A multi-nuclide approach to constrain landscape evolution and past erosion rates in previously glaciated terrains

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Abstract

Cosmogenic nuclides are typically used to either constrain an exposure age, a burial age, or an erosion rate. Constraining the landscape history and past erosion rates in previously glaciated terrains is, however, notoriously difficult because it involves a large number of unknowns. The potential use of cosmogenic nuclides in landscapes with a complex history of exposure and erosion is therefore often quite limited. Here, we present a novel multi-nuclide approach to study the landscape evolution and past erosion rates in terrains with a complex exposure history, particularly focusing on regions that were repeatedly covered by glaciers or ice sheets during the Quaternary. The approach, based on the Markov Chain Monte Carlo (MCMC) technique, focuses on mapping the range of landscape histories that are consistent with a given set of measured cosmogenic nuclide concentrations. A fundamental assumption of the model approach is that the exposure history at the site/location can be divided into two distinct regimes: i) interglacial periods characterized by zero shielding due to overlying ice and a uniform interglacial erosion rate, and ii) glacial periods characterized by 100 % shielding and a uniform glacial erosion rate. Two different approaches are proposed to incorporate the exposure history into the model framework, one of which relies on the application of different threshold values to the global marine benthic $\delta^{18}O$ record. However, any information on the glacial-interglacial history at the sampling location, in particular the timing of the last deglaciation event, is readily incorporated to constrain the inverse problem. Based on the MCMC technique, the model delineates the most likely exposure history, including the glacial and interglacial erosion rates, which, in turn, makes it possible to reconstruct an exhumation history at the site. The model framework, which currently includes the following nuclides ^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne, is highly flexible and can be adapted to many different landscape settings. As such, the algorithm requires specification of the combination of nuclides used in the study, the measured concentrations and associated uncertainties, and whether samples have been collected from the surface or from a depth profile. The model framework presented here may also be used in combination with physics-based landscape evolution models to predict nuclide concentrations at different locations in the landscape. This may help validate the landscape models via comparison to measured nuclide concentrations or to devise new effective sampling strategies.

Keywords: Cosmogenic-nuclide geochronology, Markov Chain Monte Carlo inversion, glacial landscape history, erosion rate reconstructions, Quaternary climate

1. Introduction

As global climate cooled during the late Neogene, the surface processes eroding mountain ranges seem to have accelerated substantially (Zhang et al., 2001; Herman et al., 2013). The dramatic fluctuations between glacial and interglacial periods characteristic of the Quaternary altered the erosional dynamics in most of Earth's mountain ranges, in part due to changes in river discharge, shifting vegetation regimes, and the advent of cold-climate processes, including frost weathering and the development of extensive ice masses (Shuster et al., 2005; Thomson et al., 2010). These processes played an important role in shaping many of the remarkable, first-order topographic features observed today, such as the spectacular fjord and valley landscapes of e.g. Norway, Greenland, and New Zealand.

Efforts to understand the evolution of mountain ranges and their complex links to climate are often hampered by difficulties in quantifying past denudation rates, a parameter that may vary over several orders of magnitude depending on the geological setting and processes that govern the removal of mass (e.g. von Blanckenburg, 2005). In non-glaciated terrains, steady-state denudation rates are typically estimated by use of "in-situ" produced cosmogenic nuclides (Lal, 1991), in particular ^{10}Be. This approach is unviable in terrains periodically covered by large ice masses during glacial periods, because the surface rocks were shielded for unknown lengths of time. In such settings, the concentration of cosmogenic nuclides is generally interpreted as an exposure age, typically a deglaciation age, assuming substantial glacial erosion prior to the deglaciation and that the denudation rate since the time of exposure is negligible or that it can be inferred from independent evidence.

Inherited nuclides often represent a problem when estimating exposure ages in landscapes that were repeatedly covered by ice in the past. The problem arises in landscapes characterized by low denudation rates, where a significant amount of the cosmogenic nuclides produced during previous periods of exposure remains in the surface bedrock. This problem may be overcome by collecting paired samples in the field, i.e. sampling both boulders and bedrock, in which case it is often possible to estimate the deglacial age and the amount of inheritance in the bedrock sample. An extension of this approach involves the use of paired cosmogenic nuclides, primarily ^{10}Be and ^{26}Al, which allows estimates of the minimum-limiting exposure duration and minimum-limiting burial duration (e.g. Bierman et al., 1999; Corbett et al., 2013). In this case, the different half-lives and production rates of the nuclides can be used to constrain a total landscape history. Concentrations of ^{10}Be and ^{26}Al have additionally been used to study glacial-interglacial variations in denudation rate in non-glaciated terrains (Hidy et al., 2014).

Several studies have employed paired nuclides to date buried sediments (e.g. Granger and Smith, 2000; Granger, 2006; Haeuselmann et al., 2007) and depositional landscape surfaces (Anderson et al., 1996). Most of these studies rely on ^{10}Be and ^{26}Al, but burial dating schemes involving ^{26}Al-^{10}Be-^{21}Ne also exist (Balco and Shuster, 2009). Braucher et al. (2009) showed that it is possible to determine both an exposure time and denudation rate from an in-situ produced ^{10}Be depth profile and a versatile Monte Carlo simulator for modeling depth profiles of ^{10}Be or ^{26}Al in sediments is available online (Hidy et al., 2010). None of these approaches, however, focus on resolving landscape history and past denudation rates from bedrock samples collected in terrains that were repeatedly covered by glacial ice in the past.

