Markov modeling, discrete-event simulation – Exercises module B5

5XIE0 Computational Modeling

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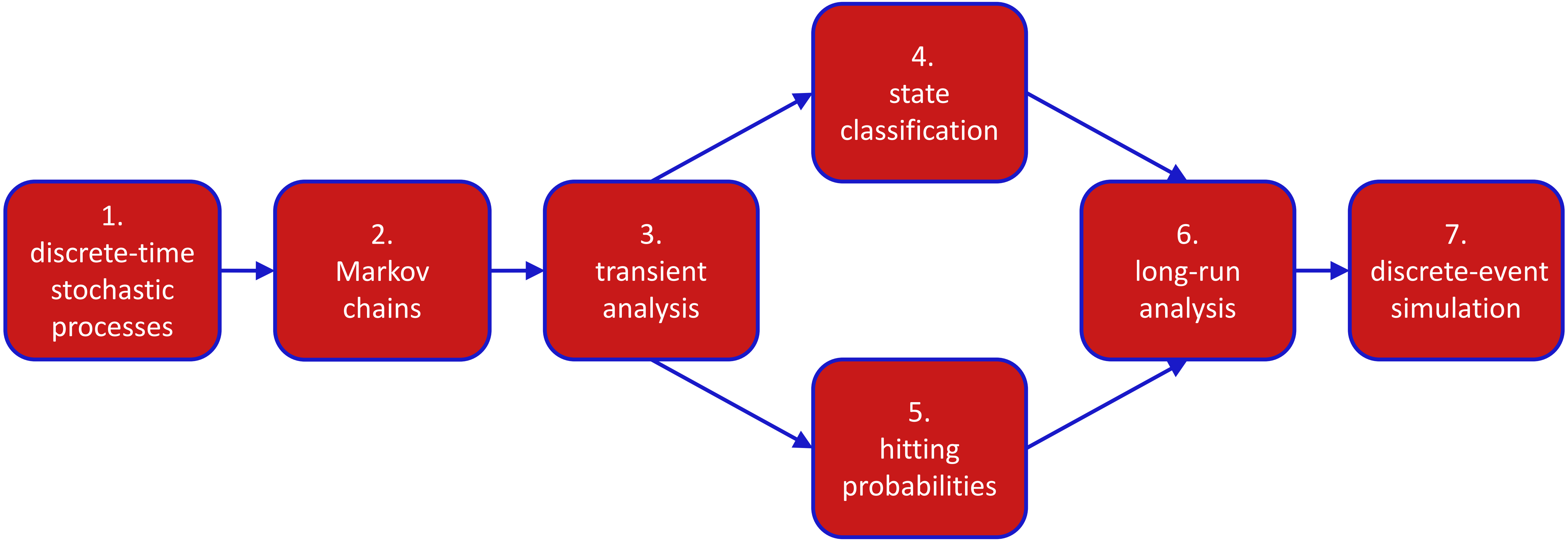
module B - submodules and dependencies



$$G_b = \begin{bmatrix} 1 & \infty & 2 \\ 1 & -\infty & 2 \\ -\infty & 3 & -\infty \end{bmatrix}$$

B.5 – hitting probabilities

hitting probabilities – exercises

- Section B.5 in the course notes
  - verify Example B.11 (Hitting probabilities – hitting a state) (same as example previous slide)
    - use CMBW (DTMC) to check answer
      1. create the model (same model as Example B.7)
      2. select ‘Hitting Probability’ and enter state to hit
  - Exercise B.22 (Computing return probabilities through equations)
    - use CMBW (DTMC) to check answer
      1. create the model corresponding to the given probability matrix (same model as Exercise B.15)
      2. select ‘Hitting Probability’ and enter state to hit
  - Exercise B.23 (Infinite closed classes are not necessarily recurrent)
- answers are provided in Section B.8 of the course notes

