Components in wireless technology, 5XTC0

Module 4
Lecture: Amplifier design

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Where innovation starts

Outline

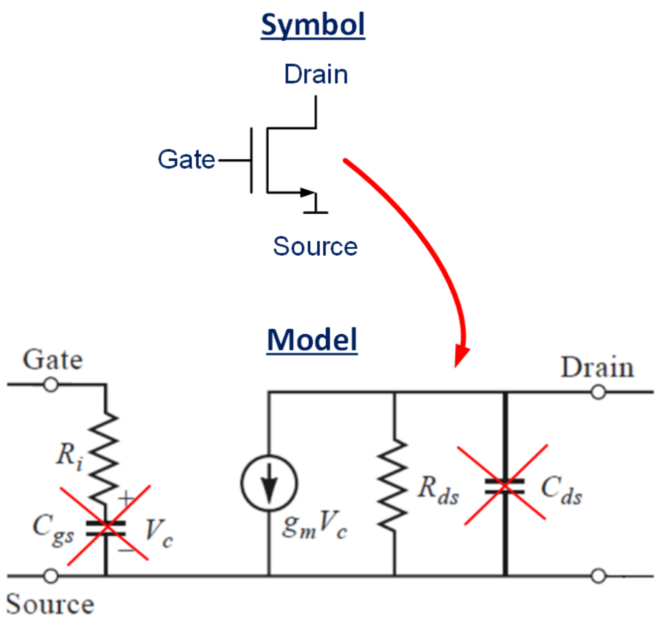
- Design methods
- Two-port network descriptions
- Introduction of S-parameters
- Definitions
  - Reflection coefficient
  - Power
  - Power gain
- Gain circles
- Stability
- Stability circles

Learning Objectives

- Understand design methods at low and high frequency
- Understand S-parameters definition
- Calculate and understand the different gains of an amplifier
- Know the basic amplifier architecture
- Understand gain circles
- Understand the concept of stability
- Be able to evaluate whether an amplifier is unconditionally stable
- Be able to evaluate the stability of an amplifier by calculating the stability circles

Design methods: low versus high frequency

- Example: single MOS transistor



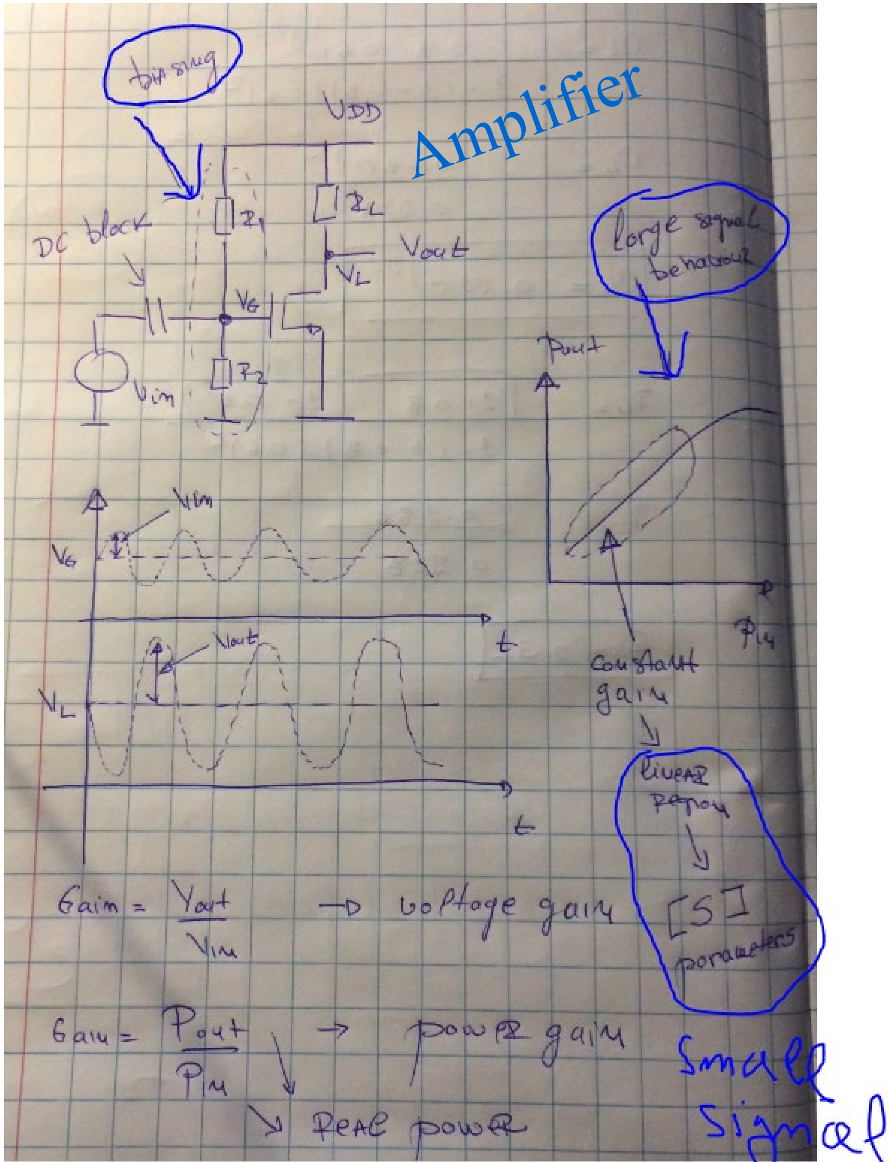
$$Z_C = \frac{1}{2*pi*freq*C}$$

Most analog circuit have more than one transistor !!!