In this study, we aim to develop a robust and flexible multi-nuclide approach to study landscape evolution in areas characterized by a complex exposure history. The main focus is to constrain the most likely glacial-interglacial landscape history of an area and estimate average glacial and interglacial erosion rates by exploiting the different half-lives and production rates of the various cosmogenic nuclides. For this purpose, a Markov Chain Monte Carlo (MCMC) approach is developed in order to systematically delineate the most likely landscape evolution within the framework of the model.

2. Approach and Methods

2.1 Model concept and framework

The basic idea underlying this model framework is to systematically simulate the production and loss of in-situ terrestrial cosmogenic nuclides (TCNs) associated with the glacial-interglacial cycles of the Quaternary and to map the glacial-interglacial landscape histories that are consistent with a given set of measured TCN concentrations. We use the MCMC technique to simulate TCN concentrations associated with a large number of different glacial-interglacial landscape histories, including highly varying glacial and interglacial erosion rates. Based on comparisons to measured concentrations, it is possible to determine the most likely landscape history and associated uncertainties. A key aspect of this approach is to select the right set of model parameters and maintain an optimal balance between the number of observations and model parameters in order to be able to constrain the problem. As too many model parameters will render the problem intractable, some simplifications are required to formulate a balanced and viable computational framework for the forward model, i.e. the computation of TCN concentrations over multiple glacial-interglacial cycles.

In general, the production of TCNs occurs during times of exposure when there is no glacial ice to shield the surface bedrock, whereas the loss of TCNs is due to radioactive decay and erosion (Lal, 1991). A key principle introduced in this study is what we refer to as "two-stage uniformitarianism", meaning that the processes that operated during the Holocene also operated during earlier interglacials with comparable intensity. This simplifying assumption implies that the denudation processes that dominated during the Holocene also dominated during earlier interglacials, and similarly that the denudation processes that dominated during the last glacial period, the so-called Weichsel/Wisconsin glacial period, also dominated during earlier glacial periods. The model concept consequently operates with two erosion rates, an interglacial erosion rate, ε_{int}, and a glacial erosion rate, ε_{gla}. As a starting point, the model concept also assumes that interglacial periods were characterized by 100 % exposure and zero shielding due to overlying glaciers, whereas glacial periods were characterized by 100 % shielding and no exposure. However, the timing of glacial-interglacial transitions at any specific location is often poorly constrained in time, or completely unknown, and these transition times must therefore be incorporated in the model framework as free parameters that vary among the simulations. In this study, we propose two different models to constrain the forward problem and address these unknowns: a periodic two-stage model and a two-stage model constrained by the marine $\delta^{18}O$ record.

2.1.1 Periodic two-stage model (Model 1)

In the periodic two-stage model, present nuclide concentrations are computed as a function of periodic transitions between glacial and interglacial intervals throughout the Quaternary with the exception of the last deglaciation (Fig. 1), which is often reasonably well constrained in time through ^{10}Be dating of allochthonous boulders (glacial erratics) or studies of terminal moraines. For the purpose of the inverse analysis, we parameterize the erosion and exposure history in Model 1 as follows

$$\mathbf{m} = \left\lfloor \varepsilon_{int}; \varepsilon_{gla}; t_{degla}; \Delta t_{gla}; \Delta t_{rat} \right\rfloor$$

where ε_{int} is the erosion rate during all non-glaciated time intervals, including the present and past interglacial periods, and ε_{gla} is the erosion rate during all glaciated periods. t_{degla} is the time elapsed since the last deglaciation and Δt_{gla} is the duration of each individual glacial period, which is constant for all glaciations, and Δt_{rat} is the ratio between the duration of glacial periods and interglacial periods (Fig. 1).

2.1.2 Two-stage model inspired by the global marine $\delta^{18}O$ stack (Model 2)

Although past transitions between glacial and interglacial periods at specific locations are often unconstrained, there is reliable information from marine $\delta^{18}O$ records regarding large-scale climatic changes during the Quaternary, indicating when glaciations were widespread and likely to occur. Marine benthic oxygen isotope records have been used in previous studies to infer the most likely glacial-interglacial history by identifying a transition threshold value based on the timing of the last deglaciation. (e.g. Kleman and Stroeven, 1997; Fabel et al., 2002). Fabel et al. (2002) use a value of 3.7‰, but such a Pleistocene-Holocene threshold value may not have applied to past glacial-interglacial transitions. In this study, we therefore estimate the most likely glacial-interglacial transitions by incorporating the $\delta^{18}O$ threshold value as a free parameter that is applied iteratively to the stacked marine benthic $\delta^{18}O$ record of Liesicki & Raymo (2005). We apply this prior information in our Model 2 as follows

$$\mathbf{m} = \lfloor \varepsilon_{int}; \varepsilon_{gla}; t_{degla}; \delta^{18}O_{threshold} \rfloor$$

where the $\delta^{18}O_{threshold}$ defines the timing of glacial and interglacial periods via the $\delta^{18}O(t)$ time history (Fig. 2). The degree of smoothing applied to the $\delta^{18}O$ record influences the number of transitions and the duration of individual glacial-interglacial periods. It is therefore possible to specify the degree of smoothing applied to the $\delta^{18}O$ record, although the difference between present-nuclide concentrations obtained with various degrees of smoothing is relatively small compared to the influence of the four model parameters. In principle, the timing of the last deglaciation, defined by model parameter t_{degla}, is redundant because it is defined by the $\delta^{18}O$ threshold value. However, it makes sense to decouple the last deglaciation from the $\delta^{18}O$ threshold because t_{degla} is often known with reasonable precision and accuracy.

