Mathematics II (5EMA0)

First Optimization Assignment

Group Number: 17

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Matrix A:

$$\begin{bmatrix} -3.95090873631221 & 3.70903082235421 & -8.42572558465403 & -8.01662771942462 \\ 1.40559785757351 & 7.81021901595398 & -12.8889287027324 & -3.22803455180356 \\ -15.1722933731061 & -13.5992613820094 & -13.9599970340248 & 17.1624543364314 \\ -18.8198533786435 & 3.47564705523433 & 8.95743858855331 & 7.36568725619399 \\ 7.96513128621806 & 0.842829723816281 & -3.96093333653340 & -12.6839583082234 \\ 2.41270907334958 & 2.32106019361867 & -9.41365059230125 & 19.0191761587253 \\ 2.86825773860921 & -0.437271769032225 & 14.2160665339664 & 1.86148288969507 \\ 2.68611697794152 & -0.934200791895131 & 2.37499550294005 & 9.83343793080708 \\ -21.6821785015943 & 1.94749703435965 & -5.40198186585973 & 1.63555720845322 \\ -1.56163799425792 & -1.17952525063512 & 1.16419111828735 & 7.96551582975236 \\ 17.9738121189033 & 7.15341409636909 & -10.9863822483929 & -6.98140584531242 \\ -14.6666172561678 & -2.24060937531757 & 7.69878474638357 & 14.5861261275561 \end{bmatrix}$$

Vector b:

$$\begin{bmatrix} 55.7508475724632 \\ 113.938363092844 \\ 41.6237410589720 \\ -82.8054265546733 \\ 33.1470668935391 \\ 96.7555918096100 \\ -93.2487514234031 \\ -6.16814446511481 \\ 2.25281582407217 \\ -6.51051028238259 \\ 127.580266693737 \\ -74.7860810339641 \end{bmatrix}$$

Problem 1:

Extended CVX script:

```
rng(17)
n = 4;
m = 12;
xt= [2,3,-7,1];
A = random('Normal',0,10,m,n);
b = A*xt'+ random('Normal', 0,1, m,1);

[t, x] = bisection(A,b)

function [t, x] = bisection(A,b)
      tmin = 0;
      tmax = 2;
      z = 0;
      while (tmax - tmin) > 0.001 || z ~= 1
            t = (tmax + tmin) / 2;
            [z, x] = feasible(t,A,b);
            if z == 0
                  tmin = t;
            else
                  tmax = t;
            end
      end
      t = (tmax + tmin) / 2;
end

function [z, x] = feasible(t,A,b)
      cvx_begin
            variable x(4)
            minimize 0
            max(abs(b-A*x))<=t;
      cvx_end
      z = 0;
      if strcmp(cvx_status, 'Solved')
            z = 1;
      end
end
```

Solutions:
Lowest value of t* = 1.0383;
Solutions for x* : 1.9447; 3.0912; -7.0171; 0.9356

Problem 2:

Extended CVX script:

```
rng(17)
n = 4;
m = 12;
xt= [2,3,-7,1];
A = random('Normal',0,10,m,n);
b = A*xt'+ random('Normal', 0,1, m,1);

cvx_begin
      variables x(n) t(m);
      dual variables y w;
      minimize(sum(t));
      subject to
            y: -t<=b-A*x
            w: b-A*x<=t
cvx_end

t, x, [y; w]
```

Solutions:

Lowest value of t* = 6.10812

Solutions for x* : 1.9757; 3.1527; -6.9656; 1.0042

Dual problem solution =

$$\begin{bmatrix} 8.0787e-09 \\ 0.3336 \\ 0.8280 \\ 0.9999 \\ 0.9999 \\ 0.5220 \\ 0.9999 \\ 0.9999 \\ 0.9999 \\ 2.3436e-08 \\ 0.9762 \\ 4.8394e-08 \\ 0.9999 \\ 0.6663 \\ 0.1719 \\ 8.5674e-09 \\ 1.5480e-07 \\ 0.4779 \\ 3.0149e-08 \\ 5.4272e-09 \\ 6.1785e-08 \\ 0.9999 \\ 0.0238 \\ 0.9999 \end{bmatrix}$$

Written dual problem:

- Original problem:
  $f_2(x) = max\{|b_i - b(a_{i_1}, \ldots, a_{in}))| : i = [1, m]\}$
- Linear optimization problem:
  - Introduce variable t
    $t \geq b_i - Ax \rightarrow t + Ax \geq b_i$
    $t \geq -b_i + Ax \rightarrow t - Ax \geq -b_i$
  - Rewrite into linear optimization problem: size(y) = size(c) = size(x) + 1 = (n + 1,1), size(d) = (2*m,1), size(B) = (2*m,m+1)

