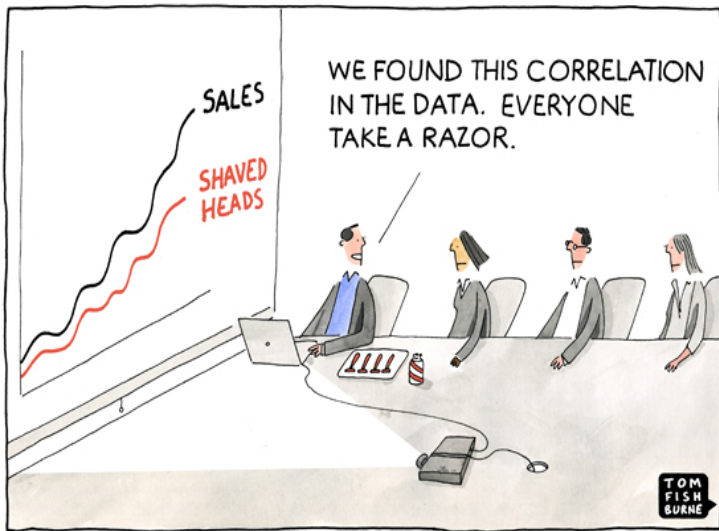


# Introduction to Structural Causal Models

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# Motivation



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Table 1: (Pearl 2012)

Association-based Concepts	Causal Concepts
Correlation	Randomization
Regression	Confounding
Conditional Independence	Disturbance
Likelihood	Error Terms
Odds Ratio	Structural Coefficients
Propensity Score	Spurious Correlation

# Table of Contents

- Foundations of SCMs
  - Assumptions
  - Comparing Causal Tools
- Pearl's Causal Hierarchy
  - Prediction
  - Intervention
  - Counterfactuals
- Graphical Models
- Causality and Time

- Topic 1: **Structural Causal Models**
- Topic 2: Linking d-separation with (conditional) independence & Causal Structure Learning
- Topic 3: do-calculus: estimating the effect of interventions & counterfactuals
- Topic 4: Rubin's Potential Outcome Framework & Causal Trees
- Topic 5: Algorithmic Recourse
- Topic 6: Causal Interpretation of Black-Box Models

# Foundations of SCMs (1)

- Flexible simulations for higher order problems (intervention, counterfactual)
- based on (elementary) 'noise'/latent variables
- Graphical visualization via directed acyclic graph
- Nonparametric SEM
- System of equations with functions

# Foundations of SCMs (2)

- **Assignments**  $':='$  (non-symmetric)
- algebraic equation  $'='$  (symmetry)
- **Error in Regression:** Omittable outside factor
- **Error in SCM:** Latent (influential) factor that is pivotal for the model

$$C := N_C$$

$$E := f_E(C, N_E)$$

Source: Peters, Janzing, and Schölkopf (2017)

# Assumptions - Independence

- Noise terms independent ( $N_c, N_e$ )
- Mechanisms independent (other variables invariant)(local changes)
- Change in distribution stems from change in mechanism
- Causal Markov Condition



## SEM:

- Conflict whether to use **graphs** or not and to use **parameters** in SEM
- SEM is a parametric model used in applied sciences

## BCN:

- A Bayesian causal network (BCN) use conditional probabilities instead of functions
- Every SCM has the same information as respective probabilistic model + more (DGP)
- Computationally less efficient and cannot cast counterfactuals

## Potential Outcome Framework:

- PO is mostly equivalent but has not graphical framework
- SCMs can derive potential outcomes but PO cannot derive SCMs

# Table of Contents

- Foundations of SCMs
  - Assumptions
  - Comparing Causal Tools
- **Pearl's Causal Hierachy**
  - Prediction
  - Intervention
  - Counterfactuals
- Graphical Models
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# Pearls Causal Hierachy

Table 2: Pearls Hierachy of Causation (2009)

Method	Action	Example	Usage
Association $P(a b)$	Co-occurrence	What happened. . .	(Un-)Supervised ML, BN, Reg.
Intervention $P(a do(b), c)$	Do- manipulation	What happens if . . .	CBN,MDP,RL
Counterfactual $P(a_b a', b')$	Hypotheticals	What would have happened if. . .	SCM ,PO

- Mathematical Tool: do-calculus
- The do-calculus enables us to study the manipulation of parent nodes (direct effect)
- **Atomic intervention:** where we set a variable to a constant
- **Policy intervention:** we specify a different function for an equation
- CBN , MDP and reinforcement learning model intervention.

# Example Intervention (1)

## Atomic Intervention:

- replacing function with a constant

$$C_1 := f_{c_1}(p, N_{c_1}) \rightarrow C_1 := 600$$

$$C_2 := f_{c_2}(a, N_{c_2})$$

$$E := f_E(C_1, C_2, N_E)$$

# Example Intervention (2)

## Policy Intervention:

- replacing function with a different conditional probability

$$C_1 := f_{c_1}(p, N_{c_1}) \rightarrow C_1 := f(\pi)$$

$$C_2 := f_{c_2}(a, N_{c_2})$$

$$E := f_E(C_1, C_2, N_E)$$

- Missing data problem in PO framework
- SCM Process is described as follows:
  - Ⓐ Abduction: Cast probability  $P(u)$  as conditional probability  $P(u|\epsilon)$
  - Ⓑ Action: Exchange ( $X = x$ )
  - Ⓒ Prediction: Compute ( $Y = y$ )

