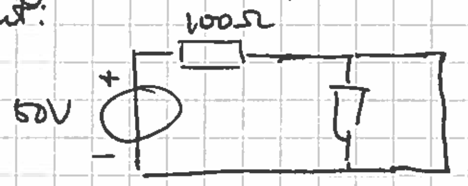
Midterm May 24, 2022 – Solution

Question 1 — Transmission Lines

1A Steady state, before $t=0$. $\Rightarrow$ TL's "do not matter"

Circuit:



$$V_{steady,Z_0} = V_{steady,Z_1} = 0\,[V]$$

$$I_{steady_{Z_0,Z_1}} = \frac{50}{100} = 0.5\,[A]$$

(note: all the current flowing in the short.)

1B The decomposition can be done in two ways:

1. Using the "standard" equations:

$$V^+_{steady} = \frac{Z_L+Z}{2(Z_L+Z_G)}V_g \qquad V^-_{steady} = \frac{Z_L-Z}{2(Z_L+Z_G)}\cdot V_g$$

$\Rightarrow$ for Z_0:

$$V^+_{steady} = \frac{100}{200}\cdot 50 = 25\,[V]$$

$$V^-_{steady} = \frac{-100}{200}\cdot 50 = -25\,[V]$$

$(\Sigma = 0 \checkmark)$

$\Rightarrow$ for Z_1:

$$V^+_{steady} = \frac{200}{200}\cdot 50 = 50\,[V]$$

$$V^-_{steady} = \frac{-200}{200}\cdot 50 = -50\,[V]$$

$(\Sigma = 0 \checkmark)$

2. Using the decomposition matrix

$$\begin{pmatrix} V^+_{steady} \\ V^-_{steady} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & Z \\ 1 & -Z \end{pmatrix}\begin{pmatrix} V_{steady} \\ I_{steady} \end{pmatrix}$$

$\Rightarrow$ for Z_0: $\begin{pmatrix} V^+_{steady} \\ V^-_{steady} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 100 \\ 1 & -100 \end{pmatrix}\begin{pmatrix} 0 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 25\,V \\ -25\,V \end{pmatrix}$

$\Rightarrow$ for Z_1: $\begin{pmatrix} V^+_{steady} \\ V^-_{steady} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 200 \\ 1 & -200 \end{pmatrix}\begin{pmatrix} 0 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 50\,V \\ -50\,V \end{pmatrix}$

$\underline{I_{steady}}$: $\Rightarrow$ for Z_0:

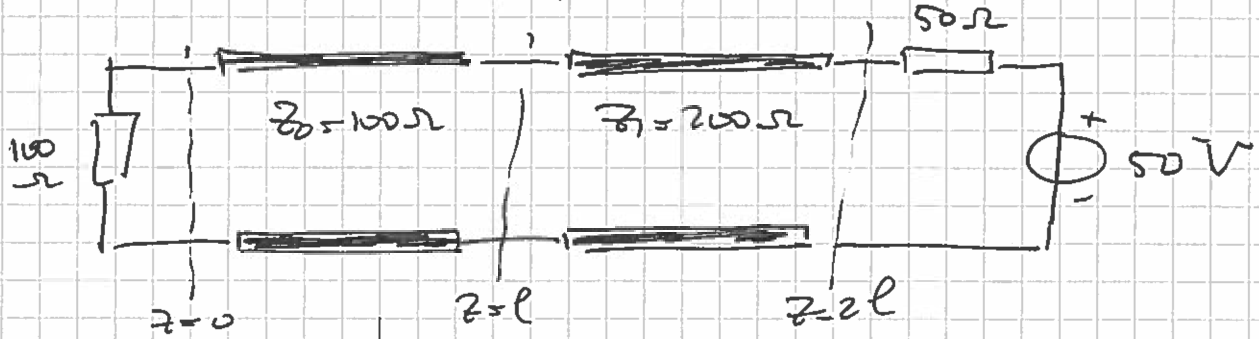
$$I^+_{steady} = \frac{V^+_{steady}}{Z} = \frac{25}{100} = 0.25\,[A]$$

$$I^-_{steady} = \frac{-V^-_{steady}}{Z} = \frac{-(-25)}{100} = 0.25\,[A]$$

$(\Sigma = 0.5\,[A] \checkmark)$

$\Rightarrow$ for Z_0: $I^+_{steady} = 0.25\,[A]$ ~~&~~ $I^-_{steady} = 0.25\,[A]$

