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Communication Theory (5ETB0) 2024-2025

Assignment 3

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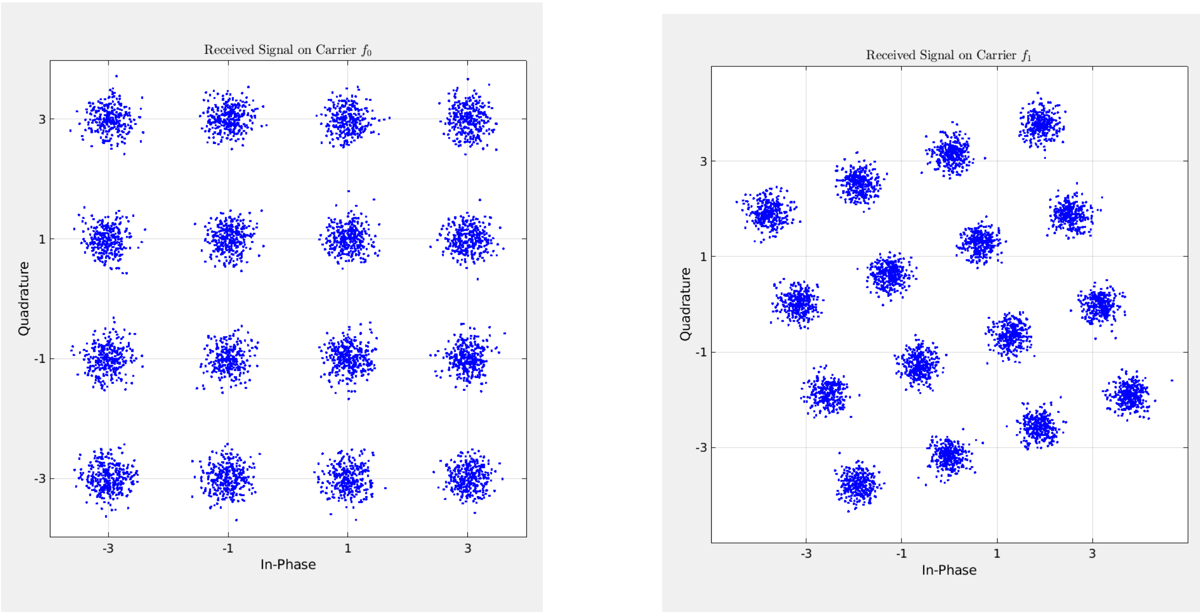
Where innovation starts

1 Problem 1: Quadrature Multiplexing with Multiple Carriers

1.1 Q1.2

The constellation diagrams for 16-QAM visually showcase a close distance between each symbol, this implies a high signal-to-noise ratio (SNR) for both f_0 and f_1, however these is a noticeable tilt for the f_1 constellation which implies a phase rotation. The consequence of which increases a degraded SNR compared to f_0 and the difficulty in further demodulation. This could be caused by residual frequency offset between transmitter and receiver or distortion through the optical fiber channel, which is mentioned because the fiber has various frequency-dependent impairments.



(a) Received Symbols at f_0 (b) Received Symbols at f_1

Figure 1.1: Comparison of received symbols at f_0 and f_1.

1.2 Q1.3

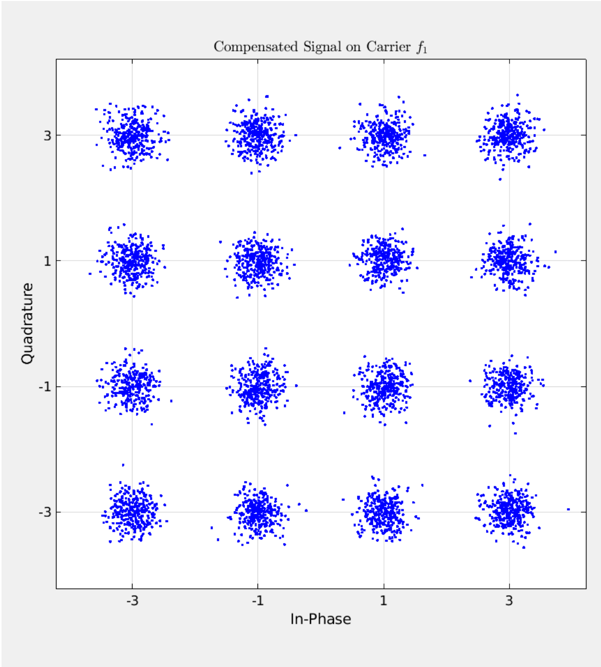


Figure 1.2: Received Symbols at f_1 Compensated

The received QAM signal for carrier f_1:

$$r(t) = s^{(0)}(t) + s^{(1)}(t - \Delta\tau) + n_w(t), \tag{1.1}$$

where $s^{(1)}(t - \Delta\tau)$ is a time-delayed signal due to the reported irregularities added by the fiber optic cable channel, with $\Delta\tau = 0.25$ ps. The time delay adds phase shift θ:

$$\theta = 2\pi f_1 \Delta\tau. \tag{1.2}$$

To compensate for the phase shift, $(r_{I,k})$ and $(r_{Q,k})$ are rotated using the inverse of the rotation matrix:

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}. \tag{1.3}$$

The compensated components:

$$\begin{bmatrix} r'_{I,k} \\ r'_{Q,k} \end{bmatrix} = \mathbf{R}(\theta) \begin{bmatrix} r_{I,k} \\ r_{Q,k} \end{bmatrix}. \tag{1.4}$$

1.3 Q1.4

Table 1.1: Received Symbols Carrier: f_0

	-3	**-1**	**+1**	**+3**
-3	331	304	305	285
-1	318	273	323	316
+1	308	335	333	305
+3	322	312	321	309

Table 1.2: Received Symbols Carrier: f_1

	-3	**-1**	**+1**	**+3**
-3	193	356	383	201
-1	381	313	314	354
+1	369	313	310	412
+3	189	382	358	172

2 Problem 2: Data Transmission with Error Correcting Codes

2.1 Q2.1

$(7, 4)$ Hamming parity matrix P:

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}. \tag{2.1}$$

Generator matrix G:

$$G = [I_k \mid P] \tag{2.2}$$

I_k is 4×4 identity matrix. Substituting P, generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}. \tag{2.3}$$

G maps each 4-bit information sequence to a 7-bit codeword using the Hamming $(7, 4)$ coding scheme.

2.2 Q2.5

The parity check matrix H for a $(7, 4)$ Hamming code is derived from G:

$$G = \begin{bmatrix} I_4 & P \end{bmatrix}, \tag{2.4}$$

The parity check matrix H:

$$H = \begin{bmatrix} P^T \\ I_3 \end{bmatrix}, \tag{2.5}$$

where P^T is transpose of parity matrix P, and I_3 since $n_c - k_c = 7 - 4 = 3$:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}. \tag{2.6}$$

2.3 Q2.8

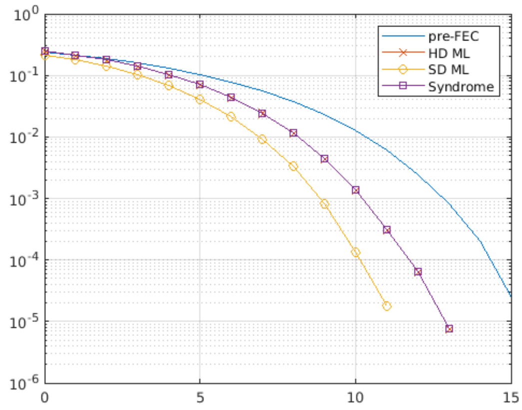


Figure 2.1: BER of Decoders

Performance Comparison::

- SD-ML decoder achieves a BER of 10^{-3} at $\text{SNR}_{SD} = 9$ dB.

- HD-ML decoder achieves a BER of 10^{-3} at $\text{SNR}_{HD} = 10.2$ dB.

Net Coding Gain (ΔNCG):

$$\Delta NCG = \text{SNR}_{HD} - \text{SNR}_{SD} = 10.2 - 9 = 1.2 \text{ dB}. \tag{2.7}$$

Explanation: The SD-ML decoder has all channel information, using the distances between received and codeword symbols. The HD-ML decoder thresholds the signal, losing information which degrades quality.

Computational Complexity:

- SD-ML: $O(n \cdot 2^k)$ distance calculations for 2^k codewords.

- HD-ML: $O(n \cdot 2^k)$ simpler comparison operations.

Conclusion: SD-ML has a higher performance but similar complexity to HD-ML.

2.4 Q2.9

Net Coding Gain (ΔNCG):

$$\Delta NCG = \text{SNR}_{HD} - \text{SNR}_{Ham} = 0 \text{ dB}. \tag{2.8}$$

Explanation: Both algorithms use hard decisioning. However, the Hamming decoder simplifies decoding using a parity-check matrix.

Computational Complexity:

- Hamming: $O(n)$ syndrome calculation and table lookup.

- HD-ML: $O(n \cdot 2^k)$