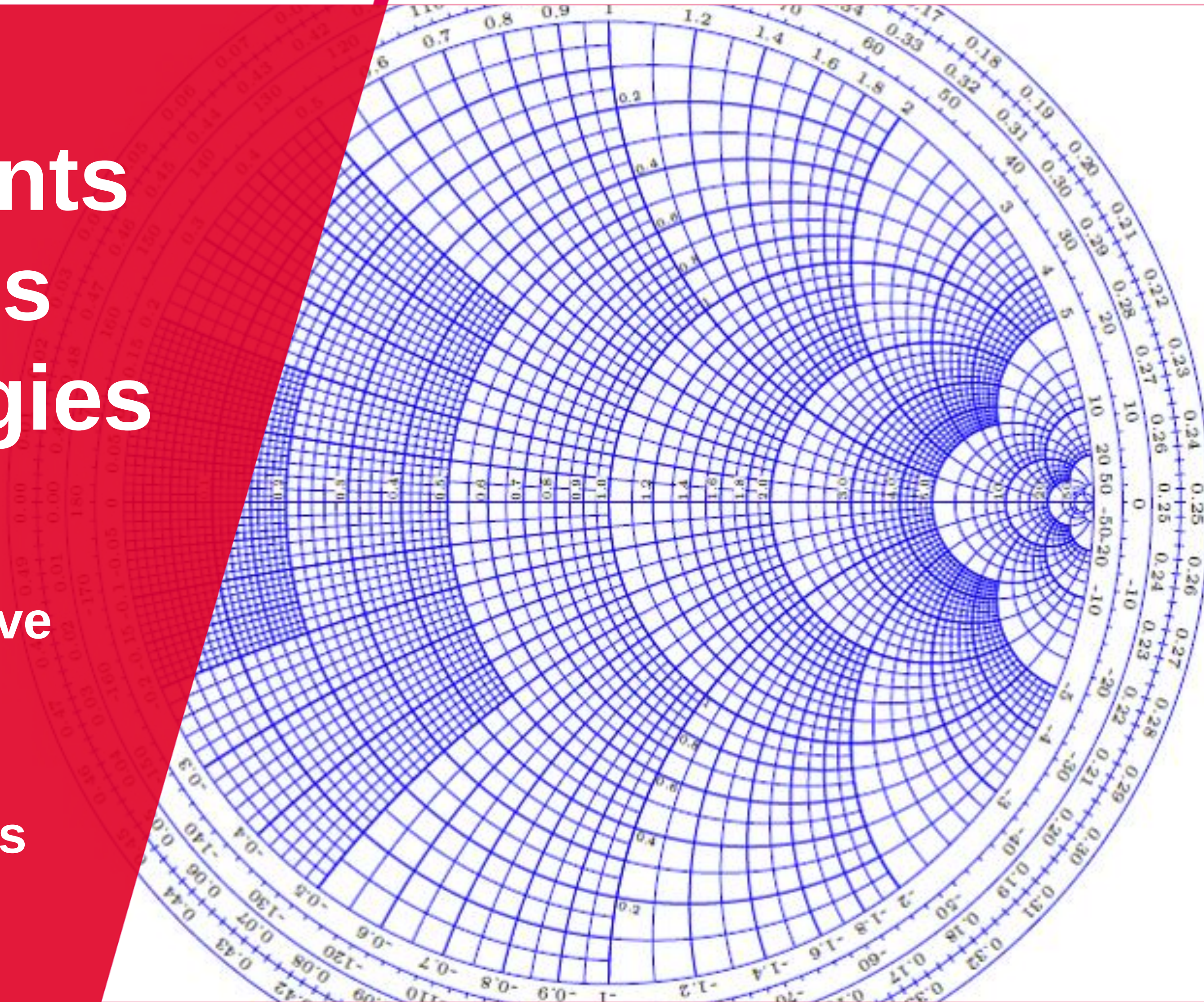
Technische Universiteit
Eindhoven
University of Technology

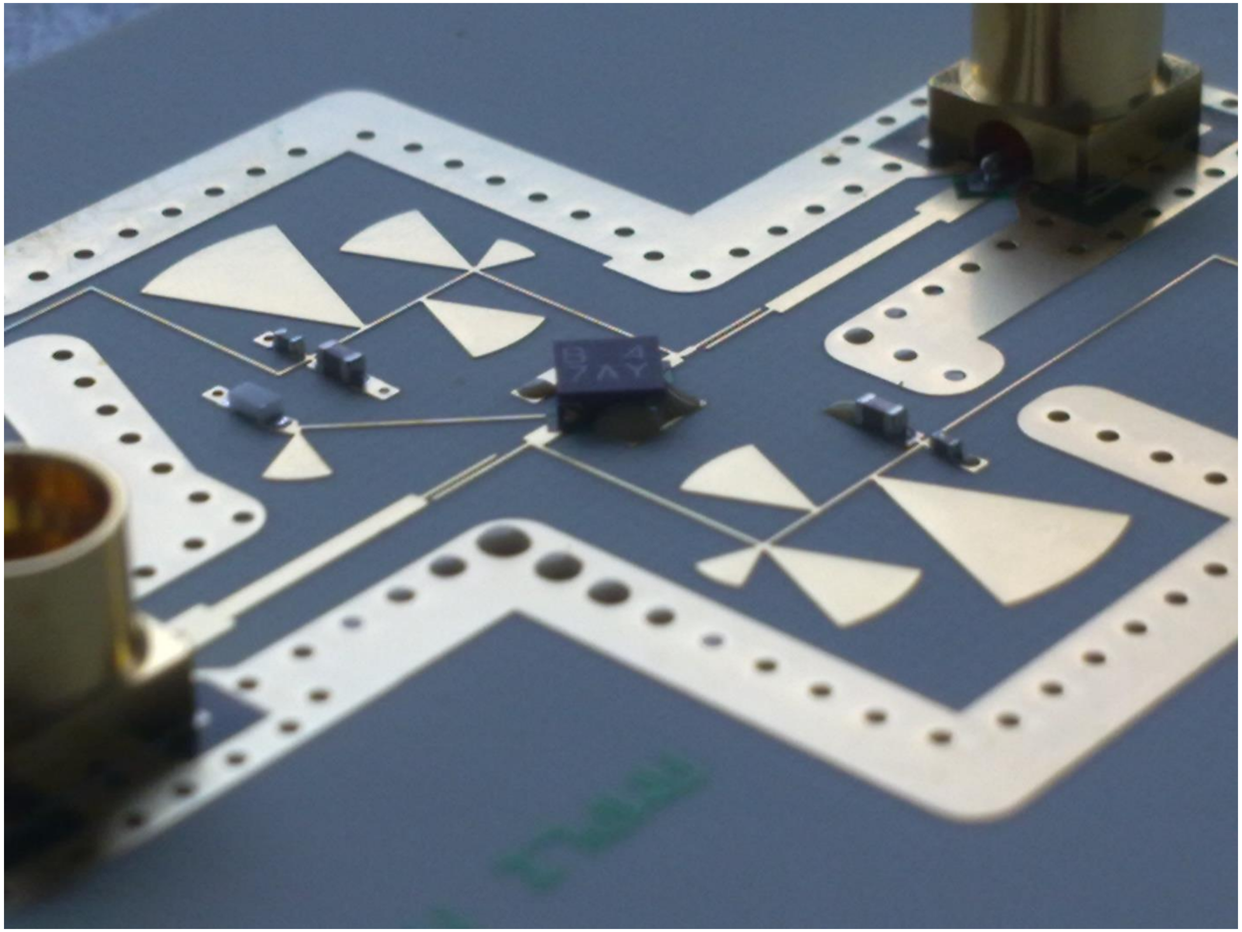
Components in Wireless Technologies

Module 2:
Passive Microwave Networks

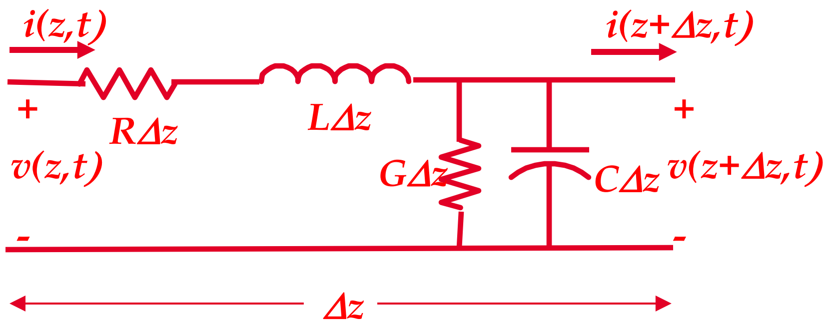
Sander Bronckers



Transmission Lines of a 24 GHz Low-Noise Amplifier



Telegrapher equations (Frequency Domain)



$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

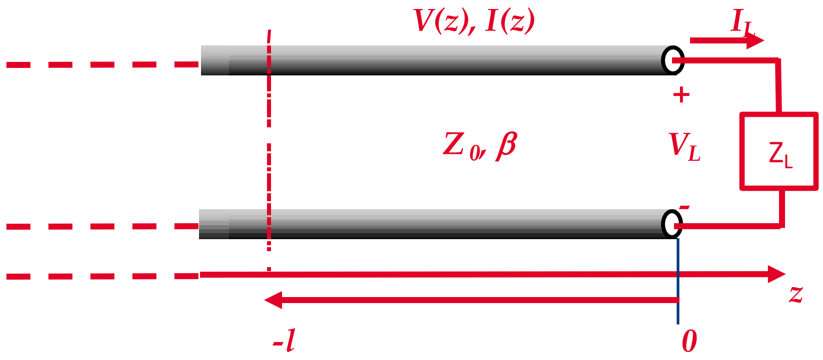
$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

$$\boxed{\frac{\partial v(z,t)}{\partial t} \leftrightarrow j\omega V(z)}$$

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

The terminated lossless Transmission Line



Total voltage and current at the load at $z = 0$:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

Rewriting gives:

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$

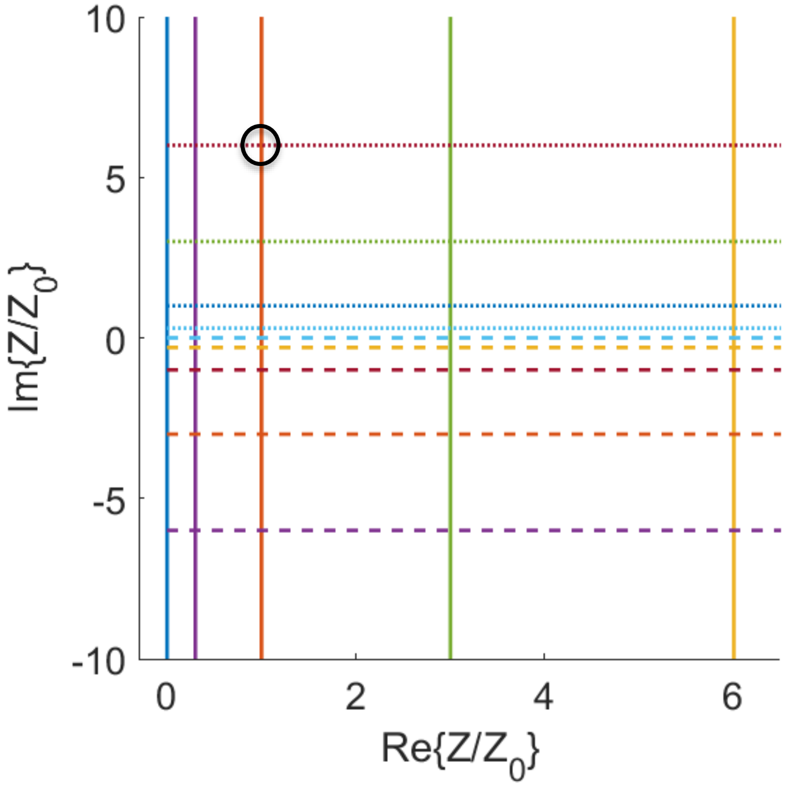
$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection coëfficiënt