- **Low frequency method**
  - Capacitors can be neglected
  - Circuit complexity reduced
  - Working with gm and Rds is sufficient
  - Complexity is manageable

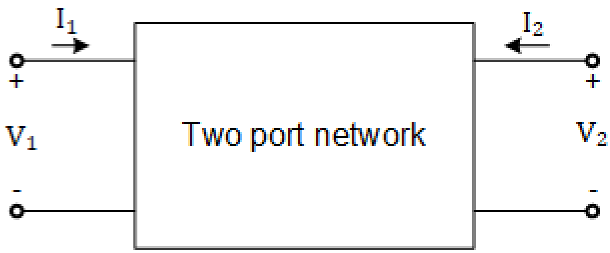
- **High frequency method**
  - Capacitors most be included
  - Circuit complexity increases
  - Black box approach
  - In general, every circuit can be described with N-port element
  - Most of circuits have 2 ports
  - Z, Y, S parameters used for N- port description

Design methods: large vs small signal



2-port network description: Z,Y parameters

- Z parameters



- Y parameters

$$V_1 = Z_{11} I_1 + Z_{22} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

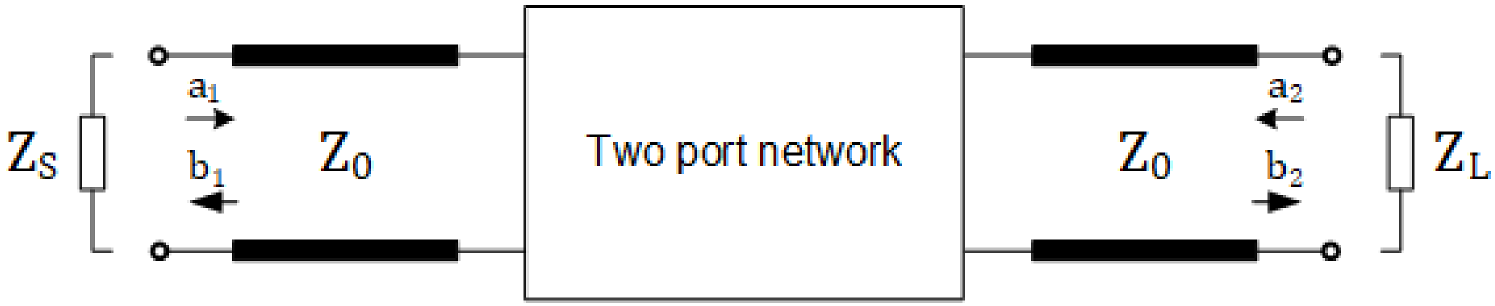
$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} \qquad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} \qquad \left.\frac{I_1}{V_1}\right|_{V_2 = 0} = Y_{11} \qquad \left.\frac{I_1}{V_2}\right|_{V_1 = 0} = Y_{12}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} \qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} \qquad \left.\frac{I_2}{V_1}\right|_{V_2 = 0} = Y_{21} \qquad \left.\frac{I_2}{V_2}\right|_{V_1 = 0} = Y_{22}$$

- **I=0 means open, V=0 means short**
- To measure Z and Y parameters open and short terminations are required
- At high frequencies there are no perfect open and short terminations

2-port network description: S parameters



$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ (input reflection coefficient with output properly terminated)

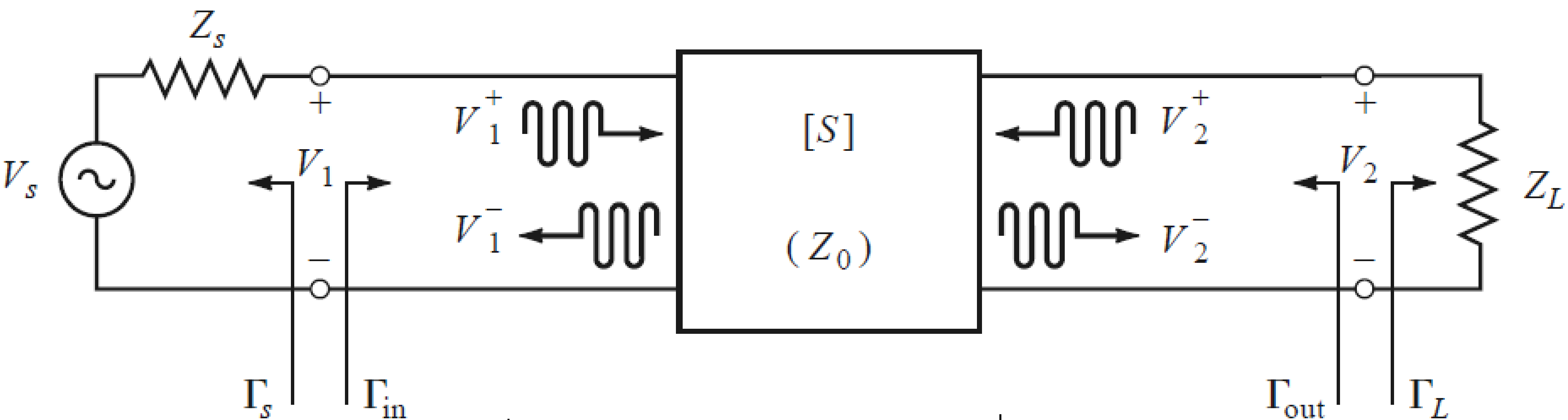
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$ (forward transmission coefficient with output properly terminated)

