bar{A}\bar{B}$- and $A\bar{B}$-worlds should become again impermissible (according to *Recovery*) when P is forbidden, that is, $A{\wedge}{\neg}B$ is commanded. But the $A\bar{B}$-worlds are going to remain, indeed they are all that remains.[51] Permitting and then forbidding P thus makes worlds permissible that were originally forbidden.

(5) *Overlap* ✓*   Are worlds rendered permissible by *You may A or B* always rendered permissible either by *You may A* or *You may B*? Let $D = A{\vee}B$. $[\mathbf{s}]{\Diamond}[\mathbf{d}] = [\mathbf{s} \uparrow (\mathbf{a}{\cup}\mathbf{b})]$, since D's truthmakers $\mathbf{d}$ are (by the van Fraassen rules) A's truthmakers together with B's ($\mathbf{a}{\cup}\mathbf{b}$). $\mathbf{s} \uparrow (\mathbf{a}{\cup}\mathbf{b}) = \mathbf{s} \uparrow \mathbf{a} \cup \mathbf{s} \uparrow \mathbf{b}$ by Factorizability. Thus $[\mathbf{s} \uparrow (\mathbf{a} \cup \mathbf{b})] = [\mathbf{s} \uparrow \mathbf{a} \cup \mathbf{s} \uparrow \mathbf{b}] = [\mathbf{s} \uparrow \mathbf{a}]{\cup}[\mathbf{s} \uparrow \mathbf{b}] = [\mathbf{s}]{\Diamond}[\mathbf{a}] \cup [\mathbf{s}]{\Diamond}[\mathbf{b}]$. It follows that $[\mathbf{s}]{\Diamond}[\mathbf{d}]$, when D is $A{\vee}B$, is a subset of $[\mathbf{s}]{\Diamond}[\mathbf{a}] \cup [\mathbf{s}]{\Diamond}[\mathbf{b}]$.[52]

(6) *Enclusion* ✓*   Suppose a world is permissible after A is permitted. Is it also permissible when *A or B* (for short, R) is permitted? This follows from Factorizability, given that $A{\vee}B$'s truthmakers ($\mathbf{r}$) are A's truthmakers together with B's ($\mathbf{a}{\cup}\mathbf{b}$). Proof: $[\mathbf{s}]{\Diamond}[\mathbf{a}] = [\mathbf{s} \uparrow \mathbf{a}] \subseteq [\mathbf{s} \uparrow \mathbf{a}] \cup [\mathbf{s} \uparrow \mathbf{b}] = [\mathbf{s} \uparrow (\mathbf{a}{\cup}\mathbf{b})] = [\mathbf{s} \uparrow \mathbf{r}] = [\mathbf{s}]{\Diamond}[\mathbf{r}]$.[53]

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[51] Admittedly no update rules have been given yet for acts of forbidding/commanding. But talk of forbidding P is really just a rhetorical flourish at this point. *Recovery* concerns the results of removing all P-worlds from $\mathbf{B}{\Diamond}\mathbf{P}$; it doesn't care if the worlds were removed because P was forbidden or for some other reason. See section 15 and following for commands and their relation to permissions. Thanks here to an anonymous referee.

[52] Suppose we think of S's truthmakers not recursively as van Fraassen does, but as its "prime implicants" = the minimal conjunctions of literals still strong enough to imply S. (Prime implicants are a semantic analogue of Mackie's INUS conditions; for details and discovery procedures, see **?**.) Then *Overlap* fails. Let $\mathbf{s}$ and P be $\{\bar{A}\bar{B}\}$ and $B{\supset}(A{\wedge}B)$. $B{\supset}(A{\wedge}B)$'s recursive truthmakers are $\bar{B}$ and AB, but its prime implicants are $\bar{B}$ and A. $\mathbf{s} \uparrow \mathbf{p}$ thus becomes (on the prime implicant picture) $\{\bar{A}\bar{B}, A\}$ instead of $\{\bar{A}\bar{B}, AB\}$, which means that $A\bar{B}$-worlds are now permissible along with AB worlds. This is a failure of *Overlap* since $A\bar{B}$-worlds are not vouched for either by the licit-makers in $\{\bar{A}\bar{B}\} \uparrow \{\bar{B}\}$ ($= \{\bar{A}\bar{B}\}$) or those in $\{\bar{A}\bar{B}\} \uparrow \{AB\}$ ($= \{\bar{A}\bar{B}, AB\}$).

[53] *Enclusion* too fails if truthmakers are prime implicants. Imagine that we start out with $\mathbf{s} = \{\bar{A}\}$, where A is atomic.

Our first reason for distinguishing the problem of permissive update from that of AGM-style belief revision is that permissive update proceeds along different lines than AGM allows. You may say that "real" belief revision doesn't follow the AGM rules either, or that it follows a different version of the rules. But the problem does not really depend on these details. It's that the inference patterns that formed our basic data do not hold in the case of belief revision.

Suppose you have never seen a naked mole-rat, but believe they are all green on the basis of testimony. You discover that the naked mole-rat in the zoo near your house might be white. How do your revise your beliefs?

Surely the most natural response is *not* to think that every other naked mole-rat, except the one at the zoo, is green, while the one at the zoo might be either be green or white. By contrast if you are required to paint every house green, and then allowed to paint a particular house white, it is natural to think that you are still required to paint the other houses green. Permissive update is digital, not holistic like belief revision.

Semanticists like to distinguish between modular, informationally encapsulated processes, and the higher-level thinking that sometimes overrides modular processes (the detection of conversational implicatures is a prime example). No one doubts that subjects may be in a position to tell that they can get away with more than what has explicitly been permitted. But permissive update, if it's to line up in any meaningful way with the semantics of modals, should be a modular, quasi-mechanical business. Semantics becomes unnecessarily difficult if meaning-detection is not kept separate from the epistemic project of guessing at what others really had in mind, or what they should have had in mind— in this case, what Madge really wants Simon to do, and would have allowed him to do if she was thinking more clearly.[54]

1 0   CONNECTION TO EXCEPTIVES

There is a prima facie analogy between permissive updates and the use of exceptives in quantifiers, such as in these examples:

(3)   Everyone but/except Ted is dead.

(4)   Except Ted, everyone is dead.

We will take it for granted that the function of exceptives in quantifier phrases is to cut down the quantificational domain.[55] So given that (3) and (4) say the same, they both say that the everyone in the domain stripped of Ted is dead.

Permissive updates have a similar flavor, but the subtractive element extends more widely than one would expect from the quantificational model. Start with some that fit that model:

(5)   (Context: you must wash every pot.) You don't have to wash the saucepan.

Permission to A leads by POSSIBILITY-ADDING to the sphere $s \uparrow \{A\} = \{\overline{A}, A\}$. Permission to $A \lor \neg A$ leaves s unchanged, however, since a tautology's only prime implicant is $\top$. Permission to A thus renders A-worlds permissible while permission to $A \lor B$ leaves them (in the case where B is $\neg A$) impermissible, which runs contrary to *Enclusion*.

[54] This relates to the question raised in note 4 of whether the inference in example (1) (*You must work Monday-Friday*, *You may take Friday off* ∴ *You may stay away Friday but must still work Monday-Thursday*) constitutes a genuine data point. To suppose that Madge's Friday-permission extends in certain contexts to earlier days lets worldly reasoning intrude into what ought to be a narrowly semantic process.

[55] See **?** for a classic discussion. von Fintel makes several important distinctions, arguing that free exceptives such as those in (4) are semantically distinct from the *but*-phrases in (3). (See also **?**.)

The resulting permissibility sphere is similar to what one gets when giving a permission with an exceptive:

(6)   You must wash every pot except the saucepan.

As Philippe Schlenker notes (p.c.), we get a similar effect with descriptive exceptions made to past statements about what was required:

(7)   a) They were required to wash every pot. Except the saucepans.
        b) He was required to work odd or even numbered days. Except Mondays.

This leads to two related questions. First, why don't we use an off-the-shelf semantics for exceptives to handle permissive updates generally? Second, what is the relationship between the semantics of exceptives and our story about permissive updates?

On the first question, to posit quantifier domain restriction as the sole mechanism of permissive update does not seem to cover all of the cases. Disjunctive and existential updates, as in (2), do not have the right kind of quantificational structure to be analyzed in any obvious way in terms of domain restriction.[56] That does not mean that we should not aspire to a unified view of the two phenomena. An adequate account of exceptives will have itself to deal with cases that go beyond the nominal realm, such as this:

(8)   It was a lovely meal, except for the food.

There may be *some* kind of pruning going on in these cases, but it's not obviously pruning of a domain, or anyway not a quantificational domain. When you are allowed to take Fridays off, the previous command holds *except insofar as it concerns Fridays*. It is not a domain that is shrinking here, but the condition that defines the domain. But we don't know yet how to generalize the notion of shrinkage beyond quantifiers.

