Components in wireless technology, 5XTC0

Module 4
Exercise: Amplifier design

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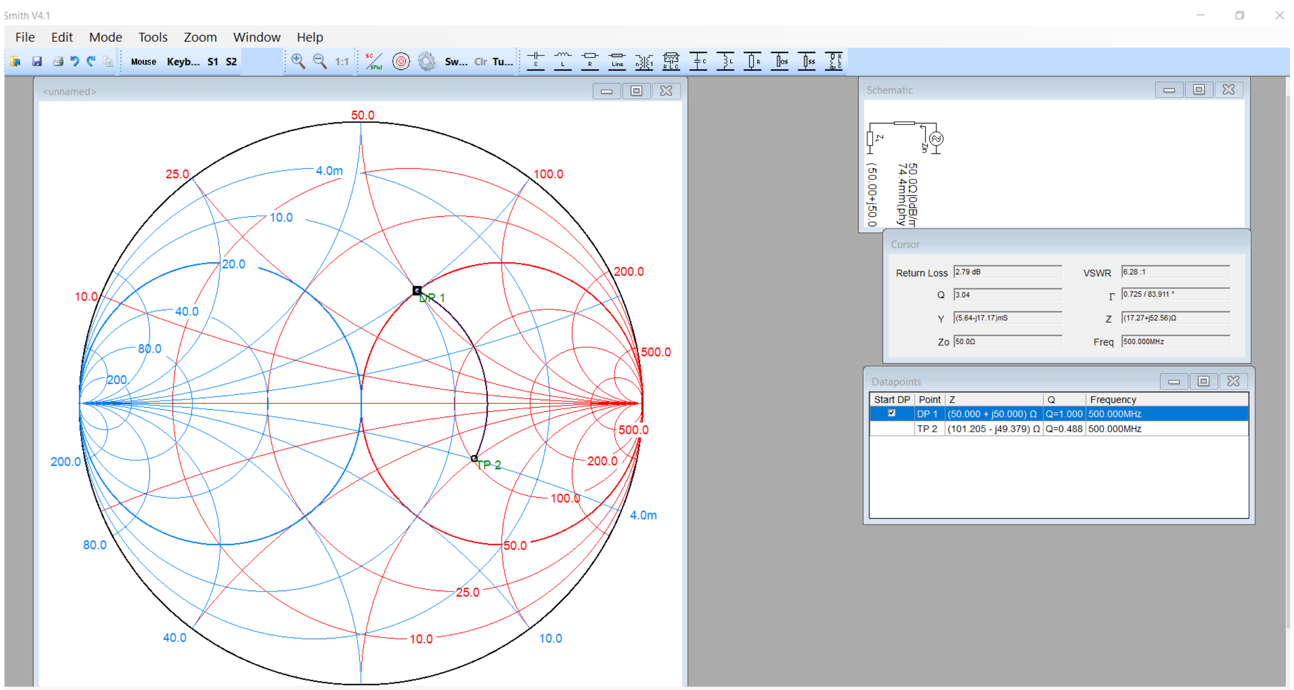
Where innovation starts

Outline

- Introduction to Smith Chart tool
- Introduction to formula list that can be used during exam
- Exercise:
  - Smith Chart
  - Transmission lines
  - Matching networks
  - Amplifier design

Smith Chart Tool

- Very good to practice and check analytical expressions
- Link to download the tool:
  https://www.fritz.dellsperger.net/smith.html
- Smith 4.1

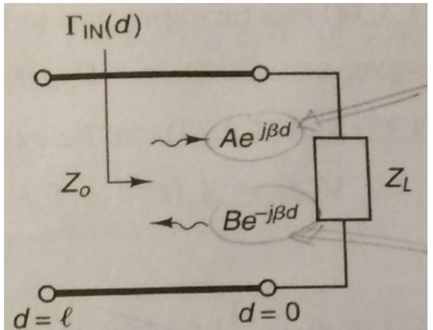


- Link to download Smith Chart: https://leleivre.com/rf_smith.html

Formula list available for exam

Power delivered to the load	$P_L = \frac{\lvert V_2^-\rvert^2}{2Z_0}\left(1-\lvert\Gamma_L\rvert^2\right)$
Input power to the network	$P_{in} = \frac{\lvert V_1^+\rvert^2}{2Z_0}\left(1-\lvert\Gamma_{in}\rvert^2\right)$
Input and output reflection coefficients of a transistor with a source and load: general case	$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$ $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$
Input and output reflection coefficients of a transistor with a source and load: unilateral case	$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$ $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22}$
Gain of the input matching network	$G_S = \frac{1-\lvert\Gamma_S\rvert^2}{\lvert 1-\Gamma_{in}\Gamma_S\rvert^2}$
Gain of the output matching network	$G_L = \frac{1-\lvert\Gamma_L\rvert^2}{\lvert 1-S_{22}\Gamma_L\rvert^2}$
Gain of the transistor (unilateral case)	$G_0 = \lvert S_{21}\rvert^2$
Transducer gain of the basic amplifier circuit (input matching, unilateral transistor, output matching)	$G_T = G_S G_0 G_L$ $G_{T,dB} = G_{S,dB} + G_{0,dB} + G_{L,dB}$
Maximum gain of the input and output matching networks	$G_{S_{max}} = \frac{1}{1-\lvert S_{11}\rvert^2},$ $G_{L_{max}} = \frac{1}{1-\lvert S_{22}\rvert^2}.$
Maximum transducer power gain, unilateral case	$G_{TU_{max}} = \frac{1}{1-\lvert S_{11}\rvert^2}\lvert S_{21}\rvert^2\frac{1}{1-\lvert S_{22}\rvert^2}$
Normalized gain factors g_S and g_L	$g_S = \frac{G_S}{G_{S_{max}}} = \frac{1-\lvert\Gamma_S\rvert^2}{\lvert 1-S_{11}\Gamma_S\rvert^2}(1-\lvert S_{11}\rvert^2),$ $g_L = \frac{G_L}{G_{L_{max}}} = \frac{1-\lvert\Gamma_L\rvert^2}{\lvert 1-S_{22}\Gamma_L\rvert^2}(1-\lvert S_{22}\rvert^2).$
Center and radius of the constant gain circle for the input matching network	$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)\lvert S_{11}\rvert^2},$ $R_S = \frac{\sqrt{1-g_S}\,(1-\lvert S_{11}\rvert^2)}{1-(1-g_S)\lvert S_{11}\rvert^2}$
Center and radius of the constant gain circle for the output matching network	$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)\lvert S_{22}\rvert^2},$ $R_L = \frac{\sqrt{1-g_L}\,(1-\lvert S_{22}\rvert^2)}{1-(1-g_L)\lvert S_{22}\rvert^2}$
Condition for "unconditionally stable" device, general case	for all $\lvert\Gamma_L\rvert<1$ and $\lvert\Gamma_S\rvert<1$ $\Rightarrow \begin{cases} \lvert\Gamma_{in}\rvert = \left\lvert S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right\rvert < 1 \\ \lvert\Gamma_{out}\rvert = \left\lvert S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}\right\rvert < 1 \end{cases}$

Conditions for "unconditionally stable" device, unilateral case	$\lvert\Gamma_{in}\rvert = \lvert S_{11}\rvert < 1$ $\lvert\Gamma_{out}\rvert = \lvert S_{22}\rvert < 1$
Center and radius of the stability circles, load side	$C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{\lvert S_{22}\rvert^2-\lvert\Delta\rvert^2}$ (center), $R_L = \left\lvert\frac{S_{12}S_{21}}{\lvert S_{22}\rvert^2-\lvert\Delta\rvert^2}\right\rvert$ (radius). $\Delta = S_{11}S_{22} - S_{12}S_{21}$
Center and radius of the stability circles, source side	$C_S = \frac{(S_{11}-\Delta S_{22}^*)^*}{\lvert S_{11}\rvert^2-\lvert\Delta\rvert^2}$ (center), $R_S = \left\lvert\frac{S_{12}S_{21}}{\lvert S_{11}\rvert^2-\lvert\Delta\rvert^2}\right\rvert$ (radius) $\Delta = S_{11}S_{22} - S_{12}S_{21}$
Test for unconditional stability, general case	$\lvert\Delta\rvert = \lvert S_{11}S_{22} - S_{12}S_{21}\rvert < 1$ **and** $K = \frac{1-\lvert S_{11}\rvert^2-\lvert S_{22}\rvert^2+\Delta^2}{2\lvert S_{12}S_{21}\rvert} > 1$
Test for unconditional stability, unilateral case	$\lvert S_{11}\rvert < 1$ $\lvert S_{22}\rvert < 1$
Two-stage amplifier: Output noise and noise figure	$P_{N,total} = G_{A2}(G_{A1}P_{N,in} + P_{n1}) + P_{n2}$ $\Rightarrow F_{total} = \frac{P_{N,total}}{P_{N,in}G_{A1}G_{A2}} = 1 + \frac{P_{n1}}{P_{N,in}G_{A1}} + \frac{P_{n2}}{P_{N,in}G_{A1}G_{A2}}$ $F_{total} = F_1 + \frac{F_2-1}{G_{A1}},$ with $F_j = 1 + \frac{P_{nj}}{P_{N,in}G_{Aj}}$, j=1,2
Noise figure of a 2-port amplifier	$F = F_{min} + \frac{r_N}{g_S}\lvert y_S - y_{opt}\rvert^2$ $F = F_{min} + 4r_N\frac{\lvert\Gamma_S-\Gamma_{opt}\rvert^2}{\left(1-\lvert\Gamma_S\rvert^2\right)\lvert 1+\Gamma_{opt}\rvert^2}$
Constant noise circles	$C_F = \frac{\Gamma_{opt}}{1+N}$ $R_F = \frac{1}{1+N}\sqrt{N^2+N(1-\lvert\Gamma_{opt}\rvert^2)}$ $\Delta F_n' = N = (F-F_{min})\frac{\lvert 1+\Gamma_{opt}\rvert^2}{4r_n} = \frac{\lvert\Gamma_S-\Gamma_{opt}\rvert^2}{1-\lvert\Gamma_S\rvert^2}$

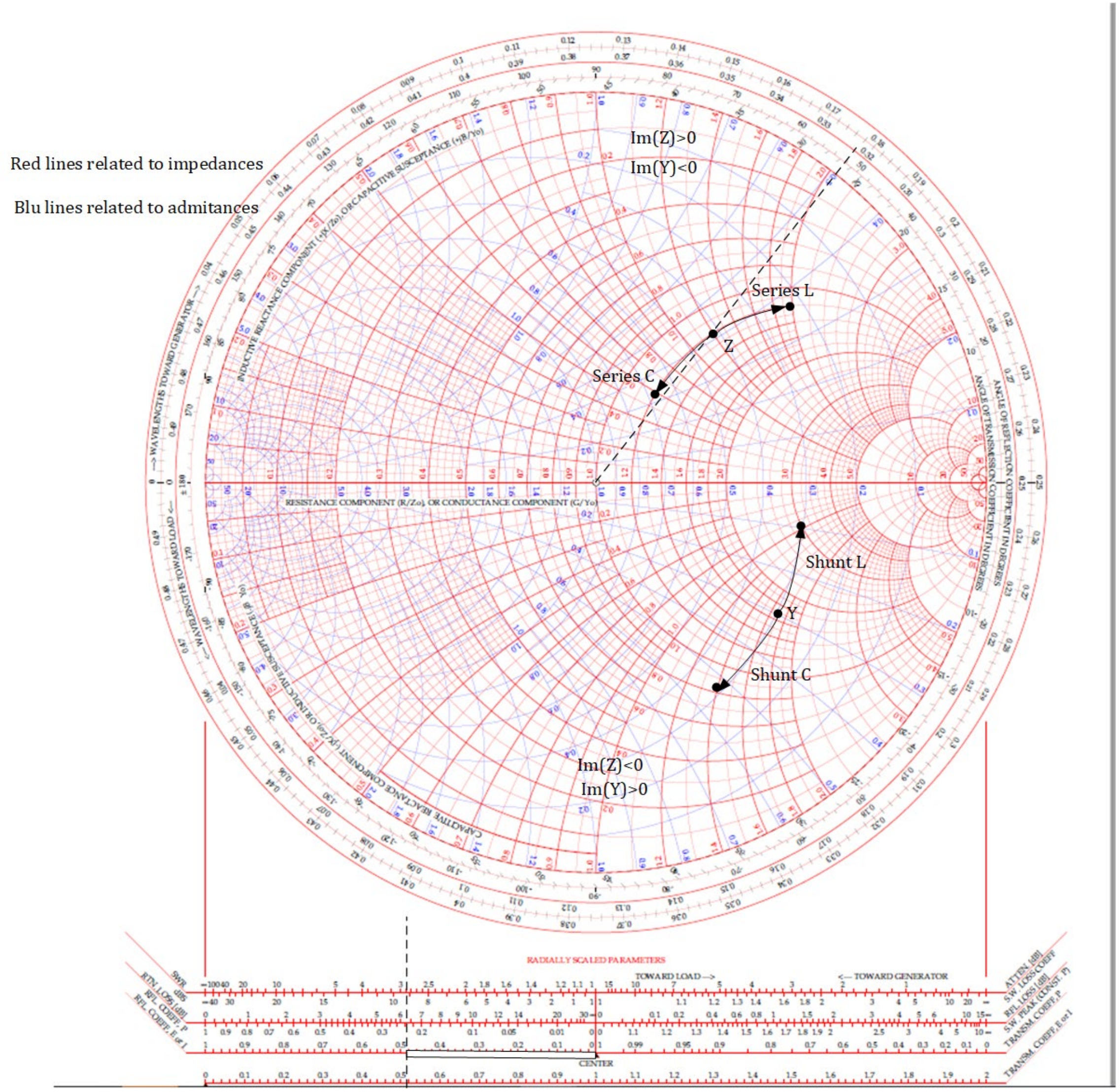


$$V(d) = A_1 e^{j\beta d} + B_1 e^{-j\beta d}$$

$$I(d) = \frac{A_1}{Z_o}e^{j\beta d} - \frac{B_1}{Z_o}e^{-j\beta d}$$

$$\Gamma_0 = \Gamma_{IN}(0) = \frac{B_1}{A_1}$$

Smith Chart Explanation



Smith Chart: Problem 1

Problem 1: Locating impedances and admittances in the Smith chart, reading of reflection coefficients

a) Find the following impedances in the Smith chart (normalized to 50 Ω) and determine the corresponding reflection coefficient:

$Z_1 = 50\ \Omega$ (matched load)	$Z_2 = 0\ \Omega$ (short circuit)	$Z_3 = \infty$ (open circuit)
$Z_4 = 20\Omega + j\ 25\Omega$	$Z_5 = 20\Omega - j\ 30\Omega$	$Z_6 = 150\Omega - j\ 150\Omega$

b) Find the following admittances in the Smith chart (normalized to 0.02 S) and determine the corresponding reflection coefficient:

$Y_1 = 0.02$ S (matched load)	$Y_2 = \infty$ (short circuit)	$Y_3 = 0$ (open circuit)
$Y_4 = 0.02 + j\ 0.02$ S	$Y_5 = 0.05 - j\ 0.1$ S	$Y_6 = 0.01 + j\ 0.05$ S

Smith Chart: Solution for Problem 1a (1/2)

$Z_1 = 50\ \Omega$ (matched load)	$Z_2 = 0\ \Omega$ (short circuit)	$Z_3 = \infty$ (open circuit)
$Z_4 = 20\Omega + j\ 25\Omega$	$Z_5 = 20\Omega - j\ 30\Omega$	$Z_6 = 150\Omega - j\ 150\Omega$

Smith chart

Problem 1a

$Z_1 = 50 \qquad \Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} = 0$

$\boxed{\text{Smith: } z_1 = \frac{50}{50} = 1}$

$Z_2 = 0 \qquad \Gamma_2 = \frac{0 - Z_0}{Z_0} = -1$

$\boxed{\text{Smith: } z_2 = 0}$

$Z_3 = \infty \qquad \Gamma_3 = \frac{\infty - Z_0}{\infty} = 1$

$\boxed{\text{Smith: } z_3 = \infty}$

$Z_4 = 20 + j25$

$z_4 = \frac{20 + j25}{50}$

$\boxed{\text{Smith: } z_4 = 0.4 + j0.5}$

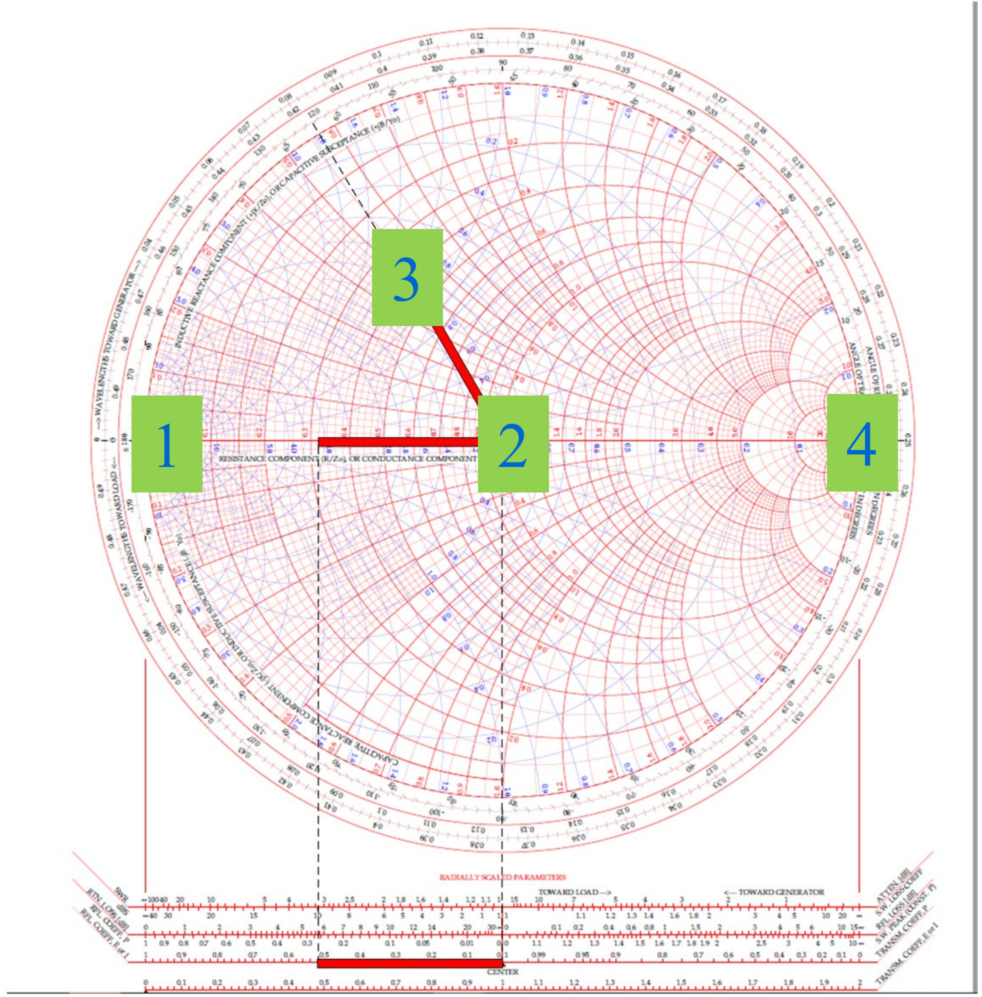
$\Gamma_4 = \frac{20 + j25 - 50}{20 + j25 + 50}$

$\Gamma_4 = \frac{-30 + j25}{70 + j25}$

$\Gamma_4 = \frac{(-30 + j25)(70 - j25)}{70^2 + 25^2}$

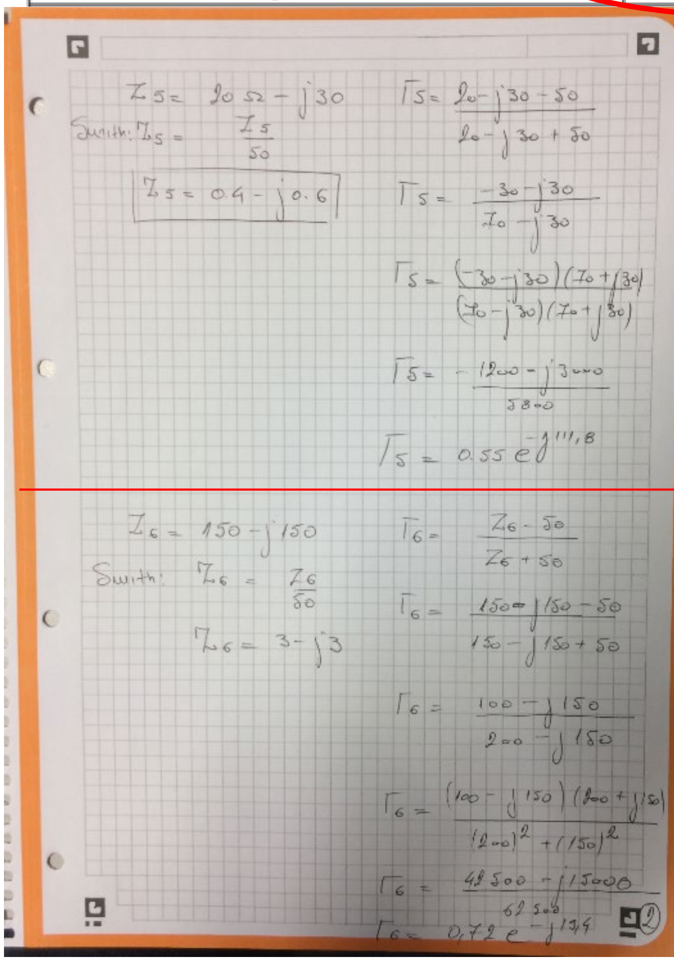
$\Gamma_4 = \frac{-1475 + j2500}{5525}$

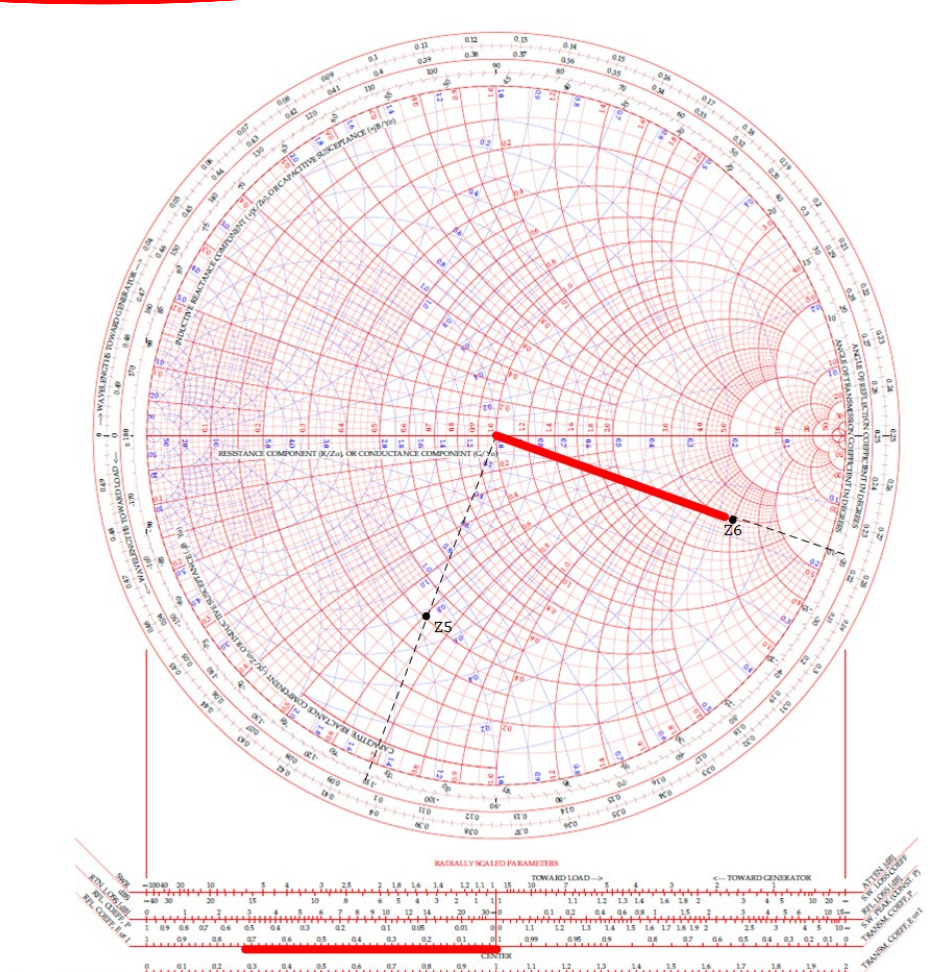
$\Gamma_4 = 0.52\, e^{j\, 120.55}$



Smith Chart: Solution for Problem 1a (2/2)

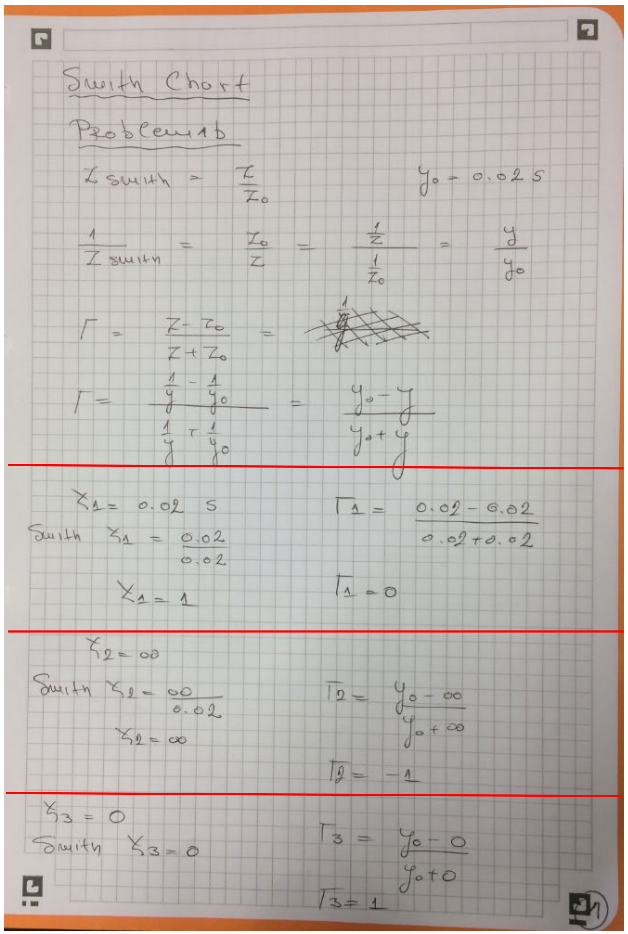
$Z_1 = 50\ \Omega$ (matched load)	$Z_2 = 0\ \Omega$ (short circuit)	$Z_3 = \infty$ (open circuit)
$Z_4 = 20\Omega + j\,25\Omega$	$Z_5 = 20\Omega - j\,30\Omega$	$Z_6 = 150\Omega - j\,150\Omega$

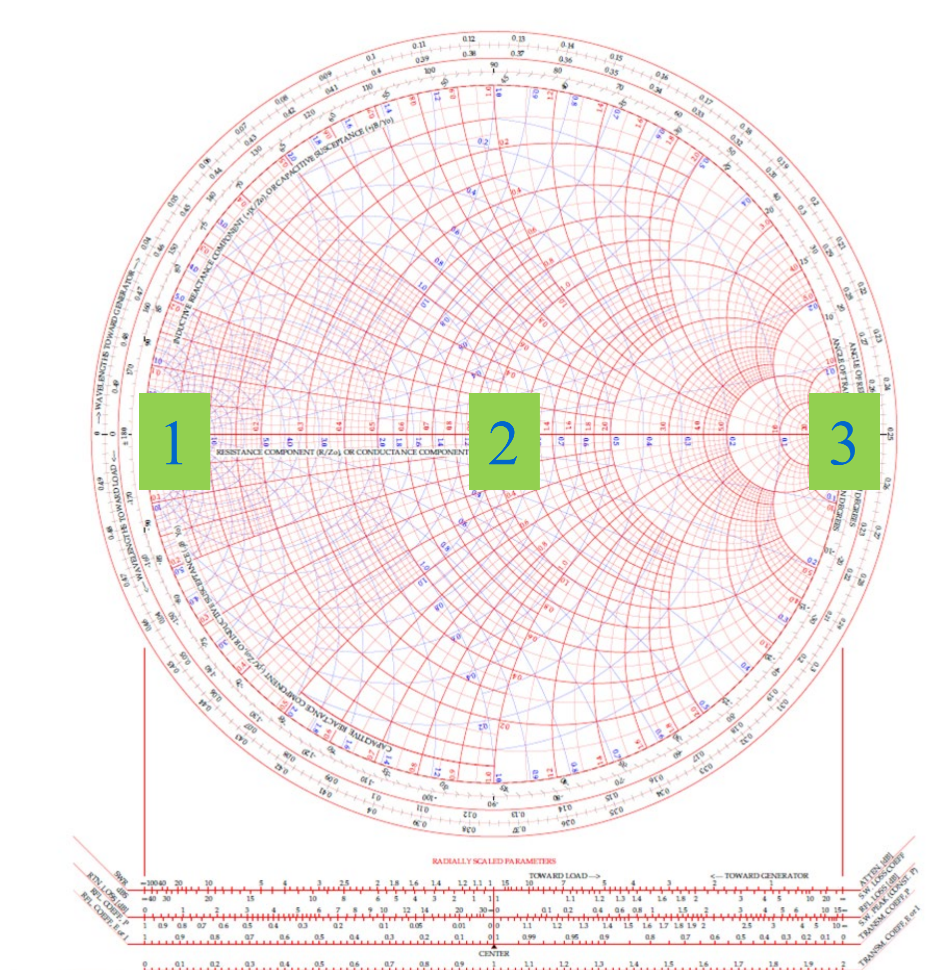




Smith Chart: Solution for Problem 1b (1/3)

