

A Gentle Introduction into Structural Causal Models

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Abstract The interest in understanding relationships of variables beyond co-occurrence has increased the popularity of causal modelling. To provide a comprehensive understanding of causal modelling, I introduce two prominent causal model specifications namely (1) Bayesian Causal Networks (BCN) and (2) Structural Causal Models (SCM), focusing on the latter. Probabilistic specifications such as a BCN cast a model based on conditional probabilities. SCMs cast a model based on assignment functions and extend probabilistic models by specifying the data generating process rather than solely utilizing conditional probabilities. Another difference between these models is their ability to address different queries such as *predictions*, *interventions* and *counterfactuals*. These queries are part of Pearl’s causal hierarchy (2009). Pearl matches these queries with their respective actions namely *observing*, *doing* and *imagining*. I compare the feasibility of addressing these queries and undertaking respective actions for both specifications. To contextualize SCMs within the field of causality, I also discuss the role of time in causality. This paper uses various directed acyclic graphs to highlight the differences in these modelling approaches. The insights of this paper can be used as a baseline for subsequent research on structural causal models.

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1 Introduction

Most students have heard the phrase ‘correlation does not imply causation.’ While correlation implies co-occurrence, for many problems correlation is not enough. Algorithmic decision making based on co-occurrence is insufficient in high stake settings [1]. For many problems we want to understand causal relationships between variables. There are different approaches on how to model causal relationships. This paper focuses on Structural Causal Models and Bayesian Causal Networks. Bayesian Causal Networks (BCN) cast a model based on **conditional probabilities**. Structural Causal Models (SCM) specify relationships based on **functional equations** [2]. Summarized, ‘they are simulators that generate a distribution based on noise variables (independent) and a unique joint distribution...’ [3]. - noise variables help generate joint distribution [4].

- latent factors great [5][6].
- untestable assumptions [2] frowned upon by statistics for long time
- graphical framework better for modeling [7] and reason why this approach is better than others

SCMs are non-parametric structural equations models with added features. Existing literature has provided an excellent introduction to SCMs and BCNs. [2] provided a comprehensive introduction into this topic with his book ‘Causality.’ His work addressed misconceptions in social sciences, causality and statistics. His contribution on the hierarchy of causation advanced the comprehensive understanding of causality. [8] address the relationship of causality, physical sciences, and causal inference. Their work highlights the importance of time in causality and inference based on observational data alone. E.g. [1] worked on the application of causal methods in machine learning. [9] discuss the history of SEMs and SCMs. The aim of this paper is to summarise SCMs and it’s intersection with social sciences and physics. I structure the rest of the paper as follows: Section 2 introduces the assumptions in causal modelling. Further, section 2 discusses different causal differences, fundamental differences and a brief history of the methods. Section 3 addresses Pearls Causal Hierachy

2 Foundations of Structural Causal Models

Consists of graph and assignments: Baseline:

$$C := N_c$$

$$E := f_E(C, N_E)$$

source: [8]

2.1 Assumptions in Causality

Definition: Independence

The causal generative process of a system's variables is composed of autonomous modules that do not inform or influence each other. In the probabilistic case, this means that the conditional distribution of each variable given its causes (i.e., its mechanism) does not inform or influence the other conditional distributions, In case we have only two variables, this reduces to an independence between the cause distribution and the mechanism producing the effect distribution [8].

If we specify the causal structure correctly:

- (a) possible to undertake local intervention \rightarrow change $f(x)$, regardless of $f(y|x)$
- (b) these components are autonomous objects \rightarrow set of autonomous equations

Further: independence of noise \rightarrow

$$N_T, N_A$$

Example conditional probabilities: independence conditional probabilities/distribution and mechanism (ICM)

[8]

- noise independent \rightarrow called **causal sufficiency** clause.
- Algorithmic independence

SCM fulfills via :

- dependency only via vertice connection (noise)
- condition on noise, variables become independent

- Independence of mechanisms
- conditional independence:
- joint distribution

Density factorizes and their expectation

Causal assumptions differentiate causal models from association learning methods.

These causal concepts are not expressible based on distribution functions/statistical associations. (Pearl 2010)

Disturbance in SCM: Correlated and causal factor ; responsible for variation Disturbance in Regression: Uncorrelated [2]

Causal assumption not testable (e.g. $Cov(U_a, U_b) = 0$). d-seperation to test assumptions in totality (cannot make assumptions in isolation).

exploit invariant characteristics of SEM without committing to shape. structural if function autonomous and invariant to change in form of other functions.

