ULTIMATE 2WBB0 PAST PAPER

Topical Question Bank

↳ Sorted by top **12** most common topics from past paper exam (Canvas)

Credit goes to:

Credit goes to:

resources from Professor Michiel Hochstenbach

See Page 36,37

Uncommon Topic	Frequency
even/odd function	1
derivative	2
double "	1
Linearization	1
composite f(x)	2
polynomials (partial fraction)	2
critical point	1

Common Topic ①: Simplify trig eq.

2017 Final Q.2

[4] 2. The value of $\sin(\arctan(\frac{1}{9}))$ is equal to:

(a) $\frac{1}{\sqrt{82}}$ (b) $\frac{1}{81}$ (c) $\frac{2}{9}$ (d) $-\frac{1}{9}$

2018. Final Q.1a

2.5p **1a** $\tan(\arccos(-\frac{5}{9})) =$

- $\frac{1}{5}\sqrt{14}$
- $\frac{1}{9}\sqrt{56}$
- $\frac{56}{81}$
- $-\frac{2}{5}\sqrt{14}$

2018. resit Q.1

1. $\cos\left(\arcsin\left(\frac{2}{5}\right)\right)$ is equal to:

(a) $\frac{1}{7}\sqrt{17}$ (b) $\frac{1}{9}\sqrt{19}$ (c) $\frac{1}{5}\sqrt{21}$ (d) $\frac{1}{7}\sqrt{29}$

2019. Resit Q.1b

2.5p **1b** $\cos\left(\arctan\left(\frac{1}{2}\right)\right) =$

- $\frac{1}{\sqrt{5}}$
- $\sqrt{\frac{3}{5}}$
- $\frac{4}{5}$
- $\sqrt{\frac{4}{5}}$

2024 Resit Q.1

2p **1** $\sin\left(\arctan\left(\frac{-3}{4}\right)\right) =$

- a $-\frac{3}{5}$
- b $-\frac{4}{5}$
- c $\frac{3}{5}$
- d $\frac{4}{5}$
- e -3

2020 Resit Q.1a

2.5p **1a** $\arccos(\cos(-\frac{\pi}{3}))$ is equal to/is gelijk aan

- $-\frac{1}{3}$
- $-\frac{\pi}{3}$
- $\frac{1}{3}$
- $\frac{\pi}{3}$
- $\frac{1}{2}$

Common Topic ②: Circle / Ellipse f(x)

2017 Final Q.1

[4] 1. The points (x, y) in the $\mathbb{R}^2$ plane, described by the equation

$$25(x^2+y^2) - 100(x-y) = -184$$

form a circle with center M and radius r. Give M and r.

(a) $\begin{aligned} r &= 2/5 \\ M &= (2,2) \end{aligned}$ (b) $\begin{aligned} r &= 4/5 \\ M &= (-2,2) \end{aligned}$ (c) $\begin{aligned} r &= 4/5 \\ M &= (2,2) \end{aligned}$ (d) $\begin{aligned} r &= 4/5 \\ M &= (2,-2) \end{aligned}$

2024 Resit Q.5

2p **5** The curve with equation $x^2 + 2x + 3 = 2y^2 - 4y + 5$ is:

a an ellipse with center $(1, -\sqrt{2})$
b an ellipse with center $(1, -1)$
c a hyperbola
d a parabola
e an ellipse with center $(-1, 1)$

Common Topic ③ Domain and Range

2018 Resit Q.7

7. Consider the function $f(x) = \dfrac{3}{\ln(x) - 1}$ with the interval (e, ∞) as its domain.
This function is 1-to-1 and has an inverse.

[3] (a) Determine the inverse function $f^{-1}(y)$.
[2] (b) What is the domain and range of $f^{-1}(y)$?

2019 Resit Q.4

Exercise 4

4p **4** Consider the function $f(x) = |2\sin x - 1|$ on the interval $[0, 2\pi]$.
Determine the local and absolute (global) extreme values as well as the range of f

2019 Final Q.1c

2.5p **1c** Let $g(x) = \arcsin(x) + \arccos(x)$. The domain of g is:

Zij $g(x) = \arcsin(x) + \arccos(x)$. Het domein van g is:

- $[-1, 1]$
- $[0, 1]$
- $[-\pi, \pi]$
- $[-\frac{\pi}{2}, \frac{\pi}{2}]$
- $[0, \pi]$

2019 Final Q.4a

Exercise 4
Consider the function / Beschouw de functie $h(x) = x^{17} + 16x - 1$.

3p **4a** Show that h is one-to-one, and determine the domain and the range of h. Explain your answers.

2019 Final Q.6a

Exercise 6
Reconsider the function $g(x) = \arcsin(x) + \arccos(x)$ of question 1(c).
(NB: This question can be answered independently of question 1(c).)

Beschouw opnieuw de functie $g(x) = \arcsin(x) + \arccos(x)$ uit vraag 1(c).
(NB: Deze vraag kan onafhankelijk van vraag 1(c) gemaakt worden.)

2p **6a** Compute $g'(x)$, and use the result to argue why the function g is constant on its domain: $g(x) = \alpha$
for a constant α.

2020 Resit Q.1b

2.5p **1b** Let $f(x) = \sqrt{x+2}$ and $g(x) = 2x - 4$. The domain D of $f \circ g(x)$ is:
Stel $f(x) = \sqrt{x+2}$ en $g(x) = 2x - 4$. Het domein D van $f \circ g(x)$ is:

- $\mathbb{R}$
- $(1, \infty)$
- $[-2, \infty)$
- $(-2, \infty)$
- $[1, \infty)$

2023 Final Q.2

2p **2** Given $f(x) = \ln(x)$ and $g(x) = 2\sin(x)$, give domain and range of the composite function $(g \circ f)(x)$.

a. domain: $x \in \mathbb{R}$; range $-2 \le x \le 2$
b. domain: $x > 0$; range $-2 \le x \le 2$
c. domain: $x \in \mathbb{R}$; range $-2 < x < 2$
d. domain: $x > 0$; range $-2 < x < 2$
e. domain: $x > 0$; range $-1 \le x \le 1$

2024 Resit Q.2

2p **2** For $f(x) = \arctan\left(\frac{1}{x}\right)$ give the derivative $f'(x)$ and the range R_f.

a. $f'(x) = \frac{-1}{1+x^2}$, $R_f = (-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2})$
b. $f'(x) = \frac{x^2}{1+x^2}$, $R_f = (-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2})$
c. $f'(x) = \frac{x^2}{1+x^2}$, $R_f = (-\infty, -\frac{2}{\pi}) \cup (\frac{2}{\pi}, \infty)$
d. $f'(x) = \frac{x^2}{1+x^2}$, $R_f = (-\infty, 0) \cup (0, \infty)$
e. $f'(x) = \frac{-1}{1+x^2}$, $R_f = (-\infty, 0) \cup (0, \infty)$

Common Topic ④ Inverse derivative

2017 Final Q.5

[4] 5. The function f given by $f(x) = x^3 + x$ is invertible. If $g(x) = f^{-1}(x)$, determine $g'(2)$.

(a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

2018 Final Q 3b

Exercise 3
Consider the function $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = x^2 + x\cos(x)$ for all $x \geqslant 0$.
Gegeven is de functie $f : [0, \infty) \to \mathbb{R}$ met $f(x) = x^2 + x\cos(x)$ voor alle $x \geqslant 0$.

2p **3a** Show that the function f is one-to-one.
Bewijs dat de functie f injectief is.

2p **3b** Determine $(f^{-1})'(\frac{\pi^2}{4})$.
Bepaal $(f^{-1})'(\frac{\pi^2}{4})$.

2019 Resit Q.2c

Exercise 2

The function $f_a : \mathbb{R} \to \mathbb{R}$ is given by $f_a(x) = x^3 + ax^2 + x$, with parameter $a \in \mathbb{R}$.
De functie $f_a : \mathbb{R} \to \mathbb{R}$ is gegeven door $f_a(x) = x^3 + ax^2 + x$, met parameter $a \in \mathbb{R}$.

2p **2a** Specify for every $a \in \mathbb{R}$ how many zeros the function f_a has.

