Communication Theory (5ETB0)
Module 12.2

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Module 12.2

Presentation Outline

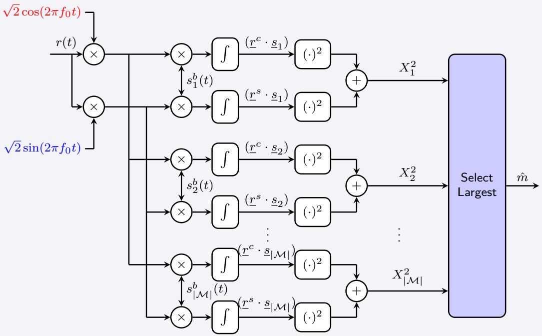
Part I Envelope Detection

Part II Error Probability

Envelope Detection: Receiver Structure

Correlation/Direct Receiver for Random Phase Equal-energy Signals

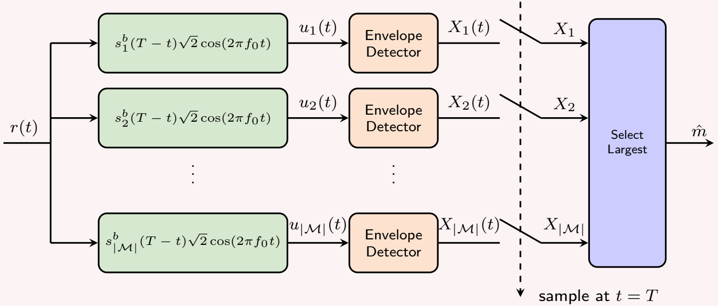
$$\hat{m}^{\mathsf{MAP}}(\underline{r}) = \underset{m\in\mathcal{M}}{\operatorname{argmax}} \left\{X_m^2\right\}, \quad X_m \triangleq \sqrt{(\underline{r}^c\cdot\underline{s}_m)^2 + (\underline{r}^s\cdot\underline{s}_m)^2}$$



Envelope Detection: Receiver Structure

Envelope-detector receiver for equal energy signals with random phase

Received signal: $r(t) = s_m^b(t)\sqrt{2}\cos(2\pi f_0 t - \theta) + n_w(t)$



Questions

Is this an optimum receiver? Are the X_m here the same as before?

Envelope Detection: Optimally Proof (1/3)

Proof Sketch (Details in Sec. 12.4)

- Output of filters can be expressed as

$$u_m(t) \quad = \quad u_m^c(t)\underbrace{\cos(2\pi f_0 t)}_{\text{high-freq.}} + u_m^s(t)\underbrace{\sin(2\pi f_0 t)}_{\text{high-freq.}}$$

with

$$\underbrace{u_m^c(t)}_{\text{baseband}} \triangleq \int_{-\infty}^{\infty} r(\alpha)\sqrt{2}\cos(2\pi f_0 \alpha)\underbrace{s_m^b(T-t+\alpha)}_{\text{baseband}}\,d\alpha$$

$$\underbrace{u_m^s(t)}_{\text{baseband}} \triangleq \int_{-\infty}^{\infty} r(\alpha)\sqrt{2}\sin(2\pi f_0 \alpha)\underbrace{s_m^b(T-t+\alpha)}_{\text{baseband}}\,d\alpha$$

- Envelope detectors: (i) squaring signal, (ii) low-pass filter, and (iii) square root. The outputs are

$$X_m(t) = \sqrt{(u_m^c(t))^2 + (u_m^s(t))^2}$$

Envelope Detection: Optimally Proof (2/3)

Proof Sketch (Details in Sec. 12.4)

Note that

$$
\begin{aligned}
(\underline{r}^c \cdot \underline{s}_m) = \sum_{i=1}^{N} r_i^c s_{mi} &= \sum_{i=1}^{N} \left(\int_{-\infty}^{\infty} r(t)\varphi_i(t)\sqrt{2}\cos(2\pi f_0 t)dt \right) s_{mi} \\
&= \int_{-\infty}^{\infty} r(t)\sqrt{2}\cos(2\pi f_0 t) \sum_{i=1}^{N} s_{mi}\varphi_i(t)dt \\
&= \int_{-\infty}^{\infty} r(t)\sqrt{2}\cos(2\pi f_0 t) s_m^b(t)dt.
\end{aligned}
$$

