MAT135H1F – Quiz 2

TUT5102

May 28 , 2015

FAMILY NAME: .... GIVEN NAME: ....

Mark your lecture and tutorial sections: STUDENT ID: ....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below! Each problem is worth 1 point. Good luck!

Question 1. *What is the 3rd derivative of* $\cos(x)$*?*

(a) $\sin(x)$

(b) $-\sin(x)$

(c) $\cos(x)$

(d) $-\cos(x)$

Answer: (a) $\sin(x)$. We can simply compute the first, second and third derivatives. We have $\frac{d}{dx}\cos(x) = -\sin(x)$ so $\frac{d^2}{dx^2}\cos(x) = -\frac{d}{dx}\sin(x) = -\cos(x)$. Then $\frac{d^3}{dx^3}\cos(x) = -\frac{d}{dx}\cos(x) = \sin(x)$.

Question 2. *Let* $f(x) = e^x \sin(x)$*. What is* $f'(\frac{\pi}{2})$*?*

(a) $\frac{\pi}{2}e^{\frac{\pi}{2}}$

(b) $\frac{\pi}{2}$

(c) $\frac{2+\pi}{2}e^{\frac{\pi}{2}}$

(d) $e^{\frac{\pi}{2}}$

Answer: (d) $e^{\frac{\pi}{2}}$. First we compute the derivative of f with respect to x by using the product rule. We obtain $f'(x) = e^x\sin(x) + e^x\cos(x) = e^x(\sin(x) + \cos(x))$. Then $f'(\frac{\pi}{2}) = e^{\frac{\pi}{2}}(\sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})) = e^{\frac{\pi}{2}}$.

Question 3. *Let* $f(x) = \frac{1}{x}+3x$*. The equation of the tangent line to the graph of* $f(x)$ *at* $x = \frac{1}{\sqrt{3}}$ *is:*

(a) $y = -x + 2\sqrt{3}$

(b) $y = -x - 2\sqrt{3}$

(c) $y = 0$

(d) $y = 2\sqrt{3}$

Answer: (d) $y = 2\sqrt{3}$. We start by computing $f'(\frac{1}{\sqrt{3}})$ which is the slope of the desired tangent line. We have $f'(x) = \frac{-1}{x^2} + 3$ so $f'(\frac{1}{\sqrt{3}}) = 0$. Thus the tangent line is horizontal with equation $y = b$ for some constant b. We know $(\frac{1}{\sqrt{3}}, f(\frac{1}{\sqrt{3}}))$ is a point on the line so we conclude $y = f(\frac{1}{\sqrt{3}}) = 2\sqrt{3}$.

Question 4. *What is the 9th derivative of* $1 + x + x^2 + \cdots + x^8$*?*

(a) $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

(b) 1

(c) 0

(d) None of the above.

Answer: (c) 0. We know that the derivative of a sum is equal to the sum of derivatives. This means that we may consider each term of $1 + x + \cdots + x^8$ independently. Differentiating each monomial term once reduces the power of the monomial by one. Thus, differentiating n times a monomial x^n reduces the power by n and leaves us with a constant. Then, differentiating once more gives 0. Namely, differentiating a monomial more times than the degree gives 0. For example $\frac{d^8}{dx^8}x^8 = 8!$ so $\frac{d^9}{dx^9}x^8 = 0$. Therefore, we conclude that $\frac{d^9}{dx^9}(1 + x + \cdots + x^8) = 0$.

Question 5. *What is* $\lim\limits_{h\to 0} \frac{e^h-1}{h}$*?*

(a) -1

(b) 0

(c) 1

(d) ∞

Answer: (c) 1. Let $f(x) = e^x$. We know $f'(a) = f(a)$ for any $a \in \mathbf{R}$. By definition, $f'(a) = \lim\limits_{h\to 0} \frac{e^{a+h}-e^a}{h}$. Taking $a = 0$, we obtain $f'(0) = \lim\limits_{h\to 0} \frac{e^h-1}{h}$ and $f'(0) = f(0) = 1$.

Question 6. *Sketch the graph of an everywhere continuous function that is not differentiable at $x = 0$ nor at $x = 3$.*

Answer: A sharp point on a graph is an example of an instance where the left and right limits exist and and equal, the function is defined but there is no derivative. We can draw a graph with a sharp point at $x = 0$ and $x = 3$.