

Module 6
Lecture: Basics of Power Amplifiers and Mixers

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Where innovation starts

#### **Outline**

- RF specifications
  - Gain
  - Noise
    - Recap: thermal noise, noise figure, constant noise circles
  - Linearity
- Transceiver functions
- Basics of mixers
  - RX case
  - TX case
- Basics of power amplifiers

#### **Learning Objectives**

- Understand RF specifications
- Recap <u>noise</u>, <u>noise figure and constant noise circles</u>
- Be able to explain <u>transceiver functions</u>
- Understand operation of <u>mixers in time and frequency</u> domain for
  - RX case
  - TX case
- Understand <u>principle of switching mixer</u>
- Understand principle of <u>power generation</u>
- Be able to explain <u>power amplifier classes</u>

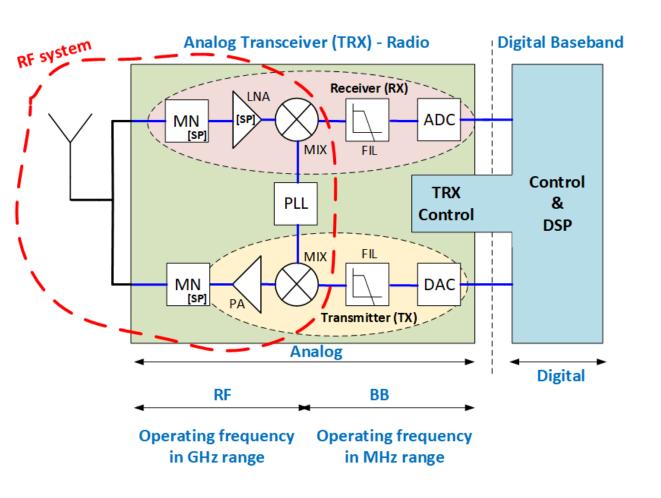
#### Literature

- Slides from this lecture have been based on books:
  - Behzad Razavi RF Microelectronics
    - RF specifications chapter 2
    - Mixers -> chapter 6
    - Power amplifiers -> chapter 12
  - Vojkan Vidojkovic Adaptive Multistandard Front-ends
    - Image rejection, transceiver functions chapter 2

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#### RF systems – transceiver block diagram



#### **TRX functions**

Data conversion
Filtering/selectivity
Frequency conversion
Amplification
Frequency synthesis

#### Legend

RF - radio frequency

BB - baseband

MN – matching netwok

LNA - low noise amplifier

MIX - mixer

FIL - filter

PA – power amplifier

ADC - analog to digital converter

DAC - digital to analog converter

DSP - digital signal processing

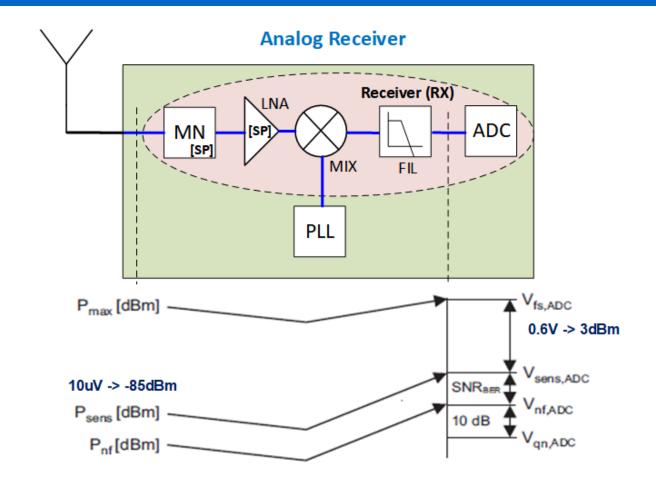
PLL – phased locked loop

#### RF specifications

- Small signal
  - Gain
  - Noise figure (NF)
  - Input third-order intercept point
  - Input second order intercept point
- Large signal
  - 1-dB compression point

Linearity specifications

## Why do we need gain?



- Gain is required for signal conditioning
- Module 4 was dedicated to amplifier gain

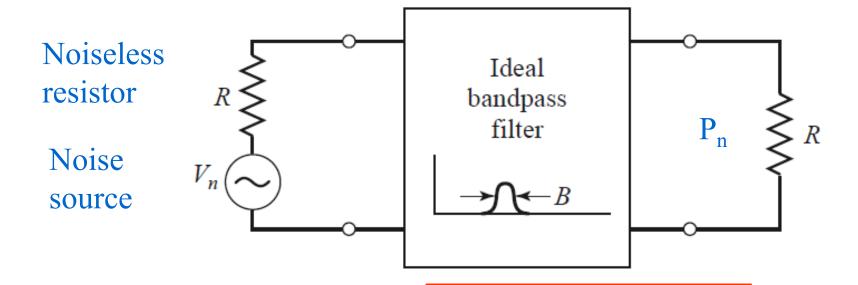
## Why do we need to analyze noise?

- Link budget allows to calculate received signal power S across a wireless link
- To transmit information across a wireless link, the received signal power must be significantly larger than the noise power N.
- The ratio between the signal power and the noise power is called "Signal-to-noise ratio" SNR:

$$SNR = \frac{S}{N}$$

 If we cannot distinguish the signal from the noise we cannot extract the information!

