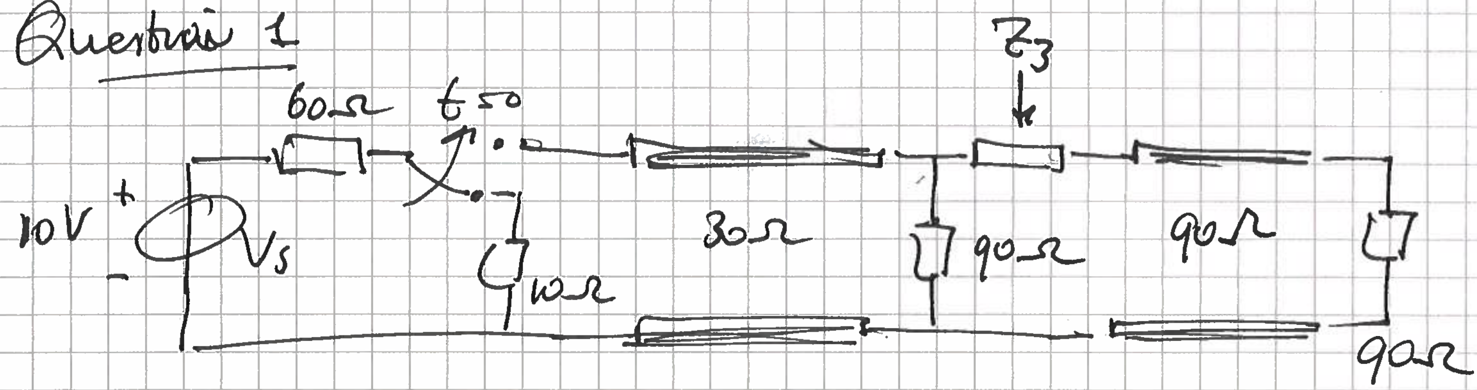
Midterm MAY 26, 2023

Question 1



(1A) Steady state at $z=0$ just after closing the switch

voltage divider $\rightarrow$ $V_{0^+, z=0} = \frac{30}{30+60} \cdot 10 = 3\frac{1}{3}$ V

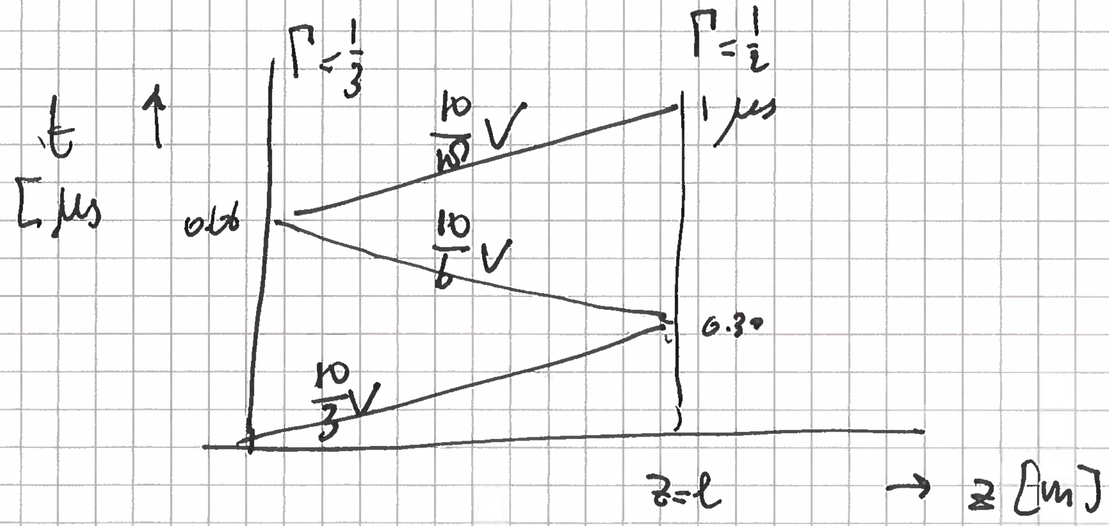
current $\Rightarrow$ $I_{0^+, z=0} = \frac{V_{0^+, z=0}}{Z_0} = \frac{3\frac{1}{3}}{30} = \frac{1}{9}$ A

(1B) $Z_3 = \infty$, so only TL Z_0 has a propagating wave.