One way to understand this paper is to see it as moving us towards such a generalization. That a theory of sentence-level subtraction would be useful has been widely noted in recent years.[57] How else to account for examples like the following?

(9)   Her version of events is correct, except regarding the dog.

A domain of objects (even fancy ones) does not seem to be involved here, or in

(10)   He is a standup guy, except when it comes to paying debts.

But the same word 'except' is present, performing to all appearances a similar function. Our algorithms for permissive update suggest a way of generalizing the phenomenon of exception-making beyond the quantificational paradigm that features so prominently in the literature.

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[56]**?** notes the strangeness of *Someone except for Bob is present* and suggests an explanation of it.
[57]**?, ?, ?**, chapter 5 of **?**, and **?**

11 CONTEXT CHANGE AND STATIC COMPOSITIONAL SEMANTICS

So far we have treated the problem of permission as a problem about context change; the question has been how to update what is allowed in light of a new permission. We argued that the truthmaker framework allows us to frame simple rules that capture a good deal of the structure of these sorts of updates. The rest of the paper will ask how this proposal relates to the semantics of the expressions used to make permissions, modals such as 'may' and 'can'. There seem to be four broad options.

The first is to completely semanticize the update rule(s) given above. We would do this by giving a dynamic semantics for modals according to which their semantic values are functions from sets of truthmakers to sets of truthmakers. This option is briefly reviewed in the next section. We do not pursue it for two reasons. (i) It is not clear how to provide plausible semantic entries that respect basic facts about the modals, such as must/may duality. (ii) If we semanticize permissive updates, we account at best for outright performative uses of modals. No light is shed on reportative uses, or "mixed" uses where φ-ing is permitted somehow or other, but Simon is not yet told which particular ways of φ-ing the permission covers.

The second option is to have a standard (Kratzer-style) truth-conditional analysis of deontic modals. Permissive updates would be treated entirely pragmatically, and the sets of truthmakers needed for update operations would have to be recovered from context in each case.

On the third option, we build truthmakers into our *static* semantic theory. The semantics does not itself involve an update procedure—-it merely tells us what the truth- and falsemakers of modal statements are—but it may be suggestive where updates are concerned, since the point of permitting φ is presumably to transform $\mathbf{s}$ into an $\mathbf{s}'$ that verifies $\diamondsuit\varphi$.

Our fourth option uses the truthmaker-based static theory to justify our (so far stipulative) update procedures. Various "strengths" of static verification are distinguished, and deontic speech acts are classified teleologically according to the type of static verification intended for $\diamondsuit\varphi$. Different speech acts call for different update rules, yielding from the same starting point $\mathbf{s}$ a sphere verifying $\diamondsuit\varphi$ in the sought-after way.

12 SEMANTICIZING THE UPDATES (OPTION 1)

Nothing in principle prevents us from trying to build rules like RR or PA into the meaning of 'may' and 'might'.[58] In the dynamic framework [e.g. **?**], we associate sentences not with propositions but rather with CONTEXT CHANGE POTENTIALS, functions from contexts to contexts. We could associate sentences of the form *might p* with functions from contexts to contexts, where contexts are represented by either sets of worlds or sets of truthmakers. The functions could take the form of one of the permissive update operations discussed above.[59]

There are reasons to be careful, however, about organizing the semantics of modals around update rules. Some uses of 'may' do not usher in a new sphere in the first place. There are reportative uses in which the speaker attempts only to convey what was permitted already, rather than push the boundary. There are promissory uses which convey that φ is no longer forbidden, but without indicating (as a sphere would) which ways of φ-ing are now OK. We will have

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[58]**?** suggests a way of doing just that.

[59]If contexts are just sets of worlds, then we will need a way of translating between sets of worlds and sets of truthmakers: we discuss one such method in the next section.

trouble accounting for these if we build update rules into the meaning of 'may.' [60]. Reportive uses are easily accounted for with static verification rules. A report that φ *is permitted* is true just if the going sphere statically verifies $\diamond\varphi$.

A larger problem with going completely dynamic, repeated here from section 3.2, is this. The following seem equivalent:

(11)   You may not — you are not permitted to — eat any apples. ($\neg\diamond A$)

(12)   You must not — you are required not to — eat any apples. ($\square\neg A$)

That $\neg\diamond$ lines up with $\square\neg$, and $\neg\square$ with $\diamond\neg$, is called *duality*. It falls out easily on the standard quantificational semantics for modals, which aligns $\square$ and $\diamond$ with $\forall$ and $\exists$. Neither of the permissive update operations sketched above, however (REQUIREMENT-REDUCTION and POSSIBILITY-ADDING), can be captured with anything as simple as existential quantification. Nor can the operation of adding a new requirement be captured with universal quantification.

If we cannot appeal to the fact that $\neg\forall$ is equivalent to $\exists\neg$ to explain the duality of *may* and *must*, how should we explain it? A dynamic semantics that respects duality may not be impossible, but how to devise one is seriously unclear. The first thing one presumably needs is an update rule $\boxplus$ for *must*, to run alongside the operation(s) $\oplus$ for permission sketched in section 7. Recalling that $\oplus$ was definable from $\uparrow$ —

$\mathbf{s} + \diamond\varphi = \mathbf{s}\oplus\|\varphi\| = \mathbf{s}\uparrow\mathbf{p}$, where $\mathbf{p}$ is the set of φ's truthmakers

— let's postulate an operation $\downarrow$ such that

$\mathbf{s} + \square\varphi = \mathbf{s}\boxplus\|\varphi\| = \mathbf{s}\downarrow\mathbf{p}$, where $\mathbf{p}$ is the set of φ's truthmakers.

Is *must* the dual of *may*, when these are understood in terms of $\downarrow$ and $\uparrow$? We have still not succeeded in making sense of that question. Duality is essentially to do with *negated* modals, and neither rule tells us what the update effect should be of *not* commanding (permitting) that φ.

If we had an operation $\bar{\boxplus}$ such that $\mathbf{s} + \neg\square\varphi = \mathbf{s}\,\bar{\boxplus}\,\|\varphi\|^+$, we could ask: is it the case for all spheres $\mathbf{s}$ that $\mathbf{s}\,\bar{\boxplus}\,\|\varphi\|^+ = \mathbf{s}\oplus\|\neg\varphi\|^+$? But we have no idea how to define such an operation. One thought would be just to *stipulate* that $\mathbf{s}\,\bar{\boxplus}\,\|\varphi\|^+ =_{df} \mathbf{s}\oplus\|\neg\varphi\|^+$. But dynamic semantics, like static, is supposed to be compositional. One would need to show how the stipulated context change potential for $\neg\square$ was obtainable from preexisting rules for $\neg$ and $\square$. Which, once again, looks difficult to impossible.

This is all rather discouraging. But hang on. Didn't we ourselves mount a defense of "dynamic duality" (in section 6) that bypassed these compositionality issues, that in fact bypassed $\bar{\boxplus}$ entirely? It went like this:

$\mathbf{s} + \diamond\neg\varphi$
$= \mathbf{s} \oplus \|\diamond\neg\varphi\|^+ \ (= \mathbf{s} \oplus \|\neg\varphi\|^+)$
$= \mathbf{s} \oplus \|\neg\square\varphi\|^+ \ (\text{assuming } \|\diamond\neg\varphi\|^+ = \|\neg\square\varphi\|^+)$
$= \mathbf{s} + \neg\square\varphi.$

The argument here does not assume that we have on hand an operation $\bar{\boxplus}$ such that $\mathbf{s} + \neg\square\varphi$

$= \mathbf{s} \boxminus \|\varphi\|^+$, still less that $\boxminus$ is thus and so derivable from the rules for $\neg$ and $\Box$. It assumes only that (i) we have in hand interpretations of $\|\Diamond\neg\varphi\|^+$ and $\|\neg\Box\varphi\|^+$ whereby the two come out identical, and (ii) these interpretations determine the effects on $\mathbf{s}$, the code of good conduct, of uttering $\neg\Box\varphi$ or $\Diamond\neg\varphi$.

But then, the case for duality runs essentially through *static* semantic values (like $\|\Diamond\neg\varphi\|^+$). $\neg\Box\varphi$ has similar update effects to $\Diamond\neg\varphi$ because

(1) the point of uttering ψ is to bring about a sphere that verifies ψ
(2) $\neg\Box\varphi$ and $\Diamond\neg\varphi$ are verified by the same spheres
(3) utterances of $\neg\Box\varphi$ and $\Diamond\neg\varphi$ aim at the same results

If this is right, then it makes no *sense* to semanticize our update rules. A static semantics is needed in any case, and the update rules are rationalized pragmatically in terms of their intended effects.

