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REVIEW ARTICLE

Laser Doppler anemometry: recent developments and future challenges

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Abstract. The availability of new optical and electronic components and the increasing demands on measurement accuracy have led to a continuous development of the laser Doppler measurement technique in recent years. This article reviews the developments in optical systems, signal processing, data processing and in the application of LDA systems. Particular emphasis is placed on examining how well present instruments meet the changing needs of the fluid mechanics community and what improvements would be desirable in the near future.

Cam Tropea was born in Toronto, Canada, where he also attended the University of Toronto to earn a BASc in Engineering Science and an MASc in Mechanical Engineering. He continued his studies in Karlsruhe, Germany, where he completed a Dr-Ing, in 1982, and after which he moved to the University of Erlangen-Nürnberg, Germany as a research associate. With one year as a Visiting Assistant Professor at the University of Waterloo, Canada, he completed his Habilitation in 1991 at Erlangen, before joining the company Invent GmbH, also in Erlangen, where he stayed for almost two years. He returned to the University in Erlangen as Professor of Fluid Mechanics in 1992, where he is presently employed. He has worked extensively on the development of LDA and PDA measurement systems and their application to complex flow situations.

1. Introduction

Expectations regarding the measurability and measurement accuracy of single- and multi-phase flow properties have increased dramatically in recent years. In part this arises from improvements in instrumentation but also in part out of necessity, as numerical flow computations reach or exceed previous experimental capabilities. The laser Doppler anemometer (LDA) has been perhaps the single most important instrumental technique to have contributed to the investigation of complex flows of fundamental and practical interest and, therefore, it is instructive to review it periodically and to identify areas worthy of further development. The present review concentrates on recent instrumentation developments in the field of laser Doppler anemometry, rather than on the broad range of applications. No new results are presented, but some attempt is made to evaluate the importance of the respective developments, implicitly through the choice of material presented and explicitly by examining current experimental needs in fluid mechanics.

The volume of literature on the LDA technique is immense and, indeed, is one of the problems facing the interested researcher. In the present review, reference will be made to a large number of specific works, but general reference sources are also valuable, especially for those readers interested in the principles and the evolution of the technique. For example, the pioneering developments of LDA have been collected in a recent volume edited by Adrian [1]. Unfortunately, most textbooks on the LDA technique are somewhat older and no longer reflect the state of the art in equipment; however, they remain valuable as an introduction to the principles [2–6]. Several conference series have been instrumental in promoting the development of the laser Doppler technique and the respective proceedings are important reference sources. For LDA, these include a series of eight conferences held biennially in Lisbon [7–13] and a conference series alternating between North America and Europe [14–18].

The present review assumes a good knowledge of the principles of LDA. The discussion has been divided into four sections on optical systems, signal processing, data processing and system application. Unavoidably, the selection of material is very subjective. Some effort has been made, however, to elucidate the reasons for particular developments, especially with respect to the needs of the fluid mechanics researcher. Furthermore, to relativize *recent* in the term recent developments, emphasis has been

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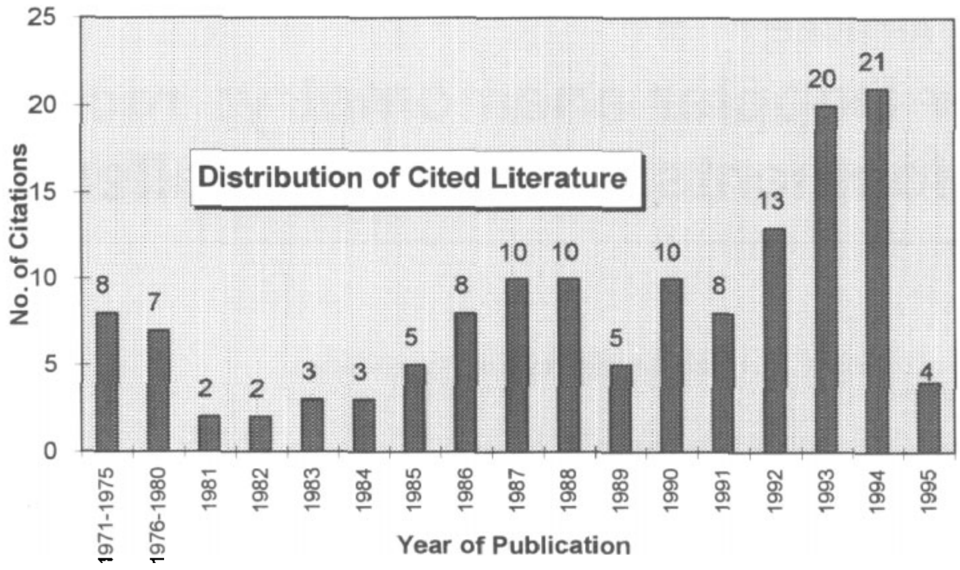

Figure 1. Distribution of publication year for literature cited in this review.

placed on developments that are not yet, or perhaps have just recently become, available in commercial form. This is illustrated in figure 1, in which the distribution of the year of publication of the cited literature is presented.

Finally, the subjectivity of the choice of cited literature can also be qualified. One purpose of this review paper is to direct readers with a specific interest as efficiently as possible to relevant literature. For this reason, the most recent citation was chosen when several were available and they were equivalent in content, since in this way the reader will have a more complete glance into the available, uncited literature. Of course with this scheme, credit does not always fall on the originator of an idea and for taking this liberty, the present author would ask to be excused.

2. Recent developments in laser Doppler anemometry optical systems

Unquestionably, the ease of handling of LDA systems was a prime motivation for the development of fibre-optic probes, which now make up over 90% of the new system market. Not only the factory alignment of the probe but also the remoteness of the large and cumbersome laser source with optics were convincing arguments for this rather rapid move away from traditional optical systems. An alternative means to achieve the same flexibility is a reduction in size of the laser source, which possibility is afforded by the laser diode.

2.1. Systems using semiconductor components

Pioneering work using laser diodes for LDA was, and continues to be, carried out by the research group of Dopheide [19–21] at the Physikalische Technische Bundesanstalt in Braunschweig. In combination with semiconductor detectors, such as avalanche diodes, with high quantum efficiencies in the infrared, high-power, single-mode laser diodes can be considered equivalent or even superior to gas lasers for many LDA applications, particularly if mobility is required [22, 23]. Two inhibiting features of early laser diode LDA systems were the relatively large size of conventional frequency shifting devices and the extension of the system to two or even three velocity components. Several novel solutions have been presented for each of these tasks.

One involves the use of alternately pulsed diode lasers, one laser for each velocity component [24]. By synchronizing the laser pulse sequences with the signal digitization, a technique known as 'coherent sampling', a single photodetector and data acquisition chain can be used for all velocity components. De-multiplexing of the electrical signal into pulses corresponding to each laser is done by re-ordering the ADC memory via the software, which technique had already been introduced some years previously [25]. In this way, lasers with identical wavelengths can still be used to distinguish various velocity components, since only one set of fringes exists at any one time in the measurement volume. More recently, coherent sampling has been replaced by switch de-multiplexing or time-division multiplexing, as is applied also in communication systems [26]. The need for multiple laser diodes for multiple component measurements has also since been alleviated by using time-delay techniques, either through additional path lengths or by using fibres of different lengths. The principle of such a system based on path-length differences and using a pulse rate of 240 MHz is illustrated in figure 2, taken from [27].

