Components in Wireless Technologies

Module 3:

Antenna Theory – Fundamental Antenna Parameters

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Where innovation starts

Course Planning 2024

Module 3: Antennas	Lecture:	Antenna parameters & theory	Feb 27, 5+6	Atlas 10.330	Johannsen	-
	Exercises:	Antenna properties	Feb 27, 7+8	Atlas 10.330	Johannsen	Telluri, Yadav
	Lab:	CST antenna simulations	Feb 29, 1+2	Atlas 6.208	Bronckers	Telluri, Yadav
	Lab:	VNA measurements of antennas	Feb 29, 3+4	Atlas 6.208	Bronckers	Telluri, Yadav

- Learning objectives of today:
  - Fundamental antenna parameters:
    - Be able to explain them (in your own words)
    - Be able to derive them
    - Be able to evaluate them
  - Be able to calculate the link budget for a basic transceiver system
- Recommended further reading:

  C.A. Balanis, *Antenna Theory – Analysis and Design*, John Wiley&Sons, Inc.

Fundamental Antenna Parameters

Content

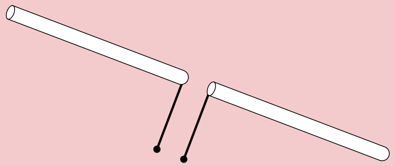
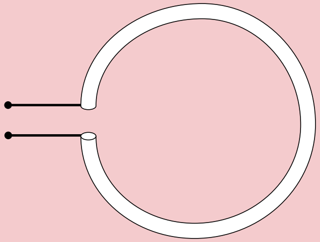
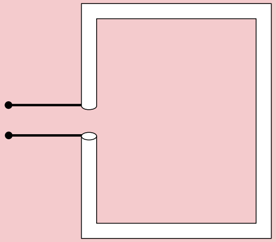
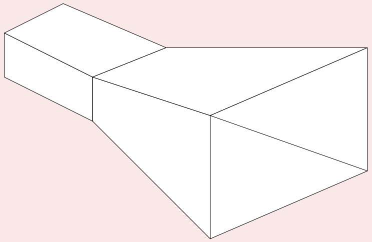
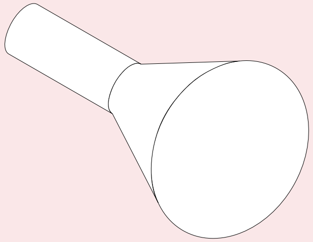
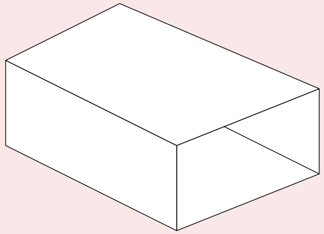
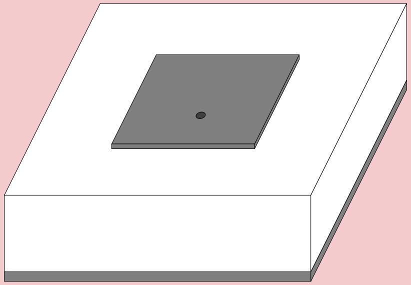
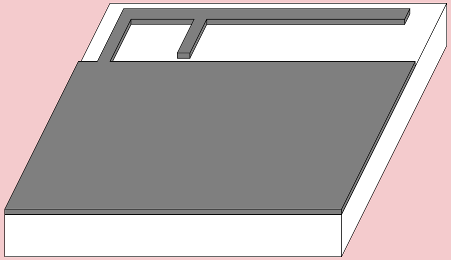
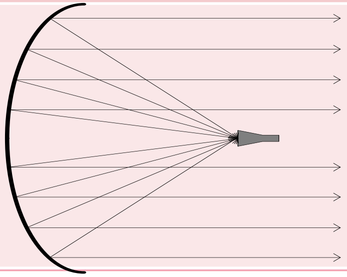
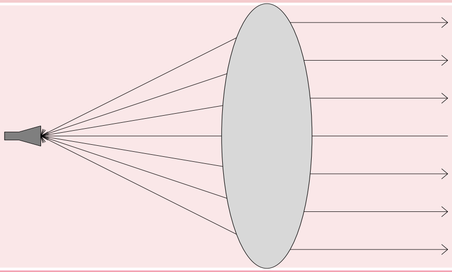
- Radiation Parameters
- Circuit Level Representation
- Propagation Channel Basics

Antenna (Aerial) Definition:

"That part of a transmitting or receiving system that is designed to radiate or to receive electromagnetic waves."

[IEEE Standard for Definitions of Terms for Antennas," in *IEEE Std 145-2013 (Revision of IEEE Std 145-1993)* , vol., no., pp.1-50, March 6 2014]

➔ Transforms guided waves into free space waves and vice versa

Antenna types	Schematic view
Wire antennas	  
Aperture antennas	  
Printed antennas	 
Reflector and lens antennas	 

- Radiation Parameters
  - Radiation Pattern
  - Directivity vs. Gain
  - Polarisation
  - Pattern Bandwidth

- Circuit Level Representation

- Propagation Channel Basics

Radiation Pattern – Definition

“The spatial distribution of a quantity that characterizes the electromagnetic field generated by an antenna.

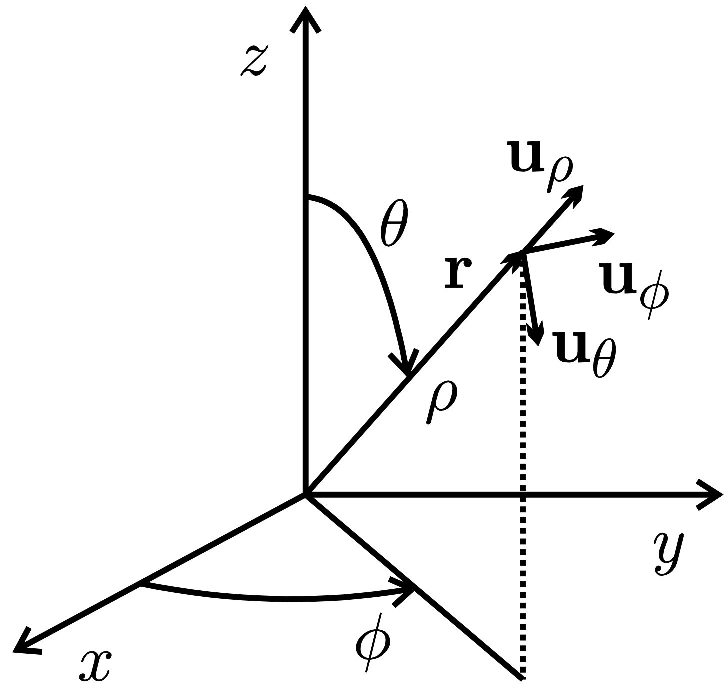
NOTE 1— The distribution can be expressed as a mathematical function or as a graphical representation.