## White noise: Representation of a resistor as a noiseless resistor and a noise voltage source



Available noise power:

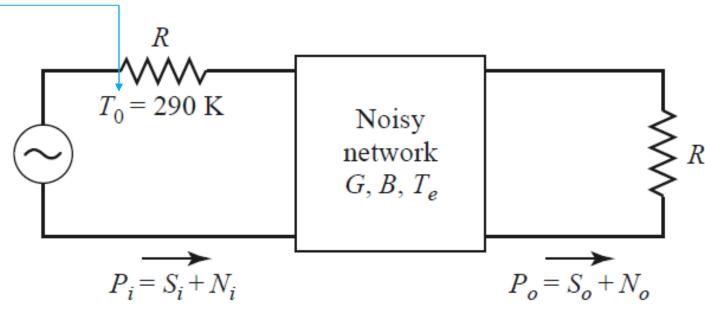
$$P_n = \frac{V_N^2}{4R_N} = k_B T B$$

What is the noise power at room temperature (300 K) for a bandwidth of 1Hz? Calculate it in W as well as in dBm.

$$k_B = 1.38 \cdot 10^{-23} \frac{kgm^2}{s^2 K}$$
  $T(K) = T({}^{o}C) + 273$ 

#### **Definition: Noise figure**

For room temperature input noise level!

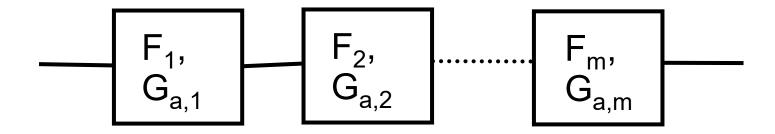


$$F = \frac{\frac{S_{i}}{N_{i}}}{\frac{S_{o}}{N_{o}}} = \frac{S_{i}}{S_{o}} \frac{N_{o}}{N_{i}} = \frac{1}{G} \frac{Gk_{B} (T_{0} + T_{e})B}{k_{B}T_{0}B} = 1 + \frac{T_{e}}{T_{0}} > 1$$

NF = 10 Log (F) 
$$\begin{cases} NF -> \text{ noise figure (dB)} \\ F -> \text{ noise factor} \end{cases}$$



#### Cascaded NF: Friis' formula



System with cascaded sub-systems with noise figure  $F_{m}$  and available gain  $G_{a,m}$ 

$$F_{total} = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_{a,1}} + \dots + \frac{F_m - 1}{G_{a,1}G_{a,2}\dots G_{a,(m-1)}}$$

## Noise figure of an amplifier (1)

- Noise figure of a 2-port amplifier: Normalized equivalent noise resistor:

$$r_n = \frac{R_n}{Z_0}$$

$$F = F_{\min} + \frac{r_N}{g_S} \left| \underline{y}_S - \underline{y}_{opt} \right|^2$$

Source admittance:  $\underline{Y}_S = g_S + jb_S$ 

Minimum noise figure for the chosen bias point:  $F_{\min} = \min(F)$ 

- Expression with the reflection coefficients  $\Gamma_{S}$  and  $\Gamma_{opt}$ 

Offset to optimum value

Scaling factor "sensitivity to offest"

$$F = F_{\min} + 4r_{N} \frac{\left|\underline{\Gamma}_{S} - \underline{\Gamma}_{opt}\right|^{2}}{\left(1 - \left|\underline{\Gamma}_{S}\right|^{2}\right) \cdot \left|1 + \underline{\Gamma}_{opt}\right|^{2}}$$



#### **Constant noise circles**

Centers: 
$$\underline{C}_F = \frac{\Gamma_{opt}}{1+N}$$

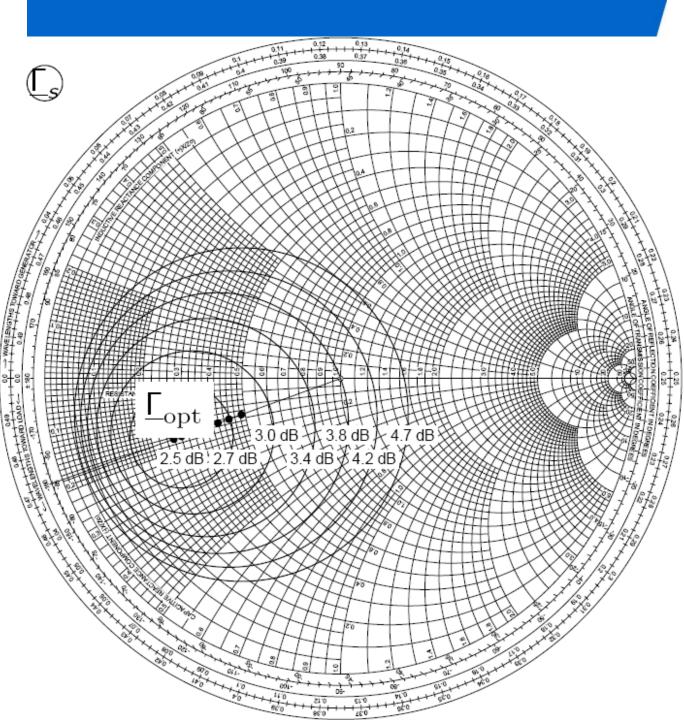
Radii: 
$$R_{F} = \frac{1}{1+N} \sqrt{N^2 + N(1 - \left|\Gamma_{opt}\right|^2)}$$

With the "Noise figure parameter N" defined as:

$$\Delta F_n' = N = \left(F - F_{\min}\right) \frac{\left|1 + \underline{\Gamma}_{opt}\right|^2}{4r_n} = \frac{\left|\underline{\Gamma}_S - \underline{\Gamma}_{opt}\right|^2}{1 - \left|\underline{\Gamma}_S\right|^2}$$



Constant noisecircles in the source-reflectioncoefficient Smith Chart



#### Design for specific noise figure

Typically the values of  $\Gamma_{opt}$ ,  $r_n$  and  $F_{min}$  are known for the transistor.

