Photonics

Geometric Optics

Ray concept

Refraction and propagation of light

Matrix formalism

Optical systems

Ray theory - geometric optics

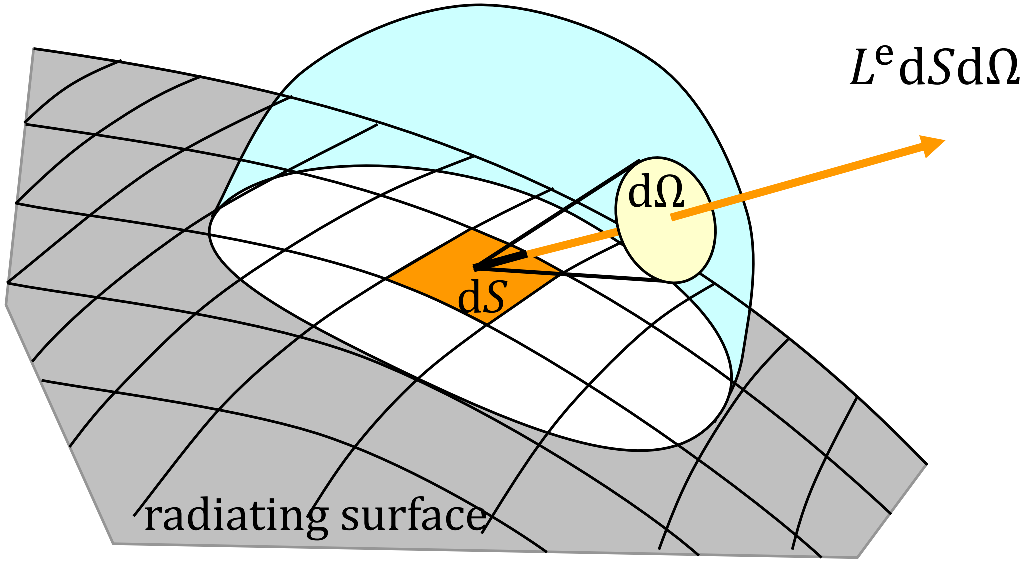
- Light ≈ bundle of “rays”
- Ray:
  - straight in a uniform medium
  - refracts or reflects at an interface between media
- Validity of the ray concept
  - ray ~ local plane wave
  - approx. valid in structures $\gg \lambda$
  - high frequency approximation
    $\rightarrow$ wavelength-independent
  - no diffraction and interference effects
- Useful for analysis of large, complex optical systems, in particular, lens systems

Ray presentation of radiating objects

- Radiance L^{e} (or luminance L) is known in each point

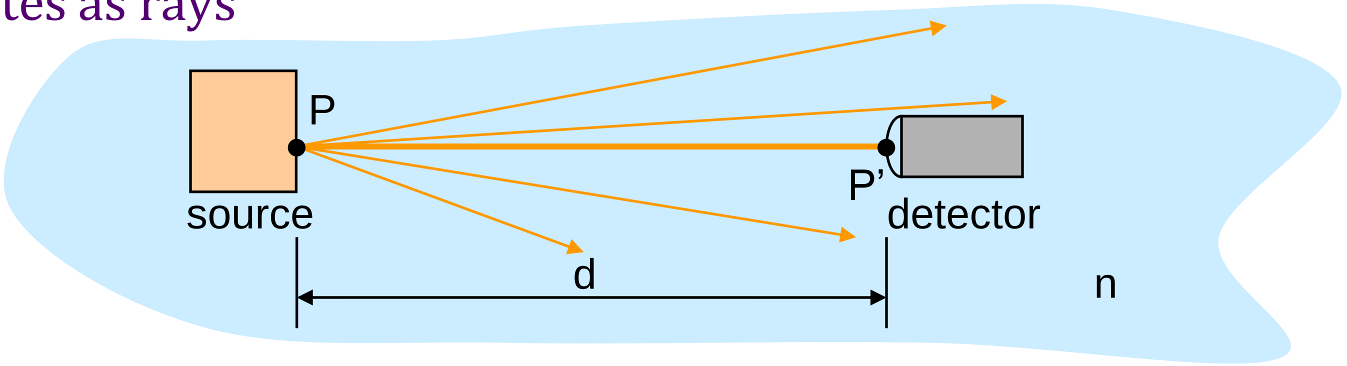
- Discretize
  - surface S
  - solid angle Ω

- Discretize
  - as finely as possible (high accuracy)
  - smaller than $\mathrm{d}S\mathrm{d}\Omega \approx \lambda^2$ is useless (diffraction theory)



Postulates of ray optics (1)

- Light propagates as rays

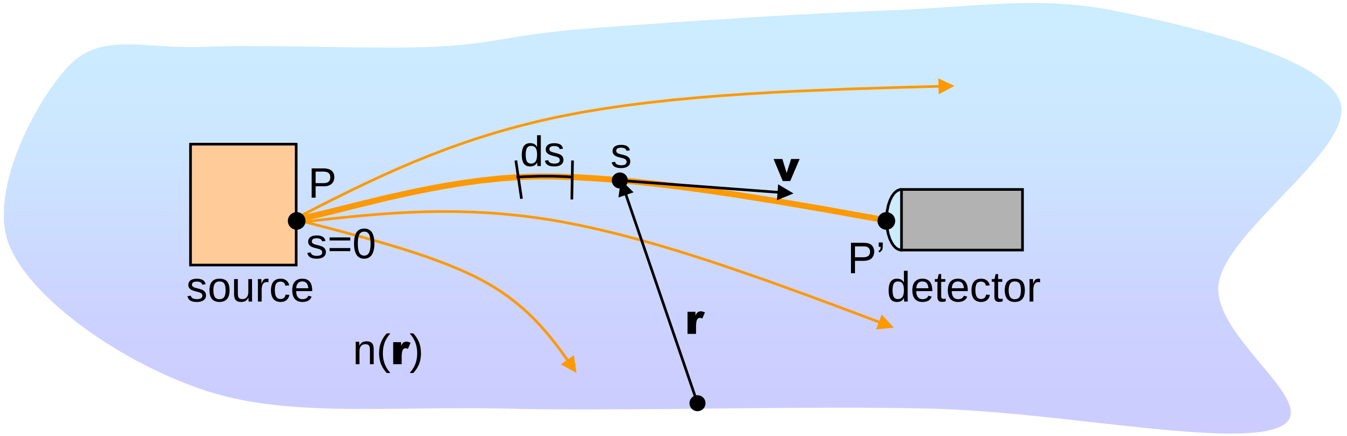


- Homogeneous optical medium: refractive index n
  - speed of light in vacuum $c = 2.998\ 10^8$ m/s
  - speed of light in a medium $v = {}^{c}/_{n}$
  - Time required to go from P to P':

$$\Delta t_{P\to P'} = \frac{d}{v} = \frac{nd}{c}$$

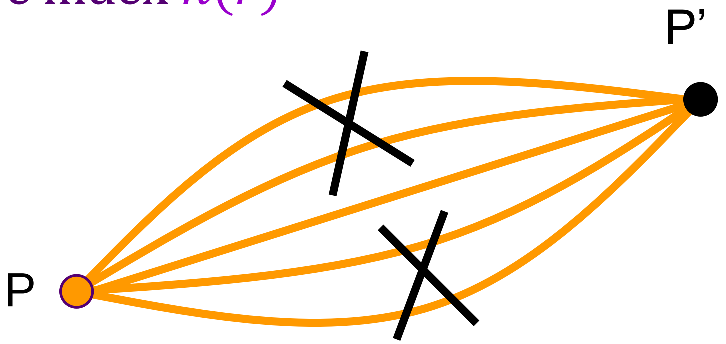
  - Optical path length $L_\mathrm{o} = nd$

Postulates of ray optics (2)



- Inhomogeneous optical medium: refractive index $n(r)$
  - Optical path length

$$L_o = \int_P^{P'} n(r)\mathrm{d}s$$
$$= c\Delta t$$



- Fermat's principle:

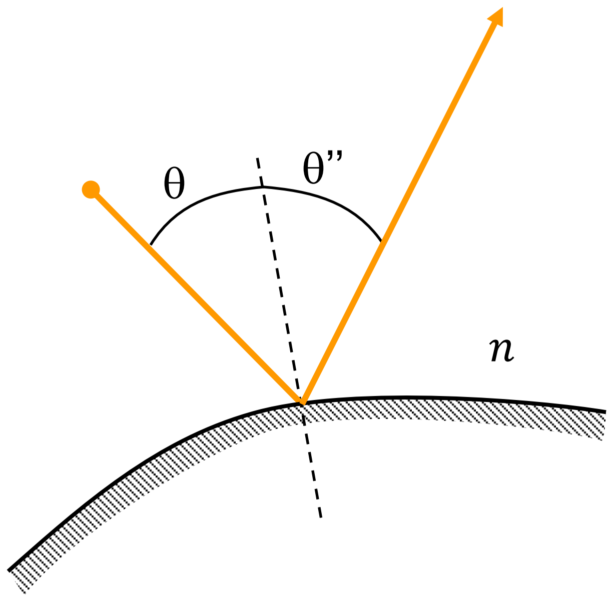
> The path taken by a ray of light has a minimum optical path length with respect to neighboring paths

Reflection at the surface

- Metallic mirror:
  light is reflected

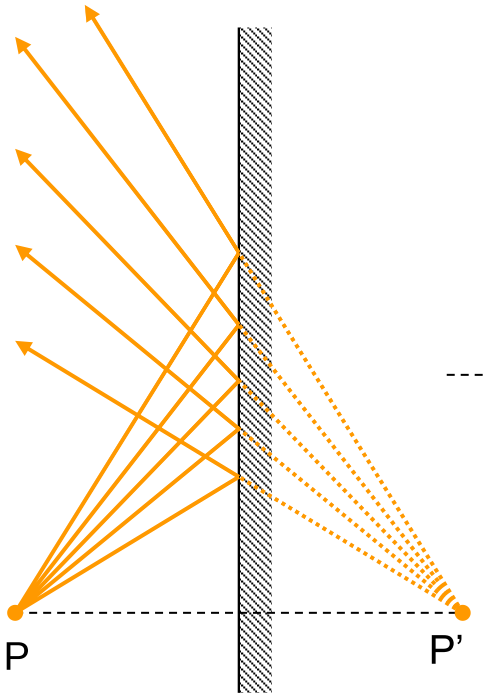
- Reflected ray
  - is in plane with
    - the incident ray
    - the normal to the surface
  - at the same angle with the normal as the incident ray
    $\theta'' = \theta'$

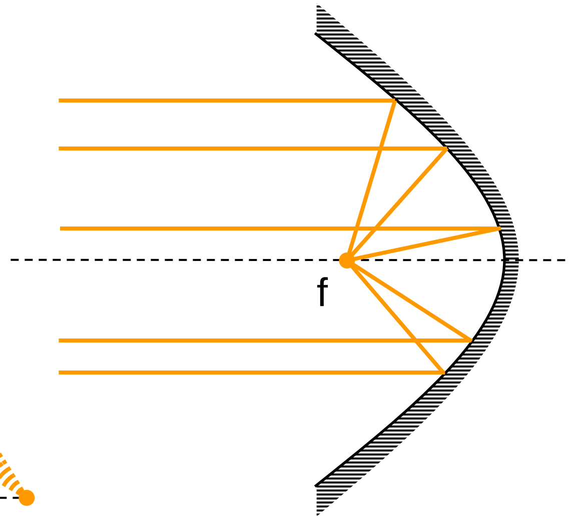
    (independent of the local curvature of the surface)

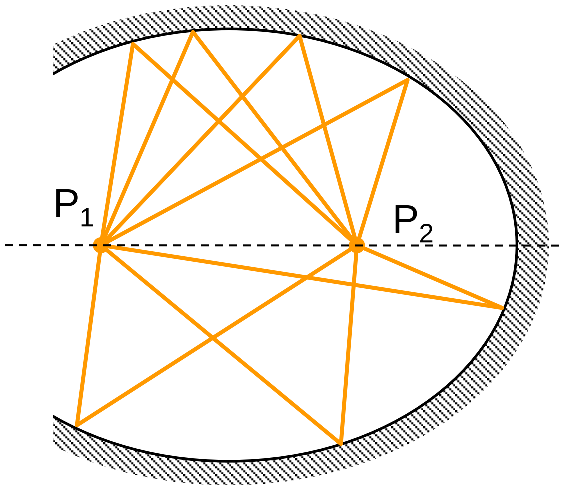


(Metallic) Mirrors

- Flat mirrors:
  Reflected light rays from P converge in P'
- Parabolic mirror:
  rays parallel to the axis converge to the focus f
- Elliptic mirror:
  Rays from focus P_1 are focused in P_2







Refraction at an interface between media

- Optical system: piecewise constant n
  - refraction at an interface
  - reflection at an interface

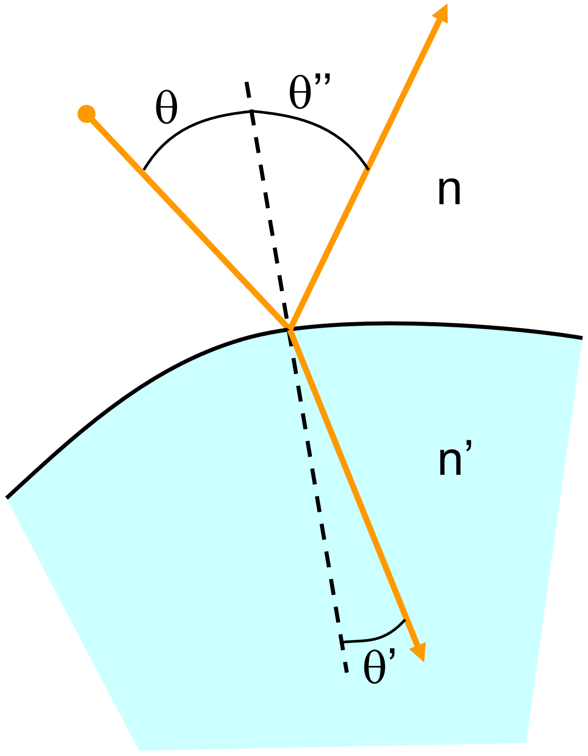
- Interface between media:
  - refracted and reflected rays are in the plane of the incident ray and the normal
  - refraction is according to **Snell's law**

$$\boxed{n \sin\theta = n' \sin\theta'}$$

  - reflection is at the same angle as the incident ray

$$\theta'' = \theta$$

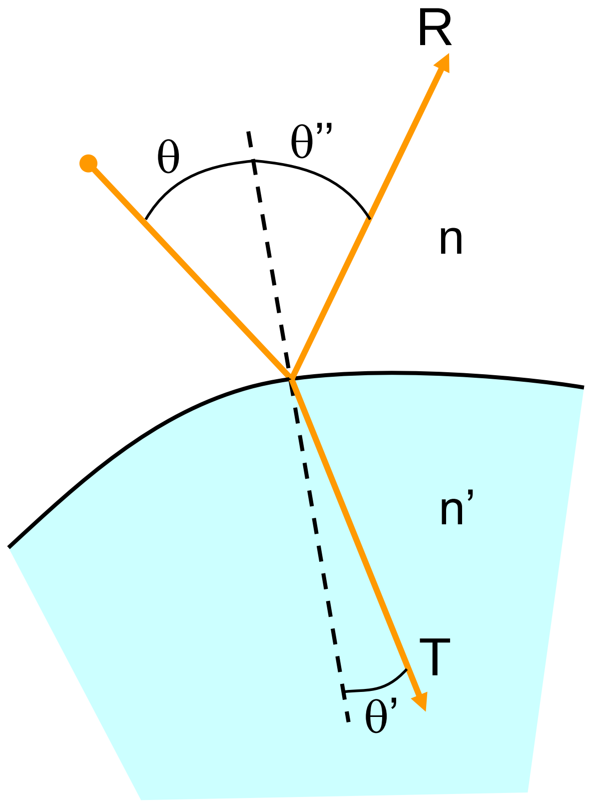
  - (independent on the local curvature of the surface)



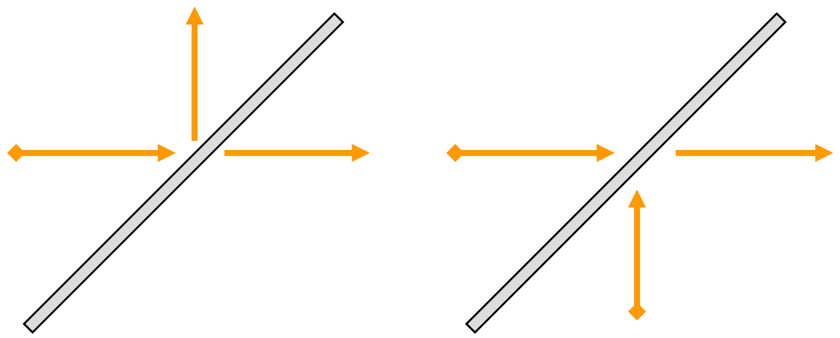
Reflection and transmission

- Reflection and Transmission at the interface
  - Depends on the angle of incidence and polarization (see chapter *Electromagnetism*)
  - Perpendicular incidence ($\theta = 0$):

$$R = \left(\frac{n - n'}{n + n'}\right)^2 \qquad T = \frac{4nn'}{(n + n')^2}$$

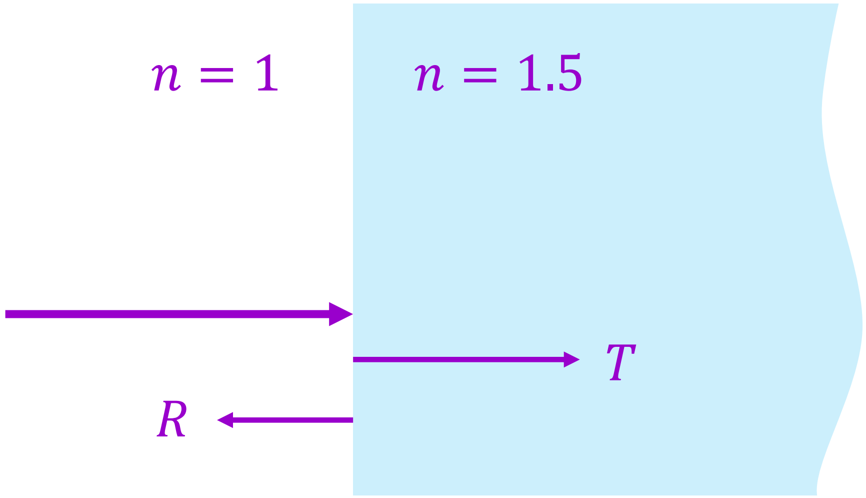


- Application:
  - beamsplitter
  - beam combiners



Exercise: calculate reflection and transmission

- Calculate the power fraction of light (in air, $n = 1$) reflecting from glass ($n = 1.5$)
- Also calculate the power transmission fraction



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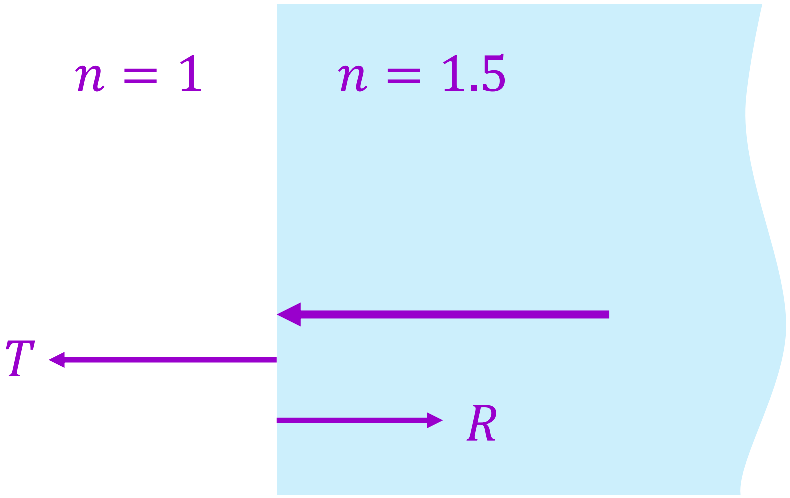


- What about the inverse?

