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PERMISSIVE UPDATES

1 INTRODUCTION

This paper is about a problem at the border of semantics and pragmatics, the problem of how to interpret new permissions that contradict previous commands [Lewis, 1979]. The problem is semantic in that its treatment requires an account of the semantics of the linguistic expressions used to convey permissions. It is pragmatic in that its treatment requires a story about how the utterances of these commands and permissions alters the conversational context, sometimes in ways that may not be semantically encoded.

Our focus is at the level of CONVERSATIONAL UPDATE: we are interested in how conversational participants update their understanding of what is permissible as new permissions are granted (we'll call these PERMISSIVE UPDATES). Lewis [1979] showed that the problem is non-trivial; in particular, it cannot be handled by any simple variation on the standard account of the conversational updates associated with assertion [e.g., Stalnaker, 1978].

In the main part of the paper, sections 2 to 5, we argue that the framework of truthmakers as developed by Yablo [2014] and Fine [2017c] provides the right resources for capturing permissive updates.[1] It allows us, in particular, to capture inference patterns such as the following:

(1) Starting context: You must work every day of the week.
New permission: You can take Tuesday off.
Resulting context: You must work the other days of the weeks.

(2) Starting context: You must interview all the students in School A or in School B.
New permission: You do not need to interview adults.
Resulting context: You must interview all the non-adults in School A or School B.

Standard stories such as those based on AGM [Alchourrón et al., 1985] or (the nearly equivalent) closest possible worlds approaches do not capture these inferences.[2] (To be precise, while they do not invalidate them, they also do not validate them.)

[1] This framework bears important similarities to the inquisitive semantics tradition [e.g., Groenendijk and Roelofsen, 2009] as well as the older literature on alternatives [Rooth, 1985, e.g.].

[2] For the connection between AGM and the closest world approach, see Grove [1988].

The next, shorter, bit of the paper, sections 6 to 7, discusses the connections between permissives updates and two other phenomena: belief revision and exceptive-expressions.

The remainder of the paper, sections 8 to 13 places the truthmaker account of permissive updates in the overall semantic and pragmatic picture of permissions. Here we discuss three different options for relating the semantics of permission statements to the pragmatics of permissive updates. The first option is to build our proposed update functions (the ones associated with permission-grantings) into the semantics of permission statements. We argue this approach is unworkable. The second option is to have a standard truth-conditional semantics of permissive modals and treat the truthmaker update method as pragmatically triggered. This option is theoretically conservative: it maintains an orthodox static semantics and invokes truthmakers only as part of a pragmatic update procedure. An intermediate option between is to give a truth-maker semantics for permission statements (and probably for all expression in our language), but to treat the update mechanism for permissions separately, rather than as falling automatically out of the semantics. We explore this last option in some detail, examining its benefits and difficulties.

2 THE PROBLEM OF PERMISSIVE UPDATES

We are interested here in *conversational dynamics*: how our statements affect what is taken for granted in a conversation. We are working within the framework described by Lewis [1983]. Conversation is viewed as a game where speech acts are moves that change the current state of the game (the *score*).

A story about assertions and common ground due to Stalnaker [1970, 1978] will be a useful starting point. On this story, we model a conversation as a process by which participants make *assertions* in order to exchange information by altering the *common ground*. Simplifying a bit, we can model both assertions and common grounds as sets of worlds; such sets in Stalnaker-ese are propositions.[3] Assertions are associated with the set of worlds in which they are true, while the common ground is associated with the set of worlds in which all the commonly held assumptions of the conversational participants hold.

This model, highly idealized, yields a simple account of the effect an assertion has on the common ground. If the assertion is accepted, the new common ground is simply the old common ground *intersected* with the proposition asserted, as in figure 1. In other words, the effect of an assertion is to remove all those worlds from the common ground that are incompatible with it.

[3]Officially the common ground in Stalnaker is a set of propositions, while the context set is the set of worlds where all the propositions are true. A set of propositions is informationally richer, clearly, than a set of worlds, but the extra information is mostly ignored by Stalnaker. A different sort of second-order information will be drawn on by the update procedures sketched below.

No doubt there is something right about this model—and it remains at the core even of many revisionary accounts of conversational update.[4] However, other types of conversational update are not naturally explicable in these terms. Asking questions, for instance, is not usefully modelled as adding propositional information, or dropping the worlds where that information fails. Similarly, as we shall see, in the deontic dimension of conversation, the granting of permissions cannot be treated as piling on information. The set of permissible worlds grows, after all, when permission is granted to do something once forbidden; you can't expand a set by dropping worlds.[5]

The problem we will focus on is essentially that identified by David Lewis in his 'A problem about permission' [1979]. Nearly forty years later, the problem looks much the same as it did to Lewis, though we will present it in more contemporary terms (following also Yablo [2009], which this paper takes off from).

Consider a situation in which one person, whom Lewis calls the Master, issues commands to another, whom Lewis calls the Slave. If the Slave is obedient, then he will do whatever the Master says. At any stage in the conversation, we can model what the Slave is allowed to do by a set of worlds where he is obeying the Master's orders, which we will call the permissibility sphere.[6]

The problem of permission is the problem of modeling how the permissibility sphere changes as commands and permissions are given. With respect to commands, there does not seem to be much of a problem. A command behaves like a Stalnakerian assertion. Consider the following simple example. The Master orders the Slave to work every day of the week. The propositional content of the command is the set of worlds where the

[4]See Rothschild and Yalcin [2017, 2016] for a discussion of how this model relates to dynamic theories.

[5]Likewise Stalnaker's context set—roughly the worlds we might be in as far as shared presuppositions go—when a might-claim is accepted. A larger set cannot be obtained by intersection from a smaller one.

[6]It will not change things much to replace sets of worlds with sets of precise courses of action for the Slave, or something similar: though more radical departures might change things [see Fine, 2014, 2018a,b, for one proposal].

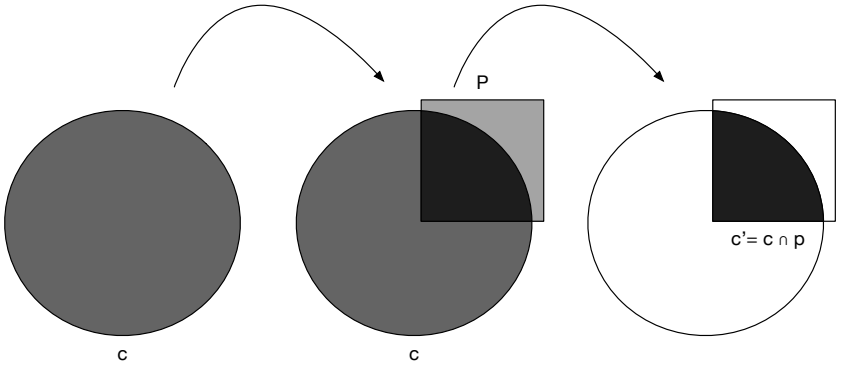

Figure 1: Stalnaker's picture

Slave works every day of the week. The effect of the command on the permissibility sphere is to remove all those worlds in which the Slave does not work every day of the week. This is, of course, just intersection.

Permission is more complex. It will be useful to distinguish some different ways in which permissions can be expressed. At one end of the spectrum are purely *reportative* cases. Here what was antecedently permitted is described as possessing that status. The person expressing the permission, e.g., by saying 'You are permitted to eat', need not be one with normative authority. They can be merely stating what *is* allowed in a given situation, without intending by their speech act to affect what is allowed. These kinds of statements are fairly well handled by a standard semantics of assertion combined with the standard truth-conditional treatment of deontic modals.[7]

We will focus instead on cases where permission statements are used to grant permissions, not just report on them. There are two relevant sub-cases here: One is when the permission granted does not contradict any previous commands (implicit or explicit), but also is not obviously redundant, i.e. what is permitted was not clearly licit prior to the permission. For example, there might be uncertainty about whether shoes are allowed in the room, and this uncertainty can be resolved by someone in charge explicitly permitting shoe wearing. This complex and important case is *not* our primary focus—or Lewis's.[8]

Rather, the case that we (and Lewis) are interested in is permission *granting*: where something clearly forbidden is subsequently allowed by a new permission.

For instance suppose the Master has previously required the Slave to work every day but then permits the Slave to take Wednesdays off. What is the new permissibility sphere after this permission has been given? If the permission is genuinely allowing what was previously forbidden then there can be no overlap between the proposition covering the states where the newly permitted action is taken and the previous permissibility sphere. So, intersection is clearly not an option.

A natural thought is that one might just union in all the worlds where the newly permitted actions occur, as in figure 2. Quick reflection on the structure of the problem shows that this solution will not work. After all, in our simple example, the worlds where Slave does not work Wednesday include worlds where he does not work *any* day. But, surely, granting permission to take Wednesday off is does not make it permissible to take all days off.

Another natural thought is that we should add the *closest* worlds in which the newly permitted actions occur, as in figure 3. There are several challenges to this proposal.

[7]For instance, as in Kratzer [1981].

[8]To address them we would need a nuanced representation of the current state of permissions that distinguishes (at the least) between what is explicitly permitted and what has simply not been forbidden. We

One challenge is that of figuring out what kind of closeness relationship is needed. Structurally speaking, we need not a closeness relationship between worlds, but rather a relationship that given a set of worlds tells you what the closest worlds to that set are.[9] As the permissibility sphere changes which worlds are closer can change in any number of ways on this approach. It would be good to have some sort of handle on what determines closeness.

Surely 'closeness' in whatever sense is typically used in the counterfactuals literature will not work, as Lewis notes. The closest worlds—in that sense—where the Slave works on Wednesday may be ones where he stays at his place of work, but simply doesn't conduct any business there.

Following a suggestion of Lewis, due originally to Stalnaker, we might be able to find the closest worlds by using the relationship of 'comparative permissibility.' The basic idea is that, relative to a given sphere of permissibility, we can order any two worlds outside the sphere in terms of how impermissible they are. Some worlds outside the sphere will naturally be worse violations than others of the norms governing the sphere. On this proposal when a new permission is given, we only let in the least impermissible worlds outside the sphere.[10]

Lewis criticizes this solution on explanatory grounds, but we do not think this is the ultimate problem with it. The problem is, as Yablo [2009] points out, that the solution is not intuitively correct. Having been forbidden to eat meat on environmental grounds, the least impermissible world in which you eat meat is one where you eat guinea pigs (these have a particularly low environmental impact). But a new permission to eat meat intuitively extends to worlds in which you eat other types of meat. (Similarly if you are

do sketch some options in section 11. For some other ideas see [Willer, 2013, Starr, 2016, Fine, 2018a,b].

