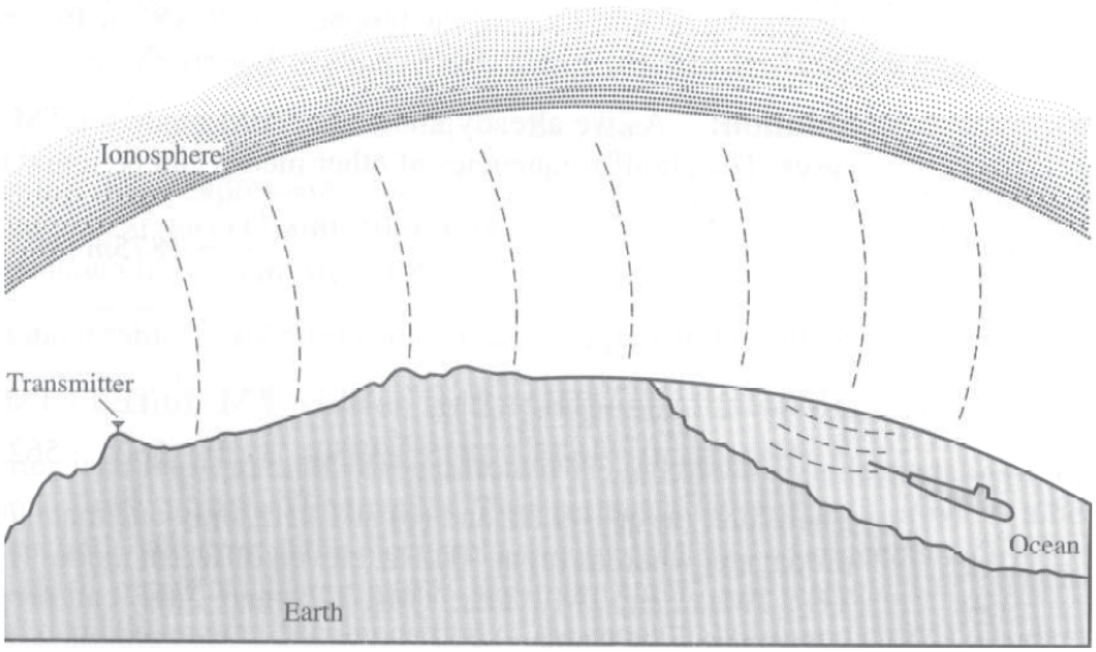
Proposed Solutions EM II (5EPB0) - Parallel-Plate Waveguide

ELF propagation in the earth-ionosphere WG. We consider the propagation of radiowaves in the extremely low frequency range (3 Hz - 3kHz). The ionosphere and the surface of the earth can act as a waveguide and are assumed to be perfect conductors. We neglect the curvature of the WG and assume it to be flat. We can consider the height of the ionosphere to be 80 km.



a) How many propagating modes are possible at 100 Hz and at 10 kHz (VLF)?

$$k_m = \frac{m\pi}{a} = \frac{\omega_m}{c} = \frac{2\pi f_m}{c} \quad \rightarrow \quad f_m = \frac{c}{2a}m$$

A mode will propagate is $f_m \leq f \quad \Rightarrow \quad$ the maximum modal index is:

$$m_{max} = \left[\frac{2af}{c}\right]$$

@ 100 Hz $\quad m_{max} = 0 \quad \Rightarrow \quad$ one mode propagates (TM_0) (or TEM)
@ 10 kHz $\quad m_{max} = 5 \quad \Rightarrow \quad$ eleven modes propagate: TEM, five TM, five TE

b) Due to the lack of ionizing UV light, the ionosphere rises by night to the height of 90 km. How many *additional* modes can now propagate at a frequency of 10 kHz?

$m_{max} = 6 \quad \Rightarrow \quad$ Two additional modes will propagate: TM_6, TE_6

Book. Solve problems 13.11 - 13.16.

13.12)

$$f < f_1 \quad \Rightarrow \quad f < \frac{c}{2\sqrt{\varepsilon_r} d} \quad \Rightarrow \quad d_{max} = \frac{c}{2\sqrt{\varepsilon} f_{max}} = 3.45\text{cm}$$

13.14)

$$f > f_m \quad \Rightarrow \quad f > \frac{mc}{2\sqrt{\varepsilon_r} d} \quad \Rightarrow \quad m < \frac{2f\sqrt{\varepsilon_r} d}{c} = 3.09 \quad \Rightarrow \quad m_{max} = 3$$

$\Rightarrow$ Seven modes will propagate: TM_0 (TEM), TM_1, TM_2, TM_3, TE_1, TE_2, TE_3

13.16)

$$f_2 = 2f_1 = 15 \text{ GHz} \qquad f = \frac{c}{\lambda} = 20 \text{ GHz}$$

$$v_{g2} = c_0 \sqrt{1 - \left(\frac{f_2}{f}\right)^2} = 2 \cdot 10^8 \text{ m/s} = \frac{2}{3} c_0$$

eq. (57)