**A multi-nuclide approach to constrain landscape evolution and past erosion rates in previously glaciated terrains**

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**Abstract**

Cosmogenic nuclides are typically used to either constrain an exposure age, a burial age, or an erosion rate. Constraining the landscape history and past erosion rates in previously glaciated terrains is, however, notoriously difficult because it involves a large number of unknowns. The potential use of cosmogenic nuclides in landscapes with a complex exposure history is therefore often quite limited. Here, we present a novel multi-nuclide approach to study the landscape evolution and past erosion rates in terrains with a complex exposure history, particularly focusing on regions that repeatedly were covered by glaciers or ice sheets during the Quaternary. The approach, which is based on the Markov Chain Monte-Carlo (MCMC) technique, focuses on mapping the range of landscape scenarios that are consistent with a given set of measured cosmogenic nuclide concentrations. A fundamental assumption of the model approach is that the exposure history at the site/location can be divided into two distinct modes: i) interglacial periods characterized by zero shielding due to overlying ice and a uniform interglacial erosion rate, and ii) glacial periods characterized by 100 % shielding and a uniform glacial erosion rate. Two different approaches are proposed to incorporate the exposure history into the model framework, one of which relies on the application of different threshold values to the global marine benthic 18O record. However, any prior information on the glacial-interglacial history at the sampling location, in particular the timing of the last deglaciation event which is often known from studies of boulders, is readily incorporated into the model framework to constrain the inverse problem. Based on the MCMC technique, the model delineates the most likely exposure history, including the glacial and interglacial erosion rates, which, in turn, makes it possible to reconstruct an exhumation history at the site. The model framework, which currently includes the following nuclides 10Be, 26Al, 14C, and 21Ne, is highly flexible and can be adapted to many different landscape settings. As such, the user must specify which nuclides are used in the study (e.g. 10Be, 26Al or 10Be, 26Al, 14C, 21Ne), the measured concentrations and associated uncertainties, and whether samples have been collected from the surface only or, alternatively, from a depth profile. The model framework presented here may also be used in combination with physics-based landscape evolution models to compute expected nuclide concentrations at different locations in the landscape. This may help validate the landscape models via comparison to measured nuclide concentrations or to devise new effective sampling strategies.

**1. Introduction**

As the global climate cooled during the Quaternary Period (last ~2.6 Myr), the surface processes eroding mountain ranges seem to have accelerated dramatically (Zhang et al., 2001; Herman et al., 2013). The dramatic fluctuations between glacial and interglacial periodscharacteristic of the Quaternary Period altered the erosional dynamics in most of Earth’s mountain ranges, in part due to changes in river discharge, new vegetation regimes, and the advent of cold-climate processes, including frost weathering and the development of extensive ice masses (Shuster et al., 2005; Thomson et al., 2010). These processes played an important role in shaping many of the remarkable, first-order topographic features observed today, such as the spectacular fjord and valley landscapes of Norway and Greenland. However, the processes controlling the evolution of mountain ranges and their complex links to long-term changes in global climate are currently poorly understood.

Efforts to understand the evolution of mountain ranges are often hampered by difficulties in quantifying past denudation rates, a parameter that may vary over several orders of magnitude depending on the geological setting and processes that govern the removal of mass (e.g. von Blanckenburg, 2005). In non-glaciated terrains, steady-state denudation rates are typically estimated by use of “in-situ” produced cosmogenic nuclides, in particular 10Be. This approach is unviable in terrains that were covered by large ice volumes during glacial periods, because the surface rocks were shielded for unknown lengths of time. In such settings, the concentration of cosmogenic nuclides is generally interpreted as an exposure age, typically a deglaciation age, assuming that the denudation rate since the time of exposure is negligible or that it can be inferred from independent evidence.

Inherited nuclides often represent a problem when estimating exposure ages in landscapes that were repeatedly covered by ice in the past. The problem arises in landscapes characterized by low denudation rates, where a significant amount of the cosmogenic nuclides produced during previous periods of exposure remains in the surface bedrock. This problem may be overcome by collecting paired samples in the field, i.e. collecting samples from both boulders and bedrock, in which case it is often possible to estimate the deglacial age and the amount of inheritance in the bedrock sample. An extension of this approach involves the use of paired cosmogenic nuclides, primarily 10Be and 26Al, which allows estimates of the minimum-limiting exposure duration and minimum-limiting burial duration (e.g. Biermann et al., 1999; Corbett et al., 2013). In this case, the different half-lives and production rates of the nuclides can be used to constrain a total landscape history.

Several studies have employed paired nuclides to date buried sediments (e.g. Granger and Smith, 2000; Granger, 2006) and depositional landscape surfaces (Anderson et al., 1996). Most of these studies rely on 10Be and 26Al, but burial dating schemes involving 26Al-10Be-21Ne also exist (Balco and Shuster, 2009). Braucher et al. (2009) showed that it is possible to determine both an exposure time and denudation rate from an in-situ produced 10Be depth profile and a versatile Monte Carlo simulator for modeling depth profiles of 10Be or 26Al in sediments is available online (Hidy et al., 2010). None of these approaches, however, focus on resolving landscape history and past denudation rates from bedrock samples collected in terrains that repeatedly were covered by glacial ice in the past.

In this study, we aim to develop a robust and flexible multi-nuclide approach to study the landscape evolution in areas characterized by a complex exposure history. The main focus is to constrain the most likely glacial-interglacial landscape history of an area and estimate average glacial and interglacial erosion rates by exploiting the different half-lives and production rates of the various cosmogenic nuclides. For this purpose, a Markov-Chain Monte-Carlo approach is developed in order to systematically delineate the most likely scenarios within the framework of the model.

