Communication Theory (5ETB0)
Module 7.1

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Module 7.1

Presentation Outline

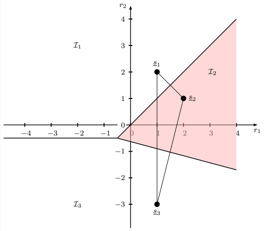
Rotation of Signal Structures

Energy and Average Energy

$$E_{s_m} = \int_0^T s_m^2(t)dt = \|\underline{s}_m\|^2, \qquad E_{\text{av}} \triangleq \sum_{m\in\mathcal{M}} \Pr\{M=m\}E_{s_m} = E[\|\underline{S}\|^2]$$

Error Probability and Energy of Rotated Signals

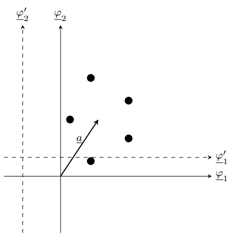
- $\mathcal{S} = \{\underline{s}_1, \underline{s}_2, \ldots, \underline{s}_{|\mathcal{M}|}\}$ is *rotated*
- $\mathcal{I}_1, \mathcal{I}_2, \cdots, \mathcal{I}_{|\mathcal{M}|}$ rotated same way
- AWGN vector is spherically symmetric
- P_{e} will not change
- Av. signal energy will not change

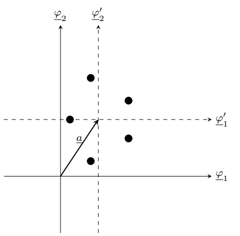


Rotation of Signal Structures: Matlab Example

Translating a Signal Structure

Translating a signal structure does not change P_e



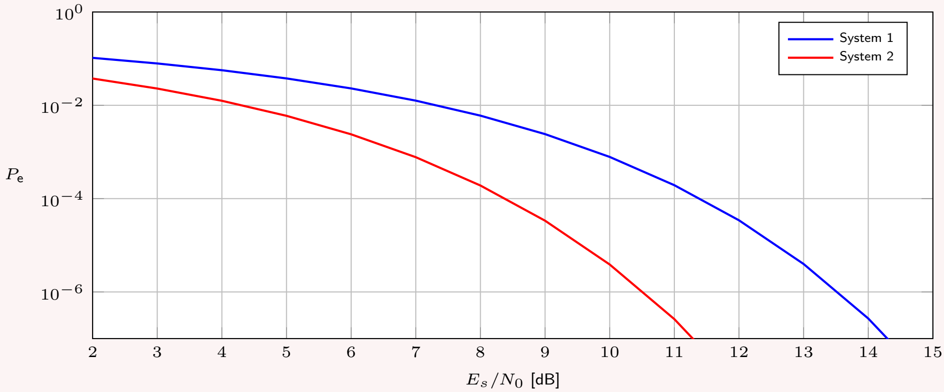


Minimizing the average signal energy

To minimize the average signal energy we should choose the **center of gravity** of the signal structure as the origin of the coordinate system. If the center of gravity of the signal structure $\underline{a} \neq \underline{0}$ we can decrease the average signal energy by $\|\underline{a}\|^2$ by moving the origin of the coordinate system to $\underline{a}$.

Translating a Signal Structure

Translating a signal structure does not change P_e



Module 7.1

Presentation Outline

Binary Orthogonal Signaling

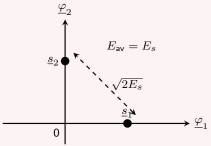
Orthogonal Waveforms (FSK)

Let $|\mathcal{M}| = \{1, 2\}$ and $\Pr\{M = 1\} = \Pr\{M = 2\} = 1/2$.
Consider two orthogonal waveforms:

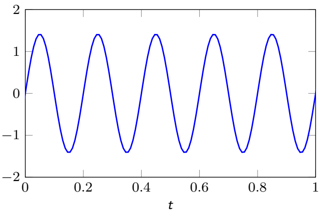
$$s_1(t) = \sqrt{2E_s}\sin(10\pi t), 0 \leq t < 1$$
$$s_2(t) = \sqrt{2E_s}\sin(12\pi t), 0 \leq t < 1$$

Vector representation of signals:

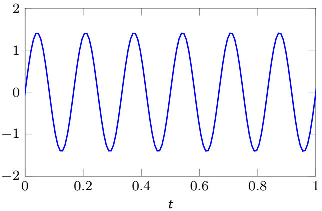
$$\underline{s}_1 = (\sqrt{E_s}, 0),\ \underline{s}_2 = (0, \sqrt{E_s})$$



