Evaluation of efficiency and steady-state torque-speed of electrical machines

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***Abstract*—This study represents a comprehensive analysis concerning three different types of electrical machines which enclose separately excited DC machines, PM synchronous machines, and induction machines under fixed operational conditions. The focus concerns the validation of the theoretical models that link the key performance metrics as the torque, speed, and efficiency both in motor and generator modes. These models are central to predicting their behavioral tendencies when integrated with various mechanical elements. Validation involves comparison with the empirical data, hence it gives faith to practically implement these models into any application over which it is studied.**

I. Introduction

ELECTRIC machines are crucial in the conversion of energy, either during the generation of electrical energy or of mechanical engineering processes. The importance of this lies in the fact that electric machines double up as both motors and generators in deploying electrical energy. This paper is concerned with theoretical model development and validation of three different types of electrical machines: separately excited direct current machines, permanent magnet synchronous machines as well as induction machines. This study aims at the development of predictive models for each machine type accommodating performance characteristics like the torque-speed relationship, efficiency, and in particular for alternating current machines the power factor.

These models play a major role in designing a system since they help find out the points of the operational efficiency and performance of the machine under variant configurations. Recent years observed the rise of importance of PMSMs in the area and signified that they are highly applicable in applications such as electric vehicles due to high power and torque density as well as fast dynamic response [1].

This paper also relies on the fundamental concepts and analysis presented in works such as "Electric Machines: Analysis and Design Applying MATLAB" by J.J. Cathey [2], which provided important approaches for the analyses performed in this study, based on MATLAB. A systematic review of online parameter estimation in PMSMs for enhanced control performance and operational reliability has similarly drawn great attention within the literature [3]. Research work done on machines with mixed poles contributed to the analysis for electromechanical torque and power dynamics [4].

Lab 1: setup 9, Khanh Lo
Lab 2: setup 10, Khanh Lo
Lab 3: setup 11, Khanh Lo

II. Methods

THIS study applied laboratory setups involving different configurations of electric machines connected in dual configurations, namely Permanent Magnet Synchronous Motor or Induction Motor load driven by the Direct Current Motor. Voltage Source Inverter energizes these systems capable of sensing the current and voltage to enable the running of either voltage or current source modes. A set of sensors senses RMS line voltages, and key operational parameters such as currents, and power factor. Moreover, the speed of rotation is monitored using an incremental encoder coupled to one of the VSIs. The experimental facility does not have a dedicated sensor to obtain this torque; its estimate is feasible through the electrical and mechanical data analysis. The analysis throughout the paper were performed on the basis of motor operation.

With the specially built torque sequencer, the torque dynamics of the machines are controlled by application. Here, controlling the torque output of either machine is facilitated, thereby providing controlled torque operation. The specifications of the torque sequencer program in table I offer the facilities.

The frictional forces with the entire system including the static Coulomb friction and velocity-proportional dynamic friction are consolidated into a single model given by:

$$T_{FW} = \mu_1 + \mu_2\omega \tag{1}$$

where T_{FW} is the total friction torque, ω is the angular velocity and μ_1, μ_2 are the static and dynamic friction coefficients. No-load tests involve the PMSM or IM running idle, where it is driven only at a constant speed while the DCM is also active. The frictional torque developed in this instance is typically regarded as totally attributed to the DCM from where the above regression models can be developed. As proposed, torque distribution between DCM and ACM (either PMSM or IM) shall obey the relations:

$$T_{FW,DCM} = 0.8T_{FW} \tag{2}$$
$$T_{FW,ACM} = 0.2T_{FW} \tag{3}$$

They shall be treated in the analysis as in motor mode concerning convention for direction of shaft speed and armature current. The model accuracy is evaluated concerning

TABLE I: Torque Sequencer Configuration

Parameter	Value	Unit
Torque Limit Lower Bound	-7	Nm
Torque Limit Upper Bound	7	Nm
Step Duration	3	s
Total Steps	43	-

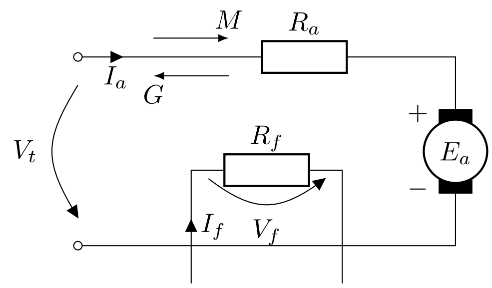

Fig. 1: Equivalent circuit of a separately excited DC motor.