$Y_1 = 0.02$ S (matched load)	$Y_2 = \infty$ (short circuit)	$Y_3 = 0$ (open circuit)
$Y_4 = 0.02 + j\,0.02$ S	$Y_5 = 0.05 - j\,0.1$ S	$Y_6 = 0.01 + j\,0.05$ S





Smith Chart: Solution for Problem 1b (2/3)

$Y_1 = 0.02$ S (matched load)	$Y_2 = \infty$ (short circuit)	$Y_3 = 0$ (open circuit)
$Y_4 = 0.02 + j\,0.02$ S	$Y_5 = 0.05 - j\,0.1$ S	$Y_6 = 0.01 + j\,0.05$ S

$y_4 = 0.02 + j0.02$

Smith $y_4 = \frac{0.02 + j0.02}{0.02}$

$\boxed{y_4 = 1 + j1}$

$\Gamma_4 = \frac{y_0 - y}{y_0 + y}$

$\Gamma_4 = \frac{0.02 - 0.02 - j0.02}{0.02 + 0.02 + j0.02}$

$\Gamma_4 = \frac{-j0.02}{0.04 + j0.02}$

$\Gamma_4 = \frac{-j}{2 + j}$

$\Gamma_4 = \frac{-j(2 - j)}{2^2 + 1^2}$

$\boxed{\Gamma_4 = \frac{-1 - j2}{5} = 0.44\,e^{-j116,5}}$

$y_5 = 0.05 - j0.1$

Smith $y_5 = \frac{0.05 - j0.1}{0.02}$

$\boxed{y_5 = 2.5 - j5}$

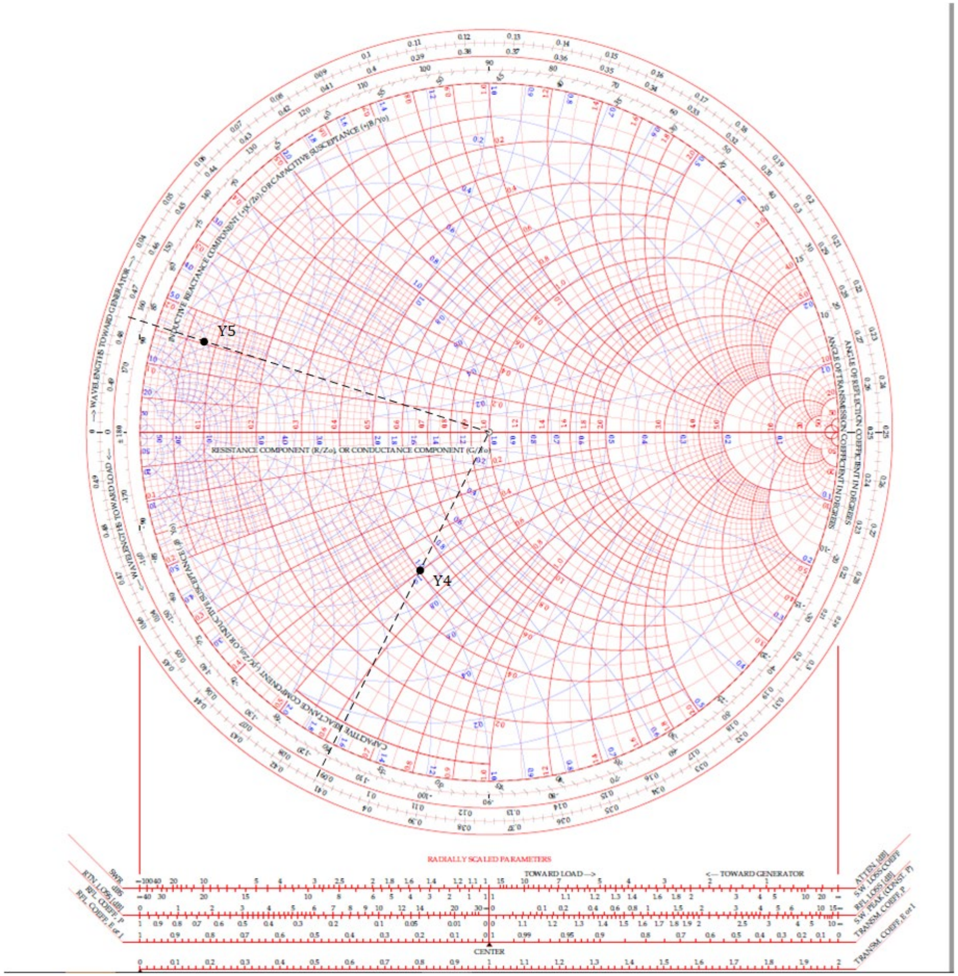
$\Gamma_5 = \frac{y_0 - y_5}{y_0 + y_5}$

$\Gamma_5 = \frac{0.02 - 0.05 + j10^{-1}}{0.02 + 0.05 - j10^{-1}}$

$\Gamma_5 = \frac{-0.03 + j1}{0.07 - j1}$ / $\cdot 100$

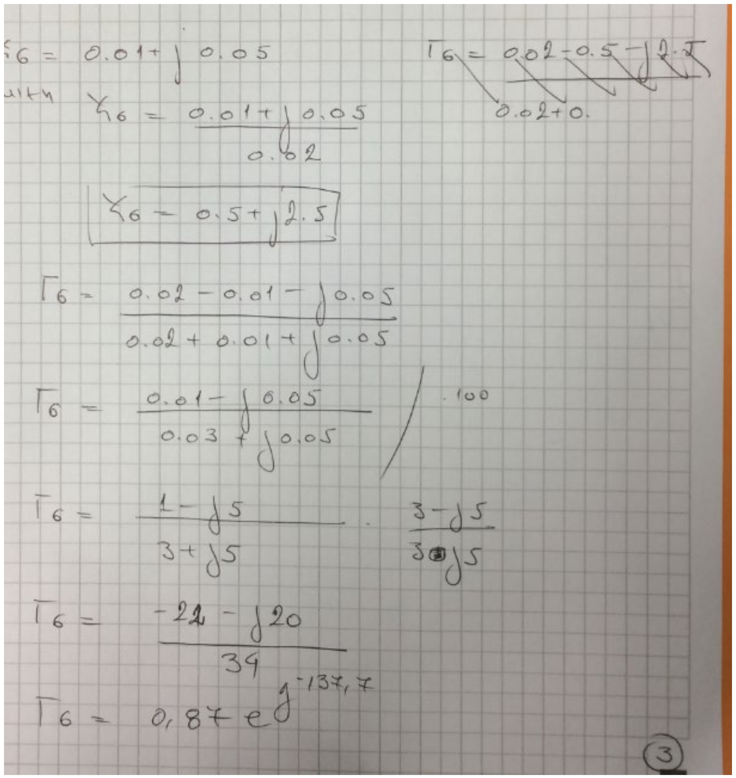
$\Gamma_5 = \frac{-121 + j40}{149}$

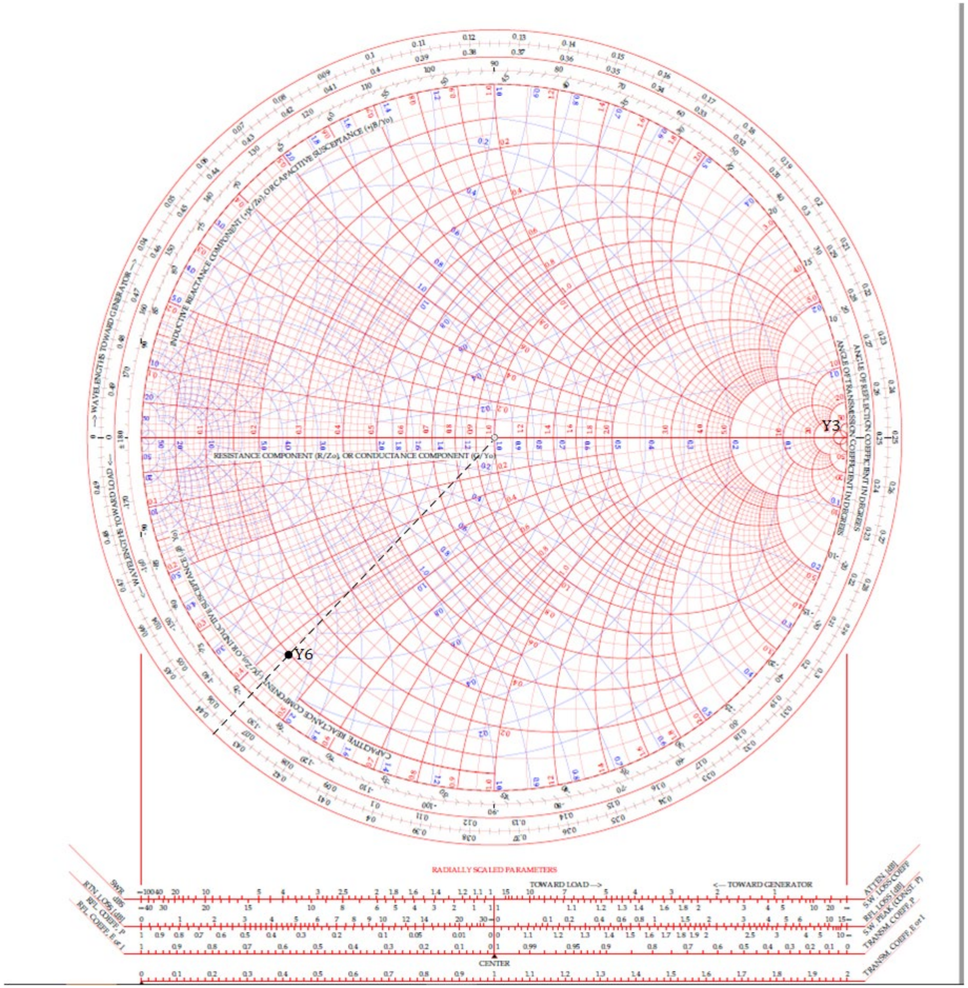
$\boxed{\Gamma_5 = 0.85\,e^{j161,5}}$



Smith Chart: Solution for Problem 1b (3/3)

$Y_1 = 0.02$ S (matched load)	$Y_2 = \infty$ (short circuit)	$Y_3 = 0$ (open circuit)
$Y_4 = 0.02 + j\,0.02$ S	$Y_5 = 0.05 - j\,0.1$ S	$Y_6 = 0.01 + j\,0.05$ S





Smith Chart: Problem 2

Problem 2: Transforming impedances into admittances

a) Find the corresponding normalized admittance values for the normalized impedance of $z = 1 + j1$.

b) Analytically calculate the normalized admittance value and compare it to your reading of the Smith chart from question a).

Smith Chart: Solution for Problem 2

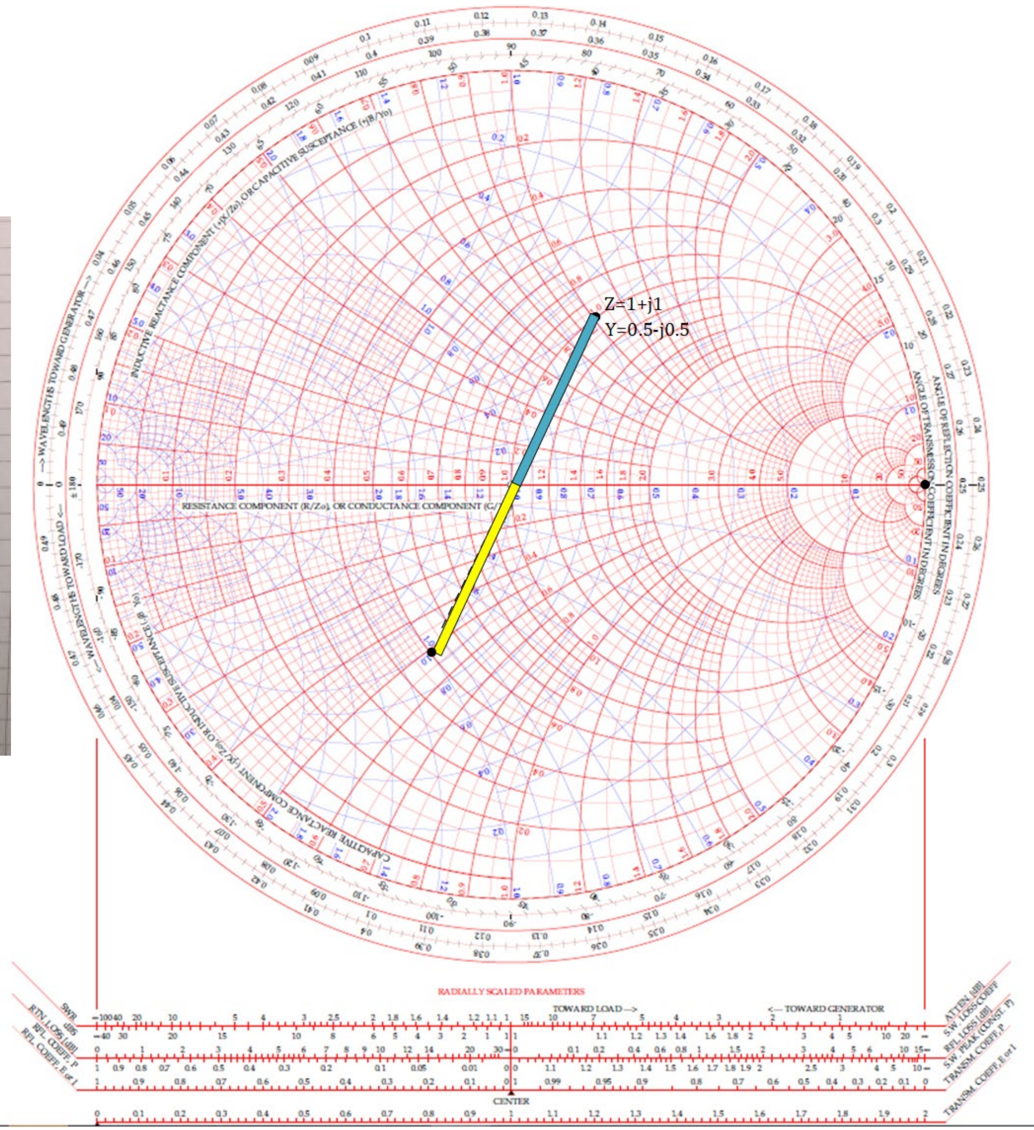
Smith Chart : Problem 2

$Z = 1 + j1$

$Y = \frac{1}{Z}$

$Y = \frac{1}{1+j1} = \frac{1 \text{[redacted]} j1}{(1-j1)(1+j1)} = \frac{1 \text{[redacted]} j1}{2}$

$Y = 0.5$ [redacted] $j0.5$



Smith Chart: Problem 3

Problem 3: From reflection coefficient to impedance and VSWR

Find the normalized input impedance whose reflection coefficient is $0.5 \sphericalangle 26^{\circ}$.
Determine the corresponding VSWR for this reflection coefficient in the Smith chart and by analytical calculation. Hint: Use the scale (SWR) in the Smith chart.

