IFN501 - System Modeling and Simulation

Session 6: Introduction to Statistics

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Outline

Inference for Numerical Data

Non-parametric Tests

References

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Inference for Numerical Data
- t-test
- Analysis of Variance

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  - One-Sample t-Test
  - Two-Sample t-Test
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  - Two-sample t-test
    - Paired sample
    - Independent sample
- Please note that this is a very short description about t-test. You should read more to gain better understanding about this method.

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- First, let us assign the scores to a variable:

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scores <- c(65, 78, 88, 55, 48, 95, 66, 57, 79, 81)
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```r
scores <- c(65, 78, 88, 55, 48, 95, 66, 57, 79, 81)
```

- Then we use the `t.test(data, mean)` function.

```r
t.test(scores, mu = 75)
```

t-Test

One-Sample t-Test

```
t.test(scores, mu = 75)

##
##  One Sample t-test
##
## data:  scores
## t = -0.783, df = 9, p-value = 0.454
## alternative hypothesis: true mean is not equal to 75
## 95 percent confidence interval:
##  60.2219 82.1781
## sample estimates:
## mean of x
##      71.2
```

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- If the p-value is higher than the α value, the we must accept the null hypothesis: the average of the test scores is significantly similar with population average, otherwise we accept the alternate hypothesis.
- In our case, the p-value is 0.453721, which is higher than the α value, therefore we accept the null hypothesis.

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Two-Sample t-Test
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Two-Sample t-Test: Matched Samples

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library(MASS)
head(immer)

##   Loc Var    Y1    Y2
## 1  UF   M  81.0  80.7
## 2  UF   S 105.4  82.3
##  [ reached getOption("max.print") -- omitted 4 rows ]
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t.test(immer$Y1, immer$Y2, paired = TRUE)
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t.test(immer$Y1, immer$Y2, paired = TRUE)

##
##  Paired t-test
##
## data:  immer$Y1 and immer$Y2
## t = 3.324, df = 29, p-value = 0.00241
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##   6.12195 25.70471
## sample estimates:
## mean of the differences
##                 15.9133
```

- The p-value in the function output is less than our α value[3].

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- The p-value in the function output is less than our α value[3].
- Therefore we have a strong evidence to reject null hypothesis and accept the alternate hypothesis: the yields of years 1931 and 1932 are significantly different.

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mtcars[, c("mpg", "am")]

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## Mazda RX4 Wag     21.0  1
## Datsun 710        22.8  1
## Hornet 4 Drive    21.4  0
## Hornet Sportabout 18.7  0
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- Particulary, these 2 columns are independent data population.
- **Problem:** Assuming that the data in mtcars follows the normal distribution, find the 95% confidence interval estimate of the difference between the mean gas mileage of manual and automatic transmissions.

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- First, we must split the data into 2 set of data, one for the automatic transmission model, and one for the manual transmission model[5].

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L = mtcars$am == 0  # select the automatic transmission model (0)
mpg.auto = mtcars[L, ]$mpg  # select automatic transmission mileage
mpg.auto

##  [1] 21.4 18.7 18.1 14.3 24.4 22.8 19.2 17.8 16.4 17.3
##  [ reached getOption("max.print") -- omitted 9 entries ]
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```
mpg.manual = mtcars[!L, ]$mpg
mpg.manual

