Components in wireless technology, 5XTC0

Module 5
Lecture: Low-Noise Amplifier design

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Where innovation starts

Outline

- Recap
  - S parameters
  - Power gain definitions
  - Transducer gain
  - Gain circles
  - Stability circles
- Noise
- Characterization of noise in amplifiers
- Noise in multi-stage amplifiers
- Noise circles

Learning Objectives

- Recap
  - Understand S-parameters definition
  - Understand the different gains of an amplifier
  - Understand gain circles andstability circles
- Understand thermal noise
- Understand how to characterize noise in amplifiers
- Noise in multi-stage amplifiers
- Understand noise circles

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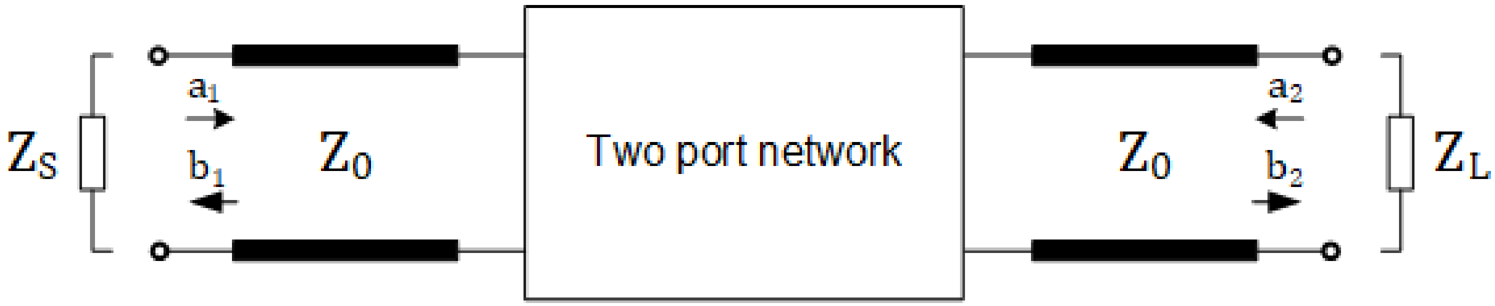
2-port network description: S parameters



$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$ (input reflection coefficient with output properly terminated)

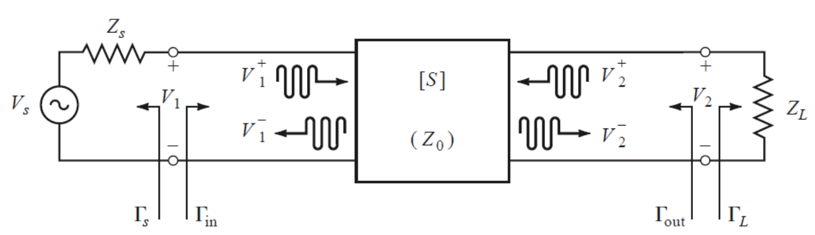
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$ (forward transmission coefficient with output properly terminated)

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$ (output reflection coefficient with input properly terminated)

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$ (reverse transmission coefficient with input properly terminated)

- To measure S parameters matched terminations are required: $Z_L=Z_0$ and $Z_S=Z_0$
- At high frequencies matched terminations could be realized much easier compared to short and open terminations

Gain definitions



Power gain G: Ratio of the power dissipated in the load Z_L to the power delivered to the input of the two-port network

Available power gain G_A: Ratio of the power available from the two-port network to the power available from the source. Assumes conjugate matching of source and load impedance.

Transducer power gain G_T: Ratio of the power delivered to the load to the power available from the source. Assumes a matched source impedance.

Unilateral transducer power gain G_{TU}:
Transducer power gain for a device with $S_{12}=0$

More info: book of Gonzalez, page 92

Amplifier gains: Equations

Power gain:

$$G = \frac{P_L}{P_{\text{in}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{\text{in}}|^2\right) |1 - S_{22}\Gamma_L|^2}$$

Available power gain:

$$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{\text{out}}|^2\right)}$$

Transducer power gain:

$$G_T = \frac{P_L}{P_{\text{avs}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_S\Gamma_{\text{in}}|^2 \, |1 - S_{22}\Gamma_L|^2}$$