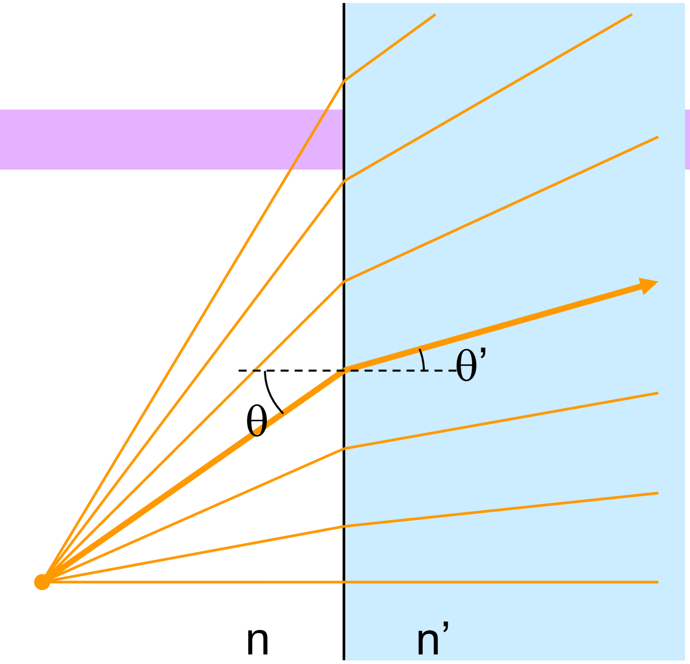
Total Internal Reflection

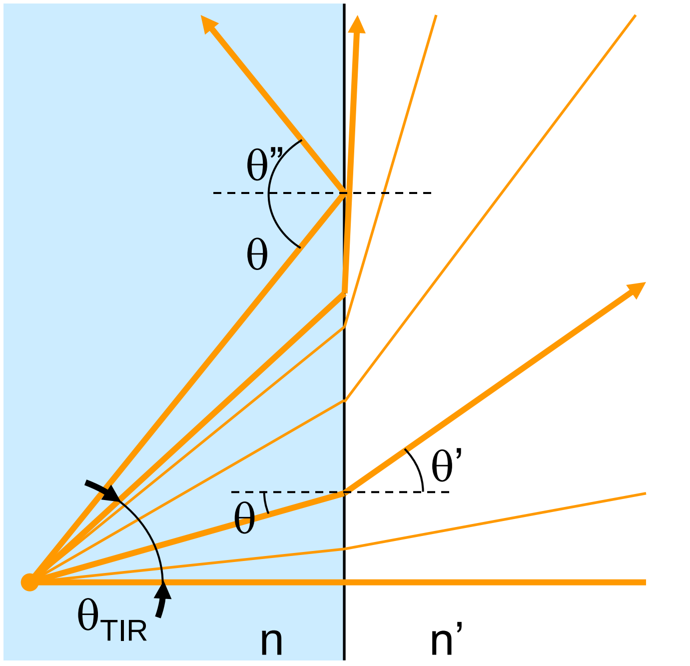
- Snell's law: $n \sin\theta = n' \sin\theta'$
  - If $n' > n$: $\sin\theta' < \sin\theta < 1$
    refraction is always possible
  - If $n' < n$: $\sin\theta' > \sin\theta$

    Refraction is only possible if $\sin\theta \leq \frac{n'}{n}$
  - Total internal reflection if $\theta > \theta_{\mathrm{TIR}}$

    with $\theta_{\mathrm{TIR}} = \arcsin\frac{n'}{n}$

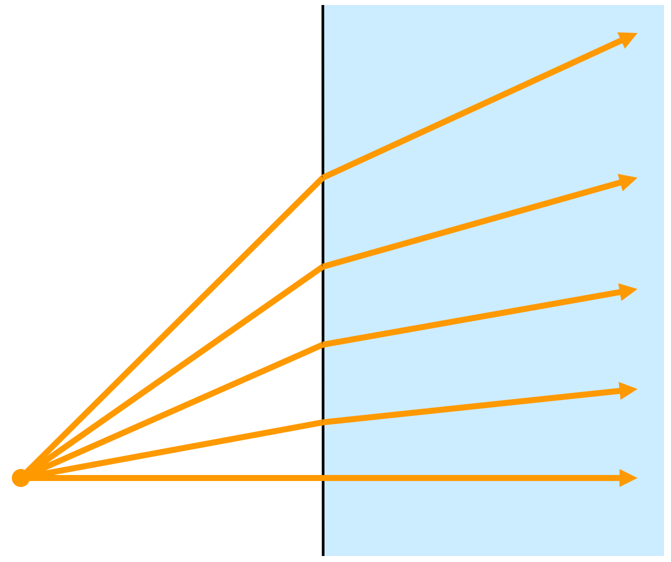
    Reflection at angle $\theta'' = \theta$

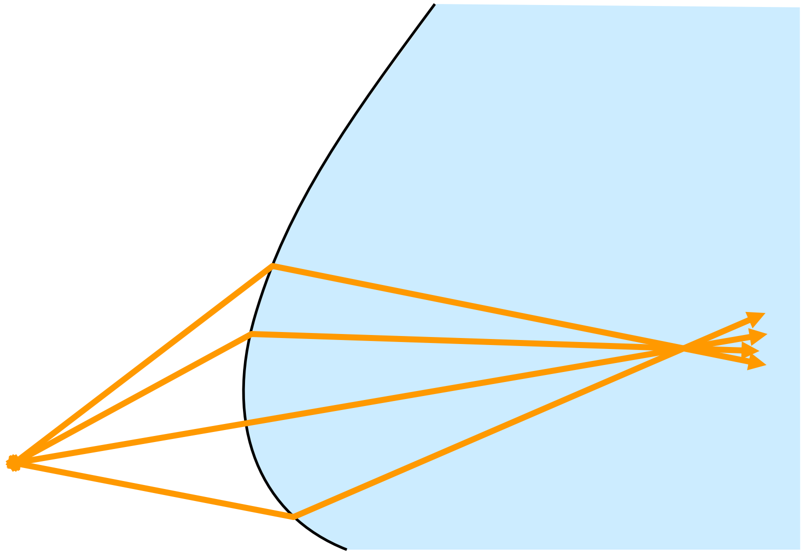




Curved surfaces

- Flat surface: rays remain divergent
- Curved surfaces: possible to focus rays
  - Application: lenses





Imaging systems

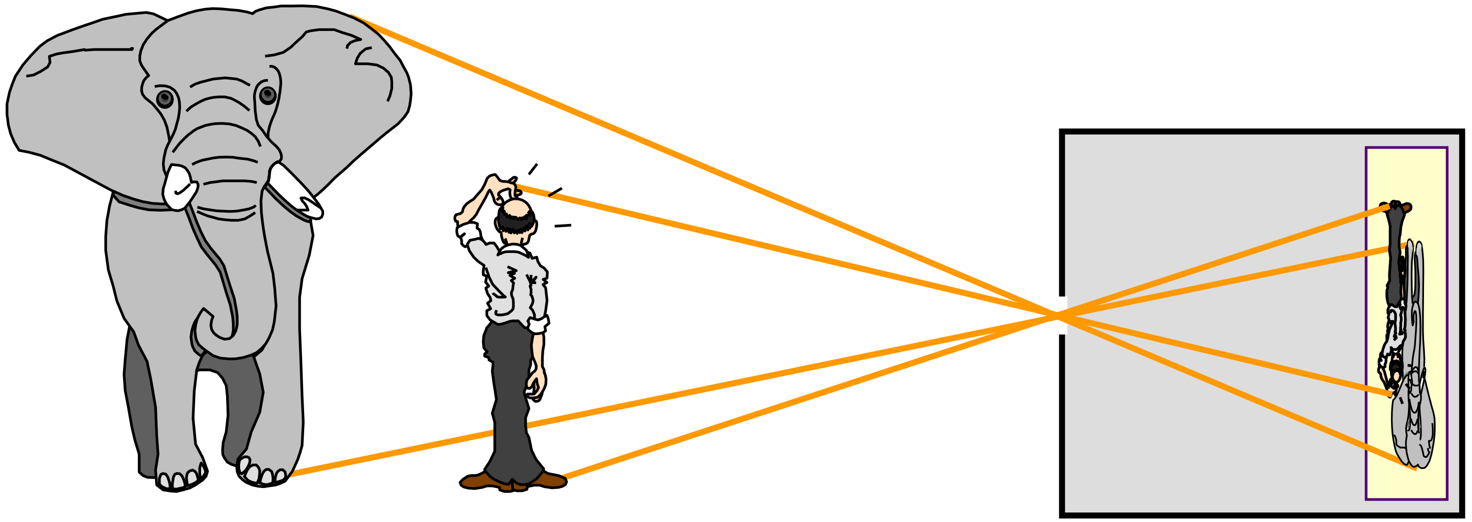
- Imaging systems: rays from one point of an object are converging to one point at some distance (="stigmatic imaging")
- Any deviation: aberrations
- Most imaging systems:
  - 3D is projected on the 2D image plane
  - Loss of information about depth
- Humans: information about depth from the parallax

Camera obscura

- Perfect projecting imaging system
  - Just "one" ray from each point

- Advantage
  - everything is imaged sharply

- Disadvantage
  - Very small amount of light is collected
    → Add a lens

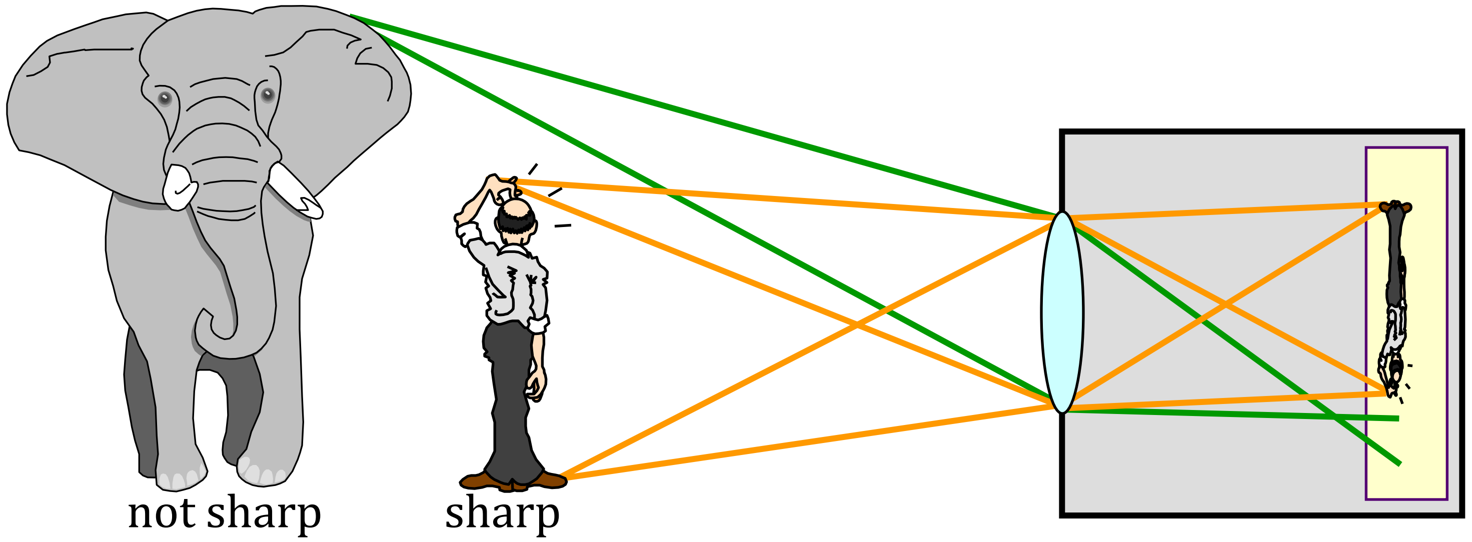


Camera obscura with lens

- Lens:
  - Focuses rays from the point on the object to the film surface
  - Possible only for the points at a certain distance from the camera

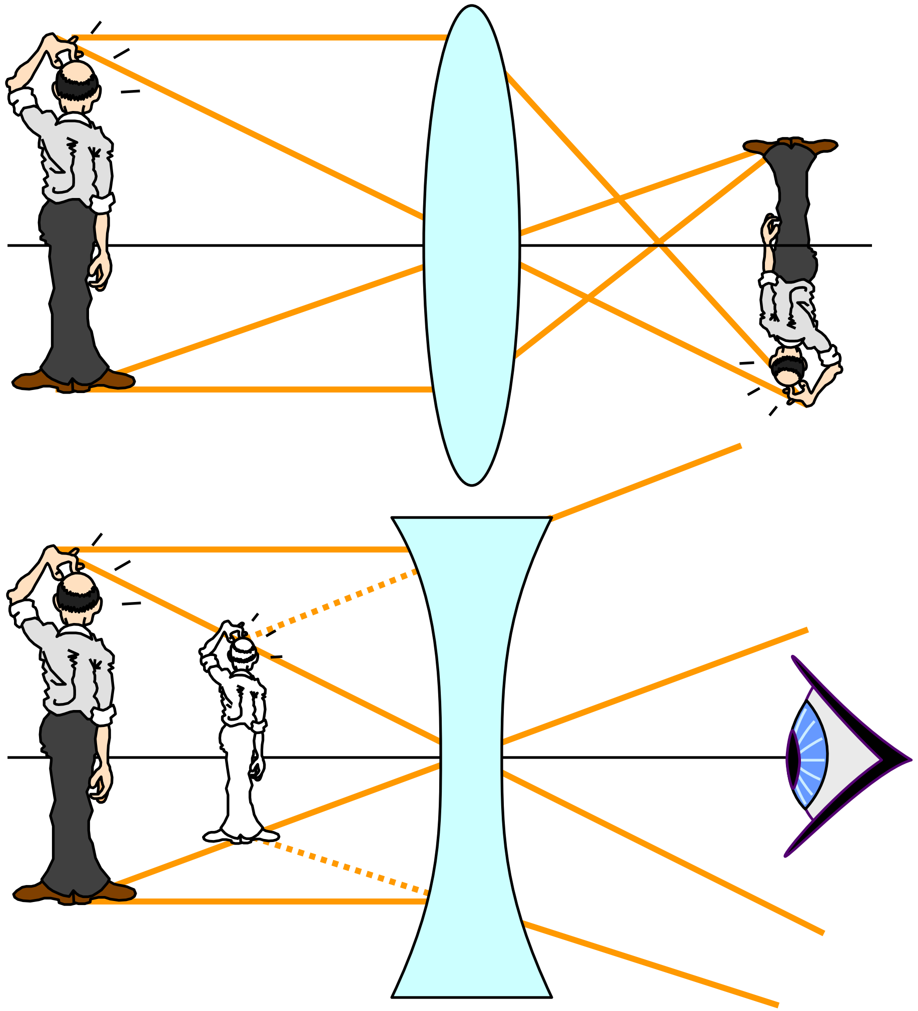
- Advantage
  - Better light efficiency

- Disadvantage
  - lower focus depth



Real and virtual image

- Real image
  - divergent rays recombine
  - Real light present
  - e.g. photo camera
- Virtual image
  - rays remain divergent
  - Object seems to be at a different location
  - e.g. magnifying glass



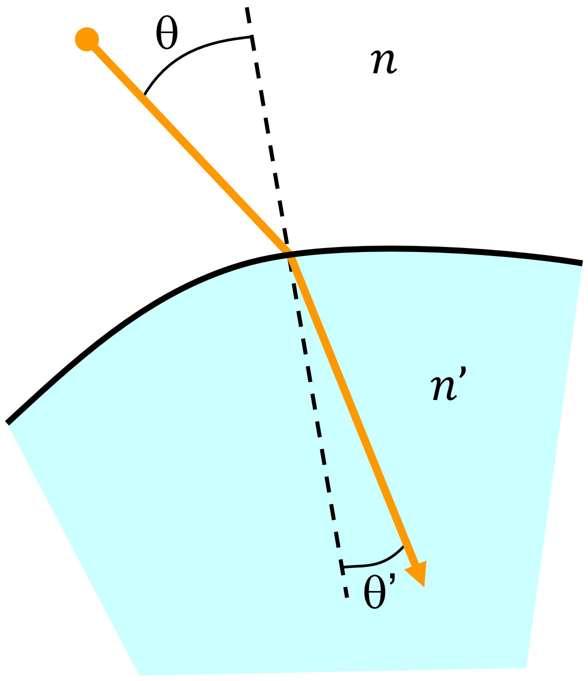
Graphical construction

Paraxial approximation

- Paraxial = small angle with the optical axis
  - $\sin\theta \simeq \theta$ (series expansion)
  - perfect stigmatic imaging is possible with a spherical refracting surface

- Snell's law

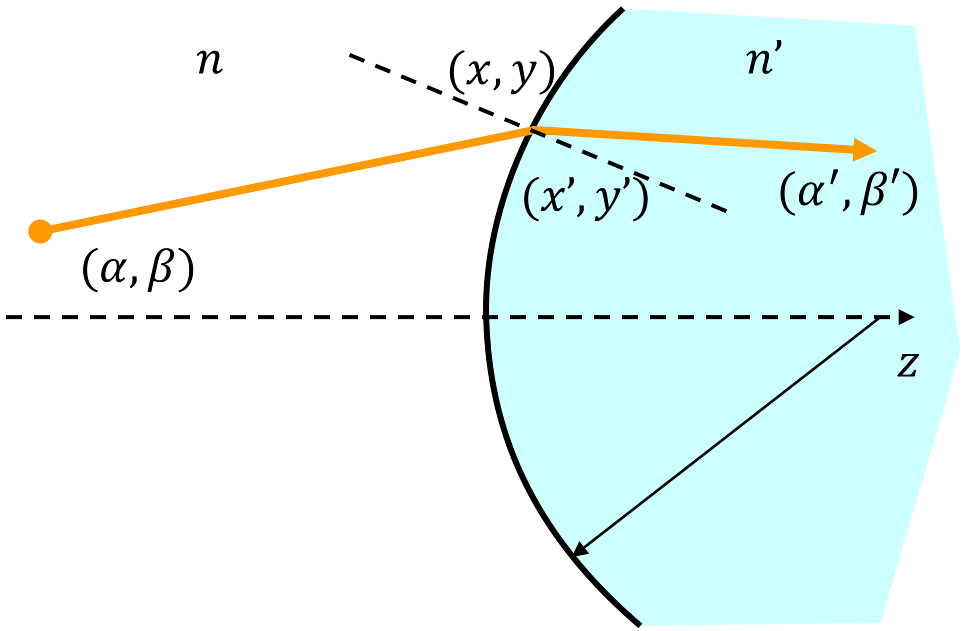
$$n\theta = n'\theta'$$



Refraction in the paraxial approximation

- Incident ray
  - Angle with z-axis (α, β)
  - Incidence point (x, y)
- After refraction
  - Angle with z-axis (α', β')
  - Starting point (x', y')

$$x' = x$$
$$y' = y$$
$$n'\alpha' = n\alpha + \frac{n - n'}{R} x$$
$$n'\beta' = n\beta + \frac{n - n'}{R} y$$

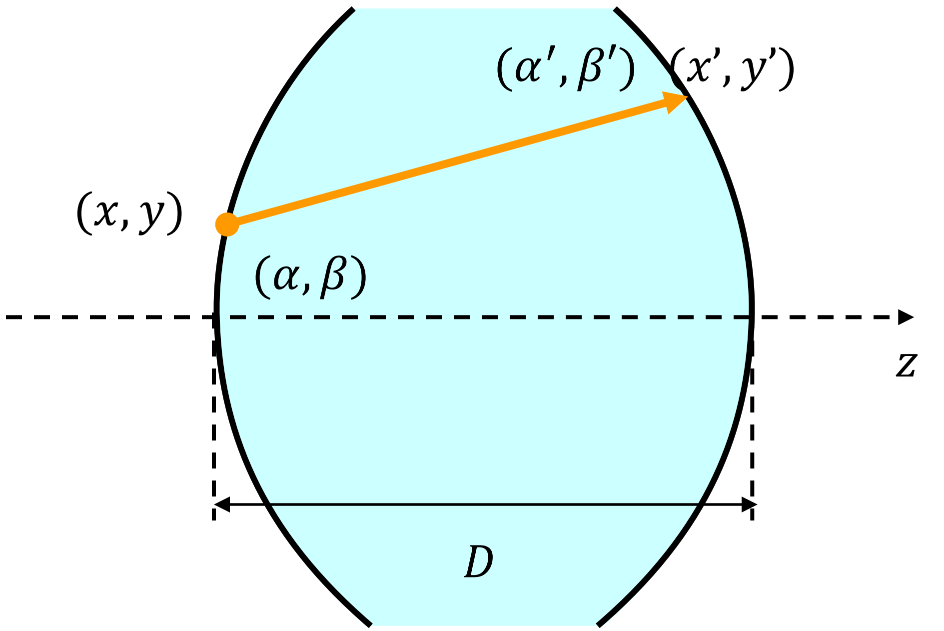


photonics reader section 3.3.1

Propagation in the paraxial approximation

- Original ray
  - Angle with z-axis (α, β)
  - Starting point (x, y)
- Ray after propagation
  - Angle with z-axis (α', β')
  - End point (x', y')

$$\alpha' = \alpha$$
$$\beta' = \beta$$
$$x' = x + D\alpha$$
$$y' = y + D\beta$$



Paraxial approximation

- Refraction:
  change of angle

$$x' = x$$
$$y' = y$$
$$n'\alpha' = n\alpha + \frac{n - n'}{R}x$$
$$n'\beta' = n\beta + \frac{n - n'}{R}y$$

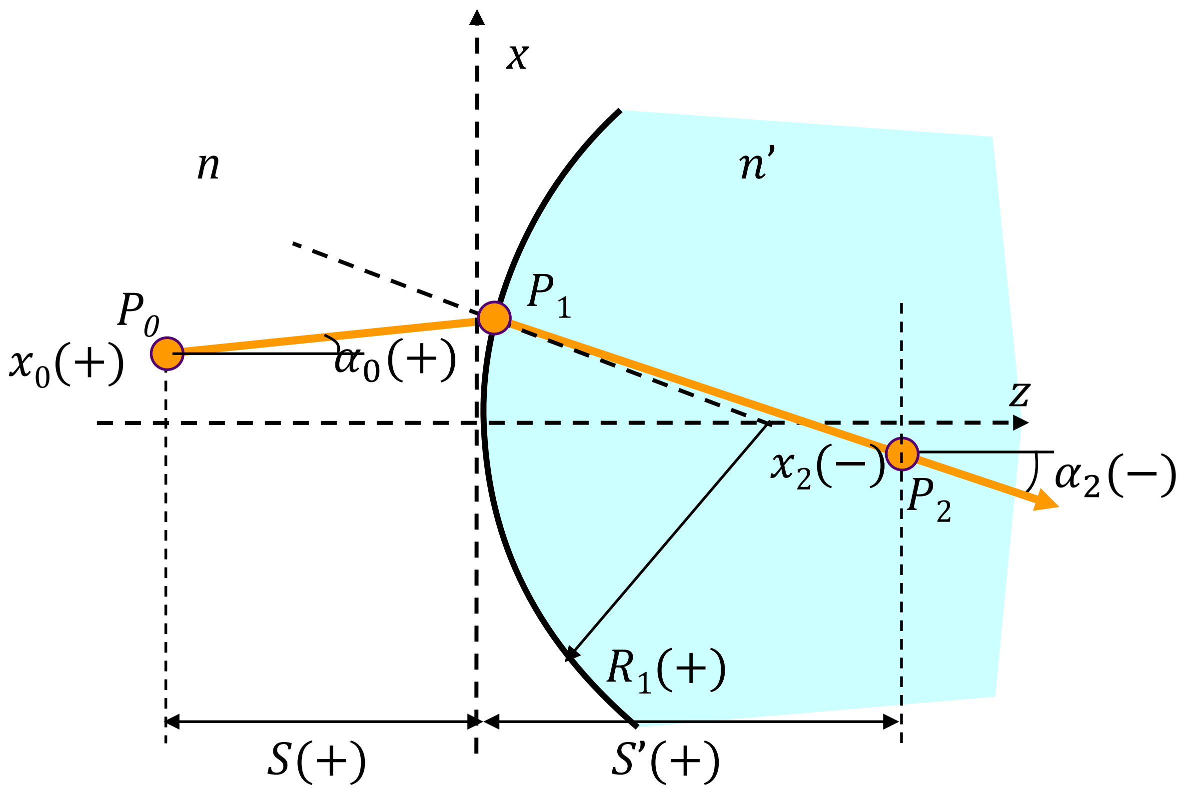
- Propagation:
  change of position

$$\alpha' = \alpha$$
$$\beta' = \beta$$
$$x' = x + D\alpha$$
$$y' = y + D\beta$$

- Linear equations
- Separated for (x, α) and (y, β)

Paraxial approximation

- A ray from P_0
  - starting angle α_0
  - starting point x_0
- Ray undergoes
  - propagation in n
  - refraction at P_1
  - propagation in n'



$$\alpha_2 = \left(\frac{n-n'}{n'R_1}\right)x_0 + \left(\frac{n}{n'} + \frac{(n-n')S}{n'R_1}\right)\alpha_0$$

$$x_2 = \left(\frac{(n-n')S'}{n'R_1} + 1\right)x_0 + \left(S + \frac{nS'}{n'}\frac{(n-n')SS'}{n'R_1}\right)\alpha_0$$