[9]That is we need the same kind of structure as used by Lewis [1973] and Stalnaker [1968] in their accounts of counterfactuals.

[10]See Stalnaker [2014], chapter 6, for a related proposal.

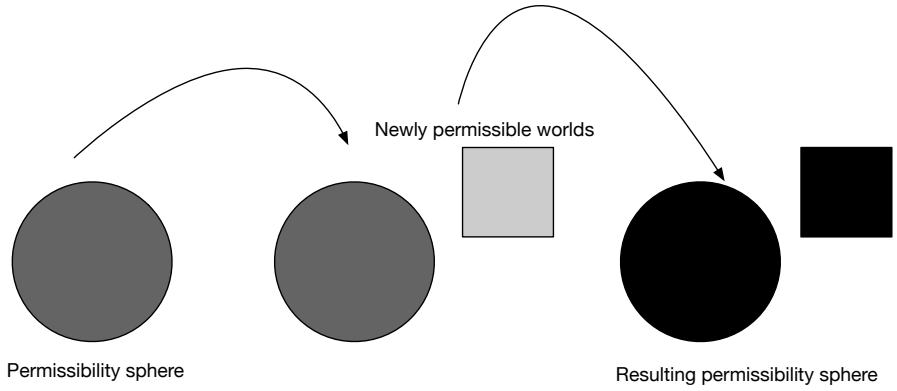

Figure 2: Unioning in all worlds

permitted to drive a car, this does not mean you can only drive for one minute, which would be the least impermissible thing from the perspective of a prohibition on car driving.)

The point that there is not a clear way of defining closeness is, of course, not a devastating one. It could be made just as well with the notion of closeness used in the Lewis/Stalnaker account of counterfactuals.[11] However, we want to suggest that, in this case, closeness gives us *much less* than we want from a theory of permission-adding. The relevant inferences in our example cases, such as (1) and (2), are simple: if you have been required to work every day and you are then given Tuesday off you are a) not required to work on Tuesday, and b) still required to work every other day of the week. The closeness account captures a) but says nothing at all useful about b). This is not good enough.[12]

In addition to being insufficiently explanatory, the closeness account might seem just extensionally incorrect, by virtue of adding in only *P*-worlds when the Master permits that *P*. Consider the following example. First, the Slave is forbidden from eating any chocolates. Later, the Master decides to lighten up a bit, and permits the Slave to eat a dozen chocolates. It's at least plausible that granting this permission also lets the Slave eat just seven chocolates, even though that does not happen at a world where the slave eats a dozen chocolates.

One natural reaction to the problem of permission is to think that the framework of possible worlds is not useful for framing this problem. Another framework for viewing content takes propositions rather than worlds as basic. The permissibility sphere becomes

[11] Lewis [1973] was himself keen to defend a reductive account of closeness, but we do not think he was successful in his reduction. Stalnaker [1968], by contrast, is more interested in the logical features of the account and is willing to take closeness as a context-dependent primitive notion in his theory.

[12] Since the standard theory of belief revision, AGM [Alchourrón et al., 1985], is essentially equivalent to the closeness account [Grove, 1988] it is also too weak.

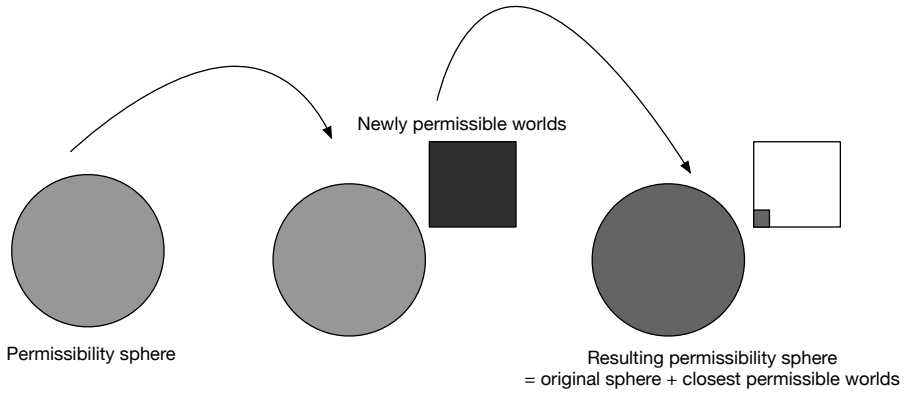

Figure 3: Unioning closest world

a list (or set) of propositions: those that have been commanded.[13] So, rather than a permissibility sphere, we have a list of propositions (a "prescription list") that defines the sphere. When a new permission is granted, we strike from this list all the propositions incompatible with the new permission, or better (since propositions individually compatible with *P* might still rule it out *en masse*), cut the list down to a set of propositions jointly consistent with *P*.

We will call this the *proposition-list* approach. Figure 4 applies it to our work-all-week example. [14]

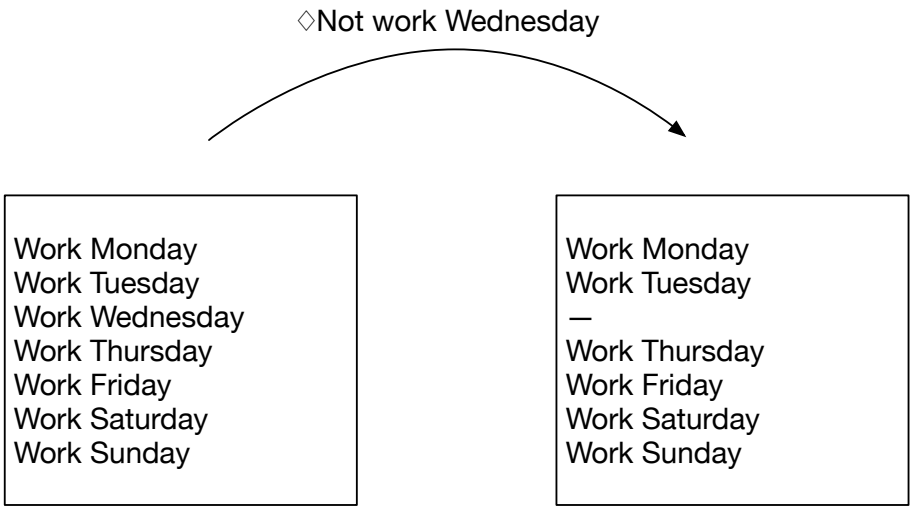

Figure 4: Removing propositions

Like the closeness solution, the proposition-list approach has an explanatory problem. Different ways of drawing up the list will result in different changes to the sphere of permissibility. One response here is to claim that some divisions of the sphere into propositions are more *natural* than others. So the naturalness of certain ways of turning permissibility spheres into lists determines how updates go. The proposition list approach, combined with a story about what propositions are natural, would provide the sought after explanation. Since we make an analogous move in the truthmaker account, claiming priority for some propositions over others, we do not object to the proposition-list approach on explanatory grounds.

The real problem with the proposition-list approach is that it takes requirements to be categorical. In our standard weekday example, the requirements are simply *work Monday*, *work Tuesday*, *work Thursday*, ... But what if, instead, the Slave was required to

[13] Compare Stalnaker's official notion of common ground as a set of propositions conjunctively defining the context set.

[14] A fuller specification of the approach would require a means of specifying how a list of propositions is reduced to make it compatible with a new permission. Here work in the AGM tradition [Alchourrón et al., 1985] tradition is relevant.

work either all odd *or* all even-numbered dates (figure 5). What is the list of propositions that defines the sphere of possibilities? Clearly, *work Monday* cannot be one of them. But then how do we update the sphere, if permission is granted to take Mondays off?

If we treat the propositional list as simply consisting of a single disjunctive requirement, [*work even or odd days*], then our only option is to permit not working at all after Mondays are given off. We could gerrymander a proposition-list to get the right answer. For example, we could use the list [*work Monday if you work any day*, *work odd days except Mondays or work even days except Monday*]. However, stipulating that the proposition list has this structure is blatantly ad hoc. A similar point can be made about example (2) above.

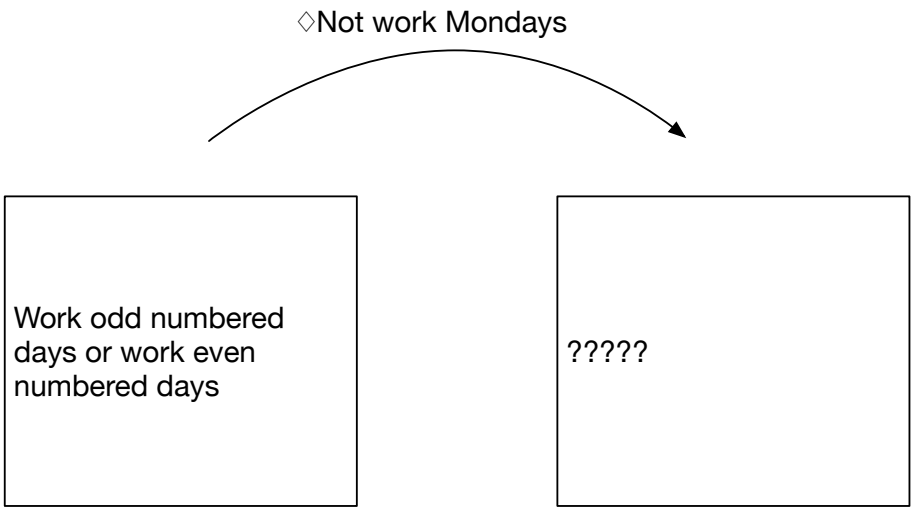

Figure 5: Problems with proposition lists

3 TRUTHMAKERS

Both Yablo [2009] and Fine [2014] have argued that the apparatus of truthmakers is helpful in modeling permissive updates. We begin by briefly introducing the notion of (semantic rather than metaphysical) truthmakers.[15] The guiding idea is that a sentence is associated not just with the worlds it is true in, but also the *ways* in which it could be true (or false). We will follow Yablo in taking ways in which things can be true or false (truthmakers or falsemakers) to be set-of-world propositions themselves.[16]

Not all such propositions are truthmakers, however. We will begin by exploring a simple model of which propositions are truthmakers for which sentences in a language.