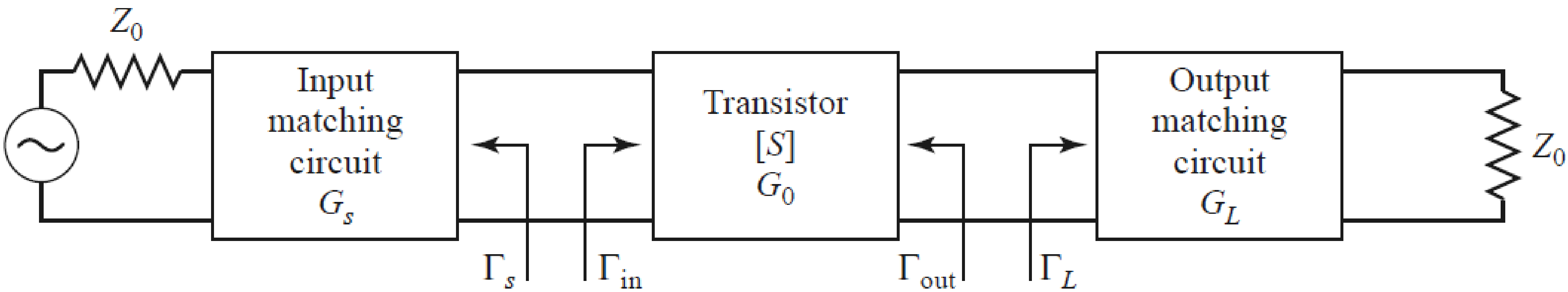
Unilateral transducer power gain:

$$G_{TU} = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{\text{in}}\Gamma_S|^2} \, |S_{21}|^2 \, \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$$

More info: book of Gonzalez, page 92

General transistor amplifier circuit



$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

Unilaterial case

$$G_T = G_S G_0 G_L$$

$$G_{T,dB} = G_{S,dB} + G_{0,dB} + G_{L,dB}$$

Remark: If S_{12}=0 then: $\Gamma_{out} = S_{22}$ and $\Gamma_{in} = S_{11}$

Circles of constant power gain

Unilateral transducer power gain G_{TU} :

$$G_{TU} = \left.\frac{P_L}{P_{AVS}}\right|_{\underline{S}_{12}=0}$$

$$= \underbrace{\frac{1-\left|\underline{\Gamma}_S\right|^2}{\left|1-\underline{\Gamma}_S\underline{S}_{11}\right|^2}}_{G_S}\left|\underline{S}_{21}\right|^2\underbrace{\frac{1-\left|\underline{\Gamma}_L\right|^2}{\left|1-\underline{\Gamma}_L\underline{S}_{22}\right|^2}}_{G_L}$$

$$= G_S \cdot G_0 \cdot G_L$$

G_S: Impact of the input matching network on the gain

G_0: Transistor gain

G_L: Impact of the output matching network on the gain

For which values of Γ_S do we achieve the desired value of G_S?

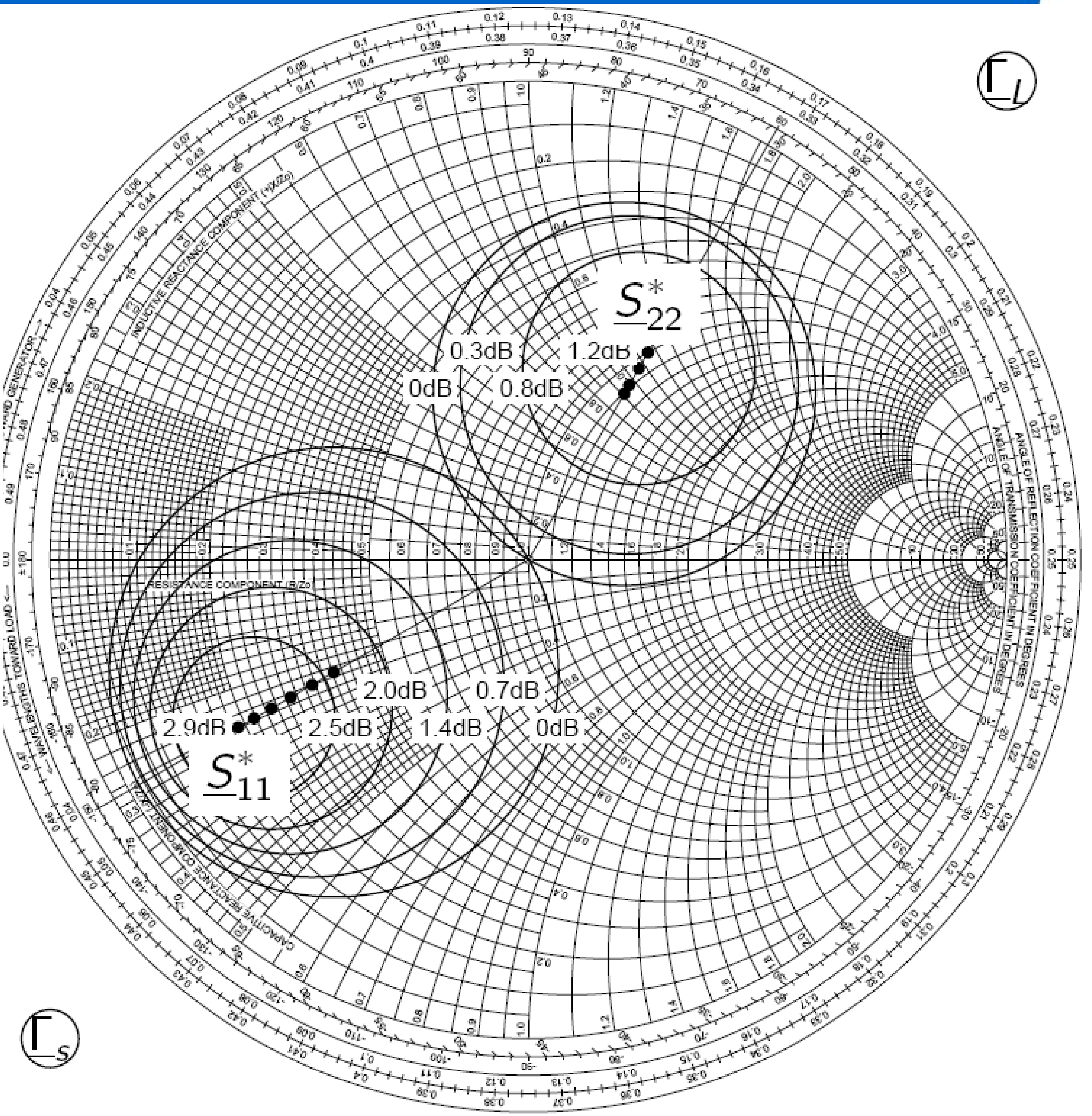
For which values of Γ_L do we achieve the desired value of G_L?

