Ejercicios 2.5

1) $(x-y)\,dx + x\,dy = 0$

$$(x-y)\,dx = -x\,dy$$

$$\frac{(x-y)}{-x} = \frac{dy}{dx}$$

$$-\frac{\cancel{x}}{\cancel{x}} + \frac{y}{x} = \frac{dy}{dx}$$

$$-1 + \frac{y}{x} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x} - 1$$

$$u = \frac{y}{x} \rightarrow xu = y$$

$$\left(u + x\frac{du}{dx}\right) = \frac{dy}{dx}$$

$$u + x\frac{du}{dx} = \frac{y}{x} - 1$$

$$\cancel{u} + x\frac{du}{dx} = \cancel{u} - 1$$

$$\frac{du}{dx} = -\frac{1}{x} \rightarrow du = -\frac{dx}{x}$$

$$\int du = -\int \frac{dx}{x} \rightarrow u = -\ln|x| + c$$

$$\frac{y}{x} = -\ln|x| + c$$

$$y = x\left[-\ln|x| + c\right]$$

5) $(y^2 + yx)\, dx - x^2 dy = 0$

$$(y^2 + yx)\, dx = x^2 dy$$

$$y^2 + yx = x^2 \frac{dy}{dx}$$

$$\frac{y^2}{x^2} + \frac{y}{x^2}x = \frac{dy}{dx}$$

$$\left[\frac{y}{x}\right]^2 + \frac{y}{x} = \frac{dy}{dx}$$

$u = \frac{y}{x}$

$xu = y$

$\left[u + x\frac{du}{dx}\right] = \frac{dy}{dx}$

$$u^2 + \cancel{u} = \cancel{u} + x\frac{du}{dx} \rightarrow x\frac{du}{dx} = u^2$$

$$\frac{du}{u^2} = \frac{dx}{x} \rightarrow \int \frac{du}{u^2} = \int \frac{dx}{x}$$

$$\int \frac{du}{u^2} = \int \frac{dx}{x} \rightarrow \int u^{-2}\,du = \int \frac{dx}{x}$$

$$\int u^{-2}\,du = \ln|x| + c$$

$$\frac{-u^{-1}}{-1} = \ln|x| + c \rightarrow -u^{-1} = \ln|x| + c$$

$$-\frac{1}{u} = \ln|x| + c$$

$$-\frac{1}{\ln|x| + c} = u$$

$$-\frac{1}{\ln|x| + c} = \frac{y}{x} \rightarrow \boxed{y = \frac{-x}{\ln|x| + c}}$$

9) $-y\,dx + (x + \sqrt{xy})\,dy = 0$

$$(x + \sqrt{xy})\,dy = y\,dx$$

$$x + \sqrt{xy} = y\,\frac{dx}{dy}$$

$$\frac{x}{y} + \frac{\sqrt{xy}}{y} = \frac{dx}{dy} \rightarrow \frac{x}{y} + \frac{\sqrt{x}}{\sqrt{y}} = \frac{dx}{dy}$$

$$\frac{x}{y} + \sqrt{\frac{x}{y}} = \frac{dx}{dy}$$

$$u = \frac{x}{y} \rightarrow uy = x \rightarrow \left(u + y\frac{du}{dy}\right) = \frac{dx}{dy}$$

$$\cancel{u} + \sqrt{u} = \cancel{u} + y\frac{du}{dy}$$

$$dy = \frac{du}{\sqrt{u}} \rightarrow \frac{du}{\sqrt{u}} = dy$$

$$\int \frac{du}{\sqrt{u}} = \int dy$$

$$\frac{u^{-1/2+1}}{-\frac{1}{2}+1} = y + c \rightarrow 2u^{1/2} = y + c$$

$$u^{1/2} = \frac{1}{2}(y+c) \rightarrow u = \sqrt{\frac{y+c}{2}}$$

$$\frac{x}{y} = \sqrt{\frac{y+c}{2}} \rightarrow \boxed{x = y\sqrt{\frac{y+c}{2}}}$$