**2. Approach and Methods**

**2.1 Model concept and framework**

The basic idea underlying this model framework is to systematically simulate the production and loss of in-situ terrestrial cosmogenic nuclides (TCNs) associated with the glacial-interglacial cycles of the Quaternary and to map the glacial-interglacial landscape histories that are consistent with a given set of measured TCN concentrations. The production of TCNs occurs during times of exposure when there is no glacial ice to shield the surface bedrock, whereas the loss of TCNs is due to radioactive decay and erosion. A key principle in this approach is what we refer to as “two-stage uniformitarianism”. This concept implies that the processes that operated during the Holocene also operated during earlier interglacials, implying that the denudation processes that dominated during the Holocene also dominated during earlier interglacials. Similarly, it is assumed that the denudation processes that dominated during the last glacial period, the so-called Weichsel or Wisconsin glacial period, also dominated during earlier glacial periods. The model concept consequently operates with two erosion rates, an interglacial erosion rate, *int*, and a glacial erosion rate, *gla*. The model concept assumes that interglacial periods were characterized by 100 % exposure and zero shielding due to overlying glaciers, whereas glacial periods were characterized by 100 % shielding and no exposure. However, the timing of glacial-interglacial transitions at any specific location is often unknown, and these transition times are therefore incorporated in the model framework as free parameters that vary among the simulations.

The model framework can accommodate any number of TCNs, but is currently based on the following four: 10Be, 26Al, 14C, and 21Ne. In some respect, this combination of nuclides is ideal because of their different half-lives (Table 1), implying that their concentrations reflect different aspects of the glacial-interglacial history. All four nuclides are produced at reasonably well-constrained rates in quartz, which is a very common mineral that is highly resistant to weathering and loss of nuclides after production. Quartz is usually the preferred target mineral for all four nuclides. Beryllium-10 and 26Al are routinely measured with high precision using Accelerator Mass Spectrometry (AMS), whereas 21Ne, which is stable, is measured with high precision using a noble gas mass spectrometer. The main challenge with 21Ne is that significant amounts of a non-cosmogenic component may be present in the samples. This interfering component is usually identified and corrected for using a neon three-isotope diagram (Niedermann, 1994; Niedermann, 2002) or via samples that contain no cosmogenic 21Ne, such as samples from road cuts. The inclusion of in-situ produced 14C is important due to its short half-life of 5730 years, which make this nuclide particularly sensitive Holocene erosion rates. It remains challenging to measure in-situ produced 14C, however, but it is currently achieved at several AMS laboratories around the world.

**2.2 Cosmogenic nuclide production rates**

The cosmogenic nuclides are produced when the surface rock is exposed to a shower of secondary cosmic-ray particles (Lal and Peters, 1967), including neutrons and muons. When these particles reach the Earth’s surface they interact with atoms in the minerals in various processes that produce cosmogenic nuclides. In this study, we include the three most important production mechanisms for the four cosmogenic nuclides mentioned above: nucleonic spallation (spal), negative muon capture (nmc), and fast muons (fm). The production of cosmogenic nuclides decays exponentially with depth for all three mechanisms, although at different rates because neutrons have considerably shorter attenuation lengths () than muons, which penetrate much deeper into the ground. However, the theoretical production of TCNs due to muons does not behave as a simple exponential function with depth (Heisinger et al., 2002a, 2002b), but it has been shown that reasonable approximations to the theoretical production can be made with multiple exponential terms for muon production mechanisms (Granger and Smith, 2000; Schaller et al., 2002, Hidy et al., 2010). In this study, we adopt the approach of Schaller et al. (2002) to calculate the TCN production as a function of depth (z) given by the following equations:

(1)

(2)

(3)

(4)

where  is the density of the rock (here we use a value of 2.65 g/cm3), the attenuation lengths, and *ai, bj, ck* are dimensionless coefficients. The values used in this study for the dimensionless coefficients and attenuation lengths associated with spallation (spal), negative muon capture (nmc), and fast muons (fm) are adopted from Schaller et al. (2002) and given in Table 1. Figure 1 shows how the total production rate, along with the various production pathways, changes with depth for 10Be in quartz. We assume that variations in nuclide production rate with depth can be approximated with Eq. 1-4 for all four nuclides. The surface production rates *P(0)spal , P(0)nmc , P(0)fm*must be specified for the study site by the user, for instance by use of the CRONUS-Earth on-line calculator. We note, however, that the current version of the model can only incorporate nuclide production rates that are constant in time. This implies that scaling schemes taking into account solar and geomagnetic field effects must be disregarded at the moment. In this study, the spallogenic production rate is scaled to the sample site by use of the Lal (1991) and Stone (2000) scaling scheme. Similar to Hidy et al. (2010), we obtain the muonic components of the surface production rate from Heisinger et al. (2002a, 2002b), and we scale the muon production rates for elevation using the approach of Balco et al. (2008).

**2.3 Computation of present nuclide concentrations**

When erosion rates and TCN production rates at the surface vary as a step function in time due to the waxing and waning of ice sheets, the present-time nuclide concentration may be calculated using a Lagrangian approach, in which a layer is tracked as it slowly is advected towards the surface. Consider a rock sample at the present depth of burial, zobs, which for samples collected at the surface will be zero (z = 0 m). Owing to the varying erosion rates, , this rock sample has followed a depth track given by



(5)



Therefore, this sample has experienced a production rate that has varied in time according to

(6)



where the production rate at the surface, , varies due to changes in the shielding associated with an overlying ice cover, i.e. 0 % shielding during interglacials and 100 % shielding during glacials. Modulations of the cosmic-ray flux due to variations in the solar magnetic and geomagnetic fields are neglected, as are potential variations in the production rate associated with elevation changes, such as isostatic-rebound effects. In general, the differential equation for the nuclide concentration is



(7)



where  is the radioactive decay constant of the nuclide. For a given erosion history and given glacial-interglacial exposure history, defining variations in the surface production rates, it is possible to solve Eq. 7 numerically with standard techniques. However, when the erosion rates and surface production rates are piecewise constant, this differential equation (7) can be solved analytically by a sum of recursive exponential terms, which makes the calculation of present nuclide concentrations particularly fast for the two-stage uniformitarian models considered here. As the computational cost represents a critical aspect due to the number of iterations required by the MCMC analysis, we use the analytical solution in the forward model computations.

**2.4 Estimating glacial-interglacial exposure histories in the model setup**

A major obstacle for disentangling a complex landscape evolution history relates to the fact that past intervals characterized by glacial cover are often poorly constrained in time, or, as in most cases, completely unknown prior to the last deglaciation event. In this paper, we propose two different approaches to address this obstacle: a periodic two-stage model and a two-stage model constrained by the marine δ18O record.