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$ (output reflection coefficient with input properly terminated)

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$ (reverse transmission coefficient with input properly terminated)

- To measure S parameters matched terminations are required: $Z_L=Z_0$ and $Z_S=Z_0$
- At high frequencies matched terminations could be realized much easier compared to short and open terminations

2-port network with source and load



$$V_1^+ = \Gamma_S V_1^-$$

$$V_2^+ = \Gamma_L V_2^-$$

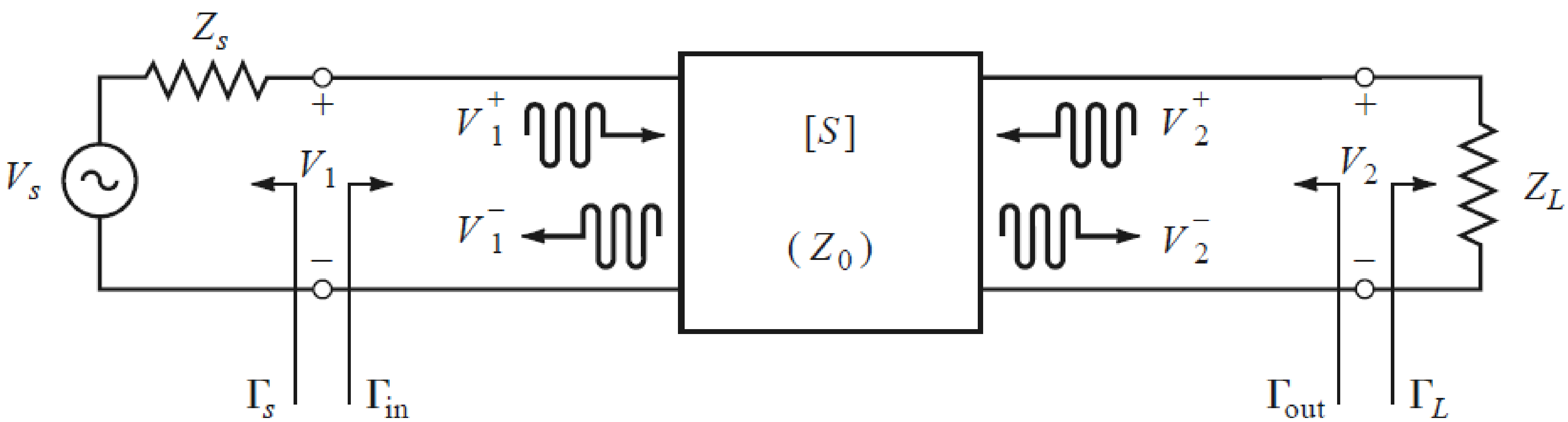
$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

How to calculate:

Γ_{in} and Γ_{out}

Derivation of the input/output reflection coefficient



$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

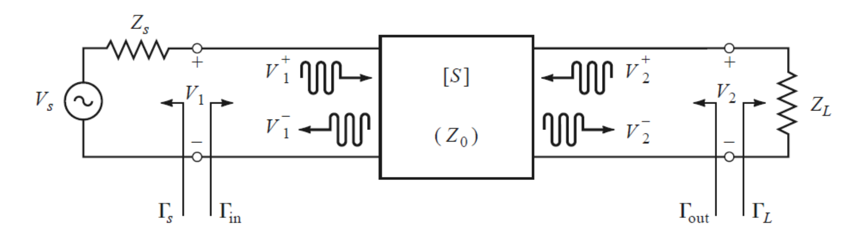
Unilaterial case: $S_{12} = 0$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22}$$

More info: book of Pozar, page 607

Derivation of the input reflection coefficient – guidelines



- Derivation for ~~S_{11}~~ Γ_{in}

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+ \quad \cdots (1)$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+ \quad \cdots (2)$$

$$\Gamma_L = \frac{V_2^+}{V_2^-} \quad \cdots (3)$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+}$$ Definition

Let's use (1) to express V_1^-

$$\Gamma_{in} = \frac{S_{11} V_1^+ + S_{12} V_2^+}{V_1^+}$$

$$\Gamma_{in} = S_{11} + S_{12} \frac{V_2^+}{V_1^+} \quad \cdots (4)$$

Needs better expression

Let's combine (2) and (3)

$$\frac{V_2^+}{\Gamma_L} = S_{21} V_1^+ + S_{22} V_2^+$$

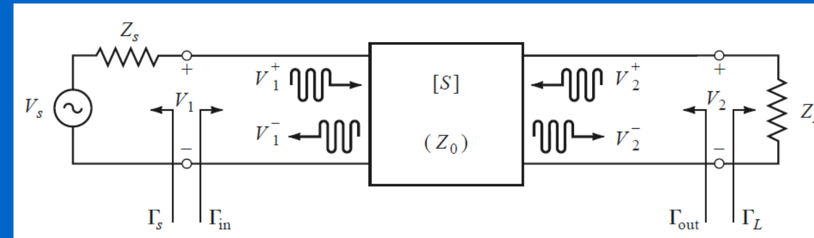
$$V_2^+ \left(\frac{1}{\Gamma_L} - S_{22} \right) = S_{21} V_1^+$$

$$\frac{V_2^+}{V_1^+} = \frac{S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \quad \cdots (5)$$

