

Ejercicios 2.3

$$5) y' + 3x^2y = x^2$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$e^{\int 3x^2 dx} \rightarrow e^{3\int x^2 dx} \rightarrow e^{\frac{3x^3}{3}} \rightarrow e^{x^3} \rightarrow e^x \rightarrow x^3$$

$$e^{x^3}y' + e^{x^3}3x^2y = x^2$$

$$\frac{d}{dx}(e^{x^3} \cdot y) = x^2 e^{x^3}$$

$$e^{x^3} \cdot y = \int x^2 e^{x^3} dx$$

$$u = x^3 \quad du = 3x^2 dx$$

$$du = 3x^2$$

$$\int x^2 \cdot \frac{du}{3x^2} = \int x^2 \cdot \frac{1}{3} du$$

$$e^{x^3} \cdot y = \frac{1}{3} \int \frac{x^2 e^u}{x^2} du$$

$$e^{x^3} \cdot y = \frac{1}{3} \int e^u du \rightarrow e^{x^3} \cdot y = \frac{1}{3} e^u + C$$

$$e^{x^3} \cdot y = \frac{1}{3} e^{x^3} + C \rightarrow y = \frac{e^{x^3}}{3 e^{x^3}} + \frac{C}{e^{x^3}}$$

$$y = \frac{1}{3} + \frac{C}{e^{x^3}}$$

$$7) x^2y' + xy = 1$$

$$x^2y' + xy = 1 \rightarrow \frac{x^2y'}{x^2} + \frac{xy}{x^2} = \frac{1}{x^2}$$

$$y' + \frac{y}{x} = \frac{1}{x^2}$$

$$\int \frac{1}{x} dx \rightarrow \ln x$$

$$xy' + \frac{y}{x} = \frac{x}{x^2} \rightarrow xy' + y = \frac{1}{x}$$

$$xy' + y = \frac{1}{x} \rightarrow \frac{d(xy)}{dx} = \frac{1}{x}$$

$$xy = \int \frac{1}{x} dx$$

$$xy = \ln x + c$$

$$y = \frac{\ln x}{x} + \frac{c}{x} \rightarrow y = \frac{1}{x}(\ln x + c)$$

$$9) x \frac{dy}{dx} - y = x^2 \sin x$$

$$xy' - y = x^2 \sin x \rightarrow \frac{xy'}{x} - \frac{y}{x} = \frac{x^2 \sin x}{x}$$

$$y' - \frac{y}{x} = x \sin x \rightarrow e^{\int \frac{1}{x} dx}$$

$$e^{\ln x} \rightarrow x$$

$$xy' - \frac{xy}{x} = (x \sin x) x$$

$$xy' - y = x^2 \sin x$$

$$\frac{d}{dx}(xy) = x^2 \sin x \Rightarrow xy = \int x^2 \sin x dx$$

$$u = x^2$$

$$v = \int \sin x dx$$

$$du = 2x dx$$

$$v = -\cos x$$

$$= -x^2 \cos x - 2 \int x \cos x dx$$

$$u = x$$

$$v = \int \cos x dx$$

$$du = dx$$

$$v = \sin x$$

$$= -x^2 \cos x - 2 \left[x \sin x - \int \sin x dx \right]$$

$$= -x^2 \cos x - 2 [x \sin x + \cos x]$$

$$(xy) = -x^2 \cos x - [x \sin x + \cos x]$$

$$(xy) = -x^2 \cos x - x \sin x - \cos x$$

$$y = -x \cos x - \sin x - \frac{\cos x}{x} + C$$

$$(11) \quad x \frac{dy}{dx} + 4y = x^3 - x$$

$$x y' + 4y = x^3 - x \rightarrow y' + \frac{4y}{x} = x^2 - 1$$

$$e^{\int \frac{4}{x} dx} \rightarrow c \rightarrow e^{4 \ln x} \rightarrow x^4$$

$$x^4 y' + \frac{x^4 y}{x} = x^4 (x^2) - x^4$$

$$x^4 y' + 4x^3 y = x^6 - x^4$$

$$\frac{d}{dx}(x^4 y) = x^6 - x^4$$

$$x^4 y = \int x^6 - x^4 dx \rightarrow x^4 y = \int x^6 dx - \int x^4 dx$$

$$x^4 y = \frac{x^7}{7} - \frac{x^5}{5} + C \rightarrow y = \frac{x^7}{7x^4} - \frac{x^5}{5x^4} + \frac{C}{x^4}$$

