ACTSC 445: Assignment 1

Daniel Matheson: 20270871

May 29^{th} 2017

Question 1:
You are given the following information on the AIG Credit Default Swap (CDS) scandal:
AIG Liquidity crisis and government bailout (Wikipedia, 31 May, 2015)
American International Group, Inc. also known as AIG is an American multinational insurance corporation headquartered in New York City. It operated a relatively small Financial Products division, headed by Joseph Cassano, in London, England.
Cassano sold hundreds of billions of dollars of credit default swaps (CDS), which provided insurance against default of structured debt securities, known as CDOs.
The CDOs were created by combining large numbers of subprime mortgages into a fund, and then selling units of that fund. Each individual mortgage had a relatively high default probability, but because large numbers were combined, AIG believed that the diversification benefit would make the rates of default very predictable, and easily covered by the premiums they received. In 2007 Cassano told analysts "It is hard for us, and without being flippant, to even see a scenario within any realm of reason that would see us losing $1 in any of those transactions."
Cassano was not required to hedge, reinsure or hold additional capital for this business, as a result of the deregulation of banks through the early 2000s.
In the financial crisis of 2008, investors started to request the insurance money as the began to default, following widespread failures of subprime loans. But AIG had no capital set aside for paying out on these insurance policies.
AIGs credit rating was downgraded and it was required to post additional collateral with its trading counterparties, leading to a liquidity crisis that began on September 16, 2008 and essentially bankrupted all of AIG.
The United States Federal Reserve Bank stepped in, announcing the creation of a secured credit facility of up to US$85 billion to prevent the companys collapse, enabling AIG to deliver additional collateral to its credit default swap trading partners. It was the largest government bailout of a private company in U.S. history.

(a) Identify four types of risk that were significantly involved in the AIG scandal.
(b) For each risk identified in part (a), suggest ways in which the risk management could have been improved.

Solution: (a)

1. The first - and most obvious - type of risk significantly involved in the AIG scandal was Stock Market risk. AIG had exposed themselves to significant risk in selling hundreds of billions of dollars of Credit Default Swaps. These CDS were providing insurance for Collateral Debt Obligations (CDOs) that were created with a large amount of subprime mortgages, and hence this move came with very large Stock Market risk.
2. The second significant type of risk involved was Strategic/Pricing Risk. AIG's Financial Products division, lead by Cassano, were assuming that diversification could make the

default rates predictable, and hence the risk was easily managed by the premiums they received. Their predictions were clearly wrong. Cassano confidently told analysts: *"It is hard for us, and without being flippant, to even see a scenario within any realm of reason that would see us losing $1 in any of those transactions."*. This could also be considered Operational Risk, if Cassano was being very dominant in the quantitative decisions of the Financial Products division.

3. The third type of significant risk involved was Liquidity Risk, which nearly bankrupted AIG. They were not setting aside any capital for large (or a large number of) claims. Hence, when the financial crisis occurred and the subprime loans began their widespread failure, AIG had no liquidity to cover the requests for insurance policy payouts.
4. Lastly, and most importantly, was Systemic Risk. While AIG, from the information given, seem to have been aware that the subprime loans involved were risky - they could not have known the so-called "unknown unknown" of the 2008 financial crisis. It was a $\geq 50\sigma$ event which was not accounted for in both their models and financial regulations in the United States.

(b)

1. *Stock Market Risk.* AIG could have lowered the impact of the stock market risk arising from these insurance products by hedging or reinsuring, which Cassano was not required to do, and did not do. Perhaps this was again a failure of a dominant CEO, but AIG also thought that diversification was enough to lower their risk.
2. *Strategic/Pricing Risk.* There isn't much that AIG could have done differently in their failure to price the insurance policies correctly. They were pricing them under the assumption that nothing like the financial crisis would occur. Therefore, the only way that they could have avoided such a catastrophe through pricing could have been to factor in such rare ($\geq 50\sigma$) events. But this would likely make their insurance products much more expensive than competing firms, as most firms (pre-2008) were not going to be so "paranoid" as to include such rare events.
3. *Liquidity Risk.* This type of risk was easily mitigated: AIG only had to hold more capital. Even without the 2008 financial crisis, it was foolish not to hold any capital, considering the sheer nominal amounts involved in the insurance contracts. Even if a lesser crisis would have happened, they could have had a liquidity crisis; it just wouldn't have been as bad as needing to be bailed out by the Federal Reserve.
4. *Systemic Risk.* Not very much could have been done to avoid this risk. Since 2008, there have been attempts by both firms and government to reduce this type of risk; through regulations like Dodd-Frank[1] (which Trump is now repealing parts of right now)[2] and through practices such as (and not limited to) Systemic Risk Regulation, Capital Requirements, the establishment of the Financial Stability Oversight Council, and the Volcker Rule (which prohibits proprietary trading).

