The Game of Stock: Play and Learn

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Chaofan Tao
ct214@duke.edu

Jingxian Huang
jh654@duke.edu

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1 Introduction

Game theory is widely seen in our daily lives, and consciously or unconsciously, we apply the concepts of "gaming" in making strategic decisions. One of the most obvious and relevant examples of the applications of gaming strategies and game theory could be the stock market. Starting from the beauty contest mentioned by John Keynes in his book *The General Theory of Employment, Interest and Money*[1], stock market has long been related to game theory. It is by nature a zero sum game. For buyers, buying a stock represents the belief that the stock price would go higher. For sellers, it's the opposite, and the order could only be fulfilled with both sides. Since the price could only go one direction, one's gain must be the other's loss. It is also by nature an imperfect information game. One agent in the market normally would have no information about other's strategy, and could only make assumptions about the price and other's behavior. There has also been numerous research on the topic of game theory in the market. Greenwood and Tymerski[2], for instance, evaluated the stock market trading strategies using a game-theoretical approach consisting of fuzzy membership functions. Alos-Ferrer and Ania[3], on the other hand, were trying to formulate the market as a non-cooperative game and find the unique Nash Equilibrium of the game.
However, although a lot of research effort has been spent on finding the models or strategies that could always beat the market and win the stock game, none has been proven correct and it seems impossible to always find the optimal strategy due to the irrational nature of human beings and other uncontrollable factors, such as policies or a global pandemic. Consequently, this project is not intended to find any solution to the game. Rather, in this project we are exploring from an empirical game theory approach, and are planning to simulate, compare, and analyze the price trend of a certain stock, and investigate the difference among different trading strategies, and how the distribution of different strategies could affect the price trend of the stock.
In the stock market, there have been two major schools of thought on developing the trading strategy. In fundamental approach decisions, buyers and sellers are using analysis about the company's revenue, performance, and potential to determine the "true value" of the stock, and invest in terms of the true value. However, in the technical approach decisions, buyers and sellers are studying each other. They make decisions based on the trend of price and the historical data. In other words, they make trading strategies based on the other agents in the market. Game theory is involved more heavily in the second approach, and the technical approach would be the main focus of the project.

2 Simulation Setup

In this project, we build an agent-based model to analyze the market between financial exchanges. Agent based modeling has been believed to be able to capture the essence of the market since it represents the agent behavior in a complex enough manner. It is also widely used in research[4][5][6].

In our simulation, we divide the continuous flow of trading into discrete rounds of order proposing. In each round, each agent is allowed to propose one order, either buy or sell stock. We also limit the simulation to only one stock such that we could better analyze the price trend as well as agent decisions. On the agent side of the simulation, each agent follows one trading strategy throughout the simulation, and proposes orders according to the price chart of the stock as well as their own order history. The proposed orders would include the price they want to buy or sell at, and the number of shares they would like to buy or sell. One of the key differences between our simulation and other research is the lack of background traders, or in other words, the type of traders who serve as market facilitators, which are often included in research [7].We believe that with the ability for each agent to learn from their previous orders, market facilitators are not necessary and would introduce unwanted noise when analyzing the trading behaviors.
On the market side of the simulation, an order is fulfilled according to the order of profitability with regard to the market maker. In each round, we would first sort the buy and sell order and match the highest buy price with the lowest sell price. The final price for each round is calculated according to the average price of the fulfilled orders in that round. With the most profitable order being paired first, we are assuming that the market facilitator would greedily match the most profitable orders, and we believe by doing so we could simulate the real life market without including background traders.
Moreover, our simulation is conducted in a closed system, which excludes all outside incentives and influence over the price of the stock, and there would not be any joining or quitting for agents. For each trading period, an agent could only see his own trading history and stock price within that trading period, and cannot change strategies within that trading period.

3 Agents

In our simulation, we have four types of strategies which belong to two genres.

3.1 Zero intelligence

Mentioned by Gode and Sunder in 1993[8], Zero Intelligent agents were first used to refer to the program that would submit random bids and orders. Since then, researchers have extended the concept and have conducted a lot of experiments on the variations of ZI agents[4]. In this simulation, we applied an extension of the Zero Intelligence Agents to represent the agents that would not learn from the current price chart. In other words, they do not take the current price trend into consideration when they are making trading decisions.

3.1.1 Fundamental

The agents following Fundamental strategy would each have a target price, i.e. the "true value" they believe about the company. For any price lower than their target price, they would propose to buy in the stock. Once the price reaches their target price, they would start to sell off their shares. The target price for each agent is randomly chosen independently within a range.

