EINDHOVEN UNIVERSITY OF TECHNOLOGY

Communication Theory (5ETB0)
Module 8.2

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Module 8.2

Presentation Outline

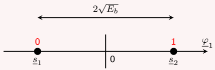
Recap: Bit-by-Bit Signaling

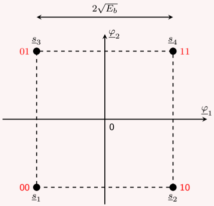
Bit-by-Bit Signaling

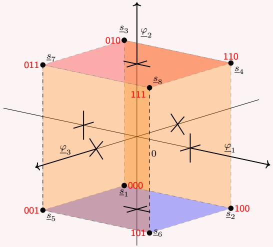
The signals are

$$s(t) = \sum_{i=1}^{K} (-1)^{b_i+1} \sqrt{E_b} \varphi_i(t)$$

The vectorial representation is $\underline{s}_m = \sqrt{E_b}((-1)^{b_1+1}, (-1)^{b_2+1}, \ldots, (-1)^{b_N+1})$







Conclusion: Reliability **cannot** be increased by increasing T

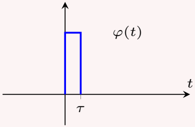
Block-Orthogonal Signaling: Description

System Description

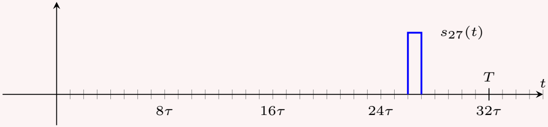
- Transmit K bits in T seconds. Rate is $R = K/T$
- Send one out of 2^K orthogonal pulses every T seconds
- Focus on pulse-position modulation (PPM) where $|\mathcal{M}| = 2^K$ signals are

$$s_m(t) = \sqrt{E_s}\varphi(t-(m-1)\tau), \text{ for } m = 1, \ldots, 2^K,$$

  with $\varphi(t)$ a unit-energy pulse and duration τ less than $T/2^K$
- All signals *within the block* $[0, T)$ are orthogonal and have energy E_s.



$\int_{-\infty}^{\infty} \varphi^2(t)dt = 1$



$\int_{-\infty}^{\infty} s_{27}^2(t)dt = E_s = TP_s$

Block-Orthogonal Signaling: Error Probability

Error Probabilities Considerations

We assume that the energy per transmitted bit of information is

$$E_b/N_0 = (1+\epsilon)^2 \ln 2,$$

with $0 \leq \epsilon \leq 1 \Rightarrow$ We are willing to spend slightly more than $N_0 \ln 2$ [Joule]
Using $\log_2 |\mathcal{M}| = RT$ in the upper bound in P_{e} for $E_b/N_0 \geq \ln 2$:

$$\begin{aligned} P_{\mathsf{e}} &\leq 2\exp(-[\sqrt{E_b/N_0} - \sqrt{\ln 2}]^2 \log_2 |\mathcal{M}|) \\ &= 2\exp(-[\sqrt{(1+\epsilon)^2 \ln 2} - \sqrt{\ln 2}]^2 RT) = 2\exp(-\epsilon^2 RT \ln 2) \end{aligned}$$

We now use $E_b = P_s/R$ to get

$$\boxed{R = \frac{1}{(1+\epsilon)^2}\frac{P_s}{N_0 \ln 2}, \qquad P_{\mathsf{e}} \leq 2\exp(-\epsilon^2 RT \ln 2)}$$

What happens if we change ϵ and T?

Block-Orthogonal Signaling: Capacity Result

Channel Capacity

With available average power P_s we can achieve rates R smaller than but arbitrarily close to

$$C_\infty \triangleq \frac{P_s}{N_0 \ln 2} \quad \left[\frac{\text{bit}}{\text{second}}\right]$$

while the error probability P_{e} can be made arbitrarily small by increasing T.

Extra Comments

- The reliability can be increased not only by increasing the power P_s or decreasing the rate R (bit-by-bit signalling) but also by increasing the “codeword-lengths” T.
- The channel **capacity** C_∞ depends only on the available power P_s and power spectral density $N_0/2$ of the noise.
- Only rates up to the capacity can be achieved (not possible to go beyond)
- Warning: Is this the end of the story? No. We have ignored the dimensionality of the signal sets...

Dimensions Needed for Bit-by-Bit and for Block-Orthogonal Signaling

Bit-by-Bit and Block-Orthogonal

Signaling	Dimensions per block	Dimensions per second
Bit-by-Bit	$K = RT$	$K/T = R$
Block Orthogonal	$2^K = 2^{RT}$	$2^K/T = 2^{RT}/T$

Block-orthogonal signaling: Pros and Cons

For a given rate R:

- Arbitrarily high reliability can be achieved by increasing T (not the case for bit-by-bit signaling)
- The number of dimensions per second explodes by increasing T
- Bad because a channel with a finite bandwidth cannot accommodate all these dimensions

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Presentation Outline

Bandwidth, time, and dimensions

The Dimensionality Theorem

Let $\varphi_i(t)$, for $i = 1, ..., N$ denote any set of orthonormal waveforms. Assume that for all waveforms $\varphi_i(t)$ for $i = 1, \ldots, N$

- $\varphi_i(t) = 0$ for all t outside $[0, T)$, and
- its Fourier transform satisfies $\int_{-W}^{+W} |\Phi_i(f)|^2 df \approx 1$.

Then the number of orthogonal waveforms (dimensions) N is (roughly) upper-bounded by $2WT$. The parameter W is called **bandwidth** (in Hz).

Comments

- Number of waveforms cannot be much more than approximately $2WT$
- The number of dimensions per second is not much more than $2W$
- It can be shown that instead of $2^K/T$, we could have $(2K+1)/T$ dimensions per second.

Summary Module 8.2

Take Home Messages

- Block orthogonal sinaling leads to a capacity result
- The number of dimensions per second explodes
- Dimensionality theorem tell us how good/bad this is w.r.t. bandwidth

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