Of course, direction-sensitivity is almost always mandatory in LDA systems and the traditional use of Bragg cells restricts the degree of miniaturization that is possible. Pulsed diode lasers, in conjunction with quadrature processing, constitute one alternative method of sensing direction. In such systems, a path-length delay is used to induce a phase shift between two sets of parallel but

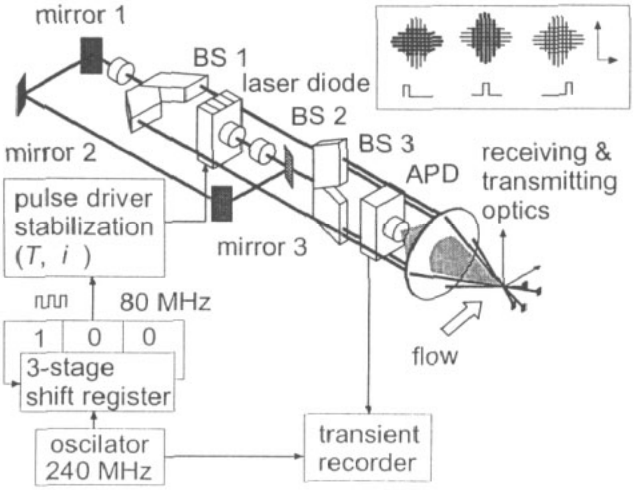

Figure 2. A schematic diagram of a HF-pulsed two-dimensional laser Doppler anemometer with a single laser diode and coherent sampling (from [27]).

displaced fringes appearing alternately in the measurement volume [28]. Using quadrature de-modulation [29], the flow velocity direction can be measured if the two equidistant fringe systems are locally shifted relative to each other by a quarter fringe spacing. One deficiency of this technique is, however, that particles moving parallel to the fringes, namely with zero flow velocity, cannot be measured. Lower limits of measurable velocity have not yet been reported. A good overview of novel frequency-shift mechanisms with quadrature signal de-modulation is given in [30].

The above examples, employing pulsed laser diodes, exploit the enhanced optical power of high-frequency pulsing at repitition rates much higher than the Doppler frequencies. However, multi-mode emissions can occur due to the short injection current pulses, according to Wang *et al* [31]. In this sense, solutions for multi-component, direction-sensitive systems using CW laser diodes may be preferrable. Such systems have been demonstrated using integrated optical (IO) devices to take on the task of high-frequency optical multiplexing between orthogonal fringe patterns [32, 33]. At present, these systems use an existing base optical unit employing Bragg cells [22] to achieve directional sensitivity. However, phase modulators embedded on lithium niobate ($LiNbO_3$) substrates have also been successfully demonstrated as a means of imposing a frequency shift [34]. Such modulators can be either used with a sawtooth-like driving signal, similar to earlier usage of Pockels cells or fibre expanders for the same purpose [35], or with a single side-band modulation, not unlike that applied directly to the laser diode supply in previous systems [36]. In the latter case, a sinusoidal modulation at two different frequencies is simultaneously applied to the phase modulators. Good side-band suppression can be achieved; however, at most 48% of the ingoing power is available after de-modulation. Nevertheless, the combination of multiplexing and frequency shifting on a single integrated optical device, as suggested in [34] is enticing and opens the way to sensor-like LDA systems.

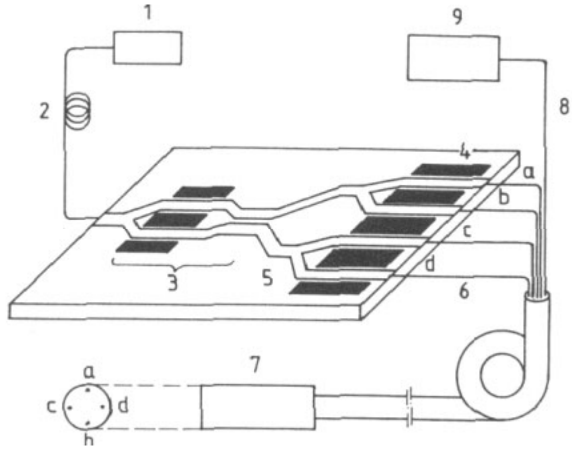

Figure 3. A schematic representation of a directionally sensitive, two-dimensional laser Doppler anemometer based on an integrated optical device (from [34]): 1, laser diode; 2, single-mode, polarization-perserving fibre; 3, *y*-fed balanced bridge; 4, phase modulators; 5, *y*-branch; 6, single-mode, polarization-preserving fibres; 7, fibre-optic laser Doppler anemometer probe; 8, GRIN receiving fibre; and 9, avalanche diode.

Such a system concept is pictured in figure 3, taken from [34].

At present there exist some limitations with IO devices. Those most suitable for LDA applications use a proton exchange method for generating waveguides on $LiNbO_3$ substrates. In contrast to TiO_2 diffused waveguides [37, 38] that were used earlier, the proton-exchange devices can withstand higher power densities without distortion and have excellent polarization and mode stability properties. Although transmitted power exceeding 1 W has been demonstrated, commercial devices typically specify 5–10 mW.

Another alternative to realize directional sensitivity applies two frequency-stabilized monomode diode lasers [39]. In this case the frequency shift is given directly by the optical difference frequency of the two laser diode beams, which are focused into the measuring volume. Small variations of the optical frequencies are compensated using a reference signal with hetrodyne detection. A further refinement, which maintains the directional sensitivity, uses quadrature signal pairs from a hybrid coupler, as pictured in figure 4, taken from [39]. Similar techniques have recently been used employing two orthogonal polarized modes of a diode pumped Nd:YAG micro-crystal laser [40]. In the light of some of the other systems now proposed, however, such solutions no longer appear so economical and may be suitable only for very specific applications.

In much of the work with laser diodes, fibre optics and integrated optics cited above, the measurement accuracy has not been addressed. More common is an evaluation of the signal-to-noise ratio (SNR) of the obtained Doppler signals. Although the SNR can be an important factor in determining the achievable measurement accuracy (see below), optical effects inherent in beams originating from small waveguides can also become influential. Such effects have been examined for the case of laser diode LDA systems in [41]. In this study, as in several previous studies from

