MAT135H1F - Calculus 1(A)

May - June 2015

What We Expect You To Know

The purpose of this document is to outline the basic concepts with which students starting MAT135H1F should be familiar. It does not purport to be a complete list, but it covers anything essential. Some of these concepts will be briefly covered in lecture, and in particular in the first lecture, but many will not be.

Wherever possible and appropriate, the list includes references to one or two websites or sections of the textbook where the student can find a refresher on the material in question. The Department of Mathematics has also prepared an excellent website for this purpose, and the student is encouraged to take some time to visit it and complete the Self-Tests in each subject area. This website can be found here:

`http://www.math.toronto.edu/preparing-for-calculus`

This document may be updated as better sources for covering the listed topics are discovered.

- Basic algebra
  - Solving simple equations like $3x + 4 = 2$ for x.
  - Manipulating fractions correctly. In particular:
$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{but} \quad \frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$
  - Dealing with exponents:
    - $a^{-b} = \frac{1}{a^b}$,
    - $(a^b)(a^c) = a^{b+c}$,
    - $(a^b)^c = a^{bc}$,
    - $a^{\frac{1}{n}} = \sqrt[n]{a}$, etc.
  - Expanding and factoring algebraic expressions. For example:
    - Expanding:
$$(x+5)(x-3) = x^2 - 3x + 5x - 15 = x^2 + 2x - 15;$$
    - Factoring:
$$2x^2 - x - 15 = (x-3)(2x+5);$$
    - Completing the square:
$$\begin{aligned} 2x^2 - 6x + 1 &= 2(x^2 - 3x) + 1 \\ &= 2(x^2 - 3x + \tfrac{9}{4}) - \tfrac{9}{2} + 1 \\ &= 2(x - \tfrac{3}{2})^2 - \tfrac{7}{2} \end{aligned}$$

- Intervals, Inequalities and Absolute Values
  - Standard notation for intervals:

$$\begin{aligned}(3,7) &= \{\, x \in \mathbb{R} : 3 < x < 7 \,\} \\ [3,7) &= \{\, x \in \mathbb{R} : 3 \leq x < 7 \,\} \\ [3,7] &= \{\, x \in \mathbb{R} : 3 \leq x \leq 7 \,\} \\ (-\infty,3] &= \{\, x \in \mathbb{R} : x \leq 3 \,\}\end{aligned}$$

    Some of you who learned this material in other countries may be used to the notation $]3,7[$ instead of $(3,7)$, but we will always use the latter.
  - Solving linear inequalities, such as $-4 < -2x + 8 < 5$.
  - Solving inequalities with higher-degree polynomials, such as $x^2 + 2x + 1 > 4$.
  - Solving linear inequalities involving absolute values, such as $2 < |2x + 5| \leq 3$.
  - See Appendix A in your textbook, and

    `http://www.math.toronto.edu/preparing-for-calculus/2_inequalities/inequalities.html`.

- Functions
  - The definition of a function.
  - What it means for a function to be "one-to-one" (also called "injective") and "onto" (also called "surjective").
  - The domains and ranges of basic functions, like $\sqrt[n]{x}$, e^x, $\ln(x)$, $\sin(x)$, $\tan(x)$, etc.
  - The graphs of basic functions, such as the ones above.
  - How changing a function in simple ways changes its graph. For example if you know the graph of $f(x)$, you should be able to graph $f(x-2)$ and $f(3x)$.
  - How to compose functions, and to find the domains and ranges of compositions of functions. For example, what is the domain of $\ln(\sin(x))$?
  - What the inverse of a function is, what functions have inverses, and how to find inverses of simple functions. (To be covered in lecture.)
  - See for example Sections 1.2 and 1.6 of your text, and

    `http://www.math.toronto.edu/preparing-for-calculus/4_functions/functions.html`

- Polynomials are a particular class of functions with which you should be familiar.
  - Graphing linear functions and quadratics.
  - The quadratic formula. That is, given a quadratic equation $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- How to find the equation for a polynomial given its roots. That is for example if a polynomial has roots 2, -1, and 4 with multiplicity 2, then its equation is

$$(x-2)(x+1)(x-4)^2.$$

- Long dividing polynomials. (In general, not just "synthetic division".)
- See the following for as many long division problems as you could ever want.

`http://www.math.toronto.edu/preparing-for-calculus/5_polynomials/we_3_dividing.html`

- High School Trigonometry
  - How to work in radians rather than degrees.
  - The definitions of sine, cosine and tangent in terms of the sides of a right-angle triangle.
  - The graphs of these functions, as well as simple modifications of them. For example, how to graph functions like

$$y = 1 + 3\cos(\tfrac{1}{2}x + \pi)$$

  - The definitions of secant, cosecant and cotangent in terms of sine, cosine and tangent.
  - Sine Law and Cosine Law.
  - The fundamental trigonometric identities and how to manipulate them to prove other simple identities, such as:
    - The Pythagorean Identity: $\sin^2(x) + \cos^2(x) = 1$
    - The double-angle identities: $\sin(2x) = 2\sin(x)\cos(x)$ and $\cos(2x) = 1 - 2\sin^x 2(x)$.
    - Be able to show for example that:

$$\tan^2(x) - \sin^2(x) = \tan^2(x)\sin^2(x).$$

  - How to solve simple trigonometric equations in a given range. For example, solve

$$\sin(2x) = \cos(x) \quad \text{for } x \text{ in } [0, 2\pi].$$

  - See for example Appendix D in your textbook, or

`http://www.math.toronto.edu/preparing-for-calculus/8_trigonometry/trigonometry.html`