2.2 Cosmogenic nuclides included in the model framework

The model framework can accommodate any number of TCNs, but is currently based on the following four: ^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne. In some respect, this combination of nuclides is advantageous because of their different half-lives and production rates (Gosse and Phillips, 2001), which enable their concentrations to integrate different aspects of the glacial-interglacial history. All four nuclides are produced at reasonably well-constrained rates in quartz, which is a very common mineral that is highly resistant to weathering and loss of nuclides after production. Quartz is usually the preferred target mineral for all four nuclides. Beryllium-10 and ^{26}Al are routinely measured with high precision using Accelerator Mass Spectrometry (AMS), whereas ^{21}Ne, which is stable, is measured with high precision using a noble gas mass spectrometer. The main challenge with ^{21}Ne is that significant amounts of a non-cosmogenic component may be present in the samples. This interfering component is usually identified and corrected for using a neon three-isotope diagram (Niedermann, 1994; Niedermann, 2002) or via deeply shielded samples that contain no cosmogenic ^{21}Ne, such as from road cuts. The inclusion of in-situ produced ^{14}C is important due to its short half-life of 5730 years, which make this nuclide an especially sensitive Holocene chronometer. It remains challenging to measure in-situ produced ^{14}C, but it is currently achieved at several AMS laboratories around the world.

2.3 Cosmogenic nuclide production rates

The cosmogenic nuclides are produced when the surface rock is exposed to a shower of secondary cosmic-ray particles, including neutrons and muons (Lal and Peters, 1967). Upon reaching Earth's surface, these particles interact with atoms in the minerals to produce cosmogenic nuclides. In this study, we include the three most important production mechanisms for the four cosmogenic nuclides mentioned above: nucleonic spallation (spal), negative muon capture (nmc), and fast muons (fm). The production of cosmogenic nuclides decays near-exponentially with depth for all three mechanisms, although at different rates because neutrons have considerably shorter attenuation lengths (Λ) than muons, which penetrate much deeper into the ground (Gosse and Phillips, 2001). Although the theoretical production of TCNs due to muons does not behave as a simple exponential function with depth (Heisinger et al., 2002a, 2002b), it has been shown that reasonable approximations to the theoretical production can be made with multiple exponential terms for muon production mechanisms (Granger and Smith, 2000; Schaller et al., 2002). In this study, we follow the general approach of Hidy et al. (2010) to calculate the TCN production as a function of depth (z), albeit with some minor modifications. We use one exponential term to calculate the spallogenic production rate and three terms for each of the muonic components:

$$P_{spal}(z) = P_{spal}(0) \times e^{-\frac{\rho z}{\Lambda_{spal}}} \quad (1)$$

$$P_{nmc}(z) = P_{nmc}(0) \times \sum_{i=1}^{3} a_i \times e^{-\frac{\rho z}{\Lambda_{nmc,i}}} \quad (2)$$

$$P_{fm}(z) = P_{fm}(0) \times \sum_{j=1}^{3} b_j \times e^{-\frac{\rho z}{\Lambda_{fm,j}}} \quad (3)$$

$$P_{Total}(z) = P_{spal}(z) + P_{nmc}(z) + P_{fm}(z) \quad (4)$$

where ρ is the density of the rock (here we use a value of 2.65 g/cm^3), Λ the attenuation lengths (Λ_{spal}=160g/cm^{-2}), and a_i, b_j, are dimensionless coefficients. The values used in this study for the dimensionless coefficients and attenuation lengths associated with negative muon capture (nmc) and fast muons (fm) are adopted from Schaller et al. (2002). We assume that variations in nuclide production rate with depth can be approximated with Eq. 1-4 for all four nuclides. The surface production rates $P(0)_{spal}$, $P(0)_{nmc}$, $P(0)_{fm}$ must be specified for the study site, for instance by use of the CRONUS-Earth on-line calculator (Balco et al., 2008). We note, however, that the current version of our model only incorporates nuclide production rates that are constant in time. This means that scaling schemes accounting for solar and geomagnetic field effects (e.g. Dunai, 2001; Lifton et al., 2005) are excluded for now.

2.4 Computation of present nuclide concentrations

When erosion rates and TCN production rates at the surface vary as a step function in time due to the waxing and waning of ice sheets, the present-time nuclide concentration may be calculated using a Lagrangian approach in which a layer is tracked as it is slowly advected towards the surface. Consider a rock sample at the present depth of burial, z_{obs}, which for samples collected at the surface will be zero (z = 0 m). Owing to the varying erosion rates, $\varepsilon(t)$, this rock sample has followed a depth track given by

$$z(t) = z_{obs} + \int_0^t \varepsilon(t')dt' \qquad (5)$$

Therefore, this sample has experienced a production rate that has varied in time according to

$$P(t,z(t)) = P_{spal}(t,0) \times e^{-\rho z(t)/\Lambda_{spal}} + P_{nmc}(t,0) \times e^{-\rho z(t)/\Lambda_{nmc}} + P_{fm}(t,0) \times e^{-\rho z(t)/\Lambda_{fm}} \qquad (6)$$