Smith Chart: Solution for Problem 3

Smith Chart: Problem 3

$\Gamma = 0.5\, e^{j26}$

$VSWR = \dfrac{1+|\Gamma|}{1-|\Gamma|}$

$VSWR = \dfrac{1+0.5}{1-0.5} = 3$

$Z = 2.2 + j1.2$ → Reading from Smith chart → normalized impedance

$\Gamma = \dfrac{Z - Z_0}{Z + Z_0}$

$\Gamma \cdot Z + \Gamma Z_0 = Z - Z_0$

$Z(1-\Gamma) = Z_0(1+\Gamma)$

$\boxed{Z = Z_0 \dfrac{1+\Gamma}{1-\Gamma}}$

$Z = Z_0 \cdot \dfrac{1+0.5e^{j26}}{1-0.5e^{j26}}$

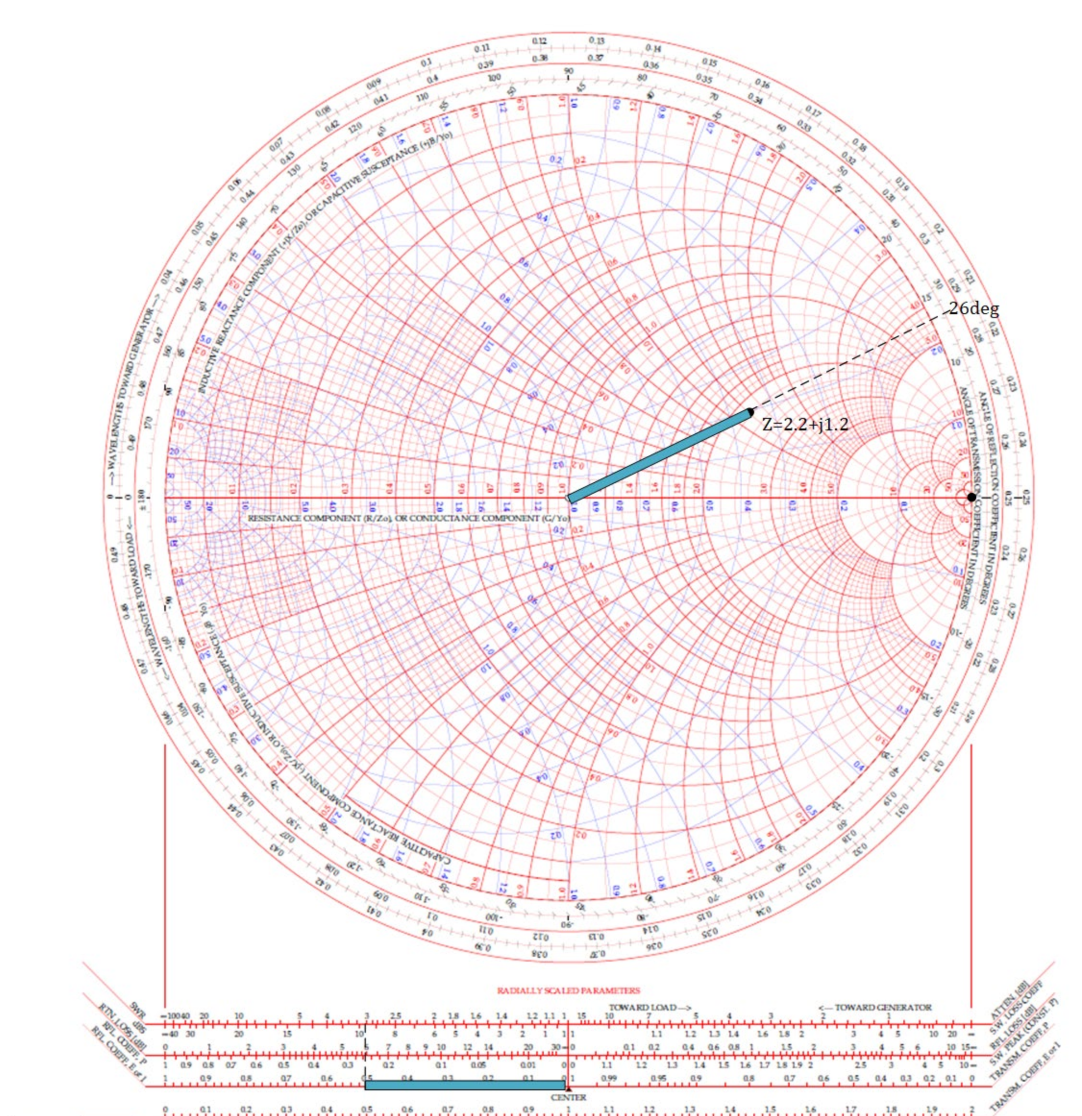
$\dfrac{Z}{Z_0} = \dfrac{1+0.5(\cos 26 + j\sin 26)}{1-0.5(\cos 26 + j \sin 26)}$

$\dfrac{Z}{Z_0} = \dfrac{1+0.45+j0.22}{1-0.45-j0.22}$

$\dfrac{Z}{Z_0} = \dfrac{(1.45+j0.22)(0.55+j0.22)}{0.55^2+0.22^2}$

$\dfrac{Z}{Z_0} = \dfrac{0.75+j0.44}{0.35}$

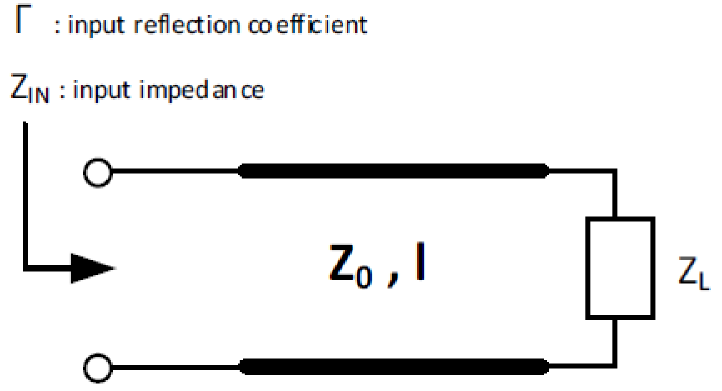
$\boxed{\dfrac{Z}{Z_0} = 2.14 + j1.25}$



Transmission lines: Problem 4

a) Use the Smith chart to find the input reflection coefficient, the input impedance and the VSWR for a transmission line having an electrical length of 45° and characteristic impedance of 50 Ω and terminated in a load of Z_L=50 + j50 Ω.

b) Compare the graphical solution of a) to an analytical calculation of the values.



Transmission lines: Solution for Problem 4 (1/2)

smith chart - problem 4

a) b) $\beta l = \frac{\pi}{4} \rightarrow$ electrical length $\rightarrow \beta d = \pi/4$

$$\frac{2\pi}{\lambda} l = \frac{\pi}{4}$$

$$l = \frac{\lambda}{8} = 0.125\,\lambda$$

$$Z_L = 50 + j50$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta d}{Z_0 + j Z_L \tan \beta d} \quad / \; d = l$$

$$Z_{in} = Z_0 \frac{50 + j50 + j50}{50 + j(50 + j50)}$$

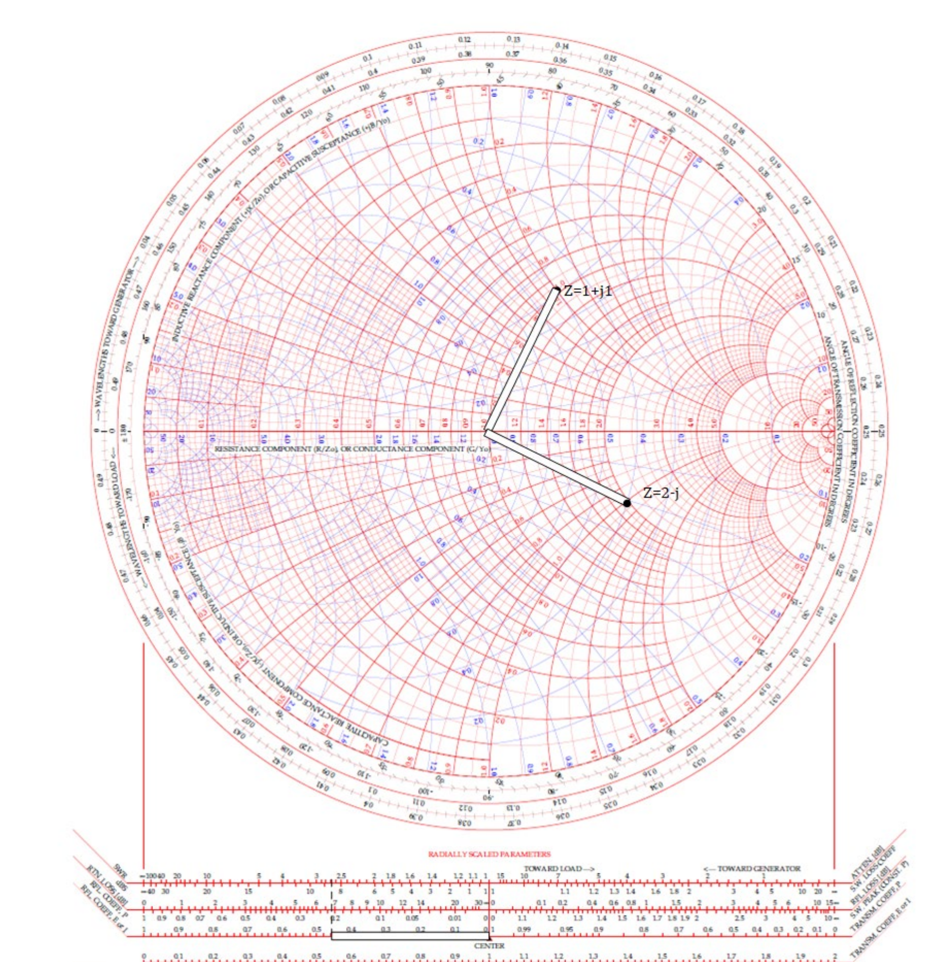
$$Z_{in} = Z_0 \frac{50 + j100}{j50}$$

$$Z_{in} = Z_0 \frac{1 + j2}{j} \quad / \cdot -j$$

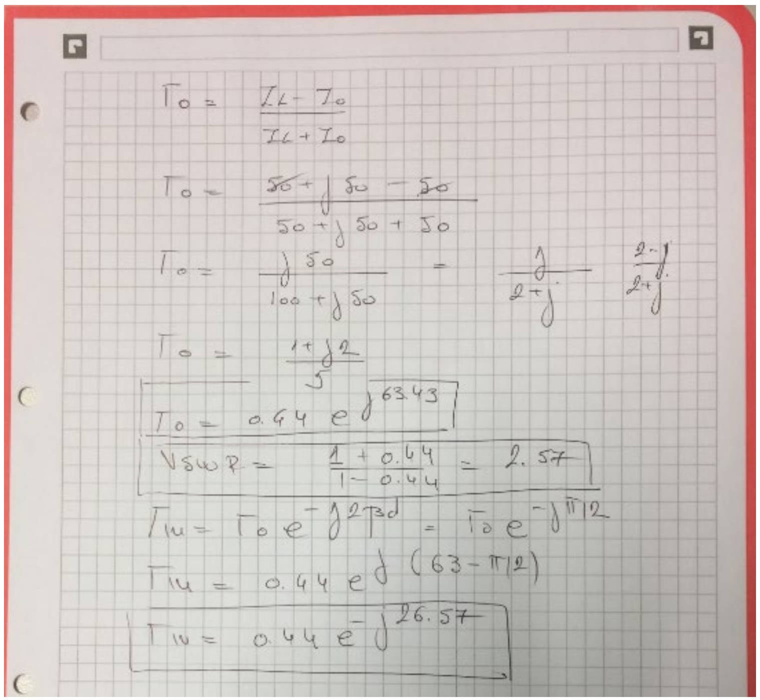
$$\cancel{Z_{in} = Z_0 + 2 + j}$$

$$\boxed{Z_{in} = Z_0 \, (2 - j)}$$

1



Transmission lines: S



$$\Gamma_o = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\Gamma_o = \frac{50 + j50 - 50}{50 + j50 + 50}$$