**2.4.1 Periodic two-stage model**

In the periodic two-stage model, present nuclide concentrations are computed as a function of periodic transitions between glacial and interglacial intervals throughout the Quaternary with the exception of the last deglaciation (Fig. 2), which is often reasonably well constrained in time through studies of e.g. boulders. For the purpose of the inverse analysis, we parameterize the erosion and exposure history in Model 1 as follows



where is the erosion rate during all interglacial periods, including the present interglacial as well as prior to the first glaciation, and is the erosion rate during all glaciated periods. Δ*tdegla* is the time elapsed since the last deglaciation and Δ*tgla* is the duration of each individual glacial period, which is constant for all glaciations, and Δ*trat* is the ratio between the duration of glacial periods and interglacial periods (Fig. 2).



**2.4.2 Two-stage model inspired by the global marine δ18O stack**

Although past transitions between glacial and interglacial periods at specific locations are often unconstrained, there is reliable information from marine δ18O records regarding large-scale climatic changes during the Quaternary, indicating when glaciations were widespread and likely to occur. Marine benthic oxygen isotope records have been used in previous studies to infer the most likely glacial-interglacial history by identifying a transition threshold value based on the timing of the last deglaciation. (e.g. Fabel et al., 2002). However, such a Pleistocene-Holocene threshold value may not have applied to past glacial-interglacial transitions. In this study, we estimate the most likely glacial-interglacial transitions by incorporating the δ18O threshold value as a free parameter that is applied iteratively to the stacked marine benthic δ18O record of Liesicki & Raymo (2005). We apply this prior information in our Model 2 as follows



where the *18Othreshold* defines a *18O(t)* time history that, in turn, defines the timing of glacial and interglacial periods together with the associated erosion rates (Fig. 3). The degree of smoothing applied to the *18O*curve influences the number of transitions and the duration of individual glacial-interglacial periods. It is consequently possible for the user to specify the degree of smoothing applied to the *18O*curve, although the difference between present-nuclide concentrations obtained with various degrees of smoothing is small compared to the influence of the four model parameters. In principle, the timing of the last deglaciation, defined by model parameter *tdegla*, is redundant because it is defined by the *18O* threshold value. However, it makes sense to decouple the last deglaciation from the *18O* threshold because *tdegla* is often known with reasonable precision and accuracy from studies of e.g. boulders.

**2.5 Validation of the forward model**

The forward model based on an analytical Lagrangian approach is easily evaluated for simple equilibrium scenarios, but it is more difficult to assess if the nuclide concentrations are correctly computed for complex landscape histories characterized by repeated glacial-interglacial cycles and varying erosion rates. In order to validate the forward model described above, we developed a completely different approach to compute nuclide concentrations for complex landscape scenarios. This validation model is based on a Eulerian approach in which the nuclide concentrations are computed at specific depths while the rock layers are advected towards the surface due to erosion. In this approach, changes in nuclide concentrations are computed in small incremental time steps (*t*) as follows



where the production term includes the number of nuclides produced within the time step , some of which are lost due to decay, and the second term calculates the loss due to decay of the existing nuclides. Changes in concentration due to erosion are handled as an upward advection of layers from below that brings layers with smaller concentrations towards the surface. This is a very different, and computationally expensive, way of calculating changes in nuclide concentrations over glacial-interglacial cycles as the time steps must be relatively small (typically 100 years) to maintain computational stability when computing the advection.



Comparison of nuclide concentrations obtained with the analytical model and the step-wise advective validation model confirms that the analytical forward model is a fast and accurate method to calculate nuclide concentrations for complex landscape histories involving multiple glacial-interglacial cycles. Figure 4a demonstrates how the analytical (Lagrangian) model (red) and the step-wise Eulerian validation model (blue) reach the same 10Be surface concentrations after simulation of 10 glacial-interglacial cycles, lasting 100 kyr and 10 kyr, respectively (Model 1). The initial concentrations used in these simulations were the depth-dependent 10Be equilibrium concentrations obtained for an erosion rate of 10 m/Myr. Figure 4b demonstrates how the two approaches yield completely similar concentrations as a function of depth after simulation of the 10 glacial-interglacial cycles.

**3. Inverse Markov Chain Monte Carlo resolution analyses**

Based on the forward model described above, we have developed an inverse Markov Chain Monte Carlo approach to constrain the most likely landscape history from a combination of cosmogenic nuclides with different production rates and half-lives. The forward models allow for observations of a number of nuclide concentrations and associated uncertainties at a number of depths. The observation data vector consists of the measured nuclide concentrations:



We analyze this problem using a conventional Metropolis-Hastings Markov Chain Monte Carlo technique (Metropolis et al., 1953, Hastings, 1970) where model parameters are constrained between fixed bounds specified by the user. Erosion rates (*int*, *gla*), which may vary over orders of magnitude, are specified with uniform probability across the logarithmic parameter interval. The time parameters (*tdegla 18Othreshold*) are specified with uniform probability across the linear parameter interval. After the user has specified the bounds of the model parameters, which define the model space that is searched with the MCMC technique, a forward response is computed based on an initial set of model parameters that is proposed using the Metropolis-Hastings technique. A burn-in phase of 1000 iterations is used to make a crude initial search of the model space. This step is followed by a more detailed and local search of the model space that uses the set of model parameters from the burn-in phase that provides the smallest misfit when compared to the observation data vector. At each iteration step, the current model is perturbed by a fraction of the prior interval. This fraction is updated every 1000 iterations so that an acceptance ratio of about 0.4 is achieved. To ensure that the set of model parameters providing the best fit to the observed data does not depend on the starting position of the random search through the model space, a number of “random walks” (e.g. 4) are started at different positions in the model space (e.g. different corners or edges). If these completely independent “random walks” achieve similar distributions for the best-fitting model parameters, it is highly unlikely that there are global misfit minima that remain undetected. Based on the combination of model parameters that provide the best fit to the data, it is possible to compute the most likely exhumation history for the site and/or study area.

