Comparing the Classical and Nonclassical Symmetries of Nonlinear Partial Differential Equations

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2023

Introduction

Our research objective for this project was to calculate the classical and nonclassical symmetry groups for the reduced Gibbons-Tsarev equation and the Born-Infeld equation and compare them.

What is a symmetry?

Definition

A symmetry is a transformation that leaves an object invariant.

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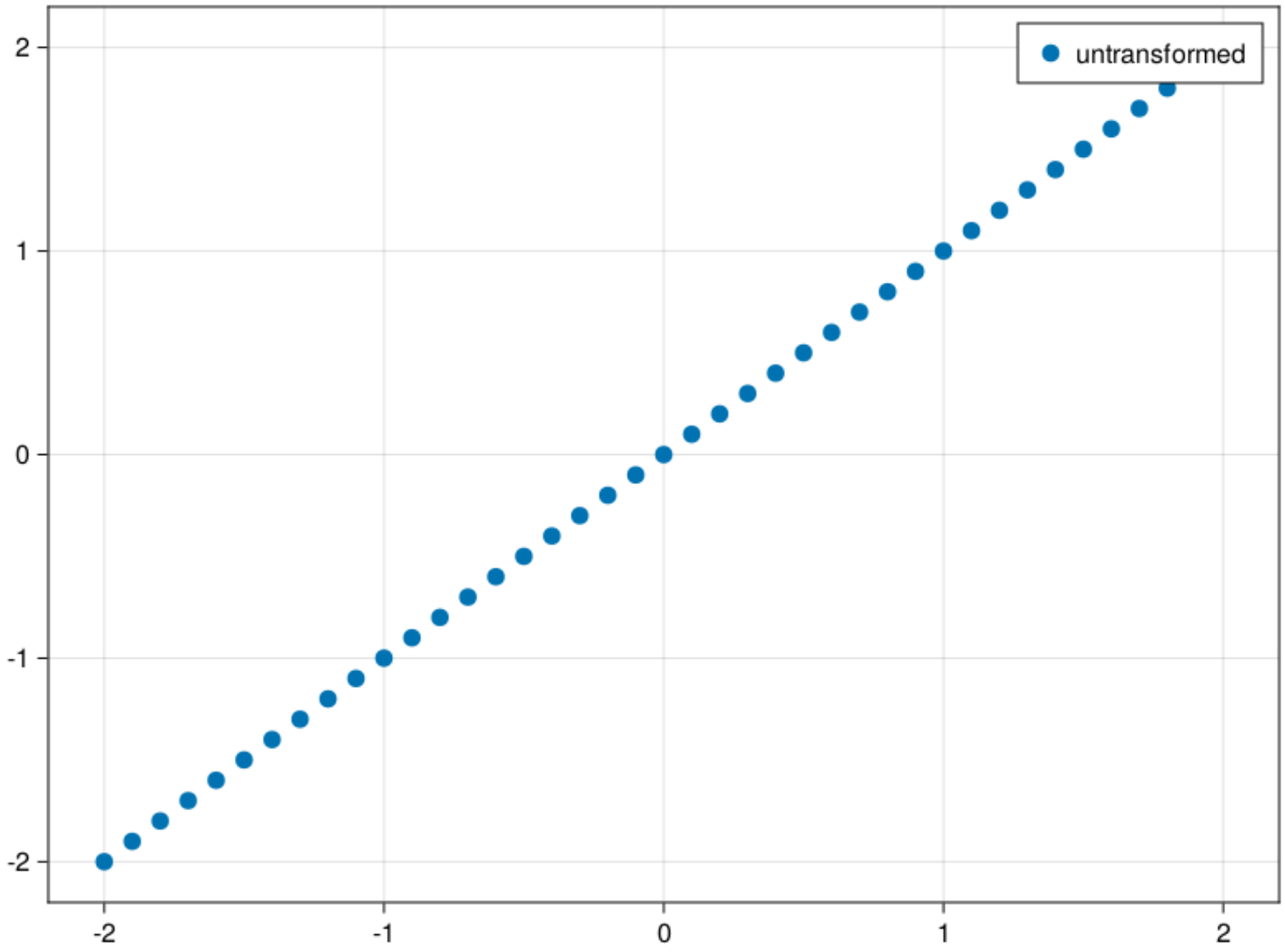
Definition

A symmetry is a change that doesn't change anything.

What is a symetry?

Let's see this in action using the simple linear equation $x - y = 0$.



What is a symmetry?

Example (A Non-Example)

- For our first transformation, let's define new variables $\bar{x} = x + 1$ and $\bar{y} = y$.
- Now we rewrite our equation using these new variables.

$$\begin{aligned} \bar{x} - \bar{y} &= 0 && \text{by definition} \\ x + 1 - y &= 0 && \text{by substitution} \\ y &= x + 1 && \text{by rewriting in slope-intercept form} \end{aligned}$$

- This is transformation is not a symmetry: $x - y + 1 \neq x - y$

What is a symmetry?