13 $(x + ye^{y/x})\,dx - xe^{y/x}\,dy = 0, \quad y(1) = 0$

$$(x + ye^{y/x})\,dx = xe^{y/x}\,dy$$

$$1 + \frac{y}{x}e^{y/x} = e^{y/x}\frac{dy}{dx} \rightarrow \frac{1}{e^{y/x}} + \frac{y}{x} = \frac{dy}{dx}$$

$$\frac{1}{e^{y/x}} + \frac{y}{x} = \frac{dy}{dx} \qquad u = \frac{y}{x}$$

$$\frac{1}{e^{u}} + \cancel{u} = \cancel{u} + x\frac{du}{dx} \qquad ux = y$$

$$\frac{1}{e^{u}} = x\frac{du}{dx} \qquad \left(u + x\frac{du}{dx}\right) = \frac{dy}{dx}$$

$$\frac{dx}{x} = du\, e^{u}$$

$$du\, e^{u} = \frac{dx}{x} \rightarrow \int du\, e^{u} = \int \frac{dx}{x}$$

$$e^{u} = \ln|x| + c$$

$$\ln e^{u} = \ln[\ln|x| + c]$$

$$u\,(1) = \ln[\ln|x| + c]$$

$$\frac{y}{x} = \ln[\ln|x| + c]$$

$$\boxed{y = x\ln[\ln(x) + c]}$$

$$0 = 1\ln[\cancel{\ln(1)} + c]$$

$$0 = \ln[c]$$

$$e^{0} = e^{\ln c}$$

$$\boxed{1 = C} \qquad y = x\ln[\ln(x) + 1]$$

(17) $\frac{dy}{dx} = y(xy^3 - 1)$

$u = y^{1-4} \to u = y^{-3}$

$\frac{dy}{dx} = xy^4 - y$

$u = \frac{1}{y^3}$

$\frac{dy}{dx} + y = xy^4$

$y^3 = \frac{1}{u}$

$-\frac{1}{3} u^{-4/3} \frac{du}{dx} + u^{-1/3} = x[u^{-1/3}]^4$

$y = u^{-1/3}$

$\frac{dy}{dx} = -\frac{1}{3} u^{-4/3} \frac{du}{dx}$

$\frac{du}{dx} + \frac{(-3) u^{-1/3}}{u^{-4/3}} = x[u^{-4/3}]$

$\frac{du}{dx} - 3u = -3x$

$\frac{du}{dx} + (-3)u = -3x$

$e^{\int -3dx} \to e^{-3x}$

$e^{-3x} \frac{du}{dx} - 3u e^{-3x} = -3x e^{-3x}$

$\frac{d}{dx}(e^{-3x} \cdot u) = -3x e^{-3x}$

$u = -3x$

$du = -3\, dx$

$\frac{du}{-3} = dx$

$e^{-3x} \cdot u = -3 \int x e^{-3x} dx$

$u = x \qquad v = \int e^{-3x} dx$

$du = dx \qquad v = -\frac{1}{3} \int e^{u} du \to v = -\frac{1}{3} e^{u} \to -\frac{1}{3} e^{-3x}$

$$\int u\,dv = -\frac{x}{3}e^{-3x} - \int\left(-\frac{1}{3}\right)e^{-3x}\,dx$$

$$\int u\,dv = -\frac{e^{-3x}x}{3} + \frac{1}{3}\int e^{-3x}\,dx$$

$$\int u\,dv = -\frac{e^{-3x}x}{3} - \frac{1}{9}e^{-3x} + c$$

$$e^{-3x}u = -\frac{e^{-3x}}{3}x - \frac{1}{9}e^{-3x} + c$$

$$u = -\frac{\cancel{e^{-3x}}x}{3\cancel{e^{-3x}}} - \frac{1\,\cancel{e^{-3x}}}{9\,\cancel{e^{-3x}}} + \frac{c}{e^{-3x}}$$

$$u = \frac{1}{3}x - \frac{1}{9} + \frac{c}{e^{-3x}}$$

$$y^{-3} = \frac{1}{3}x - \frac{1}{9} + \frac{c}{e^{-3x}}$$

$$y = \left[\frac{1}{3}x - \frac{1}{9} + \frac{c}{e^{-3x}}\right]^{-1/3}$$

21 $x^2 \frac{dy}{dx} - 2xy = 3y^4, \quad y(1) = \frac{1}{2}$

$\frac{dy}{dx} - \frac{2y}{x} = \frac{3}{x^2} y^4$ $\qquad u = y^{-3}$

$u^{-1/3} = y$

$-\frac{1}{3} u^{-4/3} \frac{du}{dx} - \frac{2u^{-1/3}}{x} = \frac{3}{x^2}(u^{-4/3})$ $\qquad \frac{dy}{dx} = -\frac{1}{3} u^{-4/3} \frac{du}{dx}$