13 WORLDLY SEMANTICS + TRUTHMAKER PRAGMATICS (OPTION 2)

A couple of reasons have been given for treating the rules of permissive update (section 7) as part of the pragmatics of update, rather than the semantics of modals. The second way of relating permissive update rules to the semantics of modals is to treat the update rules as driven by pragmatic processes.

A natural starting point here is a standard intensional semantics along the lines of **??**. (This will help us to identify the point at which needs arise that that semantics cannot easily meet.[61]) Deontic modals express on this view quantification over a contextually given set of permissible worlds. Permission statements existentially quantify over these worlds (as do negated imperatives).[62] The problem of permissive updates arises when we are told that φ is permitted, but the current sphere of permissibility $\mathbf{S}$ does not include any φ-worlds. Once again we ask how to remake $\mathbf{S}$ to reflect the permissibility of φ.

To bring our proposed algorithms to bear, we will need to lay our hands somehow on a truthmaker representation of the situation. That is, we will need to move from standard intensional representations $\mathbf{S}$ and $|\varphi|$ to a representation in terms of truthmakers. The van Fraassen mechanism tells us how to get truthmakers out of a sentence of propositional logic, not out of a set of worlds. Granted, permissions are generally expressed in sentences. But one might non-verbally permit something, or permit it verbally while distancing oneself in some way from the sentence employed. And the sphere $\mathbf{S}$ will not be verbally represented even if the original commands and permissions were. The verbal record is long gone, and wouldn't be that useful anyway, since earlier commands have been partly undone by subsequent permissions, and permissions by subsequent commands.

But, while the commands and permissions may be gone, or superseded, the language in which they were formulated is not, and the space of worlds is not; the worlds are (or correspond up

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[61]Later, in section 15, we will consider a semantics for modals expressed directly in terms of truthmakers. But for now it is worth emphasizing that our basic idea about permissive update can be pragmatically grafted onto non-truthmaker-based semantic theories as well.

[62]We do not strongly distinguish Veltman's consistency-test semantics from the standard story.

one-one to) the total valuations, the assignments of truth-value to each atomic letter. Surprising as it may seem, this is enough to get us from **S** to a set **s** of licit-makers.[63]

How will this work? The set **S** of S-worlds lends itself intrinsically to a representation in terms of "prime implicants," where a prime implicant of S is a partial valuation just large enough to ensure S's truth (it has no proper subvaluations that also ensure S's truth). More or less equivalently, p is a prime implicant of S if (i) it's a conjunction of literals that implies S, and (ii) none of whose subconjunctions imply S (**?**). $\overline{A}$ and B, for instance, are $\neg A \vee (A \wedge B)$'s only prime implicants, and hence their only truthmakers on the present plan. The null hypothesis about **s** is that it is the set of all ps such that p primely implies S.

Of course, the space of possibilities is not usually given as the set of valuations of some propositional language. The above can still serve as a *model*, though, of the process of truthmaker-detection, with facts salient in context taking on the role formerly played by partial valuations. S is made true by any such facts strong enough to imply S but not so strong that weaker facts drawn from the same pool imply it. van Fraassen-type truthmakers make a limited comeback here, as the words and sentences in play are likely to be a factor in contextual salience.

What we don't get, by the stated method, is stronger and weaker truthmakers for the same sentence, for instance A and AB as verifiers for $A \vee (A \wedge B)$. There are a number of things we might try here. Perhaps certain S-implying facts qualify as truthmakers just by virtue of salience, even if weaker S-implying facts are salient too. Or perhaps the salient facts *change* as we survey the sentence, so that AB *is* the weakest salient fact when we get to the second disjunct (A has fallen out of view). S's truthmakers include s in context C, on this view, iff s is the weakest salient implier in *any subcontext* of C. These are shots in the dark, but they aim at a worthy target: allowing syntax to be relevant to truthmaker structure but not determinative. It's a factor that is not always even present, and where present has to duke it out with other factors.[64]

To see how this might work in practice, suppose Madge permits something φ that had previously been forbidden. She clearly intends to alter the sphere of permissibility so as to make it true that φ *is permitted* ($\Diamond \varphi$). To work out the shape of the alteration, Simon must

  a) find the *old* licit-makers s, the truthmakers pre-update of *You are behaving properly* (guided by linguistic antecedents and other features of context),

  b) extract from the newly permitted φ the φ-ish ways p_1, p_2,... of behaving properly (here the linguistic form of the permission is going to be crucial), and

  c) update the first set of truthmakers **s** with the second, **p**, using one of the $\uparrow$ operations sketched in section 7.

If we update with $\uparrow_{RR}$, we obtain a sphere $\mathbf{s}\uparrow_{RR}\mathbf{p}$ that verifies $\Diamond \varphi$ in the (weak) sense that each **p** is *consistent* with one or more licit-makers. If we update with $\uparrow_{PA}$, we obtain a sphere $\mathbf{s}\uparrow_{PA}\mathbf{p}$ that verifies $\Diamond \varphi$ in the (strong) sense that each **p** is *implied* by one or more licit-makers.

14 WEAK AND STRONG PERMISSION

Before getting on to the third option, which involves a truthmaker semantics for deontic modals, let's note an advantage of the second option which carries over. There may, as we saw, be more

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[63]**?** discusses issues in this neighborhood.

[64]For a fuller discussion, see **?**, chapter 4.

than one set of truthmakers corresponding to a given set of worlds, e.g., {*A*} picks out the same worlds as {*A*, *AB*}, and {*A*, *AB*} the same as {*AB*, *AB̄*}. This shows up empirically in the distinction between *weakly* permitted options and options that are *strongly* permitted.[65]

Consider the "constitutional principle of English law" which says that *everything which is not forbidden is allowed*. The principle is absurd, if "allowed" means *expressly* allowed (e.g., protected by law). It is just false that we are expressly entitled, say, to part our hair on the right (though this is not forbidden). "Allowed" in the constitutional principle has therefore got to express *weak* permission.

A jokey contrast is sometimes drawn with crueler jurisdictions in which *whatever is not allowed is forbidden*. "Allowed" as understood in the joke is meant to express the stronger notion; you are to do nothing which is not expressly allowed. Note, this means we are damned either way when it comes to a matter the law takes no interest in? We can't part our hair on the right, since this is not expressly allowed, but we can't fail to do so either, for the same reason. But that is the point of the joke. The law takes an interest in everything.

The strong/weak distinction falls out naturally on the truthmaker approach. *A* is WEAKLY permitted relative to a set of states **s** if there is at least one *s* in **s** with which *A* is consistent; this is enough for *A* to hold in some permitted world. *A* is STRONGLY permitted if it is not only compatible with an *s* in **s**, but *implied* by *s*. *A* not only holds in a permitted world, it holds thanks to the property of that world that *makes* it permitted.

One can draw further distinctions: φ is DEFINITELY WEAKLY permitted, for instance, if it is consistent with *every* licit-maker. It is THOROUGHLY WEAKLY permitted if each of its truthmakers is consistent with some licit-maker or other. φ is THOROUGHLY STRONGLY permitted if each of its truthmakers is implied by a licit-maker. So, for instance, in a typical classroom we might find that

  coughing is weakly permitted
  breathing is thoroughly weakly permitted
  vocalizing is strongly permitted
  informing others the class is on fire is thoroughly strongly permitted

Why is coughing only weakly permitted? No licit-makers *require* you to cough. There may even be some that require you NOT to cough (say, when singing), which is why the (weak) permission is not thorough. Breathing is thoroughly weakly permitted insofar as no licit-makers involve not breathing (as might occur at a swimming school). Vocalizing is strongly permitted insofar as there are licit-makers (to do with singing, and asking questions) that require you to vocalize. If it is not *thoroughly* strongly permitted, that is because there are ways of vocalizing (screeching, maybe?) that no licit-maker ever requires. Assuming that any old way of alerting others to danger is somehow licitated, raising valid alarms is thoroughly strongly permitted.

A similar distinction can be drawn between the expressly and in effect obligatory. An option *A* is in-effect obligatory, relative to a set **s** of licit-makers, if each *s* in **s** implies *A*. Those who don't *A* somehow or other will be misbehaving. An option *A* is expressly obligatory if each *s* in **s** implies a way *a* of *A*-ing. To be clothed is only in effect obligatory if, although nakedness always

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[65]See **?**, **?** and **??** for related discussions.

prevents a from holding, no specific form a of non-nakedness is singled out for approval by any s. Alternatively it might be that some permitted conduct s involves wearing the school uniform, some must be carried out in gym clothes, and so on. If every s is like this, then non-nakedness is expressly obligatory.

One can again ramify these notions. φ is *thoroughly* expressly obligatory if not only does each s imply a way a of A-ing, each way of A-ing is implied by an s.[66] Wearing the *proper* attire—the school uniform in class, gym clothes during phys ed, the team colors at games, and so on—might be thoroughly expressly obligatory in this sense.