The values of Γ_S that lead to a constant G_S are situated on circles in the complex Γ plane.

The values of Γ_L that lead to a constant G_L are situated on circles in the complex Γ plane.

These circles are called: **Constant gain circles**

For $\Gamma_S = S^*_{11}$ maximum G_S is obtained.
For $\Gamma_L = S^*_{22}$ maximum G_L is obtained.



Circles of constant power gain

Maximum gain of the input and output matching networks

$$G_{S_{\max}} = \frac{1}{1-|S_{11}|^2}, \quad \text{for } \Gamma_S = S^*_{11}$$

$$G_{L_{\max}} = \frac{1}{1-|S_{22}|^2}. \quad \text{for } \Gamma_L = S^*_{22}$$

Normalized gain factors g_s and g_L

$$g_S = \frac{G_S}{G_{S_{\max}}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}(1-|S_{11}|^2),$$

$$g_L = \frac{G_L}{G_{L_{\max}}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}(1-|S_{22}|^2).$$

Center and radius of the constant gain circle for the input and output matching network

$$C_S = \frac{g_S S^*_{11}}{1-(1-g_S)|S_{11}|^2},$$

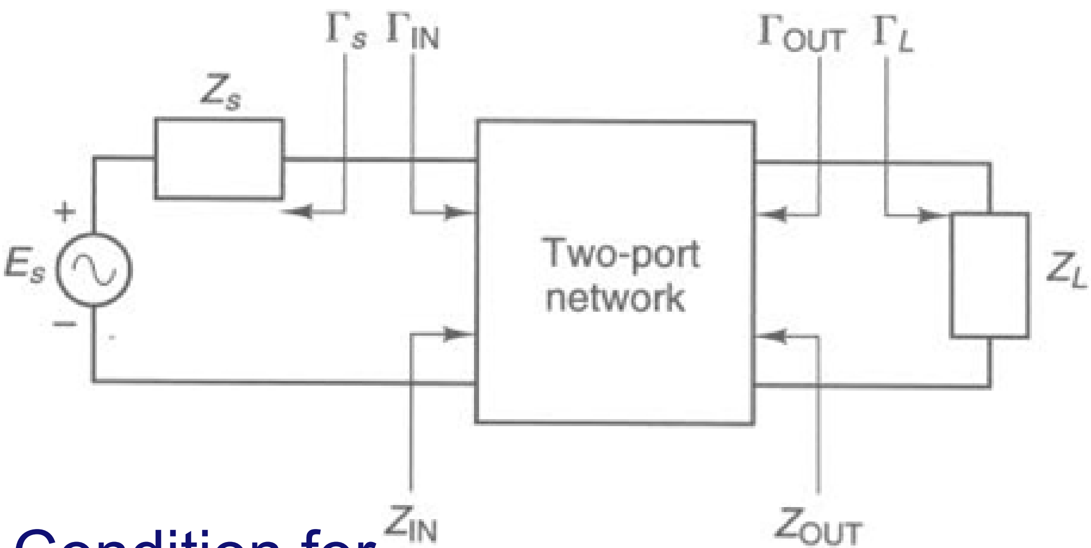
$$R_S = \frac{\sqrt{1-g_S}\,(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}$$

$$C_L = \frac{g_L S^*_{22}}{1-(1-g_L)|S_{22}|^2},$$

$$R_L = \frac{\sqrt{1-g_L}\,(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$

More info: book of Pozar, page 624, book of Gonzalez, page 103

Stability discussion of 2-port circuits



Stability analysis
of an amplfier means:
Investigation whether
there can be oscillations

Condition for
"unconditionally stable" device:

$$\text{for all} \left|\underline{\Gamma}_L\right| < 1 \ \text{ and } \left|\underline{\Gamma}_S\right| < 1$$

$$\Rightarrow \begin{cases} \left|\Gamma_{in}\right| = \left|S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| < 1 \\ \left|\Gamma_{out}\right| = \left|S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}\right| < 1 \end{cases}$$

If at a given frequency
there are source and load
reflection coefficients, for which
this condition does not hold
the device is called
"potentially unstable".