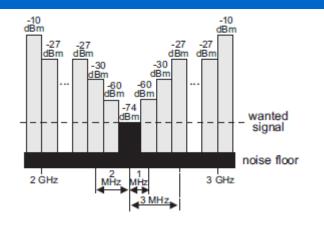
The amplifier specification requires a noise figure F and a gain G.

#### **Procedure:**

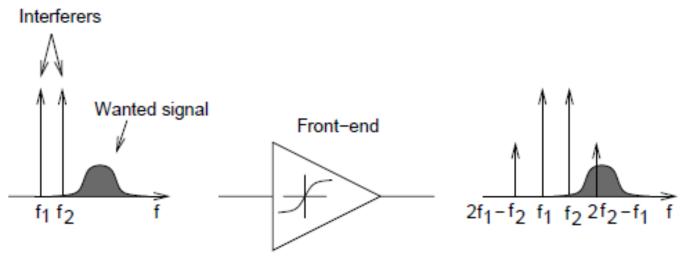
- 1. Calculate N
- 2. Calculate C<sub>F</sub> and R<sub>F</sub>
- Draw the constant noise circle for the required F in the Smith chart as well as the input section constant gain circle for several G<sub>S</sub>
- 4. Choose a value for  $\Gamma_S$  that is on the desired noise circle and a certain gain circle
- 5. The remaining gain must come from the transistor and the output matching stage



## Why do we need to analyse linearity?



- Typical scenario in wireless communications
- Very weak wanted signal in vicinity of strong interferers



SNR degradation due to nonlinearity



#### Effect of nonlinear devices on the signal

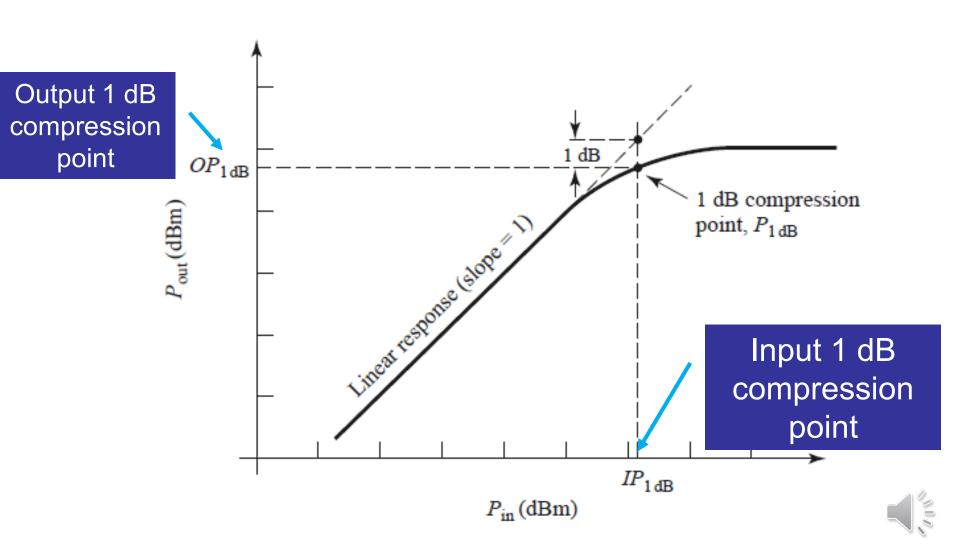
#### E.g. transistors, diodes



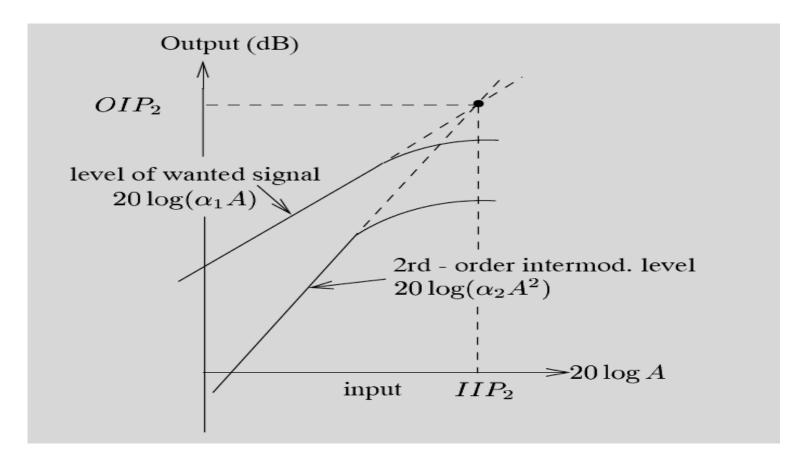
$$v_{out}(t) = v_{out,DC} + a_1 v_{in}(t) + a_2 v_{in}^2(t) + a_3 v_{in}^3(t) + \dots$$

- Harmonic generation (multiples of a fundamental signal)
- Saturation (gain reduction in an amplifier)
- Intermodulation distortion (products of a two-tone input signal)
- Cross-modulation (modulation transfer from one signal to another)
- AM-PM conversion (amplitude variation causes phase shift)
- Spectral regrowth (intermodulation with many closely spaced signals)

## 1 dB compression point



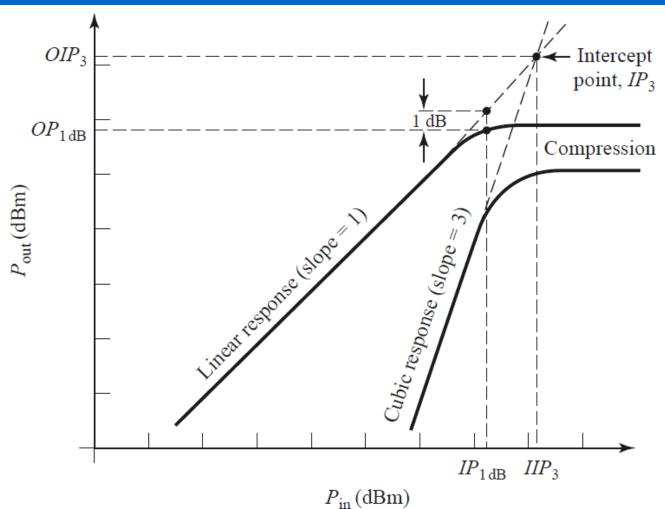
#### Second order intercept point: IP2



IIP2: input power where wanted power = second order power (extrapolated point).