**4. Investigating possible landscape scenarios**

Many mountain ranges experienced the advent of glaciers as the global climate cooled during the Quaternary Period. This marked a transition from a landscape that essentially was free of ice to a landscape that saw the waxing and waning of glacial ice during glacial-interglacial cycles. Depending on latitude, elevation, and regional climate regime, these glaciers may have been warm-based and highly erosive, or cold-based and non-erosive. In the following, we illustrate the model approach and associated MCMC technique by simulating landscape-evolution scenarios that resemble the onset of widespread glaciations, both erosive and non-erosive, during the Quaternary. In the first two scenarios, we simulate the “observed” nuclide concentrations (10Be, 26Al, 14C, and 21Ne), which form the basis of the MCMC inversion, based on a set of “true” model parameters. The third scenario is based on real data from Upernavik, West Greenland.

**4.1 Synthetic landscape scenario 1**

In scenario 1, we simulate a landscape evolution in which the glacial erosion rate (*gla* = 1x10-6 m/yr) is considerably smaller than the inter-glacial erosion rate (*int* = 5x10-5 m/yr), which leaves a large inherited signal from earlier periods of exposure. The glacial-interglacial history is obtained by applying a threshold value of 4.0 ‰ to the marine benthic oxygen isotope record (Liesicki & Raymo, 2005) (Fig. 3), whereas the timing of the last glacial-interglacial transition (*tdegla* = 11000 ± 1000 ka) is assumed to be well constrained from studies of boulders. The reference value of 4.76 atoms/g/yr for sea level and high latitudes (Stone [2000] recalibrated according to Nishiizumi et al. [2007]) is used for the spallogenic 10Be production rate, whereas the muonic component is obtained from Heisinger et al., (2002a, and 2002b). The production rates of the other nuclides are estimated from the literature (e.g. Dunai, 2010; Braucher et al., 2011; Braucher et al., 2013). An uncertainty of 5 % was assumed for all four nuclides.

For the inverse MCMC approach, the bounds of the model parameters must be specified. For scenario 1, the bounds were as follows: *int* =10-3 to 10-7 m/yr ; *gla* =10-3 to 10-7 m/yr; *18Othreshold* = 3.7 to 4.3 ‰; *tdegla* = 10000 to 12000 years (Fig. 5). The model parameters that provide the best fit to the “observed” nuclide concentrations are indicated by yellow/reddish colors in Fig. 5. The histograms in Fig. 6 show the range of possible solutions for the four model parameters and the skewness of the histograms reflects the non-linearity of the problem. The interglacial erosion rate is very well constrained in this scenario, whereas the low erosion rate associated with glacial periods is less well defined. Note, however, that all four model parameters are correctly identified by the MCMC technique. Figure 7 shows the exhumation histories that are computed from the best-fitting erosion rates and glacial-interglacial transitions (*18Othreshold*) provided by the distributions in Fig. 6. Note that the change in exhumation rate around 1 Myr ago, which is linked to the advent of cold-based, non-erosive ice sheets, is captured by the MCMC approach.

**4.2 Synthetic landscape scenario 2**

In this scenario, the glacial erosion rate (*gla* = 1x10-5 m/yr) is an order of magnitude higher than the interglacial erosion rate (*int* = 1x10-6 m/yr), whereas the other model parameters remain unchanged compared to scenario 1 (*18Othreshold* = 4.0; *tdegla* = 11000 ± 1000 ka). The production rates and bounds of the model space, which define the range of possible model parameters investigated with the MCMC technique, also remain unchanged.

The MCMC inversion approach correctly identifies the true model parameters, but the interglacial erosion rate and *18Othreshold* are not as well constrained as in scenario 1. As a result, the ensemble of possible exhumation rates is less well defined, although the majority falls within a relatively limited band over the past 500 kyr (Fig. 8). In particular, the uncertainty associated with the *18Othreshold* value gives rise to relatively large uncertainties in the exhumation rate prior to 500 kyr. This is a direct result of applying the assumption of two-stage uniformitarianism 2 Myr back in time, which clearly may be problematic because the *18Othreshold* value may have changed in sync with the overall changes in climate during the Quaternary. Nevertheless, the majority of the best-fitting model parameters correctly capture the change in exhumation rate associated with the onset of glaciations around 1 Myr ago.

For scenario 2, we also investigate how different sample and measuring strategies may influence the estimated exhumation history. It is currently common practice to measure concentrations of 10Be and 26Al in boulder and bedrock samples (e.g. Corbett et al., 2013). Figure 9 shows the estimated exhumation history for scenario 2 based solely on 10Be and 26Al concentrations, i.e. the concentrations of 14C and 21Ne have been excluded in the MCMC analysis. In this scenario, the estimated exhumation history based on 10Be and 26Al is very similar to that based on four nuclides (10Be, 26Al, 14C, and 21Ne) and there are almost no differences between best estimates (median values) and the degree to which the exhumation history is constrained (first and third quartiles). In a quite different, and somewhat extreme, experiment, we investigate how well constrained the exhumation history becomes for scenario 2 if all four nuclides are measured in a depth profile (z = 0, 0.3, 1, 3, and 10 m). The experiment shows how a depth profile potentially provides a much more well-constrained exhumation history (Fig. 10).

**4.3 Landscape scenario 3 – Upernavik, West Greenland**

In the third scenario, the model framework is applied to real data derived from a sample collected in the area around Upernavik, West Greenland (Corbett et al., 2013). This bedrock sample (GU110), which was collected at an elevation of 745 m a.s.l., was chosen for this analysis because the measured 10Be and 26Al concentrations indicate an old total history (989 kyr). No boulders were sampled at this site, but boulders collected from lower elevations in this area suggest that the last deglaciation event occurred around 11-12 kyr ago, but this age may not apply to higher elevations. The 10Be and 26Al production rates at the sample site were obtained by use of the northeastern North American production rates and the CRONUS-Earth on-line calculator. The muonic production rates were calculated according to Balco et al. (2008). The uncertainties associated with the measured 10Be (5.67x105 atoms/g) and 26Al (2.67x106 atoms/g) concentrations were 2.6 % and 4.0 %, respectively.

