HOT-WIRE ANEMOMETRY ✳8088

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1 INTRODUCTION

1.1 *Operating Principle*

Hot-wire and hot-film anemometers are devices used to measure the variables occurring in turbulent flows, such as mean- and fluctuating-velocity components, mean and fluctuating temperature, etc. The sensors are thin metallic elements heated by an electric current (Joule effect) and cooled by the incident flow, which acts by virtue of its mass flux and its temperature (through various effects, but with forced convection usually predominant). From the temperature (or resistance) attained by the sensor, it is then possible to deduce information on the flow. More than one sensor, or more than one value of the heating current, is often necessary to investigate thoroughly a turbulence field.

1.2 *Brief Historical Survey*

The exact origin of hot-wire anemometry is difficult to ascertain, but it seems to go back to the beginning of the present century. In fact, according to King (1915), preliminary experiments were carried out by Shakespear at Birmingham as early as 1902, but were discontinued because of the difficulty involved in the erection of a suitable whirling table for calibration of the wires. Kennelly, Wright & Van Bylevelt (1909) and also Riabouchinsky (1909) clearly defined the concept of electrical anemometry. Some of the earliest independent publications are those of Bordoni (1912), Morris (1912), Retschy (1912), Gerdien (1913), and Kennelly & Sanborn (1914). King's work was published a little later in 1914, 1915, and 1916, and was important both for the design of hot-wire anemometers and for the theory of the convection of heat from cylinders immersed in a stream of fluid. Some theoretical work in this area was done earlier by Boussinesq (1905) and Russel (1910), and Ser (1888) seems to have been the first experimenter to show that the forced-convection loss is proportional to the temperature difference and to the square root of the velocity.

Although generally limited to the measurement of mean velocities, this early research provided many sophisticated ideas, such as the use of a glass coating for wires working in an active chemical medium (Thomas 1920), auxiliary wires to correct the velocity measurements for the effects of mean temperature drift (Fay 1917), hot wires in liquids (Davis 1922), two hot wires set in a narrow "V," or even three hot wires

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grouped in a pyramid with a small vertex angle for the purpose of detecting flow direction (Simmons & Bailey 1927). In these early papers two different methods emerged. In one case, the heating current through the wire was kept constant. Thus the changes in wire resistance were a measure of the flow velocity (the so-called constant-current method). In the other case, the wire was placed in a Wheatstone bridge that was kept balanced by manual adjustment of the heating current, the magnitude of which determined the mean velocity (the so-called constant-resistance method). Practical details concerning these early experiments can be found in Richardson (1934).

Emphasis shifted in later years to the measurement of velocity fluctuations. An early experimental example is cited by Huguenard, Magnan & Planiol (1926), who were investigating wind gusts in the atmosphere. Truly quantitative measurements of velocity fluctuations were made possible by Dryden & Kuethe (1929), who developed an electronic technique in conjunction with the constant-current method to compensate for the thermal time lag of the wire. Ziegler (1934) was the first to utilize the advantages of the constant-resistance technique for unsteady measurements by a unique application of a feedback that maintains a constant wire resistance (and hence constant temperature) at all times. This was another method for overcoming the basically inadequate temporal response of the wire.

Later, a considerable amount of work was devoted to understanding in more detail the response of the wire and to improving the associated electronic circuitry. Furthermore, measurement of flow variables other than velocity was suggested, such as temperature, or even density and concentration, through application of the fact that the cooling of the wire is a function of the mass flux rather than of the velocity.

The rms value u' of the velocity fluctuations occurring in turbulent flows covers a large range of values. When expressed as a percentage of the local mean velocity $\bar{U}$, it is of the order of 0.05% in good wind tunnels, 0.2–2% in grid turbulence, 2–5% in wakes, 3–20% in boundary layers or pipe flows, and over 20% in jets. The energy spectrum of the fluctuations also covers a large range of frequencies, from about $\bar{U}/L$ (where L is an integral length scale or the approximate width of the flow when appropriate) to $\bar{U}/\eta$ [where η is the Kolmogorov dissipative cutoff, with $\eta/L \simeq (u'L/v)^{-3/4}$]. For instance, for fully developed pipe flow with a mean Reynolds number of 10^5, the upper cutoff is about 30 kHz. Similar ranges are encountered for the other fluctuations (temperature, etc) occurring in turbulent flows.

1.3   Plan of the Paper

The various current areas of interest in hot-wire anemometry can be separated into the following categories:

1. Hot-wire response, both in the steady and unsteady cases and for small and large fluctuations.
2. Electronic circuitry complementary to the wire sensor in both constant-current and constant-resistance (or constant-temperature) anemometers.

3. Special signal-processing techniques necessary in hot fluids, compressible flows, etc.

4. Particular applications, such as wires operating in non-Newtonian fluids, liquid metals, or two-phase flows, and also unusual wire arrangements.

For each area the problems that appear unsolved are also discussed. The literature cited can be separated into two sections: the original papers that are landmarks in the development of hot-wire anemometry, and the already available surveys that provide descriptions of the techniques (e.g. Kovasznay 1954a, Hinze 1959, Comte-Bellot 1961, Corrsin 1963, Kovasznay 1965, Comte-Bellot 1974, 1975).

2   HOT-WIRE RESPONSE

2.1   Sensors

Hot-wire sensors are thin metallic wires, with a typical diameter of 0.5–5 μm and a typical length of 0.1–1 mm; the material used is usually platinum or tungsten, or sometimes platinum-rhodium or iridium. When drawn by the Wollaston process (with a sheath of silver), the wires are usually soft soldered onto two prongs and then etched at the desired length. When bare wires are available (as in the case of tungsten), the wires are usually electrically soldered onto the prongs. However, a large variety of complete probes are available commercially. Hot wires are employed mostly in gases.

Hot-film sensors consist of a very thin film of platinum (typically 1 μm) deposited by sputtering on a quartz support. The support is usually a cylinder (with a typical diameter of 25–100 μm) or sometimes a wedge or a cone. Hot films are used mostly in liquids. An additional quartz coating then protects the film from electrochemical effects.

The metallic elements are heated by an electric current above the ambient temperature of the fluid up to about 300°C for wires (the limit is due not only to the mechanical properties of the material but also to nonlinear effects) (see Section 2.3), and 20°C for films (the limit comes mostly from the occurrence of vapor bubbles). The usual arrangement of hot wires with respect to the flow is shown in Figure 1. A single wire placed normal to the flow is used when investigating the longitudinal component of the velocity. Two wires, forming an "X," and at a small
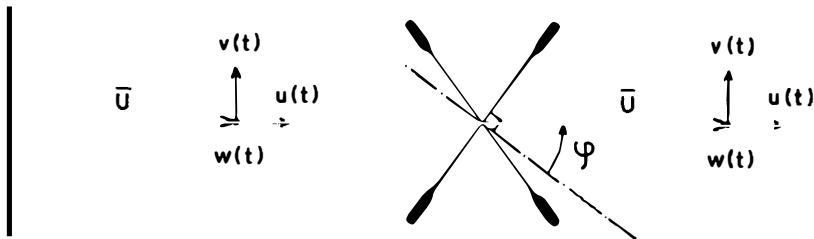
Figure 1   Arrangement of single wire and "X" wire with respect to the mean flow direction.

distance apart along their common normal, are required to obtain the longitudinal and transverse components of the velocity.