(1c) The new circuit for $t>0$ is:



We have to calculate the compensating voltage amplitude
The voltage over the line for $t<0$ $= \underline{0V}$!

at $\boxed{z=0}$ ⇒ not a lot will happen. It was zero and no additional wave is generated
→ still 0V, so no wave

at $\boxed{z=2\ell}$ ⇒ There will be a step of voltage 50V over $Z_{S_2} = 50\Omega$ and the transmission line with $Z_1 = 200\Omega$

→ gives a wave of $\frac{200}{200+50} \cdot 50 = \underline{40\,[V]}$

The wave is propagating from $z=2\ell$ in the negative z-direction

(1d) <u>Bouncing diagram</u> for $0<t<2\mu s$

<u>first</u>: timing. $\ell = 90m \Rightarrow 0.3\mu s$
$2\ell = 180m \Rightarrow 0.6\mu s$

<u>second</u>: reflection and transmission coefficients.

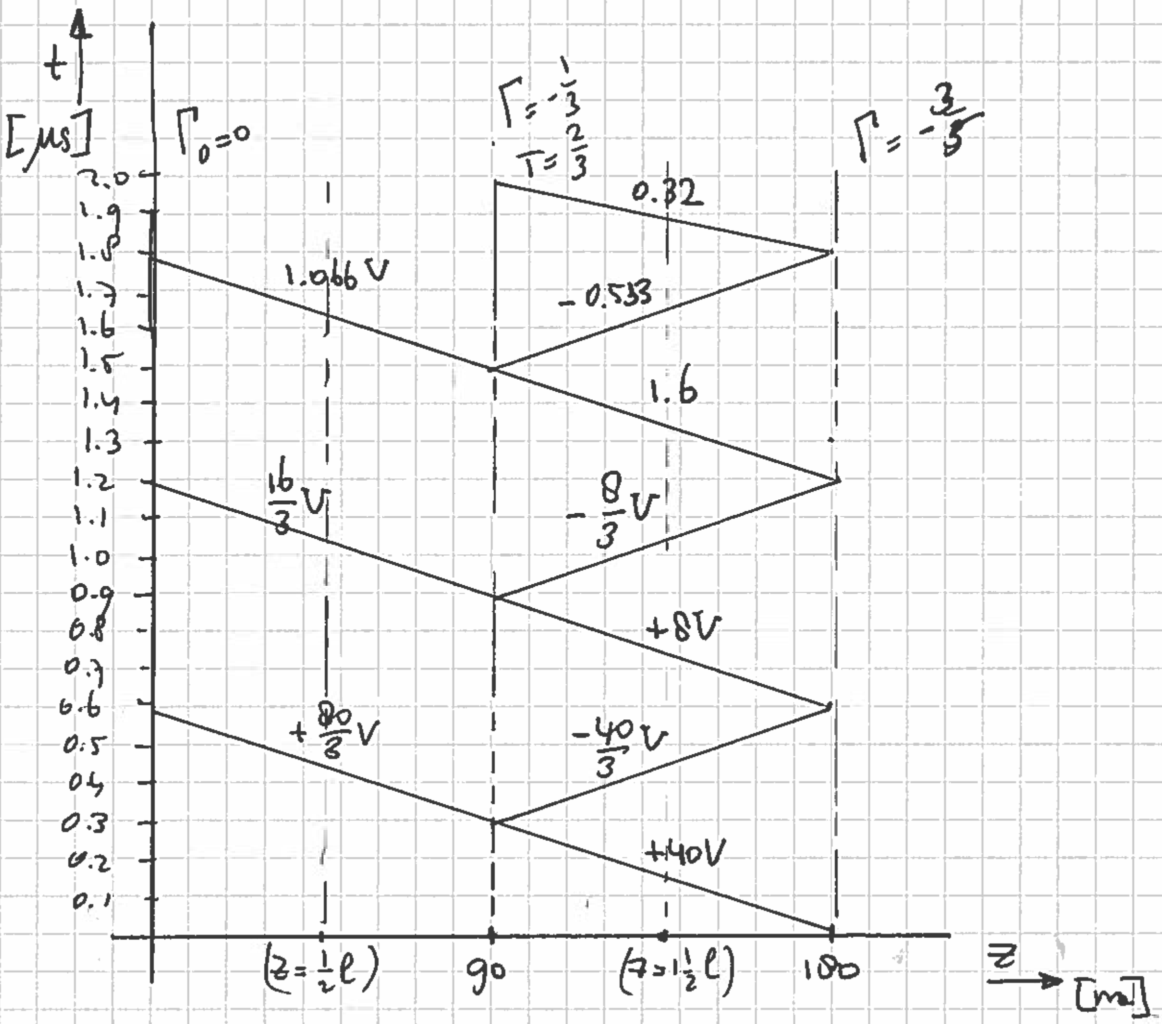
$$\Gamma_0 = \frac{100-100}{100+100} = \phi \quad \text{(matched)}$$