Input stability circles

Boundary between stability and instability is given by:

$$|\Gamma_{OUT}| = 1$$

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| = 1$$

$$\left| \Gamma_s - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

Circle equation
in the complex Γ-plane

$$r_s = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}$$

$$C_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \qquad \text{(center)}$$

This circle is called the **output stability circle**. It is the boundary between the region Γ_S that lead to a stable or an unstable reflection amplifier.

An equivalent derivation of the output reflection coefficient
leads to the **input stability circle**.

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Output stability circles

Boundary between stability and instability is given by:

$$|\underline{\Gamma}_{in}| = 1$$

$$\Leftrightarrow \left| \underline{S}_{11} + \frac{\underline{S}_{12}\underline{S}_{21}\underline{\Gamma}_L}{1-\underline{S}_{22}\underline{\Gamma}_L} \right| = 1$$

$$\left| \underline{\Gamma}_L - \frac{\underline{S}_{22}^* - \underline{\Delta}^* \underline{S}_{11}}{|\underline{S}_{22}|^2 - |\underline{\Delta}|^2} \right|^2 = \left| \frac{\underline{S}_{12}\underline{S}_{21}}{|\underline{S}_{22}|^2 - |\underline{\Delta}|^2} \right|^2$$

Circle equation in the complex Γ-plane

$$|\underline{\Gamma}_L - \underline{C}_L|^2 = |R_L|^2$$

This circle is called the **output stability circle**. It is the boundary between the region Γ_L that lead to a stable or an unstable reflection amplifier.

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \qquad \text{(center)},$$

$$R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}.$$

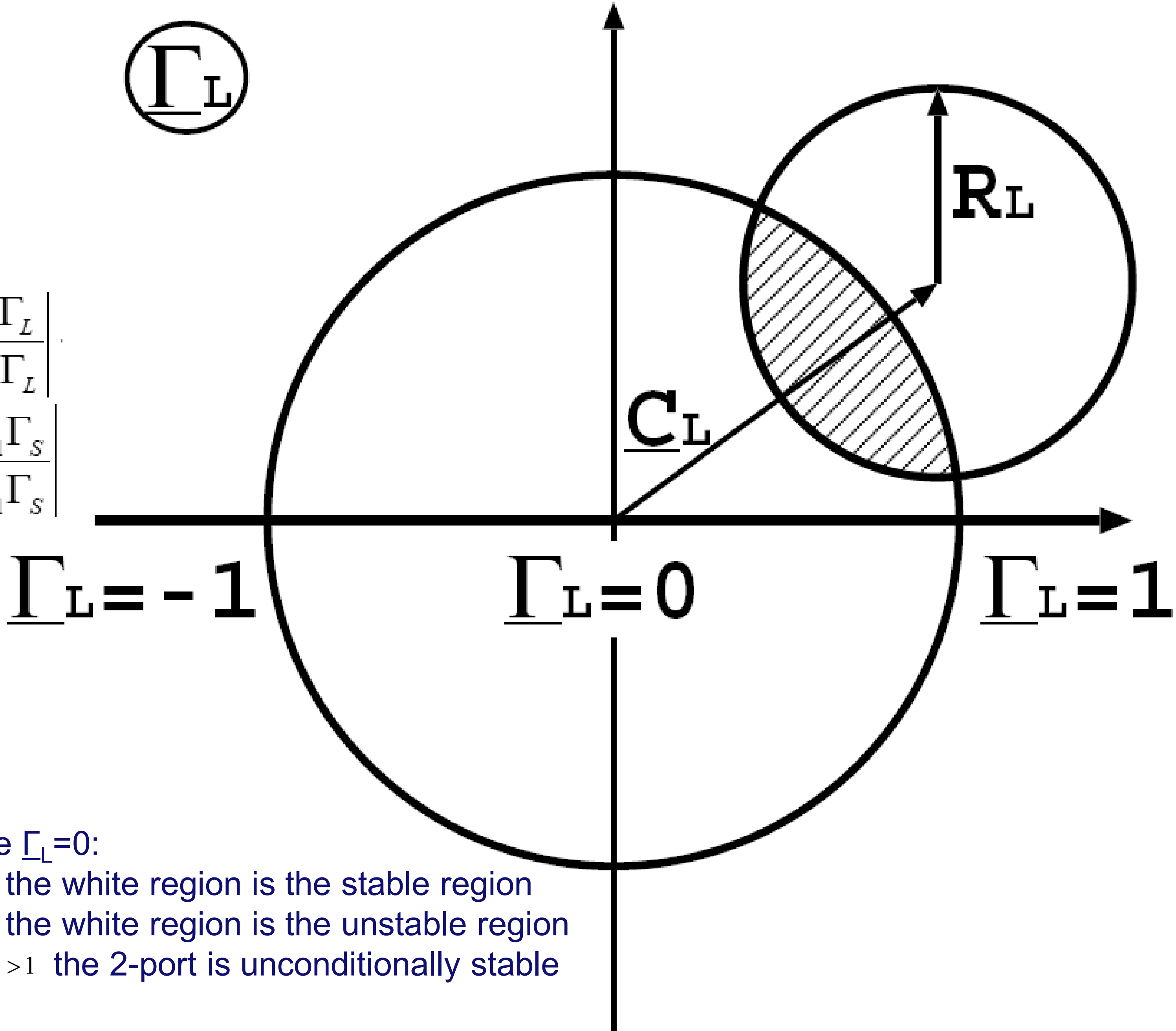
$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

