HW5

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1 Problem 1

First prove the base case. When n=1, $1 = 1^2$, the statement holds.

Then assume the statement holds for n=k, namely $\sum_1^k (2n-1) = k^2, k \geq 1$.

When n=k+1, $\sum_1^{k+1}(2n-1) = \sum_1^k (2n-1) + 2k + 1 = k^2 + 2k + 1 = (k+1)^2$

By induction, the statement is TRUE

Q.E.D.

2 Problem 2

First consider the base cases. When n=1, n=F_1; when n=2, n=F_3, when n=3, n=F_4

Assume the statement holds $\forall n \leq k, k \geq 3$

When n=k+1, if there is a fibonacci number F_p that is equal to n, n $= F_p$. Else, there must be a $0 < F_t < n < F_{t+1}$. Let m = n-F_t. Since the statement holds $\forall n \leq k$ and $m < n$, m could be expressed as the sum of distinctive non-consecutive Fibonacci numbers $\implies n = F_t + m$

Meanwhile, F_{t-1} is not in m since $m = n - F_t < F_{t+1} - F_t = F_{t-1} \implies$ n could be represented as the sum of distinctive non-consecutive Fibonacci numbers.

By induction, the statement is true

Q.E.D.

3 Problem 3

We prove it using induction. We first prove the base case. When n=1, i=j=k=1, we do not need to cut the cake at all, which means the minimum cut is 1-1=0. The statement holds.

Then we assume that for all $n \leq k, k \geq 1$, the statement holds. When n=k+1, we let the longest edge be i, and divide the rectangle into 2 smaller rectangles, Denote them as R_1, R_2. Since the rectangle has integer sides, the volumes r_1 and r_2 of R_1 and R_2 are smaller than n $\implies$ it took $r_1 - 1 + r_2 - 1$ in total to cut the two slices.

Since $r_1 + r_2 = n$, and it took 1 cut to divide the rectangle into 2 pieces, so in total is $n - 1$ cuts.

By induction, the statement holds.

Q.E.D.