Let's combine (4) and (5)

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L}$$

Power definitions



- **Power delivered to the load:** $$P_L = \frac{|V_2^-|^2}{2Z_0}\left(1 - |\Gamma_L|^2\right)$$

- **Input power to the network:** $$P_{in} = \frac{|V_1^+|^2}{2Z_0}\left(1 - |\Gamma_{in}|^2\right)$$

- **Power available from the source:** P_{avs}
  P_{in} when the source impedance is conjugately matched to the input impedance $$Z_{in} = Z_S^*$$

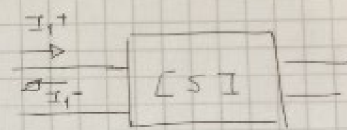
- **Power available from the network:** P_{avn}
  P_L when the load impedance is conjugately matched to the output impedance $$Z_{out} = Z_{load}^*$$

More info: book of Pozar, page 607, More practice: assignment 2

Power definitions – derivation guidelines (1/2)

Derivation of power delivered to network

$$V_1 = V_1^+ + V_1^-$$



$$I_1 = \frac{V_1^+ - V_1^-}{Z_0}$$

→ Gonzalez Chapters 1.3, 1.4

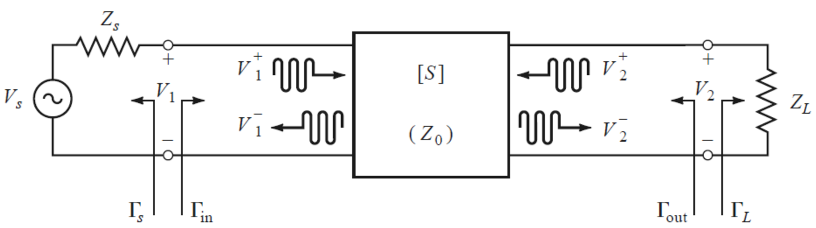
$$P_{in} = \frac{V_1 \cdot I_1^*}{2}$$

$$P_{in} = \frac{(V_1^+ + V_1^-) \cdot (V_1^+ - V_1^-)}{2 \cdot Z_0}$$

$$P_{in} = \frac{|V_1^+|^2 - |V_1^-|^2}{2 \cdot Z_0}$$

$$P_{in} = |V_1^+|^2 \cdot \frac{(1 - |\Gamma_{in}|^2)}{2 \cdot Z_0} \quad \ldots (1)$$

Power definitions – derivation guidelines (2/2)



$$V_1 = V_s \cdot \frac{Z_{in}}{Z_s + Z_{in}} \quad \dots (2)$$

$$V_1 = V_1^+ + V_1^- = V_1^+ \cdot (1 + \Gamma_{in}) \quad \dots (3)$$

After ~~continuing~~ combination (2) and (3)

$$V_1^+ = V_s \cdot \frac{Z_{in}}{Z_{in} + Z_s} \cdot \frac{1}{1 + \Gamma_{in}} \quad \dots (4)$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \quad (5)$$

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} \Rightarrow Z_s = Z_0 \frac{1 + \Gamma_s}{1 - \Gamma_s} \quad (6)$$

After combination (1), (5), (6)

$$V_1^+ = \frac{V_s}{2} \frac{1 - \Gamma_s}{1 - \Gamma_s \Gamma_{in}} \quad \dots (7)$$

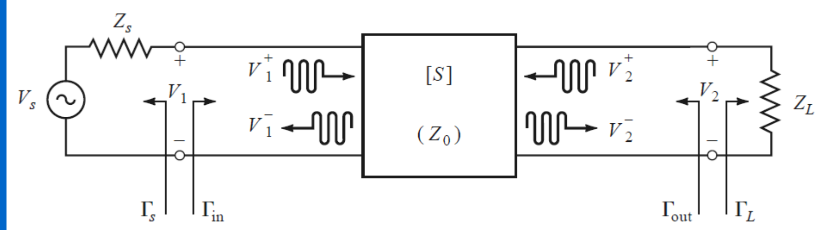
After combination (1) and (7)

$$P_{in} = \frac{|V_s|^2}{8 Z_0} \frac{|1 - \Gamma_s|^2}{|1 - \Gamma_s \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

Meaning of the word gain

- **Requires additional specifications:**
  - ***What is the load / source impedance***
  - ***Voltage, current or power gain***
  - ***Which power gain?***
- **Analog designers often use voltage gain.**
- **RF / microwave designers often use power gain.**

Gain definitions



Power gain G: Ratio of the power dissipated in the load Z_L to the power delivered to the input of the two-port network

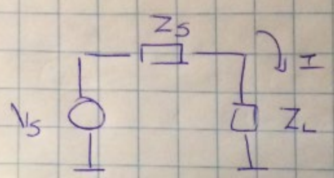
Available power gain G_A: Ratio of the power available from the two-port network to the power available from the source. Assumes conjugate matching of source and load impedance.