3.1.2 Bear

The agents following Bear strategy believe that the stock is overpriced, and they would love to get rid of their shares with some cash in hand. They will propose to sell their stock no matter what happens.

3.2 Heuristic Belief Learning

We also include some agents that follow the heuristic belief learning model. Strategies under this category have a heuristic belief function based on market observation over a period of time T[6]. We have also tuned

T and analyze the behaviors accordingly.

3.2.1 Speculator

This class of agents would try to buy low and sell high. They will try to time the market, and they will buy when they see a downward trend in stock price, and sell when there is an uprising trend.

3.2.2 Chaser

This category of agents try to ride the tide. They will sell if the stock starts to go down, and buy when the stock price just starts rising. We have also forced arbitrary randomness to individual agents behaviors as well. As argued by David Byrd, each agent should have his individual preference for holding and selling assets, even among those of the same strategy class[9]. By enforcing arbitrary randomness, our simulation is closer to real life situations

4 Experiments and Results

There are apparently many parameters that could be tuned in the simulation. In the following sections, we would start with an explanation of the simulation algorithm and basic setups, followed by an easy simulation. We would then introduce a closer simulation to the real world scenarios, and find the empirical Nash Equilibrium in terms of proportions of strategies in the agent population. At last, we will analyze the performance of each strategy in the NE.

4.1 Initial Setup

In this project, we simulate a stock market with number of agents $N = 500$. At the beginning of the trading period, each agent holds a certain amount of money $M = 10000$ and shares of stock $S = 10$. For each time step t, each agents will submit a proposal of buy or sell, then we match the proposal and execute the orders and go to the next time step. Each trading period will last $T = 200$ time steps unless the trading price reaches zero, at which time we will end the period early and set all agent's stock value to zero. The simulation process is in Algorithm 1.

To compare the net gain of each agent, we approximately evaluate the stock value at t as $n_t \times \frac{p_t}{2}$, where n_t is the total shares of stock that are traded at t, and p_t is the current stock price. The utility at the end of each period is calculated as the total asset at the end of the trading period minus the total asset at the beginning of the trading period.

We will stick to this setup unless otherwise stated.

Algorithm 1 Simulation within each trading period

Data: initial price trend at the start of the trading period
Result: price trend and utility at the end of the trading period
Spawn N agents, each individually chooses one strategy to follow according to the strategy probability distribution
for *t in T* **do**
  each agent proposes order
  sort buy orders and sell orders
  match orders
  calculate market price at t
  if *market price* $\leq$ *0* **then**
    set agents share value to zero
    return
  end
end

4.2 Basic model

The model will be very complicated if we start by simulating with all four types of agents and all the possible variance, so we start with a very simple model and dig deeper in later experiments.
In the basic model, we set up a simple situation where there are only Chaser(buy on increasing trend and sell on decreasing) and Speculator(buy on decreasing trend and sell on increasing), these two are the most popular strategies in the stock market in real life and they act opposite, which makes them very interesting to the study.
We fix the number of Chaser and Speculator to 250. They will propose at each time step and only propose to buy or sell 1 share.
The simulation result in terms of the utility for each trading period is shown in Table 2. We can see that Chaser gain much higher utility than Speculator. This is because chaser will never miss out on the rising trends of the stock, while speculators would likely to miss most of those opportunities.

Strategy	Mean	Std	95 CI (low, high)
Speculator	-1457	1069	-1669, -1244
Chaser	1552	1089	1136, 1768

Table 1: Mean and Std of utilities for Chaser and Speculator in Basic Model.

4.3 Real-life Simulation

In the next step, we simulate more complicated model that is closer to the real word situation. All of the four types of agents will involve, and the strategies share an equal possibility to be chosen. Instead of only one share per propose, they can propose a random number of shares, ranging 0 to max number of shares s/he holds. The performance of each types is summarized in the following table.

Strategy	Mean	Std	95 CI (low, high)
Speculator	-1021	1241	-1267, -774
Bear	32	1.6	31.6, 32.2
Fundamental	-1827	1693	-2163, -1491
Chaser	2082	423	1998, 2166

Table 2: Mean and Std of utilities for Agents in Real-life Simulation.

4.4 Nash Equilibrium for Strategy Distribution

We know that in real life, people are not always fixed to one strategy. They can learn to adjust to the best. So, now we treat the stock market as a repeated game.

At the start of each trading period, the agents would independently pick one of the four strategies to follow throughout the trading periods. The probability distribution of picking the strategy is updated at the end of each trading period, according to the result average utilities for each type of strategies. For the strategy with the highest utility return at each trading period, we increase its probability of being chosen and decrease the others. In this way, we are allowing the agents to change strategies and pick the most profitable strategy to follow. We start the simulation with a uniform probability distribution over the four strategies.