[15]Fine [2017c], chapter 4 of Yablo [2014]. Rumfitt [2015] pursues similar ideas. Antecedents of this work include van Fraassen [1969] and Gemes [1997, 2007, 2006].

[16]Yablo also has fine-grained propositions, sets of ways rather than worlds. Fine argues for a framework that replaces possible worlds with truthmakers as the basic semantic primitive and defines propositions directly in terms of truthmakers (Fine [forthcoming]).

The model we will start with assumes a language with the syntactic and semantic structure of propositional logic.[17]

The term 'truthmaker' is used in two related senses: There's first the monadic notion of being the kind of thing that can play the role of making sentences true.[18] There is then relational notion of being a truthmaker *for* a particular sentence. In our simple model, the propositions which can play the role of making sentences true (or false) are those which can be expressed as conjunctions of literals (negated and unnegated atoms). Such propositions correspond in an obvious way to partial valuations of the language. A special notation will be used in which truthmakers are encoded as concatenations of the relevant literals. Concatenation is expressing here a kind of commutative conjunction, so order is not relevant; AB is a different string than BA, but they express the same proposition. The concatenation will go proxy for the proposition where this doesn't cause confusion. A truthmaker $A\bar{B}$ is said to be *composed* of A and $\bar{B}$, and A and $\bar{B}$ are parts of the truthmaker. The possible truthmakers composed of atomic sentences A, B are A, B, $\bar{A}$, $\bar{B}$ AB, $\bar{A}B$, $A\bar{B}$, $\bar{A}\bar{B}$.[19] We can also speak of the trivial truthmaker $\top$, a tautological proposition with no proper parts. $\top$ will be treated as part of every truthmaker.

We will follow van Fraassen's [1969] method for associating these kinds of truthmakers with sentences, that is, in determining which truthmakers in the monadic sense are truthmakers for which sentences.[20] The plural here is important for a sentence may have more than one truthmaker.[21] The disjunction $A \vee \neg B$ has two truthmakers A and $\bar{B}$. By contrast the conjunction $A \wedge \neg B$ has just the one truthmaker $A\bar{B}$. This contrast reflects the intuitive notion that there are two ways for a disjunction to be true, whereas there is just one way for a conjunction to be true. Disjunctions have likewise one falsemaker while conjunctions tend to have more than one.

There are two natural constraints on what the set of truthmakers for a sentence can look like. For $\mathbf{s}$ to be the set of truthmakers for a sentence S, it must be the case that

a) every member of $\mathbf{s}$ entails S, and
b) the truth of S guarantees that at least one member of $\mathbf{s}$ obtains.

A set of truthmakers $\mathbf{s}$ that satisfies and a) and b) with respect to a sentence S will be called EXTENSIONALLY ADEQUATE for S. There clearly exist extensionally equiva-

[17] Thus there is a language $\mathscr{L}$ with a set of atomic sentences $\mathscr{A}$ and each sentence is true or false with respect to a model $\mathscr{M}$ which consists in a valuation of all the members of $\mathscr{A}$. (A valuation is just a function from $\mathscr{A}$ to the truth values $0, 1$. Partial valuations are partial functions of this type.) Valuations, of course, correspond to worlds. We can speak of propositions in this model as sets of valuations (i.e. models), and with the usual semantics for propositional logic; every sentence expresses the proposition which is the set of valuations in which it is true.

[18] Fine uses the term 'state' for this sense.

[19] Fine calls such composition *fusion*, emphasizing the mereological aspect of truthmakers.

[20] His paper differs in terminology and notation from us in inessential ways.

[21] It can also have more than one in a world; but here we are talking about *potential* truthmakers.

lent truthmaker-sets for any sentence in the language because each PC sentence can be rewritten in disjunctive normal for; the truthmakers are essentially the disjuncts. However, just as sentences have oftentimes more than one disjunctive normal form, we cannot expect for each S a unique set of extensionally adequate truthmakers. For instance, the sentence $(A \wedge B) \vee C$ has as extensionally adequate truthmaker sets $\{AB, C\}$, $\{AB\bar{C}, C\}$, $\{AB, C, CE\}$, and so on. One cannot expect even a unique *minimal* set of this kind. $\{AB, \bar{B}\bar{C}, \bar{A}C\}$ is extensionally adequate for $A \equiv B \vee B \equiv C$, but so is $\{BC, \bar{A}\bar{B}, A\bar{C}\}$; and neither can be reduced further by dropping a truthmaker or shrinking one.[22]

This is where van Fraassen's rules come in. He associates with every sentence in $\mathscr{L}$ a *unique* set of truthmakers. The rules are as follows, writing $|\varphi|^+(|\varphi|^-)$ for the set of truthmakers (falsemakers) of φ, and writing ss' for the union of truthmakers s and s' (= the set of all value-assignments made either by s or s').[23]

Van Fraassen's Recursive Rules

$$|A|^+ = \{A\} \text{ (where } A \text{ is any atomic formula)}$$
$$|A|^- = \{\bar{A}\} \text{ (where } A \text{ is any atomic formula)}$$
$$|\neg\varphi|^+ = |\varphi|^-$$
$$|\neg\psi|^- = |\psi|^+$$
$$|\varphi \wedge \psi|^+ = \{ss' : s \in |\varphi|^+, s' \in |\psi|^+\}$$
$$|\varphi \wedge \psi|^- = |\varphi|^- \cup |\psi|^-$$
$$|\varphi \vee \psi|^+ = |\varphi|^+ \cup |\psi|^+$$
$$|\varphi \vee \psi|^- = \{ss' : s \in |\varphi|^-, s' \in |\psi|^-\}$$

These rules yield, for instance, that

$$|A \wedge \neg B \wedge C|^+ = \{A\bar{B}C\}$$
$$|A \wedge (B \vee C)|^+ = \{AB, AC\}$$
$$|A \equiv B|^+ = \{AB, \bar{A}\bar{B}\}$$
$$|A \wedge \neg A|^+ = \{A\bar{A}\}$$

van Fraassen's mapping of sentences to truthmakers (and his particular understanding of truthmakers as conjunctions of literals) is a starting point; it does not represent our final story about the truthmakers of permissions and commands. There are a variety of reasons for this. For one, to capture natural language permissions and commands

[22] Since sets of truthmakers are understood 'disjunctively' we will sometimes speak of them as being true when one of their members is true and false otherwise. Thus, we can speak of sets of truthmakers, like sentences, being extensionally equivalent.

[23] Alternatively, the concatenation of all literals corresponding to assignments made by s or s'. This is sharply to be distinguished from a different use of unions, one level up: the truthmaker set for $\varphi \vee \psi$ is the union of the truthmaker set for φ with the truthmaker set for ψ. Unions at the truthmaker level are conjunctive; unions at the level of truthmaker-sets are disjunctive, just as the sets themselves are disjunctive, by encoding disjunctions of ways for φ to be true.

one needs additional operators not present in propositional logic. Second, permissions and commands will need a different kind of truthmaker. A third and more fundamental reason we are not content with this model is that commands and permissions can be generated in non-linguistic ways. What is permitted in a given case, for example, may have evolved over time without explicit linguistic commands. Even when there is a linguistic command to analyze, the truthmaker structure behind it may not reflect what is there syntactically. A command to water the plants while I am gone might be satisfied by the complex truthmaker consisting of an act of watering the plant each day I am away, but those days themselves don't seem to be in the syntactic structure of the command. Or again, *Help yourself to anything in the fridge* may express one permission when the question is *What am I supposed to do with myself when you're away?* another when it's *What if I'm too broke to order out?* We address later in section 5 the problem of how to find truthmakers without syntactic antecedents. For now we simply defer the issue and assume a perfect mapping (à la van Fraassen) between the syntactically given commands and permissions and the truthmakers associated with them. Putting these qualifications aside, let us begin to employ the apparatus of truthmakers to treat permissive updates.

To start with we will assume that the set of permissible worlds (the worlds we're allowed to realize) is given by a set of truthmakers. If we think of the set as the proposition that *Everyone is behaving* or *It's all OK*, these truthmakers are the modes of conduct that *make* it OK. Truthmakers thus conceived are *licit-makers* and that is a word we will sometimes use.

Crucially the sphere for us is not the set of permitted worlds, but the set of licit-makers; these, of course, determine which worlds are permitted. Where do the licit-makers come from? That remains to be discussed. Just for orientation, though, one might identify them with the van Fraassen truthmakers of all φ such that φ has been permitted. Or, one might try to derive them from the van Fraassen *falsemakers* of all prior *proscriptions*. The set of permissible worlds in the second case is *Ain't misbehaving* (as opposed to *Am behaving*, and the licit-makers would be the different ways of staying out of trouble. (See the discussion of strong and weak permission below; behavior that is not proscribed is *weakly* permitted.) Either way, each licit-maker lays out a permitted option for the Slave.

In some respects our model is close to the proposition-list approach canvassed above, except that we now have a set of truthmakers (each in turn a combination of literals), rather than a set of propositions. Where in the first approach, the set of propositions *conjunctively* represented what was required, the set of truthmakers *disjunctively* represents what is required.

So far we are talking about the sphere $\mathbf{s}$ of permissibility. What about the new

permissions φ that expand the sphere? These too are associated with a set of truthmakers. Each truthmaker in the set represents one way of executing the newly permitted φ.

The question now is, how does one alter the sphere of permissibility, with its truthmaker structure, to reflect the (similarly articulated) permission just granted? A method is needed to go from the two sets of truthmakers that we have been given (those defining the preexisting permissibility sphere, and those representing the new permission) to a new set of truthmakers that define between them the updated sphere of permissibility.

Let's take a simple case. Suppose there are just four potential atomic truthmakers: 'You eat a cookie' (C), 'You eat an orange' (O), and their negations ($\bar{C}, \bar{O}$). Suppose the Slave is required to eat a cookie and to eat an orange. Then the sphere of permitted worlds contains just those in which the Slave eats a cookie and the Slave eats an orange. The one truthmaker associated with this is CO and so the set of currently operative licit-makers is the singleton set $\{CO\}$.

