SLT-C

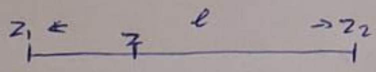
1/ a/ $$J(\vec{r};t) = \int_0^{\infty} \sigma(\vec{r};t')\, E(\vec{r};t-t')\,dt'$$

Key properties:

- $J(\vec{r};t)$ depends at time t only on the E values, leading ~~to~~ to t, aka those at the present (@ $t=0$) and past (@ $t<0$)
  Called "causality"
- Linearly reacting - can sum them up freely
  $$(J_1 + J_2) = \int_0^{\infty} \sigma(E_1 + E_2)\,dt' = \int_0^{\infty} \sigma . E_1 dt' + \int_0^{\infty} \sigma . E_2 . dt'$$
- Time invariat - can delay the $\vec{E}$ field and $\vec{J}$ will have the same time delay as $\vec{E}$, with the exact function shape

b/ The voltage standing wave ratio is the ratio between the maximum and minimum voltages at the line. It describes how much reflections are there along a transmission line.

c/



$\rightsquigarrow \{V_{s1}^+ ; Y V_{s1}^+\}$ $\quad$ $\rightsquigarrow \{V_{s2}^+ ; Y . V_{s2}^+\}$

$\leftsquigarrow \{V_{s1}^- ; -Y V_{s1}^-\}$ $\quad$ $\leftsquigarrow \{V_{s2}^- ; -Y . V_{s2}^-\}$

$$V_{s1}^+ = V_{s2}^+ . e^{jkl}$$
$$V_{s1}^- = V_{s2}^- . e^{-jkl}$$
$$V_s(z) = V_s^+ + V_s^- = V_{s2}^+ . e^{jk(l-z)} + V_{s2}^- . e^{-jk(l-z)}$$

| z - arbitary distance from 0 (z_1) to l (z_2)

$$I_s(z) = \cancel{Y V_s(z)}\ Y . V_s^+(z) - Y V_s^-(z) =$$
$$= Y\left(V_{s2}^+ e^{jk(l-z)} - V_{s2}^- . e^{-jk(l-z)}\right)$$

2/ ~~μ~~ Medium 1: $\varepsilon_1; \mu_1; \eta_1 = 100\,\Omega$ | Medium 2: $\varepsilon_2; \mu_2; \eta_2 = 300\,\Omega$

$$\vec{E_i}(z) = E^+ . e^{-jk_1 z} . \hat{a}_x \; ; \; E^+ = 100 \tfrac{V}{m}$$

a/ $\eta = \sqrt{\frac{\mu}{\varepsilon}}$; $\mu_1 = \mu_2 = 1$ (non-magnetic)

$$\Rightarrow \cancel{\varepsilon} \; \eta_1^2 = \frac{\mu}{\varepsilon_1} = \frac{1}{\varepsilon_1}$$

$$100^2 . \varepsilon_1 = 4\pi . 10^{-7}$$

$$\varepsilon_1 = \cancel{0{,}01\ F/m} \; 1{,}257.10^{-10} \tfrac{F}{m}$$

$$\cancel{\eta_2^2 = \frac{\mu}{\varepsilon_2} \rightarrow (\eta_2 . \varepsilon_2) = 4\pi . 10^{-7}}$$

$$\cancel{\Rightarrow \varepsilon_2 = \frac{1}{300} = 3{,}33.10^{-3}\ F/m}$$

$$\varepsilon_2 = \frac{4\pi . 10^{-7}}{300^2} = 1{,}396.10^{-11} \tfrac{F}{m}$$

$$k = \beta - j\alpha = \omega\sqrt{\varepsilon . \mu}$$

$$V_{phase} = \cancel{\frac{\omega}{k}} \; \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\varepsilon . \mu}} = \frac{1}{\sqrt{\varepsilon . \mu}}$$

$$\Rightarrow V_{p1} = \frac{1}{\sqrt{\varepsilon_1 . \mu_1}} = \cancel{\frac{1}{\sqrt{0{,}01 . 4\pi . 10^{-7}}}} = \frac{1}{\sqrt{1{,}257.10^{-10} . 4\pi . 10^{-7}}} =$$

$$= 7{,}95.10^{7} \ m/s$$

$$V_{p2} = \frac{1}{\sqrt{\varepsilon_2 . \mu_2}} = \cancel{2{,}387} \; 2{,}387.10^{8} \ m/s$$

b/ $\nabla \times \vec{E} = -\frac{\partial B}{\partial t} - K \quad (K = 0)$

$\vec{E} = E^+ . e^{-jk_1 z} . \hat{a}_x \rightarrow$ E field is travelling along the $\vec{z}$ axis, and its polarized along the $\vec{x}$ axis