Similarly,

$$
(\underline{r}^s \cdot \underline{s}_m) = \int_{-\infty}^{\infty} r(t)\sqrt{2}\sin(2\pi f_0 t) s_m^b(t)dt.
$$

Envelope Detection: Optimally Proof (3/3)

Proof Sketch (Details in Sec. 12.4)

$$\begin{aligned}
u_m^c(t) &= \int_{-\infty}^{\infty} r(\alpha)\sqrt{2}\cos(2\pi f_0 \alpha) s_m^b(T-t+\alpha)d\alpha\Big|_{t=T}\\
u_m^c(T) &= \int_{-\infty}^{\infty} r(\alpha)\sqrt{2}\cos(2\pi f_0 \alpha) s_m^b(\alpha)d\alpha\\
u_m^s(t) &= \int_{-\infty}^{\infty} r(\alpha)\sqrt{2}\sin(2\pi f_0 \alpha) s_m^b(T-t+\alpha)d\alpha\Big|_{t=T}\\
u_m^s(T) &= \int_{-\infty}^{\infty} r(\alpha)\sqrt{2}\sin(2\pi f_0 \alpha) s_m^b(\alpha)d\alpha
\end{aligned}$$

Thus, at $t = T$

$$\begin{aligned}
u_m^c(T) &= (\underline{r}^c \cdot \underline{s}_m),\\
u_m^s(T) &= (\underline{r}^s \cdot \underline{s}_m),
\end{aligned}$$

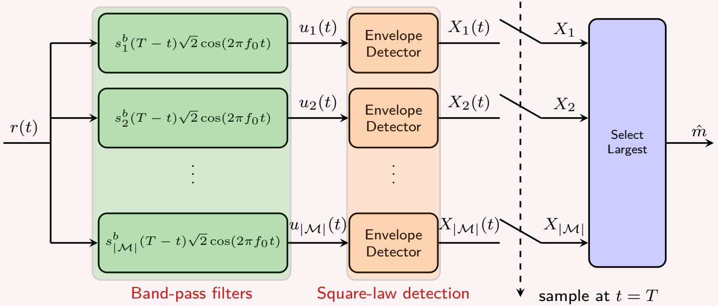
and therefore

$$X_m(T) = \sqrt{(\underline{r}^c \cdot \underline{s}_m)^2 + (\underline{r}^s \cdot \underline{s}_m)^2}.$$

Envelope Detection: Receiver Structure

Envelope-detector receiver for equal energy signals with random phase

Received signal: $r(t) = s_m^b(t)\sqrt{2}\cos(2\pi f_0 t - \theta) + n_w(t)$

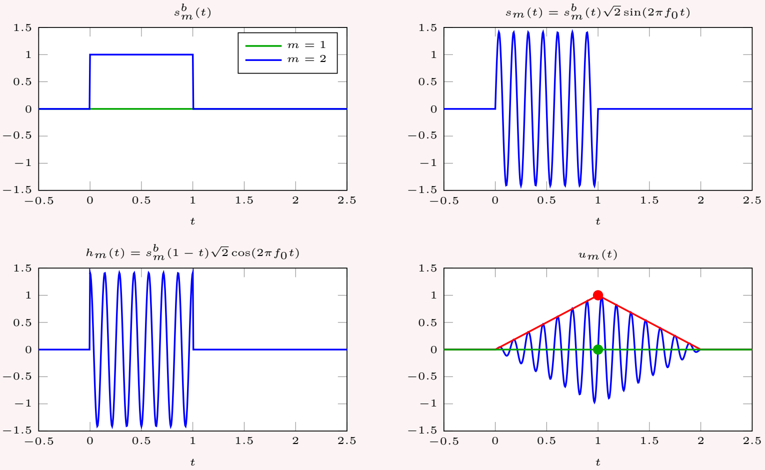


Questions

Is this an optimum receiver? Yes Are the X_m here the same as before? and Yes!

