1 $P(5) = 2P(0)$ $t=5 \rightarrow P_0 = C$

$$\frac{dP}{dt} = kP$$

$$\frac{dP}{P} = k\,dt \rightarrow \int \frac{dP}{P} = \int k\,dt$$

$$\ln(P) = k \int dt$$

$$\ln(P) = kt + C$$

$$e^{\ln(P)} = e^{(kt+C)}$$

$$P = e^{kt} e^{C} \rightarrow P = Ce^{kt}$$

$$P = Ce^{k(0)}$$

$$P = C$$

$$2C = Ce^{k(5)}$$

$$2 = e^{k(5)}$$

$$\ln(2) = 5k$$

$$\frac{\ln(2)}{5} = k \rightarrow 0.1386$$

$P = C e^{0.1386t}$

$3C = C e^{0.1386t}$

$3 = e^{0.1386t} \rightarrow \ln 3 = 0.1386t$

$t = \frac{\ln(3)}{0.1386} = 7.9264$ (años)

$\ln 4 = 0.1386t \rightarrow \frac{\ln(4)}{0.1386} = t$

$t = 10.0021$ (años)

2) $P(3) = 10{,}000$ $\qquad P_0 = ?$

$10{,}000 = C e^{0.1386(3)} \rightarrow \frac{10{,}000}{1.5155} = C$

$\boxed{6598.48 = C} = P_0$

a)

La población inicial es 6598.48

$P = 6598.48 e^{0.1386t}$

$\frac{dP}{dt} = (6598.48)(0.1386) e^{0.1386t}$

$$\frac{dP}{dt} = (914.55)\, e^{0.1386t}$$

$$\frac{dP}{dt} = (914.55)\, e^{0.1386(10)} =$$

$$\frac{dP}{dt} = 3657.12 \text{ P/año}$$

3

$P_0 = 500$ $\quad 15\% = 75$

$P_{(10)} = 575$

$500 = C\cancel{e^{K(0)}} \rightarrow 500 = C$

$575 = 500\, e^{K\,10} \rightarrow \ln(1.15) = 10K$

$0.01397 = K$

$$P_{(t)} = 500\, e^{0.01397t}$$

$$P_{(t)} = 500\, e^{0.01397(30)} = \boxed{760.30}$$

$$\frac{dP}{dt} = 500(0.01397)\, e^{0.01397t}$$

$$\frac{dD}{dt} = 6.985\left(e^{0.01397t}\right)$$

$$= 6.985\left(e^{0.01397(30)}\right)$$

$$\boxed{= 10.62\ \$/\text{año.}}$$

4) $P_{(3)} = 400$

$P_{(10)} = 2000$

$400 = Ce^{k(3)}$ $\quad$ $2000 = Ce^{k(10)}$

$$\frac{2000 = \cancel{C}e^{k(10)}}{400 = \cancel{C}e^{k(3)}}$$

$$5 = e^{7k}$$

$\ln(5) = 7k$

$\frac{\ln(5)}{7} = k \rightarrow k = 0.23$

$\ln(400) = Ce^{0.23(3)}$

$$\frac{\ln(400)}{(1.9937)} = C\ (1.9937)$$

$C = 3.005 \quad 1 = 3$

$P = 3e^{0.23(0)}$

$\boxed{P_0 = 3}$

5

$P(3.3) = \frac{P}{2} \qquad P_0 = 1$

$P_2 = 0.1$

$-\frac{dP}{dt} = kP$

$\frac{dP}{P} = -k\,dt \Rightarrow \int \frac{dP}{P} = -k \int dt$

$\ln(P) = -kt + c \Rightarrow P = e^{-kt+c}$

$P = Ce^{-kt}$

$1 = C\cancel{e^{-k(0)}}\ 1$

$1 = C \qquad \frac{1}{2} = e^{-k(3.3)}$

$\frac{\ln(\frac{1}{2})}{3.3} = -k$

$0.21 = k$

$P = e^{-0.21t}$

$0.1 = e^{-0.21t} \rightarrow \ln(0.1) = -0.21t$

$\frac{\ln(0.1)}{-0.21} = t$

$10.96 = t$

Decae en 10.96 horas.

