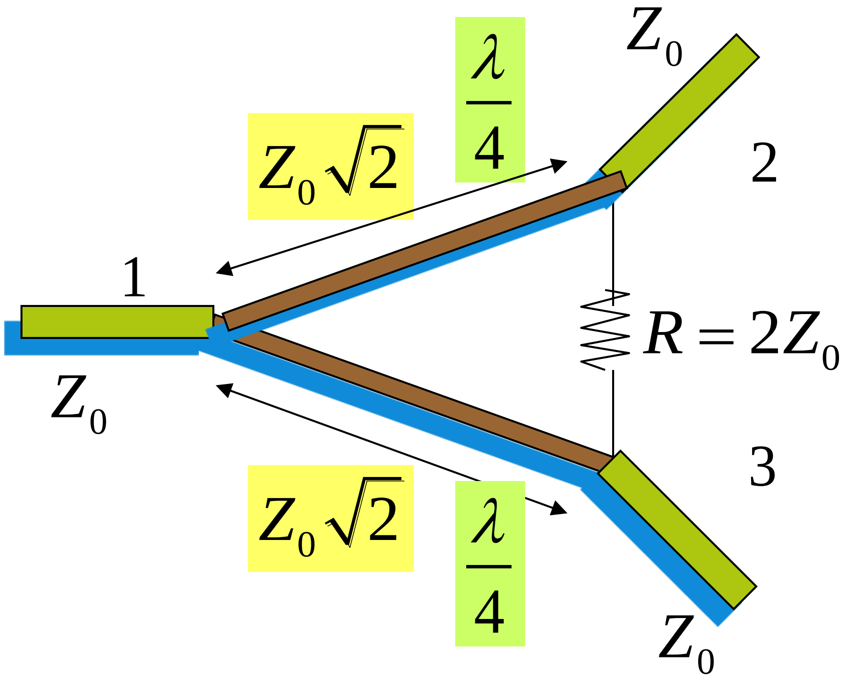
Transmission line components

EE4C05 Track specific lecture

The Wilkinson power divider

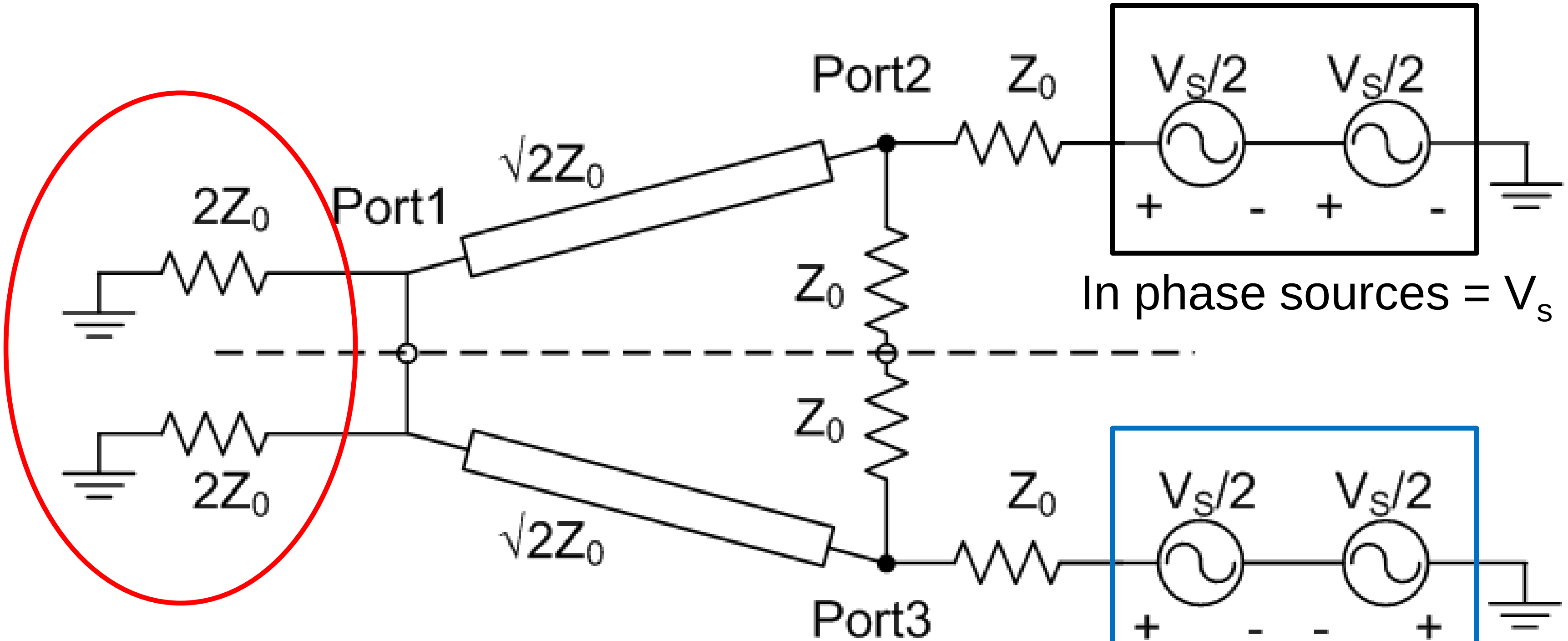
The Wilkinson power divider is a transmission line based component which can be designed lossless (low-loss with real components) when the output port are matched.
The divider can be designed with arbitrary power division, we will start considering the equal power case first (e.g. 3 dB).



- Parameters:
  - Insertion loss:
    IL[dB]=20 log $|S_{21}|$
    IL[dB]=20 log $|S_{31}|$
  - Isolation:
    Iso[dB]=-20 log $|S_{23}|$

The Wilkinson power divider

- We will analyze the circuit between port 1 and 2 by reducing it to two simpler circuits driven by symmetric (even-mode) and asymmetric (odd-mode) sources at the output ports.



In phase sources = V_s

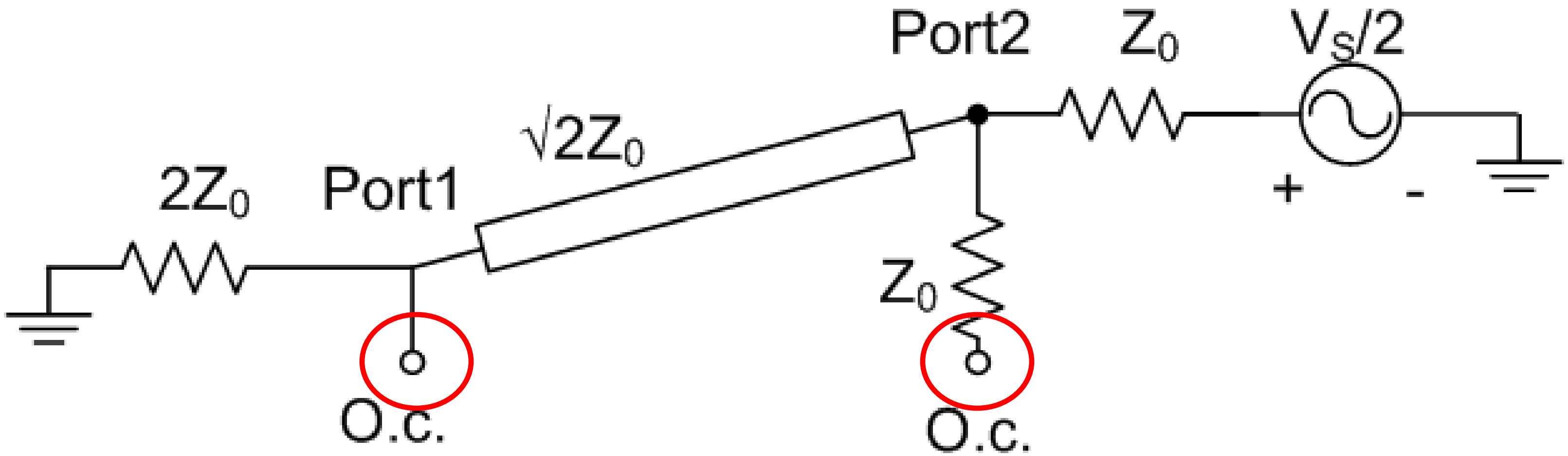
Input Z_0 replaced by the parallel of two 2 Z_0

Out of phase sources = 0

The Wilkinson power divider

Even mode

When the previous circuit is driven by two in phase signals at port 3 and 4, thus there is no current flowing along the symmetry line which can be replaced by an open circuit.

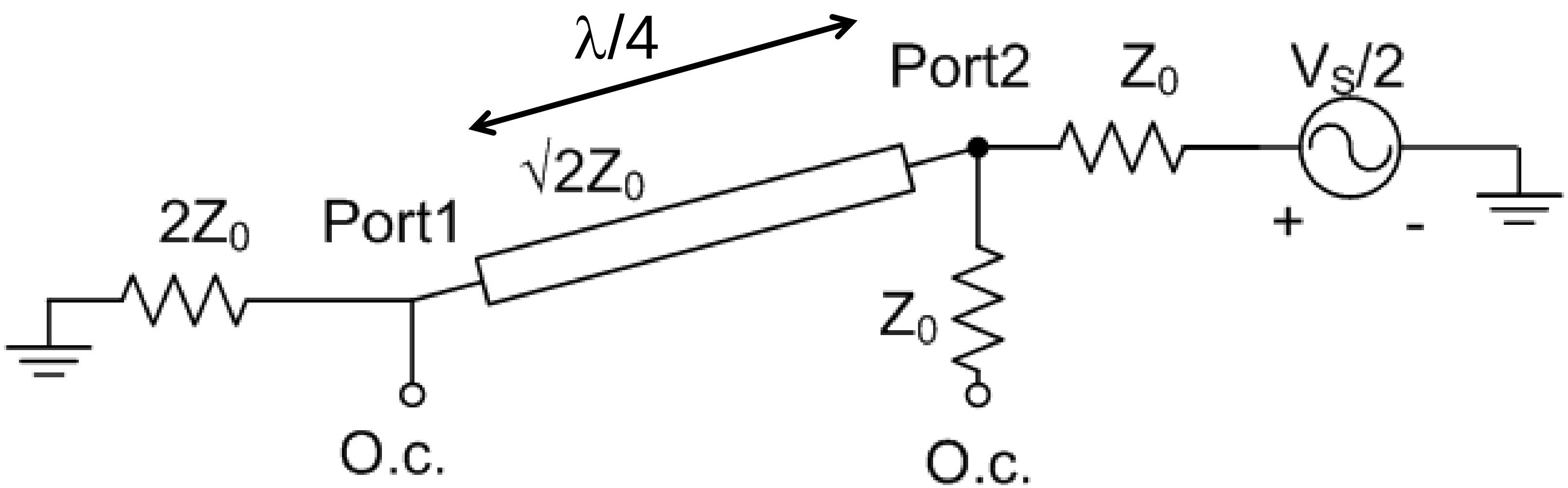


The Wilkinson power divider

- The impedance seen by port 2 is the impedance of a Ö2Z_0 quarter lambda transformer terminated with a 2Z_0 load.

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{\left(\sqrt{2}\cdot Z_0\right)^2}{2\cdot Z_0} = Z_0$$

Port 2 is matched under even mode excitation



The Wilkinson power divider

The voltage at port 2 will be

$$V_2^{even} = \frac{1}{2}\left(\frac{V_S}{2}\right) = \frac{V_S}{4}$$

If we define x=0 at port 1 and x=-λ/4 at port 2 we have

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x})$$

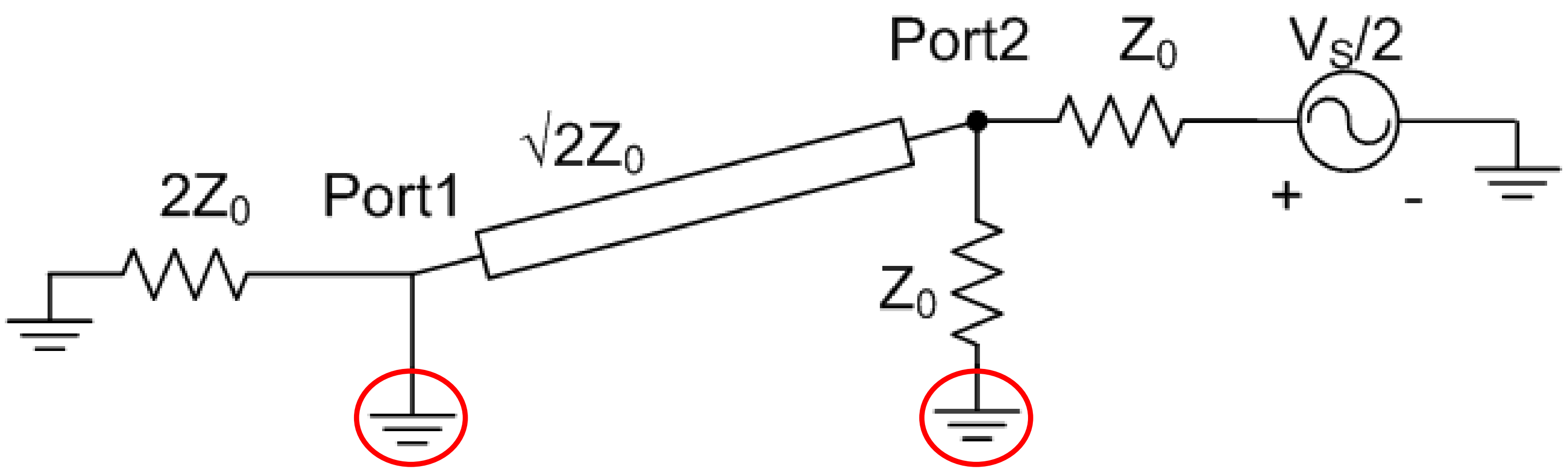
$$V_2^{even} = V\left(-\frac{\lambda}{4}\right) = jV^+(1-\Gamma) = V_0$$

$$V_1^{even} = V(0) = V^+(1+\Gamma) = jV_0\frac{\Gamma+1}{\Gamma-1}$$

$$\Gamma_{port1} = \frac{2Z_0 - \sqrt{2}Z_0}{2Z_0 + \sqrt{2}Z_0} \quad \Rightarrow \quad V_1^{even} = \frac{-j\sqrt{2}}{4}V_S$$

The Wilkinson power divider

- *Odd mode*
  In this mode of operation port 2 and 3 have opposite polarities, and there is zero potential along the middle of the circuit, this means that the middle is a virtual ground.

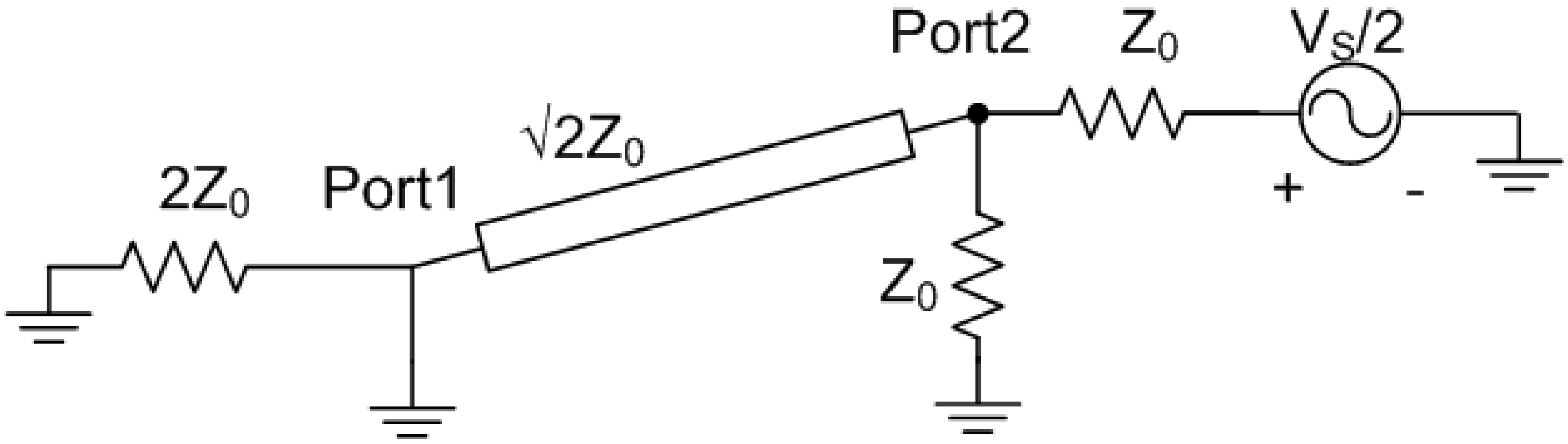


The Wilkinson power divider

- The input seen at port 2 is still Z_0 (match), then the voltage will again be:

$$V_2^{odd} = \frac{1}{2}\left(\frac{V_S}{2}\right) = \frac{V_S}{4}$$

- While V at port 1 will be 0.



The Wilkinson power divider

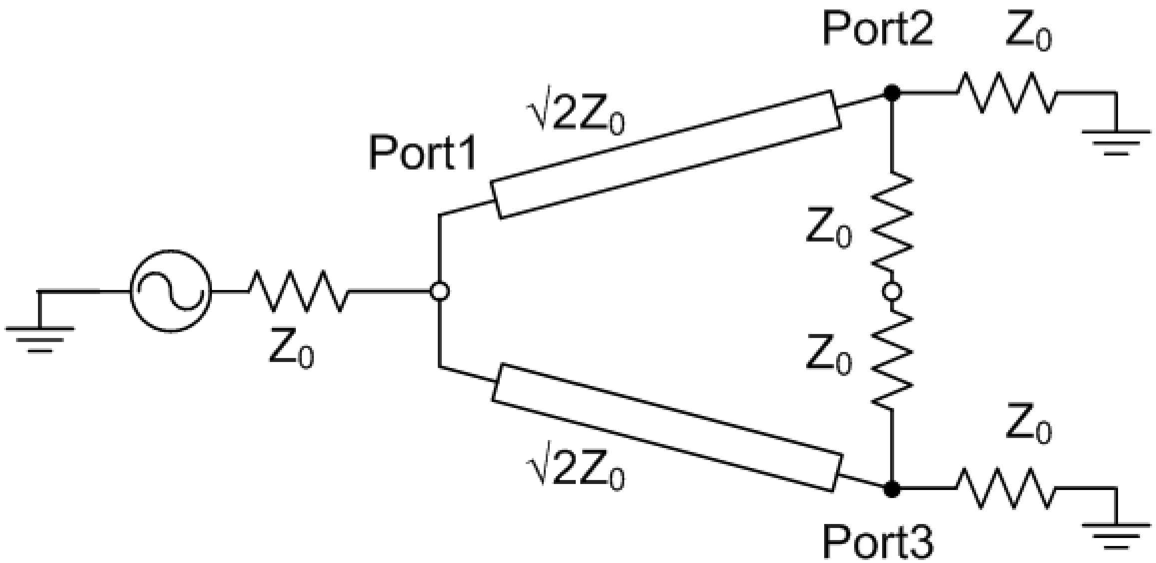
We can now calculate the voltages at port 1 and 2 by adding the even and odd mode voltages.

$$S_{12} = \frac{V_1}{V_2} = \frac{V_1^{even} + V_1^{odd}}{V_2^{even} + V_2^{odd}} = -\frac{j}{\sqrt{2}}$$

- An identical analysis can be done for port 3 and 1 leading to the result that $S_{12}=S_{13}$.
- Since the device is passive it will be reciprocal and $S_{12}=S_{21}$ and $S_{13}=S_{31}$.
- We have also seen that port 2 and 3 are isolated (open in even mode and short in odd mode), thus $S_{23}=S_{32}=0$.
- Since port 2 (for analogy port 3) was matched in both even and odd mode we have that $S_{22}=S_{33}=0$.

The Wilkinson power divider

We now have to only find the impedance seen by port 1, so we can consider the case when the source is at port 1.