[…]

NOTE 4— When the amplitude or relative amplitude of a specified component of the electric-field vector is plotted graphically, it is called an <u>amplitude pattern</u>, <u>field pattern</u>, or <u>voltage pattern</u>. When the square of the amplitude or relative amplitude is plotted, it is called a <u>power pattern</u>.” [1]

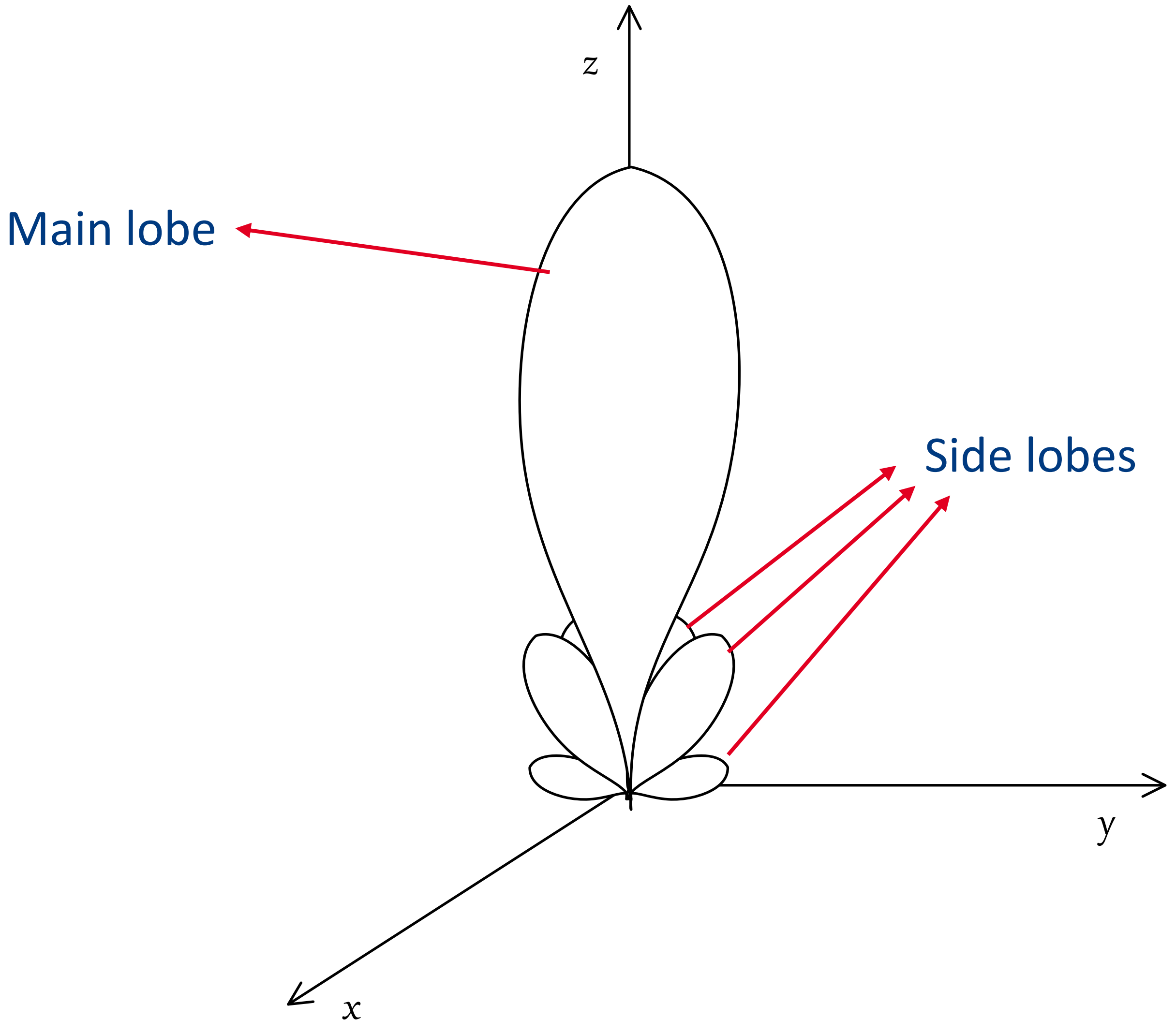
[1] IEEE Standard for Definitions of Terms for Antennas," in IEEE Std 145-2013 (Revision of IEEE Std 145-1993) , vol., no., pp.1-50, March 6 2014

Radiation Pattern – Coordinate System

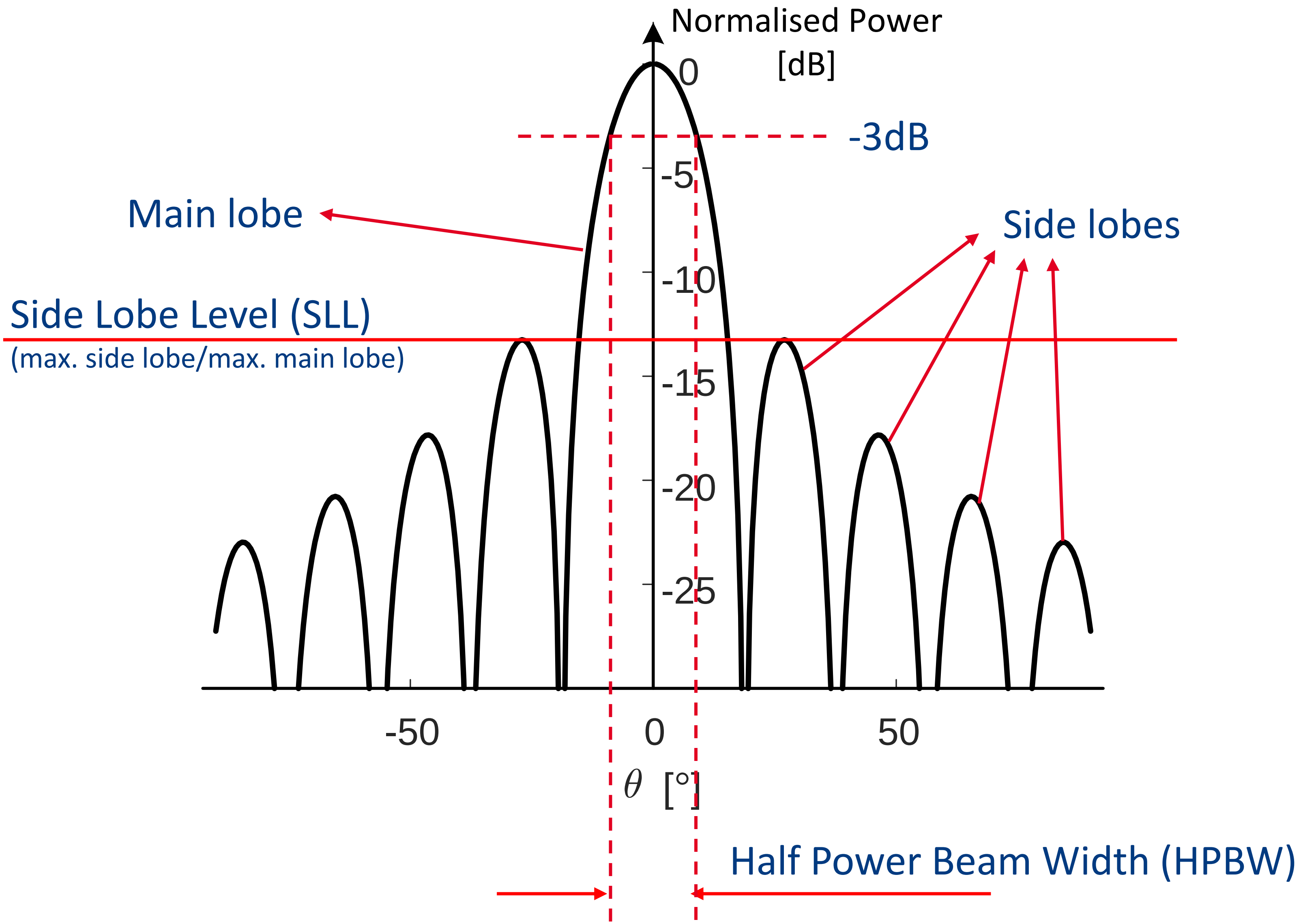


Typically, the <u>power pattern</u> is measured in the far-field. In the far-field, the power pattern is independent of the distance, r. Hence, the pattern is a function/plot of θ and ϕ only.