$$\cancel{e^{-jk_2} = e^{-j(\omega\sqrt{\varepsilon\mu})z}}$$

$$\cancel{e^{j\omega\frac{z}{v}} . e^{j\omega t} = e^{j\omega(t + \frac{z}{v})}}$$

$$E(\vec{z};t) = \cancel{Re\{E_i e^{-jk_1 z} . \hat{a}_x\} = E^+ \cos(}$$

$$= Re\{ E_i(z) . e^{j\omega t} \} =$$

$$= E^+ . Re\{ e^{-jk_1 z} . e^{j\omega t} \} . \hat{a}_x =$$

$$= E^+ . \hat{a}_x . \cos(\omega t - k_1 . z)$$

~~c/ $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{200}{400} = \frac{1}{2}$~~

~~⇒ half of the wave is reflected, and so $|\langle S_r \rangle_T| = |\langle S_i \rangle_T| \cdot \Gamma = 50W$~~

~~d/ $T = 1 + \Gamma = 1,5$~~

~~$|\langle S_t \rangle_T| = 150W$~~

c/ $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{200}{400} = \frac{1}{2} \Rightarrow \vec{E}_r = \Gamma \cdot \vec{E}_i = \frac{1}{2}\vec{E}_i$

$$|\langle S_r \rangle_T| = \frac{(E_r^+)^2}{2\eta} = \frac{(\frac{1}{2}E^+)^2}{2\eta} = \frac{50^2}{2 \cdot 100} = 12,5W$$

d/ $T = 1 + \Gamma = 1,5 \Rightarrow E_t(z) = E^+ \cdot T \cdot e^{-jk_2 z} \cdot \hat{a}_x = 150 \cdot e^{-jk_2 z} \cdot \hat{a}_x$

$$\vec{H}_t(z) = \frac{E_t^+}{\eta_2} \cdot e^{-jk_2 z} \cdot \hat{a}_y = 0,5 \cdot e^{-jk_2 z} \cdot \hat{a}_y$$

$$\Rightarrow |\langle S_t \rangle_T| = \frac{(T \cdot E^+)^2}{2\eta_2} = \frac{150^2}{2 \cdot 300} = 37,5 \frac{W}{m^2}$$

3/ a/

→ ~~The pointing vector is $\vec{p} = \vec{S} - \vec{e}_n$, with $\vec{e}_n$ being an arbitrary point on the surface.~~

$P = 50kW$

$r = 100km$

$\langle \vec{S} \rangle = \frac{P}{\pi r^2} \cdot \vec{n}$ with $\vec{n}$ being the propagation direction, aka from the satelite to a point on Earth

b/ $|\langle \vec{S} \rangle| = \left|\frac{P}{\pi r^2} \cdot \vec{n}\right| = \left|\frac{50000}{\pi \cdot (100 \cdot 10^3)^2}\right| = 1,5915 \cdot 10^{-6} \frac{W}{m^2}$

c/ $A_{antena} = \pi r_a^2 = 31415\ m^2$

$W_a = t \cdot |\langle \vec{S} \rangle| \cdot A_{antena} = 5 \cdot 60 \cdot 1,5915 \cdot 10^{-6} \cdot 31415 =$

$= 15J$

d/ $P_{new} = 4 \cdot P_{old}$

⇒ for $|\vec{S}|$ to keep constant, r_{new} should be $2r_{old}$ or 200km

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad ; \quad \nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\rightarrow \nabla \times \vec{E} = -j\omega\mu \cdot \vec{H} \quad ; \quad \nabla \times \vec{H} = j\omega\epsilon \vec{E}$$

$$\Rightarrow \vec{H} = \frac{1}{j\omega\mu}(\nabla \times \vec{E}) \quad ; \quad \vec{E} = \vec{E}(z;t) = E^{+} \cdot e^{-jk_1 z} \cdot \hat{a_x}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z) & 0 & 0 \end{vmatrix} = -\left(\frac{\partial}{\partial z} E_x(z)\, \vec{a_y} - \underbrace{\frac{\partial}{\partial y} E_x(z)\, \vec{a_z}}_{=0}\right) =$$

$$= -\frac{\partial E(z)}{\partial z} \cdot \vec{a_y} =$$

$$= -E^{+} \cdot \hat{a_y} \cdot \frac{\partial}{\partial z} \cdot e^{-jk_1 z} =$$