the experimental results by the median and maximal absolute relative errors expressed in:

$$\epsilon_1 = \text{median}\left(\left|\frac{y_1 - y_2}{y_1}\right|\right) \tag{4}$$

$$\epsilon_2 = \max\left(\left|\frac{y_1 - y_2}{y_1}\right|\right) \tag{5}$$

where ϵ_1 and ϵ_2 represent median and maximum absolute relative errors respectively with y_1 and y_2 representing experimental and theoretical values respectively. General equation to quantify efficiency evaluation across the system:

$$\eta = \frac{P_o}{P_i} \tag{6}$$

A. Direct Current Machine Analysis

This subsection details the experimental procedure and analysis for a Direct Current Motor (DCM) functioning in tandem with a Permanent Magnet Synchronous Motor (PMSM). The DCM's operational characteristics are deduced through its equivalent circuit in figure figure 1 and empirical data obtained in a controlled laboratory environment.

Operational Principle: The operational principle of the DCM is using brushes with armatures to ensure that the generated torque is only in one direction. Mathematically, the armature electromotive force (EMF) is denoted as E_a, the function of the motor's physical parameters concerning the motor's operational speed.

$$E_a = C_m \Phi \omega_r \tag{7}$$

Here C_m is the machine constant of the motor, at Φ is the magnetic flux, and ω_r is the rotation speed.

Voltage and Torque Relationships: Apart from the internal resistance R_a, the function is dominantly dictated by the armature circuit's terminal voltage V_t. Therefore, the relationship can be stated as:

$$V_t = E_a + I_a R_a \tag{8}$$

TABLE II: DCM Quantities in the DCM-PMSM

Quantities	Symbols	Values	Units
Field Voltage	V_f	340	V
Field Current	I_f	0.48	A
Armature Resistance	R_a	1.63	Ω
Constant	C_m	0.97	Vsrad^{-1}

TABLE III: Friction Quantities in the DCM-PMSM

Quantities	Symbols	Values	Units
Static Friction	μ_1	0.49	Nm
Dynamic Friction	μ_2	0.0016	Nmsrad^{-1}

where I_a is the armature current. The generated torque of the DCM, T_g, is directly proportional to the armature current:

$$T_g = C_m \Phi I_a \tag{9}$$

Efficiency Calculation: DCM efficiency needs to be calculated following energy management to find its input and output powers. Power input, P_i, includes an armature contribution as also that of field which can be summed together, and power output, P_o, is mechanical:

$$P_i = V_t I_a + V_f I_f \tag{10}$$

$$P_o = T_g \omega_r \tag{11}$$

V_f and I_f being the field values of the separately excited circuit showcased in figure 1 Thus, the DCM efficiency η_{DCM} is given by:

$$\eta_{DCM} = \frac{P_o}{P_i} \tag{12}$$

Experimental Set Up: The parameters of DCM listed into table II have been used. Field winding parameters are set at maximum value to bring out optimal efficiency based on equation (12). From the control of the armature current and then measuring the corresponding voltage, a constant C_m of the machine was deduced. This hence simplifies directly equation (8) to yield C_m. Under steady-state conditions, the armature resistance R_a is then deduced thereafter.

The friction coefficients present to carry out the accurate torque analysis are tabulated in table III. These were calculated by performing a no-load test based on which these frictional losses are isolated from the operational torques.

Data Acquisition and Analysis: Torque was varied in a systematic manner using a torque sequencer such that monitoring of armature voltage and current is done simultaneously to obtain the torque-speed characteristic of DCM. This data is then processed to derive the torque-speed curve based on equation (9) and equation (8).

B. Permanent Magnet Synchronous Machine Analysis

Analysis of the PMSM operational characteristics in a DCM-PMSM setup is performed in this subsection as established in the DCM section.

Operational Principle Unlike DCM, the PMSM utilizes three-phase alternative sources to generate torque on the rotor.