$$\Gamma_o = \frac{j50}{100 + j50} = \frac{j}{2+j} \cdot \frac{2-j}{2-j}$$

$$\Gamma_o = \frac{1+j2}{5}$$

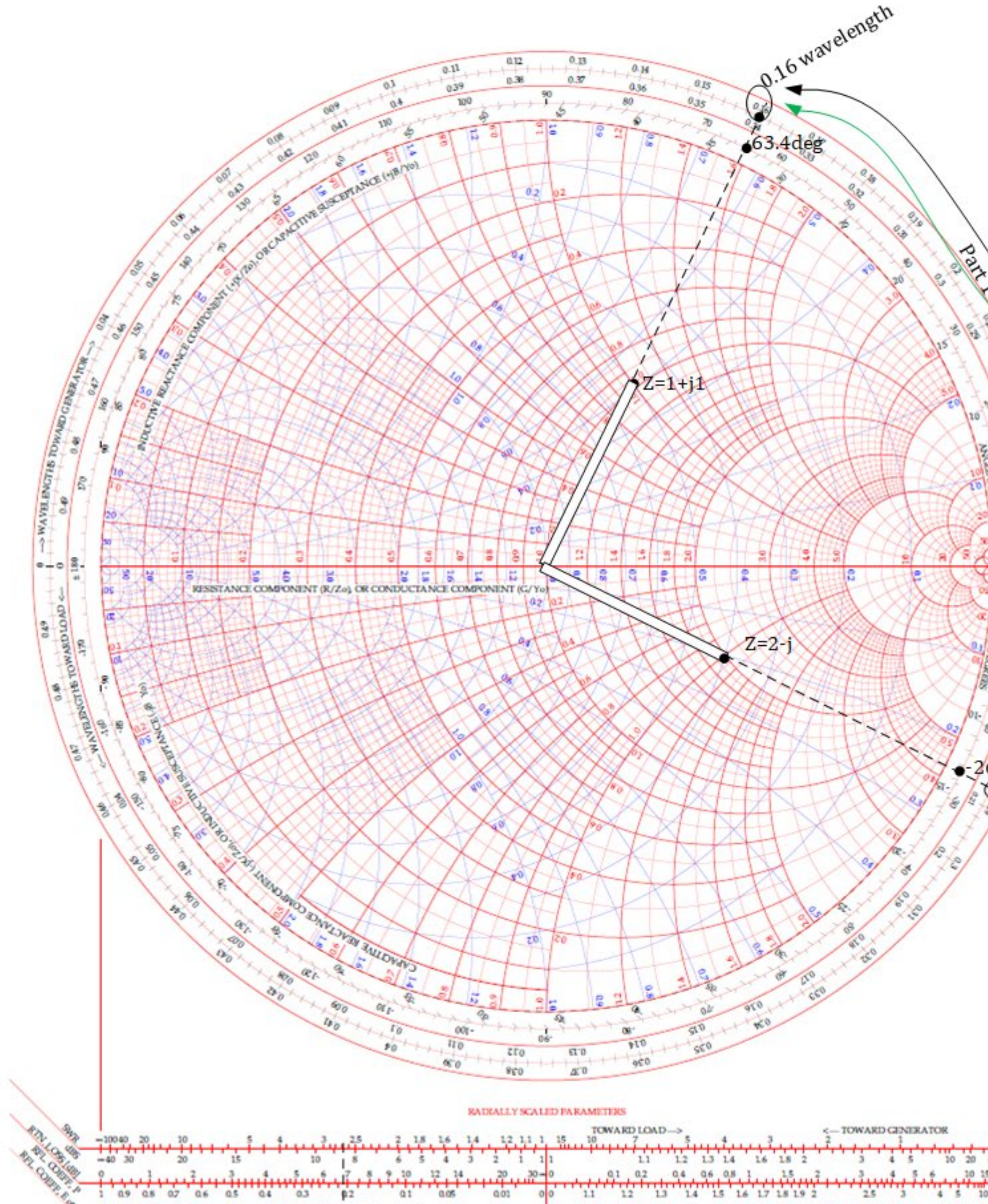
$$\boxed{\Gamma_o = 0.44\, e^{j63.43}}$$

$$\boxed{VSWR = \frac{1+0.44}{1-0.44} = 2.57}$$

$$\Gamma_{IN} = \Gamma_o e^{-j2\beta d} = \Gamma_o e^{-j\pi/2}$$

$$\Gamma_{IN} = 0.44\, e^{j(63-\pi/2)}$$

$$\boxed{\Gamma_{IN} = 0.44\, e^{-j26.57}}$$



Transmission lines: Problem 5

5) Analytically calculate the required length l of a 50 Ω short circuited transmission line to obtain an input impedance of $Z_{in}(l) = j\,100\ \Omega$. Express the length in terms of the wavelength λ. Determine the required length in the Smith chart and compare to your calculation.

Transmission lines: Solution for Problem 5

$$Z_{in} = Z_0 \left(\frac{Z_0 + jZ_L \tan \beta d}{Z_L + jZ_0 \tan \beta d} \right)^{-1}$$

$$Z_L = 0$$

$$Z_{in} = Z_0 \left(\frac{Z_0}{jZ_0 \tan \beta d} \right)^{-1}$$

$$\cancel{Z_{in} = -jZ_0 \cot \beta d}$$

$$Z_{in} = jZ_0 \tan \beta d$$

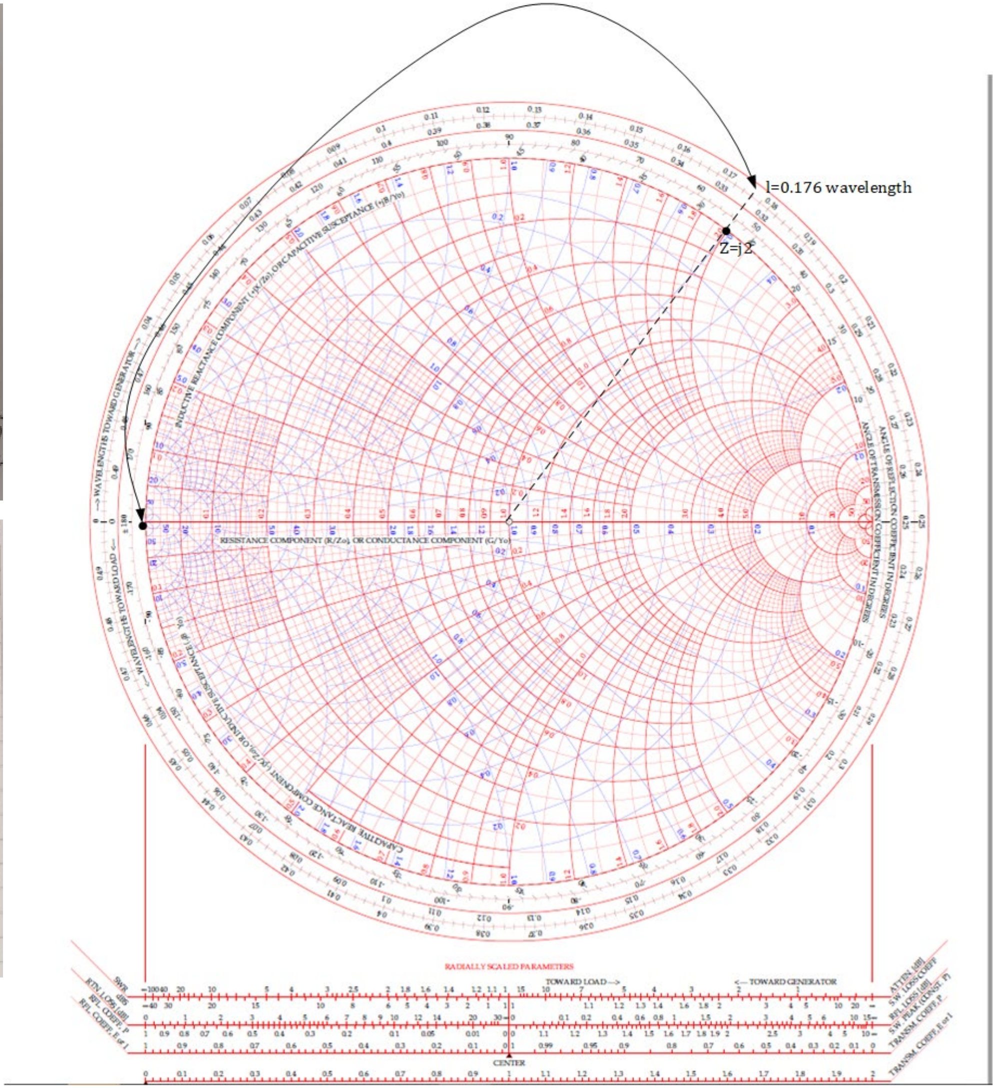
$$j100 = j50 \tan \frac{2\pi}{\lambda} \cdot d$$

$$(\pi = 180^\circ)$$

$$2 = \tan \frac{2\pi}{\lambda} \cdot d$$

$$\tan^{-1} 2 = 63.43^\circ = 2 \cdot 180^\circ \cdot \frac{d}{\lambda}$$

$$\boxed{d = 0.176\,\lambda}$$



Transmission lines: Problem 6

6) Analytically calculate the required length l of a 50 Ω open circuited transmission line to obtain an input impedance of $Z_{in}(l) = j\,100\ \Omega$. Express the length in terms of the wavelength λ. Determine the required length in the Smith chart and compare to your calculation.

Transmission lines: Solution for Problem 6

d) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$

$Z_L = \infty$

$Z_{in} = Z_0 \dfrac{1}{j\tan\beta d}$

$Z_{in} = Z_0(-j\cot\beta d)$

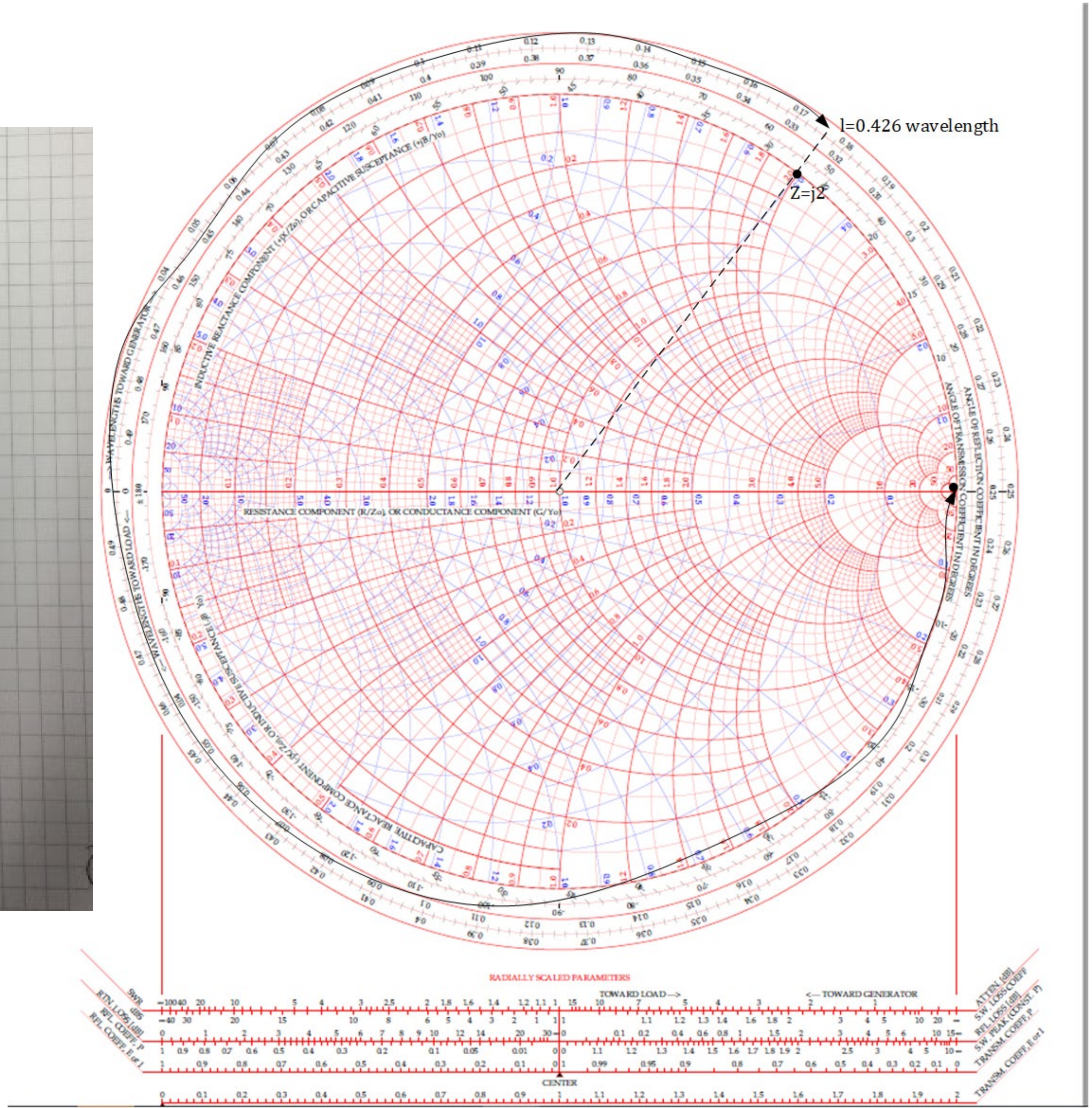
$j100 = -50\, j \cot\beta d$

$-2 = \cot\beta d$

$\beta d = \cot^{-1}(-2)$

$\frac{2\pi}{\lambda} \cdot d = 153.44$

$\boxed{d = 0.426\lambda}$



Transmission lines: Problem 7

- Book of Gonzalez: Example 1.3.1

(a) Find the load reflection coefficient, the input impedance, and the VSWR in the transmission line shown in Fig. 1. The length of the transmission line is $\lambda/8$ and the characteristic impedance is 50 Ω.

(b) Evaluate $V(\lambda/8)$, $I(\lambda/8)$, $P(\lambda/8)$, $V(0)$, $I(o)$ and $P(0)$.

(c) Write the expression for $v(d,t)$ and $i(d,t)$ along the transmission line.

(d) Find the length in centimeters of the $\lambda/8$ transmission line at f=1GHz.

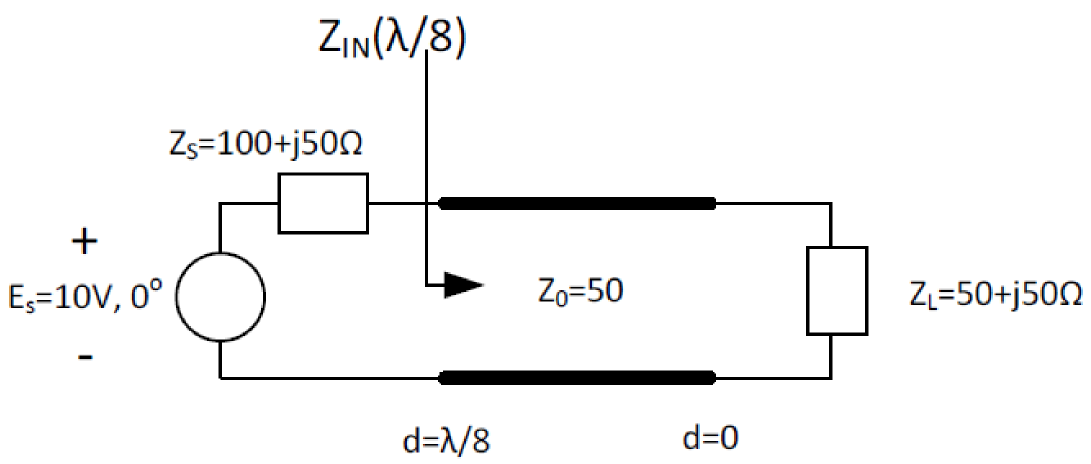
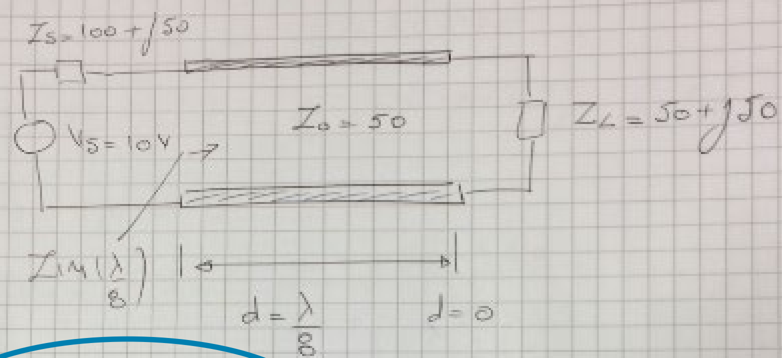


Figure 1 Transmission line circuit example problem 1.