$\varphi_1(t) = s_1(t)/\sqrt{E_s}$



$\varphi_2(t) = s_2(t)/\sqrt{E_s}$



Module 7.1

Presentation Outline

Binary Antipodal Signaling

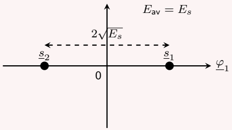
Antipodal Waveforms (PSK)

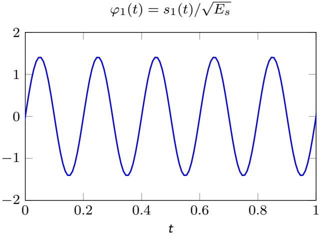
Let $|\mathcal{M}| = \{1, 2\}$ and $\Pr\{M = 1\} = \Pr\{M = 2\} = 1/2$.
Consider two antipodal waveforms:

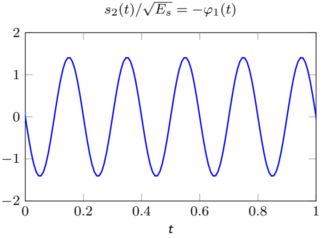
$$s_1(t) = \sqrt{2E_s}\sin(10\pi t), 0 \leq t < 1$$
$$s_2(t) = -\sqrt{2E_s}\sin(10\pi t), 0 \leq t < 1$$

Vector representation of signals:

$$\underline{s}_1 = (\sqrt{E_s}, 0),\ \underline{s}_2 = (-\sqrt{E_s}, 0)$$







Comparison of Orthogonal and Antipodal Signaling

AGN Vector Channel

For the AGN vector channel, the probability that the noise pushes a signal to the wrong side of a hyperplane is

$$P_{\mathcal{I}} = Q\left(\frac{\Delta}{\sigma}\right),$$

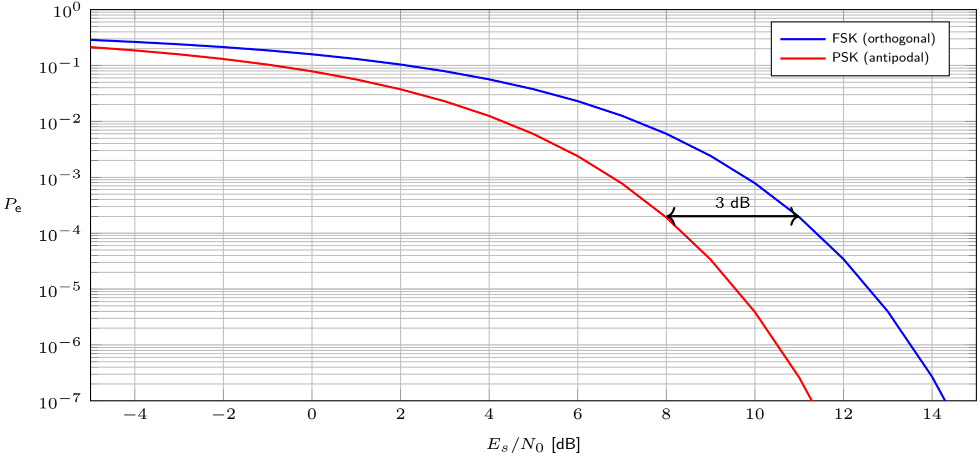
where Δ is the distance from the signal-point to the hyperplane and σ^2 is the variance of each noise component.

Error Probability Comparison

With $E_{av} = E_s$ and power spectral density $N_0/2$, the error probabilities are:

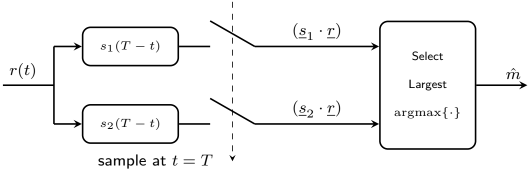
$$P_{\mathrm{e}}^{orth.} = Q\left(\sqrt{E_s/N_0}\right), \quad P_{\mathrm{e}}^{antip.} = Q\left(\sqrt{2E_s/N_0}\right).$$

Comparison of Orthogonal and Antipodal Signaling: Example

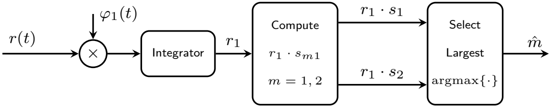


Receivers for Antipodal Signaling

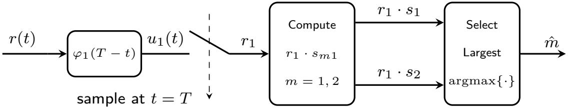
- Direct Receiver



- Correlation Receiver



- Match-filter Receiver



Summary Module 7.1

Take Home Messages

- Rotations do not change the error probability
- Translations save you energy
- Two binary signaling schemes: orthogonal and antipodal.
- Analysis based on building-block waveforms and geometric interpretation of signals

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