Reliable sensors have to meet several requirements. For instance, hot wires must not be soldered too tightly between their supports because of internal stress. Thus slack wires are more reliable than straight wires. It is also recommended to overheat the wires before use. Dust particles have to be removed from the flow, whenever possible, to avoid drift in the sensor response [i.e. increase with time of the wire resistance (Collis 1952)]. For wind tunnels, filters that have an efficiency of about 97% for particles of 3 μm are usually sufficient (Comte-Bellot 1961). Investigation of velocity turbulence in water is also made easier if the temperature is kept constant with time, within about $\pm 0.05°$C.

2.2 Steady-Heat-Loss Laws

For flow varying from free-molecular to continuum motion, and speeds from those of natural convection to supersonic, the Nusselt number Nu for an electrically heated wire has the general form

$$Nu = Nu(Re, Gr, Ma, Pr, \gamma, 2l/d, \phi, a), \tag{2.1}$$

where the dimensionless parameters involved are respectively

Re: Reynolds number based on the wire diameter;
Gr: Grashof number based on the wire diameter;
Ma: Mach number of the flow;
Pr: Prandtl number of the fluid;
γ: ratio of specific heats of the fluid ($\gamma \simeq$ const);
$2l/d$: aspect ratio of the wire;
$2l$: length of the wire;
d: diameter of the wire;
ϕ: angle between the velocity and the normal to the wire;
a: overheat ratio of the wire, $a \equiv (\Theta - \Theta_a)/\Theta_a$ for incompressible fluid, and
$a \equiv (\Theta - \Theta_r)/\Theta_0$ for compressible fluid;
Θ: wire temperature;
Θ_a: air temperature;   } as indicated by an unheated wire immersed in a free
Θ_r: recovery temperature;} stream.
Θ_0: stagnation temperature. [$\Theta_r \approx \Theta_0$ for fine wires; Kovasznay (1950).]

Radiative effects have not been included in (2.1) because they are usually negligible (about 10^{-4} times the rate of heat input to the wire). Only the case of rarefied gases requires a correction.

For the parameters $2l/d$ and ϕ occurring in (2.1), it is usually assumed as a first step that the wire is infinitely long so that its heat-loss rate depends only on the velocity component U_n normal to the wire ($U_n = U \cos \phi$). As noted by Corrsin (1963), such an assumption is exact only for laminar flow past an infinitely long cylinder placed in a uniform, undisturbed velocity field, since the key to the demonstration is that the derivatives of the velocity and thermal fields with respect to distance along the wire axis can be set to zero. Real hot wires are, of course,

finite and, in fact, of short length to make possible local measurements. Typical values are $2l/d \simeq 100$ for hot wires and $2l/d \simeq 15$ for cylindrical hot films. Effects due to the finite length of the sensors are examined in Section 2.5.

Empirical expressions for (2.1) are available depending on the flow range of interest. For forced convection, when compressibility effects are negligible, a frequently used expression is

$$Nu_f = 0.42 \, Pr_f^{0.26} + 0.57 \, Pr_f^{0.33} \, Re_f^{0.50} \qquad (2.2)$$

as proposed by Kramers (1946) and based on the experimental data of Ulsamer, which cover the range $0.71 \lesssim Pr \lesssim 525$ and $2 \lesssim Nu \lesssim 20$. Hence expression (2.2) is applicable for a great variety of fluids but is mostly used for liquids. The subscript f means that the physical properties of the fluid are evaluated at the "film temperature" $(\Theta + \Theta_a)/2$. For gases, a recent survey by Andrews, Bradley & Hundy (1972) lists about forty experiments. When restricted to air a useful expression is given by Collis & Williams (1959) and recently confirmed by Bradbury & Castro (1972):

$$Nu_f = (A + B \, Re_f^n)[(\Theta + \Theta_a)/2\Theta_a]^{0.17}, \qquad (2.3)$$

with $n = 0.45$, $A = 0.24$, and $B = 0.56$ for $0.02 \lesssim Re \lesssim 44$; and $n = 0.51$, $A = 0$, and $B = 0.48$ for $44 \lesssim Re \lesssim 140$. The discontinuity at $Re \simeq 44$ is, of course, due to the vortex shedding behind the wire. In many experiments with hot wires, the Reynolds number stays in the range $2 \lesssim Re \lesssim 30$ so that the viscous effects dominate around the wire. However, for cylindrical hot films placed in liquids, the value $Re = 44$ is reached at a relatively low speed, for example at $U \simeq 1$ m sec^{-1} in water. For films supported on wedges or cones, a separated flow will, of course, always exist behind the probes, and the above discontinuity disappears.

Buoyancy effects have to be taken into account when $Re \lesssim Gr^{1/n}$, where n is about 3 for air (Collis & Williams 1959) and between 2 and 3 for fluids having a Prandtl number between 6.3 and 63 (Gebhart & Pera 1971). Nets of curves giving $Nu = Nu \, (Re, Gr, Pr)$ in mixed convection are available in Nakai & Okazaki (1975). For air, Baille (1971) suggested introducing into (2.3) an effective Reynolds number given by

$$Re_{\text{eff}}^2 = Re_f^2 + [0.9 \, Gr_f^{0.418}]^2. \qquad (2.4)$$

In the case of free convection only, the following law stated by Hatton, James & Swire (1970) is often used:

$$Nu_f = 0.525 + 0.422 \, Gr_f \, Pr_f^{0.315}[(\Theta + \Theta_a)/2\Theta_a]^{-0.154}. \qquad (2.5)$$

When compressibility effects are present, the Nusselt number has (for a given gas) the functional form $Nu_0 = Nu_0 \, (Re_0, Ma_\infty, a)$. The subscripts 0 and ∞ refer to the stagnation and free-stream conditions, respectively (for the viscosity and conductivity of the gas). Empirical data for the high subsonic and transonic ranges are usually given in terms of plots showing Nu_0 as a function of Re_0 with mainly Ma_∞ as a curve parameter, since the effects of the wire overheat are small (Baldwin, Sandborn & Laurence 1960, Dewey 1965). For physical clarity, the Knudsen

number based on the wire diameter may be used: $Kn \equiv \lambda/d = (\pi\gamma/2)^{1/2}(Ma/Re)$, with λ the molecular mean free path and γ the ratio of specific heats (Springer 1971, Corrsin 1961). For example, slip flow occurs when $0.1 \gtrsim Kn \gtrsim 0.01$. Correlation of the Nusselt-number data is then expressed in terms of the product of two analytical functions that approach the free-molecule flow and the continuum flow, respectively (Dewey 1965, Vrebalovich 1967). When the Knudsen number increases, the Nu_0 curves show a shift to a first-power dependence on Re_0 instead of $Re_0^{1/2}$ as in King's law.

For supersonic flows, the results are simpler in the sense that they become independent of the Mach number. Here, as suggested, apparently for the first time, by Kovasznay (1950), we have

$$Nu_0 = (A\,Re_0^{1/2} - B)(1 - Ca), \tag{2.6}$$

with $A = 0.580$, $B = 0.795$, $C = 0.18$ (for air). The range usually tested is $1.2 \lesssim Ma_\infty \lesssim 5$. The rationalization of the result comes from the fact that the Mach number behind the normal shock created by the wire does not vary greatly with the free-stream conditions. However, the shock is close to the wire and its position can be slightly dependent on the overheat ratio of the wire, which could explain the presence of a in (2.6). More recent experimental work confirms the above Mach-number independence, although a scatter of about $\pm 10\%$ appears in the data (Laufer & McClellan 1956, Dewey 1965). Extension to hypersonic flows has also been attempted (Dewey 1961, Doughman 1972, Laderman & Demetriades 1973).