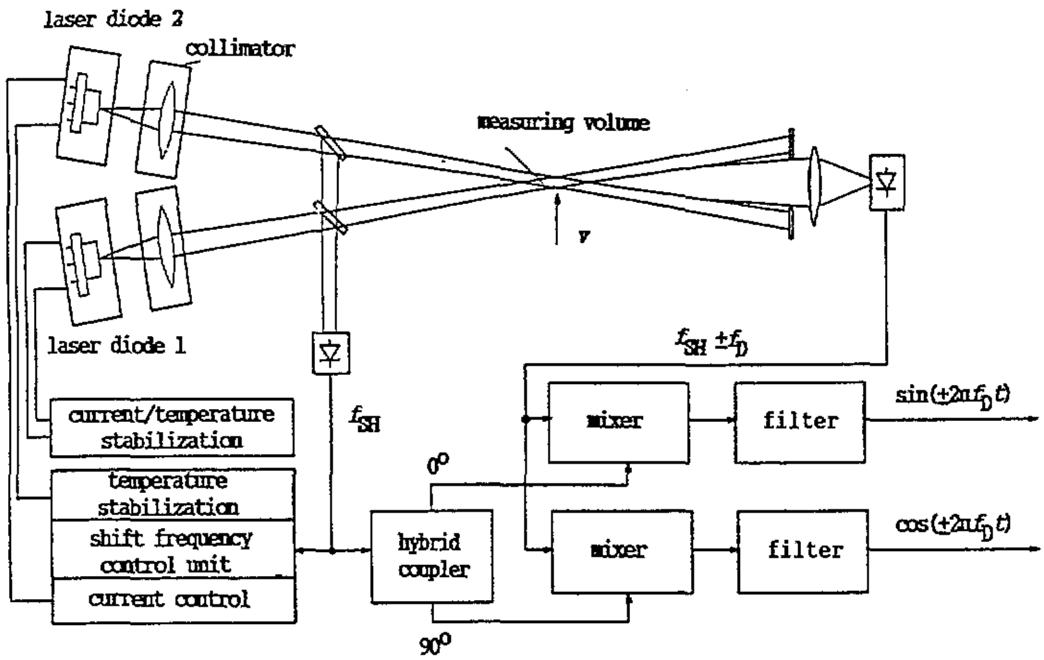

Figure 4. The block diagram of a laser Doppler anemometer using monomode laser diodes for frequency-shift generation (from [39]).

the same group [42], the non-parallelity of fringes in the measurement volume has been examined for typical optical arrangements. The essential arguments fall back on the basic equations describing Gaussian beam propagation, as outlined also in figure 5. From the relation

$$\omega_0\theta_0/\lambda = 1/\pi \quad (1)$$

it is clear that a smaller waist is inherently accompanied by a higher beam divergence, since the above quotient must remain constant. Such small waists occur in LDA systems not only in the measurement volume, but also in the output plane of fibres and/or laser diodes. For situations of high beam divergence, however, collimation is such that $g \approx f$ in the equation

$$b = \frac{f[z_R^2 + g(g-f)]}{z_R^2 + (g-f)^2} \quad (2)$$

which leads to a very high sensitivity of b to the exact position g when ω_0 is small, that is, when z_R is small. If, due to poor alignment, the position b no longer coincides with the measurement volume, then fringe gradients occur.

Basically, these equations confirm that which is more *completely treated in* [41], *namely that alignment is tricky* when a small beam waist occurs anywhere in the LDA optical chain. In figure 6 for instance, results showing the effect of small collimator misalignments on the effective fringe distortion in a laser diode system are illustrated. Lens movements of as little as 1.5 μm already lead to fringe variations of 2%. Such lens movement is conceivable from thermal expansion effects alone. These results indicate that fibre-optic and laser diode systems will, in general, be susceptible to systematic errors due to changes in the fringe spacing. This variation in frequency/fringe spacing will also place a lower limit on measurable turbulence levels. This is also one motivation for establishing a simple and reliable optical calibration procedure for LDA, which topic is discussed in more detail below.

Before leaving the discussion of LDA systems based on semiconductor lasers, it is appropriate to mention also the difficulties of working with these lasers. The fact that the beams are not visible demands extra safety precautions and usually a visible, direction-giving beam, typically of much lower power. Attenuation coefficients of light in water rise by approximately two orders of magnitude between λ = 480 nm and λ = 800 nm [43], making laser diode-based LDA systems unattractive for use in liquid flows. The frequency-doubled Nd:YAG layer at λ = 532 nm is preferable in this sense, but the cost is currently still high and the performance of semiconductor detectors decreases at this wavelength [44]. Early work stressed the need for temperature- and current-stabilizing of laser diodes, in *part to maintain a constant wavelength.* This wavelength-stability problem can be overcome by using diffraction elements, typically holographic ones, which compensate for wavelength variations by changing the intersection angle [45]. More recent work has shown that certain laser diodes exhibit much more stable plateaux in the wavelength–temperature relation and thus do not demand the same degree of temperature stabilization [46]. Nevertheless, these laser diodes must be selected and their plateaus determined. A minimal temperature stabilization is then required to keep the diode operating within its plateau.

2.2. Multi-velocity-component systems

Whereas the above discussion centred on novel LDA systems employing newly available components, an increasing need for more flexible optical arrangements

$$\frac{\omega_o \theta}{\lambda} = \frac{1}{\pi}$$

$$z_R = \frac{\pi \omega_o^2}{\lambda}$$

$$\omega_o' = \omega_o \frac{f}{\sqrt{(f-g)^2 + (\pi \omega_o^2 / \lambda)^2}}$$

$$b = \frac{f\left[z_R^2 + g(g-f)\right]}{z_R^2 + (g-f)^2}$$

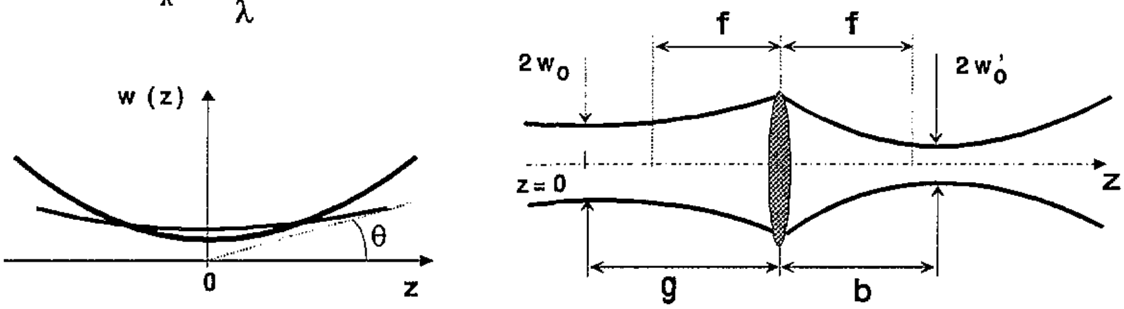

Figure 5. A summary of relations describing Gaussian beam propagation.

