HW1

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1 Problem 1

(a) Prove the last statement above; that is, prove that any positive integer n that is not prime has a factor (other than 1) less than or equal to $\sqrt{n}$.

here is the proof

If n is a prime, then n = pq, where p and q are positive integers. Since $n = \sqrt{n} * \sqrt{n}$, p can be larger than, smaller than, or equal to $\sqrt{n}$

If $p \leq \sqrt{n}$, Q.E.D.

If $p > \sqrt{n}$, $\quad q = \frac{n}{p} < \frac{n}{\sqrt{n}} = \sqrt{n} \implies q < \sqrt{n}$. Q.E.D.

(b) see python

2 Problem 2

(a) Prove that the prove a number n is divisible by 7 if and only if the result from performing the procedure above on n is divisible by 7.

here is the proof

Denote the result from performing the procedure as r, the last digit of n as m
$\implies n = 10(r + 2 * m) + m = 10r + 21m$
First prove that if r is divisible by 7, then n is divisible by 7. Because $r = 7k$, where k is a positive integer, therefore 10r=70k is also divisible by 7. $\implies$ n=7(10k+3m) is divisible by 7.
Then prove that if n is divisible by 7, then r is divisible by 7. Let $n = 7k = 10r + 21m$, then $10r = 7k - 21m = 7(k - 3m) \implies$ 10r is divisible by 7.
Since 10 is not divisible by 7, then r must be also divisible by 7. Q.E.D.

(b) Show that for any positive integer x with at least two digits, repeating the procedure above will eventually result in a number with one digit.

here is the proof

Denote the number we get from performing the operation on x as y, and the last digit of x as a. $x = 10y + 21a \implies y = \frac{x-21a}{10}$ Because $log(y) = log(x - 21a) - log(10) = log(x - 21a) - 1 < log(x - 21a) \leq log(x)$, since log function is strictly increasing and a is a non-negative integer. Therefore, y has fewer digits than x. If we repeat the process, we would thus eventually end up with a number with one digit. Q.E.D.

3 Problem 3

Suppose a, b, and n are positive integers such that b $>$ a. Prove the following: If b$-$a is even then $a^n + b^n$ is even.

here is the proof

If $b - a$ is even, then $b - a = 2k$, where k is a non-negative integer.

$b + a = a + 2k + a = 2(a + k)$ is also even.

Since $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 ...)$, and $a + b$ is even $\implies$ $a^n + b^n$ is even.   Q.E.D.