[1] https://corpgov.law.harvard.edu/2010/11/20/the-financial-panic-of-2008-and-financial-regulatory-reform/
[2] https://www.theatlantic.com/business/archive/2017/02/trump-dodd-frank/515646/

Question 2: In the Assignments tab in LEARN there is an Excel File called MC Output Assignment 1. This gives the results of a simulation of 10,000 values from a loss distribution. The values are not sorted. In each question below involving calculations, you should explain your work, with formulas as appropriate.

(a) Estimate the 97% VaR
(b) Construct a 95% confidence interval for the 97% VaR using the order statistics method
(c) Estimate the 95% CTE
(d) Estimate the standard error of your CTE estimate using the Manistre and Hancock formula
(e) Use your estimates to calculate an approximate 95% confidence interval for the 95% CTE
(f) Now treat the simulated losses as 10 samples of size 1000
  (i) Estimate the 97% VaR and its standard error using the repeated simulation approach
  (ii) Estimate the 95% CTE and its standard error using the repeat simulation approach
(g) Comment on the differences in your estimates
  (i) for the VaR
  (ii) for the CTE

Solution: Note: for these questions, results are provided, and refer to code posted in Appendix A to see how the results were obtained. When possible, results will be explained in terms of formulae.

(a) $\widehat{\text{VaR}}_{97\%} = L_{(9700)} = 1362.4$

(b) In class, we derived that the q-Confidence Interval for VaR_α. given N observations is

$$\left(L_{(N\alpha-k)}, L_{(N\alpha+k)}\right)$$
$$\text{such that } k = Z_{\frac{1+q}{2}}\sqrt{N\alpha(1-\alpha)}$$
$$\text{where } \Phi\left(Z_{\frac{1+q}{2}}\right) = \frac{1+q}{2} \qquad (\Phi \text{ is the std. normal CDF})$$

Where we round $\{\alpha N - k, \alpha N + k\} \to \{\lfloor \alpha N - k \rfloor, \lceil \alpha N + k \rceil\}$
$\Rightarrow$ the 95% confidence interval for $\text{VaR}_{97\%}$ is $[1306, 1444]$.

(c) We estimate the 95% CTE by using the formula

$$\widehat{\text{CTE}}_\alpha(L) = \frac{1}{N(1-\alpha)} \sum_{j=N\alpha+1}^{N} L_{(j)}$$

The result is $\widehat{\text{CTE}}_{95\%} = 1762.389$

(d) We estimate the standard error of $\widehat{\text{CTE}}_{95\%}$ using the Manistre & Hancock method:

$$\text{Var}\left[\widehat{\text{CTE}}_\alpha\right] = \frac{1}{N(1-\alpha)}\left(S_n^2 + \alpha\left(\widehat{\text{CTE}}_\alpha - \hat{\text{VaR}}_\alpha\right)^2\right)$$

where $S_n^2 = \frac{1}{N(1-\alpha)-1}\sum_{j=N\alpha+1}^{N}\left(L_{(j)} - \widehat{\text{CTE}}_\alpha\right)^2$

We get that $\text{Var}(\widehat{\text{CTE}}_{95\%}) = 2287.864 \Rightarrow \text{SE}(\widehat{\text{CTE}}_{95\%}) = \sqrt{\text{Var}(\widehat{\text{CTE}}_{95\%})} = 47.83162$

(e) Due to the large number of observations, and the large number of random variables involved in determining the CTE we can assume that it follows a normal distribution, but we know that $\widehat{\text{CTE}}_{95\%} = 1762.389$ and $\text{SE}(\widehat{\text{CTE}}_{95\%}) = 47.83162$, hence $\text{CTE}_{95\%} \sim \mathcal{N}(1762.389, 47.83162^2)$.
Therefore the 95% confidence interval is

$$\begin{aligned}&(1762.389 - 1.96 \times 47.83162, 1762.389 + 1.96 \times 47.83162)\\ &= (1668.639, 1856.139)\end{aligned}$$