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  - verify Example B.13 (Hitting probabilities – hitting a set)
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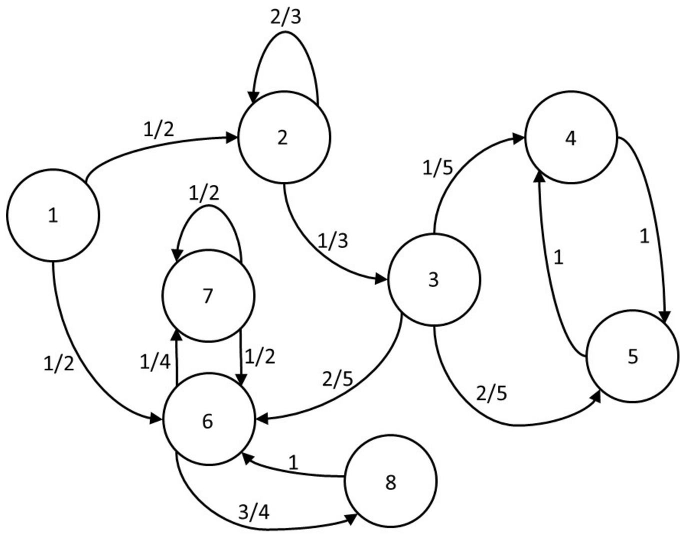
      1. create the model (same model as Example B.7)
      2. select ‘Hitting Probability Set’ and enter states to hit
  - verify Example B.14 (Gambler’s ruin – win probability) and Example B.15 (Gambler’s ruin - expected number of spins until win)
    - use CMWB (DTMC) to check answer
      1. use Gambler’s ruin model and adapt rewards
      2. select ‘Hitting Probability Set’ / ‘Reward Until Hit Set’ and enter states to hit
  - Exercise B.25 (Hitting a state versus hitting a singleton state set)
  - Exercise B.26 (Hitting recurrent states with probability 1)
  - Exercise B.27 (Rover in a maze)
    - use CMWB (DTMC) to check the answer
- answers are provided in Section B.8 of the course notes

Example B.11 (Hitting probabilities – hitting a state)



What is f_{16}?

$$f_{ij} = \sum \begin{array}{l} \{P(i, i_1, \cdots, i_{n-1}, j) \mid i, i_1, \cdots, i_{n-1}, j \text{ is a path} \\ \text{of length } n \geq 1 \text{ such that } i_k \neq j \text{ for all } k = 1, \cdots, n-1\} \end{array} \quad \text{(B.34)}$$

The hitting probabilities f_{ij} form the least non-negative solution to the following set of equations: $x_i = P_{ij} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{ik} x_k$ (B.36)

Use CMBW (DTMC) to check answer

1. create the model (same model as Example B.7)
2. select 'Hitting Probability' and enter state to hit

- probability to hit state 6 from 1
  - $x_1 = \frac{1}{2} x_2 + \frac{1}{2}$
  - $x_2 = \frac{2}{3} x_2 + \frac{1}{3} x_3$
  - $x_3 = \frac{1}{5} x_4 + \frac{2}{5} x_5 + \frac{2}{5}$
  - $x_4 = x_5, \ x_5 = x_4$
  - $x_6 = \frac{1}{4} x_7 + \frac{3}{4} x_8, \ x_7 = \frac{1}{2} x_7 + \frac{1}{2}, x_8 = 1$
- least non-negative solution
  - if state 6 is not accessible from i: set $x_i = 0$
  - $x_4 = x_5 = 0$
  - $x_3 = \frac{2}{5}, x_2 = \frac{2}{5}$
  - $x_6 = x_7 = x_8 = 1$
  - $x_1 = \frac{7}{10}$, so $f_{16} = \frac{7}{10}$

Exercise B.22 (Computing return probabilities through equations)

Exercise B.22 (Computing return probabilities through equations). Consider a Markov chain with state space $\{1, 2, 3, 4, 5\}$ and transition probability matrix $P =$

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Compute f_{11} by solving the system of linear equations in (B.36). Compare the result to the answer of Exercise B.19(a).

(b) Compute f_{22} by solving the system of linear equations in (B.36). Compare the result to the answer of Exercise B.19(b).

$$f_{ij} = \sum \begin{array}{l} \{P(i, i_1, \cdots, i_{n-1}, j) \mid i, i_1, \cdots, i_{n-1}, j \text{ is a path} \\ \text{of length } n \geq 1 \text{ such that } i_k \neq j \text{ for all } k = 1, \cdots, n-1\} \end{array} \quad \text{(B.34)}$$

The hitting probabilities f_{ij} form the least non-negative solution to the following set of equations: $x_i = P_{ij} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{ik} x_k$ (B.36)

Use CMBW (DTMC) to check answer

1. create the model corresponding to the given probability matrix (same model as Exercise B.15)
2. select ‘Hitting Probability’ and enter state to hit

Exercise B.22 (Computing return probabilities through equations).