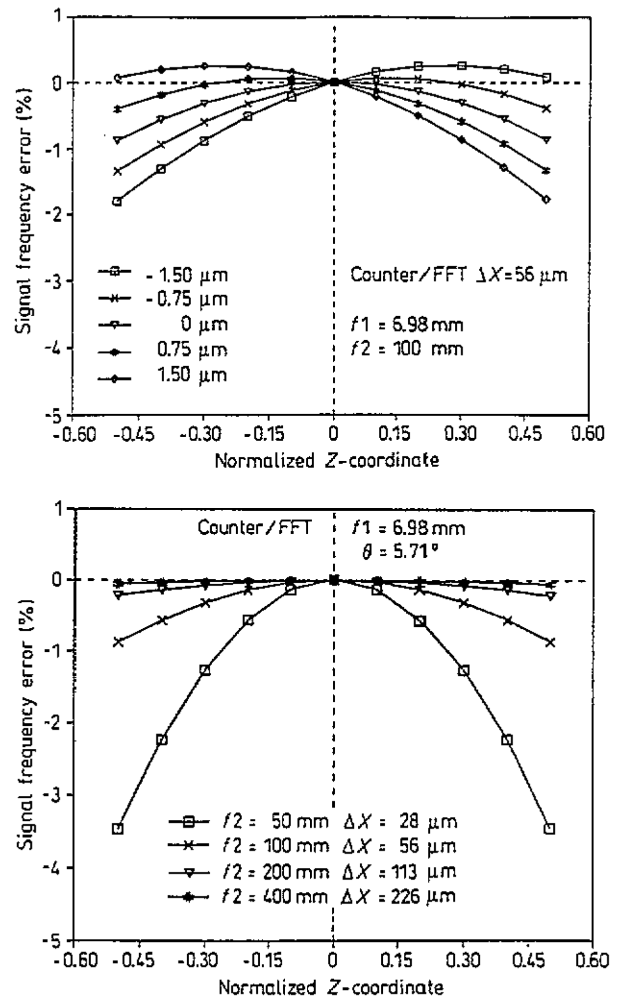

Figure 6. The computed frequency error for a laser diode laser Doppler anemometry system: (a) the effect of collimator misalignment and (b) the effect of front lens focal length (f_c) (from [41]).

for three or more velocity components or for velocity measurements at two points simultaneously has emerged. Many fundamental flow studies, which previously dealt with two-dimensional flow fields to stay within the realm of 'computable' flows, are now examining fully three-dimensional flows. In such cases, in which advanced turbulence models (such as algebraic stress models and Reynolds stress models) are capable of simulating high degrees of turbulence anisotropy, measurements must be made available to confirm and verify model assumptions. Especially in separated flows, where anistropic conditions undermine the effective viscosity hypothesis that is often used in simulations, the triple correlations can reveal much about the return to isotropy or about the entrainment of low-speed fluid and ejection of high-speed fluid that occurs in a shear layer [47]. In three-dimensional boundary layers, acquisition of all velocity components is equally important, since many near-wall models fail in the presence of a strong cross stream shear stress. In particular, the relationship between angles of the turbulent shear stress and the mean velocity gradient vectors becomes quite complex [48]. Excellent examples of a five-velocity-component LDA and a two-velocity-component scanning LDA used to investigate three-dimensional turbulent boundary layers have been presented recently [49, 50].

Conventional three-velocity-component LDA systems employ an orthogonal arrangement for two of the three components and a non-orthogonal alignment for the third velocity measurement. The propagation of alignment errors or intersection angle errors to final turbulence quantities has been examined in detail in [51] and previously in [52], leading to the conclusion that the angle to the third component should be maximized and not less than 60°. This is a typical example of the *difference-of-two-large-numbers* problem. Unfortunately, many applications, such as turbomachinery, provide limited optical access, and lower third-component angles are unavoidable.

A recent study has examined the case of three-component LDA systems using a single-lens approach (five beams) and also here it is shown that the propagation of misalignment errors, even to the mean flow quantities, becomes intolerable [53]. The very realistic approach of calibrating for such systematic errors is put forward, which is essentially the same conclusion reached by Karlsson *et al* [54] for non-orthogonality in a two-component LDA system. In the latter case, a calibration was not directly used, but the condition $\overline{V} > 0$ immediately above the wall was invoked to determine the misalignment of the components.

Covariance terms (Reynolds stresses) are especially sensitive to coincidence detection with such optical arrangements, which is presumably related to the virtual particle effect identified for three-dimensional LDA systems [55, 56]. In three-component LDA systems, cross detection is strongly advisable when forward scatter is not possible. This refers to the detection of light scattered from one set of beams through the transmitting lens of the off-set set of beams, and vice versa. This minimizes the virtual particle effect, which arises when two channels simultaneously have signals, but from different particles, not necessarily with the same flow velocity. However, even with cross detection, a further bias factor has recently been identified, which is related to the coincidence window [57]. Previous experience has underlined the importance of choosing very short coincidence windows in three-dimensional LDA systems, due to there being strong amplification in error propagation. Depending on the exact criteria for signal validation in the signal processors, the exact time of arrival assigned to a particle may vary from channel to channel. This time discrepancy will, however, in general be larger for slower moving particles, thus increasing the probability of coincidence rejection. This leads to a preferential acceptance of rapidly moving particles for a fixed coincidence time window and is of particular importance in flow regions with broad velocity distributions. One solution to this difficulty would be to assign the time of arrival to the centre of the signal, necessitating a modification to existing processors. Alternatively, a coincidence distance rather than a coincidence time could be invoked, of course requiring all three velocity components.

In three-dimensional flow fields, but also in transient flows, the need for two-point, multi-point or scanning LDA has become apparent. For well-directed flows, Taylor's hypothesis can be invoked to transform from a single point-velocity measurement in time to an estimate of local spatial velocity fluctuations and thus to length scale estimates [58]. This approach breaks down for three-dimensional flows or for flows with strong mean flow changes with time, such as in engine flows [59]. Indeed, direct length-scale measurements have direct applicability to verification of large-eddy simulation (LES) computations and can also be related directly to results of direct numerical simulations (DNS). This increased need for spatial distributions of turbulent fluctuations is similarly reflected in the rapid acceptance of particle-image velocimetry (PIV) [60] and particle-tracking velocimetry [61, 62] as investigative tools.

From the optical point of view, a large number of two-point LDA systems have been proposed [63–68]. More recent systems have recognized the benefits of operating in forward scattering mode, for the signal quality, but especially for the increased data rate and its influence on the spatial correlation estimators [69–71]. The spatial correlation function of velocity fluctuations is given by a cross correlation of velocity measurements taken simultaneously at two points in the flow field. This can be extended to a space–time correlation function by cross correlating the velocity measurements at two positions but at different times [72]. Whereas the spatial correlation can provide estimates of Eulerian length scales, only the space–time correlations can provide Langrangian length scales. Such information will undoubtably be in greater demand in the near future. Interestingly, no manufacturer of LDA equipment presently offers a standard, two-point measurement system.

3. Recent developments in laser Doppler anemometry signal processing

A distinction is to be made between signal processing and data processing. Signal processing involves estimation of the primary measurement quantities (for example frequency, phase, residence time and arrival time), whereas data processing estimates secondary quantities related to the flow field (for example mean velocity and turbulence spectra) [73]. Specifically, for the estimation of signal frequency, the Cramer–Rao lower bound (CRLB) represents the lower bound of variance when the Doppler signal consists of a single sinusoid [74]. For the case of LDA signal processors operating with a sample frequency of f_s on a set of N samples extending the length of the Doppler signal, the CRLB takes the form [75]

$$\sigma_f^2 = \frac{3}{\pi^2 \mathrm{SNR} \cdot N(N^2-1)} f_s^2. \tag{3}$$

This limit has been used to show that current LDA processors operating in frequency domain are very close to reaching an optimum estimation [76, 77], indeed some exceeding the limit if narrow-band filters are employed [78], effectively using *a priori* knowledge of the signal. Equation (3) indicates that this variance should be insignificant for reasonable values of SNR. Note, however, that, if the particle residence time (effectively N/f_s) in the measurement volume (MV) is reduced, either due to very high velocities or due to a decrease in the size of the volume, then the CRLB can become large. Thus, a higher spatial resolution necessarily introduces a larger scatter in the primary measurement quantities, effectively imposing a lower limit on the magnitude and the scales of measurable turbulence. This aspect will be discussed further in connection with the estimation of turbulence spectra.

