MAT135H1F – Quiz 1

TUT0101

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FAMILY NAME: ..... GIVEN NAME: .....

Mark your lecture and tutorial sections: STUDENT ID: .....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below!

Question 1. *Suppose that the function f is invertible with inverse f^{-1}. If $g(x) = (f^{-1} \circ f)(x)$ then what is $g(3)$?*

(a) 1

(b) Depends on f.

(c) 3

(d) $\frac{1}{3}$

Answer: (b) It depends on f. If f is defined at $x = 3$ then $g(3) = f^{-1}(f(3)) = 3$. However, if f is undefined at $x = 3$ then g is also undefined at $x = 3$. In general, if f is invertible with inverse f^{-1} then $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ for any x from the appropriate domains.

Question 2. *What is $\lim_{x\to-\infty} 2^{\frac{1}{x}}$?*

(a) 1

(b) 0

(c) $-\infty$

(d) -1

Answer: (a) 1. We have $\lim_{x\to-\infty} \frac{1}{x} \ln(2) = \ln(2) \lim_{x\to-\infty} \frac{1}{x} = 0$ so $\lim_{x\to-\infty} 2^{\frac{1}{x}} = \lim_{x\to-\infty} e^{\frac{1}{x}\ln(2)} = 1$.

Question 3. *Suppose that f satisfies*

$$e^{t^2+t} \le f(t) \le t^2 + t + 1$$

for all $t \in \mathbb{R}$. What is $\lim_{t\to0} f(t)$?

(a) Not determined by this information.

(b) ∞

(c) 1

(d) 0

Answer: (c) 1. We have $\lim_{t\to0}(t^2 + t) = 0$ so $\lim_{t\to0} e^{t^2+t} = 1$. We also have $\lim_{t\to0}(t^2 + t + 1) = 1$. Thus, by the Squeeze Theorem, $\lim_{t\to0} f(t) = 1$.

Question 4. *What is $\lim_{y\to1} |e^y - 5|$?*

(a) $5 - e$

(b) $e - 5$

(c) 5

(d) -5

Answer: (a) 5 - e. We know

$$|e^y - 5| = \begin{cases} e^y - 5 & \text{if } e^y \ge 5 \\ -(e^y - 5) & \text{if } e^y \le 5. \end{cases}$$

Close to $y = 1$, e^y is close to e which is smaller than 5. Thus $\lim_{y\to1} |e^y - 5| = \lim_{y\to1} -(e^y - 5) = 5 - e$.

Question 5. *What is* $\lim_{s\to 3} \dfrac{s^2-9}{3-s}$*?*

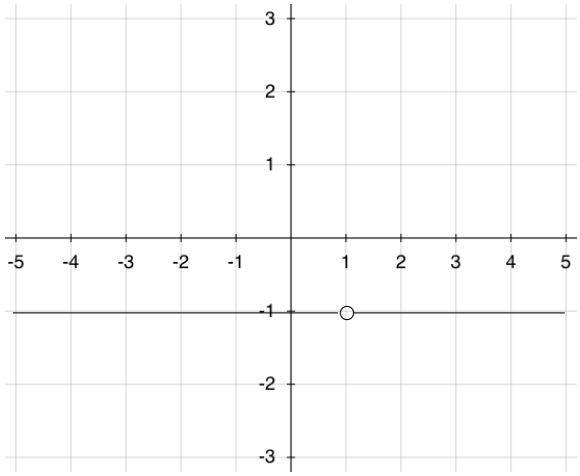
(a) ∞

(b) 3

(c) -6

(d) -3

Answer: (c) -6. Factor the numerator to obtain $\lim_{s\to 3} \dfrac{s^2-9}{3-s} = \lim_{s\to 3} \dfrac{(s+3)(s-3)}{3-s} = \lim_{s\to 3} -(s+3) = -6$.



Question 6. *Sketch the graph of a function with removable discontinuity at* $x = 1$*.*

Answer: Any function with a hole at $x = 1$ will do. This means the function is undefined at $x = 1$ but the limit as x approaches 1 exists. A simple example is $f(x) = \frac{1-x}{x-1}$.