As is shown in Figure 1, the x axis represents the trading periods, and the y axis represents the probability of

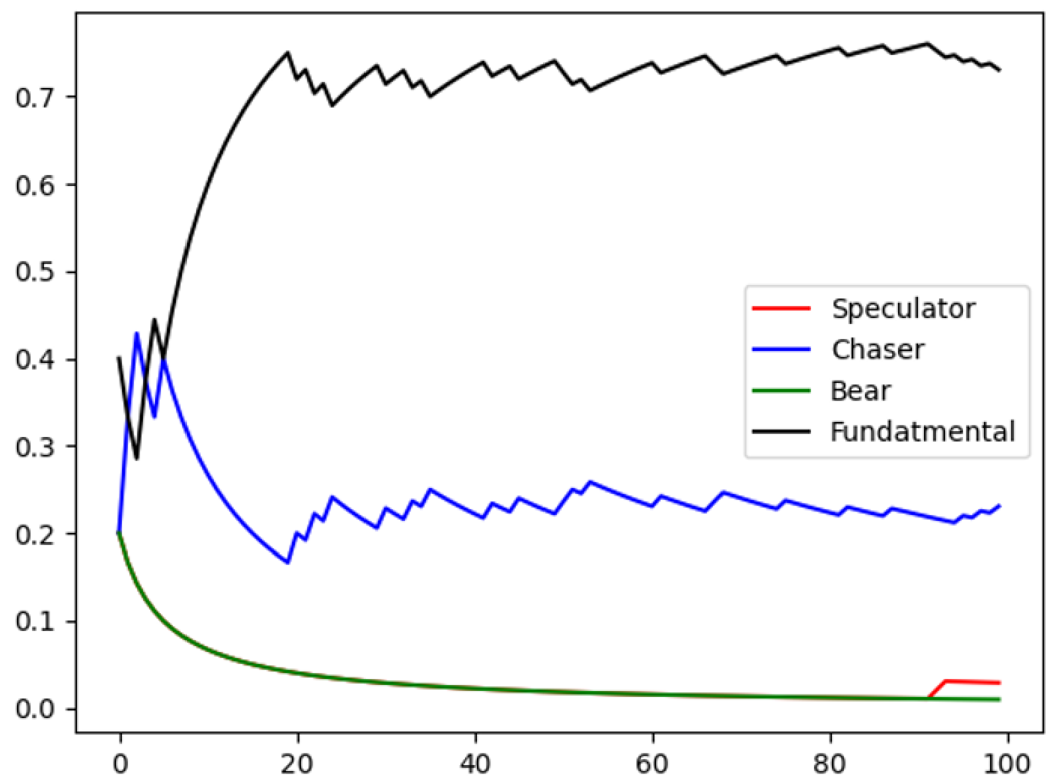

Figure 1: Strategy probabilities over trading periods.

an agent choosing a corresponding trading strategy. We can see that in the simulation within a closed system, most agents would prefer Fundamental strategy, and they tend to move away from Bear and Speculator since they are not as profitable. The probability converges to an empirical NE of {Speculator: 0.05, Bear: 0.05, Chaser: 0.2, Fundamental: 0.7}. Fundamental and chaser seem to be the winning strategies because in most of our simulations, the stock price would end up higher than its starting price, which would benefit the ones with stock in hand rather than those who sold early in the simulation.

4.5 Performance under NE

Now that we have the empirical NE for distribution of strategies, it's time to evaluate the performance of each type of strategy. In the following simulations, we set the probability distribution of choosing strategies to be the same as shown in the Nash Equilibrium, and spawn 500 agents in each trading period. The utility is calculated using the same formula defined above. We ran the simulation for 100 trading periods and calculated the average and standard deviation. The results are summarized in Table 3.

Strategy	Mean	Std	95 CI (low, high)
Bear	41	2.42	40.4, 41.4
Speculator	-3181	2510	-3679, -2682
Chaser	75	1365	-196, 345
Fundamental	884	436	797, 970

Table 3: Mean and Std of utilities for different strategies.

5 Conclusion and Future Work

In our work, we built a model to simulate the stock market using four different types of strategies. We concluded an empirical NE regarding the population distribution of strategies where people tend to choose Fundamental strategies over others, and analyzed the return of each performance.
While we believe that our work is a good starting point for simulating the stock market, there are many ways that we could improve if given more time. For instance, right now our simulation is based entirely on a closed system but in real life there would always be outside information that could change agent strategies dramatically. Meanwhile, manipulation or "spoofing" is a hot topic in the game of stock market, and due to time limits, we are unable to further investigate the problem. We expect to dive deeper in the future.

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