where the production rate at the surface, $P(t,0)$, varies due to changes in the shielding associated with an overlying ice cover, i.e. 0 % shielding during interglacials and 100 % shielding during glacials. Modulations of the cosmic-ray flux due to variations in the solar magnetic and geomagnetic fields are neglected, as are potential variations in the production rate associated with elevation changes via glacioisostatic-rebound effects. In general, the differential equation for the nuclide concentration is

$$\frac{\partial C(t)}{\partial t} = -\lambda C(t) + P\left(t, z(t)\right) \qquad (7)$$

where λ is the radioactive decay constant of the nuclide. For a given erosion history and glacial-interglacial exposure history, governing variations in the surface production rates, it is possible to solve Eq. 7 numerically with standard techniques (see below). However, when the erosion rates and surface production rates are piece-wise constant, this differential equation (7) can be solved analytically by a sum of recursive exponential terms, which makes the calculation of present nuclide concentrations particularly fast for the two-stage uniformitarian models considered here.

2.4.1 A Eulerian approach to compute present nuclide concentrations

The present TCN concentrations may also be determined using a Eulerian approach, in which the nuclide concentrations are computed at specific depths while the rock layers are advected towards the surface due to erosion. In this approach, changes in nuclide concentrations are computed in small incremental time steps (dt) as follows

$$\frac{\partial C(t)}{\partial t} = P_{Total}\,(z) \times dt \times e^{(-\lambda \times dt)} - C(t) \times \lambda \times dt + \varepsilon \times \nabla C(t) \qquad (8)$$

where the production term (P_{Total}) includes those nuclides produced within the time step dt, some of which are lost due to decay, and the second term calculates the loss due to decay of existing nuclides. Changes in concentration due to erosion are handled as an upward advection of layers (with $\varepsilon>0$) with lower concentrations towards the surface, as described by the last term of Eq. 8. This Eulerian approach is a very different, and computationally expensive, way of calculating changes in nuclide concentrations over glacial-interglacial cycles as the time steps must be relatively small (typically 100 years) to maintain computational stability when computing the advection. Nevertheless, the two different methods yield virtually identical TCN concentrations. Figure 3a demonstrates how the Lagrangian (analytical) approach (red) and the step-wise Eulerian approach (blue) reach the same ^{10}Be surface concentrations after simulation of 10 glacial-interglacial cycles, lasting 100 kyr and 10 kyr, respectively (periodic Model 1). Figure 3b demonstrates how the two approaches yield similar concentrations as a function of depth after simulation of the 10 glacial-interglacial cycles. As the computational cost represents a critical aspect due to the number of iterations required by the inverse MCMC analysis, we use the analytical solution in the forward computation of TCN concentrations.

3. Inverse Markov Chain Monte Carlo resolution analyses

Based on the forward model described above, we have developed an inverse MCMC approach to constrain the most likely landscape history from a combination of cosmogenic nuclides with different production rates and half-lives. The forward models allow for observations of a number of nuclide concentrations and associated uncertainties at a range of depths. The observation data vector consists of the measured nuclide concentrations, which for surface-based measurements (z = 0 m) of four nuclides is given by

$$\mathbf{d}_{obs} = \left[C_{10Be}, C_{26Al}, C_{14C}, C_{21Ne}\right]$$

We analyze this problem using a conventional Metropolis-Hastings MCMC technique (Metropolis et al., 1953, Hastings, 1970) where model parameters are constrained between fixed bounds specified by the user. Erosion rates (ε_{int}, ε_{gla}), which may vary over orders of magnitude, are specified with uniform probability across the logarithmic parameter interval. The time parameters (e.g. t_{degla} and $\delta^{18}O_{threshold}$ in Model 2) are specified with uniform probability across the linear parameter interval. After the user has specified the bounds of the model parameters, which define the model space that is searched with the MCMC technique, a forward response is computed based on an initial set of model parameters that is proposed using the Metropolis-Hastings technique. A burn-in phase of 1,000 iterations is first used to make a crude initial search of the model space. This step is followed by a more detailed and local search of the model space using the set of model parameters from the burn-in phase with the smallest misfit when compared to the observation data vector. At each iteration step, the current model is perturbed by a fraction of the prior interval. This fraction is updated every 1000 iterations so that an acceptance ratio of about 0.4 is maintained (Gelman et al., 1996). To ensure that the set of model parameters providing the best fit to the observed data does not depend on the starting position of the random search through the model space, a number of "random walks" (e.g. 4) are started at different positions in the model space (e.g. different corners or edges). If these completely independent "random walks" achieve similar distributions for the best-fitting model parameters, it is highly unlikely that there are global misfit minima that remain undetected. Based on the combination of model parameters that provide the best fit to the data, it is possible to compute the most likely exhumation history for the site and/or study area.

4. Investigating possible landscape scenarios

Many mountain ranges developed large ice masses as the global climate cooled during the late Neogene. This marked a transition from essentially ice-free conditions to a time of repeated growth and decay of glacial ice over tens of glacial cycles. Depending on latitude, elevation, and regional climate regime, these glaciers may have been warm-based and highly erosive, or cold-based and non-erosive (e.g. Fabel et al., 2002; Kleman et al., 2008). In the following, we illustrate the model approach and associated MCMC technique by simulating landscape-evolution scenarios that resemble the onset of widespread glaciations, both erosive and non-erosive, during the Quaternary. In the first two scenarios, we simulate the "observed" nuclide concentrations (^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne), which form the basis of the MCMC inversion, based on a set of "true", synthetic model parameters. The third scenario is based on real ^{10}Be and ^{26}Al data from Upernavik, West Greenland (Corbett et al., 2013). In all three scenarios, we use Model 2, based on the global marine $\delta^{18}O$ stack, and a 30-kyr running mean of the $\delta^{18}O$ record.