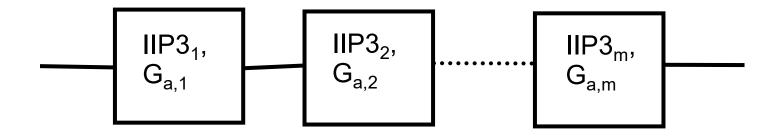
#### Third order intercept point



IIP3: input power where wanted power = the third order power (extrapolated point).



#### Cascaded IIP3 formula



System with cascaded sub-systems with input IIP3 IIP3<sub>m</sub> and available gain G<sub>a,m</sub>

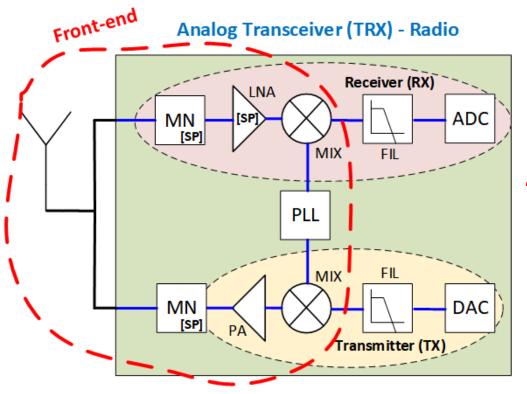
$$\frac{1}{\text{IIP3}_{\text{total}}} = \frac{1}{\text{IIP3}_{1}} + \frac{G_{a,1}}{\text{IIP3}_{2}} + \dots + \frac{G_{a,1}G_{a,2}...G_{a,(m-1)}}{\text{IIP3}_{m}}$$



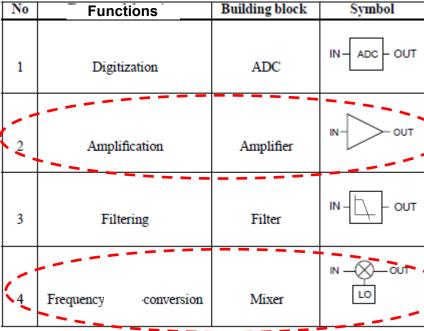
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#### **Transceiver functions**



#### Analog transceiver functions





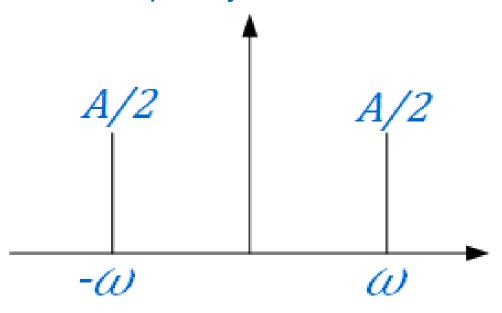
#### Spectra of cosine wave

Time domain

$$x(t) = A\cos(\omega t)$$

$$x(t) = \frac{A}{2}(e^{j\omega t} + e^{-j\omega t})$$

Frequency domain

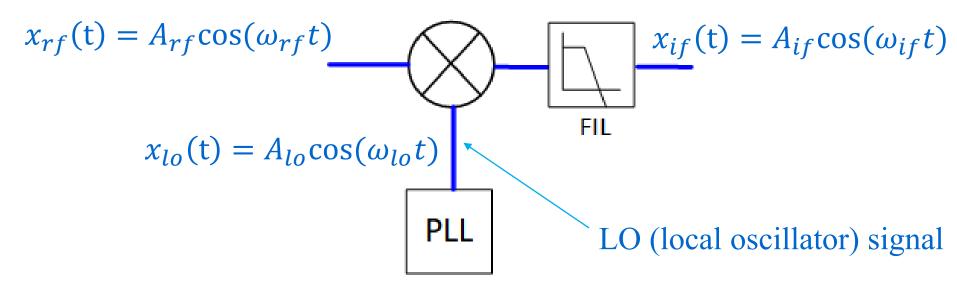




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#### Mixing function in time domain – RX case



$$x_{if}(t) = A_{rf}\cos(\omega_{rf}t) A_{lo}\cos(\omega_{lo}t)$$

$$x_{if}(t) = \frac{1}{2} A_{rf} A_{lo} \left(\cos\left(\left(\omega_{rf} + \omega_{lo}\right)t\right) + \cos\left(\left(\omega_{rf} - \omega_{lo}\right)t\right)\right)$$

