Introduction to Structural Causal Models

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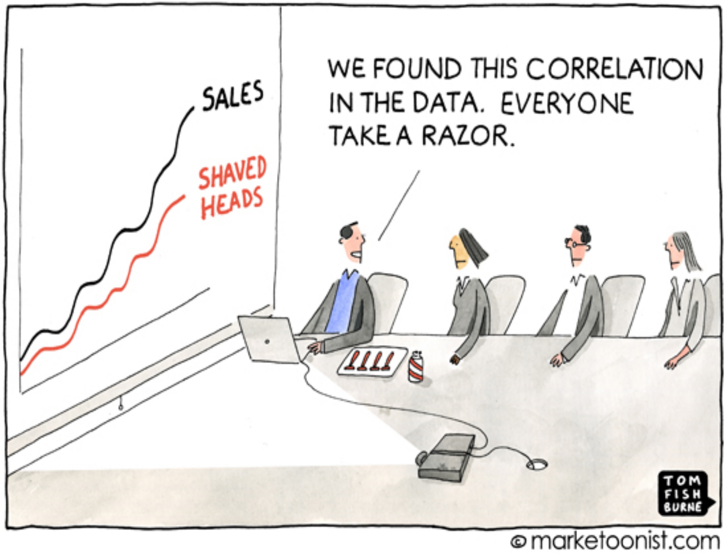
Motivation


SALES
SHAVED HEADS
WE FOUND THIS CORRELATION IN THE DATA. EVERYONE TAKE A RAZOR.
TOM FISH BURNE
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Introduction

Table 1: (Pearl 2012)

Association-based Concepts	Causal Concepts
Correlation	Randomization
Regression	Confounding
Conditional Independence	Disturbance
Likelihood	Error Terms
Odds Ratio	Structural Coefficients
Propensity Score	Spurious Correlation

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Foundations of SCMs

- Nonparametric SEM
- System of equations with functions
- **Assignments** ':=' (non-symmetric)
- algebraic equation '=' (symmetry)
- **Error in Regression:** Omittable outside factor
- **Error in SCM:** Latent (influential) factor that is pivotal for the model

$$C := N_c$$

$$E := f_E(C, N_E)$$

source: Peters, Janzing, and Schölkopf (2017)

Assumptions in Causality (1)

Independence:

- Noise terms independent (N_c, N_e)
- Mechanisms independent (other variables invariant)(local changes)
- Change in distribution stems from change in mechanism
- Causal Markov Condition

Assumptions under which causal inference is possible (2)

- **SUTVA** 'The treatment that one unit receives does not change the effect of treatment for any other unit.'
- **Consistency** The outcome Y agrees with the potential outcome corresponding to the treatment indicator.'
- **Ignorability** The potential outcomes are conditionally independent of treatment given some set of de-confounding variables. (perfect RCT)
- First tow hold for Counterfactuals in SCM
- third not testable but can check via backdoor criterion in SCM (Hardt and Recht 2021)

SCM Applications:

- Flexible simulations for higher order problems (intervention, counterfactual) based on elementary noise variables
- Graphical visualization via directed acyclic graph
- Example: SCM for fairness in dynamical system (Creager et al. 2020) : credit loan approval, time allocation, college admission

Fundamental Differences (1)

- Conflict whether to use **graphs** or not and to use **parameters** in SEM
- SEM is a parametric model used in applied sciences
- A Bayesian causal network (BCN) use conditional probabilities instead of functions
- PO is mostly equivalent but has not graphical framework
- SCM and PO are only frameworks that allow for counterfactuals
- Every SCM has the same information as respective probabilistic model + more (DGP)
- SCMs can derive potential outcomes but PO cannot derive SCMs

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Pearls Causal Hierachy

Table 2: Pearls Hierachy of Causation (2009)

Method	Action	Example	Usage
Association $P(a\|b)$	Co-occurrence	What happened...	(Un-)Supervised ML, BN, Reg.
Intervention $P(a\|do(b), c)$	Do-manipulation	What happens if ...	CBN,MDP,RL
Counterfactual $P(a_b\|a', b')$	Hypotheticals	What would have happened if...	SCM ,PO

Prediction

- ML, BN and regression are at the lowest level in the causal hierarchy
- Prediction methods demand the least information and depend on association alone
- Association-based methods ignore external changes outside of our data
- Intervention distribution needed for higher level information

Intervention

- Mathematical Tool: do-calculus
- The do-calculus enables us to study the manipulation of parent nodes (direct effect)
- **Atomic intervention:** where we set a variable to a constant
- **Policy intervention:** we specify a different function for an equation
- CBN , MDP and reinforcement learning model intervention.