$$= -E^{+} \cdot \hat{a_y} \cdot -jk_1 \cdot e^{-jk_1 z} =$$

$$= E^{+} \cdot jk_1 \cdot e^{-jk_1 z} \cdot \hat{a_y}$$

$$\Rightarrow \vec{H} = \frac{1}{j\omega\mu} \cdot E^{+} j k_1 \cdot e^{-jk_1 z} \cdot \hat{a_y} =$$

$$= E^{+} \cdot \frac{\omega\sqrt{\epsilon\mu}}{\omega\mu} \cdot e^{-jk_1 z} \cdot \hat{a_y} = E^{+} \cdot \sqrt{\frac{\epsilon}{\mu}} \cdot e^{-jk_1 z} \cdot \hat{a_y} = \frac{E^{+}}{\eta} \cdot e^{-jk_1 z} \cdot \hat{a_y}$$

$$\Rightarrow H(z;t) = Re\{H_i(z) \cdot e^{j\omega t}\} = \frac{E^{+}}{\eta} \cdot \hat{a_y} \cdot \cos(\omega t - k_1 z)$$

Power balance:

$$\langle S \rangle_T = \frac{1}{2} Re\{E_s \times H_s^{*}\} = \quad \Big| \quad H_s^{*} = \left(\frac{E^{+}}{\eta} \cdot e^{-jk_1 z} \cdot \hat{a_y}\right)^{*} = \frac{E^{+}}{\eta} \cdot e^{jk_1 z} \cdot \hat{a_y}$$

$$= \frac{1}{2} Re\left\{E^{+} \cdot e^{-jk_1 z} \cdot \hat{a_x} \times \frac{E^{+}}{\eta} e^{jk_1 z} \cdot \hat{a_y}\right\} =$$

$$= \frac{1}{2} Re\left\{\frac{E^{+2}}{\eta} \cdot e^{-jk_1 z} \cdot e^{jk_1 z} \cdot \hat{a_z}\right\} = \frac{1}{2} Re\left\{\frac{(E^{+})^2}{\eta} \cdot \hat{a_z}\right\} = \frac{(E^{+})^2}{2\eta} \cdot \hat{a_z}$$

$$\Rightarrow \langle S \rangle_T = \frac{100^2}{2 \cdot 100} \cdot \hat{a_z} = 100 \cdot \hat{a_z} \left[\frac{W}{m^2}\right] \rightarrow |\langle S \rangle_T| = 100\, W$$

4/ a/ The complex part of the permitivity relates to the energy lost within the material when exposed to AC field

b/ $$Z_{frog} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r - \frac{j\sigma}{\omega}}} =$$

$$= \sqrt{\frac{4\pi \cdot 10^{-7}}{8,85 \cdot 10^{-12} \cdot 2,44 \cdot 10^{3} - \frac{j \cdot 86,5 \cdot 10^{-3}}{290 \cdot 10^{3} \cdot 2\pi}}} =$$

$$= 3,58 + 2,54j \; [\Omega]$$

c/ The real part is the resistive properties of the load (dissipation via heat), whilst the imaginary part is assosiated with inductive behavior.
In more detail, the inductance is caused by the bundles of nerves through which current flows, which opposses the current in neighboring nerves.

d/ Voltage source - ~~OS~~ SC ~~note~~ modeled as

$$\Rightarrow \overleftarrow{\Gamma_0} = \frac{0-50}{0+50} = -1$$

$$\overrightarrow{\Gamma_e} = \frac{Z_{frog} - 50}{Z_{frog} + 50} = 0,867 \angle 174° = 0,867 \cdot e^{j \cdot 3,037}$$

e/ $$V^- = \Gamma \cdot V^+ \Rightarrow |V^-| = |\Gamma \cdot V^+| = |\Gamma| \cdot |V^+|$$

$$\arg(V^-) = \arg(\Gamma \cdot V^+) = \arg(\Gamma) + \arg(V^+)$$

$\Rightarrow$ The ~~regressive~~ reflected wave is scaled by $|\Gamma|$ and phase shifted by $\arg(\Gamma)$ compared to V^+, aka the incident wave

f/ $$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 14,04$$

g/ Assumed the lighting struck the frog leg directly:

$$\Delta T = \frac{Q}{c \cdot m} = \frac{1}{cm} \cdot \frac{V_L^2}{Re\{Z_{frog}\}} \cdot t = \frac{I}{3613 \cdot 10 \cdot 10^{-3}} \ldots = 9889 [K]$$

$V_L = I_e \cdot Z_{air} =$
$= 30 \cdot 10^3 \cdot 377 =$
$\approx 1,131 \cdot 10^6 V$

Save to say, "what frog legs? I do not see any legs."