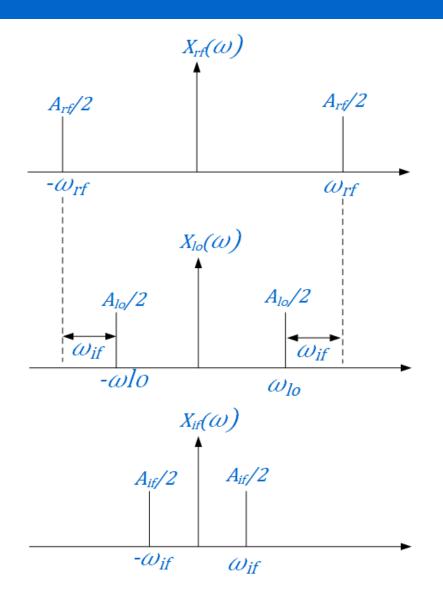
$$A_{if} = \frac{1}{2} A_{rf} A \square lo$$

$$\omega_{if} = \omega_{rf} - \omega_{lo}$$

Due to low-pass filter



#### Mixing function in frequency domain – RX case



#### Time domain

$$x_{if}(t) = x_{rf}(t) x_{lo}(t)$$
multiplication

#### Frequency domain

$$X_{if}(\omega) = X_{rf}(\omega) * X_{lo}(\omega)$$
convolution



#### Image problem – time domain (info only)

$$x_{rf}(t) = 2A_w \cos(\omega_w t) + 2A_i \cos(\omega_i t)$$
 (2.7)

 $\underline{A_w}$  and  $\underline{A_i}$  are half of the amplitudes of the <u>wanted</u> and <u>unwanted</u> signal, respectively. The sinusoidal LO signal can be expressed as:

$$x_{lo}(t) = 2A_{lo}\cos(\omega_{lo}t) \tag{2.8}$$

The signal at the mixer output (x(t)) is obtained by multiplying the signals  $x_{rf}(t)$  and  $x_{lo}(t)$ :

$$x(t) = x_{rf} \cdot x_{lo} \tag{2.9}$$

Substituting (2.7) and (2.8) into (2.9), x(t) can be expressed as:

$$x(t) = 2[A_w A_{lo} \cos((\omega_w + \omega_{lo})t) + A_w A_{lo} \cos((\omega_w - \omega_{lo})t)] + 2[(A_i A_{lo} \cos((\omega_i + \omega_{lo})t) + A_i A_{lo} \cos((\omega_i - \omega_{lo})t)]$$
(2.10)

The high frequency components that are located at the frequencies  $\omega_w + \omega_{lo}$  and  $\omega_i + \omega_{lo}$  have to be filtered out. After the filtering, the down-converted signal becomes:

$$x_{if}(t) = 2A_{wd}\cos((\omega_w - \omega_{lo})t) + 2A_{id}\cos((\omega_i - \omega_{lo})t)$$
 (2.11)

 $A_{wd}$  ( $A_{wd} = A_w A_{lo}$ ) and  $A_{id}$  ( $A_{id} = A_i A_{lo}$ ) are half of the amplitudes of the down-converted wanted and unwanted signals, respectively. In the case that

$$\omega_i \le \omega_{lo} \le \omega_w$$
 (2.12)

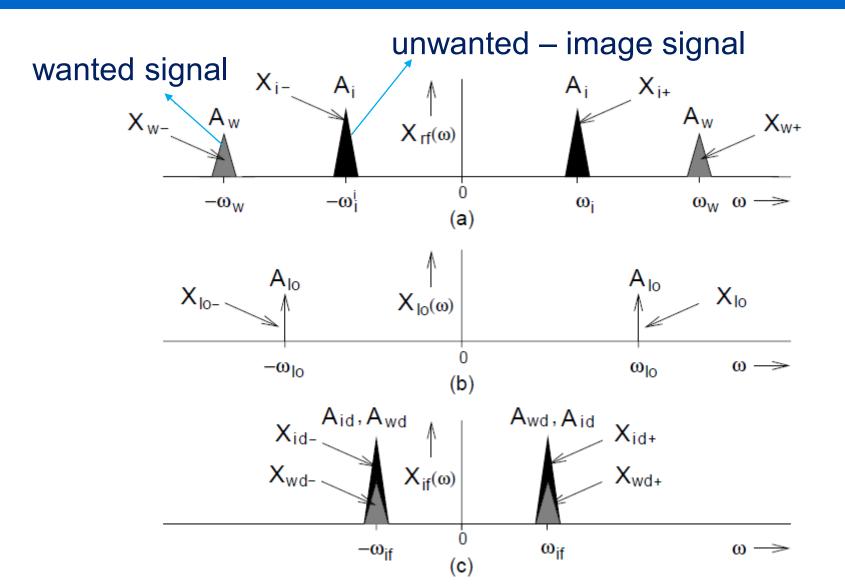
and

$$\omega_{lo} - \omega_i = \omega_w - \omega_{lo} \tag{2.13}$$

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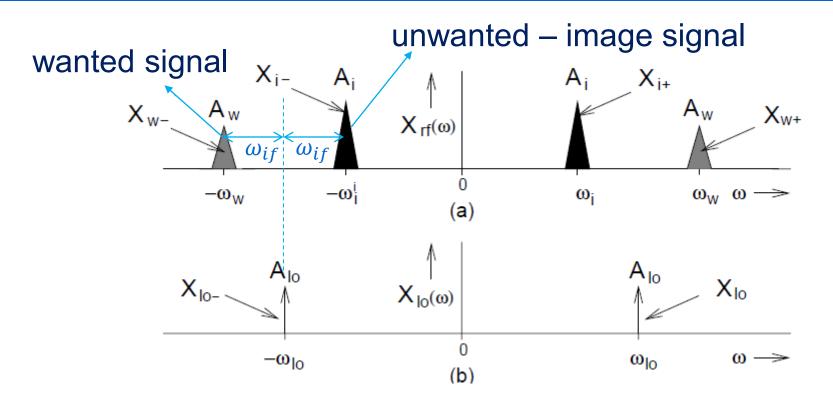
the unwanted signal is down-converted to the same IF as the wanted signal.