(6) $P_0 = 100$ 3:1

$P_6 = 97$

$P_{24} = ?$

$P = Ce^{-kt}$

$100 = C\cancel{e^{-k(0)}}$

$C = 100$

$97 = 100\, e^{-k(6)}$

$0.97 = e^{-k6}$

$\frac{\ln(0.97)}{6} = -k$

$-0.00507 = -k$

$P = 100\, e^{-0.00507t}$

$P = 100\, e^{-0.00507(24)} = \underline{88.54}$

7

$50 = 100\, e^{-0.00507t}$

$\ln\frac{1}{2} = -0.00507t$

$-\frac{\ln(0.5)}{0.00507} = t$

136.71 horas $= t$

8 a)

$\frac{dA}{dt} = KA$

$\frac{dA}{A} = K\,dt$

$\ln(A) = Kt + c$

$e^{\ln(A)} = C e^{kt}$

$A = C e^{kt}$

$A = C\,\cancel{e^{k(0)}} \Rightarrow A = C$

$A = A e^{kt}$

$1 = e^{kt}$

$\frac{1}{2} = e^{kt} \rightarrow \ln\left(\frac{1}{2}\right) = kt$

$\ln(1) - \ln(2) = kt$

$-\frac{\ln(2)}{k} = t$

9

$$-\frac{dI}{dt} = kI \qquad t = 3 \to I \; .25$$

$$I(3) = 0.25\, I_0$$

$$\frac{dI}{I} = -k\, dt \qquad I(0) = I_0$$

$$\ln(I) = -kt + C$$

$$I = Ce^{-kt} \to I(0) = C\cancel{e^{-k(0)}}$$

$$I(0) = C$$

$$\frac{\cancel{I_0}}{4} = \cancel{I_0}\, e^{-k(3)}$$

$$\ln\frac{1}{4} = -3k \to \frac{\cancel{\ln(1)} - \ln(4)}{-3} =$$

$$\frac{-\ln(4)}{-3} = k$$

$$0.4620981204 = k$$

$I(15) = I_0 e^{-k(15)}$

$I(15) = I_0 (0.000976)$

aproximadamente 0.1%

$\frac{0.1}{100} = 0.001$

10

$\frac{dS}{dt} = rS \qquad S(0) = 5{,}000$

$S = ce^{rt}$

$S = 5000 e^{0.0575t}$

$S = 5000 e^{0.0575(5)}$

$S = 6665.45$ a)

$10{,}000 = 5000 e^{0.0575t}$

$\frac{\ln(2)}{0.0575} = t$

$12.05 = t$ b)

11 85%

$t = 5,600$ años $\rightarrow \frac{A_0}{2}$

$$\frac{1}{2} = e^{k(5,600)}$$

$$\frac{\ln\left(\frac{1}{2}\right)}{5,600} = k$$

$$-0.000124 = k$$

$$(0.145)\, A_0 = A_0\, e^{-0.000124t}$$

$$\frac{\ln(0.145)}{-0.000124} = t$$

$$\boxed{15572.75 \text{ años} = t}$$

12 $I(660) = I_0\,(e^{-0.000124(660)})$

$= I_0\,(0.92)$

Que aun el 92.14 %.

