Additive and Multiplicative Effects Model[*]

Seminar on Statistical Modeling of Social Networks

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Abstract

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1 Introduction

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2 Theory

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2.1 Network Effects & Social Structure

- 1st-order: Sender (Receiver) Effects
- 2nd-order: Reciprocity
- 3rd-order: Homophily & Stochastic Equivalence
- System-level - changing actor composition

2.2 The Model

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2.3 The SRM Model: The Additive Part

$$y_{ij} = \mu + e_{ij}$$
$$e_{ij} = a_i + b_j + \epsilon_{ij}$$
$$\{(a_1, b_1), \ldots, (a_n, b_n)\} \sim N(0, \Sigma_{ab})$$
$$\{(\epsilon_{ij}, \epsilon_{ji}) : \ i \neq j\} \sim N(0, \Sigma_\epsilon), \ \text{where}$$
$$\Sigma_{ab} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix} \quad \Sigma_\epsilon = \sigma_\epsilon^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

2.4 The Latent Factor Model

$$\mathbf{u}_i, \mathbf{v}_j \in \mathbb{R}^k \ \ i, j \in \{1, \ldots, n\}$$

$$\gamma(\mathbf{u}_i, \mathbf{v}_j) = \mathbf{u}_i^T D \mathbf{v}_j$$
$$= \sum_{k \in K} d_k u_{ik} v_{jk}$$
$$D \text{ is a } K \times K \text{ diagonal matrix}$$

2.5 Additive and Multiplicative Effects (AME) Model

$$y_{ij,t} = g(\theta_{ij,t})$$
$$\theta_{ij,t} = \beta^T \mathbf{X}_{ij,t} + e_{ij,t}$$
$$e_{ij,t} = a_i + b_j + \epsilon_{ij} + \alpha(\mathbf{u}_i, \mathbf{v}_j) \text{ , where}$$
$$\alpha(\mathbf{u}_i, \mathbf{v}_j) = \mathbf{u}_i^T \mathbf{D} \mathbf{v}_j = \sum_{k \in K} d_k u_{ik} v_{jk}$$

2.6 Goodness of Fit

2.7 Parameter Estimation

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Donec quam felis, ultricies nec, pellentesque eu, pretium quis, sem. Nulla consequat massa quis enim. Donec pede justo, fringilla vel, aliquet nec, vulputate eget, arcu. In enim justo, rhoncus ut, imperdiet a, venenatis vitae, justo. Nullam dictum felis eu pede mollis pretium. Integer tincidunt. Cras dapibus. Vivamus elementum semper nisi. Aenean vulputate eleifend tellus. Aenean leo ligula, porttitor eu, consequat vitae, eleifend ac, enim. Aliquam lorem ante, dapibus in, viverra quis, feugiat a, tellus. Phasellus viverra nulla ut metus varius laoreet. Quisque rutrum. Aenean imperdiet. Etiam ultricies nisi vel augue. Curabitur ullamcorper ultricies nisi. Nam eget dui. Etiam rhoncus. Maecenas tempus, tellus eget condimentum rhoncus, sem quam semper libero, sit amet adipiscing sem neque sed ipsum. Nam quam nunc, blandit vel, luctus pulvinar, hendrerit id, lorem. Maecenas nec odio et ante tincidunt tempus. Donec vitae sapien ut libero venenatis faucibus. Nullam quis ante. Etiam sit amet orci eget eros faucibus tincidunt. Duis leo. Sed fringilla mauris sit amet nibh. Donec sodales sagittis magna. Sed consequat, leo eget bibendum sodales, augue velit cursus nunc,

3   Application case: The Alliances Data Set 2000

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The analysis was done with the R-Software (R Core Team (2021)) and the AMEN R-Package (P. Hoff, Fosdick, and Volfovsky (2020)).

3.1   Network Statistics

Table 1: Goodness of Fit Statistics for the Year 2000

sd.rowmean	sd.colmean	dyad.dep	cycle.dep	trans.dep
0.057	0.057	1	0.389	0.389

3.2   Modeling without network effects

We first fit the model without the SRM Terms. As illustrated in Fig, the model performs quite bad.

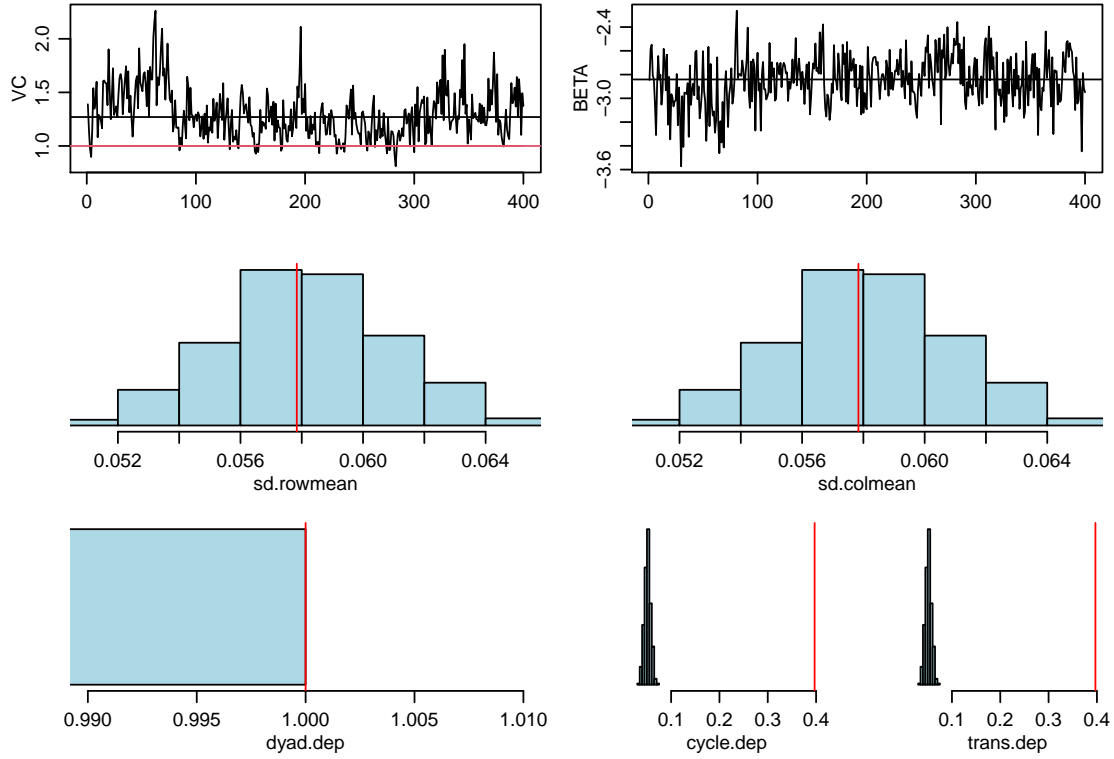Figure 1: The Alliances Data Set in the Year 2000.

