Comparing the Classical and Nonclassical Symmetries of Nonlinear Partial Differential Equations

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Introduction

Our research objective for this project was to calculate the classical and nonclassical symmetry groups for the reduced Gibbons-Tsarev equation and the Born-Infeld equation and compare them.

What is a symmetry?

Definition

A symmetry is a transformation that leaves an object invariant.

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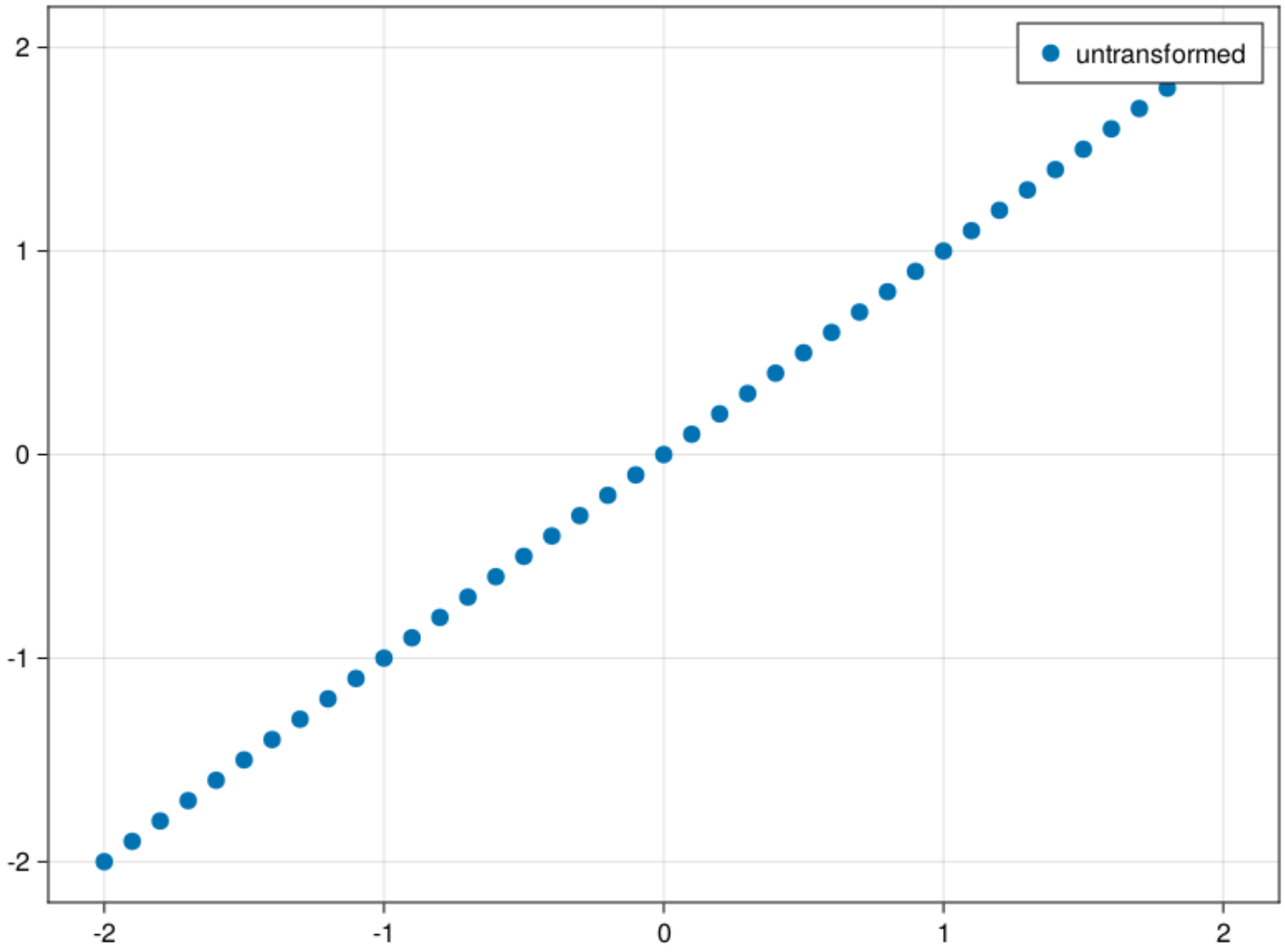
Definition

A symmetry is a change that doesn't change anything.

What is a symetry?

Let's see this in action using the simple linear equation $x - y = 0$.



What is a symmetry?

Example (A Non-Example)

- For our first transformation, let's define new variables $\bar{x} = x + 1$ and $\bar{y} = y$.
- Now we rewrite our equation using these new variables.

$$\begin{aligned} \bar{x} - \bar{y} &= 0 && \text{by definition} \\ x + 1 - y &= 0 && \text{by substitution} \\ y &= x + 1 && \text{by rewriting in slope-intercept form} \end{aligned}$$

- This is transformation is not a symmetry: $x - y + 1 \neq x - y$

What is a symmetry?