Geef voor iedere $a \in \mathbb{R}$ het aantal nulpunten van f_a.

2p **2b** Show that f_0 is injective.
Toon aan dat f_0 injectief is.

2p **2c** Determine/Bepaal $(f_0^{-1})'(-10)$.

2019 Final Q.4b

Exercise 4

Consider the function / Beschouw de functie $h(x) = x^{17} + 16x - 1$.

3p **4a** Show that h is one-to-one, and determine the domain and the range of h. Explain your answers.

3p **4b** In view of **(a)**, the function h has an inverse h^{-1}. Determine $(h^{-1})'(-18)$.

2020 Resit Q.1d

2.5p **1d** The function $f : (0, \infty) \to \mathbb{R}$ with $f(x) = \sin(x-1) + x^2$ for all $x > 0$, is invertible. The derivative of the inverse f^{-1} at $x = 1$ equals

De functie $f(0, \infty) \to \mathbb{R}$ met $f(x) = \sin(x-1) + x^2$ voor alle $x > 0$ is inverteerbaar. De afgeleide van de inverse f^{-1} in $x = 1$ is gelijk aan

- ○ 1
- ○ $\frac{1}{3}$
- ○ 2
- ○ $-\frac{5}{2}$
- ○ $\frac{6}{7}$

2023 Final Q.5

2p **5** Given the one-to-one function $f(x) = \dfrac{e^{-3x}}{x^4 + a}$, determine $(f^{-1})'(1/a)$

- (a) $-3a$
- (b) 0
- (c) $-\frac{a}{3}$
- (d) $\frac{3}{a}$
- (e) $\frac{1}{a}$

2024 Resit Q.3

2p **3** The function $f(x) = xe^x$ is one-to-one (injective) on the interval $(-1, \infty)$. Determine the derivative of the inverse function f^{-1} in e.

- (a) $2e$.
- (b) $-\dfrac{1}{2e}$
- (c) $\dfrac{1}{2e}$
- (d) $\dfrac{1}{e}$
- (e) $\dfrac{1}{e^e + e^{e+1}}$

PAST EXAM QUESTIONS INVERSE FUNCTIONS, CALCULUS 2WBB0

(1) Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = (x^2+1)\arctan(x) - x$. Show that $f'(x) > 0$ for $x \neq 0$. Therefore, f is invertible. Determine $(f^{-1})'(1 - \frac{\pi}{2})$.

(2) The function $g : [\frac{3}{2}, \infty) \to \mathbb{R}$ is defined by $g(x) = 4x^3 - 7x^2 - 3$. Determine the derivative g' and show that g has an inverse. Determine the domain of g^{-1}, and $(g^{-1})'(1)$.

(3) The function $h : (0, \infty) \to \mathbb{R}$ is defined by $h(x) = \frac{1}{x+\arctan(x)}$. Show that $h'(x) = -\frac{x^2+2}{(1+x^2)(x+\arctan(x))^2}$. Prove that h has an inverse, and determine $(h^{-1})'(\frac{4}{4+\pi})$.

(4) Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = x^3 + 2x + \sin(\frac{\pi x}{2})$. Show that f has an inverse and determine $(f^{-1})'(4)$.

(5) Consider the functie $g(x) = \dfrac{3}{\ln(x) - 1}$ with the interval (e, ∞) as its domain.

This functie is 1-to-1 and has an inverse.

(a) Determine the inverse functie $g^{-1}(y)$.

(b) What are domain and range of $g^{-1}(y)$?

(9) Let $f : \mathbb{R} \to \mathbb{R}$ be the functie given by $f(x) = \dfrac{4x^3}{1+x^2}$.

(a) Determine the derivative $f'(x)$ and show that $f'(x) > 0$ for all $0 \neq x \in \mathbb{R}$.

(b) From **(a)** it follows that f is invertible. Determine $(f^{-1})'(2)$.

(6) The inverse $h^{-1}(x)$ of the function $h(x) = \ln(x^2 + 1)$ with $D_h = [0, \infty)$ is:

(a) $\sqrt{e^x - 1}$ **(b)** $e^{\sqrt{x^2-1}}$ **(c)** $e^{\sqrt{x^2+1}}$ **(d)** $e^{\frac{1}{2}x} - 1$

(7) The inverse $g{-}1(x)$ of the function $g(x) = \frac{1+e^x}{2+e^x}$ is:

(a) $g^{-1}(x) = \ln\left(-\frac{1-2x}{1-x}\right)$ with its domain equal to $(\frac{1}{2}, 1)$

(b) $g^{-1}(x) = \ln\left(\frac{1-x}{1-2x}\right)$ with its domain equal to $(-\infty, \frac{1}{2}) \cup (1, \infty)$

(c) $g^{-1}(x) = \ln\left(-\frac{1-x}{1-2x}\right)$ with its domain equal to $(\frac{1}{2}, 1)$

(d) $g^{-1}(x) = \ln\left(\frac{1-2x}{1-x}\right)$ with its domain equal to $(-\infty, \frac{1}{2}) \cup (1, \infty)$

(10) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \dfrac{e^{2x} - 1}{e^{2x} + 1}$.

(a) Determine the derivative $f'(x)$ and show that $f'(x) > 0$ for all $x \in \mathbb{R}$.

(b) From **(a)** it follows that f is invertible. Determine the inverse f^{-1} of f.

(8) Given is the function $f(x) = \dfrac{e^{2x}}{e^{2x} + 1}$ for all $x \in \mathbb{R}$.

(a) Show that f is an increasing functie.

(b) Determine the inverse functie f^{-1}.

(c) Give domain and range of f^{-1}, and explain your answer.

Common Topic 5. Continuity / Differentiability

2018 Final Q 1c

2.5p **1c** In $x = 1$ the piecewise defined function
In $x = 1$ is de stuksgewijs gedefinieerde functie

$$f(x) = \begin{cases} x^2 + 3x - 4 \text{ if } x > 1 \\ x^2 + 2x - 3 \text{ if } x \leqslant 1 \end{cases}$$

- ○ is not continuous and not differentiable / niet continue en niet differentieerbaar
- ○ is continuous but not differentiable / continue, maar niet differentieerbaar
- ○ is not continuous but differentiable / niet continu maar wel differentieerbaar
- ○ is continuous and differentiable / continu en differentieerbaar

2020 Resit Q 1f

2.5p **1f** Let $f(x)$ be continuous and differentiable on the interval $[-7, 0]$. Suppose $f(-7) = -3$ and $f'(x) \leq 2$. The largest possible value for $f(0)$ is:

Zij $f(x)$ continu en differentieerbaar op het interval $[-7, 0]$. Neem aan dat $f(-7) = -3$ en $f'(x) \leq 2$. De grootst mogelijke waarde voor $f(0)$ is:

- ○ -10
- ○ -11
- ○ 11
- ○ -1
- ○ 14

2019 Resit Q 1f

2.5p **1f** The function / De functie

$$f(x) = \begin{cases} \dfrac{1 - \cos(x)}{2x^2} & x \neq 0 \\ \dfrac{1}{2} & x = 0 \end{cases}$$

is

- ○ continuous and differentiable / continu en differentieerbaar
- ○ continuous but not differentiable / continu maar niet differentieerbaar
- ○ differentiable but not continuous / differentieerbaar maar niet continu
- ○ neither continuous nor differentiable / noch continu noch differentieerbaar

2020 Resit Q 1h

2.5p **1h** For which value of the constant c is the function f continuous on $(-\infty, \infty)$?
Voor welke waarde van de constante c is de functie f continu op $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ 2x + 4 & \text{if } x \geq 2 \end{cases}$$

- ○ 4
- ○ 3
- ○ 2
- ○ 1
- ○ 0

2019 Final Q. 1f

2.5p **1f** The function $f(x) = |x + 2|\,(x + 2)$ is in $x = -2$:
De functie $f(x) = |x + 2|\,(x + 2)$ is in $x = -2$:

- ○ continuous and differentiable; continu en differentieerbaar
- ○ continuous and **not** differentiable; continu en **niet** differentieerbaar
- ○ **not** continuous and differentiable; **niet** continu en wel differentieerbaar
- ○ **not** continuous and **not** differentiable; **niet** continu en **niet** differentieerbaar

2024 Resit Q. 7

2p **7** The function $f(x)$ is defined by $f(x) = \begin{cases} e^{-1/|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

In $x = 0$ the function f is:

- (a) not continuous but differentiable.
- (b) both continuous and differentiable.
- (c) not continuous and not differentiable.
- (d) continuous but not differentiable.