$$\Gamma_{2\ell} = \frac{50-200}{50+200} = -\frac{3}{5}$$

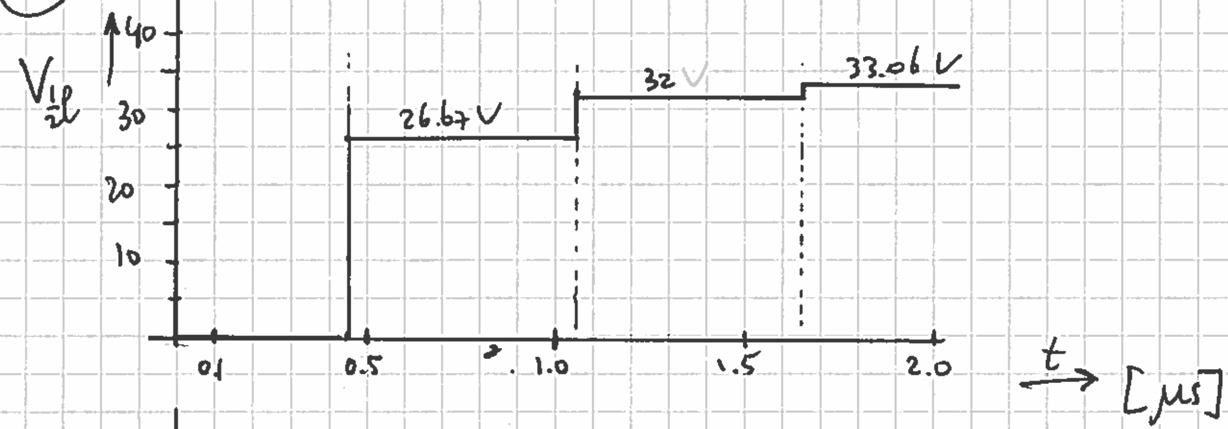
$$\Gamma_{\rightarrow\ell} = \frac{200-100}{200+100} = \frac{1}{3} \quad \Rightarrow T_{\rightarrow\ell} = \frac{4}{3}$$

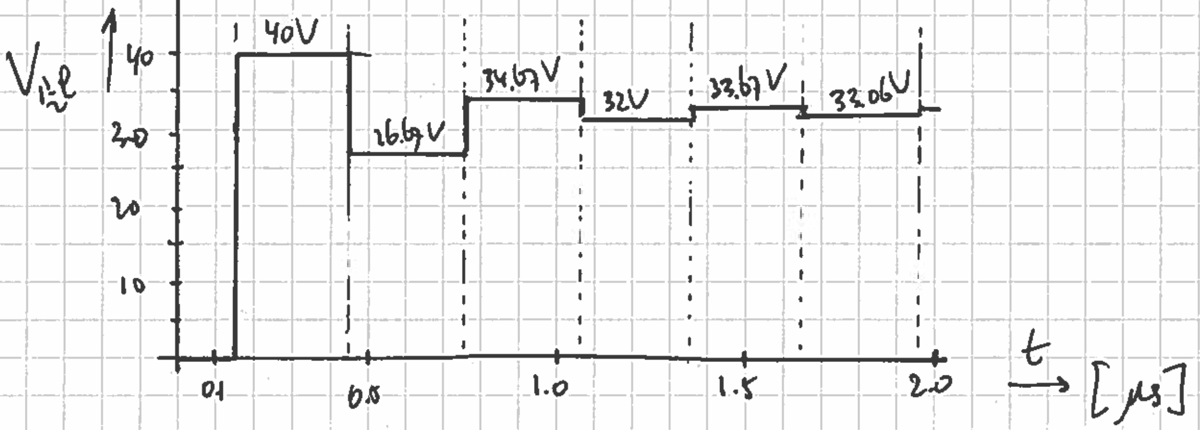
$$\Gamma_{\leftarrow\ell} = \frac{100-200}{100+200} = -\frac{1}{3} \quad \Rightarrow T_{\leftarrow\ell} = \frac{2}{3}$$

Now we can draw the bounce diagram.

