MAT135H1F – Quiz 3

TUT0101

June 4, 2015

FAMILY NAME: .... GIVEN NAME: ....

Mark your lecture and tutorial sections: STUDENT ID: ....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below! Each problem is worth 1 point. Good luck!

Question 1. *What is the slope of the tangent to $f(x) = \arctan(2x)$ at $x = 1$?*

(a) $\frac{2}{5}$

(b) $\frac{1}{5}$

(c) 0

(d) $\frac{1}{\sqrt{5}}$

Answer: (a) $\frac{2}{5}$. First we determine $f'(x)$. Using the chain rule and our knowledge of differentiating arctan, we obtain $f'(x) = 2\frac{1}{1+4x^2}$. The slope of the line tangent to f at $x = 1$ is given by $f'(1) = \frac{2}{1+4} = \frac{2}{5}$.

Question 2. *Let $g'(1) = 1$ and $g(1) = 2$. What is $\dfrac{d}{dx}\ln(g(x))$ at $x = 1$?*

(a) 1

(b) 2

(c) $\frac{1}{2}$

(d) 0

Answer: (c) $\frac{1}{2}$. Differentiating $\ln(g(x))$ with the chain rule gives $\dfrac{d}{dx}\ln(g(x)) = \frac{1}{g(x)}g'(x)$. Thus, evaluating at $x = 1$ gives $\frac{g'(1)}{g(1)} = \frac{1}{2}$.

Question 3. *Let $x^2y + y^2x = 2$. Find y' by implicit differentiation when $x = y = 1$.*

(a) 0

(b) 1

(c) -1

(d) Does not exist.

Answer: (c) -1. First we differentiate both sides of the equation using the chain rule and the product rule on the left side. We obtain $2xy + x^2\dfrac{dy}{dx} + 2yx\dfrac{dy}{dx} + y^2 = 0$. This implies $\dfrac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$ which gives $\dfrac{dy}{dx} = \frac{-y(2x+y)}{x(2y+x)}$. Lastly, we evaluate at $(x, y) = (1, 1)$. We have $\left.\dfrac{dy}{dx}\right|_{(1,1)} = \frac{-3}{3} = -1$.

Question 4. *Where is the horizontal tangent of $f(x) = x\ln(x)$?*

(a) At $x = 0$

(b) At $x = -1$

(c) At $x = e$

(d) At $x = e^{-1}$

Answer: (d) at $x = e^{-1}$. We solve for $f'(x)$ using the product rule to obtain $f'(x) = \ln(x) + \frac{x}{x} = \ln(x) + 1$. Note that $\ln(0)$ is undefined so we know $x \neq 0$. Horizontal lines have a slope of 0, so we solve $f'(x) = 0$ which gives $\ln(x) = -1$. This implies $x = e^{-1}$.

Question 5. *What is* $\frac{d}{dx}\frac{1}{\ln(e+x)}$ *at* $x = 0$*?*

(a) $-\frac{1}{\ln(e)^2}$

(b) $-e$

(c) $-e^{-1}$

(d) $\frac{1}{e}$

Answer: (c) $-e^{-1}$. We differentiate $(\ln(e+x))^{-1}$ using the chain rule. We have $\frac{d}{dx}(\ln(e+x))^{-1} = -(\ln(e+x))^{-2}\frac{1}{e+x} = \frac{-1}{(e+x)(\ln(e+x))^2}$. At $x = 0$, we obtain $\frac{-1}{e(\ln(e))^2} = -e^{-1}$.

Question 6. *What is* $\frac{d^2}{dx^2}\sin(2x)$ *at* $x = \frac{\pi}{4}$*?*

(a) -2

(b) 4

(c) -4

(d) 2

Answer: (c) -4. First, we differentiate with the chain rule to obtain $\frac{d}{dx}\sin(2x) = 2\cos(2x)$. Differentiating again gives $\frac{d^2}{dx^2}\sin(2x) = \frac{d}{dx}2\cos(2x) = -4\sin(2x)$. Then we evaluate the second derivative at $x = \frac{\pi}{4}$ which is $-4\sin(2\frac{\pi}{4}) = -4\sin(\frac{\pi}{2}) = -4$.