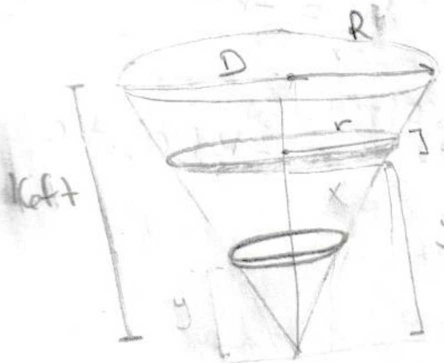
(56) En el tiempo $t=0$ se retira el tapón del fondo (en el vértice) de un tanque cónico de 16ft de altura, lleno de agua. Después de 1h el agua del tanque tiene una altura de 9ft, ¿cuándo quedará vacío?



$t = 1h \Rightarrow y = 9ft$

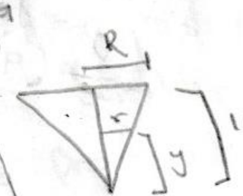
$A = \pi r^2$

$V = L A \to 16 * \pi r^2$

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

$$A(y)\frac{dy}{dt} = -a\sqrt{2g}\sqrt{y} \to A(y)\frac{dy}{\sqrt{y}} = -a\sqrt{2g}\,dt$$

$$\frac{R}{16} = \frac{r}{y} \to \boxed{r = \frac{yR}{16}}$$



$$A = \pi\left(\frac{yR}{16}\right)^2 \to A = \pi\frac{y^2R^2}{256} \to A = \pi r^2$$

$$\frac{\pi y^2 R^2}{256}\cdot\frac{dy}{\sqrt{y}} = -a\sqrt{2g}\,dt$$

esto es constante → K

$$\frac{y^2\,dy}{\sqrt{y}} = \boxed{-\frac{256\,a\sqrt{2g}}{\pi R^2}}\,dt$$

$$\frac{y^2\,dy}{\sqrt{y}} = \frac{-256(a)\sqrt{2g}}{\pi R^2}\,dt$$

$$\frac{y^2\,dy}{\sqrt{y}} = -k\,dt \rightarrow \int \frac{y^2\,dy}{\sqrt{y}} = -k\int dt$$

$$\int y^{3/2}\,dy = -k\int dt \rightarrow \frac{2y^{5/2}}{5}\,dy = -kt + c$$

$$\frac{2}{5}y^{5/2} = -kt + c \rightarrow \frac{2}{5}(16)^{5/2} = -k\cancel{(0)} + c$$

$$\frac{2}{5}(16)^{5/2} = c \rightarrow \frac{2048}{5} = c$$

$$\frac{2}{5}y^{5/2} = -\frac{256}{\pi}\sqrt{2}\,kt + \frac{2048}{5}$$

$$\frac{2}{5}(9)^{5/2} = -\frac{256}{\pi}\sqrt{2}\,k + \frac{2048}{5}$$

$$-\frac{1562}{5} = \frac{-256}{\pi}\sqrt{2}\,k$$

$$+\frac{1562}{5} \cdot \frac{\pi}{256\sqrt{2}} = k$$

$$2.71 = k$$

$$\frac{2}{5}(y)^{5/2} = -\frac{256}{\pi}\sqrt{2}(2.71)t + \frac{2048}{5}$$

$$\frac{2}{5}(y)^{5/2} = -312.30t + \frac{2048}{5}$$

$$0 = -312.30t + \frac{2048}{5}$$

$$-\frac{2048}{5} = -312.30t$$

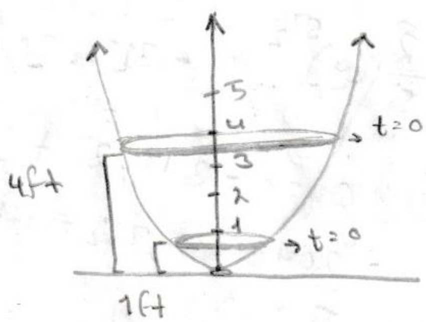
$$\frac{-2048}{-(5)(312.30)} = t$$

$$1.31 = t$$

El tanque quedara vacio en 1.31 horas.

(59) Un tanque de agua tiene la forma obtenida al girar la parabola $x^2 = by$ alrededor del eje y. La profundidad del agua es de 4ft a las 12 del día, cuando se quita el tapon circular del fondo del toque. A la 1 P.M. la profundidad del agua 1ft (a) ¿cúal es la profundidad del agua y(t) que permanece despues de t h? b) ¿cuando queda vacio el tanque? c) Si el radio inicial de la superficie superior del agua es de 2ft, ¿cual es el ~~de~~ radio del orificio circular en el fondo?

$$A(y)\frac{dy}{dt} = -a\sqrt{2yg}$$



$$\frac{A(y)\,dy}{\sqrt{y}} = -a\sqrt{2g}\,dt$$

$$\frac{(\pi b y)\,dy}{\sqrt{y}} = -a\sqrt{2g}\,dt$$

$A(y) = \pi r^2 \rightarrow$ $r = x$

$A(y) = \pi(\sqrt{by})^2$ $x^2 = by$

$A(y) = \pi b y$ $x = \sqrt{by}$

$$\frac{y\,dy}{\sqrt{y}} = \frac{-a\sqrt{2g}}{\pi b}\,dt$$

$$\sqrt{y}\,dy = -K\,dt$$

$$\int \sqrt{y}\,dy = -K\int dt \rightarrow \frac{2y^{3/2}}{3} = -Kt + C$$

$$\frac{2(4)^{3/2}}{3} = \cancel{-K(0)} + C \rightarrow C = \frac{16}{3}$$

$$\frac{2y^{3/2}}{3} = -Kt + \frac{16}{3} \quad (a)$$

$$\frac{2(1)^{3/2}}{3} = -K + \frac{16}{3} \rightarrow \frac{2}{3} - \frac{16}{3} = -K$$

$$-\frac{14}{3} = -K$$

$$\frac{2y^{3/2}}{3} = -\frac{14}{3}t + \frac{16}{3} \rightarrow 2y^{3/2} = -14t + 16$$

$$y^{3/2} = -7t + 8 \quad (a)$$

$$\boxed{y = (-7t+8)^{2/3}}$$

(b)

