A Gentle Introduction into Structural Causal Models

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April 5th, 2021

Abstract The interest in understanding relationships of variables beyond co-occurrence has increased the popularity of causal modelling. To provide a comprehensive understanding of causal modelling, I introduce two prominent causal model specifications namely (1) Bayesian Causal Networks (BCN) and (2) Structural Causal Models (SCM), focusing on the latter. Probabilistic specifications such as a BCN cast a model based on conditional probabilities. SCMs cast a model based on assignment functions and extend probabilistic models by specifying the data generating process rather than solely utilizing conditional probabilities. Another difference between these models is their ability to address different queries such as *predictions*, *interventions* and *counterfactuals*. These queries are part of Pearl's causal hierarchy (2009). Pearl matches these queries with their respective actions namely *observing*, *doing* and *imagining*. I compare the feasibility of addressing these queries and undertaking respective actions for both specifications. To contextualize SCMs within the field of causality, I also discuss the role of time in causality. This paper uses various directed acyclic graphs to highlight the differences in these modelling approaches. The insights of this paper can be used as a baseline for subsequent research on structural causal models.

Contents

1   Introduction

Most students have heard the phrase 'correlation does not imply causation.' While correlation implies co-occurrence, for many problems correlation is not enough. Algorithmic decision making based on co-occurrence is insufficient in high stake settings (Bareinboim et al. 2020). For many problems we want to understand causal relationships between variables. There are different approaches on how to model causal relationships. This paper focuses on Structural Causal Models and Bayesian Causal Networks. Bayesian Causal Networks (BCN) cast a model based on **conditional probabilities.** Structural Causal Models (SCM) specify relationships based on **functional equations** (Pearl 2009). SCMs are non-parametric structural equations models with added features. Existing literature has provided an excellent introduction to SCMs and BCNs. Pearl (2009) provided a comprehensive introduction into this topic with his book 'Causality.' His work addressed misconceptions in social sciences, causality and statistics. His contribution on the hierarchy of causation advanced the comprehensive understanding of causality. Peters, Janzing, and Schölkopf (2017) address the relationship of causality, physical sciences, and causal inference. Their work highlights the importance of time in causality and inference based on observational data alone. E.g. (Bareinboim et al. 2020) worked on the application of causal methods in machine learning. Tarka (2018) discuss the history of SEMs and SCMs. The aim of this paper is to summarise SCMs and it's intersection with social sciences and physics. I structure the rest of the paper as follows: Section 2 introduces the assumptions in causal modelling. Further, section 2 discusses different causal differencecs, fundamental differences and a brief history of the methods. Section 3 addresses Pearls Causal Hierachy

2   Foundations of Structural Causal Models

Consists of graph and assignments: Baseline:

$$C := N_c$$

$$E := f_E(C, N_E)$$

source: Peters, Janzing, and Schölkopf (2017)

2.1   Assumptions in Causality

*Definition:***Independence**

If we specify the causal structure correctly:

(a) possible to undertake local intervention -> change f(x), regardless of f(y|x)

(b) these components are autonmous objects -> set of autonmous equations

Further: independence of noise ->

$$N_T, N_A$$

Example conditional probabilities: independence conditional probabilities/distribution and mechanism (ICM)

(Peters, Janzing, and Schölkopf 2017)

- noise independent -> called **causal sufficiency** clause.
- Algorithmic independence

SCM fulfills via :

- dependecy only via vertice connection (noise)
- condition on noise, variables become independent

- Independence of mechanisms
- conditional independence:
- joint distribution

Density factorizes and their expectation

Causal assumptions differentiate causal models from association learning methods.

These causal concepts are not expressible based on distribution functions/statistical associations. (Pearl 2010)

Disturbance in SCM: Correlated and causal factor ; responsible for variation Disturbance in Regression: Uncorrelated

Causal assumption not testable (e.g. $Cov(U_a, U_b) = 0$). d-seperation to test assumptions in totality (cannot make assumptions in isolation).

exploit invariant characteristics of SEM whithout committing to shape. structural if function autonomous and invariant to change in form of other functions.

Definition: **Reichenbachs common causal Principle**

2.2 Comparing Causal Tools

A structural equation model (SEM) is a set of autonous equations. This set represents the 'state of the world.' SEMs are popular in fields like economics, psychology and sociology (Pearl 2009). SCM are also defined by a (sub-)set of structural equations. Additionally, SCMs feature mathematical components from graphical models and the potential-outcome framework (Pearl 2012). There is a lot of controversy around SEMs. Many scholars challenge the parameter-specification in a SEM (Pearl 2012).[1] SCMs specify an underlying data generating process without the computational effort of creating a parametric model. Note, that structural equations refer to assignment equations, used in computer science. Equations in causal models did not always have a concise notation (Pearl, 2009). First, there was no sign to express the assignment equation and people used the '=' and one would e.g. write $A = B$. Treating an equation as a **algebraic equation** led to confusion because those have no causal information. This algebraic equation would imply that $B = A$ because the order has no concrete meaning in algebraic equations. The problem is that the equation is symmetric. The initial $'='$ sign was replaced with the ':=' which is asymmetric (Pearl, 2009) and called an **assignment**. This misconception has caused a lot of challenges which I will address further on in this paper. As mentioned, we define variables as functions e.g. $A = f_A(B, U_A)$. B defines A and the latent factor U_A. To summarize, a SCM consists of a set of (autonomous) equations to generate (a) endogenous variables and (b) exogenous variables.