(a) The equations are: $x_1 = \frac{1}{3} + \frac{2}{3}x_2$, $x_2 = \frac{1}{2}x_2 + \frac{1}{2}x_3$, $x_3 = x_4$, $x_4 = x_2$ and $x_5 = x_5$. Solving for x_1, x_2, x_3 and x_4 yields $x_1 = \frac{1}{3}$, $x_2 = 0$, $x_3 = 0$ and $x_4 = 0$. The equation $x_5 = x_5$ has infinitely many solutions, but the least non-negative solution is 0. Notice that we can quickly determine the values of x_2, x_3, x_4 and x_5 to be zero by realising that state 1 is not accessible from states 2, 3, 4 and 5 respectively. We thus have that $f_{11} = \frac{1}{3}$; the same as the answer to Exercise B.19(a).

(b) The equations are $x_1 = \frac{1}{3}x_1 + \frac{2}{3}$, $x_2 = \frac{1}{2} + \frac{1}{2}x_3$, $x_3 = x_4$, $x_4 = x_2$ and $x_5 = x_5$. Solving for x_1, x_2, x_3 and x_4 yields $x_1 = 1$, $x_2 = 1$, $x_3 = 1$ and $x_4 = 1$. The least non-negative solution to $x_5 = x_5$ is 0. Hence $f_{22} = 1$, similar to the outcome of Exercise B.19(b).

Exercise B.23 (Infinite closed classes are not necessarily recurrent)

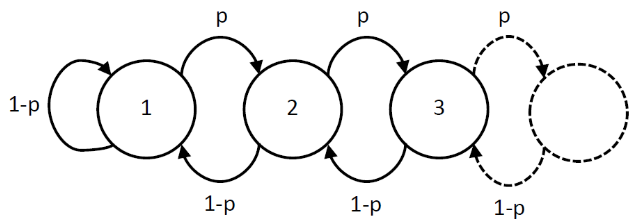


Figure B.7: Transition diagram of infinite Markov chain

Exercise B.23 (Infinite closed classes are not necessarily recurrent). Equation (B.28) states that every closed class is recurrent. This is true for finite-state systems, but not necessarily for infinite-state systems, as will see in this exercise. To this end consider the transition diagram in Figure B.7 and assume $0 < p < 1$. Since all states communicate, we are dealing with a single class of states which is closed.

(a) Assume $p = \frac{1}{2}$. Show that the class is recurrent by proving that $f_{11} = 1$. Hint: use Equation B.36 and exploit the fact that the hitting probability forms the least non-negative solution.

(b) Assume $p = \frac{2}{3}$. Show that the class is transient by proving that $f_{11} < 1$. Hint: use Equation B.36 and show that $x_1 = \frac{2}{3}$ and $x_n = \frac{1}{2}^n$ (for $n \geq 2$) is a solution.

$$C \text{ is recurrent if and only if for all } i \in C \text{ and } j \in \mathcal{S} \setminus C, P_{ij} = 0 \qquad \text{(B.28)}$$

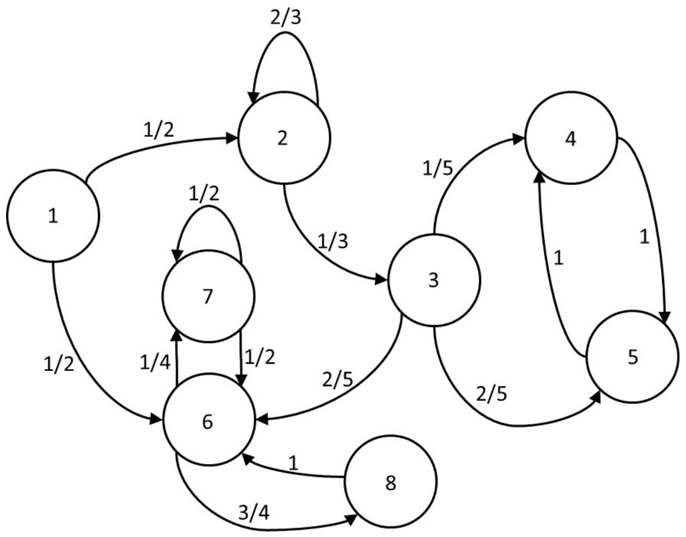
Exercise B.23 (Infinite closed classes are not necessarily recurrent).

(a) Following Equation B.36 we obtain the following equations: $x_1 = \frac{1}{2} + \frac{1}{2}x_2$, $x_2 = \frac{1}{2} + \frac{1}{2}x_3$, and $x_n = \frac{1}{2}x_{n-1} + \frac{1}{2}x_{n+1}$ (for $n \geq 3$). Notice that we obtain a solution when all variables are set to 1, in which case $x_1 = 1$. However, the hitting probability f_{11} forms the *least non-negative* solution. To show that no smaller solution than $x_1 = 1$ exists, assume that $x_1 = 1 - \epsilon$ for some $\epsilon > 0$. Then $x_2 = 1 - 2\epsilon$, $x_3 = 1 - 4\epsilon$ and in general $x_n = 1 - 2(n-1)\epsilon$ (for $n \geq 2$). But then for large enough n, x_n would be negative and thus $x_1 = 1 - \epsilon$ does not result in an overall non-negative solution. Hence for $x_1 = 1$ the least non-negative solution is obtained and thus $f_{11} = 1$.