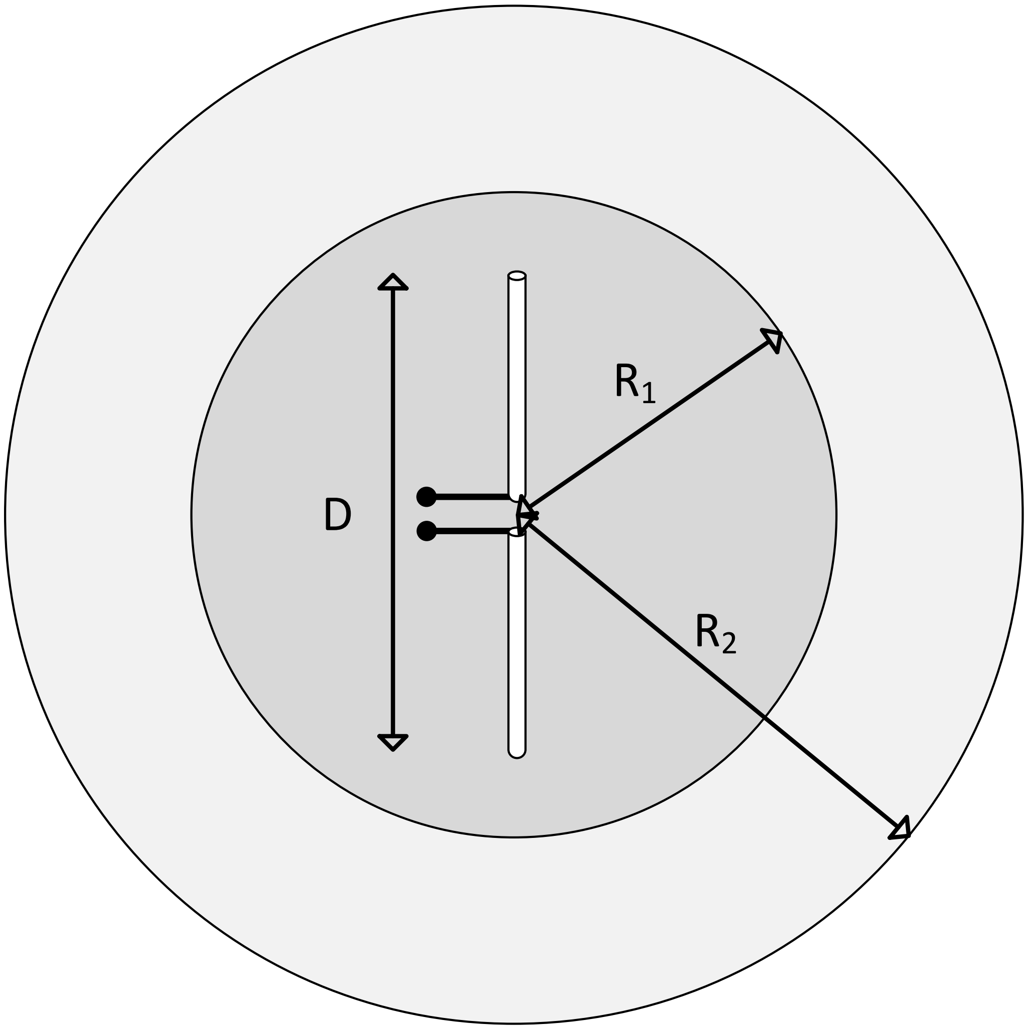
Radiation Pattern – 3D



Radiation Pattern – 2D
(ϕ constant)



Field Regions



D: Largest dimension of antenna

Reactive near-field region:

- $R < R_1 = 0.62\sqrt{D^3/\lambda}$
- Reactive fields predominate
- Radiation pattern changes with R

Radiating near-field (Fresnel) region:

- $R_1 < R < R_2 = 2D^2/\lambda$
- Radiation fields predominate
- Radiation pattern changes with R

Far-field (Fraunhofer) region:

- $R > R_2 = 2D^2/\lambda$
- $R \gg D, R \gg \lambda$
- Radiation fields predominate
- Radiation pattern independent of R

Directivity

The power associated with an electromagnetic wave is given by the instantaneous Poynting vector:

$$\vec{\mathcal{W}}(\vec{r},t) = \vec{\mathcal{E}}(\vec{r},t) \times \vec{\mathcal{H}}(\vec{r},t)$$

$\vec{\mathcal{W}}(\vec{r},t)$: instantaneous Poynting vector [W/m²]
$\vec{\mathcal{E}}(\vec{r},t)$: instantaneous electric field intensity [V/m]
$\vec{\mathcal{H}}(\vec{r},t)$: instantaneous magnetic field intensity [A/m]

$$\vec{\mathcal{E}}(\vec{r},t) = \mathcal{R}e\left\{\vec{E}(\vec{r})\, e^{j\omega t}\right\} = \tfrac{1}{2}\left(\vec{E}(\vec{r})\, e^{j\omega t} + \vec{E}^*(\vec{r})\, e^{-j\omega t}\right)$$
$$\vec{\mathcal{H}}(\vec{r},t) = \mathcal{R}e\left\{\vec{H}(\vec{r})\, e^{j\omega t}\right\} = \tfrac{1}{2}\left(\vec{H}(\vec{r})\, e^{j\omega t} + \vec{H}^*(\vec{r})\, e^{-j\omega t}\right)$$

$$\Rightarrow \vec{\mathcal{W}}(\vec{r},t) = \frac{1}{4}\Big[\vec{E}^*(\vec{r}) \times \vec{H}(\vec{r})\ e^{0} + \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})\, e^{0} + \vec{E}(\vec{r}) \times \vec{H}(\vec{r})\ e^{j2\omega t} + \vec{E}^*(\vec{r}) \times \vec{H}^*(\vec{r})\ e^{-j2\omega t}\Big]$$

$$\vec{\mathcal{W}}(\vec{r},t) = \frac{1}{2}\mathcal{R}e\left\{\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})\right\} + \frac{1}{2}\mathcal{R}e\left\{\vec{E}(\vec{r}) \times \vec{H}(\vec{r})\, e^{j2\omega t}\right\}$$

Directivity

This means that for the time average Poynting vector we obtain:

$$\overrightarrow{W}_{av}(\vec{r}) = \frac{1}{T}\int_{t}^{t+T} \overrightarrow{\mathcal{W}}(\vec{r},t)dt$$

$$= \frac{1}{T}\int_{t}^{t+T} \frac{1}{2}\mathcal{R}e\{\vec{E}(\vec{r}) \times \vec{H}^{*}(\vec{r})\} + \overbrace{\frac{1}{2}\mathcal{R}e\{\vec{E}(\vec{r}) \times \vec{H}(\vec{r})\, e^{j2\omega t}\}dt}^{=\,0}$$

$$\boxed{\overrightarrow{W}_{av}(\vec{r}) = \frac{1}{2}\mathcal{R}e\{\vec{E}(\vec{r}) \times \vec{H}^{*}(\vec{r})\}}$$

Power density [W/m²]