5/ a/ $T = \begin{pmatrix} \cos(k.\ell) & jZ.\sin(k.\ell) \\ jY\sin(k.\ell) & \cos(k.\ell) \end{pmatrix}$

T_{11} = Voltage transfer coeff. when output is OC (@ $I_2 = 0$)

T_{12} = Transfer impedance - voltage at input due to current at output with $V_2 = 0$

T_{21} = Transfer admitance - current at input due to voltage at output with $I_2 = 0$

T_{22} = Current trasfer when output is SC (@ $V_2 = 0$)

Appenantly the test conditions are using OC and SC to isolate the effects of either V_2 or I_2

b/ $T_1 = \begin{bmatrix} 1 & 0 \\ Z & 1 \end{bmatrix} \to \begin{pmatrix} V_{S1} \\ I_{S1} \end{pmatrix} = T_1 \begin{pmatrix} V_{S2} \\ I_{S2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ Z & 1 \end{pmatrix} \begin{pmatrix} V_{S2} \\ I_{S2} \end{pmatrix} = \begin{pmatrix} V_{S2} \\ Z.V_{S2} + I_{S2} \end{pmatrix}$

$\Rightarrow V_{S1} = V_{S2}$

$I_{S1} = Z.V_{S2} + I_{S2}$ → units are clashing, so probably a mistake and Y was meant instead of Z, so

$$I_{S1} = Y.V_{S2} + I_{S2} = Y.V_{S1} + I_{S2} = I_{S1} + I_{S2} \Rightarrow I_{S2} = 0$$

Only thing I can think of OC - keeps the voltage, but no current flows

$T_2 = \begin{bmatrix} 1+ZY & Y \\ Z & 1 \end{bmatrix} \to \begin{pmatrix} V_{S1} \\ I_{S1} \end{pmatrix} = T_2 \begin{pmatrix} V_{S2} \\ I_{S2} \end{pmatrix} = \begin{pmatrix} V_{S2}(1+ZY) + Y.\cancel{V_{S2}} \\ V_{S2}.Z + I_{S2} \end{pmatrix}$

Again units do not match?

$V_{S1} \to [V]$; $V_{S2}(1+ZY) \to [V]$; $Y.\cancel{V}_{S2} \to [S.\cancel{A}]$?

$V_{S2}.Z \to [V.\Omega]$?

c/ $T_1 . T_2 = \begin{bmatrix} 1 & 0 \\ Z & 1 \end{bmatrix} . \begin{bmatrix} 1+ZY & Y \\ Z & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Y \\ 2Z+ZY & ZY+1 \end{bmatrix}$

Transfer matrixes are useful for easily translating voltages and currents accross a transmission line ~~netwok~~, basically simplifying the needed calculations. They also can provide great insight of the network quickly.

d/

$Z_{in} = \frac{V_{S1}}{I_{S1}}$; $Z_L = \frac{V_{S2}}{I_{S2}}$

$$\begin{pmatrix}V_{S1}\\ I_{S1}\end{pmatrix} = T\begin{pmatrix}V_{S2}\\ I_{S2}\end{pmatrix} = \begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}V_{S2}\\ I_{S2}\end{pmatrix} = \begin{pmatrix}A V_{S2} + B I_{S2}\\ C.V_{S2} + D I_{S2}\end{pmatrix} =$$

$$V_{S1} = A.V_{S2} + B.I_{S2} = A.V_{S2} + B.\frac{V_{S2}}{Z_L} = V_{S2}\left(A + \frac{B}{Z_L}\right)$$

$$I_{S1} = I_{S2}.D + V_{S2}.C = \frac{V_{S2}}{Z_L}.D + C.V_{S2} = V_{S2}\left(C + \frac{D}{Z_L}\right)$$

$$\frac{V_{S1}}{I_{S1}} = Z_{in} = \frac{\cancel{V_{S2}}(A + B/Z_L)}{\cancel{V_{S2}}(C + D/Z_L)} = \frac{A + B/Z_L}{C + D/Z_L} = \frac{A.Z_L + B}{C.Z_L + D}$$

e/

$$Z_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \to 45\pi = \sqrt{\frac{\mu_2.\mu_0}{\varepsilon_0.\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}.\sqrt{\frac{\mu_2}{9}} = 120\pi.\frac{\sqrt{\mu_2}}{3} = 40\pi.\sqrt{\mu_2}$$

$$\Rightarrow \cancel{\mu_2^2 = \left(\frac{45\pi}{40\pi}\right)^2 \varepsilon_2}$$

$$\Rightarrow \mu_2 = 1{,}061$$

$$\varepsilon_r ; \mu_r \in \mathbb{R} \Rightarrow v = \frac{1}{\sqrt{\varepsilon.\mu}} = \frac{1}{\sqrt{\varepsilon_0.\mu_0}} \cdot \frac{1}{\sqrt{\mu_r.\varepsilon_r}} =$$

$$= c.\frac{1}{3{,}09} = 9{,}7.10^{7} \text{ m/s}$$

$$f = \frac{v}{\lambda} = 1{,}552 \text{ GHz}$$

$k = \omega\sqrt{\varepsilon.\mu} = \cancel{0{,}1.10^{-3} \approx 0}$ ~~(lets be honest, close enough)~~