Plausible theoretical approaches to the heat-loss rate mainly follow an Oseen approximation in the case of an incompressible fluid (Cole & Roshko 1954, Wood 1968, Hieber & Gebhart 1968). Fair results are indeed obtained in the limit $Re \to 0$ (Collis & Williams 1959). The effect of buoyancy has been included by Wood (1972). Approaches using an inviscid potential flow have no logical basis and should be avoided. At the other extreme (free-molecule flow), kinetic theory leads to an explicit expression of the Nusselt number (Stalder, Goodwin & Creager 1952, Springer 1971).

For practical anemometry, expressions in which physical variables appear are preferred. For example, expression (2.3) can be transformed into

$$RI^2/(R - R_a) = A(1 + 0.49a) + B(1 + 0.12a)U^{0.45}, \tag{2.7}$$

where A and B depend on the physical properties of the fluid and on the wire characteristics at temperature Θ_a (Corrsin 1963). It is worth noting that (2.7) is similar to the relation originally given by King (1914). At small overheat, (2.7) leads to

$$RI^2/(R - R_a) = A + BU^{0.45}, \tag{2.8}$$

which is sometimes referred to, as an oversimplification, as King's law.

2.3   Dynamic Response of the Wires

A prerequisite to this section concerns the relative order of magnitude of the various time scales relevant to the unsteadiness of the flow around the wire. These are as follows (Corrsin 1963): $\tau_1 \simeq d/\bar{U}$, the time for the flow to pass the wire;

$\tau_2 \simeq d^2/v$ and $\tau_3 \simeq d^2/\mathscr{D}$, the times for a shear disturbance and a thermal disturbance respectively to diffuse through the viscous layer around the wire (v is the kinematic viscosity of the fluid and $\mathscr{D}$ its thermal diffusivity); $\tau_4 \simeq d^2/\mathscr{D}'$, the time of the radial conduction in the wire ($\mathscr{D}'$ is the thermal diffusivity of the wire material); and $\tau \simeq \bar{U}^{-1}(v^3/\bar{\varepsilon})^{1/4}$, the smallest time scale occurring in a turbulent velocity field (the Kolmogorov time scale, where $\bar{\varepsilon}$ is the mean viscous-dissipation rate). For metallic wires in air, one usually has

$$\underset{[2.5\times10^{-7}\,s]}{\tau_4} \quad\simeq\quad \underset{[2.5\times10^{-7}\,s]}{\tau_1} \quad\simeq\quad \underset{[2.8\times10^{-7}\,s]}{\tau_3} \quad\simeq\quad \underset{[4\times10^{-7}\,s]}{\tau_2} \quad\lll\quad \underset{[1\times10^{-5}\,s]}{\tau}. \tag{2.9}$$

The orders of magnitude in a typical laboratory situation of "incompressible" turbulence at $\bar{U} = 10$ m s^{-1} and for a platinum wire of 2.5 μm are given between brackets. The situation in supersonic flows seems to have been considered only by Kovasznay (1950). Additional investigation concerning the change undergone by the turbulence in passing through the shock (a sort of rapid distortion), together with the eventual distortions of the shock itself by the turbulence, would be useful.

Because of (2.9), the instantaneous heat-loss term in the heat balance of the wire

$$\underset{\substack{\text{(rate of change} \\ \text{of energy stored} \\ \text{in the wire)}}}{mc\,\frac{d\Theta}{dt}} \quad=\quad \underset{\substack{\text{(heat-input} \\ \text{rate)}}}{RI^2} \quad-\quad \underset{\substack{\text{(heat-loss} \\ \text{rate to} \\ \text{the flow)}}}{\Phi} \tag{2.10}$$

can be approximated by the steady-state loss rate at the same velocity. This is usually called the "quasi-steady approximation." [In (2.10) mc is the thermal capacity of the wire, R its resistance, and I the current intensity.] The lag exhibited by the wire when following the fluctuation of the incident flow stems mainly from the fact that the viscous flow around the wire has to pass the information on to the wire.

The differences between the responses given by a real wire (with thermal lag) and an "ideal" wire (without thermal lag and hence in equilibrium with the flow at all times, i.e. $d\Theta/dt = 0$) must be examined. An analytical approach requires tractable heat-loss laws. Ziegler (1934) and Corrsin (1963) have chosen the case of forced convection with small overheat ratio (2.8). The resistance R of the real wire is then given in terms of the resistance R^* of the ideal wire by

$$\frac{M}{\bar{R^*} - R_a}\,\frac{d(R - R_a)}{dt} + \frac{R - R_a}{R^* - R_a} = 1, \tag{2.11}[1]$$

where R_a is the wire resistance at the ambient temperature Θ_a. This is a first-order linear differential equation, one coefficient of which, $R^* - R_a$, is time dependent. For small velocity fluctuations, (2.11) reduces to the following first-order linear differential equation with constant coefficients, as found by Dryden & Kuethe (1929):

$$M\,\frac{dr}{dt} + r = r^*, \tag{2.12}$$

[1] Overbars denote time averages.

where r^* is the resistance fluctuation of the ideal wire and r that of the real wire. M is the so-called time constant of the wire, which depends on the characteristics of the wire (diameter, material, etc) and on the operating conditions (velocity, overheat, etc). Approximately, we have $M \sim d^2$, $M \sim \bar{U}^{-1/2}$, and $M \sim (\bar{R} - R_a)/R_a$. As an order of magnitude estimate we may take for a platinum wire $M \simeq 0.1$ ms when $d \simeq 2.5 \mu$m, $\bar{U} \simeq 10$ m sec^{-1}, and $(\bar{R} - R_a)/R_a \simeq 0.5$.

On the other hand, in the case of large fluctuations, the solution of (2.11) contains higher harmonics because $R^* - R_a$ is time dependent (parametric excitation). These harmonics have been shown to exist by Corrsin (1963) for a sinusoidal input (Figure 2) and were later measured for a random input by resolving (2.11) with an analog computer (Comte-Bellot & Schon 1969). Additional information is given in Sections 3.1 and 3.2. It should further be emphasized that R^* is itself a nonlinear function of velocity [equation (2.8)]. This "static" nonlinearity, at variance with the above "dynamic" nonlinearity, has been known for a long time (cf Hinze 1959). Various cases depending on the associated electronics have later been analyzed. The tools used are simply Taylor's-series expansions up to the third order around the operating point. Numerical estimates are limited, however, to Gaussian isotropic turbulence.

Other situations would seem to point to a "dynamic" nonlinearity, but apparently no information is as yet available. For example, in temperature measurements with nearly cold hot wires acting as thermometers (Section 2.4), the inequality $r \ll \bar{R} - R_a$ or $\theta \ll \bar{\Theta} - \Theta_a$ cannot be realized. Indeed, if $d = 2 \mu$m, $I \simeq 0.3$ mA, the mean temperature difference is estimated as $\bar{\Theta} - \Theta_a \simeq 0.02°$C, whereas the amplitude of the temperature fluctuations $\theta(t)$ can reach several degrees centigrade.