Directivity

The power density decays with ${}^{1}/_{R^2}$. Hence, the following quantity

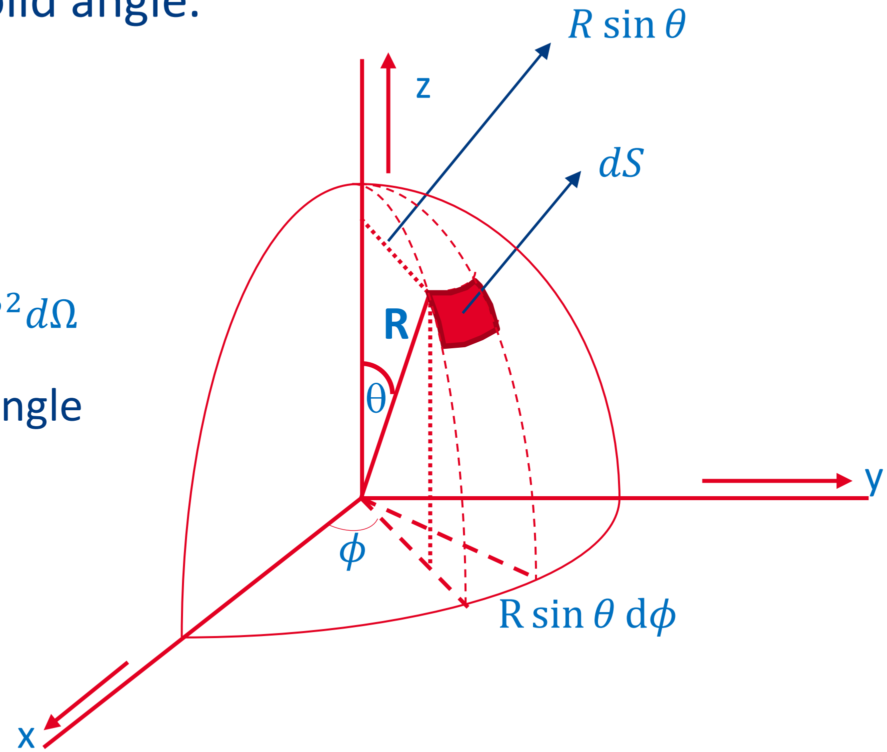
$$\boxed{\vec{U}(\theta, \phi) = R^2\, \overrightarrow{W_{av}}(\vec{r}),}$$

called the radiation intensity, is defined as distance independent parameter. It provides the radiated power per unit solid angle.

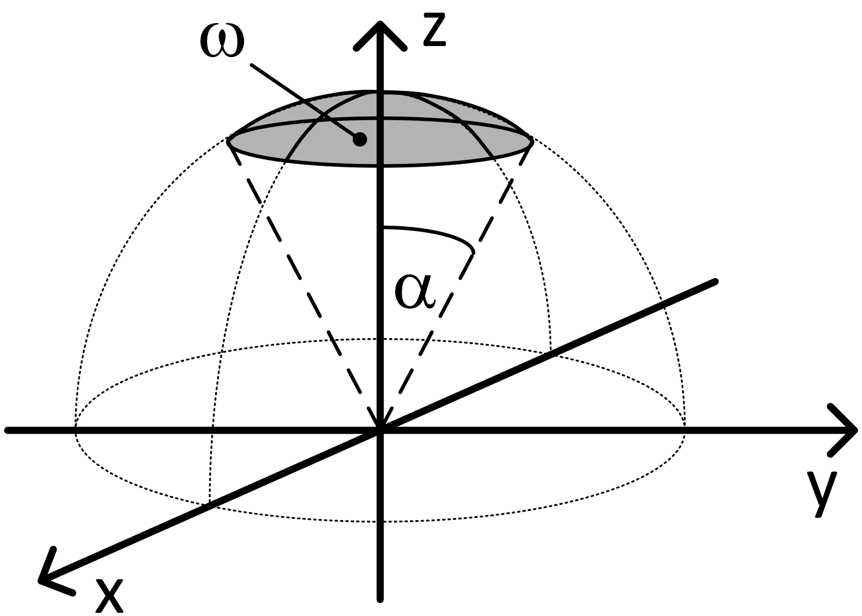
Recap of spherical coordinates:

$$dS = (R\ d\theta)(R \sin\theta\ \ d\phi) = R^2 \sin\theta\ d\theta d\phi = R^2 d\Omega$$

Ω: solid angle



Note on Unit Solid Angle



Definition of solid angle ω:
That part of space which is bounded by the rays emerging from one point, the vertex, to all points of a closed curve*.

$$\omega = \int_0^{2\pi} \int_0^{\alpha} \sin\theta \, d\theta d\phi$$

$$\omega = \int_0^{2\pi} 1 - \cos\alpha \, d\phi$$

$$\omega = 2\pi(1 - \cos\alpha)$$

$$\alpha = \cos^{-1}\left(1 - \frac{\omega}{2\pi}\right)$$

$$\xrightarrow{\omega=1} \alpha \approx 32.77°$$

* F. Kasten, "Note on the Unit Solid Angle," Journal of the Optical Society of America, 1 June 1964, Volume 54, Issue 6, pp. 845-846

Directivity

The radiation intensity still depends on the totally radiated power, since

$$P_{rad} = \oiint \vec{W}(\theta,\phi) d\vec{S} = \oiint \vec{W}(\theta,\phi)\overrightarrow{u_\rho} R^2 \sin\theta \, d\theta d\phi = \oiint U(\theta,\phi) \sin\theta \, d\theta\phi.$$

In order to achieve a power independent figure, we normalise the radiation intensity with the respective radiation intensity of the (ideal) isotropic radiator:

$$\boxed{D_g = \frac{U(\theta,\phi)}{U_0}}$$

D_g is called <u>directivity</u>. The radiation intensity of the isotropic radiator is given by $U_0 = \frac{P_{rad}}{4\pi} \left(= R^2 \frac{P_{rad}}{4\pi R^2}\right)$ and is angle independent (by definition).

Gain

In order to determine the directivity, we need to know the totally radiated power. This quantity is difficult to measure (requires a complete hemispherical scan in the far-field). It is much easier to determine the power, P_{in}, that enters the antenna at its input terminal/port. In analogy to the directive gain we then define the gain as:

$$G_g = \frac{4\pi\, U(\theta, \phi)}{P_{in}}.$$

The input power is related to the radiated power by the efficiency, e_t, of the antenna:

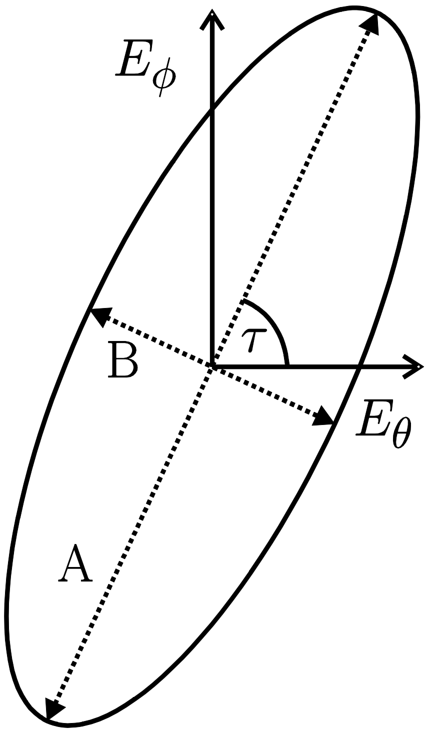
$$P_{rad} = e_t P_{in} = e_r e_{cd} P_{in}.$$

with $e_r = 1 - |\Gamma|^2$ the reflection (mismatch) efficiency, e_{cd} the efficiency related to conduction and dielectric loss. Hence, we have

$$\boxed{G_g = e_t D_g}$$

Polarisation – Definition

"In a specified direction from an antenna and at a point in its far field, the polarization of the (locally) plane wave that is used to represent the radiated wave at that point." [1]



