Introduction to Structural Causal Models

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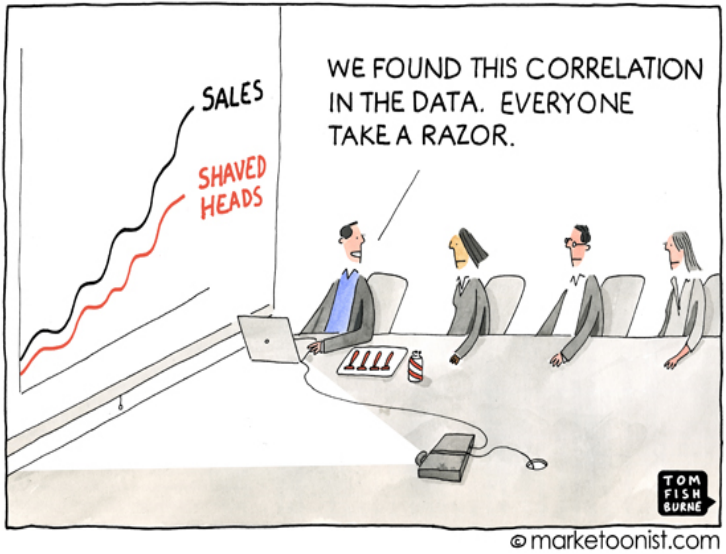
Motivation



SALES
SHAVED HEADS
WE FOUND THIS CORRELATION IN THE DATA. EVERYONE TAKE A RAZOR.
TOM FISH BURNE
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Introduction (1)

- Machine learning has provided many insights into different problems
- One issue is the consideration of 'What are we actually predicting?'
- Mainstream tools are build on association-based learning
- Associations are not enough for high stake settings
- We want causation and not correlation

Introduction (2)

Causal assumptions differentiate causal models from association learning methods.

Association-based Concepts	Causal Concepts
Correlation	Randomization
Regression	Confounding
Conditional Independence	Disturbance
Likelihood	Error Terms
Odds Ratio	Structural Coefficients
Propensity Score	Spurious Correlation

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Foundations of SCMs (1)

- System of equations
- Assignment equation ':=' rather than regular equation '='
- Nonparametric SEM
- Functional form rather than using conditional probabilities
- SCM = PO + Graphical Tools
- Exogenous factors are part of the model specification

SCM vs SEM

Error terms

Regression: Omittable outside factor SCM/SEM; Latent influential factor that is pivotal for the model specification but not observable

Consists of graph and assignments: Baseline:

$$C := N_c$$
$$E := f_E(C, N_E)$$

source: Peters, Janzing, and Schölkopf (2017)

Assumptions

- First, there was no sign to express the assignment equation and people used the '$=$' and one would e.g. write $A = B$.
- Treating an equation as a **algebraic equation** led to confusion because those have no causal information.
- This algebraic equation would imply that $B = A$ because the order has no concrete meaning in algebraic equations.
- The problem is that the equation is symmetric.
- The initial $'='$ sign was replaced with the '$:=$' which is asymmetric (Pearl, 2009) and called an **assignment**.
- This misconception has caused a lot of challenges.[1]
- As mentioned, we define variables as functions e.g. $A = f_A(B, U_A)$.

[1] For more information see Pearl (2009)

Assumptions in Causality

Independence:

- noise terms independent
- mechanisms independent (other variables invariant)(local changes)
- change in distribution stems from change in mechanism
- Causal Markov Condition allows to factorize
- Each Conditional is a causal mechanism

Source: (Peters, Janzing, and Schölkopf 2017)

Assumptions in Causality (2)

Established Conditions:

Definition: **SUTVA**

'The treatment that one unit receives does not change the effect of treatment for any other unit.'(Hardt and Recht 2021)

Definition: **Consistency**

The outcome Y agrees with the potential outcome corresponding to the treatment indicator.' (Hardt and Recht 2021)

Definition: **Ignorability**

The potential outcomes are conditionally independent of treatment given some set of de-confounding variables. (Hardt and Recht 2021) (perfect RCT)

- First tow hold for SCM counterfactuals
- third not testable but can check via backdoor criterion in SCM

SCM Applications:

- Flexible simulations for higher order problems (intervention, counterfactual)
- Graphical visualization via directed acyclic graph

Fundamental Differences (1)

- conflict whether to use graphs or not
- A SEM is a parametric specification used in applied sciences (parameters contested)
- A Bayesian causal network is another popular causal model using conditional probabilities and NO functions
- Differences in performance between BCN and SCM# Performance Evaluation