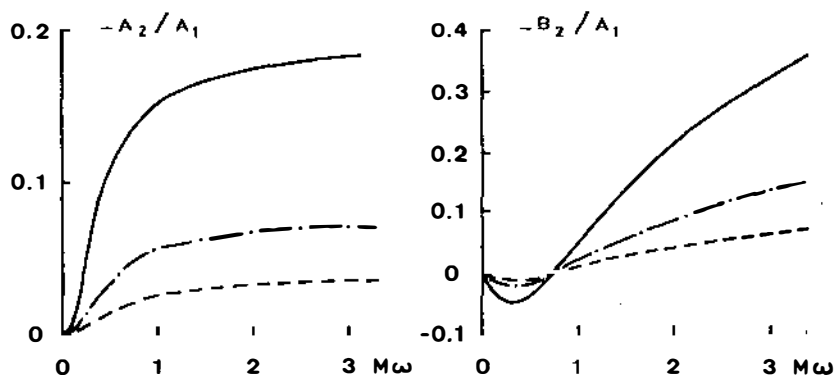Finally, in the case of hot films the material backing the film also contributes

Figure 2   Relative amplitude of second harmonic introduced by nonlinear "dynamic" response of hot wire. Input function for equation (2.11) is $R^* - R_a = \bar{R}^* - R_a + \hat{r}^* \sin \omega t$. Output response is $R - R_a = \bar{R} - R_a + \sum_{n=1,2} (A_n \cos n\omega t + B_n \sin n\omega t)$ with $\hat{r}^*/(\bar{R}^* - R_a) = 0.10$ (- - - -), 0.20 (— · — · —), and 0.40 (———). $\hat{r}^*/(\bar{R}^* - R_a) = 0.10$ corresponds to $u/\bar{U} \simeq 0.25$ (Corrsin 1963).

to the thermal response. This point was not perceived in the early measurements. It was merely observed experimentally that intensities of turbulence measured in water were correct, while those measured in air were too low by a factor of about two (Bankoff & Rosler 1962). The reason was later found by Bellhouse & Schultz (1967), and the error was estimated for a one-dimensional model. The results show clearly the importance of thermal exchange with the glass substrate whenever the external fluid has a low conductivity. The shape of the probe is a relevant parameter of the problem.

2.4   Sensitivity Coefficients of the Wires

These coefficients are defined as the (local) values of the first partial derivatives of the heat-transfer law with respect to the different flow variables (velocity components, density, temperature, concentration, etc). Sometimes, as for compressible flows, the mass flux ρU is worth considering because it appears directly in the equations governing the flow and in the response of the hot wire (Kovasznay 1950). For the important area of "incompressible" turbulence, the dependence of these coefficients on the wire characteristics and on the operating conditions has been analyzed (Comte-Bellot 1961). Concerning the relative orders of magnitude of the sensitivity coefficient α for the normal-velocity fluctuations and the sensitivity coefficient β for the temperature fluctuations, Corrsin (1947) has shown that $\beta \gg \alpha$ at very small overheat ratios, so that the wire can work as a resistance thermometer. On the other hand, high overheat ratios do not allow the wire to become only velocity sensitive.

In practical anemometry, the sensitivity coefficients are found by direct calibrations of the wire placed in its electronic circuitry. They then become the "sensitivity coefficients of the anemometer." The calibration procedure (Section 4.1) is therefore simple in the sense that the exact characteristics of the wire and the corrections for taking into account its finite length do not have to be known.

2.5   Wire of Finite Length

Measurements of local variables require the use of short wires, for which the end supports (prongs or larger wire) are at the ambient temperature. Heat is therefore conducted along the wire, and the corresponding term has to be taken into account in the thermal balance of the wire. Most of the available theoretical deductions are given in Betchov (1948, 1949) and Corrsin (1963): 1. temperature distribution along the wire for uniform incident velocity normal or inclined to the wire, and also for nonuniform incident velocity; 2. heat loss to the support (typically 7-10%); 3. deviation from the "cosine law," or cooling by the velocity normal to the wire; and 4. modification of the time constant of the wire, which depends, even for small perturbations, on either a change in the incident velocity or in the heating current of the wire.

Experimental data are also available. Champagne, Sleicher & Wehrmann (1967) have investigated the temperature distribution along the wire by means of an infrared detector. This technique was later used by Gessner & Moller (1971) to study the case of a constant-mean-shear flow. It appeared that the sensitivity

coefficient of the wire was reduced with respect to the value corresponding to a uniform flow. The resulting error in $\overline{U}$ and u' are respectively -4.3 and -5.8% for $2l/d \simeq 400$, $(\overline{R} - R_a)/R_a = 0.8$, $\overline{U} \simeq 50$ m sec^{-1}, and $(\Delta\overline{U}/\overline{U}_c)(d/2l) \simeq 2.10^{-3}$ (where $\overline{U}_c$ is the mean velocity at the wire center and $\Delta\overline{U}$ the velocity difference between its two extremities). For the departure from the cosine law, empirical expressions are given in the form (Hinze 1959, and also Champagne, Sleicher & Wehrmann 1967)

$$U_{\text{eff}} = U[\cos^2 \varphi + b^2 \sin^2 \varphi]^{1/2}. \tag{2.13}$$

Of course, b increases when $2l/d$ decreases, with $b = 0.20$ approximately at $2l/d = 200$. For most present hot wires, the error will not therefore exceed a few percent. The situation seems worse for hot films, for which $2l/d$ does not exceed 20 (Friehe & Schwarz 1968). More attention should certainly be paid to this point by experimenters using hot films.

2.6 Spatial Resolution of Hot Wires

When the wire length is not negligible compared with the characteristic length of interest in turbulence, an imperfect spatial resolution occurs because the wire "smears out" the Fourier components whose wave number along the wire direction is greater than $2\pi/2l$. This problem has been thoroughly investigated by Uberoi & Kovasznay (1953) for the case of negligible conduction along the wire and isotropic turbulence. Analytical expressions for the errors, especially for the one-dimensional velocity spectra, are obtained. For a given wire length, the numerical values of the errors depend on the way the three-dimensional velocity spectrum $E(k)$ varies with the modulus k of the wave number $\mathbf{k}$. For high Reynolds numbers the form $E = \frac{3}{2}\bar{\varepsilon}^{2/3} k^{-5/3} \exp\left[-\frac{9}{4}(k\eta)^{4/3}\right]$ can be used (Pao 1965, Wyngaard & Pao 1972). The errors ε_{E_1} and ε_{Ω_1} in the one-dimensional velocity spectra $E_1(k_1)$ and the one-dimensional vorticity spectra $\Omega_1(k_1)$ are then as follows:

$\eta/2l$	$k_1\eta$	ε_{E_1}	ε_{Ω_1}
1	0.01	$\simeq 0\%$	1.4%
	0.5	4.3%	5.0%
0.2	0.01	0.6%	8.4%
	0.5	20.5%	24.5%

(The quantity k_1 is the component of $\mathbf{k}$ along the flow direction, which is normal to the wire.) The error ε_{Ω_1} is large even for small $k_1\eta$ because the vorticity involves the first spatial derivative of the velocity field so that its spectra are proportional to $k^2 E$ in isotropic turbulence. More weight is then given to the attenuated high wave numbers. One should be cautious of this error when investigating the spectrum of higher moments of the velocity field.

