Comparing the Classical and Nonclassical Symmetries of Nonlinear Partial Differential Equations

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Project Goal

Our research objective for this project was to calculate the classical and nonclassical symmetry groups for the reduced Gibbons-Tsarev equation and the Born-Infeld equation and compare them.

What is a symmetry?

Definition

A symmetry is a transformation that leaves an object invariant.

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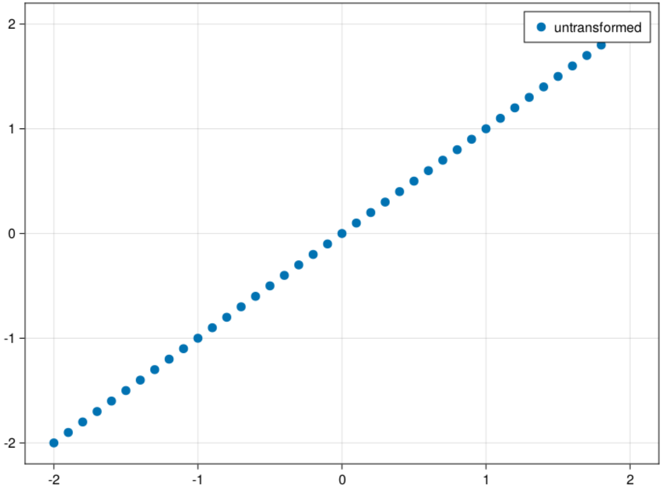
A symmetry is a transformation that leaves an object invariant.

Definition

A symmetry is a change that doesn't change anything.

What is a symetry?

Let's see this in action using the simple linear equation $x - y = 0$.



What is a symmetry?

Example (A Non-Example)

- For our first transformation, let's define new variables $\bar{x} = x + 1$ and $\bar{y} = y$.
- Now we rewrite our equation using these new variables.

$$
\begin{aligned}
\bar{x} - \bar{y} &= 0 && \text{by definition} \\
x + 1 - y &= 0 && \text{by substitution} \\
y &= x + 1 && \text{by rewriting in slope-intercept form}
\end{aligned}
$$

- This is transformation is not a symmetry: $x - y + 1 \neq x - y$

Example (2)

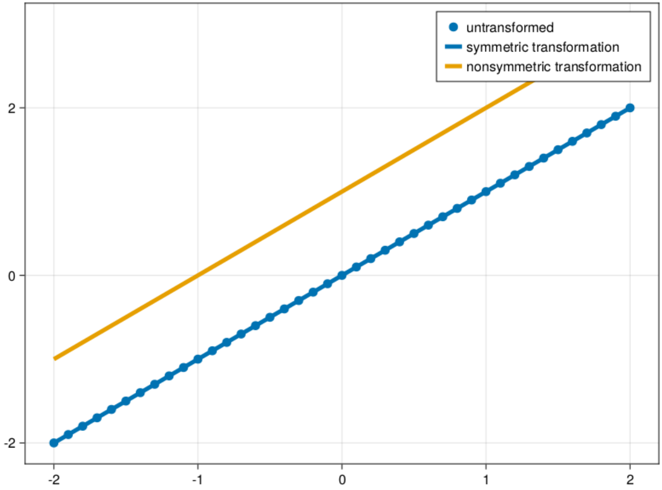
- Let's define some new variables again
  $\bar{x} = x + 1$ and $\bar{y} = y + 1$.
- Now we rewrite our equation using these new variables.

$$
\begin{aligned}
\bar{x} - \bar{y} &= 0 && \text{by definition} \\
(x+1) - (y+1) &= 0 && \text{by substitution} \\
(x-y) + (1-1) &= 0 && \text{by algebra} \\
x - y &= 0 && \text{by algebra} \\
y &= x && \text{by rewriting in slope-intercept form}
\end{aligned}
$$

- This transformation is a symmetry:
  $x - y = x - y$

What is a symetry?

The graphs of our three equations.



What is a symmetry?

Who cares?

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- Symmetries help us understand and solve equations that we wouldn't normally be able to.
- Symmetries encode physically meaningful aspects of equations, like conservation laws in physics.
- They're cool.

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- A differential equation is an equation that contains both an unknown function and information about how that function relates to its rates of change.
- Differential equations show up everywhere we model something using information about how that thing changes over time. This includes everything from population dynamics to the motion of the planets.
- Differential equations are different from algebraic equations, and can't be solved in the same ways.

What is a Differential Equation?

As an example, we'll use the equation for a spring.

$$m\ddot{x} + \gamma\dot{x} + \kappa x = 0$$

Here m is the mass, $\ddot{x}$ is the acceleration, γ is the damping constant (friction), $\dot{x}$ is the velocity, κ is the spring constant, and x is the position.

The History of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Classical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

The Nonclassical Symmetries of the Born-Infeld and the reduced Gibbons-Tsarev Equations

Future Work: Does Integrability Imply Equivalence of Classical and Nonclassical Symmetries?

Future Work: Does Equivalence of Classical and Nonclassical Symmetries Imply Integrability?