Digital Signal Processing Fundamentals (5ESC0)

Introduction

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Books

* The course is based on the book:

  Schaum's Outline of Theory and Problems
  "Digital Signal Processing"; Monson H. Hayes
  ISBN13: 97890071635097

* Stochastic Signal Processing is based on:

  Chapter 3 from the book
  "Statistical and Adaptive Signal Processing"
  Dimitris G. Manolakis, Vinay K. Ingle and Stephan M. Kogon
  ISBN: 1-58053-610-7

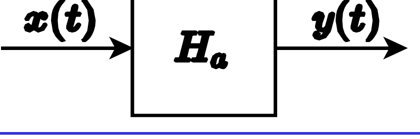
Introduction Content

* Global course content
* Organization
* Preknowledge

Global Content

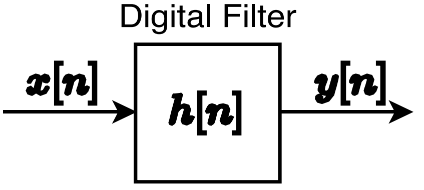
* In practice, many signals are analog signals
* Analog signals are continuous time signals and in this course we denote them by using round brackets: $x(t)$
* In the figure below, an analog filter is used to obtain output signal $y(t)$ from input signal $x(t)$
* A filter with an in- and output signal is an example of what we cover under Systems and Signals

Analog Filter



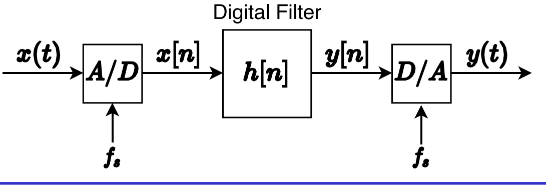
Global Content

* Manipulating signals is mainly done in the digital domain, where we talk about discrete time signals
* In this course, we denote discrete time signals by using square brackets: $x[n]$
* In the figure below, a digital filter is used to obtain output signal $y[n]$ from input signal $x[n]$

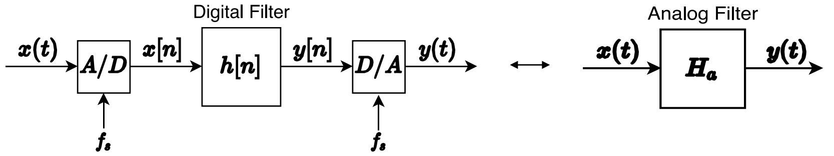


Global Content

* There are benefits to using the digital domain, but this implies we have to convert an analog signal to a digital signal, which we do by Sampling
* Sampling was treated in Signals I for periodic signals. In this course, we will look at what is happening in these blocks in detail
* The block scheme below shows the same digital filter, but now with conversion from and to analog

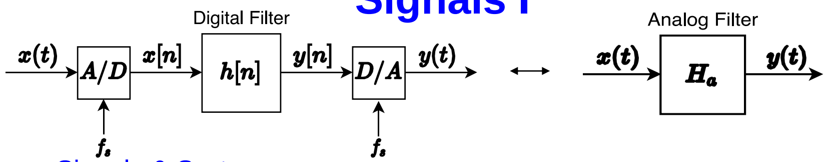


Content Outline



- To explain Signals & Systems and Sampling, we need mathematical analysis tools which are often in the form of Transforms
- Many transforms are related to Fourier: Fourier Series (FS), Fourier Transform for Continuous time signals (FTC), Fourier Transform for Discrete time signals (FTD), Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), Z-Transform (ZT)
- On the next slide, the outline of the chapters that we will cover is shown

Content Outline: red means new w.r.t. Signals I



- Signals & Systems
  - Periodic vs a-periodic signals : Ch 1
  - Deterministic vs Stochastic signals : Ch 1 + Ch 10
  - LTI systems (Linear Time Invariant) : Ch 1
  - System function : Ch 5
  - Filter structures : Ch 8
  - Filter design : Ch 9
- Sampling
  - A/D, D/A and multirate : Ch 3
- Mathematical analysis tools, Transforms
  - FS, FTC, FTD, DFT/FFT, ZT : Ch 2 + Ch 4 + Ch 6/7

Organization

- Course hours: 8 hours per week
  - 4 hours lectures
  - 4 hours instructions/ labs

We strongly advise you to prepare at home and use the contact hours for assistance. You learn most from your own efforts and mistakes.