##  [1] 21.0 21.0 22.8 32.4 30.4 33.9 27.3 26.0 30.4 15.8
##  [ reached getOption("max.print") -- omitted 3 entries ]
```

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Two-Sample t-Test: Independent Samples

- After having data for both models in separated variables, now we can use t-test to compute difference of means between the automatic and manual transmission models.

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```
t.test(mpg.auto, mpg.manual)

##
##  Welch Two Sample t-test
##
## data:  mpg.auto and mpg.manual
## t = -3.767, df = 18.33, p-value = 0.00137
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -11.28019  -3.20968
## sample estimates:
## mean of x mean of y
##   17.1474   24.3923
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- The result shows that the p-value is lower than the predefined α, therefore we can reject H_0 and conclude that the mileages of automatic transmission and the manual transmission are significantly different.

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t.test(mpg~am, data=mtcars)

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## data:  mpg by am
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## sample estimates:
## mean in group 0 mean in group 1
##         17.1474         24.3923
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Item1	22, 42, 44, 52, 45, 37

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Item1	22, 42, 44, 52, 45, 37
Item2	52, 33, 8, 47, 43, 32

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Item2	52, 33, 8, 47, 43, 32
Item3	16, 24, 19, 18, 34, 39

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Item1	22, 42, 44, 52, 45, 37
Item2	52, 33, 8, 47, 43, 32
Item3	16, 24, 19, 18, 34, 39

- **Problem:** At .05 level of significance, test whether the mean sales volume for the 3 new menu items are all equal.

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```
df1 <- data.frame(
        Item1 = c(22, 42, 44, 52, 45, 37),
        Item2 = c(52, 33,  8, 47, 43, 32),
        Item3 = c(16, 24, 19, 18, 34, 39)
)
```

Analysis of Variance

The solution includes several steps of data preparation.

1. First, let's create a data frame to store the data.

```
df1 <- data.frame(
        Item1 = c(22, 42, 44, 52, 45, 37),
        Item2 = c(52, 33,  8, 47, 43, 32),
        Item3 = c(16, 24, 19, 18, 34, 39)
)
```

2. Check the content

```
df1

##   Item1 Item2 Item3
## 1    22    52    16
## 2    42    33    24
## 3    44     8    19
## 4    52    47    18
## 5    45    43    34
## 6    37    32    39
```

3. Concatenate the data rows of `df1` into a single vector `r`.

```
r = c(t(as.matrix(df1)))  # response data
r
```

```
##  [1] 22 52 16 42 33 24 44  8 19 52
##  [ reached getOption("max.print") -- omitted 8 entries ]
```

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```

4. Assign new variables for the treatment levels and number of observations.

```
f = c("Item1", "Item2", "Item3")  # treatment levels
k = 3  # number of treatment levels
n = 6  # observations per treatment
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5. Create a vector of treatment factors that corresponds to each element of `r` in step 3 with the `gl()` function.

```
tm = gl(k, 1, n * k, factor(f))  # matching treatments
tm

##  [1] Item1 Item2 Item3 Item1 Item2 Item3 Item1 Item2 Item3
## [10] Item1
##  [ reached getOption("max.print") -- omitted 8 entries ]
## Levels: Item1 Item2 Item3
```

Analysis of Variance

6. Apply the function `aov` to a formula that describes the response `r` by the treatment factor tm and assign it to a new variable `av`.

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Analysis of Variance

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```
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7. Print out the ANOVA table using the `summary()` function.

```
summary(av)

##             Df Sum Sq Mean Sq F value Pr(>F)
## tm           2    745     373    2.54   0.11
## Residuals   15   2200     147
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8. The p-value in the output is greater than the significance level ($\alpha = 0.05$). Therefore we accept the null hypothesis: <u>The mean sales volume of the new menu items are equal.</u>

Outline

Inference for Numerical Data

Non-parametric Tests

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- However, there are several assumptions that have to be satisfied before using the parametric methods.
- Distribution normality is one of the assumptions in parametric methods.
- Unfortunately, not all data that normally distributed.
- If we can not satisfy the assumption for parametric methods, then we can use the non-parametric methods.

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- R already has all these tests, you just need to browse the Internet. There a lot of tutorials there.

- Normality Test
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- Mann-Whitney-Wilcoxon Test
- Kruskal-Wallis Test

After learned some statistical tests and know that these tests have assumptions that have to be fulfilled, we can write down steps for data analysis:

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3. If the data is NOT normally distributed, we have to use the non-parametric methods.

Next session..

- Cellular Automaton

Outline

Inference for Numerical Data

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References

References I