$y_2 = [x_2; t_2]$

$c_2 = [0; \ldots; 0; 1]$

$B_2 = \begin{bmatrix} A' & 1 \\ -A' & 1 \end{bmatrix}, \ A' = [A; \ldots; A]$

$d_2 = \begin{bmatrix} b \\ -b \end{bmatrix}$

Primal and dual problem:

$$min\{c_2^T y_2 : B_2 y_2 \geq d_2, y_2 \in \mathbb{R}\} = max\{d_2^T u_2 : B_2^T u_2 = c, u_2 \geq 0, u_2 \in \mathbb{R}\}$$

Problem 3:

Extended CVX script:

```
rng(17);
n = 4;
m = 12;
xt = [2, 3, -7, 1];
A = random('Normal', 0, 10, m, n);
b = A * xt' + random('Normal', 0, 1, m, 1);

cvx_begin
variables x(n) t(m)
      dual variables y1 y2 y3 y4
      minimize(sum(t))
      subject to
            y1:  2 * (A * x - b) <= t + 1
            y2: -2 * (A * x - b) <= t + 1
            y3: A * x - b  <= t
            y4: -A * x + b  <= t
cvx_end

x_star = x
min_value = cvx_optval
opt_dual_solution = [y1; y2; y3; y4]
```

Solutions:

Lowest value of t* = 7.2714

Solutions for x* : 1.9928, 3.1397, -6.9824, 0.9953

Dual problem solution =

$$\begin{bmatrix} 6.4503e-10 \\ 3.4896e-09 \\ 2.9058e-09 \\ 0.1513 \\ 4.8797e-09 \\ 2.9056e-09 \\ 3.5521e-09 \\ 0.9999 \\ 2.9058e-09 \\ 1.1546e-09 \\ 5.3852e-09 \\ 9.24016e-10 \\ 0.9999 \\ 1.9346e-09 \\ 2.9056e-09 \\ 7.2643e-10 \\ 1.3127e-09 \\ 2.9058e-09 \\ 1.8796e-09 \\ 4.0535e-10 \\ 2.9056e-09 \\ 5.8767e-09 \\ 1.2202e-09 \\ 1.0192e-08 \\ 1.2216e-09 \\ 0.9999 \\ 0.9819 \\ 0.8487 \\ 0.9999 \\ 0.0076 \\ 0.9999 \\ 3.6683e-09 \\ 0.9853 \\ 2.8738e-09 \\ 0.9999 \\ 2.0323e-09 \\ 2.3025e-08 \\ 8.6831e-09 \\ 0.0180 \\ 1.4529e-09 \\ 5.5915e-09 \\ 0.9924 \\ 7.9844e-09 \\ 6.6396e-10 \\ 0.0147 \\ 0.9999 \\ 3.1555e-09 \\ 0.9999 \end{bmatrix}$$

Written solution:

Original problem:

$$min\{f(3) : x \in \mathbb{R}\},\ f_3(x) = \sum_{i=1}^{m} \phi(b_i - b(a_{i_1}, \ldots, a_{in})),\ \phi(z) = max\{2|z| - 1, |z|\}$$

Linear optimization problem:

- Introduce m variable t_i

$t_i \geq z_i \rightarrow t_i + Ax \geq b_i,\ i = [1, m]$

$t_i \geq -z_i \rightarrow t_i - Ax \geq -b_i,\ i = [1, m]$

$t_i \geq 2z_i - 1 \rightarrow t_i + 2Ax \geq 2b_i - 1,\ i = [1, m]$

$t_i \geq -2z_i - 1 \rightarrow t_i - 2Ax \geq -2b_i - 1,\ i = [1, m]$

- Rewrite into linear optimization problem: size(y) = size(c) = size(x) + size(t) = (n + m,1), size(d) = (4*m,1), size(B) = (4*m,m+n)

$y_3 = [x_3; t_3]$

$c_3 = [0; \ldots; 0; 1; \ldots 1]$

$$B = \begin{bmatrix} A' & I_t \\ -A' & I_t \\ 2A & I_t \\ -2A & I_t \end{bmatrix},\ A' = [A; \ldots; A]$$

$$d = \begin{bmatrix} b \\ -b \\ 2b - 1 \\ -2b - 1 \end{bmatrix}$$

Primal and dual problem:

$$min\{c_3^T y_3 : B_3 y_3 \geq d_3, y_3 \in \mathbb{R}\} = max\{d_3^T u_3 : B_3^T u_3 = c, u_3 \geq 0, u_3 \in \mathbb{R}\}$$