Although frequency estimation is doubtlessly much improved over that of early systems, even a few 'bad' data points can be particularly influential if higher order moments are being computed [79]. Higher moments, while not regularly reported, are being used increasingly as a more sensitive validation of numerical computations, specifically for DNS. Athough undesirable, identification and removal of 'bad samples' appears to be mandatory for refined turbulence measurements and several methods have been successfully applied [80–83].

The goals of past years have, however, in many ways been achieved with current instruments. High-speed DSPs allow complex frequency-domain analysis, either FFT-based or employing the autocorrelation function, to be performed digitally on-line and with little user intervention. Certainly a miniaturization (and reduction in cost) of signal processing units is now desirable, to accompany the respective size reduction of optical systems that has been outlined in the previous section, even if some features of accuracy or bandwidth are sacrificed. A notable step in this direction is the minimal cross correlator (MCC), which was introduced as part of an LDA-sensor concept [84, 85]. In this system, massively parallelized 1-bit correlators on an ASIC simultaneously observe the signal coherency over a very broad bandwidth by cross correlating with the shortest possible signal signature (0101010...), hence the term *minimal* cross correlation. First results with such a processor are encouraging, but recent examination of processing with 1-bit quantization indicates that the system must remain sub-optimal [86].

It might be pointed out that the largest efforts in developing and analysing LDA signal processors in recent years have been restricted to the case of single-realization anemometry, that is, under the assumption that the probability of more than one particle appearing simultaneously in the measurement volume is very small. This is in contrast to several early examinations of achievable resolution of turbulence in LDA [87, 88] and to a more recent examination of the application of LDA to the velocity measurement of solid surfaces [89]. Indeed, in the latter study, the laser transit velocimeter (LTV) is shown to be, at least in theory, preferable for such measurements, although the measurement of zero velocity still remains a practical problem with the use of LTV.

No doubt for the user, guidelines for the choice of processor for a particular application, or information about the accuracy of a given processor would be valuable. Studies abound on this subject [90–94, 73], but the reader must be warned about the universality of conclusions. A very large number of factors influence the frequency estimation in a processor, so many that any study will of necessity be restricted in its scope to a sub-set. User adjustment alone can outweigh in its influence that of inherent estimator merits. In the final analysis, it is the total system that determines the accuracy and reliability of the results, not just the processor. Therefore it is difficult to put the discussion of processor merits or pitfalls on an objective footing that has some degree of universality.

Certainly improvements have been achieved in the past years, as any experienced user can testify. Nevertheless, at present it is very difficult to estimate the accuracy of an LDA signal processor or for that matter of an LDA system, and be confident of the value. What may now benefit particularly from the greatly enhanced and affordable computation capabilities are improved signal detection schemes. Indeed, several instruments have appeared that work on-line in frequency domain, using the coherency or spectral power as a means for signal detection.

4. Recent developments in laser Doppler anemometry data processing

Data processing deals with the estimation of relevant flow properties from the primary measurement quantities. Basically, two properties of LDA signals have made data processing in LDA an issue at all. The first is the fact that the rate of particle arrivals in the measuring volume is usually correlated to some degree with the measured velocity component, leading for instance to a difference between the particle mean velocity and the time mean flow velocity. This effect is responsible for a bias of many estimators. The second property is the irregular arrival times of particles in the measurement volume, which introduces some differences with respect to estimation from a regularly sampled time series. The severity of potential errors in choosing an appropriate estimator increases, almost without exception, with increasing local turbulence intensity. Thus, the present discussion has particular relevance to the measurement of highly turbulent flows or of separated and/or recirculating flows, in which mean flow velocities can approach zero, and hence the local turbulence intensity can be very large.

Two questions must be addressed when examining any estimator.

(i) What is the expectation of the estimator?
(ii) What is the variance of the estimator?

The latter question can be answered by applying conventional rules concerning the statistical uncertainty of an estimator [95, 96], and, while not wanting to under-emphasize these aspects, they will not be discussed further here. It is sufficient to note that averaging times only have meaning when expressed in terms of integral time scales and *not* in terms of number of particles.

Considering the large volume of literature addressing the first question (the expectation of the estimator) it is encouraging to find consensus on at least one point, namely that *higher estimator accuracies are possible with higher particle densities.* Most of the literature, therefore, deals either with defining a sufficiently high particle density for a specific estimator or examining the expectation error of estimators (bias) if the particle density is less than optimal. At least for simple moment and spectrum/autocorrelation estimators, this limit appears to be about ten particle arrivals per integral time scale.

Any deviation of the expectation from the true value is known as a statistical bias. It is noteworthy that turbulence measurements dealing with deviations from the mean can be in error not only due to the bias of their estimator but also because the mean value subtracted from the measurement values may in itself be biased!

Another common point of consensus is the important parameters of the problem. These are

(i) $\dot{N} = 1/\tau_m$, the mean valid signal arrival rate (single realization),
(ii) T_u, the integral time scale of the measured quantity,
(iii) T_s the sample interval time (applicable for a controlled processor) and
(iv) τ_i, the residence time of a particle in the MV

The mean data density is given by $N_D = T_u/\tau_m$.

One reason that verification of bias is so difficult is that the true answer is seldom known and other errors may be larger than the expected bias. For this reason, and also so as to be in a position to examine estimators that are theoretically intractable, simulations are being used increasingly often [97, 98]. Some results from these simulations will be cited below.

4.1. Moment estimators

An interim report on the status of moment estimators was given in 1987 [99], in which at least one moment estimator was judged to be valid for all situations, namely a transit time weighting [88]

$$u^n = \sum u^n \tau_i \Big/ \sum \tau_i. \qquad (4)$$

If the transit time of each particle is not available, then there appears to be only sub-optimal solutions and solutions that in general require a mean data density exceeding 10. The most concise summary of such solutions, including experimental verification, is given in [100, 101]. Incidently, these results have been duplicated in the simulations of [98]. For the case of moment estimators, therefore, the question of bias appears to be well understood for most common situations.

One exception is the case of inhomogeneously seeded flows, in which a correlation exists between the particle density and the flow velocity. The simplest example is the free jet, in which the seeded air issues into non-seeded ambient air [102], but many other flows exhibit such behaviour [103]. Although this problem has been recognized by several authors [104–106], very few investigations have attempted to quantify the errors incurred [107, 108].

4.2. Spectral estimation

Considerable effort is being directed towards the estimation of turbulent spectra from LDA data. Perhaps one reason for the resurgence of interest in measuring the spectrum of turbulence is the opportunity to estimate the rate of dissipation of turbulent kinetic energy from the spectrum [109], which is one of the more elusive quantities experimentally and at the same time the least secured quantity in numerical modelling. In estimating the turbulence spectrum, the bias is generally secondary in importance to the variance, since at high frequencies the variance of the estimator can completely mask the shape of the spectrum. In fact, the main motivation for investigating new spectral estimators is to minimize this variance.