(b) Following Equation B.36 we obtain the following equations: $x_1 = \frac{1}{3} + \frac{2}{3}x_2$, $x_2 = \frac{1}{3} + \frac{2}{3}x_3$, and $x_n = \frac{1}{3}x_{n-1} + \frac{2}{3}x_{n+1}$ (for $n \geq 3$). It is easy to check through substitution that $x_1 = \frac{2}{3}$ and $x_n = \frac{1}{2}^n$ (for $n \geq 2$) are a solution to these equations. The hitting probability f_{11} is the least non-negative solution which is therefore at most $\frac{2}{3}$. Hence $f_{11} < 1$.

$$\text{The hitting probabilities } f_{ij} \text{ form the least non-negative solution to the following set of equations: } x_i = P_{ij} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{ik} x_k \qquad \text{(B.36)}$$

Example B.12 (Expected cumulative reward – hitting a state)



What is f^r_{16}/f_{16}?

The expected cumulative reward earned until state j is hit, starting from state i, is defined as $\frac{f^r_{ij}}{f_{ij}}$	(B.37)

$f^r_{ij} = \sum \; \{P(i, i_1, \cdots, i_{n-1}, j) \cdot (r(i) + r(i_1) + \cdots + r(i_{n-1})) \mid i, i_1, \cdots, i_{n-1}, j$ is a path of length $n \geq 1$ such that $i_k \neq j$ for all $k = 1, \cdots, n-1\}$	(B.38)

The probability-weighted averages of the cumulative rewards f^r_{ij} form the least non-negative solution to the following system of equations: $x_i = r(i) \cdot f_{ij} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{ik} x_k$	(B.39)

use CMWB (DTMC) to check answers

1. create the model (same model as Example B.7) and add rewards
2. select 'Reward Until Hit' and enter state to hit

- expected number of steps to hit state 6, starting from 1
- assign reward 1 to each state
- compute f^r_{16} by solving
  - $x_1 = 1 \cdot f_{16} + \frac{1}{2}\, x_2$
  - $x_2 = 1 \cdot f_{26} + \frac{2}{3}\, x_2 + \frac{1}{3} x_3$
  - $x_3 = 1 \cdot f_{36} + \frac{1}{5}\, x_4 + \frac{2}{5} x_5$
  - $x_4 = x_5,\ x_5 = x_4$
- solution
  - $x_1 = \frac{15}{10}$, so $f^r_{16} = \frac{15}{10}$
  - the expected number of steps to hit state 6 from state 1 is

    thus $\frac{f^r_{16}}{f_{16}} = \frac{\frac{15}{10}}{\frac{7}{10}} = \frac{15}{7}$

Exercise B.24 (Hiccups in a video application)

Exercise B.24 (Hiccups in a video application). A video application displays a streaming movie with a rate of 60 frames per second. Upon a clock event, a movie frame (480×640 pixels) is read and removed from a buffer (if the buffer is non-empty) and is being displayed. The buffer can store up to 3 frames. Between two clock events new frames arrive from a network with probability p ($0 < p < 1$). In that case the buffer has completely refilled when the next clock event is raised. If the buffer is empty at the moment the clock event raised, a hiccup occurs.

(a) Model this video application as a Markov chain. Explain the time domain, the states and the transitions chosen.

(b) Compute (possibly making use of symbolic solver such as Mathematica) the expected number of video frames that are displayed (as function of p) before a hiccup occurs, when the video application starts with a full buffer.

(c) Estimate (by using a solver) the values of p for which the expected time before a hiccup occurs (starting from a full buffer) is at least an hour.

Exercise B.24 (Hiccups in a video application).

(a) We can model this system as a Markov chain $X_0, X_1, \cdots$ with state-space $\{0, 1, 2, 3\}$, where X_n represents the number of frames present in the buffer just before the next clock event is raised. A transition diagram is shown in Figure B.16. From each state a transition with probability p exists to state 3, corresponding to the refill of the buffer. In case the buffer is not refilled (with probability $1-p$), the buffer occupancy decreases by 1 (except when the buffer is empty).

