Communication Theory (5ETB0)
Module 12.1

Alex Alvarado
a.alvarado@tue.nl

Information and Communication Theory Lab
Signal Processing Systems Group
Department of Electrical Engineering
Eindhoven University of Technology, The Netherlands

www.tue.nl/ictlab/

Module 12.1

Presentation Outline

Part I Motivation and Model

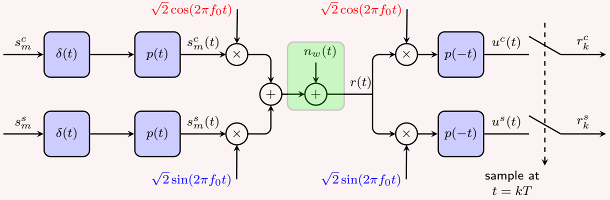
Part II Optimum Incoherent Reception

Part III Equal Energy Signals

Motivation: Coherent vs. Incoherent Communications

Coherent Reception

- *Coherent* reception: Phase of Tx and Rx are equal
- Very popular case: Serial QAM



Potential Problems and Solution

- Frequency mismatch: Diffent Tx an Rx oscilators
- Phase mismatch can be caused by propagation delay

Incoherent reception: Phase difference is unknown and/or cannot be compensated

Model

System Model

- Transmitted signal $s(t) = s^b(t)\sqrt{2}\cos(2\pi f_0 t - \theta)$
- Signal vectors are $\underline{s}_m = (s_{m1}, s_{m2}, \ldots, s_{mN})$, where

$$s_m^b(t) = \sum_{i=1}^{N} s_{mi}\varphi_i(t).$$

- Phase θ is assumed random and uniform over $[0, 2\pi)$:

$$p_\Theta(\theta) = \frac{1}{2\pi}, \text{ for } 0 \leq \theta < 2\pi.$$

A Few Assumptions...

- Phase difference assumed at the transmitter (without loss of generality)
- Spectrum of baseband signal within $[-W, W]$
- Only one baseband signal (for simplicity)

Quadrature-multiplexed Transmitted Waveform

Connection to QAM: N or $2N$ Dimensions?

$$\begin{aligned}
s_m(t) &= s_m^b(t)\sqrt{2}\cos(2\pi f_0 t - \theta)\\
&= s_m^b(t)\cos(\theta)\sqrt{2}\cos(2\pi f_0 t) + s_m^b(t)\sin(\theta)\sqrt{2}\sin(2\pi f_0 t)\\
&= \sum_{i=1}^{N} s_{mi}^c \varphi_i(t)\sqrt{2}\cos(2\pi f_0 t) + \sum_{i=1}^{N} s_{mi}^s \varphi_i(t)\sqrt{2}\sin(2\pi f_0 t)\\
&= \sum_{i=1}^{N} s_{mi}^c \phi_{c,i}(t) + \sum_{i=1}^{N} s_{mi}^s \psi_{s,i}(t)
\end{aligned}$$

where

$$\begin{aligned}
\underline{s}_m^c &= (s_{m1}, s_{m2}, \ldots, s_{mN})\cos(\theta) = \underline{s}_m \cos(\theta)\\
\underline{s}_m^s &= (s_{m1}, s_{m2}, \ldots, s_{mN})\sin(\theta) = \underline{s}_m \sin(\theta)
\end{aligned}$$

Module 12.1

Presentation Outline

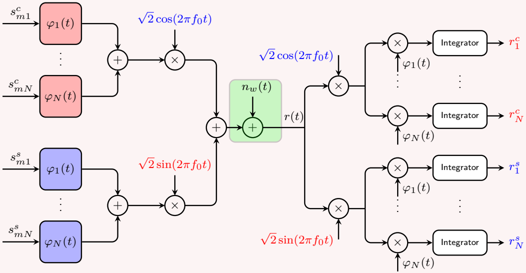
Part I Motivation and Model

Part II Optimum Incoherent Reception

Part III Equal Energy Signals

Optimum Incoherent Reception (1/3)

Forming the Vectors



Forming the Vectors

An optimum-receiver forms $\underline{r}^c$ and $\underline{r}^s$ using $r(t) = s_m(t) + n_w(t)$:

$$\begin{aligned} \underline{r}^c &= \underline{s}^c_m + \underline{n}^c = \underline{s}_m \cos\theta + \underline{n}^c \\ \underline{r}^s &= \underline{s}^s_m + \underline{n}^s = \underline{s}_m \sin\theta + \underline{n}^s \end{aligned}$$

Optimum Incoherent Reception (2/3)

Optimum Receiver for Equally Likely Messages (Result 12.1)

The optimum receiver for "random-phase transmission" (incoherent detection) is:

$$\hat{m}^{\mathsf{MAP}}(\underline{r}) = \underset{m \in \mathcal{M}}{\operatorname{argmax}} \left\{ I_0\left(\frac{2X_m}{N_0}\right) \exp\left(-\frac{E_m}{N_0}\right) \right\}$$

where

$$X_m \triangleq \sqrt{(\underline{r}^c \cdot \underline{s}_m)^2 + (\underline{r}^s \cdot \underline{s}_m)^2}$$