#### Image problem – frequency domain (info only)



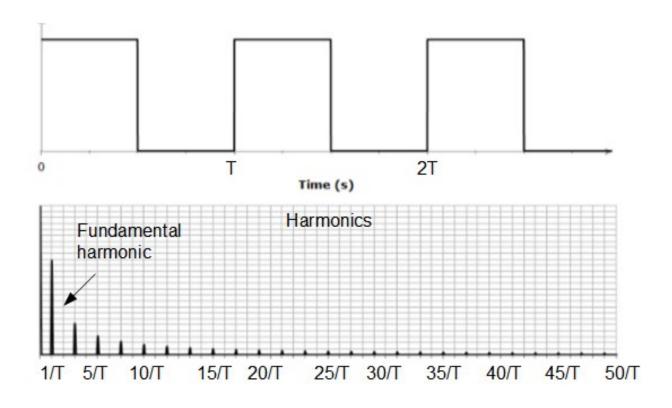
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#### Image problem – summary (info only)



- Image signal is unwanted signal
- Image signal is located at frequency  $\omega_{lo}-\omega_{if}$
- Image signal is down-converted to the same  $\omega_{if}$  as wanted signal
- Image signal degrades SNR

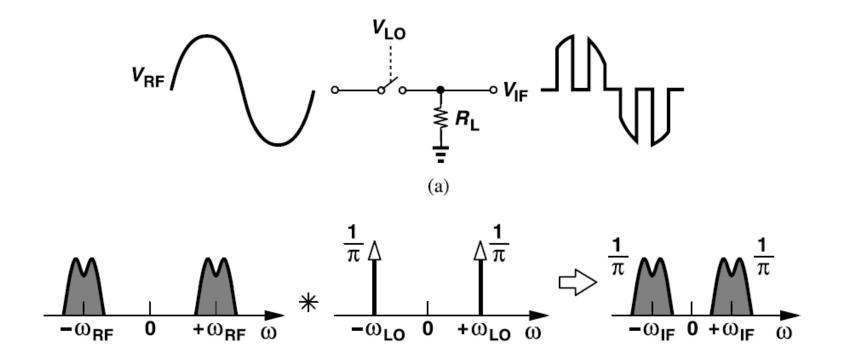
## Spectrum of square wave (info only)



Fundamental harmonic can be used for mixing as LO

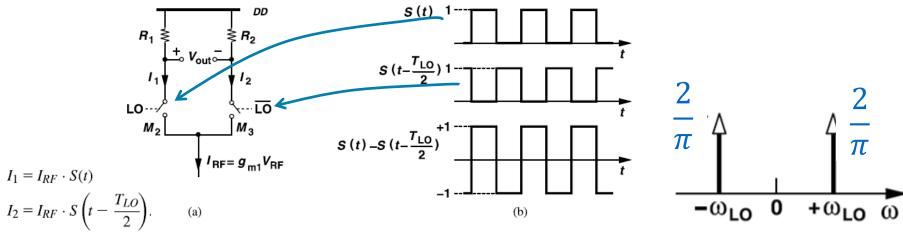


# Implementation of mixing by switching (info only)



 Mixing can be implemented by real multiplication, but using switching is the most common way how mixing is implemented in practice

#### Gilbert cell mixer – operating principle (info only)



**Figure 6.44** (a) Equivalent circuit of active mixer, (b) switching waveforms.

Since  $V_{out} = V_{DD} - I_1R_1 - (V_{DD} - I_2R_2)$ , we have for  $R_1 = R_2 = R_D$ ,

$$V_{out}(t) = I_{RF}R_D \left[ S\left(t - \frac{T_{LO}}{2}\right) - S(t) \right]. \tag{6.57}$$

From Fig. 6.44(b), we recognize that the switching operation in Eq. (6.57) is equivalent to multiplying  $I_{RF}$  by a square wave toggling between -1 and +1. Such a waveform exhibits a fundamental amplitude equal to  $4/\pi$ , 4 yielding an output given by

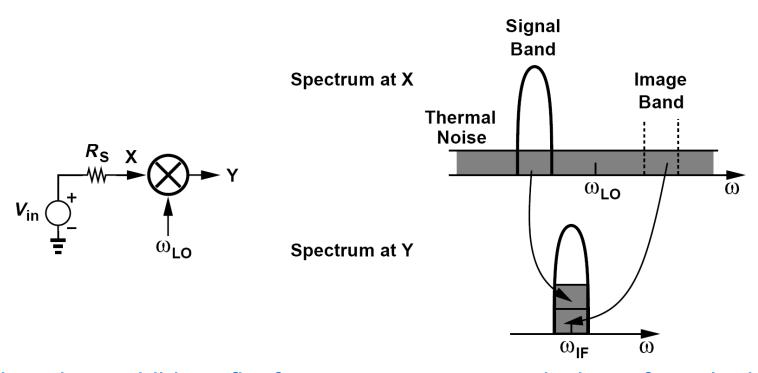
$$V_{out}(t) = I_{RF}(t)R_D \cdot \frac{4}{\pi}\cos\omega_{LO}t + \cdots$$
 (6.58)

If  $I_{RF}(t) = g_{m1}V_{RF}\cos\omega_{RF}t$ , then the IF component at  $\omega_{RF} - \omega_{LO}$  is equal to





## Mixer noise figure (info only)



- The mixer exhibits a flat frequency response at its input from the image band to the signal band.
- The noise figure of a noiseless mixer is 3 dB. This quantity is called the "single-sideband" (SSB) noise.
- In practice, taking into account mixer noise, noise figure is much higher, around 10dB