(f) (and (g)) In order to estimate the losses with 10 samples of 1000, we calculate the $\text{VaR}_{97\%}$ and $\text{CTE}_{95\%}$ for every 100 elements, then take the mean of the 10 values for the estimate; and the standard deviation of these 10 values for the standard error. The results are:
  (i) $\widehat{\text{VaR}}_{97\%} = 1351.57$ with $\text{SE}(\widehat{\text{VaR}}_{97\%}) = 137.0208$. The previously calculated 97% VaR was 1362.4 with a 95% confidence interval $[1306, 1444]$ which contains this new estimate.
  (ii) $\widehat{\text{CTE}}_{95\%} = 1754.0708$ with $\text{SE}\left(\widehat{\text{CTE}}_{95\%}\right) = 157.1584$. The value for CTE is relatively close (it was 1762.389 previously) but the standard error is larger (157.1584 compared to 47.83162). This is not unexpected due to the small sizes of the samples that were used.

This result should have been expected: when we work with the VaR confidence intervals we have $k = Z_{\frac{1+q}{2}}\sqrt{N\alpha(1-\alpha)}$; and $\sum_{i=1}^{10}\sqrt{100} = 100 > 31.62 = \sqrt{1000}$. Hence taking 10 samples of 1000 observations results in larger error.
The same principle applies to the standard error of the CTE since it is calculated based on the VaR values for each sample of 1000.

Question 3: The variance principle is the risk measure

$$\rho(X) = E[X] + \alpha\sigma_X^2$$

where $\alpha > 0$ and σ_X is the standard deviation of X.
Identify which of the coherency criteria, if any, are satisfied by the variance principle. Justify your answers

Solution: We will simply check each criterion for coherence one by one:

1. Translation Invariance:
   Let $c \in \mathbb{R}$. Then

$$\begin{aligned}\rho(X+c) &= E[X+c] + \alpha\,\text{Var}(X+c)\\ &= E[X] + c + \alpha\,\text{Var}(X)\\ &= (E[X] + \alpha\,\text{Var}(X)) + c\\ &= \rho(X) + c\end{aligned}$$

   Where $\text{Var}(X) = \text{Var}(X+c)$ since Var is translation invariant.
   Hence $\rho(X)$ is translation invariant.
2. Linear Homogeneity.
   Let $\lambda > 0$. Then

$$\begin{aligned}\rho(\lambda X) &= E[\lambda X] + \alpha\,\text{Var}(\lambda X)\\ &= \lambda E[X] + \alpha\lambda^2\,\text{Var}(X)\\ &= \lambda\left(E[X] + \alpha\lambda\,\text{Var}(X)\right) \neq \lambda\rho(X)\end{aligned}$$

   So $\rho(X)$ does not exhibit linear homogeneity. This was expected since Var is not linearly homogeneous.
3. Sub-additivity.

Let X, Y be random losses with standard deviations σ_x, σ_y and covariance σ_{xy}. Then

$$\begin{aligned}\rho(X+Y) &= E[X+Y] + \alpha \operatorname{Var}(X+Y) \\ &= E[X] + E[Y] + \alpha\left(\operatorname{Var}(X) + \operatorname{Var}(Y) + 2\operatorname{Cov}(X,Y)\right) \\ &= (E[X] + \alpha \operatorname{Var}(X)) + (E[Y] + \alpha \operatorname{Var}(Y)) + 2\alpha \operatorname{Cov}(X,Y) \\ &= \rho(X) + \rho(Y) + 2\alpha \operatorname{Cov}(X,Y)\end{aligned}$$

but $\rho(X) + \rho(Y) + 2\alpha \operatorname{Cov}(X,Y) \le \rho(X) + \rho(Y)$

$$\iff 2\alpha \operatorname{Cov}(X,Y) \le 0$$

Hence $\rho(X)$ is sub-additive if for every choice of X, Y we have $\operatorname{Cov}(X,Y) \le 0$ (since $\alpha > 0$) which is clearly not always the case since we can easily have positive covariance. Hence $\rho(X)$ is not sub-additive.

4. Monotonicity. Assume $P[X \le Y] = 1$.
   $\rho(X) = E[X] + \alpha\sigma_X^2$ and $\rho(Y) = E[Y] + \alpha\sigma_Y^2$.
   $P[X \le Y] = 1 \Rightarrow E[X] \le E[Y]$ but this tells us nothing about the variance of X and Y.;
   i.e $\operatorname{Var}(X) = E[X^2] - E[X]^2$, but $E[X] \le E[Y] \nRightarrow E[X^2] \le E[Y^2]$. So we cannot say that $\rho(X)$ is Monotonic without more information.