Now suppose that the Master allows the Slave to not eat an orange. From the worldly perspective, it is clear that this permission adds worlds where the Slave eats a cookie but not an orange (as well as keeping the worlds where she eats both). From the truthmaker perspective, on the other hand, we have choices: the new sphere of permissibility could be $\{C\}$, but it could also be something extensionally equivalent such as $\{CO, C\bar{O}\}$. So we go from the permissibility sphere $\{CO\}$ and the new permission $\{\bar{O}\}$ either to $\{C\}$ or $\{C\bar{O}, CO\}$. There are quite a lot of natural update functions that will yield a set of truthmakers extensionally equivalent to $\{C\}$. We will have to cut the range down somehow. In the next section we will outline three specific ways of doing this, that is, three proposals for updating permissibility spheres based on their truthmaker structure and that of the newly permitted φ.

4 TRUTHMAKER-BASED PERMISSIVE UPDATES

Truthmakers in our model these are essentially just sums of literals in $\mathscr{L}$. But the ideas in this section can be generalized to other models of truthmakers.[24] If the set all truthmakers is $\mathscr{T}$, An update rule is a function of the type $2^{\mathscr{T}} \times 2^{\mathscr{T}} \to 2^{\mathscr{T}}$.[25] An upward pointing arrow $\uparrow$ is used for permissive update functions. Using infix notation, $\mathbf{p} \uparrow \mathbf{q} = \mathbf{r}$ means that the permissibility sphere $\mathbf{p}$ updated by $\mathbf{q}$ is equal to $\mathbf{r}$, where $\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$ are all sets of truthmakers.

We have so far explicitly postulated one fact about $\uparrow$ given our examples above, namely that $\{CO\} \uparrow \{\bar{O}\} = \{C\}$ — or another set of truthmakers extensionally equivalent to $\{C\}$. Of course, one such observation (even if generalized) puts few constraints on

[24] See, in particular, Kit Fine's work for a very general account of the structure of truthmakers.

[25] $2^{\mathscr{T}}$ is the powerset of the set of truthmakers— we are talking then about the functions that take a pair of sets of truthmakers and return a new set of truthmakers.

the shape of ↑. But if we make some natural structural assumptions about the way in which permissions are updated we can immediately see some salient, plausible ways of defining ↑ that grow out of this one observation.

Let us begin with the assumption that updates are trivial when what is being permitted is antecedently allowed. This TRIVIALITY assumption says that a new permission, to change the sphere of permissibility, must allow something that was antecedently forbidden, or else the update is trivial.[26] Our assumption amounts to the following constraint on ↑:

TRIVIALITY: $\mathbf{p} \uparrow \mathbf{a} = \mathbf{p}$ if there exists a $p \in \mathbf{p}$ and a $q \in \mathbf{q}$ such that p and q are compatible (in which case we say that $\mathbf{p}$ and $\mathbf{q}$ are compatible).[27]

With the assumption in hand, we reduce the problem to that of defining ↑ in the non-trivial cases, when the permission is genuinely new. Here we make another assumption that will significantly simplify the problem.

FACTORIZABILITY: There is a function $\Uparrow$: $\mathscr{T} \times \mathscr{T} \to 2^{\mathscr{T}}$ such that when $\mathbf{p}$ and $\mathbf{q}$ are not compatible $\mathbf{p} \uparrow \mathbf{q} = \bigcup\{p \Uparrow q : p \in \mathbf{p}, q \in \mathbf{q}\}$

All this is rather abstract; it would be nice to have an example of a function $\Uparrow$ on truthmakers that might serve to complete a definition of the update function ↑ on sets of truthmakers. The first such function we consider is

REQUIREMENT REDUCTION

$p \Uparrow_{\text{RR}} q = \{p \backslash \bar{q}\}$, where $p \backslash \bar{q}$ is the largest part of p compatible with q.

What we need now, to make sense of REQUIREMENT REDUCTION (RR), is a notion of "largest part". Following Fine [2017c], parthood should be a partial order $\sqsubset$ on truthmakers (or on facts suited to serve as truthmakers). In our simple model $p \sqsubset q$ if p is composed only of literals figuring also in q.[28]

What RR does is take the set of truthmakers defining the permissibility sphere, and the set of truthmakers making up the permission, and give back a pointwise reduction, or paring back, of the first set of truthmakers by the truthmakers in the second set. For every way in which the old requirements could be satisfied, and every way in which the new permission could be exercised, we get a weakening of the former to allow in the latter.

[26] This assumption is provisional and we will consider modifying it later. Two reasons to modify it: the case of a permission making things explicit that were only implicit before, and the case where a disjunctive permission is used to let in both disjunctions when only one was allowed before.

[27] As should be clear, given that truthmakers are propositions, two truthmakers are *compatible* if they can both be true at one valuation.

[28] If we think of truthmakers as sets of literals, then $\sqsubset$ is just the subset relation $\subset$. Fine does not assume that truthmakers have an atomic structure.

Suppose for instance that you are required to work either Monday, Wednesday and Friday, or Tuesday, Wednesday and Thursday. Then the going sphere of permissibility is $\{MWF, TWH\}$ (M is a literal corresponding to working Monday, and so on with the other letters as expected). Now you are told you can take Wednesday off; this is has only a single truthmaker, so the permission granted is $\{\bar{W}\}$. Note first that this permission is genuinely new, as it is incompatible with both elements of the going sphere of permissibility. Applying the REQUIREMENT REDUCTION rule we get the following:

$$\{MWF, TWH\} \uparrow_{\text{RR}} \{\bar{W}\} = \{MF, TH\}$$

This seems a good result. It can easily be seen that this situation is analogous to the even/odd day example that was problematic for the proposition-list approach discussed above and in figure 5. It is clear too that REQUIREMENT-REDUCTION will give a correct answer to the very simple case we used as our basic touchstone:

$$\{CO\} \uparrow_{\text{RR}} \{\bar{O}\} = \{C\}$$

So far so good, but we will want to consider other rules besides RR as well. Here is a kind of example where RR might seem inadequate. Suppose you are not allowed to eat peanut butter or chocolate. Then you are permitted to eat peanut butter and to exercise. If we represent this in the obvious way, we get the following:

$$\{\bar{P}\bar{C}\} \uparrow_{\text{RR}} \{PE\} = \{\bar{C}\}$$

You are permitted then to eat peanut butter (full stop). This might seem too liberal an understanding of what the new permission is. It might seem rather that the resulting sphere should be $\{\bar{P}\bar{C}, PE\bar{C}\}$. That is, you are allowed to eat peanut butter only *contingent* on exercising. We can get this result by giving a different definition of $\uparrow$:

CONSERVATIVE POSSIBILITY-ADDING:

$p \Uparrow_{\text{CP}} q = \{p, q \star p\}$, where $q \star p$ is qr, for r the largest part of p compatible with q.[29]

How does this block the permissibility of "cheating"-worlds, with peanut-butter unredeemed by exercise? The newly permissible worlds are those defined by q (peanut butter and exercise) combined with the largest q-compatible part of p (no chocolate and no peanut butter), viz. no chocolate. The combination that results—-no chocolate, peanut butter, exercise—doesn't cheat, so the definition to that extent succeeds. Our earlier examples come out much the same as before.[30] Here it is written out explicitly:

[29] In other words, $q \star p$ is the truthmaker composed of every literal in q and as many of the literals in p as are compatible with q.

[30] The leading candidates for $\{CO\} \uparrow \{\bar{O}\}$ were $\{C\}$ and $\{CO, C\bar{O}\}$. RR picked the first, CP the second.

$$\{MWF, TWH\} \uparrow_{\text{CP}} \{\bar{W}\} = \{MWF, TWH, MF\bar{W}, TH\bar{W}\}$$
$$\{CO\} \uparrow_{\text{CP}} \{\bar{O}\} = \{CO, C\bar{O}\}$$
$$\{\bar{P}\bar{C}\} \uparrow_{\text{CP}} \{PE\} = \{\bar{P}\bar{C}, PE\bar{C}\}$$

Permission to take Wednesday off, after being required to work either Monday, Wednesday, and Friday or Tuesday, Wednesday, and Thursday, leaves it open to us (in addition to what was formerly permitted) to work Monday, Friday, and *not* Wednesday, or Tuesday, Thursday, and not Wednesday. Permission not to eat an orange, following on an order to eat an orange and chocolate, leaves it open to us (in addition to what was formerly permitted) to eat chocolate and *not* an orange, Permission to eat peanut butter and exercise, after a ban both on chocolate and peanut butter, allows us to: avoid chocolate, eat peanut butter, and exercise.

A third possible definition of $\Uparrow$ is motivated by the thought that we sometimes have *choices* about how much of our freedom to exercise. It is strange to think that one must always choose between hewing entirely to the old requirements, and *defying* the old requirements to the largest extent possible. This is the idea behind

LIBERAL POSSIBILITY ADDING:
$p \Uparrow_{\text{LP}} q = \{p\} \cup \{x : x \sqsupset p \backslash \bar{q} \text{ and } x \sqsubseteq q \star p\}$

You can see the respect in which this is more liberal by comparing its response to "You may B and you may C," when A, B, and C had all been previously forbidden, to the conservative rule's response:

$$\{\bar{A}\bar{B}\bar{C}\} \uparrow_{\text{CP}} \{BC\} = \{\bar{A}\bar{B}\bar{C}, \bar{A}BC\}$$
$$\{\bar{A}\bar{B}\bar{C}\} \uparrow_{\text{LP}} \{BC\} = \{\bar{A}\bar{B}\bar{C}, \bar{A}B, \bar{A}C, \bar{A}BC\}$$

For an example with this structure, suppose you are initially required to work Monday to Wednesday, but then are told you can take Tuesday and Wednesday off. It would seem that you might still be allowed to work one of those days, that is, you do not have to choose between either taking neither off or both off. LIBERAL POSSIBILITY ADDING allows this, while CONSERVATIVE POSSIBILITY ADDING does not.