A Transformation that is a Symmetry

Example (2)

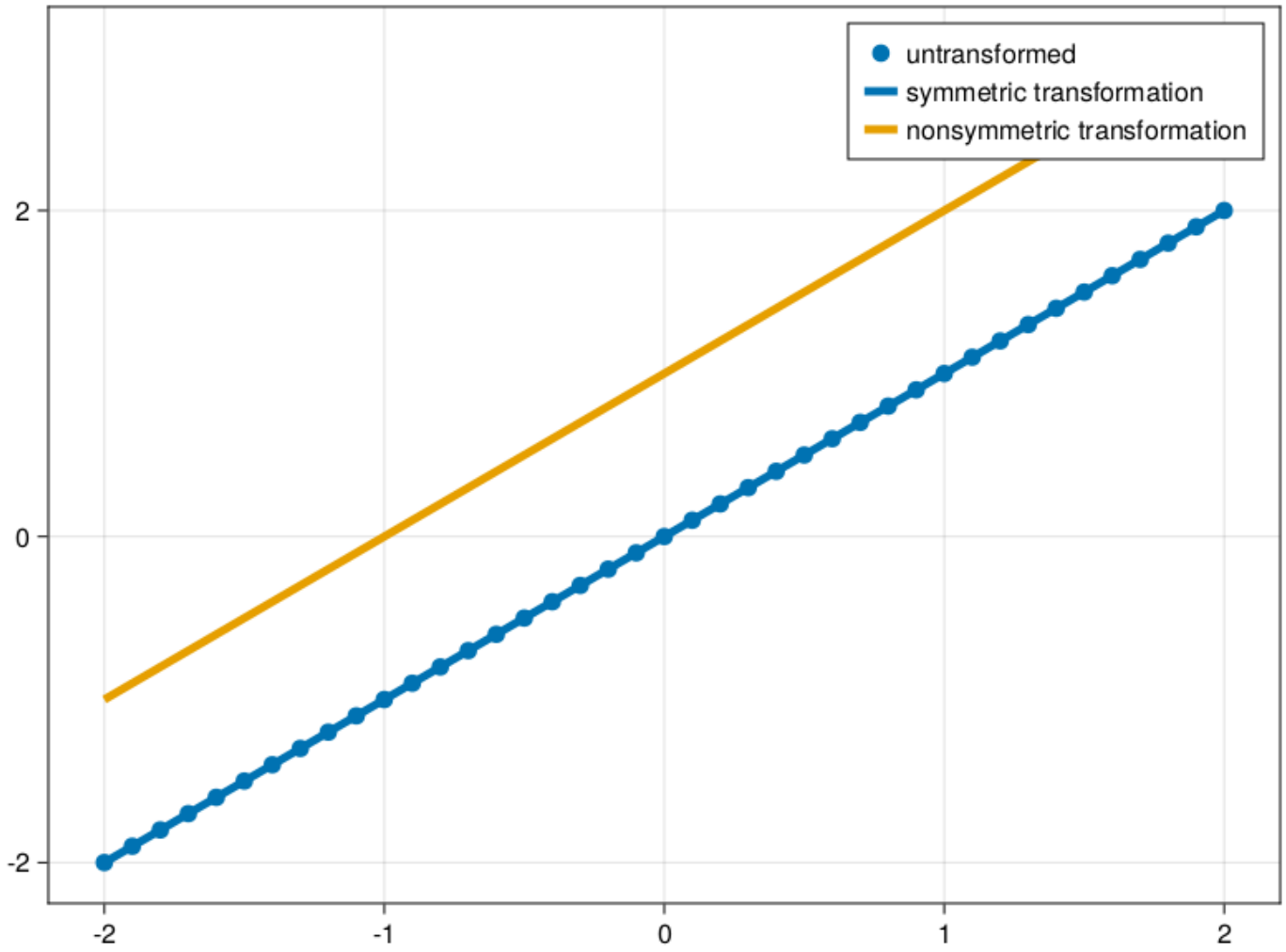
- Let's define some new variables again
  $\bar{x} = x + 1$ and $\bar{y} = y + 1$.
- Now we rewrite our equation using these new variables.

$$\begin{aligned}
\bar{x} - \bar{y} &= 0 && \text{by definition} \\
(x + 1) - (y + 1) &= 0 && \text{by substitution} \\
(x - y) + (1 - 1) &= 0 && \text{by algebra} \\
x - y &= 0 && \text{by algebra} \\
y &= x && \text{by rewriting in slope-intercept form}
\end{aligned}$$

- This transformation is a symmetry:
  $x - y = x - y$

What is a symetry?

The graphs of our three equations.



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Who cares?

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- Symmetries help us understand and solve equations that we wouldn't normally be able to.
- Symmetries encode physically meaningful aspects of equations, like conservation laws in physics.
- They're cool.

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- A differential equation is an equation that contains both an unknown function and information about how that function relates to its rates of change.
- Differential equations show up everywhere we model something using information about how that thing changes over time. This includes everything from population dynamics to the motion of the planets.

What is a Differential Equation?

As an example, we'll use the equation for a spring.

$$m\ddot{x} + \gamma\dot{x} + \kappa x = 0$$

Here m is the mass, $\ddot{x}$ is the acceleration, γ is the damping constant (friction), $\dot{x}$ is the velocity, κ is the spring constant, and x is the position.

The History of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Classical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Nonclassical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

Future Work: Does Integrability Imply Equivalence of Classical and Nonclassical Symmetries?

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