Circuit Representation The PMSM is analyzed as a three-phase balanced in wye configuration machine, the circuit for

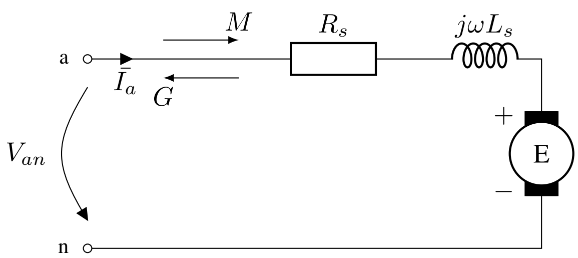

Fig. 2: Equivalent Circuit of a Permanent Magnet Synchronous Motor

TABLE IV: PMSM Quantities in the DCM-PMSM

Quantities	Symbols	Values	Units
Number of Poles	p	8	-
Synchronous Inductance	L_s	3.15	mH
Stator Resistance	R_s	0.85	Ω

this setup is represented in figure 2. The PMSM's behavior is characterized by its phase voltage and current relationship, with the synchronous inductance (L_s) and coil resistance (R_s) being key components:

$$\overline{V}_{an} - \overline{E}_f = \overline{I}_a R_s + \overline{I}_a j\omega L_s \tag{13}$$

The induced EMF ($\overline{E}_f$) in the PMSM results from flux linkage changes and is given by:

$$\overline{E}_f = \overline{\lambda}_m \frac{p}{2} \omega_m \tag{14}$$

where $\overline{\lambda}_m$ is the RMS flux linkage.

Power Transfer and Efficiency: The developed torque ($T_{d,PMSM}$) and shaft torque ($T_{s,PMSM}$) are pivotal for understanding the machine's performance:

$$T_{d,PMSM} = \frac{E_f I_a \cos\beta}{\omega_m} \tag{15}$$

$$T_{s,PMSM} = 3T_{d,PMSM} - 0.2T_{FW,tot} \tag{16}$$

Efficiency in motor mode is then derived by using the developed torque for which the equation was derived by using the developed power:

$$\eta_{PMSM,m} = \frac{E_f I_a \cos\beta - 0.2T_{FW,tot}}{3V_{an} I_a \cos\theta} 100\% \tag{17}$$

Determination of the Parameters: This analysis will involve the parameters of the PMSM (table IV) as well as coefficients (table V) of friction. Adequate determination for these parameters is allowed by the no-load test as described in section II.

Operational Analysis: The performance of the PMSM based on power factor is experimented with different load conditions. Hence, the functioning of the PMSM to set in mode torque control and that of the DCM to set in speed control mode which makes it focus into the analysis of the system intensively.

TABLE V: Friction Quantities in the DCM-PMSM

Quantities	Symbols	Values	Units
Static Friction	μ_1	0.52	Nm
Dynamic Friction	μ_2	0.0024	Nmsrad^{-1}

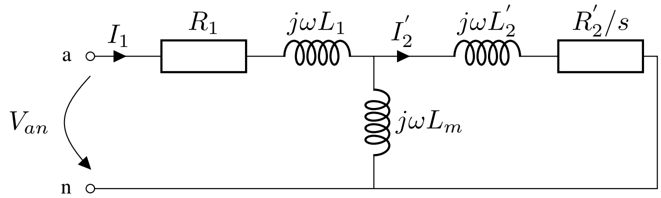

Fig. 3: Scaled Equivalent Circuit of an Induction Motor (IM)

C. Induction Machine Analysis

This subsection deals with the operational mechanics of an Induction Machine (IM) in DCM-IM configuration, bringing out its unique attributes over synchronous machines.

Operational Principle Similar to PMSM, IM generates torque with alternative power. The fundamental difference is the rotor magnetic flux is generated through induction, hence the speed at which mechanical rotor rotates (ω_m) need to be different from the speed of the stator, termed as synchronous speed (ω_s). The difference between these two speeds commonly referred to as slip (s), is given by:

$$s = \frac{\omega_s - \omega_m}{\omega_s} \tag{18}$$

The synchronous speed is related to the stator's electrical frequency (f) as:

$$\omega_s = 2\pi f \tag{19}$$

Circuit Representation: The figure 3 representing the IM equivalent circuit simplifies hiding complex interaction in manageable components. Resistances (R_1 and R_2') and inductances (L_1, L_2', and L_m) represent stator and rotor circuit elements. Stator current (I_1') and rotor current (I_2') are important concerning the analysis of power transfer as well as $j\omega L_m$ was reduced from the full model of R_c in parallel to $j\omega L_m$:

$$I_1 = \frac{V_{an}}{Z_{total}} \tag{20}$$

$$I_2' = I_1 \left[\frac{j\omega L_m}{(j\omega L_m + j\omega L_2') + \frac{R_2'}{s}} \right] \tag{21}$$

where Z_{total} is the total impedance of the stator and the transformed rotor circuit.