(b) We have to compute the expected number of steps to transition from state 3 to state 0. We do so by assigning a reward value of 1 to each state and compute f^r_{30}/f_{30}. Now it is not hard to find out that $f_{i0} = 1$ for each i. To compute f^r_{30} we solve the following system of equations (see (B.39)): $x_0 = 1 + px_3$, $x_1 = 1 + px_3$, $x_2 = 1 + (1-p)x_1 + px_3$, $x_3 = 1 + (1-p)x_2 + px_3$. Solving yields $x_0 = \frac{1}{(1-p)^3}$, $x_1 = \frac{1}{(1-p)^3}$, $x_2 = \frac{2-p}{(1-p)^3}$ and $x_3 = \frac{3-3p+p^2}{(1-p)^3}$. Hence $f^r_{30} = \frac{3-3p+p^2}{(1-p)^3}$.

(c) Each frame takes $\frac{1}{60}$ seconds. So the expected time until a hiccup occurs is $\frac{3-3p+p^2}{60(1-p)^3}$ seconds. For $p = 0.983239$, this expression is approximately equal to 3600. Since the expression is increasing in p, the answer is $[0.983239, 1]$.

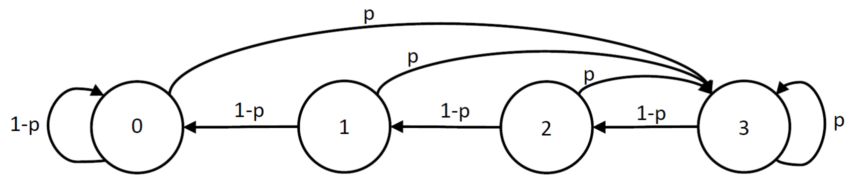


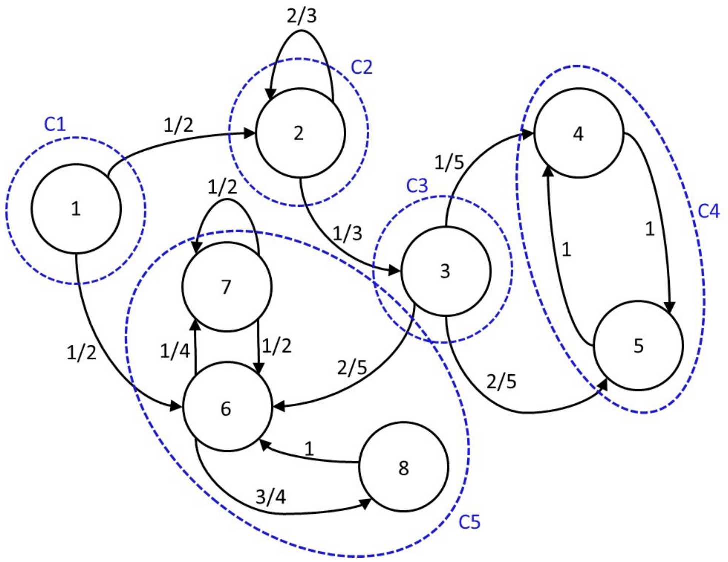
Figure B.16: Transition diagram of movie application

The probability-weighted averages of the cumulative rewards f^r_{ij} form the least non-negative solution to the following system of equations: $x_i = r(i) \cdot f_{ij} + \sum_{k \in \mathcal{S}\setminus\{j\}} P_{ik} x_k$ (B.39)

Use CMWB (DTMC) to check answers

1. create the Markov reward model for different values of parameter p
2. select ‘Reward Until Hit’ and enter state to hit

Example B.13 (Hitting probabilities – hitting a set)



What is h_{1C_5}?

$$h_{iH} = \sum \; \{P(i, i_1, \cdots, i_n) \mid i, i_1, \cdots, i_n \text{ is a path of length } n \geq 0 \text{ such that } i \in H \text{ implies } n = 0, \text{ and } i \notin H \text{ implies } n \geq 1, i_n \in H \text{ and } i_k \notin H \text{ for all } k = 1 \cdots n-1\} \quad \text{(B.40)}$$

The hitting probabilities h_{iH} form the least non-negative solution to the following set of equations

