Communication Theory (5ETB0)
Module 11.2

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Module 11.2

Presentation Outline

Quadrature Multiplexing: Optimum Receiver (1/3)

Passband Transmitted Waveform

$$s_m(t) = \sum_{i=1}^{N_c} s_{mi}^c \phi_{c,i}(t) + \sum_{j=1}^{N_s} s_{mj}^s \psi_{s,j}(t)$$

Optimum Receiver

The optimum receiver applies the rule

$$\hat{m}^{\mathsf{MAP}}(\underline{r}) = \underset{m \in \mathcal{M}}{\operatorname{argmax}} \{ (\underline{r} \cdot \underline{s}_m) + c_m \}$$

where

$$c_m = \frac{N_0}{2} \ln \Pr\{M = m\} - \frac{E_m}{2}$$

and E_m is the energy of the waveform $s_m(t)$, for $m \in \mathcal{M} = \{1, 2, \ldots, |\mathcal{M}|\}$.

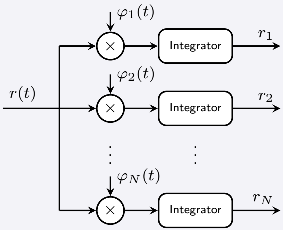
Recap: Correlation Receiver

Correlation Receiver

The transmitted waveform is

$$s_m(t) = \sum_{i=1}^{N} s_{mi}\varphi_i(t)$$

where $\varphi_i(t)$ are N building-block waveforms.
The received waveform is $r(t) = s_m(t) + n_w(t)$.



Q1: What is the structure of the corresponding correlation receiver that gives us the r-values $\underline{r} = (r_1, r_2, \ldots, r_N)$?

Answer: N multipliers and integrators

Quadrature Multiplexing: Optimum Receiver (2/3)

Computing the r-values

$$
\begin{aligned}
\int_{-\infty}^{\infty} r(t)\phi_{c,i}(t)dt &= \int_{-\infty}^{\infty} r(t)\sqrt{2}\cos(2\pi f_0 t)\phi_i(t)dt = r_i^c, \quad i = 1, 2, \ldots, N_c \\
\int_{-\infty}^{\infty} r(t)\psi_{s,j}(t)dt &= \int_{-\infty}^{\infty} r(t)\sqrt{2}\sin(2\pi f_0 t)\psi_j(t)dt = r_j^s, \quad i = 1, 2, \ldots, N_s
\end{aligned}
$$

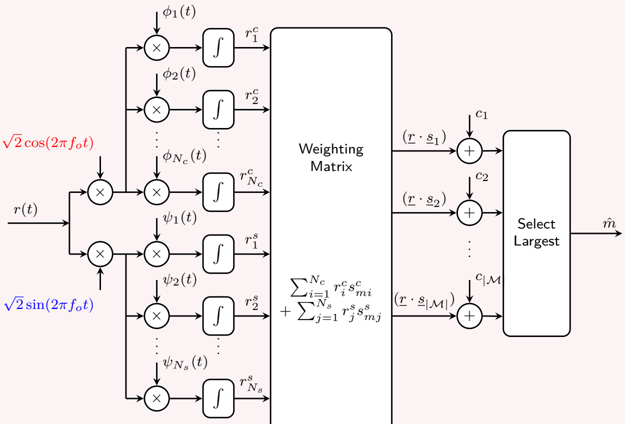
Computing the dot products and constants

$$
(\underline{r} \cdot \underline{s}_m) = \sum_{i=1}^{N_c} r_i^c s_{mi}^c + \sum_{j=1}^{N_s} r_j^s s_{mj}^s
$$

$$
c_m = \frac{N_0}{2} \ln \Pr\{M = m\} - \frac{\|\underline{s}_m\|^2}{2}, \qquad \|\underline{s}_m\|^2 = \|\underline{s}_m^c\|^2 + \|\underline{s}_m^s\|^2
$$

Quadrature Multiplexing: Optimum Receiver (3/3)

Computing the r-values



Quadrature Multiplexing: Short Pause

What have we done so far?