$\square$

Question 4: A trader has a \$0 $\mathrm{VaR}_{0.99}$ constraint for her 10-trading day losses. That is, her trading strategy must have a 99% VaR less than or equal to 0 for a 10-day horizon. Assume 256 trading days in a year.

She has constructed a portfolio of derivatives on a non-dividend paying stock with initial value $S_0 = 100$. The value at $T = \frac{10}{256}$ of $\frac{S_T}{S_0}$ follows a log-normal distribution with parameters $T\mu$ and $T\sigma^2$ where $\mu = 0.05, \sigma = 0.3$.

She has decided on the following strategy:

- Purchase 1 million ten-day call options with a strike price of 87
- Sell short 1 million ten-day call options with a strike price of 91
- Finance the cost of the call options by selling short M ten-day put options with a strike price of 87.

All the options are priced using the Black Scholes formulas, with risk free rate $r = 0.04$, and volatility $\sigma = 0.30$ per year. You are given that the price of a T-year call option with strike K is

$$c = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

and the price of a T-year put option with strike K is

$$p = Ke^{-rT}\Phi(-d_2) - S_0\Phi(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

and $\Phi(x)$ is the CDF of the standard normal distribution.

(a) Show that M, the number of put options sold such that the strategy has zero initial cost is 240 million, to the nearest million. You should calculate M to the nearest 100,000

(b) Complete the table.

(c) Calculate the 99% VaR for the loss at T and verify that it complies with the trader's constraint.

(d) Comment on the advantages and disadvantages of using the 99% VaR as a trading constraint. Do you think this is a good way to manage the traders risk exposure? Would it be better to use a 99.5% VaR? Or would you recommend a CTE constraint? Justify your answer.

Solution: (a) To find M we have to find the value of the the ten-day call options with $K_1 = 87$ (denoted C_1) and the ten-day call options with $K_2 = 91$ (denoted C_2):

$$\begin{aligned} C_1 &= S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2) \\ \ldots &= 13.15203 \\ \text{and } C_2 &= 9.268234 \end{aligned}$$

We purchase 1 million C_1 and short 1 million C_2, and finance the cost with M P_1 put options where

$$\begin{aligned} P_1 &= Ke^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \\ &= 0.01619608 \end{aligned}$$

Hence $M = \frac{1000000\times C_1 - 1000000\times C_2}{P_1} = \frac{13152030-9268234}{0.01619608} = 239,798,539 \sim 240,000,000$

(b)

Stock Price at T	Prob. $S_T \leq$ this value	Loss on Portfolio (millions)
90	3.5155947%	-3
89	2.2839947%	-2
88	1.4301630%	-1
87	0.8617351%	0
86	0.4988276%	239.7985
85	0.2769433%	479.5971

(c) 99% VaR for loss at T:
Since the loss is a decreasing function of S_T, from the course notes we know that $Q_\alpha(g(S_T)) = g(Q_{1-\alpha}(S_T))$. i.e: the 99% VaR for Loss is the Loss at the 1% quantile of S_T. So we consider that $Z_{0.01} = -2.325$ to find the value of the 1% quantile of S_T:

$$\Rightarrow \frac{\ln\left(\frac{S_T}{S_0}\right) - 0.05T}{0.3\sqrt{T}} = -2.325$$

$$\Rightarrow \ln(S_T) = -2.325(0.3)\sqrt{\frac{10}{256}} + 0.05\left(\frac{10}{256}\right) + \ln 100$$

$$\Rightarrow S_T = \exp\left(-2.325(0.3)\sqrt{\frac{10}{256}} + 0.05\left(\frac{10}{256}\right) + \ln 100\right)$$

$$\Rightarrow S_T = 87.293$$

And the loss for $S_T = 87.293$ is \$293,000. Which complies with the trader's \$0 10-day 99% VaR.

(d) The advantage of using 99% VaR is that it is easy to calculate, and we can also easily find a confidence interval for VaR. However, in this instance; it is clear that 99% VaR was not sufficient in actually explaining the amount of risk involved in this trading strategy. As we can see in the table from part (b); if we were to use a 99.5% VaR which is not that much higher of a quantile; it would be close to a loss of \$240 million which is huge; much larger than the original investments at time zero.