Respect for this distinction between the in effect, and the expressly, permitted was mentioned earlier as a reason for skepticism about the TRIVIALITY condition. TRIVIALITY says that permission to φ leaves **s** unchanged ($\mathbf{s}{\uparrow}\{p\} = \mathbf{s}$) if φ is compatible with each of **s**'s members But the form of permissibility defined by compatibility with each licit-maker in **s**, although thorough, is weak. Piling each $p*s$ into **s**—as is done by POSSIBILITY-ADDING—does not bring in any new worlds, but it accomplishes something nevertheless; it raises the status of φ-type behavior in the existing worlds, by jumping a weak permission up into one that is strong.

The possibility of this kind of status upgrade helps to explain a distinction which world-based accounts cannot easily capture. To begin with, let's say, nothing is forbidden, nor is anything explicitly permitted. One thing Madge can say is

(13)   You may eat an orange.

Another is

(14)   You may eat either an orange, or an orange and an apple.

Why does (14) seem stronger? (13) arranges, if we update by POSSIBILITY-ADDING, for orange-eating to be strongly permissible. Eating them with apples, however, remains only unforbidden (weakly permitted). (14) puts orange-and-apple eating on the same deontic plane as orange-eating. The claim so far is that truthmakers (licit-makers) create space for two important distinctions—between weak and strong permission, and between intensionally equivalent permissions—that are difficult to make sense of with worlds. For this reason alone (others will be given later), it is worth pursuing a full-blown static truthmaker semantics for deontic modals. This is explored in the next section.

15   TRUTHMAKER SEMANTICS FOR MODALS (OPTION 3)

Can deontic modals be equipped with a truthmaker semantics à la van Fraassen? Our goal is to sketch one, and to show how such a semantics can capture

  a) the weak vs. strong distinction,

  b) free-choice effects, and

  c) the duality of 'may' and 'must',

all in a unified manner. While **??** covers the first two, we do not know of a semantics in the truthmaker tradition that also covers duality.[67] As already noted (section 13), a truthmaker-based

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[66]This is later called "intailment" by **s**.

[67]**?** has the potential to do so, if the spheres of permissibility and obligation are suitably coordinated. Handling

understanding of permissive updates does not presuppose a truthmaker semantics for 'may' and 'must.' It is nonetheless worth spelling out what such a semantics might look like, since the truthmaker version makes things clearer.

A reminder first of why a)-c) seem difficult to combine. Free choice suggests that *You may feed the cat or kick it* —

(15) $\Diamond(F \vee K)$

ought to be false. Therefore its negation

(16) $\neg\Diamond(F \vee K)$

ought to be true. $\neg\Diamond\varphi$ is equivalent, given duality, to $\Box\neg(F \vee K)$, hence to

(17) $\Box(\neg F \wedge \neg K)$

But it seems just false that one must neither feed it nor kick it; to feed the cat is perfectly OK. (The strong/weak distinction is relevant to the extent that one could block the argument by insisting that free choice holds only for strong (weak) permission, and duality only for weak (strong).)

Now the formalities. The language we work in is that of propositional modal logic—-$\mathscr{L}$ with unary $\Box$ and $\Diamond$ operators—except that $\Box\varphi$ and $\Diamond\varphi$ count as well-formed only if φ is $\Box$- and $\Diamond$-free.[68] We want to assign to each sentence of $\mathscr{L}$ a set of truthmakers. $\Box$-free sentences have the familiar old truthmakers we get from van Fraassen (described in section 5). A special new class of truthmakers will be introduced for modal sentences.

In any given context there is a sphere **s** of permissibility made up of states s. These spheres will play the role of deontic truth- and falsemakers, the items conferring truth on sentences like $\Box\varphi$. As always, we think of the sphere's members s_j as licit-makers. They constitute together a code GC of "good conduct," with each licit-maker marking out a different way of behaving properly.[69]

What should a code **s** be like to make $\Box\varphi$ true or false? There are a number of options here, but let's start with $\vDash^+$ and $\vDash^-$, what we will call *strict* verification and falsification. $\Box\varphi$ is strictly verified by **s** if

    (a) every licit-maker (every $s \in \mathbf{s}$) implies a truthmaker for φ *and*

    (b) every way of φ-ing is implied by a licit-maker (an s in **s**).

free-choice effects pragmatically tends to allow a simple treatment of necessity and possibility modals as duals. There are systems of dynamic semantics that cover free choice and duality, such as **??**.

[68]This is to sidestep issues raised by iterated deontic modals, which do sometimes occur. (**?** mentions *There ought to be a law*.) See Moss for iterated epistemic modals (**?**). Anglberger et al make room for iterated deontic modality by complicating the model; they associate with each action a distinct sphere which it "triggers" when performed (**?**). Interesting as this is, it assumes what we are trying to explain: why one sphere would be triggered by a directive rather than another.

[69]To repeat a point from earlier, codes of conduct GC in a structural sense resemble Stalnakerian common grounds (CGs). Both are sets of sets of worlds. But where common grounds are understood conjunctively—the believable worlds are those belonging to *each* set in the CG—codes of conduct are disjunctive—the permissible worlds are those where *some* licit-maker (some member of GC) holds.

$\Box\varphi$ is strictly falsified by **s** just if each falsemaker for φ is implied by a licit-maker (an $s\epsilon$**s**).[70] Putting these clauses together and adding their analogues for $\Diamond\varphi$, we get the following as our first static notion of modal truthmaking:

STRICT VERIFICATION ($\vDash^+$)

($\Box^+$) **s** $\vDash^+ \Box\varphi$ iff each $p\epsilon\|\varphi\|^+$ is implied by an $s\epsilon$**s**, and each $s\epsilon$**s** implies a $p\epsilon\|\varphi\|^+$

($\Box^-$) **s** $\vDash^- \Box\varphi$ iff each $p\epsilon\|\varphi\|^-$ is implied by an $s\epsilon$**s** .

($\Diamond^+$) **s** $\vDash^+ \Diamond\varphi$ iff each $p\epsilon\|\varphi\|^+$ is implied by an $s\epsilon$**s**

($\Diamond^-$) **s** $\vDash^- \Diamond\varphi$ iff each $p\epsilon\|\varphi\|^-$ is implied by an $s\epsilon$**s**, and each $s\epsilon$**s** implies a $p\epsilon\|\varphi\|^+$

Note that the property **s** needs in order to verify a $\Box$-claim is stronger than the property it needs to falsify one. $\Box\varphi$ is falsified by **s** if every way of $\overline{\varphi}$-ing is implied by a licit-maker. Verification requires not only (a) each way of φ-ing is implied by a licit-maker, but (b) each licit-maker implies a way of φ-ing.

What goes wrong if (a) is omitted? We get Ross's Paradox: *You must post the letter* winds up implying *You must post the letter or burn it*. $\Box A$ implies $\Box(A\lor B)$, if we use only (b), because a sphere whose members all imply truthmakers for A is a sphere whose members all imply truthmakers for $A\lor B$. Clause (a) blocks this reasoning by requiring φ to be *included* in **s**; each of φ's truthmakers has to be implied by an $s \in$ **s**. That all of A's truthmakers have this property does not remotely suggest that all of $A\lor B$'s truthmakers have it, because $A\lor B$ has additional truthmakers inherited from B. This is why strictly verifying A does not suffice for strictly verifying $A\lor B$. The disjunction has additional truthmakers; and these may not implied by licit-makers, as A's truthmakers were. It is also why truth-value gaps arise. $\Box\varphi$ is not verified *or* falsified if, for instance, some but not all ways of φ-ing are implied by licit-makers.

Clauses like ($\Box^+$) and ($\Box^-$) are prominent in recent work on relevant entailment and fine-grained content. Van Fraassen showed that P (tautologically) entails Q iff

each of P's truthmakers implies a truthmaker for Q.[71]

P includes Q ($P\sqsupseteq Q$), let us say, iff

each of Q's truthmakers is implied by a truthmaker for P.[72]

P inclusively entails Q ($P\geq Q$) iff P both entails Q and includes it. Inclusive entailment will be called "intailment."

Inclusive entailment—intailment— is the relation that **s** must bear to φ if **s** is to strictly verify $\Box\varphi$. To put it another way, **s** makes an option φ obligatory if it intails that option. (To φ is *part of what it is* to behave yourself, as opposed merely to being entailed by good behavior.) This enables us to characterize $\Box\varphi$'s strict truth- and falsemakers more simply as follows:

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[70]$\Box\varphi$ isn't strictly falsified by **s** unless **s** strongly permits $\neg\varphi$. Should $\neg\varphi$ be only weakly permitted, $\Box\varphi$ is not strictly true, but not strictly false either,

[71]Proved in **?**. Tautological entailment is a well known form of relevant entailment; Belnap's 4-valued semantics superseded van Fraassen's.

[72]For inclusion see **?** and **?**. Yablo sometimes requires implication as well for inclusion; we drop this further requirement, as it comes for free with entailment.