At present, spectral estimation takes one of two approaches, as indicated in figure 7. Either estimates are based directly on the available velocity samples, their arrival times and possible further information such as residence time, or an interpolation of the velocity signal between the measured values is performed in order to *reconstruct* a continuous time series, which can then be re-sampled at equidistant time intervals, followed by a spectral estimation using a fast Fourier transform (FFT) or which can be a basis for parametric spectral estimation.

Regardless of the estimator used, there are some limiting factors to resolving the turbulence spectrum. First there is the spatial averaging incurred through the use of a finite-sized measurement volume. Side-scatter collection can improve this spatial resolution, but only at the cost of a lower data rate (for a given seed density). A large number of factors can lead to scatter in the measured frequency, which, in general, are all stochastic in nature and thus characterized by a white noise floor in the spectrum, below which no estimator can yield meaningful results. These factors include non-parallel fringe spacing, fringe distortion due to interfering particles in the path of the laser beams before the measurement volume [110], laser instabilities or poor optical alignment, the variance of the frequency estimator itself, most notably influenced by the signal-to-noise ratio and the residence time of the particle, and the scatter due to the quantization error that is inherently involved in digital data transfer (of all primary quantities), although the latter effect has been shown to be insignificant for a data resolution in the frequency above 4 bits [111].

It is interesting to note that the minimization of this noise floor can be considered independently of the estimator optimization problem. Most of the above factors can be influenced to some degree by the user in his choice of equipment and experimental design. Some compromises are unavoidable, however, for instance a higher spatial resolution (smaller measurement volume) leads to lower residence times and thus to higher variance in the frequency estimate. Although most of these parameter relationships have been recognized in the past and are well understood, to date there exist no general guidelines for system design. Only recently has this optimization problem be considered in depth for specific frequency estimators [112]. It is also noteworthy that this same noise floor represents a lower limit to measurable turbulence, that is, the limit at which it is no longer possible to distinguish with certainty between a turbulent flow fluctuation and the frequency scatter due to one of the above-mentioned causes. Typical lower limits to turbulence measurements with standard equipment are 0.5–1.0%, which is still considerably higher than that of a well-adjusted hot-wire anemometer. Incidentally, one immediate conclusion that can be drawn from these remarks on the variance of frequency estimation and their effect on the measurable spectrum is that a forward scattering system will always be superior to a backscattering system, due to an increase in both SNR and data rate.

Estimation methods using direct transforms were developed in a series of papers [113–115], which showed in principle that the random sampling nature of the LDA allowed the power above the equivalent Nyquist frequency to be estimated. This estimator was refined by using residence-time weighting to remove the bias [116] and was shown to perform favourably in comparison with various other reconstruction approaches [117]. However, at higher frequencies the variance of the estimates increases dramatically, necessitating inordinately high averaging times, as shown theoretically in [118].

Less work has pursued the slot correlation as an alternative means of spectral estimation, although the

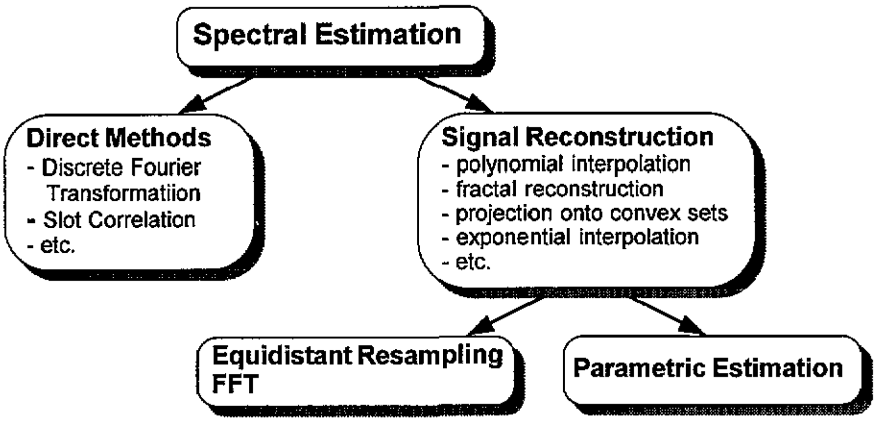

Figure 7. An overview of approaches for spectral estimation.

expectation and variance of the estimator have been derived since its introduction [119–122]. The slot correlation has recently been considered as a means of obtaining space–time correlations [70, 72], which is an application that will require increased attention as two-point LDA systems become more widely used. At present, two approaches are commonly used for two-point measurements. The first involves the use of a coincidence window. However, since the two measurement volumes are not coincident themselves, the data rate after coincidence detection lies far below the data rates of the individual channels. Effectively, a large amount of velocity information is being discarded. The slot correlation therefore allows more efficient use of the available data.

Signal reconstruction in its most primitive form, the zeroth-order polynomial interpolation (known as sample and hold, S + H, and equivalent to arrival time weighting), has been used implicitly since the earliest days of LDA. The analogue output of a tracker or the equidistant acquisition of data from a counter processor exhibits the statistics of a zeroth-order signal reconstruction. A very complete treatment of this reconstruction approach, especially in terms of spectral estimation, is given in [123]. In this study it is shown that the measured spectrum $G_m(\omega)$ is related to the true spectrum $G(\omega)$ through the expression

$$G_m(\omega) = \underbrace{\frac{1}{1+\omega^2/\dot{N}^2}}_{\text{filter}} \left(G(\omega) + \underbrace{\frac{2\sigma_u^2}{\dot{N}^3 T_\lambda^2}}_{\text{step noise}} \right). \quad (5)$$

This equation illustrates the added noise due to the step-like character of the zeroth-order reconstruction and the filter effect due to the particle rate ($\dot{N}$). Both influences disappear for high ratios of the particle rate to the Taylor microscale T_λ. Note that the 3 dB cut-off of the filter occurs at the frequency $\dot{\omega}/2\pi$, a frequently used estimation of the upper validity limit of a spectrum. As an example, a spectral measurement taken with a particle rate of 700 Hz should be undistorted up to about 100 Hz.

Equation 5 illustrates, however, another feature of all reconstruction approaches, namely the inherent filtering due to the particle arrival rate. Figure 8 (from [123]), which is the estimated spectrum of a simulated white-noise process using various data rates, illustrates how similar, and thus misleading, the estimated spectrum of white noise can be to a turbulence spectrum. More than once, this similarity has led to misinterpretations of measured spectra or to false conclusions about the merits of a signal reconstruction scheme, as pointed out in [117].