Power-Torque Dynamics: The air-gap power (P_g) transferable from the stator to the rotor and the torque developed (T_d) constitute two roots for IM performance equations:

$$3P_g = 3\left(I_2'\right)^2 \frac{R_2'}{s} \tag{22}$$

$$T_d = \frac{3\left(I_2'\right)^2 R_2'(1-s)}{\omega_s s} \tag{23}$$

TABLE VI: IM Quantities in the DCM-IM

Quantities	Symbols	Values	Units
Rotor Resistance	R_2	0.96	Ω
Magnetizing Inductance	L_m	0.31	H
Rotor Leakage Inductance	L_2	6.3	mH
Voltage-Frequency Coefficient	a	4.63	VHz^{-1}
Stator Resistance	R_1	1.84	Ω
Stator Leakage Inductance	L_1	12.7	mH
Offset Voltage Coefficient	b	7.32	V

Voltage-Frequency Control: Voltage-frequency (V/f) control is usually in control of the response of IM to the changing load conditions. The terminal voltage (V_{an}) is relative to the frequency of the stator controlled with a constant offset:

$$V_{an} = af + b \tag{24}$$

Efficiency Evaluation: Mechanical to electrical power comparisons should be made for the correct efficiency assessment of the IM in motor mode:

$$\eta_{IM,motor} = \frac{T_s \omega_m}{\Re\left\{\overline{V}_a \overline{I}_a\right\}} 100\% \tag{25}$$

Experimental Procedure: Parameters of IM detailed in table VI are important to go ahead with proper analysis. A no-load test explained in section II gives an initial idea on machine friction coefficients and how machine behaves operationally. The IM was set in V/f control mode while DCM was set in torque control mode to enable a more detailed evaluation. Configuration of this nature is necessary in helping capture accurately parameters such as torque-speed characteristics and efficiency parameters of the IM done during the study.

III. Results

THIS section discusses a detailed analysis of the performance of Machines against the wide variation of operational conditions applied herein. The performance here relates to the torque-speed relationship and efficiency metrics with a keen eye on model accuracy.

A. Direct Current Machine Performance Evaluation

DCM operation was focused in the report under armature voltages of 50V, 150V, and 250V to correspond with the torque sequencer settings indicated in table I. The main comparison of this study prototype lies between the empirical behavior of torque speed and the theoretical model.

Efficiency Analysis: The efficiency trends, as depicted in figure 5, illustrates that the median relative errors of (3.429%, 0.481%, and 1.403%) and maximum of relative errors of (18.354%, 26.360%, and 52.386%) respectively. At the higher order, compensating for torque-speed discrepancies and unconsidered frictional aspects may mask out the lower error margins of efficiency.

Torque-Speed Characterization: Contrast against the theoretical model exhibits significant deviations in torque-speed characteristics (figure 4).

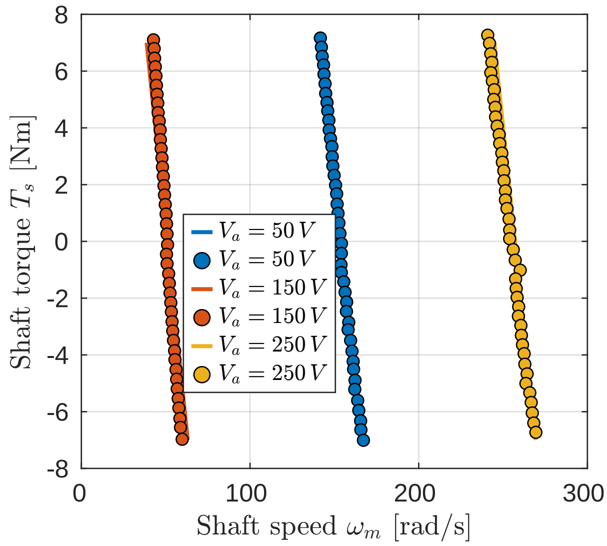

Fig. 4: Torque-Speed of Direct Current Machine. (line: generated, circle: recorded)