The effect of conduction along the wire has been investigated by Corrsin (1963). The main deduction is that the heat-conduction term precludes the customary separate sequential correction for space and time resolution. The above analyses concern the small-perturbation linear response. Even with this limitation, they have

been useful for other configurations such as "X" wires or vorticity meters (Section 5.2) and pressure transducers (Corcos 1963).

2.7 *Aerodynamic Perturbations of the Probes*

Because hot-wire anemometers have to compete with laser velocimeters (which are advantageously located outside the flows), it is important to know the aerodynamic perturbations that are generated by the probes holding the wires. Although quite obvious in high-speed flows, this problem has been ignored for a long time at low speeds. Hoole & Calvert (1967) were apparently the first to observe the systematic error occurring in mean-velocity measurements, depending on whether the stem of the probe was aligned with or normal to the flow (up to 20% according to the probe used). Gilmore (1967) and Dahm & Rasmussen (1969) undertook more detailed investigations. The former used a model 10-times larger than the probe, with removable prongs, while the latter used prongs of varying diameter, length, and spacing on given stems. However, the same probe acted at the same time as the perturbing probe and the measuring probe. Later, Comte-Bellot, Strohl & Alcaraz (1971) and Strohl & Comte-Bellot (1973) investigated the perturbations by means of an auxiliary probe used as a reference. This probe itself introduced negligible aerodynamic disturbance [as checked by the global-rotation test suggested by Hoole & Calvert (1967)] and could detect the perturbations created by a variety of stems and prongs, or by complete probe units that were advanced toward it from various directions. Whenever possible, predictions were also made on the basis of a nonviscous irrotational flow around the probes. For example, for "X"-wire probes the errors in $\overline{v^2}$ and $\overline{uv}$ have their origins in the perturbation of these statistical quantities and in the change of the u- and v-sensitivity coefficients of the wires. Systematically, low values of $\overline{uv}$ are obtained. Values too low by about 16% result when two pairs of prongs (diameter tapering from 0.4 to 0.15 mm over 8 mm) are mounted on a stem (diameter tapering from 7 to 3 mm over 26 mm) with a 0.6-mm space between the two pairs of prongs and with a 1-mm space between the prongs of a single pair.

However, the case of hot films (either cylinders, wedges, or cones) does not seem to have retained attention, despite the obvious large size of the sensor itself. An investigation would be of considerable interest. Even the cases in which probes are mounted on airplanes for research in atmospheric turbulence, or on towed bodies for research in oceanic turbulence, should be considered since the support members can be of considerable size.

3 ELECTRONIC CIRCUITRY

3.1 *Constant-Current Anemometer*

In this setup (Figure 3), the hot wire is heated by an electrical current of quasi-constant intensity and the wire resistance, or the wire voltage, is the electrical variable used to detect the changes occurring in the flow variables.

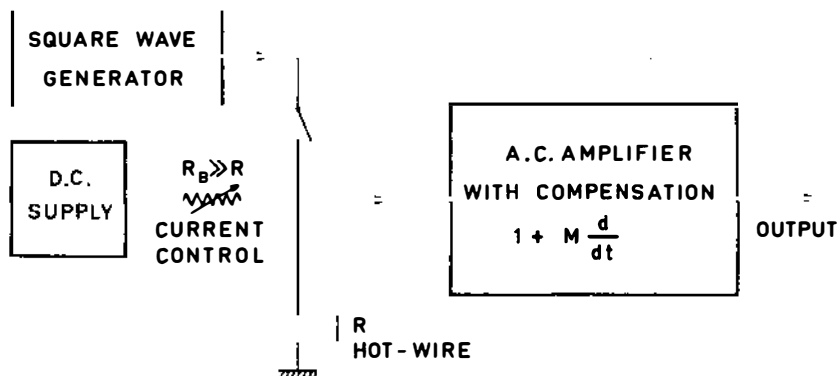Constant-current anemometers are normally based on the small-perturbation

Figure 3   Constant-current anemometer.

linear theory of the hot-wire response [equation (2.12)]. Therefore, $r^*(t)$ has to be deduced from $r(t)$, so that the compensating amplifier must include a differentiator working in a large frequency range with high gain and low noise. Interest in supersonic flows has led to significant improvements in circuitry (Kovasznay 1954b). The frequency response of actual constant-current anemometers extends to about $Mw \simeq 200$, i.e. to about 300 kHz for $M = 0.1$ ms (Demetriades 1970). The noise is also reduced down to an equivalent turbulence level of about 0.05%. Adjustment of the compensation is done by means of a square-wave signal added to the heating current of the wire. The adjustment is correct when the square-wave signal is recovered at the output of the amplifier. The gain of the amplifier is of course measured with the compensating circuit off.

In the case of large fluctuations (in practice for turbulence levels larger than 5%), the use of constant-current anemometers leads to errors, because of the "static" and "dynamic" nonlinearities occurring in the wire response (Section 2.3). The higher harmonics generated are of course not suppressed, but amplified, by the compensating circuit. Comte-Bellot & Schon (1969) investigated the effect of the "dynamic" nonlinearity on the skewness factors of the velocity fluctuations and on the spectra by simulating the compensating circuit on the analog computer used to solve (2.11). For the skewness factors, the error is about half as large as the error due to the "static" nonlinearity and is opposite in sign. For sake of simplicity, these errors have been separated, but in fact they act together. The determination of R^* from R through (2.11) has apparently not yet been attempted.

3.2   Constant-Temperature Anemometer

In this setup the hot wire is included in a Wheatstone bridge (Figure 4), and the unbalance voltage is amplified and fed back to the bridge to suppress the resistance (and temperature) changes of the wire. This feedback signal is then able to represent the changes occurring in the flow variables. In principle, the global response of the anemometer would no longer be differential but simply algebraic [(2.10) in which

$d\Theta/dt = 0$]. The problem caused by the thermal time lag of the wire would then be suppressed. Furthermore, since no assumption is made concerning the order of magnitude of the perturbation, this anemometer would cope with large fluctuations, and a "linearizer" could be set at the output in order to obtain linear calibration curves (usually, the bridge mean voltage versus the mean-velocity component normal to the wire). In turbulence this velocity component includes, as a second-order term, the transverse velocity fluctuation. This is a "static" nonlinearity according to the terminology used in Sections 2.3 and 3.1. This impossibility of perfectly linearizing the constant-temperature anemometer was first observed by Rose (1962). The construction of electronic linearizers does not raise serious problems. Biased diodes are often used as suggested by Betchov (1954), who measured skewness factors. Comte-Bellot (1965) also employed biased diodes and, in order to avoid tedious adjustments, built a set of 11 nonlinear resistances. Ten exponents ranging from 2.2 to 3.3 were then available by simple switching. Wire diameters between 3 and 6 μm and mean velocities from 3 to 100 m sec^{-1} could be used. Transistor chains are, of course, presently available (Kovasznay & Chevray 1969).

