(20) $y' = x - y$; $y(x) = ce^{-x} + x - 1$, $y(0) = 10$

$10 = ce^{-0} + 0 - 1$

$10 = c(1) - 1$

$9 = c$

24 $y(x) = 9e^{-x} + x - 1$

24 $y'(x) = -9e^{-x} + 1$

$-9e^{-x} + 1 = x - (9e^{-x} + x - 1)$

$-9e^{-x} + 1 = \cancel{x} - 9e^{-x} \cancel{- x} + 1$

24 $-9e^{-x} + 1 = -9e^{-x}$

(21) $y' + 3x^2 y = 0$; $y(x) = Ce^{-x^3}$. $y(0) = 7$

$7 = Ce^{-(0)^3}$

$7 = C$

$y(x) = 7e^{-x^3}$

$\ln y = \ln 7e^{-x^3} \rightarrow \ln y = \ln 7 + \ln e^{-x^3}$

$\ln y = \ln 7 - x^3 \ln e$ 1

$\ln y = \ln 7 - x^3$

$\frac{y'}{y} = 0 - 3x^2 \rightarrow y' = -3x^2 y$

$y' = -3x^2 (7e^{-x^3})$

$y' = -21x^2 e^{-x^3}$

$-21x^2 e^{-x^3} + 3x^2 (7e^{-x^3}) = 0$

$-21x^2 e^{-x^3} + 21x^2 e^{-x^3} = 0$

$0 = 0$

(24) $xy' - 3y = x^3$; $y(x) = x^3(c + \ln x)$,

$y(1) = 17$

$17 = 1(c + \ln(1))$

$17 = c$

$y(x) = x^3(17 + \ln x)$

$y(x) = x^3(17 + \ln x)$

$y'(x) = 3x^2(17 + \ln x) + x^3\left(\frac{1}{x}\right)$

$y'(x) = 3x^2(17 + \ln x) + x^2$

$x(3x^2(17 + \ln x) + x^2) - 3(x^3(17 + \ln x)) = x^3$

$3x^3(17 + \ln x) + x^3 - 51x^3 - 3x^3 \ln x = x^3$

$51x^3 + 3x^3 \ln x + x^3 - 51x^3 - 3x^3 \ln x = x^3$

$x^3 = x^3$ ✓

(25) $y' = 3x^2(y^2+1)$; $y(x) = \tan(x^3+C)$, $y(0)=1$

$1 = \tan(0+C)$

$1 = \tan C$

$\tan^{-1}(1) = C \rightarrow \frac{1}{4}\pi = C$

$y(x) = \tan(x^3 + \frac{1}{4}\pi)$

$y'(x) = \sec^2(x^3+\frac{1}{4}\pi) \cdot 3x^2$

$y' = 3x^2(\tan^2(x^3+\frac{1}{4}\pi)+1)$

$y' = 3x^2(\sec^2\theta - 1 + 1)$

$y' = 3x^2(\sec^2\theta)$

$3x^2\sec^2(x^3+\frac{1}{4}\pi) = 3x^2(\sec^2(x^3+\frac{1}{4}\pi))$

$1+\tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

(25) $y' + y\tan x = \cos x$; $y(x) = (x+C)\cos x$,

$y(\pi) = 0$

$0 = (\pi + C)\cos\pi$

$0 = (\pi + C)(-1) \rightarrow 0 = -\pi - C$

$\pi = -C$

$-\pi = C$

$y(x) = (x - \pi)\cos x$

$y(x) = x\cos x - \pi\cos x$

$y'(x) = [\cos x - x\,\text{sen}\,x] + \pi\,\text{sen}\,x$

$y'(x) = \cos x - x\,\text{sen}\,x + \pi\,\text{sen}\,x$

$\cos x - x\,\text{sen}\,x + \pi\,\text{sen}\,x + (x\cos x - \pi\cos x)\tan x = \cos x$

$\cos x - x\,\text{sen}\,x + \pi\,\text{sen}\,x + x\cos x\left(\frac{\text{sen}\,x}{\cos x}\right) - \pi\cos x\left(\frac{\text{sen}\,x}{\cos x}\right) = \cos x$

$\cos x - x\,\text{sen}\,x + \pi\,\text{sen}\,x + x\,\text{sen}\,x - \pi\,\text{sen}\,x = \cos x$

$\cos x = \cos x$ //