$$y = \frac{x^3}{7} - \frac{x}{5} + \frac{C}{x^4}$$

$$(13) \quad x^2 y' + x(x+2)y = e^x$$

$$\frac{x^2 y'}{x^2} + \frac{x(x+2)y}{x^2} = \frac{e^x}{x^2}$$

$$y' + \frac{(x+2)}{x} y = \frac{e^x}{x^2}$$

$$e^{\int \frac{(x+2)}{x} dx} \rightarrow \int \frac{(x+2)}{x} dx \rightarrow \int \frac{x}{x} dx + \int \frac{2}{x} dx$$

$$\int dx + 2 \ln x \rightarrow x + 2 \ln x$$

$$e^{x+2\ln x} \rightarrow e^x \cdot e^{2\ln x} \rightarrow [e^x x^2]$$

$$c x^2 y' + \frac{(e^x x^2)(x+2)}{x} y = \frac{e^x x^2 e^x}{x}$$

$$e^x x^2 y' + e^x x (x+2) y = e^{2x} x^3$$

$$\frac{d}{dx} (e^x x^2 \cdot y) = e^{2x} \rightarrow e^x x^2 y = \int e^{2x} dx$$

$$u = 2x \quad e^x x^2 y = \int \frac{e^u du}{2}$$

$$\frac{du}{2} = x \quad e^x x^2 y = \frac{1}{2} e^u + C$$

$$e^x x^2 y = \frac{1}{2} e^{2x} + C$$

$$c^x x^2 y = \frac{1}{2} c^x + c$$

$$y = \frac{c^{2x}}{2x^2 e^x} + \frac{c}{e^x x^2}$$

$$y = \frac{c^x}{2x^2} + \frac{c}{e^x x^2} \Rightarrow y = \frac{1}{x^2} \left(\frac{c^x}{2} + \frac{c}{e^x} \right)$$

(15) $y dx - 4(x+y^4)dy = 0$

$$y dx = 4(x+y^6)dy$$

$$\frac{y dx}{dy} = 4x + 4y^6$$

$$\frac{y dx}{dy} = 4x = 4y^6$$

$$y x' - 4x = 4y^6 \Rightarrow \frac{y x'}{y} - \frac{4x}{y} = \frac{4y^6}{y}$$

$$x' - \frac{4x}{y} = 4y^5$$

$$e^{\int -\frac{4}{y} dy} \rightarrow e^{-4 \ln y} \rightarrow y^{-4} \rightarrow \boxed{\frac{1}{y^4}}$$

$$y \frac{dy}{y^4} - \frac{4x}{y^5} = \frac{4y^3}{y^4} \rightarrow -\frac{x^1}{y^4} + \frac{4x}{y^5} = 4y$$

$$\frac{d}{dx} \left(\frac{1}{y^4} x \right) = 4y$$

$$\frac{1}{y^4} x = \int 4y dy \rightarrow \frac{1}{y^4} x = \frac{4y^2}{2} + C$$

$$\frac{1}{y^4} x = 2y^2 + C \rightarrow x = 2y^6 + Cy^4$$

(17) $\cos x \frac{dy}{dx} + (\sin x) y = 1$

$$\cos x y' + \sin x y = 1$$

$$\frac{\cos x y'}{\cos x} + \frac{\sin x y}{\cos x} = \frac{1}{\cos x}$$

$$y' + y \tan x = \sec x$$

$$e^{\int \tan x dx} \rightarrow \int \sec x dx \rightarrow \ln |\sec x|$$

$$e^{\ln \sec x} \rightarrow \sec x$$

$$y \sec x + y \sec x \tan x = \sec^2 x$$

$$\frac{d}{dx}(\sec x \cdot y) = \sec^2 x \rightarrow$$

$$y \sec x = \int \sec^2 x \, dx$$

$$y \sec x = \tan x + C$$

$$y = \frac{\tan x}{\sec x} + \frac{C}{\sec x}$$

$$y = \left[\frac{\sec x}{\cos x} + \frac{C}{\sec x} \right] \rightarrow y = \frac{\cos x \sec x}{\cos x} + \frac{C}{\sec x}$$