PAST EXAM QUESTIONS ON CONTINUITY AND DIFFERENTIABILITY
CALCULUS 2WBB0

(1) The piecewise defined function $f(x) = \begin{cases} -\frac{\pi x}{2} & \text{for } x < -1 \\ \arcsin(x) & \text{for } -1 \leq x \leq 1 \\ \frac{\pi x}{2} & \text{for } x > 1 \end{cases}$ is :

(a) continuous in $x = -1$ and continuous in $x = 1$
(b) **dis**continuous in $x = -1$ and continuous in $x = 1$
(c) continuous in $x = -1$ and **dis**continuous in $x = 1$
(d) **dis**continuous in $x = -1$ and **dis**continuous in $x = 1$

(2) The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{x+a}{2\pi} & \text{for } x < 2\pi \\ b & \text{for } x = 2\pi \\ \frac{\sin(2x)}{x-2\pi} & \text{for } x > 2\pi \end{cases}$

is continuous. What are the values of a and b ?
(a) $a = 0$ and $b = 1$ (b) $a = 0$ and $b = 2$
(c) $a = 2\pi$ and $b = 1$ (d) $a = 2\pi$ and $b = 2$

(3) The function $f(x) = \begin{cases} \frac{x^2-4}{2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$ is in $x = 2$:

(a) continuous and differentiable
(b) **dis**continuous and differentiable
(c) continuous and **not** differentiable
(d) **dis**continuous and **not** differentiable

(4) The function $f(x) = \begin{cases} 1 - |x| - \frac{\sin(x)\cos(x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is in $x = 0$:

(a) continuous and differentiable
(b) **dis**continuous and differentiable
(c) continuous and **not** differentiable
(d) **dis**continuous and **not** differentiable

(5) The function $f(x) = |x+2|\,(x+2)$ is in $x = -2$:

(a) continuous and differentiable
(b) **dis**continuous and differentiable
(c) continuous and **not** differentiable
(d) **dis**continuous and **not** differentiable

Common Topic 6 Limit / Taylor

2017 Final

8. Calculate the following limits:

[4] (a) $\lim_{x\to 0}\left(\frac{e^x}{e^x-1}-\frac{1}{x}\right)$

[4] (b) $\lim_{x\to 0^+}\sin(x)\ln(x)$

2018 Final

Exercise 4

Determine the following limits.

Bepaal de volgende limieten.

4p **4a** $\lim_{x\to 0}\frac{e^{(x^4)}-\cos(x^2)}{x^2(\cos(x)-1)}$.

2018 Resit

8. Compute the following limits (give a good argumentation):

[4] (a) $\lim_{x\to 0} x\sin\left(\frac{1}{x^2}\right)$

[4] (b) $\lim_{n\to\infty}\left(1+\frac{1}{\sqrt{n}}\right)^n$

2018 Final

4p **4b** $\lim_{x\to\infty}\frac{\ln(f(x))}{x^2-x}$,

where f is a function satisfying $\frac{1}{2}e^{3x^2-x}\leqslant f(x)\leqslant 2e^{3x^2+5x}$ for all $x>0$.

waarbij f een functie is met $\frac{1}{2}e^{3x^2-x}\leqslant f(x)\leqslant 2e^{3x^2+5x}$ voor alle $x>0$.

2019 Resit

2.5p **1e** The limit / de limiet

$$\lim_{x\to\infty}\left(\frac{x^2}{x+1}-\sqrt{x^2+1}\right)$$

- ○ doesn't exist (neither as a real number, nor as $\pm\infty$) / bestaat niet (noch als reëel getal, noch als $\pm\infty$)
- ○ exists and has value -2 / bestaat en heeft waarde -2
- ○ exists and has value -1 / bestaat en heeft waarde -1
- ○ exists and has value 2 / bestaat en heeft waarde 2

2020 Resit

2.5p **1c**

$$\lim_{x\to 0}\frac{\frac{1}{x+2}-\frac{1}{2}}{x}$$ equals/is gelijk aan:

- ○ $-\frac{1}{4}$
- ○ ∞
- ○ $\frac{1}{2}$
- ○ 0
- ○ $\frac{1}{4}$

2019 Final

2.5p **1e**

$$\lim_{x\to 0}\frac{x^2\sin(2x)-\ln(1+2x^3)+\frac{4}{3}x^5}{\arctan(x^2)-x^2}=$$

- ○ -6
- ○ -2
- ○ 2
- ○ -4
- ○ 0

2020 Resit

5p **4** Determine the following limit:
Bepaal de volgende limiet:

$$\lim_{x\to 0}\frac{\cos(x)-1+\frac{1}{2}x\sin(x)}{[\ln(1+x)]^4}$$

2023 Final

2p **3** Find

$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x}}-\sqrt{x}\right)$$

- (a) 1
- (b) $-\infty$
- (c) $\frac{1}{2}$
- (d) 0
- (e) ∞

2023 Final

4p **10** Determine

$$\lim_{x\to 0}\frac{\sin(x^2)-x^2+\frac{x^6}{6}}{x^{10}}.$$

2024 Resit

2p **6** Find $\lim_{x\to\infty}\frac{x-2\sin(x)}{x+3\cos(x)}$.

- (a) $\frac{-2}{3}$
- (b) 1
- (c) Does not exist, also not as $\pm\infty$.
- (d) ∞
- (e) 0

2024 Resit

2p **13a** Prove that $\ln\left(\frac{1+x}{1-x}\right)=2x+\frac{2}{3}x^3+\frac{2}{5}x^5+O(x^7)$.

2024 Resit

3p

13b Compute $\lim_{x\to 0}\frac{\ln\left(\frac{1+x}{1-x}\right)-(2+x^2)\sin(x)}{x\left(1-\cos\left(x^2\right)\right)}$.

Hint: do not try l'Hopital's rule.

2017 Final

[4] 7. Determine the Taylor polynomial of degree 3 around $a = 0$ of the function

$$f(x) = \ln(1 + \sin(x)).$$

2019 Resit

4p **3** Determine the Taylor polynomial of degree 2 around $x = \frac{\pi}{3}$ of the function

Bepaal het Taylorpolynoom van graad 2 rond $x = \frac{\pi}{3}$ van de functie

$$f(x) = e^{\cos(x)}.$$

Hint: Use the definition.

2018 Resit

3. Determine the Taylor series in $a = 0$ up to and including the 2nd-order term of the function

$$f(x) = (\sin(x) - \cos(x))^2$$

(a) $2 - x + \frac{1}{2}x^2$ **(b)** $1 - 2x$ **(c)** $2 + x$ **(d)** $1 + 2x + x^2$

LIMITS: EXTRA EXERCISES, CALCULUS 2WBB0

This document contains **8 limits, all of a different type.**
Try to use the step-by-step plan consistently. In particular: before you start, determine the type of limit for each exercises:

$$\frac{0}{0},\ \frac{\neq 0}{0},\ \frac{\infty}{\infty},\ \infty-\infty,\ \frac{\text{“oscillate”}}{\infty},\ 0\cdot\infty,\ 1^{\infty},\ \infty^{0}$$

Good luck!

(1) $\displaystyle\lim_{x\to\infty} \frac{\cos(x)}{x}$

(2) $\displaystyle\lim_{x\to-2} \frac{x}{2x^2+16x+24}$

(3) $\displaystyle\lim_{x\to\infty} \left(1+\frac{5}{x}\right)^x$

(4) $\displaystyle\lim_{x\to0} \frac{\tan(x)}{x}$

(5) $\displaystyle\lim_{x\to\infty} \frac{x^3+2^x}{x^2+3^x}$

(6) $\displaystyle\lim_{x\to\infty} \sqrt{x^2+10x}-\sqrt{x^2-10x}$

(7) $\displaystyle\lim_{x\to\infty} \sqrt[x]{x}$

(8) $\displaystyle\lim_{x\to0^+} x\ln^2(x)$

PAST EXAM QUESTIONS LIMITS, CALCULUS 2WBB0

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Compute the following limits:
(Good luck! Answers at the end of the document, but try first yourself for quite some time!)