Transmission lines: Solution for Problem 7 a

Gonzales : Example 1.3.1



a)

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_0 = \frac{50 + j50 - 50}{50 + j50 + 50} = \frac{j50}{100 + j50} = \frac{j}{2 + j}$$

$$\Gamma_0 = \frac{j(2-j)}{(2+j)(2-j)} = \frac{1 + j2}{5}$$

$$|\Gamma_0| = \sqrt{\frac{1 + 2^2}{5}} = \frac{1}{\sqrt{5}} = 0.447$$

$$\angle \Gamma_0 = \tan^{-1}(2/1) = 63.43$$

$$\boxed{\Gamma_0 = 0.447 \angle 63.43}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}$$

$$\beta = \frac{2\pi}{\lambda}, \quad d = \frac{\lambda}{8} \Rightarrow \beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$$

$$\tan \pi/4 = 1$$

$$Z_{in} = Z_0 \frac{50 + j50 + j50}{50 + j \cdot (50 + j50)}$$

$$Z_{in} = Z_0 \frac{50 + j100}{j50}$$

$$Z_{in} = \cancel{\text{ß}}\ 100 - \frac{50}{j}$$

$$\boxed{Z_{in} = 100 - j50}$$

$$V = A_1 e^{j\beta d} + B_1 e^{-j\beta d}$$

$$V = A_1 e^{j\beta d} \cdot (1 - \Gamma_0 e^{-j2\beta d})$$

$$|V_{max}| = |A_1| \, (1 + |\Gamma_0|)$$

$$|V_{min}| = |A_1| \, (1 - |\Gamma_0|)$$

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{1 + |\Gamma_0|}{1 - |\Gamma_0|}$$

$$\boxed{VSWR = \frac{1 + 0.447}{1 - 0.447} = 2.62}$$

2

Transmission lines: Solution for Problem 7 b (1/3)

b) $V = A_1 e^{j\beta d} + B_1 e^{-j\beta d}$

$V = A_1 e^{j\beta d}\left(1 + \frac{B_1}{A_1} e^{-j2\beta d}\right)$

$\Gamma_0 = \frac{B_1}{A_1}$

$V(d) = A_1 e^{j\beta d}\left(1 + \Gamma_0 e^{-j2\beta d}\right)$

$I(d) = \frac{A_1}{Z_0} e^{j\beta d}\left(1 - \Gamma_0 e^{-j2\beta d}\right)$

$d = \lambda/8 \qquad \beta d = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{8} = \pi/4$

$V\left(\frac{\lambda}{8}\right) = A_1 e^{j\pi/4}\left(1 + 0.447 e^{j63.43} e^{-j\pi/2}\right)$

$V\left(\frac{\lambda}{8}\right) = A_1 e^{j\pi/4}\left(1 + 0.447 e^{-j26.57}\right)$

$V\left(\frac{\lambda}{8}\right) = A_1 \left(e^{j\pi/4} + 0.447 e^{j18.43}\right)$

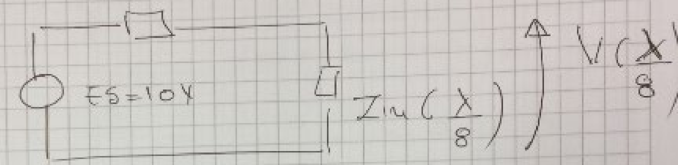
$V\left(\frac{\lambda}{8}\right) = A_1 \left(0.71 + j0.71 + 0.42 + j0.14\right)$

$\boxed{V\left(\frac{\lambda}{8}\right) = A_1 \left(1.13 + j0.85\right)}$

3

Equivalent circuit:

$Z_s = 100 + j50$



$Z_{in}\left(\frac{\lambda}{8}\right) = 100 - j50$

$V\left(\frac{\lambda}{8}\right) = E_s \cdot \frac{Z_{in}\left(\frac{\lambda}{8}\right)}{Z_s + Z_{in}(\lambda/8)}$

$V\left(\frac{\lambda}{8}\right) = E_s \cdot \frac{100 - j50}{100 + \cancel{j50} + 100 - \cancel{j50}}$

$V\left(\frac{\lambda}{8}\right) = E_s \, \frac{100 - j50}{200}$

$V\left(\frac{\lambda}{8}\right) = 10\,(0.5 - j0.25)$

$V\left(\frac{\lambda}{8}\right) = 10 \cdot \sqrt{0.5^2 + 0.25^2} \; \angle -\tan^{-1}\frac{0.25}{0.5}$

$V\left(\frac{\lambda}{8}\right) = 5.59 \angle -26.57$

4

Transmission lines: Solution for Problem 7 b (2/3)

$$I\left(\frac{\lambda}{8}\right) = \frac{E_s}{Z_s + Z_{in}\left(\frac{\lambda}{8}\right)}$$

$$I\left(\frac{\lambda}{8}\right) = \frac{10}{100 + j50 + 100 - j50}$$

$$I\left(\frac{\lambda}{8}\right) = \frac{10}{200} = 0.05\ A$$

$$P\left(\frac{\lambda}{8}\right) = Re\left(V_{rms}\left(\frac{\lambda}{8}\right) \cdot I_{rms}\left(\frac{\lambda}{8}\right)^*\right)$$

$$P\left(\frac{\lambda}{8}\right) = \frac{1}{2} Re\left(V\left(\frac{\lambda}{8}\right) \cdot I\left(\frac{\lambda}{8}\right)\right)$$

$$P\left(\frac{\lambda}{8}\right) = \frac{1}{2} Re\left(10 \cdot (0.5 - j0.25) \cdot 0.05\right)$$

$$P\left(\frac{\lambda}{8}\right) = \frac{1}{2}\, 10 \cdot 0.5 \cdot 0.05$$

$$\boxed{P\left(\frac{\lambda}{8}\right) = 125\ mW}$$

$V\left(\frac{\lambda}{8}\right) = 10\,(0.5 - j0.25) \rightarrow$ calculated on page 4

$V\left(\frac{\lambda}{8}\right) = A_1\,(1.13 + j0.85) \rightarrow$ calculated on page 3

$A_1 = ?$

5

$$10\,(0.5 - j0.25) = A_1\,(1.13 + j0.85)$$

$$A_1 = 10\,\frac{0.5 - j0.25}{1.13 + j0.85}$$

$$A_1 = 10\,\frac{(0.5 - j0.25)(1.13 - j0.85)}{(1.13)^2 + (0.85)^2}$$

$$A_1 = 10\,\frac{0.3525 - j0.7075}{2}$$

$$A_1 = 3.95\angle{-63.45}$$

$$V(d) = 3.95\angle{-63.45}\, e^{j\beta d}\left(1 + 0.447\angle 63.43\, e^{-j2\beta d}\right)$$

$$\cancel{V(0) = 3.95\angle{-63.45}\,(1+}$$

$$V(d) = 3.95\angle{-63.45}\, e^{j\beta d} + 1.77\, e^{-j\beta d}$$

$$I(d) = \frac{3.95\angle{-63.45}\, e^{j\beta d} - 1.77\, e^{-j\beta d}}{50}$$

6

Transmission lines: Solution for Problem 7 b (3/3)

$V(d) = 3.95 \angle{-63.45}\, e^{j\beta d} + 1.77\, e^{-j\beta d}$

$V(0) \Rightarrow d = 0$

$V(0) = 3.95\, e^{-j63.45} + 1.77$

$V(0) = 3.95 \cos(63.45)$ [redacted] $j3.95 \sin(63.45) + 1.77$

$V(0) = 3.53$ [redacted] $j\, 3.53$

$\boxed{V(0) = 3.53\,(1$ [redacted] $j) = 5\, e^{-j45}}$

$$I(0) = \frac{V(0)}{50 + j50} = \frac{3.53(1-j)}{50(1+j)}$$

$$P(0) = \frac{1}{2} \mathrm{Re}\left\{ V(0) \cdot I(0)^{*} \right\}$$

$$P(0) = \frac{1}{2} \mathrm{Re}\left\{ 5 e^{-j45^\circ} \cdot 0.0706\, e^{j\pi/2} \right\}$$

$$P(0) = \frac{1}{2}\, 0.353 \cos(45^\circ)$$

$P(0) = 125$ mW

Transmission lines: Solution for Problem 7 c,d

c) $v(d,t) = \mathrm{Re}\left\{\left(3.55\angle -63.45\, e^{j\beta d} + 1.77 e^{-j\beta d}\right) \cdot e^{j\omega t}\right\}$

$$V(d,t) = \mathrm{Re}\left\{3.55\, e^{j\beta d - j63.45 + j\omega t} + 1.77\, e^{-j\beta d + j\omega t}\right\}$$

$$V(d,t) = 3.55\cos(\omega t + \beta d - 63.45)$$
$$1.77\cos(\omega t - \beta d)$$

$$V(d,t) = 3.55\left[\cos(\omega t + \beta d)\cdot\cos(63.45) + \sin(\omega t + \beta d)\sin(63.45)\right] + 1.77\cos(\omega t - \beta d)$$

$$V(d,t) = \cancel{3.55}\ 1.76\cos(\omega t + \beta d) + 3.53\sin(\omega t + \beta d) + 1.77\cos(\omega t - \beta d)$$

$$V(d,t) = 1.76\left(\cos\omega t \cdot \cos\beta d - \cancel{\sin\omega t \sin\beta d}\right) + 3.53\left(\sin\omega t\cos\beta d + \sin\beta d\cos\omega t\right) + 1.77\left(\cos\omega t\cos\beta d + \cancel{\sin\omega t\sin\beta d}\right)$$

$$V(d,t) = 3.54\cos\beta d\cos\omega t + \cancel{3.54} + 3.54\sin\beta d\cos\omega t + 3.54\cos\beta d\sin\omega t$$

7

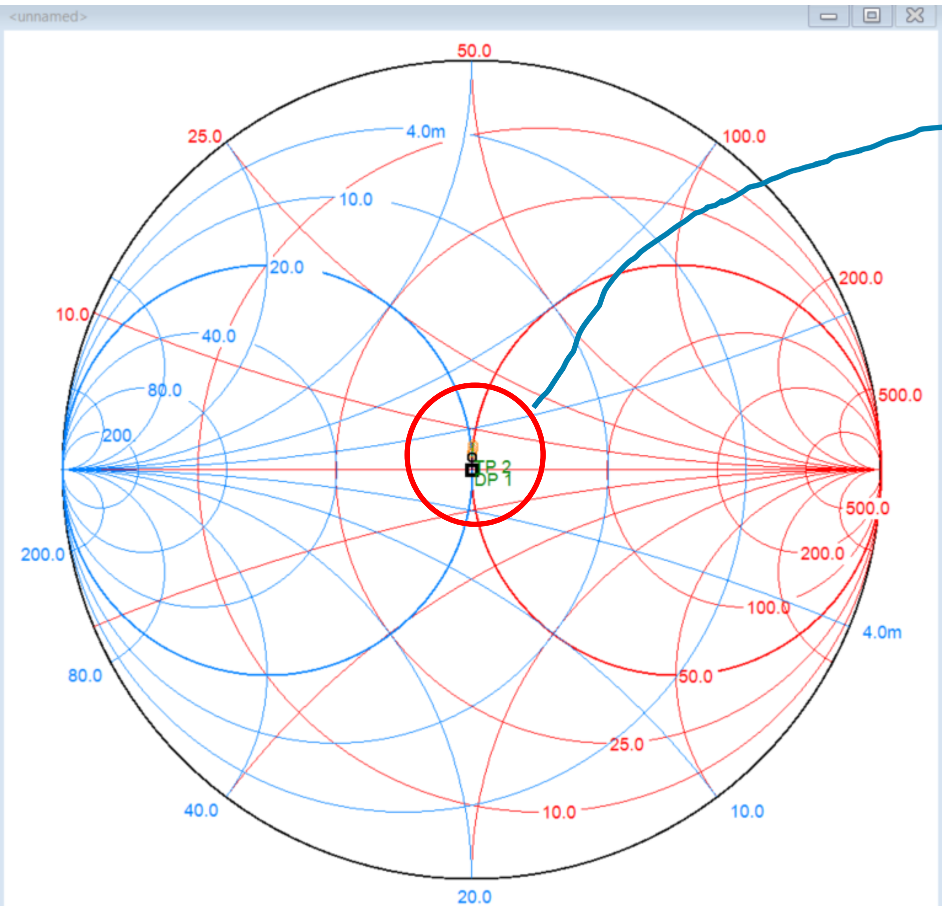
d) $f = 1\ \text{GHz}$

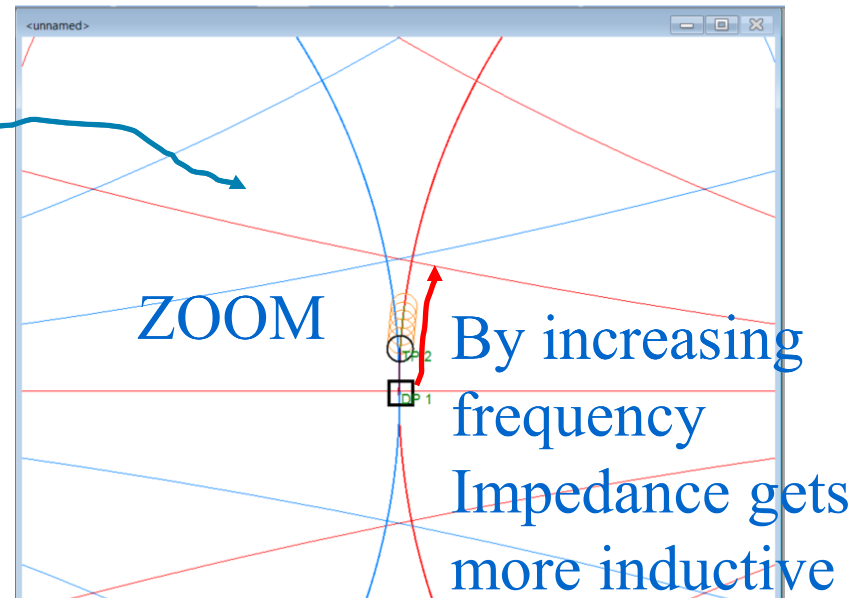
$\lambda = c/f = 0.3$

$\frac{\lambda}{8} = 3.75\ \text{cm}.$

Matching network: Problem 9

For a series R-L network draw the path in the ZY-Smith chart for the frequency rage 500 MHz to 1 GHz for R=50 Ω and L=1 nH.