Example:

(#TODO change to own example) Joint factorization sprinkler

$$P_{X_3=\text{On}}(x_1, x_2, x_4, x_5) = P(x_1) P(x_2 | x_1) P(x_4 | x_2, X_3 = \text{On}) P(x_5 | x_4)$$

$$P_x(v) = \prod_{\{i | v_i \notin X\}} P(v_i | pa_i)$$

for all v consistent with x

Definition: **Reichenbachs common causal Principle**

2.2 Comparing Causal Tools

A structural equation model (SEM) is a set of autonomous equations. This set represents the ‘state of the world.’ SEMs are popular in fields like economics, psychology and sociology [2]. SCM are also defined by a (sub-)set of structural equations. Additionally, SCMs feature mathematical components from graphical models and the potential-outcome framework [10]. There is a lot of controversy around SEMs. Many scholars challenge the parameter-specification in a SEM [10].¹ SCMs specify an underlying data generating process without the computational effort of creating a parametric model. Note, that structural equations refer to assignment equations, used in computer science. Equations in causal models did not always have a concise notation (Pearl, 2009). First, there was no sign to express the assignment equation and people used the ‘=’ and one would e.g. write $A = B$. Treating an equation as a **algebraic equation** led to confusion because those have no causal information. This algebraic equation would imply that $B = A$ because the order has no concrete meaning in algebraic equations. The problem is that the equation is symmetric. The initial ‘=’ sign was replaced with the ‘:=’ which is asymmetric (Pearl, 2009) and called an **assignment**. This misconception has caused a lot of challenges.² As mentioned, we define variables as functions e.g. $A = f_A(B, U_A)$. B defines A and the latent factor U_A . To summarize, a SCM consists of a set of (autonomous) equations to generate (a) endogenous variables and (b) exogenous variables.

¹Inconsistency refers to inherent nature of creating parameters based of observational data. Observational data is seldom consistent. Henceforth respective estimates are often questioned. For further information see [11]

²For more information see [2]

In Figure 1 we can see an example of a probabilistic model depicted as a directed acyclic graph (DAG). The nodes are the white circles. The edges are the arrows, defined by the conditional probabilities. C is our **collider** variable because the affect of A on C and T on C collide [2].

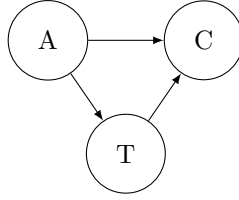


Figure 1: Probabilistic Model

In figure 2 we can see a vanilla structural causal model. The square nodes represent the latent variables.

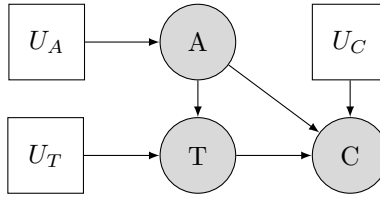


Figure 2: Structural Causal Model

The circle nodes represent the observed variables. The arrows are our structural equations and depict the relationship between variables. ^[Note that there are also cyclic structural causal models but no cyclic bayesian causal networks. For further information see [2]. Due to the confined scope of this paper, I will not explore cyclic structures.

Potential outcomes treat outcomes merely as potential outcomes rather than true outcomes. - Only observe outcomes for units that are treated - ATE is a typical query - Every SCM can generate PO - Cannot derive SCM from PO alone - SCM has more concepts : causal graphs, mediating paths, counterfactuals for every variable etc. - PO Easier to apply because apply to observational data [3]

2.3 A Brief History of Causal Modeling

Path analysis is the foundation of modern structural causal models. This path analysis is a structural equation model with one variable per indicator. Sewall Wright, a Statistician and Geneticist, introduced the topic in the 1920s. (Pearl, 2012) Various other disciplines such as econometrics, psychometrics and sociology adopted path analysis. Aside from path diagrams also introduced graph rules to formalise relationships. (tbc. Cowles, Haavelmo, Aldrich,)

3 Pearl's Causal Hierachy

To contextualise causal methods, [2] introduced the hierarchy of causation. Pearl focuses on three layers: association, intervention and counterfactuals (Table 1).

Method	Action	
Association $P(a b)$	Co-occurrence	(Un-)Supervised ML, BN, Reg.
Intervention $P(a do(b), c)$	Do-manipulation	CBN,MDP,RL
Counterfactual $P(a_b a', b')$	Hypotheticals	SCM ,PO

A higher level implies more detailed knowledge of the relationship between the variables. [2] introduced the do-calculus. The do-calculus is a mathematical framework allowing for active substitution (E.g. $X := 10$) [3]. The first query is association, where we examine relationships based on observations.

Table: Pearls Hierachy of Causation (2009)

3.1 Association

Vanilla machine learning (ML), bayesian networks (BN) and regression models (Reg) are at the lowest level in the causal hierarchy (see table 1). These methods demand the least information and depend on association alone. Associational methods ignore external changes outside of our data. The interventional distribution has information on these external changes. The interventional distribution is only defined in high level causal methods.