B

$T_0 = 70$

$T_A = 10$

$T(t) = Ce^{-kt} + 10$

$70 = C\cancel{e^{-k(0)}} + 10 \Rightarrow 60 = C$

$T(t) = 60e^{-kt} + 10$

$50 = 60e^{-k(1/2)} + 10$

$\frac{40}{60} = e^{-k(1/2)} \rightarrow \frac{2}{3} = e^{-k(1/2)}$

$\ln(2/3) = -\frac{k}{2} \rightarrow -2(\ln(2/3)) = k$

$0.81 = k$

$T(t) = 60e^{-0.81(4)} + 10$

$\boxed{T(t) = 36.69\,°F}$

(14) $T_A = 5\,°F$

$T(1) = 55\,°F$

$T(5) = 30\,°F$

$$\frac{dT}{dt} = k(T - T_A)$$

$$\frac{dT}{(T-T_A)} = k\,dt \rightarrow \int \frac{dT}{(T-T_A)} = \int k\,dt$$

$$T = Ce^{-kt} + 5$$

$$T(1) 55 = Ce^{-k} + 5 \rightarrow 50 = Ce^{-k}$$

$$30 = Ce^{-5k} + 5 \rightarrow 25 = Ce^{-5k}$$

$$\frac{50}{e^{-k}} = C \rightarrow \frac{25}{e^{-5k}} = C$$

$$\frac{50}{e^{-k}} = \frac{25}{e^{-5k}} \rightarrow 50e^{k} = 25e^{5k}$$

$$2 = e^{4k}$$

$$\ln(2) = 4k \rightarrow \frac{\ln(2)}{4} = k$$

$$\boxed{0.1732 = k}$$

$$\frac{25}{e^{-5(0.1732)}} = C \qquad \frac{50}{e^{-(0.1732)}} = C$$

$$59.43 = C \qquad 59.45 = C$$

$$T = 59.43e^{-0.1732t} + 5$$

$$T = (59.43)\cancel{e^{0}} + 5 \rightarrow T = 64.43$$

15 $T_0 = 20$ El agua hierve a $100°$

$T(1) = 22$

$\frac{dT}{dt} = k(T-100) \rightarrow \frac{dT}{(T-100)} = k\,dt$

$\ln(T-100) = kt + C$

$T - 100 = Ce^{kt} \rightarrow T = Ce^{kt} + 100$

$20 = C\cancel{e^{0t}} + 100$

$-80 = C$

$22 = -80e^{k \cdot 1} + 100$

$-78 = -80e^{k}$

$\frac{39}{40} = e^{k} \rightarrow \ln\left(\frac{39}{40}\right) = k$

$-0.025 = k$

$T = -80e^{-0.025t} + 100$

a)

$-10 = -80e^{-0.025t}$

$\frac{1}{8} = e^{-0.025t} \rightarrow \frac{\ln\left(\frac{1}{8}\right)}{-0.025} = t$

$\underline{t = 83.18 \text{ s}}$

$$98 = -80\, e^{-0.025t} + 100$$

$$\frac{-2}{-80} = e^{-0.025t} \rightarrow \frac{\ln\left(\frac{2}{80}\right)}{-0.025} = t$$

$$t = 147.56 \text{ (s)}$$

16 A B

$T_A = 0$ $T_A = 100$

$T_0 = 100$

$T(1) = 90$

$$T = Ce^{-kt}$$

$$100 = Ce^{-k(0)} \rightarrow C = 100$$

$$90 = 100e^{k(1)}$$

$$\ln\left(\frac{9}{10}\right) = k \rightarrow k = -0.10536$$

$T = C e^{-0.1053t}$

2

$T = 100 e^{-0.1053(2)} = \underline{81.00}$

B

$T_0 = 81.00$

$T_A = 100$

$T = C e^{kt} + 100$

$81 - 100 = C \rightarrow C = -19$

$T = -19 e^{kt} + 100$

$91 = -19 e^{k} + 100 \rightarrow -9 = -19 e^{k}$

1 $\frac{9}{19} = e^{k} \rightarrow \ln\left(\frac{9}{19}\right) = k$

$k = -0.74$

$99.9 = -19 e^{-0.74t} + 100$

$\frac{-0.1}{-19} = e^{-0.74t} \rightarrow \frac{1}{190} = e^{-0.74t}$

$\frac{\ln\left(\frac{1}{190}\right)}{-0.74} = t \rightarrow t = 7.09$

alcanzara la temperatura de 99.9
a los 10 minutos.