An equivalent derivation of the output reflection coefficient leads to the **input stability circle**.

Construction Of the Output Stability circle

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right|$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right|$$



1) Consider the case Γ_L=0:

if $|\Gamma_{in}|=|S_{11}|<1$, then the white region is the stable region

if $|\Gamma_{in}|=|S_{11}|>1$, then the white region is the unstable region

2) If $|S_{11}|<1$ and $\left||C_L|-R_L\right|>1$ the 2-port is unconditionally stable

Tests for unconditional stability

If $$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$$

Rollet stability factor K

and $$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + \Delta^2}{2|S_{12}S_{21}|} > 1$$

than the 2-port is unconditionally stable.

Unilateral case: $S_{12}=0$

Conditions for unconditional stability:

$$|\underline{S}_{11}| < 1$$

$$|\underline{S}_{22}| < 1$$

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Why do we need to analyze noise?

- Link budget allows to calculate received signal power S across a wireless link
- To transmit information across a wireless link, the received signal power must be significantly larger than the noise power N.
- The ratio between the signal power and the noise power is called "Signal-to-noise ratio" SNR:

$$SNR = \frac{S}{N}$$

- If we cannot distinguish the signal from the noise we cannot extract the information!

Origin of the noise power

All electronic devices:

1. **Thermal noise:** Thermal agitation of electrons
2. **Shot noise:** Fluctuation of current due to number of discrete charges

Semiconductor devices have additionally:

3. **Flicker noise:** 1/f noise caused by impurities in the channel region, recombination and generation of charges, …
4. **Burst noise:** Charge trapping at semiconductor interfaces e.g. FET channel bias that is randomly changed
5. **Avalanche noise:** Free carrier generation in strong electric fields due to carrier acceleration (also this is a statistic process)

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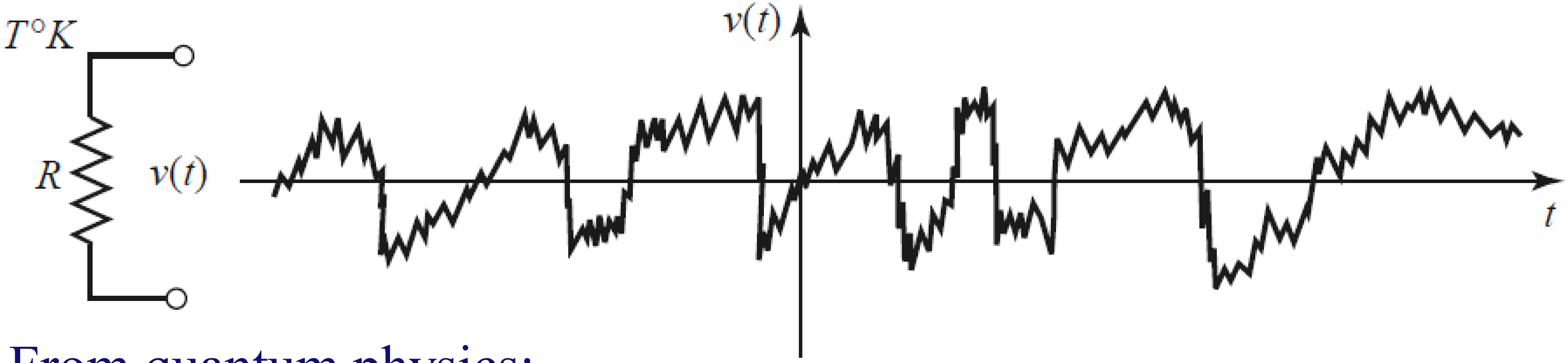
Thermal noise

- Electrons at a temperature T have thermal energy
- They move randomly inside the material and generate random voltage drops
- Example: Resistor at temperature T:



From quantum physics:

$$P_n = k_B \cdot T \cdot B$$

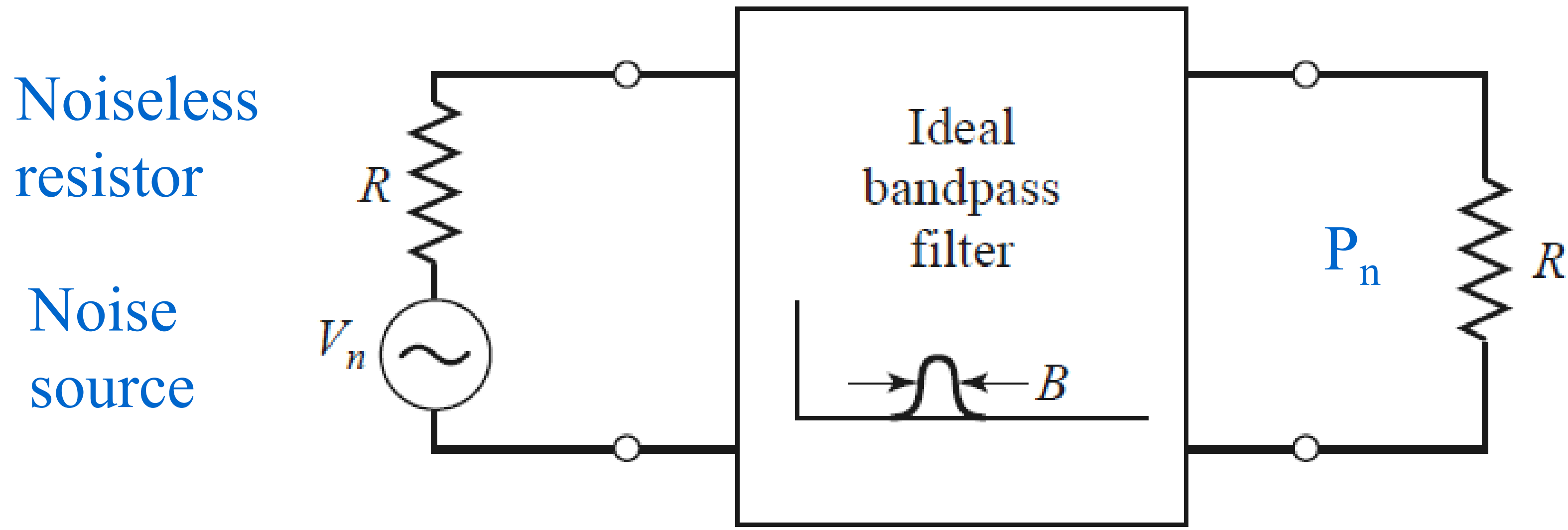
k_B : Boltzmann constant

T : temperature in Kelvin

B : Bandwidth of the system

Average voltage is zero.
But: Non-zero average power!

White noise: Representation of a resistor as a noiseless resistor and a noise voltage source



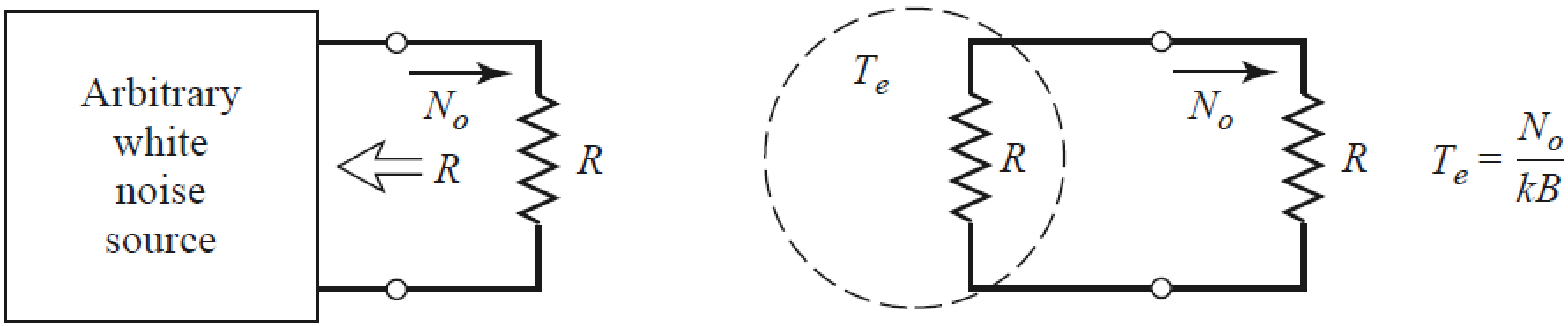
Available noise power:

$$\boxed{P_n = \frac{V_N^2}{4R_N} = k_B T B}$$

What is the noise power at room temperature (300 K) for a bandwidth of 1Hz? Calculate it in W as well as in dBm.

$$k_B = 1{,}38 \cdot 10^{-23} \frac{kgm^2}{s^2 K} \qquad T(K) = T\left({}^oC\right) + 273$$

Noise of an arbitrary source



Measure the noise power N_0 and input resistance R.