Imaging

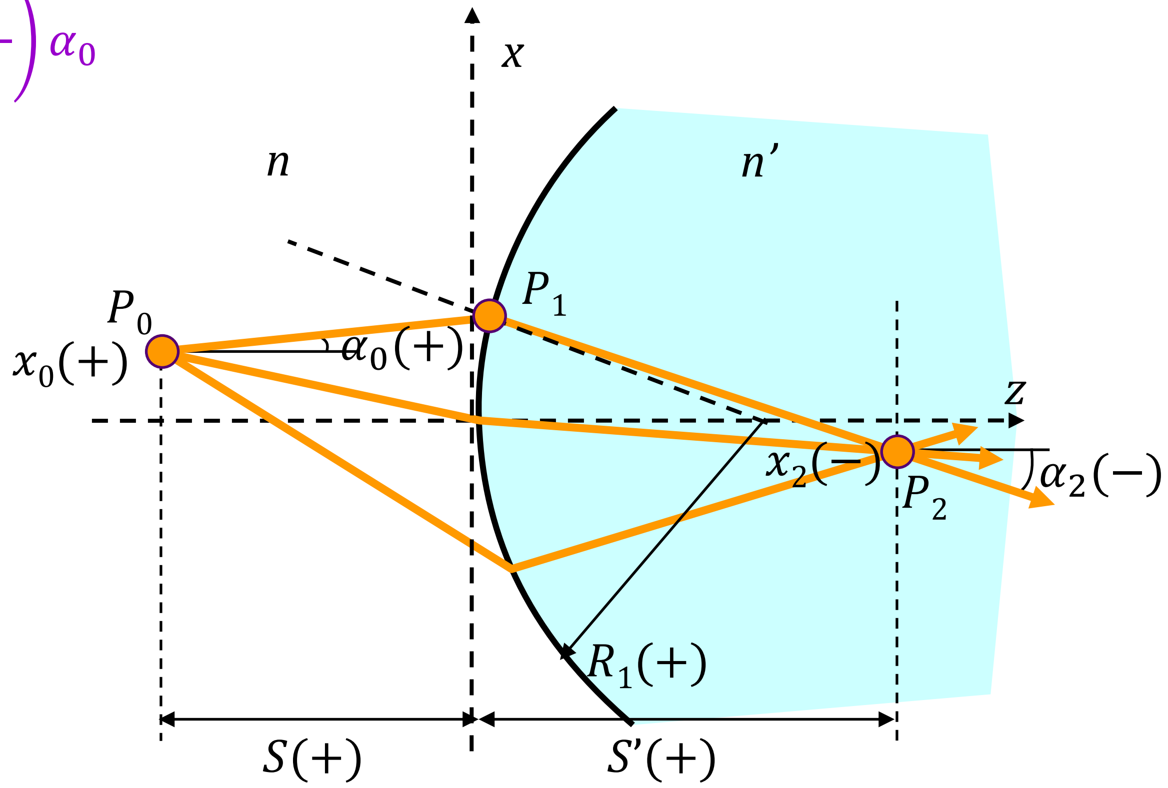
- Transformation:

$$\alpha_2 = \left(\frac{n-n'}{n'R_1}\right)x_0 + \left(\frac{n}{n'} + \frac{(n-n')S}{n'R_1}\right)\alpha_0$$
$$x_2 = \left(\frac{(n-n')S'}{n'R_1} + 1\right)x_0 + \left(S + \frac{nS'}{n'}\frac{(n-n')SS'}{n'R_1}\right)\alpha_0$$

- Imaging:
  all rays from x_0 arrive at x_2, regardless of the angle α_0

$$\Rightarrow S + \frac{nS'}{n'} + \frac{(n-n')SS'}{n'R_1} = 0$$

$$\Leftrightarrow \frac{n}{S} + \frac{n'}{S'} = \frac{n'-n}{R_1}$$



Magnification

- Lateral magnification m_x

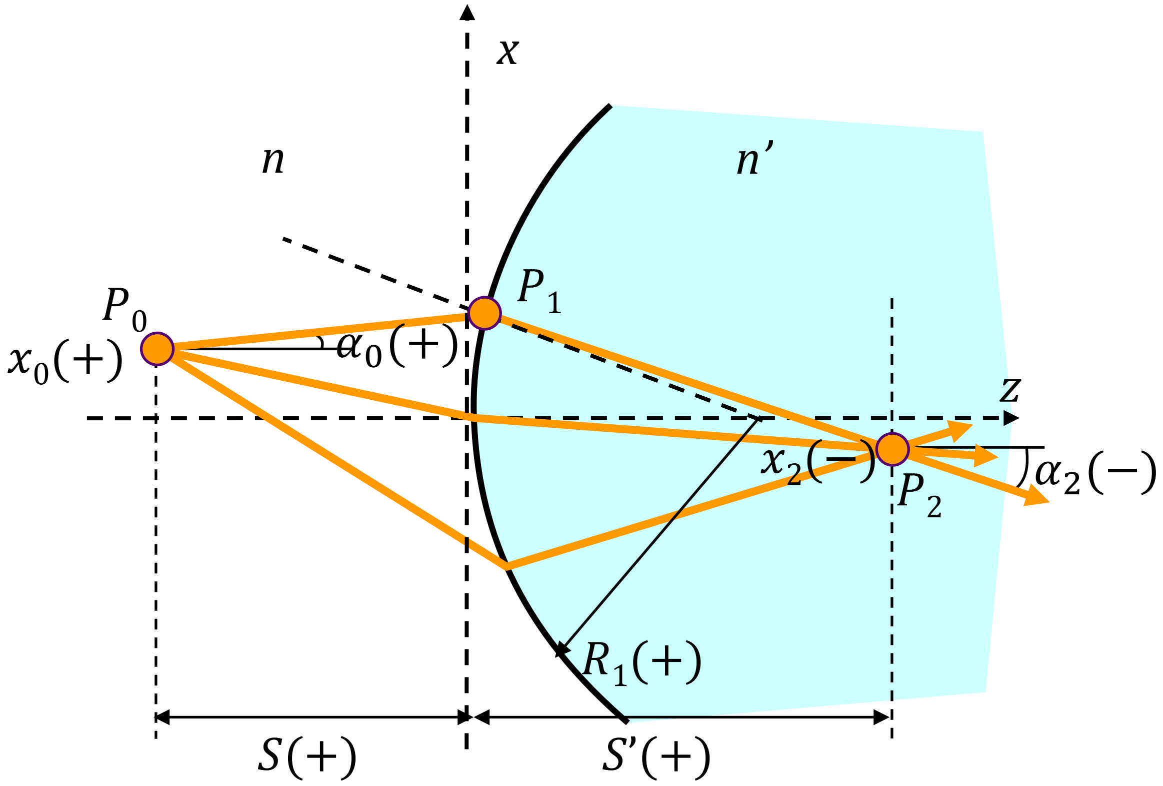
$$m_x \triangleq \frac{x_2}{x_0} = -\frac{nS'}{n'S}$$

- Angular magnification m_α

$$m_\alpha \triangleq \frac{\Delta\alpha_2}{\Delta\alpha_0} = -\frac{S}{S'}$$

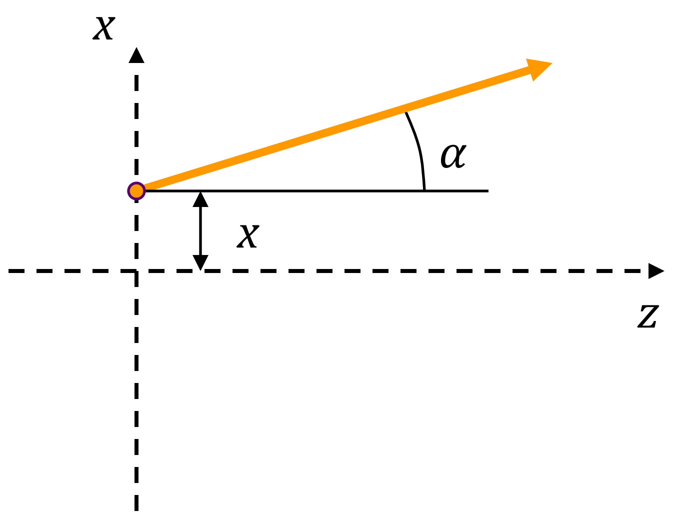
- We see that

$$m_x \cdot m_\alpha = \frac{n}{n'}$$



Matrix formalism

- Ray equations
  - 2 variables: x and α
  - linear



- Matrix form
  - variables: 2 × 1 column matrix

$$\boldsymbol{r} = \begin{bmatrix} x \\ n\alpha \end{bmatrix}$$

  - Transformations: 2 × 2 matrix

Refraction in the matrix formalism

- Incident ray

$$\mathbf{r} = \begin{bmatrix} x \\ n\alpha \end{bmatrix}$$

- After refraction

$$\mathbf{r}' = \begin{bmatrix} x' \\ n'\alpha' \end{bmatrix}$$

- Transformation

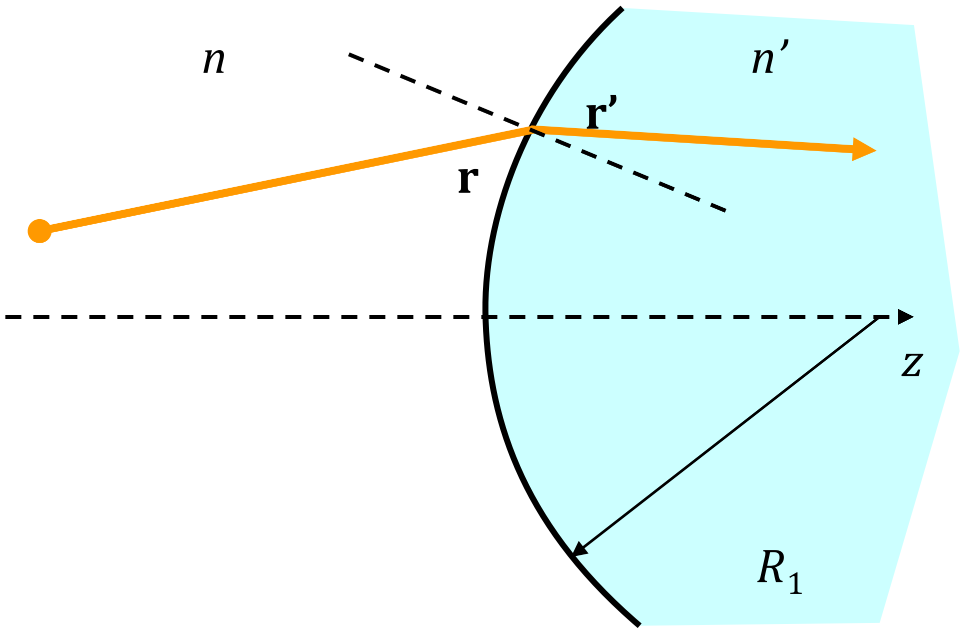
$\mathbf{r}' = \mathbf{R}\mathbf{r}$

with $\mathbf{R} = \begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix}$

- Refractive power $P = \frac{n'-n}{R}$

$$x' = x$$

$$n'\alpha' = n\alpha + \frac{n-n'}{R}x$$



Translation in the matrix formalism

- Original ray

$$\mathbf{r} = \begin{bmatrix} x \\ n\alpha \end{bmatrix}$$

- Ray after propagation

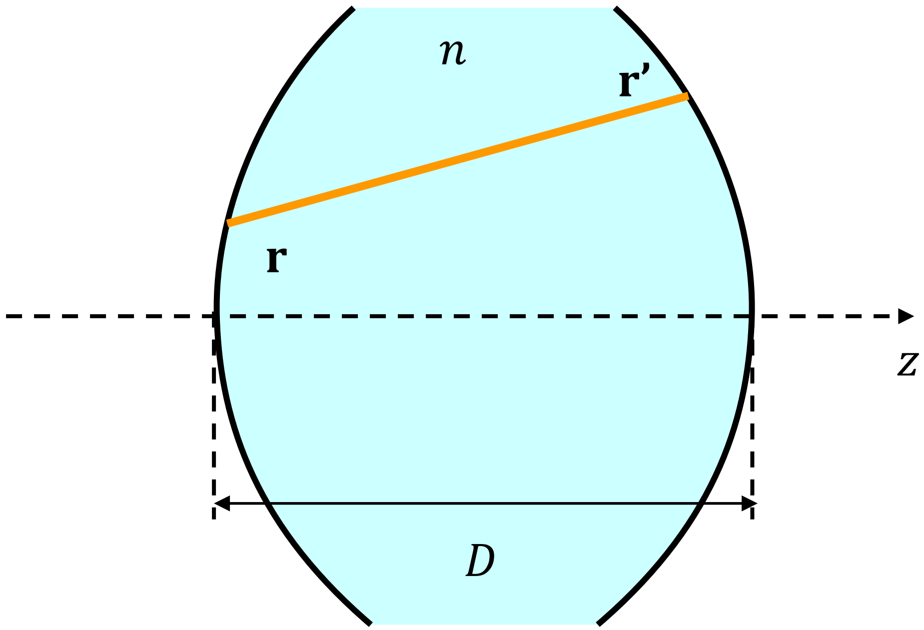
$$\mathbf{r}' = \begin{bmatrix} x' \\ n'\alpha' \end{bmatrix}$$

- Transformation

$$\mathbf{r}' = \mathbf{T}\mathbf{r}$$

with $\mathbf{T} = \begin{bmatrix} 1 & D/n \\ 0 & 1 \end{bmatrix}$

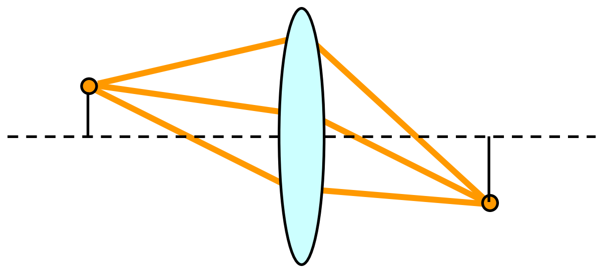
$$x' = x + D\, n\alpha/n$$
$$n'\alpha' = n\alpha, \quad n' = n$$



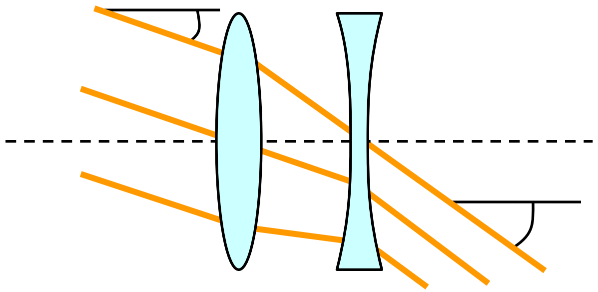
Imaging

- $\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ with $\det(\mathbf{M}) = 1$

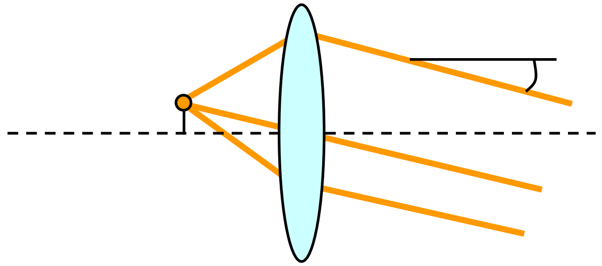
- Position-position map: $M_{12} = 0$



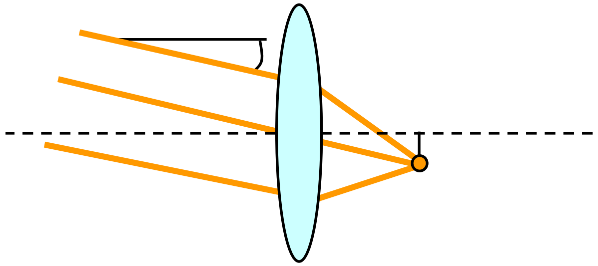
- Angle-angle map: $M_{21} = 0$



- Position-angle map: $M_{22} = 0$

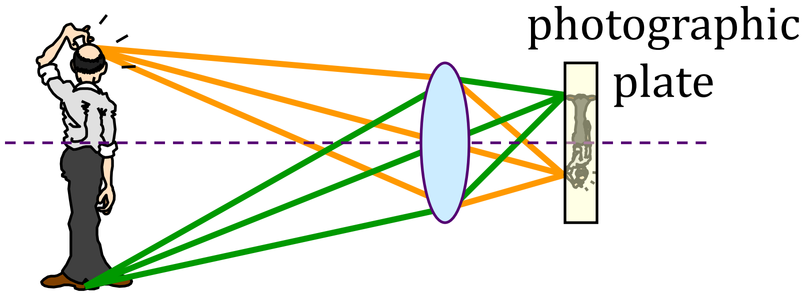


- Angle-position map: $M_{11} = 0$

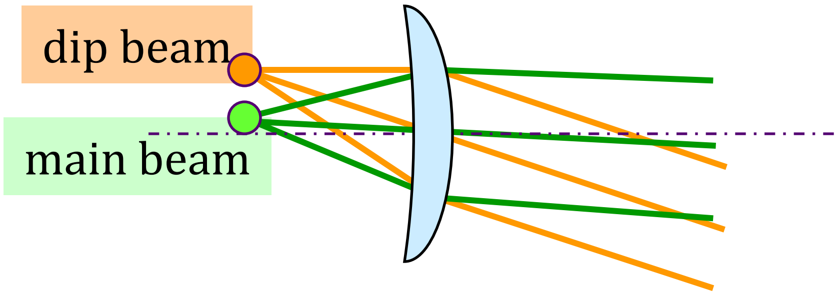


Examples of imaging

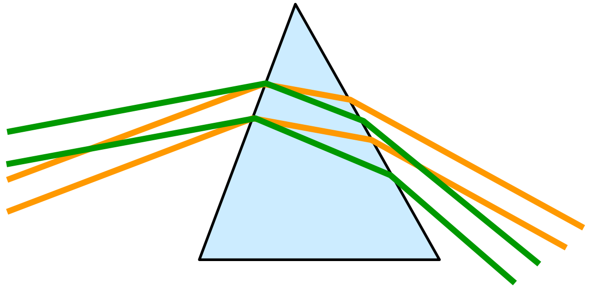
- $\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ with $\det(\mathbf{M}) = 1$
- Photo camera: position-position: $M_{12} = 0$



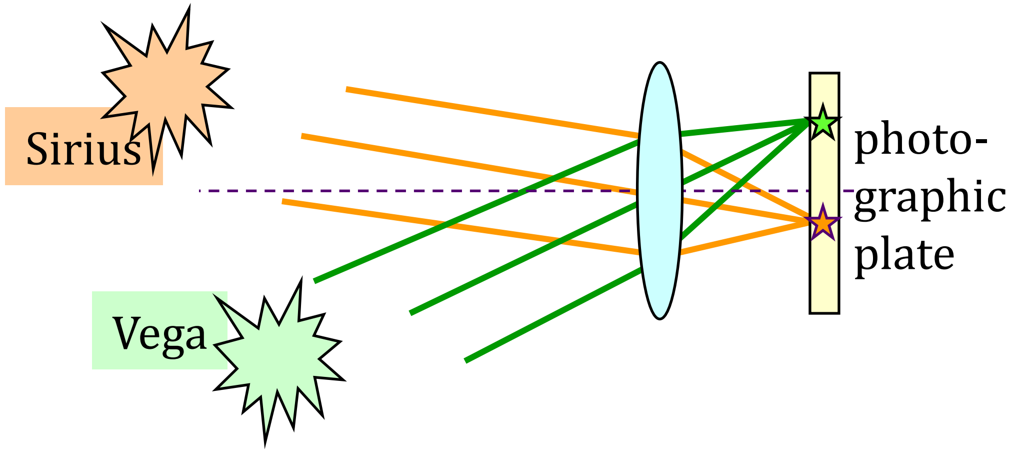
- Headlight: position-angle: $M_{22} = 0$



- Prism: angle-angle: $M_{21} = 0$



- Telescope: angle-position: $M_{11} = 0$



A lens in the matrix formalism

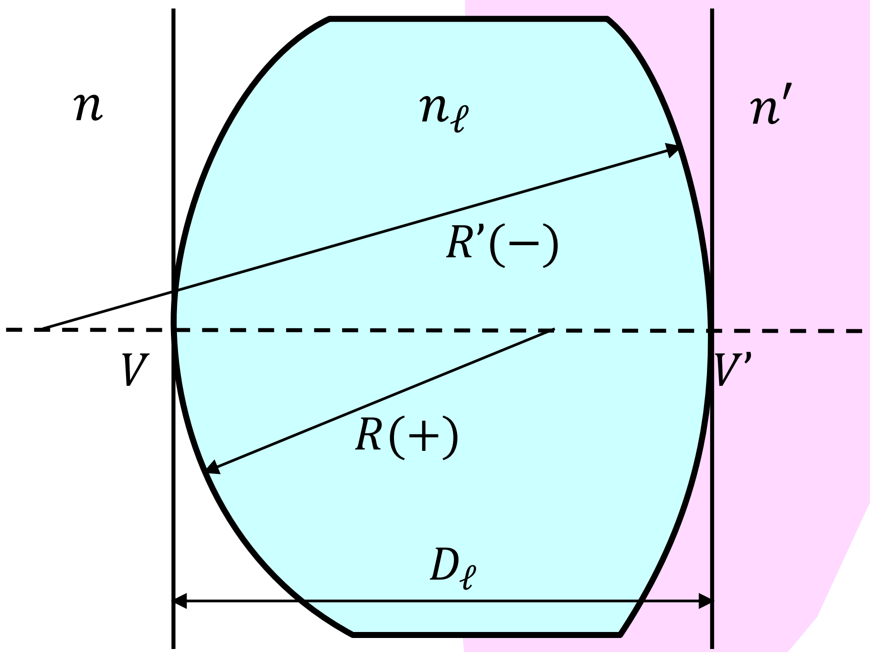
- Refractive power of the interfaces

$$P = \frac{n_\ell - n}{R} \qquad P' = \frac{n' - n_\ell}{R'}$$

- Vertices V and V'

- System matrix $\mathbf{M}$

$$\begin{aligned}\mathbf{M} &= \mathbf{R}'\mathbf{T}\mathbf{R}\\ &= \begin{bmatrix} 1 & 0 \\ -P' & 1 \end{bmatrix}\begin{bmatrix} 1 & D_\ell/n_\ell \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix}\\ &= \begin{bmatrix} 1 - P\,D_\ell/n_\ell & D_\ell/n_\ell \\ PP'\,D_\ell/n_\ell - P - P' & 1 - P'\,D_\ell/n_\ell \end{bmatrix}\end{aligned}$$



A thin lens in the matrix formalism (1)

- System matrix **M**

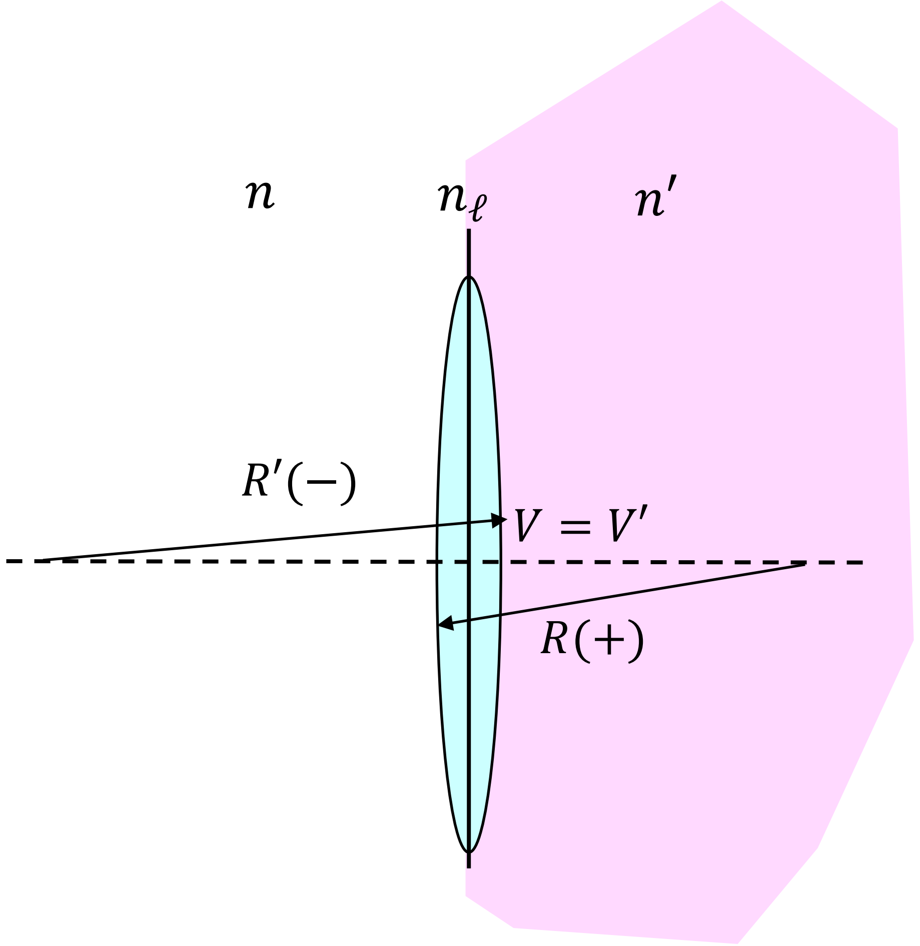
$$\mathbf{M} = \begin{bmatrix} 1 - P\,D_\ell/n_\ell & D_\ell/n_\ell \\ PP'\,D_\ell/n_\ell - P - P' & 1 - P'\,D_\ell/n_\ell \end{bmatrix}$$