A Transformation that is a Symmetry

Example (2)

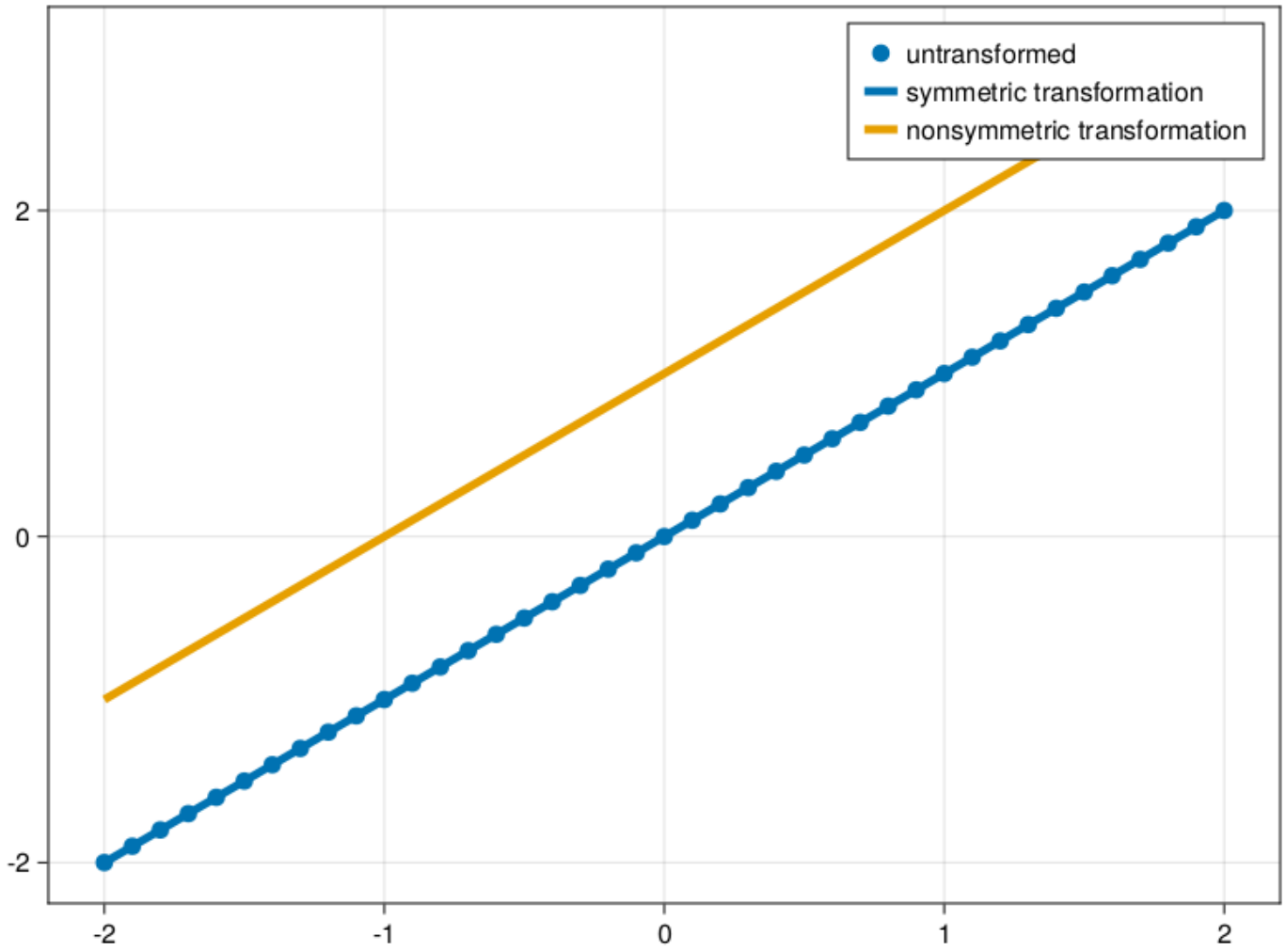
- Let's define some new variables again
  $\bar{x} = x + 1$ and $\bar{y} = y + 1$.
- Now we rewrite our equation using these new variables.

$$\begin{aligned}
\bar{x} - \bar{y} &= 0 && \text{by definition} \\
(x + 1) - (y + 1) &= 0 && \text{by substitution} \\
(x - y) + (1 - 1) &= 0 && \text{by algebra} \\
x - y &= 0 && \text{by algebra} \\
y &= x && \text{by rewriting in slope-intercept form}
\end{aligned}$$

- This transformation is a symmetry:
  $x - y = x - y$

What is a symetry?

The graphs of our three equations.



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Who cares?

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- Symmetries help us understand and solve equations that we wouldn't normally be able to.
- Symmetries encode physically meaningful aspects of equations, like conservation laws in physics.
- They're cool.

What is a Differential Equation?

The History of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Classical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Nonclassical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

Future Work: Does Integrability Imply Equivalence of Classical and Nonclassical Symmetries?

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