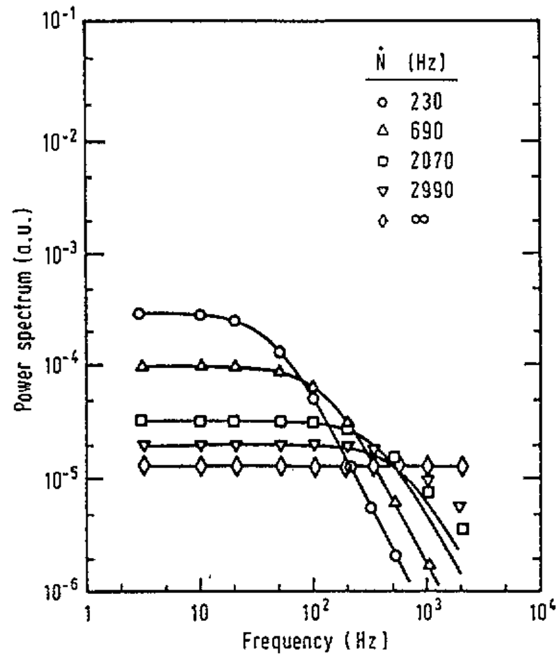

Figure 8. Spectral estimation of a white-noise process with various particle rates (from [123]).

Relatively new to LDA are the large number of alternative interpolation schemes to reconstruct the signal, including the non-uniform Shannon reconstruction [124–126], projection onto convex sets (POCS) [126–128] and fractal reconstruction [129–131]. It should be emphasized that much of the work with signal reconstruction has concentrated on the case of relatively low particle densities, namely less than ten particles per integral scale. This of course exaggerates the filter effect mentioned above, although at the same time the problem of aliasing is reduced. A recent comparison of these techniques for low data rates yielded the result that these reconstruction techniques, in their present form, offer no significant improvement over a simple sample-and-hold approach [117].

A theoretical framework for the analysis of reconstructed LDA signals has recently been presented, together with various consequences for spectral estimation [118]. The exponential interpolation is shown to cover both direct estimation and sample-and-hold interpolation, depending on the exponential decay parameter. From this study, therefore, it becomes relatively easy to assess the performance of the various approaches and, as expected, there are no universally valid conclusions. It becomes evident that the best choice of estimator will depend on the actual spectrum to be measured, the SNR and the choice of window. For many common situations, however, it is shown that an exponential interpolation can be found, which outperforms both the direct estimation and the sample-and-hold method in terms of bias and variance.

A final area of spectral estimation will be briefly discussed, because of its importance for a large number of applications. This is the estimation of frequency content in flow fluctuations from short data records, a situation which is encountered in transient flows, one-shot experiments and near-periodic flows, like measurements in internal combustion engines or wave-induced flow fields. Analytically, the flow is decomposed into a time mean velocity, a time (or phase)-dependent velocity and a turbulent fluctuation

$$U(t) = \overline{U} + \tilde{U}(t, \phi) + u'(t). \tag{6}$$

With this formulation, a distinction can be made between turbulence and cycle-to-cycle variations in an engine flow [132], or between the *wave-associated* part and the turbulent part of the shear stresses in an oscillatory flow field such as in a wave tank [133]. Without the aid of two-point LDA measurements or other spatial velocity information, such a velocity decomposition can only be made on the basis of an instantaneous or short-time record spectrum of the flow fluctuations, in which the latter two components in equation (6) are distinguished according to their frequency content. From other areas of signal processing it is well known that nonlinear spectral estimators can, under certain circumstances, outperform a windowed FFT approach for short data records [134]. Some first applications of non-linear techniques to LDA data have been presented [125], but this remains an area requiring further investigation.

5. System applications

Although the LDA is generally considered to be a technique not requiring calibration, this is not true if very high absolute accuracies are required. Therefore, in recent years, there have been some first attempts at system calibration, although it has also become apparent that this is an area requiring considerable further development. The need for calibration is varied. In some instances the LDA is to serve as a reference for the calibration of other velocity-measuring instruments or volume flow rate devices [135, 136]. Especially at low velocities, the LDA is one of the few available alternatives. On the other hand, the increased accuracy demanded of turbulence measurements also leads to the need for calibration of LDA systems.

The calibration of individual components of LDA systems has often been discussed. Boutier [137] points out that few calibration techniques for the intersection angle exceed an accuracy of 1%. A procedure with a theodolite is proposed, whereby the beam direction, also for three-dimensional systems, can be determined to within 2×10^{-4} [138]. The accuracy obtained in fringe spacing is better than 0.1% and, in any one component direction, 0.1°. Further aspects of optical system calibration are discussed in [139]. Many 'benchmark' tests for the calibration of signal processors have also been presented, albeit usually with artificially generated Doppler signals, as mentioned briefly in section 3.

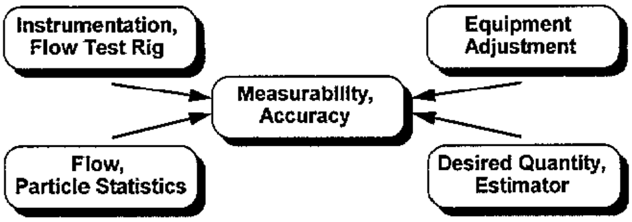

Figure 9. Factors influencing the measurability and accuracy of measured flow quantities using laser Doppler anemometry (from [140]).

If, however, the LDA system is considered as a whole, including the user adjustments, the seeding, the flow to be measured, the optical access and so on, then the propagation of individual component errors (systematic and stochastic) becomes very complex and probably in the long run intractable. These influences have been considered superficially in [140] and are shown schematically in figure 9. Very quickly, the conclusion is drawn that the calibration of LDA systems must be performed *in situ*, allowing no system adjustments between the calibration and subsequent measurements. Indeed, this is not unlike hot-wire anemometry, with the exception that the temperature is no longer a significant factor and seeding particles must be present.

While such an approach is conceivable for low levels of turbulence, for instance using a calibration nozzle, for high levels of turbulence no such calibration facility is available and one can only rely on theoretical predictions of system performance, in particular with respect to the influence of the particle statistics on the chosen estimator. Indeed, in many instances a calibration may be pointless, if the system has to be dismanteled and re-adjusted between calibration and measurement. In any case, the topic of system calibration has not been well treated to date, and deserves further attention in the near future.

Indirect calibration is possible for some selected flow fields, in particular boundary layers. Either physical reasons or simulation results can provide information about the flow velocities near the wall. These can be used to correct for systematic errors in the LDA measurement systems. One good example of this application of *a priori*