Define the equivalent noise temperature as: $$T_e = \frac{N_0}{k_B B}$$

For a noise analysis: use a resistor of temperature T_e.
This resistor produces the same noise power as the original source.

Noise analysis is always aiming at noise power.

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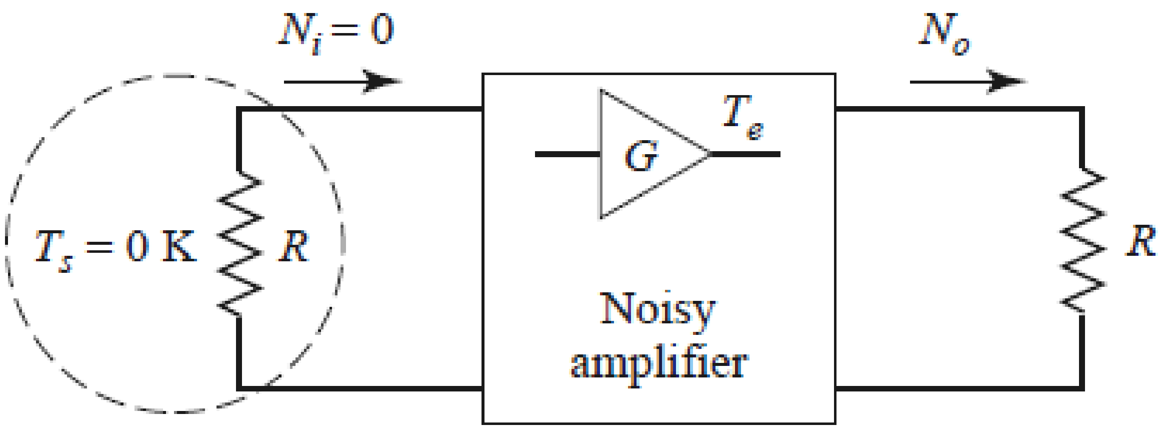
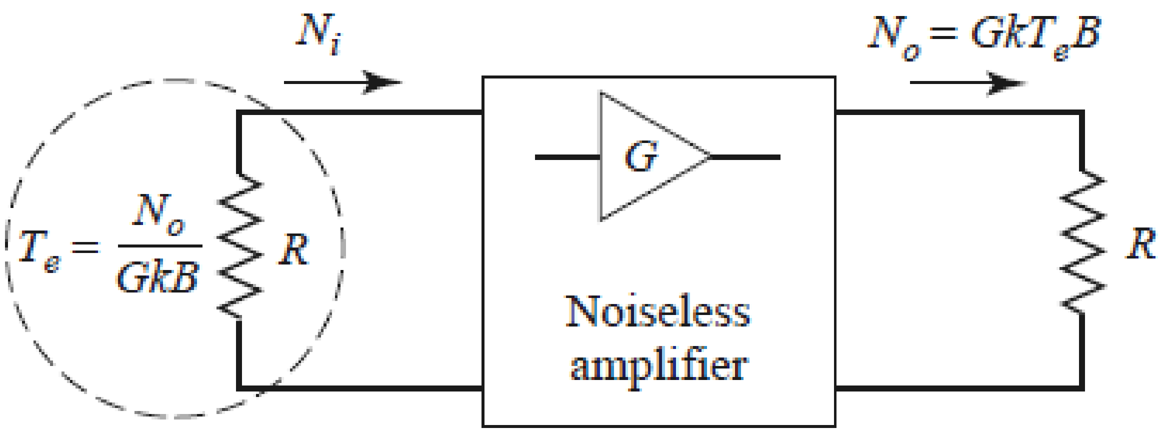
Noise temperature of an amplifier



(a)