In practice, the feedback efficiency (i.e. its stability and frequency response) is the determining factor, since $d\Theta/dt$ can only be maintained approximately at zero. Early experimenters were very conscious of the problem, and almost all of them attempted to develop their own solutions, a tendency that unfortunately has disappeared with the availability of ready-to-use anemometers. One can refer to Weske (1943), Ossofsky (1948), Hubbard (1957), Janssen, Ensing & Van Erp (1959), Kovasznay, Miller & Vasudeva (1963), Davies, Tanner & Day (1966), Andersen (1966), Freymuth (1967), Wyngaard & Lumley (1967), Davis (1970), Sheih, Tennekes & Lumley (1970), Perry & Morrison (1971), and Wood (1975). In all of these investigations, however, the perturbations are apparently assumed to be small. This fact was pointed out by Freymuth (1967, 1969), in particular, who analyzed
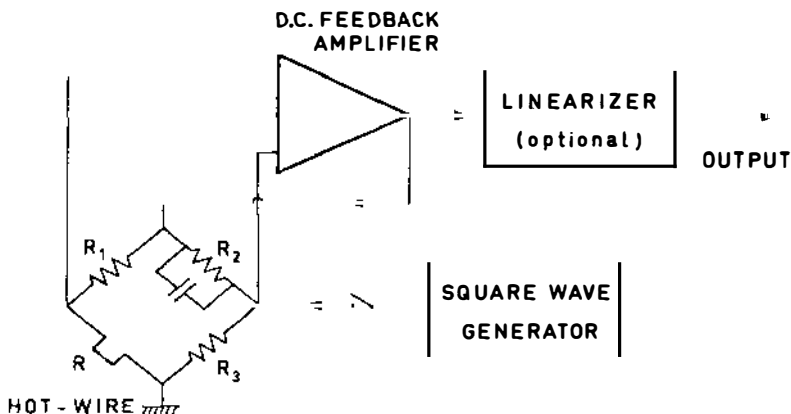

Figure 4   Constant-temperature anemometer.

the general differential equation that governs the loop output and then took the small-perturbation linear approximation. The general equation is obtained from the relation governing the three components of the feedback, i.e. the hot-wire that follows King's law, the Wheatstone bridge that includes an adjustable capacity, and the amplifier that is represented by a second-order linear differential equation. In the linear approximation, the optimal aperiodic case of the equation is then determined. For the adjustment, a square wave can be fed into the bridge (Figure 4), and the pulse corresponding to the response of the system to the front step must be of as short duration as possible, without a marked overshoot. The frequency response usually attained is about 100 kHz for $d = 3$ μm, $\bar{U} = 30$ m sec^{-1}, and $(\bar{R} - R_a)/R_a = 0.8$ (Davis 1970, Sheih, Tennekes & Lumley 1970). Direct checks using flow perturbations are worth making, however.

Regarding the wire overheat, it is of interest to observe that the feedback efficiency is diminished when the overheat decreases. This appeared clearly in Hubbard (1957), although the amplifier was simply assumed to have a gain G independent of frequency. The time constant of the feedback loop is then given by

$$M' = M \left[1 + 2G \frac{\bar{R} - R_a}{R_a} \frac{\bar{R}}{\bar{R} + R_1} \frac{R_2}{R_2 + R_3} \right]^{-1}, \tag{3.1}$$

where M is the time constant of the wire, $\bar{R}$ the mean wire resistance, and R_1, R_2, R_3 the other resistances of the Wheatstone bridge. Hence M' approaches M when $(\bar{R} - R_a)/R_a \to 0$. For real amplifiers, careful checks of the frequency response are therefore a prerequisite to the measurements as soon as low overheats have to be used.
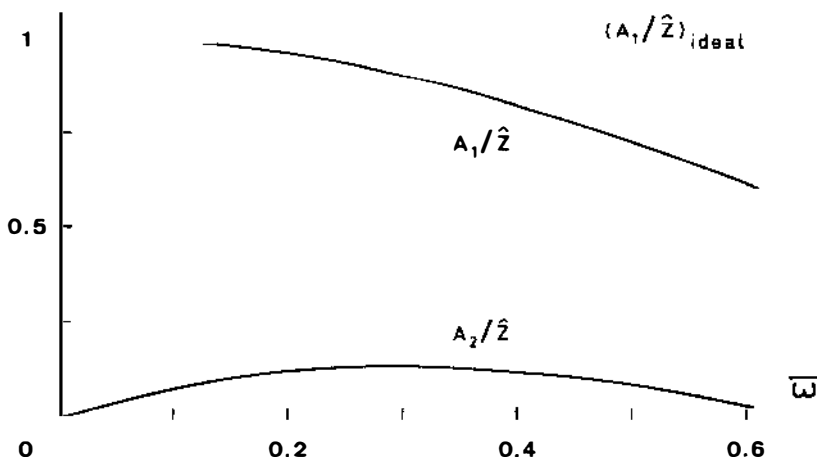

Figure 5   Fundamental $A_1/\hat{z}$ and second harmonic $A_2/\hat{z}$ introduced by nonlinear "dynamic" response of constant-temperature anemometer for a dimensionless velocity function $z = \hat{z} \sin \bar{\omega} x$ (x, dimensionless time; $\bar{\omega}$, dimensionless pulsation; $\hat{z} \simeq 0.40$ corresponds to $u/\bar{U} \simeq 0.64$) (Freymuth 1969).

The nonlinear case seems to have been studied only by Freymuth (1969). The general differential equation governing the loop is a nonlinear third-order equation with variable coefficients. It is solved by iteration, and Figure 5 shows the second harmonic A_2 generated in the case of a sinusoidal change in the flow velocity. As a consequence, the upper useful frequency has to be increased in order to reduce the relative importance of A_2. If a maximum amplitude of 2% is tolerated for $A_2/\hat{z}$, this will correspond to velocity fluctuations up to 12%. Furthermore, the square-wave test cannot be used as a check for the large-amplitude case, because the differential equations for the velocity signal and for the test pulse differ from each other if their nonlinearities are considered. Additional information concerning, for example, the skewness factors or the spectra would be very useful in the context of the nonlinear approach. Unexplained differences still exist in the measurements of skewness factors obtained by constant-temperature anemometers and by constant-current anemometers, even at low turbulence levels (Van Atta & Chen 1968). High- or low-frequency filtering seems to be without effect (Stegen 1969, Helland & Stegen 1970). Sophisticated digital processing would give more information on these problems.

4   SIGNAL PROCESSING

4.1   Calibration of the Anemometers

Calibration is the procedure used to determine the sensitivity coefficients of the wires (associated with their electrical setups) with respect to the different flow variables. The variables are those quantities to which the wires will directly respond when submitted to an unsteady situation. Thus, for a given fluid and for a wire normal to the flow, the variables are the velocity and the ambient temperature if the fluid is incompressible, or the velocity, the density, and the stagnation temperature if the fluid is compressible. The flow variables are measured by conventional equipment (Pitot tube, thermocouple, etc). For a wire set obliquely to the flow, the angular sensitivity is found by simulating a small additional transverse velocity by a rotation of the wire around the normal to the wire and the longitudinal flow direction ($d\phi = v/\overline{U}$, Figure 1). This is a geometric and absolute calibration that can be done at supersonic speeds as well as at low speeds. [However, care should be taken when the individual wires are along Mach waves (Kovasznay 1950).] Although simple, this type of measurement appears never to have been attempted. Reynolds-stress measurements are also infrequently reported (Demetriades & Laderman 1972, Sandborn 1974).

These direct calibrations avoid having to specify the exact characteristics of the wire (length, diameter, resistivity, etc) and also allow for some of the deficiencies of the setups (intensity change in the constant-current anemometer due to the finite value of the ballast resistance, deviation from the "cosine law" due to the finite length of the wire, etc).