Transducer power gain G_T: Ratio of the power delivered to the load to the power available from the source. Assumes a matched source impedance.

Unilateral transducer power gain G_{TU}:
Transducer power gain for a device with S_{12}=0

More info: book of Gonzalez, page 92

Load condition for max real power



$$I = \frac{V_S}{Z_S + Z_L}$$

$$V_L = \frac{Z_L}{Z_L + Z_S} \cdot V_S$$

$$P_{Re} = Re\{V_L \cdot I^*\}$$

We are interested in Real power

$$P_{Re} = Re\left\{\frac{Z_L}{Z_L + Z_S} \cdot \frac{1}{(Z_S + Z_L)^*} \cdot V_S^2\right\}$$

$$P_{Re} = Re\left\{\frac{Z_L}{|Z_L + Z_S|^2} V_S^2\right\}$$

$$P_{Re} = Re\{Z_L\} \frac{V_S^2}{|Z_L + Z_S|^2}$$

$$P_{Re} = \frac{R_L}{(R_L + R_S)^2 + (X_L + X_S)^2} \cdot V_S^2$$

$$\max P_{Re} \Rightarrow (1)\ \boxed{X_L = -X_S}$$

$$(2)\ \left(\frac{R_L}{(R_L + R_S)^2}\right)' = 0$$

$$(R_L + R_S)^2 - R_L \cdot 2 \cdot (R_L + R_S) = 0$$

$$R_L + R_S - 2R_L = 0$$

$$\boxed{R_S = R_L}$$

$$\boxed{Z_L = Z_S^*}$$

conjugate match leads to max real power on load.

Amplifier gains: Equations

Power gain:

$$G = \frac{P_L}{P_{\text{in}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{\text{in}}|^2\right) |1 - S_{22}\Gamma_L|^2}$$

Available power gain:

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{\text{out}}|^2\right)}$$

Transducer power gain:

$$G_T = \frac{P_L}{P_{\text{avs}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right) \left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_S\Gamma_{\text{in}}|^2 |1 - S_{22}\Gamma_L|^2}$$

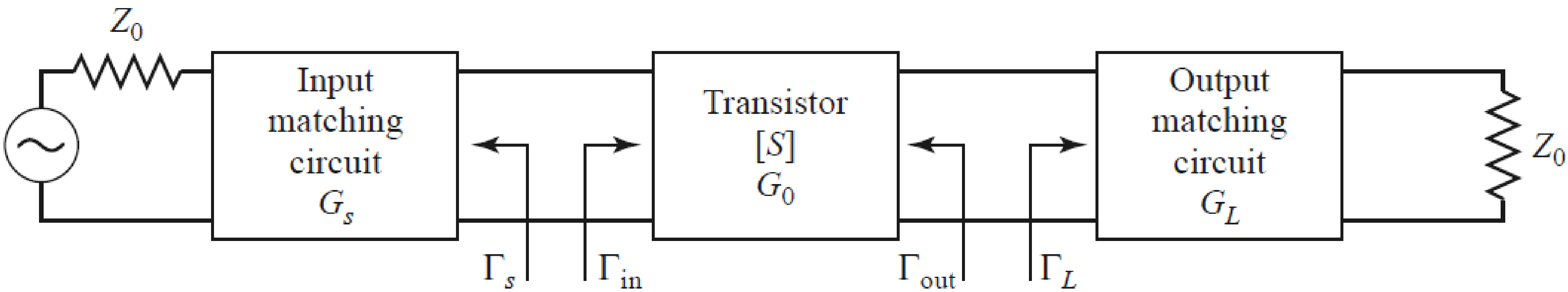
Unilateral transducer power gain:

$$G_{TU} = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{\text{in}}\Gamma_S|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$$

More info: book of Gonzalez, page 92

General transistor amplifier circuit



$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

Unilaterial case

$$G_T = G_S G_0 G_L$$

$$G_{T,dB} = G_{S,dB} + G_{0,dB} + G_{L,dB}$$

Remark: If $S_{12}=0$ then: $\Gamma_{out} = S_{22}$ and $\Gamma_{in} = S_{11}$

Circles of constant power gain

Unilateral transducer power gain G_{TU} :

$$G_{TU} = \left.\frac{P_L}{P_{AVS}}\right|_{\underline{S}_{12}=0}$$

$$= \underbrace{\frac{1-\left|\underline{\Gamma}_S\right|^2}{\left|1-\underline{\Gamma}_S \underline{S}_{11}\right|^2}}_{} \left|\underline{S}_{21}\right|^2 \underbrace{\frac{1-\left|\underline{\Gamma}_L\right|^2}{\left|1-\underline{\Gamma}_L \underline{S}_{22}\right|^2}}_{}$$

$$= G_S \cdot G_0 \cdot G_L$$

Impact of the input matching network on the gain

Transistor gain

Impact of the output matching network on the gain

For which values of Γ_S do we achieve the desired value of G_S?