$[\Box^+]$ $\mathbf{s} \vDash^+ \Box\varphi$ iff $\mathbf{s} \geq \|\varphi\|^+$ $\qquad$ ($\mathbf{s}$ intails $\|\varphi\|^+$)

$[\Box^-]$ $\mathbf{s} \vDash^- \Box\varphi$ iff $\mathbf{s} \sqsupseteq \|\varphi\|^-$ $\qquad$ ($\mathbf{s}$ includes $\|\varphi\|^-$)

$[\Diamond^+]$ $\mathbf{s} \vDash^+ \Diamond\varphi$ iff $\mathbf{s} \sqsupseteq \|\varphi\|^+$ $\qquad$ ($\mathbf{s}$ includes $\|\varphi\|^+$)

$[\Diamond^-]$ $\mathbf{s} \vDash^- \Diamond\varphi$ iff $\mathbf{s} \geq \|\varphi\|^-$ $\qquad$ ($\mathbf{s}$ intails $\|\varphi\|^-$)

This clarifies how $\mathbf{s}$ can leave $\Box\varphi$ undefined. There is no reason why $\mathbf{s}$ should have to either intail $\|\varphi\|^+$ or include $\|\varphi\|^-$. Suppose for instance that $\mathbf{s}$ is $\|A\|^+$ and φ is $A\lor B$. A doesn't intail $A\lor B$ (since it doesn't include it), nor does it include $\neg(A\lor B)$ (since the latter is made true by $\bar{A}\bar{B}$, which is not implied by a truthmaker (or falsemaker) for A.

There is no reason, in fact, why $\mathbf{s}$ should always meet the (even less demanding) condition of *including* one or the other of $\|\varphi\|^+$, $\|\varphi\|^-$. For φ may have truthmakers not implied by members of $\mathbf{s}$, and also falsemakers not implied by members of $\mathbf{s}$. (emphFeed the cat or kick it! has a truthmaker (to do with kicking) that is not implied by any decent licit-maker; it also has a falsemaker (starve the cat and be gentle with it) that is not implied by any decent licit-maker. Decent spheres do not make feeding-or-kicking either obligatory, or starving-and-not kicking permissible.[73]

16 GRADES OF VERIFICATION (OPTION 4)

Now we begin to throw some additional grades of verification into the mix. The hope eventually is to link these up with permissive update rules via principles like the following: top-quality, grade A verification goes with permissive update rule $\uparrow$, since $\uparrow_A$ is the rule to pick if we want to transform $\mathbf{s}$ into a sphere that A-ly verifies $\Diamond\varphi$. Let's first remind ourselves how the verification relation already on the table works.

STRICT VERIFICATION ($\vDash^+$)

($\Box^+$) $\mathbf{s} \vDash^+ \Box\varphi$ iff each $p \in \|\varphi\|^+$ is implied by an $s \in \mathbf{s}$, and each $s \in \mathbf{s}$ implies a $p \in \|\varphi\|^+$

($\Box^-$) $\mathbf{s} \vDash^- \Box\varphi$ iff each $p \in \|\varphi\|^-$ is implied by an $s \in \mathbf{s}$.

($\Diamond^+$) $\mathbf{s} \vDash^+ \Diamond\varphi$ iff each $p \in \|\varphi\|^+$ is implied by an $s \in \mathbf{s}$

($\Diamond^-$) $\mathbf{s} \vDash^- \Diamond\varphi$ iff each $p \in \|\varphi\|^-$ is implied by an $s \in \mathbf{s}$, and each $s \in \mathbf{s}$ implies a $p \in \|\varphi\|^-$

Strict verification asks too much in certain cases. Sometimes $\Box\varphi$ expresses that Simon is to φ somehow or other, but in a way or ways as yet unspecified. Some ways of φ-ing fulfill the stated obligation, but others may not.[74] *You must post the letter or burn it* ($\Box(P\lor B)$) comes out true on such a reading. The letter has got to be posted or burned, because it has to be posted. But strict verification fails here; $\Box(P\lor B)$ is *not* strictly verified by $\mathbf{s}$, since burning the letter is not implied by any licit-maker.

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[73] $[\Diamond^+]$ requires each of φ's truthmakers to be *implied* by a licit-maker, for $\Diamond\varphi$ to hold. Later we'll introduce a less demanding rule $[\Diamond_+]$ asking only that each p in $\|\varphi\|^+$ be *consistent* with a licit-maker. $\Diamond$ expresses strong permission if verification is strict ($\vDash^+$) and weak if verification is loose ($\vDash_+$). But it holds for both of these two grades of verification that *each* of φ's truthmakers must be certified by a licit-maker, in order for $\Diamond\varphi$ to be true. The difference is only that certification takes one form (implication) if verification is $\vDash^+$, another (compatibility) if verification is $\vDash^-$. Free choice effects as we're understanding them do not depend on whether the permission is strong or weak.

[74] The idea of quantifying over obligations, viewed as entities in their own right, figures prominently in Moltmann's work, e.g., **?**.

If $\Box(P \lor B)$ is nevertheless to be verified, It will have to be in some less demanding sense. The idea behind *loose* verification ($\vDash_+$) is that $\Box \varphi$ holds iff there are particular ways p_i of φ-ing such that to fulfill the obligation, one needs to φ in one of *those* ways. This will be so if every licit-maker s implies a p_i, regardless of whether each p_i is implied conversely by an s. The preferred, obligation-fulfilling, p_is are the s-implied ones for some s in **s**. All p_is are implied if s is inconsistent, so each s is assumed not to be inconsistent.

Now, if s is consistent, and implies a p_i, then it is consistent with that p_i. Thus at least some ways of φ-ing are weakly permissible, given that **s** $\vDash_+ \Box \varphi$. Homogeneity, to be introduced in section 17), assures us that *every* way of φ-ing is weakly permissible (consistent with a licit-maker). This defines **s** $\vDash_+ \Diamond \varphi$, and so, if duality is to be preserved, also gives the condition under which **s** $\vDash_- \Box \neg \varphi$. Thus we arrive at the following clauses:

LOOSE VERIFICATION ($\vDash_+$)

($\Box_+$) **s** $\vDash_+ \Box \varphi$ iff each $s \epsilon$**s** implies a $p \epsilon \|\varphi\|^+$

($\Box_-$) **s** $\vDash_- \Box \varphi$ iff each $p \epsilon \|\varphi\|^-$ is compatible with an $s \epsilon$**s**

($\Diamond_+$) **s** $\vDash_+ \Diamond \varphi$ iff each $p \epsilon \|\varphi\|^+$ is compatible with an $s \epsilon$**s**

($\Diamond_-$) **s** $\vDash_- \Diamond \varphi$ iff each $s \epsilon$**s** implies a $p \epsilon \|\varphi\|^-$

To see how the two grades of verification relate, let the sphere be $\{A, B\}$. It holds both for strict verification and loose that **s** $\vDash \Box(A \lor B)$ and **s** $\vDash \Diamond A$. But now let a further disjunct C be brought into the picture. **s** $\nvDash^+ \Box(A \lor B \lor C)$, since C is not implied by A or B. But **s** $\vDash_+ \Box(A \lor B \lor C)$, since every s in **s** (each of A, B) implies one of A, B, and C. There is a corresponding difference on the side of permission. $\{A, B\} \nvDash^+ \Diamond(A \lor C)$, since C is not implied either by A or B. But **s** $\vDash_+$ $\Diamond(A \lor C)$, since both disjuncts are compatible with an s in $\{A, B\}$.

17 STRENGTH, DUALITY, AND FREE CHOICE

That *each* of φ's truthmakers must be implied by (consistent with) a licit-maker, for $\Diamond \varphi$ to be strictly (loosely) verified, has an important consequence. Consider inferences like the following:[75]

(18)　　You may eat an apple or an orange

　　　　⤳ You may eat an apple *and* you may eat an orange.

The strict semantics validates (18), since for $\Diamond(A \lor O)$ to be strictly verified by **s**, each of A, O — each of $A \lor O$'s truthmakers — must be implied by members of **s**, whence $\Diamond A$ and $\Diamond O$ are strictly verified by **s**. The loose semantics yields (18), since each of A, O must be consistent with licit-makers for **s** to loosely verify $\Diamond(A \lor O)$; and that is all it takes for $\Diamond A$ and $\Diamond O$ to be loosely verified.

Trouble does seem to arise, however, with "mixed" disjunctions, in which one disjunct is permissible and the other not. Consider the permissive analogue (19) of Ross's *You must post the letter or burn it*:

(19)　　You may post the letter or burn it ($\Diamond(P \lor B)$)

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[75]An early reference is **?**; **?**, and **?** are important later discussions.