- Thin lens: $D_\ell = 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -P' - P & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P_{\text{thin}} & 1 \end{bmatrix}$$

- Refractive power P_{thin} (if $n = n' = 1$)

$$P_{\text{thin}} = (n_\ell - 1)\left(\frac{1}{R} - \frac{1}{R'}\right)$$



A thin lens in the matrix formalism (2)

- Refractive power P_{thin} (if $n \neq n'$)

$$P_{\text{thin}} = \frac{n_\ell - n}{R} - \frac{n_\ell - n'}{R'}$$

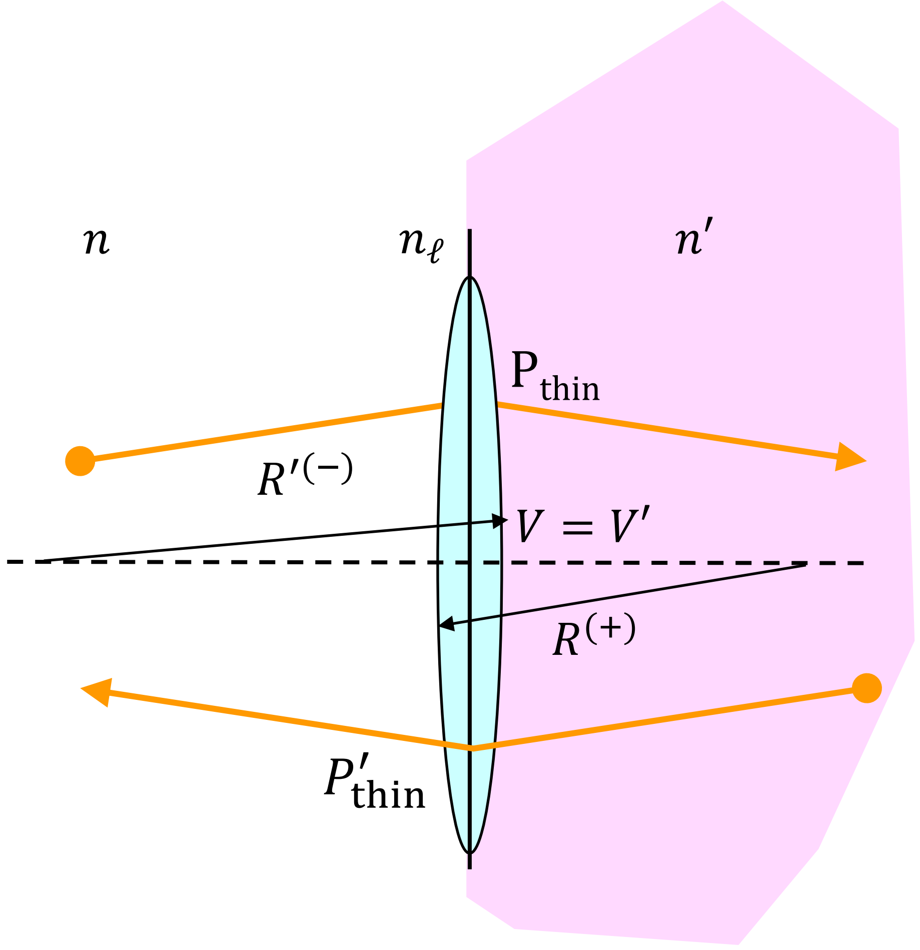
from medium n into medium n'

- Refractive power P'_{thin} (if $n \neq n'$)

$$P'_{\text{thin}} = \frac{n_\ell - n'}{-R'} - \frac{n_\ell - n}{-R}$$

from medium n' into medium n
(radius R and R' have opposite sign)

- $P_{\text{thin}} = P'_{\text{thin}}$



Focal length of a thin lens

- Refractive power
  - the only quantity which characterizes a thin lens
  - unit: diopter = $1/m$
- Focal length
  - A point to which all rays with $\alpha = 0$ or $\alpha' = 0$ converge

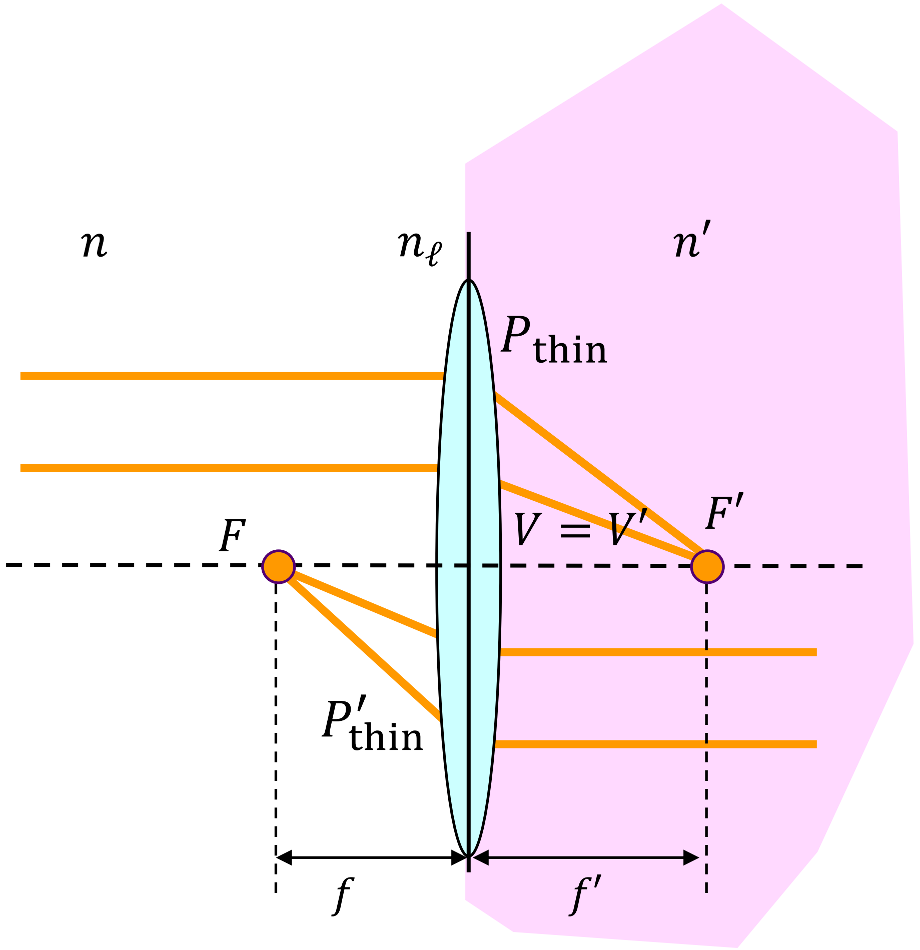
$$n'\alpha' = -P_{\text{thin}}x + \cancel{n\alpha}, \qquad \alpha = 0$$

$$\alpha' = -x'/f', \qquad x' = x$$

- Focal length f and f'

$$f' = \frac{n'}{P_{\text{thin}}} \qquad f = \frac{n}{P'_{\text{thin}}}$$

$$\begin{bmatrix} x' \\ n'\alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P_{\text{thin}} & 1 \end{bmatrix} \begin{bmatrix} x \\ n\alpha \end{bmatrix} = \begin{bmatrix} x \\ -P_{\text{thin}}x + n\alpha \end{bmatrix}$$



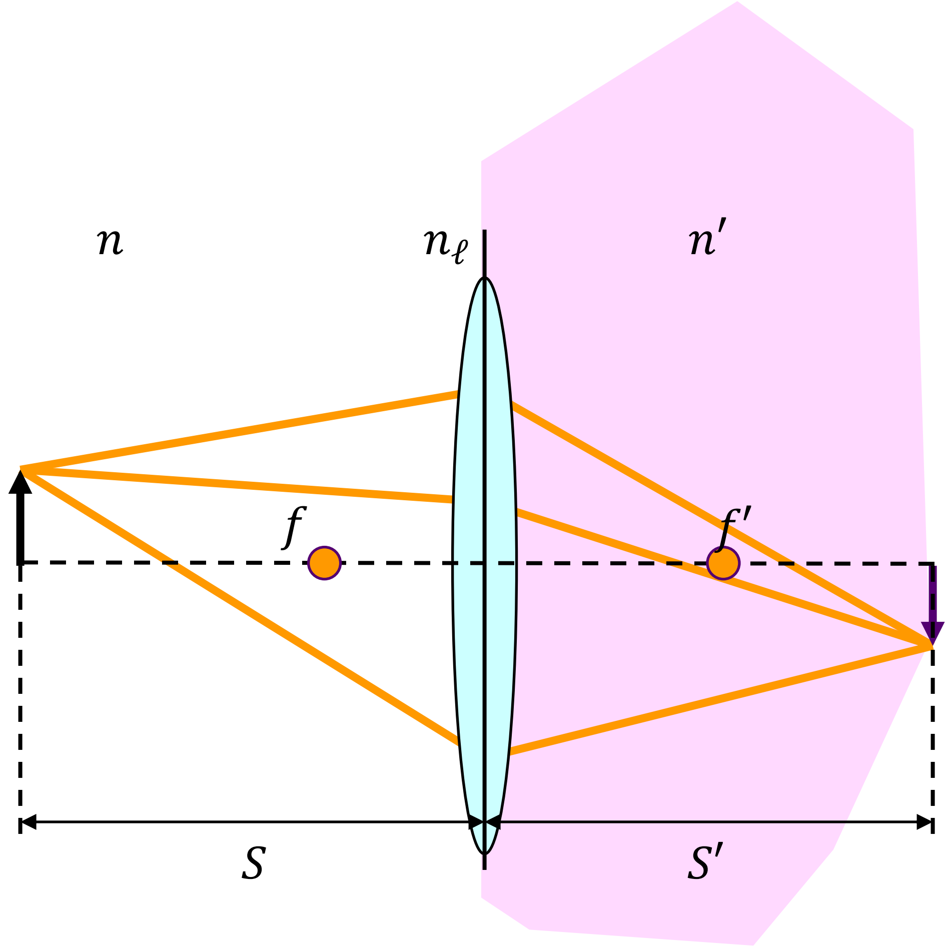
Imaging with a thin lens

- Transformations:
  - translation over a distance S
  - refraction at the lens (power P)
  - translation over a distance S'

$$\mathbf{M} = \begin{bmatrix} 1 & S'/n' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \begin{bmatrix} 1 & S/n \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 - PS'/n' & S/n + S'/n' - P\dfrac{SS'}{nn'} \\ -P & 1 - PS/n \end{bmatrix}$$

- Imaging: $M_{12} = 0$

$$\boxed{\begin{aligned} \frac{n}{S} + \frac{n'}{S'} &= P_{\text{thin}} = \frac{n'}{f'} \\ &= P'_{\text{thin}} = \frac{n}{f} \end{aligned}}$$



Complex lens system = thin lens (1)

- System matrix $\mathbf{M}$

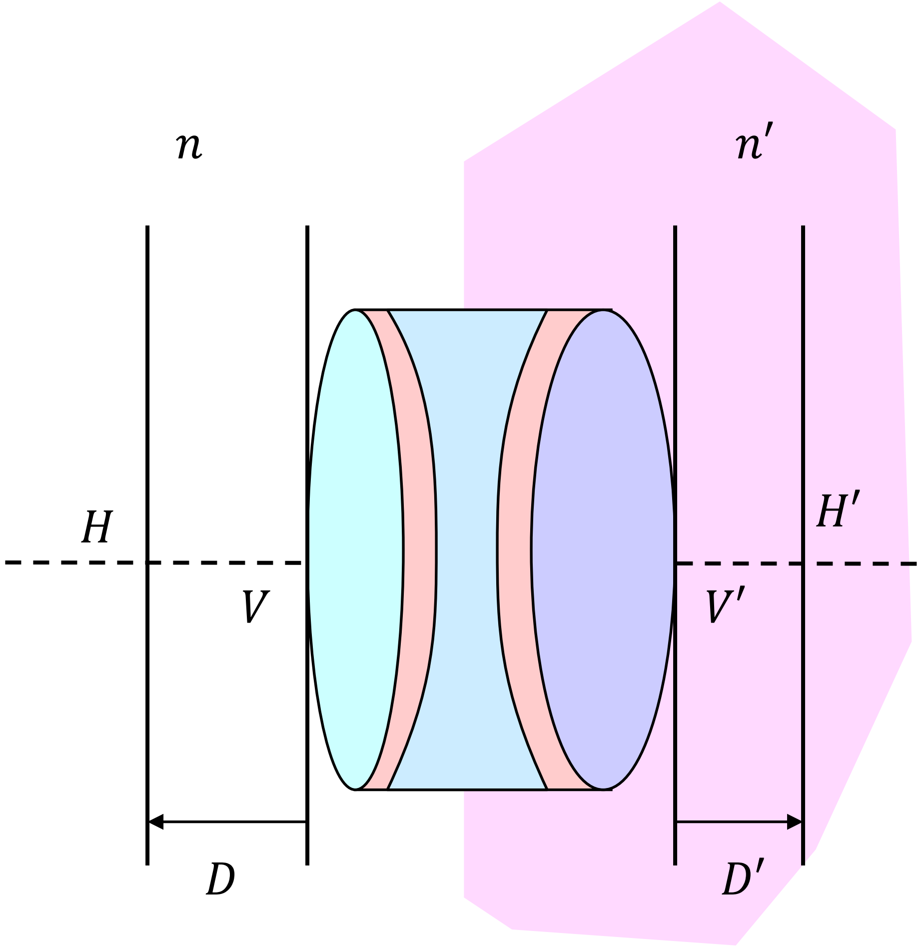
$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

- Transformation into a "thin lens" by the translations $\mathbf{T}$ and $\mathbf{T}'$

$$\mathbf{M}' = \mathbf{T}'\mathbf{M}\mathbf{T}$$

with $\mathbf{T} = \begin{bmatrix} 1 & D/n \\ 0 & 1 \end{bmatrix}$

and $\mathbf{T}' = \begin{bmatrix} 1 & D'/n' \\ 0 & 1 \end{bmatrix}$



Complex lens system = thin lens (2)

- System matrix **M’**

$$\mathbf{M}' = \begin{bmatrix} M_{11} + M_{21}\, D'/n' & M_{22}\, D'/n' + M_{21}\, D/n \cdot D'/n' + M_{12} + M_{11}\, D/n \\ M_{21} & M_{22} + M_{21}\, D/n \end{bmatrix}$$

- Thin lens matrix

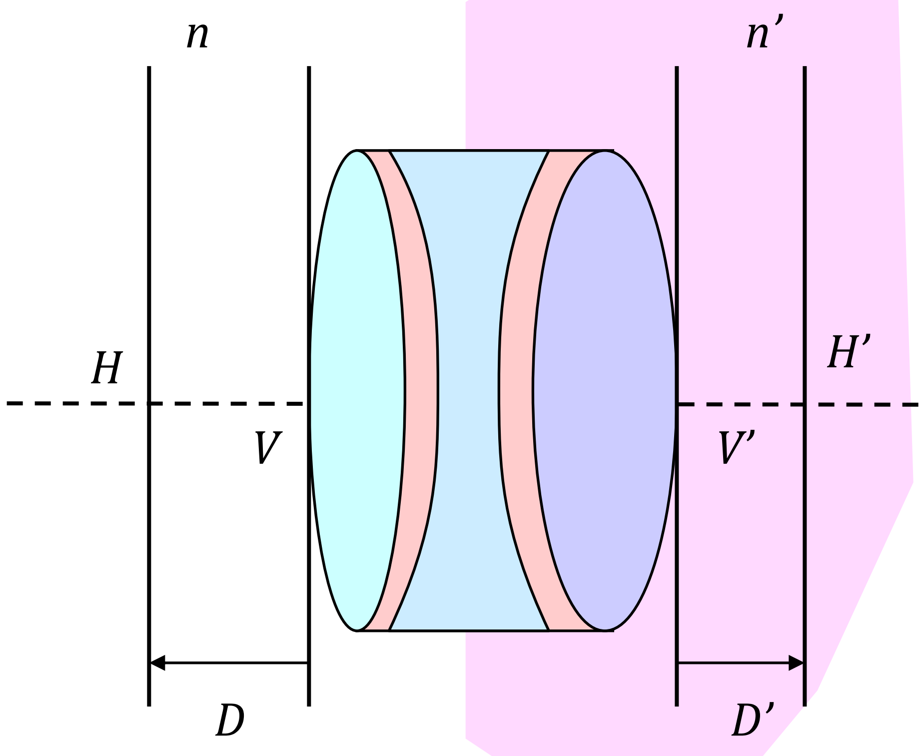
$$\mathbf{M}' = \begin{bmatrix} 1 & 0 \\ M_{21} & 1 \end{bmatrix}$$

- Solution for D and D'

$$D/n = \frac{1 - M_{22}}{M_{21}}$$

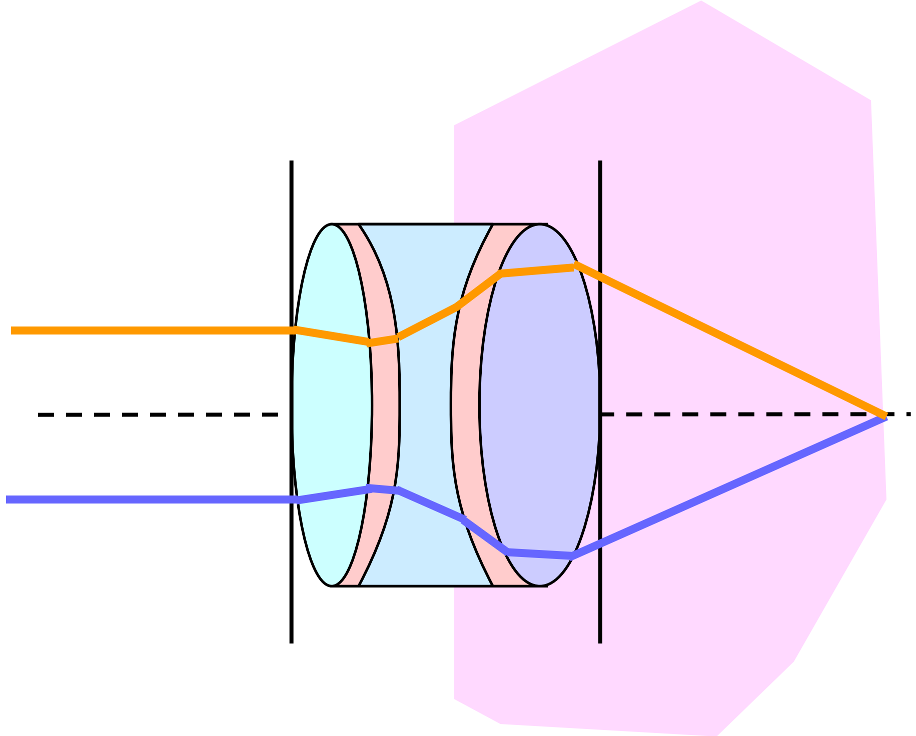
$$D'/n' = \frac{1 - M_{11}}{M_{21}}$$

- Principal planes H and H'

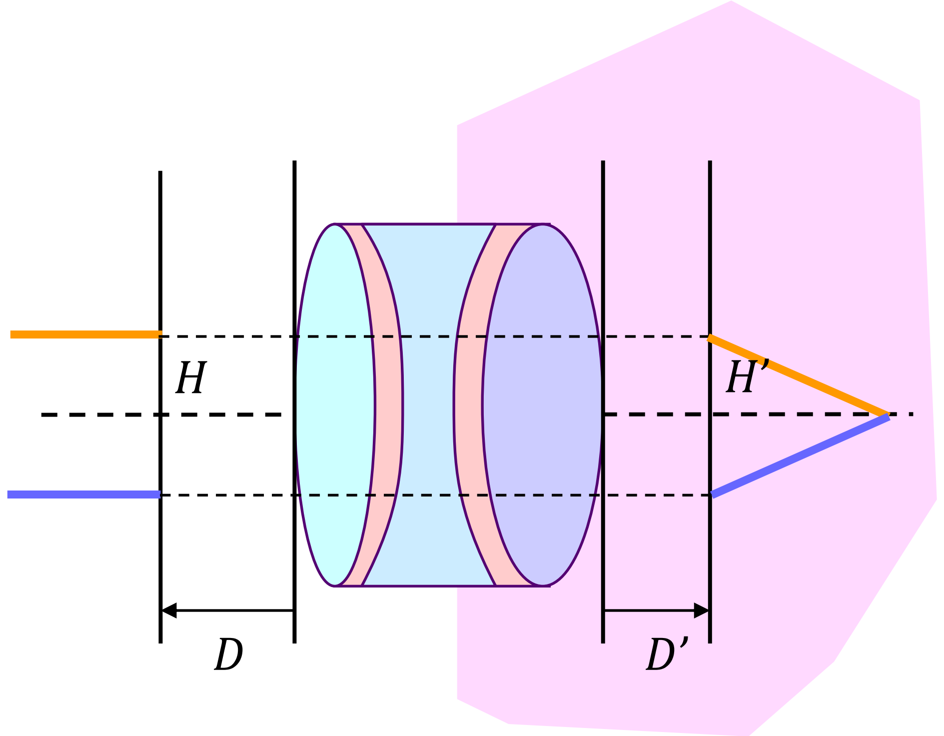


Principal planes

- Physical
  - Propagation through the optical system



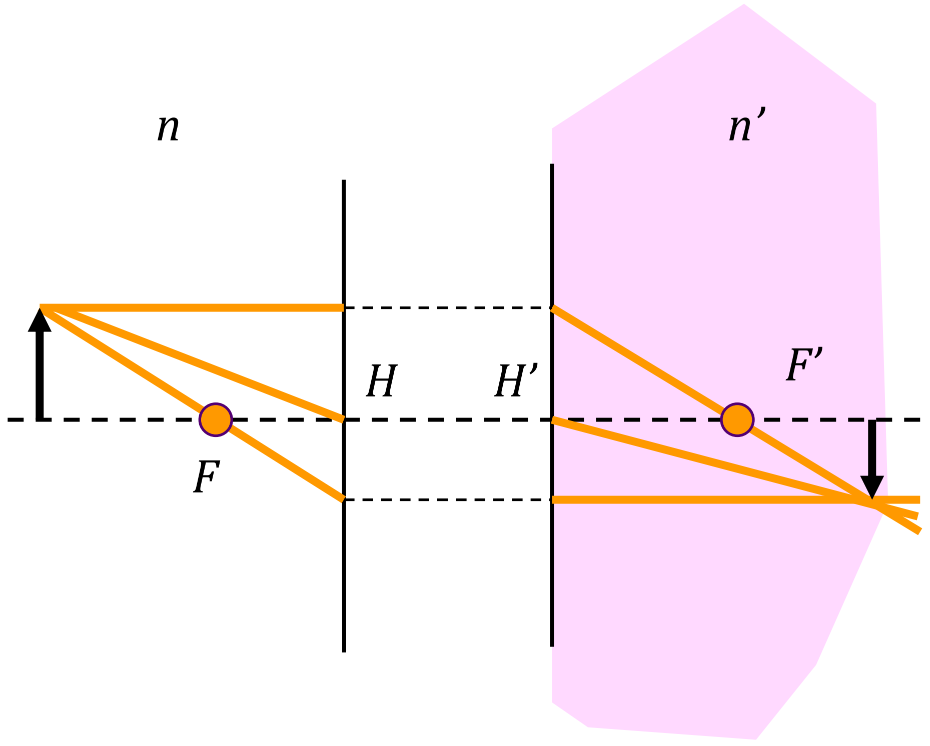
- Mathematical
  - Incoming ray in *H*
  - Thin lens
  - Outgoing ray from *H'*



