Student-led tutorials 5EPB0 'EM II'

SLT-B

May 6th 2025

Use of AI for solving SLT questions:

You are allowed to use generative AI, specifically to create content (like images, (source) code, video, text or any other kind). You are allowed to use AI tools as a source for inspiration, e.g. to enhance design and/or research activities, or to improve/enhance the quality of products, but you are not allowed to copy-paste pieces of writing from the AI tool into your assignment. You should be able to explain why you opted to use specific tools and evaluate their usefulness.

Philosophy

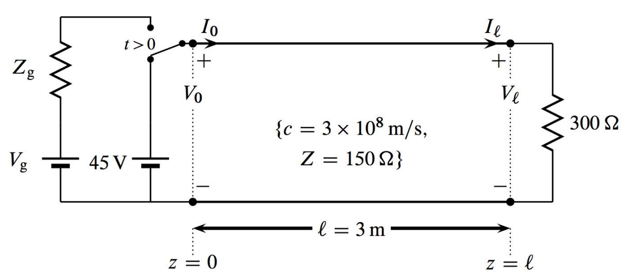
1. (a) A vacuum is often considered an empty space with no physical matter. Still, when considering electromagnetic waves, a vacuum has a characteristic impedance. How is this possible? Describe in your own words what the characteristic impedance of a vacuum represents and whether energy is dissipated when an EM wave propagates through it.

   (b) There exist materials with frequency-dependent impedance. In your own words, describe what it means for impedance to vary with frequency and how this affects the propagation of electromagnetic waves in such materials.

   (c) Transmission lines exhibit standing wave patterns when there is impedance mismatch. Can the standing wave ratio (SWR) be infinite? If so, could you describe a situation in which this would occur?

Bouncing Waves

Consider the example below with an undefined battery voltage and internal impedance that have produced the steady state current amplitude (t<0 at z=0) of 60 mA. Also the source impedance is given as $Z_g = 100\Omega$



2. At t = 0, the switch has been flicked (switched), thus connecting the transmission line to a 45 V battery. As a consequence, in addition to the alternative steady-state voltage and current amplitudes, a compensating TEM wave, with voltage amplitude $V_1^+(0,t) = V_{comp}U(t)$ at z = 0, starts to propagate along the transmission line towards the 300 Ω load. (This amounts to a linear superposition of field constituents.)

   (a) Determine the source voltage V_g and the steady state voltage V_0 at $t \leq 0$

   (b) Decompose the steady state voltage and current into forward and backward propagating waves for $t \leq 0$ at $z = 0$

   (c) What is the total voltage amplitude at z = 0 after t = 0?

(d) Determine the compensating voltage amplitude V_{comp} by considering an arbitrary moment in time just after the switch has been activated, but before possible reflections from the end of the line have returned.

Because the line has not been terminated properly at either end, the compensating waves that arrive at the terminations will be partially reflected.

(e) Give the reflection coefficients at z = 0 and z = ℓ, denoted as Γ_0 and Γ_ℓ respectively.

(f) For $t \to \infty$ a new steady-state is reached for the compensating forward and backward compensating propagating waves. Determine the new steady-state voltage and current amplitudes of these compensating forward and backward propagating waves.

$$\left\{ V^+_{\text{comp}}\ (z,t)|_{t\to\infty}\,, V^-_{\text{comp}}\ (z,t)|_{t\to\infty}\,, I^+_{\text{comp}}\ (z,t)|_{t\to\infty}\,, I^-_{\text{comp}}\ (z,t)|_{t\to\infty} \right\}$$