The axial ratio, AR, is defined as:

$$AR = \frac{A}{B} = \frac{\left|E_\theta \overrightarrow{u_\theta} + E_\phi \overrightarrow{u_\phi}\right|_{max}}{\left|E_\theta \overrightarrow{u_\theta} + E_\phi \overrightarrow{u_\phi}\right|_{min}}$$

Linear polarisation: $AR = \infty$
Circular polarisation: $AR = 1$

[1] IEEE Standard for Definitions of Terms for Antennas," in IEEE Std 145-2013 (Revision of IEEE Std 145-1993) , vol., no., pp.1-50, March 6 2014

Polarisation

The wave is propagating towards $\overrightarrow{u_r}$ and its electric field can be decomposed in the following two components (in time domain):

$$E_\theta = \widehat{E_\theta}\cos\left(\omega t - \vec{k}R\overrightarrow{u_r} + \varphi_\theta\right)$$
$$E_\phi = \widehat{E_\phi}\cos\left(\omega t - \vec{k}R\overrightarrow{u_r} + \varphi_\phi\right)$$

Depending on the phase difference between the two components, we can observe different polarisation behaviour:

Linear polarisation: $\Delta\varphi = \varphi_\theta - \varphi_\phi = n\pi, n \in \mathbb{N}_0$

Circular polarisation: $\widehat{E_\theta} = \widehat{E_\phi}$ and $\Delta\varphi = \varphi_\theta - \varphi_\phi = \pm\left(\frac{1}{2} + 2n\right)\pi, n \in \mathbb{N}_0$

Elliptical polarisation: $\widehat{E_\theta} \neq \widehat{E_\phi}$ and $\Delta\varphi = \varphi_\theta - \varphi_\phi = \pm\left(\frac{1}{2} + 2n\right)\pi, n \in \mathbb{N}_0$

or $\Delta\varphi = \varphi_\theta - \varphi_\phi \neq \pm\frac{n}{2}\pi, n \in \mathbb{N}_0$ (tilted ellipse)

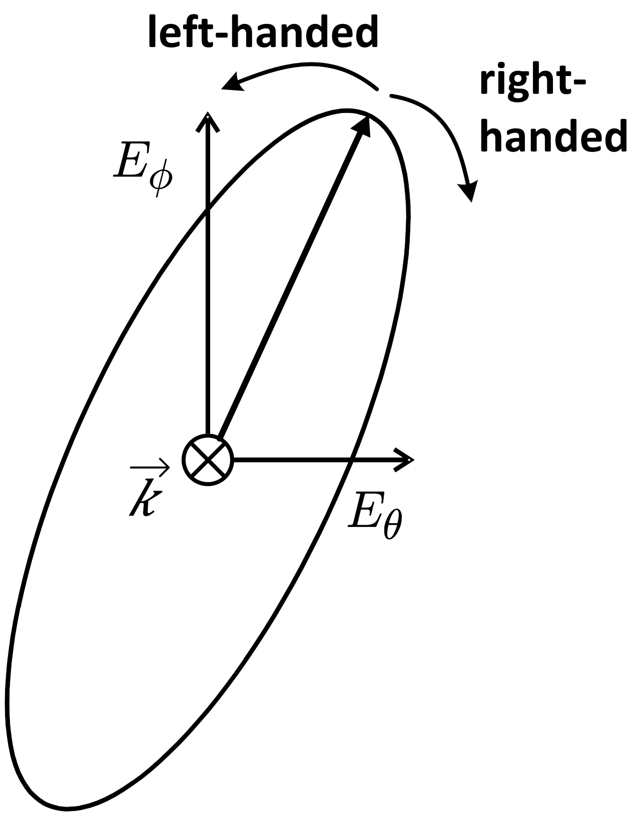
Determines rotation direction

Polarisation

<u>Sense of polarisation:</u>

"For an elliptical or circularly polarized field vector, the sense of rotation of the extremity of the field vector when its origin is fixed.

NOTE—When the plane of polarization is viewed from a specified side, if the extremity of the field vector rotates clockwise [counterclockwise] the sense is right-handed [left-handed]. For a plane wave, the plane of polarization is viewed looking in the direction of propagation." [1]



[1] IEEE Standard for Definitions of Terms for Antennas," in IEEE Std 145-2013 (Revision of IEEE Std 145-1993) , vol., no., pp.1-50, March 6 2014

Pattern Bandwidth – Definition

"The range of frequencies within which the performance of the antenna conforms to a specified standard with respect to some characteristic." [1]

The term pattern bandwidth is typically used when referred to variations in parameters associated with the radiation pattern, e.g.

- Gain
- Side lobe level
- Beamwidth
- Polarisation
- Main beam direction

Bandwidth ($BW = f_{max} - f_{min}$) designations for

- Broadband antennas: $\boldsymbol{X}:\mathbf{1}$ (f_{max} is X times larger than f_{min})
- Narrowband antennas: $\boldsymbol{p}$**%** ($p = BW/f_c$, with f_c the centre frequency)

[1] IEEE Standard for Definitions of Terms for Antennas," in IEEE Std 145-2013 (Revision of IEEE Std 145-1993) , vol., no., pp.1-50, March 6 2014

- Radiation Parameters

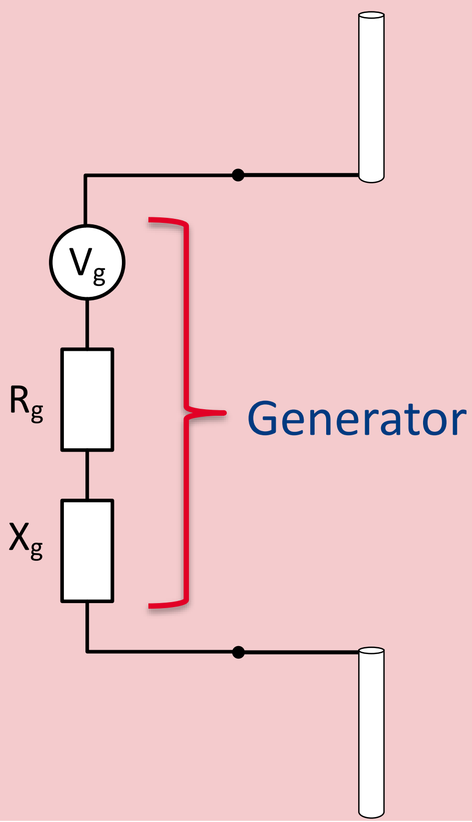
- Circuit Level Representation
  - Input impedance
  - Radiation efficiency
  - Impedance Bandwidth

- Propagation Channel Basics

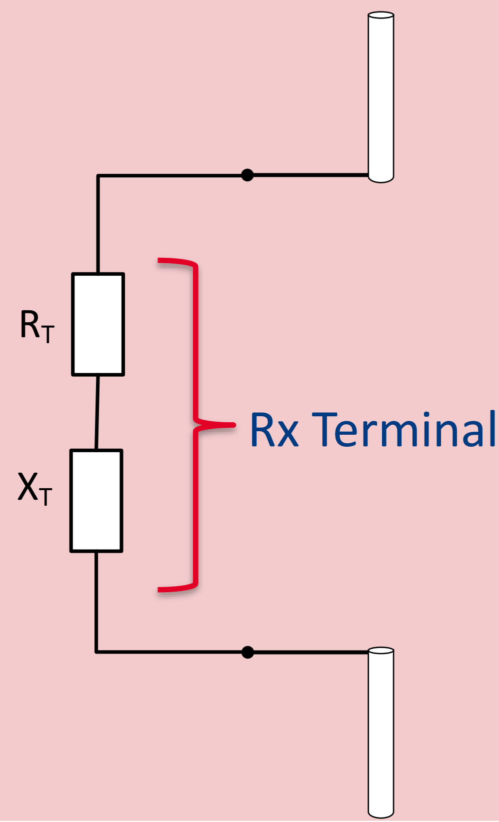
Input Impedance

Depending on its mode of operation, an antenna from a circuit point of view can be represented as a load or a generator, respectively. Starting point are the schematic views below:

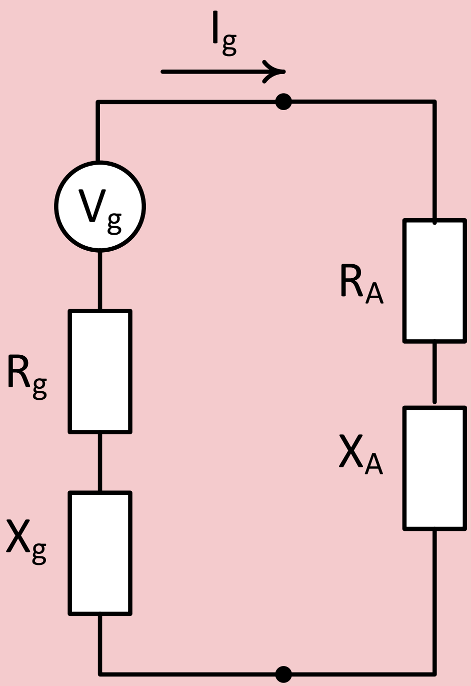
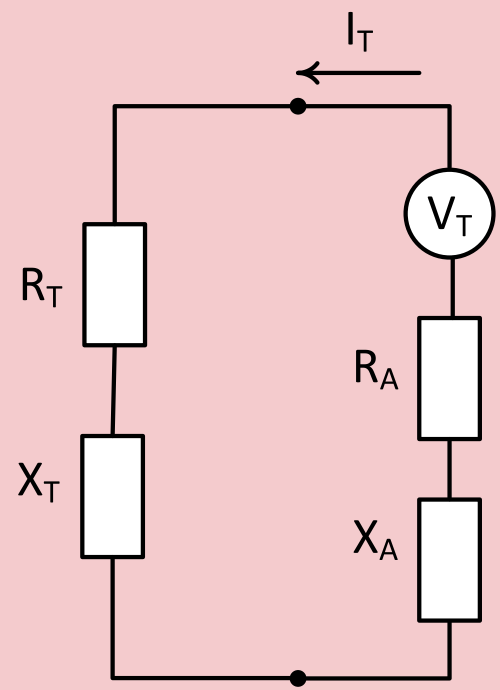
Transmitting mode



Receiving mode



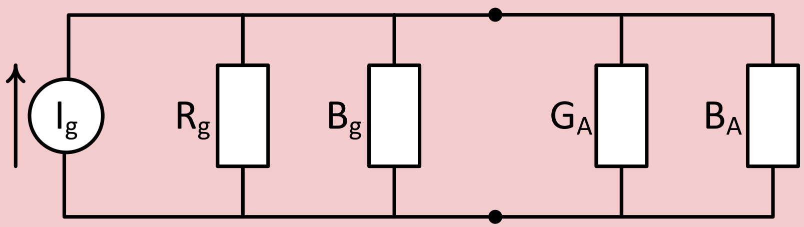
Input Impedance

Transmitting mode	Receiving mode
	

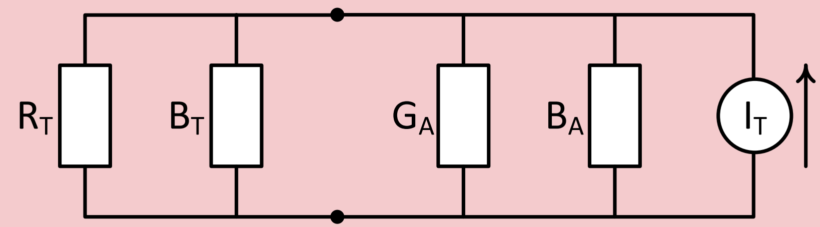
Thevenin equivalent model

Input Impedance and Radiation Efficiency

Transmitting mode



Receiving mode



Norton equivalent model

Thevenin: $Z_A = R_A + jX_A = R_{rad} + R_{loss} + jX_A \rightarrow \frac{R_{rad}}{R_{rad}+R_{loss}}$

Norton: $Y_A = G_A + jB_A = G_{rad} + G_{loss} + jB_A \rightarrow \frac{G_{rad}}{G_{rad}+G_{loss}}$

$$= \frac{P_{rad}}{P_{in}} = e_t$$

Input Impedance and Effective Aperture

In receive mode, the power, P_T, accepted by the Rx terminal can be linked to the power density, $W_{inc} = \left|\vec{W}_{av}(\vec{r})\right|$, of the incident electromagnetic wave:

$$P_T = W_{inc}\, A_e$$

A_e is called the effective aperture of the antenna. Reformulating above equation yields

$$A_e = \frac{P_T}{W_{inc}} = \frac{|V_T|^2}{2W_{inc}} \left(\frac{R_T}{(R_{rad} + R_{loss} + R_T)^2 + (X_T + X_A)^2} \right)$$

For the case of maximum power transfer, i.e. $R_T = R_A$ and $X_T = -X_A$:

$$\boxed{A_{e,max} = \frac{|V_T|^2}{8W_{inc}} \left(\frac{1}{R_{rad} + R_{loss}} \right)}$$

Input Impedance and Effective Aperture - Example -

A very short ($l \ll \lambda$) lossless dipole (R_{loss} =0) exhibits the radiation resistance $R_{rad} = 80(\pi l/\lambda)^2$. The maximum effective aperture is then given by:

$$A_{e,max} = \frac{|V_T|^2}{8W_{inc}}\left(\frac{1}{80(\pi l/\lambda)^2}\right)$$

As the dipole is very short, the voltage is given by $V_T = \left|\vec{E}(\vec{r})\right| l$ (assuming that the incident electric field is directed along the dipole). Furthermore, we know that for a plane wave in free space it holds that

$$W_{inc} = \left|\overrightarrow{W_{av}}(\vec{r})\right| = \left|\frac{1}{2}\mathcal{R}e\{\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})\}\right|$$
$$= \left|\frac{1}{2}\mathcal{R}e\{\left|\vec{E}(\vec{r})\right|\left|\vec{H}^*(\vec{r})\right| \sin \perp \overrightarrow{u_k}\}\right| = \frac{1}{2}\left|\vec{E}(\vec{r})\right|\left|\vec{H}(\vec{r})\right| = \frac{\left|\vec{E}(\vec{r})\right|^2}{2\eta}$$

with $\eta = 120\pi\ \Omega$ the impedance of free space.

