Student-led tutorials 5EPB0 ‘EM II’

SLT-C

May 13th 2025

Use of AI for solving SLT questions:

You are allowed to use generative AI, specifically to create content (like images, (source) code, video, text or any other kind). You are allowed to use AI tools as a source for inspiration, e.g. to enhance design and/or research activities, or to improve/enhance the quality of products, but you are not allowed to copy-paste pieces of writing from the AI tool into your assignment. You should be able to explain why you opted to use specific tools and evaluate their usefulness.

Philosophy

1. (a) In the context of electromagnetic wave theory, the constitutive relation between the electric field $E(\mathbf{r}, t)$ and the resulting current density $J(\mathbf{r}, t)$ is given by:

$$J(\mathbf{r}, t) = \int_0^\infty \sigma(\mathbf{r}, t')E(\mathbf{r}, t - t')\, dt' \tag{1}$$

where $\sigma(\mathbf{r}, t)$ is the conduction relaxation function.

From the specific form of this equation, we can infer certain properties of this constitutive relation. List and describe three of the four key properties that define this constitutive relation.

(b) What is the voltage standing wave ratio? Explain in your own words (not just the formula).

(c) Consider a homogeneous layer (or a uniform section of a transmission line) that includes the section $[z_1, z_2]$ of the real z-axis. Assume that at $z = z_2 = z_1 + \ell$ with $\ell \geq 0$, the voltage $V_{s2} = V_s(z_2)$ and the current $I_{s2} = I_s(z_2)$ are known.

Fill in the provided diagram1 with the correct description of the propagating waves and derive the general expressions for the voltage $V_s(z)$ and current $I_s(z)$ at an arbitrary position z within this section in terms of the given values V_{s2}, I_{s2}.

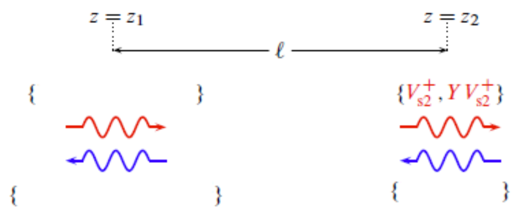

Figure 1: Backward propagation of the progressive voltage and current amplitudes

Normal incidence plane wave

2. A plane wave is normally incident from Medium 1, which has parameter μ_1, ϵ_1, and intrinsic impedance $\eta_1 = 100\ \Omega$, onto a semi-infinite medium 2 with parameters μ_2, ϵ_2 and $\eta_2 = 300\ \Omega$. The incident electric field in phasor form for z<0 is given by:

$$\vec{E}_i(z) = E^{+} e^{-jk_1 z}\, \hat{a}_x, \quad \text{where } E^{+} = 100 \text{ V/m}$$

   (a) Determine the phase velocity and relative permittivity of the wave in both media. Hint: Assume that the media are non-magnetic.
   (b) Calculate the incident electric and magnetic field waves, along with the magnitude of the average incident power density.
   (c) Determine the reflected wave and compute the average reflected power density.
   (d) Derive the expression for the transmitted wave and compute the average transmitted power density.

Pointing Vector

3. A solar power satellite in geostationary orbit transmits energy to Earth using a microwave beam at a frequency of 2.45 GHz. The satellite emits a total power of 50 kW. The beam is directed toward a circular area on Earth's surface with a radius of $r =$ 100 km, representing the irradiated footprint of the beam. Assume uniform power distribution over the area and ideal free-space propagation.
   (a) Define the Poynting Vector and its significance.
   (b) Calculate the average magnitude of the Poynting vector over the irradiated circular area. Also determine the electric and magnetic field amplitudes.
   (c) A ground-based receiving antenna has a radius of 100 m. How much energy is collected by the antenna over a period of 5 minutes?

(d) If the satellite increases its output power to 200 kW, what beam radius would produce the same average intensity as in part (b)?

Galvani

4. Luigi Galvani was a pioneer in bioelectricity. He discovered that nerve tissue reacts to electric pulses. Today, we are going to study one of his world-famous experiments.

   During a thunderstorm, Galvani attached a long metal cable to his house and the other end to severed frog's legs. When lightning struck nearby, the legs moved. Galvani summarized the response as "not so little". See Figure 2.