Our aim is not to push one of these rules over the others. Which update procedure is followed in a given situation may well depend on pragmatics. Our aim rather is to show how the truthmaker framework allows us to easily formulate a number of plausible update rules; it's then a further question whether to choose once and for all or on a case by by case basis. There may, for instance, be two kinds of permission, package-deal and trickle-down. Or perhaps trickle-down permissions should be explained away as conjunctions of simpler permissions; the Slave has the option of taking just Tuesday off because no two-day permission was ever given, rather a *pair* of (separately exercisable)

permissions, one for each day.[31]

Before ending this section, it's worth highlighting the fact that our model is based only on a propositional fragment. Luckily, nothing in principle prevents us from extending the truthmaker model to a quantificational fragment. The simplest way to do so is to assume a fixed domain, and a name for every object in the domain, and simply treat the truthmakers as composed of any atomic sentences in the language. This effectively reduces a logic with quantifiers to a propositional logic. For more sophisticated models of truthmakers in a quantificational setting, see Yablo [2014] and Fine [2017a], Fine [2017b])

5 DETERMINING TRUTHMAKERS

Three styles of permissive update have been distinguished: requirement reduction, conservative possibility-adding, and liberal possibility-adding. Table 1 illustrates their differences in the case where $\Box(AB)$ is updated with $\Diamond(\bar{B}C)$:

Rule	Calculates $\{p\}\uparrow\{q\}$ like this:	Yielding
RR	$\{p\backslash\bar{q}\}$, where $p\backslash\bar{q}$ is the part of p compatible with q	$\{A\}$
CP	$\{p, q^*p\}$, where q^*p is q fused, or conjoined, with $p\backslash\bar{q}$	$\{AB, A\bar{B}C\}$
LP	$\{p\} \cup \{x\mid x$ lies between some y in $p\backslash\bar{q}$ and z in $q^*p\}$	$\{A, AB, A\bar{B}, AC, A\bar{B}C\ \}$

Table 1: Three ways of updating $\Box(AB)$ with $\Diamond(\bar{B}C)$

One needs truthmaker structure, of course, for the rules to get a grip, and this may give us pause. Recall a problem we identified with the proposition-list and closest-world approaches. These approaches make predictions only relative to a determination of what propositions make up the list, or what the closeness relation should be. Have we simply replaced stipulations about closeness and propositions with stipulations about truthmakers? Does truthmaker structure have to simply be reverse engineered from the results it is meant to generate?

This is indeed a worry, but the cases are different. One can get somewhat of an independent grip on truthmaker structure, more anyway than on the requisite closeness relation or proposition list. For one thing a compositional story exists about the truthmakers of complex sentences, so—to a certain extent—we ought be able to read truthmakers off syntactic structures. In this respect the truthmaker approach shares the virtues of alternative semantics [Rooth, 1985] and inquisitive semantics [Groenendijk

[31] van Rooij [2000] has an interesting discussion of package-deal vs trickle-down permissions; we get the term "package-deal" from that paper. van Rooy argues that for most "conjunctive permission sentences, [the] package-deal prediction is empirically wrong, ... these other conjunctive permissions allow also for the conjuncts to be done separately." p.133.

and Roelofsen, 2009]. This relationship to syntactic form differs from anything available to the closest worlds or AGM approaches.

Also there are other, less syntax-bound, ways of coming to grips with truthmaker structure. One is intuitive judgments about agreement. If I think Mary is in Paris Monday to Wednesday, and Bill thinks she is there Tuesday to Thursday, then we agree she is in Paris Tuesday and Wednesday. We do not, however, agree that she is in Paris either on Monday or on Thursday. These agreement judgments track what the natural truthmakers are for propositions and sentences. (Bill and I agree if the content S of my belief overlaps the content T of his, that is, S and T have a part in common.P is part of S if it's implied by S and has no "stray" truthmakers, ones not implied by any truthmaker for S.) Yablo [2014] reviews a number of other phenomena— assertive content, partial truth, presupposition failure, logical priority, inductive confirmation, and relevant implication among others—that at least seem to be locking on to the same "natural kind" of factual verification.[32]

What the relevant truthmakers are may vary with context: the toy model does not allow for this and oversimplifies in other ways. For all its faults the model suggests that truthmakers are implicated in a wide variety of natural language phenomena. The notion is not just conjured up for the problem of permissive updates.

6 CONNECTION TO BELIEF REVISION

Structurally speaking the problem of permissive updates seems much like that of belief revision as understood in AGM and related literature.[33] The problem of belief revision can be stated as follows: suppose we model our beliefs by a set of possible worlds B. Then we learn a new proposition P such that there are no worlds compatible with both B and P. The question is how to update B to reflect the new information. This is not precisely structurally analogous to the problem of permissive update. In permissive updates we have a set S (the sphere of permissibility) and when we add P (worlds in which the new permissions are exercised) we want to find a new set S' *compatible* with P. Whereas in the belief update case we want the new set, B' to *entail* P. However, since at least Levi [1977], and in the AGM tradition, it is common to divide such belief update into two operations: first, what is called *contraction*, cutting B back so that it no longer entails $\neg P$, (i.e.adding P worlds to B), and then *expansion*, (removing all $\neg P$ worlds

[32]The idea goes back at least to van Fraassen [1969] and has been explored by Kit Fine in recent work (Fine [2017a]), Fine [2017b]). See the recent Phil Studies symposium (Yablo [2017]) and also a number of papers by Gemes (Gemes [1997], Gemes [2007], Gemes [2006]. van Rooij [2017] applies the truthmaker idea to implicatures.

[33]Gärdenfors [1984] is among the many to draw this connection. See also the aforementioned paper by van Rooy.

from the result).[34] Levi and AGM's contraction operation plays the same structural role as what is needed for permissive updates.

Despite these structural similarities there are at least two good reasons for treating belief revision differently from permissive update.

The first is that the inference patterns that formed our basic data do not seem to hold in the case of belief revision. Suppose you have never seen a naked mole-rat, but believe they are all green on the basis of testimony. You then discover that the naked mole-rat in the zoo near your house might be white. How do your revise your beliefs? Surely the most natural response is not to think that every naked mole-rat except the one at the zoo must be white, while the one at the zoo might be white or green. Examples of this form are easy to come by. By contrast if you are required to paint every house green, and then allowed to paint a particular house white, it is natural to think think that you are still required to paint the other houses white.

The second reason to dissociate belief revision from the problem of permissive updates is that the former but not the latter can be understood probabilistically. Beliefs clearly come in degrees, and the best way of understanding the structure of these degrees is Bayesianism. It is natural to think that many problems about belief revision are best thought of as problems about how to update credences, and thus Bayesian resources have much to say about how to answer these problems. It is, however, much less clear that Bayesians will have much useful to say about the problem of permission.[35] The sphere of permissibility given by a command may not be categorical, but it is also not (when clearly expressed) probabilistic. Probability theory seems to have little to offer here.[36] Perhaps clever artificial ways may be found to treat permissive updates as conditioning on new evidence, but the prospects do not look good. So, the structural analogies between belief revision and permissive updates notwithstanding, there is reason to think that ultimately our accounts of these two kinds of update operation will be different.

7 CONNECTION TO EXCEPTIVES

There is a clear connection between permissive updates and the use of exceptives in quantifiers, such as in these examples:

(3) Everyone but/except Ted is dead.

(4) Except Ted, everyone is dead.

[34] In the AGM tradition one generally treats belief sets as sets of propositions or sentences rather than worlds, hence what looks like expansion in worldly terms is called contraction in the AGM tradition.

[35] Though see Shear and Fitelson [forthcoming] for an attempt to wring constraints on qualitative AGM-type update rules out of Bayesian update rules.

[36] This is not to take a stand on the view of Jackson and others that the normative is (in other ways) up to its neck in probability (Jackson [1986]).

Without going into details, we will take it for granted that the function of exceptives in quantifier phrases is to subtract from the domain of the quantificational statement.[37] So given that (3) and (4) say the same, they both say that the everyone in the domain stripped of Ted is dead.

Permissive updates have a similar flavor. But it extends more widely than you would expect from the quantificational model. Start with some that fit that model:

(5) (Context: you must wash every dish.) You don't have to wash the saucepan.

The resulting permissibility sphere is similar to what one gets when simply giving a permission with an exceptive:

(6) You must wash every dish except the saucepan.

Similarly as Philippe Schlenker notes (p.c.), the problem of permissive updates is similar to explicit exceptions made about past statements, as in these examples:

(7) a) They were required to wash every dish. Except saucepans.
b) He was required to work odd or even numbered days. Except Sundays.

This leads to two related questions. First, why don't we use an off-the-shelf semantics for exceptives to handle permissive updates generally? Second, what is the relationship between the semantics of exceptives and our story about permissive updates?

In reply to the first question, positing quantifier domain restriction as the mechanism of permissive update does not cover all of the cases we need to. In particular, the crucial disjunctive form of permissive update, as in (2) and figure 5, does not have a universal quantificational structure at all and so cannot be analyzed in any obvious way as quantifier domain restriction.[38] That does not mean that we should not aspire to a unified view of the two phenomena. Indeed an adequate account of exceptives will have itself to deal with cases that go beyond the nominal realm, such as this:

(8) It was a lovely meal, except for the food.

There might still be some kind of shrinkage going on in these cases, but it's not obviously shrinkage of a domain, or anyway not a quantificational domains. So when you are allowed to take Tuesdays off, the previous command holds *except insofar as it concerns Tuesdays*. It is not a domain that is shrinking here but the condition that defines a domain.

[37] See von Fintel [1993] for a classic discussion of exceptives in quantifiers. Von Fintel makes several important distinctions, arguing that free exceptives such as those in (4) are semantically distinct from the *but*-phrases in (3). (See also Moltmann [1995].)

[38] von Fintel [1993] notes the strangeness of *Someone except for Bob is present* and suggests an explanation of it.

But to see things this way, one must generalize the notion of reduction or shrinkage beyond quantifiers.

One way of looking at what we are doing here is providing such a generalization. Yablo [2014] argues that there is need for a general story about content subtraction.[39] This is to account for examples such as the following:

(9) Her version of events is correct except when it comes to the dog.

A domain of objects (even fancy ones) does not seem to be involved here, or in

(10) He's a standup guy except when it comes to bowling.

But the same word 'except' is present, performing a similar function. Our algorithms for permissive update could be seen as generalizing the phenomenon of exception-making beyond the quantificational paradigms that feature so prominently in the literature on exceptives.

8 CONTEXT CHANGE AND COMPOSITIONAL SEMANTICS

So far we have treated the problem of permission entirely as a problem about context change; the question has been how to update what is allowed in light of a new permission. We argued that the truthmaker framework allows us to frame simple rules that capture the structure of permissive updates. The rest of the paper will ask how this proposal relates to the semantics and pragmatics of the expressions used to make permissions, the modals such as 'may' and 'can'.