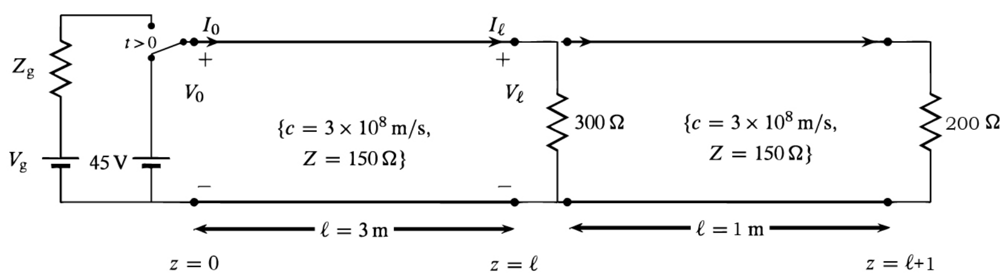
Bounce Diagram

3. We are now going to construct the bounce diagram and voltage diagram of the previously given circuit in Q2.

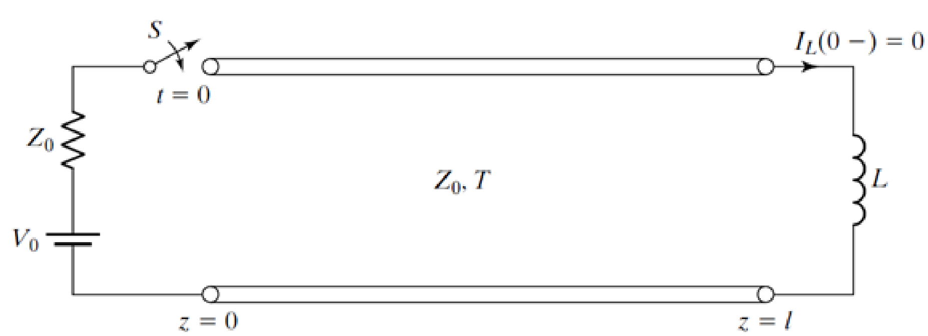
(a) Give the bounce diagram for $0 \leq t \leq 40ns$ for the compensating wave

(b) Give the **total** voltage amplitude for $0 \leq t \leq 40ns$ at $t = 2m$

(c) Now we are going to add a parallel resistance to the existing 300Ω with a piece of transmission line of $1m$ and $Z_0 = 150\Omega$. The resistance has an impedance of 200Ω. Recalculate the reflection and transmission coefficients and draw the bounce diagram for the compensating wave for $0 \leq t \leq 30ns$. The new circuit is shown below:



Transmission line with Inductor and Capacitor



4. Consider the transmission line system shown below, with a total length l and characteristic impedance Z_0. At time t = 0, the switch is closed, connecting the transmission line to a constant voltage source V_0 through a series internal resistance equal to Z_0. Assume V_0 = 20V (DC), Z_0 = 50 Ω, and T= 1 μs.

(a) A forward-traveling wave originates at z=0 when the switch is closed at t=0.
Determine the forward voltage V^+ and current I^+.

(b) Find the reflection coefficient at the load when wave reaches z = L at t=T, and under steady state conditions as $t \to \infty$

(c) Determine the backward voltage V^- and current I^- resulting from reflections

(d) Sketch intuitive waveforms for the voltage and current across the inductor as functions of time. Calculate the time constant of the response, and determine the voltage across an inductor of 0.1 mH at time t = 2μs.

(e) Replace the inductor with a capacitor C at the load. Determine V^+, V^-, I^+, I^- and reflection coefficient at time t = T when the wave first reaches the capacitor, and also at $t \to \infty$ in steady state. Sketch the current and voltage across the capacitor as a function of time, using a qualitatively accurate shape.

Selling Time

5. *In the 1830s to 1880s, the "new world" was still being formed. The newly independent country the United States of America consisted of large areas of nature and every now and then a village. A relatively unknown issue with keeping such a large country together, is time keeping. Many local initiatives estimated the time, and there was no law that decided which time was the correct one. This minor issue led to multiple train crashes with dozens of deaths*[1]*.*

   (a) Thus the railroads required second-accurate timekeeping between distant cities. A common means to synchronize the time keeping, was to sync two identical mechanical clocks. You would then move one of these to a new location, usually a village hundreds of kilometres away. But upon arrival, the time difference between the clocks would usually be minutes or more. Why?