4.1 Synthetic landscape scenario 1

In scenario 1, we simulate landscape evolution in which the glacial erosion rate ($\varepsilon_{gla} = 1x10^{-6}$ m/yr) is considerably lower than the inter-glacial erosion rate ($\varepsilon_{int} = 5x10^{-5}$ m/yr), thereby leaving a large, inherited cosmogenic inventory from earlier periods of exposure. The glacial-interglacial history is obtained by applying a threshold value of 3.8 ‰ to the marine benthic oxygen isotope record (Liesicki & Raymo, 2005) (similar to Fig. 2), whereas the timing of the last local glacial-interglacial transition ($t_{degla} = 11{,}000 \pm 1{,}000$ kyr ago) is assumed to be well constrained from studies of glacial erratics. In this synthetic scenario, we use the spallogenic ^{10}Be production rate of 5.33 atoms/g/yr for sea level and high latitudes (Dunai, 2000), whereas the muonic surface production components are obtained from Heisinger et al., (2002a, and 2002b). The production rates of the other nuclides are estimated from the literature (e.g. Gosse and Phillips, 2001; Dunai, 2010; Braucher et al., 2011; Braucher et al., 2013), and a total uncertainty of 5 % was assumed for each nuclide.

For the inverse MCMC approach, the bounds of the model parameters must be specified. For scenario 1, the bounds were as follows: $\varepsilon_{int} = 10^{-3}$ to 10^{-7} m/yr ; $\varepsilon_{gla} = 10^{-3}$ to 10^{-7} m/yr; $\delta^{18}O_{threshold}$ = 3.7 to 4.3 ‰; t_{degla} = 10,000 to 12,000 years (Fig. 5). The model parameters that provide the best fit to the "observed" nuclide concentrations are indicated by yellow/reddish colors in Fig. 4. The histograms in Fig. 5 show the range of possible solutions for the four model parameters and the skewness of the histograms reflects the non-linearity of the problem. The interglacial erosion rate is very well constrained in this scenario (Fig. 5a), whereas the low erosion rate associated with glacial periods is less well defined (Fig. 5b). The model parameters are generally identified correctly by the MCMC technique, as indicated by the peak distribution of the histograms. Figure 6 shows the exhumation histories that are computed from the best-fitting erosion rates and glacial-interglacial transitions ($\delta^{18}O_{threshold}$) provided by the distributions in Fig. 5. The underlying density plot in Fig. 6 reflects the exhumation history of the 10,000 simulations of the MCMC analysis. Note that the general change in exhumation rate around 1.5-1 Myr ago, which is linked to the emerging predominance of cold-based, non-erosive ice sheets, is captured by the MCMC approach.

4.2 Synthetic landscape scenario 2

In this scenario, the glacial erosion rate ($\varepsilon_{gla} = 1x10^{-5}$ m/yr) is an order of magnitude higher than the interglacial erosion rate ($\varepsilon_{int} = 1x10^{-6}$ m/yr), and the other model parameters are as follows; $\delta^{18}O_{threshold} = 4.0$ ‰ and $t_{degla} = 11{,}000 \pm 1{,}000$ kyr ago. The production rates and bounds of the model space, which define the range of possible model parameters investigated with the MCMC technique, remain unchanged compared to scenario 1.

The MCMC inversion approach correctly identifies the true model parameters, but the interglacial erosion rate and $\delta^{18}O_{threshold}$ are not as well constrained as in scenario 1 (not shown). The glacial erosion rate, however, which is an order of magnitude higher than the interglacial erosion rate, is better constrained. The full range of possible exhumation paths, encompassing all 10,000 MCMC simulations, displays some scatter (Fig. 7), particularly prior to 500 kyr ago due to the uncertainty associated with the estimated $\delta^{18}O_{threshold}$ value. This is a direct result of applying our two-stage uniformitarianism 2 Myr back in time. Also, by holding the $\delta^{18}O_{threshold}$ value constant, we neglect the realistic possibility of variations occurring in sync with the overall changes in climate during the Quaternary. Nevertheless, the majority of simulated exhumation rates falls within a relatively limited band (Fig. 7), and the median value tracks the true exhumation history (magenta line). The majority of the best-fitting model parameters correctly capture the change in exhumation rate associated with the onset of warm-based glaciations around 1 Myr ago.

For scenario 2, we also investigate how different sample and measuring strategies may influence the estimated exhumation history. It is currently common practice to measure concentrations of ^{10}Be and ^{26}Al in boulder and bedrock samples (e.g. Corbett et al., 2013). Figure 8 shows the estimated exhumation history for scenario 2 based solely on ^{10}Be and ^{26}Al concentrations, i.e. the concentrations of ^{14}C and ^{21}Ne have been excluded in the MCMC analysis. In this scenario, the median exhumation history based on ^{10}Be and ^{26}Al is very similar to that based on four nuclides (^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne) and it also tracks the true exhumation rate closely (Fig. 8). The scatter among the 10,000 individual MCMC simulations is, however, somewhat higher and more arbitrary. Also, the sensitivity to the starting position of the random search of the model space is higher when the MCMC analysis is based on two nuclides compared to four nuclides, underlining the importance of using multiple random walkers. We also investigate how well constrained the exhumation history becomes for scenario 2 if all four nuclides are measured in a depth profile (z = 0, 0.3, 1, 3, and 10 m). The experiment shows how a depth profile potentially provides a much more well-constrained exhumation history (Fig. 9).