$= 100$ rad/m

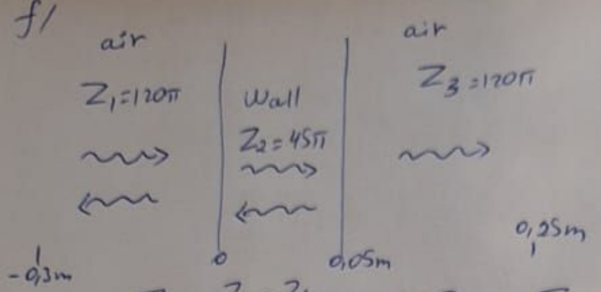
~~$\Rightarrow T_2 \approx \begin{pmatrix}\sim 1 & 0\\ 0 & \sim 1\end{pmatrix}$ → almost perfect transmission through wall~~

$$T_2 = \begin{pmatrix}\cos(5)^{(\text{rads})} & jZ\sin(5)\\ jY\sin(5) & \cos(5)\end{pmatrix} =$$

~~$k.\ell$ = unitless?~~

$$= \begin{pmatrix}0{,}28 & -135j\\ -6{,}48.10^{-3}j & 0{,}28\end{pmatrix}$$

f/



$$\Gamma_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = -0,45 \rightarrow T_{12} = 0,55$$

$$\Gamma_{23} = 0,45 \rightarrow T_{23} = 1,45$$

$$\Gamma_{21} = 0,45 \rightarrow T_{21} = 1,45$$

g/ k_0 - free space wave number, used for air mediums

$$k_0 = \omega\sqrt{\varepsilon_0 . \mu_0} = 2\pi f\sqrt{\varepsilon_0 . \mu_0} = 32,52$$

$$T_0(-0,3;0) = \begin{pmatrix} \cos(32,52 . 0,3) & j . 120\pi . \sin(9,756) \\ j\frac{1}{120\pi}\sin(9,756) & \cos(9,756) \end{pmatrix} =$$

$$= \begin{pmatrix} -0,946 & -122j \\ -8,63.10^{-4}j & -0,946 \end{pmatrix}$$

$$T_2(0,05;0,25) = \begin{pmatrix} \cos(6,5) & j120\pi\sin(6,5) \\ j\frac{1}{120\pi}\sin(6,5) & \cos(6,5) \end{pmatrix} =$$

$$= \begin{pmatrix} 0,976 & 81,1j \\ 5,71.10^{-4}j & 0,976 \end{pmatrix}$$

$$T_{tot} = T_0 . T_1 . T_2 =$$

$$= \begin{pmatrix} -1,1 & 93,55j \\ 6,11.10^{-3}j & -0,38 \end{pmatrix} . T_2 =$$

$$= \begin{pmatrix} -1,13 & 2,09j \\ 5,8.10^{-3}j & -0,87 \end{pmatrix}$$

h/ $f = 2{,}1\ GHz$

~~[illegible]~~

~~[illegible]~~

$\lambda = \frac{v}{f}$; $v = \frac{1}{\sqrt{\mu_r . \varepsilon_r}} \cdot c = \frac{c}{6}$ (assuming non-magnetic plastic)

$= 50.10^6\ m/s$

$\lambda = \frac{1}{42} m$

$\Rightarrow \frac{\lambda}{4} = \frac{1}{168} m \approx 5{,}95\ mm$

i/ $\begin{pmatrix} V_{s1}^- \\ V_{s2}^+ \end{pmatrix} = S \begin{pmatrix} V_{s1}^+ \\ V_{s2}^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_{s1}^+ \\ V_{s2}^- \end{pmatrix}$

S_{11} - reflexion coeff. at input node (port 1)

S_{12} - transmission coeff at input node (from port 1 to 2)

S_{21} - transmission coeff from port 2 to 1

S_{22} - reflexion coeff. at port 2

Scatter matrixes relate decomposed wave amplitudes. The parameters relate only to reflexion and transmission coefficients.

Transfer matrixes relate voltages and currents, and their params relate to the medium impedances and physical characteristics such as lenght, ε and μ, etc...