The graphical formalism

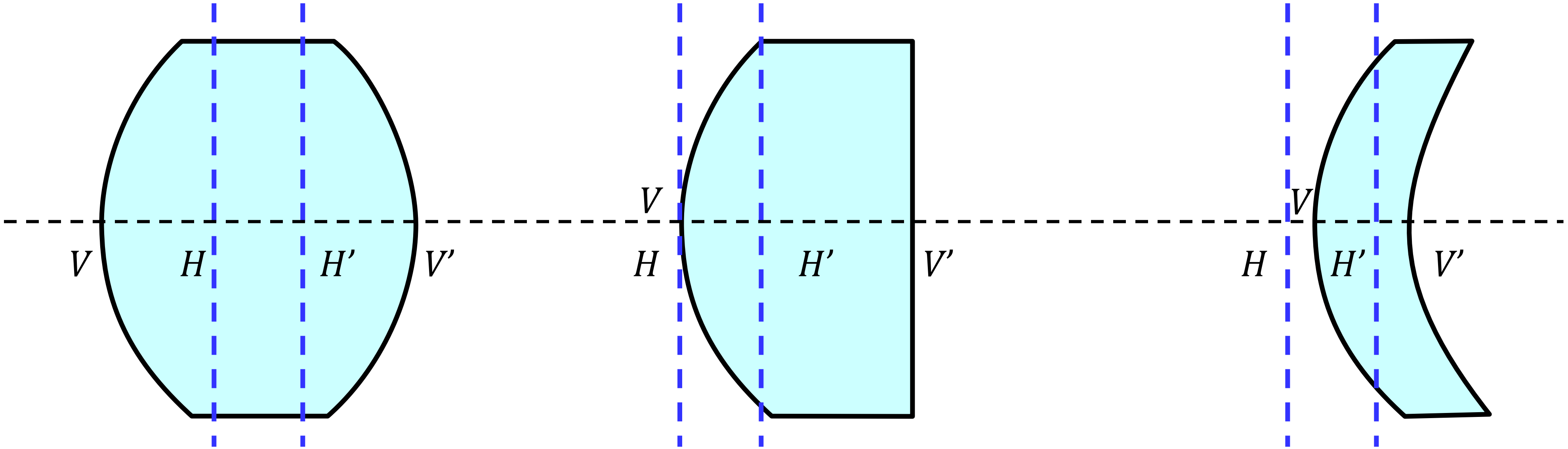
- Definition of the principal planes
  - The area between H and H' is not considered

- Rules for rays
  - An incident ray parallel to the axis passes through F'
  - A ray coming through point H leaves from H' and has the same direction (apart from a factor n/n'): *chief ray*
  - A ray coming through F leaves plane H' parallel to the axis



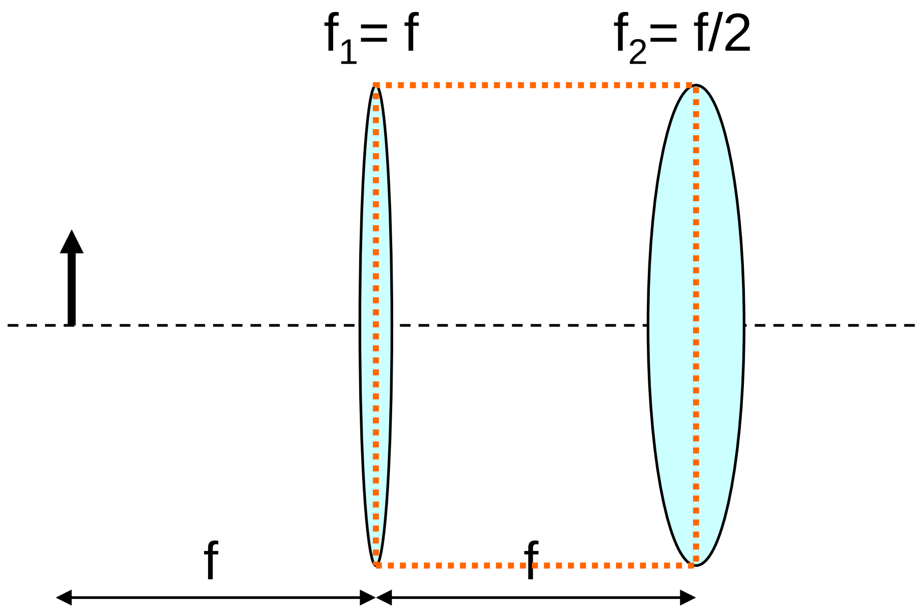
Location of the principal planes

- Thick convex lens
- Plano-convex lens
- Meniscus lens



Exercise: double lens

- Optical system with 2 lenses in air:
  - lens 1 with focal length f
  - lens 2 with focal length $f/2$
  - distance f between the lenses



- Wanted:
  - The system matrix of the optical system
  - Location of the principal planes
  - Magnification of an object at a distance f from the first lens
  - Construct this image with and without the principal planes

Spherical mirrors

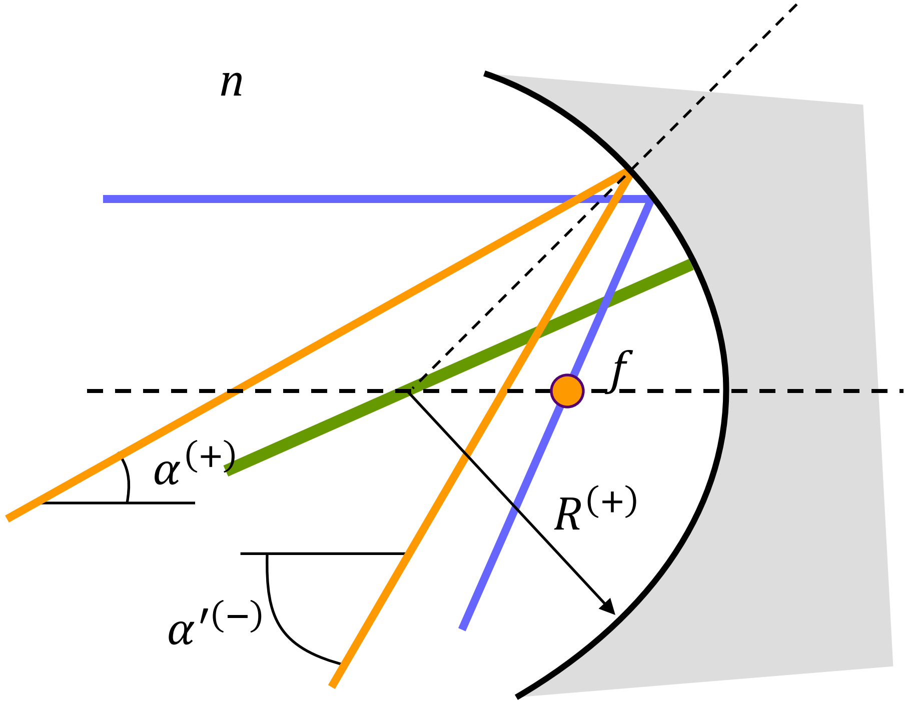
- Paraxial approximation:

$$\begin{bmatrix} x' \\ n\alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \begin{bmatrix} x \\ n\alpha \end{bmatrix}$$

with $P = \dfrac{2n}{R}$

- Expansion of the sign convention
  - concave mirror: $R > 0$
  - take into account the propagation direction
- Focal length $f = n/P$

$$f = \frac{R}{2}$$



Numerical aperture and *f*-number

- *f*-number (relative aperture)

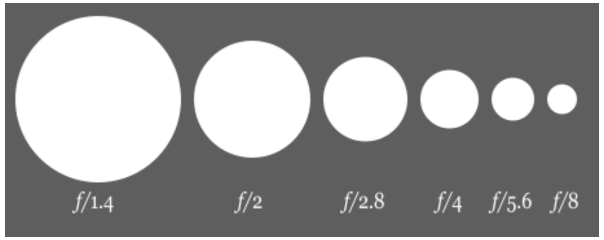
$$f\text{−number} = \frac{f}{D}$$

$$D = \frac{f}{f\text{−number}}$$

Example *f*-number:

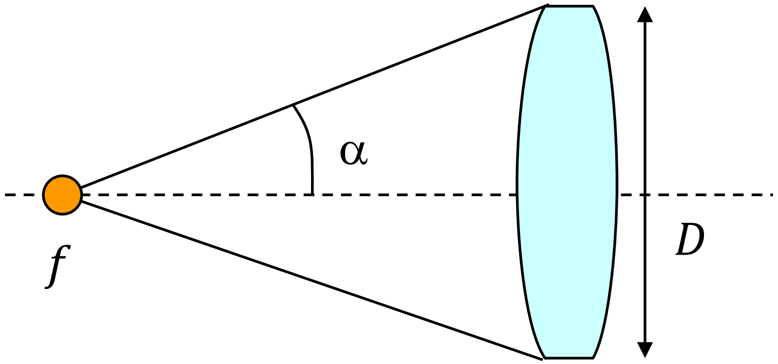
2, 2.8, 4, 5.6, 8, 11

Denoted as: $f/2$, $f/2.8$, $f/4$, …



- Numerical aperture

$$\begin{aligned} \text{NA} &= \sin\alpha \\ &= \frac{1}{2(f\text{−number})} \end{aligned}$$



Aberrations

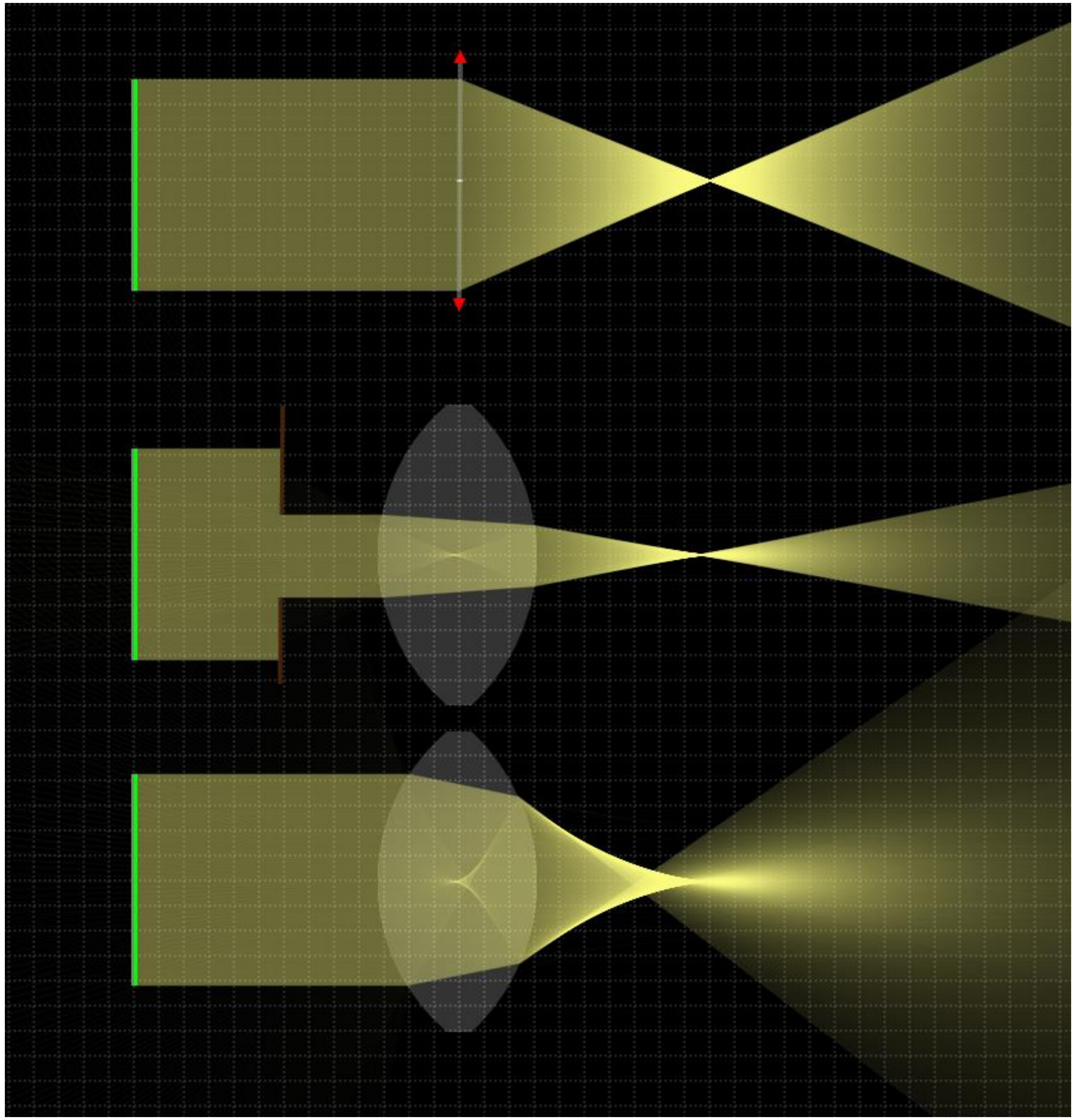
- Imaging differs from the paraxial imaging
  - paraxial = 1st order approximation of sine
- Seidel: 3rd order approximation
  - aberrations, which result in non-stigmatic image (spherical aberration, astigmatism, coma)
  - aberrations with distorted stigmatic image (field curvature, distorsion)
  - chromatic aberrations (dispersion of the material)
- Aberrations depend on
  - the lens system
  - choice of the object plane (or magnification): optimization is only possible for one magnification

Spherical aberration (1)

Ideal lens

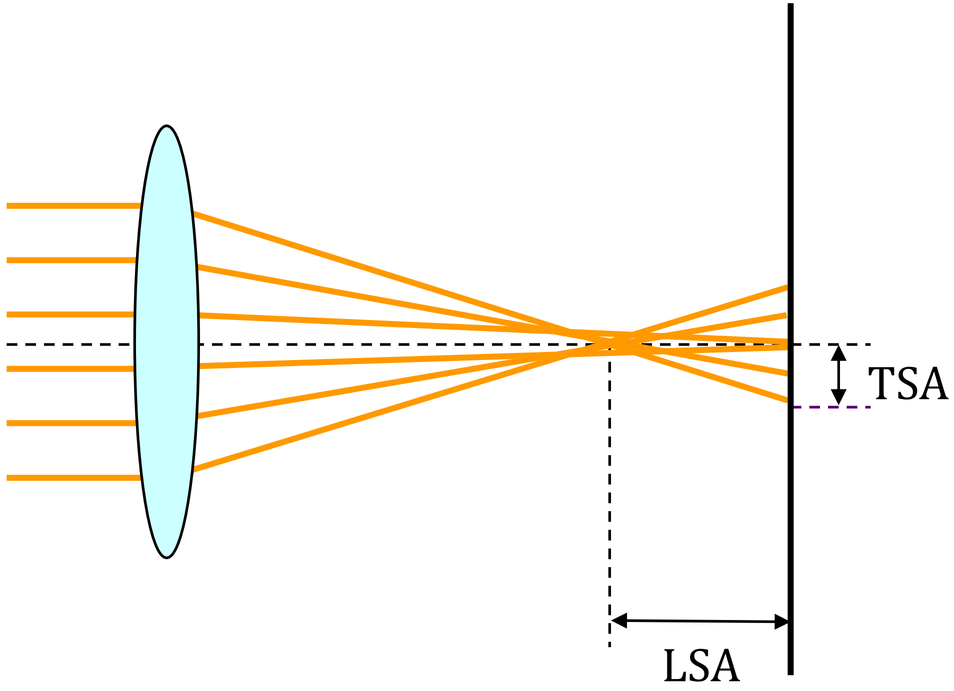
Spherical lens
small diameter
large *f*-number

Spherical lens
large diameter
small *f*-number



Spherical aberration

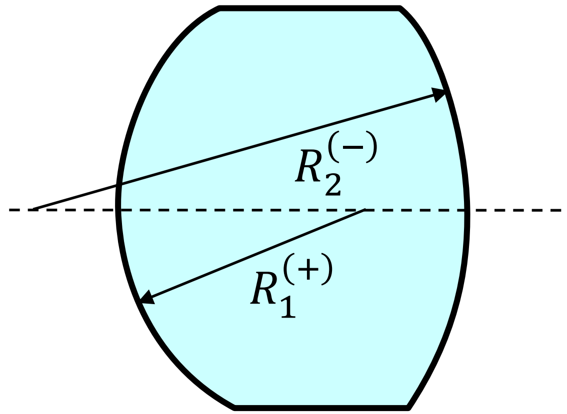
- Imaging on the optical axis:
  - focus point on the axis ~ angle (= Longitudinal S.A.)
  - deviation in the focal plane (= Transversal S.A.)
- ~ (Lens diameter)^2
  - strong for small *f*-numbers
- Solution
  - lens with the best shape
  - combination of lenses
  - aspherical lens