(1) $\lim_{x\to\infty} (\sqrt{x})^{1/x}$

(2) $\lim_{x\to -2} \frac{x}{2x^2+16x+24}$

(3) $\lim_{x\to 0} \frac{\tan(x)-\sin(x)}{x^3}$

(4) $\lim_{x\to 0} \frac{\cos(x)}{x^2-x}$

(5) $\lim_{x\to\infty} \frac{x^3+2^x}{x^2+3^x}$

(6) $\lim_{x\to\infty} \sqrt{x^2+7x}-\sqrt{x^2+2x}$

(7) $\lim_{x\to 4} \frac{(x-4)\,\sin(x-4)}{(\sqrt{x}-2)(x^2-5x+4)}$

(8) $\lim_{x\to 0} \frac{2x(\cos(2x)-1)}{\sin(3x)-3x}$

(9) $\lim_{x\to 0} \frac{x-\tan(x)}{x^3}$

(10) $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right)$

(11) $\lim_{x\to 4} \frac{(x-4)e^{x-4}}{\sqrt{x}-2}$

(12) $\lim_{x\to\infty} \frac{1}{2+\cos(x)}$

(13) $\lim_{x\to -2} \frac{x^2}{x^2+3x+2}$

(14) $\lim_{x\to\infty} \left(1-\frac{2}{x}\right)^x$

(15) $\lim_{x\to\infty} \sqrt{x+1}-\sqrt{x-1}$

(16) $\lim_{x\to 0} \frac{\sin(2x)}{\tan(5x)}$

(17) $\lim_{x\to\infty} \sqrt{x^2+10x}-\sqrt{x^2-10x}$

(18) $\lim\limits_{x\to\infty} x^{1/\sqrt{x}}$

(19) $\lim\limits_{x\to 0} \dfrac{\ln(1+x) - x\,e^x}{x^2}$

(20) $\lim\limits_{x\to\infty} \dfrac{\cos^2(x)}{x}$

(21) $\lim\limits_{x\to\infty} \dfrac{2^{-4x+6} + 5^{-2x+1}}{(\frac{1}{25})^{x-1} + (\frac{1}{4})^{2x}}$

(22) $\lim\limits_{x\to 0} \dfrac{e^{x^3} - \cos(x^3) - \frac{1}{4}\ln(1+4x^3)}{\sin(x^2) + \arctan(x^2) - 2x^2}$

(23) $\lim\limits_{x\to\infty} \dfrac{7x^3 + 8x^5 + 2^{2x}}{1 + 4^{x-1}}$

(24) $\lim\limits_{x\to 0} \dfrac{\frac{1}{2}\ln(1+2x^2) + \frac{1}{1+x^2} - 1}{\sin(x^2) - x^2\cos(x^2)}$

(25) $\lim\limits_{x\to\infty} \dfrac{\ln x + 8x^{10} - 3e^{x-1}}{x^7 + (\sqrt{e})^{2x-4}}$

(26) $\lim\limits_{x\to-\infty} \dfrac{x^2-2}{x^2-1}$

(27) $\lim\limits_{x\to 1} \dfrac{x^2-2}{x^2-1}$

(28) $\lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$

(29) $\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}}$

(30) $\lim\limits_{x\to\infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

(31) $\lim\limits_{x\to-\infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

(32) $\lim\limits_{x\to\infty} \dfrac{2e^x}{e^{2x} - 1}$

(33) $\lim\limits_{x\to-\infty} \dfrac{2e^x}{e^{2x} - 1}$

PAST EXAM QUESTIONS TAYLOR SERIES, CALCULUS 2WBB0
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Here are some past exam questions about Taylor series. Practise them until you master the topic! (Good luck! Answers at the end of this document, but first try yourself for quite some time!)

(1) Determine the terms to degree 3 of the Taylor series of the functions:
  (a) $f(x) = \ln(x) + \sin(\pi x) + \dfrac{1}{1+(x-1)^2}$ around $x = 1$.
  (b) $g(x) = \cos(2x) - x^3 + 4x^4$ around $x = 0$.

(2) **(a)** Give the Taylor polynomial of order 5 of $f(x) = \cos(x)$ around $x = 0$.
  (b) Give the Taylor polynomial of order 5 of $g(x) = \dfrac{e^x + e^{-x}}{2}$ around $x = 0$.
  (c) Compare (a) and (b): what do you see?

(3) Give the Taylor polynomial of order 5 of $f(x) = \sin(x)$ and $g(x) = \dfrac{e^x - e^{-x}}{2}$ in $x = 0$. Compare them: what do you see?

(4) Give the Taylor polynomial of order 3 of $f(x) = e^{x^2}$ around $x = 0$.

(5) Consider the function f, defined by $f(x) = e^{x-2}\, x^2$.
  The tangent to the graph of f through the point $(2, 4)$ is indicate by ℓ.
  (a) Determine the Taylor polynomial of order 2 of f around $x = 2$.
  (b) Determine the equation of the tangent ℓ.
  (c) Compare (a) and (b): what do you see?

(6) Determine the terms up to and including degree 3 of the Taylor series of the functions:
  (a) $f(x) = \dfrac{e^x - 1}{x}$ around $x = 0$.
  (b) $g(x) = e^{(x^3)} - \ln(1 + x^3) - \cos(x^4)$ around $x = 0$.

(7) Determine the third-order Taylor polynomial around $x = 0$ of the function f given by $f(x) = \ln\left((1 + 2x)^2\right) + e^{-x+1}$.

(8) Determine the terms up to and including degree 3 of the Taylor series of the functions:
  (a) $f(x) = (1 + x)\sqrt{1 + x}$ around $x = 0$.
  (b) $g(x) = \dfrac{\cos(x) - 1}{x^2}$ around $x = 0$.

(9) Determine the terms up to including degree 3 of the Taylor series of the functions:
  (a) $f(x) = \dfrac{1}{(1 - x)(1 + x)}$ around $x = 0$.
  (b) $g(x) = \tan(x)$ around $x = 0$.

(10) Determine the terms up to and including degree 4 of the Taylor series of the function $f(x) = \dfrac{1}{2 + 4x^2} + \ln(1 - x^2) + \cos(x + \frac{\pi}{2})$ around the point $a = 0$.

(11) Determine the terms up to and including degree 3 of the Taylor series of the functions:

(a) $f(x) = \dfrac{3}{2+x}$
(b) $g(x) = \sin^2(x)$
all around the point $a = 0$.

(12) Determine the terms up to and including degree 3 of the Taylor series of the functions:
(a) $f(x) = \sin(x + \pi)$
(b) $g(x) = \sqrt[3]{x+1}$
(c) $h(x) = \ln(2 + 4x^2)$
all around the point $a = 0$.

(13) Determine the terms up to and including degree 3 of the Taylor series of the functions:
(a) $f(x) = x^4 + 2^x$
(b) $g(x) = \cos^3(x)$
(c) $h(x) = \ln\left(\dfrac{1+x}{1-x}\right)$
all around the point $a = 0$.

(14) Determine the 7th-order Maclaurin polynomial (= Taylor polynomial around 0) of the function f given by $f(x) = \sqrt[3]{1+3x^2} + 1 + x^5 + \dfrac{1}{1+x^3}$.

(15) Determine the 9th-order Maclaurin polynomial (= Taylor polynomial around 0) of the function f given by $f(x) = \sqrt{1+4x^3} + 6 + e^{3x^4}$.