In the initial MCMC analysis, the last deglaciation event was constrained to the period 10-12 kyr, in agreement with studies of boulders from lower elevations in the area. The MCMC analysis reveals that the interglacial erosion rate was less than 6 m/Myr, and most likely in the range 0.6-6 m/Myr. The glacial erosion rate is more well constrained with the most likely erosion rate falling within the interval 1.25-2.5 m/Myr, depending on the exposure history constrained by the model. Interestingly, the *18Othreshold* value is very well constrained, suggesting that the exposure history is defined by values in the range 3.81-3.84 ‰. Such *18Othreshold* values imply that the total amount of exposure within the last million years was limited to the range 80-110 kyr, of which ~26 kyr of exposure occurred during marine isotope stage 11 around 400 ka. This limited amount of exposure is highly consistent with the minimum-limiting exposure duration (88 kyr) and burial duration (901 kyr) found by Corbett et al. (2013) based on the two-isotope burial-exposure diagram. The most likely exhumation history can be computed from the best fitting model parameters (Fig. 11, black line). Note that the shady lines in Fig. 11, representing the 10,000 simulations used to constrain the best fitting model parameters, do no reflect the actual uncertainty. The reason is that, intrinsic to the MCMC technique, the method is forced to also accept models that do not provide an optimal fit to the observations in order to map out the range of model parameters that actually do provide optimal fits. The upper and lower quartiles provide a more accurate representation of the uncertainty associated with the exhumation history.

A slightly different MCMC analysis, which assumed no prior information regarding the timing of the last deglaciation event, was also carried out. This analysis yielded very similar results, including a slightly lower glacial erosion rate (1.25 m/Myr compared to 1.8 m/Myr on average) and slightly less total exposure over the last Myr. These differences result from an estimated age of the last deglacial event of 30-35 kyr, which clearly is different from the deglacial age inferred from nearby boulders found at lower elevations. Although the exhumation histories are very similar in this case, it demonstrates the importance of including prior information regarding glacial-interglacial transitions. The inclusion of an extra nuclide, particularly 14C, or a depth profile would help constrain the model parameters in the MCMC analysis.

**5. Discussion**

The framework presented here provides a highly flexible multi-nuclide approach to delineate likely landscape histories and past erosion rates in terrains previously covered by glaciers or ice sheets. The approach is designed to be easily applicable to a wide range of very specific settings or problems. As such, the user may specific the production rates due to spallation and muons at the study site, the attenuation lengths, the rock density, the number of cosmogenic nuclides used in the study (e.g. 10Be, 26Al or 10Be, 26Al, 14C, 21Ne), the measured concentrations and associated uncertainties as well as the sample depths (e.g. z=0 m or z=0, 0.5, 1 m). It is also possible to specify various kinds of prior information regarding past glacial-interglacial transition times. Likewise, if any of the model parameters are well constrained from other studies, a narrow bound should be specified for these parameters for the MCMC inversion. Nevertheless, the approach relies on some very fundamental assumptions concerning past glacial-interglacial transition times and past erosion rates that are subject to debate.

**5.1 Estimating the glacial-interglacial exposure history**

In many cases, it is possible to estimate the timing of the last deglacial transition, e.g. via studies of boulders, but no information regarding earlier transitions is available. For such scenarios, this study provides two possible solutions: a) the periodic two-stage model, and b) the two-stage model inspired by global climatic changes during the Quaternary. The latter approach is appealing, because the regional extent of glaciations is likely to correlate with changes in global climate, but this may not be true locally as the occurrence of past glaciations depends on local climate and altitude. It is also unknown if the assumption regarding 100 % shielding during glacial periods is reasonable, as it would require >10m of ice to render the production due to spallation negligible (> 50 m for muons). Another aspect concerns the temporal resolution at which glaciations occur, e.g. was the Eemian and subsequent interstadials characterized by one interglacial period or punctuated by a series of relatively brief glaciations. A recent study by Mangerud et al. (2011) shows that the stadials following the Eemian was characterized by a series of glaciations lasting 5-10 kyr. These changes are very consistent with the glacial-interglacial history obtained by applying a 5-kyr running mean to the marine 18O record, whereas the 30-kyr running mean used in Fig. 2 and scenarios 1-3 produces a somewhat simpler exposure history with fewer glacial-interglacial transitions. In many cases, however, the resolution of the marine 18O record has a limited effect on the concentration of cosmogenic nuclides, typically less than 1% for 10Be when comparing Quaternary exposure histories based on 5-kyr and a 30-kyr running means. The application of different *18Othreshold* levels result in much larger differences in concentration because changes in the *18Othreshold* level significantly influence the ratio between glacial and interglacial times. A change in the *18Othreshold* level from 4.0 ‰ to 3.8 ‰ thus results in a 50% decrease in the 10Be and 26Al concentrations, whereas the 14C concentration remains unchanged due to its short half-life. It remains an open question, however, whether it is meaningful to define the glacial history throughout the Quaternary based on a constant *18Othreshold* level. Changes in the long-term climate trend during the Quaternary may have influenced this threshold level, implying that the exposure history estimated for the early and middle parts of the Quaternary are highly uncertain. However, in the majority of geological settings this is unlikely to represent a large problem because the observed concentrations are dominated by the most recent glacial-interglacial cycles, where the long-term trend is relatively small.

With respect to the exposure history, we emphasize that any prior information on past glacial-interglacial transition times, similar to those compiled for Norway by Mangerud et al. (2011), is easily be incorporated in the model framework. This will minimize the uncertainty associated with the application of a *18Othreshold* level and significantly improve estimates of past erosion rates.

**5.2 The concept a glacial and an interglacial erosion rate**

The assumption of one uniform erosion rate across all interglacial periods and another uniform erosion rate across all glacial periods is obviously too simplistic, but it is difficult to assess the magnitude of the error introduced by this assumption. It is clear that the growth and decay of glaciers and ice sheets throughout the Quaternary must have been accompanied by a variety of climate-dependent erosion processes. For instance, the importance of periglacial processes, which are likely to dominate in the absence of glacial activity, depend strongly on the mean annual temperature, which changed considerably throughout the Quaternary. However, the introduction of additional model parameters in order to include such aspects would render the problem intractable, because the inclusion of additional free parameters linked to erosion rate would also imply additional free parameters linked to the associated timespans. We thus believe the current model framework provides a reasonable balance between observations and number of free model parameters. Moreover, we emphasize that the concept of a glacial and an interglacial erosion rate implies that the estimated rates represent gross averages across glacials and interglacials, respectively, and thus should be interpreted within this framework.