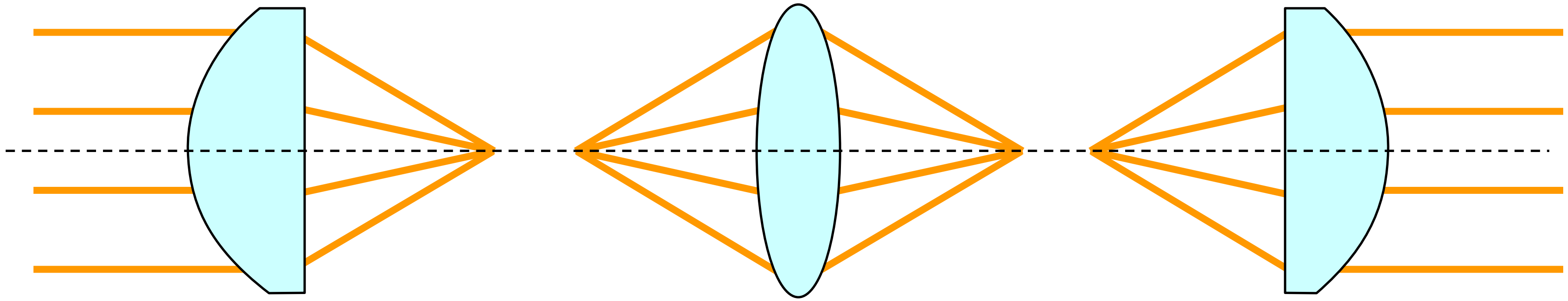
Lens with the best shape

- Shape factor q

$$q = \frac{R_2 + R_1}{R_2 - R_1}$$



- Best shape factor depends on the required magnification

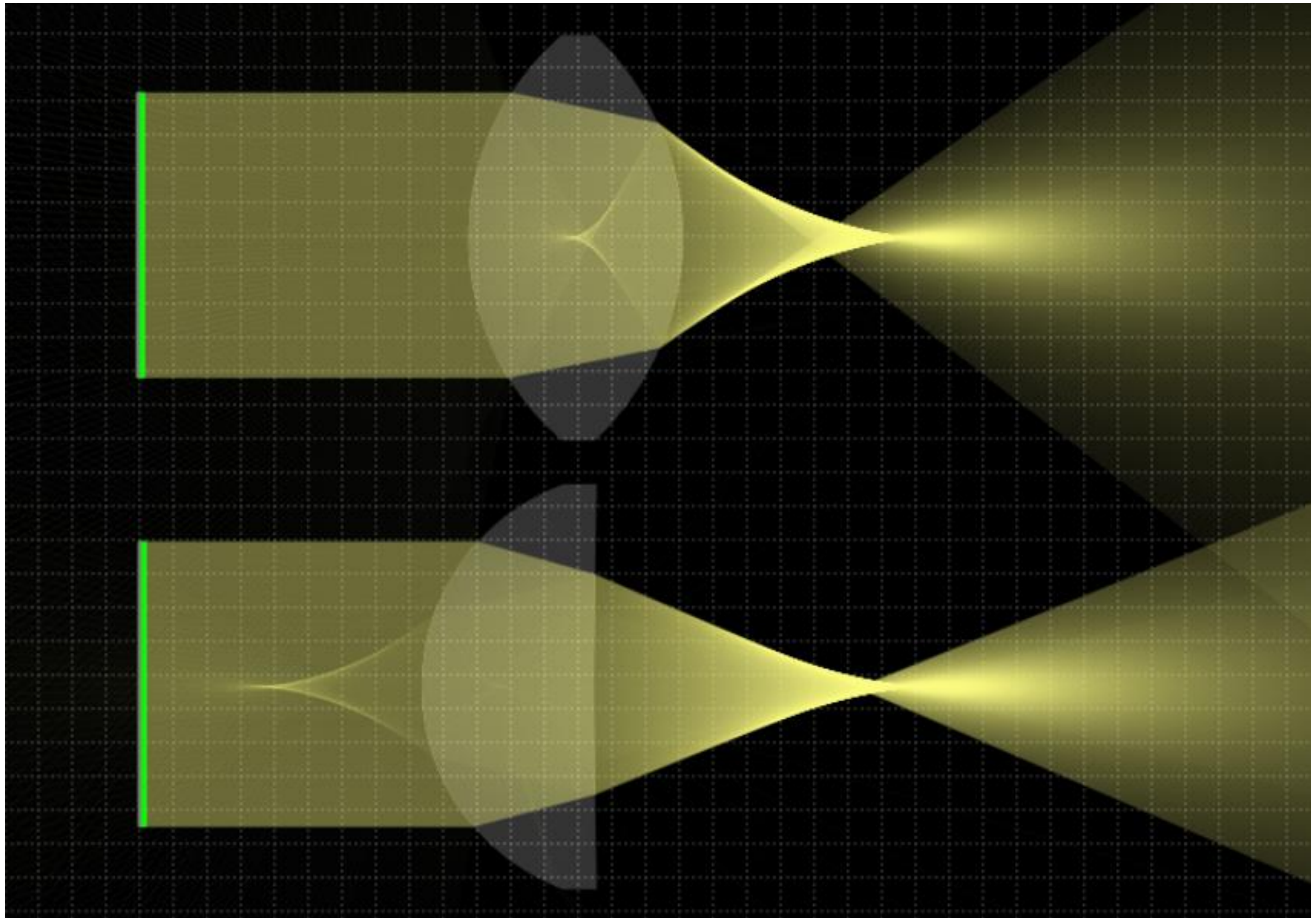


magnification 0
shape factor $q = +1$

magnification 1
shape factor $q = 0$

magnification ∞
shape factor $q = -1$

Lens with the best shape



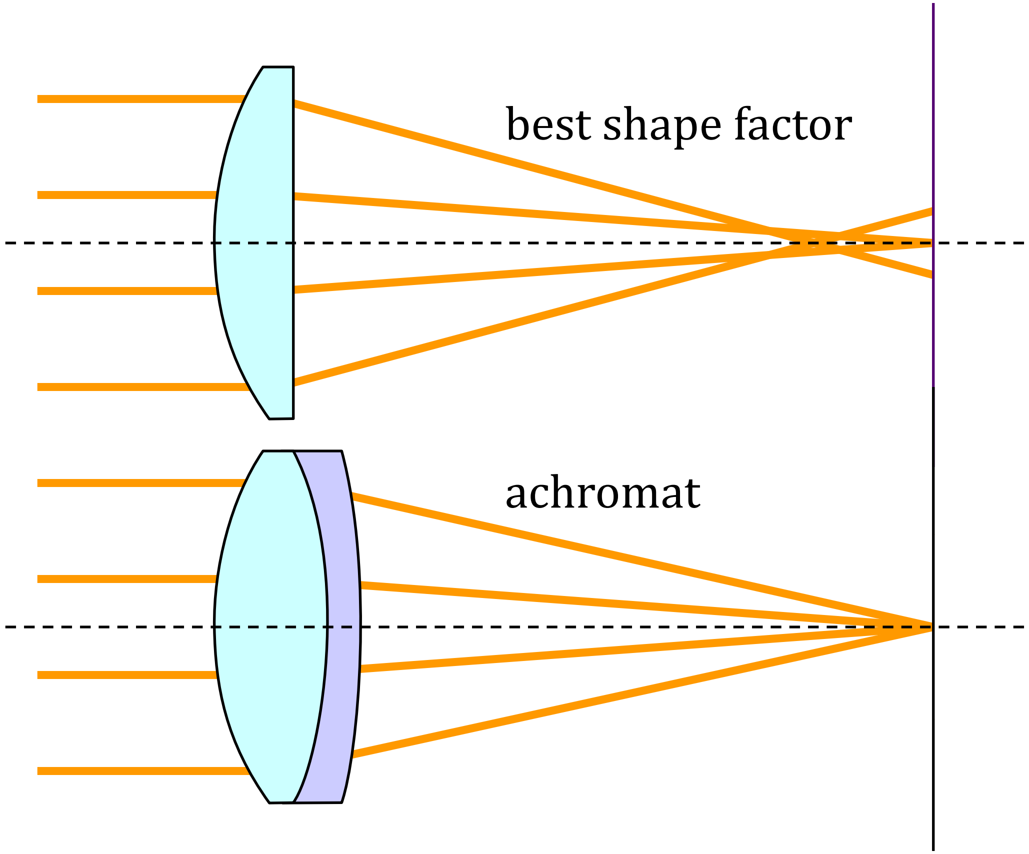
Achromatic doublet

- Achromatic doublet
  - positive lens
  - meniscus with other n

- Spherical aberrations of lenses compensate each other

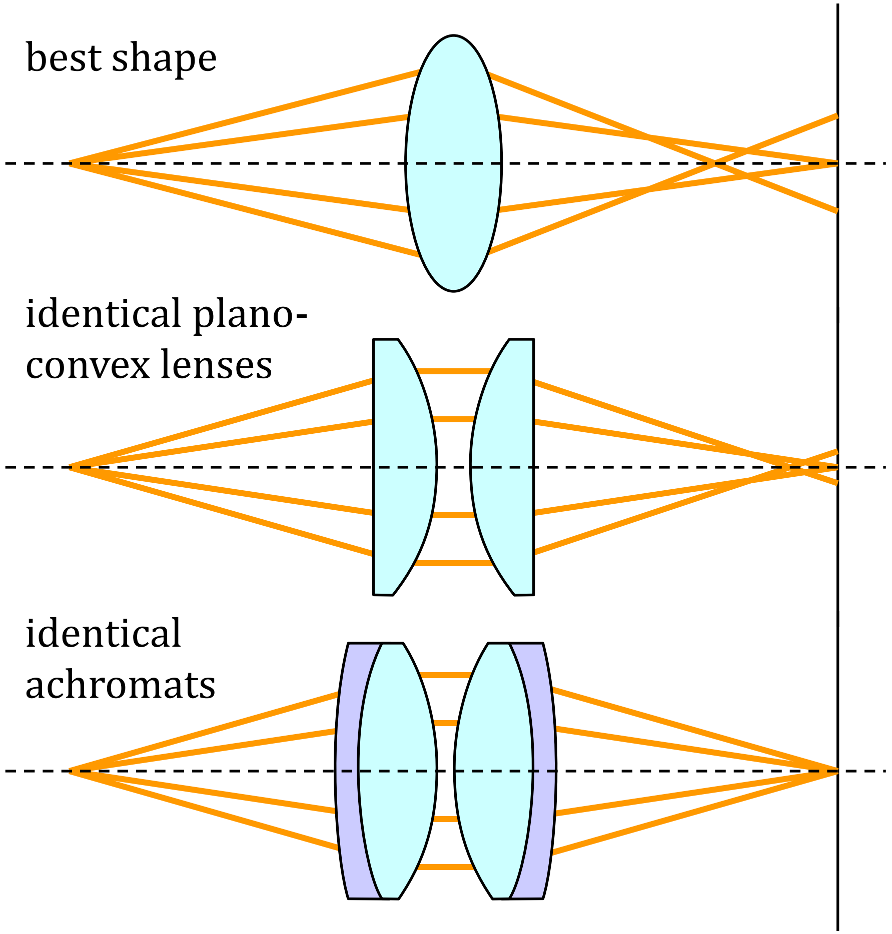
- Chromatic aberration can also be corrected

- Good for magnification of 0 and ∞
  - Parallel beam is incident on the most convex side.



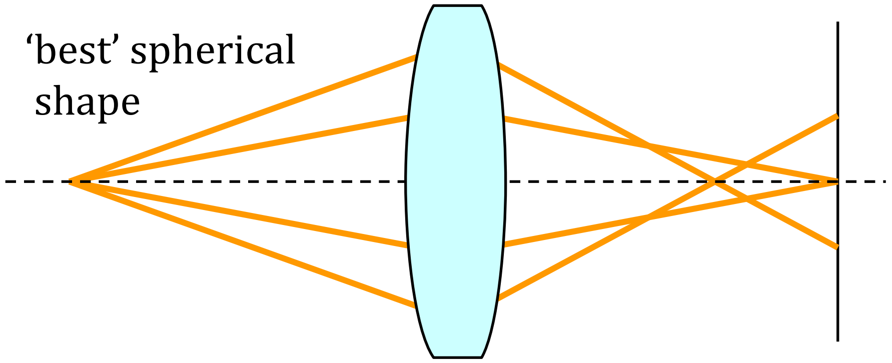
Symmetric doublets

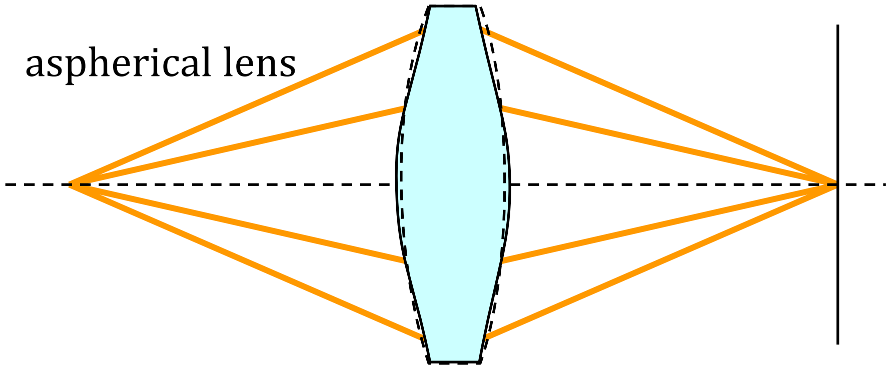
- For replacement of a symmetric double-convex lens
  - magnification 1
- Identical plano-convex lenses:
  - convex sides facing each other
- Identical achromats
  - correction of the chromatic aberration



Aspheric lenses

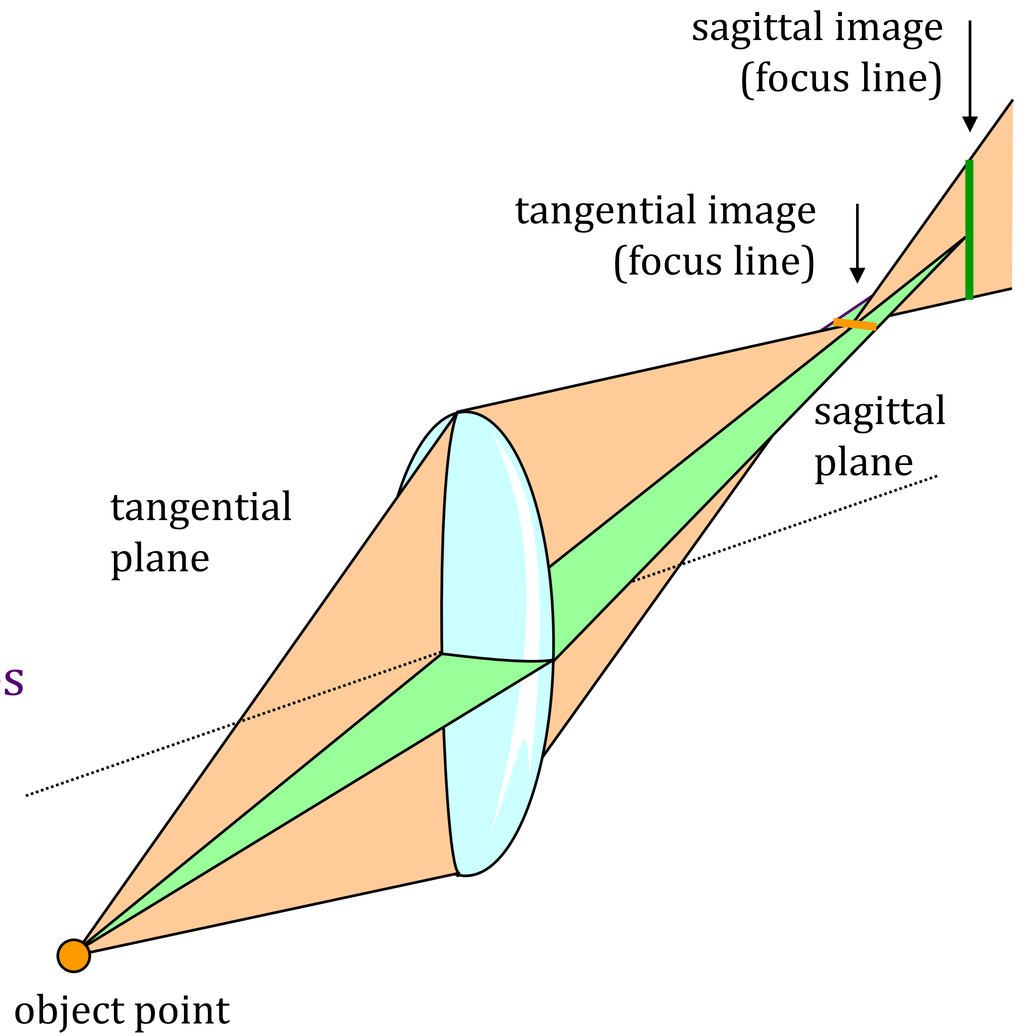
- Spherical aberration can be completely eliminated
  - works only for one specific magnification
- Technologically different
  - poured in a mold instead of polishing
- Used also for mirrors





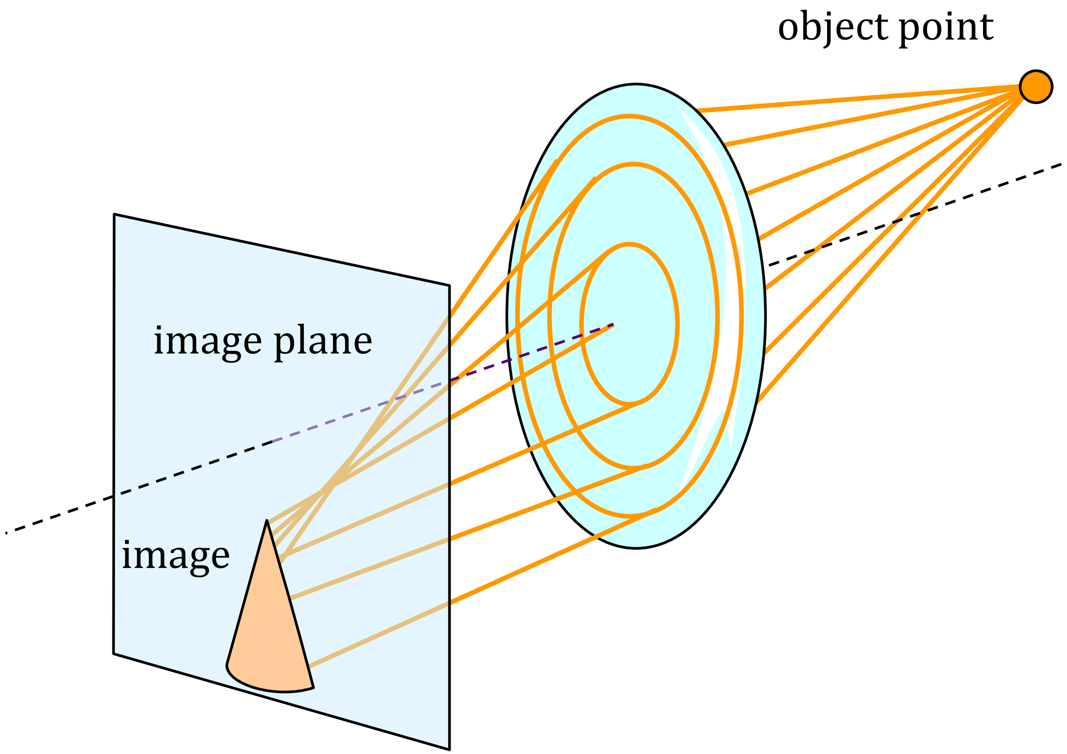
Astigmatism

- Non-meridional ray does not behave as a superposition of the meridional rays
- Tangential plane:
  plane through the object point and optical axis
  - meridional rays
- Sagittal plane:
  plane through both object and image planes perpendicular to the tangential plane



Coma

- Object points located not on the optical axis:
  - rays through the edge of the lens have other lateral magnification
    $\rightarrow$ image point moves
  - meridional rays
    have other magnification
    than sagittal rays
    $\rightarrow$ image point enlarges
- Comet-like image

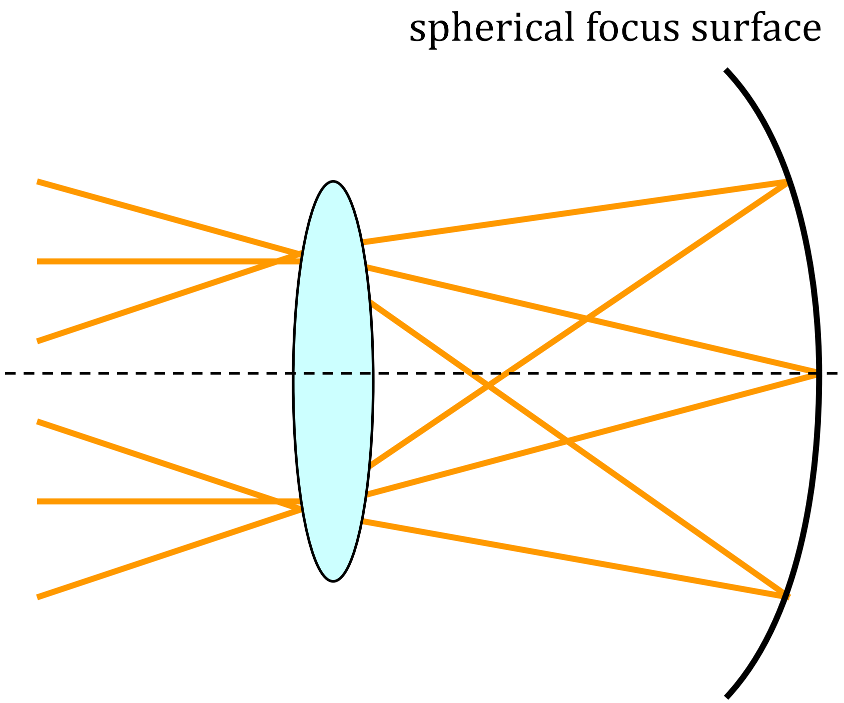


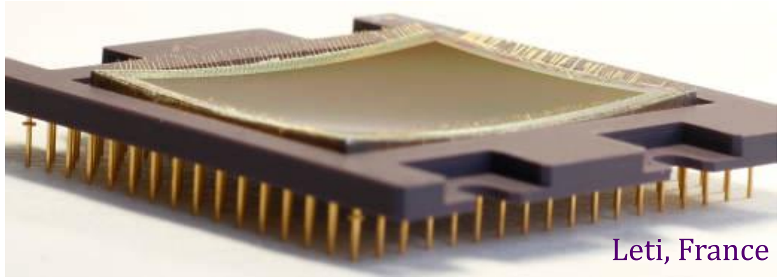
Field curvature

- Stigmatic lens system
  - no spherical aberrations
  - no coma
  - no astigmatism

  - still deviate from paraxial imaging:
    - longitudinal: field curvature
    - lateral: distortion

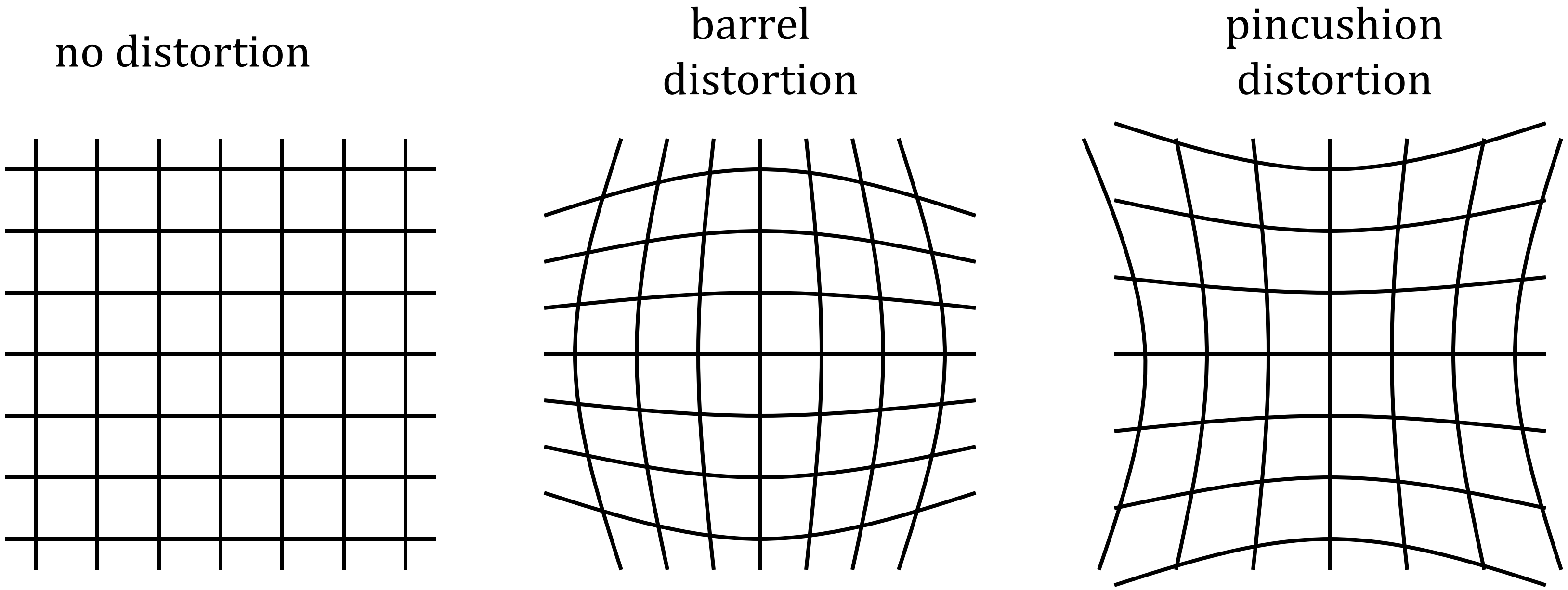
- Flat object is imaged on a curved surface (Petzval surface)





Distortion

- Variation of the lateral magnification of the image:
  - symmetric lens system (1:1 magnification):
    no distortion
  - pincushion or barrel shape

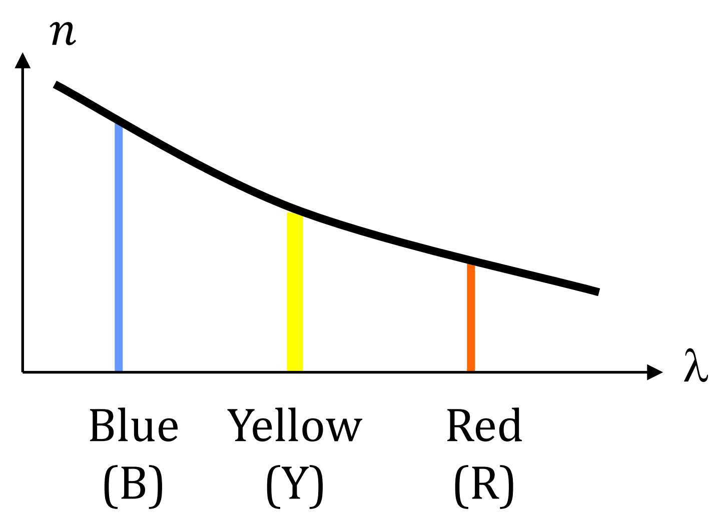


Chromatic aberration (1)

- Refractive index n depends on the wavelength (material dispersion)