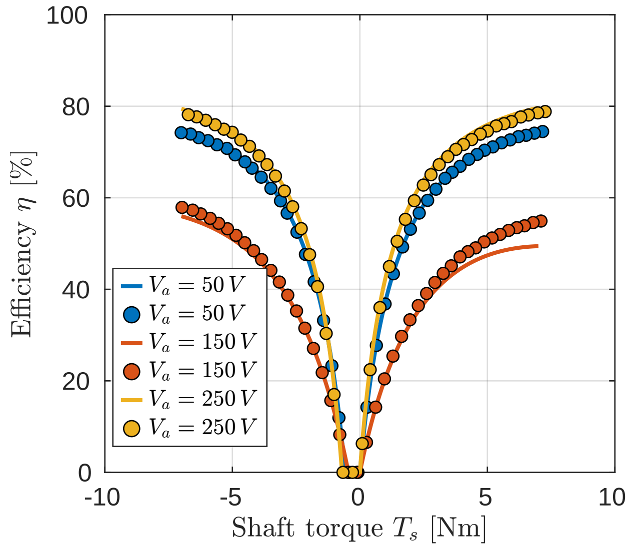

Fig. 5: Efficiency-Torque of Direct Current Machine. (line: generated, circle: recorded)

There exists a lot of systematic errors in this experimental setup. Across the voltage settings, the observed median and maximum relative errors are (36.575% 4.523% 50.540%) and (152.147% 102.435% 1847%). These variations imply deviation in the DCM internal dynamics, probably due to armature reaction effects which change machine constant and torque-speed slope.

Further Insights: The study brings out a fact that although the theoretical model gives rudimentary knowledge, with the actual operational subtleties like armature reaction and variable friction torque, the impact upon the DCM performance is not inconsiderable. These have to find their place within the newer versions of the model to improve its accuracy. The above analysis underlines the empirical validation conducted to take forward the understanding of the dynamic behavior of electrical machines and adjustments in the model formulation

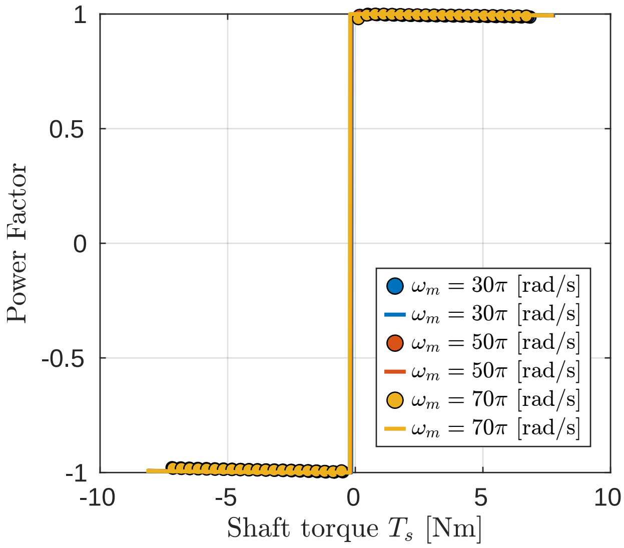

Fig. 6: Power Factor-Torque of Permanent Magnet Synchronous Machine. (line: generated, circle: recorded)