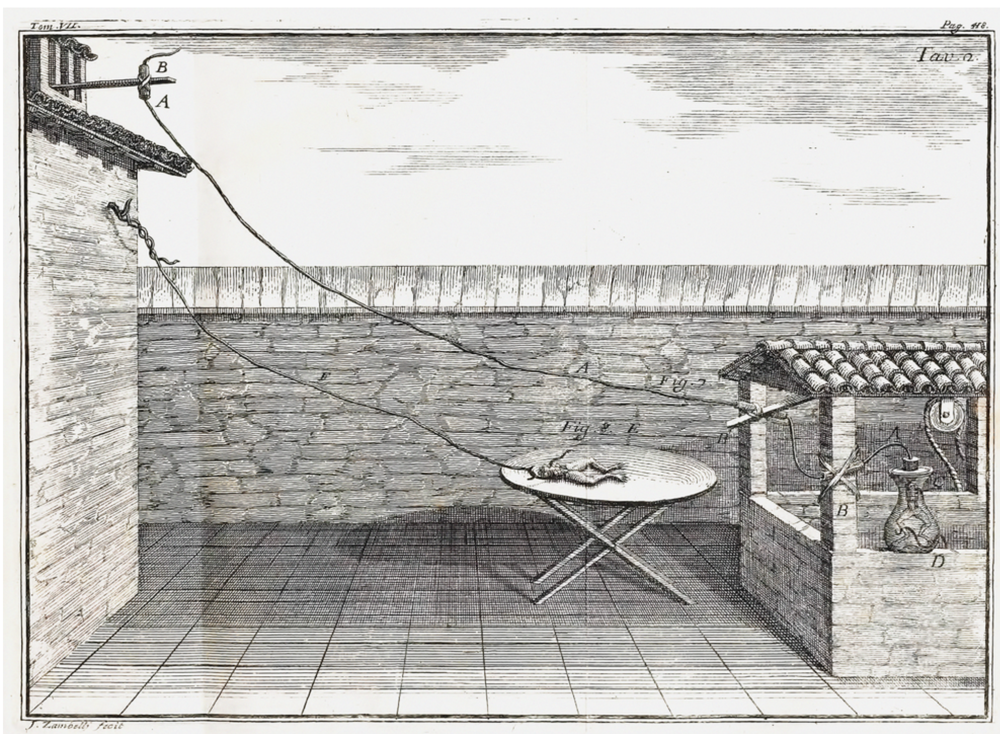

Figure 2: Galvani's actual plans. Please try this at home and send us pictures of the results.

   In Figure 2, you can see an approximation of the system that consists of an open-ended cable that is terminated by frog legs[1]. We hence model the system as a transmission line with a switch attached to an AC source that rapidly opens and closes. See Figure 3.

[1]This system is an approximation. The cable Galvani used is too short for the transmission line model to fit.