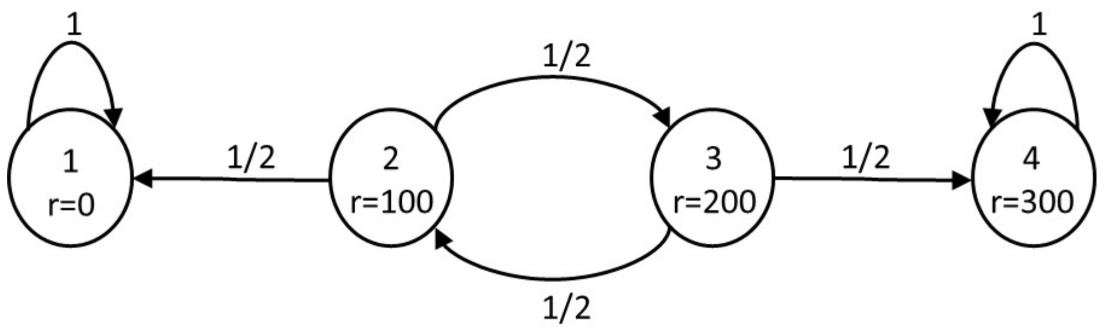
$$x_i = \begin{cases} 1 & \text{if } i \in H \\ \sum_{k \in \mathcal{S}} P_{ik} x_k & \text{if } i \notin H \end{cases} \quad \text{(B.41)}$$

use CMWB (DTMC) to check answer

1. create the model (same model as Example B.7)
2. select 'Hitting Probability Set' and enter states to hit

- probability to hit C_5 from 1
  - $x_1 = \frac{1}{2} x_2 + \frac{1}{2} x_6$
  - $x_2 = \frac{2}{3} x_2 + \frac{1}{3} x_3$
  - $x_3 = \frac{1}{5} x_4 + \frac{2}{5} x_5 + \frac{2}{5} x_6$
  - $x_4 = 0, \; x_5 = 0$
  - $x_6 = x_7 = x_8 = 1$
- solution
  - $x_1 = \frac{7}{10}$
  - $h_{1C_5} = \frac{7}{10}$

Example B.14 (Gambler's ruin – win probability)



What is win probability, starting with a capital of €100?

What is $h_{2\{4\}}$

$$h_{iH} = \sum \{P(i, i_1, \cdots, i_n) \mid i, i_1, \cdots, i_n \text{ is a path of length } n \geq 0 \text{ such that } i \in H \text{ implies } n = 0, \text{ and } i \notin H \text{ implies } n \geq 1, i_n \in H \text{ and } i_k \notin H \text{ for all } k = 1 \cdots n-1\} \quad \text{(B.40)}$$

The hitting probabilities h_{iH} form the least non-negative solution to the following set of equations

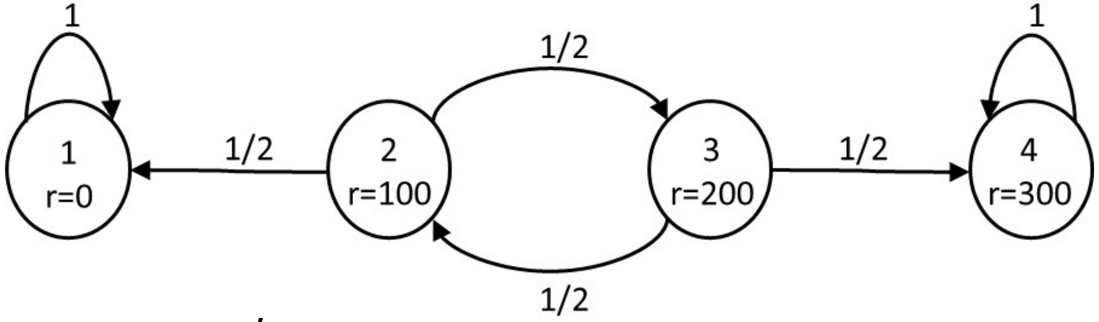
$$x_i = \begin{cases} 1 & \text{if } i \in H \\ \sum_{k \in \mathcal{S}} P_{ik} x_k & \text{if } i \notin H \end{cases} \quad \text{(B.41)}$$

Use CMWB (DTMC) to check answer

1. use Gambler's ruin model and adapt rewards
2. select 'Hitting Probability Set'

- equations
  - $x_1 = x_1$
  - $x_2 = \frac{1}{2} x_1 + \frac{1}{2} x_3$
  - $x_3 = \frac{1}{2} x_2 + \frac{1}{2} x_4$
  - $x_4 = 1$
- solution
  - $x_1 = 0$
  - $x_2 = \frac{1}{3}$ thus $h_{2\{4\}} = \frac{1}{3}$
  - $x_3 = \frac{2}{3}$
  - $x_4 = 1$
- we will see later that: $P^n_{2,4} \to \frac{1}{3}$ for $n \to \infty$

Example B.15 (Gambler's ruin – expected number of spins until win)



What is $h^{r'}_{2\{4\}}/h_{2\{4\}}$ - the expected number of spins until the gambler wins, starting with a capital of €100?