Answers:

(1) **(a)** Define $y = x - 1$, then $x = y + 1$. In 3 parts:
$\ln(y+1) = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 + \mathcal{O}(y^4) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + \mathcal{O}((x-1)^4)$
$\sin(\pi(y+1)) = \sin(\pi y)\cos(\pi) + \cos(\pi y)\sin(\pi) = -\sin(\pi y)$
$= -(\pi y - \frac{1}{6}\pi^3 y^3 + \mathcal{O}(y^5)) = -\pi(x-1) + \frac{1}{6}\pi^3(x-1)^3 + \mathcal{O}((x-1)^5))$
$\frac{1}{1+y^2} = \frac{1}{1-(-y^2)} = 1 - y^2 + \mathcal{O}(y^4) = 1 - (x-1)^2 + \mathcal{O}((x-1)^4)$
So together: $1 + (1-\pi)(x-1) - \frac{3}{2}(x-1)^2 + (\frac{1}{3} + \frac{1}{6}\pi^3)(x-1)^3 + \mathcal{O}((x-1)^4)$
This can also be done with the definition; this requires a bit more work.

(b) Taylor series of a polynomial is the polynomial itself, x^4 is not of importance, so $1 - \frac{1}{2}(2x)^2 + \mathcal{O}(x^4) - x^3 + \mathcal{O}(x^4) = 1 - 2x^2 - x^3 + \mathcal{O}(x^4)$.
This can also be done with the definition; this requires a bit more work.

(2) **(a)** $1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$
(b) Use the Taylor series of e^x and e^{-x} and check that this is $1 + \frac{1}{2}x^2 + \frac{1}{24}x^4$.
(c) The difference is one minus sign. The function in (b) is called cosh = cosine hyperbolic!

(3) **(a)** $x - \frac{1}{6}x^3 + \frac{1}{120}x^5$
(b) Use the Taylor series of e^x and e^{-x} and check that this is $x + \frac{1}{6}x^3 + \frac{1}{120}x^5$.
(c) The difference is one minus sign. The function in (b) is called sinh = sine hyperbolic!

(4) $1 + x^2$; the rest is of higher order.

Common Topic ⑦ Mean-Value Theorem

2018 Final

3p **6** Show that for all a, b in $\mathbb{R}$ we have:
Bewijs dat voor alle a, b in $\mathbb{R}$ geldt dat

$$|\sin^2(b) - \sin^2(a)| \leqslant |b - a|.$$

Hint: Mean-Value Theorem.

2019 Resit

4p **7** Show with the Mean Value Theorem
Bewijs met de Middelwaardestelling

$$1 + 2\ln(a) < a^2 \qquad \text{for all / voor alle } a > 1.$$

Hint: interval $[1, a]$.

2019 Final

2.5p **1b** Let $f(x) = x^3 - 16x$ and let c be the number whose existence is guaranteed by the Mean Value Theorem for f on the interval $[-4, 2]$. Determine c.

Zij $f(x) = x^3 - 16x$ en zij c het getal waarvan de existentie gegarandeerd wordt door de Middelwaardestelling voor f op het interval $[-4, 2]$. Wat is c?

- o -3
- o -2
- o -1
- o 0
- o 1

2020 Resit

4p **3** Show with the help of the Mean Value Theorem that $|\cos(a) - \cos(b)| \leq |a - b|$.

2023 Final

4p **14** Prove using the Mean Value Theorem that $\frac{1}{x} < \frac{\ln(x)}{x-1} < 1$ for $x > 1$.

Common Topic ⑧ Implicit

2017 Final Q.6

[4] 6. Determine an equation for the tangent line to the curve given by the equation

$$y + x\ln(y) - 2x = 0$$

in the point $(\frac{1}{2}, 1)$.

2018 Resit Q 4

4. On the curve given by $x^3 + xy^2 + y^3 = 1$ it holds that $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{3x^2 + xy}{2xy + 3y^2}$ (b) $-\dfrac{3x^2 + xy}{2xy + 3y^2}$ (c) $\dfrac{3x^2 + y^2}{2xy + 3y^2}$ (d) $-\dfrac{3x^2 + y^2}{2xy + 3y^2}$

2018 Final Q 1b

2.5p **1b** The slope of the tangent line at point $(-3, 1)$ of curve $x^2y + xy^2 = 6$ is:
De richtingscoëfficient van de raaklijn in het punt $(-3, 1)$ aan de kromme $x^2y + xy^2 = 6$ is:

- ○ 1
- ○ $-\dfrac{3}{5}$
- ○ $\dfrac{5}{3}$
- ○ -2

2019 Resit Q.1c

2.5p **1c** The slope of the tanget line through the point $(3, 1)$ at the curve $yx^2 + e^y = x + 6 + e$ equals
De richtingscoëfficient van raaklijn door $(3, 1)$ aan de kromme $yx^2 + e^y = x + 6 + e$ is

- ○ $-\dfrac{5}{9+e}$
- ○ $-\dfrac{5+e}{9}$
- ○ $\dfrac{1}{9+e}$
- ○ $\dfrac{5+e}{9}$

2019 Final Q.3

5p **3** Consider the equation for the circle with radius r: $x^2 + y^2 = r^2$.
Show by **implicitly differentiating twice** that y'' satisfies $y'' = -\frac{r^2}{y^3}$.

2020 Resit Q.1e

2.5p **1e** The equation of the tangent line to the curve $\sin(x) + \cos(y) = 1$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is:

De vergelijking van de raaklijn aan de kromme $\sin(x) + \cos(y) = 1$ in het punt $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is:

- ○ $y = \frac{\pi}{2} + 4\left(x - \frac{\pi}{2}\right)$
- ○ $y = \pi$
- ○ $y = \frac{\pi}{2}$
- ○ $y = x - \frac{\pi}{2}$
- ○ $y = \frac{\pi}{2} - 4\left(x - \frac{\pi}{2}\right)$

2023 Final Q 7

2p **7** Find the equation of the **normal** line to $y^2 + 3xy + x^2 = 20$ at the point $(2, 2)$.

- (a) $y = -2x + 2$
- (b) $y = x + 2$
- (c) $y = -2x - 2$
- (d) $y = -x + 2$
- (e) $y = x$

PAST EXAM QUESTIONS IMPLICIT DIFFERENTIATION CALCULUS 2WBB0

Solve the following exercises on implicit differentiation:
(Good luck! Answers at the end of the document, but first try yourself for quite some time!)

Determine the equation of the tangent to the curve:

(1) $x^3 + x^2\,y + y^3 + 5x - \frac{2}{\pi}\cos(\frac{\pi}{2}\,y) = 4$ in the point $(1, -1)$.

(2) $x\,y^4 + x^3\,y^2 - 6x - 3y = 25$ in the point $(-2, -1)$.

(3) $(x^2 + y^2)^{3/2} = x^2 - y^2$ in the point $(x, y) = (-\frac{\sqrt{3}}{4}, \frac{1}{4})$.

(4) $\arctan(xy) + y^2 = x^2 - 4$ in the point $(2, 0)$.

(5) $\frac{1}{x^2} + \tan(xy) = 1$ in the point $(1, 0)$.

(6) $x^2y^3 + 2x^3y^2 - 8x + y = 20$ in the point $(3, -1)$.

(7) $\sin(x^2y^2) + x^3y = x^2 - 4$ in the point $(2, 0)$.

(8) $xy^2 = 4y^3 - \dfrac{4y}{x}$ in the point $(2, 1)$.

(9) $x^2 + xy + 2y^3 = 4$ in the point $(-2, 1)$.

(10) $xy^3 - x^4y^2 + y - 8x^3 + 2 = 0$ in the point $(-1, 2)$.

(11) Given is the curve $3x^2 + y^2 + y^4 = 5$ and the point $(1, -1)$ on this curve.

