Comparing the Classical and Nonclassical Symmetries of Nonlinear Partial Differential Equations

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Introduction

Our research objective for this project was to calculate the classical and nonclassical symmetry groups for the reduced Gibbons-Tsarev equation and the Born-Infeld equation and compare them.

What is a symmetry?

Definitions

Definition

A symmetry is a transformation that leaves an object invariant.

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A symmetry is a transformation that leaves an object invariant.

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A symmetry is a change that doesn't change anything.

Example (1)

Let's take the equation $x - y = 0$.
In slope-intercept form, this is the line $y = x$.

- For our first transformation, let's define new variables $\bar{x} = x + 1$ and $\bar{y} = y$.
- Now we rewrite our equation using these new variables.

$$\begin{aligned} \bar{x} - \bar{y} &= 0 && \text{by definition} \\ x + 1 - y &= 0 && \text{by substitution} \\ y &= x + 1 && \text{by rewriting in slope-intercept form} \end{aligned}$$

- As we can see, this is NOT a transformation: $x - y + 1 \neq x - y$

Example (2)

- Let's define some new variables again
  $\bar{x} = x + 1$ and $\bar{y} = y + 1$.
- Now we rewrite our equation using these new variables.

$$\begin{aligned}
\bar{x} - \bar{y} &= 0 && \text{by definition} \\
(x+1) - (y+1) &= 0 && \text{by substitution} \\
(x-y) + (1-1) &= 0 && \text{by algebra} \\
x - y &= 0 && \text{by algebra} \\
y &= x && \text{by rewriting in slope-intercept form}
\end{aligned}$$

- As we can see, this IS a transformation:
  $x - y = x - y$

What is a Differential Equation?

The History of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Classical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Nonclassical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

Future Work: Does Integrability Imply Equivalence of Classical and Nonclassical Symmetries?

Future Work: Does Equivalence of Classical and Nonclassical Symmetries Imply Integrability?