- Abbe number V:

$$V = \frac{n_\mathrm{Y} - 1}{n_\mathrm{B} - n_\mathrm{R}}$$

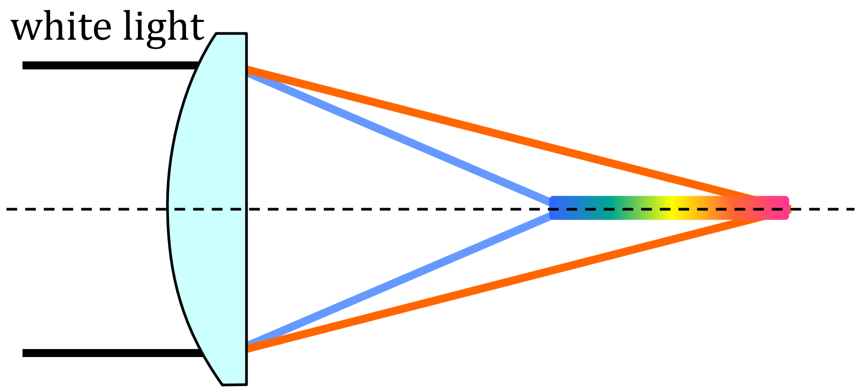


- Glass:
  - stronger refraction for smaller λ
  - $V > 0$

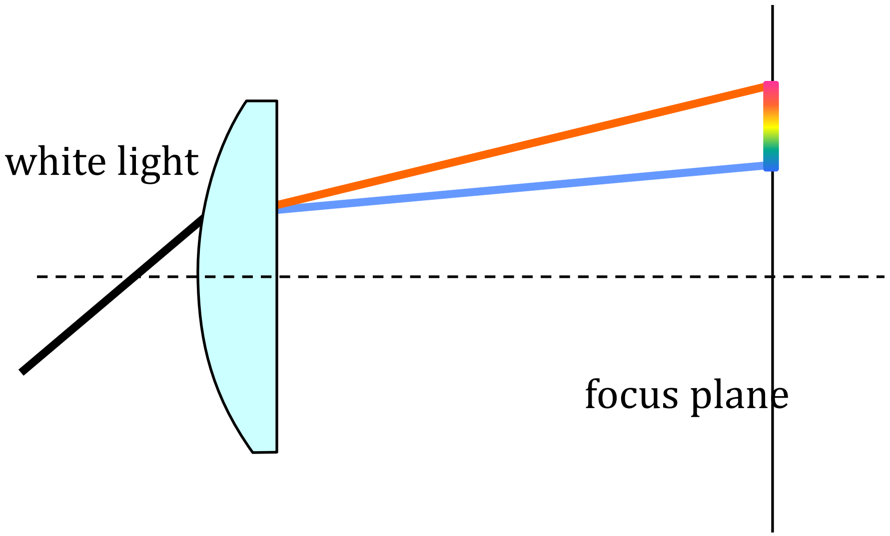
- Compensated with achromatic doublets

Chromatic aberration (2)

- Longitudinal chromatic aberration
  - focus point on the axis depends on the wavelength



- Transversal chromatic aberration
  - lateral magnification depends on the wavelength

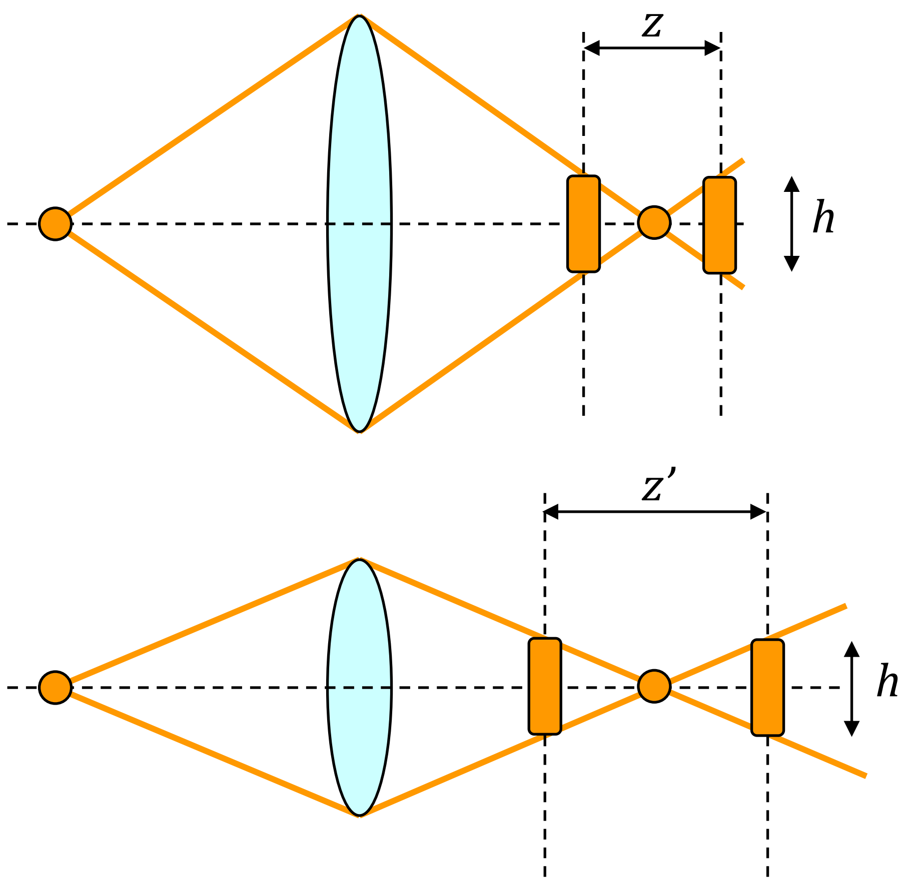


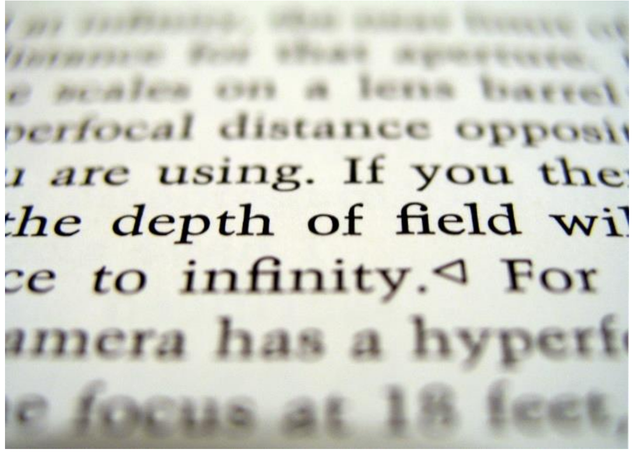
Depth of field

- Lens system:
  sharp image for one object surface

- Depth of field *z*:
  distance through which one may move the image plane to view the object still sharply.

- Larger aperture
  - gathering more light
    (= larger intensity)
  - more aberrations
  - lower focus depth



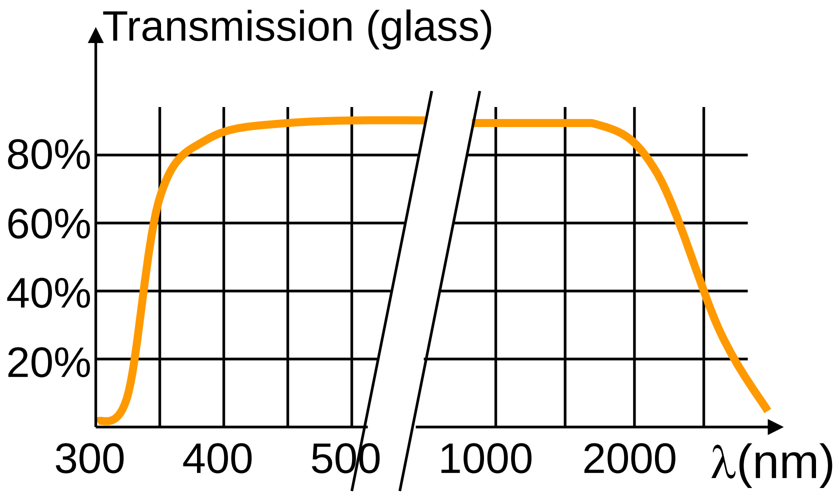


Material properties

- Important material properties
  - refractive index n
    - direction dependence (anisotropy)
    - wavelength dependence (dispersion)
  - absorption
  - hardness
  - uniformity
  - thermal expansion coefficient
  - chemical resistance

Absorption (1)

- Glass:
  - Visible light: low absorption
  - UV: quickly increasing absorption
  - IR: strong absorption from $\lambda = 2 - 3\ \mu\text{m}$
  - $n = 1.4 - 1.8$
- Synthetic quartz
  - Amorphous SiO_2
  - Harder, low thermal expansion
  - Good between $\lambda = 200\ \text{nm}$ and $\lambda = 3.5\ \mu\text{m}$
  - Absorption peaks in IR range
  - $n = 1.46$



Absorption (2)

- Sapphire:
  - Crystalline Al_2O_3
  - Hard, strong, chemically inert
  - Good between $\lambda = 200$ nm and $\lambda = 5$ μm
  - $n = 1.76$

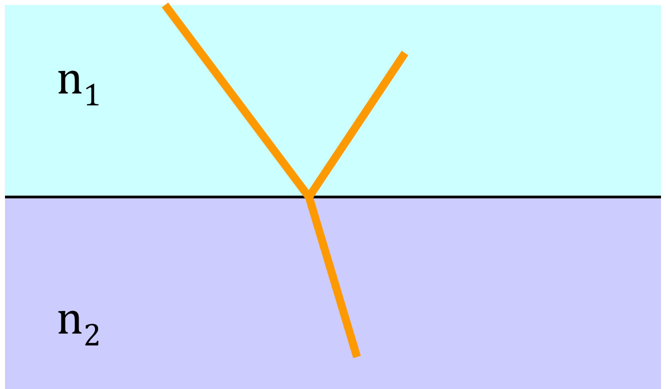
- Semiconductors
  - Si or Ge, mono- or polycrystalline
  - Good for IR: $\lambda = 1$ μm - 5 μm
  - High refractive index: $n > 3$

- Zinc selenide
  - Good for visible and IR
  - $n = 2.5$

Anti-reflection coating

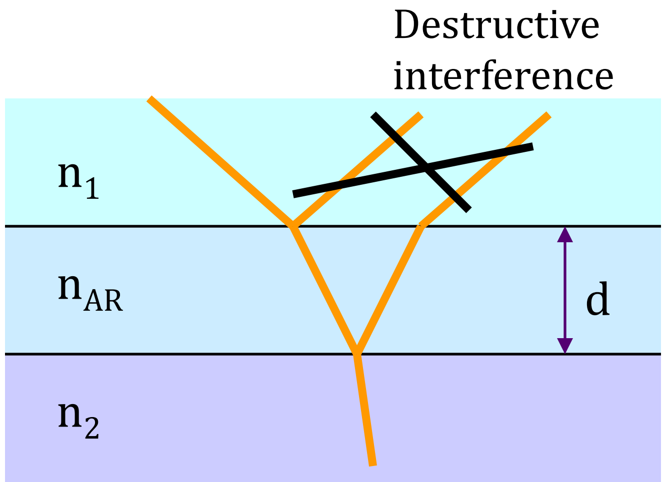
- High refractive index
  - Strong refraction
  - Large reflection

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$



- Anti-reflection coating (1 layer)
  - Thickness $d = \frac{\lambda}{4}$
  - Refractive index $n_{AR} = \sqrt{n_1 n_2}$

- Better results achieved with multiple layers

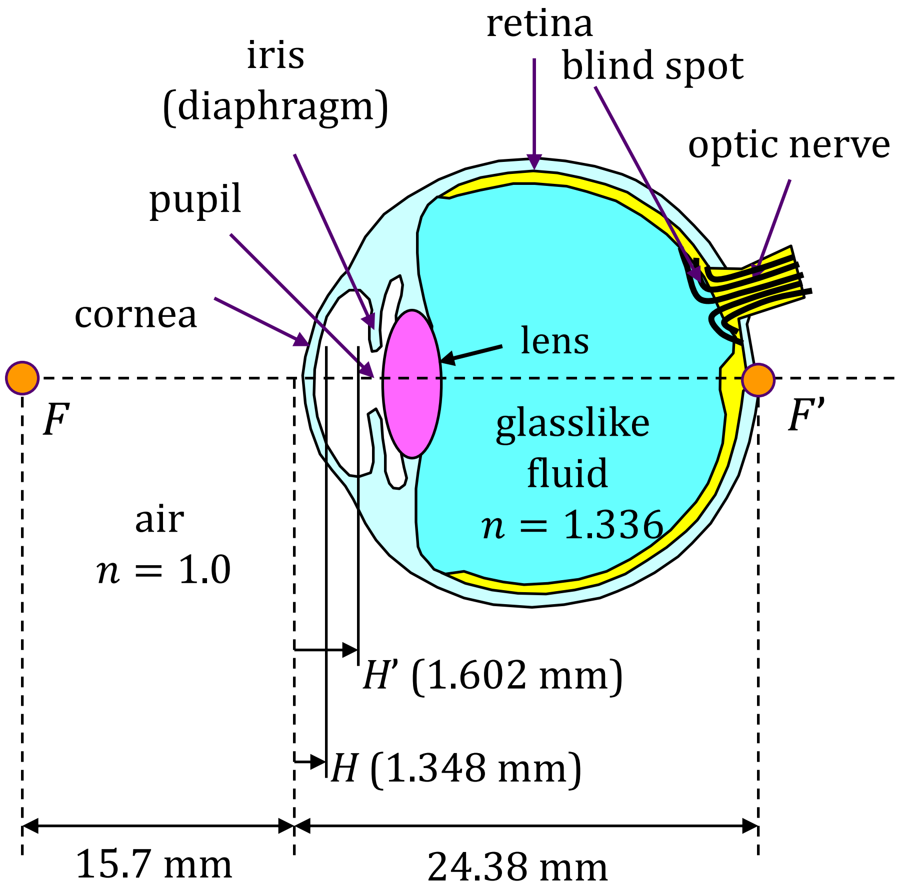


Imaging systems

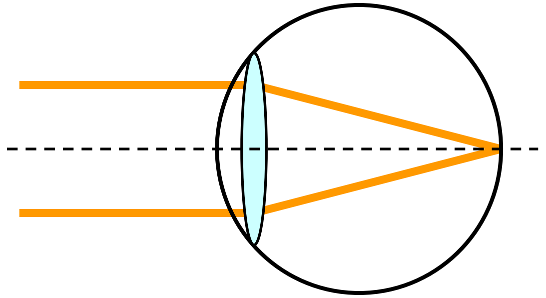
- Theoretical parameters
  - magnification
  - real or virtual image
- Parameters depending on the application
  - constant or variable magnification
  - field of view
  - brightness
  - monochromatic and chromatic aberrations
  - size and shape of the system
  - sensitivity to changes in geometry (resulting from thermal expansion, shocks,...)

The eye

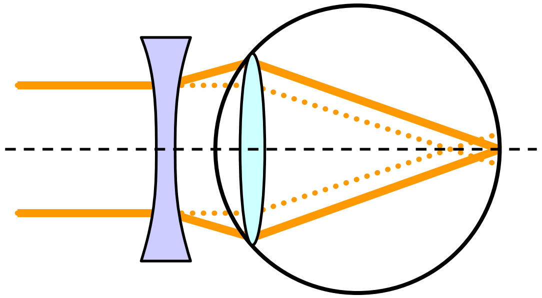
- Refraction
  - Curved cornea ($n = 1.34$)
  - Lens ($n = 1.37 - 1.42$)
  - Refractive power: $P_{\mathrm{eye}} = 58\ \mathrm{m}^{-1}$ (58 diopter)
- Lens
  - Adjustable: extra 10 diopter
  - Adaptive power decreases with age
- Adjustment to intensity
  - Size of the iris
  - Two types of receptors



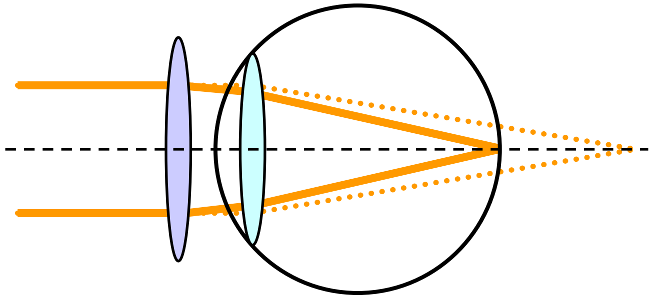
Nearsightedness and farsightedness



- Nearsightedness
  - refraction too strong
  - focusing in front of retina
  - correction with a negative lens



- Farsightedness
  - refraction too weak
  - focusing behind retina
  - correction with a positive lens



Magnifying glass and eyepiece

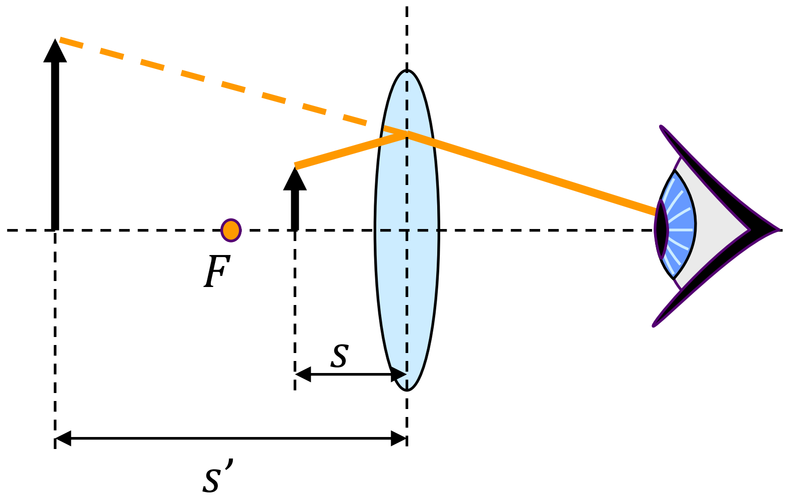
- Eyepiece (ocular):
  - a magnifying glass which is held closely to the eye
  - In optical instruments:
    magnification of a real image

- Object between the focal point and lens system → virtual image

- Magnification depends on object distance s

$$M = -\frac{s'}{s} = \frac{|s'|}{s} = 1 + \frac{|s'|}{f}$$

Not useful definition: says nothing about *perceived* magnification



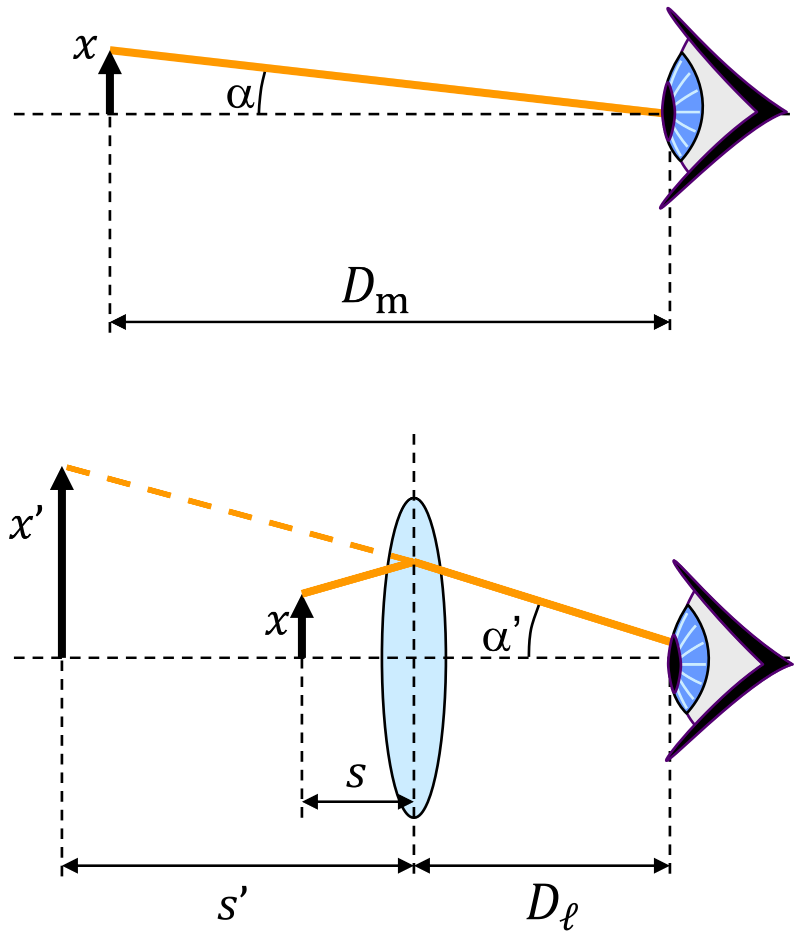
Magnification of an eyepiece (1)

- Visual magnification:
  Relation between α and α’
  - α: Maximum angle of the object without lens
  - α’: Angle for the virtual image
    (eyepiece: lens in front of eye: $D_\ell = 0$)

$$\alpha = \frac{x}{D_\mathrm{m}}$$

$$\alpha' = \frac{x'}{D_\ell + |s'|} = \frac{x\frac{|s'|}{s}}{D_\ell + |s'|} = \frac{x|s'|}{\cancel{D_\ell} + |s'|}\left(\frac{1}{f} + \frac{1}{|s'|}\right)$$

$$M = \frac{\alpha'}{\alpha} = D_\mathrm{m}\left(\frac{1}{f} + \frac{1}{|s'|}\right)$$

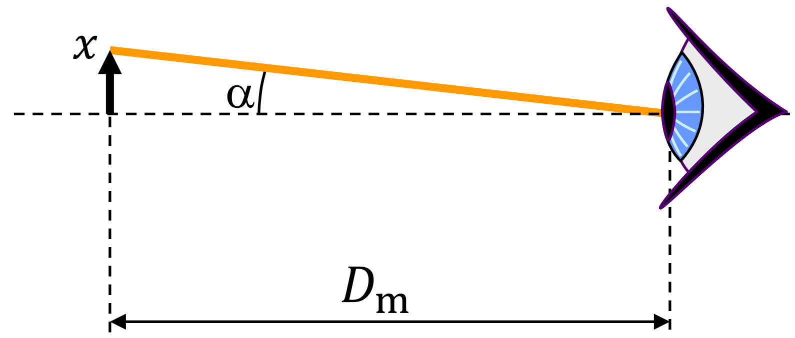


Magnification of an eyepiece (2)

- Magnification M:

$$M = \frac{\alpha'}{\alpha} = D_\mathrm{m}\left(\frac{1}{f} + \frac{1}{|s'|}\right)$$