4.3 Landscape scenario 3 – Upernavik, West Greenland

In the third scenario, the model framework is applied to real data derived from a sample collected in the area around Upernavik, West Greenland (Corbett et al., 2013). This bedrock sample (GU110), which was collected at an elevation of 745 m a.s.l., was chosen for this analysis because the measured ^{10}Be and ^{26}Al concentrations indicate a long total exposure history (~989 kyr). No allochthonous boulders were sampled at this site, but boulders collected from lower elevations in this area suggest that the last deglaciation event occurred around 11-12 kyr ago, and we note that this age may not apply to higher elevations. The ^{10}Be and ^{26}Al production rates at the sample site were estimated by use of the northeastern North American production rates (Balco et al., 2009) and the CRONUS-Earth on-line calculator. The muonic production rates were calculated according to Balco et al. (2008). The uncertainties associated with the measured ^{10}Be (5.67×10^5 atoms/g) and ^{26}Al (2.67×10^6 atoms/g) concentrations were 2.6 % and 4.0 %, respectively.

In the initial MCMC analysis, the last deglaciation event was constrained to the period 10-12 kyr ago, in agreement with ^{10}Be ages from boulders sampled at lower elevations in the area. The MCMC analysis reveals that the interglacial erosion rate is poorly constrained, but less than 6 m/Myr and most likely in the range 0.6-6 m/Myr. The glacial erosion rate is better constrained with the most likely erosion rate falling within the interval 1.25-2.3 m/Myr, depending on the exposure history. Interestingly, the $\delta^{18}O_{threshold}$ value is very well constrained, suggesting that the exposure history is defined by values in the range 3.81-3.84 ‰. Such $\delta^{18}O_{threshold}$ values imply that the total amount of exposure within the last million years was limited to the range 80-110 kyr, of which ~26 kyr of exposure occurred during MIS (marine isotope stage) 11 around 400 ka. This limited amount of exposure is highly consistent with the minimum-limiting exposure duration (88 kyr) and burial duration (901 kyr) found by Corbett et al. (2013) based on the two-isotope burial-exposure diagram. The most likely exhumation history can be computed from the best-fitting model parameters with an estimated exhumation rate of 1.5 ± 0.5 m/Myr (Fig. 10). This uncertainty estimate is not based on Gaussian distributions, but the fact that 50% of the simulated, best-fitting exhumation histories fall within this range. The interquartile range (middle 50 %) provides a robust representation of the uncertainty associated with the estimated exhumation history based on the MCMC technique.

A slightly different MCMC analysis, which assumed no prior information regarding the timing of the last deglaciation event, was also carried out. This analysis yielded very similar results, including a slightly lower glacial erosion rate (1.25 m/Myr compared to 1.8 m/Myr on average) and slightly less total exposure over the last 1 Myr. These differences result from an estimated last deglacial event at 30-35 kyr, which is clearly different from the deglacial age inferred from nearby erratic boulders found at lower elevations. Although the exhumation histories are very similar in this case, it demonstrates the importance of including prior information regarding glacial-interglacial transitions. The inclusion of an extra nuclide, particularly ^{14}C, or a depth profile would help constrain the model parameters in the MCMC analysis.

5. Discussion

The framework presented here provides a highly flexible multi-nuclide approach to delineate likely landscape histories and past erosion rates in terrains previously covered by ice masses. The approach is designed to be easily applicable to a wide range of very specific settings or problems. As such, the user may specify the production rates due to spallation and muons at the study site, the attenuation lengths, the rock density, the number of cosmogenic nuclides used in the study (e.g. ^{10}Be, ^{26}Al or ^{10}Be, ^{26}Al, ^{14}C, ^{21}Ne), the measured concentrations and associated uncertainties as well as the sample depths (e.g. z = 0 m or z = 0, 0.5, 1 m). It is also possible to specify various kinds of information regarding past glacial-interglacial transition times. Likewise, if any of the model parameters are well constrained from other studies, a narrow bound should be specified for these parameters for the MCMC inversion.

Nevertheless, the general approach contains some rather simplistic assumptions concerning the choice of model parameters related to past glacial-interglacial transition times and past erosion rates that are subject to debate.

5.1 Estimating the glacial-interglacial exposure history

In many cases, it is possible to estimate the timing of the last deglacial transition, e.g. via ^{10}Be dating of erratic boulders or terminal moraines, but no information regarding earlier transitions is available. For such scenarios, this study provides two possible solutions: a) the periodic two-stage model, and b) the two-stage model calibrated to proxy records of global climatic changes. The latter approach is appealing, because the regional extent of glaciations is likely to correlate with changes in global climate. This may not be true locally, however, as the occurrence of past glaciations depends on local climate and altitude. It is also unknown if the assumption regarding 100 % shielding during glacial periods is reasonable, as it would require >10m of ice to render the production due to spallation negligible (> 50 m for muons). The cumulative effects of snow shielding during interglacials, as discussed by Schildgen et al. (2005), are also uncertain and potentially important. It is possible to correct for such effects by introducing a correction factor in the model, provided the effect is well constrained.