This circuit is similar to the even mode case, since $V_2=V_3$, thus there will be no current through vertical resistor and it can be "removed".

The Wilkinson power divider

We are then left with two quarter lambda transformer in parallel, terminated with Z_0.

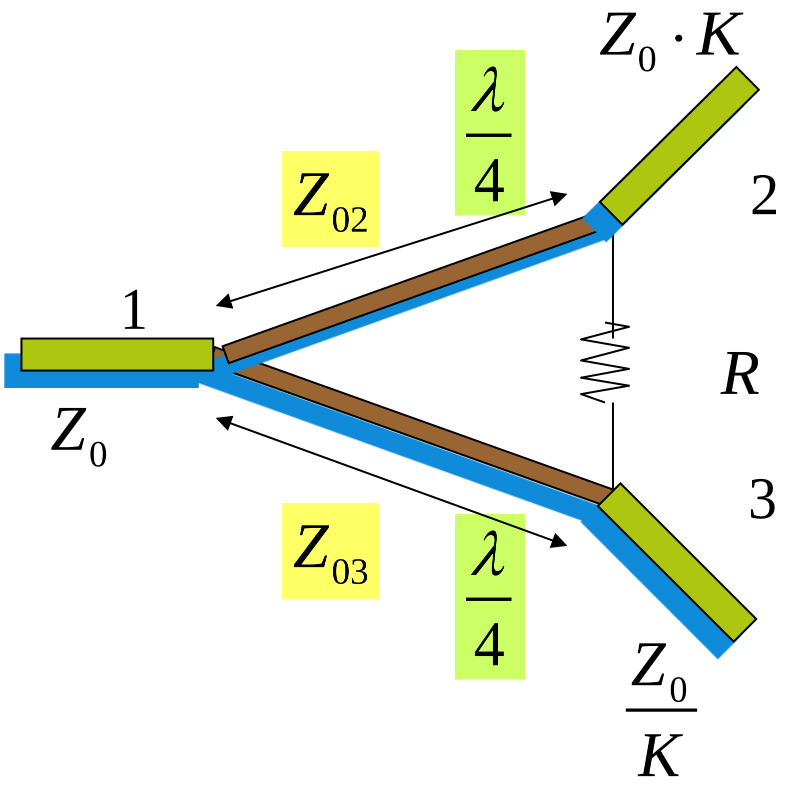
$$Z_1 = \frac{Z_0^2}{Z_L} = \frac{1}{2}\frac{\left(\sqrt{2}\cdot Z_0\right)^2}{Z_0} = Z_0$$

The scattering matrix of the Wilkinson power divider is then

$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & j \\ j & 0 & 0 \\ j & 0 & 0 \end{bmatrix}$$

The Wilkinson power divider

The Wilkinson power divider can also be made with unequal power splits.



We then have to define the power ratio between port 2 and 3.

$$K^2 = P_3 / P_2$$

then

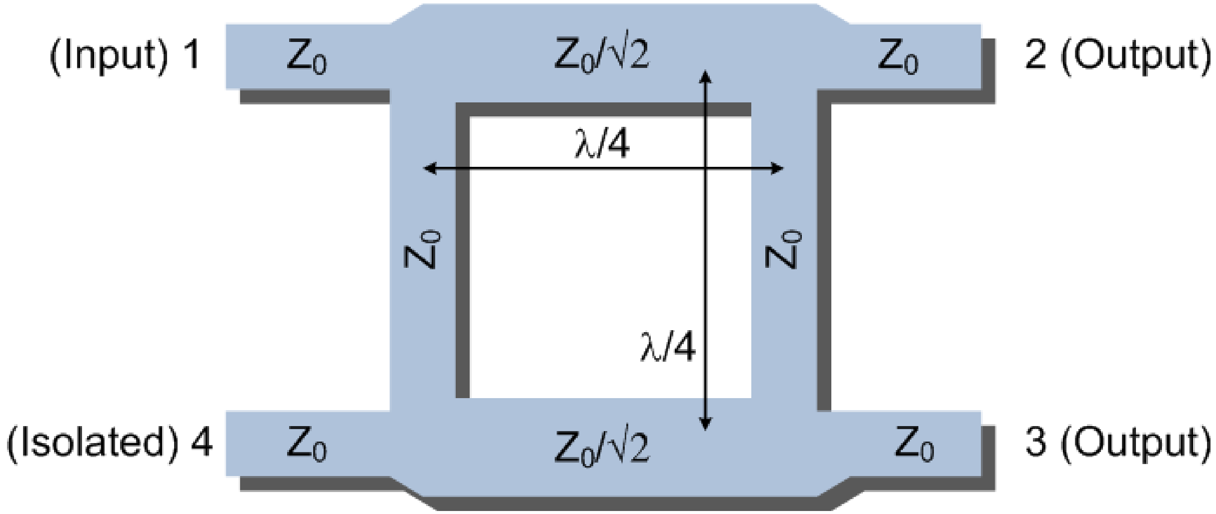
$$Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^3}}$$

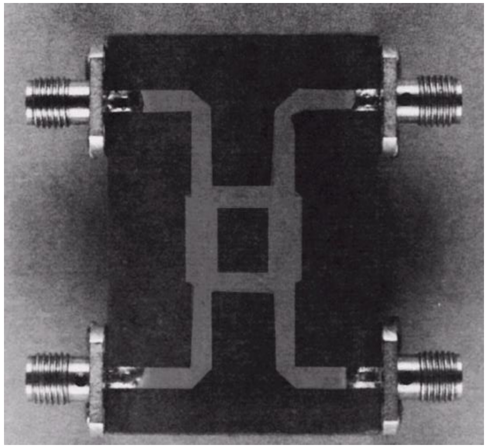
$$Z_{03} = K^2 \cdot Z_{03} = Z_0 \sqrt{K \cdot 1 + K^2}$$

$$R = Z_0 \left(K + \frac{1}{K} \right)$$

The Branch line coupler

- Quadrature hybrids are 3dB directional couplers with a 90° phase difference in the output of the through and coupled arms.



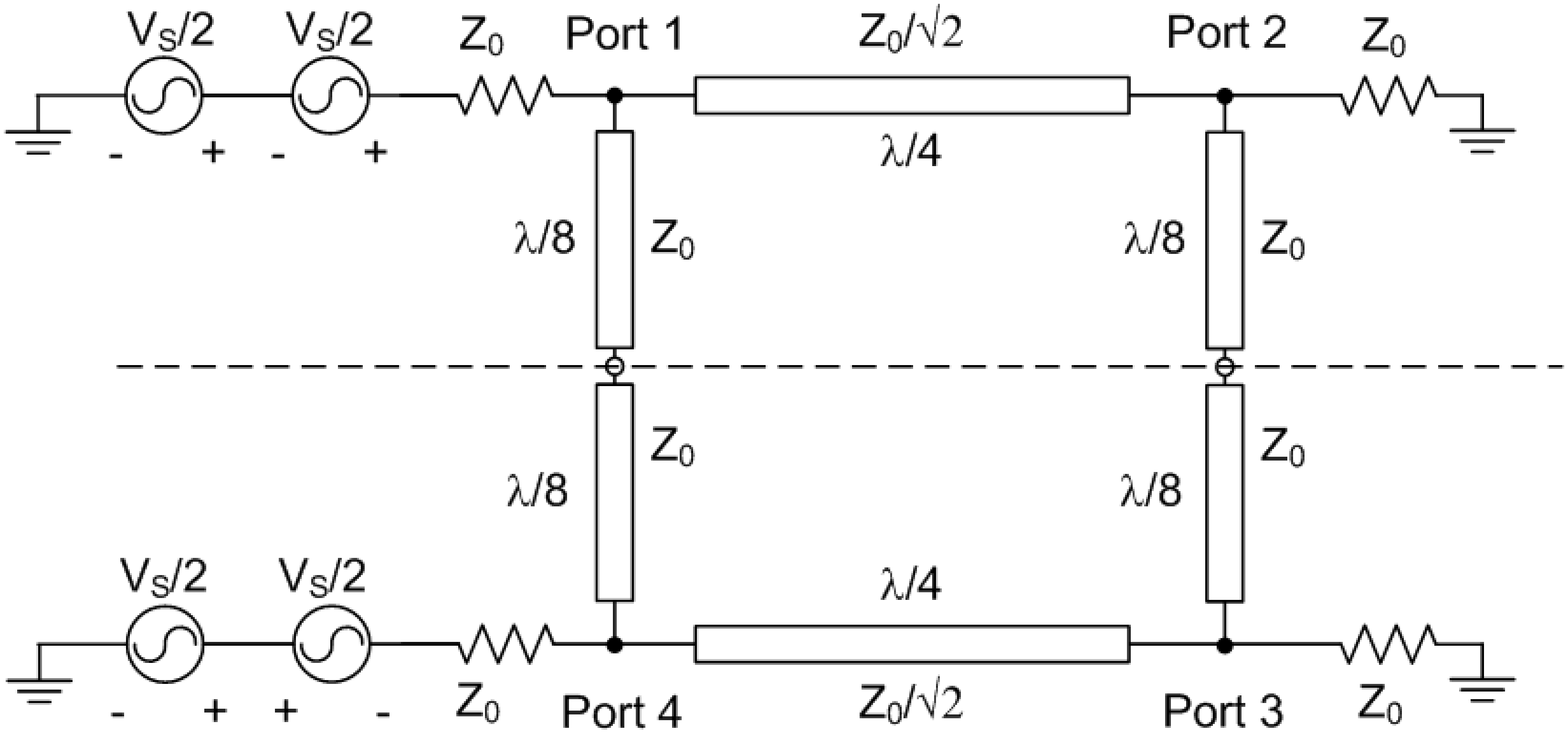


An important parameter for the quadrature hybrid is the directivity:

$D[dB] = -20 \log |S_{41} / S_{31}|$

The Branch line coupler

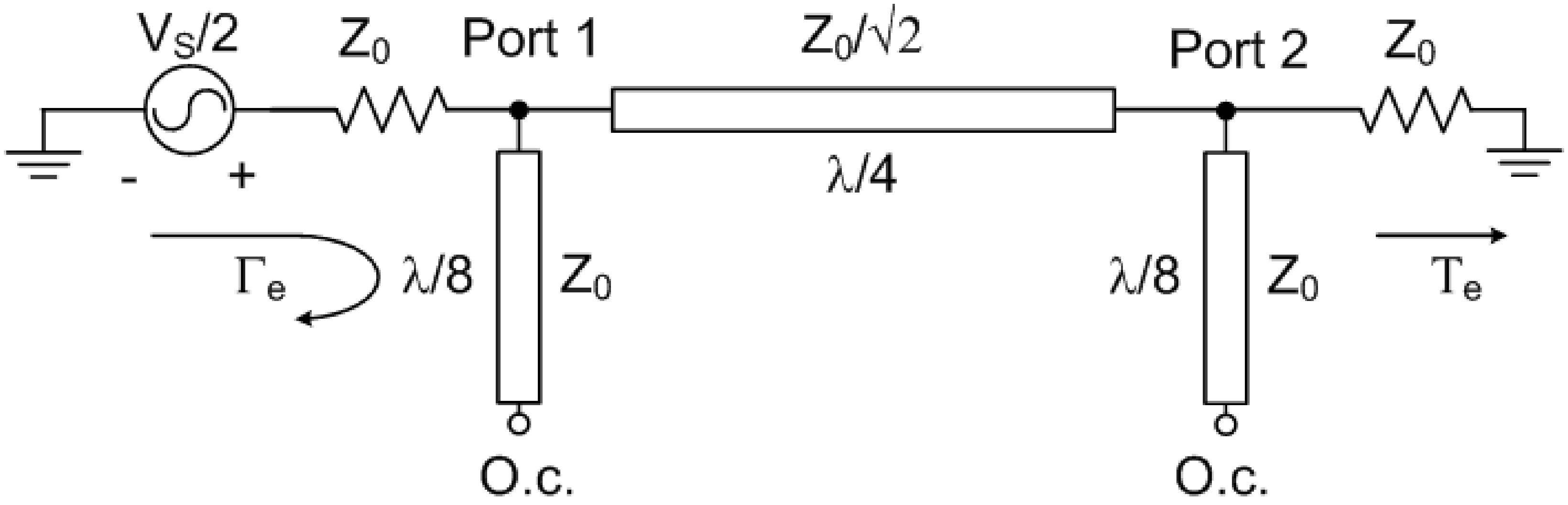
We will analyze the operation of the branch line coupler using the even and odd mode decomposition technique (as seen in the Wilkison power divider).



The Branch line coupler

Even mode analysis:

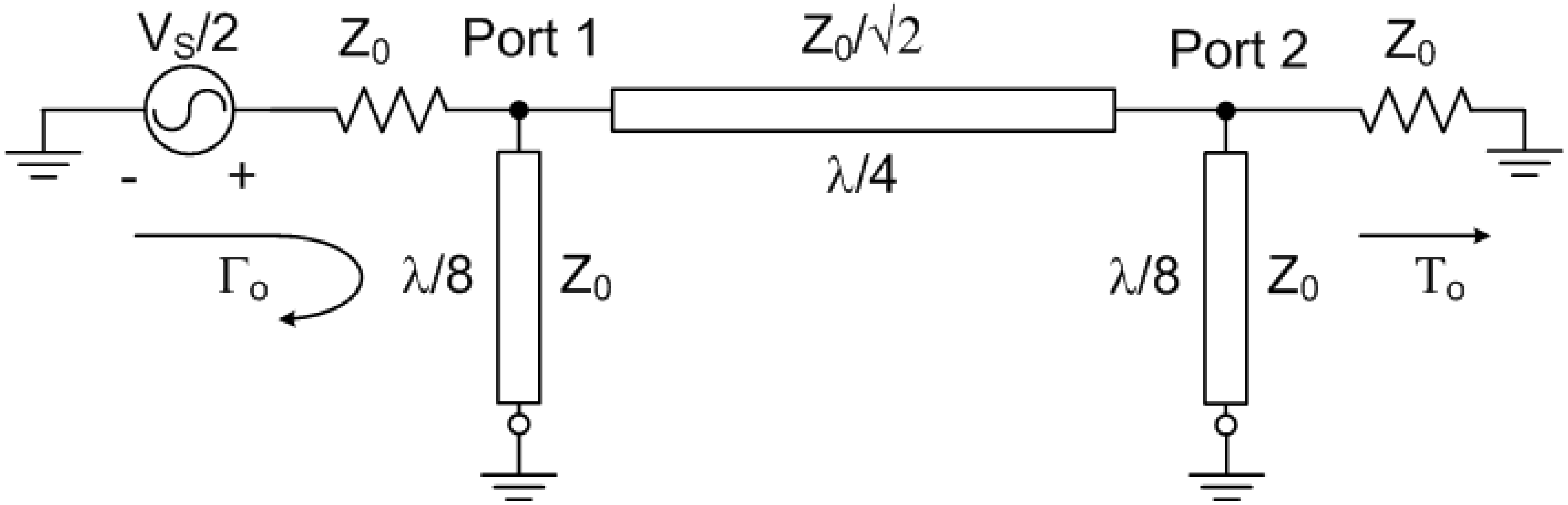
on port 1 and 4 we consider two in phase sources of Vs/2. Since the signals are in phase the line of symmetry can be replaced with an open circuit.



The Branch line coupler

- Odd mode analysis:

  on port 1 and 4 we consider two out of phase sources of Vs/2.
  Since the signals are in opposite phase the line of symmetry can be replaced with an short circuit.



The Branch line coupler

The emerging waves can be then expressed as:

$$B_1 = \frac{V_S}{2}\Gamma_e + \frac{V_S}{2}\Gamma_o$$

$$B_2 = \frac{V_S}{2}T_e + \frac{V_S}{2}T_o$$

$$B_3 = \frac{V_S}{2}T_e - \frac{V_S}{2}T_o$$

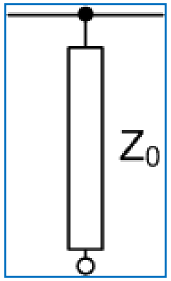
$$B_4 = \frac{V_S}{2}\Gamma_e - \frac{V_S}{2}\Gamma_o$$

$\Gamma_{e,o}$ and $T_{e,o}$ represent the even and odd reflection coefficient and the even and odd transmission coefficient.

The Branch line coupler

We will first consider the calculation of the even mode coefficients using the ABCD matrices.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{shunt} = \begin{bmatrix} 1 & 0 \\ Y_{even} & 1 \end{bmatrix}$$



$$Y_{even} = \frac{1}{Z_0} j\tan(\beta l) = \frac{1}{Z_0} j\tan(\frac{\pi}{4})$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\lambda/4line} = \begin{bmatrix} \cos(\beta l) & j\cdot\frac{Z_0}{\sqrt{2}}\sin(\beta l) \\ j\cdot Y_0\cdot\sqrt{2}\sin(\beta l) & \cos(\beta l) \end{bmatrix} = \begin{bmatrix} 0 & j\cdot\frac{Z_0}{\sqrt{2}} \\ j\cdot Y_0\cdot\sqrt{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total\_even} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & j\cdot Z_0 \\ \frac{j}{Z_0} & -1 \end{bmatrix}$$

The Branch line coupler

Recalling that:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{A+B/Z_0 - C\cdot Z_0 - D}{A+B/Z_0 + C\cdot Z_0 + D} & \dfrac{2(A\cdot D - B\cdot C)}{A+B/Z_0 + C\cdot Z_0 + D} \\ \dfrac{2}{A+B/Z_0 + C\cdot Z_0 + D} & \dfrac{-A+B/Z_0 - C\cdot Z_0 + D}{A+B/Z_0 + C\cdot Z_0 + D} \end{bmatrix}$$

We have for the even mode:

$$\Gamma_{even} = \frac{(-1+j-j+1)\sqrt{2}}{(-1+j+j-1)\sqrt{2}} = 0$$

$$T_{even} = \frac{2}{(-1+j+j-1)\sqrt{2}} = \frac{-1}{\sqrt{2}}(1+j)$$

The Branch line coupler

Following the same procedure we obtain for the odd-mode:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total\_odd} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \cdot Z_0 \\ \dfrac{j}{Z_0} & 1 \end{bmatrix}$$

$$\Gamma_{odd} = \frac{(1 + j - j - 1)\sqrt{2}}{(1 + j + j + 1)\sqrt{2}} = 0$$

$$T_{odd} = \frac{2}{(1 + j + j + 1)\sqrt{2}} = \frac{1}{\sqrt{2}}(1 - j)$$

The Branch line coupler

- We can now calculate the B waves by superimposing the solutions found for the two set of excitations.

$$B_1 = \frac{V_S}{2}\Gamma_e + \frac{V_S}{2}\Gamma_o = 0$$

Port 1 is matched

$$B_2 = \frac{V_S}{2}T_e + \frac{V_S}{2}T_o = -V_S\frac{j}{\sqrt{2}}$$

Half power with -90° phase shift

$$B_3 = \frac{V_S}{2}T_e - \frac{V_S}{2}T_o = -V_S\frac{1}{\sqrt{2}}$$

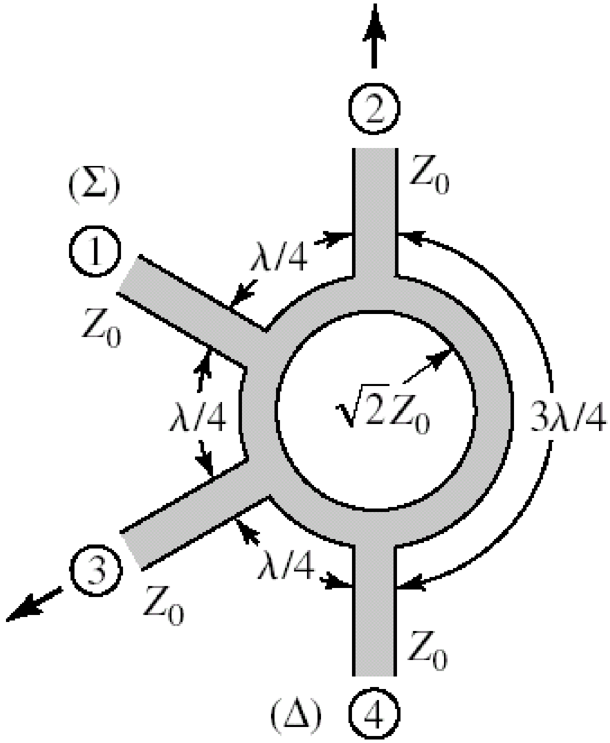
Half power with -180° phase shift

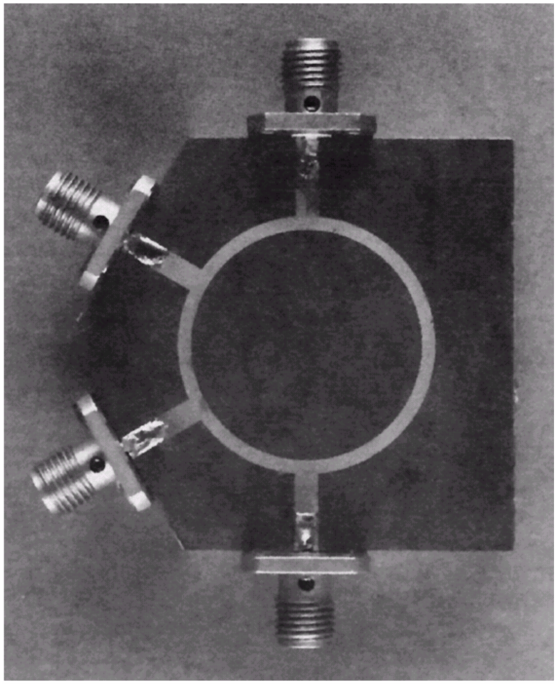
$$B_4 = \frac{V_S}{2}\Gamma_e - \frac{V_S}{2}\Gamma_o = 0$$

Port 4 is isolated

The 180° hybrid junction (rat race)

This component is a four port network providing an equal power division with a 0° phase shift between port 2 and 3 when using port 1 is as the input and 180° phase shift when port 4 is used as the input.