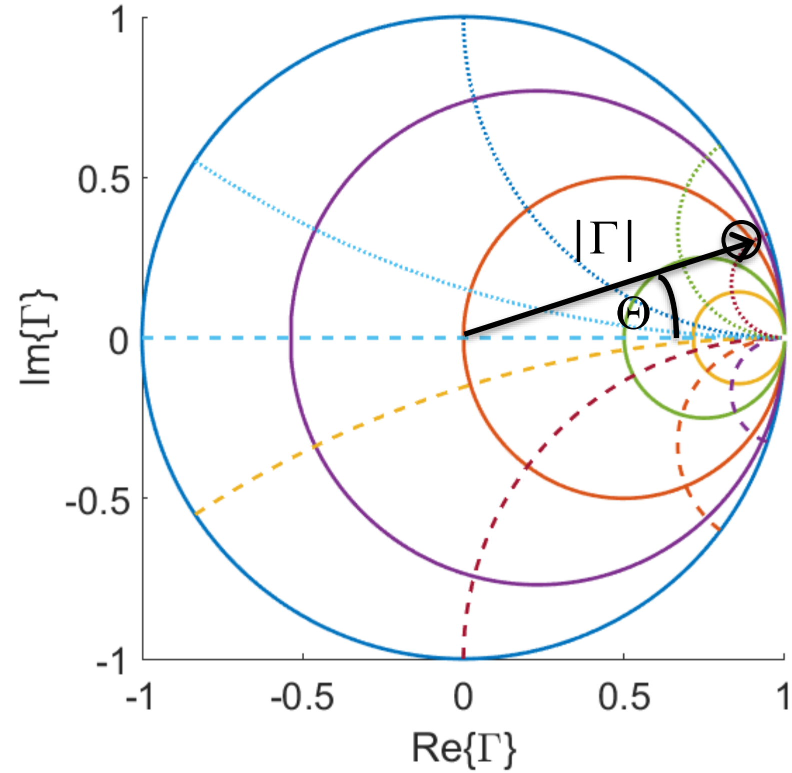
“Derivation” of the Smith Chart

Im{Z} = const.

Z domain



Γ domain



Microwave Networks

Learning goals

- Be able to calculate and interpret S-parameter matrices of microwave networks.
- Be able to explain basic microwave networks.

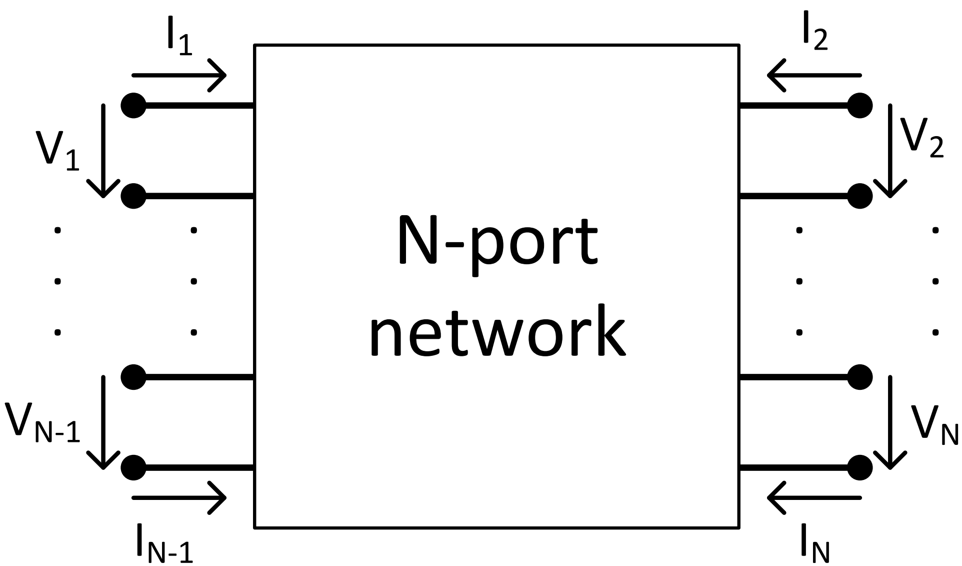
Microwave Networks

Content

- Microwave network matrices
- Impedance Matching and Tuning
- Power Dividers and Directional Couplers
- Application example: Vector Network Analyser

Impedance and Admittance Matrices

A recall of electronic circuit theory



$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

and

$$[I] = [Y][V]$$

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0, k\neq j}$$

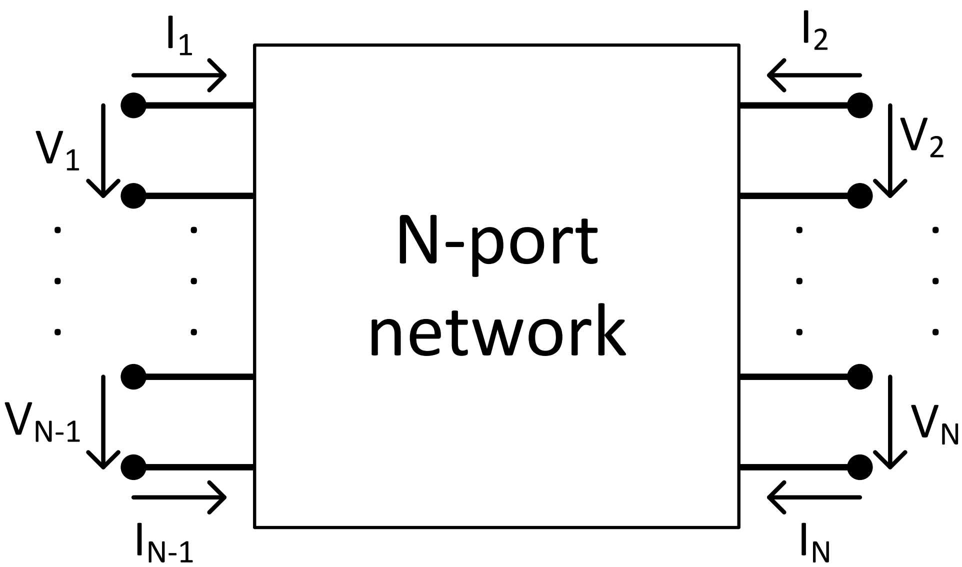
$$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0, k\neq j}$$

hence

$$[Y] = [Z]^{-1}$$

Impedance and Admittance Matrices

Microwave circuits



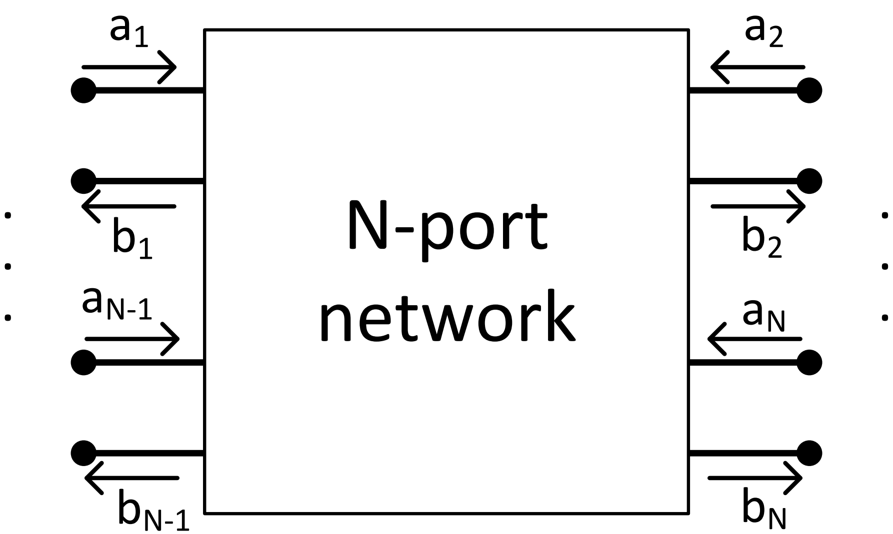
$$V_n = V_n^+ + V_n^-$$
$$I_n = I_n^+ - I_n^-$$

- Voltage and current difficult to measure (either separate measurement of V_n^+ and V_n^- or of standing wave pattern required)
- Voltage and current difficult to define for non-TEM transmission lines.

➔ Introduction of Scattering Parameters

Scattering Parameters

Using power relations



The power delivered to a port is equal to the power of the incident wave minus the power of the reflected wave!

- Define wave amplitudes such that we obtain physically meaningful power relations:

$$a_n = \frac{V_n^+}{\sqrt{Z_{0n}}}, \quad b_n = \frac{V_n^-}{\sqrt{Z_{0n}}}$$

- With this definition and

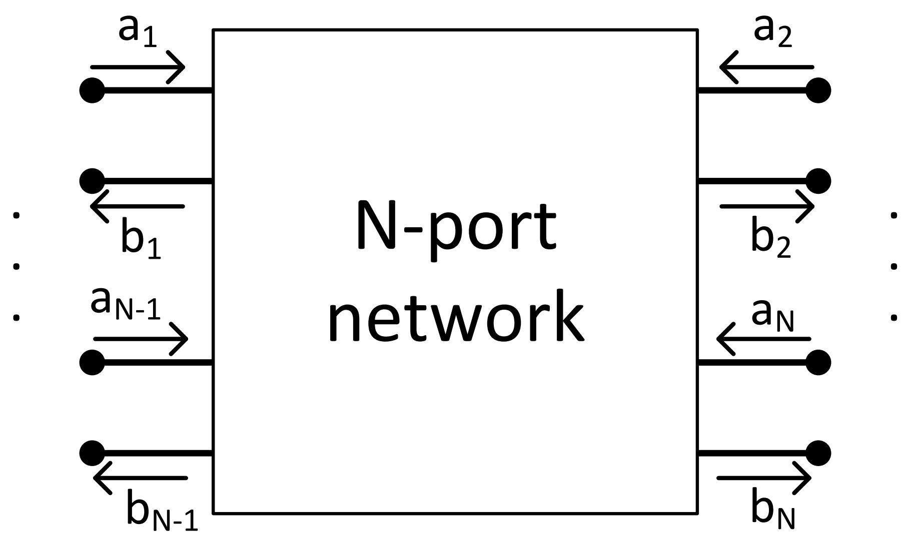
$$V_n = V_n^+ + V_n^- = \sqrt{Z_{0n}}\,(a_n + b_n)$$
$$I_n = I_n^+ - I_n^- = \frac{1}{\sqrt{Z_{0n}}}(a_n - b_n)$$

we obtain

$$P_n = \frac{1}{2}\Re\{V_n I_n^*\} = \frac{1}{2}|a_n|^2 - \frac{1}{2}|b_n|^2$$

Scattering Parameters

Scattering matrix



- S-matrix relates incident and reflected waves between all ports:

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$$

- The elements of the matrix are given by

$$S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0,\, k\neq j}$$

Only port j excited, all other ports are perfectly matched!

For a reciprocal network, $[S]$ is symmetrical, i.e.

$$[S] = [S]^t$$

Scattering Parameters

Scattering matrix for lossless networks

For a lossless network, no power can be delivered to the network. Hence,

$$P = \tfrac{1}{2}\Re\mathrm{e}\{[V]^t[I]^*\} = \tfrac{1}{2}[a]^t[a]^* - \tfrac{1}{2}[b]^t[b]^* = 0,$$

$$[a]^t[a]^* - [a]^t[S]^t[S]^*[a]^* = 0.$$

This can only be satisfied for

$$[S]^t[S]^* = [U] = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix},$$

$$[S]^* = \{[S]^t\}^{-1}$$

For a lossless network, $[S]$ is a unitary matrix!

