M

(7) $(x^2-y^2)\,dx + (x^2-2xy)\,dy = 0$

$\frac{\partial M}{\partial y} = (x^2-y^2) \Rightarrow (0-2y) \Rightarrow -2y$

$\frac{\partial M}{\partial x} (2x-2y) \rightarrow 2x-2y$

No es exacta.

(9) $(x-y^3+y^2\operatorname{sen}x)\,dx = (3xy^2+2y\cos x)\,dy$

$(x-y^3+y^2\operatorname{sen}x)\,dx + (3xy^2+2y\cos x)\,dy = 0$

$\frac{\partial M}{\partial y} = 0 - 3y^2 + 2y\operatorname{sen}x$

$\frac{\partial M}{\partial x} = -3y^2 - 2y\operatorname{sen}x$

$-3y^2 + 2y\operatorname{sen}x$

Exacta.

$\int (x-y^3+y^2\operatorname{sen}x)\,dx = \frac{x^2}{2} - y^3x - y^2\cos x$

$\int (-3xy^2 - 2y\cos x)\,dy =$

$= -\frac{3y^3}{3} - \frac{2y^2\cos x}{2}$

$= -y^3 - y^2\cos x$

$\frac{x^2}{2} - y^3 - y^2\cos x + C = 0$

(11) $(y\ln y - e^{-xy})dx + \left(\frac{1}{y} + x\ln y\right)dy = 0$

$\frac{M}{2y} = \left((1\ln y + y\frac{1}{y}) + x e^{-xy}\right)$

$\frac{M}{\partial y} = \left[\ln y + y\cdot\frac{1}{y} + x e^{-xy}\right] \rightarrow \ln y + 1 + x e^{-xy}$

$\frac{M}{2x} = (0 + [(1)\ln y + x] = \ln y + x$

NO es exacta.

(13) $x\frac{dy}{dx} = 2xe^x - y + 6x^2$

$x\,dy = (2xe^x - y + 6x^2)dx$

$0 = (2xe^x - y + 6x^2)dx + (-x\,dy)$

$\frac{M}{2y} = 2xe^x - y + 6x^2 \rightarrow -1$

exacto

$\frac{M}{2x} = -x\,dy \rightarrow -1$

$$\int (2xe^x - y + 6x^2)\,dx =$$

$$\int (2xe^x)\,dx - y\int dx + \int 6x^2\,dx$$

$u = 2x \qquad v = \int e^x\,dx$

$du = 2\,dx \qquad v = e^x$

$du = 2\,dx$

$$= 2xe^x - \int e^x (2)\,dx$$

$$= 2xe^x - 2e^x$$

$$(2xe^x - 2e^x) - yx + \frac{6x^3}{3} + C$$

$$\int -(x)\,dy = -xy + C$$

$$\boxed{-xy + 2x^3 + C = 0}$$

15) $$\left(x^2y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$$

$$\left[x^2y^3 - \frac{1}{1+9x^2}\right]\frac{dx}{dy} = -x^3y^2$$

$$\left[x^2y^3 - \frac{1}{1+9x^2}\right]dx = -x^3y^2\,dy$$

$$\left[x^2y^3 - \frac{1}{1+9x^2}\right]dx + x^3y^2\,dy = 0$$

$$\left[x^2y^3 - \frac{1}{1+9x^2}\right]dx + x^3y^2\,dy = 0$$

$$\frac{M}{2y}\left[x^2y^3 - \frac{1}{1+9x^2}\right]$$

$$3x^2y^2 - 0 \Rightarrow \frac{M}{2y}(3x^2y^2)$$

$$\frac{M}{2x} = x^3y^2 \rightarrow (3x^2y^2)$$

$$\int\left(x^2y^3 - \frac{1}{(1+9x^2)}\right)dx$$

$$\int (x^2y^3)\,dx - \int \frac{1}{(1+(3x)^2)}\,dx$$

$$\frac{x^3}{3}y^3 - \int \frac{1}{1+u^2}\left(\frac{du}{3}\right)$$

$u = 3x$

$du = 3\,dx$

$\frac{du}{3} = dx$

$$\frac{x^3}{3}y^3 - \frac{1}{3}\int \frac{1}{1+u^2}\,du$$

$$\frac{x^3}{3}y^3 - \frac{1}{3}\tan^{-1}(u) + C$$

$$\frac{x^3}{3}y^3 - \frac{1}{3}\tan^{-1}(3x)$$

$$\int x^3 y^2 \, dy$$

$$x^3 \int y^2 \, dy \Rightarrow \frac{x^3 y^3}{3}$$

$$\frac{x^3 y^3}{3} - \frac{1}{3}\tan^{-1}(3x)$$

$$\frac{1}{3}\left[x^3 y^3 - \tan^{-1}(3x)\right] + C = \emptyset$$

17 $(\tan x - \operatorname{sen} x \operatorname{sen} y)\,dx + \cos x \cos y \, dy = \emptyset$