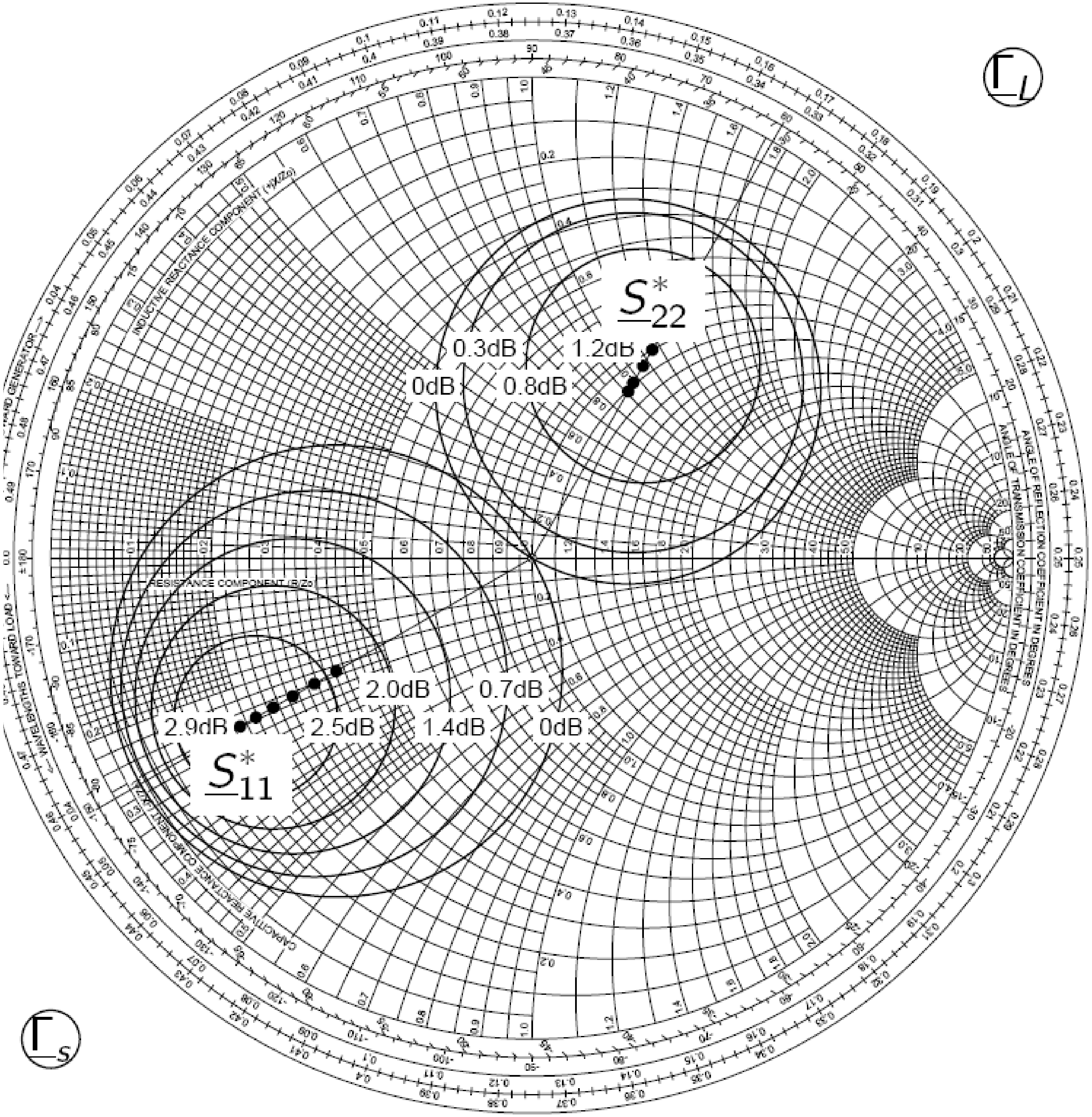
For which values of Γ_L do we achieve the desired value of G_L?

The values of Γ_S that lead to a constant G_S are situated on circles in the complex Γ plane.

The values of Γ_L that lead to a constant G_L are situated on circles in the complex Γ plane.

These circles are called: **Constant gain circles**

For $\Gamma_S = S^*_{11}$ maximum G_S is obtained.
For $\Gamma_L = S^*_{22}$ maximum G_L is obtained.



Circles of constant power gain

Maximum gain of the input and output matching networks

$$G_{S_{\max}} = \frac{1}{1 - |S_{11}|^2}, \quad \text{for } \Gamma_S = S^*_{11}$$

$$G_{L_{\max}} = \frac{1}{1 - |S_{22}|^2}. \quad \text{for } \Gamma_L = S^*_{22}$$

Normalized gain factors g_s and g_L

$$g_S = \frac{G_S}{G_{S_{\max}}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}(1 - |S_{11}|^2),$$

$$g_L = \frac{G_L}{G_{L_{\max}}} = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}(1 - |S_{22}|^2).$$