The expected cumulative reward earned until a state in H is hit, starting from state i, is defined as $\frac{h^r_{iH}}{h_{iH}}$	(B.42)

$$h^r_{iH} = \sum \{P(i, i_1, \cdots, i_n) \cdot (r(i) + r(i_1) + \cdots + r(i_{n-1})) \mid i, i_1, \cdots, i_n \text{ is a path of length } n \geq 1 \text{ such that } i \notin H, i_n \in H \text{ and } i_k \notin H \text{ for all } k = 1, \cdots, n-1\} \quad \text{(B.43)}$$

The probability-weighted averages of the cumulative rewards h^r_{iH} form the least non-negative solution to the following set of equations (B.44)

$$x_i = \begin{cases} 0 & \text{if } i \in H \\ r(i) \cdot h_{iH} + \sum_{k \in S} P_{ik} x_k & \text{if } i \notin H \end{cases}$$

Use CMWB (DTMC) to check answer

1. use Gambler's ruin model and adapt rewards
2. select 'Reward Until Hit Set' and enter states to hit

- define new reward *r'* taking value 1 in each state
- equations
  - $x_1 = 0$
  - $x_2 = 1 \times \frac{1}{3} + \frac{1}{2} x_3$
  - $x_3 = 1 \times \frac{2}{3} + \frac{1}{2} x_2 + \frac{1}{2} x_4$
  - $x_4 = 0$
- solution
  - $x_1 = 0$
  - $x_2 = \frac{8}{9}$ thus $\mathrm{h}^{r'}_{2\{4\}} = \frac{8}{9}$
  - $x_3 = \frac{10}{9}$
  - $x_4 = 1$
- hence $\frac{\mathrm{h}^{r'}_{2\{4\}}}{\mathrm{h}_{2\{4\}}} = \frac{\frac{8}{9}}{\frac{1}{3}} = 2\,\frac{2}{3}$

Exercise B.25 (Hitting a state versus hitting a singleton state set)

Exercise B.25 (Hitting a state versus hitting a singleton state set). Show that for all $i, j \in \mathcal{S}$ with $i \neq j$, $f_{ij} = h_{i\{j\}}$, based on equations (B.36) and (B.41).

The hitting probabilities f_{ij} form the least non-negative solution to the following set of equations: $x_i = P_{ij} + \sum_{k \in \mathcal{S}\setminus\{j\}} P_{ik} x_k$ (B.36)

The hitting probabilities h_{iH} form the least non-negative solution to the following set of equations (B.41)

$$x_i = \begin{cases} 1 & \text{if } i \in H \\ \sum_{k \in \mathcal{S}} P_{ik} x_k & \text{if } i \notin H \end{cases}$$

Exercise B.25 (Hitting a state versus hitting a singleton state set). Fix $j \in \mathcal{S}$. Let x_i and y_i ($i \neq j$) denote the variables corresponding to the linear equations in (B.36) and (B.41) respectively (where $H = \{j\}$ in (B.41)). Then $x_i = P_{ij} + \sum_{k \in \mathcal{S}\setminus\{j\}} P_{ik} x_k$ and $y_i = \sum_{k \in \mathcal{S}} P_{ik} y_k$. Since $y_j = 1$ (see (B.41)), the latter equations can be rewritten to $y_i = P_{ij} + \sum_{k \in \mathcal{S}\setminus\{j\}} P_{ik} y_k$. Hence the systems of equations are the same, up to the use of variable names, containing all variables except for x_j and y_j. Hence their least solutions are the same as well.

Exercise B.26 (Hitting recurrent states with probability 1)

Exercise B.26 (Hitting recurrent states with probability 1). By definition, a state i is recurrent if $f_{ii} = 1$ (see (B.24)). When i is in class C, this implies that $f_{jj} = 1$ for each $j \in C$ (B.27). Show that the following generalized property also holds: if C is a recurrent class, then for all $i, j \in C$, $f_{ij} = 1$. Hint: use the system of equations in (B.36).