Input Impedance and Effective Aperture - Example -

Such that we finally end up with:

$$A_{e,max} = \frac{\left|\vec{E}(\vec{r})\right|^2 l^2 \lambda^2 2\eta}{8\left|\vec{E}(\vec{r})\right|^2 80\pi^2 l^2} = \frac{3\lambda^2}{8\pi}$$

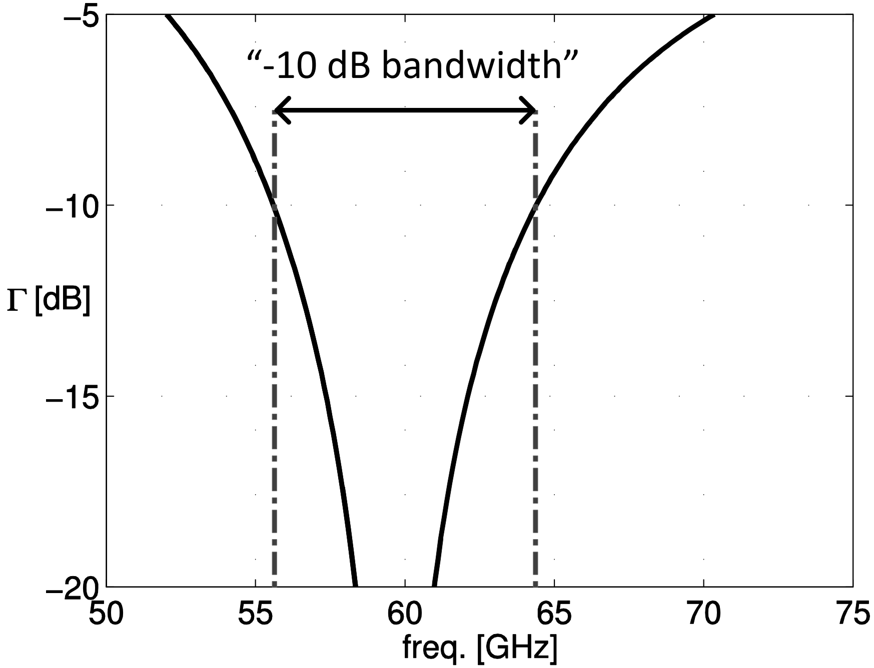
This result will later be very useful for the derivation of the Friis Equation!

Impedance Bandwidth – Definition

"The range of frequencies within which the performance of the antenna conforms to a specified standard with respect to some characteristic." [1]

The term <u>impedance bandwidth</u> is typically used when referred to variations in parameters associated with the input impedance, e.g.

- Reflection coefficient
- Radiation efficiency



[1] IEEE Standard for Definitions of Terms for Antennas," in IEEE Std 145-2013 (Revision of IEEE Std 145-1993) , vol., no., pp.1-50, March 6 2014

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- Radiation Parameters
- Circuit Level Representation
- Propagation Channel Basics
  - Friis Transmission Equation
  - Other propagation effects

Friis Transmission Equation

The Friis transmission equation relates the power received by an antenna 2 to the radiated power by antenna 1 and vice versa.

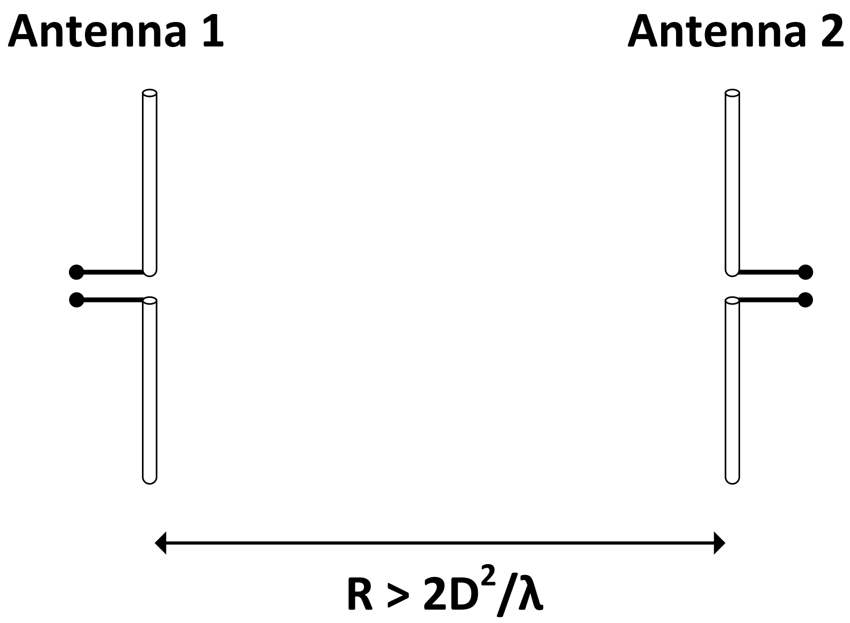
The power density of an isotropic radiator at distance R is given by:

$$W_{inc,iso} = \frac{P_{rad}}{4\pi R^2}.$$

Hence, for a real antenna we obtain:

$$W_{inc} = D_{g1}\frac{P_{rad}}{4\pi R^2} = G_1\frac{P_{in}}{4\pi R^2},$$

with G_1 the gain of antenna 1.



Friis Transmission Equation

The power received by antenna 2 depends on its effective aperture:

$$P_{rec} = W_{inc}\, A_{e,2} = A_{e,2} G_1 \frac{P_{in}}{4\pi R^2}$$

$$\frac{P_{rec}}{P_{in}}(4\pi R^2) = A_{e,2} G_1.$$

If we used antenna 2 in transmitting mode and antenna 1 as receiving antenna, we would get:

$$\frac{P_{rec}}{P_{in}}(4\pi R^2) = A_{e,1} G_2.$$

Hence, the following relation must hold:

$$\boxed{\frac{A_{e,1}}{G_1} = \frac{A_{e,2}}{G_2}}$$

Friis Transmission Equation

Apparently, the ratio of gain to effective aperture in a certain direction is the same for all antennas! We know that for a very short lossless dipole $A_{e,max} = \frac{3\lambda^2}{8\pi}$ (see previous example). Its maximum gain can be determined to be 1.5. Therefore:

$$\boxed{\frac{A_{e,max}}{G_{max}} = \frac{\lambda^2}{4\pi}.}$$

Using this result, we can re-write for the received power:

$$P_{rec} = W_{inc}\, A_{e,2} = A_{e,2} G_1 \frac{P_{in}}{4\pi R^2} = G_1 G_2 \frac{\lambda^2}{(4\pi R)^2} P_{in}$$

$$\boxed{\frac{P_{rec}}{P_{in}} = \left(\frac{\lambda}{4\pi R}\right)^2 G_1 G_2}$$