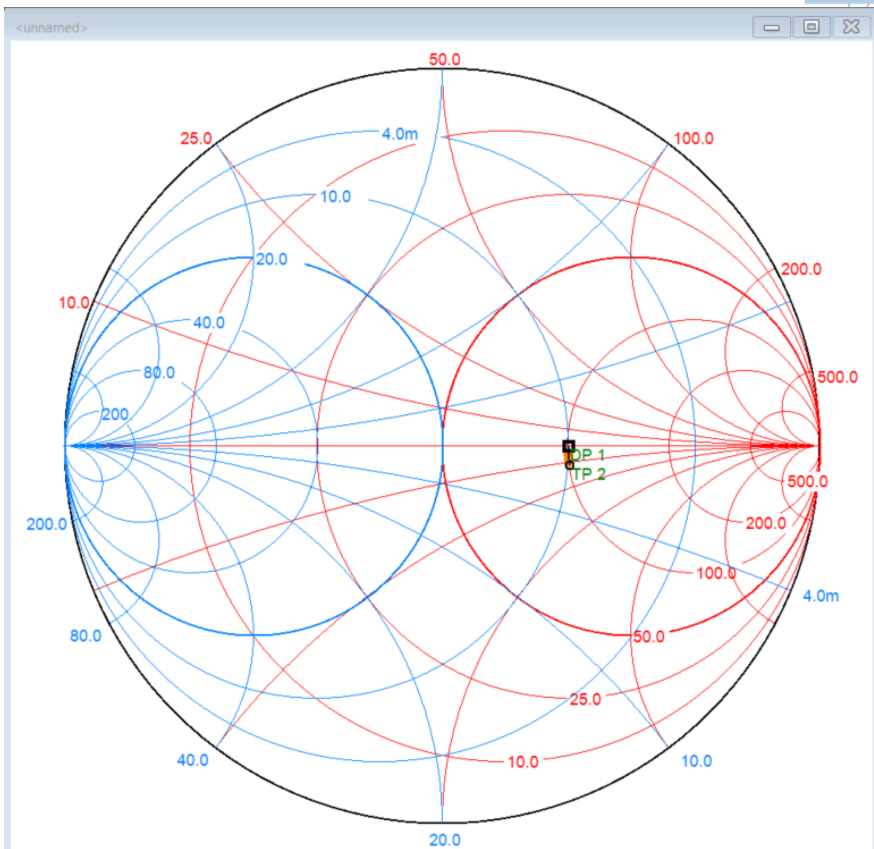


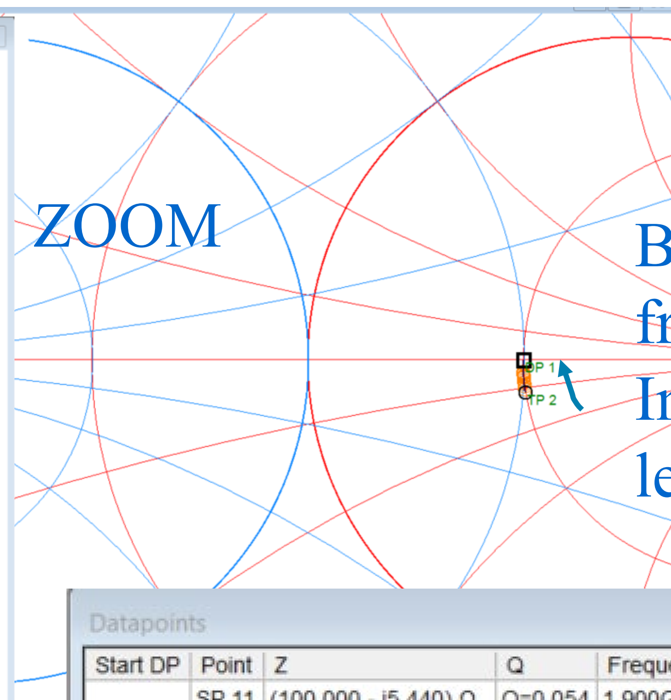
Datapoints

Start DP	Point	Z	Q	Frequency
	TP 2	(50.000 + j3.250) Ω	Q=0.065	500.000MHz
	SP 1	(50.000 + j3.250) Ω	Q=0.065	500.000MHz
	SP 2	(50.000 + j3.899) Ω	Q=0.078	600.000MHz
	SP 3	(50.000 + j4.549) Ω	Q=0.091	700.000MHz
	SP 4	(50.000 + j5.199) Ω	Q=0.104	800.000MHz
	SP 5	(50.000 + j5.849) Ω	Q=0.117	900.000MHz
	SP 6	(50.000 + j6.499) Ω	Q=0.130	1.000GHz

Matching network: Problem 10

For a series R-C network draw the path in the 50 Ω ZY-Smith chart for the frequency rage 900 MHz to 2.5 GHz for R=100 Ω and C=1 5 pF.





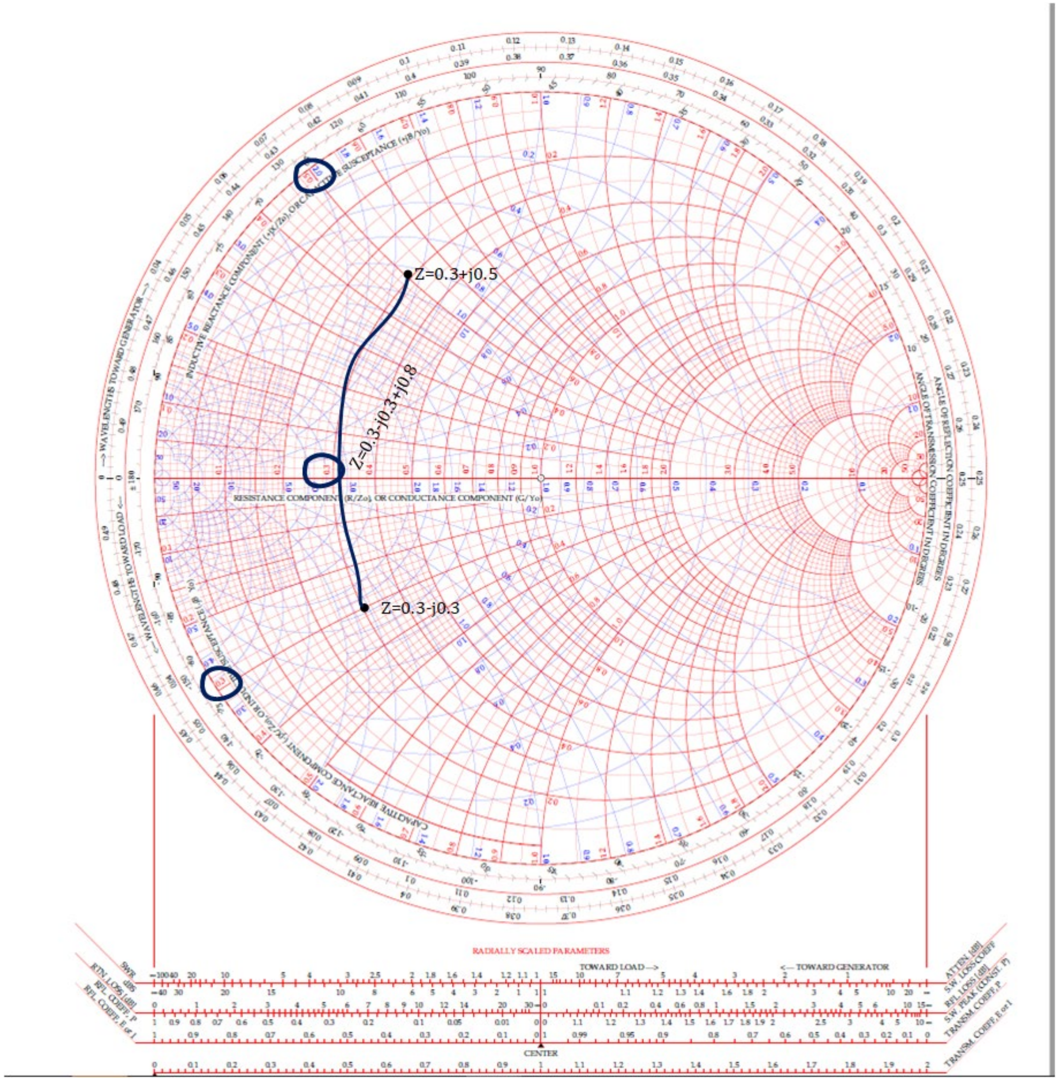
By increasing frequency Impedance gets less capacitive

Datapoints

Start DP	Point	Z	Q	Frequency
	SP 11	(100.000 - j5.440) Ω	Q=0.054	1.900GHz
	SP 12	(100.000 - j5.168) Ω	Q=0.052	2.000GHz
	SP 13	(100.000 - j4.922) Ω	Q=0.049	2.100GHz
	SP 14	(100.000 - j4.698) Ω	Q=0.047	2.200GHz
	SP 15	(100.000 - j4.494) Ω	Q=0.045	2.300GHz
	SP 16	(100.000 - j4.307) Ω	Q=0.043	2.400GHz
	SP 17	(100.000 - j4.135) Ω	Q=0.041	2.500GHz

Matching: Problem 11

In the Smith chart, illustrate the effect of adding a series inductor L (z_L=j 0.8) to an impedance z=0.3 - j0.3.

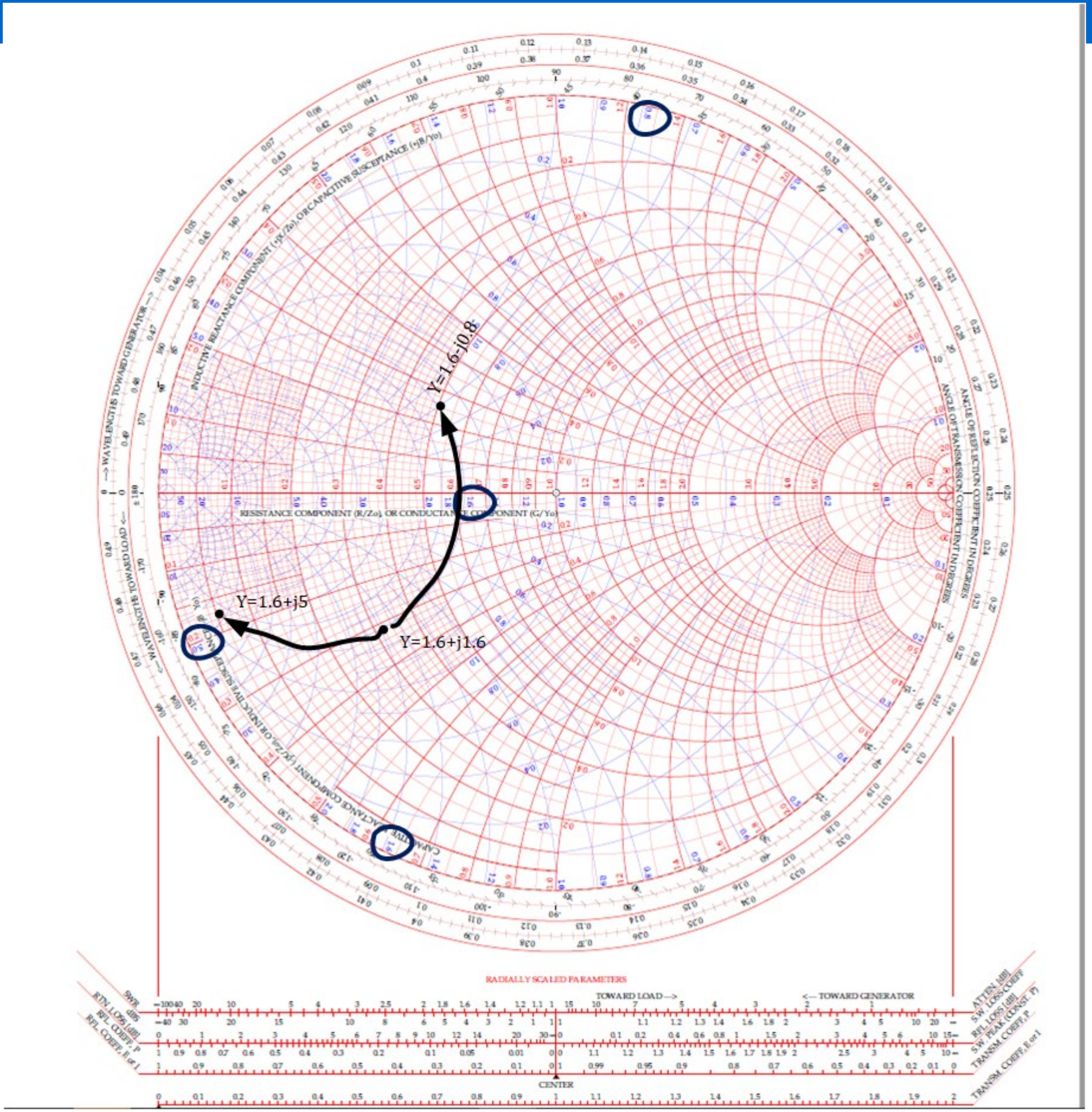


Matching: Problem 12

d) Illustrate the effect of adding a shunt inductor L (y_L=-j2.4) to an admittance y = 1.6 + j1.6 in the ZY Smith chart.

e) Illustrate the effect of adding a shunt capacitor C (y_{LC}= j3.4) to an admittance y = 1.6 + j1.6 in the ZY Smith chart.