17 $T_0 = 70$

$T_{0.5} = 110$

$T_1 = 145$

$T = Ce^{xt} + T_A \Rightarrow T_0 = C + T_A$

$110 = Ce^{0.5t} + T_A$

$145 = Ce^{t} + T_A \rightarrow$

$70 = C + T_A \rightarrow$

$70 - T_A = C$

$145 = (70 - T_A)e^{t} + T_A$

$110 = (70 - T_A)e^{1/2 t} + T_A$

$\frac{145 - T_A}{70 - T_A} = e^{t}$

$$\frac{110 - T_A}{70 - T_A} = e^{1/2 t} \rightarrow \left(\frac{110 - T_A}{70 - T_A}\right)^2 = e^{t.}$$

$$\frac{(110 - T_A)^2}{(70 - T_A)^2} = \frac{145 - T_A}{70 - T_A}$$

$$\frac{(110 - T_A)^2}{70 - T_A} = 145 - T_A$$

$$12{,}100 - 220 T_A + \cancel{T^2} = 10150 - 70 T_A + 145 T_A + \cancel{T_A^2}$$

$$12{,}100 - 220 T_A = 10150 - 215 T_A$$

$$12{,}100 - 10150 = -215 T_A + 220 T_A$$

$$1950 = 5 T_A$$

$$\boxed{390 = T_A}$$

18 $T_o = 80\,°F$

$$T_m = 100 - 40 e^{-0.1 t} \quad , t \geq 0$$

$$T(t) = c e^{-0.1 t} + (100 - 40 e^{-0.1 t})$$

$$T(t) = c e^{-0.1 t}(1 - 40) + 100$$

$$Tt = -39 c e^{-0.t} + 100$$

$T(t) = -39ce^{-0.1t} + 100$

$80 = -39\cancel{ce}^{\cancel{-0.1(0)}\,1} + 100$

$80 = -39c + 100$

$\dfrac{80 - 100}{-39} = c$

$\dfrac{20}{39} = c$

$T(t) = (\cancel{-39})\dfrac{20}{\cancel{39}}e^{-0.1t} + 100$

$$\underbrace{T(t) = -20e^{-0.1t} + 100}$$

1a $T_A = 70°F$ 98.6

$T = 85°F$

$T_1 = 80°F$

$T = Ce^{-kt} + 70$

$98.6 = C + 70$

$28.6 = C$

$T = 28.6\, e^{kt} + 70$

$85 = 28.6\, e^{kt} + 70$

$\frac{15}{28.6} = e^{kt} \rightarrow \ln\left(\frac{75}{143}\right) = kt$

$\frac{-0.6453}{t} = k$

$\frac{10}{28.6} = e^{k(t+1)} = \frac{-1.0508}{t+1}$

$\frac{-0.6453}{t} = \frac{-1.0508}{t+1}$

$\frac{t+1}{t} = \frac{-1.0508}{-0.6453}$

$1 + \frac{1}{t} = 1.6284$

$\frac{1}{t} = 0.6283 \rightarrow \frac{1}{0.6283} = t$

$\boxed{1.5915 = t}$

20 $T_0 = 150°$

$$\frac{dT}{dt} = k5(T - T_m)$$

$$\frac{1}{T - T_m} = k5\, dt$$

$$\ln(T - T_m) = k5t + C \rightarrow$$

$$T - T_m = Ce^{2k5t}$$

$$T = Ce^{2k5t} + T_m$$

$$150 = \cancel{C}e^{\cancel{0}} + 70$$

$$T_A = 80e^{k5t} + 70$$

$$T_B = 80e^{2k5t} + 70$$

(A) $100 = 80e^{k5(30)} + 70$

(B) $T_B = 80e^{5k60} + 70$

(A) $\frac{3}{8} = e^{k5(30)} \rightarrow \frac{\ln(\frac{3}{8})}{30} = k5$

$$-0.0326 = k5$$

(3) $\dfrac{T_B - 70}{80} = e^{5k60}$

$\dfrac{\ln(T_B - 70) - \ln 80}{60} = k5$

$\dfrac{\ln(T_B - 70)}{60} - 0.0730 = 5k$

$\dfrac{\ln(T_B - 70)}{60} - 0.0730 = -0.0326$

$\ln(T_B - 70) = 0.0404\ (60)$

$\ln(T_B - 70) = 2.424$

$T_B = e^{2.424} + 70$

$\boxed{T_B = 81.29\,°F}$