$$\frac{M}{2y} = \emptyset - (\operatorname{sen} x \cos y)$$

$$\frac{M}{2x} = -\operatorname{sen} x \cos y$$ ← Exacta

$$\int (\tan x - \operatorname{sen} x \operatorname{sen} y)\,dx$$

$$\int \tan x \, dx - \int \operatorname{sen} x \operatorname{sen} y \, dx$$

$$\ln|\sec x| - (\operatorname{sen} y)(-\cos x) + C$$

$$\ln|\sec x| + \operatorname{sen} y \cos x + C$$

$\int \cos x \cos y \, dy$

$\cos x \int \cos y \cdot dy \Rightarrow \cos x \operatorname{sen} y + c$

$\cos x \operatorname{sen} y + \ln|\sec x| + c = \emptyset$

19) $(4t^3y - 15t^2 - y)dt + (t^4 + 3y^2 - t)dy = \emptyset$

$\frac{M}{2y} = 4t^3 + \emptyset - 1 \rightarrow 4t^3 - 1$

$\updownarrow$ Exacta

$\frac{M}{2t} (4t^3 + \emptyset - 1 \rightarrow 4t^3 - 1$

$\int (4t^3y - 15t^2 - y)\, dt$

$\frac{4t^4}{4}y - \frac{15t^3}{3} - yt \Rightarrow t^4y - 5t^3 - yt$

$\int (t^4 + 3y^2 - t)\, dy$

$t^4y + \frac{3y^3}{3} - ty \Rightarrow t^4y + y^3 - ty$

$t^4 y - yt + y^3 - 5t^3 + c = 0$

(21) $(x+y)^2 dx + (2xy + x^2 - 1) dy = 0, \quad y(4) = 1$

$(x^2 + 2xy + y^2) dx + (2xy + x^2 - 1) dy = 0$

$\frac{M}{2y}(0 + 2x + 2y)$

$\frac{M}{2x}(2y + 2x - 0)$ ↖ Exacto

$\int (x^2 + 2xy + y^2) dx$

$\frac{x^3}{3} + \frac{2yx^2}{2} + y^2 x \Rightarrow \frac{x^3}{3} + yx^2 + y^2 x$

$\int (2xy + x^2 - 1) dy$

$\frac{2xy^2}{2} + x^2 y - y \Rightarrow xy^2 + x^2 y - y$

$\frac{x^3}{3} + xy^2 + x^2 y - y + c = 0$

$\frac{1}{3} + 1 + 1 - 1 = -c \rightarrow -\frac{4}{3} = c$

$$\frac{x^3}{3} + xy^2 + x^2y - y - \frac{4}{3} = 0$$

(23) $(4y + 2t - 5)\,dt + (6y + 4t - 1)\,dy = 0$

$\frac{M}{2y}(4 + 0 - 0) = 4$

$\frac{3M}{2t}(0 + 4 - 0) = 4$

$$\int (4y + 2t - 5)\,dt$$

$4yt + \frac{2t^2}{2} - 5t$

$4yt + t^2 - 5t$

$(-1) = t$

$2 = y$

$$\int (6y + 4t - 1)\,dy$$

$\frac{6y^2}{2} + 4ty - y \Rightarrow 3y^2 + 4ty - y$

$4ty + 3y^2 + t^2 - 5t - y + c = 0$

$4(-1)(2) + 3(2)^2 + (-1)^2 - 5(-1) + 2 = C$

$8 = C$

$4ty + 3y^2 + t^2 - 5t - y - 8 = 0$

(25) $(y^2 \cos x - 3x^2 y - 2x)\,dx +$

$(2y \operatorname{sen} x - x^3 + \ln y)\,dy = 0 \quad y(0) = c$

$\frac{M}{\partial y} = 2y \cos x - 3x^2 - 0$ ← Exacto

$\frac{N}{\partial x} = 2y \cos x - 3x^2 + 0$ ←

$\int (y^2 \cos x - 3x^2 y - 2x)\,dx$

$y^2 \operatorname{sen} x - \frac{3x^3}{3} y - \frac{2x^2}{2}$

$y^2 \operatorname{sen} x - x^3 y - x^2 + c$

$\int (2y \operatorname{sen} x - x^3 + \ln y)\,dy$

$\frac{2y^2 \operatorname{sen} x}{2} - x^3 y + \cdots$

$u = \ln y \qquad dv = dy$

$u = \ln y \qquad du = dy$

$du = \frac{1}{y}\,dy \qquad v = \int dy$

$v = y$

$= y \ln y - \int y \frac{1}{y}\,dy$

$= y \ln y = \int dy \qquad y(0) = e$

$= y \ln y - y$

$y^2 \operatorname{sen} x - x^3 y + y \ln y - y$

$y^2 \operatorname{sen} x - x^3 y + y \ln y - y - x^2 + c = 0$

$(e)^2(\operatorname{sen}(0)) - (0)^3(e) + e \ln e - e - (0)^2 = -C$

$+ e(1) - e - 0 = -C$

$0 = C$

$y^2 \operatorname{sen} x - x^3 y + y \ln y - y - x^2 = 0$