Scattering Parameters

Scattering matrix for reciprocal networks

Re-cap from circuit theory:

$$[I] = [Y][V]$$
$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix}$$

$$I_m = Y_{mn} V_n,$$
$$I_n = Y_{nm} V_m$$

A circuit is reciprocal if $\forall\, m, n$

$$I_m = I_n \quad \text{for} \quad V_m = V_n.$$

Hence $\quad Y_{nm} = Y_{mn}$

S-parameters:

$$[b] = [S][a]$$
$$\begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}$$

$$b_m = S_{mn} a_n,$$
$$b_n = S_{nm} a_m$$

A circuit is reciprocal if $\forall\, m, n$

$$b_m = b_n \quad \text{for} \quad a_m = a_n.$$

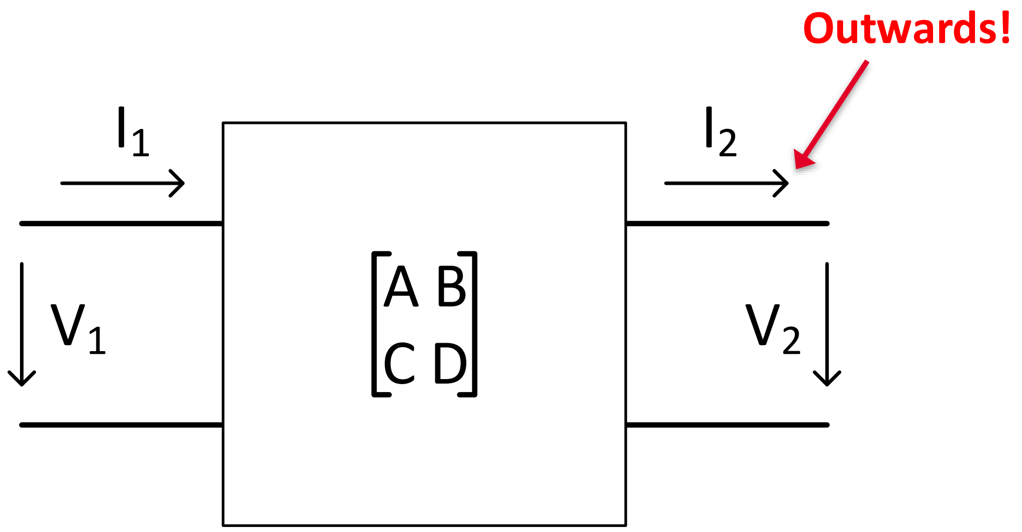
Hence $\quad S_{nm} = S_{mn}$

For a reciprocal network, $[S]$ is a symmetrical matrix!

Transmission (ABCD) Matrix

- Z, Y and S parameters good for microwave networks with **arbitrary number of ports**.
- In practice many microwave networks consist of **cascaded 2-port networks**.
  → Transmission (ABCD) matrix allows easy calculation of overall properties.

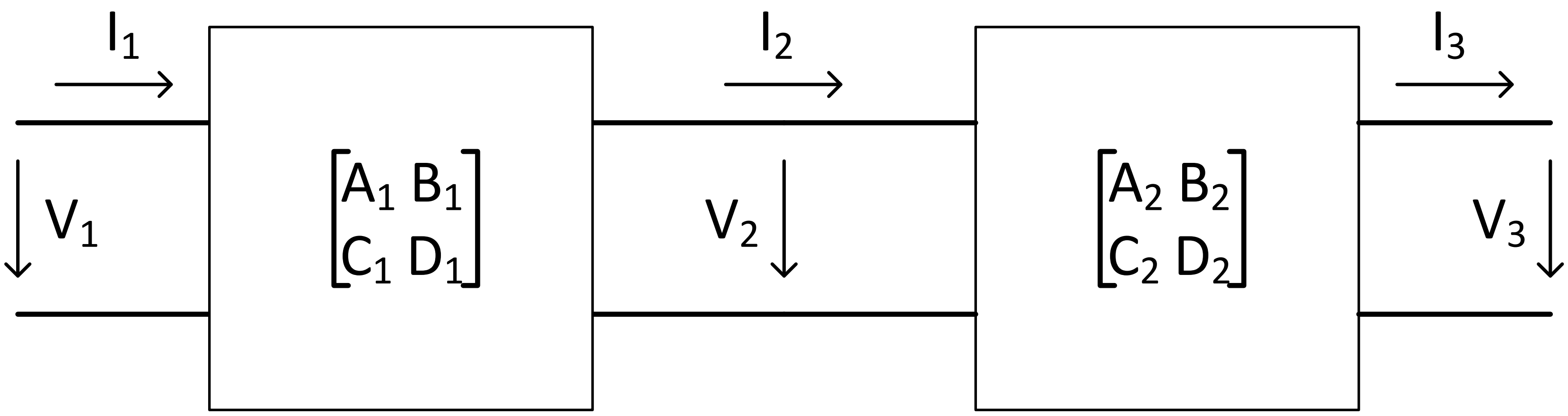
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



Transmission (ABCD) Matrix

- Z, Y and S parameters good for microwave networks with **arbitrary number of ports**.
- In practice many microwave networks consist of **cascaded 2-port networks**.
  → Transmission (ABCD) matrix allows easy calculation of overall properties.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$



Microwave Networks

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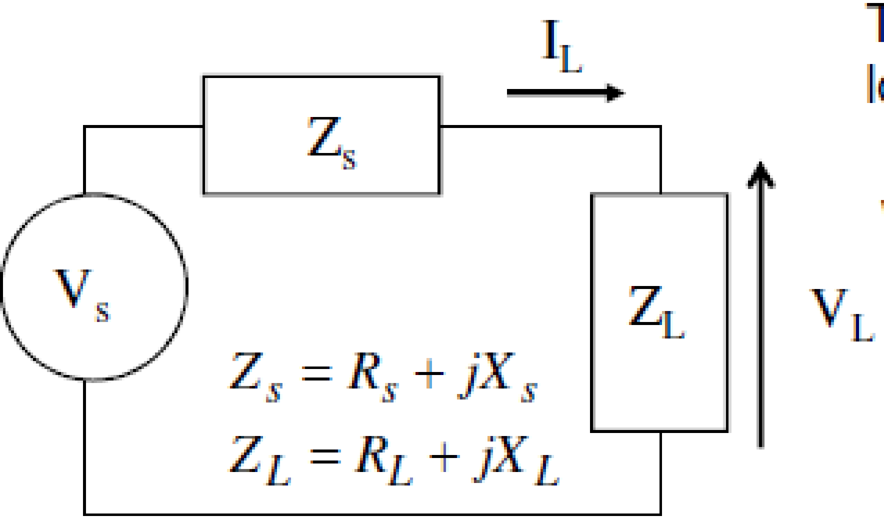
- Application example: Vector Network Analyser

Maximum Power Transfer



Time averaged power dissipated across load Z_L:

$$P_L = \frac{1}{2}\mathrm{Re}\left\{V_L I_L^{\,*}\right\}$$

where

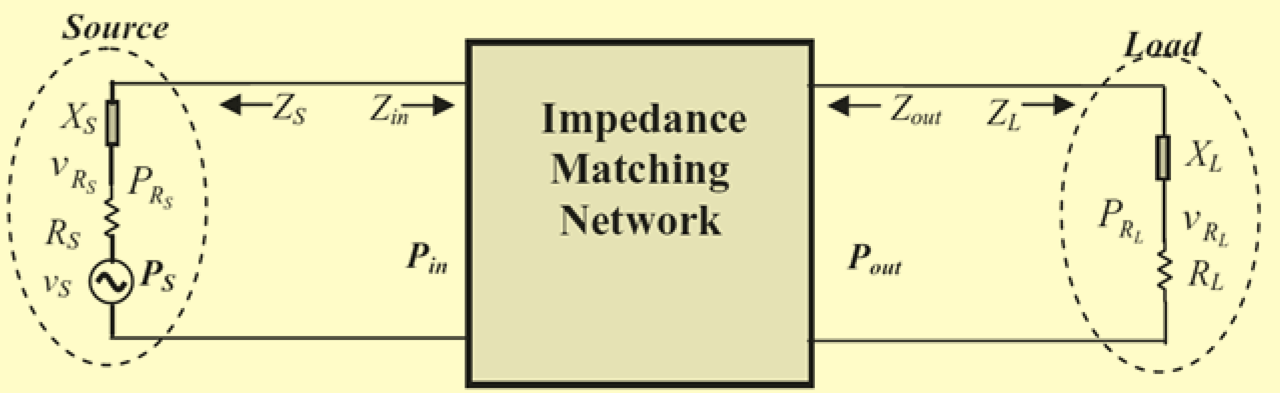
$$V_L = \frac{V_s Z_L}{Z_s + Z_L} \qquad I_L = \frac{V_s}{Z_s + Z_L}$$

$$P_L = \frac{1}{2}\mathrm{Re}\left\{\frac{V_s Z_L}{Z_s + Z_L}\cdot\left(\frac{V_s}{Z_s + Z_L}\right)^*\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|V_s|^2 Z_L}{|Z_s + Z_L|^2}\right\}$$

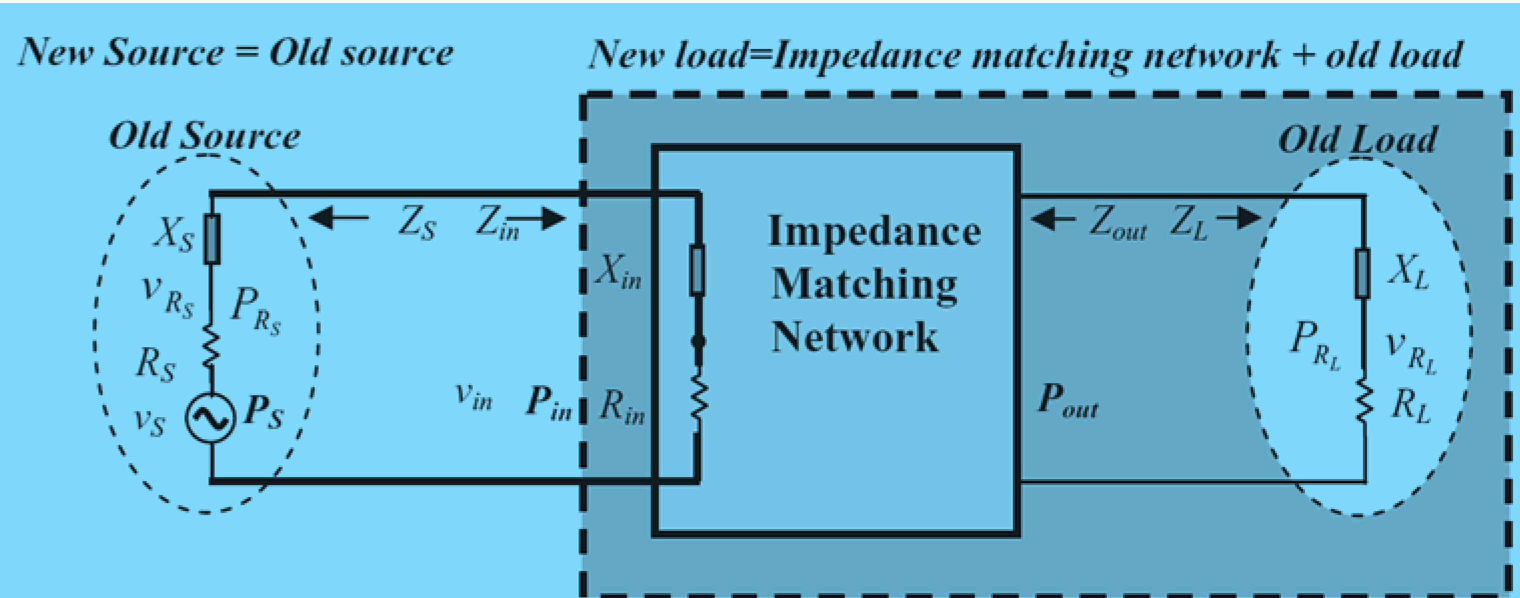
$$\Rightarrow P_L = \frac{1}{2}\frac{|V_s|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