Envelope Detection: Example

Example 12.1 ($\theta = \pi/2$, $T = 1$)



Module 12.2

Presentation Outline

Error Probability for Two Orthogonal Signals (1/3)

Model and Assumptions

Signal vectors are $\underline{s}_m = (s_{m1}, s_{m2}, \ldots, s_{mN})$, where

$$s_m^b(t) = \sum_{i=1}^{N} s_{mi}\varphi_i(t).$$

Two messages and two dimensions ($|\mathcal{M}| = N = 2$), equally likely. Waveforms are

$$\begin{aligned} s_1^b(t) &= \sqrt{E_s}\varphi_1(t), \text{ hence } \underline{s}_1 = (\sqrt{E_s}, 0), \\ s_2^b(t) &= \sqrt{E_s}\varphi_2(t), \text{ hence } \underline{s}_2 = (0, \sqrt{E_s}). \end{aligned}$$

All vectors are two-dimensional: $\underline{r}^c = \underline{s}_m \cos(\theta) + \underline{n}^c$, $\underline{r}^s = \underline{s}_m \sin(\theta) + \underline{n}^s$. Optimum receiver rule chooses $\hat{m} = 1$ if

$$(\underline{r}^c \cdot \underline{s}_1)^2 + (\underline{r}^s \cdot \underline{s}_1)^2 > (\underline{r}^c \cdot \underline{s}_2)^2 + (\underline{r}^s \cdot \underline{s}_2)^2$$
$$(r_1^c)^2 + (r_1^s)^2 > (r_2^c)^2 + (r_2^s)^2$$

Error Probability for Two Orthogonal Signals (2/3)

Error Probability Result (Sec. 12.5)

The error probability for an incoherent receiver for two equally likely orthogonal signals, both having energy E_s, is

$$P_{\mathrm{e}}^{in} = \frac{1}{2} \exp\left(-\frac{E_s}{2N_0}\right).$$

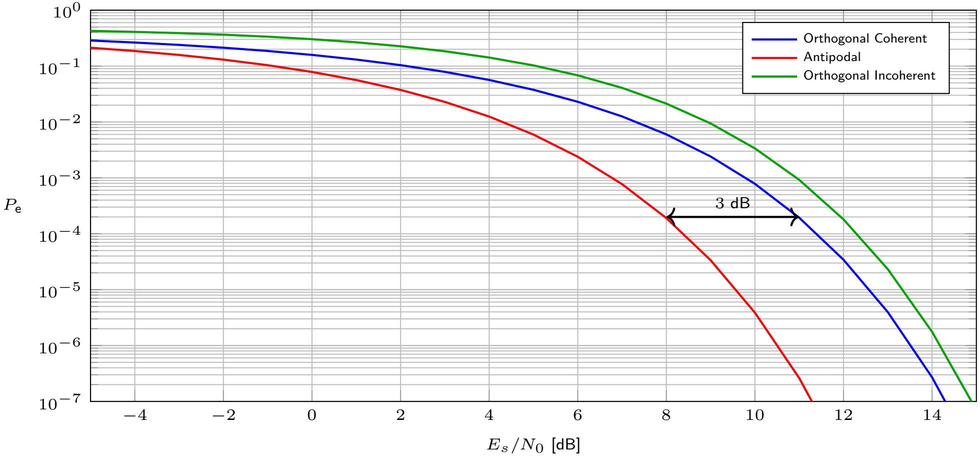
Comparison vs. Coherent and Antipodal

For coherent **(orthogonal)** reception and antipodal signaling we have obtained:

$$P_{\mathrm{e}}^{orth.} = Q\left(\sqrt{\frac{E_s}{N_0}}\right) \leq \frac{1}{2} \exp\left(-\frac{E_s}{2N_0}\right)$$

$$P_{\mathrm{e}}^{antip.} = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$

Error Probability for Two Orthogonal Signals (3/3)



Pros and Cons of Incoherent Transmission

Pros and Cons

Advantages

- Simpler and cheaper implementation
- No need to track and compensate phase

Disadvantages

- Worse error probability for binary transmission and any SNR
- More than 3 dB loss compared to antipodal signaling

Summary Module 12.2

Take Home Messages

- Optimum incoherent receiver can be implemented easily
- Analysis based on building-block waveforms
- Error probability analysis: performance degradation

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