Center and radius of the constant gain circle for the input and output matching network

$$C_S = \frac{g_S S_{11}^*}{1 - (1 - g_S)|S_{11}|^2},$$

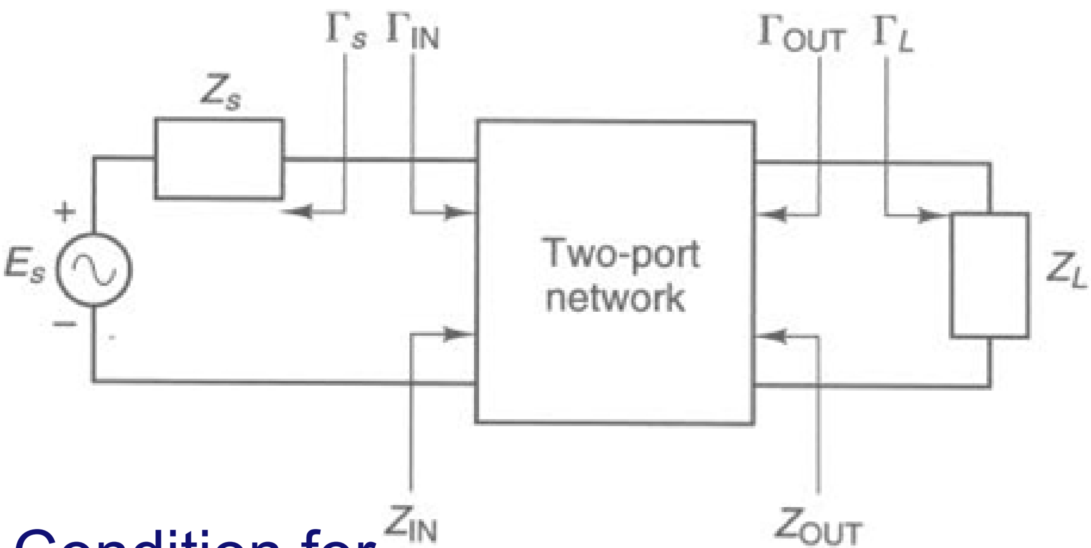
$$R_S = \frac{\sqrt{1 - g_S}\,(1 - |S_{11}|^2)}{1 - (1 - g_S)|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L)|S_{22}|^2},$$

$$R_L = \frac{\sqrt{1 - g_L}\,(1 - |S_{22}|^2)}{1 - (1 - g_L)|S_{22}|^2}$$

More info: book of Pozar, page 624, book of Gonzalez, page 103

Stability discussion of 2-port circuits



Stability analysis of an amplfier means: Investigation whether there can be oscillations

Condition for **"unconditionally stable"** device:

$$\text{for all} \left|\underline{\Gamma}_L\right| < 1 \text{ and } \left|\underline{\Gamma}_S\right| < 1$$

$$\Rightarrow \begin{cases} \left|\Gamma_{in}\right| = \left|S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1 \\ \left|\Gamma_{out}\right| = \left|S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}\right| < 1 \end{cases}$$

If at a given frequency there are source and load reflection coefficients, for which this condition does not hold the device is called **"potentially unstable"**.

Simplification: Unilateral case

$$\text{for all } |\underline{\Gamma}_L| < 1 \text{ and } |\underline{\Gamma}_S| < 1$$

General case:

$S_{12} \neq 0$

$$\Rightarrow \begin{cases} |\Gamma_{in}| = \left| S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \\ |\Gamma_{out}| = \left| S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1 \end{cases}$$

Unilateral case:

$S_{12} = 0$

$$\Rightarrow \begin{cases} |\Gamma_{in}| = |S_{11}| < 1 \\ |\Gamma_{out}| = |S_{22}| < 1 \end{cases}$$

Stability - background

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$Z = R + jX$$

$$\Gamma = \frac{R + jX - Z_0}{R + jX + Z_0}$$

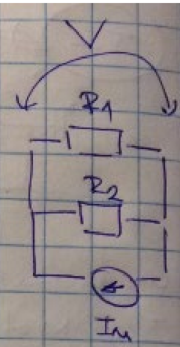
$$Z_0 = 50\,\Omega = R_0$$

$$\Gamma = \frac{R - R_0 + jX}{R + R_0 + jX}$$

$$|\Gamma|^2 = \frac{(R - R_0)^2 + X^2}{(R + R_0)^2 + X^2}$$

$$R > 0 \Rightarrow |\Gamma| < 1$$

$$R < 0 \Rightarrow |\Gamma| > 1$$



$$R_e = R_1 \parallel R_2$$

$$R_e = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$R_1 = -R_2 \Rightarrow R_e = \infty$$

$$V = R_e \cdot I_n$$

$$\boxed{R_1 = -R_2 \Rightarrow V = \infty}$$

oscillations

Input stability circles

Boundary between stability and instability is given by:

$$|\Gamma_{OUT}| = 1$$

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| = 1$$

$$\left| \Gamma_s - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

Circle equation
in the complex Γ-plane

$$r_s = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}$$

$$C_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \qquad \text{(center)}$$

This circle is called the **output stability circle**. It is the boundary between the region Γ_S that lead to a stable or an unstable reflection amplifier.

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

An equivalent derivation of the output reflection coefficient
leads to the **input stability circle**.