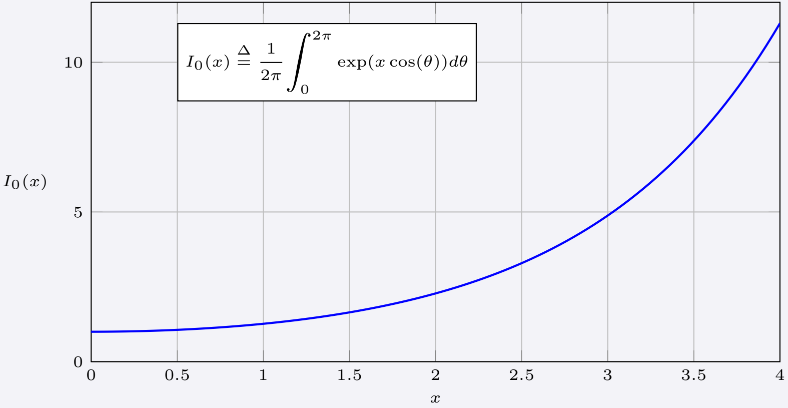
and E_m is the energy of the m-th message

Proof Sketch

- Compute MAP decision variables for a given θ, average over the distribution of Θ
- Simplify expressions (details in Sec. 12.2 for details)
- Express result using the zero-order modified Bessel function of the first kind

Zero-order modified Bessel function of the first kind

$I_0(x)$ is an increasing function of x

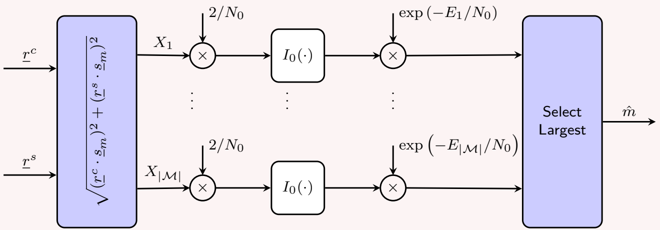


Optimum Incoherent Reception (3/3)

Optimum Receiver for Equally Likely Messages

$$I_0\left(\frac{2X_m}{N_0}\right)\exp\left(-\frac{E_m}{N_0}\right), \quad X_m \triangleq \sqrt{(\underline{r}^c\cdot\underline{s}_m)^2+(\underline{r}^s\cdot\underline{s}_m)^2}$$

General Structure



Module 12.1

Presentation Outline

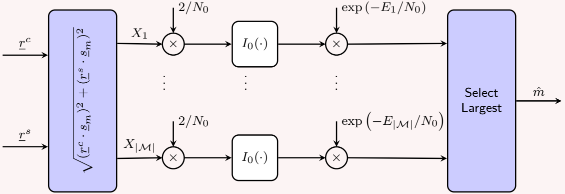
Part I Motivation and Model

Part II Optimum Incoherent Reception

Part III Equal Energy Signals

Equal Energy Signals: Optimum Reception

General Structure

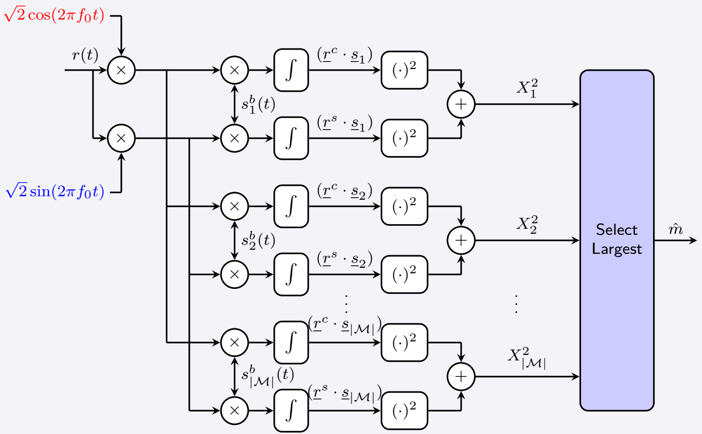


Equal Energy Signals

$$\begin{aligned}
\hat{m}^{\text{MAP}}(\underline{r}) &= \operatorname*{argmax}_{m\in\mathcal{M}} \left\{ I_0\left(\frac{2\sqrt{(\underline{r}^c \cdot \underline{s}_m)^2 + (\underline{r}^s \cdot \underline{s}_m)^2}}{N_0}\right) \exp\left(-\frac{E_m}{N_0}\right) \right\} \\
&= \operatorname*{argmax}_{m\in\mathcal{M}} \left\{ (\underline{r}^c \cdot \underline{s}_m)^2 + (\underline{r}^s \cdot \underline{s}_m)^2 \right\}
\end{aligned}$$

Equal Energy Signals: Receiver Implementation

Correlation/Direct Receiver for Random Phase Equal-energy Signals



Summary Module 12.1

Take Home Messages

- Coherent vs. incoherent reception
- General structure of incoherent receiver: no knowledge of θ
- Equal energy signals are simpler to detect

Communication Theory (5ETB0)
Module 12.1

Alex Alvarado
a.alvarado@tue.nl

Information and Communication Theory Lab
Signal Processing Systems Group
Department of Electrical Engineering
Eindhoven University of Technology, The Netherlands

www.tue.nl/ictlab/