(a) Determine $y'(x)$ for this point $(x, y) = (1, -1)$

(b) Determine $y''(x)$ for this point $(x, y) = (1, -1)$

Common Topic 9. Integeration

2017 Final

9. Calculate the following integrals:

[4] **(a)** $\displaystyle\int_0^{\pi} 5(5-4\cos(x))^{1/4}\sin(x)\,dx$

[5] **(b)** $\displaystyle\int x^7\sin(2x^4)\,dx$

2018 Resit

9. Determine the following integrals (show your answer in clear steps):

[4] **(a)** $\displaystyle\int x^5\sqrt{x^2+1}\,dx$

[4] **(b)** $\displaystyle\int e^{2x}\sin(x)\,dx$

2018 Final

2.5p **1f** For $0 \leqslant x < 1$ we have
Voor $0 \leqslant x < 1$ geldt

$$f(x) = \int_0^{\sin(x)} \arcsin(t)\,dt.$$

Then $f'(x) =$
Dan is $f'(x) =$

- ○ x
- ○ $x\cos(x)$
- ○ $\dfrac{x}{\sqrt{1-x^2}}$
- ○ $x\sin(x)$

2018 Final

2.5p **1h** $\displaystyle\int_{e^2}^{e^6} \frac{1}{x\ln(x)}\,dx =$

- ○ $\ln(\frac{1}{4})$
- ○ $\ln(\frac{1}{3})$
- ○ $\ln(3)$
- ○ $\ln(4)$

2018 Final

Determine the following integrals.
Bepaal de volgende integralen.

4p **5a** $\displaystyle\int_0^{\pi} x^2\cos(x)\,dx.$

2018 Final

5b $\displaystyle\int_0^{\infty} \frac{1}{x^2+3x+2}\,dx.$

2019 Resit

2.5p **1g** Consider the function $f(x) = \displaystyle\int_{x^3}^{2} \frac{\sin(t)}{3t}\,dt$. The derivative of f is

Gegeven is de functie $f(x) = \displaystyle\int_{x^3}^{2} \frac{\sin(t)}{3t}\,dt$. De afgeleide van de functie f is

- ○ $-\dfrac{\sin(x^3)}{x}$
- ○ $\dfrac{\sin(x^3)}{x}$
- ○ $-\dfrac{\sin(x^3)}{3x^3}$
- ○ $\dfrac{\sin(x^3)}{3x^3}$

2019 Resit

2.5p **1h** The integrals / de integralen

$$\mathrm{I}: \int_1^{\infty} \sqrt{\frac{x^7}{e^{3x}}}\,dx, \qquad \mathrm{II}: \int_0^1 \frac{1}{x^{3/2}}\,dx$$

are / zijn

- ○ I: convergent, II: convergent
- ○ I: divergent, II: convergent
- ○ I: convergent, II: divergent
- ○ I: divergent, II: divergent

2019 Resit

Determine the following integrals.
Bepaal de volgende integralen.

4p **5a** $\displaystyle\int_3^5 \frac{13-5x}{x^3-2x^2+2x+5}\,dx$

2019 Final

2.5p **1d** $\displaystyle\int_0^{\ln(6)} \frac{e^x}{\sqrt{10-e^x}}\,dx =$

- 1
- 2
- 3
- $\frac{3}{2}$
- $\frac{5}{2}$

2019 Resit

4p **5b** $\displaystyle\int \sin^4(x)\cos^5(x)\,dx.$

2019 Final

4p **5** Compute/Bepaal $\displaystyle\int \frac{\ln(x)}{x^2}\,dx.$

2019 Resit

5p **5a** Compute/Bepaal $\int_1^9 \frac{\ln(x)}{\sqrt{x}}\,dx.$

2019 Resit

6p **5b** Compute/Bepaal $\int \frac{\sin(x)}{\cos^2(x)+5\cos(x)+6}\,dx.$

2023 Final

2p **4** The integral $I = \int_e^\infty \frac{1}{x\sqrt{\ln(x)}}\,dx$ is:

- (a) convergent, with $I = 2$
- (b) convergent, with $I = \frac{1}{2}$
- (c) convergent, with $I = 0$
- (d) divergent
- (e) convergent, with $I = -\frac{1}{4}$

2023 Final

2p **6** Determine the integral

$$\int_0^6 (2+5x)e^{\frac{x}{3}}\,dx$$

- (a) 90
- (b) $-51e + 39$
- (c) $51e^2 + 39$
- (d) $51e^2$
- (e) $96e^2$

2023 Final

4p **11** What is the area between the curve $f(x) = \sin^8(x)\cos^5(x)$ and the x-axis from $x = 0$ to $x = \pi/2$?

2024 Resit

2p **4** $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\, dx =$

- a ∞
- b $\frac{\pi}{2}$
- c 0
- d $\frac{\pi}{4}$
- e π

2023 Final

4p **12** Evaluate the integral

$$\int \frac{x^3 - 4x - 10}{x^2 - x - 6}\, dx.$$

2024 Resit

3p **12** Let $f(x) = \int_e^{e^{3x}} \frac{1}{\ln t}\, dt$ for $x > 0$.

Determine $f'(x)$. Simplify your answer as much as possible.

2024 Resit

4p **14b** Compute $\int \frac{x^5 - x^4 + x^3 - 4x^2 + 3x}{x^3 - x^2 - x - 2}\, dx$.

Hint: the denominator has a small integer as root.

PAST EXAM QUESTIONS INTEGRATION, CALCULUS 2WBB0

Compute the following integrals. Practise them until you master the topic!
(Good luck! Answers at the end of the document, but first try yourself for quite some time!)

(1) $\int x^2 (1+x)^{37}\, dx$

(2) $\int_{-\ln(2)}^{0} \frac{x}{e^{2x}}\, dx$

(3) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin(x)\cos(x)\, e^{\sin(x)}\, dx$

(4) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1-\cos(3t))\sin(3t)\, dt$

(5) $\int 2x \arctan(x)\, dx$

(6) $\int_0^{\frac{\pi}{2}} \frac{\cos(t)}{1+\sin^2(t)}\, dt$

(7) $\int \frac{\ln(x)}{x^2}\, dx$

(8) $\int_0^1 x\sin(x)\, dx$

(9) $\int_{-1}^{7} \frac{\cos(\sqrt{x+2})}{\sqrt{x+2}}\, dx$

(10) $\int \frac{1}{x^2+4x}\, dx$

(11) $\int (x^2+1)\ln(x)\, dx$

(12) $\int_0^1 2x^3\sqrt{x^2+1}\, dx$

(13) $\int \frac{1}{x^3+9x}\, dx$

(14) $\int \arcsin(x)\, dx$

(15) $\int_0^3 \sqrt{9-x^2}\, dx$ **Hint: there is an "easy" way.**

(16) $\int \frac{1}{x^2+9}\, dx$

(17) $\int 2x^3 e^{x^2}\, dx$

(18) $\int 2^x \sin(x)\, dx$

(19) $\int \frac{1}{x^3+x^2}\, dx$

(20) $\int \frac{e^x}{e^{2x}+4}\, dx$

(21) $\int (x^2-1)\, e^{2x}\, dx$

(22) $\int \ln(x^2)\, dx$

(23) $\int_0^{\sqrt{2}} -\sqrt{2-x^2}\, dx$

(24) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \arctan(x)\, dx$

(25) Sometimes we cannot compute a difficult integral exactly, but we *can* estimate its value. Consider $\int_0^{\pi/2} \sin^3(x)\, dx$.

(a) Show, *without* computing the integral, that $0 \le \int_0^{\pi/2} \sin^3(x)\, dx \le \frac{1}{64}\pi^4$.

Hint: you have to show 2 inequalities; compare the integral with 2 "easy" functions on the interval $[0, \frac{\pi}{2}]$.

(b) In this case we can still compute this integral exactly; show this.

Hint: use a well-known trigonometric identity and possibly a substitution.

(26) $\int_0^1 12x^2(x^3+1)^7\, dx$

(27) $\int \sqrt{x\sqrt{x\sqrt{x}}}\, dx$

(28) $\int \frac{\ln(x)}{x}\, dx$

(29) $\int \ln^2(x)\, dx$

(30) Compute the average height $\int_1^2 f(x)\, dx$ of the function $f(x) = \frac{x+1}{x^2+3x}$ on the interval $[1,2]$.

(31) $\int \frac{\cos^5(x)+\cos^3(x)\sin^2(x)}{1-\sin^2(x)}\, dx$

(32) $\int \arccos(x)\, dx$

(33) $\int x\, e^{\alpha x}\, dx$ (where $\alpha > 0$ is a constant)

(34) $\int \frac{11x+17}{(2x-1)(x+4)}\, dx$

(35) $\int \frac{1}{\tan(x)}\, dx$

(36) $\int 4\sin^2(x)\cos^2(x)\, dx$

(37) $\int \frac{x+4}{x^2-5x+6}\, dx$

(38) $\int \frac{\tan(x)}{\cos^2(x)}\, dx$

(39) $\int (1\cdot \sin^2(3x+4)+5)\, dx$

Common Topic (10) Differential Eq

2017 Final

[5] 10. Determine the solution y of the differential equation

$$\frac{dy}{dx} = -2xy + 3x^2e^{-x^2}$$

with initial value $y(0) = 1$.