**5.3 Other applications of the model framework**

The potential application of the model framework presented here is not limited to constraining past erosion rates in previously glaciated terrains. This approach can be applied to any complex exposure history with alternating periods of exposure and shielding, and non-uniform erosion rates. Importantly, the model framework may also be combined with numerical landscape simulations that produce virtual, process-dependent, landscape histories, which may be difficult to relate to real landscapes. The forward model presented in this study makes it possible to calculate the virtual concentration of cosmogenic nuclides at any point in the simulated landscape, which then may be compared to measured concentrations based on field studies. This application does not involve any assumptions regarding past glacial-interglacial transitions and associated erosion rates, because the exhumation history and ice cover can be tracked through time at any point in the model simulation. In this way, the presented model framework provides a tool that potentially can be used in conjunction with physics-based models to explore and identify dominant landscape processes. It may also be used to design sampling strategies based on expected patterns in cosmogenic nuclide concentrations linked to local variations in exposure history and erosion rates.

**6. Future perspectives**

We have developed a model framework that is designed to constrain the most likely landscape scenario and past erosion rates, based on multiple cosmogenic nuclides, in regions characterized by a complex exposure history. The approach focuses mainly on terrains that experienced the waxing and waning of thick glacial ice covers during glacial-interglacial cycles, but the method is highly flexible and can be applied to a wide range of geological settings. Currently, the model framework includes the following cosmogenic nuclides 10Be, 26Al, 14C, and 21Ne, but it is possible to incorporate other nuclides, such as 36Cl and 3He, in the future in order to further constrain the inverse problem. Other additions and improvements to the model can be incorporated to meet specific demands, including new approaches to constrain complex exposure histories during the Quaternary.

**References**

Anderson, R. S., Repka, J.L., Dick, G.S., 1996. Explicit treatment of inheritance in dating depositional surfaces using in situ 10Be and 26Al. Geology, 24, 47–51, doi:10.1130/ 0091-7613(1996)024<0047:ETOIID>2.3.CO;2.

Balco, G., Stone, J.O., Lifton, N.A., Dunai, T., 2008. A complete and easily accessible means of calculating surface exposure ages or erosion rates from 10Be and 26Al measurements. Quaternary Geochronology 3, 174-195.

Balco, G., Shuster, D.L., 2009. 26Al–10Be–21Ne burial dating. Earth Planet. Sci.

Lett. 286, 570-575.

Bierman, P.R., Marsella, K.A., Patterson, C., Davis, P.T., and Caffee, M., 1999. Mid-Pleistocene cosmogenic minimum-age limits for pre-Wisconsinan glacial surfaces in southwestern Minnesota and southern Baffin Island; a multiple nuclide approach. Geomorphology 27, p. 25-39.

Braucher, R., Del Castillo, P., Siame, L., Hidy, A.J., Bourlés, D.L., 2009. Determination of both exposure time and denudation rate from an in situ-produced 10Be depth profile: a mathematical proof of uniqueness. Model sensitivity and applications to natural cases. Quat. Geochronol. 4 (1), 56–67.

R. Braucher, D. Bourlès, S. Merchel, J. Vidal Romani, D. Fernadez-Mosquera, K.

Marty, L. Léanni, F. Chauvet, M. Arnold, G. Aumaître, K. Keddadouche,

Determination of muon attenuation lengths in depth profiles from in situ

produced cosmogenic nuclides, Nucl. Instr. and Meth. in Phys. Res. B. 294

(2013) 484-490.

Braucher, R., Merchel, S., Borgomano, J., Bourlès, D.L., 2011.Production of cosmogenic

radionuclides at great depth: A multi element approach. Earth Planet. Sci.

Lett. 309, 1-9.

Corbett, L.B., Bierman, P.R., Graly, J.A., Neumann, T.A., Rood, D.H., 2013. Constraining landscape history and glacial erosivity using paired cosmogenic nuclides in Upernavik, northwest Greenland. Geologicl Society of America Bulletin, doi:10.1130/B30813.1.

Desilets, D., Zreda, M., 2003. Spatial and temporal distribution of secondary cosmic-ray nucleon intensities and applications to in-situ cosmogenic dating. Earth and Planetary Science Letters 206, 21–42.

Desilets, D., Zreda, M., Prabu, T., 2006. Extended scaling factors for in situ cosmogenic nuclides: New measurements at low latitude. Earth and Planetary Science Letters 246, 265–276.

Dunai, T.J., 2001. Reply to comments on “Scaling factors for production rates of in situ produced cosmogenic nuclides: a critical evaluation“ by Darin Desilets, Marek Zreda and Nathaniel Lifton. Earth Planet. Sci. Lett. 188, 289-298.

Dunai, T., 2010. Cosmogenic nuclides: Principles, Concepts and Applications in the Earth Surface Sciences. Cambridge University Press, Cambridge, UK.

Fabel, D., Stroeven, A.P., Harbor, J., Kleman, J., Elmore, D., Fink, D., 2002. Earth and Planetary Science Letters 201, 397-406.

Gosse, J.C., Phillips, F.M., 2001. Terrestrial in situ cosmogenic nuclides: theory and application, Quat. Sci. Rev. 20, 1475-1560.

Granger, D.E., Smith, A.L. 2000. Dating buried sediments using radioactive decay and muogenic production of 26Al and 10Be, Nucl. Instrum. Methods Phys. Res., Sect. B, 172, 822–826, doi:10.1016/S0168-583X(00)00087-2.

Granger, D. E., (2006) A review of burial dating methods using 26Al and 10Be, in Siame, L., Bourlès, D. L., and Brown, E. T., eds., In situ-produced cosmogenic nuclides and quantification of geological processes, *Geological Society of America Special Paper 415*, p. 1-16.

Hastings, W.K. (1970). Monte Carlo Sampling Methods Using Markov Chains and Their Applications. [Biometrika](http://en.wikipedia.org/wiki/Biometrika) 57 (1): 97–109. [doi](http://en.wikipedia.org/wiki/Digital_object_identifier):[10.1093/biomet/57.1.97](http://dx.doi.org/10.1093%2Fbiomet%2F57.1.97).

