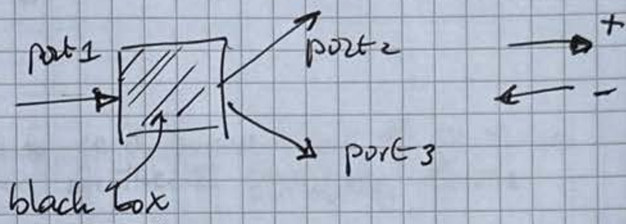
Power splitter problem

Q: Find the S-matrix of an (ideal) power splitter

The question is, of course, what is the circuit behind a power splitter. There are 4 (!) (and perhaps even more) different versions of a power splitter/divider.

1 The perfect case



if port 1 is the input of the black box, and port 2 and 3 are the outputs. you want ½ the power in port 2 and the other half in port 3.

→ since $P (\because) V^2$ ⇒ voltage in port 2 and 3 is $\frac{1}{\sqrt{2}}$ $(= \frac{1}{2}\sqrt{2})$ that of port 1

no reflection

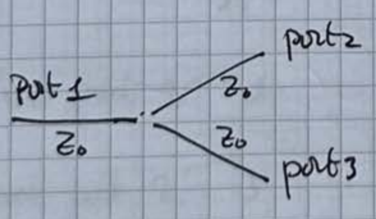
so

$$\begin{pmatrix} V_1^- \\ V_2^+ \\ V_3^+ \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{2}\sqrt{2} & 0 & 0 \\ \frac{1}{2}\sqrt{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^- \\ V_3^- \end{pmatrix}$$

However, how to make this ?? No reflections, perfect matching

.... little bit difficult.

2 The very simple splitter:



suppose $Z_0 = 50\ \Omega$ (why not?)

Now .. we do have lots of reflections

For instance at port 1

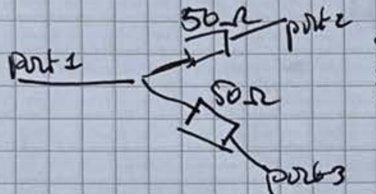
$$\Gamma_{11} = \frac{Z_0 // Z_0 - Z_0}{Z_0 // Z_0 + Z_0} = \frac{-0.5 Z_0}{1.5 Z_0} = -\frac{1}{3}$$

system is symmetrical → $\Gamma_{22} = \Gamma_{33} = -\frac{1}{3}$

That also means that the Transmission is equal $T = 1 + \Gamma = 1 - \frac{1}{3} = \frac{2}{3}$.

$$\Rightarrow \begin{pmatrix} V_1^- \\ V_2^+ \\ V_3^+ \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^- \\ V_3^- \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^- \\ V_3^- \end{pmatrix}$$

3) The power splitter.



So two real impedances of 50 Ω in series in the 2 out going lines

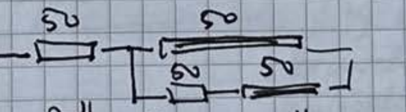
again we have to look at the progressive and regressive voltages.

$$\Gamma_{11} : \frac{(50+50)/\!/(50+50) - 50}{(50+50)/\!/(50+50) + 50} = 0 \Rightarrow \text{no input reflection}$$

↑ serie Z + characteristic impedance !!

lets have a look on the reflection at port 2 (= reflection at port 3)

looking in port 2, we 'see':



$= 50 + 33\frac{1}{3} = 83\frac{1}{3}\ \Omega$

$$\text{So: } \Gamma_{22} = \frac{83\frac{1}{3} - 50}{83\frac{1}{3} + 50} = \frac{33\frac{1}{3}}{133\frac{1}{3}} = \underline{1/4}$$

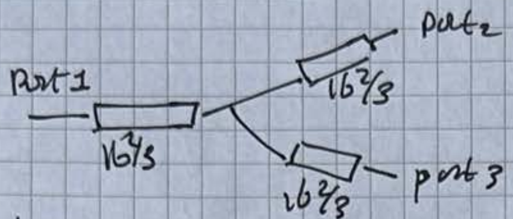
So: $\Gamma_{33} = 1/4$

From port $2 \rightarrow 1$: $V_1^- = \frac{50}{50+50} = \frac{1}{2} V_2^-$ same for port $3 \rightarrow 1$

From port $2 \rightarrow 3$: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (two times voltage divider)

$$\Rightarrow \begin{pmatrix} V_1^- \\ V_2^+ \\ V_3^+ \end{pmatrix} = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^- \\ V_3^- \end{pmatrix}$$

4 The power divider (part symmetric)



Looks symmetrical :)

First

$$\Gamma_{11} = \frac{[(16\tfrac{2}{3}+50)/\!/(16\tfrac{2}{3}+50)+16\tfrac{2}{3}]-50}{[(16\tfrac{2}{3}+50)/\!/(16\tfrac{2}{3}+50)+16\tfrac{2}{3}]+50}$$

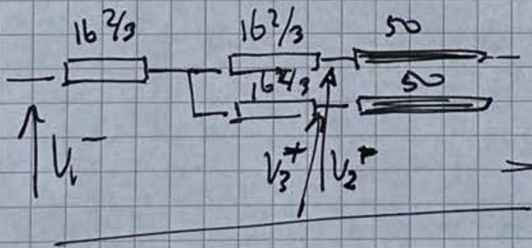
$$= \frac{66\tfrac{2}{3}/\!/66\tfrac{2}{3} + 16\tfrac{2}{3} - 50}{66\tfrac{2}{3}/\!/66\tfrac{2}{3} + 16\tfrac{2}{3} + 50}$$

$$= \frac{33\tfrac{1}{3} + 16\tfrac{2}{3} - 50}{33\tfrac{1}{3} + 16\tfrac{2}{3} + 50} = \phi$$

jipie... no reflection

$$\Rightarrow \Gamma_{22} = \Gamma_{33} = \phi$$

now the relation between, for instance V_1^- and V_2^+



→ just a division of the $16\frac{2}{3}$ impedance in one branch and a $16\frac{2}{3}\,\Omega$ impedance in the other branch.

→ $\frac{1}{2}$

So,

$$\begin{pmatrix} V_1^- \\ V_2^+ \\ V_3^+ \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^- \\ V_3^- \end{pmatrix}$$