   (b) The new telegraph network provided a solution. Harvard university started sending synchronization pulses through the telegraph. Listeners could read these sync pulses and use it to synchronize their clocks. Selling time even became a nice business over the years.

   A 1 ms 1V sync pulse is sent to a city 1400 km away. The pulse is created by connecting and disconnecting a voltage source through a circuit breaker. See Figure 1.

   Consider the following diagram.

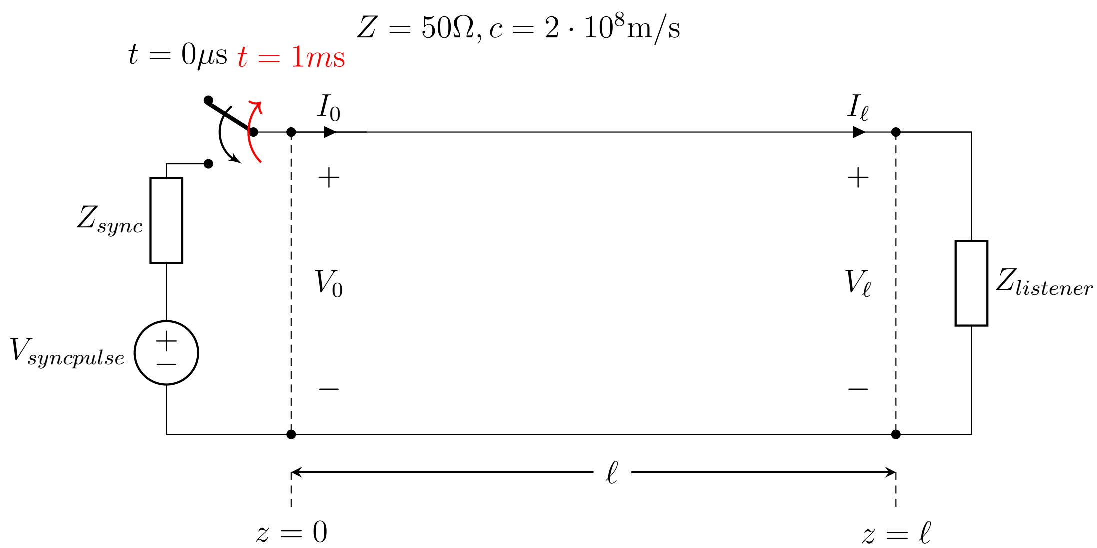


Figure 2: A schematic of a mother clock sending a sync pulse to a single listener.

   Draw the voltage across the line at $T = 5$ms when $Z_{sync} = 100\ \Omega$.

[1] Note that train rails at the time were often bi-directional to save costs. So train collisions were a lot more common: 97 accidents were registered between 1831 and 1853. Source: https://news.harvard.edu/gazette/story/2011/11/americas-first-time-zone/