The mth transmitted waveform is

$$s_m(t) = \sum_{i=1}^{N_c} s_{mi}^c \phi_i(t)\sqrt{2}\cos(2\pi f_0 t) + \sum_{j=1}^{N_s} s_{mj}^s \psi_j(t)\sqrt{2}\sin(2\pi f_0 t)$$

- Symbols $\underline{s}_m^c$ and $\underline{s}_m^s$ modulated using $\phi_i(t)$ and $\psi_j(t)$
- The resulting signal is BW-limited (to W)
- Then up-conversion around frequency f_0
- At Rx: down-conversion, followed by correlation receiver

Questions:

Q1: What is the dimensionality of the signal space in the general case above?
Q2: How do we make this complex system to be baseband PAM?
Q3: What is the dimensionality of the signal space now?

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Presentation Outline

Quadrature Amplitude Modulation

Simplifying general model gives us Quadrature Amplitude Modulation (QAM)

The mth transmitted waveform is

$$s_m(t) = \sum_{i=1}^{N_c} s_{mi}^c \phi_i(t)\sqrt{2}\cos(2\pi f_0 t) + \sum_{j=1}^{N_s} s_{mj}^s \psi_j(t)\sqrt{2}\sin(2\pi f_0 t)$$

- Make $N_c = N_s = N$ and the baseband in-phase and quadrature building-block waveforms to be equal ($\phi_i(t) = \psi_i(t)$ for all $i = 1, \ldots, N$):

$$s_m(t) = \sum_{i=1}^{N} \phi_i(t)\left(s_{mi}^c \sqrt{2}\cos(2\pi f_0 t) + s_{mi}^s \sqrt{2}\sin(2\pi f_0 t)\right)$$

- Take a single dimension ($N = 1$) with a and b amplitudes for in-phase and quadrature:

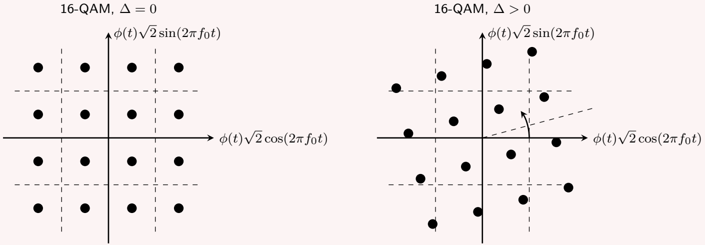
$$s_m(t) = a\phi(t)\sqrt{2}\cos(2\pi f_0 t) + b\phi(t)\sqrt{2}\sin(2\pi f_0 t)$$

Quadrature Amplitude Modulation

Time Delay ⇒ Phase Rotation

What happens if the channel introduces delay? If we observe a slightly delayed version of the signal $r(t) = s(t-\Delta) + n_w(t)$:

$$\begin{aligned} s(t-\Delta) \quad &\approx \quad [a\cos(2\pi f_0\Delta) - b\sin(2\pi f_0\Delta)]\,\phi(t)\sqrt{2}\cos(2\pi f_0 t) \\ &\quad + [b\cos(2\pi f_0\Delta) + a\sin(2\pi f_0\Delta)]\,\phi(t)\sqrt{2}\sin(2\pi f_0 t). \end{aligned}$$



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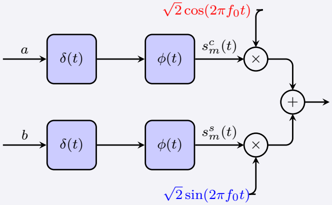
Presentation Outline