Figure 2: Goodness of Fit Statistics for the model without SRM Terms (above) and with SRM Terms (below). As indicated by the dependence structure, the second model performs better, taking sender and receiver effects into account.

3.3 Adding Covariates

3.4 Adding Time

3.5 Extension: Modeling longitudinal data

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```
fit_AME_Rep_R3 <- readRDS(file = "models/fit_AME_Rep_R3.rds")
stargazer(cbind(fit_AME_Rep_R3$BETA,fit_AME_Rep_R3$VC), type = "latex", header = FALSE, summary
```

Table 2: Estimated Effects for AME Replicated Data

Statistic	N	Mean	St. Dev.
intercept	400	−6.461	0.796
cinc.node	400	105.681	10.316
polity.node	400	0.010	0.003
contigMat.dyad	400	1.086	0.052
lNet.dyad	400	0.457	0.237
LSP.dyad	400	0.062	0.016
warNet.dyad	400	−0.136	0.082
va	400	5.358	1.793
ve	400	1.000	0.000

```
fit_AME_Rep_R3 <- readRDS(file = "models/fit_AME_Rep_R3.rds")
stargazer(cbind(fit_AME_Rep_R3$BETA,fit_AME_Rep_R3$VC), type = "latex", header = FALSE, summary
```

```
fit_AME_Rep_R3 <- readRDS(file = "models/fit_AME_Rep_R3.rds")
stargazer(table_ame(fit_AME_Rep_R3), type = "latex", header = FALSE,
          summary = FALSE, title = "Estimated Effects for AME Replicated Data",
          notes = "MCMC Estimates of 500 Burn-In and 10000 Draws.")
```

3.6 Things that may be included:

Table 3: Estimated Effects for AME Replicated Data

Statistic	N	Mean	St. Dev.
intercept	400	−6.461	0.796
cinc.node	400	105.681	10.316
polity.node	400	0.010	0.003
contigMat.dyad	400	1.086	0.052
lNet.dyad	400	0.457	0.237
LSP.dyad	400	0.062	0.016
warNet.dyad	400	−0.136	0.082
va	400	5.358	1.793
ve	400	1.000	0.000

Table 4: Estimated Effects for AME Replicated Data

	pmean	psd	z-stat	p-val
intercept	-6.4606	0.7957	-8.1197	0
cinc.node	105.6805	10.3157	10.2446	0
polity.node	0.0099	0.003	3.2562	0.0011
contigMat.dyad	1.0858	0.0518	20.9772	0
lNet.dyad	0.4569	0.2369	1.9284	0.0538
LSP.dyad	0.0624	0.0165	3.7894	2e-04
warNet.dyad	-0.1361	0.0823	-1.6533	0.0983
va	5.3579	1.7935	-	-
ve	1	0	-	-

MCMC Estimates of 500 Burn-In and 10000 Draws.

Table 5: Network Statistics x of the Alliances Network 2000

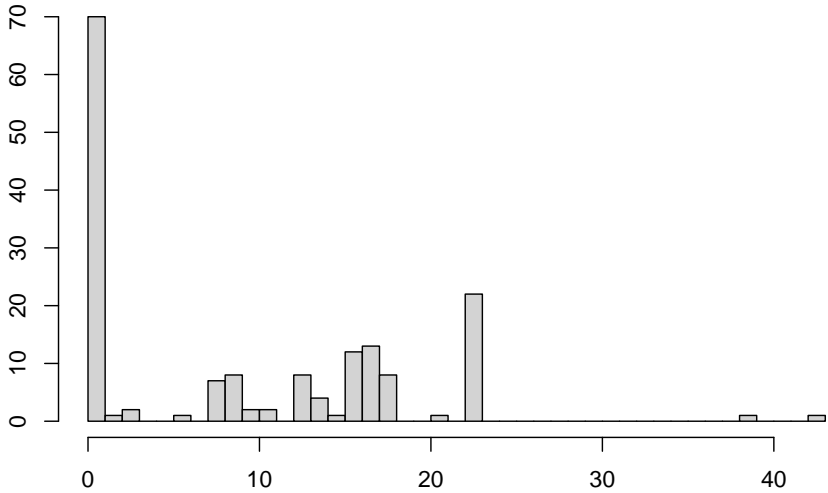|	x
Size	164.00
Edgecount	767.00
Dyadcount	13366.00
Density	0.06

Figure 3: The Degree Distribution.

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