Example Intevention (1)

Atomic Intervention:

- replacing function with a constant

$$C_1 := f_{c_1}(p, N_{c_1}) \rightarrow C_1 := 600$$
$$C_2 := f_{c_2}(a, N_{c_2})$$
$$E := f_E(C_1, C_2, N_E)$$

Example Intevention (2)

Policy Intervention:

- replacing function with a different conditional probability

$$C_1 := f_{c_1}(p, N_{c_1}) \rightarrow C_1 := f(\pi)$$

$$C_2 := f_{c_2}(a, N_{c_2})$$

$$E := f_E(C_1, C_2, N_E)$$

Counterfactuals

- missing data problem in PO framework

Process is described as follows:

a) Abduction: Cast probability $P(u)$ as conditional probability $P(u|\epsilon)$
b) Action: Exchange $(X = x)$
c) Prediction: Compute $(Y = y)$

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Graphical Tools

- Nodes -> Variables (endogenous/exogenous)
- Edges -> relationship (equations)
- Parents/Ancestors/Descendants
- No need to specify exact parametric shape
- Highlight colliders
- Estimation back door criterion
- Test theoretical model structure via causal algorithms to detect structure in data (IC/PC Algo.)
- More information: (Morgan and Winship 2014)

Graphical Illustration - Probabilisitic Model

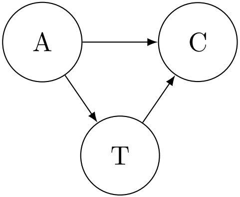

Figure 1: Probabilistic Model

Graphical Illustration - Structural Causal Model

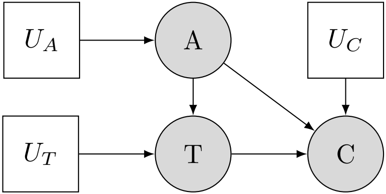

Figure 2: Structural Causal Model

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Causal Modelling with Differential Equations

- Time in Social Sciences: Often Vague
- Time in Physical Sciences: Mechanical via **Differential equations**
- dependence on prior time point and change in time contribute to the value at time point

Initial Value:

$$\mathbf{x}(t_0) = \mathbf{x}_0$$

Derivative of function x with respect to time t:

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}), \mathbf{x} \in \mathbb{R}^d$$

Value of Function at time t + dt:

$$\mathbf{x}(t + dt) = \mathbf{x}(t) + dt \cdot f(\mathbf{x}(t))$$

Graphical Overview

model	IID setting	changing distributions	counter-factual questions	physical insight
mechanistic model	Y	Y	Y	Y
structural causal model	Y	Y	Y	N
causal graphical model	Y	Y	N	N
statistical model	Y	N	N	N

Source: (Peters, Janzing, and Schölkopf 2017)

Extensions

- Mediation Analysis
- PO-Framework
- Causal Algorithms
- IV-Estimation
- Causal Constraints Model
- Causal Trees

Concluding Remarks

- Association-based learning is easy to model because of lower information neccessary
- but not always appropriate in high stake settings
- SCM as simulator for causal modelling based on noise variables
- SCMS entail a lot of information (DGP, intervention distribution)
- Note: *"Garbage in, Garbage out"*
- computational advantage casting causal model as assignments
- Enables modelling of higher order concepts like counterfactuals
- Extensions through differential equations for concise modelling of time

References

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Appendix

Method	CBN	SCM
Prediction	· Unstable · Volatile to parameter changes · Re-Estimate entire model	· Stable · More Natural Specification · Only estimate Δ CM
Intervention	· Costly for Non-Markovian Models · Unstable(Nature CP) · Only generic estimates(Δ CP)	· Pot. Cyclic Representation · Stable · Context specific
Counterfactuals	· **Impossible** · no information on latent factors(ϵ)	· Possible · Inclusion of latent factors