Heisinger, B., D. Lal, A. J. T. Jull, P. Kubik, S. Ivy‐Ochs, S. Neumaier, K. Knie, V. Lazarev, and E. Nolte (2002a), Pro- duction of selected cosmogenic radionuclides by muons: 1. Fast muons, Earth Planet. Sci. Lett., 200, 345–355, doi:10.1016/S0012-821X(02)00640-4.

Heisinger, B., D. Lal, A. J. T. Jull, P. Kubik, S. Ivy‐Ochs, K. Knie, and E. Nolte (2002b), Production of selected cos- mogenic radionuclides by muons: 2. Capture of negative muons, Earth Planet. Sci. Lett., 200, 357–369.

Herman, F., Seward, D., Valla, P. G., Carter, A., Kohn, B., Willett, S. D., and Ehlers, T. A.: Worldwide acceler- ation of mountain erosion under a cooling climate, Nature, 504, 423–426, 2013.

Hidy, A.J., Gosse, J.C., Pederson, J.L., Mattern, J.P., Finkel, R.C., 2010. A geologically constrained Monte Carlo approach to modeling exposure ages from profiles of cosmogenic nuclides: An example from Lees Ferry, Arizona. Geochemistry, Geophysics, Geosystems 11, 1-18, doi:10.1029/2010GC003084.

Hidy, A.J., Gosse, J.C., Blum, M.D., Gibling, M.R., 2014. Glacial-interglacial variation in denudation rates from interior Texas, USA, established with cosmogenic nuclides. Earth and Planetary Science Letters 390, 209-221.

Lal, D., 1991. Cosmic ray labeling of erosion surfaces: in situ nuclide production rates and erosion models, Earth Planet. Sci. Lett. 104, 424-439, doi:10.1016/0012-821X(91)90220-C.

Lisiecki, L.E., Raymo, M.E., 2005. A Pliocene-Pleistocene stack of 57 globally distributed benthic 18O records. Paleoceanography 20 (1), PA1003, doi:10.1029/2004PA001071.

Mangerud, J., Gyllencreutz, R., Lohne, Ø., Svendsen, J.I., 2011. Glacial history of Norway. Developments in Quaternary Science, Vol. 15, 279-298, doi: 10.1016/B978-0-444-53447-7.00022-2.

[Metropolis, N.](http://en.wikipedia.org/wiki/Nicholas_Metropolis), Rosenbluth, A.W., [Rosenbluth, M.N.](http://en.wikipedia.org/wiki/Marshall_N._Rosenbluth), Teller, A.H., [Teller, E.](http://en.wikipedia.org/wiki/Edward_Teller), 1953. [Equations of State Calculations by Fast Computing Machines](http://en.wikipedia.org/wiki/Equations_of_State_Calculations_by_Fast_Computing_Machines). [Journal of Chemical Physics](http://en.wikipedia.org/wiki/Journal_of_Chemical_Physics) 21 (6): 1087–1092.

Niedermann, S., Graf, T., Kim, J.S., Kohl, C.P., Marti, K., Nishiizumi, K., 1994. Cosmic-ray produced 21Ne in terrestrial quartz: the neon inventory of Sierra Nevada quartz separates. Earth Planet. Sci. Lett. 125, 341-355.

Niedermann, S., 2002. Cosmic-ray-produced noble gases in terrestrial rocks: dating tools for surface processes. Rev. Mineral Geochem. 47, 731-784.

Nishiizumi, K., M. Imamura, M. W. Caffee, J. R. Southon, R. C. Finkel, and J. McAninch (2007), Absolute calibration of 10Be AMS standards, Nucl. Instrum. Methods Phys. Res., Sect. B, 258, 403–413, doi:10.1016/j.nimb.2007.01.297.

Schaller, M., von Blanckenburg, F., Veldkamp, A., Tebbens, L.A., Hovius, N., Kubik, P.W., 2002. A 30000 yr record of erosion rates from cosmogenic 10Be in Middle European river terraces. Earth Planet. Sci. Lett. 204, 307-320.

Shuster et al. Rapid glacial erosion at 1.8 Ma revealed by 4He/3He thermochronometry. Science, 310, 1668-1670 (2005).

Thomson *et al*. Glaciations as a destructive and constructive control on mountain building. Nature, 467, 313-317 (2010).

Stone, J.O., 2000. Air pressure and cosmogenic isotope production. Journal of Geophysical Research 105 (B10), 23753–23759.

von Blanckenburg, F., 2005. The control mechanisms of erosion and weathering at basin scale from cosmogenic nuclides in river sediment. Earth and Planetary Science Letters 237, 462-479.

Zhang *et al*. Increased sedimentation rates and grain sizes 2-4 Myr ago due to the influence of climate change on erosion rates. Nature 410, 891-897 (2001).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | T½  [Kyr] | Pspal(0)  [atoms/g/yr] | Pnmc(0)  [atoms/g/yr] | Pfm(0)  [atoms/g/yr] | Λspal  [g/cm2] | Λnmc  [g/cm2] | Λfm  [g/cm2] |
| 10Be | 1390 | 5.1 | 0.11 | 0.1 | 180.5 | 1510 | 4320 |
| 26Al | 705 | 31.1 | 0.7 | 0.6 | 180.5 | 1510 | 4320 |
| 14C | 5.730 | 14.6 | 2.3 | 2.1 | 180.5 | 1510 | 4320 |
| 21Ne | Stable | 20.8 | 0.4 | 0.35 | 180.5 | 1510 | 4320 |

Table 1: Half-lives, surface production rates, and attenuation lengths for the cosmogenic nuclides used in this study. The publicly available software is flexible and users can apply values of their own choice.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Model parameters | | | | | |
| Model 1 : | *int* | *gla* | *tdegla* | *tgla* | *trat* |
| Model 2 : | *int* | *gla* | *tdegla* | *18Othreshold* |  |

Table 2: Model parameters associated with the two models

**Figure Captions**

**Figure 1**

The total production of 10Be as a function of depth (black) in the model framework results from production due to spallation (red), negative muon capture (green), and fast muons (blue). The production rates of the other nuclides included in this study (26Al, 14C, 21Ne) are calculated in a similar way. The relative contributions to the production rate from spallation and muonic processes are based the currently available literature (e.g. Dunai, 2010; Braucher et al., 2011; Braucher et al., 2013).