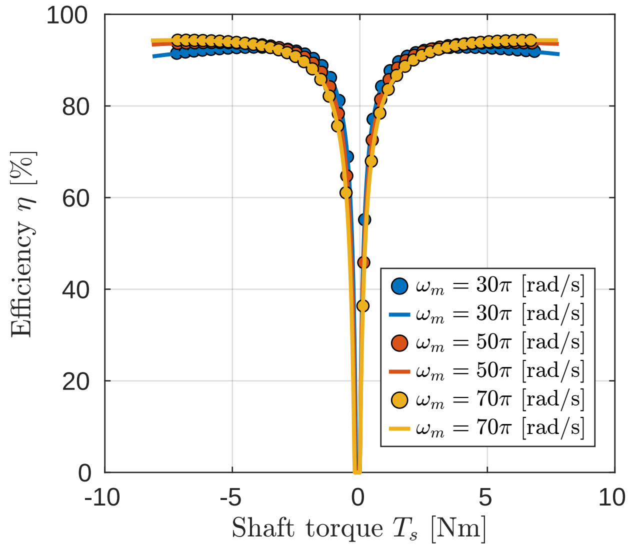

Fig. 7: Efficiency-Torque of Permanent Magnet Synchronous Machine. (line: generated, circle: recorded)

needed to include the complex operational realities.

B. Permanent Magnet Synchronous Machine Performance Evaluation

The section provides the functional efficiency of PMSM, noting its power factor and efficiency at different shaft speeds being 30π, 50π, to be 70π rad/s.

A systematic investigation was made on the performances of PMSM to seek a relation between its power factor, efficiency, and speeds on its operational shaft.

Efficiency Analysis: The efficiency correlation, as depicted in figure 7, reveals very low median relative errors (0.089%, 0.016%, and 0.006%) interspersed with higher maximum relative errors (2.154%, 2.252%, and 3.324%). These observations do support the model prediction accuracy of anticipating efficiency trends under diversified operational conditionality.

Trends of Power Factor: Similar to the case illustrated in figure 6, the unity power factor always carries it close to 1.000% at all measured shaft speeds, implying more efficient operational performance. The median absolute relative errors of 0.965%, 0.848%, and 0.744%, with maximum errors of 1.346%, 1.236%, and 1.373%, respectively, are a testimony to the high consistency of the model to actual performance.

Further Insights: The PMSM has a very strong correlation to theoretical power factor predictions and is highly efficient. The results are also verified by the robustness of the PMSM in retaining high efficiency besides verifying the model. In further improvements, adjustments will be required that are to be made to the prediction of margins of the efficient performance especially at higher points in the operational speed spectrum. Essentially, a PMSM emerges as a high-efficiency machine whose performance comes very close to the theoretically predicted one reinforcing its suitability in applications that demand high precision and efficiency.

C. Induction Machine Performance Evaluation

The subsection determines characteristics evident from operation with different stator frequency settings of the Induction Machine (IM). It shall describe the torque-speed performance of the IM, power factor, and efficiency analyses.

The IM was tested on each of the stator frequency set points and evaluated in terms of torque-speed characteristics, power factor, and efficiency at various shaft speeds.

Efficiency Analysis: Looking at figure 10, the efficiency trends of IM have medium errors (3.797%, 5.292%, and 23.553%) with maximum errors of (3.920%, 12.427%, and 133.760%). The substantial errors imply that the model simplifies the operational behavior of the complex IM.

Torque-Speed Characterization: IM's torque-speed curves as depicted by figure 8 indicate linearity relationship characteristics of small slip operations. The median absolute of relative errors found was (8.068%, 5.092%, and 3.695%) while reaching maximum error levels at (172.734%, 891.832%, and 148.360%). Errors, especially at lower torque ranges, are probably due to the omission from the model of some empirical losses.

Trends of Power Factor: As the IM's power factor trends presented in figure 9, the median errors are (2.138%, 10.777%, and 32.583%), and the maximum errors are (151.465%, 47.469%, and 4185.9%). Partial representation of the machine dynamics is likely to be contributed in part to these errors.

Further Insights: Analysis brings forth the IM's operational linearity under small slip conditions but reiterates the relative limitations of the model for capturing a true picture of the real machine, especially concerning the power factor and efficiency. This indicates that advanced modeling approaches are necessarily needed to enable a more accurate reflection of the IM performance functioning under varying load and speed conditions.