$$P_L = P_L(R_L, X_L)$$

An impedance matching network

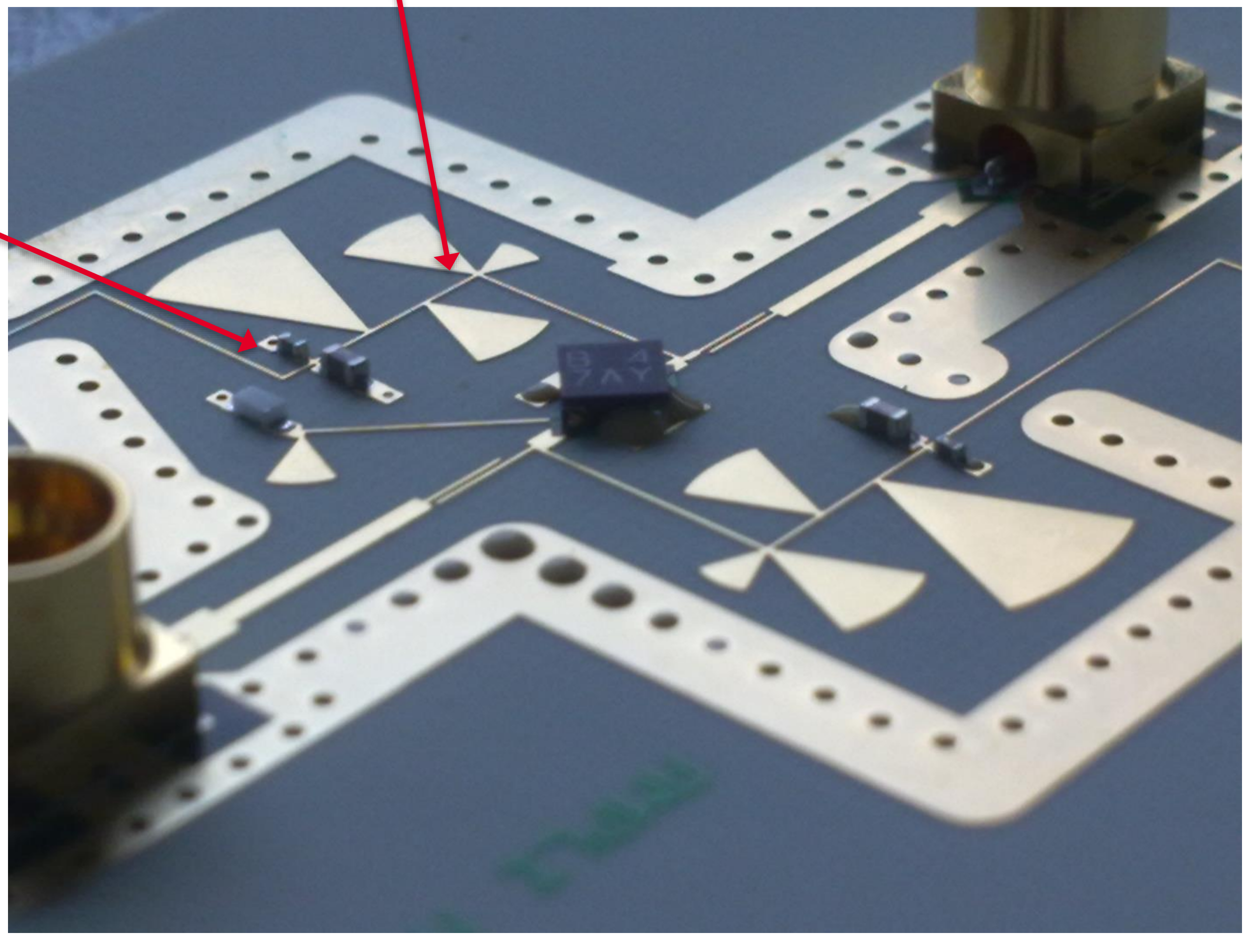


An impedance matching network is inserted between source and load when $Z_S \neq Z_L^*$



Distributed
elements
(transmission lines and stubs)

Lumped
elements
(RLC)

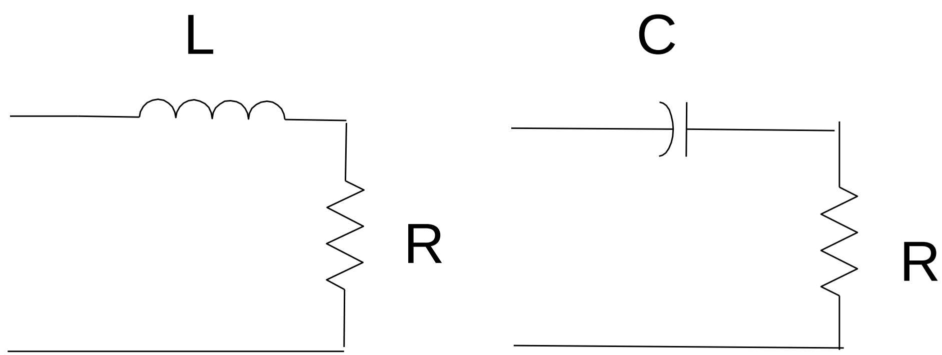


Impedances for serial lumped elements

Serial circuit

$Z = R + j*X$

R : Resistance, X: Reactance



$X > 0$

$$jX = j\omega_0 L$$

$$L = \frac{X}{\omega_0} = \frac{X}{2\pi f_0}$$

$X < 0$

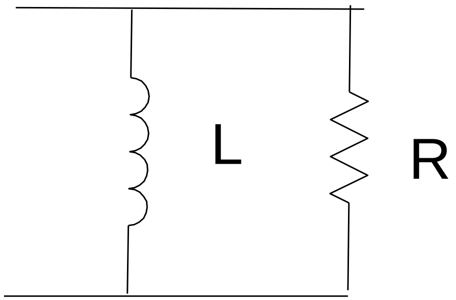
$$jX = \frac{1}{j\omega_0 C} = j\left(-\frac{1}{\omega_0 C}\right)$$

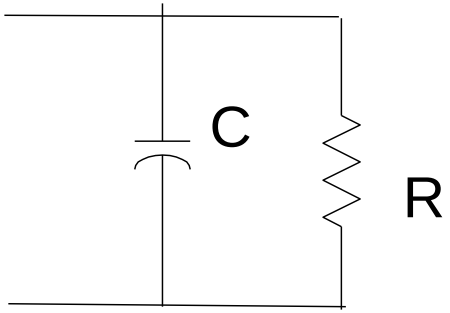
$$C = \frac{1}{\omega_0 |X|} = \frac{1}{2\pi f_0 |X|}$$

Impedances for parallel lumped elements

Parallel circuit   Y = G + j*B

G : Conductance, B: Suspectance





$B < 0$

$$jB = \frac{1}{j\omega_0 L} = j\left(-\frac{1}{\omega_0 L}\right)$$

$$L = \frac{1}{\omega_0 |B|} = \frac{1}{2\pi f_0 |B|}$$

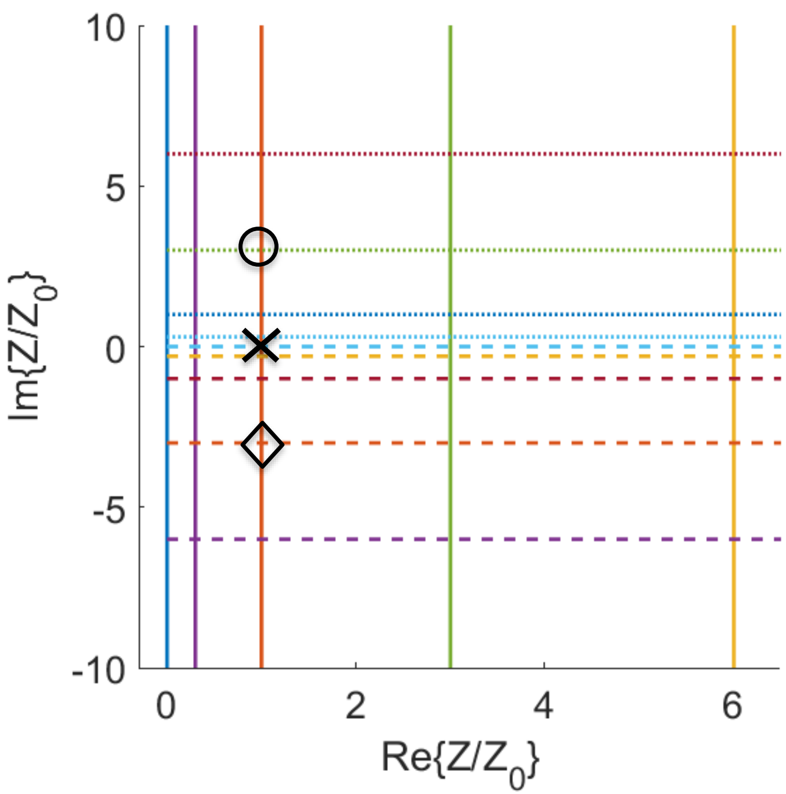
$B > 0$

$$jB = j\omega_0 C$$

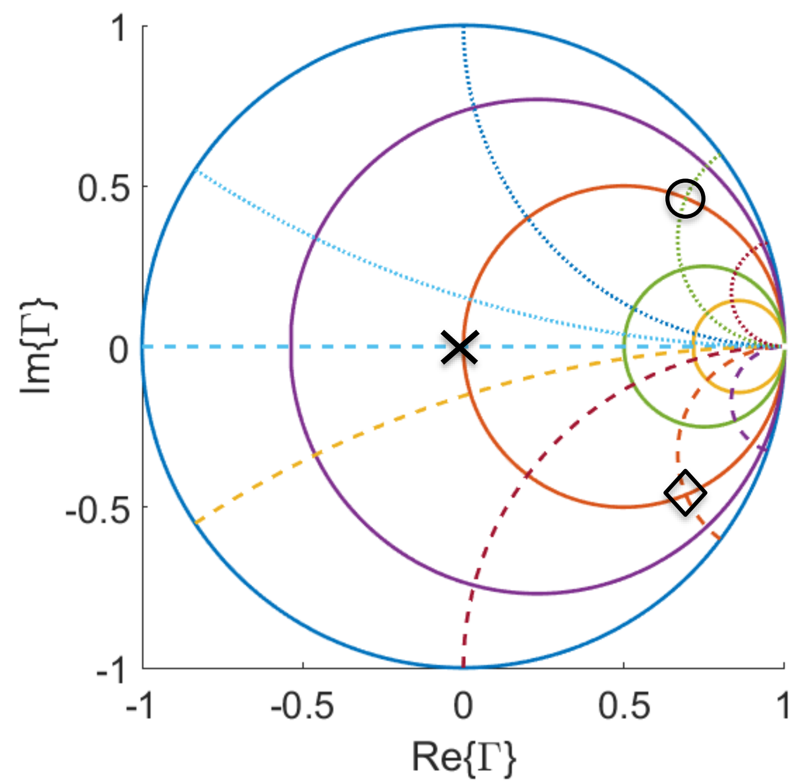
$$C = \frac{B}{\omega_0} = \frac{B}{2\pi f_0}$$

