EINDHOVEN UNIVERSITY OF TECHNOLOGY

# Communication Theory (5ETB0)
# Module 11.1

Alex Alvarado
a.alvarado@tue.nl

Information and Communication Theory Lab
Signal Processing Systems Group
Department of Electrical Engineering
Eindhoven University of Technology, The Netherlands

www.tue.nl/ictlab/

# Module 11.1

## Presentation Outline

# Passband Transmission: Motivation (1/2)

## Baseband Transmission

- For a baseband pulse $p(t)$ with Fourier transform $P(f)$, the BW of the transmitted signal is at least $1/2T$, where $1/T$ is the symbol rate
- Best case scenario, we use pulses sinc pulses and the BW is exactly $1/2T$
- Baseband signals can be sent over a telephone line or a coaxial cable

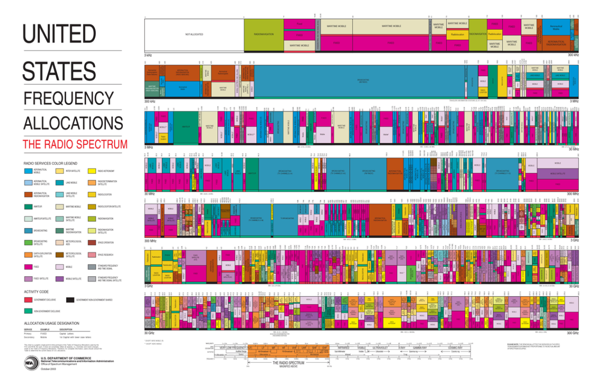
## Passband Transmission

- FM radio operates at 100 [MHz], $\lambda = v/f \approx 3$ m $\Longrightarrow$ Large antennas
- WiFi operates at 2.4 [GHz] or 5 [GHz], $6 \lessapprox \lambda \lessapprox 12.5$ cm $\Longrightarrow$ Small antennas
- More transmission bandwidth available at higher frequencies.

# Passband Transmission: Motivation (2/2)

## Other Reasons for Passband Transmission

- Makes best use of the channel
- Allows us to assign different users to different frequencies



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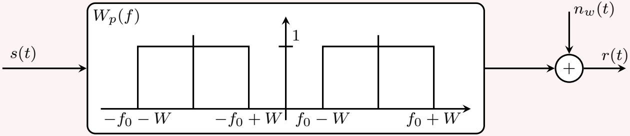
## Presentation Outline

# System Model

## Ideal passband channel with AWGN

- Ideal passband filter




$$W_p(f) \triangleq \begin{cases} 1 & \text{if } f_0 - W < |f| < f_0 + W \\ 0 & \text{elsewhere} \end{cases}$$

- Noise is AWGN, PSD is:

$$S_{N_w}(f) = N_0/2 \text{ for } -\infty < f < \infty$$

# Quadrature Multiplexing: Model

## Quadrature Multiplexing Transmitter

- Consider two set of baseband waveforms having bandwidth smaller than $W$:

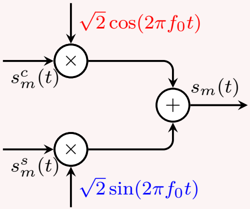
$$\{s_1^c(t), s_2^c(t), \dots, s_{|\mathcal{M}|}^c(t)\}$$

$$\{s_1^s(t), s_2^s(t), \dots, s_{|\mathcal{M}|}^s(t)\}$$

- Corresponding building-block waveforms are:

$$\phi_i(t), i = 1, 2, \dots, N_c, \quad N_c \leq |\mathcal{M}|$$

$$\psi_j(t), j = 1, 2, \dots, N_s, \quad N_s \leq |\mathcal{M}|$$




Transmitted signal is:

$$s_m(t) = s_m^c(t)\sqrt{2}\cos(2\pi f_0 t) + s_m^s(t)\sqrt{2}\sin(2\pi f_0 t)$$

# Quadrature Multiplexing: Orthogonality (1/2)

## Transmitted waveform

Building-block waveforms $\phi_i(t)$ and $\psi_j(t)$:

$$s^c_m(t) = \sum_{i=1}^{N_c} s^c_{mi}\phi_i(t) \quad \text{and} \quad s^s_m(t) = \sum_{j=1}^{N_s} s^s_{mj}\psi_j(t)$$