D_m is standardized at 25 cm



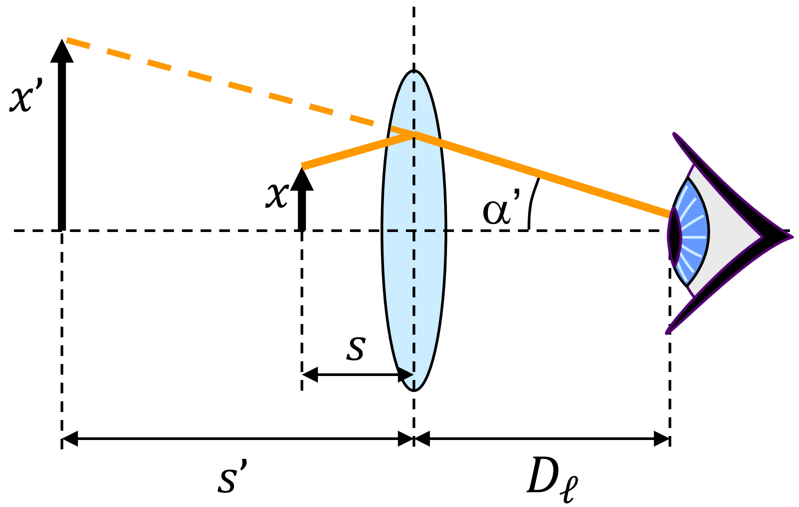
- Selection of the image distance: $|s'| = D_\mathrm{m}$
  - minimal distance:

$$M = \frac{D_\mathrm{m}}{f} + 1 \approx \frac{D_\mathrm{m}}{f}$$

  - infinite distance: $|s'| = \infty$

$$M = \frac{D_\mathrm{m}}{f} = \frac{25\ \mathrm{cm}}{f}$$

= nominal magnification



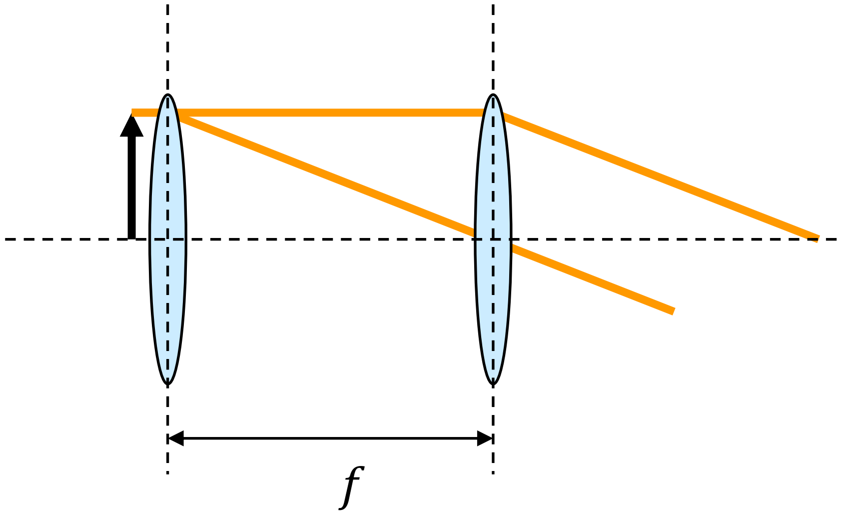
Ramsden eyepiece

- Eyepiece with one lens
  - Very strong chromatic aberration

- Ramsden eyepiece:
  - two identical lenses at a focal length from each other

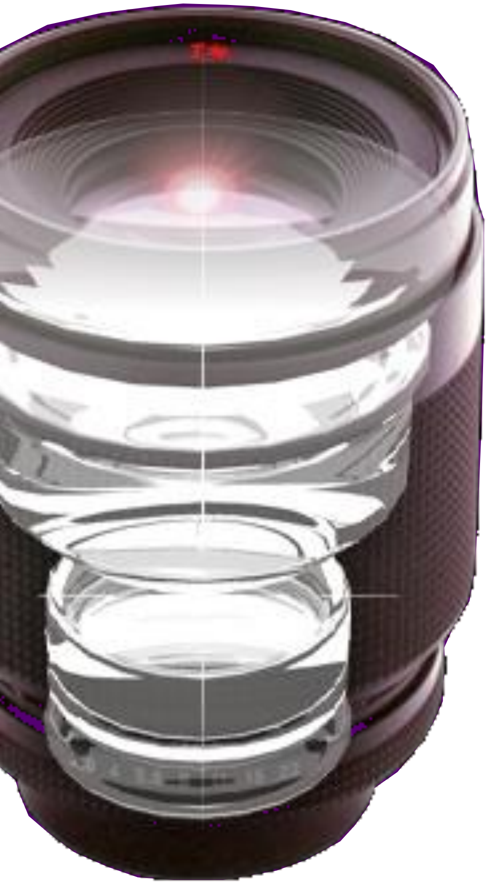
- Two lenses (focal length f_1 and f_2) behave achromatically if distance between them D is an average of f_1 and f_2:

$$D = \frac{f_1 + f_2}{2}$$

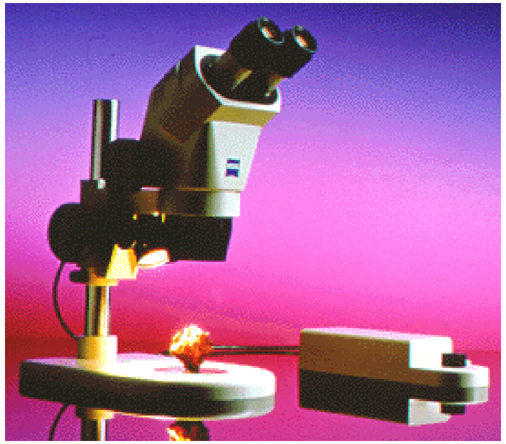


Disadvantage: dust on the first lens is imaged sharply

Objectives



- Create a real inverted image of the object
  - on the film surface
  - or can be viewed by an eyepiece
- Microscope
  - magnification of the object
- Telescope
  - making the object smaller
  - magnification of the angles



Microscope

- Magnification of the objective

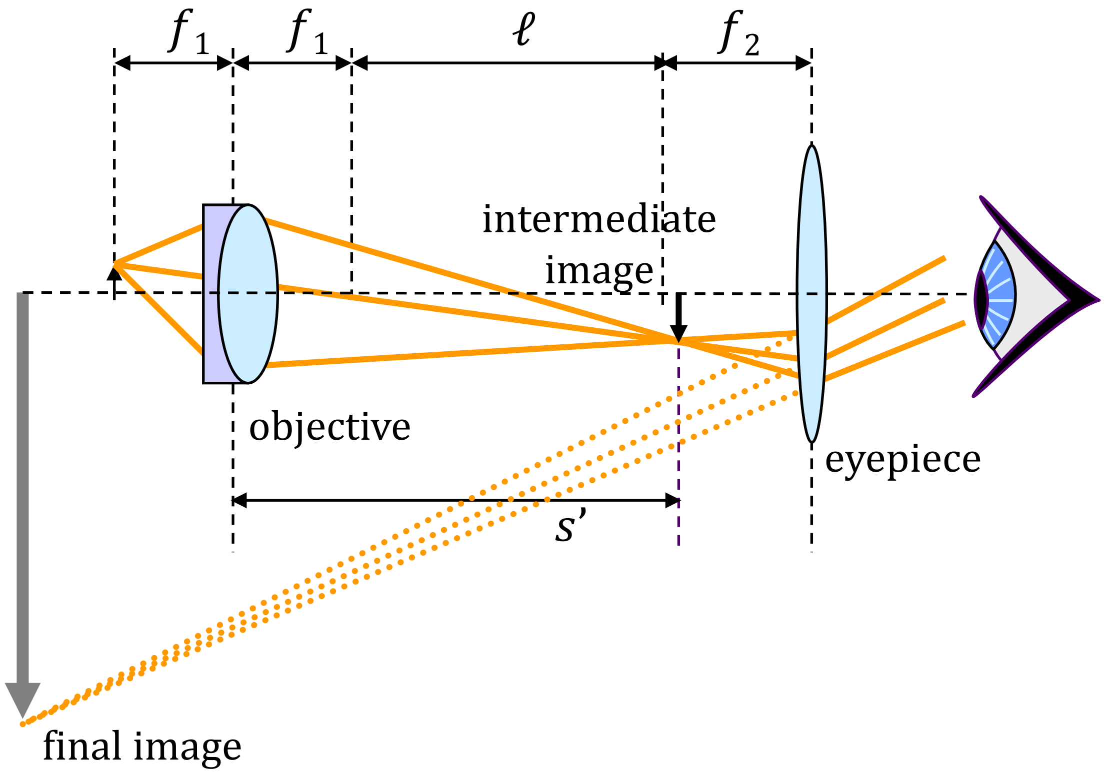
$$M_{\mathrm{ob}} = -\frac{s'}{s} = 1 - \frac{s'}{f_1}$$

  - strong magnification: $s' \gg f_1$
    object is located close to the focal plane
  - standard: $s' = 16$ cm

$$M_{\mathrm{ob}} \approx -\frac{s'}{f_1} = -\frac{16}{f_1}$$

- Total magnification

$$M_{\mathrm{tot}} = M_{\mathrm{ob}} \cdot M_{\mathrm{oc}} = -\frac{16}{f_1} \cdot \frac{25}{f_2}$$



Telescope

- Magnification of the angles:

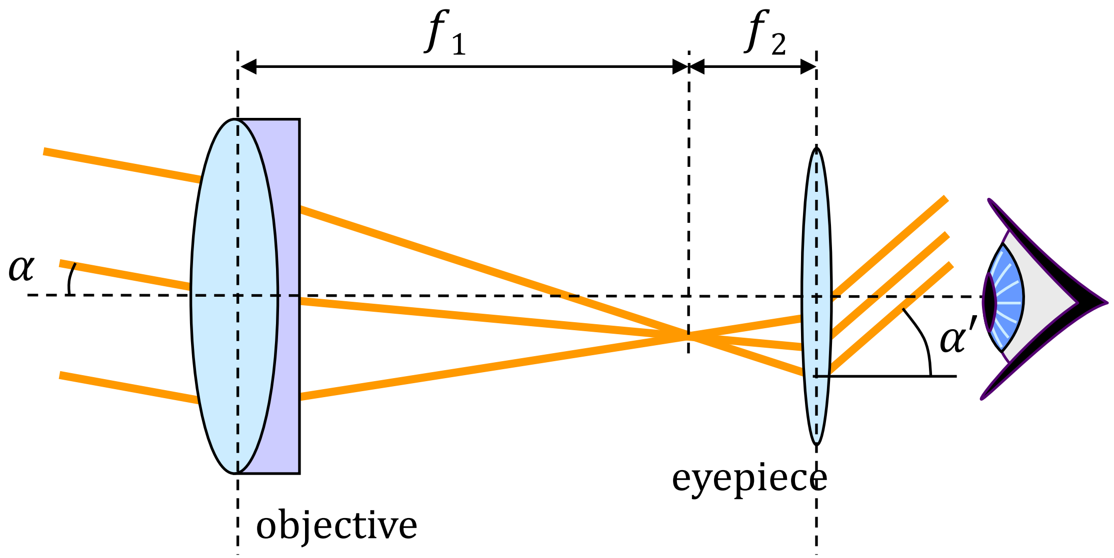
$$M = \frac{\alpha'}{\alpha}$$

- Intermediate image:
  in the common focal plane

- Eyepiece:
  virtual image at a large distance

$$M = \frac{f_1}{f_2}$$

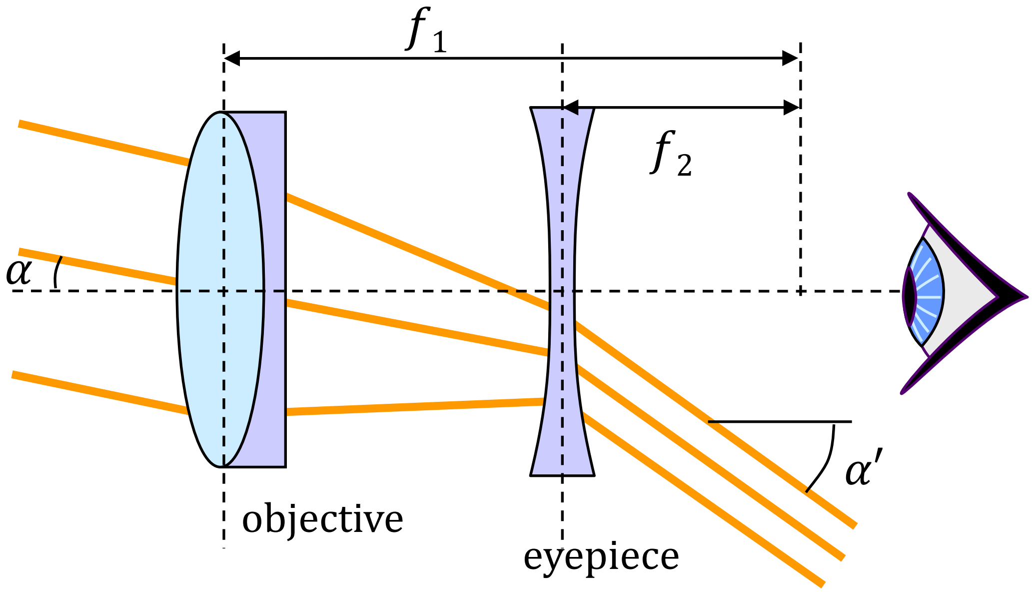
- Total refractive power of zero (angle-angle transformation)



Galilean telescope

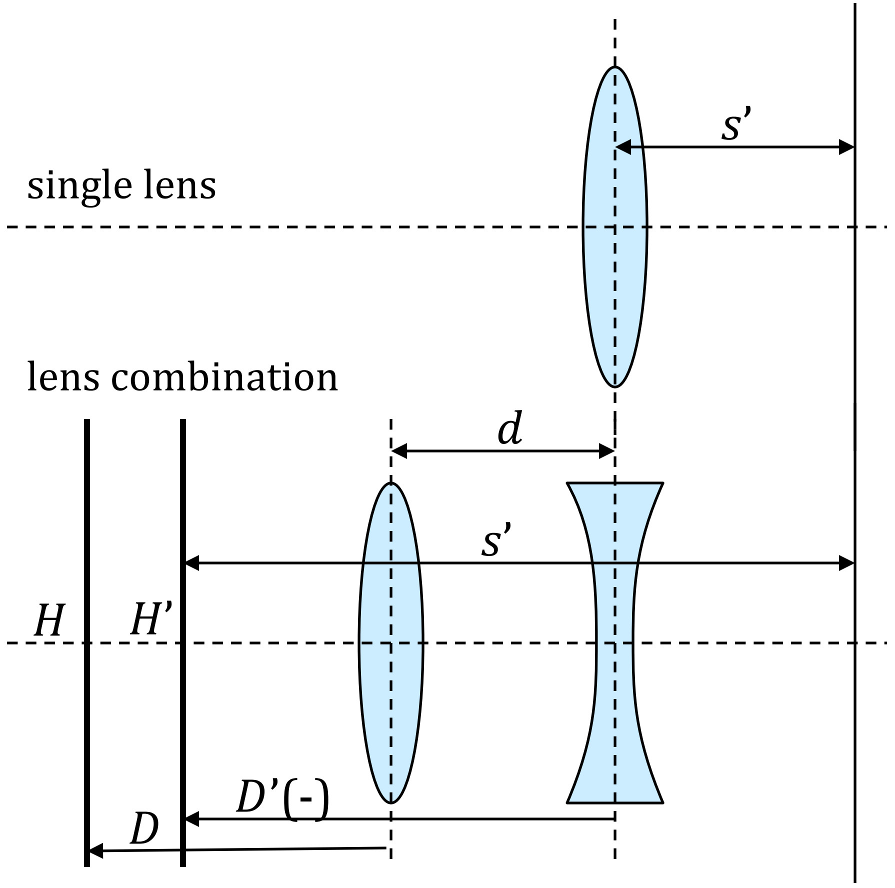
- Regular telescope:
  - inverted image

- Galilean telescope:
  - negative eyepiece
  - eyepiece is located in front of the focal plane of the objective
  - gives positive magnification



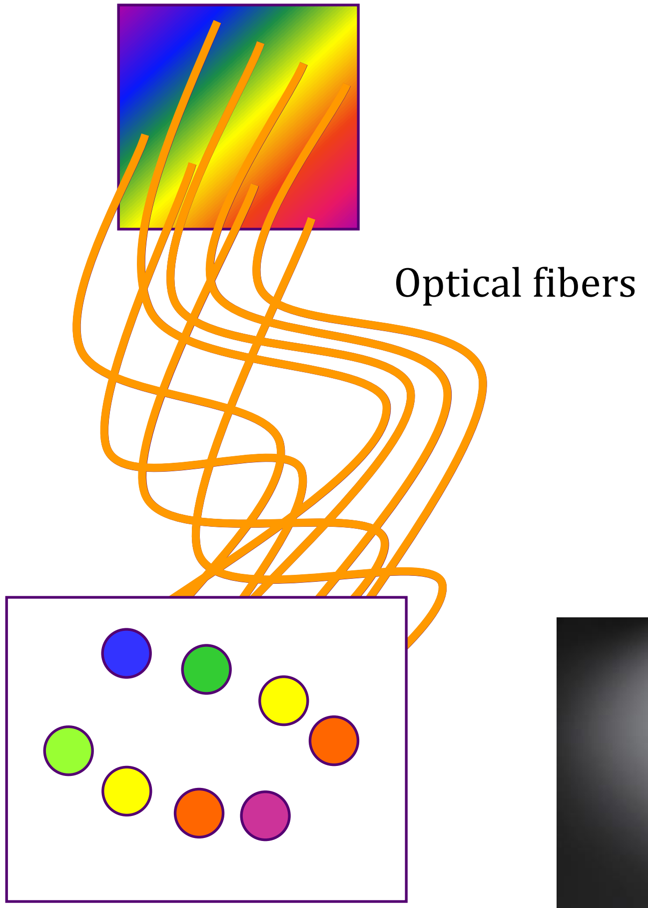
Camera objective

- Similar to the telescope
  - object distance ≫ image distance
  - typical: $f = 50$ mm
- Teleobjective
  - large focal distance → impractical
  - adding more lenses
- Additional lenses:
  - larger focal length
  - short system: principal planes are located in front of the objective



Fiber bundles

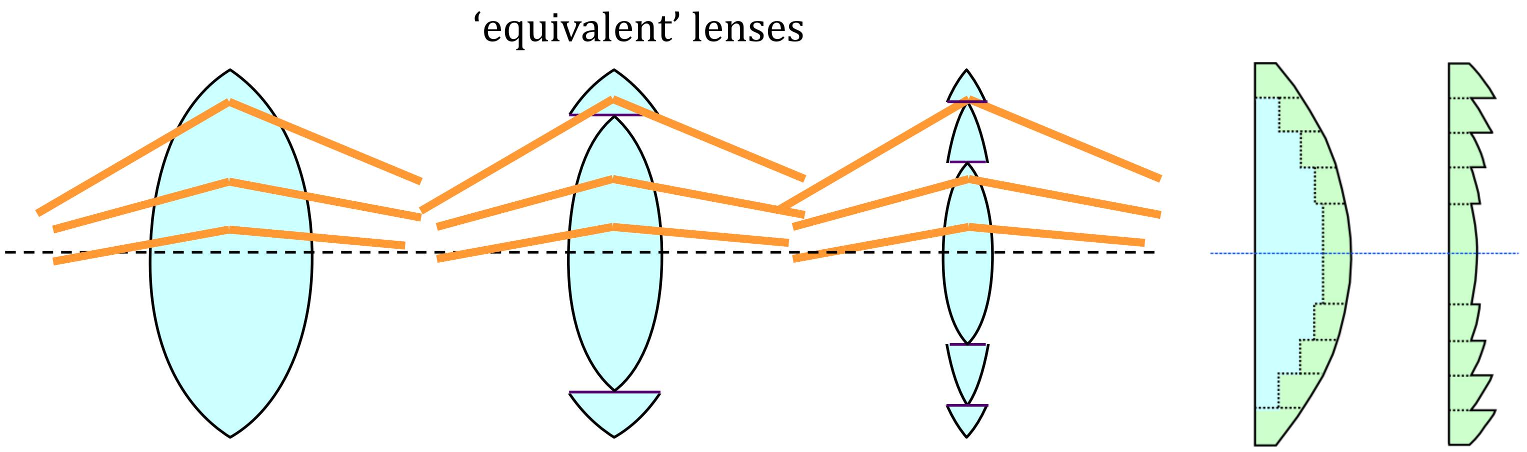
- Challenging circumstances:
  - limited space
  - flexible system
- Limitation in resolutions:
  number of fibers = number of pixels
- Application:
  - Endoscope (medicine)
  - Transformation of a light source shape



Fresnel lenses

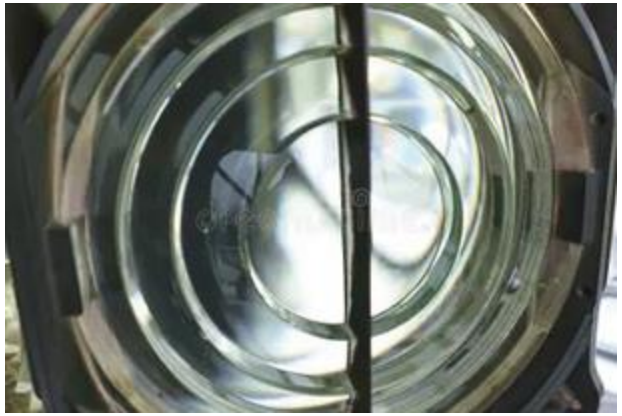
- Refraction:
  only the angle with the surface
  is important

- Fresnel lens:
  - discontinuous surface
  - angle changes continuously



- Application:
  - large lenses (which otherwise would be too thick)
  - concentration of light, where fine imaging is not important

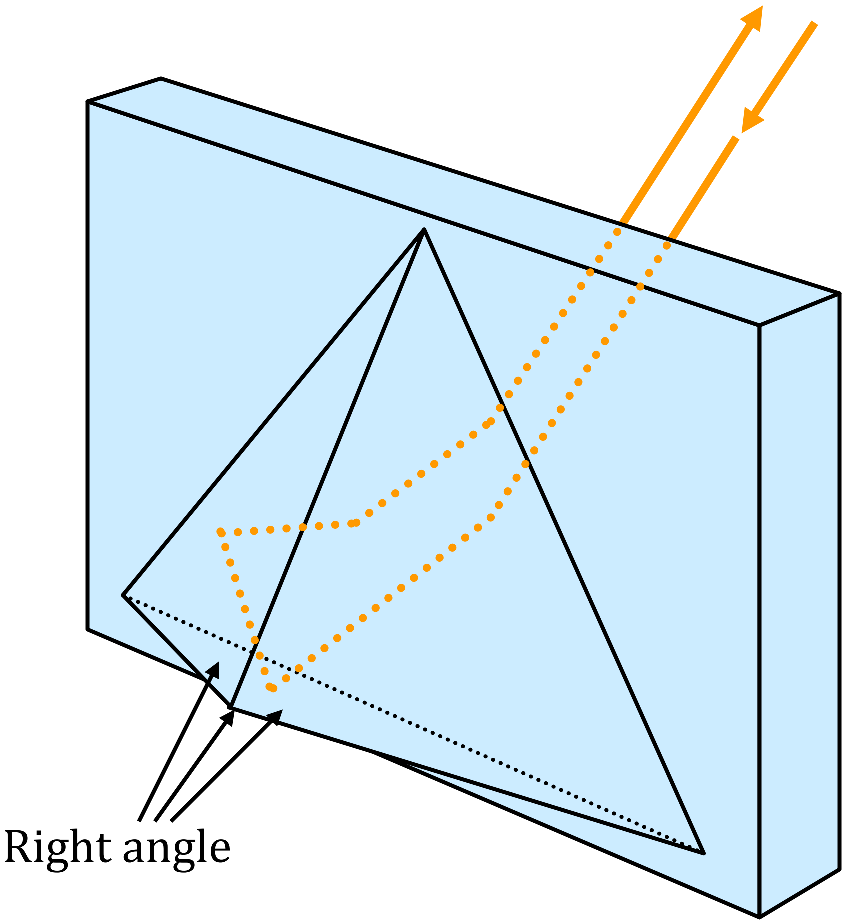
- e. g. light house, car lights, traffic lights, overhead,...



"Cat's eye"

- "Cat's eye" = Corner Cube Reflector
  - Reflects (almost) all light back to the source
  - Three mirrors at right angles (or total internal reflection)
  - Different CCR near each other
  - Phase front is distorted

- Applications:
  - reflectors in traffic
  - distance measurements e.g. Earth-Moon



Distance to the moon

- APOLLO: Apache Point Observatory Lunar Laser-ranging Operation