In Figure 1 we can see an example of a probabilistic model depicted as a directed acyclic graph (DAG). The nodes are the white circles. The edges are the arrows, defined by the conditional probabilities. C is our **collider** variable because the affect of A on C and T on C collide (Pearl 2009).
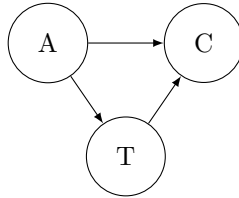

Figure 1: Probabilistic Model

In figure 2 we can see a vanilla structural causal model. The square nodes represent the latent variables.
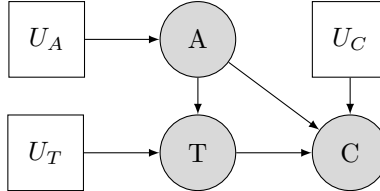

Figure 2: Structural Causal Model

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[1] Inconsistency refers to inherent nature of creating parameters based of observational data. Observational data is seldom consistent. Henceforth respective estimates are often questioned. For further information see Hernán and Taubman (2008)

The circle nodes represent the observed variables. The arrows are our structural equations and depict the relationship between variables.[2]

2.3 A Brief History of Causal Modeling

Path analysis is the foundation of modern structural causal models. This path analysis is a structural equation model with one variable per indicator. Sewall Wright, a Statistican and Geneticist, introduced the topic in the 1920s. (Pearl, 2012) Various other disciplines such as econometrics, psychometrics and sociology adopted path analysis. Aside from path diagrams also introduced graph rules to formalise relationships. (tbc. Cowles, Haavelmo,Aldrich,)

3 Pearl's Causal Hierachy

To contextualise causal methods, Judea Pearl (2009) introduced the hierarchy of causation. Pearl focuses on three layers: association, intervention and counterfactuals (Table 1).

Method	Action	
Association $P(a|b)$	Co-occurrence	(Un-)Supervised ML, BN, Reg.
Intervention $P(a|do(b), c)$	Do-manipulation	CBN,MDP,RL
Counterfactual $P(a_b|a', b')$	Hypotheticals	SCM ,PO

A higher level implies more detailed knowledge of the relationship between the variables. The first query is association, where we examine relationships based on observations.

Table: Pearls Hierachy of Causation (2009)

3.1 Prediction

Vanilla machine learning (ML), bayesian networks (BN) and regression models (Reg) are at the lowest level in the causal hierarchy (see table 1). These methods demand the least information and depend on association alone. Associational methods ignore external changes outside of our data. The interventional distribution has information on these external changes. The interventional distribution is only defined in high level causal methods.

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[2]Note that there are also cyclic structural causal models but no cyclic bayesian causal networks. For further information see Pearl (2009). Due to the confined scope of this paper, I will not explore cyclic structures.

3.2 Intervention

The second query deals with interventions. Here we can use Pearl (2009) do-calculus. The do-calculus enables us to study the manipulation of parent nodes. There are various types of intervention. One example is **atomic intervention,** where we set a variable to a constant. In **policy intervention** we specify a different function for an equation. off-policy intervention models different intervention that is not in our historical data (Oberst and Sontag 2019). Causal bayesian networks , Markov Decision Processes (MDP) and reinforcement learning model intervention. (tbc.) Hypothetical interventions:

3.3 Counterfactuals

The third query is counterfactual modelling. Here we deal with hypothetical settings. SCMs and potential outcome models allow for counterfactual modelling. These models can model counterfactuals because they include a interventional distribution (Oberst and Sontag 2019). BCNs only entail conditional probabilities. There is no information on relationships outside of the observations in the data. Henceforth, they cannot create counterfactuals (Pearl 2009).

Process is described as follows:

(a) Abduction: Cast probability $P(u)$ as conditional probability $P(u|\epsilon)$
(b) Action: Exchange $(X = x)$
(c) Prediction: Compute $(Y = y)$

3.4 Implications

BCN:

SCM:

4 Graphical Models

4.1 Features

A graph in a SCM contains endogenous and exogenous variables. In a directed graph all edges have arrows. Directed arrows are direct (causal) effects. No edge between variables means that there is no causal effect. If we have some edges without arrows, we call that graph semi-directed. If we have all edges without arrows, we call that graph un-directed. The most famous graphical model is the acyclic graph. An acyclic graph has

no roots that cause itself (directly and indirectly). Because we do not specify changes over time we typically only deal with acyclic models. This is partially because we assume the relationship to be constant over time. There are various tools to derive graphical models.[3]

- d-seperation
- vertices variables and arrows are direct effects.

4.2 Implications

Any DAG can be written as SCM. given joint disrtribution can be discover grahp? Markov condition

5 Causal Inference and Time

Another issue in this paper is understanding the relationship of time and causality. SCMs make the assumption that causal relationships hold over time(Peters, Janzing, and Schölkopf 2017). These vague definitions of time is more prevalent in social sciences. Meanwhile hard sciences deal with time in a more concise manner. Differential equations model time on a more mechanic manner (Mooij, Janzing, and Schölkopf (2013)).

model	predict in IID setting	predict under changing distributions / interventions	answer counter-factual questions	obtain physical insight	automatically learn from data
mechanistic model	Y	Y	Y	Y	?
structural causal model	Y	Y	Y	N	Y??
causal graphical model	Y	Y	N	N	Y?
statistical model	Y	N	N	N	Y

Table: Source: Peters, Janzing, and Schölkopf (2017)

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[3]The PC-Algorithm and the IC-Algorthm are two prominent examples.

6   Conclusion

This paper provides a gentle introduction into structural causal models. SCMs entail many features, complementing research on association learning by providing depth. This in turn, is of particular benefit for high stake decision settings. SCMs differentiate from other methods through the specification of endogenous and exogenous variables, treating the exogenous factors as pivotal components of the actual model rather than assuming they are ommitable errors that are uncorrelated. As suggested by Pearls Hierachy, there are different levels to learning and each higher step can do anything the prior step can but with more detail and information. Henceforth, as machin

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