$$t = \frac{100 \text{ meter}}{c} = 0.33\,\mu s.$$

$$\Gamma_{z=0} = \frac{60-30}{60+30} = \frac{1}{3}$$

$$\Gamma_{z=\ell} = \frac{90-30}{90+30} = \frac{1}{2}$$



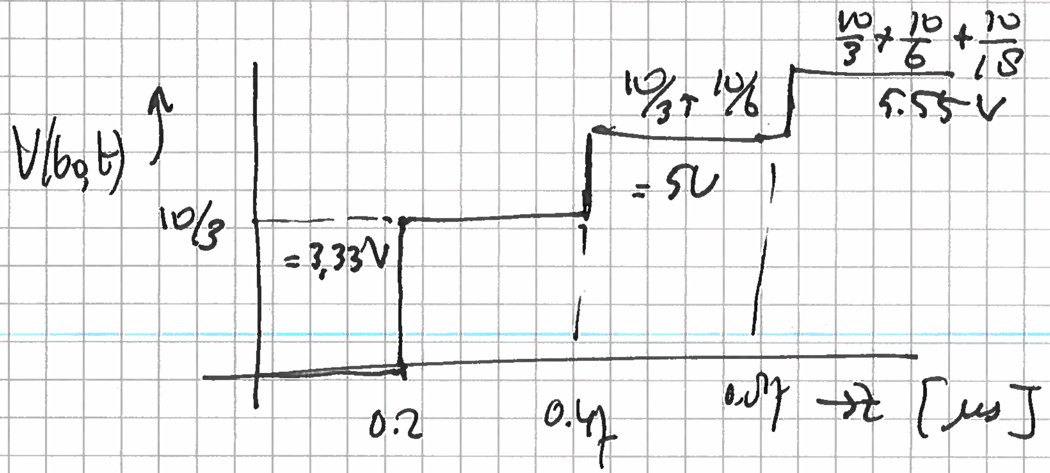
②

1C Make a plot at $z = 60m$

$$t_1 = \frac{60}{c} = 0.20\ \mu s$$

$$t_2 = \frac{60 + 2*40}{c} = 0.47\ \mu s$$

$$t_3 = \frac{60 + 2*40 + 2*60}{c} = 0.57\ \mu s$$



1D steady state for $Z_3 = \infty$ and $Z_3 = 0$

$\boxed{Z_3 = 0}$

$$U_{steady} = \frac{90 // 90}{90//90 + 60} = \frac{45}{105} \cdot 10 = \frac{30}{7}\ V$$

4.2857

$$I_{steady} = \frac{U_{steady}}{45} = \frac{2}{21}\ [A]$$

$= 0.095$

$\boxed{Z_3 = \infty}$

$$U_{steady} = \frac{90}{90 + 60} = \frac{90}{150} \cdot 10 = 6\ V$$

$$I_{steady} = \frac{U_{steady}}{90} = \frac{6}{90}\ A$$

$= 0.0666\ V$

1E Decomposition

$$\begin{pmatrix} V^+ \\ V^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & Z_0 \\ 1 & -Z_0 \end{pmatrix} \begin{pmatrix} U_{steady} \\ I_{steady} \end{pmatrix}$$

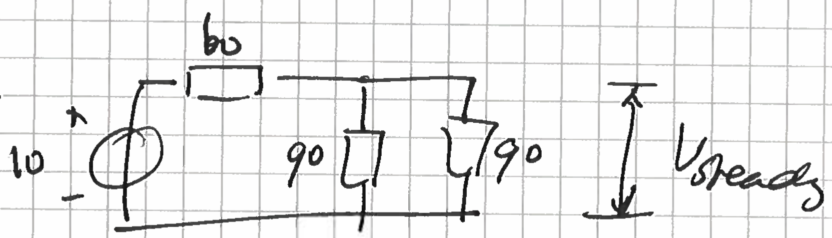
$$\begin{pmatrix} V^+ \\ V^- \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 30 \\ 1 & -30 \end{pmatrix}\begin{pmatrix} 6 \\ \frac{6}{90} \end{pmatrix}$$

$$= \begin{pmatrix} 4V \\ 2V \end{pmatrix}$$

$$\begin{pmatrix} I^+ \\ I^- \end{pmatrix} = \begin{pmatrix} 4/30 \, A \\ -2/30 \, A \end{pmatrix}$$

1f Power to load Z_2 for $Z_3 = 0$ and $Z_3 = \infty$

$\boxed{Z_3 = 0}$



$$P_{Z_2} = \frac{V_{steady}^2}{Z_2} = \frac{(30/7)^2}{90} = \underline{\underline{0.20 \text{ W}}}$$

$\boxed{Z_3 = \infty}$

$$P_{Z_2} = \frac{V_{steady}^2}{Z_2} = \frac{6^2}{90} = 0.4 \text{ W}$$

Question 2

2A limitation is $\rightarrow$ the bandwidth is limited because it is designed for a specific λ.