There are three broad options for integrating our account of permissive updates with a semantics of the relevant modal expressions:

The first option is to completely semanticize the update rule we have given above. We would do this by giving a dynamic semantics for modals that according to which their semantic values are functions from sets of truthmakers to sets of truthmakers. We briefly review this option in the next section, but do not pursue it because of two serious problem. The first problem is that it is not at all clear how to provide plausible and simple-enough semantic entries that respect basic facts about the modals, including their duality. The second problem is that by semanticizing permissive updates, one may provide an account of performative uses of modals, but only by losing sight of descriptive uses.

The second, most conservative option, is to have a standard (Kratzer-style) truth-conditional analysis of deontic modals. Permissive updates would then be treated entirely

[39] See also Fuhrmann [1999], Humberstone [2000], and chapter 5 of Humberstone [2011].

pragmatically, and the relevant sets of truth-makers needed for the operations would need to be recovered from context in each case.

Our third option is in between the first and second in the extent to which it semanticizes the suggested update procedures. On this option we give a semantics for modal statements in terms of truthmakers, thus explicitly building truthmakers into our static semantic theory. The semantics does not itself define an update procedure—-it merely tells us what the truth and falsemakers of our modal statements are—but it may be suggestive where updates are concerned. We explore some of the advantages and difficulties of this approach in the final sections of the paper. A fourth option, linking the semantics and pragmatics more closely still, is briefly sketched in the appendix.

9 SEMANTICIZING UPDATES

Nothing in principle prevents us from trying to build rules like RR or CP into the meaning of 'may' and 'might'.[40] In the dynamic framework [e.g. Heim, 1982], we associate sentences not with propositions but rather with CONTEXT CHANGE POTENTIALS, functions from contexts to contexts. We could associate with sentences of the form *might p* with functions from contexts to contexts, where contexts are represented by either sets of worlds or sets of truthmakers. The functions could take the form of one of the permissive update operations discussed above.[41]

There are, however, serious obstacles to putting the particular operations discussed above into the compositional semantics of modals. The most immediate challenge is capturing the duality of 'can' and 'must'. Such a duality comes out easily in most of the standard semantics for modals; these are quantificational semantics à la Kripke which align $\Box$ and $\Diamond$ with $\forall$ and $\exists$. None of the operations for possibility addition we outlined above, however, can be captured with anything as simple as existential quantification over worlds. And indeed the operation of adding a new requirement cannot be captured with universal quantification.[42] Thus we cannot simply appeal to the fact that $\neg\forall$ is equivalent to $\exists\neg$, to explain the duality of *can* and *must*. But again, it seems a pretty basic datum about deontic modals that *can* and *must are* duals. The following, for example, seem equivalent:

(11) You cannot (you are not permitted to) eat any apples.
$\neg\Diamond A$

(12) You must not (you are required not to) eat an apple.
$\Box\neg A$

[40] Indeed Asher and McCready [2007] suggests a way of doing just that.

[41] If contexts are just sets of worlds, then we will need a way of translating between sets of worlds and sets of truthmakers: we discuss one such method in the next section.

[42] A new requirement augments one by one each licit-maker in the going sphere of permissibility.

This is not to say that devising a semantics that respects duality is impossible, just that it is a serious challenge.

A perhaps more serious challenge to giving a dynamic update semantics for deontic uses of 'can' and 'may' that yield permissive updates is to expand that semantics to other uses of 'can' and 'may' that do not. As we observed earlier, deontic modals also have purely reportative uses in which the speaker only has the authority to report on what is allowed, not to determine it. If we build the permissive update into the meaning of 'can' or 'may' we will have trouble accounting for those uses. (See however the appendix.) In addition we may run into trouble treating the other flavors of modality that can be used with word such as 'can' and 'may', such as epistemic and ability modals.[43]

10 TRUTH-CONDITIONAL SEMANTICS + UPDATE PRAGMATICS

We have just seen two reasons for treating the truthmaker methods of permissive update discussed in section 4 as part of the pragmatics of updates, rather than the semantics of modals. The second way of relating permissive updates to the meaning of modals is to treat the updates as arising entirely for pragmatic reasons. We will sketch this option in this section.

A natural starting point nevertheless is a standard intensional semantics along the lines of Kratzer [1981, 2012]. (This will help us to identify the point at which needs arise that semantics is not able to meet.) Later, in section 12, we will consider a semantics for modals expressed directly in terms of truthmakers, but for now it is worth emphasizing that our basic idea of how permissive updates work can be pragmatically grafted onto standard, not truthmaker-based, semantic theories as well.

As we saw in the discussion of duality, deontic modals on standard accounts semantically express quantification over a contextually given set of permissible worlds. Permission statements existentially quantify over these worlds (as do negations of requirements).[44] The problem of permissive updates arises when we are told that P is permitted but the current sphere of permissions S does not include any P worlds. The problem is how to remake the sphere S to reflect the permissibility of P.[45]

In order to use our proposed algorithms above, we will need to somehow get a truthmaker representation of the situation. That is, we face the problem of how to move from standard intensional meanings for P and S to a representation in terms of truthmakers. The van Fraassen mechanism tells us how to get truthmakers out of a sentence in propositional logic, not out of a set of worlds. It is true that permissions are

[43] Yablo [2009] reports important parallels between epistemic and deontic context changes, but there are also distinctions. In the case of ability modals, no natural non-divine uses create new possibilities.

[44] Again we do not strongly distinguish Veltman's dynamic semantics from the standard story.

[45] An analogous problem arises for Bayesians: how do we change the priors to reflect the occurrence of an event that was assigned zero probability. The word "update" may be confusing in this respect. Bayesian update is business as usual, while permissive update is from a Kratzerian perspective making a new start.

generally expressed in sentences. But this should not be written into the update rules, for one might non-verbally permit something, or permit it verbally while distancing oneself in some way from the sentence employed. The real problem, however, is that the sphere of permissibility will not generally be verbally represented even if the original commands and permissions were. The verbal record is long gone, and wouldn't be that useful anyway, since commands have been partly undone by subsequent permissions and permissions by commands.

But, while the commands and permissions may be gone, the language in which they they were formulated is not, and the space of worlds is not; the worlds are (or line up one-one with) the total valuations, the assignments of truth-value to each atomic letter. How does this endow S with truthmakers? The set of S-worlds lends itself intrinsically to a representation in terms of "prime implicants."[46] S's prime implicants are the partial valuations big enough to ensure S's truth, but not so big that a proper subvaluation also ensures S's truth. The null hypothesis about S's truthmakers is: s makes S true iff it primely implies S.

Of course the space of possibilities is not usually given as the set of valuations of some propositional language. The above can still serve as a *model* though of the process of truthmaker-detection, with facts salient in context taking on the role formerly played by partial valuations. S is made true by any such facts strong enough to imply S but not so strong that weaker facts drawn from the same pool imply it. van Fraassen-type truthmakers make a limited comeback here, as the words and sentences in play are likely to be a factor in contextual salience. What we don't get by the method as stated is stronger and weaker truthmakers for the same sentence, for instance A and AB as verifiers for $A \vee AB$. There are a number of options at this point. Perhaps certain S-implying facts qualify as truthmakers just by virtue of salience, even if weaker S-implying facts are salient too. Or perhaps the salient facts *change* as we survey the sentence, so that AB *is* the weakest salient fact when we get to the second disjunct (A has fallen out of view). S's truthmakers include s in context C, on this view, iff s is the weakest salient implier in *any subcontext* of C. These are admittedly shots in the dark, but they aim at a worthy target: allowing syntax to be relevant to truthmaker structure but not determinative. It's a factor that is not always even present, and where present has to duke it with other factors.

Let us briefly describe the process of permissive update as we see it. Consider a case in which the Master gives a new permission for something Q that had previously been forbidden. The Slave reasons that the Master intends to alter the sphere of permissibility to make it true that Q *is permitted*. To work out how this is done—the shape of the

[46] Yablo [2014].

alteration— the Slave must

a) find the *old* licit-makers p, the truthmakers back then of *You are behaving yourself* (guided by linguistic antecedents and other features of context),
b) find the novel licit-makers q, representing the new ways of behaving yourself (here the linguistic form of the permission is likely to be crucial), and
c) update the first set of truthmakers $\mathbf{p}$ with the second, $\mathbf{q}$, using one of the $\uparrow$ operations sketched in section 4.

11 WEAK AND STRONG PERMISSION: MOTIVATING TRUTHMAKERS

We will now turn to the third option for integrating our account of permissive updates. On this option we give a non-standard semantics for deontic modals in terms of truthmakers. In this section we begin by independently motivating such a semantics.

The truthmaker approach offers new resources for distinguishing between different types of permission. Having laid out ways to get from sets of worlds to sets of truthmakers, we saw that there may be more than one set of truthmakers corresponding to the same set of worlds. This shows up empirically in the distinction between weakly permitted options and options that are strongly permitted.[47]

Consider the "constitutional principle of English law" which says that *everything which is not forbidden is allowed*. The principle is absurd if "allowed" means *expressly* allowed, e.g., protected by law, or written into the bill of rights. It's just false that we are expressly entitled, say, to part our hair on the right (though this is not forbidden). "Allowed" in the constitutional principle has thus got to express *weak* permission. A jokey contrast is sometimes drawn with crueler jurisdictions in which *whatever is not allowed is forbidden*. "Allowed" as understood in the joke expresses the stronger notion; you are to do nothing which is not expressly allowed. Doesn't this mean that we are damned either way when it comes to a matter the law takes no interest in? We can't part our hair, since this is not expressly allowed, but we can't leave it unparted either, for the same reason. The point of the joke is that the law takes an interest in everything.

A distinction of this sort falls out naturally on the truthmaker approach. An option A is WEAKLY permitted relative to a set of truthmakers if there is at least one truthmaker — licit-maker — in the set with which it's consistent; this is enough for it to hold in some permitted world. An option is STRONGLY permitted if it is not just compatible with a licit-maker, but implied by a licit-maker. It not only holds in a permitted world, it holds because of the property of that world that *makes* it permitted.

One can drawn subtler distinctions even than this: A is THOROUGHLY WEAKLY permitted, for instance, if it's consistent with *every* licit-maker. A is THOROUGHLY STRONGLY

[47] See Moltmann [2017], Moltmann [to appear] and Fine [2018a,b] for related discussions.

permitted if, in addition to A being implied by a licit-maker, each of A's truthmakers is implied by a licit-maker. So, for instance, in a typical classroom we might find that

nose-blowing is weakly permitted
breathing is thoroughly weakly permitted
vocalizing is strongly permitted
answering teacher's questions is thoroughly strongly permitted

Nose-blowing is only weakly permitted, because no licit-makers require you to blow your nose. There may even be some that require you NOT to blow your nose (say, when asking a question); that's why the permission isn't thorough. No licit-makers, though, I assume, require you not to breathe. Vocalizing is strongly permitted insofar as there are licit-makers (to do with asking questions) that require you to vocalize. It is because there are ways of speaking (sharing private jokes, say) that are not implied by licit-makers that the permission isn't as thorough as that for answering questions.