2018 Resit

[5] 10. Determine the solution y of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = 2\frac{e^{x^2}}{x},$$

with initial value $y(1) = 2$.

2018 Final

4p 7 Find a solution to the following differential equation satisfying the given initial condition:
Vind een oplossing voor de volgende differentiaalvergelijking die voldoet aan de gegeven beginvoorwaarde:

$$\frac{dy}{dx} = x^2 - 1 + \frac{y}{x+1}, \quad y(0) = 2.$$

2018 Resit

2. The number of hungry bacteria on a toilet seat at time t is modeled by the differential equation $y'(t) = \frac{1}{10}y(t)$ (or, in other words, $\frac{dy}{dt} = \frac{1}{10}y$). Here, t is measured in seconds. At time $t = 0$ there are 10 hungry bacteria. After approximately how many seconds are there 4000 of them? (**Hint:** $e^6 \approx 403$).

(a) 29 secs. **(b)** 60 secs. **(c)** 108 secs. **(d)** 114 secs.

2019 Resit

4p **6** Determine the solution y of the differential equation
Bepaal de oplossing y van de differentiaalvergelijking

$$\frac{dy}{dx} = \frac{y^{2/3}}{(x-2)^2}, \qquad y(1) = 0.$$

2019 Final

2.5p **1g** The differential equation $2^{y-1}\frac{dy}{dx} = e^{x^2y+2x^2} + y + 1$ with initial value $y(0) = \beta$ has as solution a constant function $y(x) = \beta$ when β is equal to:

De differentiaalvergelijking $2^{y-1}\frac{dy}{dx} = e^{x^2y+2x^2} + y + 1$ met beginwaarde $y(0) = \beta$ heeft een constante functie $y(x) = \beta$ als oplossing als β gelijk is aan:

- ○ -2
- ○ -1
- ○ 0
- ○ 1
- ○ 2

2019 Final

4p **7a** Determine the general solution y of the differential equation:

Bepaal de algemene oplossing y van de differentiaalvergelijking

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{3}{x^2}.$$

2019 Final

2p **7b** Determine the solution of this differential equation that has a maximum in $x = 1$.

2020 Resit

5p **6** Determine the general solution y of the differential equation
Bepaal de algemene oplossing y voor de differentiaalvergelijking

$$\frac{dy}{dx} + \frac{2}{x}y - \sin(x) = 0.$$

2024 Resit

4p **15** Solve the differential equation $y' - \frac{y}{x} = x\cos(x)$ for the function $y = y(x)$, and find the solution with $y\left(\frac{\pi}{2}\right) = 0$.

2023 Final

4p **13** Solve the differential equation $x\frac{dy}{dx} + 2y - 8x^2 = -10$ for $x > 0$ and find the solution for y satisfying $y(1) = 0$.

PAST EXAM QUESTIONS DIFFERENTIAL EQUATIONS
CALCULUS 2WBB0

Solve the following differential equations:
(Good luck! Answers at the end of the document, but first try yourself, do not look too fast!)

(1) $x^2 \dfrac{dy}{dx} = 2x^3 e^{-\frac{1}{x}} + y, \quad y(-1) = 2e$

(2) $e^{-t^2} \dfrac{dy}{dt} = 3t^2 + 2t\, y\, e^{-t^2}$

(3) $x \dfrac{dy}{dx} + 3y = \dfrac{1}{x^2\,(1+x^2)}$

(4) $\dfrac{dy}{dx} = 2x^3 - 2xy$

(5) $\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{1}{x^2}$

(6) $e^x \dfrac{dy}{dx} = (x+1)\,(y+1)$

(7) $\dfrac{dy}{dx} - \dfrac{2xy}{x^2+1} = 1$

(8) Consider the following "rabbit differential equation" $\dfrac{dx}{dt}(t) = x\,(100 - x)$.
- **(a)** Explain briefly why this might be a plausible model for the number of rabbits in a certain area.
- **(b)** Give an explicit expression for the general solution for this ODE.
Hint: you have to integrate a rational function.

(9) $x \dfrac{dy}{dx} = 2x + 3y$

(10) $\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\cos(x)}{x^2}, \quad y(\pi) = 1$

(11) $\dfrac{dy}{dx} = 1 + x + y + xy, \quad y(2) = 1$

(12) The number of rabbits in your backyard after t weeks is given by the ODE $y'(t) = y(t)(10 - y(t))$.
At the initial time $t = 0$ there are 2 rabbits. After how many weeks are there 5?
- **(a)** First give a rough estimate only using $y'(0)$.
- **(b)** Now determine the exact answer by solving the ODE.
(Hint: begin with $\frac{dy}{dt} = y\,(10 - y)$, partial fraction expansion)

(13) The number of hungry bacteria on your toilet seat after t minutes is given by the ODE $y'(t) = 10\,y(t)$. At $t = 0$ there are 100 of them. After how many minutes are there 10000?
- **(a)** Provide first a rough estimate only using $y'(0)$.
- **(b)** Now determine the exact solution by solving the ODE.

Common Topic (11) Vector

2017 Final

5. The angle between 2 planes in $\mathbb{R}^3$ is defined by the angle between their normal vectors.
What is the angle between plane V in $\mathbb{R}^3$ given by the equation $3x+2y+z=0$ and plane W in $\mathbb{R}^3$ given by the equation $2x-y+3z=0$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

2018 Final

2.5p **1g** The plane V is given by the vector representation
Het vlak V is gegeven door parameter voorstelling

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}.$$

The distance of the piont $(4,2,3)$ to V equals
De afstand van het punt $(4,2,3)$ tot V is gelijk aan

- ○ 3
- ○ $\sqrt{14}$
- ○ $\sqrt{5}$
- ○ 2

2018 Resit

[4] 6. Let $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $y = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$. The number α is chosen such that $x - \alpha y \perp y$.

Determine α. (As you know, "$\perp$" means "is perpendicular to".)

2018 Final

3p **2** Give a vector representation for the intersection line ℓ of the planes V, given by the equation $3x-2y+4z=5$, and W, given by the equation $x+y+3z=5$.

2019 Resit

2.5p **1d** Determine the distance from $(1,2,2)$ to the line with parameter representation
Bepaal de afstand van het punt $(1,2,2)$ tot de lijn met parametervoorstelling

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

- ○ 0
- ○ $\sqrt{5}$
- ○ $\sqrt{7}$
- ○ 3

2019 Final

2.5p **1h** Let $\mathbf{a}, \mathbf{b}$ be two vectors in $\mathbb{R}^3$ with norm (length) equal to 1. The vector $\mathbf{c} = \mathbf{a} + \alpha \cdot \mathbf{b}$ is orthogonal to $\mathbf{b}$ if α is equal to:

Zij $\mathbf{a}, \mathbf{b}$ twee vectoren in de $\mathbb{R}^3$ met norm (lengte) gelijk aan 1. De vector $\mathbf{c} = \mathbf{a} + \alpha \cdot \mathbf{b}$ staat loodrecht op $\mathbf{b}$ als α gelijk is aan:

- ○ 1
- ○ $\mathbf{a} \bullet \mathbf{b}$
- ○ $-\mathbf{a} \bullet \mathbf{b}$
- ○ $\frac{1}{\mathbf{a} \bullet \mathbf{b}}$
- ○ $-\frac{1}{\mathbf{a} \bullet \mathbf{b}}$

2019 Final

2.5p **1a** The line ℓ in $\mathbb{R}^3$ passes through the points $(5,2,1)$ and $(3,1,-1)$. The distance of ℓ to the point $(0,2,3)$ is equal to:

De lijn ℓ in $\mathbb{R}^3$ gaat door de punten $(5,2,1)$ en $(3,1,-1)$. De afstand van ℓ tot het punt $(0,2,3)$ is gelijk aan:

- ○ 3
- ○ $\frac{7}{2}$
- ○ 4
- ○ $\frac{9}{2}$
- ○ 5

2020 Resit

5p **2** Determine the distance of the point $P(1,1,1)$ to the line given by the equations:
Bepaal de afstand van het punt $P(1,1,1)$ tot de lijn gegeven door de vergelijkingen:

$$\begin{cases} x + y + z = -1 \\ x + 2y + 3z = -4 \end{cases}$$

2023 Final

2p **8a** **This question is NOT for students from Biomedical Engineering and Medical Sciences and Technology, Electrical and Automotive Engineering (retakers from these studies can do this exercise or the next one.)**

Consider the two planes V given by the equation $ax - 5z = 3$ and W given in parametric form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 0 \\ a \end{bmatrix}.$$

For which value of $a < 0$ are the two planes perpendicular?