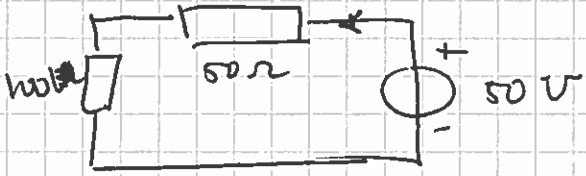

t [μs]
Γ₀=0
Γ = -1/3
T = 2/3
Γ = -3/5
0.32
1.066 V
-0.533
1.6
16/3 V
-8/3 V
+8V
+80/3 V
-40/3 V
+40V
(z = ½ℓ)
90
(z = 1½ℓ)
100
z [m]

1e


V½ℓ
26.67 V
32 V
33.06 V
t [μs]


V1½ℓ
40V
26.67V
34.67V
32V
33.67V
33.06V
t [μs]

1f) for $t \to \infty$ the circuit will be:



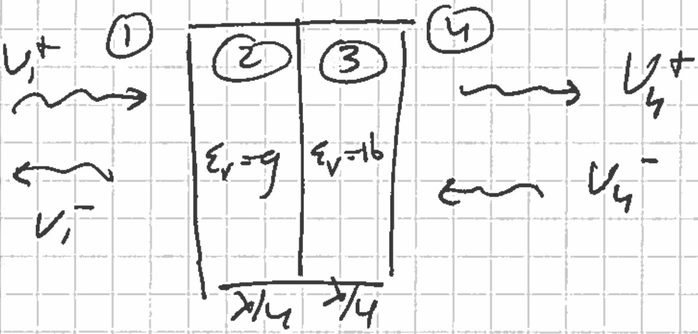
$$V_{steady} = \frac{100}{150} \cdot 50 = 33\tfrac{1}{3}\ [V]$$

$$I_{steady} = \frac{50}{150} = 0.33\ [A]$$

This current is flowing in the negative z-direction so, formally it is -0.33 [A].

You can also see this V_{steady} in the plots of 1e

Question 2



2A
$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = T \begin{pmatrix} V_4 \\ I_4 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} V_4 \\ I_4 \end{pmatrix}$$

mention something like: a transfer matrix gives the voltage and current at one side (~~pos~~ medium 1) given the voltage and current of another side (medium 4)

2B given voltages: V_1^+, V_1^-, V_4^+ and V_4^-

Transfer matrix $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = T_2 T_3 \begin{pmatrix} V_4 \\ I_4 \end{pmatrix}$

now $\begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & Z_1 \\ 1 & -Z_1 \end{pmatrix} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$ (decomposition matrix)

and $\begin{pmatrix} V_4 \\ I_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ Y_4 & -Y_4 \end{pmatrix} \begin{pmatrix} V_4^+ \\ V_4^- \end{pmatrix}$ (composition matrix)

filling in:
$$\begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & Z_1 \\ 1 & -Z_1 \end{pmatrix} T_2 T_3 \begin{pmatrix} 1 & 1 \\ Y_4 & -Y_4 \end{pmatrix} \begin{pmatrix} V_4^+ \\ V_4^- \end{pmatrix}$$

(or
$$\begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = D_1 T_2 T_3 C_4 \begin{pmatrix} V_4^+ \\ V_4^- \end{pmatrix}$$

2c Both medium 2 and 3 are $\lambda/4$ long

$$\Rightarrow \quad k\ell = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \frac{1}{2}\pi \quad \text{(in both media)}$$

So:
$$T_2(0,\ell_1) = T_2\left(0,\frac{\lambda}{4}\right) = \begin{pmatrix} 0 & jZ_2 \\ jZ_2^{-1} & 0 \end{pmatrix}$$

$$T_3(\ell_1,\ell_2) = T_3\left(\ell_1, \ell_1+\frac{\lambda}{4}\right) = \begin{pmatrix} 0 & jZ_3 \\ jZ_3^{-1} & 0 \end{pmatrix}$$

$$\Rightarrow \quad T_2T_3 = \begin{pmatrix} 0 & jZ_2 \\ jZ_2^{-1} & 0 \end{pmatrix}\begin{pmatrix} 0 & jZ_3 \\ jZ_3^{-1} & 0 \end{pmatrix} = \begin{pmatrix} -Z_2/Z_3 & 0 \\ 0 & -Z_3/Z_2 \end{pmatrix}$$