From QAM to Serial QAM

Quadrature Amplitude Multiplexing so far

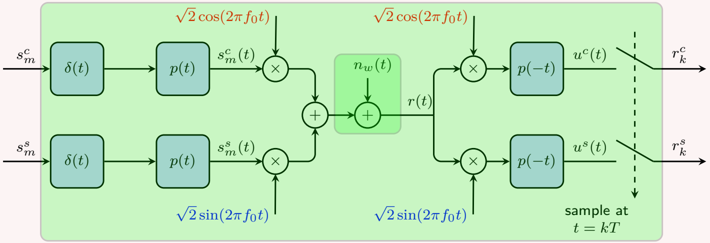


Serial QAM

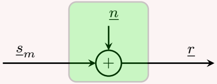
- Use serial PAM with a pulse $\phi(t) = p(t)$ satisfying the Nyquist criterion
- Use same pulse in cosine and sine branches
- Use sinc pulses as baseband building blocks with T [s] shifts

Serial QAM Transceiver

Serial QAM System



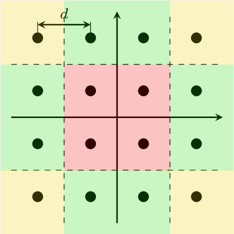
The whole transmission chain can be replaced by a Vector (2D) DICO channel



$$\underline{r} = \underline{s}_m + \underline{n}$$
$$\underline{s}_m = (s_m^c, s_m^s)$$
$$n_i \sim \mathcal{N}(0, N_0/2), i = 1, 2$$

Error Probability for 16-QAM (1)

Serial QAM Error Probability



Union bound

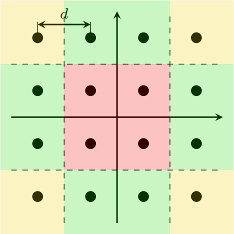
- Yellow points: $P_{\mathrm{e}}^{Y} \leq 2Q\left(\frac{d}{2\sigma}\right)$
- Green points: $P_{\mathrm{e}}^{G} \leq 3Q\left(\frac{d}{2\sigma}\right)$
- Red points: $P_{\mathrm{e}}^{R} \leq 4Q\left(\frac{d}{2\sigma}\right)$

Exact error probability for each point

- Yellow points: $P_{e}^{Y} = 2Q\left(\frac{d}{2\sigma}\right) - Q^{2}\left(\frac{d}{2\sigma}\right)$
- Green points: $P_{e}^{G} = 3Q\left(\frac{d}{2\sigma}\right) - 2Q^{2}\left(\frac{d}{2\sigma}\right)$
- Red points: $P_{e}^{R} = 4Q\left(\frac{d}{2\sigma}\right) - 4Q^{2}\left(\frac{d}{2\sigma}\right)$

Error Probability for 16-QAM (2)

Serial QAM Error Probability



Upper bound on the total average error probability:

$$P_e \leq \frac{4}{16} \cdot 2Q\left(\frac{d}{2\sigma}\right) + \frac{8}{16} \cdot 3Q\left(\frac{d}{2\sigma}\right) + \frac{4}{16} \cdot 4Q\left(\frac{d}{2\sigma}\right)$$

$$= \frac{48}{16} Q\left(\frac{d}{2\sigma}\right)$$

Who Cares?

Modulation Enhancements

Like most recent wireless specification, 802.11ac uses Orthogonal Frequency-Division Multiplexing (OFDM) to modulate bits for transmission over the wireless medium. While the modulation approach is identical to that used in 802.11n, 802.11ac optionally allows the use of 256 QAM in addition to the mandatory Quadrature Phase Shift Keying (QPSK), Binary PSK (BPSK), 16 QAM and 64 QAM modulations. 256 QAM increases the number of bits per sub-carrier from 6 to 8, resulting in a 33% increase in PHY rate under the right conditions. It should be noted however that 256 QAM can only be used in high signal-to-noise ratio (SNR) scenarios (across the used spectrum and desired streams); i.e. for very favorable channel conditions. The support of 256 QAM will increase the maximum PHY rate that can be supported by the system, but will have no effect in typical scenarios and will not lead to any reach increase for the service. Also, supporting 256 QAM requires transmitter and receiver to be designed such that the inherent SNR (transmit and receive

Summary Module 11.2

Take Home Messages

- Receiver structure for quadrature multiplexing
- Quadrature amplitude modulation
- Serial QAM System: Continuous-time vs. discrete-time

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