- (a) -5
- (b) $-\frac{2}{5}$
- (c) -10
- (d) -3
- (e) -25

2024 Resit

2p **8** **This question is NOT for students from Biomedical Engineering and Medical Sciences and Technology, Electrical and Automotive Engineering (retakers from these studies can do this exercise or the next one.)**

Determine the angle between the vector $\mathbf{a} = (1,2)$ and a vector $\mathbf{b}$ that is perpendicular to the line with equation $y = -3x + \frac{3}{2}$.

- (a) $\arccos\left(\frac{\sqrt{2}}{10}\right)$
- (b) $\frac{\pi}{2}$
- (c) $\arccos\left(\frac{1}{10}\right)$
- (d) $\arccos\left(\frac{1}{50}\right)$
- (e) $\frac{\pi}{4}$

2023 Final

4p **9** **NOT for students from Biomedical Engineering and Medical Sciences and Technology, Electrical and Automotive Engineering (retakers from these studies can answer this question or the one below.)**
Calculate the distance between the point $(7, -1, 4)$ and the plane V given by the vector representation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}.$$

2024 Resit

4p **10** **This question is NOT for students from Biomedical Engineering and Medical Sciences and Technology, Electrical and Automotive Engineering (retakers from these studies can do this exercise or the next one.)**

The line ℓ in $\mathbb{R}^3$ is given by the two equations

$$\begin{cases} x + y - z = 2 \\ 2x - y = 3 \end{cases}$$

Determine the distance from the point $P = (0, -1, 3)$ to the line ℓ.

Common Topic (12) One-to-one / Injective

2018 Final

Exercise 3
Consider the function $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = x^2 + x\cos(x)$ for all $x \geqslant 0$.
Gegeven is de functie $f : [0, \infty) \to \mathbb{R}$ met $f(x) = x^2 + x\cos(x)$ voor alle $x \geqslant 0$.

2p **3a** Show that the function f is one-to-one.
Bewijs dat de functie f injectief is.

2019 Final

Exercise 4
Consider the function / Beschouw de functie $h(x) = x^{17} + 16x - 1$.

3p **4a** Show that h is one-to-one, and determine the domain and the range of h. Explain your answers.

Laat zien dat h is 1-op-1 is, en geef het domein en het bereik van h. Verklaar uw antwoorden.

2019 Resit

Exercise 2
The function $f_a : \mathbb{R} \to \mathbb{R}$ is given by $f_a(x) = x^3 + ax^2 + x$, with parameter $a \in \mathbb{R}$.
De functie $f_a : \mathbb{R} \to \mathbb{R}$ is gegeven door $f_a(x) = x^3 + ax^2 + x$, met parameter $a \in \mathbb{R}$.

2p **2a** Specify for every $a \in \mathbb{R}$ how many zeros the function f_a has.

2p **2b** Show that f_0 is injective.
Toon aan dat f_0 injectief is.

* Uncommon topics

2017 Final

[4]

4. The function $P(t)$ measures the level of oxygen in a pond, where $P(t) = 1$ is the normal (un-polluted) level and $0 \leq P(t) \leq 1$. Time t is measured in weeks. At $t = 0$ waste is dumped into the pond and, while the waste material is oxidized, the level of oxygen is given by

$$P(t) = \frac{t^2 - t + 1}{t^2 + 1}, \text{ for } t \geq 0.$$

At which time t is the level of oxygen in the pond the lowest?

(a) 0 weeks (b) $\sqrt{3}$ weeks (c) 1 week (d) 3 weeks

2018 Resit

2.5p **1d** The derivative of $\arctan(x^2) - \arctan(\frac{1}{x^2})$ with respect to x equals:
De afgeleide van $\arctan(x^2) - \arctan(\frac{1}{x^2})$ naar x is gelijk aan:

- O $\dfrac{4x}{1+x^4}$
- O 0
- O $\dfrac{2x+2x^3}{1+x^4}$
- O $\dfrac{2x-2x^3}{1+x^4}$

2.5p **1e** Linearisation of $f(x) = \sqrt{x}$ in $x = 49$ yields the following approximation of $\sqrt{50}$:
Linearisatie van $f(x) = \sqrt{x}$ in $x = 49$ geeft de volgende benadering van $\sqrt{50}$:

- O 7
- O $\dfrac{50}{7}$
- O $\dfrac{99}{14}$
- O $\dfrac{197}{28}$

2019 Resit

2.5p **1a** The function/ De functie $f(x) = \dfrac{\sin(x)}{1+\cos(x)}$ is

- O even
- O odd / oneven
- O even and odd / even en oneven
- O neither even nor odd/ noch even noch oneven

2019 Final

5p **2** Consider the function $f(x) = \frac{2x^4 + x^3 - 8x^2 + 6x - 4}{x^3 - 4x}$.
Use polynomial division followed by partial fraction expansion to write $f(x)$ in the form

$$p(x) + \frac{A}{x-\alpha} + \frac{B}{x-\beta} + \frac{C}{x-\gamma},$$

where p is a polynomial, and α, β, γ, and A, B, and C are real numbers.

Beschouw de functie $f(x) = \frac{2x^4 + x^3 - 8x^2 + 6x - 4}{x^3 - 4x}$.
Gebruik staartdeling gevolgd door breuksplitsen om $f(x)$ te schrijven naar de vorm

$$p(x) + \frac{A}{x-\alpha} + \frac{B}{x-\beta} + \frac{C}{x-\gamma},$$

waar p een polynoom is, en α, β, γ, en A, B en C reële getallen zijn.

2023 Final

2p **1** Given $f(x) = \ln(x)$, find the derivative of $(f \circ f)(x)$.

- a $\frac{\ln(x)}{x}$
- b $\frac{1}{x\ln(x)}$
- c $\frac{x}{\ln(x)}$
- d $\frac{1}{\ln^2(x)}$
- e $\frac{1}{\ln(x^2)}$

2020 Resit

2.5p **1g** Let $f(x) = \frac{5}{2}x^2 - e^x$. For which value of x is $f''(x) = 0$?
Stel $f(x) = \frac{5}{2}x^2 - e^x$. Voor welke waarde van x is $f''(x) = 0$?

- $5e$
- 0
- $\ln(5)$
- e^5
- e^{-5}

2p **2** Given $f(x) = \ln(x)$ and $g(x) = 2\sin(x)$, give domain and range of the composite function $(g \circ f)(x)$.

- a domain: $x \in \mathbb{R}$; range $-2 \leq x \leq 2$
- b domain: $x > 0$; range $-2 \leq x \leq 2$
- c domain: $x \in \mathbb{R}$; range $-2 < x < 2$
- d domain: $x > 0$; range $-2 < x < 2$
- e domain: $x > 0$; range $-1 \leq x \leq 1$

2024 Resit

1p **14a** Determine polynomials $Q(x)$ and $R(x)$ such that $R(x)$ has degree at most 2 (i.e. $R(x) = ax^2 + bx + x$ for some $a, b, c \in \mathbb{R}$) with

$$\frac{x^5 - x^4 + x^3 - 4x^2 + 3x}{x^3 - x^2 - x - 2} = Q(x) + \frac{R(x)}{x^3 - x^2 - x - 2}$$