We certainly don't want to count this true, since you are not permitted to burn it. But there is a problem as well in counting it false. (19)'s negation

(20)   It is not the case that you may post the letter or burn it ($\neg\Diamond(P\lor B)$)

implies by duality that

(21)   You must not post the letter or burn it ($\Box\neg(P\lor B)$);

which means, by De Morgan's Law, that

(22)   You must neither post the letter nor burn it ($\Box(\neg P\land\neg B)$).

This last cannot be accepted, though, for while burning the letter is forbidden, posting it is perfectly OK. Where does the argument from (20) to (22) go wrong? Our suggestion is that (19) is despite appearances not really false, making (20) is not really true.

Certainly (19) is apt to *seem* false, on account of falsely implying that you may burn the letter. But all that really follows from a valid (truth-preserving) inference leading from a suspicious-looking premise to a false conclusion is that the premise *fails to be true*. And that is the situation here. $\Diamond(P\lor B)$ is left unevaluated by **s**—which let's take to be $\{P\bar{B}, \bar{P}\bar{B}\}$, as posting is only permitted this time, not commanded—whether $\vDash$ is strict or loose. $\Diamond(P\lor B)$ fails to be

(ia) strictly verified, since $P\lor B$'s truthmaker B is not implied by any licit-maker;

(ib) strictly falsified, since no licit-maker implies $P\lor B$'s sole falsemaker $\bar{P}\bar{B}$;

(iia) loosely verified, since $P\lor B$'s truthmaker B is not consistent with any licit-maker;

(iib) loosely falsified, since not all licit-makers imply $P\lor B$'s sole falsemaker $\bar{P}\bar{B}$.

How are we to understand the truth-value gaps here? They do not seem to signal presupposition failure. What would the presupposition be? That φ is either permitted or forbidden? This is not something we would likely assume, whether the modals are read strongly, in terms of $\vDash^+$, or weakly, in keeping with $\vDash_+$. Parting your hair on the right is neither strongly permitted nor strongly ruled out. Posting the letter or burning it is neither weakly permitted on that reading (**s** $\nvDash_+ \Diamond(P\lor B)$) nor weakly forbidden (**s** $\nvDash_+ \Box(\neg P\land\neg B)$).

Luckily for all involved, there is an alternative to presupposition failure that appears to give us just what we need.[76] **?** argued that *homogeneity effects* in sentences like the following give rise to truth-value gaps of a non-presuppositional variety.

(23)   The boys went to the park.

(24)   The boys didn't go to the park.

There seems a gap between these two sentences, though they are related syntactically as negation to negatum. Neither is true if half the boys went to the park, and the rest went home. But neither sentence presupposes (by the usual tests) that either all the boys went to the park or none did.

Similarly, we have seen, not all spheres of permission make either of these sentences true (assuming you are allowed to post it but not burn it):

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[76]See also **?**, to which we're indebted.

(25)   You are allowed to post the letter or burn it.

(26)   You are not allowed to post the letter or burn it.

The analogy with (23) and (24) comes out when we note that (25) and (26) correspond (taking the verification loosely) to the following, where "my options" are dancing and stealing:

(27)   My options are consistent with licit-makers (at least one)

(28)   My options are not consistent with licit-makers (not even one),

and require at least this much if we take the verification strongly. $\Diamond\varphi$ is verified, however we take the verification, only if each of φ's truthmakers (P, B) is consistent with a licit-maker, and its negation $\neg\Diamond\varphi$ holds only if none are consistent with licit-makers. Truth-value gaps arise if some of φ's truthmakers are permitted and others not. Homogeneity thus points the way to a unified treatment of free-choice effects and the strong/weak distinction which respects duality.[77]

18 HARMONY OF THE SPHERES

Madge in declaring that $\Diamond\varphi$ may be proposing to bring about a sphere $\mathbf{s}'$ such that $\mathbf{s}' \vDash^+ \Diamond\varphi$. But she might intend only that $\mathbf{s}' \vDash_+ \Diamond\varphi$. Presumably she will not want to make the same *kind* of declaration in both cases. To expect different results based on the very same course of action makes no sense.

And yet Madge has (as we have set things up so far) just the one sentence $\Diamond\varphi$ available to her. Pending some further grammatical innovation, there will have to be two different speech acts performable with that single sentence; compare the way we use the use the same sentence S both assertively, and to guess that S in a forced-choice scenario. Ideally Madge will be able to indicate by the way in which she declares that $\Diamond\varphi$ which of the two outcomes (strong or weak verification of $\Diamond\varphi$) she is seeking to bring about.

From the definition of $\vDash^+$, we see that the stricter sort of declaration aims to produce a sphere whose members imply, between them, each and every way of φ-ing, so that all ways of φ-ing comes out strongly permitted. To have a word for strictly declaring that $\Diamond\varphi$, let's say that Madge in this case is inviting Simon to φ. When Simon is invited to take Tuesday or Wednesday off, it is explicitly envisaged that he will do it the Tuesday way, or the Wednesday way, as he chooses. Both options are strongly permitted.

Declaring in that same spirit that $\Box\varphi$—pushing for a sphere that strictly verifies $\Box\varphi$— will for similar reasons be called *demanding* φ. A sphere $\mathbf{s}$strictly verifies $\Box\varphi$ if its members imply between them each way of φ-ing, and each way of φ-ing is implied by a member of $\mathbf{s}$. "Demand" is the natural term here, since if Madge demands of Simon that he take Tuesday or Wednesday off, then both options are strongly permissible, and nothing is permissible that does not involve one or the other of those options.

Declaring in a looser spirit that $\Diamond\varphi$ is angling, we see from the definition of $\vDash_+$, for a sphere all of whose members are *consistent*, for each way p of φ-ing, with φ-ing in that way. This is the

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[77]The suggested truthmakers for modal claims are a different sort of animal from the truthmakers for non-modal claims. Ordinary truthmakers *s* are valuations or sets of literals. Deontic truthmakers $\mathbf{s}$ are sets of ordinary truthmakers. What are the truthmakers supposed to be of descriptive-deontic hybrids like $P \wedge \Box Q$? Presumably some kind of *amalgam* $s\mathbf{s}$ of ordinary truthmakers and sets of ordinary truthmakers. A full-fledged truthmaker semantics will have to address these questions. The Appendix takes some first steps.

speech act to choose if one is seeking to *weakly* permit each way *p* of φ-ing. Whether or not *p* is specifically approved, the conduct that is approved never implies *p*-avoidance. Not implying *p*-avoidance, for any way *p* of φ-ing, lines up more or less with the ordinary notion of *allowing* someone to φ.[78]

Let us use the word *allowing*, then, for declaring in the second spirit that $\diamond\varphi$. And let's say that someone *requires* φ if they declare in the second spirit (with the idea of transforming **s** into a sphere that loosely verifies the sentence uttered) that $\square\varphi$. If φ is *You do this or that*, then $\square\varphi$ would normally be used to issue a command that is two-ways satisfiable. Doing this is one way of carrying out Madge's instructions, doing that is another.

Looking back at our update rules, REQUIREMENT REDUCTION produces a sphere that loosely verifies $\diamond\varphi$, while POSSIBILITY-ADDING produces a sphere that strictly verifies $\diamond\varphi$. A kind of harmony is thus emerging between deontic speech acts, update rules, and grades of verification. If the initial permissibility sphere is **s**, then *inviting* φ triggers an update operation $\mathbf{s} \uparrow_{PA} \mathbf{p}$ ($\mathbf{p} = \|\varphi\|^{+}$) whose output $\mathbf{s}'$ makes $\diamond\varphi$ *strictly* true. Whereas *allowing* φ when the sphere is **s** triggers an operation $\mathbf{s} \uparrow_{RR} \mathbf{p}$ whose output $\mathbf{s}'$ makes $\diamond\varphi$ *loosely* true.

Suppose as claimed earlier that the possibility-adding update rule for $\diamond\varphi$ yield a sphere that strictly verifies ($\vDash^{+}$) $\diamond\varphi$, while $\diamond\varphi$ is only loosely verified ($\vDash_{+}$) by spheres obtained by our other rule, the requirement-reduction rule. Then, given that strict (loose) verification goes with strong (weak) permission, possibility-adding update is appropriate to strongly permissive acts (invitations) and update by requirement-reduction is appropriate to weak permissions (allowings). Ideally one would like to make this the model everywhere. A three-sided BRIDGE PRINCIPLE should be sought linking

    (i) types of deontic act with

    (ii) types of update rule with

    (iii) hopes and plans for the new sphere or spheres.

Of course we are not there yet. Our discussion has been tentative and exploratory. Let us indicate some further lines of research before wrapping up.

19 NEXT STEPS

Madge *requires* φ, we said, if she declares that $\square\varphi$ with the idea of transforming **s** into a sphere that loosely verifies $\square\varphi$. If φ is *You do A or B*, then requiring φ is commanding it in a manner that is two-ways satisfiable. *A* gives one way of carrying out Madge's instructions, *B* another.