The $m$th transmitted waveform is

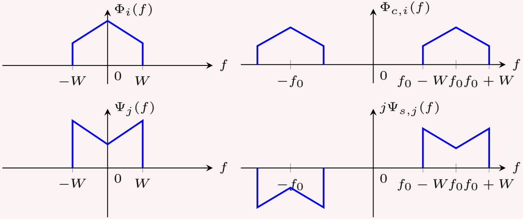
$$\begin{aligned} s_m(t) &= s^c_m(t)\sqrt{2}\cos(2\pi f_0 t) + s^s_m(t)\sqrt{2}\sin(2\pi f_0 t) \\ &= \sum_{i=1}^{N_c} s^c_{mi}\phi_i(t)\sqrt{2}\cos(2\pi f_0 t) + \sum_{j=1}^{N_s} s^s_{mj}\psi_j(t)\sqrt{2}\sin(2\pi f_0 t) \\ &= \sum_{i=1}^{N_c} s^c_{mi}\phi_{c,i}(t) + \sum_{j=1}^{N_s} s^s_{mj}\psi_{s,j}(t) \end{aligned}$$

where $\phi_{c,i}(t)$ and $\psi_{s,j}(t)$ are in-phase and quadrature building-block waveforms.

# Quadrature Multiplexing: Orthogonality (2/2)

## Building-block waveforms?

Are $\phi_{c,i}(t)$ and $\psi_{s,j}(t)$ building-block waveforms?
**Step 1** of proof is (11.9) of the reader:




**Step 2** of proof is to show (see (11.10)–(11.12)) of the reader)

$$\int_{-\infty}^{\infty} \phi_{c,i}(t)\phi_{c,i'}(t)dt = \int_{-\infty}^{\infty} \Phi_{c,i}(f)\Phi^*_{c,i'}(f)df = \delta_{i,i'}$$

$$\int_{-\infty}^{\infty} \phi_{s,i}(t)\phi_{s,i'}(t)dt = \delta_{i,i'} \qquad \text{and} \qquad \int_{-\infty}^{\infty} \phi_{s,i}(t)\phi_{c,i'}(t)dt = 0$$

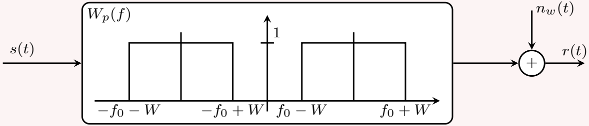
# Quadrature Multiplexing: Building-block Waveforms

## Quadrature Building-Block Waveforms

We have shown that all in-phase building-block waveforms $\phi_{c,i}(t)$ for $i = 1, \ldots, N_c$ and all quadrature building-block waveforms $\psi_{s,j}(t)$ for $j = 1, \ldots, N_s$ together form an orthonormal basis.

## Bandwidth Considerations

The spectra $\Phi_{c,i}(f)$ and $\Psi_{s,j}(f)$ of all these building-block waveforms are zero outside the passband $\pm[f_0 - W, f_0 + W]$. Therefore none of these building-block waveforms is hindered by the passband filter $W(f)$ when they are sent over our passband channel.



# Module 11.1

## Presentation Outline

# Capacity of the Passband Channel

## Passband Channel Capacity

The capacity is $C_N = 1/2 \log_2 (1 + E_N/(N_0/2))$ [bit/dimension], where $E_N = P_s/(4W)$ [Joule/dimension] and $4W$ is the number of dimension/second for the passband channel (dimensionality theorem).
The capacity of the passband channel with bandwidth $\pm[f_0 - W, f_0 + W]$ is

$$C_N = \frac{1}{2} \log_2 \left(1 + \frac{P_s}{2N_0W}\right) \left[\frac{\text{bit}}{\text{dimension}}\right]$$

The capacity in bit per second is

$$C = 2W \log_2 \left(1 + \frac{P_s}{2N_0W}\right) \left[\frac{\text{bit}}{\text{second}}\right]$$

## Connection to Baseband AWGN Capacity

The passband bandwidth is $2W$, thus

$$C = W \log_2 \left(1 + \frac{P_s}{WN_0}\right) \left[\frac{\text{bit}}{\text{second}}\right].$$

# Who Cares?

## IEEE 802.11 Standards (WiFi)

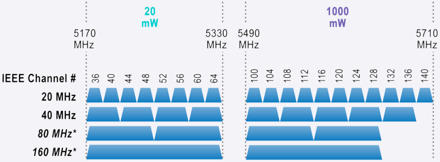




**FIGURE 2** • *European channel allocations for 20/40/80/160 MHz*

# Summary Module 11.1

**Take Home Messages**

- Motivation for passband transmission
- Quadrature transmitter using in-phase and quadrature building-blocks
- Capacity of the passband channel has the same structure of the baseband AWGN channel

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ICT LAB

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