$\frac{du}{dx} + \frac{6u}{x} = \frac{-9}{x^2}$

$x^6 \frac{du}{dx} + 6x^5 u = -9x^4$ $\qquad e^{\int \frac{6}{x} dx} \rightarrow e^{6\int \frac{1}{x} dx} \rightarrow e^{6\ln x}$

x^6

$\frac{d}{dx}(x^6 u) = -9x^4$

$x^6 u = -9 \int x^4 dx \rightarrow x^6 u = -\frac{9x^5}{5} + C$

$u = \frac{-9}{5x} + \frac{C}{x^6} \rightarrow y^{-3} = -\frac{9}{5x} + \frac{C}{x^6}$

$\left(\frac{1}{2}\right)^{-3} = -\frac{9}{5} + C$

$\left(\frac{1}{2}\right)^{-3} + \frac{9}{5} = C \rightarrow C = \frac{49}{5}$

$$y^{-3} = -\frac{9}{5x} + \frac{49}{5x^6} \Rightarrow y^{-3} = \frac{1}{5x}\left[-9 + \frac{49}{x^5}\right]$$

$$y = \left[\frac{1}{5x}\left[-9 + \frac{49}{x^5}\right]\right]^{-1/3}$$

25. $$\frac{dy}{dx} = \tan^2(x+y)$$

$$w = x+y \Rightarrow \frac{dw}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dw}{dx} - 1 = \frac{dy}{dx}$$

$$\frac{dw}{dx} - 1 = \tan^2(w) \Rightarrow \frac{dw}{dx} = \tan^2(w) + 1$$

$$\frac{dw}{dx} = \sec^2(w) \Rightarrow \frac{dw}{\sec^2(w)} = dx$$

$$\int \cos^2 w \, dw = \int dx$$

$$b = 2w$$
$$db = 2dw$$
$$\frac{db}{2} = dw$$

$$\int \left[\frac{1}{2} + \frac{\cos 2w}{2}\right] dw = x$$

$$\frac{1}{2}\int dw + \frac{1}{2}\int \cos 2w \, dw = x$$

$$\frac{1}{2}w + \frac{1}{4}\int \cos b \, db \Rightarrow \frac{1}{2}w + \frac{1}{4}\operatorname{sen} b = x$$

$$\frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u = x + C$$

$$\frac{1}{2}(x+y) + \frac{1}{4}\, 2\operatorname{sen} u \cos u = x + C$$

$$\frac{1}{2}(x+y) + \frac{1}{2}\operatorname{sen}(x+y)\cos(x+y) = x + C$$

24) $$\frac{dy}{dx} = \cos(x+y), \quad y(0) = \pi/4$$

$$u = x+y \rightarrow \frac{du}{dx} = 1 + \frac{dy}{dx} \rightarrow \frac{du}{dx} - 1 = \frac{dy}{dx}$$

$$\frac{du}{dx} - 1 = \cos u \rightarrow \frac{du}{dx} = \cos u + 1$$

$$\frac{du}{\cos u + 1} = dx \rightarrow \int \frac{du}{\cos u + 1} = x + C$$

$$\int \frac{1}{1+\cos u} \cdot \frac{1-\cos u}{1-\cos u}\, du = \int dx$$

$$\int \frac{1-\cos u}{1-\cos^2 u}\, du = x + C$$

$$\int \frac{1-\cos u}{\operatorname{sen}^2 u}\, du = x + C$$

$$\int \frac{1}{sen^2 w}\,dw - \int \frac{\cos w \, dw}{sen^2 w} = x + c$$

$$\int \csc^2 w \, dw - \int \csc w \cot w \, dw = x + c$$

$$-\cot w - (-\csc w) = x + c$$

$$-\cot w + \csc w = x + c$$

$$\underline{-\cot(x+y) + \csc(x+y) = x + c}$$