But we can also imagine a use of $\square(A \lor B)$ which presents *A* and *B* as a pair of *one*-way satisfiable requirements. It is the audience's job to figure out which of the two requirements is actually in force. Madge tells Simon that he must either post the letter or hand-deliver it, expecting that he will put this command together with other facts (e.g., it may turn out that civilians are not to deliver stamped letters by hand) to conclude that he must post it full stop. Corresponding to this looser form of command, there ought to be a looser form of verification such that commands of the new form aim at a sphere thusly verifying $\square\varphi$. Here is how the new form of verification might go:

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[78]Or, in an older terminology, "suffering" them to φ, as in, " Suffer the little children to come unto me, and forbid them not."

($_+\Box$) $\mathbf{s}$ $_+\vDash$ $\Box\varphi$ iff each $s\epsilon\mathbf{s}$ implies the disjunction of all p's in$\|\varphi\|^+$, i.e., s implies φ.

($_-\Box$) $\mathbf{s}$ $_-\vDash$ $\Box\varphi$ iff some $p\epsilon\|\varphi\|^-$ is compatible with an $s\epsilon\mathbf{s}$.

($_+\Diamond$) $\mathbf{s}$ $_+\vDash$ $\Diamond\varphi$ iff some $p\epsilon\|\varphi\|^+$ is compatible with an $s\epsilon\mathbf{s}$

($_-\Diamond$) $\mathbf{s}$ $_-\vDash$ $\Diamond\varphi$ iff each $s\epsilon\mathbf{s}$ implies the disjunction of all p's in$\|\varphi\|^-$, i.e., s implies $\neg\varphi$.

But although this makes sense in principle, what kind of directive *intuitively* speaking aims to produce a sphere that super-loosely verifies $\Box\varphi$?

Suppose that Madge tells Simon to come to work tomorrow (W). Does she tell him to either (i) come on a missile that destroys the workplace ($W\wedge M$), or (ii) come some other way ($W\wedge\neg M$)? Arriving by missile is not consistent with any licit-maker, since Simon is not permitted to destroy his place of work. Assuming licit-makers are themselves consistent, $W\wedge M$ is not *implied* by any licit-maker either. But then, by the definition of $\vDash_+$, $\Box(W\wedge M \vee W\wedge\neg M)$ is not loosely verified by $\mathbf{s}'$. $\mathbf{s}'$ does however verify $\Box(W\wedge M \vee W\wedge\neg M)$ *super*-loosely. Each licit-maker implies $W\wedge M \vee W\wedge\neg M$, just by virtue of implying W. The language of forbiddenness comes naturally here. If not coming to work is forbidden, then so is not-coming-either-by-missile-or-non-missile. "Forbidding $\neg\varphi$" can thus serve as the act that aims for a sphere that super-loosely verifies $\Box\varphi$.

Is there a correspondingly looser form of permission than allowing? φ-allowers seek a sphere that does not preclude *any* way p of φ-ing. (An $\mathbf{s}'$, in other words, such that $\mathbf{s}' \vDash_+ \Diamond\varphi$.) But we might be satisfied with a sphere whereby it is φ itself that is not forbidden, as opposed to particular ways p of φ-ing. (An $\mathbf{s}'$, that is, bearing $_+\vDash$ to $\Diamond\varphi$.) Trouble is, there are lots of successor-spheres meeting the condition of not forbidding φ, corresponding to the many sets of ways p of φ-ing one could unforbid. The act we're contemplating is best seen as identifying a certain *range* of successor spheres (say, the ones obtained by stripping one or more licit-makers in $\mathbf{s}$ of elements inconsistent with φ) as the pool from which $\mathbf{s}'$ shall be drawn.[79] The act lacks a determinate update effect — *unless* we want to reconceive deontic contexts as sets of spheres, an idea with much to be said for it, though it will not be pursued in this paper.

Why would Madge be drawn to this sort of speech act (call it, for lack of a better word, *unforbidding φ*)? Just as forbidding $\neg\varphi$ gives her a way to ban $\overline{\varphi}$-ing, without specifying how one needs to φ, unforbidding φ gives her a way to lift the ban on φing, without specifying how it is OK to φ. Madge will unforbid φ when she wants to declare it possible to φ permissibly, leaving for later the question of how to realize the possibility. Perhaps a further decision is required on her part, or it may be further investigation that's needed.

Either way, the mere fact of unforbiddenness does not give Simon license to go ahead and φ. He would have to φ in some particular way after all, corresponding to some $p\epsilon\|\varphi\|^+$; and that p might turn out not to be one of the good ones. The point of unforbidding φ is that it indicates to Simon that φ-ing appropriately is going to be OK, and encourages him to except further news on this score, or at least to be wary of φing inappropriately. The act of ban-lifting, even if it lacks a single-valued *update* rule, may still be useful on account of its *uptake* rule, e.g., accept the verdicts that all the spheres agree on, and await further instructions.

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[79][**?**] makes a similar suggestion in the context of epistemic modality. $\Diamond\varphi$ is verified by the pool if by every sphere in the pool.

Looking up now instead of down, to invite φ is to propose a sphere that strictly verifies $\Diamond\varphi$. From the definition of $\vDash^+$, to invite $A\wedge B$ is also to invite A.[80] Sometimes though, as discussed in section 7, we want to permit $A\wedge B$ only as a package, without permitting either taken alone. You may give both the children a cookie, in the unlikely event of your having two cookies. But the answer to *May I give Al a cookie?* is *no*, for then Val most likely winds up with nothing.

What we are after in these sorts of cases is a sphere that verifies $\Diamond(P\wedge Q)$ but not $\Diamond P$. This will not be possible if $\vDash$ is strict ($\vDash^+$), since strict verification is closed under inclusion. But what if $\vDash$ is super-strict?

SUPER-STRICT VERIFICATION ($^+\!\vDash$)

($^+\square$) **s** $^+\!\vDash \square\varphi$ iff each $p\epsilon\|\varphi\|^+$ is identical to an $s\epsilon\mathbf{s}$, and each $s\epsilon\mathbf{s}$ implies a $p\epsilon\|\varphi\|^+$

($^-\square$) **s** $^-\!\vDash \square\varphi$ iff each $p\epsilon\|\varphi\|^-$ is identical to an $s\epsilon\mathbf{s}$.

($^+\Diamond$) **s** $^+\!\vDash \Diamond\varphi$ iff each $p\epsilon\|\varphi\|^+$ is identical to an $s\epsilon\mathbf{s}$

($^-\Diamond$) **s** $^-\!\vDash \Diamond\varphi$ iff each $p\epsilon\|\varphi\|^-$ is identical to an $s\epsilon\mathbf{s}$, and each $s\epsilon\mathbf{s}$ implies a $p\epsilon\|\varphi\|^-$

To see the difference, let $\mathbf{s} = \{AB\}$. **s** strictly verifies $\Diamond A$, since a truthmaker for A is implied by a member of **s**. But it does not verify $\Diamond A$ super-strictly, since no truthmaker for A is *identical* to a licit-maker in **s**. The only available licit-maker, AB, is not a truthmaker for A on account of taking a stand, irrelevantly, on B. More generally if **s**'s members properly imply truthmakers for a part ψ of φ, without ψ's truthmakers belonging to **s** in their own right, we get the desired result that $\Diamond\varphi$ is super-strictly verified while $\Diamond\psi$ is not so verified.

The corresponding speech act, package-deal or in-toto inviting, aims at a sphere that super-strictly verifies $\Diamond\varphi$, and so verifies $\Diamond\psi$ ($\psi\leq\varphi$) only if truthmakers for ψ belong to **s** in addition to truthmakers for φ. Trickle-down inviting (what we have called "inviting") aims at an **s** that strictly verifies $\Diamond\varphi$, given which it also strictly verifies $\Diamond\psi$ for each $\psi\leq\varphi$. Importantly it could be the same sphere **s** in both cases. But the package-deal inviter wants **s** to be read super-strictly by Simon; he should not considered himself licensed by **s** to φ **s** verifies $\Diamond\varphi$ only strictly. Package-deal-command contrasts in a similar way with trickle-down command. The associated update rules in both cases differ, not only in how they affect the sphere, but how they affect Simon's understanding of what the sphere is telling him. He should consider himself

licensed to φ by a package-deal permission iff the resulting sphere super-strictly verifies $\Diamond\varphi$.

ordered to φ by a package-deal command iff the resulting sphere super-strictly verifies $\square\varphi$.

licensed to φ by a trickle-down permission iff the resulting sphere strictly verifies $\square\varphi$. $\Diamond\varphi$.

ordered to φ by a trickle-down permission iff the resulting sphere strictly verifies $\square\varphi$.

One last thought before concluding. We have spoken just now of the spheres "resulting" from package-deal and trickle-down commands. But no update rules for command have been given. There are a number of options here; we consider first an operation complementary to $\uparrow_{PA} =$ POSSIBILITY-ADDING.