- **SUTVA** ‘The treatment that one unit receives does not change the effect of treatment for any other unit.’
- **Consistency** The outcome  $Y$  agrees with the potential outcome corresponding to the treatment indicator.’
- **Ignorability** The potential outcomes are conditionally independent of treatment given some set of de-confounding variables. (perfect RCT)
- First two hold for Counterfactuals in SCM
- third not testable but can check via backdoor criterion in SCM
- Source: (Hardt and Recht 2021)



# Table of Contents

- Foundations of SCMs
  - Assumptions
  - Comparing Causal Tools
- Pearl's Causal Hierarchy
  - Prediction
  - Intervention
  - Counterfactuals
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- Nodes  $\rightarrow$  Variables (endogenous/exogenous)
- Edges  $\rightarrow$  relationship (equations)
- Parents/Ancestors/Descendants
- No need to specify exact parametric shape
- Focus on acyclical structures (DAG)
- Estimation back door criterion
- Test theoretical model structure via causal algorithms to detect structure in data (IC/PC Algo.)
- More information: (Morgan and Winship 2014)

# Undirected/Semidirected Graph

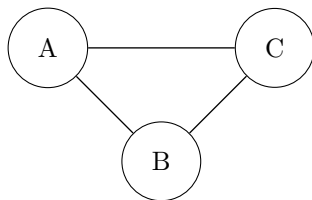


Figure 1: Undirected graph

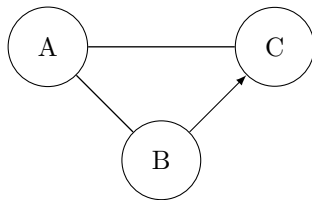


Figure 2: Semi-directed graph

# Graphical Illustration - Probabilistic Model

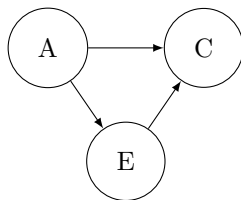


Figure 3: Probabilistic Model

# Graphical Illustration - Structural Causal Model

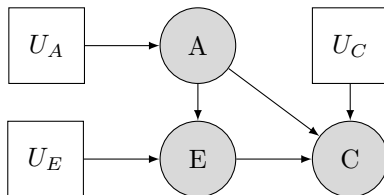


Figure 4: Structural Causal Model

# Table of Contents

- Foundations of SCMs
  - Assumptions
  - Comparing Causal Tools
- Pearl's Causal Hierarchy
  - Prediction
  - Intervention
  - Counterfactuals
- Graphical Models
- **Causality and Time**

# Causal Modelling with Differential Equations

- Time in Social Sciences: Often Vague
- Time in Physical Sciences: Mechanical via **Differential equations**
- dependence on prior time point and change in time contribute to the value at time point

Initial Value:

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

Derivative of function  $\mathbf{x}$  with respect to time  $t$ :

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}), \mathbf{x} \in \mathbb{R}^d$$

Value of Function at time  $t + dt$ :

$$\mathbf{x}(t + dt) = \mathbf{x}(t) + dt \cdot f(\mathbf{x}(t))$$

# Graphical Overview

model	IID setting	changing distributions	counter-factual questions	physical insight
mechanistic model	Y	Y	Y	Y
structural causal model	Y	Y	Y	N
causal graphical model	Y	Y	N	N
statistical model	Y	N	N	N

Source: (Peters, Janzing, and Schölkopf 2017)



# Concluding Remarks

- Association-based learning is easy to model because of lower information necessary
- but not always appropriate in high stake settings
- SCM as simulator for causal modelling based on noise variables
- SCMs entail a lot of information (DGP, intervention distribution)
- BUT: **“Garbage in, Garbage out”**
- Computational advantage casting causal model as assignments
- Enables modelling of higher order concepts like counterfactuals
- Extensions through differential equations for concise modelling of time

## If you...

- ..only have conditional probabilities
- ..are only interested in association
- ..have very limited knowledge of your actual underlying structure

→ you should probably go for a probabilistic model, association based model or a model based on conditional probabilities

## But if you

- ..have an understanding of underlying relationships in the data
- ..are dealing with high stake settings
- ..want to simulate different interventions

→ you could probably go for a SCM

# References

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- Peters, Jonas, Dominik Janzing, and Bernhard Schölkopf. 2017. *Elements of Causal Inference: Foundations and Learning Algorithms*. Adaptive Computation and Machine Learning Series. Cambridge, Massachusetts: The MIT Press.

# Appendix

Method	CBN	SCM
Prediction	<ul style="list-style-type: none"><li>• Unstable</li><li>• Volatile to parameter changes</li><li>• Re-Estimate entire model</li></ul>	<ul style="list-style-type: none"><li>• Stable</li><li>• More Natural Specification</li><li>• Only estimate <math>\Delta</math> CM</li></ul>
Intervention	<ul style="list-style-type: none"><li>• Costly for Non-Markovian Models</li><li>• Unstable(Nature CP)</li><li>• Only generic estimates(<math>\Delta</math> CP)</li></ul>	<ul style="list-style-type: none"><li>• Pot. Cyclic Representation</li><li>• Stable</li><li>• Context specific</li></ul>
Counterfactuals	<ul style="list-style-type: none"><li>• <b>Impossible</b></li><li>• no information on latent factors(<math>\epsilon</math>)</li></ul>	<ul style="list-style-type: none"><li>• Possible</li><li>• Inclusion of latent factors</li></ul>

# Questions:

Thank you so much for your time!

If you have any questions contact me via:

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