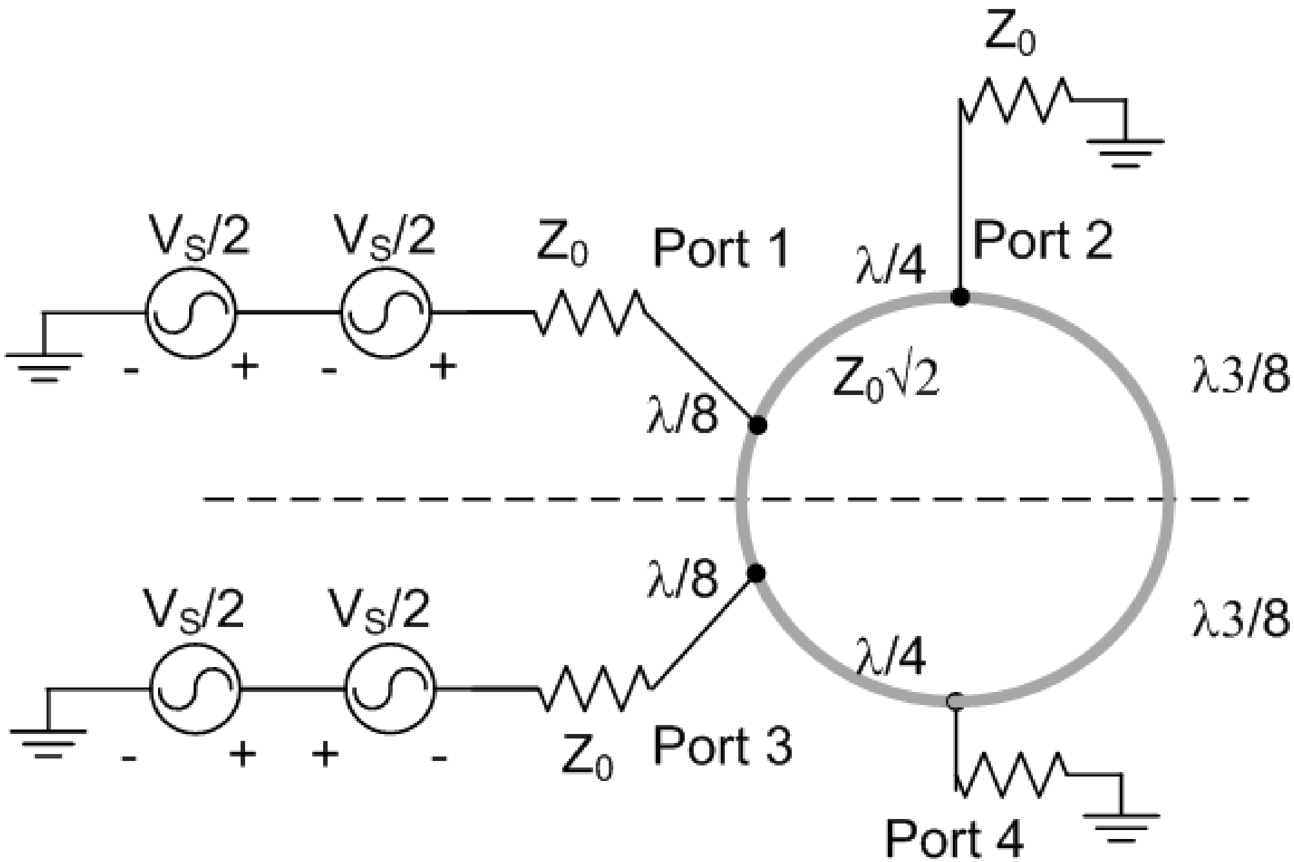


The rat race

- The rat race can also be used as a combiner in this case we apply the signals at port 2 and 3 and port 1 will provide an output proportional to the sum of the inputs while port 4 will be proportional to the difference of the inputs.
- We will study the behavior of this component using the even and odd mode analysis technique.
- We will start considering the case with power applied to port 1.

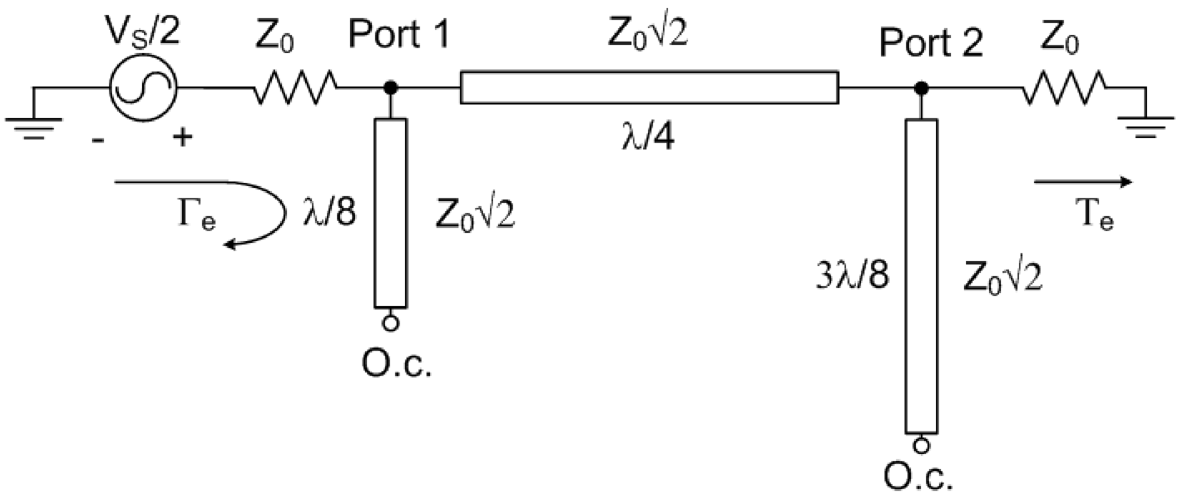
The rat race

- We can decompose the rat race in two sub-circuits considering an even and an odd mode excitation.

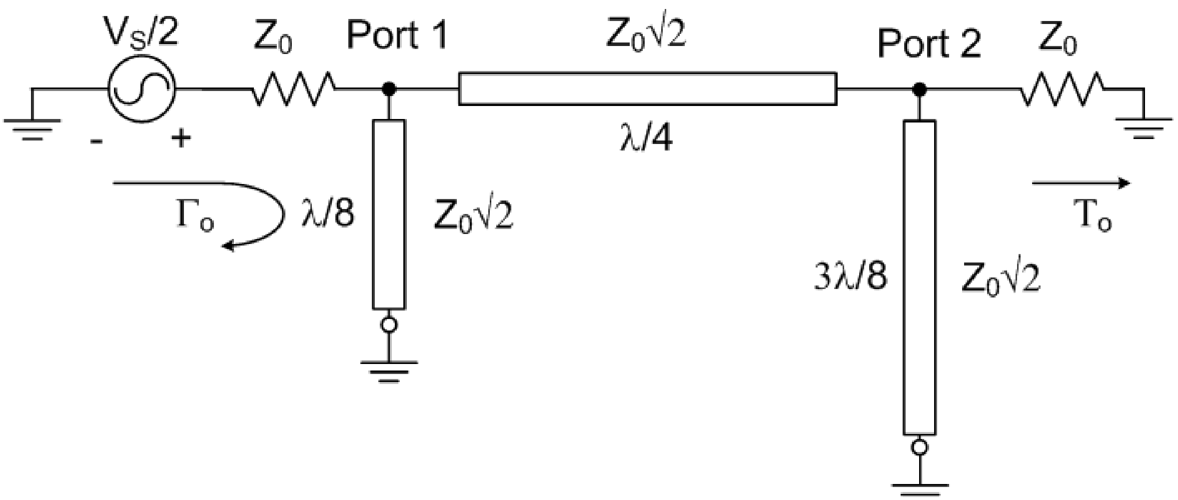


The rat race

Port 1 excitation



Even mode



Odd mode

The rat race

Again the emerging waves can be then expressed as:

$$B_1 = \frac{V_S}{2}\Gamma_e + \frac{V_S}{2}\Gamma_o$$

$$B_2 = \frac{V_S}{2}T_e + \frac{V_S}{2}T_o$$

$$B_3 = \frac{V_S}{2}\Gamma_e - \frac{V_S}{2}\Gamma_o$$

$$B_4 = \frac{V_S}{2}T_e - \frac{V_S}{2}T_o$$

$\Gamma_{e,o}$ and $T_{e,o}$ represent the even and odd reflection coefficient and the even and odd transmission coefficient.

The rat race

Now we can repeat the analysis seen for the branch line coupler with the only difference that:

$$Y_{even} = \frac{1}{Z_0\sqrt{2}} j\tan(\beta l) = \frac{1}{Z_0\sqrt{2}} j\tan(\frac{\pi}{4}) = \frac{j}{Z_0\sqrt{2}} \quad \text{for the } \lambda/8 \text{ line}$$

$$Y_{even} = \frac{1}{Z_0\sqrt{2}} j\tan(\beta l) = \frac{1}{Z_0\sqrt{2}} j\tan(\frac{3\pi}{4}) = -\frac{j}{Z_0\sqrt{2}} \quad \text{for the } 3\lambda/8 \text{ line}$$

$$Y_{odd} = -\frac{1}{Z_0\sqrt{2}} j\cot(\beta l) = -\frac{1}{Z_0\sqrt{2}} j\cot(\frac{\pi}{4}) = -\frac{j}{Z_0\sqrt{2}} \quad \text{for the } \lambda/8 \text{ line}$$

$$Y_{odd} = -\frac{1}{Z_0\sqrt{2}} j\cot(\beta l) = -\frac{1}{Z_0\sqrt{2}} j\cot(\frac{3\pi}{4}) = \frac{j}{Z_0\sqrt{2}} \quad \text{for the } 3\lambda/8 \text{ line}$$

The rat race

The total ABCD matrices for the even and odd case are:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total\_even} = \begin{bmatrix} 1 & j \cdot Z_0 \cdot \sqrt{2} \\ \dfrac{j \cdot \sqrt{2}}{Z_0} & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total\_odd} = \begin{bmatrix} -1 & j \cdot Z_0 \cdot \sqrt{2} \\ \dfrac{j \cdot \sqrt{2}}{Z_0} & 1 \end{bmatrix}$$

$$\Gamma_{even} = \frac{-j}{\sqrt{2}} \qquad \Gamma_{odd} = \frac{j}{\sqrt{2}}$$

$$T_{even} = \frac{-j}{\sqrt{2}} \qquad T_{odd} = \frac{-j}{\sqrt{2}}$$

The rat race

Substituting the values just found we end up with:

$$B_1 = \frac{V_S}{2}\Gamma_e + \frac{V_S}{2}\Gamma_o = 0$$

Port 1 is matched

$$B_2 = \frac{V_S}{2}T_e + \frac{V_S}{2}T_o = \frac{-j}{\sqrt{2}}$$

Half power with -90° phase shift

$$B_3 = \frac{V_S}{2}\Gamma_e - \frac{V_S}{2}\Gamma_o = \frac{-j}{\sqrt{2}}$$

Half power with -90° phase shift

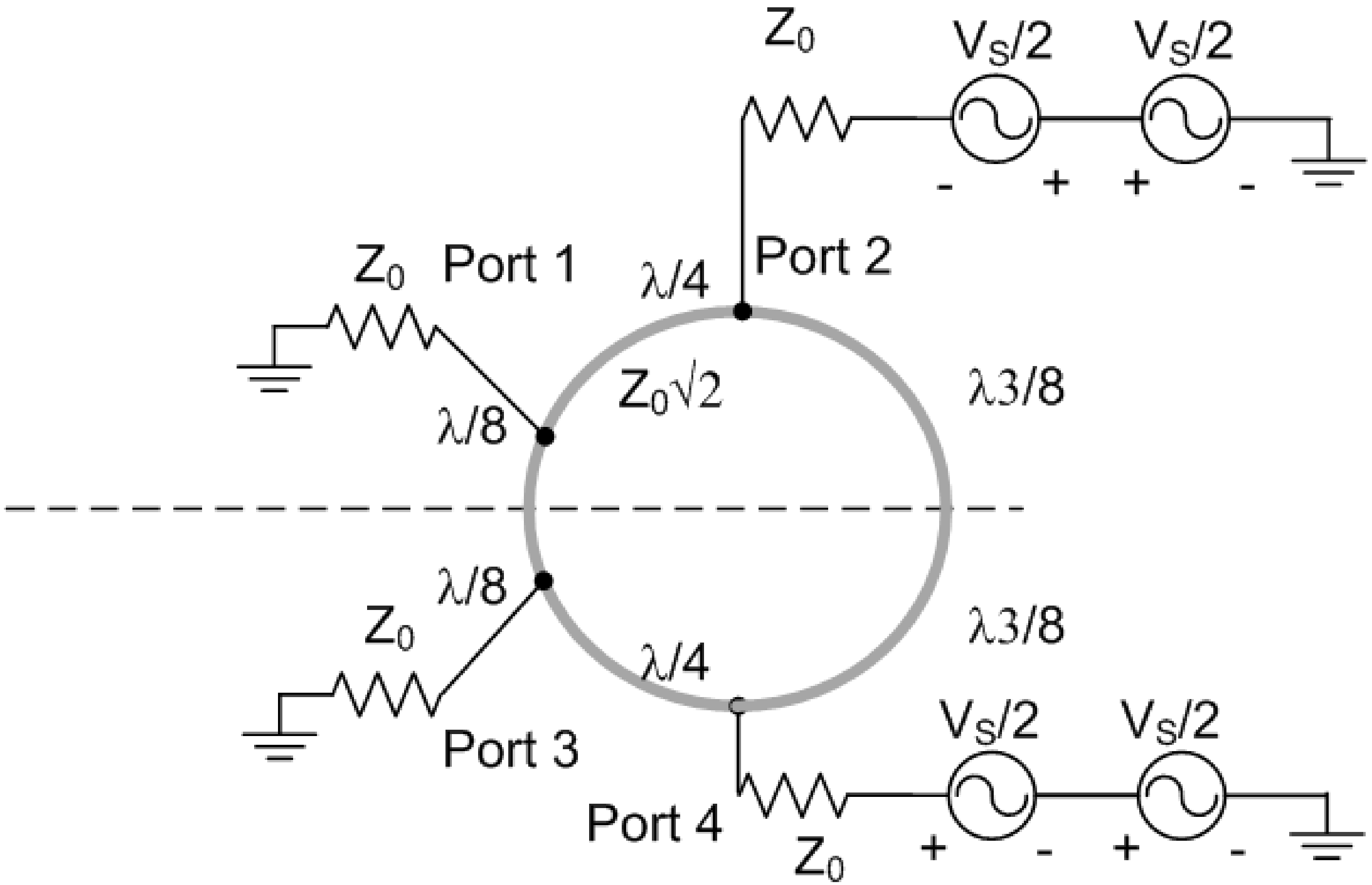
In PHASE

$$B_4 = \frac{V_S}{2}T_e - \frac{V_S}{2}T_o = 0$$

Port 4 is isolated

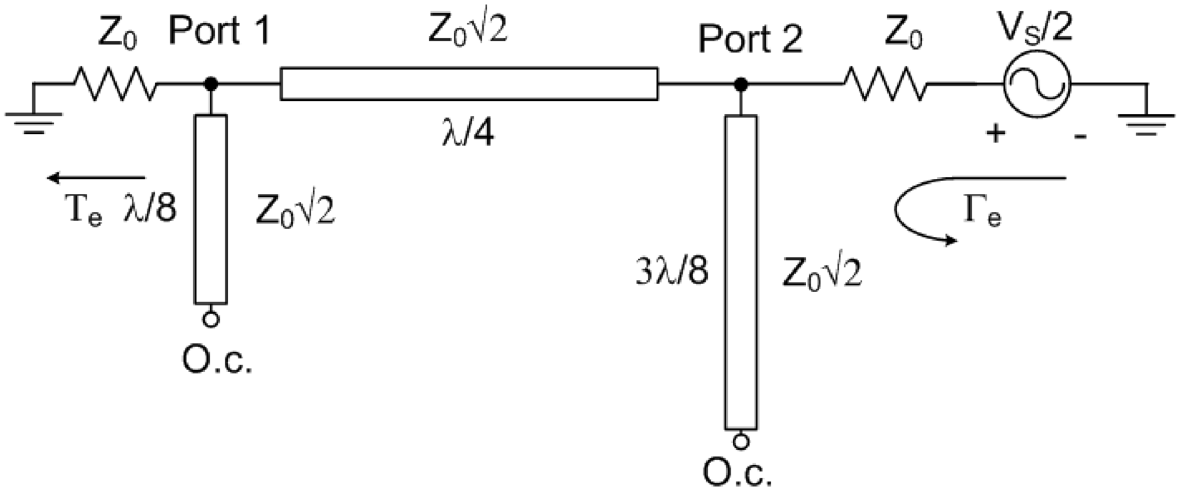
The rat race

- Now lets repeat the previous analysis with power coming from port 4.