2B

$\frac{\lambda}{2}$ piece $\Rightarrow l = \frac{\lambda}{2} \quad \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$

$\tan(\pi) = \phi$

$Z_L = \text{open} = \infty$

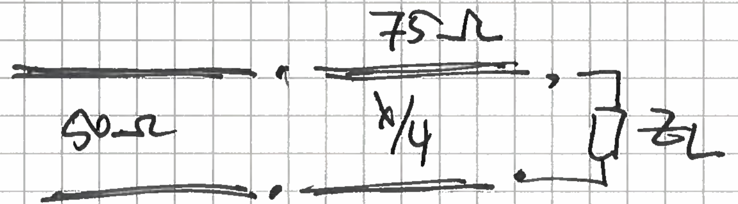
$\Rightarrow Z_{IN}(\frac{\lambda}{2} \text{ piece}) = Z_0 \cdot \frac{\infty + 0}{Z_0 + 0} = \infty$

That mean that $\underline{Z_{IN} = Z_L}$

2C shorted $\frac{\lambda}{4}$ stub $\quad \frac{\lambda}{4} \Rightarrow \tan(\beta l) = \overset{\tan}{\cancel{tg}}\left(\frac{\pi}{2}\right) = \infty$

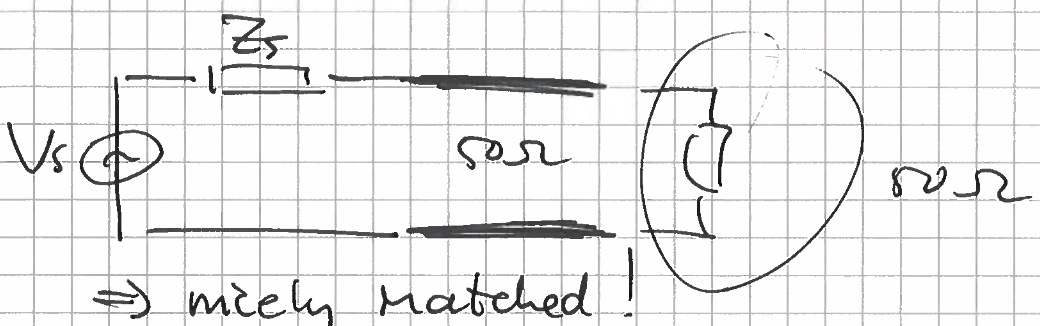
$\Rightarrow Z_{IN}(\text{shorted stub}) = Z_0 \cdot \frac{jZ_0}{jZ_L} = \frac{Z_0^2}{\underbrace{Z_L}_{=0}} = \infty$

2d new situation is:



Matched if $\underline{75 = \sqrt{Z_L \cdot 50}} \Rightarrow Z_L = \frac{75^2}{50} = \underline{112.5\ \Omega}$

2e) Now situation is like



⇒ nicely <u>matched</u>!

2f) in the stubs ⇒ ∅ W
you can calculate this via

$$P_L^{stub} = (1-|\Gamma|^2)P_{in}$$

$$\Gamma_{stub,1} = -1 \qquad \Gamma_{stub,2} = 1$$

Power = 0 [W]

<u>load</u>: for the first wave:

$$\Gamma_{LOAD} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{112.75 - 75}{112.75 + 75} = 0.2$$

$$P_L = (1-|\Gamma|^2)P_{in} = (1-0.2^2)\,100 = \underline{96\,[W]}$$

I also count 100% = 100 W ok, since it is perfectly matched after some waves :)

2g) Transfer matrix

General form: $$T(0,\ell) = \begin{pmatrix} \cos\beta\ell & jZ_0\sin\beta\ell \\ jY_0\sin\beta\ell & \cos\beta\ell \end{pmatrix}$$

$$\beta\ell = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{1}{2}\pi$$

$$\Rightarrow T\left(0,\frac{\lambda}{4}\right) = \begin{pmatrix} 0 & jZ_1 \\ jY_1 & 0 \end{pmatrix}$$

S-matrix (not asked for ☺)

$$S = \begin{pmatrix} 0 & e^{-\beta l} \\ e^{-\beta l} & 0 \end{pmatrix}$$ for a TL with length l.

$$\beta l = \pi/2 \quad \Rightarrow \quad S = \begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}$$

and (if you are really eager) $\Rightarrow$ convert $S \to T$ via the A-matrix.