# HW3

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## Contents

# 1 Problem 1

**a.** $\forall x \in \mathbb{R} \exists q \in \mathbb{Z}(q + x = x)$

**b.** $Q(n) \forall n \in \mathbb{Z} \nexists (\neg O(x) \neg O(y) \neg O(z) \forall x, y, z \in \mathbb{Z}).P(x, y, z, n)$

## 2 Problem 2

**a.** G cannot be equivalent to F. I first show F $\implies$ G

$$G = \exists x \exists y.P(x, y)$$

Then G $\implies$ F

$$G = \forall x \forall y.P(x, y)$$

**b.** F cannot be equivalent to G. First show F $\implies$ G

$$G = \forall y \exists x.P(x, y)$$

Then G $\implies$ F

$$G = \forall y \forall x.P(x, y)$$

**c.** F $\leftrightarrow$ G, $G = \forall x \forall w \exists z \exists y.P(x, w, y, z)$

# 3 Problem 3

**a.** Prove: $(A\backslash B)\backslash C = (A\backslash C)\backslash(B\backslash C)$

$(A\backslash B)\backslash C \leftrightarrow (A \cap B^c) \cap C^c \leftrightarrow A \cap B^c \cap C^c$

$(A\backslash C)\backslash(B\backslash C) \leftrightarrow (A \cap C^c) \cap (B \cap C^c)^c \leftrightarrow (A \cap C^c) \cap (B^c \cup C)$

$\leftrightarrow ((A \cap C^c) \cap B^c) \cup ((A \cap C^c) \cap C) \leftrightarrow (A \cap C^c \cap B^c) \cup (A \cap \emptyset) \leftrightarrow A \cap C^c \cap B^c$

Q.E.D.

**b.** $(C\backslash(A \cup B)) \cup (B \cap C) \cup (A \cap C) \leftrightarrow (C \cap (A \cup B)^c) \cup (B \cap C) \cup (A \cap C) \leftrightarrow$ $(C \cap (A^c \cap B^c)) \cup (B \cap C) \cup (A \cap C) \leftrightarrow (C \cap (A^c \cap B^c)) \cup ((A \cup B) \cap C) \leftrightarrow$ $((A^c \cap B^c) \cup (A \cup B)) \cap C \leftrightarrow ((A^c \cup (A \cup B)) \cap (B^c \cup (A \cup B)) \cap C \leftrightarrow \mathbb{U} \cap \mathbb{U} \cap C \leftrightarrow C$

Q.E.D

# 4 Problem 4

see attached python.