A similar distinction can be drawn between the expressly and in-effect obligatory. An option A is in-effect obligatory relative to a set of licit-makers if each λ in the set implies A. Thus if you don't A somehow or other, you'll be misbehaving. It goes further to say that particular ways of A-ing are implied by some λs. This is the difference between being non-naked and being present when attendance is taken. The first is in effect obligatory—no licit-maker allows nakedness— but not expressly obligatory unless there is a form of non-nakedness (wearing the school uniform, say) that is singled out for approval by some λ. One can again strengthen these notions, e.g., A is *thoroughly* expressly obligatory if each λ implies a way of A-ing and each way of A-ing is implied by a λ.[48] Wearing the proper attire, the school uniform or gym clothes, might be thoroughly expressly obligatory in this sense.

With these distinctions in mind, we might want to consider amending TRIVIALITY to allow updates when, although p was already permitted, the permission was weak. Adding in a truthmaker for p may not bring in any new worlds, but it upgrades the reasons certain existing worlds are permitted, by converting a previously weak permission to one that is strong. The proposed new update procedure violates TRIVIALITY on the hyperintensional side while respecting its intensional implications.[49]

STRONG POSSIBILITY-ADDING:

$\mathbf{s} \uparrow_{\mathrm{SP}} \mathbf{p} = \mathbf{s} \cup \{sp : s \in \mathbf{s}, p \in \mathbf{p}$, where s and p are compatible $\}$

[48] This is referrred to in the Appendix as "intailment" by the sphere: tautological entailment plus inclusion.

[49] Note that this is just a special case of CONSERVATIVE POSSIBILITY ADDING. We could also allow a different form of STRONG POSSIBILITY ADDING based on LIBERAL POSSIBILITY ADDING.

The new rule yields, against the background of van Frassen's recursive assignment of truthmakers, an explanation of the difference between these two permissions, which Kratzerian accounts cannot easily capture:

(13) You can eat an orange.

(14) You can eat an orange or an orange and an apple.

The first arranges that orange-eating is expressly permitted (eating them with apples is only implicitly permitted); the second arranges for orange-and-apple eating to be expressly permitted as well.

Similarly we might want to introduce a rule for imperatives that generated non-trivial update effects when p was only implicitly mandated. Demanding that p would not knock out any previously permitted worlds, but change the hoops those worlds had to jump through to achieve permissibility.

STRONG NECESSITY ADDING:
$\mathbf{s} \downarrow_{\mathrm{SN}} \mathbf{p} = \{sp : s \in \mathbf{s}, p \in \mathbf{p}$, where s and p are compatible $\}$

The new rule explains the difference between these two commands:

(15) You must eat an orange.

(16) You must either eat an orange, or an orange and an apple.

The first arranges that orange-eating is expressly required—each licit-maker implies *O*—while taking no stand at all on orange-and-apple eating. The second ensures that orange-and-apple eating is permitted, if it wasn't already, and identifies it as a way of fulfilling one's overall obligations.

Truthmakers thus allow us to make sense of two important distinctions—between weak and strong permissions (and commands), and between intensionally equivalent permissions (and commands). For this reason among others, it is worth pursuing a full-scale truthmaker semantics for deontic modals. We sketch an account in the next section.

12 TRUTHMAKER SEMANTICS FOR MODALS

The claim is that deontic modals can be given a truthmaker semantics, à la van Fraassen, though going in one way beyond van Frassen. Our goal is to show that such a semantics can capture: a) the weak vs. strong permission distinction, b) free-choice effects, and c) duality. While Fine [2018a,b] covers the first two, we do not know of a semantics in the

truthmaker tradition that also covers duality.[50] We reiterate here a point made in section 10: one can employ a truthmaker-based understanding of permissive updates *without* giving an explicit truthmaker semantics for deontic modals. It is nonetheless worth spelling out what such a semantics might look like, since the truthmaker version has the advantage of providing an elegant resolution of some of the longstanding problems about deontic modals.

The language we work in is that of propositional modal logic—-$\mathscr{L}$ with the unary $\Box$ operator—except that $\Box\varphi$ is a sentence only if φ is $\Box$-free.[51] We want to assign to each sentence a set of truthmakers. $\Box$-free sentences have the familiar old truthmakers we get from van Fraassen (described in section 3),. A special new class of truthmakers will be introduced for modal sentences. At any given context there is a sphere of permissibility, we assume, with a truthmaker structure. These spheres (essentially just sets of ordinary truth-makers) will play the role of truth- and falsemakers for sentences like $\Box\varphi$.

What should $\mathbf{s}$ be like to make $\Box\varphi$ true or false? $\Box\varphi$ is verified by $\mathbf{s}$, to a first approximation, if every state of affairs marked out as permissible by $\mathbf{s}$ (every $s{\in}\mathbf{s}$) implies φ; not to φ is impermissible. $\Box\varphi$ is falsified if every way of $\overline{\varphi}$-ing is implied by an $s{\in}\mathbf{s}$).[52]

($\Box^+$) $\mathbf{s} \vDash^+ \Box\varphi$ iff each $p \in |\varphi|^+$ is implied by an $s \in \mathbf{s}$, and each $s \in \mathbf{s}$ implies a $p \in |\varphi|^+$
($\Box^-$) $\mathbf{s} \vDash^- \Box\varphi$ iff each $p \in |\varphi|^-$ is implied by an $s \in \mathbf{s}$.

Similarly

($\Diamond^+$) $\mathbf{s} \vDash^+ \Diamond\varphi$ iff each $p \in |\varphi|^+$ is implied by an $s \in \mathbf{s}$
($\Diamond^-$) $\mathbf{s} \vDash^- \Diamond\varphi$ iff each $p \in |\varphi|^-$ is implied by an $s \in \mathbf{s}$, and each $s \in \mathbf{s}$ implies a $p \in |\varphi|^+$

Note that the property $\mathbf{s}$ needs to verify a must-claim is stronger than the property it needs to falsify one. $\Box\varphi$ is falsified by $\mathbf{s}$ if every way of $\overline{\varphi}$-ing is implied by a licit-maker. For it to be verified, not only must (i) each way of φ-ing be implied by a licit-maker, (ii) each licit-maker must imply a way of φ-ing.

What goes wrong if (ii) is omitted? We get Ross's Paradox: *Post the letter!* winds up implying *Post the letter or burn it!* $\Box A$ implies $\Box(A{\vee}B)$, given just (i), because a sphere whose members all imply truthmakers for A is a sphere whose members all imply truthmakers for $A{\vee}B$. Clause (ii) blocks this reasoning by requiring φ to be part of $\mathbf{s}$; each of φ's truthmakers has to be implied by an $s \in \mathbf{s}$. That each of A's truthmakers have

[50] Covering free-choice effects pragmatically, generally allows a simple treatment of necessity and possibility modals as duals. There are some examples in the dynamic tradition of semantics that cover free choice and duality, such as Starr [2016], Willer [2017].

[51] It is hard to see much value in giving a treatment of iterated deontic modals, though see Marcus [1966] for discussion.

[52] $\Box\varphi$ isn't falsified by $\mathbf{s}$ unless $\mathbf{s}$ strongly permits $\neg\varphi$. If $\neg\varphi$ is only weakly permitted then $\Box\varphi$ is untrue but not false.

this property does not at all mean that each of $A \vee B$'s truthmakers have it, because $A \vee B$ has more truthmakers.

13 CONTENT

Clauses like ($\Box^+$) and ($\Box^-$) are prominent in recent work on fine-grained content. *P* (*tautologically*) *entails Q* ($P \Rightarrow Q$) iff

> each of *P*'s truthmakers implies a truthmaker for *Q*.[53]

P includes Q ($P \sqsupseteq Q$) iff

> each of *Q*'s truthmakers is implied by a truthmaker for *P*. [54]

P inclusively entails Q ($P \geq Q$) iff *P* both entails and includes *Q*. Inclusive entailment will sometimes for brevity be called "intailment."

Inclusive entailment—intailment— is the relation that **s** must bear to φ if **s** is to verify $\Box\varphi$. To put it another way, **s** makes an option φ obligatory if it intails that option. (To φ is *part of what it is* to behave yourself, as opposed merely to being entailed by good behavior.) This enables us to characterize $\Box\varphi$'s truth- and falsemakers more simply as follows:

$[\Box^+]$ $\mathbf{s} \vDash^+ \Box\varphi$ iff $\mathbf{s} \geq |\varphi|^+$ (**s** intails $|\varphi|^+$)
$[\Box^-]$ $\mathbf{s} \vDash^- \Box\varphi$ iff $\mathbf{s} \sqsupseteq |\varphi|^-$ (**s** includes $|\varphi|^-$)

These rules interestingly enough allow for the possibility of **s** leaving $\Box\varphi$ undefined. There is no reason why **s** should have to either intail $|\varphi|^+$ or include $|\varphi|^-$. There is no reason even why **s** should include one or the other of $|\varphi|^+$, $|\varphi|^-$. For φ may have truthmakers not implied by members of **s**, and also falsemakers not implied by members of **s**. (*Feed the cat or kick it!* has a truthmaker (to do with kicking) that is not implied by any decent licit-maker; it also has a falsemaker (Starve the cat and be gentle with it) that is not implied by any decent licit-maker. Decent spheres do not make feeding-or-kicking either obligatory or permissibly left undone. It is this that enables the semantics to generate free-choice inferences in a way compatible with duality.

Duality says that $\Diamond\varphi$ is definable as $\neg\Box\neg\varphi$. This yields, given the rules for $\Box$ and $\neg$, the following account of possibility modals. $[\Diamond^+]$ says that **s** makes φ pemissible just if each way of φ-ing is implied by a licit-maker. $[\Diamond^-]$ says that **s** makes φ impemissible just if each licit-maker implies a way of $\overline{\varphi}$-ing, and each way of $\overline{\varphi}$-ing is implied by a licit-maker .

[53] Proved in van Fraassen [1969]. Tautological entailment is perhaps the best known form of relevant entailment.

