ELE 535 - Machine Learning and Pattern Recognition

Fall 2018

HOMEWORK 2: Theory

Q1 Let $u \in \mathbb{R}^m$, $v \in \mathbb{R}^n$ and $A \in \mathbb{R}^{m \times n}$. Find the orthogonal projection of A onto $\mathrm{span}(uv^T)$.

Q2 **Norm Invariance under Orthogonal Transformations.** Show that for any $A \in \mathbb{R}^{m \times n}$, $Q \in \mathcal{O}_m, R \in \mathcal{O}_n$, $\|QAR\|_F = \|A\|_F$. Thus the Frobenius norm is invariant under orthogonal transformations. Similarly, show the induced 2-norm of $A \in \mathbb{R}^{m \times n}$ is invariant under orthogonal transformations.

Q3 Let A, B be matrices of appropriate size and $x \in \mathbb{R}^n$. Prove that

(a) $\|Ax\|_2 \leq \|A\|_2 \|x\|_2$;

(b) $\|AB\|_2 \leq \|A\|_2 \, \|B\|_2$.

Q4 For $A, B \in \mathbb{R}^{m \times n}$. Show that $\sigma_1(A + B) \leq \sigma_1(A) + \sigma_1(B)$.

Q5 **The Moore-Penrose pseudo-inverse.** The Moore-Penrose pseudo-inverse of a matrix $A \in \mathbb{R}^{m \times n}$ is the unique matrix $A^+ \in \mathbb{R}^{n \times m}$ satisfying the following four properties:

(a) $A(A^+A) = A$

(b) $(A^+A)A^+ = A^+$

(c) $(A^+A)^T = A^+A$

(d) $(AA^+)^T = AA^+$

Let A have compact SVD $A = U\Sigma V^T$. Show that $A^+ = V\Sigma^{-1}U^T$. Give an interpretation of A^+ in terms of $\mathcal{N}(A)$, $\mathcal{N}(A)^\perp$ and $\mathcal{R}(A)$.