Implications

Method	CBN	SCM
Prediction	· Unstable · Volatile to parameter changes · Re-Estimate entire model	· Stable · More Natural Specification · Only estimate Δ CM
Intervention	· Costly for Non-Markovian Models · Unstable(Nature CP) · Only generic estimates(Δ CP)	· Pot. Cyclic Representation · Stable · Context specific
Counterfactuals	· **Impossible** · no information on latent factors(ϵ)	· Possible · Inclusion of latent factors

Pearls Causal Hierachy

Table 3: Pearls Hierachy of Causation (2009)

Method	Action	Example	Usage
Association $P(a\|b)$	Co-occurrence	What happened...	(Un-)Supervised ML, BN, Reg.
Intervention $P(a\|do(b), c)$	Do-manipulation	What happens if ...	CBN,MDP,RL
Counterfactual $P(a_b\|a`, b`)$	Hypotheticals	What would have happened if...	SCM ,PO

Prediction

- ML, BN and regression are at the lowest level in the causal hierarchy
- Prediction methods demand the least information and depend on association alone
- Associational methods ignore external changes outside of our data.
- intervention distribution has information on these external changes.
- intervention distribution is only defined in high level causal methods.

Intervention

- The second query deals with interventions
- Mathematical Tool: do-calculus
- The do-calculus enables us to study the manipulation of parent nodes
- There are various types of intervention
- One example is **atomic intervention,** where we set a variable to a constant
- In **policy intervention** we specify a different function for an equation
- Off-policy intervention models different intervention that is not in our historical data
- CBN , MDP and reinforcement learning model intervention.

Example Intevention (1)

Atomic Intervention:

- by replacing function with a constant

$$C_1 := f_{c_1}(p, u_1) \rightarrow C_1 := 600$$

$$C_2 := f_{c_2}(a, u_2)$$

$$E := f_E(C_1, C_2, N_E)$$

Example Intevention (2)

Policy Intervention:

- Intervention by replacing function with a different conditional probability

$$C_1 := f_{c_1}(p, u_1) \rightarrow C_1 := f(\pi)$$

$$C_2 := f_{c_2}(a, u_2)$$

$$E := f_E(C_1, C_2, u_1, u_2)$$

Counterfactuals

- missing data problem in PO framework

Process is described as follows:

a) Abduction: Cast probability $P(u)$ as conditional probability $P(u|\epsilon)$
b) Action: Exchange $(X = x)$
c) Prediction: Compute $(Y = y)$

Graphical Tools

- No need to specify exact parametric shape
- highlight colliders
- estimation back door criterion
-
- Nodes
- Edges
- Parents/Ancestors/Descendents
- (Missing) Arrows

Difference Path Diagramm and Pearls Causal Diagrams

Graphical Illustration - Probabilisitic Model

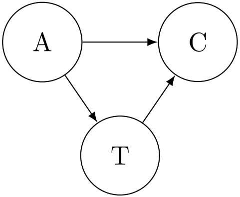

Figure 1: Probabilistic Model

Graphical Illustration - Structural Causal Model

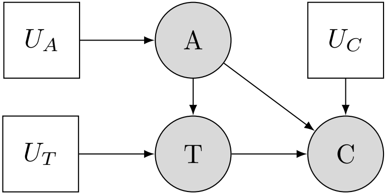

Figure 2: Structural Causal Model

Causality and Time

- Time in Physical Sciences: Mechanical and exact
- Time in Social Sciences: Often Vague
- Regular Time Specification is also more vague
- To accommodate that issue, research on differential equation based SCMs started

Causal Modelling with Differential Equations

Intial Value:

$$\mathbf{x}\left(t_0\right) = \mathbf{x}_0$$

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}), \mathbf{x} \in \mathbb{R}^d$$

$$\mathbf{x}(t + dt) = \mathbf{x}(t) + dt \cdot f(\mathbf{x}(t))$$

Graphical Overview

model	predict in IID setting	predict under changing distributions / interventions	answer counter-factual questions	obtain physical insight
mechanistic model	Y	Y	Y	Y
structural causal model	Y	Y	Y	N
causal graphical model	Y	Y	N	N
statistical model	Y	N	N	N

Source: (Peters, Janzing, and Schölkopf 2017)

References

Hardt, Moritz, and Benjamin Recht. 2021. *Patterns, Predictions, and Actions: A Story about Machine Learning*. https://mlstory.org. http://arxiv.org/abs/2102.05242.

Pearl, Judea. 2009. *Causality*. Cambridge university press.

Peters, Jonas, Dominik Janzing, and Bernhard Schölkopf. 2017. *Elements of Causal Inference: Foundations and Learning Algorithms*. Adaptive Computation and Machine Learning Series. Cambridge, Massachuestts: The MIT Press.