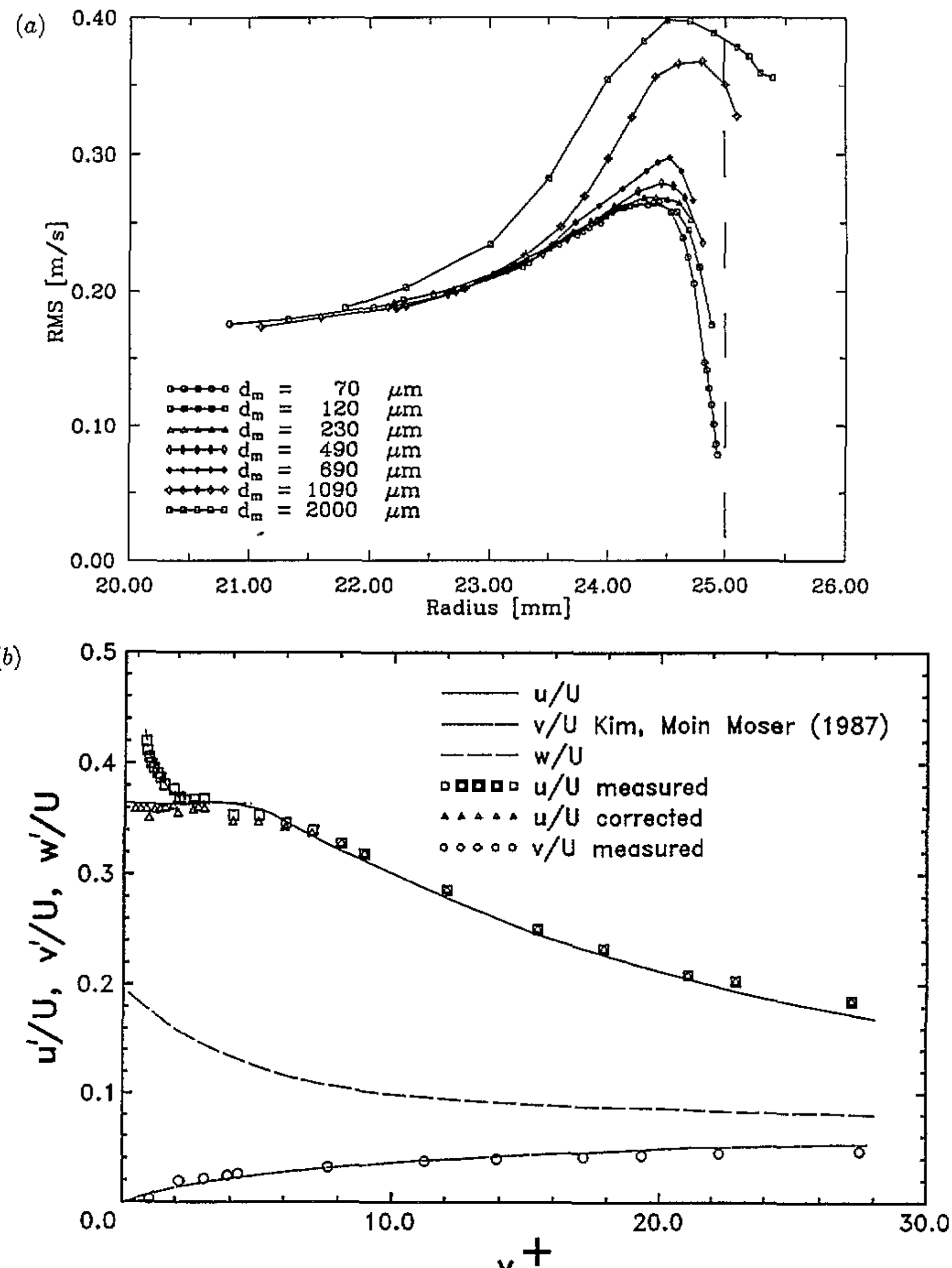

Figure 10. (a) Measured u_{rms} in a turbulent boundary layer (Re = 7500) as a function of measurement volume diameter. (b) Corrected and measured turbulence intensity (from [141]).

knowledge, is the correction for non-orthogonality in a two-component LDA system [80]. Here the fact that the wall normal mean velocity must remain positive is used to deduce the actual angle between the measured velocity components, namely the deviation from 90°. Normally such deviations are inconsequential, but near the wall the stream-wise velocity component is much larger than the normal velocity component so that small angular errors can be significant, relative to the measured normal component. The propagation of this correction to turbulence quantities has also been investigated in [80].

Similarly, the effect of spatial averaging over the measurement volume, which is easily demonstrated with near-wall measurements when the local velocity gradient is generally much higher than in other flow regions, can also be calibrated indirectly. A relatively simple model has been successfully used to express the effect of spatial averaging and to allow corrections to the measured mean and turbulence quantities to be made [141]. The spatial mean is expressed as

$$\langle U \rangle = \frac{1}{d} \int_{y_c - d/2}^{y_c + d/2} U(y) f(y)\, \mathrm{d}y \tag{7}$$

where d is the diameter of the measurement volume and $f(y)$ is a weighting function, expressing the probability of particle detection over the measurement volume. A

Gaussian distribution, corresponding approximately to a signal-amplitude-weighted detection probability, is an appropriate weighting function. As can be expected, the mean velocity is only affected through a second-order error, whereas higher moments can be much more sensitive to the spatial averaging. Results taken from [141] are presented in figure 10, showing the effect of the measurement volume diameter on the measured velocity variance and the successful correction for the finite volume size.

Such *a priori* knowledge about the flow field is sometimes used to determine the absolute position of the measurement volume with respect to the wall. Of course, positioning is possible by observing a maximum of scattered light intensity as the measurement volume is moved towards a wall or obstacle. Refinements to this technique are based on fitting the measured mean velocity profile to a straight line with intercept zero [142]. A much higher accuracy can be obtained by observing also the vanishing of u' and v' on the wall, albeit with appropriate corrections for the velocity gradient, as discussed above. Using a 70 μm measurement volume and a refractive-index-matched test section, mean velocities down to 10% of the outer velocity were obtained ($y^+ = 1.2$), with a positioning error of 10 μm in [141]. On monitoring only the mean flow velocity and working without refractive index matching, positioning uncertainties of approximately 40–50% of the measurement volume diameter are typical, with some improvement if working in forward scattering mode.

6. Closing remarks

The purpose of this review was to look at developing fields of laser Doppler anemometry and to identify areas that require increased attention in the future. Through a generous use of citations, this review should also serve the reader as a reference source to additional material on the subjects covered. Although not exhaustive, the following list summarizes some of the topics that have been discussed as future challenges in the development of LDA instrumentation.

(i) Realization of a miniaturized, three-dimensional, direction-sensitive LDA based on semiconducter lasers or solid state lasers, integrated optics and affording sufficient power to replace conventional gas laser systems.

(ii) Development of convenient, multi-point LDA systems, possibly with multiplexing to overcome the need for multiple laser sources, detectors, signal processors and so on. A more convenient approach to MV positioning/separation is desirable.

(iii) Miniature (and less expensive) signal processors are not only desirable but also necessary before LDA an be considered for sensor-like applications.

(iv) Data-processing methods for measurements from inhomogeneously seeded flow fields or from short data records may be valuable for more specific applications.

(v) Considerable uncertainty exists in the correct choice (or even implementation) of coincidence windows, both for multiple-component LDA and for multi-point LDA.

(vi) Comprehensive guidelines for system lay-out, considering the flow scales, turbulence levels, the particle seeding and the desired measurement quantitities are overdue. Specifically, estimates of achievable accuracy and a decision-making basis for compromise choices would be helpful.

(vii) Standardized calibration procedures for LDA are lacking. Without these, the LDA technique cannot be considered as a reference method and the final accuracy of LDA measurements will remain uncertain.

It is certainly interesting to note that, despite its long history and the large number of papers dealing with the development and application of laser Doppler anemometry, the efforts to improve the technique and to extend its accuracy and applicability remain undiminished. A very safe conclusion of the present review is that this activity is not likely to subside in the near future.

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