Output stability circles

Boundary between stability and instability is given by:

$$\left|\underline{\Gamma}_{in}\right| = 1$$

$$\Leftrightarrow \left| \underline{S}_{11} + \frac{\underline{S}_{12}\underline{S}_{21}\underline{\Gamma}_L}{1-\underline{S}_{22}\underline{\Gamma}_L} \right| = 1$$

$$\left| \underline{\Gamma}_L - \frac{\underline{S}_{22}^* - \underline{\Delta}^* \underline{S}_{11}}{\left|\underline{S}_{22}\right|^2 - \left|\underline{\Delta}\right|^2} \right|^2 = \left| \frac{\underline{S}_{12}\underline{S}_{21}}{\left|\underline{S}_{22}\right|^2 - \left|\underline{\Delta}\right|^2} \right|^2$$

Circle equation
in the complex Γ-plane

$$\left|\underline{\Gamma}_L - \underline{C}_L\right|^2 = \left|R_L\right|^2$$

This circle is called the **output stability circle**. It is the boundary between the region Γ_L that lead to a stable or an unstable reflection amplifier.

$$C_L = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \qquad \text{(center)},$$

$$R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}.$$

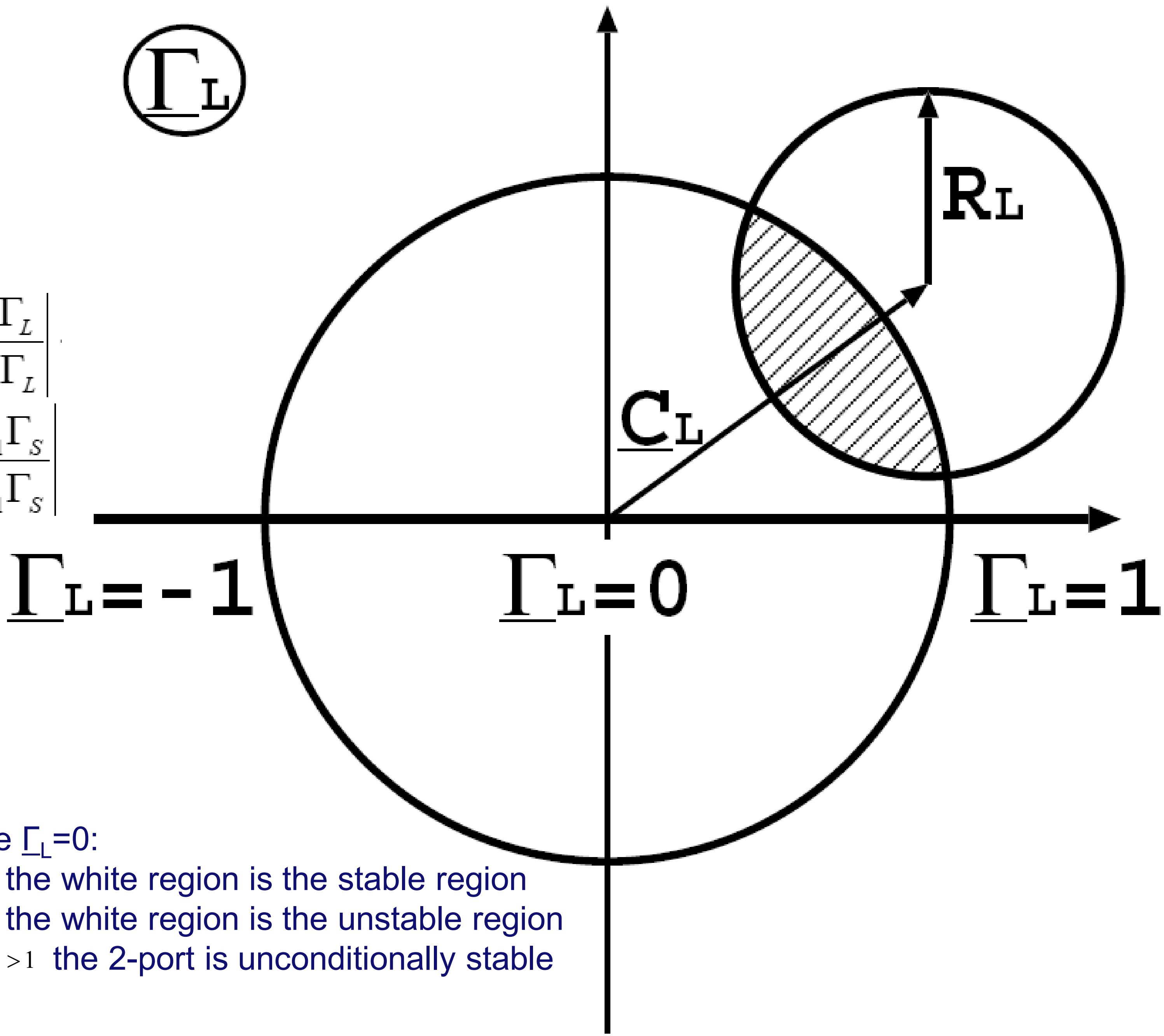
$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

An equivalent derivation of the output reflection coefficient
leads to the **input stability circle**.

Construction Of the Output Stability circle

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right|$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right|$$



1) Consider the case $\Gamma_L=0$:

if $|\Gamma_{in}|=|S_{11}|<1$, then the white region is the stable region

if $|\Gamma_{in}|=|S_{11}|>1$, then the white region is the unstable region

2) If $|S_{11}|<1$ and $\left||C_L|-R_L\right|>1$ the 2-port is unconditionally stable

Tests for unconditional stability

If $$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$$

Rollet stability factor K

and $$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + \Delta^2}{2|S_{12}S_{21}|} > 1$$

than the 2-port is unconditionally stable.

Unilateral case: $S_{12}=0$

Conditions for unconditional stability:

$$|\underline{S}_{11}| < 1$$

$$|\underline{S}_{22}| < 1$$

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