MAT135H1F – Quiz 3

TUT0201

June 4, 2015

FAMILY NAME: .... GIVEN NAME: ....

Mark your lecture and tutorial sections: STUDENT ID: ....

L0101	L5101	T0101	T0201	T5101	T5102

You have 25 minutes to solve the problems below! Each problem is worth 1 point. Good luck!

Question 1. *What is the slope of the tangent to* $f(x) = \arcsin(1-2x))$ *at* $x = \frac{1}{2}$*?*

(a) 2

(b) -2

(c) 1

(d) -1

Answer: (b) -2. First we determine $f'(x)$. Using the chain rule and our knowledge of differentiating arcsin, we obtain $f'(x) = -2\frac{1}{\sqrt{1-(1-2x)^2}} = \frac{-2}{\sqrt{4x-4x^2}} = \frac{-1}{\sqrt{x(1-x)}}$. The slope of the line tangent to f at $x = \frac{1}{2}$ is given by $f'(\frac{1}{2}) = \frac{-1}{\sqrt{\frac{1}{4}}} = -2$.

Question 2. *Let* $g'(2) = 4$. *What is* $\dfrac{d}{dx}g(x^4+1)$ *at* $x = 1$*?*

(a) 4

(b) 16

(c) 32

(d) 2

Answer: (b) 16. Differentiating $g(x^4+1)$ with the chain rule gives $\dfrac{d}{dx}g(x^4+1) = 4x^3g'(x^4+1)$. Thus, evaluating at $x = 1$ gives $4g'(2) = 16$.

Question 3. *Let* $x + x^2y + y^2 = 1$. *Find* y' *by implicit differentiation when* $x = 1$ *and* $y = 0$.

(a) -1

(b) 1

(c) $-\frac{2}{3}$

(d) $\frac{2}{3}$

Answer: (a) -1. First we differentiate both sides of the equation using the chain rule and the product rule on the left side. We obtain $1 + 2xy + x^2\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$. This implies $\dfrac{dy}{dx}(x^2+2y) = -1-2xy$ which gives $\dfrac{dy}{dx} = \frac{-1-2xy}{x^2+2y}$. Lastly, we evaluate at $(x, y) = (1, 0)$. We have $\left.\dfrac{dy}{dx}\right|_{(1,0)} = \frac{-1}{1} = -1$.

Question 4. *Where is the horizontal tangent of $f(x) = x2^x$?*

(a) At $x = \frac{-1}{\ln(2)}$

(b) At $x = \frac{1}{\ln(2)}$

(c) At $\ln(2)$

(d) At $-\ln(2)$

Answer: (a) at $x = \frac{-1}{\ln 2}$. We solve for $f'(x)$ using logarithmic differentiation. We have $\ln(f(x)) = \ln(x2^x) = \ln(x) + x\ln 2$ which we differentiate implicitly to obtain $\frac{1}{f(x)}f'(x) = \frac{1}{x} + \ln 2$. Solving for $f'(x)$ gives $f'(x) = f(x)(\frac{1}{x} + \ln 2) = x2^x(\frac{1}{x} + \ln 2) = 2^x(1 + x\ln 2)$ where $x \neq 0$. Horizontal lines have a slope of 0, so we solve $f'(x) = 0$. Since $2^x > 0$ for all $x \in \mathbf{R}$, we must have $1 + x\ln 2 = 0$ so $x = \frac{-1}{\ln 2}$.

Question 5. *What is $\frac{d}{dx}\frac{1}{\ln(x)}$ at $x = e$?*

(a) $-\frac{1}{e}$

(b) 1

(c) -1

(d) $\frac{1}{e}$

Answer: (a) $-\frac{1}{e}$. We differentiate $(\ln(x))^{-1}$ using the chain rule. We have $\dfrac{d}{dx}(\ln(x))^{-1} = -(\ln(x))^{-2}\frac{1}{x} = \frac{-1}{(x)(\ln(x))^2}$. At $x = e$, we obtain $\frac{-1}{e(\ln(e))^2} = -e^{-1}$.

Question 6. *What is $\frac{d^2}{dx^2}\cos(2x)$ at $x = \pi$?*

(a) -2

(b) 4

(c) -4

(d) 2

Answer: (c) -4. First, we differentiate with the chain rule to obtain $\dfrac{d}{dx}\cos(2x) = -2\sin(2x)$. Differentiating again gives $\dfrac{d^2}{dx^2}\cos(2x) = \dfrac{d}{dx}(-2\sin(2x)) = -4\cos(2x)$. Then we evaluate the second derivative at $x = \pi$ which is $-4\cos(2\pi) = -4$.