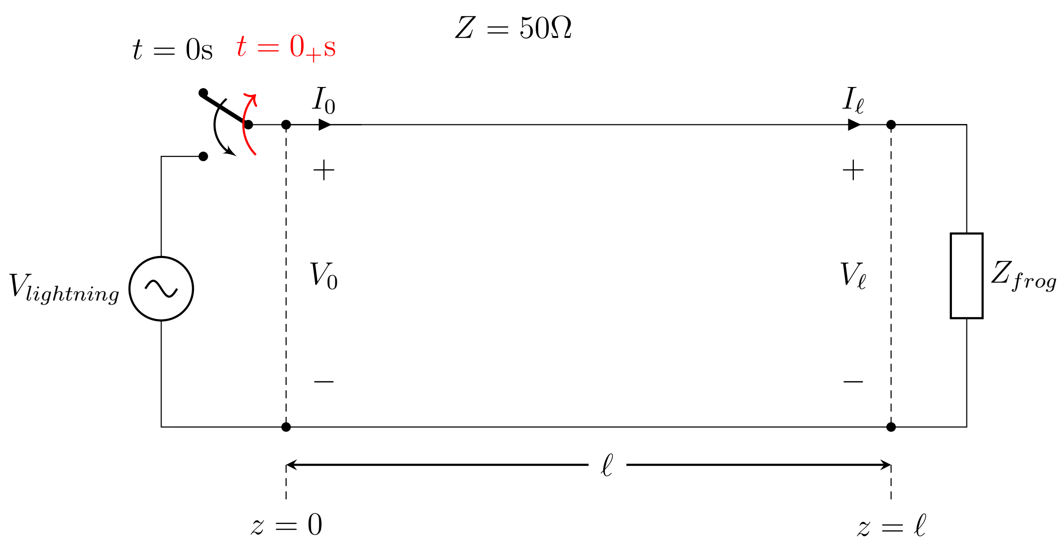

Figure 3: The equivalent circuit of Galvani's experiment. During the impact, the bolt behaves as a perfect voltage source ($Z = 0\Omega$) and enforces a signal onto the cable. A split second after the impact, the source is disconnected.

Lightning has a characteristic frequency of 250 kHz. At this frequency, nerve tissue has a complex permittivity of $\varepsilon = \varepsilon_0 \varepsilon_r - \frac{j\sigma}{\omega}$. At 250 kHz and for nerve tissue, $\varepsilon_r = 2.44\text{k}$ and $\sigma = 96.5\text{mS/m}$ and $\mu = \mu_0$.

(a) Nerve tissue has a complex permittivity. What does the imaginary part of the permittivity represent?

(b) Calculate the load Z_{frog} in Ohms.

(c) Your impedance should be of the form $Z_{frog} = a + jb$. What does the real part of the impedance represent? And the imaginary part? Why?

(d) Find Γ_0 and Γ_ℓ for $t > 0_+\text{s}$.

(e) You find a reflection coefficient with an imaginary part. What does that mean? What happens to a signal that encounters such an interface?"

(f) Calculate the VSWR at $z = \ell$.

(g) Galvani captured the EM-radiation from lightning strikes nearby, which means that $V_{lightning}$ effectively only yielded a few milliVolts. An actual lightning strike carries about 30 kA of current. Furthermore, the energy applied to an object Q [J] is equal to

$$Q = \frac{V^2}{real(Z)} \cdot t, \tag{2}$$

with V the voltage [V] exerted and t [s] the duration of the energy pulse. The temperature increase can be calculated from the applied energy

$$Q = c \cdot m \cdot \Delta T, \tag{3}$$

with c being the heat capacity of nerve tissue (3613 J kg^{-1} K$^{-1 \circ}$), m the mass of the frog's legs (10 grams) and ΔT the temperature increase in degree Kelvin.

Calculate the temperature increase of the frog's legs in case of a lightning strike of 1 us (assume a constant voltage is applied during the strike). Is the frog rare, medium-rare, well-done or burnt to a crisp?

Transfer Matrices, Impedance Networks, and Wave Propagation

You're trying to study for your Electromagnetics II exam, but your roommates are next door, busy streaming Netflix like their lives depend on it. You've had enough. As a brilliant (and slightly petty) electrical engineering student, you come up with a plan: weaken the WiFi signal in their room. To do this, you analyze how electromagnetic waves (like the 2.1 GHz WiFi signal) propagate through the wall using transfer matrices. Your mission: block the signal, save your peace, and maybe, just maybe, get some actual studying done.

5. (a) Set up the general transfer matrix that relates voltage and current between the input and output ports of a two-port network. Explain the physical meaning of each term in the matrix

   (b) Identify the network elements corresponding to the following transfer matrices:

   - $T = \begin{bmatrix} 1 & 0 \\ Z & 1 \end{bmatrix}$
   - $T = \begin{bmatrix} 1+ZY & Y \\ Z & 1 \end{bmatrix}$

   (c) Given two cascaded two-port networks with transfer matrices T_1 and T_2, write the resulting transfer matrix and explain why transfer matrices are convenient for analyzing cascaded systems.

(d) Derive the input impedance Z_{in} for a two-port network with transfer matrix $T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, when terminated with a load impedance Z_L.

(e) Now consider a real-world example. Model the wall between your rooms as a dielectric slab with thickness $\ell = \frac{1}{20}$ m and relative permittivity $\varepsilon_r = 9$. The wave impedance in the wall is $Z_2 = 45\pi\,\Omega$, and the wavelength in the wall is $\lambda = \frac{1}{16}$ m. Set up the transfer matrix $T_2(0, \ell)$ for the wall and define all necessary parameters.

