2 - Root Locus design method

Root Locus method

Analyse changes in the system parameters that modify the roots of the characteristic equation

Closed loop poles

analyse closed loop behaviour using the open loop plant

- understanding of system
- Implication on changes

But First, a side step

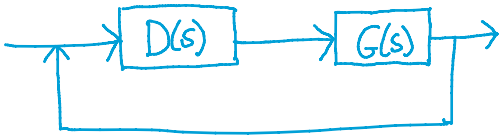
n = # Poles , m = # zeros

Product operator

$$G(s) = \frac{s^m + b_1 s^{m-1} + \dots + b_m}{s^n + a_1 s^{n-1} + \dots + a_n} = \frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)} = \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}$$

Monic: highest power has a coefficient of 1

Strictly	proper	$n > m$
Bi	Proper	$n = m$
non	Proper	$n < m$



$$\frac{D(s)G(s)}{1 + D(s)G(s)}$$

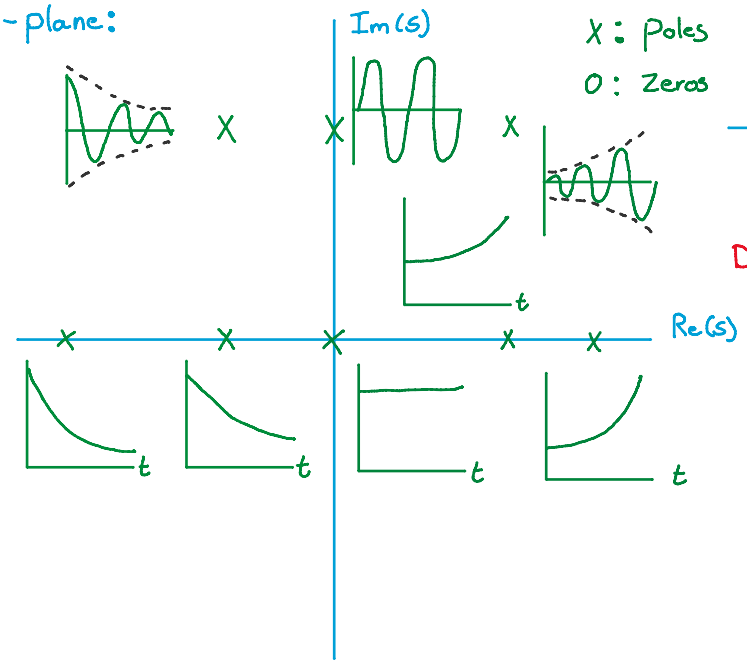
Closed loop system characteristic equation

$$\frac{b(s)}{a(s)} = \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}$$

equivalent forms

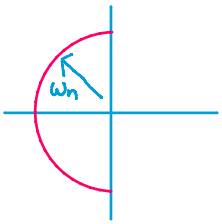
$$1 + kL(s) = 0$$
$$1 + k\frac{b(s)}{a(s)} = 0$$
$$a(s) + kb(s) = 0$$
$$L(s) = -1/k$$

S-plane:

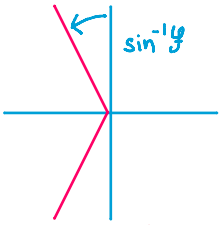


X : Poles

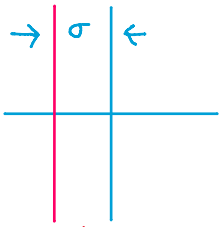
O : Zeros



Damping + Rise time



overshoot



Settling time

Waveform : e^{st}

Root Locus

plural : Loci

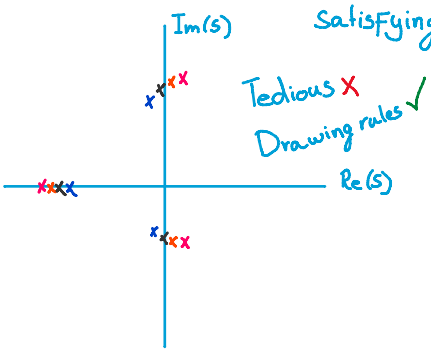
Locus : position or place where something occurs or is situated.

↳ Curve or figure formed by points satisfying an equation

Root Locus : $1 + kL(s) = 0 \iff 1 + D(s)G(s) = 0$

$k \in [0, \infty)$

open loop plant → dynamics closed loop plant



Tedious X

Drawing rules ✓

→ "But my computer can do this"

↳ - Understand change in dynamic response

- Understand consequences on response for controller design

Example:

$$G(s) = \frac{s+2C}{s(s+C)}$$

$$D(s) = 1$$

$$1 + D(s)G(s) = 1 + \frac{s+2C}{s(s+C)}$$

Characteristic equation: $s^2 + s + C(s+2) = 0$

$$L(s) = \frac{s+2}{s(s+1)} \qquad K = C$$

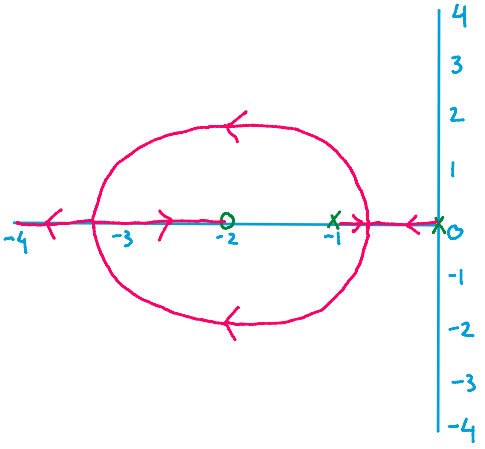
$b(s) = s+2 \quad m = 1 \quad z_i = -2$

$a(s) = s(s+1) \quad n = 2 \quad p_i = 0, -1$

Complex when < 0

$0.172 < K < 5.828$

$$r_1, r_2 = -\frac{K+1}{2} \pm \frac{\sqrt{K^2 - 6K + 1}}{2}$$



Summary

- Closed loop system analysis by varying K and using open loop system
- Solve characteristic equation $1 + KL(s) = 0$
- Changing pole location changes dynamic response
- Demonstration of simple example