Matching: Problem 12 - solution



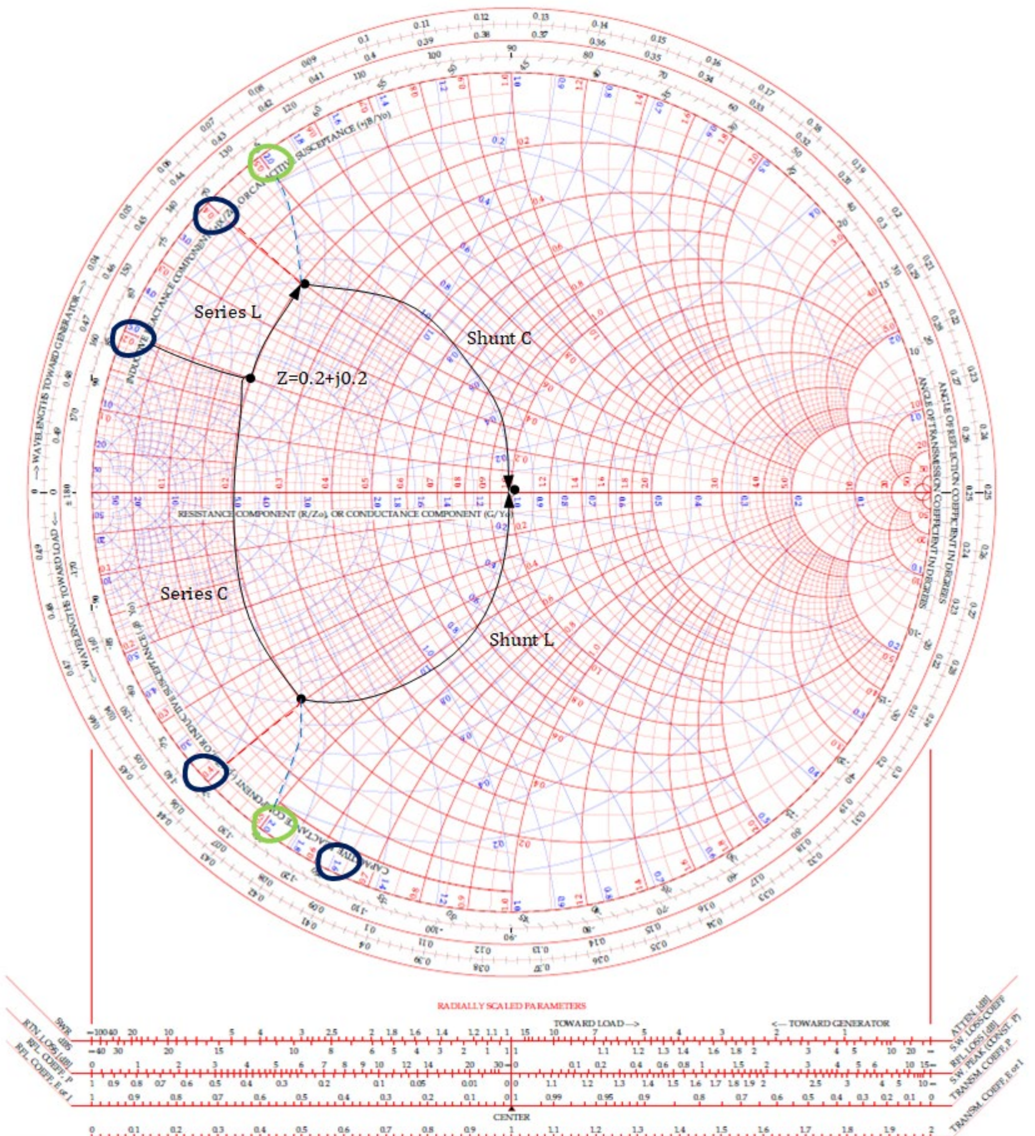
Smith Chart: Problem 13

A load of impedance $Z_{LOAD} = 10 + j10\Omega$ is to be matched to a 50 Ω line for the frequency f=500 MHz.

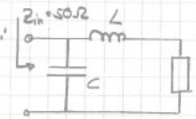
a) Use the Smith chart and design two 2-element L-C matching networks and specify the values of L and C. Illustrate your answer in the Smith chart.

b) Find the values of L and C by an analytical calculation (without the Smith chart).

Matching network: Problem 13 – solution (1/2)



a) option 1:



$Z_L = 10\Omega + j\,10\Omega$

$z_L = 0{,}2 + j\,0{,}2$

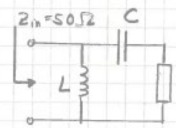
$\omega L = 0{,}4 - 0{,}2 = 0{,}2 \cdot 50\Omega$

$L = \dfrac{10\Omega}{2\pi \cdot 500 \cdot 10^6\,Hz} = 3{,}18\ nH$

$\omega C = 2 \cdot 0{,}02 = 0{,}04$

$C = \dfrac{0{,}04}{2\pi \cdot 500 \cdot 10^6} = 12{,}7\ pF$

option 2:



$Z_L = 10\Omega + j10\Omega$

$z_L = 0{,}2 + j\,0{,}2$

$\dfrac{1}{\omega C} = (0{,}2 - (-0{,}4)) \cdot 50\Omega = 30\Omega$

$\rightarrow C = \dfrac{1}{2\pi \cdot 500 \cdot 10^6\,Hz \cdot 30\Omega} = 10{,}6\ pF$

$\dfrac{1}{\omega L} = 2 \cdot 0{,}02\ S = 0{,}04\ S$

$L = \dfrac{1}{2\pi \cdot 500 \cdot 10^6 \cdot 0{,}04} = 7{,}96\ nH$

Matching network: Problem 13 – solution (2/2)

b) option 1:

$$50\,\Omega = Z_{in} = \left((Z_L + j\omega L)^{-1} + j\omega C \right)^{-1}$$

$$0{,}02 = \frac{1}{Z_L + j\omega L} + j\omega C$$

$$= \frac{1}{R + j\,Im(Z_L) + j\omega L} + j\omega C$$

$$= \frac{R - j(Im Z_L + \omega L)}{R^2 + (Im Z_L + \omega L)^2} + j\omega C$$

condition 1: $$\frac{R}{R^2 + (Im Z_L + \omega L)^2} = 0{,}02 \qquad (1)$$

condition 2: $$\frac{-Im Z_L - \omega L}{R^2 + (Im Z_L + \omega L)^2} + \omega C = 0 \qquad (2)$$

$$(1) \Rightarrow \quad R = 0{,}02\left(R^2 + (Im Z_L + \omega L)^2 \right)$$

$$\pm\sqrt{\frac{R}{0{,}02} - R^2} = (Im Z_L + \omega L)$$

$$\rightarrow \quad \omega L = \pm\sqrt{\frac{R}{0{,}02} - R^2} - Im Z_L$$

$$\omega L = (\pm 20 - 10)\,\Omega$$

$$= 10\,\Omega \quad , \text{as } \omega L > 0$$

$$L = 3{,}18 \cdot 10^{-9}\,H$$

$$= 3{,}18\,nH$$

(2) with $L = 3{,}18\,nH$: $$\omega C = \frac{Im Z_L + \omega L}{R^2 + (Im Z_L + \omega L)^2} = 0{,}04$$

$$C = 12{,}7\,pF$$

Similar procedure for option 2

Matching network: Problem 14

a) Design a matching network for f = 5 GHz composed of an inductor L and a capacitor C to transform a 50 Ω load at the input to an admittance $Y_{OUT}=(8 - j12)\cdot 10^{-3}$ S at the output.

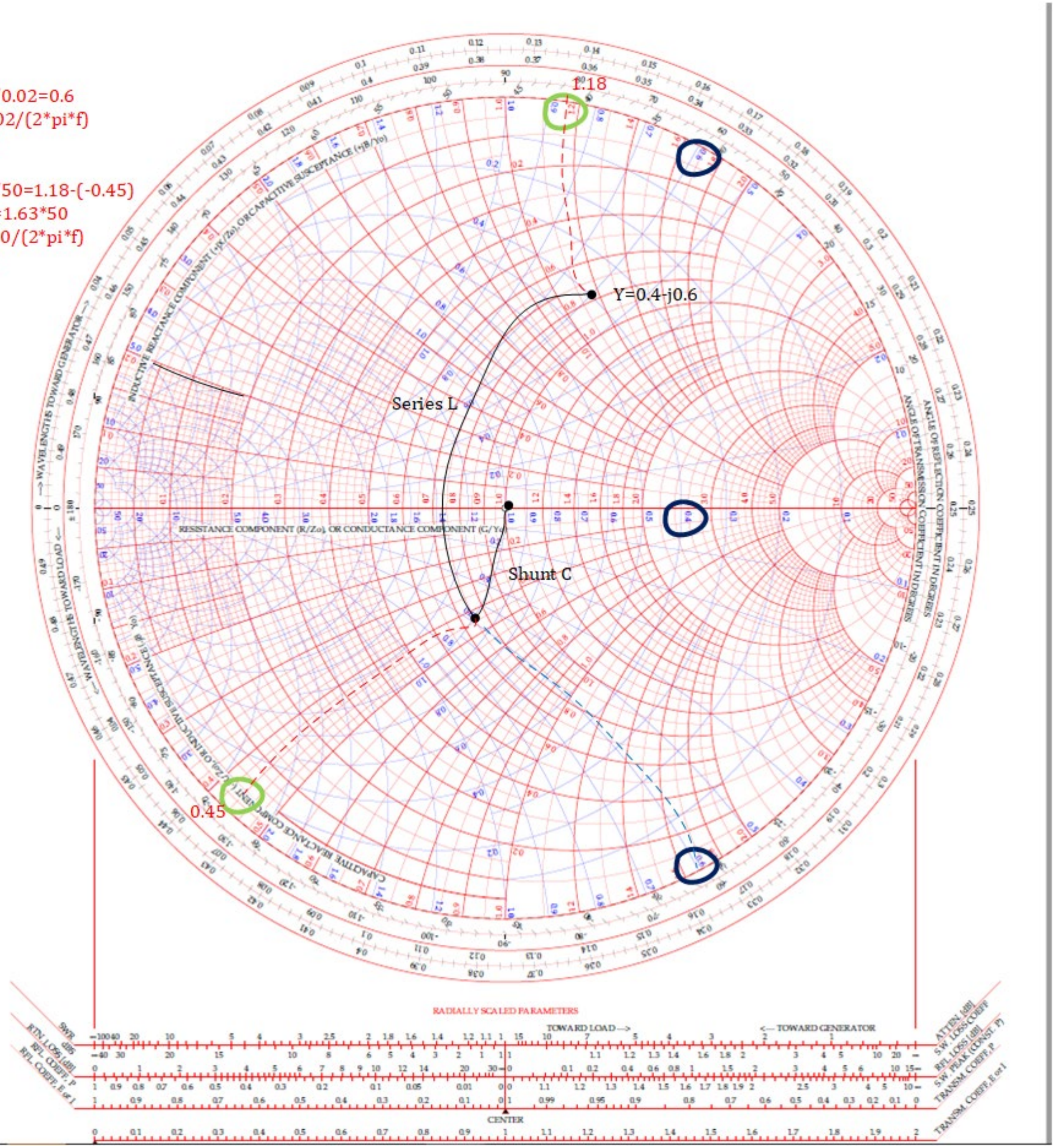
b) Illustrate your solution in the Smith chart.

Matching network: Problem 14 –solution (1/2)

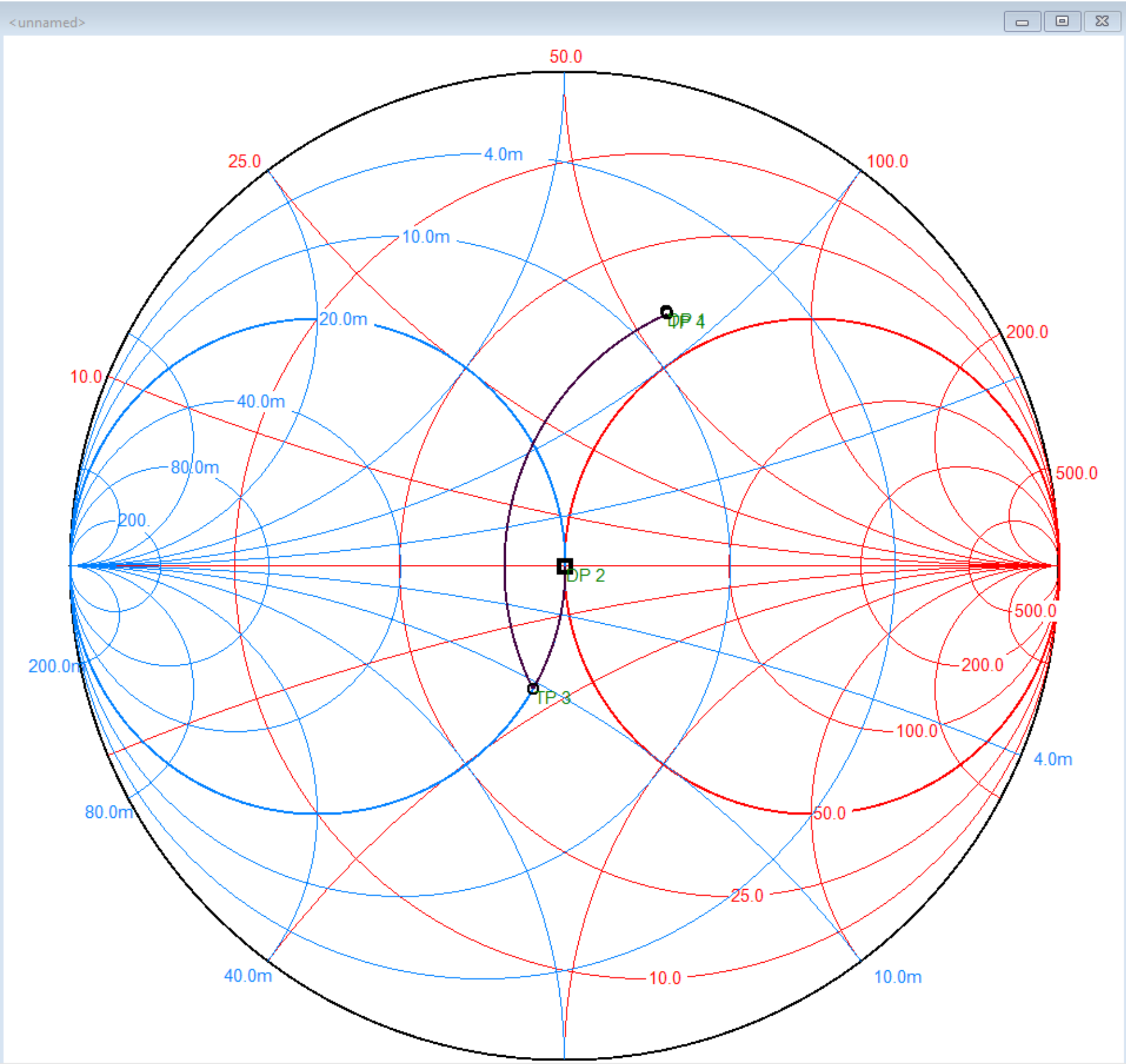
f=5GHz

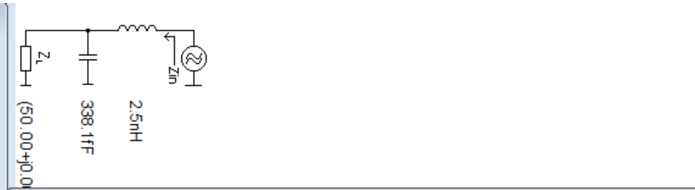
2*pi*f*C/0.02=0.6
C=0.6*0.02/(2*pi*f)
C=382fF

2*pi*f*L/50=1.18-(-0.45)
2*pi*f*L=1.63*50
L=1.63*50/(2*pi*f)
L=2.6nH



Matching network: Problem 14 –solution (2/2)





Datapoints

Start DP	Point	Z	Q	Frequency
☐	DP 1	(38.462 + j57.692) Ω	Q=1.500	5.000GHz
☑	DP 2	(50.000 + j0.000) Ω	Q=0.000	5.000GHz
	TP 3	(38.999 - j20.713) Ω	Q=0.531	5.000GHz
	TP 4	(38.999 + j57.608) Ω	Q=1.477	5.000GHz

Circles

Visible	Highlighted	Details

Cursor

Return Loss	0.16 dB	VSWR	108.95 :1
Q	23.78	Γ	0.982 / 154.118 °
Y	(3.65-j86.89)mS	Z	(0.48+j11.49)Ω
Zo	50.0Ω	Freq	5.000GHz