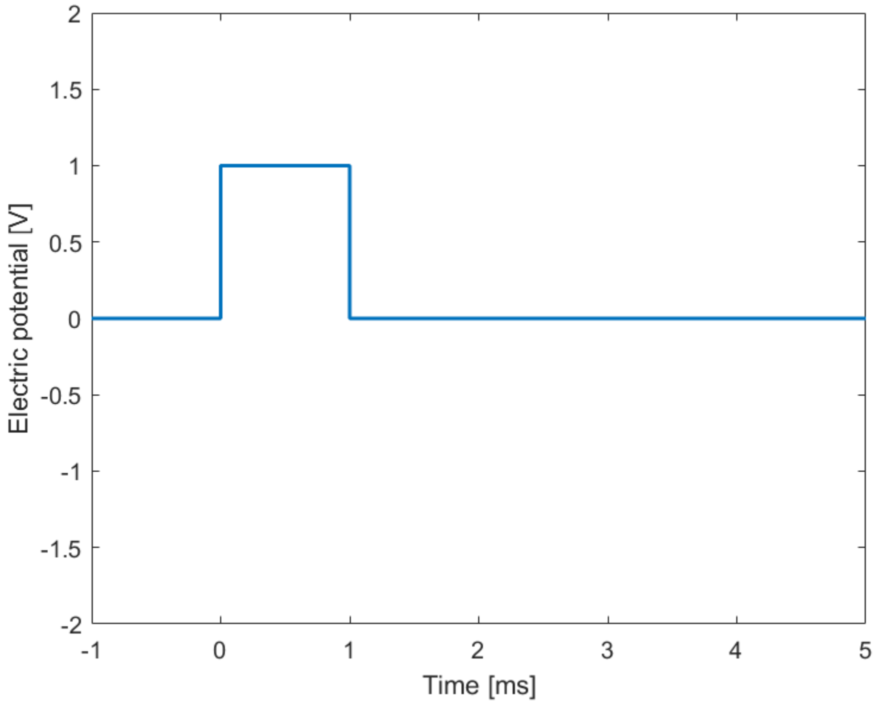


Figure 1: The synchronization pulse.

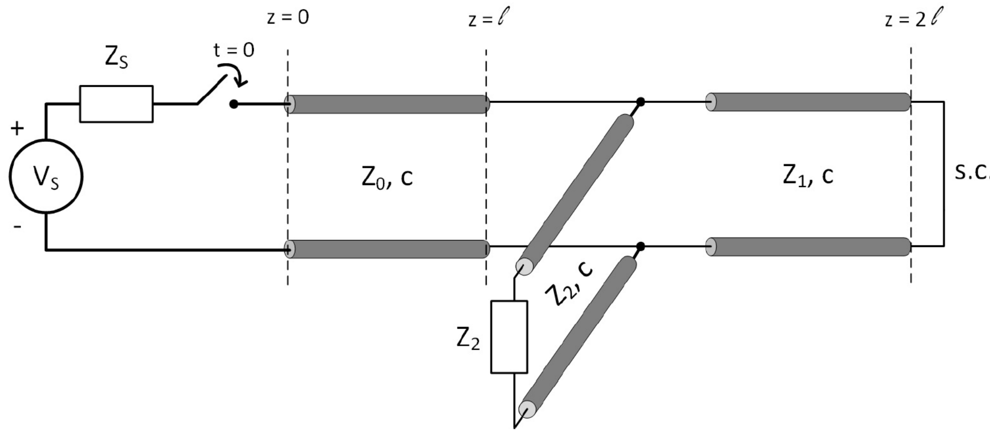
(c) The line is terminated with a impedance of $Z_{listener}$ =125 Ω. Draw voltage across the line at $T = 10$ms.

(d) Draw the bounce diagram up until T =28 ms.

(e) The mother clock is actually connected to many listeners in parallel, all with a different termination and line length. Constructive interference of reflecting pulses could be misinterpreted by a listener as a sync pulse. How could you reduce this effect?

Parallel Transmission Lines

See the transmission line in the figure below. The transmission line has the following characteristics (please note section Z_2 also has a length of l):
$V_S = 9V$
$Z_0 = 300\Omega$
$Z_1 = 150\Omega$
$Z_2 = 400\Omega$
$Z_s = 50\Omega$
$l = 450m$
$c = 3 \cdot 10^8$



6. (a) Calculate the steady state voltage and current at $z = 0$,$z = l$, and at $z = 2l$ for $t \to \infty$

   (b) Calculate the amplitude of the compensating wave which will start propagating just after closing the switch ($t = 0$)

   (c) Calculate the various reflection and transmission coefficients needed to complete a bounce diagram.

   (d) Now sketch the bounce diagram for the compensating wave. Make one drawing for sections Z_0 aned Z_1, so from $z = 0$ to $z = 2l$ and make a separate sketch for the parallel section Z_2. Sketch it for $0 \leq t \leq 6\mu s$. Also denote the amplitude of each wave (you can do this as a product of V_{comp} and reflection/transmission coefficients)

   (e) Make a plot showing the total voltage at $z = 225m$ for $0 \leq t \leq 6\mu s$