However, regardless of the VaR quantile used in practice, it would be best to use a CTE constraint in addition to the VaR, to know how large the tails really are. In our example, the trader could have felt confident in their trading strategy, but in 1% of cases, the losses would be enormous, and if they had not checked their 99% CTE - because it was not a constraint they were meant to follow - they could be setting themselves up for a disaster. With losses in the range of 240 to 480 million or more; they could be looking at bankrupting their firm (depending on their size) and getting fired.
A CTE constraint coupled with a VaR constraint would be a more thorough safety-net, and reduce potentially large fat-tail risks. And it is not that much more computationally expensive to calculate both.

Appendix A: Code

```
###############################
# ACTSC 445 Assignment 1      #
# Daniel Matheson: 20270871   #
#        May 27th 2017        #
###############################

### Question 2 Code

# Import data
setwd("C:/Users/Daniel/Dropbox/Spring 2017/ACTSC 445/Assignment 1")

# Take losses values only
losses_data <- read.csv("MC_Output_A1.csv",header=T)
losses <- losses_data$Simulated.Losses.using.Monte.Carlo

# Sorted losses for estimating VaR/CTE
sorted_losses <- sort(losses)

N <- length(losses) # number of observations

### Question 2(a):
alpha2a <- 0.97
q2a <- 0.97 * N
VaR2a <- sorted_losses[q2a]

### Question 2(b):
alpha2b <- 0.97
q <- 0.95       # 95% confidence interval
qc <- (1+q)/2  # centered quantile; as in equal tails
k <- qnorm(qc, 0,1) * sqrt(N * alpha2b * (1 - alpha2b))
k <- ceiling(k)
CI2b <- c(sorted_losses[N * alpha2b - k], sorted_losses[N * alpha2b + k])

### Question 2(c)
alpha2c <- 0.95
cte_values <- sorted_losses[(N * alpha2c + 1):N]
cte2c <- (1/(N * (1-alpha2c))) * sum(cte_values)

### Question 2(d)
alpha2d <- 0.95
q2d <- alpha2d * N
VaR2d <- sorted_losses[q2d]
```

```r
S_n <- (1/(N*(1 - alpha2d) -1)) * sum((cte_values - cte2c)^2)
varcte <- (1/(N*(1 - alpha2d))) *(S_n + alpha2d *(cte2c - VaR2d)^2)
ctese <- sqrt(varcte)

### Question 2(f)
n <- 1000
VaR2f <- rep(0,10)
VaRCI2f <- rep(0,20)
dim(VaRCI2f) <- c(10,2)
CTE2f <- rep(0,10)
alphaVaR <- 0.97
qVaR <- n * alphaVaR
alphaCTE <- 0.95
qCTE <- n * alphaCTE
losses_temp <- rep(0,n)

for(ii in 1:10){
  losses_temp <- losses[((ii-1)*n+1):(ii*n)]
  losses_temp <- sort(losses_temp)
  VaR2f[ii] <- losses_temp[qVaR]
  cte_values2f <- losses_temp[(qCTE + 1):n]
  CTE2f[ii] <- (1/(n * (1- alphaCTE))) * sum(cte_values2f)
}

VaR2fmean <- mean(VaR2f)
CTE2fmean <- mean(CTE2f)
VaR2fsd <- sd(VaR2f)
CTE2fsd <- sd(CTE2f)

### Question 4(a)
S_0 <- 100
r <- 0.04
sigma <- 0.3
T <- 10/256
K <- 87
d_1 <- (log(S_0/K) + (r + 1/2 * sigma^2) * T)/(sigma * sqrt(T))
d_2 <- d_1 - sigma * sqrt(T)
phi1 <- pnorm(d_1,0,1)
phi2 <- pnorm(d_2,0,1)
C <- S_0 * phi1 - K * exp(-r * T) * phi2

phi1put <- pnorm(-d_1,0,1)
phi2put <- pnorm(-d_2,0,1)
P <- K * exp(-r * T) * phi2put - S_0 * phi1put

### Question 4(b)
mu <- 0.05
S_T = c(90,89,88,87,86,85, 87.293)
obs <- ((log(S_T/S_0)) - mu * T)/(sigma * sqrt(T))
p_obs <- pnorm(obs,0,1)

C1_val <- pmax(S_T - 87,0)
C2_val <- - pmax(S_T - 91,0)
P1_val <- - pmax(87 - S_T,0)
M <- 239798539

portfolio_value <- 1000000 * (C1_val + C2_val) + M * P1_val
```