Another aspect concerns the temporal resolution at which glaciations occur, e.g., did MIS 5 comprise a warm, ice-free interval, including the Eemian (MIS 5e), and a series of relatively brief glaciations? A recent study by Mangerud et al. (2011) shows that in Scandinavia the stadials following the Eemian were marked by glacial advances lasting 5-10 kyr. These changes are very consistent with the glacial-interglacial history obtained by applying a 5-kyr running mean to the marine δ^{18}O record (not shown), whereas the 30-kyr running mean used in Fig. 2 and scenarios 1-3 produces a somewhat simpler exposure history with fewer glacial-interglacial transitions. In most scenarios, however, the resolution of the marine $\delta^{18}O$ record has a limited effect on the concentration of cosmogenic nuclides. The application of different $\delta^{18}O_{threshold}$ levels generally result in considerably larger differences in TCN concentrations because changes in the $\delta^{18}O_{threshold}$ level significantly influence the ratio between glacial and interglacial times. It remains an open question, however, whether it is meaningful to define the glacial history throughout the Quaternary based on a constant $\delta^{18}O_{threshold}$ level. The long-term cooling trend during the Quaternary may have influenced this threshold level, implying that the exposure history estimated for the early and middle parts of the Quaternary are highly uncertain. Yet, for the majority of geological settings this is unlikely to represent a large problem because the observed concentrations are likely to be dominated by the more recent glacial-interglacial cycles, where the long-term trend is relatively small.

With respect to the exposure history, we emphasize that any information on past glacial-interglacial transition times, similar to those compiled for Norway by Mangerud et al. (2011), is easily incorporated in the model framework. This will minimize the uncertainty associated with the application of a $\delta^{18}O_{threshold}$ level and significantly improve estimates of past erosion rates.

5.2 The concept of locally constant glacial and interglacial erosion rate

The assumption of one uniform erosion rate across all interglacial periods and another uniform erosion rate across all glacial periods is obviously simplistic, but it is difficult to assess the validity of this assumption. It is clear that multiple climate-dependent erosion processes, which varied greatly over short time spans, must have accompanied the growth and decay of ice masses throughout the Quaternary. For instance, each deglaciation yields prodigious volumes of meltwater and debris resulting in major episodes of erosion and deposition along proglacial river valleys (Ballantyne, 2002); and secondly, interglacials bring periglacial activity, which fluctuates in intensity according to mean annual temperatures that vary considerably over time (Hales and Roering, 2007). Nevertheless, it is likely that the same short-term evolution was more or less repeated over multiple glacial cycles and that average erosion rates of, for example, different glacial periods were largely similar. Furthermore, the average erosion rates over full glacial and interglacial periods, respectively, were likely dominated by fundamentally different processes and our two-stage model is designed to resolve the differences between these two regimes. Accounting for the detailed temporal variations would also render the problem intractable, because the inclusion of additional free parameters linked to erosion rate would also imply additional free parameters linked to the associated timespans. We thus believe the current model framework provides a reasonable balance between observations and number of free model parameters. We emphasize that the concept of a glacial and an interglacial erosion rate implies that the estimated rates represent gross averages across glacials and interglacials, respectively, and thus should be interpreted within this framework.

5.3 Other applications of the model framework

The potential application of the model framework presented here is not limited to constraining past erosion rates in previously glaciated terrains. This approach can be applied to a variety of landscape settings characterized by a complex exposure history and erosion rates that vary in time. For instance, it may be used to investigate temporal changes in erosion rate in non-glacial, fluvial landscapes, and help constrain whether erosion rates increased in sync with the global cooling trend, as suggested by e.g. Herman et al. (2013).

The model framework may also be combined with numerical landscape simulations (e.g Egholm et al., 2012; Egholm et al., 2013) that produce virtual, process-dependent, landscape histories, which otherwise may be difficult to link-up with real, specific landscapes. The forward model presented in this study makes it possible to calculate the virtual TCN concentrations for any simulated landscape history, which then may be compared to measured concentrations based on field studies. This application does not involve any two-stage model assumptions regarding past glacial-interglacial transitions and associated erosion rates, because the exhumation history and ice cover can be tracked through time at any point in the model simulation. In this way, the model framework offers a potentially useful tool to explore and identify dominant landscape processes by testing and calibrating physics-based models. It may also be used to design sampling strategies based on expected patterns in cosmogenic nuclide concentrations linked to local variations in exposure history and erosion rates.

6. Future perspectives

We have developed a model framework that is designed to constrain the most likely landscape history and past erosion rates, based on multiple cosmogenic nuclides, in regions characterized by a complex exposure history. The current approach focuses mainly on terrains that experienced the waxing and waning of thick glacial ice masses during numerous glacial-interglacial cycles, but the method is highly flexible and can be applied to a wide range of geological settings. Currently, the model framework includes the following cosmogenic nuclides ^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne, but it is straightforward to incorporate other nuclides, such as ^{36}Cl and ^{3}He, in the future so as to further constrain the inverse problem. Similarly, it is also possible to include isostatic rebound effects, provided the magnitude of this effect can be estimated from other sources. This may be particularly relevant for efforts to integrate the computation of landscape-wide TCN concentrations with physics-based landscape simulations where the isostatic rebound effect is known.