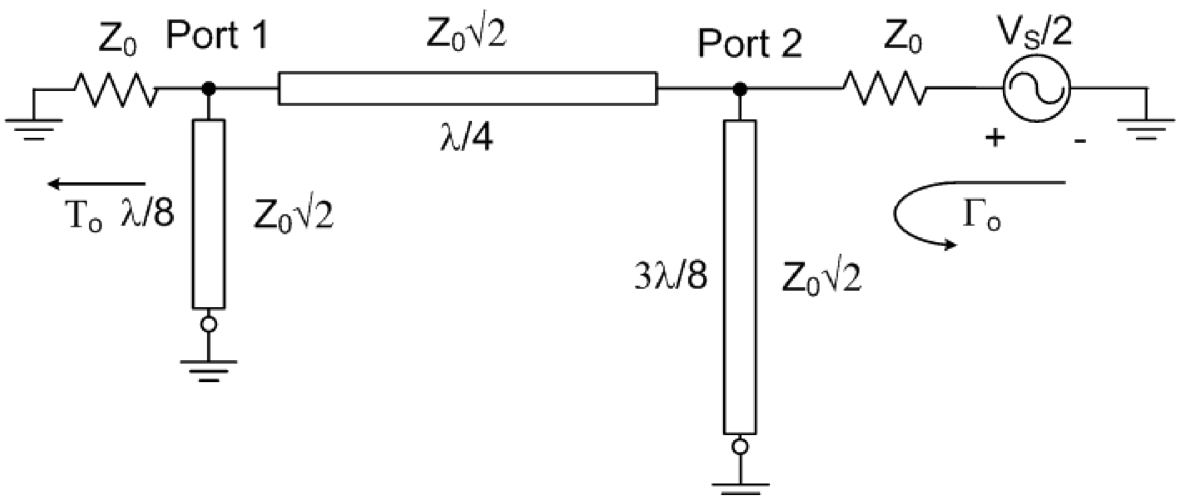


The rat race

Port 4 excitation



Even mode



Odd mode

The rat race

Resulting in:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total\_even} = \begin{bmatrix} -1 & j \cdot Z_0 \cdot \sqrt{2} \\ \dfrac{j \cdot \sqrt{2}}{Z_0} & 1 \end{bmatrix} \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total\_odd} = \begin{bmatrix} 1 & j \cdot Z_0 \cdot \sqrt{2} \\ \dfrac{j \cdot \sqrt{2}}{Z_0} & -1 \end{bmatrix}$$

$$\Gamma_{even} = \frac{j}{\sqrt{2}} \qquad \Gamma_{odd} = \frac{-j}{\sqrt{2}}$$

$$T_{even} = \frac{-j}{\sqrt{2}} \qquad T_{odd} = \frac{-j}{\sqrt{2}}$$

$B_1 = 0$ Port 1 is isolated

$$B_2 = \frac{j}{\sqrt{2}}$$

$$B_3 = \frac{-j}{\sqrt{2}}$$

Half power with 90° phase shift

Half power with -90° phase shift

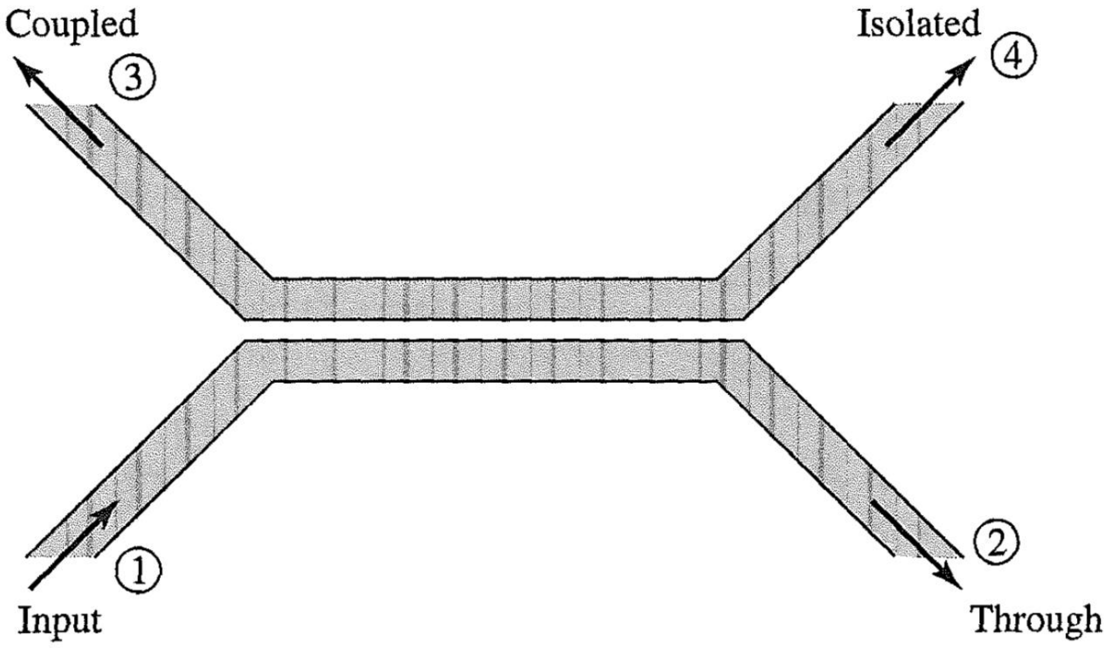
180° out of PHASE

$B_4 = 0$ Port 4 is matched

Coupled line couplers

Unshielded transmission lines couple power from one line to the other due to the interaction of the EM fields.

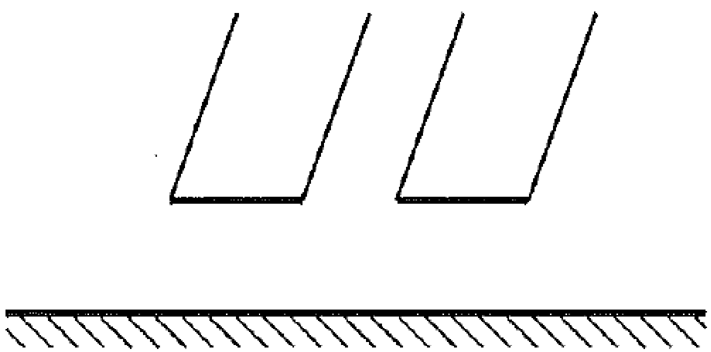
Couple line couplers are used to transfer power (with different coupling intensity) from one transmission line to another.

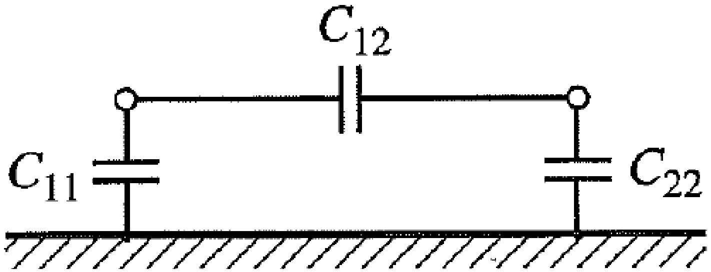


Coupled line couplers

- Assuming TEM propagation the electrical characteristics of the lines can be completely characterized by effective capacitance between the lines and the propagation velocity. Where:

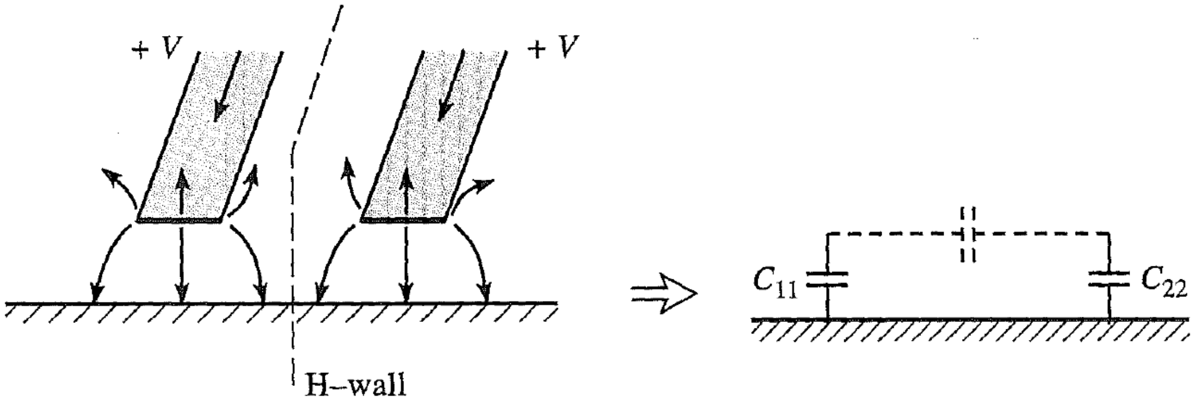
  C_{12} is the capacitance between the conductors
  C_{11}/C_{22} capacitance from the conductor to ground



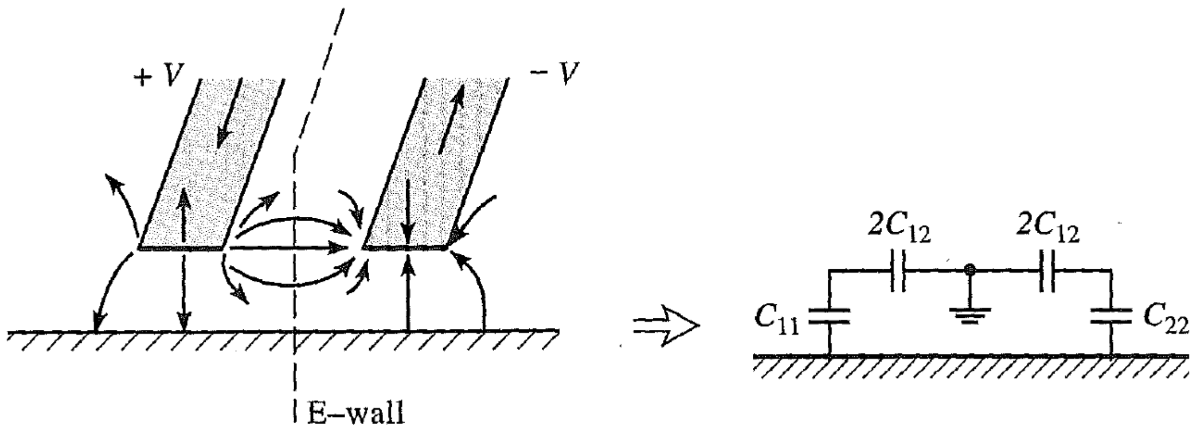


Coupled line couplers

n we have:



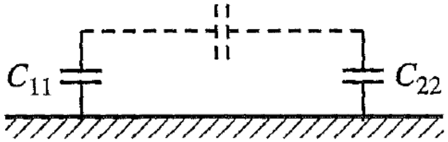
Even-mode:
symmetry to the center vertical line, no current flows



Odd-mode:
symmetry to the center horizontal (virtual) ground exists between conductors

Coupled line couplers

- Considering the two possible modes of excitation we have:

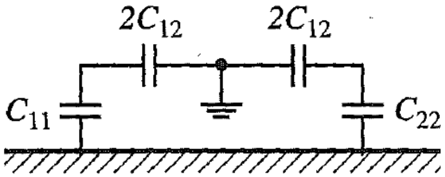


Even-mode:

$$C_e = C_{11} = C_{22}$$

$$Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{C_e L}}{C_e} = \frac{1}{v_p C_e}$$

Characteristic impedance of one conductors in even-mode operation



Odd-mode:

$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

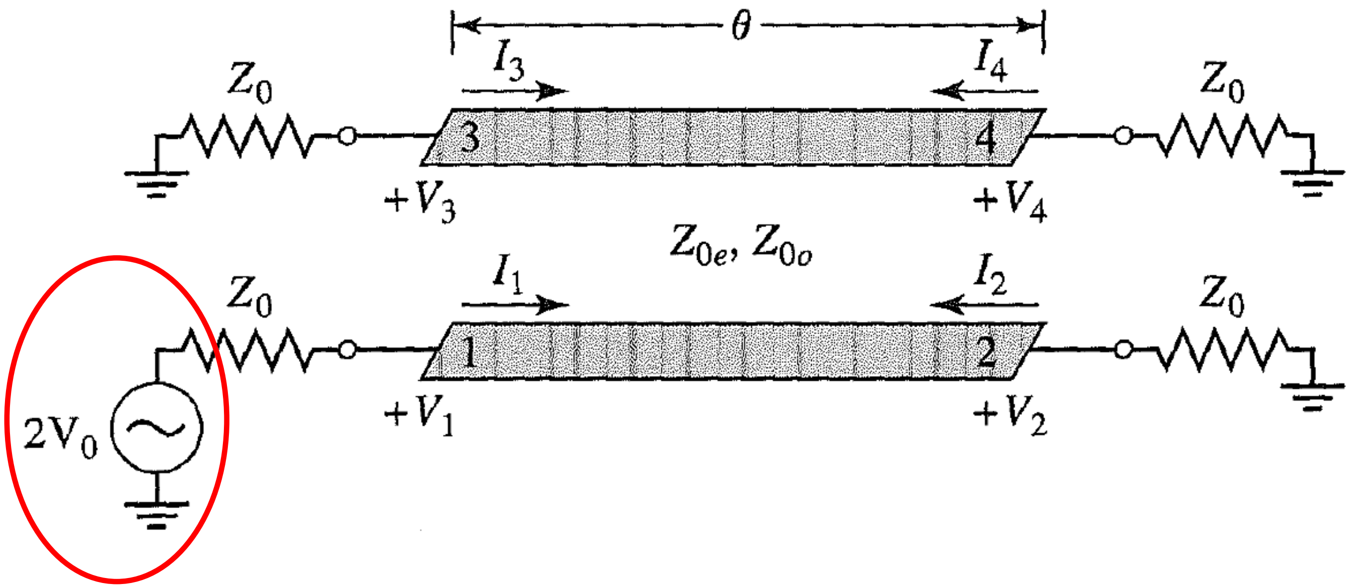
$$Z_{0o} = \frac{1}{v_p C_o}$$

Characteristic impedance of one conductors in odd-mode operation

Coupled line couplers

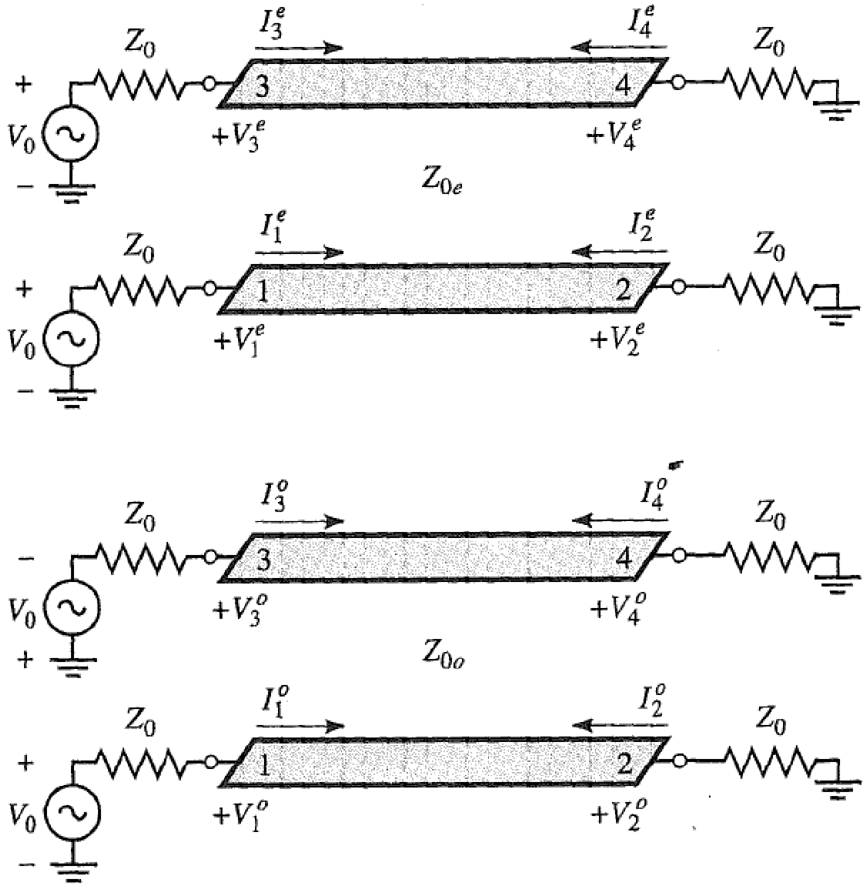
How do we design a coupled like coupler to be matched and have a specific coupling factor?

We will analyze the coupled lines structure and find design equations using even-mode and odd-mode technique.



Coupled line couplers

Using superposition, port one can be treated as the sum of even- and odd-mode excitations



Where:

$$I_1^e = I_3^e \qquad I_1^o = -I_3^o$$
$$I_4^e = I_2^e \qquad I_4^o = -I_2^o$$
$$V_1^e = V_3^e \qquad V_1^o = -V_3^o$$
$$V_4^e = V_2^e \qquad V_4^o = -V_2^o$$

Coupled line couplers

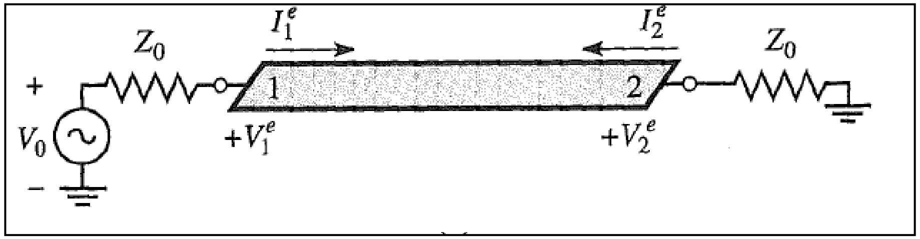
The input expression at port one can be expressed as:

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o}$$

For the even- and odd-mode we will have:

$$Z^e{}_{in} = Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta}$$

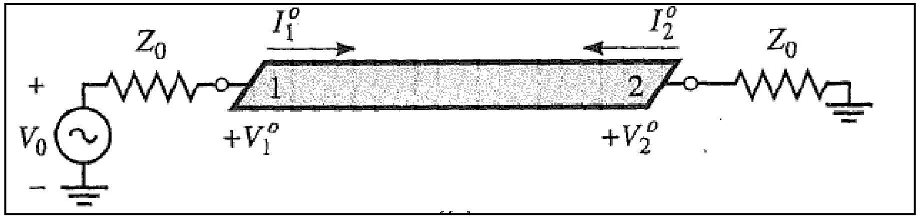
$$Z^o{}_{in} = Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta}$$



Then the I and V are:

$$V^e{}_1 = V_0 \frac{Z_{in}^e}{Z_{in}^e + Z_0} \qquad I^e{}_1 = \frac{V_0}{Z_{in}^e + Z_0}$$

$$V^o{}_1 = V_0 \frac{Z_{in}^o}{Z_{in}^o + Z_0} \qquad I^o{}_1 = \frac{V_0}{Z_{in}^o + Z_0}$$



Coupled line couplers

- Using the previous results leads to:

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = Z_0 + \frac{2\left(Z_{in}^o Z_{in}^e - Z_0^2\right)}{Z_{in}^e + Z_{in}^o + 2Z_0}$$

- Now imposing:

$$Z_0 = \sqrt{Z_{0e} Z_{0o}}$$

- We get:

$$Z^e{}_{in} = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}$$

$$Z^o{}_{in} = Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}$$

$$\Rightarrow \quad Z_{in}^e Z_{in}^o = Z_{0e} Z_{0o} = Z_0^2$$

All ports are matched

Coupled line couplers

- Since port 1 is matched V_1 there is no V_1^-

$$V_3 = V^e_3 + V^o_3 = V^e_1 - V^o_1 = V_0\left(\frac{Z^e_{in}}{Z^e_{in}+Z_0} - \frac{Z^o_{in}}{Z^o_{in}+Z_0}\right)$$

$$\boxed{\begin{aligned} Z^e_{in} &= Z_{0e}\frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta} \\ Z^o_{in} &= Z_{0o}\frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta} \end{aligned}}$$

$$V_3 = V_0\left(\frac{j(Z_{0e}-Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e}-Z_{0o})\tan\theta}\right)$$

- Defining

$$C = \left(\frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}\right)$$

$$\sqrt{1-C^2} = \frac{2Z_0}{Z_{0e}+Z_{0o}}$$

$$\Rightarrow \quad V_3 = V_0\left(\frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta}\right)$$