$\cancel{\frac{2(0)^{3/2}}{3}} = -\frac{14}{3}t + \frac{16}{3} \rightarrow 0 = -\frac{14}{3}t + \frac{16}{3}$

$-\frac{16}{3} = -\frac{14}{3}t \rightarrow \frac{-\cancel{3}(16)}{\cancel{3}(-14)} = t$

$\frac{-16}{-14} = t \rightarrow t = 8/7$

(c)

$\pi \frac{y\,dy}{\sqrt{y}} = -a\sqrt{2g}\,dt$

$\frac{y\,dy}{\sqrt{y}} = \frac{-a\sqrt{2 * 32}}{\pi}dt$ 8

$\int \sqrt{y}\,dy = \int -\frac{a\,8}{\pi}dt \rightarrow \frac{2y^{3/2}}{3} = -\frac{8a}{\pi}t + c$

$\frac{2(4)^{3/2}}{3} = \cancel{-\frac{8a(0)}{\pi}} + c$

$t = 1 \quad y = 1$

$\boxed{\frac{14}{3} = c} \rightarrow \frac{2(1)^{3/2}}{3} = \frac{-8(1)a}{\pi} + \frac{14}{3}$

$\frac{2}{3} = -\frac{8a}{\pi} + \frac{14}{3} \rightarrow \frac{-14}{3} = -\frac{8a}{\pi}$

$a = \frac{-14(\pi)}{-(3)(8)} \rightarrow \boxed{a = \frac{7}{12}\pi}$

$$\frac{7}{12}\pi = r^2\pi$$

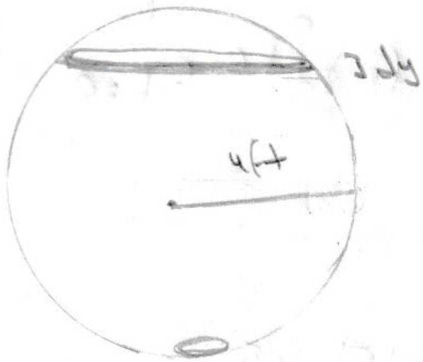
$$r^2 = \frac{7\cancel{\pi}}{12\cancel{\pi}} \rightarrow r = \sqrt{\frac{7}{12}} = \frac{\sqrt{21}}{6}$$

a) La profundidad del agua despues de t horas es igual a: $y = (-7t + 8)^{2/3}$

b) El tanque queda vacio despues de $\frac{8}{7}$ de hora despues de las 12:00

c) El radio del orificio donde sale el agua es de $\frac{\sqrt{21}}{6}$ ft

C.1 Un tanque esferico con un radio de 4 ft esta lleno de gasolina cuando se abre un orificio con un radio de 1 pulgada en la parte inferior. ¿cuanto tiempo se requiere para que toda la gasolina salga del tanque.



$$x^2 + (y-4)^2 = r^2$$

$$x^2 + (y-4)^2 = 16$$

1ft = 12in

$$1\,in * \frac{1\,ft}{12\,in} = \frac{1}{12}\,ft$$

$$a = \pi\left(\frac{1}{12}\right)^2 = \frac{1}{144}\pi$$

$$A(y)\,dy = -a\sqrt{2gy}$$

$$\frac{A(y)\,dy}{\sqrt{y}} = -a\sqrt{64}\;8 \rightarrow \frac{A_y\,dy}{\sqrt{y}} = -8a\,dt$$

$$A_y = \pi R^2$$

$$A_y = \pi(-y^2+8y)$$

$$\frac{\pi(-y^2+8y)\,dy}{\sqrt{y}} = -8a\,dt$$

$$\pi y^{3/2} + 8\pi\sqrt{y}\,dy = -2\pi$$

$$x^2 = 16 - (y-4)^2$$

$$x^2 = 16 - (y^2 - 8y + 16)$$

$$x^2 = \cancel{16} - y^2 + 8y - \cancel{16}$$

$$x = \sqrt{-y^2+8y} \rightarrow x = \sqrt{y(-y+8)}$$

$$x = \sqrt{y}\cdot\sqrt{-y+8}$$

$$\pi\left[-\int y^{3/2}\,dy + 8\int y^{1/2}\,dy\right] = -8at + c$$

$$\pi\left[-y^{5/2}\cdot\frac{2}{5} + 8y^{3/2}\,\frac{2}{3}\right] = -8at + c$$

$$-\frac{2\pi}{5}(8)^{5/2}+\frac{16\pi}{3}(8)^{3/2}=\cancel{-2\pi(0)^{0}}+c \qquad \begin{matrix} 8=y \\ t=0 \end{matrix}$$

$$151.65=c$$

$$\cancel{-\frac{2\pi}{5}(0)^{5/2}}+\cancel{\frac{16\pi}{3}(0)^{3/2}}=-8at+151.65$$

$$\frac{-151.65}{-8\left(\frac{1}{144}\pi\right)}=t$$

$$\boxed{868.89=t}$$

El tanque se vaciara en 868.89 segundos