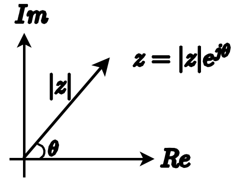
- Grading:
  1. Written exam (70%)
  2. Labs (30%)
- All relevant information on Canvas
- Register in Canvas for a lab group (maximum 2 members per group)
- If previous year average lab grade was at least a 6 → exemption

Preknowledge: Complex numbers

- Polar notation:

$$z = |z|e^{j\theta}$$

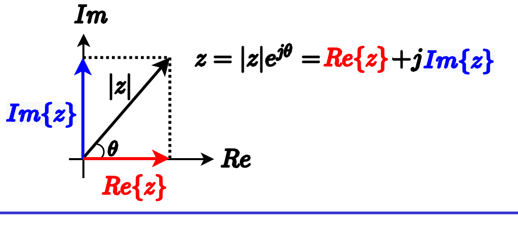
  - $|z|$ is the length of the vector and θ is the angle



- Cartesian notation:

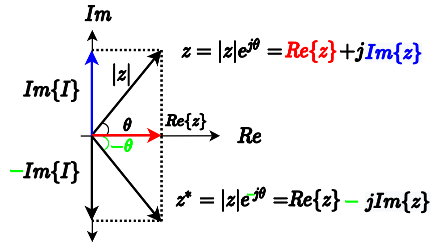
$$z = Re\{z\} + j \cdot Im\{z\}$$

  - There is a real part and an imaginary part, the imaginary part is indicated by j, $j = \sqrt{-1}$



Preknowledge: Complex numbers

- Complex conjugation: $j \rightarrow -j$



- Euler: $e^{j\theta} = \cos\theta + j\sin\theta$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

Preknowledge: Important geometric series

- With z_0 some (possibly complex) number:

$$\sum_{n=0}^{\infty}(z_0)^n = \frac{1}{1-z_0}$$

iff $|z_0| < 1$

- If $|z_0| \geq 1$, the series will not hold as divergence instead of convergence will occur

Preknowledge: Important geometric series

* Proof:
* $\sum_{n=0}^{\infty}(z_0)^n = \frac{1}{1-z_0}$ (Multiply both sides with $(1-z_0)$)
* $\sum_{n=0}^{\infty}(z_0)^n - z_0\sum_{n=0}^{\infty}(z_0)^n = 1$ (If the series holds then the left hand side has to equal 1. Now we expand the summations)
* $(1 + z_0 + {z_0}^2 + {z_0}^3 + \cdots) - z_0(1 + z_0 + {z_0}^2 + \cdots) = 1$
* $1 + z_0 - z_0 + {z_0}^2 - {z_0}^2 + {z_0}^3 - {z_0}^3 + \cdots = 1$
* All terms after the 1 cancel each other out, so the series holds.

Preknowledge: Important geometric series

* With z_0 some (possibly complex) number:

$$\sum_{n=0}^{M-1}(z_0)^n = \frac{1-{z_0}^M}{1-z_0}$$

* Note that there is no restriction on the magnitude of z_0

Preknowledge: Important geometric series

- Proof using $\sum_{n=0}^{\infty}(z_0)^n = \frac{1}{1-z_0}$:
- $\sum_{n=0}^{M-1}(z_0)^n = \sum_{n=0}^{\infty}(z_0)^n - \sum_{n=M}^{\infty}(z_0)^n$ (The series can be split in two series, mind the bounds)
- $= \sum_{n=0}^{\infty}(z_0)^n - \sum_{n=M}^{\infty}(z_0)^n = \frac{1}{1-z_0} - \sum_{n=M}^{\infty}(z_0)^n$ (now we expand the remaining summation)
- $= \frac{1}{1-z_0} - ({z_0}^M + {z_0}^{M+1} + \cdots)$ (now we can take ${z_0}^M$ out of the brackets)
- $= \frac{1}{1-z_0} - {z_0}^M(1 + {z_0}^1 + \cdots) = \frac{1}{1-z_0} - {z_0}^M \sum_{p=0}^{\infty}(z_0)^p$
- $\frac{1}{1-z_0} - \frac{{z_0}^M}{1-z_0} = \frac{1-{z_0}^M}{1-z_0}$

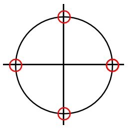
Preknowledge: Zeros of a complex equation

- With a some (complex) number, find the zeros of: $z^N - a = 0$
- We move a to the other side and then take the Nth root of both sides

$$z^N = a = ae^{jk\cdot 2\pi} \rightarrow z_k = a^{\frac{1}{N}} \cdot e^{jk\cdot\frac{2\pi}{N}} \text{ for } k = 0, 1, \dots, N-1$$

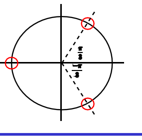
- Example: $a = 1$, $N = 4$. We fill in the equation:

$$\rightarrow z_k = e^{jk\cdot\frac{\pi}{2}}$$



- Example: $a = -1$, $N = 3$

$$\rightarrow z_k = (-1)^{\frac{1}{3}} \cdot e^{jk\cdot\frac{2\pi}{3}}$$
$$= (e^{j\pi})^{\frac{1}{3}} \cdot e^{jk\cdot\frac{2\pi}{3}}$$
$$= e^{j\frac{\pi}{3}+k\cdot\frac{2\pi}{3}}$$

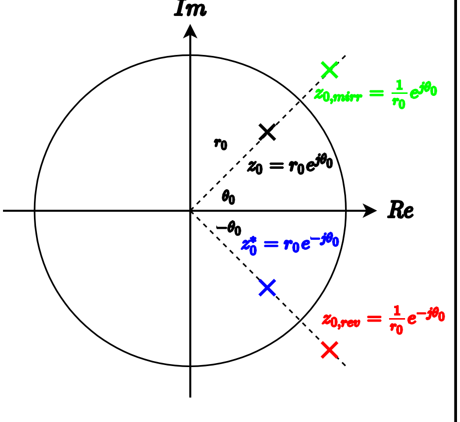


Preknowledge: Mirroring and conjugation

* Take a point $z_0 = r_0 e^{j\theta_0}$
* *Complex conjugation* is done by replacing j with $-j$: $z_0^* = r_0 e^{-j\theta_0}$
* *Mirroring* z_0 is done by taking the inverse of z_0 and then the complex conjugate: $z_{0,mirr} = (\frac{1}{z_0})^* = \frac{1}{r_0} e^{j\theta_0}$
  This is called mirroring because the point is mirrored with respect to the unit circle.
* *Reversing* z_0 is done by both mirroring and taking the complex conjugate: $z_{0,rev} = z_{0,mirr}^* = \frac{1}{r_0} e^{-j\theta_0} = \frac{1}{z_0}$

Preknowledge: Mirroring and conjugation

* Take a point $z_0 = r_0 e^{j\theta_0}$
* *Complex conjugation*: $z_0^* = r_0 e^{-j\theta_0}$
* *Mirroring*: $z_{0,mirr} = (\frac{1}{z_0})^* = \frac{1}{r_0} e^{j\theta_0}$
* *Reversing*: $z_{0,rev} = z_{0,mirr}^* = \frac{1}{r_0} e^{-j\theta_0}$



Summary

* We discussed the global content of the course
* We divided the global content in Signals and Systems, Sampling and the Mathematical analysis tools/ Transforms: the subjects we will cover in this course
* We looked at the preknowledge needed to follow the course:
  - Complex numbers
  - Geometric series
  - Finding the zeros of a complex equation
  - Mirroring, conjugation and reversal