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[80]Assuming that $\|A\wedge B\|^+$ includes $\|A\|^+$. For then a sphere including the first—a sphere strictly verifying $\Diamond(A\wedge B)$— is bound to include the second—thus strictly verifying $\Diamond A$. And so to call for a $\square(A\wedge B)$-verifying sphere (in the $\vDash^+$ sense) is to call inter alia for a sphere that verifies $\square B$.

Suppose our starting sphere **s** is made up of licit-makers $s_1,...., s_m$, and that the newly commanded φ has as its truthmakers $p_1,...., p_n$. The new licit-makers $s_1', s_2', ...$ should resemble the old ones except in holding only in φ-worlds, that is, worlds where some p_k holds. The obvious way to arrange this is to fuse s_1 successively with each p_k, then s_2 successively with each p_k, and so on through s_m. **s′** in other words should be

POSSIBILITY-ENRICHMENT **s↓p** = the set of all $s_j p_k$ (where $s_j p_k$ is the fusion of s_j with p_k).[81]

Here we are using a downward-pointing arrow for the operation on **s** and **p** corresponding to the issuing of a command, as before upward-pointing arrows were used for the operation corresponding to a permission; that is,

s + $\square\varphi$ = **s**⊞$\|\varphi\|$ = **s↓p**, where **p** is the set of φ's truthmakers.

s + $\diamond\varphi$ = **s**⊕$\|\varphi\|$ = **s↑p**, where **p** is the set of φ's truthmakers.

The reader may check that POSSIBILITY-ENRICHMENT produces a sphere that strictly verifies $\square\varphi$, as POSSIBILITY-ADDING produced one that strictly verified $\diamond\varphi$. This suggests POSSIBILITY-ENRICHMENT is the rule that is triggered by demanding that φ (demanding, recall, aims at strict verification of $\square\varphi$). The question that remains is, what is the update rule that goes with requiring φ, the act that aims at loose verification of $\square\varphi$? It will again have to be an indeterministic rule (like the rule for unforbidding), since there are lots of ways of enriching the licit-makers in **s** so that each implies a truthmaker for φ.

20 CONCLUSION

The framework of truthmaker semantics was seen to be useful for understanding permissive updates. We defined two update procedures in that framework, REQUIREMENT-REDUCTION and POSSIBILITY-ADDING. After discussing the connections between permissive update, belief revision, and the semantics of exceptives, we discussed how a truth-maker account of permissive updates might be integrated into an overall semantic/pragmatic story of the language of permission. We speculated finally about the possibility of bridge principles that would correlate deontic acts and update rules with the use envisaged for the resulting spheres.

APPENDIX: HYBRID TRUTHMAKER SEMANTICS

$\mathscr{L}$ is the language of propositional modal logic, except that modal operators cannot occur within the scope of modal operators:

$$A|B|C\dots$$
$$\neg\varphi|\varphi\wedge\psi|\varphi\vee\psi$$
$$\diamond\varphi \text{ if } \varphi \text{ is factual (free of modal operators)}$$

Other connectives are defined in the usual way, and $\square\varphi$ is short for $\neg\diamond\neg\varphi$. A formula is factual if it contains no boxes or diamonds, and deontic if either (i) of the form $\square\varphi$ or $\diamond\varphi$, or (ii) a truth-functional combination of deontic formulae. χ is *simple* if it is factual or deontic. The only compound formulas we consider are disjunctions/conjunctions of simple formulae.

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[81]If no $s_j p_k$ is consistent, φ will need to be permitted before it is commanded. **s** + $\diamond\varphi$ is in that case (**s↑p**)**↓p**. See **?**, **?**, for discussion. Let's assume that some s_j is in cases of interest consistent with some p_k.

TRUTHMAKERS are ordered pairs $<s, \mathbf{s}>$ of ordinary truthmakers s and deontic truthmakers $\mathbf{s}$. (An ordinary truthmaker is a valuation, or set of literals; a deontic truthmaker $\mathbf{s}$ is a set of ordinary truthmakers.) Likewise FALSEMAKERS. We write $\vDash^+$ and $\vDash^-$ for the TRUTHMAKING and FALSEMAKING relations. If φ is factual, then $\|\varphi\|^+$ and $\|\varphi\|^-$ are φ's ordinary truthmakers and false makers. st, as usual, is the fusion of s and t.[82]

$$<s,\mathbf{s}> \vDash^+ A \text{ iff: } s = A$$
$$<s,\mathbf{s}> \vDash^- A \text{ iff: } s = \overline{A}$$
$$<s,\mathbf{s}> \vDash^+ \neg\varphi \text{ iff: } <s,\mathbf{s}> \vDash^- \varphi$$
$$<s,\mathbf{s}> \vDash^- \neg\varphi \text{ iff: } <s,\mathbf{s}> \vDash^+ \varphi$$
$$<s,\mathbf{s}> \vDash^+ \varphi \vee \psi \text{ iff: } <s,\mathbf{s}> \vDash^+ \varphi \text{ or } <s,\mathbf{s}> \vDash^+ \psi$$
$$<s,\mathbf{s}> \vDash^- \varphi \wedge \psi \text{ iff: } <s,\mathbf{s}> \vDash^- \varphi \text{ or } <s,\mathbf{s}> \vDash^- \psi$$
$$<s,\mathbf{s}> \vDash^+ \varphi \wedge \psi \text{ iff: } s = pq, \dots \text{ where } <p,\mathbf{s}> \vDash^+ \varphi \text{ and } <q,\mathbf{s}> \vDash^+ \psi$$
$$<s,\mathbf{s}> \vDash^- \varphi \vee \psi \text{ iff: } s = pq \dots \text{where } <p,\mathbf{s}> \vDash^- \varphi \text{ and } <q,\mathbf{s}> \vDash^- \psi$$
$$<s,\mathbf{s}> \vDash^+ \Diamond\varphi \text{ iff: } \mathbf{s} \text{ includes } \|\varphi\|^+ \ (\mathbf{s} \sqsupseteq \|\varphi\|^+)$$
$$<s,\mathbf{s}> \vDash^- \Diamond\varphi \text{ iff } \mathbf{s} \text{ intails } \|\varphi\|^- \ (\mathbf{s} \geq \|\varphi\|^-)$$

From the clauses for $\Diamond$ and duality, it follows that

$$<s,\mathbf{s}> \vDash^+ \Box\varphi \text{ iff: } \mathbf{s} \text{ intails } \|\varphi\|^+ \ (\mathbf{s} \geq \|\varphi\|^+)$$
$$<s,\mathbf{s}> \vDash^- \Box\varphi \text{ iff } \mathbf{s} \text{ includes } \|\varphi\|^- \ (\mathbf{s} \sqsupseteq \|\varphi\|^-)$$

Example 1. $<A, \{B, C\}> \vDash^+ A \wedge \Diamond B$

because $<A, \{B, C\}> \vDash^+ A$ and $<A, \{B, C\}> \vDash^+ \Diamond B$

because $s = A$ and $\mathbf{s} = \{B, C\}$ includes $\|B\|^+ = \{B\}$

because .... and everything in $\|B\|^+$ is implied by something in $\mathbf{s}$

Example 2. $<A, \{B, C, D\}> \vDash^+ \Diamond(B \vee C) \wedge \Box(B \vee C \vee D)$

because $<A, \{B, C, D\}> \vDash^+ \Diamond(B \vee C)$ and $<A, \{B, C, D\}> \vDash^+ \Box(B \vee C \vee D)$

because $\{B, C, D\}$ includes $\|B \vee C\|^+ = \{B, C\}$ and intails $\|B \vee C \vee D\|^+$

because (i) everything in $\|B \vee C\|^+$ is implied by something in $\{B, C, D\}$,

(ii) everything in $\{B, C, D\}$ implies something in $\|B \vee C \vee D\|^+$, and

(iii) everything in $\|B \vee C \vee D\|^+$ is implied by something in $\{B, C, D\}$

Example 3. $<A, \{B, C, D\}> \nvDash^+ \Box(B \vee C)$

because $\{B, C, D\}$ does not intail (include and entail) $\|B \vee C\|^+$

because $\{B, C, D\}$ does not entail $\|B \vee C\|^+$

because not everything in $\{B, C, D\}$ implies something in $\|B \vee C\|^+$

because D does not imply either B or C

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[82]The TRUTHMAKERS about to be suggested are exact on the s side, and inexact on the $\mathbf{s}$ side. It may be that TRUTHMAKERS exact on both sides are obtainable by treating spheres as bicameral, with an $\mathbf{s}^+$ consisting of licit-makers and $\mathbf{s}^-$ consisting of illicit-makers. Here we stick to the simpler, unicameral treatment. **?** explore a different way of arranging for exact deontic truthmakers.