**Figure 2**

An example of the two-stage periodic model, in which the interglacial (tint) and glacial (tgla) periods have uniform durations throughout the Quaternary. In this approach, the ratio of interglacial to glacial time (trat) is a model parameter that is estimated using the MCMC technique. The other parameters related to the timespans include the time elapsed since the last deglacial event (tdegla) and the duration of the individual glacial periods (tgla). A fundamental assumption of the model approach is that interglacial periods are characterized by 100 % exposure, while glacial periods are characterized by 100 % shielding. The remaining model parameters include the interglacial erosion rate (int) and the glacial erosion rate (gla). The interglacial and glacial erosion rates are allowed to vary within bounds specified by the user, and nothing is assumed regarding their relative magnitudes.

**Figure 3**

An example of the two-stage periodic model, in which the interglacial (tint) and glacial (tgla) periods have uniform durations throughout the Quaternary. In this approach, the ratio of interglacial to glacial time (trat) is a model parameter that is estimated using the MCMC technique. The other parameters related to the timespans include the time elapsed since the last deglacial event (tdegla) and the duration of the individual glacial periods (tgla). A fundamental assumption of the model approach is that interglacial periods are characterized by 100 % exposure, while glacial periods are characterized by 100 % shielding. The remaining model parameters include the interglacial erosion rate (int) and the glacial erosion rate (gla). The interglacial and glacial erosion rates are allowed to vary within bounds specified by the user, and nothing is assumed regarding their relative magnitudes.

**Figure 4**

The forward model is validated by comparing nuclide concentrations calculated by use of two different forward models: the analytical, Lagrangian approach used in the MCMC analysis (red), and 2) a numerical, Eulerian approach (blue). Panel A illustrates the Lagragian (red) and Eulerian (blue) approaches to calculate the surface nuclide concentrations for a particular glacial-interglacial landscape scenario. While the Lagrangian approach tracks a certain layer as it moves towards the surface due to erosion, the Eulerian approach calculates the change in concentration with time at certain depths (z = 0 m in panel A). As shown in panel B, the two methods yield identical, present-time nuclide concentrations at all depths.

**Figure 5**

Density cross-plots of the four model parameters included in synthetic landscape scenario 1, where the landscape history is estimated based on the global marine δ18O stack. The density cross-plots show the bounds of the model parameters used in the MCMC inversion and the yellow-reddish colors indicate the family of parameters that provide the best fit to the observed nuclide concentrations.

**Figure 6**

Histograms showing the distribution of model parameters used in the MCMC analysis to define the models that provide the best fit to the nuclide concentrations calculated in synthetic landscape scenario 1. In this synthetic landscape scenario, the observed nuclide concentrations are computed using the forward model and a set of known model parameters (these true model parameters are indicated by red lines). In this model setup, the second step of the MCMC analysis, following the burn-in phase, is based on 10,000 simulations and one random walk. To ensure that the parameter estimation is robust, it is possible to start several, parallel random walks with different starting points in the model space.

**Figure 7**

Exhumation histories computed for synthetic landscape scenario 1 based on the family of model parameters that provides the best fit to the observed nuclide concentrations. The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations applied in the second step of the MCMC analysis. The depth and time units were binned and the shading reflects the number of simulations passing through the bins. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The MCMC analysis is based on four nuclides and observations from the surface only. The magenta line shows the true exhumation history. In landscape scenario 1, the observations consist of four nuclide concentrations (10Be, 26Al, 14C, and 21Ne) from a surface sample (z = 0 m).

**Figure 8**

Exhumation histories computed for synthetic landscape scenario 2, based on 10Be, 26Al, 14C, and 21Ne concentrations from the surface (z = 0 m). The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations used to constrain the best-fitting model parameters. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history.

**Figure 9**

Exhumation histories computed for synthetic landscape scenario 2, based solely on 10Be and 26Al concentrations from the surface (z = 0 m). The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations used to constrain the best-fitting model parameters. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history.

**Figure 10**

Exhumation histories computed for synthetic landscape scenario 2, based on 10Be, 26Al, 14C, and 21Ne concentrations obtained from a depth profile (z = 0, 0.3, 1, 3, 10 m). The shady cloud in the background demonstrates the landscape histories associated with the 10,000 simulations used to constrain the best-fitting model parameters. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles. The magenta line shows the true exhumation history.

**Figure 11**

Exhumation history computed for a sample (GU110) collected from an elevation of 745 m a.s.l. in the Upernavik area, W. Greenland. The MCMC analysis is based on 10,000 simulations and measured concentrations of 10Be and 26Al. The black line denotes the median value, whereas the red lines denote the 25% and 75% quartiles.

The forward simulation provides simulated concentrations for the four nuclides in the model, which subsequently are used in the MCMC approach as observed concentrations, each associated with a specified uncertainty (here taken to be 2% for all nuclides).

Importance of the degree of smoothing

Use of d18O-threshold values

Flexibility

* A priori knowledge of past glacial-interglacial transitions (as time vector)
* Number of nuclides
* Specify uncertainty
* Sampling depths
* Start concentrations
* Other a priori info

Flexibility of the inverse approach

Number of iterations

Figure 4 shows the histograms of each parameter in Model 1, keeping one out of 10 samples in a 10,000 step MH-MCMC computation. The observation data vector



was computed at the surface from the model shown in Figure 2. The data were noise free, but an uncertainty of 2% was assumed in the likelihood-ratio criterion of the Metropolis-Hastings iterations.

Some technical details are: The random walk was started four times with parameters at extreme corners of the prior intervals. A burn-in phase of 1000 iterations was used. At each iteration the current model is perturbed by a fraction of the prior interval. This fraction is updated every 1000 iterations so that an acceptance ratio of about 0.4 is achieved.

We note that the histograms differ but only slightly, indicating good convergence of the procedure. The widths of the histograms indicate the interval within which each parameter is constrained by the data. Skewness of the histograms reflects that the problem is nonlinear.