Recap from previous lecture

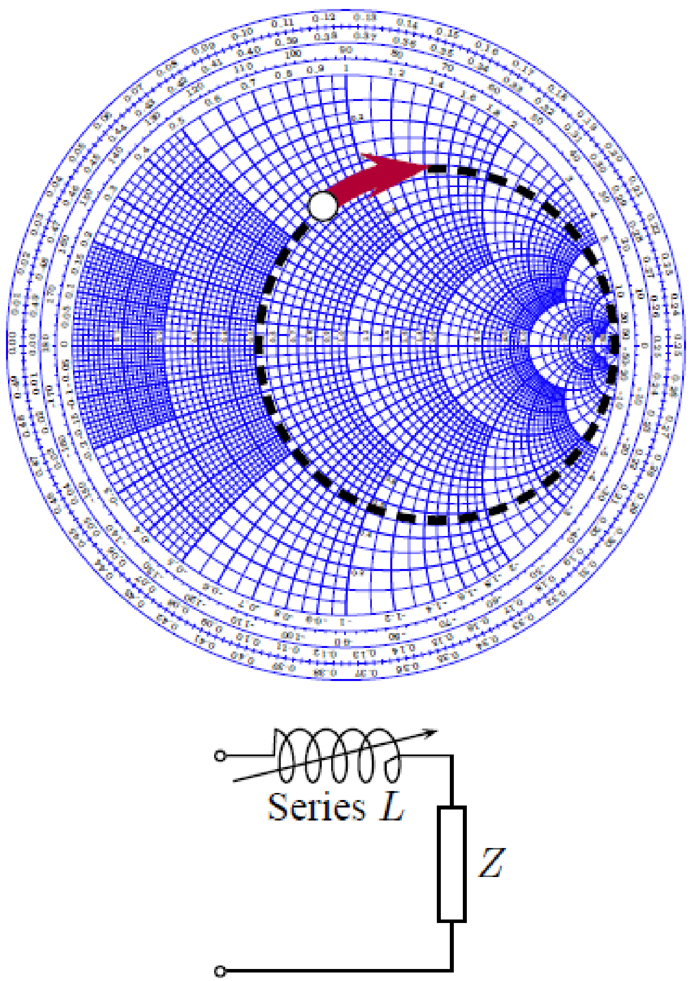
Z domain

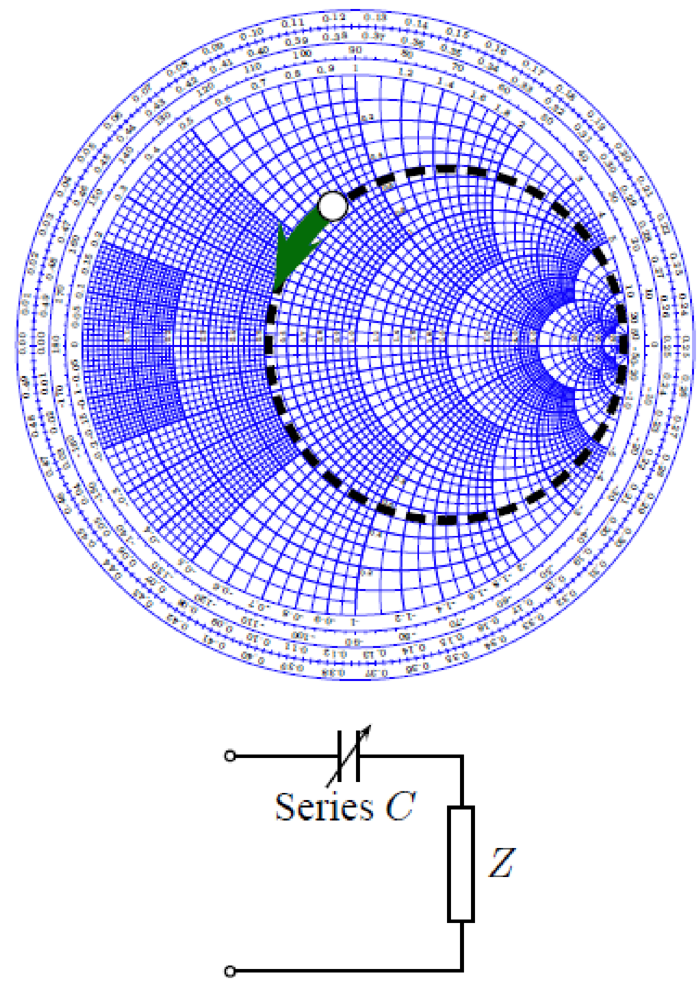


Γ domain

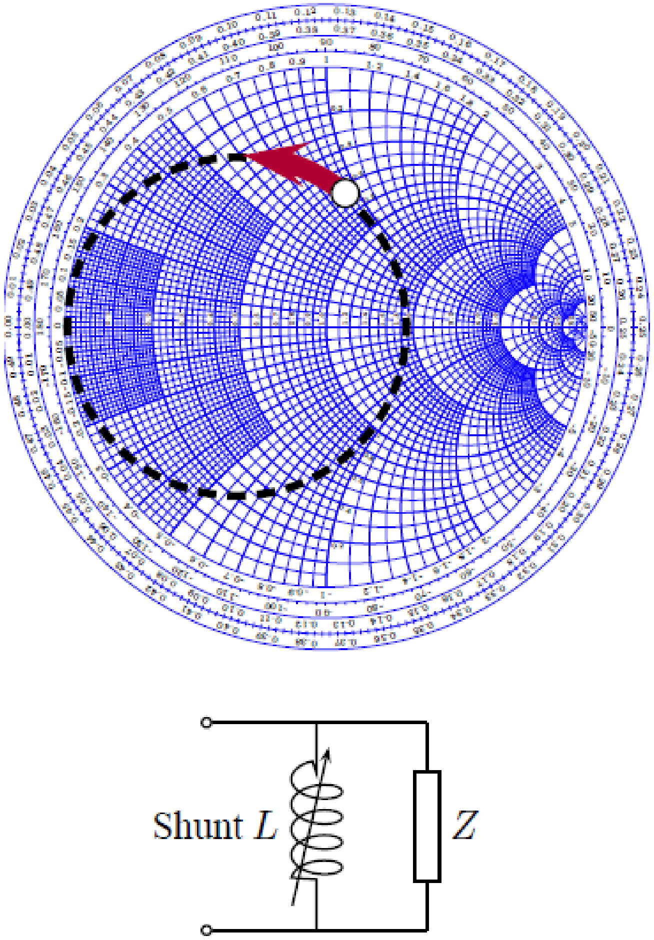


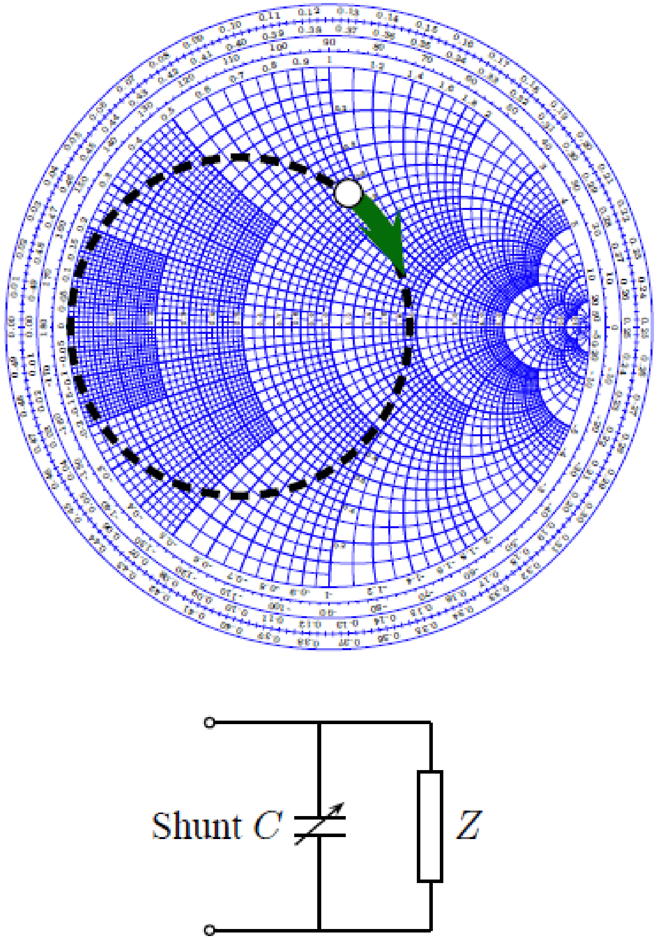
Impact of series component





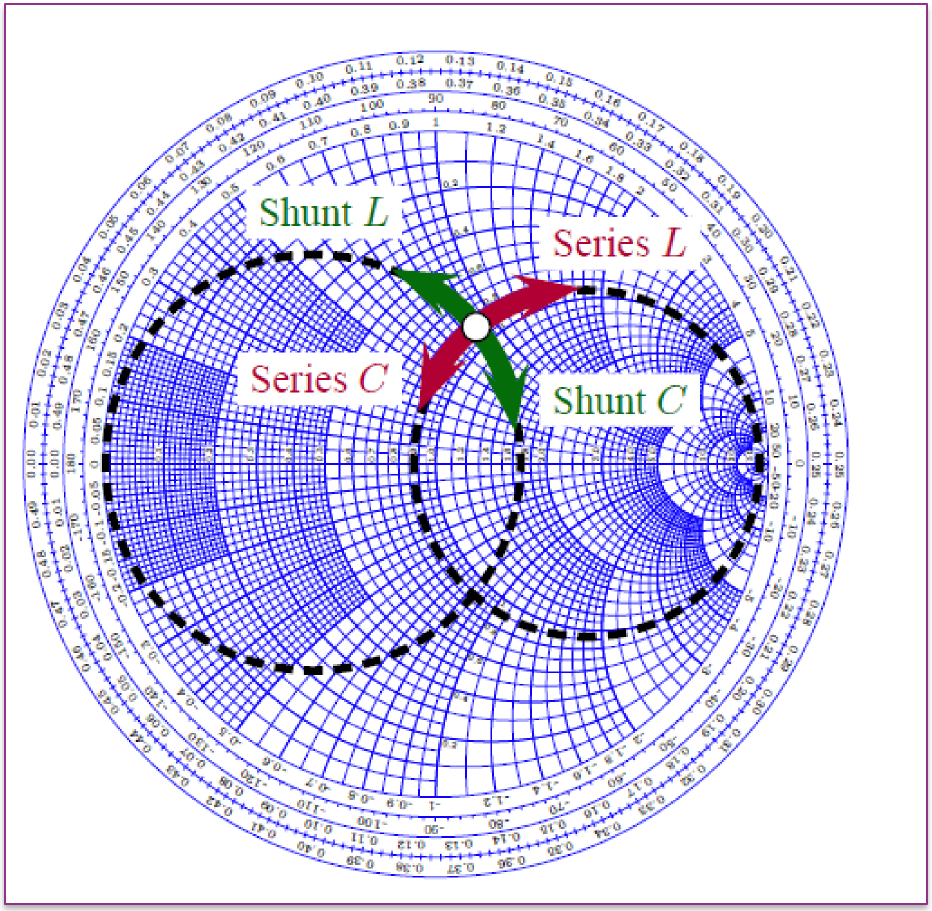
Impact of parallel component





Smith chart and matching

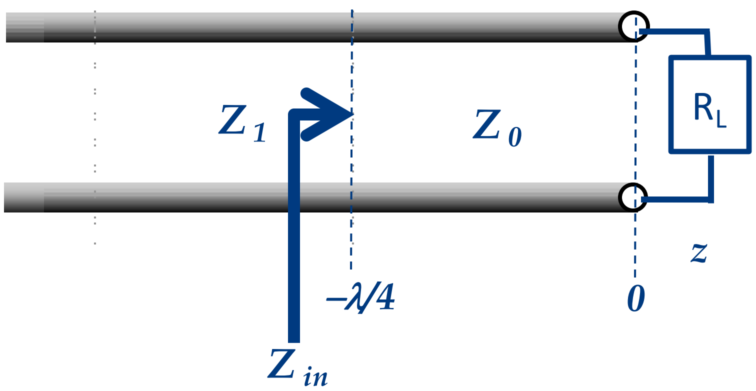
Lumped element



Distributed elements

Using Stubs to mimic lumped components

A resistive value can be achieved using a quarter-wavelength transmission line terminated into a resistor:



$$Z_{in} = Z_0 \frac{R_L + jZ_0 \tan \beta l}{Z_0 + jR_L \tan \beta l}$$

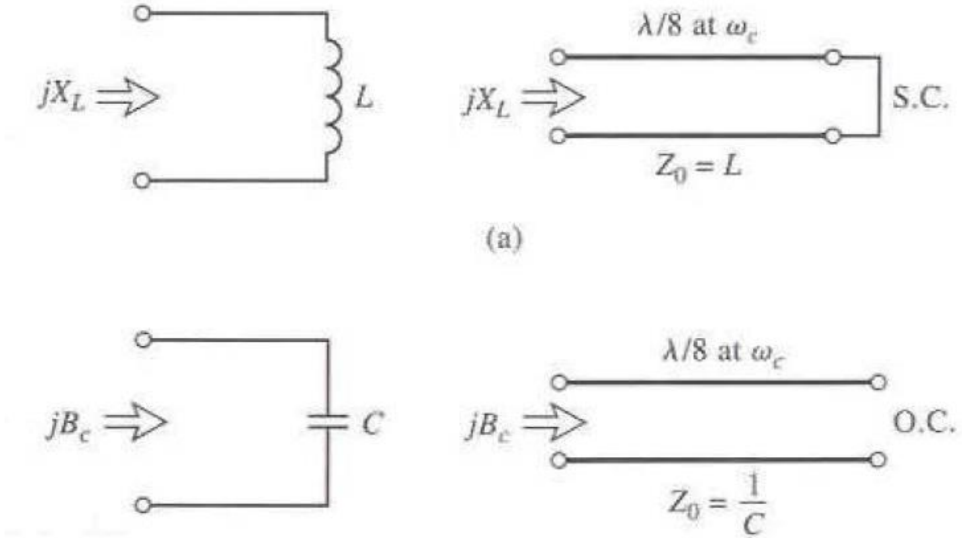
Now $\beta l = (2\pi / \lambda)(\lambda / 4) = \pi / 2$

$$Z_{in} = \frac{Z_0^2}{R_L}$$

Distributed elements

Using Stubs to mimic lumped components

Inductances and capacitances can be achieved using, e.g. λ/8 transmission lines:



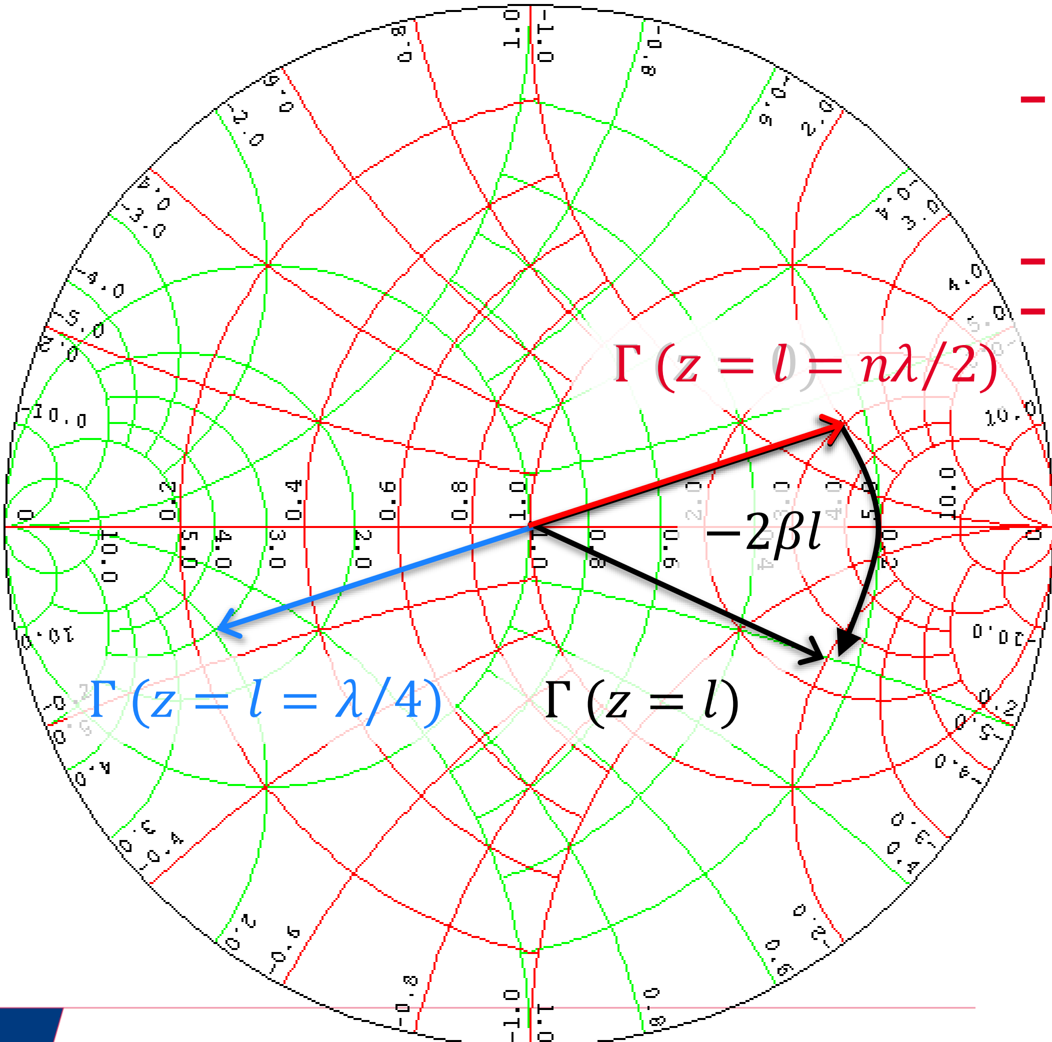
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

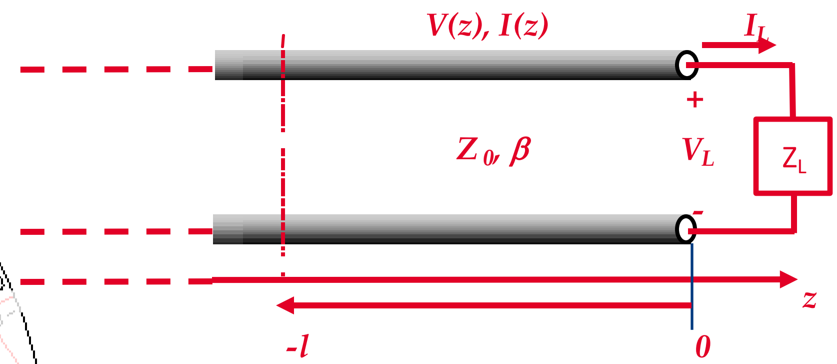
Inductance: $Z_{in}(Z_L = 0) = jZ_0 \tan \beta l = j\Omega Z_0 = j\Omega L$

Capacitance: $Z_{in}(Z_L = \infty) = \dfrac{Z_0}{j \tan \beta l} = \dfrac{Z_0}{j\Omega} = \dfrac{1}{j\Omega C}$

Distributed elements

Impedance transformation using transmission line





$$\Gamma(z=-l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma e^{-2j\beta l}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Rightarrow -2\beta l = -2\pi \quad \text{for } l = \frac{\lambda}{2}$$

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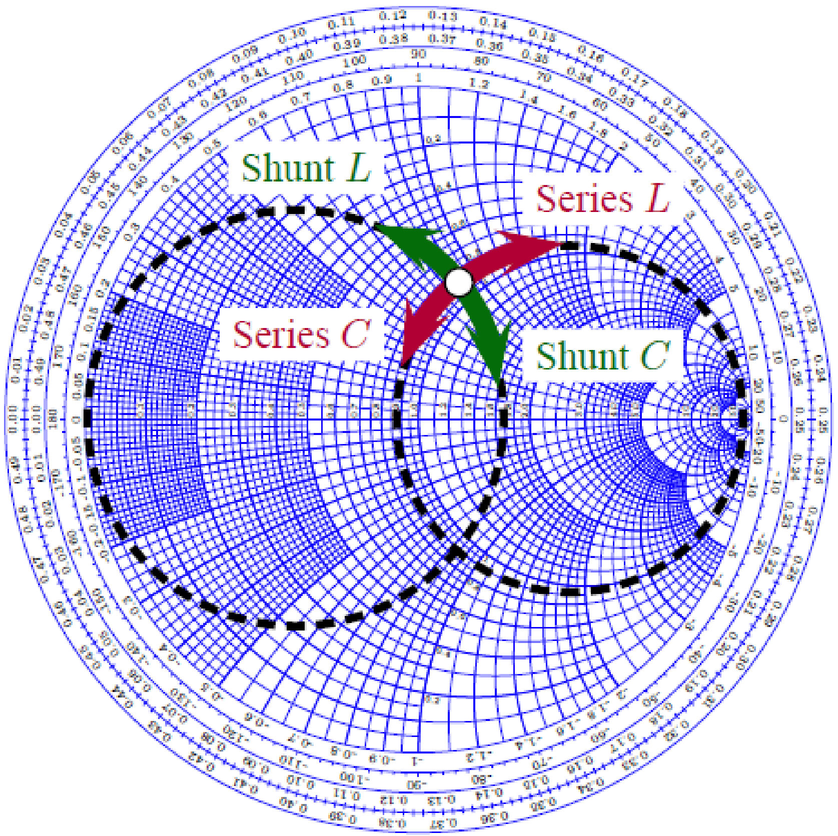
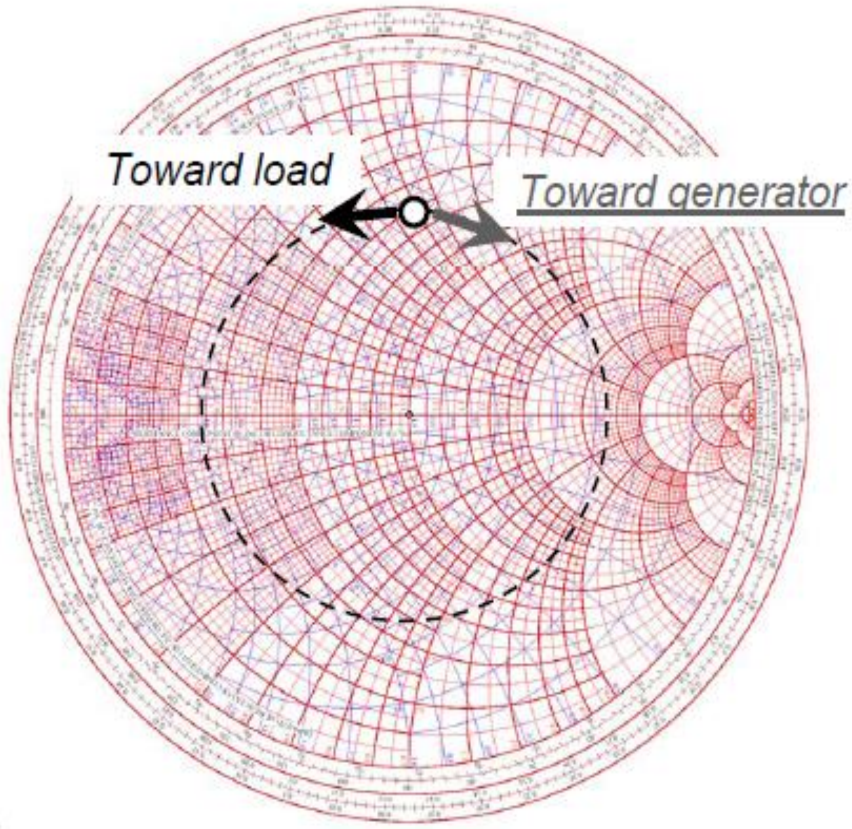
Smith Chart and Matching