(f) Assume both rooms are filled with air, i.e., $Z_1 = Z_3 = 120\pi\,\Omega$. Compute the reflection and transmission coefficients for the wall.

(g) Suppose your WiFi router is located 0.3 m from the wall and your roommate's TV is 0.2 m away on the other side. How does this affect the transfer matrix? Write the full matrix expression.

(h) You attempt to reduce reflections by adding a plastic panel with $\varepsilon_r = 36$ on your side. Determine the thickness required for this layer to act as a quarter-wave transformer at 2.1 GHz, and compute the resulting reflection and transmission coefficients.

(i) Briefly explain what scattering parameters (S-parameters) are and how they differ from transfer matrices. Then, write the general form of the scattering matrix for a two-port dielectric slab.

A super cool magnet

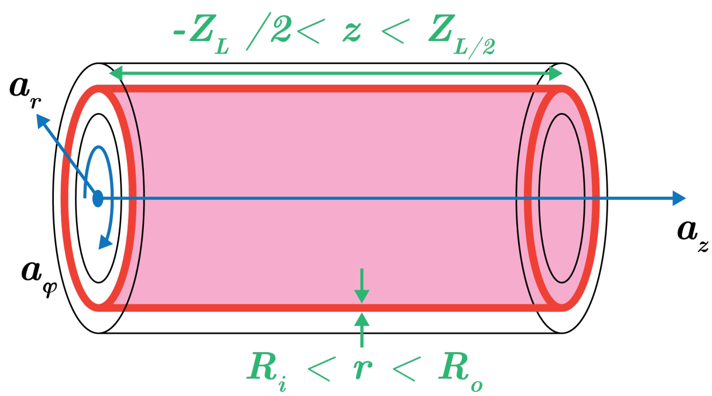

Figure 4: A schematic overview of the MRI scanner and the area of interest (pink) that ranges from $R_i < r < R_o$ and from $-Z_L/2 < z < Z_L/2$.

An MRI scanner is a super cool magnet: -270°C. Under these conditions, some interesting properties like superconductivity pop up. Let's investigate what this does with the electric and magnetic fields. We use cylindrical coordinates for this question, see Figure

4. We also consider the power flow $\nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \sigma |\boldsymbol{E}|^2 + \frac{j\omega\varepsilon}{2} |\boldsymbol{E}|^2 + \frac{j\omega\mu}{2} |\boldsymbol{H}|^2$.

To activate an MRI scanner, a current is enforced into the coil once. Because the coil is lossless, the current remains there as long as the coil remains superconductive[2].

6. (a) We assume an area inside an MRI coil: $R_i < r < R_o; -Z_L/2 < z < Z_L/2$. This area does not include any boundaries between interfaces, and the area is so small that the current density inside is homogeneous[3]:

$$\boldsymbol{J}(r, \phi, z, \omega) = J_0 \exp(-j\omega t) \boldsymbol{a_\phi}. \tag{4}$$

Use Ohm's law to derive the electric field in this area.

(b) Prove that the magnetic field in this area is equal to

$$\boldsymbol{H} = \frac{-J_0 \exp(-j\omega t)}{j\omega\mu\sigma r} \boldsymbol{a_z}. \tag{5}$$

(c) Write down the power flow equation. Describe the energy that is stored in the electric field, the energy that is stored in the magnetic field and the energy that is dissipated.

(d) The coil of an MRI scanner is lossless. Does this mean that conductivity of the coil σ approaches 0 or ∞?

(e) Use your answer in the last question to describe how much energy is stored in the electric and magnetic fields and how much energy is dissipated. So either take the limit of $\sigma \to 0$ or $\sigma \to \infty$ for your answer at question c.

(f) Calculate the gradient of the Poynting vector to check the amount of power outflow. Is this what you would expect from a physical point of view?

[2] In practice, the currents in superconductive materials persist for tens of years but do degrade, source: https://eos.org/science-updates/recording-belgiums-gravitational-history.

[3] Note that the main coil of the MRI scanner is a static field. However, due to high-frequency MR coils that scan the images, time-variant fields, thus currents, are superimposed. Therefore, a time-variant current density can occur.