$$\boxed{y = \sec x + \frac{C}{\sec x}}$$

$$(19) \quad (x+1) \frac{dy}{dx} + (x+2)y = 2x e^{-x}$$

$$(x+1)y' + (x+2)y = 2x e^{-x}$$

$$\frac{(x+1)y'}{(x+1)} + \frac{(x+2)}{x+1}y = \frac{2x e^{-x}}{x+1}$$

$$y' + \left[\frac{x+2}{x+1} \right]y = \frac{2x e^{-x}}{x+1}$$

$$C \rightarrow \int \frac{x+2}{x+1} dx \rightarrow \int \frac{x+1+1}{x+1} dx$$

$$\int \frac{x+1}{x+1} + \frac{1}{x+1} dx$$

$$\int \frac{x+1}{x+1} dx + \int \frac{1}{x+1} dx$$

$$u = \int dx + \int \frac{1}{x+1} dx + x + \int \frac{1}{u} du$$

$$u = x+1 \quad x + \ln u$$

$$e^{x+\ln u} \rightarrow e^{x+\ln(x+1)} \rightarrow e^x \cdot e^{\ln(x+1)}$$

$$e^x (x+1)$$

$$e^x (x+1)y + \left[\frac{x+2}{x+1} \right] y e^x (x+1) = \frac{2x e^x (x+1)}{(x+1)}$$

$$e^x (x+1)y + y e^x (x+2) = 2x e^x$$

$$\frac{d}{dx} (e^x (x+1)y) = 2x e^x$$

$$e^x (x+1)y = 2 \int x e^x dx$$

$$u = x \quad v = \int e^x \rightarrow v = -1 e^{-x}$$

$$du = dx \quad = -x e^{-x} + \int e^{-x} dx \rightarrow -x e^{-x} - e^{-x}$$

$$e^x(x+1)y = -xe^{-x} - e^{-x} + c$$

$$e^x(x+1)y = -e^{-x}(x+1)$$

$$y = \frac{-e^{-x}(x+1)}{e^x(x+1)} + \frac{c}{e^x(x+1)}$$

$$y = -\frac{1}{e^{2x}} + \frac{c}{e^x(x+1)}$$

(21) $\frac{dr}{d\theta} + r \sec \theta = \cos \theta$

$$e^{\int \sec \theta d\theta} \Rightarrow e^{\ln(\sec \theta + \tan \theta)}$$

$$\Rightarrow (\sec \theta + \tan \theta)$$

$$(\sec \theta + \tan \theta)r' + r(\sec \theta + \tan \theta)\sec \theta =$$

$$\cos \theta (\sec \theta + \tan \theta)$$

$$\frac{d}{d\theta} ((\sec \theta + \tan \theta)r) = \cos \theta (\sec \theta + \tan \theta)$$

$$(\sec \theta + \tan \theta)dr = \int \cos \theta (\sec \theta + \tan \theta) d\theta$$

$$(\sec \theta + \tan \theta)r = \int \left(\frac{\cos \theta}{\cos \theta} + \frac{\sin \theta \cos \theta}{\cos \theta} \right) d\theta$$

$$(\sec \theta + \tan \theta)r = \int (1 + \sin \theta) d\theta$$

$$(\sec \theta + \tan \theta)r = \int \theta + \int \sin \theta d\theta$$

$$(\sec \theta + \tan \theta)r = \theta - \cos \theta + C$$

$$\boxed{r = \frac{\theta - \cos \theta + C}{\sec \theta + \tan \theta}}$$

(23) $x \frac{dy}{dx} + (3x+1)y = e^{-3x}$

$$xy' + (3x+1)y = e^{-3x}$$

$$y' + \frac{(3x+1)y}{x} = \frac{e^{-3x}}{x}$$

$$e^{\int \frac{3x+1}{x} dx} \rightarrow e^{\int \frac{3x+1}{x} dx} \rightarrow e^{\int 3dx + \int \frac{1}{x} dx}$$

$$e^{3x + \ln x} \rightarrow e^{3x} \cdot x$$

$$e^{3x} xy' + e^{3x} (3x+1)y = 1$$

$$\frac{d}{dx}(e^{3x} xy) = 1 \rightarrow e^{3x} y = \int dx$$

$$xe^y = \int e^{3x} dx \Rightarrow xe^y = x + C$$

$$y = \frac{x + C}{xe^{3x}} \Rightarrow y = \frac{1}{e^{3x}} + \frac{C}{xe^{3x}}$$

$$y = \frac{1}{e^{3x}} \left(1 + \frac{C}{x} \right)$$