Coupled line couplers

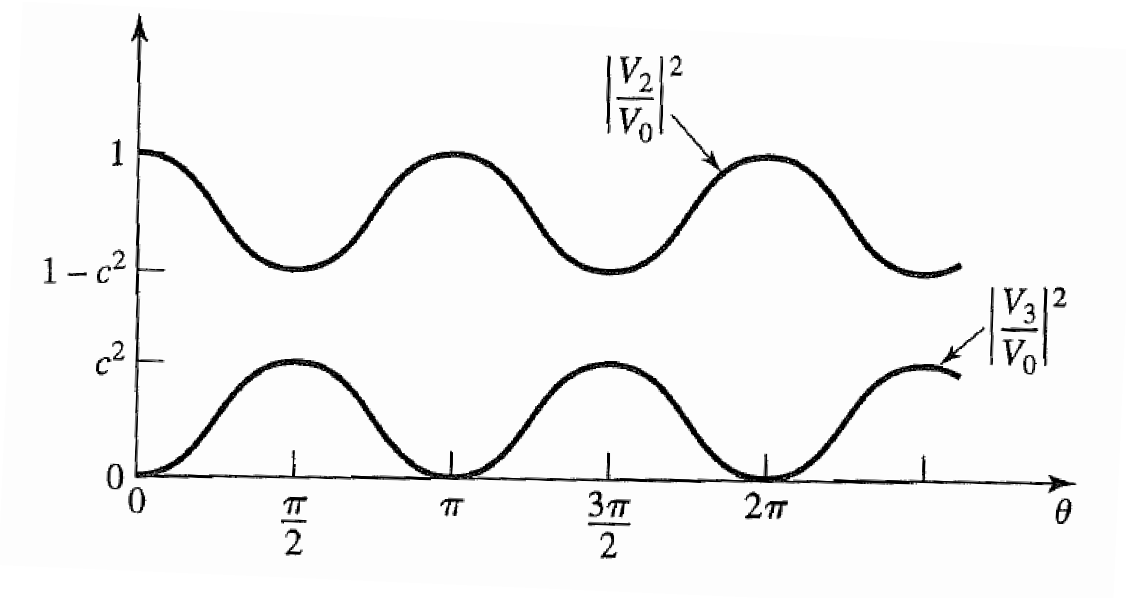
Using similar procedure it can be shown:

$$V_4 = V^e_4 + V^o_4 = V^e_2 - V^o_2 = 0$$

$$V_2 = V^e_2 + V^o_2 = V_0\left(\frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}\right)$$

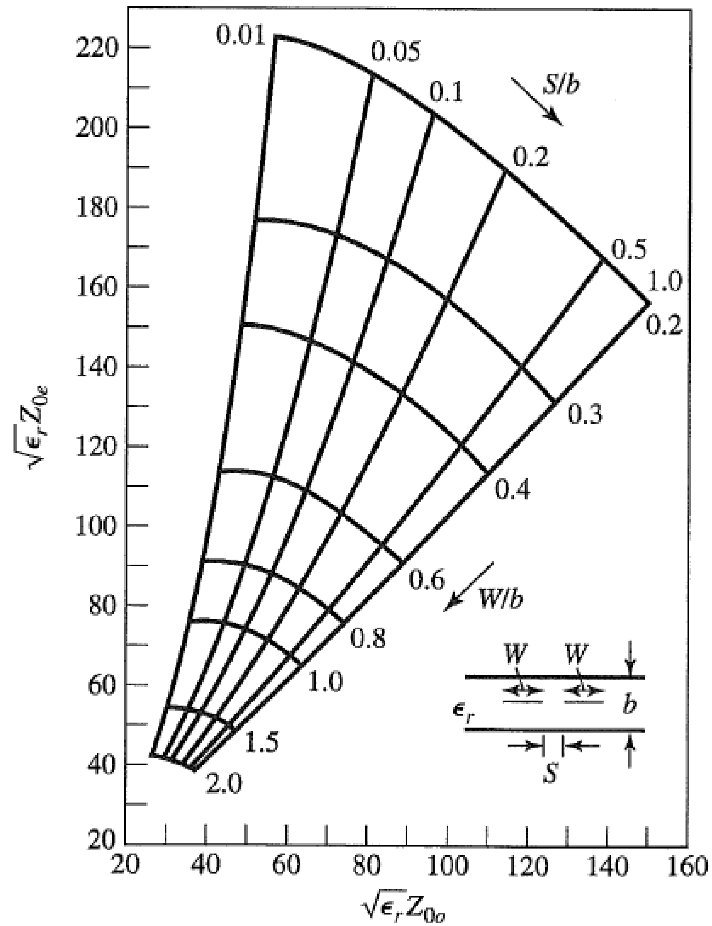
When q=p/2 we have the design equations

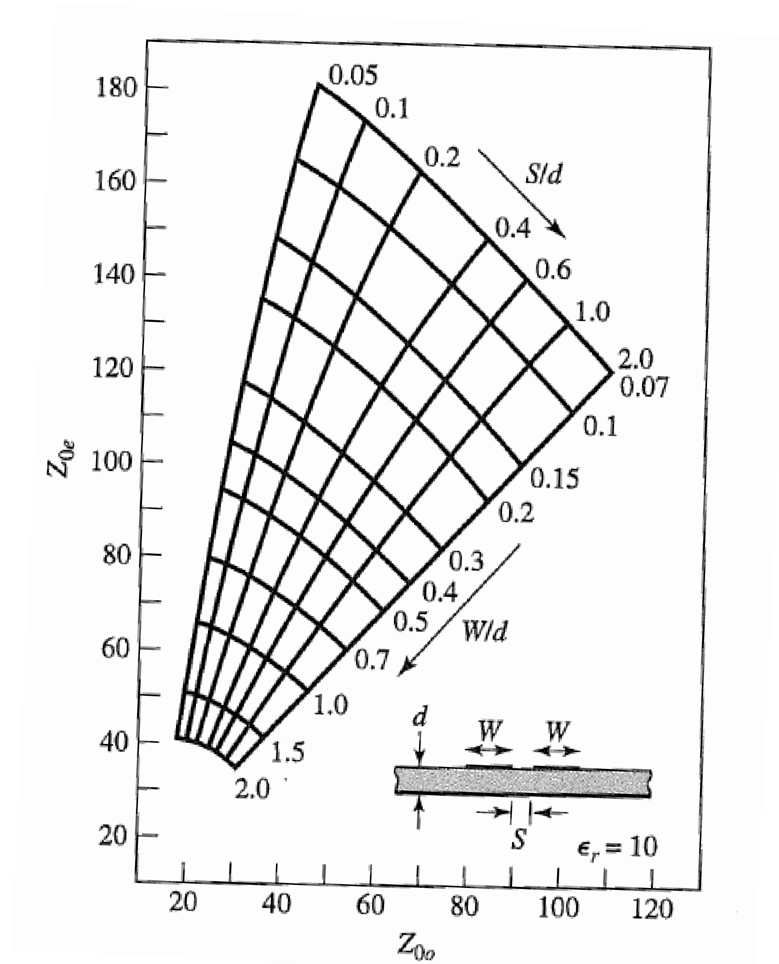
$$\boxed{\begin{aligned} \frac{V_3}{V_0} &= C \\ \frac{V_2}{V_0} &= -j\sqrt{1-C^2} \\ Z_{0e} &= Z_0\sqrt{\frac{1+C}{1-C}} \\ Z_{0o} &= Z_0\sqrt{\frac{1-C}{1+C}} \end{aligned}}$$



Coupled line couplers

- How can we extract the geometrical dimension for a given coupling factor?





Coupler design

- Let us design a edge coupled line coupler in stripline technology.
- Specification:
  f_c=5 GHz
  Coupling 10 dB
- Substrate def:

SSub

SSUB
SSub1
Er=3.6
Mur=1
B=0.5 mm
T=35 um
Cond=5.4E+7
TanD=2.7e-3

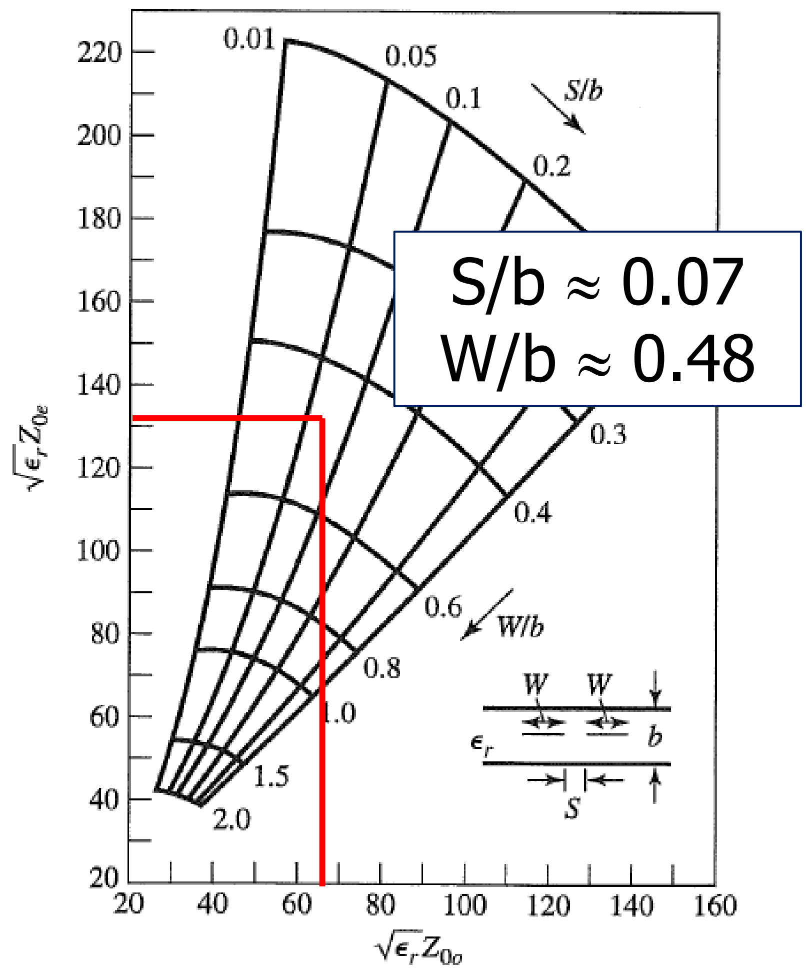
Coupler design

Step one calculate the even and odd mode characteristic impedance from the coupling factor.

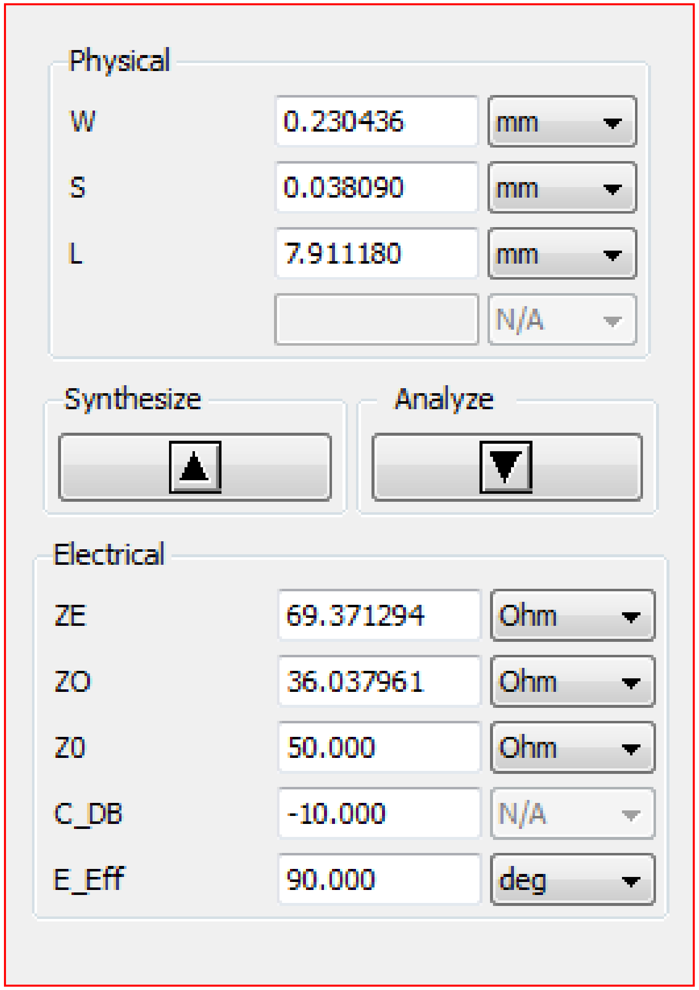
```
C=10^(-10/20);
Z0=50;
Z0e=Z0*sqrt((1+C)/(1-C));
Z0o=Z0*sqrt((1-C)/(1+C));
er=3.6
Z0e_eff=sqrt(er)*Z0e;
Z0o_eff=sqrt(er)*Z0o;
```