Lumped element



Distributed



Microwave Networks

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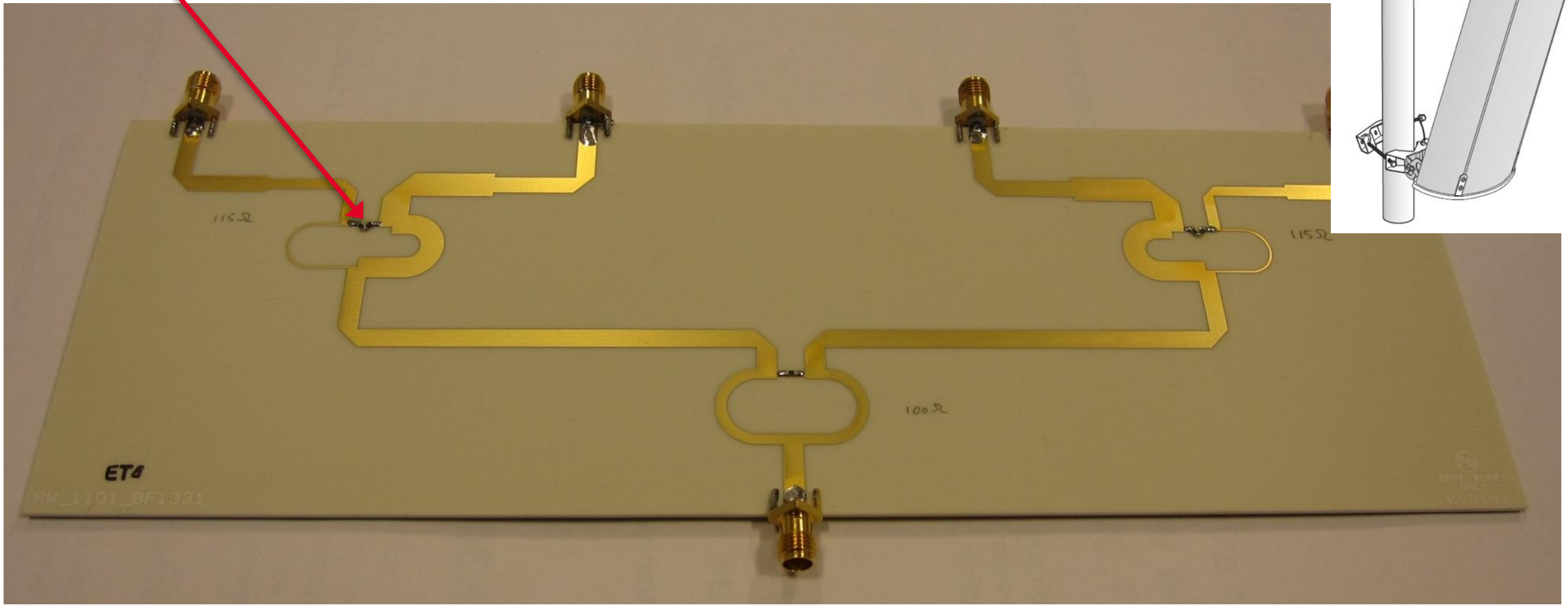
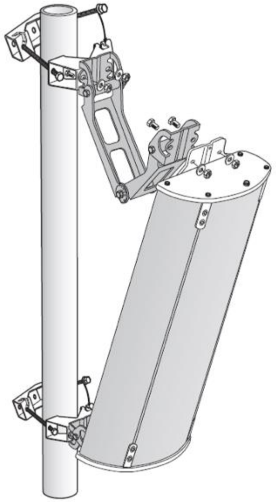
Power Divider Network
Base-station Antenna Feed Network

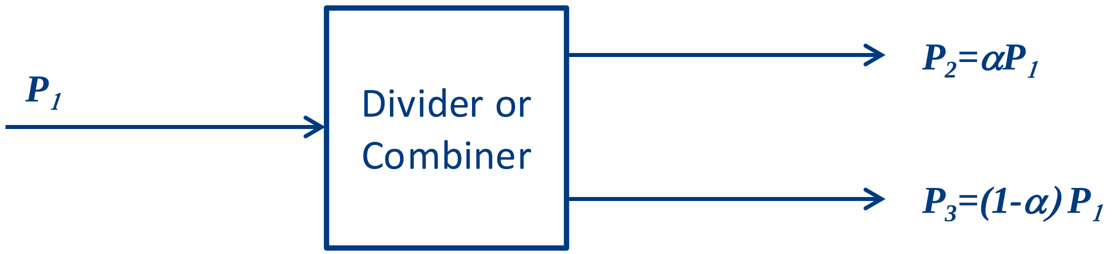
Resistor



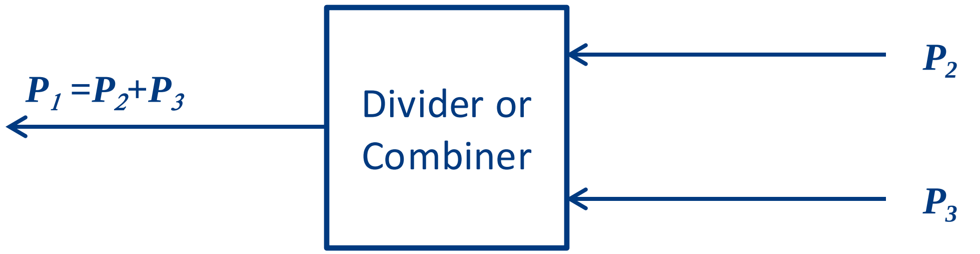


Power Divider/Combiners

Asymmetrical power divider

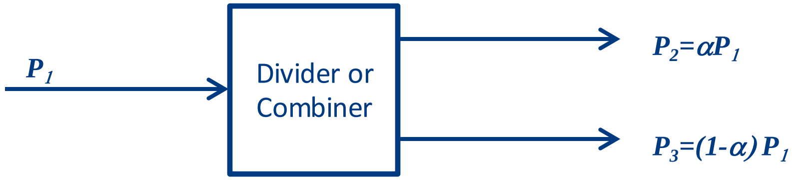


Symmetrical power Combiner



Power Divider/Combiners

Three-port network representation



$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

When properly matched at all ports : $S_{11}=S_{22}=S_{33}=0$.

For a reciprocal network:

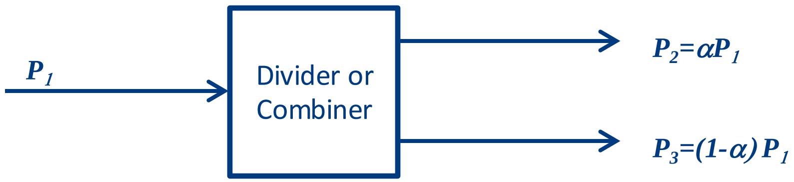
$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

If the network is lossless,

$[S]$ needs to be unitary

Power Divider/Combiners

Unitary condition of [S]:



$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{12}|^2 + |S_{23}|^2 = 1$$

$$|S_{13}|^2 + |S_{23}|^2 = 1$$

$$S_{13}^* S_{23} = 0$$

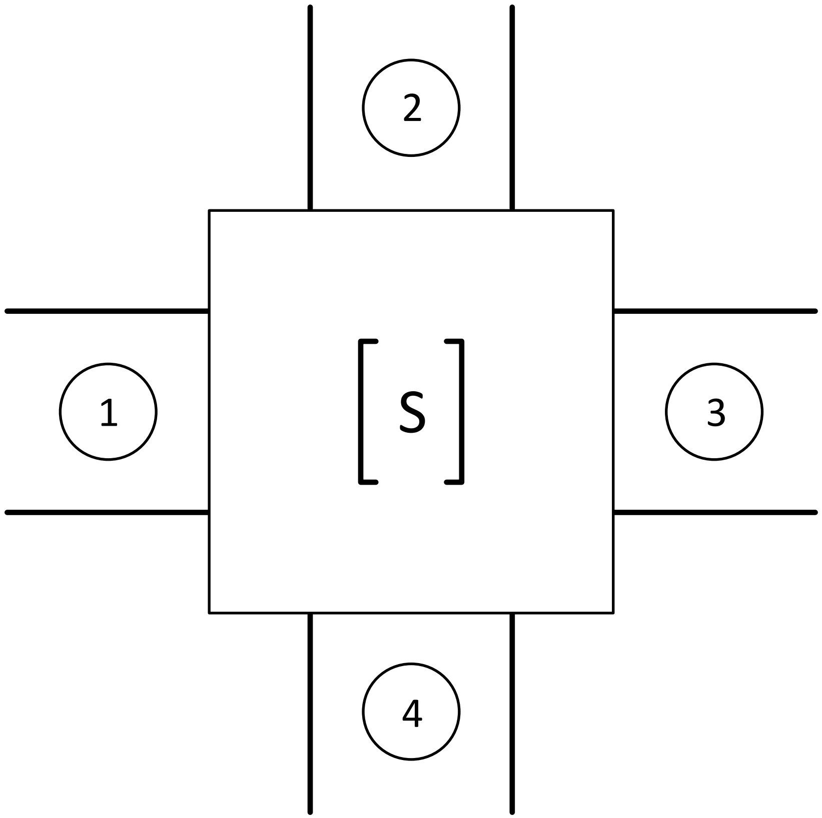
$$S_{23}^* S_{12} = 0$$

$$S_{12}^* S_{13} = 0$$

This would mean that at least 2 out of 3 parameters (S_{12},S_{13},S_{23}) must be zero!

A three-port network cannot be lossless, reciprocal and matched at all ports at the same time!

Directional Couplers (Lossless, reciprocal 4-ports)



Directional Couplers

Reciprocal:

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \rightarrow \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

As it is lossless, it must be unitary: $[S]^t[S]^* = [U]$.