The calibration curves are either obtained in a completely empirical way or partly determined from the known heat-transfer laws. In the former case they form nets with running parameters. In the latter case functional forms are used. For

example, for forced convection in isothermal fluids at low speeds, plots of $RI^2/R - R_a$ vs $u^{0.50}$ follow from (2.8), and the calibration consists of determining the values of A and B. Regarding the effect of the ambient temperature, simplifications have also been looked for (Comte-Bellot & Mathieu 1958). In supersonic flows, since the Nusselt number is not appreciably dependent on the Mach number, the sensitivity coefficients of the wire to velocity, density, and mass flux are the same, so that the wire can be directly calibrated with respect to the mass flux and the stagnation temperature (Morkovin 1956).

The calibrations are advantageously made in low levels of turbulence, which can be obtained in the same facility (potential core of jets, external flow of boundary layer, etc) or in auxiliary facilities. Highly turbulent flows have to be avoided because of the integration time needed to obtain a true mean value and because of the nonlinear errors affecting the measurements; in turn, these errors considerably affect the partial derivatives of the curves. Because of a lack of compensation between the components of the velocity fluctuations, the errors in the mean values are larger for a constant-temperature anemometer with linearizer than for a constant-temperature anemometer without linearizer or for a constant-current anemometer ($+2\%$, $+0.7\%$, and -1%, respectively, for an isotropic turbulence level of 20%).

At very low speeds ($\lesssim 2$ m sec^{-1}), Pitot tubes are not sensitive enough so that absolute direct measurements of the velocity are needed. These can be achieved by various means: 1. particle- (or hydrogen-bubble-) tracking techniques; 2. phase measurements in the wake of a sinusoidally heated wire (Walker & Westenberg 1956); 3. measurement of the shedding frequency of cylinders; 4. use of whirling arms (King 1914, Richardson 1934, Tsubouchi & Sato 1961); 5. immersion of wires into a rotating tank (Delleur, Toebes & Tin 1966, James & Acosta 1970); and 6. use of towing facilities (Deardorff & Willis 1967, Fabula 1968, Dring & Gebhart 1969, Baille 1971). With methods 4 and 5 great care must be taken concerning the perturbations that can be builtup in the container by the successive passages of the wire holder.

A method of dynamic calibration has also been suggested to obtain the sensitivity coefficients of the wires directly. The hot wires are made to oscillate in the flow, either longitudinally or transversely (Thrasher & Schaetzle 1970, Baille 1971, Morrison, Perry & Samuel 1972, Kirchhoff & Safarik 1974). The main interest is probably the realization of large velocity fluctuations whatever the magnitude of the external (subsonic) flow. However, attention has to be paid to several features: the law of motion imposed on the wire (usually a simple harmonic displacement), the extraneous vibrations despite the usual low-frequency limit ($\simeq 50$ Hz), the aerodynamic perturbations induced by the probe motion, etc.

4.2 Separation of the Fluctuations

The separation of the fluctuations is often done within a linear theory (small perturbations), for which a unique solution is readily obtained. A primary example is the separation of the longitudinal $u(t)$ and transversal $v(t)$ velocity components in an isothermal fluid by means of an "X" wire. Each wire gives a signal in which

$u(t)$ and $v(t)$ enter linearly so that an algebraic combination of the two signals can provide the two unknowns. In the early measurements, the wires were assumed identical and symmetrically placed with respect to the mean flow direction (Schubauer & Klebanoff 1946). In later work, these constraints were no longer considered to be necessary (Comte-Bellot 1960). Another example is the determination of $u(t)$ and $\theta_a(t)$ (fluctuation of the ambient temperature) by means of two single wires set side by side and normal to the flow (Townsend 1951). One wire is normally heated and responds to $u(t)$ and $\theta_a(t)$. The other is practically unheated so that it acts as a thermometer (Section 2.4). The latter then provides $\theta_a(t)$, and after substitution of this into the first signal, $u(t)$ can be obtained. This technique can be extended to a heated "X" wire and a single cold wire to obtain $u(t), v(t)$, and $\theta_a(t)$ (Johnson 1955). Operational amplifiers have been used extensively for this signal processing, and recent examples of circuitry are given by Schon & Baille (1972), Chevray & Tutu (1972), and Ali (1975). The signal contamination has still to be analyzed for high-order turbulent characteristics (Bremhorst & Bullock 1970, Mimaud-Lacoste 1972).

Prior to the availability of operational amplifiers, the mean-square values of the various signals were formed in order to get the mean-square value of the fluctuations, but at the same time the cross-correlation coefficients occur as additional variables. As an example, the determination of $\overline{u^2}$ and θ_a^2 requires the simultaneous determination of $\overline{u\theta_a}$. However, only one wire is needed if it is successively operated at three different overheats. The ways in which the u and θ_a sensitivities of the wire depend on the overheat are different enough to allow independent linear algebraic equations to be solved for $\overline{u^2}$, $\overline{\theta_a^2}$, and $\overline{u\theta_a}$ (Corrsin 1947, Mills, Kistler, O'Brien & Corrsin 1958). In fact, this technique is still in use because of the help of computational techniques. Even, more equations than unknowns are formed in order to obtain the best fit to the solution. Repeated at every frequency, this procedure allows the spectra to be obtained (Fulachier & Dumas 1971). Although relatively tedious, this is quite a useful procedure for supersonic flows when it is almost impossible to introduce more than one probe into the flow. The hot wire then gives information on $\Delta\rho U$ (fluctuation of the mass flux) and on $\Delta\Theta_0$ (fluctuation of the stagnation temperature), i.e. on two parameters, although the flow is characterized by three variables (longitudinal velocity, temperature, and density) if the transverse velocity components are not considered. A simplifying assumption is therefore needed. It is most easily expressed in terms of the three "modes" into which random fluctuations can be uniquely decomposed: vorticity, sound, and entropy (Kovasznay 1950, 1953; Morkovin 1956). In boundary layers or wakes, the sound mode is usually neglected (Kovasznay 1950, Kistler 1959, Demetriades 1970), but this seems possible only up to Mach numbers of about 5 (Kistler & Chen 1963, Gaviglio 1971, Laderman & Demetriades 1973). In these measurements, the flexibility of constant-current anemometers with respect to the overheat ratio of the wire is of great value.

In the case of gas mixtures, the use of wires of different diameter has been suggested by Corrsin (1947), so that independent equations for the velocity and concentration fluctuations can be obtained. This procedure has been followed

mainly by McQuaid & Wright (1973), who also adjust the heating current through the wires.

The assumption of small fluctuations is a constraint in some cases, and efforts should be made to avoid it. For this purpose, digital techniques appear to be a powerful tool. Tracking the instantaneous values of large fluctuations seems possible as soon as a fine-mesh net of calibration data is available from measurements and interpolation procedures. When processing an experimental run to obtain the velocity field, all possible solutions must be determined for the initial data sample given by the wires. Selection of the unique correct solution can be made later with the aid of the subsequent data. Such tracking has been suggested to attain the three velocity components in an isothermal flow (Coles & Van Atta 1966, Cheesewright 1972). Considerable attention should be paid to these techniques, which extend the range of applicability of hot wires.