In summary, though the IM exhibits predictable torque-speed behavior, its power factor and efficiency characteristics

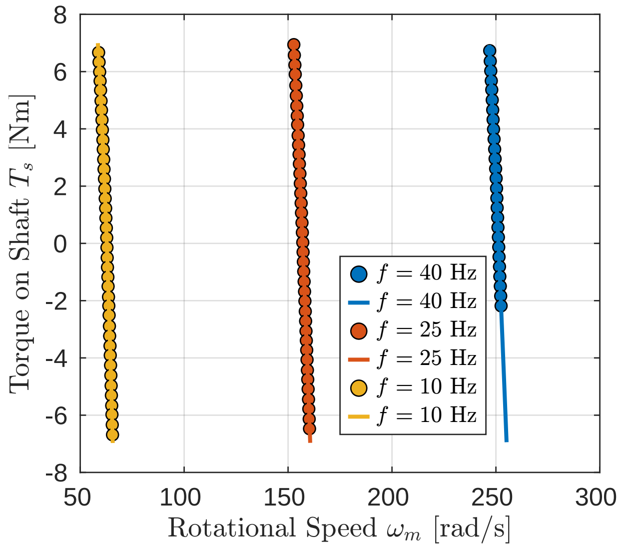

Fig. 8: Torque-Speed of Induction Machine. (line: generated, circle: recorded)

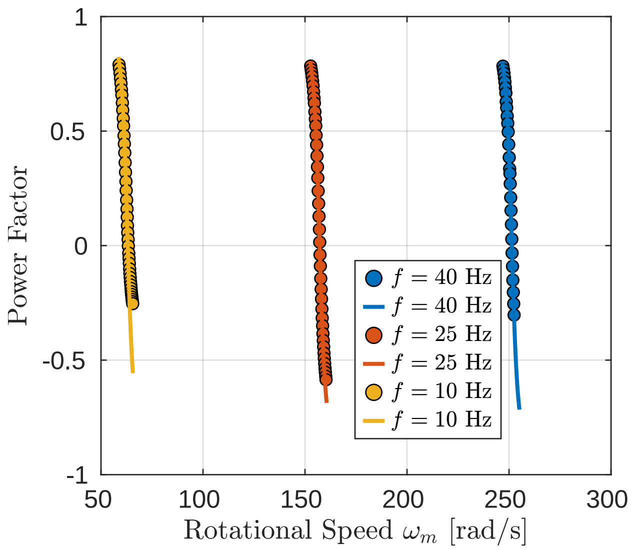

Fig. 9: Power Factor-Speed Induction Machine. (line: generated, circle: recorded)

require a much more detailed modeling approach for it to be in a position to adequately capture its features for better accuracy in industrial applications.

IV. Conclusions

MODELED and measured characteristics of Direct Current Machines, Permanent Magnet Synchronous Machines, and Induction Machines are effectively compared in this study. While DCMs showed relatively small efficiency discrepancies caused because of the armature reaction effect, PMSMs aligned closely to Its model showcasing high accuracy in power factor and efficiencies metrics. While the IMs torque-speed characteristics had a sense of linearity, power factor, and efficiency showed significant deviations and thus there is a need to model the motor more comprehensively. These results draw the importance of a study with explicit modeling

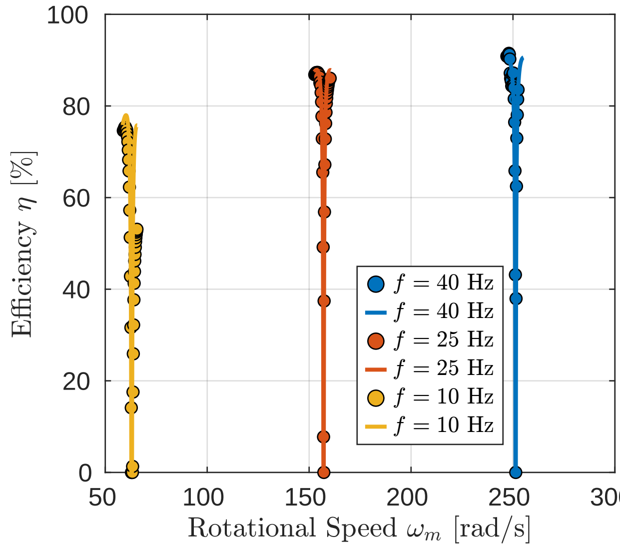

Fig. 10: Efficiency-Speed of Induction Machine. (line: generated, circle: recorded)

in capturing electric machine behavior more accurately, where future pretensions can be made toward the enhancement of the IM model and the treatment of the DCM complexities.

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