Multiplication of row 1 and row 2 as well as 4 and 3:

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \quad |\cdot (-S_{24}^*)$$
$$S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \quad |\cdot S_{13}^*$$

$$\sum = S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0$$

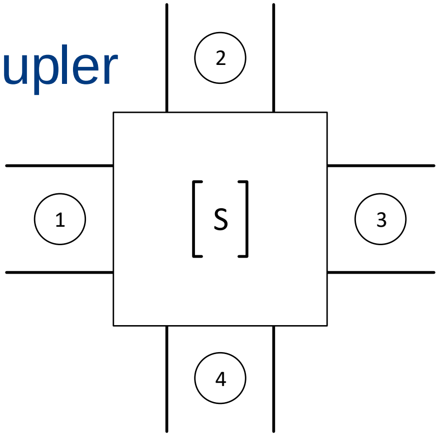
Directional Couplers

Multiplication of row 1 and row 3 as well as 4 and 2:

$$
\begin{aligned}
S_{12}^* S_{23} + S_{14}^* S_{34} &= 0 \quad |\cdot S_{12} \\
S_{14}^* S_{12} + S_{34}^* S_{23} &= 0 \quad |\cdot (-S_{34})
\end{aligned}
$$

$$\sum = S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0$$

One possible solution: $S_{14} = S_{23} = 0$ → Directional coupler



$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

Directional Couplers

Moreover, we obtain from the unitary requirement:

$$\left.\begin{array}{l}|S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{24}|^2 = 1\end{array}\right\} |S_{13}| = |S_{24}|$$

$$\begin{array}{l}|S_{13}|^2 + |S_{34}|^2 = 1 \\ |S_{24}|^2 + |S_{34}|^2 = 1\end{array} \quad |S_{12}| = |S_{34}|$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

We can simplify by choosing:

$$S_{12} = S_{34} = \alpha$$
$$S_{13} = \beta e^{j\theta}$$
$$S_{24} = \beta e^{j\phi}$$

Substitution into the product of rows 2 and 3:

$$S_{12}^* S_{13} + S_{24}^* S_{34} = \alpha\beta e^{j\theta} + \alpha\beta e^{-j\phi} = 0 \Rightarrow \theta + \phi = \pi \pm 2n\pi$$

Directional Couplers

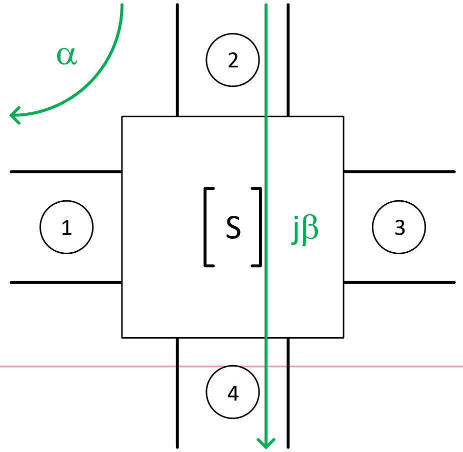
$$\theta + \phi = \pi + 2n\pi$$

$$S_{12} = S_{34} = \alpha$$
$$S_{13} = \beta e^{j\theta}$$
$$S_{24} = \beta e^{j\phi}$$

In practice, you will find two typical types of solutions:

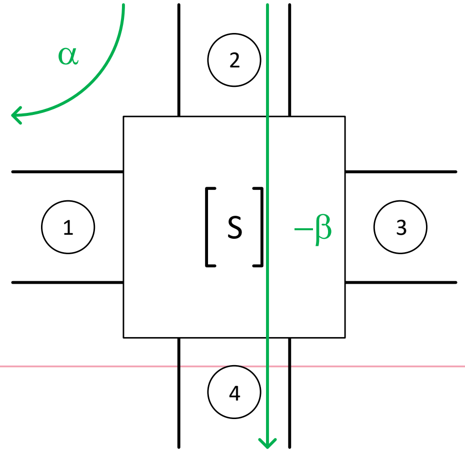
Symmetrical coupler:

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$



Anti-symmetrical coupler:

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$



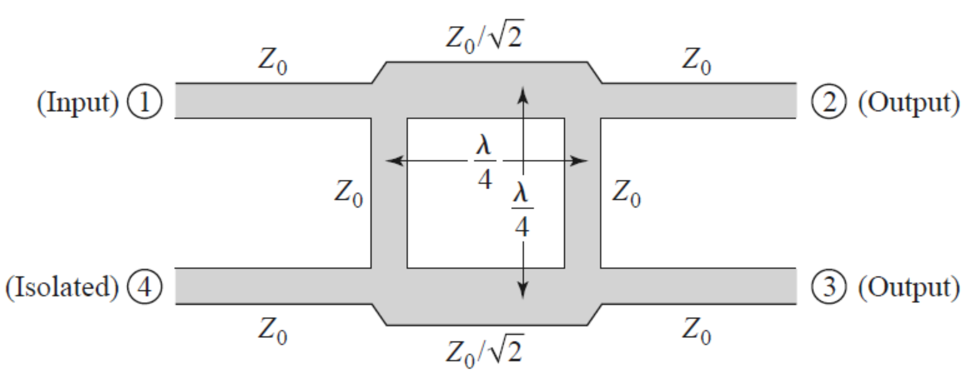
Directional Couplers

Typical examples:

Symmetrical coupler:

90° Hybrid

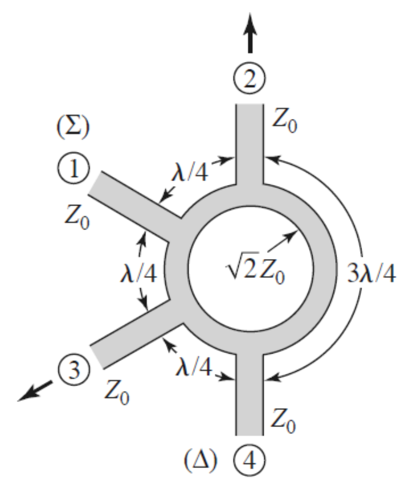
$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$



Anti-symmetrical coupler:

Rat-Race (ring hybrid)

$$[S] = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$



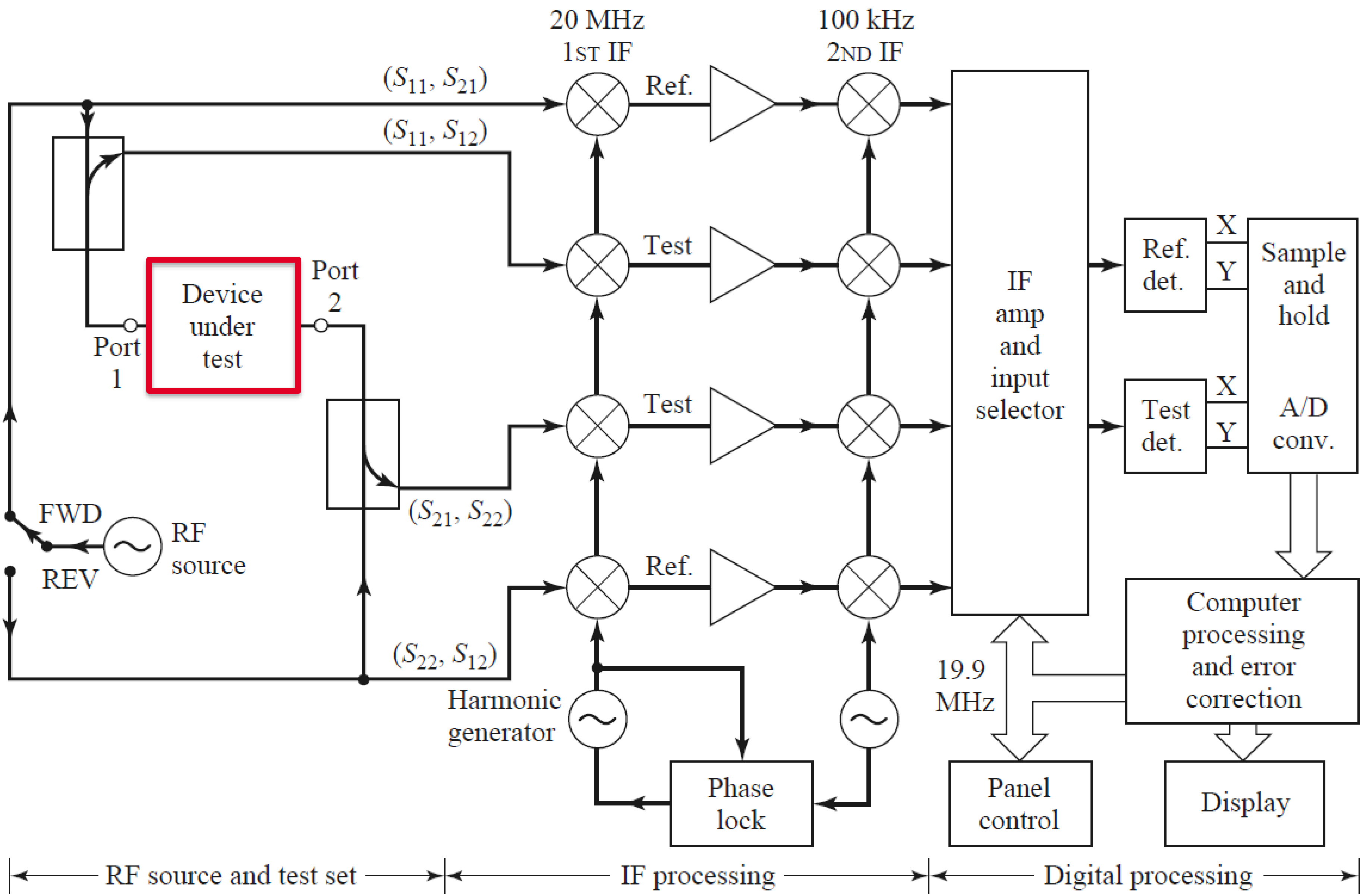
Microwave Networks

Content

- Microwave network matrices
- Impedance Matching and Tuning
- Power Dividers and Directional Couplers
- Application example: Vector Network Analyser

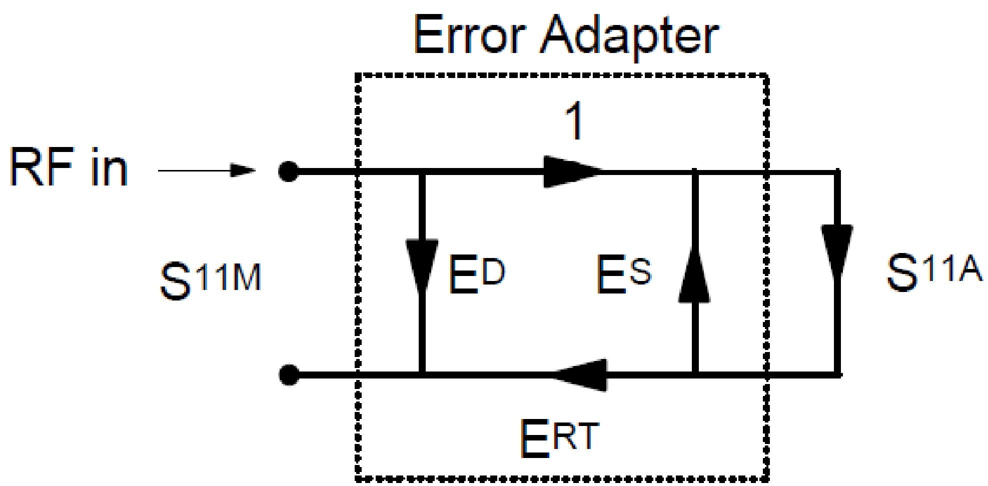
Scattering parameters can be measured with a VNA

- It is not possible to measure directly voltages and currents at high frequencies.
- In this course we will use the nanoVNA.



David M. Pozar, “Microwave Engineering,” 4th edition, John Wiley & Sons Inc., 2012

1-Port Error Model



$$S_{11M} = E_D + E_{RT} \frac{S_{11A}}{1 - E_S S_{11A}}$$

E_D = Directivity

E_{RT} = Reflection tracking

E_S = Source Match

S_{11M} = Measured

S_{11A} = Actual

To solve for S_{11A}, we have 3 equations and 3 unknowns

Next:

- Exercise session

Thursday 20.02.2025:

- Module 2 lab: bring lab kit + laptop!