5   SPECIAL TOPICS

5.1   Measurements Close to a Wall

Because of its small size, the hot wire seems a useful tool to investigate turbulence in the vicinity of walls. However, it is subject to an additional cooling from the wall (Richardson 1934, Cox 1957, Piercy, Richardson & Winny 1956, Wills 1962, Alcaraz & Mathieu 1975). The mean-velocity profile is affected up to about $yu^*/\nu \simeq 5$, where y is the distance from the wall and u^* is the friction velocity. In fact, the apparently linear part does not extrapolate towards the origin, and its slope is systematically smaller than that obtained in direct measurements of the wall shear stress. Strong reduction of the wire diameter ($\simeq 0.5$ μm) and hence of the wire overheat (heating current $\simeq 3$ mA) seems a palliative (Bogar & Willmarth 1974).

5.2   Unusual Wire Arrangements

An array of three parallel hot wires (one upstream and two downstream, set symmetrically with respect to the first wire) has been proposed to measure the lateral-velocity components (Reichardt 1938, Rey 1973). The downstream wires are unheated and sense the lateral flapping of the thermal (laminar) wake of the first wire. Because of its small overall width ($\simeq 70$ μm), this probe may be useful in strong mean-velocity gradients. Similar techniques, but with only one cold wire set downstream, have also been suggested (Walker & Bullock 1972). For concentration and velocity fluctuations, Stanford & Libby (1974) have used a special three-sensor probe with a film normal to the flow, a wire in the thermal (upstream) field of this film, and an additional swept film. Sophisticated setups, such as four single wires in line or in a triangular array, can provide information on the coincidence between certain turbulent events (Betchov 1974). Rakes of hot wires have been used to study the free edges of turbulent flows (Sunyach 1971, Paizis & Schwarz 1974).

A few measurements of the longitudinal-vorticity component were attained by a tetrahedral array of four wires, all with the same length and the same angular orientation with respect to the mean flow (Kistler 1952, Kovasznay 1954a). Later,

Wyngaard (1969) investigated the spatial resolution of this array. He found that for accurate measurements, the array size should be of the order of the Kolmogorov microscale of the turbulent field, which is a severe restriction. However, new attempts will be of interest because of the importance of the vorticity dynamics in all turbulent flows.

5.3   Case of Non-Newtonian Fluids

Hot films exhibit various kinds of anomalous behavior when used in solutions of high polymers: strong decrease of the heat-transfer rate in comparison with that of the solvent; dependence of the heat transfer on the mean strain rate as indicated by calibrations done at different distances from a wall; spectra dependent on the probe, cylinder, or cone that is used (Smith, Merrill, Mickley & Virk 1967); and, above all, the fact that a wire set normal to the flow is less cooled than a wire set obliquely (Friehe & Schwarz 1969). The explanation given by Lumley (1973) is based on the expansion of polymer molecules in pure strain (stagnation regions, cylindrical film set normal to the flow) so that the viscosity increases, the boundary layer on the film becomes thicker, and the heat transfer is reduced. When the film is oblique, the flow acquires vorticity so that the molecules rotate past the principal axes of the deformation too fast and cannot expand significantly. In non-Newtonian fluids, the laser velocimetry is a very suitable technique.

5.4   Measurements in Liquid Metals

Measurements in liquid metals, especially in mercury, apparently started with Sajben (1965), who used a thick (38 μm) enamel-insulated tungsten wire. At present, cylindrical hot films are rugged enough for the purpose. The particular problems encountered are:

1. The mercury has to wet the sensor. This is facilitated by vapor depositing of copper or gold on the sensor's quartz insulation.

2. Empirical laws fitting the low Prandtl number of the fluid (Pr $\simeq$ 0.025 for mercury at 20°C) and the low Peclet numbers that are encountered (typically $0.01 \lesssim \text{Pe} \lesssim 1$) have to be looked for. Natural convection might also be present when Pe $\simeq$ 0.01. When Pe $\gtrsim$ 0.1 the heat transfer is expressed in the simple form Nu = Nu (Pe); however, numerical factors are strongly dependent on the experimental conditions. Thermal contact resistances cause drifts that are hardly eliminated by calibration (Sajben 1965, Malcolm 1969, Hill & Sleicher 1969).

3. Because of the low Peclet numbers, the diffusive thermal zone around the sensor is much larger in liquid metals than in other fluids, and even much bigger than the diameter of the sensor itself. Therefore, the time required for a thermal disturbance to diffuse through this zone is greater than $d^2/\mathscr{D}$ (Section 2.3), and the frequency response of the sensor may be notably reduced (Illingworth 1960, also Sleicher & Lim, to be published).

4. The thickness of the diffusive thermal zone also impairs the directional sensitivity of hot-film sensors that are too short. The reason is that this zone would be quite rounded and hence almost insensitive to flow direction (Hill & Sleicher 1971). A better understanding of all these questions would be of value, although

recent measurements of turbulence in mercury pipe flow agree with the familiar results obtained with air or water (Hochreiter & Sesonske 1974).

5.5 *Measurements in Two-Phase Flows*

Hot-wire measurements in water-and-air or water-and-vapor mixtures are based on the large difference of heat-transfer rates between the liquid and the gas. A constant-temperature setup is advantageously used to avoid overheating and damaging the probes. The sensor temperature must usually be low enough ($\simeq 10°C$) to avoid film boiling on the probe. At higher temperatures ($\simeq 50°C$) nucleate boiling may take place on the probe, and this can be tolerated when the only purpose of the measurement is to determine the void fraction (Hsu, Simon & Graham 1963). The response of a probe to a bubble is the main problem encountered. Bubbles smaller than the sensor diameter tend to avoid striking the probe or tend to roll off without breaking upon impact. Bubbles larger than the sensor diameter but smaller than its length seem to behave in an orderly and predictable manner. Finally, bubbles of about the length of he sensor or larger change shape and distort into an unpredictable geometry (Chuang & Goldschmidt 1971). With conical probes, the perturbations seem smaller (Delhaye 1969), but additional information would be useful. The data processing raises problems similar to those encountered in turbulent intermittent signals. Various techniques have been suggested to analyze the peak values and the peak duration (due to the bubble) and the remainder of the signal corresponding to the turbulent liquid flow (Hsu, Simon & Graham 1963, Resch & Leutheusser 1972, Delhaye 1969).

6 CONCLUSION

Hot-wire anemometry is based on a rather simple principle but requires careful insight into the associated physics when accurate results or sophisticated measurements are looked for. Several problems that still seem unsolved have therefore been outlined. In particular, the "dynamic" nonlinearity that is relatively well known for constant-current anemometers does not seem to have been investigated sufficiently in the case of constant-temperature anemometers (Section 3.2). Such an investigation is all the more needed since these anemometers are systematically employed because of the nearly automatic compensation of the thermal time lag of the wires. In the controversy concerning constant-current and constant-temperature anemometers, both types have a role to play, depending on the investigation to be made. For instance, temperature fluctuations and measurements in compressible flows are easily obtained with constant-current anemometers, which possess a high flexibility in regard to the operating temperature of the wire. Finally, hot-wire anemometry and laser velocimetry are complementary techniques. In particular, hot wires provide signals continuous in time and do not require particles to be added to the flow. This is an additional incentive to pursue developments in hot-wire anemometry, which has already made such an important contribution to the knowledge of turbulent flows. In addition, the use of digital techniques improves the accuracy of hot-wire measurements.

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