Now we look if we can replace this with a single medium 5

$$k_5\ell_5 = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi \quad \Rightarrow \sin(k_5\ell_5) = \phi$$
$$\Rightarrow \cos(k_5\ell_5) = -1$$

$$T_5 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

So, never T_5 will be equal to T_2T_3 ....

yeah perhaps if $Z_2 = Z_3$, ... However

$Z_2 = 40\pi$ and $Z_3 = 30\pi$

$\Rightarrow$ Conclusion: this isn't possible.

2d So, we have $T_2T_3 = \begin{pmatrix} -Z_2/Z_3 & 0 \\ 0 & -Z_3/Z_2 \end{pmatrix}$

to calculate the scatter matrix, we use the A-matrix and reshuffle the parameters

$\bar{A} = D_1 T C_4$

with $Z_1 = Z_4 = 120\pi$

$Z_2 = 40\pi$

$Z_3 = 30\pi$

$$\Rightarrow \quad T = \begin{pmatrix} -4/3 & 0 \\ 0 & -3/4 \end{pmatrix}$$

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$$\bar{\bar{A}} = \underbrace{\begin{pmatrix} \frac{1}{2} & 60\pi \\ \frac{1}{2} & -60\pi \end{pmatrix} \begin{pmatrix} -\frac{4}{3} & 0 \\ 0 & -\frac{3}{4} \end{pmatrix}} \begin{pmatrix} 1 & 1 \\ \frac{+1}{120\pi} & \frac{-1}{120\pi} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} -\frac{4}{6} & -45\pi \\ -\frac{4}{6} & 45\pi \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{1}{120\pi} & \frac{-1}{120\pi} \end{pmatrix}}$$

$$\begin{pmatrix} -\frac{2}{3} - \frac{3}{8} & -\frac{2}{3} + \frac{3}{8} \\ -\frac{2}{3} + \frac{3}{8} & -\frac{2}{3} - \frac{3}{8} \end{pmatrix} = \begin{pmatrix} -\frac{25}{24} & -7/24 \\ -\frac{7}{24} & -25/24 \end{pmatrix}$$

So

$$\begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = \begin{pmatrix} -25/24 & -7/24 \\ -7/24 & -25/24 \end{pmatrix} \begin{pmatrix} V_4^+ \\ V_4^- \end{pmatrix}$$

$$\Rightarrow \quad V_1^+ = -25/24\, V_4^+ - 7/24\, V_4^-$$
$$V_1^- = -7/24\, V_4^+ - 25/24\, V_4^-$$

$$\Rightarrow \quad 24 V_1^+ = -25 V_4^+ - 7 V_4^- \quad (1)$$
$$24 V_1^- = -7 V_4^+ - 25 V_4^- \quad (2)$$

$25 * (2)$: $\quad 600 V_1^- = -175 V_4^+ - 625 V_4^- \quad (3)$

$7 * (1)$: $\quad 168 V_1^+ = -175 V_4^+ - 49 V_4^- \quad (4)$

(4) in (3) $\quad 600 V_1^- = 168 V_1^+ + 49 V_4^- - 625 V_4^-$

$$\Rightarrow \boxed{V_1^- = \frac{168}{600} V_1^+ - \frac{576}{600} V_4^-} \quad (5)$$

(5) in (3)

$$600\left(\frac{168}{600} V_1^+ - \frac{576}{600} V_4^-\right) = -175 V_4^+ - 625 V_4^-$$

$$-175 V_4^+ = 168 V_1^+ + 49 V_4^-$$

$$\Rightarrow \boxed{V_4^+ = \frac{-168}{175} V_1^+ - \frac{49}{175} V_4^-}$$

So, the scatter matrix is

$$\begin{pmatrix} V_1^- \\ V_4^+ \end{pmatrix} = \begin{pmatrix} \frac{7}{25} & \frac{-24}{25} \\ -\frac{24}{25} & -7/25 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_4^- \end{pmatrix}$$