PROBABILITY THEORY

- $Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\{-\frac{\alpha^2}{2}\} d\alpha$
- Gaussian PDF: $p_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{n^2}{2\sigma^2}\right)$
- $\Pr\{X=x, Y=y\} = \Pr\{Y=y|X=x\}\Pr\{X=x\}$
- $\Pr\{X=x, Y=y\} = \Pr\{X=x|Y=y\}\Pr\{Y=y\}$

PROPERTIES OF LOGARITHM

- $\log_2(xy) = \log_2(x) + \log_2(y)$
- $\log_2\left(\frac{x}{y}\right) = \log_2(x) - \log_2(y)$
- $\log_2(x^y) = y \log_2(x)$
- $\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)} = \frac{\ln(x)}{\ln(2)}$

TRIGONOMETRIC IDENTITIES

- $\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$ and $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$
- $\sin\alpha\ \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
- $\cos\alpha\ \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
- $\sin\alpha\ \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$
- $\cos\alpha\ \sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\sin(\alpha \pm \beta) = \sin(\alpha) cos(\beta) \pm \cos(\alpha)\sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

CHAPTERS

- $S_{N_w}(f) = \frac{N_0}{2}$ [W/Hz] for White Gaussian Noise
- $N = \frac{N_0}{2} 2B; D = E[D^2(t)]$
- Distortion $d(t) \triangleq v(t) - u(t)$
- SDR $\triangleq \frac{E[U^2(t)]}{E[D^2(t)]} = \frac{P}{N_0 W} = \text{SNR}_b$ iff AWGN channel
- $P_e = \sum_{m\in\mathcal{M}} \sum_{r: f(r)\neq m} \Pr\{M=m\}\Pr\{R=r|M=m\}$
- $\hat{m}^{\text{MAP}}(\underline{r}) \triangleq \text{argmax}_{m\in\mathcal{M}} \Pr\{M=m\} p_{\underline{R}}(\underline{r}|\underline{S}=\underline{s}_m)$
- $\hat{m}^{\text{MAP}}(\underline{r}) \triangleq \text{argmin}_{m\in\mathcal{M}} ||\underline{r}-\underline{s}_m||^2 - 2\sigma^2 \ln \Pr\{M=m\}$
- $p_{\underline{R}}(\underline{r}|\underline{S}=\underline{s}_m) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp(-\frac{||\underline{r}-\underline{s}_m||^2}{2\sigma^2})$
- $P_e = \sum_{m\in\mathcal{M}} \frac{1}{|\mathcal{M}|} \sum_{m'\in\mathcal{M}, m'\neq m} \Pr\{\underline{R} \in \mathcal{I}_{m'} | M=m\}$
- Parseval relation:
  $\int_{-\infty}^{\infty} f(t)g(t)dt = \int_{-\infty}^{\infty} F(f)G^*(f)df$
- $\underline{a} = \sum_{m\in\mathcal{M}} \Pr\{M=m\}\underline{s}_m$
- $\int_{-\infty}^{\infty} f(t)g(t)dt = (\underline{f}\cdot\underline{g})$
- $E_b \triangleq \frac{E_s}{\log_2|\mathcal{M}|} = \frac{E_s}{T}\frac{T}{\log_2|\mathcal{M}|} = \frac{P_s}{R}$ [Joule/bit]
- $R_N \triangleq \frac{\log_2|M|}{N} < C_N \triangleq \frac{1}{2}\log_2(\frac{E_N + N_0/2}{N_0/2})$ [bits/dim]
- $C_N = \frac{1}{2}\log_2(1+\frac{P_s}{N_0 W}) = \frac{1}{2}\log_2(1+\text{SNR})$ [bits/dim]
- $C = W\log_2(1+\frac{P_s}{N_0 W})$ [bits/sec]
- $C \approx \begin{cases} C_\infty = \frac{P_s}{N_0 \ln 2} & \text{if SNR} \ll 1 \text{ (power-limited)} \\ W\log_2(\text{SNR}) & \text{if SNR} \gg 1 \text{ (bandwidth-limited)} \end{cases}$
- $\text{SNR} = \frac{E_N}{N_0/2} = \frac{P_s}{N_0 W} = \frac{R}{W}\frac{E_b}{N_0}$
- Nyquist Criterion for zero intersymbol interference (ISI):
  Time domain: $\int_{-\infty}^{\infty} p(t)p(t-k\alpha)dt = \begin{cases} 0 & \text{if } k\neq 0 \\ 1 & \text{if } k=0 \end{cases}$
  Freq. domain: $\frac{1}{T}\sum_{m=-\infty}^{\infty} |P(f+\frac{m}{T})|^2 = 1$ for all f
- Sinc pulses:
  $p(t) = \frac{1}{\sqrt{T}}\frac{\sin(\pi t/T)}{\pi t/T}$
  $P(f) = \begin{cases} \sqrt{T} & \text{if } |f| < 1/(2T) \\ \sqrt{T/2} & \text{if } |f| = 1/(2T) \\ 0 & \text{if } |f| > 1/(2T) \end{cases}$
- Passband signal:
  $s_m(t) = \sum_{i=1}^{N_c} s^c_{mi}\varphi_i(t)\sqrt{2}\cos(2\pi f_0 t) + \sum_{j=1}^{N_s} s^s_{mj}\psi_j(t)\sqrt{2}\sin(2\pi f_0 t)$
- Passband capacity:
  $C_N = \frac{1}{2}\log_2(1+\frac{P_s}{2N_0W})$ [bits/dim]
  $C = 2W\log_2(1+\frac{P_s}{2N_0W})$ [bits/sec]
- Random carrier phase:
  $\underline{r}^c = \underline{s}^c_m + \underline{n}^c = \underline{s}_m\cos\theta + \underline{n}^c$
  $\underline{r}^s = \underline{s}^s_m + \underline{n}^s = \underline{s}_m\sin\theta + \underline{n}^s$
- $I_0(x) \triangleq \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos(\theta))d\theta$
- Gram-Schmidt procedure:
  1) $\varphi_1(t) = s_1(t)/\sqrt{E_1}$, $\quad s_{11} = \sqrt{E_1}$
  2) $\theta_2(t) = s_2(t) - s_{21}\varphi_1(t)$, $\quad s_{21} = \int_{-\infty}^{\infty} s_2(t)\varphi_1(t)dt$
  3) $\varphi_2(t) = \theta_2(t)/\sqrt{E_{\theta_2}}$, $\quad s_{22} = \int_{-\infty}^{\infty} s_2(t)\varphi_2(t)dt$
  4) $\theta_m(t) \triangleq s_m(t) - \sum_{i=1}^{m-1} s_{mi}\varphi_i(t)$,
  $s_{mi} = \int_{-\infty}^{\infty} s_m(t)\varphi_i(t)dt$, for $i = 1, \ldots, m-1$
  5a) If $\theta_m(t) \equiv 0$ then we stop, or
  5b) If $\theta_m(t) \not\equiv 0$ then $\varphi_m(t) = \theta_m(t)/\sqrt{E_{\theta_m}}$ and go back to step 4