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Figure Captions

Figure 1

An example of the two-stage periodic model (Model 1), in which the interglacial (Δt_{int}) and glacial (Δt_{gla}) periods have uniform durations throughout the Quaternary. In this approach, the time-related model parameters that estimated using the MCMC technique are as follows: the ratio of interglacial to glacial time (Δt_{rat}), the time elapsed since the last deglacial event (t_{degla}), and the duration of the individual glacial periods (Δt_{gla}). The remaining model parameters include the interglacial erosion rate (ε_{int}) and the glacial erosion rate (ε_{gla}). The interglacial and glacial erosion rates are allowed to vary within bounds specified by the user, and nothing is assumed regarding their relative magnitudes. It is a fundamental assumption of the model approach that interglacial periods are characterized by 100 % exposure, while glacial periods are characterized by 100 % shielding.

Figure 2

An example of the two-stage model inspired by the global marine $\delta^{18}O$ stack (Lisiecki and Raymo, 2005). In this approach (Model 2), the timing and duration of glacial and interglacial periods are defined by a threshold value ($\delta^{18}O_{threshold}$) that is applied to the global $\delta^{18}O$ record. The $\delta^{18}O$ record may be subjected to various degrees of smoothing depending on the landscape setting and prior information. The scenario presented here is based on a 30-kyr running mean of the $\delta^{18}O$ record and a threshold value of 4.0 ‰. The $\delta^{18}O_{threshold}$ level is a model parameter that is determined as part of the MCMC inversion analysis along with the glacial (ε_{gla}) and interglacial (ε_{int}) erosion rates. The timing of the last deglacial event (t_{degla}), which is also a model parameter, is decoupled from the $\delta^{18}O_{threshold}$ level, because t_{degla} is often known with reasonable precision and accuracy from ^{10}Be dating of glacial erratics or other independent evidence.

Figure 3

The forward model is validated by comparing nuclide concentrations calculated by use of two different forward models: the analytical, Lagrangian approach used in the MCMC analysis (red), and 2) a numerical, Eulerian approach (blue). The initial concentrations used in these simulations were the depth-dependent ^{10}Be equilibrium concentrations obtained for an erosion rate of 10 m/Myr. Panel A illustrates the Lagragian (red) and Eulerian (blue) approaches to calculate the surface nuclide concentrations for a particular glacial-interglacial landscape scenario consisting of 10 periodic glacial (100 kyr) and interglacial (10 kyr) cycles (Model 1). While the Lagrangian approach tracks a certain layer as it moves towards the surface due to erosion, the Eulerian approach calculates the change in concentration with time at certain depths ($z = 0$ m in panel a). As shown in panel b, the two methods yield identical, present-time nuclide concentrations at all depths.

Figure 4

Density cross-plots of the four model parameters included in synthetic landscape scenario 1, where the landscape history is estimated based on 10,000 simulations using the MCMC inversion technique and the global marine $\delta^{18}O$ stack (Model 2). The density cross-plots show the bounds of the model parameters used in the MCMC inversion and the yellow-reddish colors indicate the family of parameters that provide the best fit to the observed TCN concentrations. Note that the timing of the last deglaciation appears unconstrained because of the narrow bounds (10-12 kyr ago) in the MCMC analysis associated with this particular parameter.

Figure 5

Histograms showing the distribution of model parameters that provide the best fit to the TCN concentrations in synthetic landscape scenario 1. The interglacial erosion rate (a), glacial erosion rate (b), timing of last deglaciation (c), and $\delta^{18}O_{threshold}$ level (d) are based on one random walk of the MCMC analysis. In this synthetic landscape scenario, the observed TCN concentrations are computed using the forward model and a set of known 'true' model parameters (shown by magenta lines). The fraction indicates the number of simulations included in each bin out of the 10,000 simulations that followed the burn-in phase. To ensure that the parameter estimation is robust, it is possible to start several, parallel random walks with different starting points in the model space. The model parameters of the other random walks show very similar distributions.

Figure 6

Exhumation histories computed for synthetic landscape scenario 1 based on the family of model parameters that provides the best fit to the observed nuclide concentrations. The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations applied in the second step of the MCMC analysis. The depth and time units were binned and the shading reflects the number of simulations passing through the bins. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history. In landscape scenario 1, the observations consist of four nuclide concentrations (^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne) from a surface sample (z = 0 m).

Figure 7

Exhumation histories computed for synthetic landscape scenario 2, based on ^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne concentrations from the surface (z = 0 m). The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations used to constrain the best-fitting model parameters. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history.

Figure 8

Exhumation histories computed for synthetic landscape scenario 2, based solely on ^{10}Be and ^{26}Al concentrations from the surface ($z = 0$ m). The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations used to constrain the best-fitting model parameters. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history.

Figure 9

Exhumation histories computed for synthetic landscape scenario 2, based on ^{10}Be, ^{26}Al, ^{14}C, and ^{21}Ne concentrations obtained from a depth profile ($z = 0, 0.3, 1, 3, 10$ m). The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations used to constrain the best-fitting model parameters. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history.

Figure 10

Exhumation history computed for a sample (GU110) collected from an elevation of 745 m a.s.l. in the Upernavik area, West Greenland. The MCMC analysis is based on 10,000 simulations and measured concentrations of ^{10}Be and ^{26}Al. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The underlying color densities indicate the full range of exhumation histories based on the 10,000 MCMC simulations.