University of Toronto
Faculty of Arts and Science
Term Test
MAT135H1F - Calculus I (A)
Duration - 2 hours
No Aids Allowed

Family Name: ______________________________

Given Name: ______________________________

Student Number: ______________________________

Lecture and Tutorial section:

L0101 TR10-1	L5101 TR6-9	T0101 TR1	T0102 TR1	T5101 TR5

This exam contains 11 pages (including this cover page) and 7 problems. Check to see if any pages are missing and ensure that all required information at the top of this page has been filled in.

No aids are permitted on this examination. Examples of illegal aids include, but are not limited to textbooks, notes, calculators, or any electronic device.

Unless otherwise indicated, you are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work!** Write your answers in the spaces provided. Work scattered over the page without clear ordering will receive very little credit.
- **Justify your answers!** A correct answer without explanation or algebraic work will receive **no credit**; an incorrect answer supported by correct calculations and explanations **may** still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	25	
2	10	
3	30	
4	10	
5	10	
6	10	
7	25	
Total:	120	

1. (a) (5 points) Define what it means for a function $f(x)$ to be continuous at $a = 1$.

(b) (5 points) Give an example of a function $f(x)$ and point a such that both $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ do not exist. Sketch its graph.

(c) (5 points) Let $g(x) = x^2$ and let $f(x)$ be defined by

$$f(x) = \begin{cases} -3 & \text{when } x \geq 0, \\ 3 - x & \text{when } x < 0. \end{cases}$$

Is $f \circ g$ continuous? Is $g \circ f$ continuous?

(d) (10 points) What are the three main types of discontinuity discussed in class? For each one, give its name, a function with that type of discontinuity, and a sketch of its graph.

2. (a) (4 points) State the Intermediate Value Theorem.

   (b) (6 points) Show that the equation $\sin(t) = t^2 - 1$ has at least two solutions.

3. (a) (5 points) Evaluate $\lim_{s\to 1} \ln(\cos(2\pi s))$.
Here "ln" denotes the natural logarithm, which is sometimes also denoted by "log".

(b) (5 points) Find the horizontal asymptotes of the function $f(x) = \dfrac{1 - 8x + 2x^2}{3x^2 + 9}$.

(c) (5 points) Suppose that $\lim_{x\to 0} \dfrac{f(x)}{x} = -22$. What is $\lim_{x\to 0} f(x)$?

(d) (5 points) Evaluate $\lim_{s\to\infty}(2^s - 3^s)$.

(e) (5 points) Find a value for c such that $f(x)$ is continuous on $(-\frac{\pi}{2}, \frac{\pi}{2})$, where

$$f(x) = \begin{cases} \tan(x) & \text{when } x \geq \pi/4, \\ c \cdot x & \text{when } x < \pi/4. \end{cases}$$

(f) (5 points) Evaluate $\lim_{t\to 0}\dfrac{\tan(2t)}{\tan(3t)}$.

4. (10 points) Evaluate $\lim_{x \to \infty} \dfrac{\cos(2x) - e^x}{e^x + \arctan(x)}$.

5. (10 points) Find the derivative of $f(x) = \dfrac{1}{\sqrt{1+x}}$ at $x = 0$ using the definition of the derivative as a limit.

6. Let $g(x) = xe^{-x^2}$.

   (a) (5 points) Find all points where the graph of $g(x)$ has horizontal tangent lines.

   (b) (5 points) What is the equation of the tangent line to $g(x)$ at $x = 1$?

7. Answer the following questions about derivatives.

(a) (5 points) Find the 3rd derivative of $x^3 + x^2 + x + 1$ at $x = -1$.

(b) (5 points) Find the derivative of $f(x) = \dfrac{\sin(x)}{\cos(x) + 1}$. Simplify your answer as much as possible.

(c) (5 points) Find the derivative of $g(x) = \ln(x \ln(x^2))$.

(d) (5 points) You are falling into a black hole along a straight line. Your distance from the spaceship from which you fell is given by $e^{\sqrt{t}}$ meters after t seconds. What is your *acceleration* after 4 seconds?

(e) (5 points) Suppose that $h(x)$ is a differentiable function and that $h'(x) = 2x + 1$. Further assume that $h(1) = 2$ and $h(2) = 3$. What is $\frac{d}{dx}(h \circ h \circ h)(x)$ at $x = 1$?