Z0e_eff = 131.623
Z0o_eff = 68.377

W=0.24 mm
S=35 um

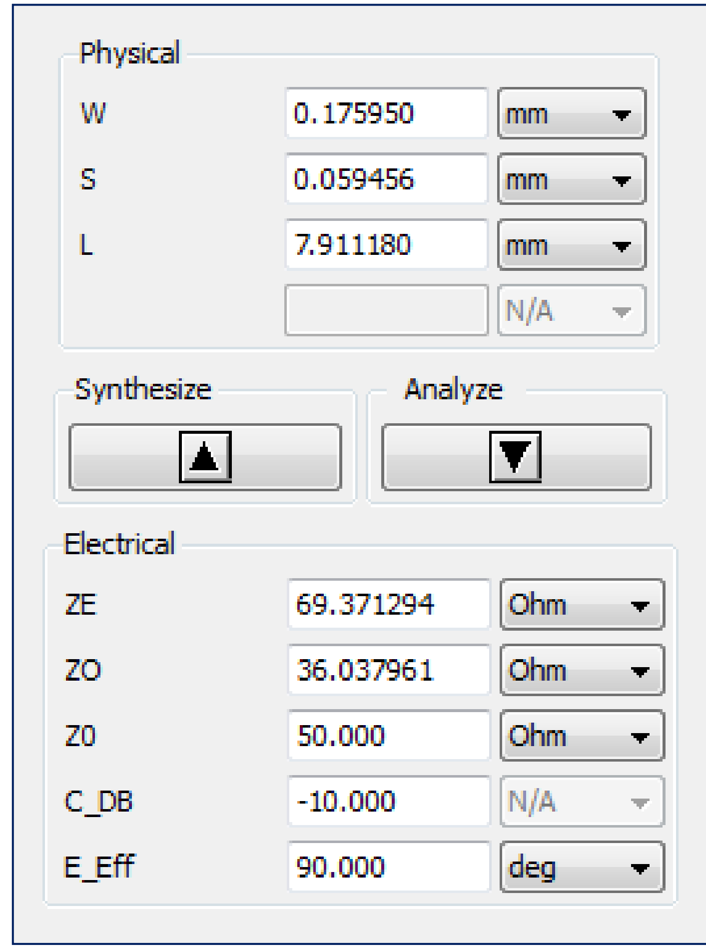


Coupler design



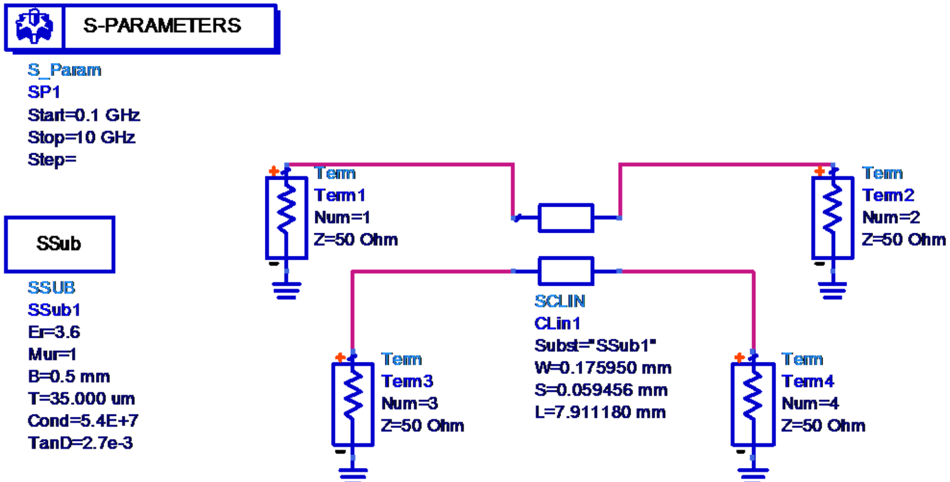
T=0um

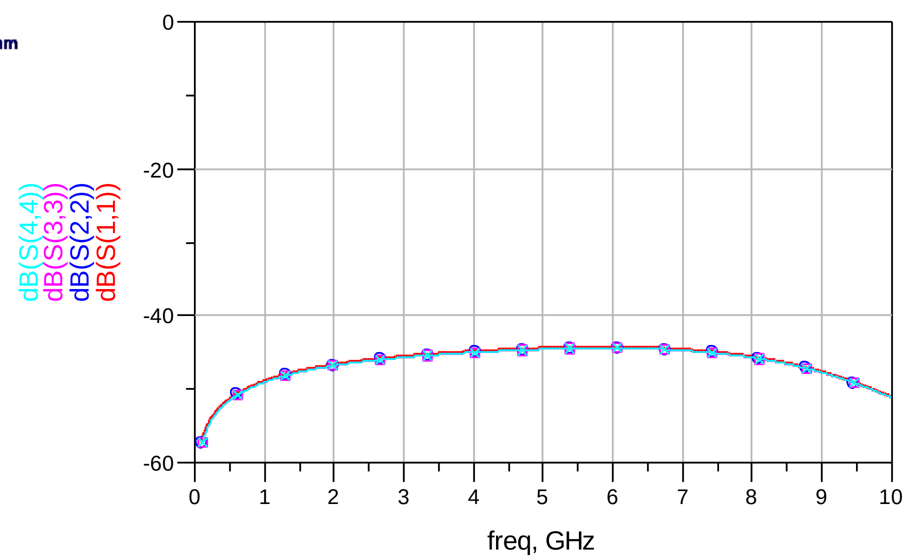
T=35um



Coupled design

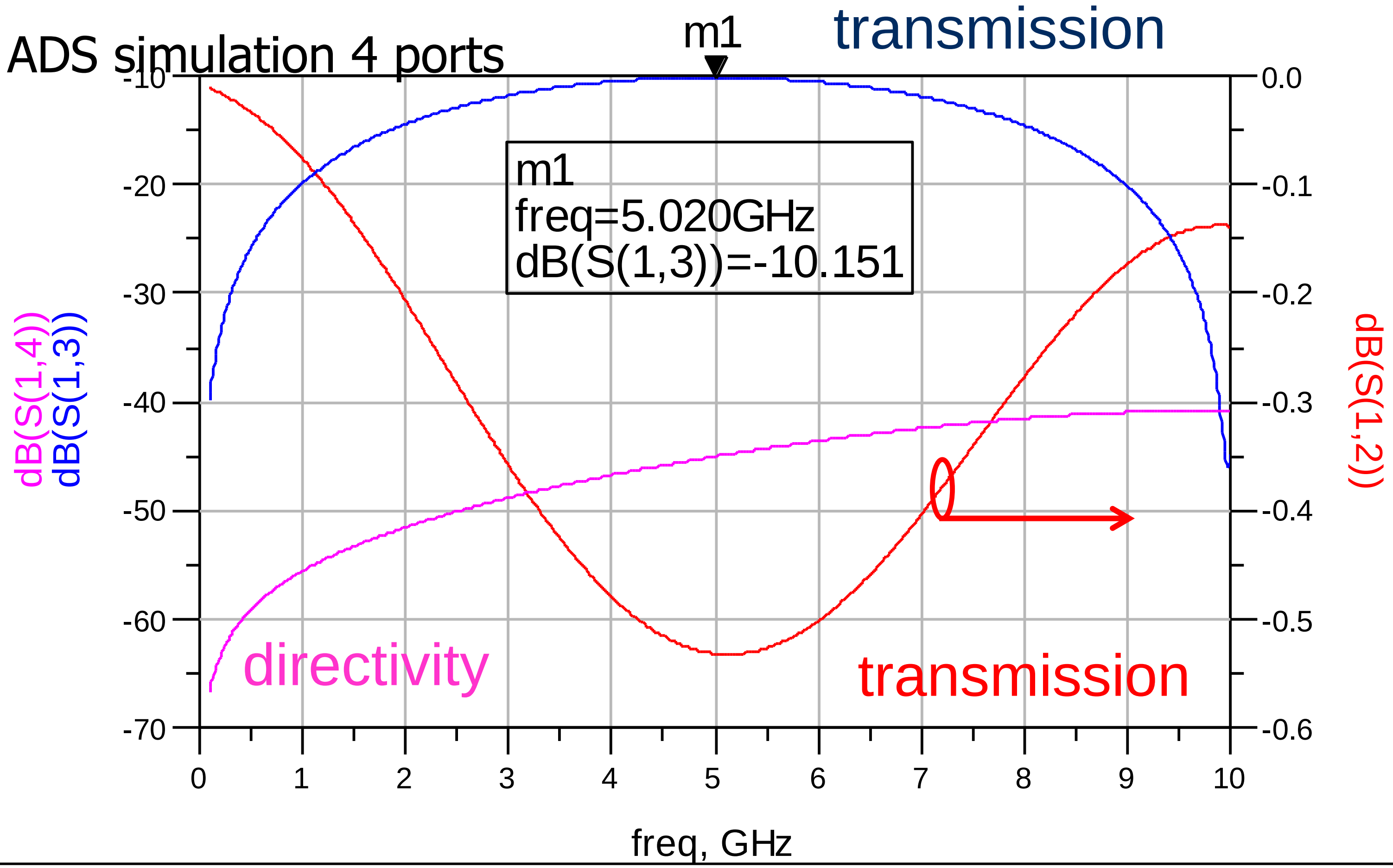
ADS simulation 4 ports





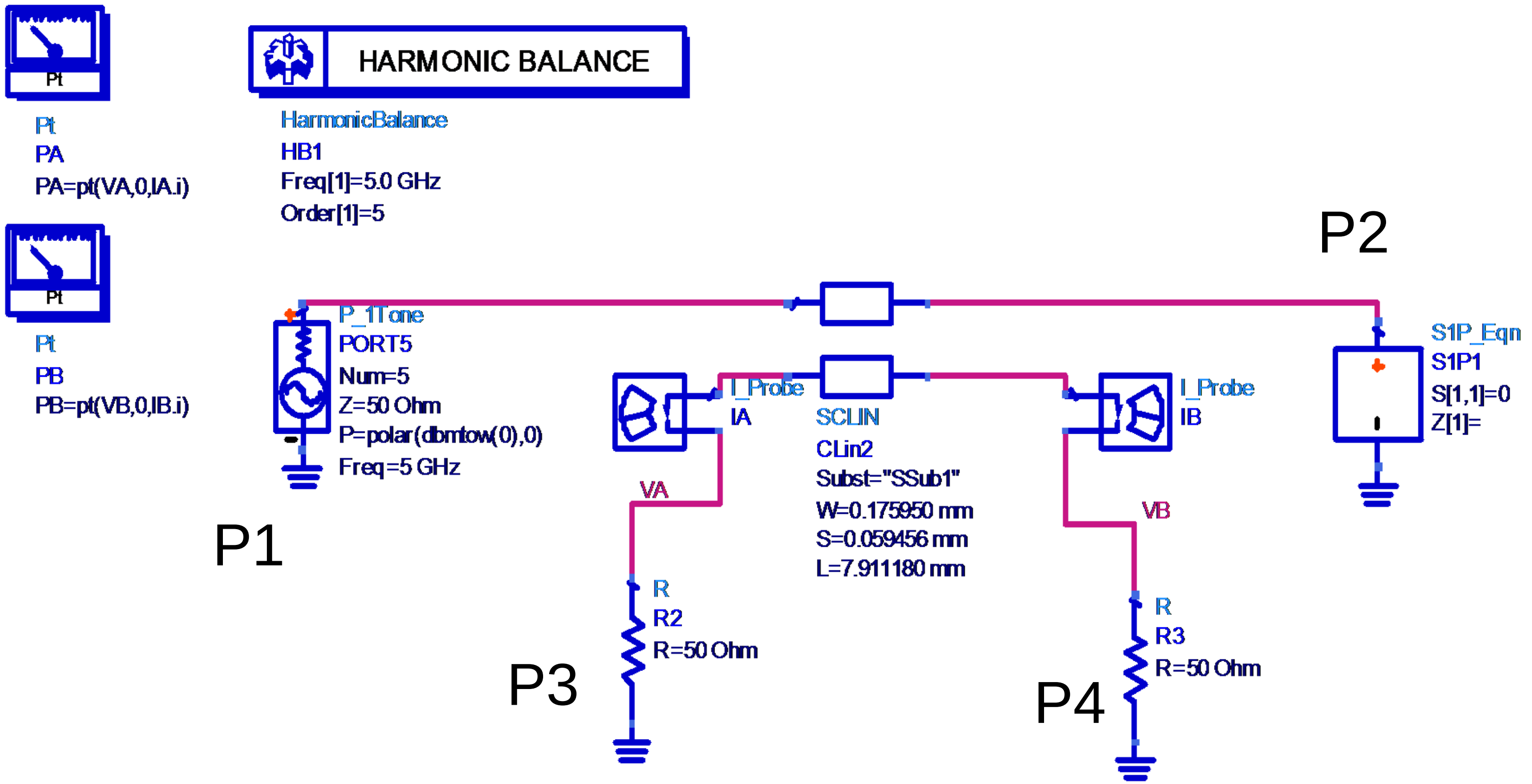
Coupled design

- ADS simulation 4 ports



Coupled design

Let us now see the powers in various conditions, use HB.



Coupled design

- Let us now see the powers in various conditions, use HB.

P3 dBm	P4 dBm	P3 W
..._HB_50Ohm..PA)	..._HB_50Ohm..PB)	..._HB_50Ohm..PA)
-10.150	-45.033	9.660E-5

Γ @ P2 =0

...erDesign_HB..PA)	...erDesign_HB..PB)	...erDesign_HB..PA)
-10.206	-12.485	9.537E-5

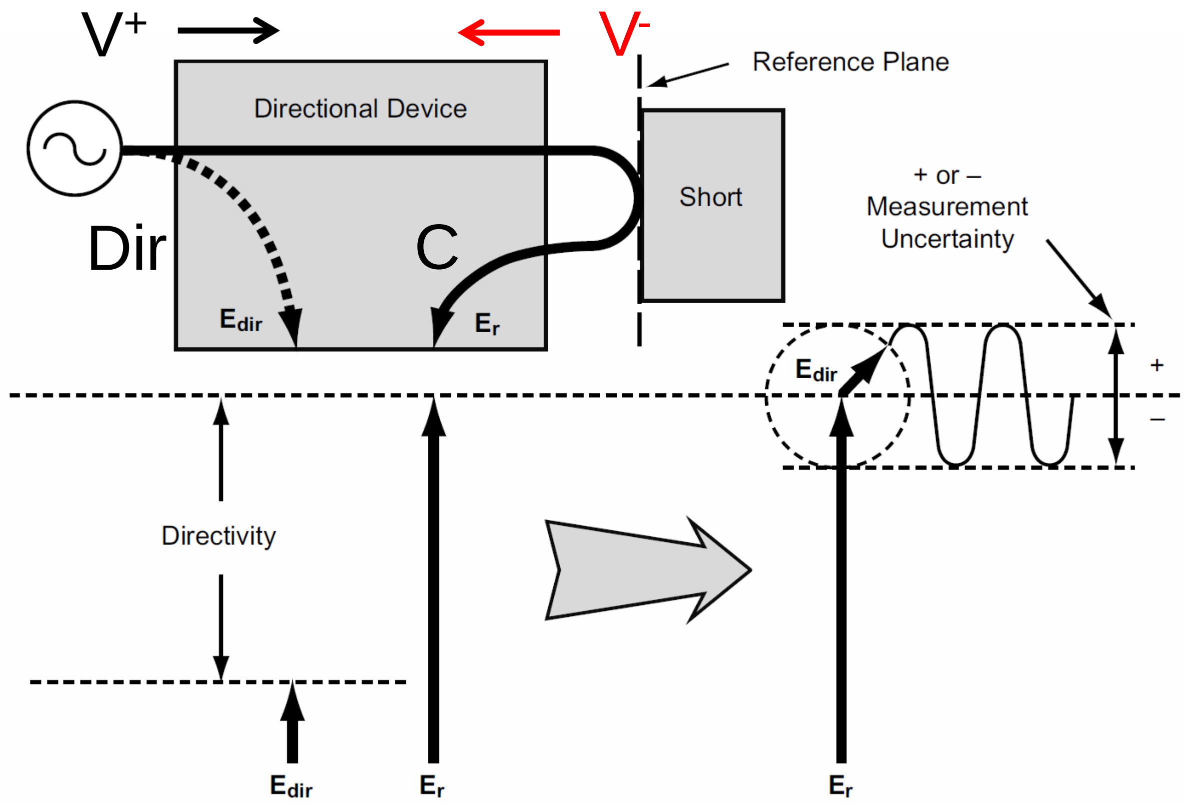
Γ @ P2 =0.8

..._30Ohm_P4..PA)	..._30Ohm_P4..PB)	..._30Ohm_P4..PA)
-8.788	-12.944	1.322E-4

Γ@ P2 =0.8
R@ P4=30 Ohm

Coupled design

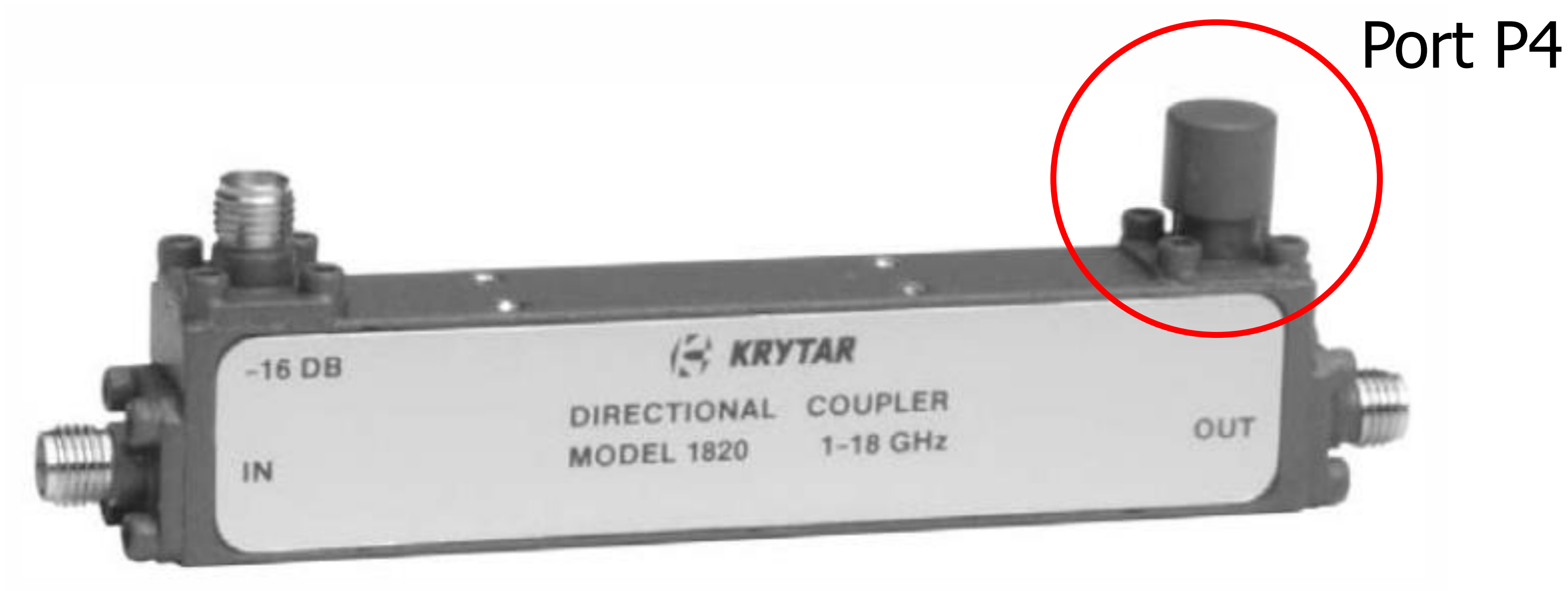
Let us now see the powers in various conditions, use HB.



$$Er = V^{-} \cdot C + V^{+} \cdot Dir + V^{+} \cdot \Gamma_{P3}$$

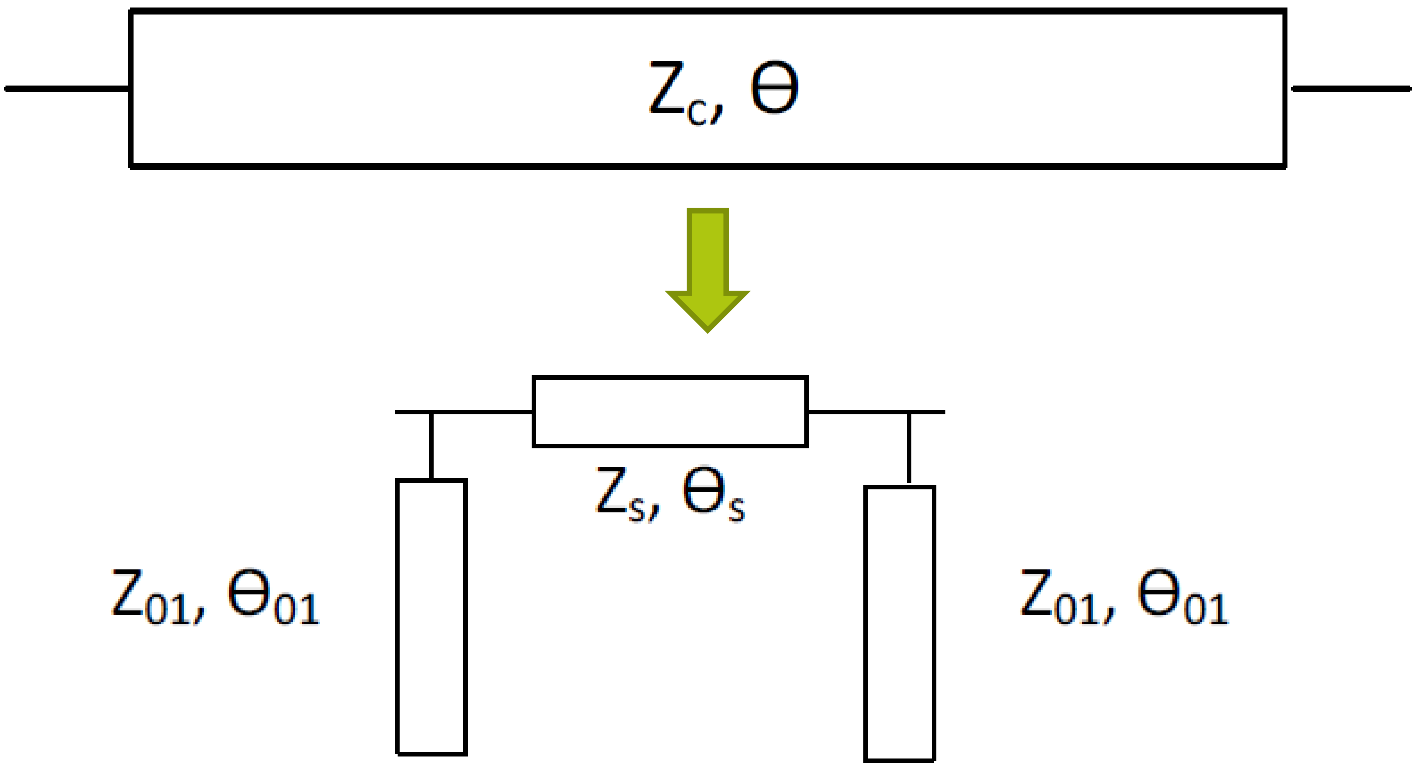
Coupled design

- For this reason commercial high performance couplers embed the 50 Ohm load in the component to ensure the high directivity value.



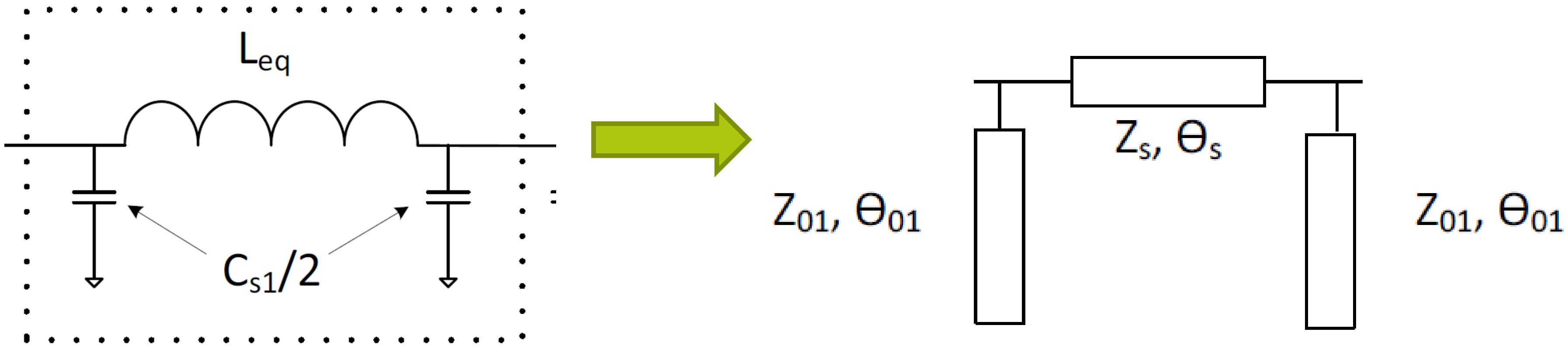
TL lumped equivalent

- Lumped distributed elements allow to reduce the size of a transmission line section (and hence of the component designed) but they lead to a reduction of the bandwidth.