A state i is said to be *recurrent* if and only if $f_{ii} = 1$ (B.24)

The properties of recurrence and transience are class properties (B.27)

The hitting probabilities f_{ij} form the least non-negative solution to the following set of equations: $x_i = P_{ij} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{ik} x_k$ (B.36)

Exercise B.26 (Hitting recurrent states with probability 1). Let C be a recurrent class and fix $j \in C$. Consider the system of equations in (B.36): $x_i = P_{ij} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{ik} x_k$. We will show that these equations have a unique solution, where $x_i = 1$ for all $i \in C$. In case $i = j$, the fact that $x_j = 1$ follows from (B.27). In that case we have $x_j = P_{jj} + \sum_{k \in \mathcal{S} \setminus \{j\}} P_{jk} x_k$. Since all successor states of j are in C we can rewrite this equation to $x_j = P_{jj} + \sum_{k \in C \setminus \{j\}} P_{jk} x_k$. Since $\sum_{k \in C} P_{jk} = 1$ and $x_j = 1$ we must have $x_k = 1$ for each state k that is accessible from j. We can perform the above steps for each of these successor states (and the successors thereof, etcetera) until all states in C have been considered. Hence $x_i = 1$ (and thus also $f_{ij} = 1$) for each $i \in C$.

Exercise B.27 (Rover in a maze)

Exercise B.27 (Rover in a maze). An autonomous rover tries to find its way of a maze consisting of 16 cells, see Figure B.8. The rover starts at the right-bottom cell, driving in the upward direction as indicated with the arrow. It can pass the dashed separation lines between cells, but not the solid lines (which represent walls). When arriving in a cell, it decides what cell it will visit next. It can choose from all neighboring cells, except for the cell it just arrived from. From the cells it can choose, it makes a decision with equal probabilities. So if it can go in three directions, each of these directions can be chosen with probability $\frac{1}{3}$. The goal of the rover is to find the exit cell (see figure), but along the way it might get stuck since it cannot turn around.

(a) What is the probability that the rover finds its way out of the maze?

(b) What is the expected number of cells the rover visits until it finds the exit cell?

(c) What is the expected number of times the rover visits the initial cell, before it finds the exit cell?

Use CMWB (DTMC) to check answer

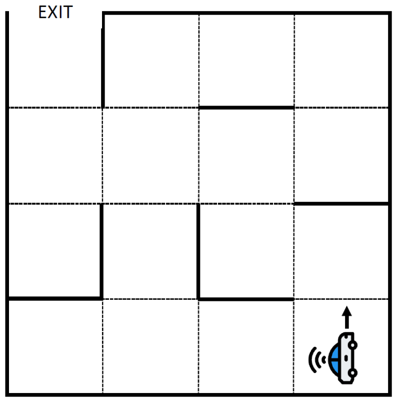


Figure B.8: An autonomous rover in a maze

Exercise B.27 (Rover in a maze). We first create a Markov chain. The basic idea is to create a state for each cell of the maze. However, since the rover cannot return to the cell it just came from, in some way information about the previous cell has to be remembered. The transition diagram in Figure B.17 presents a solution which models the decision for the next cell to visit. To this end each cell corresponds to four different states. The fact that the rover starts to ride in the upward direction, corresponds to the top-most state corresponding to initial cell (i.e. the cell in which the rover starts). When the rover arrives in the cell above the initial cell, as a next step it must move to the right. Hence a transition is made from the top-most state of the initial cell to the left-most state of the cell above it. Certain states are never used. These states are given a colour grey in the figure. The other states are given a black colour and have a number between 1 and 33. Since the outgoing transition

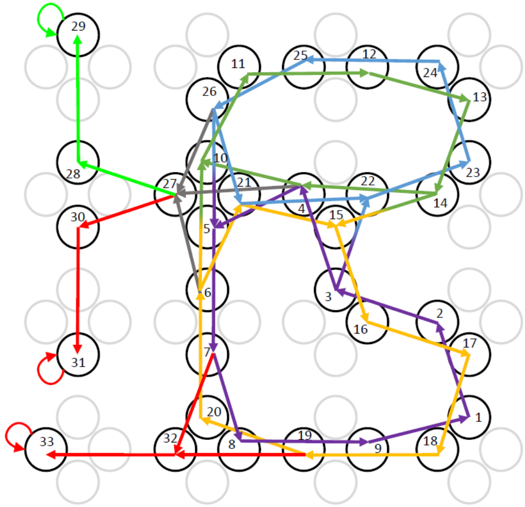


Figure B.17: Transition diagram of rover in maze problem

a) $h_{1\{29\}} = \frac{11}{38}$

b) $\frac{h^{r}_{1\{29\}}}{h_{1\{29\}}} = \frac{2978}{209}$ (r=1 in every state)

c) $\frac{h^{r'}_{1\{29\}}}{h_{1\{29\}}} = \frac{24}{19}$ (r=1 in state 1)