MAT135H1F – Quiz 2

TUT0201

May 28 , 2015

FAMILY NAME: ..... GIVEN NAME: .....

Mark your lecture and tutorial sections: STUDENT ID: .....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below! Each problem is worth 1 point. Good luck!

Question 1. *What is* $\lim_{x\to 0} \frac{\cos(x)-1}{x}$ *?*

(a) 1

(b) $\frac{1}{2}$

(c) 0

(d) Does not exist.

Answer: (c) 0. This limit is standard. It is computed in detail on page 193.

Question 2. *Let* $f(x) = \sin^2(x)$*. What is* $f'(\frac{\pi}{4})$*?*

(a) 1

(b) -1

(c) $\frac{1}{\sqrt{2}}$

(d) 0

Answer: (a) 1. We differentiate using the chain rule to obtain $\frac{d}{dx}\sin^2(x) = 2\sin(x)\cos(x)$. Thus, evaluating at $\frac{\pi}{4}$ gives $2\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) = 1$.

Question 3. *Let* $f(x) = e^x - e^2x$*. The equation of the tangent line to the graph of* $f(x)$ *at* $x = 2$ *is:*

(a) $y = e^2x + 3$

(b) $y = -x + 3(e^2 - 2)$

(c) $y = 0$

(d) $y = -e^2$

Answer: (d) $y = -e^2$. We start by computing $f'(2)$ which is the slope of the desired tangent line. We have $f'(x) = e^x - e^2$ so $f'(2) = 0$. Thus the tangent line is horizontal with equation $y = b$ for some constant b. We know $(2, f(2))$ is a point on the line so we conclude $y = f(2) = e^2 - 2e^2 = -e^2$.

Question 4. *What is the 8th derivative of* $x^7 + x^6 + 1$*?*

(a) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

(b) 1

(c) 0

(d) None of the above.

Answer: (c) 0. We know that the derivative of a sum is equal to the sum of derivatives. This means that we may consider each term of $x^7 + x^6 + 1$ independently. Differentiating each monomial term once reduces the power of the monomial by one. Thus, differentiating n times a monomial x^n reduces the power by n and

leaves us with a constant. Then, differentiating once more gives 0. Namely, differentiating a monomial more times than the degree gives 0. For example $\frac{d^7}{dx^7}x^7 = 7!$ so $\frac{d^8}{dx^8}x^7 = 0$. Therefore, we conclude that $\frac{d^8}{dx^8}(x^7 + x^6 + 1) = 0$.

Question 5. *What is* $\frac{d}{dx}\frac{x}{\cos(x)}$ *at* $x = 0$*?*

(a) 0

(b) 1

(c) -1

(d) Does not exists.

Answer: (b) 1. Differentiating using the quotient rule gives $\frac{d}{dx}\frac{x}{\cos(x)} = \frac{\cos(x)+x\sin(x)}{\cos^2(x)}$. Thus, evaluating at $x = 0$ gives $\frac{\cos(0)}{\cos^2(0)} = 1$.

Question 6. *Sketch the graph of an everywhere continuous function that is not differentiable at* $x = -1$ *nor at* $x = +1$.

Answer: A sharp point on a graph is an example of an instance where the left and right limits exist and and equal, the function is defined but there is no derivative. We can draw a graph with a sharp point at $x = -1$ and $x = 1$.