#### Mixing function in time domain – TX case

$$x_{if}(t) = A_{if}\cos(\omega_{if}t)$$

$$x_{lo}(t) = A_{lo}\cos(\omega_{lo}t)$$

$$x_{lo}(t) = A_{lo}\cos(\omega_{lo}t)$$

$$LO (local oscillator) signal$$

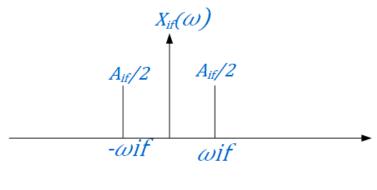
$$x_{rf}(t) = A_{if}\cos(\omega_{rf}t) A_{lo}\cos(\omega_{lo}t)$$

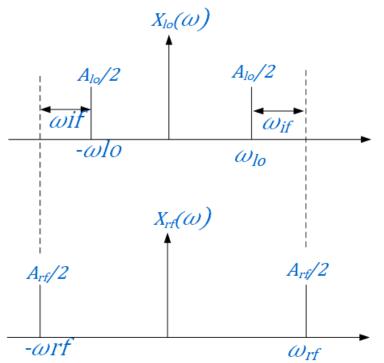
$$x_{rf}(t) = \frac{1}{2}A_{if} A_{lo}(\cos\left((\omega_{if} + \omega_{lo})t\right) + \cos\left((\omega_{if} + \omega_{lo})t\right))$$

$$A_{rf} = \frac{1}{2}A_{if} A \boxed{lo}$$
Due to band-pass filter **TU**/e Technische University of Technology

 $\omega_{rf} = \omega_{if} + \omega_{lo}$ 

### Mixing function in frequency domain – TX case





#### Time domain

$$x_{rf}(t) = x_{if}(t) x_{lo}(t)$$
multiplication

#### Frequency domain

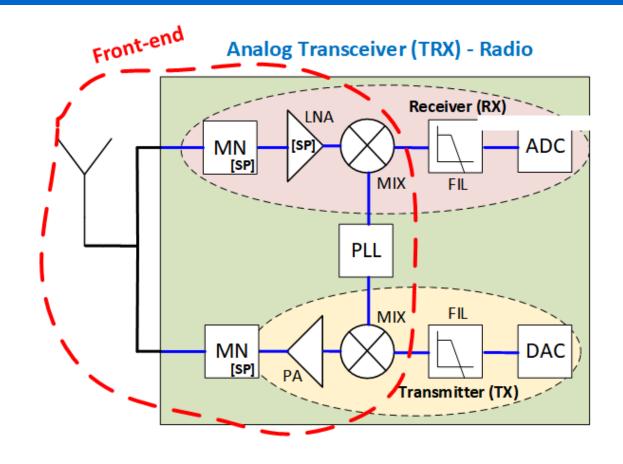
$$X_{rf}(\omega) = X_{if}(\omega) * X_{lo}(\omega)$$
convolution



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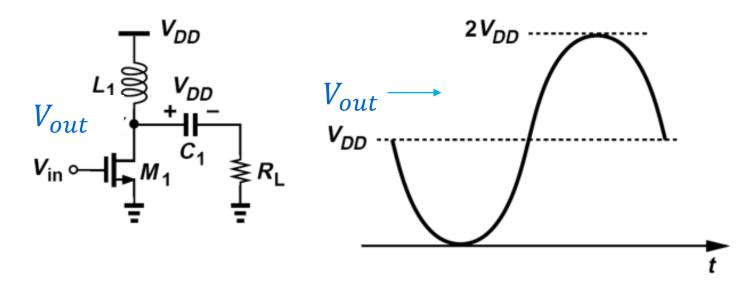
## TX – output power requirements



- Antenna is a load for a PA
- > PA is expected to deliver power to the antenna

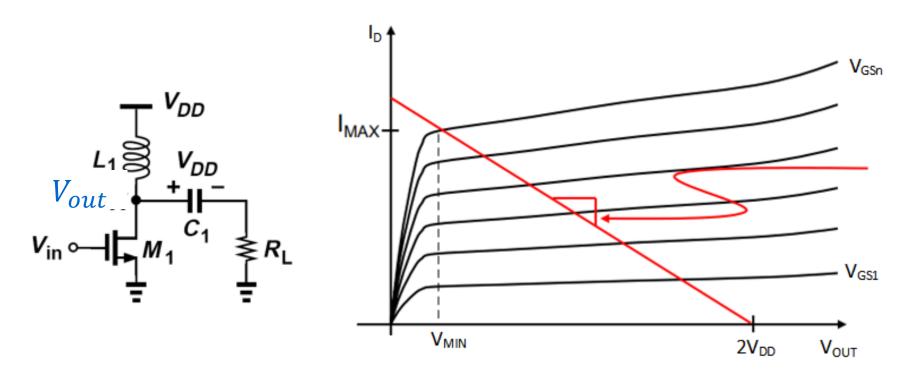
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#### **Power generation - limitation**



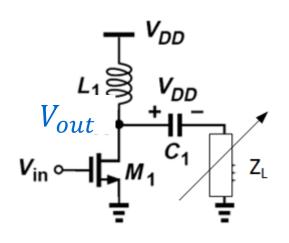
- ightharpoonup Output power  $P_{out@R_L} = \frac{V_{out}^2}{R_L}$
- Output power limited by load R<sub>L</sub> and supply V<sub>DD</sub>
- Supply V<sub>DD</sub> is limited by technology
- The only way to improve output power is selecting right R<sub>1</sub>