[54] For inclusion see Yablo [2014] and Fine [2015]. Yablo normally requires classical implication as well for inclusion; we drop this further requirement here as it comes for free with entailment.

$[\Diamond^+]\ \mathbf{s} \vDash^+ \Diamond\varphi$ iff $\mathbf{s} \sqsupseteq |\varphi|^+$ ($\mathbf{s}$ includes $|\varphi|^+$)

$[\Diamond^-]\ \mathbf{s} \vDash^- \Diamond\varphi$ iff $\mathbf{s} \geq |\varphi|^-$ ($\mathbf{s}$ intails $|\varphi|^-$)

Note that φ's truthmakers must be (not only consistent with but) *implied* by licit-makers, in order for $\Diamond\varphi$ to be true; $\Diamond$ therefore expresses strong permission rather than weak. Also that *each* of φ's truthmakers must be implied by a licit-maker, in order for $\Diamond\varphi$ to be true. This relates, as we now discuss, to free choice. By free-choice effects we have in mind inferences like the following:[55]

(17) You may eat an apple or an orange
⇝ You may eat an apple *and* you may eat an orange.

(18) You may not eat an apple or an orange
⇝ You may not eat an apple *and* you may not eat an orange.

The proposed semantics yields (7), as for $\Diamond(A \vee B)$ to be true in $\mathbf{s}$, both of $A \vee B$'s truthmakers need to be implied by licit-makers. It yields (8), as for $\Diamond(A \vee B)$ to be false in $\mathbf{s}$, every licit-maker in $\mathbf{s}$ has got to imply a falsemaker for $A \vee B$.

How are we to understand the truth-value gaps here? If φ's lack of truth-value signalled the failure of some presupposition, then we would make rather strange predictions. For example, an utterance of *John may ride a trolley* would presuppose that either John is permitted to ride a trolley or he is forbidden from riding a trolley. This seems wrong.

Luckily there is an alternative, growing out of the homogeneity literature. Križ [2015] argues that homogeneity effects such as the following give rise to truth-value gaps of a non-presuppositional variety.

(19) The boys came to the park.

(20) The boys didn't come to the park.

There seems a gap between these two sentences, which are negations of each other. But neither sentence presupposes (according to the usual tests) that either all the boys came to the park or all of them didn't come to the park.

Similarly, you might think, not all spheres of permission make either of these sentences true (in the case in which you are allowed to dance but not steal):

(21) You are allowed to dance or steal.

(22) You are not allowed to dance or steal.

[55] An early reference is Kamp [1973]; Klinedinst [2007] and Fox [2007] are important later discussions.

To bring out the analogy with (8) and (9), (10) and (11) correspond to the following, where "my options" are dancing and stealing:

(23) My options are implied by licit-makers.

(24) My options are not implied by licit-makers.

We propose that this approach based on the homogeneity literature points the way to a unified treatment of free-choice effects and the strong/weak permission distinction, while respecting duality.[56] $\Diamond\varphi$ says more or less that "the ways for φ to hold are permitted." $\neg\Diamond\varphi$ says that "they are *not* permitted." Truth-value gaps arise if some are permitted (implied by licit-makers) and others not.

A couple of points before closing. The suggested truthmakers for modal claims are a very different sort of animal than the truthmakers for non-modal claims. Ordinary truthmakers s are partial valuations or sets of literals. Deontic truthmakers $\boldsymbol{s}$ are sets of ordinary truthmakers. What are the truthmakers supposed to be of mixed claims like $P\&\Box Q$? Presumably some kind of amalgam $s\boldsymbol{s}$ of ordinary truthmakers and sets of ordinary truthmakers. Nothing has been said so far about how to derive P's amalgamated truthmakers from its ordinary truthmakers, and $\Box Q$'s amalgamated truthmakers from its deontic truthmakers. Or about how to obtain $P\&\Box Q$'s amalgamated truthmakers from those of its conjuncts. A full-fledged truthmaker semantics will have to address these questions. See part I of the Appendix.

Then there is the question of update. The proposed semantics says nothing about this—-they are static—so our update rules will have to be developed independently. Something has been said about *permissive* updates $c[\Diamond\varphi]$ but what about, say, $c[\varphi\vee\Diamond\psi]$? Even for permissive updates the pragmatic story will have to be more complex. When a permitted φ has no truthmaker in the initial sphere, this could be because it was forbidden as opposed to permitted, or because it was weakly permitted as opposed to strongly permitted. Different update procedures are called for according to which of these hypotheses is correct.

14 CONCLUSION

Our goal here has been relatively modest. We proposed that the framework of truthmaker semantics is useful for understanding permissive updates. We defined three different update procedures in that framework for capturing these updates. After discussing the connections between this permissive updates and belief revision and the semantics of exceptives, we discussed how a truth-maker account of permissive updates might be

[56] Duality turns out to be a rather weak constraint when you can (and must) define truthmakers and falsemakers independently.

integrated into an overall semantic/pragmatic story of the language of permission and obligation. Our discussion has necessarily been exploratory but we hope that it will spur further exploration.

APPENDIX: TRUTHMAKER SEMANTICS

We work in the language $\mathscr{L}$ of propositional modal logic, except that modal operators cannot occur within the scope of modal operators:

$$A|B|C\ldots$$
$$\neg\varphi|\varphi\wedge\psi|\varphi\vee\psi$$
$$\Box\varphi \text{ if } \varphi \text{ is factual (free of modal operators)}$$

Other connectives are defined in the usual way, and $\Diamond\varphi$ is short for $\neg\Box\neg\varphi$. A formula is factual if it contains no boxes or diamonds, and deontic if of the form $\Box\varphi$ or $\Diamond\varphi$ So far so normal. But the semantics is different.

TRUTHMAKERS are ordered pairs $<s, \mathbf{s}>$ of ordinary truthmakers s and deontic truthmakers $\boldsymbol{s}$. (An ordinary truthmaker is a partial valuation or set of literals; a deontic truthmaker $\boldsymbol{s}$ is a set of ordinary truthmakers.) Likewise FALSEMAKERS. We write $\models^+$ and $\models^-$ for the TRUTHMAKING and FALSEMAKING relations. $|\varphi|^+$ is $\{<s, \boldsymbol{s}>|\ <s, \boldsymbol{s}>\models^+\varphi\}$ and $|\varphi|^-$ is $\{<s, \boldsymbol{s}>|\ <s, \boldsymbol{s}>\models^-\varphi\}$. As before we write st ($\mathbf{st}$) for the union of s and t ($\mathbf{s}$ and $\mathbf{t}$).

$$<s,\boldsymbol{s}>\models^+ A \text{ iff: } s=\{A\},\ \mathbf{s}=\{\varnothing\},$$
$$<s,\boldsymbol{s}>\models^- A \text{ iff: } s=\{\overline{A}\},\ \mathbf{s}=\{\varnothing\}$$
$$<s,\boldsymbol{s}>\models^+\neg\varphi \text{ iff: } <s,\boldsymbol{s}>\models^-\varphi$$
$$<s,\boldsymbol{s}>\models^-\neg\varphi \text{ iff: } <s,\boldsymbol{s}>\models^+\varphi$$
$$<s,\boldsymbol{s}>\models^+\varphi\vee\psi \text{ iff: } <s,\boldsymbol{s}>\models^+\varphi \text{ or } <s,\boldsymbol{s}>\models^+\psi$$
$$<s,\boldsymbol{s}>\models^-\varphi\&\psi \text{ iff: } <s,\boldsymbol{s}>\models^-\varphi \text{ or } <s,\boldsymbol{s}>\models^-\psi$$
$$<s,\boldsymbol{s}>\models^+\varphi\&\psi \text{ iff: } <s,\boldsymbol{s}>=<pq,\boldsymbol{pq}> \ldots \text{ where } <p,\boldsymbol{p}>\models^+\varphi \text{ and } <q,\boldsymbol{q}>\models^+\psi$$
$$<s,\boldsymbol{s}>\models^-\varphi\vee\psi \text{ iff: } <s,\boldsymbol{s}>=<pq,\boldsymbol{pq}> \ldots \text{where } <p,\boldsymbol{p}>\models^-\varphi \text{ and } <q,\boldsymbol{q}>\models^-\psi$$
$$<s,\boldsymbol{s}>\models^+\Box\varphi \text{ iff: } s=\varnothing \text{ and } \boldsymbol{s} \text{ intails } |\varphi|^+ \ (|\varphi|^+\leq\mathbf{s})$$
$$<s,\boldsymbol{s}>\models^-\Box\varphi \text{ iff: } s=\varnothing \text{ and } \mathbf{s} \text{ includes } |\varphi|^- \ (|\varphi|^-\sqsubseteq\mathbf{s})$$

A word finally about the fourth option mentioned above, the one that integrates the semantics and pragmatics more tightly. Verification as we have defined it is "strong." $\mathbf{s}$ to verify $\Box\varphi$ must inclusively entail $|\varphi|^+$, which means that in addition to each licit-maker in $\mathbf{s}$ implying a truthmaker for φ, each truthmaker for φ has got to be implied by a licit-maker.

The idea that *every-way-of-φing-is-equally-good* makes sense if φ is commanded—the Boss knows exactly which behavior meets her requirements, and has no reason to throw a disjunct into φ that is made true by behavior she dislikes or is indifferent to. But it makes no sense at all if φ is just a report, for the Reporter may know only that if you

don't φ at all, your behavior will not be in compliance. This suggests we add a weaker form of verification in addition to

$$[\Box^+]\ \mathbf{s} \vDash^+ \Box\varphi \text{ iff } \mathbf{s} \text{ entails and includes } |\varphi|^+,$$

namely

$$[\Box_+]\ \mathbf{s} \vDash_+ \Box\varphi \text{ iff } \mathbf{s} \text{ entails } |\varphi|^+.$$

One might then articulate a BRIDGE PRINCIPLE linking different kinds of speech act (command, report) with different kinds of update rules, according to how strongly we want the output sphere to verify the prejacent. It is not hard to see, for instance, that our strong necessity-adding update rule for $\Box$ yield spheres strongly verifying $\Box\varphi$, while $\Box\varphi$ is only weakly verified if we employ a rule which leaves the sphere unchanged when φ is already mandatory, in the sense of being implied by each licit-maker. To command is to insist on the strong update rule. Bosses insist on strong update because that's the rule that yields strong results.

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