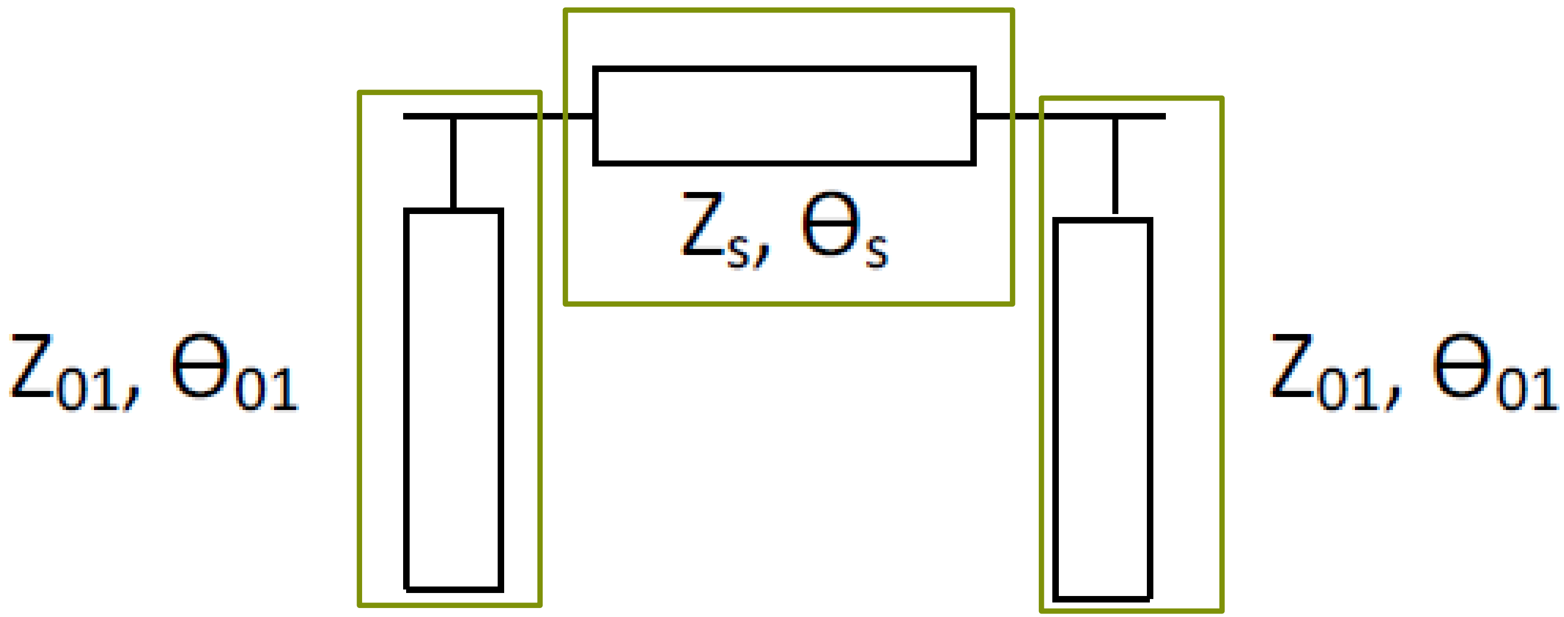
TL lumped equivalent

- How can we transform a transmission line in its equivalent lumped approximation (still using Tlines) and what are the limitations?
- Recalling the distributed model of a transmission line we know that for a short length a Tline can be approximated by:



TL lumped equivalent

- To represent the original Tline (for a given frequency) with the Tline lumped model we need to equate the ABCD parameters of the two networks



TL lumped equivalent

- We need now to equate the Tline with the cascaded of the 3 ABCD matrixes in order to find the values for Z_s, Θ_s, Z_{01} and Θ_{01}

$$\begin{pmatrix} \cos\theta & jZ_C\sin\theta \\ j\dfrac{\sin\theta}{Z_C} & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ j\beta_{01} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_S & jZ_S\sin\theta_S \\ j\dfrac{\sin\theta_S}{Z_S} & \cos\theta_S \end{pmatrix} \begin{pmatrix} 1 & 0 \\ j\beta_{01} & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\theta_S - \beta_{01}Z_S\sin\theta_S & jZ_S\sin\theta_S \\ j\dfrac{\sin\theta_S}{Z_S}\left(1 - Z_S^2\beta_{01}^2 + 2Z_S\beta_{01}\cot\theta_S\right) & \cos\theta_S - \beta_{01}Z_S\sin\theta_S \end{pmatrix}$$

$$j\beta_{01} = \frac{j\tan\Theta_{01}}{Z_{01}}$$ input admittance of the open stubs

TL lumped equivalent

- We need now to equate the Tline with the cascaded of the 3 ABCD matrixes in order to find the values for Z_s, Θ_s, Z_{01} and Θ_{01}

$$\begin{pmatrix} \cos\theta & jZ_C \sin\theta \\ j\dfrac{\sin\theta}{Z_C} & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ j\beta_{01} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_S & jZ_S \sin\theta_S \\ j\dfrac{\sin\theta_S}{Z_S} & \cos\theta_S \end{pmatrix} \begin{pmatrix} 1 & 0 \\ j\beta_{01} & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\theta_S - \beta_{01} Z_S \sin\theta_S & jZ_S \sin\theta_S \\ j\dfrac{\sin\theta_S}{Z_S}\left(1 - Z_S^2 \beta_{01}^2 + 2 Z_S \beta_{01} \cot\theta_S\right) & \cos\theta_S - \beta_{01} Z_S \sin\theta_S \end{pmatrix}$$

$$j\beta_{01} = \frac{j\tan\Theta_{01}}{Z_{01}}$$

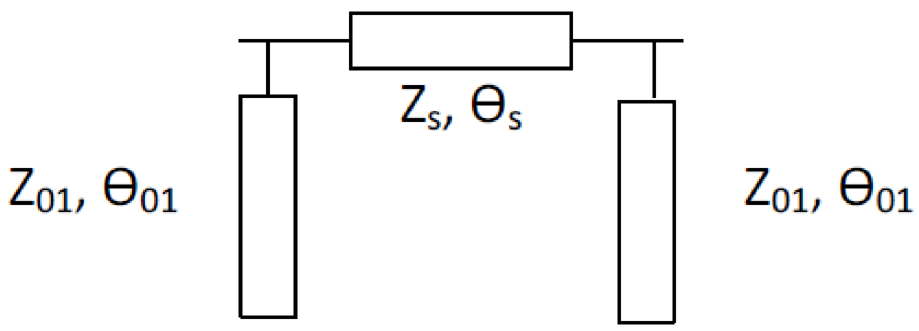
input admittance of the open stubs

TL lumped equivalent

- From the system shown in the previous slide we obtain:

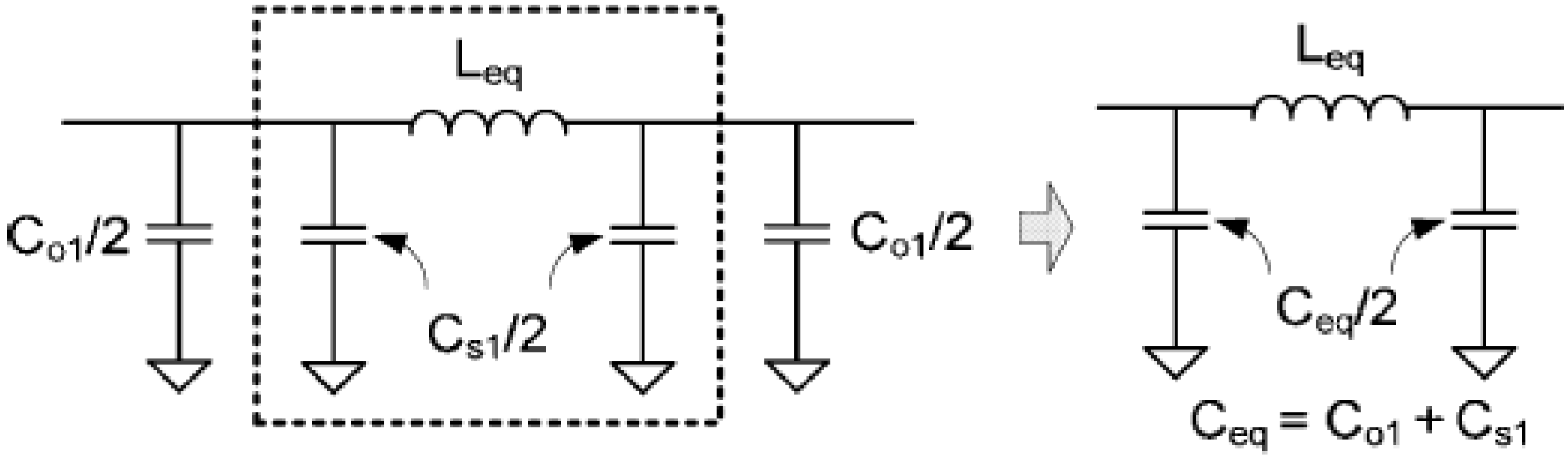
$$\beta_{01} = \frac{\cos\theta_S - \cos\theta}{Z_C \sin\theta}$$

$$Z_S = \frac{Z_C \sin\theta}{\sin\theta_S}$$

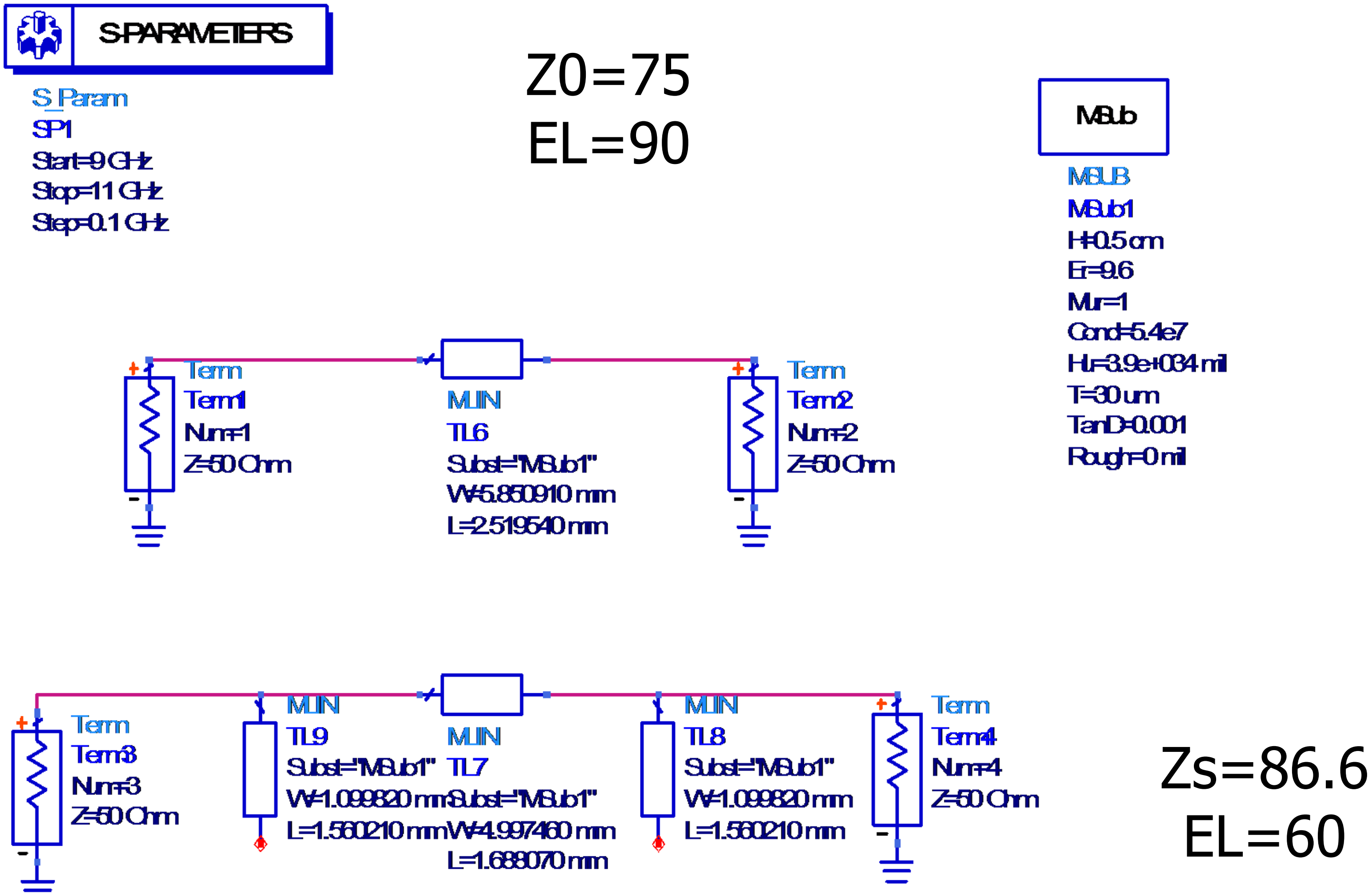


TL lumped equivalent

- Why does this approach allows us to have shorter lines?

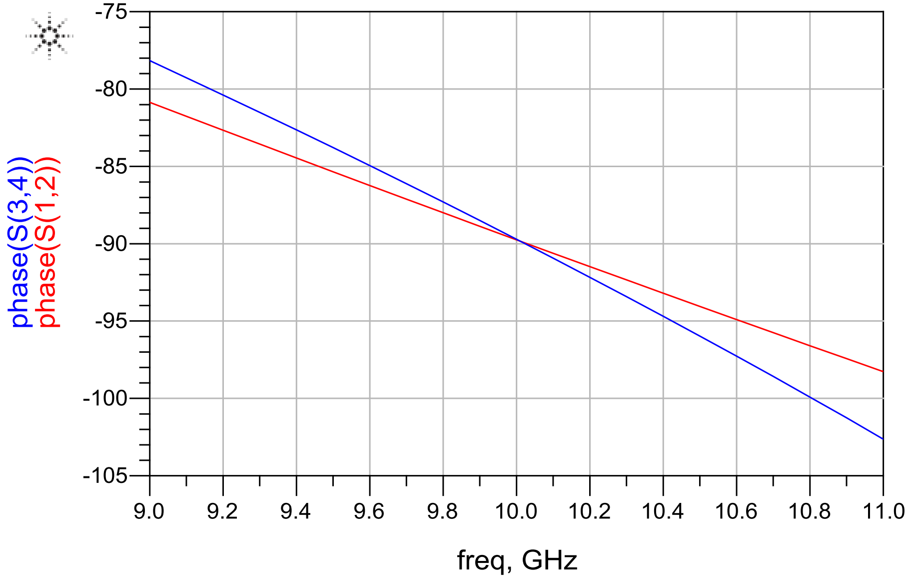


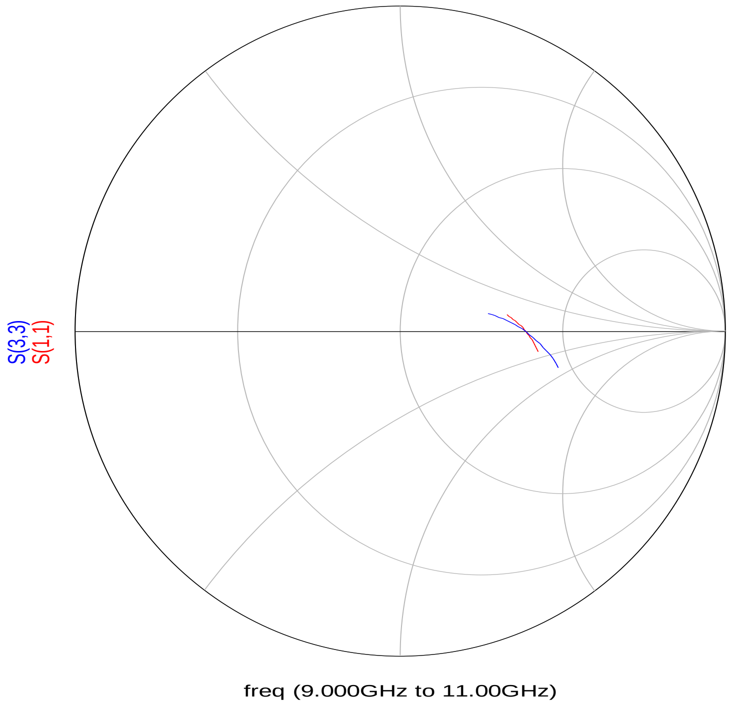
TL lumped equivalent



TL lumped equivalent

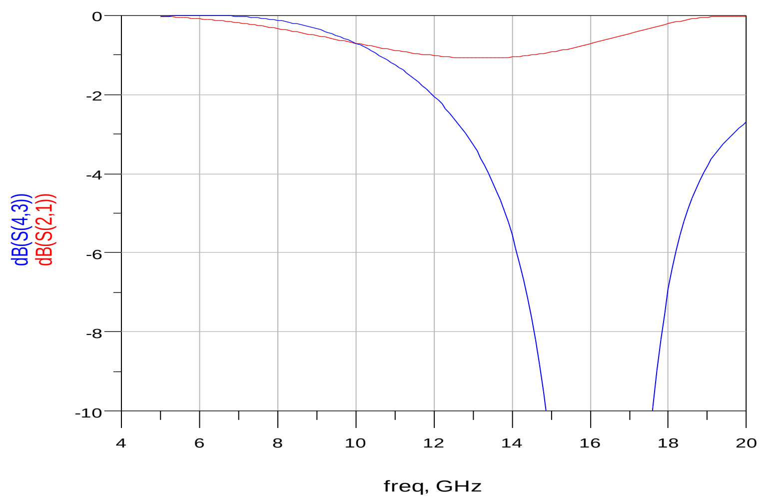
- At 10 GHz which is the design frequency the response of the two networks is identical. Also in a 20% relative BW the agreement is very close.

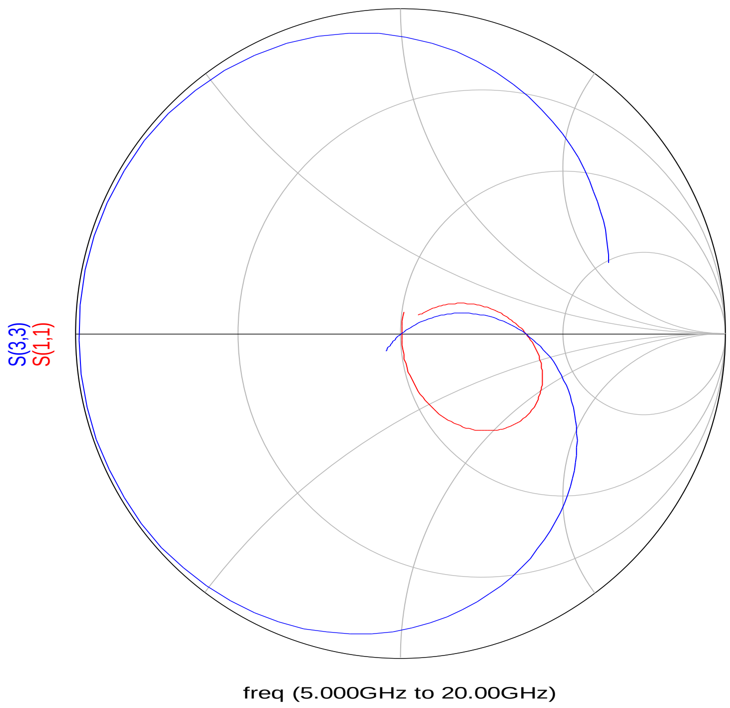




TL lumped equivalent

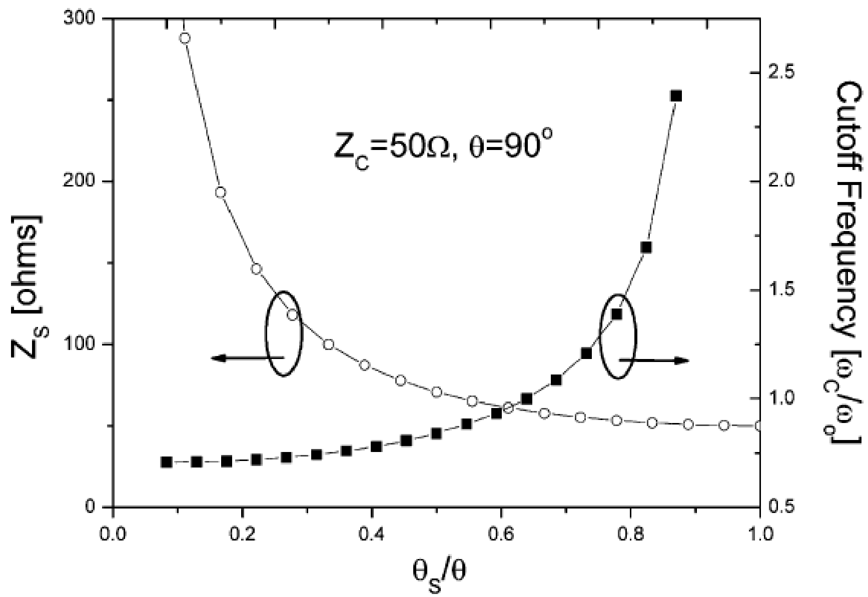
- As soon as we move consistently away from the design frequency the behaviour of the two networks becomes very different.