Friis Transmission Equation

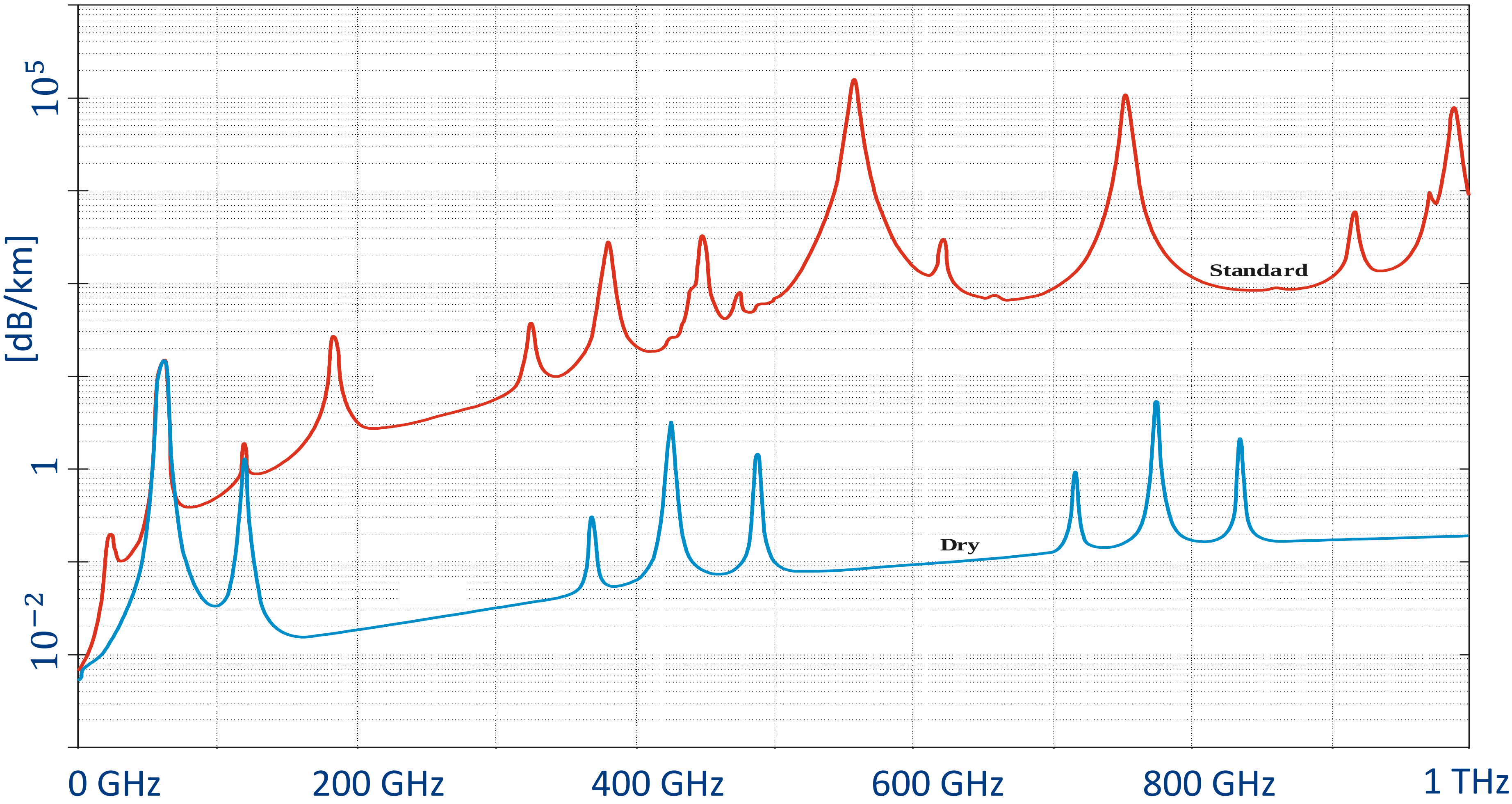
Friis Transmission Equation

Some definitions related to the Friis equation:

$$P_{rec} = \overbrace{\left(\frac{\lambda}{4\pi R}\right)^2}^{(\text{“Path Loss”})^{-1}} G_{Rx} \underbrace{G_{Tx} P_{in}}_{\substack{\text{Equivalent isotropically} \\ \text{radiated power (EIRP)}}}$$

Note:
The definition of “path loss” might indicate that the received power level decreases when increasing the frequency. This is, however, not true! As long as the antenna sizes (i.e. their effective apertures) are not changed, the received power level stays constant (when λ decreases, the gain increases).

Specific attenuation due to atmospheric gases [2]



[2] ITU Recommendation: ITU-R P.676-11 - Attenuation by atmospheric gases (International Telecommunication Union, 2016)

Earth to Satellite Communication Link

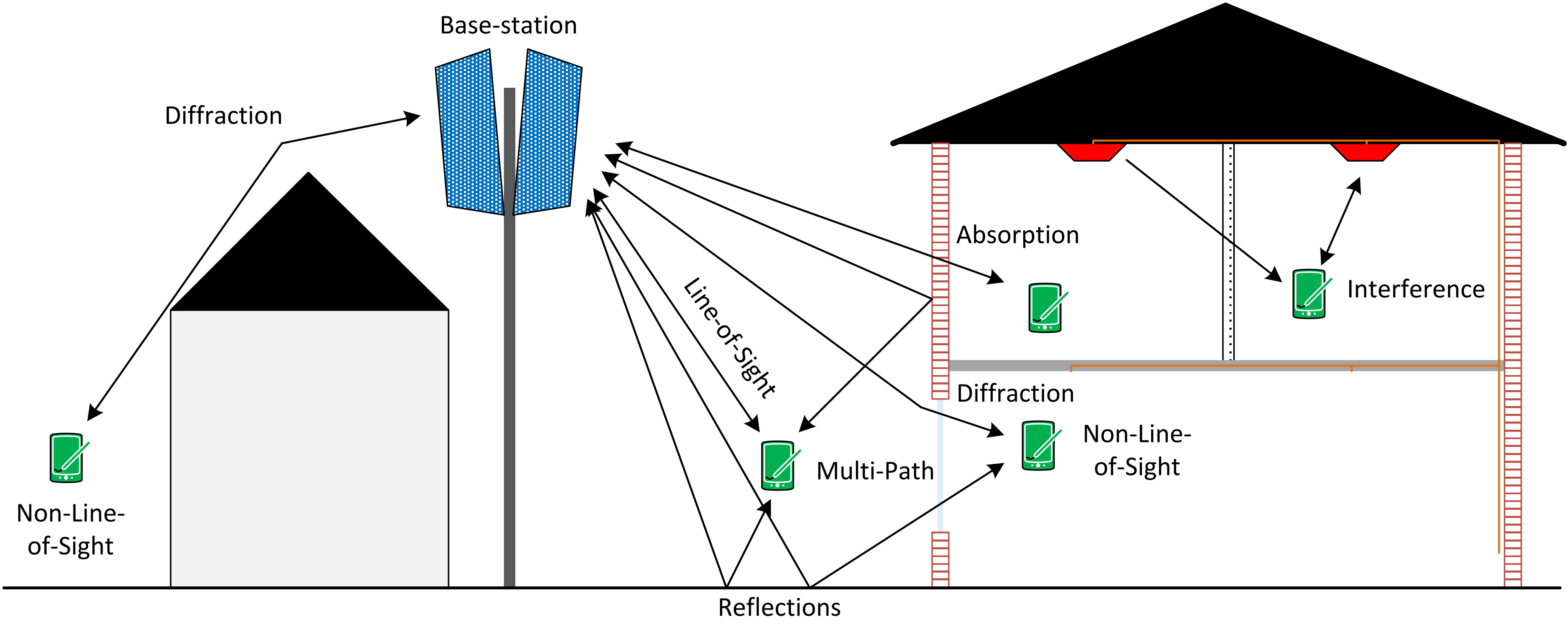
"Caused by solar radiation, the Earth's ionosphere consists of several regions of ionization. […] In each region, the ionized medium is neither homogeneous in space nor stationary in time. […] In addition to the background ionization, there are always highly dynamic, small-scale non-stationary structures known as irregularities." [3]

"The following effects may take place on an Earth-space path when the signal is passing through the ionosphere:

- rotation of the polarization […];
- group delay and phase advance of the signal […];
- rapid variation of amplitude and phase (scintillations) of the signal […];
- a change in the apparent direction of arrival due to refraction;
- Doppler effects due to non-linear polarization rotations and time delays." [3]

[3] ITU Recommendation: ITU-R P.531-13 - Ionospheric propagation data and prediction methods required for the design of satellite services and systems (International Telecommunication Union, 2016)

Other Propagation Effects



- History of Electromagnetic Waves:

- https://youtu.be/SAfIOfXl7ic