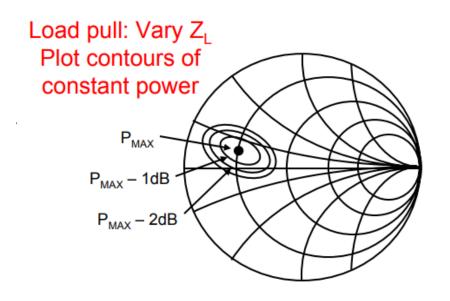
#### Optimum load for power amplifier (info only)



- ightharpoonup Optimum load  $R_L = \frac{2V_{DD} V_{MIN}}{I_{MAX}}$
- $\triangleright$  Load-pull simulations provide  $R_L$

#### **Load-pull simulations**

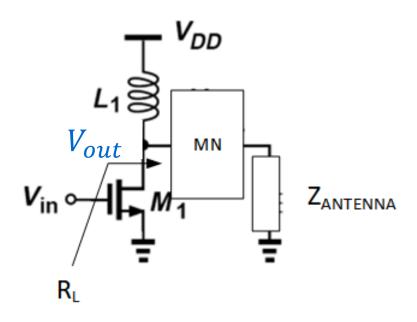




- Optimum load found by sweeping
- Load-pull simulations results in constant power contours



### **Matching network for PA**

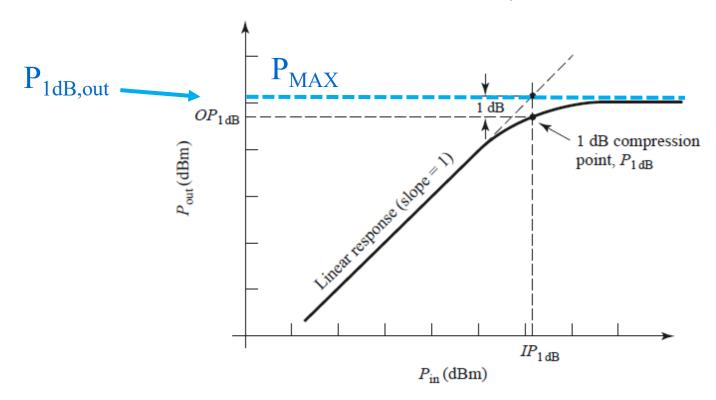


Matching network transform antenna impedance to optimum load



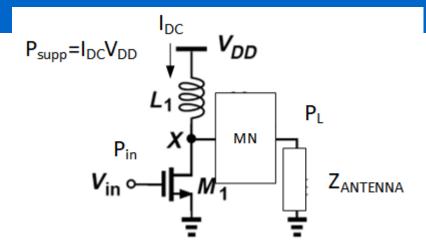
# PA performance parameters: P<sub>1dB,out</sub>, P<sub>MAX</sub>

➤ Output 1-dB compression point: P<sub>1dB,out</sub>



➤ Maximum output power: P<sub>MAX</sub>

#### PA performance parameters: efficiency



The "drain efficiency" (for FET implementations) or "collector efficiency" (for bipolar implementations) is defined as:

$$\eta = \frac{P_L}{P_{supp}}$$

where  $P_L$  denotes the average power delivered to the load and  $P_{supp}$  the average power drawn from the supply voltage.

"Power-added efficiency", PAE, defined as

$$PAE = \frac{P_L - P_{in}}{P_{supp}}$$

where Pin is the average input power

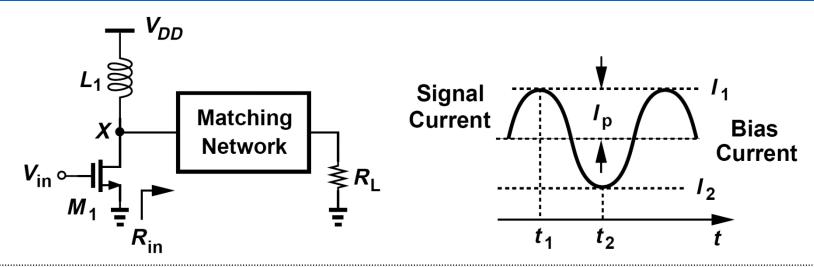
#### PA classes

- Based on operating point PAs can be divided into:
  - Class A
  - Class B
  - Class C
  - Class D
  - Class F
  - > Class E

Out of scope for this course



### PA operation in class A (info only)



- Class A amplifiers are defined as circuits in which the transistor(s) remain on and operate linearly across the full input and output range.
- If linearity is required, then class A operation is necessary.

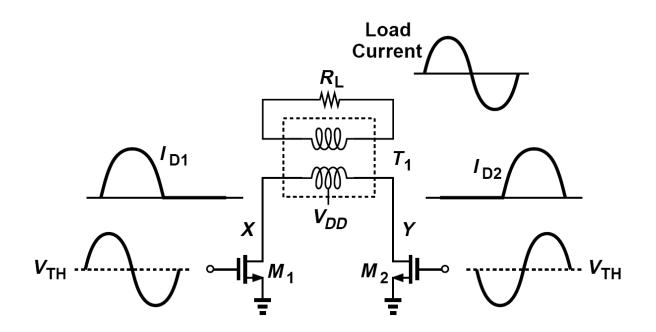
#### The maximum drain efficiency of class A amplifiers:

$$\eta = \frac{V_{DD}^2/(2R_{in})}{V_{DD}^2/R_{in}} = 50\%.$$



#### PA operation in class B (info only)

Conduction Angle is defined as the percentage of the signal period during which the transistor remain on multiplied by 360 °



➤ The traditional class B PA employs two parallel stages each of which conducts for only 180°, thereby achieving a higher efficiency than the class A counterpart.