3.2 Intervention

The second query deals with interventions. Here we can use [2] do-calculus. The do-calculus enables us to study the manipulation of parent nodes. There are various types of intervention. One example is **atomic intervention**, where we set a variable to a constant. In **policy intervention** we specify a different function for an equation. off-policy intervention models different intervention that is not in our historical data [6]. Causal bayesian networks , Markov Decision Processes (MDP) and reinforcement learning model intervention. (tbc.) Hypothetical interventions:

3.3 Counterfactuals

The third query is counterfactual modelling. Here we deal with hypothetical settings. SCMs and potential outcome models allow for counterfactual modelling. These models can model counterfactuals because they

include a interventional distribution [6]. BCNs only entail conditional probabilities. There is no information on relationships outside of the observations in the data. Henceforth, they cannot create counterfactuals [2].

Process is described as follows:

- (a) Abduction: Cast probability $P(u)$ as conditional probability $P(u|\epsilon)$
- (b) Action: Exchange ($X = x$)
- (c) Prediction: Compute ($Y = y$)

Established Conditions for counterfactuals:

Definition: **SUTVA**

‘The treatment that one unit receives does not change the effect of treatment for any other unit.’ [3]

Definition: **Consistency**

The outcome Y agrees with the potential outcome corresponding to the treatment indicator.’ [3]

Definition: **Ignorability**

The potential outcomes are conditionally independent of treatment given some set of de-confounding variables. [3] (perfect RCT)

- First tow hold for SCM counterfactuals
- third not testable but can check via backdoor criterion in SCM

3.4 Implications

BCN:

SCM:

4 Graphical Models

Also called causal diagram - collider, mediator, fork [4]

4.1 Features

A graph in a SCM contains endogenous and exogenous variables In a directed graph all edges have arrows. Directed arrows are direct (causal) effects. No edge between variables means that there is no causal effect. If

we have some edges without arrows, we call that graph semi-directed. If we have all edges without arrows, we call that graph un-directed. The most famous graphical model is the acyclic graph. An acyclic graph has no roots that cause itself (directly and indirectly). Because we do not specify changes over time we typically only deal with acyclic models. This is partially because we assume the relationship to be constant over time. [5]

There are various tools to derive graphical models.³

- d-seperation
- vertices variables and arrows are direct effects.

4.2 Implications

- no need functional dependency -> less assumptions about form
- show collider variables
- iv backdoor, estimation by mechanism [7]
- backdoor criterion non experimental way remove confounding [4]

Any DAG can be written as SCM. given joint distribution can be discover graph? Markov condition

5 Causal Inference and Time

Another issue in this paper is understanding the relationship of time and causality. SCMs make the assumption that causal relationships hold over time[8]. These vague definitions of time is more prevalent in social sciences. Meanwhile hard sciences deal with time in a more concise manner. Differential equations model time on a more mechanic manner [12]. scriptsize

³The PC-Algorithm and the IC-Algorithm are two prominent examples.

model	IID setting	changing distributions	counter-factual questions	physical insight
mechanistic model	Y	Y	Y	Y
structural causal model	Y	Y	Y	N
causal graphical model	Y	Y	N	N
statistical model	Y	N	N	N

Table: Source: [\[8\]](#)

6 Conclusion

This paper provides a gentle introduction into structural causal models. SCMs entail many features, complementing research on association learning by providing depth. This in turn, is of particular benefit for high stake decision settings. SCMs differentiate from other methods through the specification of endogenous and exogenous variables, treating the exogenous factors as pivotal components of the actual model rather than assuming they are omittable errors that are uncorrelated. As suggested by Pearls Hierachy, there are different levels to learning and each higher step can do anything the prior step can but with more detail and information. Henceforth, as machin

7 References

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8 Appendix

Table 2: Source: Pearl (2009)

Method	CBN	SCM
Prediction	<ul style="list-style-type: none">• Unstable• Volatile to parameter changes• Re-Estimate entire model	<ul style="list-style-type: none">• Stable• More Natural Specification• Only estimate Δ CM
Intervention	<ul style="list-style-type: none">• Costly for Non-Markovian Models• Unstable(Nature CP)• Only generic estimates(Δ CP)	<ul style="list-style-type: none">• Pot. Cyclic Representation• Stable(Nature Eq.)• Context specific(Invariance of Eq.)
Counterfactuals	<ul style="list-style-type: none">• Impossible• no information on latent factors(ϵ)	<ul style="list-style-type: none">• Possible• Inclusion of latent factors