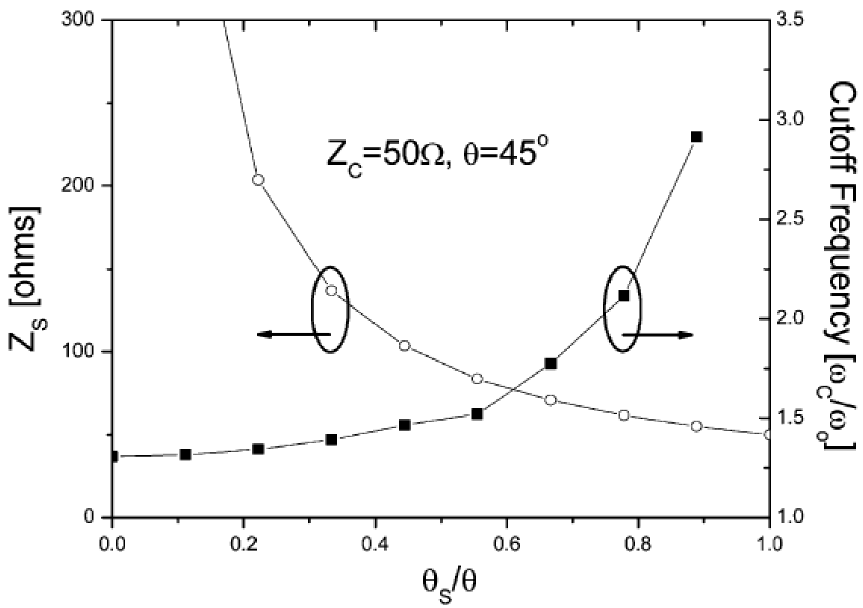




TL lumped equivalent

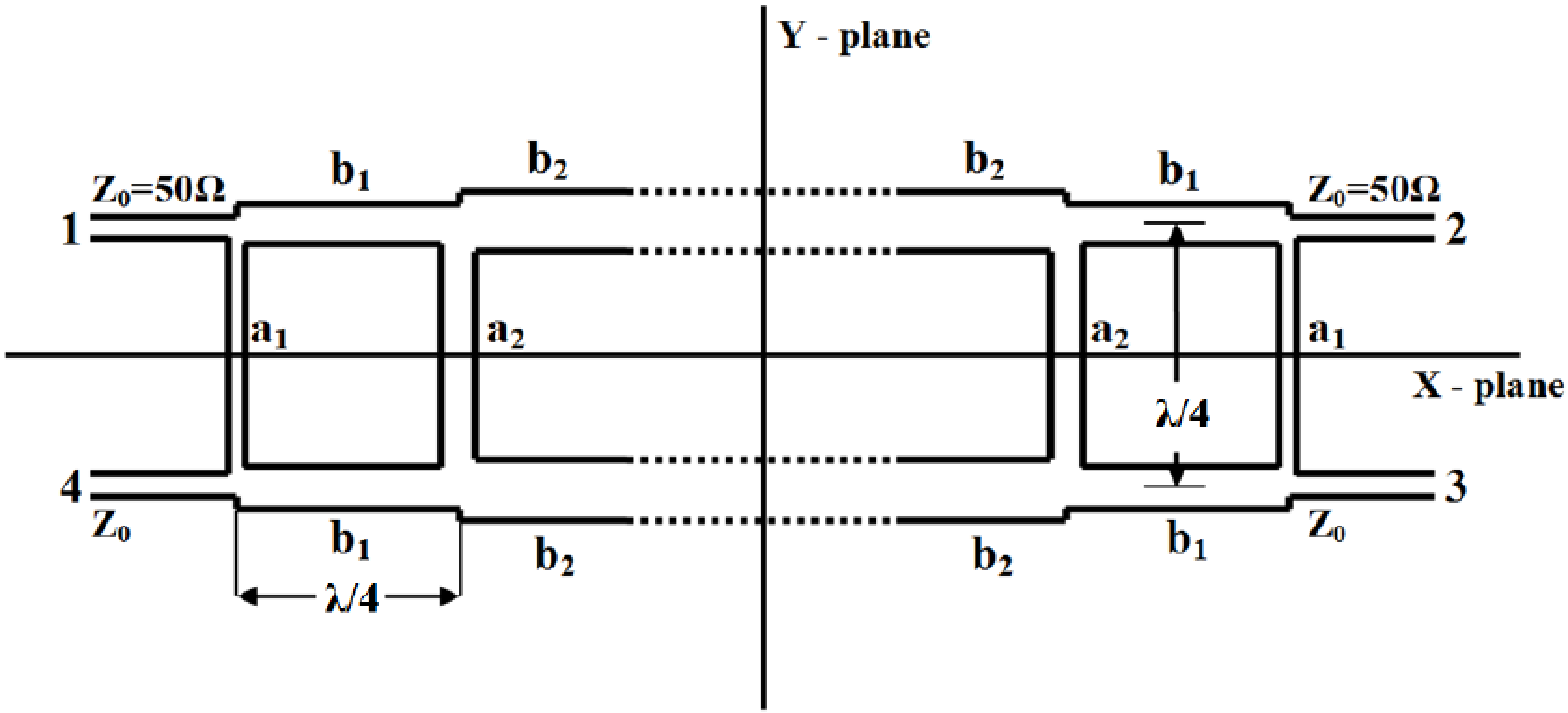
- So what is the relative BW we can expect we can use?
- We can use graphs linking the size reduction with the maximum relative BW of the equivalent line.





TL lumped equivalent

- When we need to achieve a relative BW using branch line couplers larger than 3% we need to cascade more sections with the appropriate impedance levels.



TL lumped equivalent

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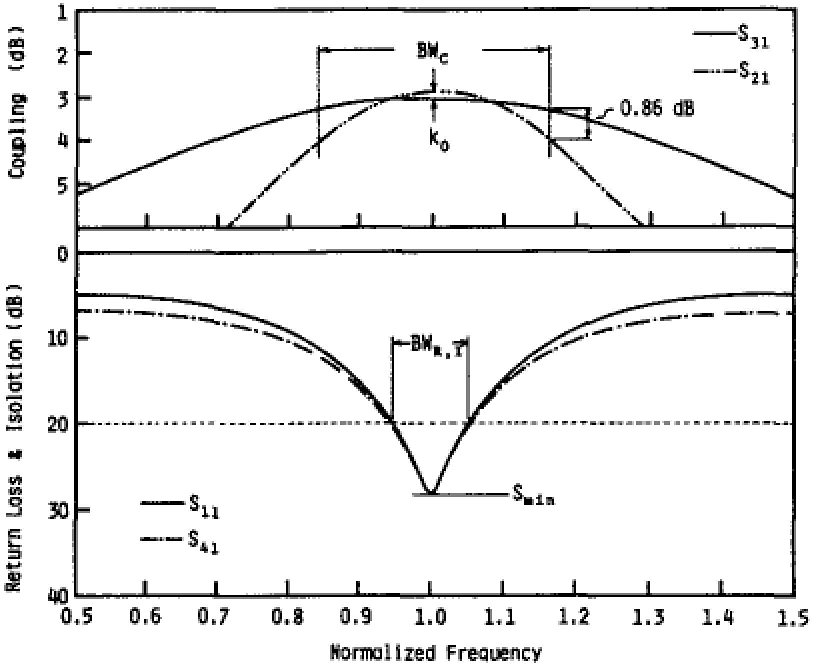
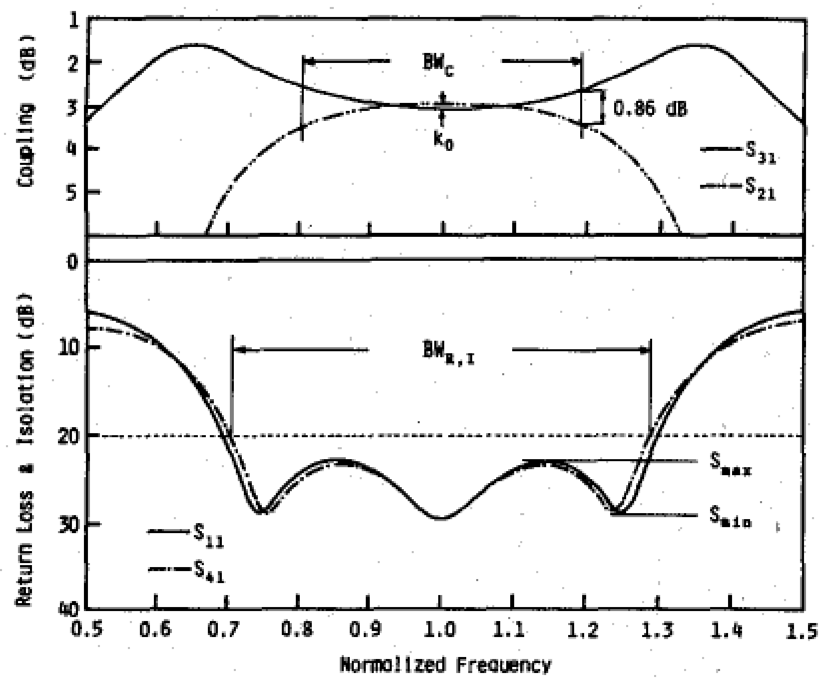


Two sections



Four sections

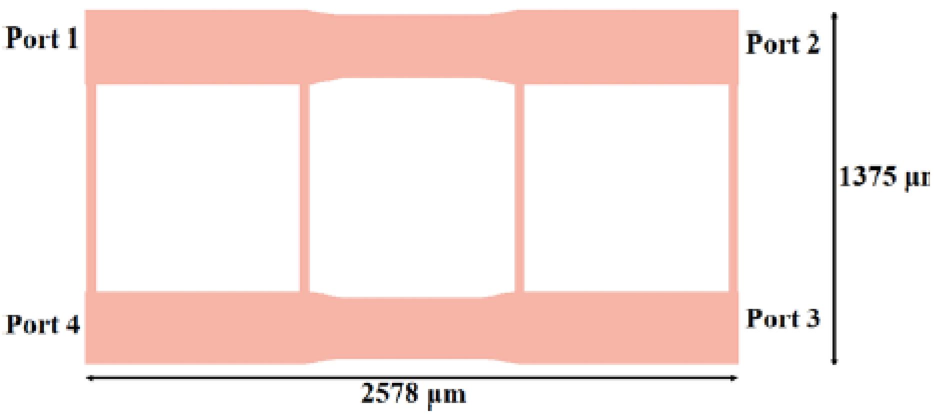
TL lumped equivalent

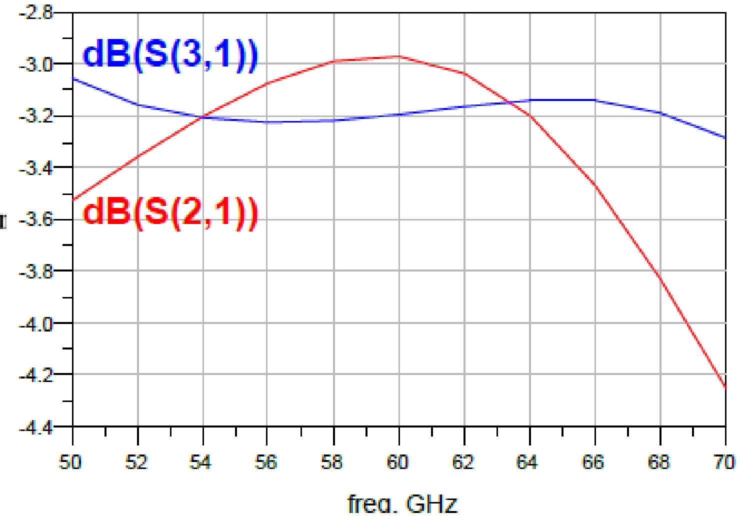
- Designing a 4 section BLC to achieve large relative BW requires very low impedance levels.

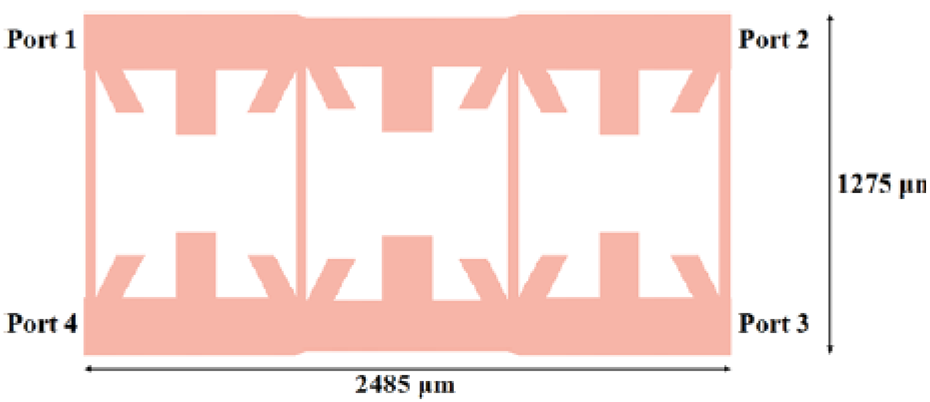
	D	b_1 (Ω)	b_2 (Ω)	a_1 (Ω)	a_2 (Ω)	$BW_{R,I}$ (%)	BW_C (%)	S_{min} (dB)	S_{max} (dB)	K_0 (dB)
4 - 1	---	39.97	30.33	263.85	52.82	43	32	∞	---	0
4 - 2	---	36.83	28.77	170.53	57.44	60	34	∞	22.14	0
4 - 3	16	44.34	39.40	152.95	90.96	63	44	36.02	20.51	0.28
4 - 4	20	44.14	39.15	162.39	85.32	60	40	36.23	22.73	0.19
4 - 5	24	45.03	41.99	157.52	91.12	58	38	29.03	23.02	0.14
4 - 6	16	39.40	30.66	166.67	67.03	62	44	31.89	20.47	0.29
4 - 7	16	42.99	36.84	156.25	83.91	63	44	47.30	20.45	0.28
4 - 8	16	47.16	44.76	147.06	106.32	61	43	28.90	20.46	0.28
4 - 9	16	51.96	54.85	138.89	136.09	57	43	23.47	20.39	0.26
4 - 10	16	57.37	67.39	131.58	174.95	50	43	22.06	20.15	0.25
4 - 11	14	53.68	56.51	142.86	142.86	57	45	25.24	20.20	0.40
4 - 12	16	53.94	58.31	142.86	142.86	55	43	24.17	21.41	0.29
4 - 13	16	52.50	55.92	138.89	138.89	56	43	23.30	20.52	0.27
4 - 14	18	52.62	57.14	138.89	138.89	54	41	24.16	21.01	0.21
4 - 15	20	52.69	58.06	138.89	138.89	52	40	25.66	21.23	0.17
4 - 16	20	51.28	55.34	135.14	135.14	53	40	26.33	20.04	0.16
4 - 17	**22**	**51.32**	**56.41**	**135.14**	**135.14**	**52**	**39**	**27.75**	**20.08**	**0.13**

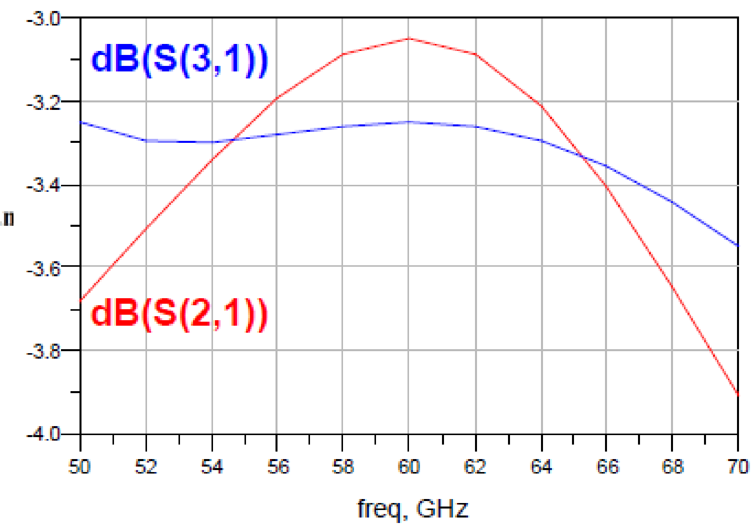
TL lumped equivalent

- Four section BLC using lumped line approximation



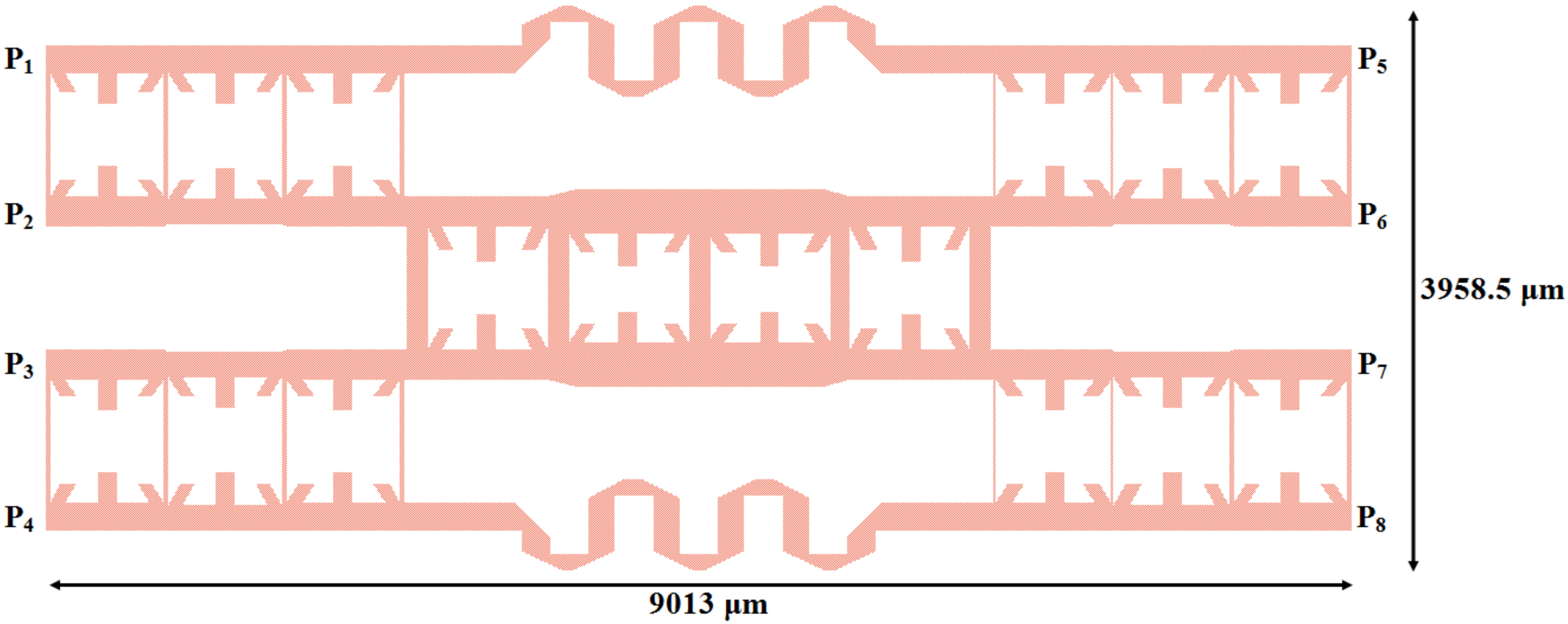






TL lumped equivalent

- Design of Butler matrix using lumped line approximation



TL lumped equivalent

- Implementation of Butler matrix using lumped line approximation