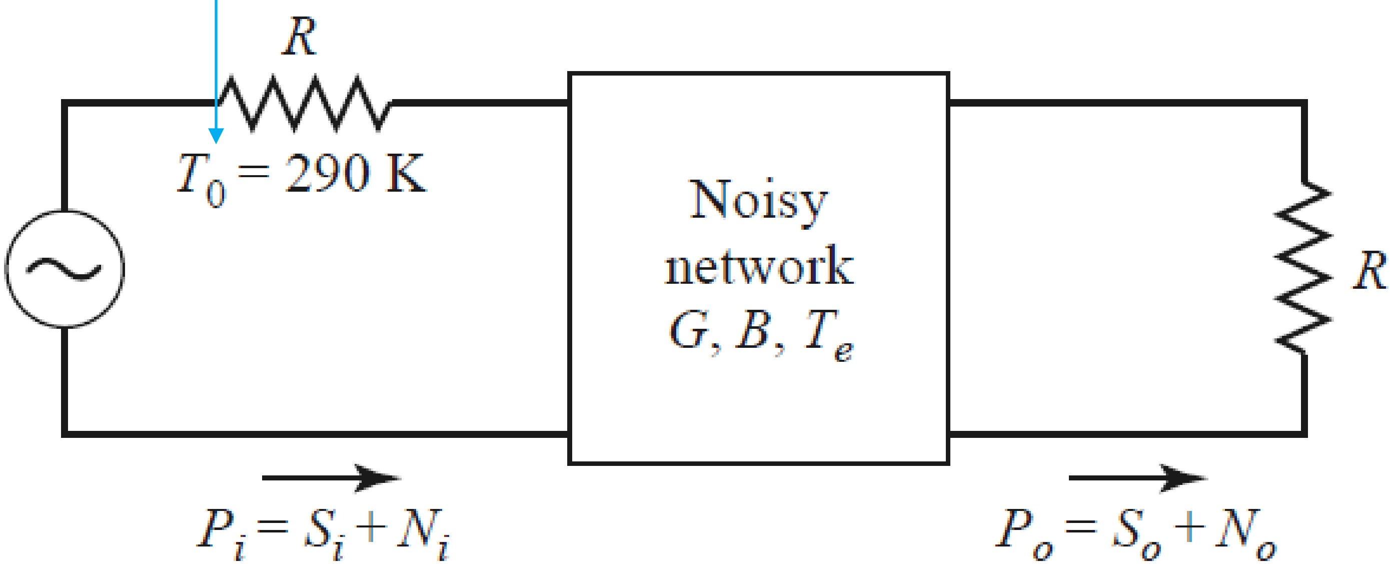


(b)

Remark:
The equivalent noise source is at the <u>input</u> of the amplifier

Definition: Noise figure

For room temperature input noise level!



$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{S_i}{S_o}\frac{N_o}{N_i} = \frac{1}{G}\frac{G k_B \left(T_0 + T_e\right) B}{k_B T_0 B} = 1 + \frac{T_e}{T_0} > 1$$

NF = *10 Log (F)*
- NF -> noise figure (dB)
- F -> noise factor

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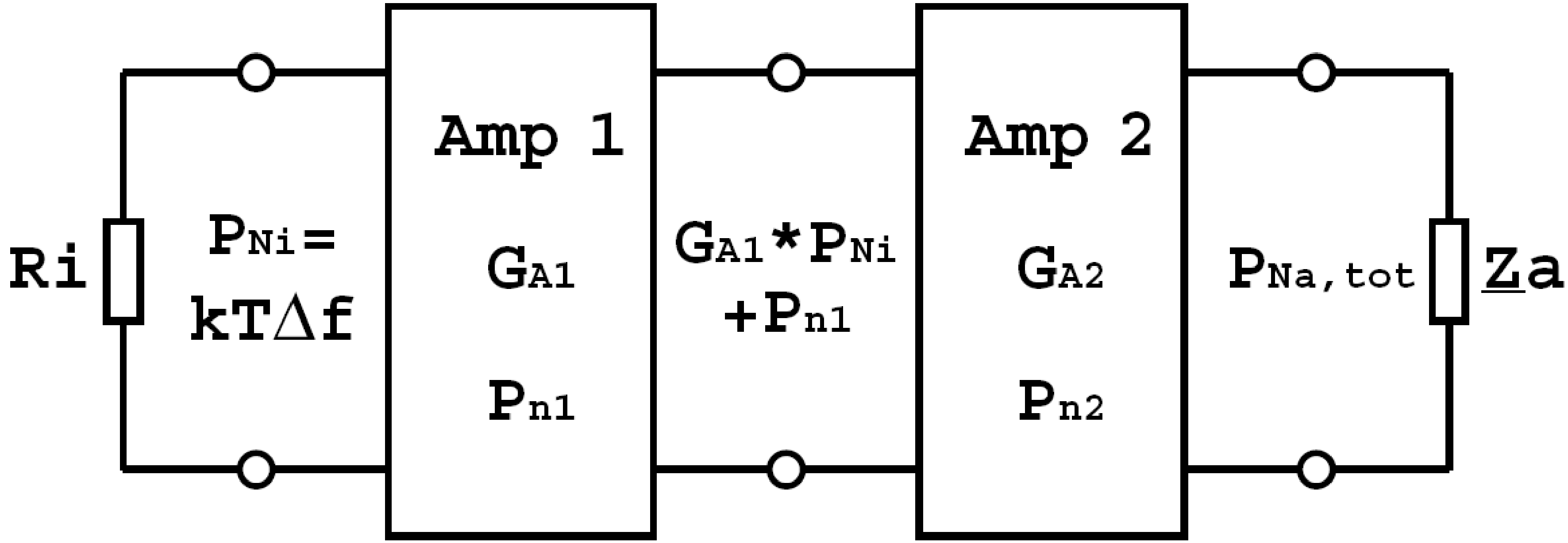
Determination of the noise figure of a two-stage amplifier



$$P_{N,total} = G_{A2}(G_{A1}P_{N,in} + P_{n1}) + P_{n2}$$

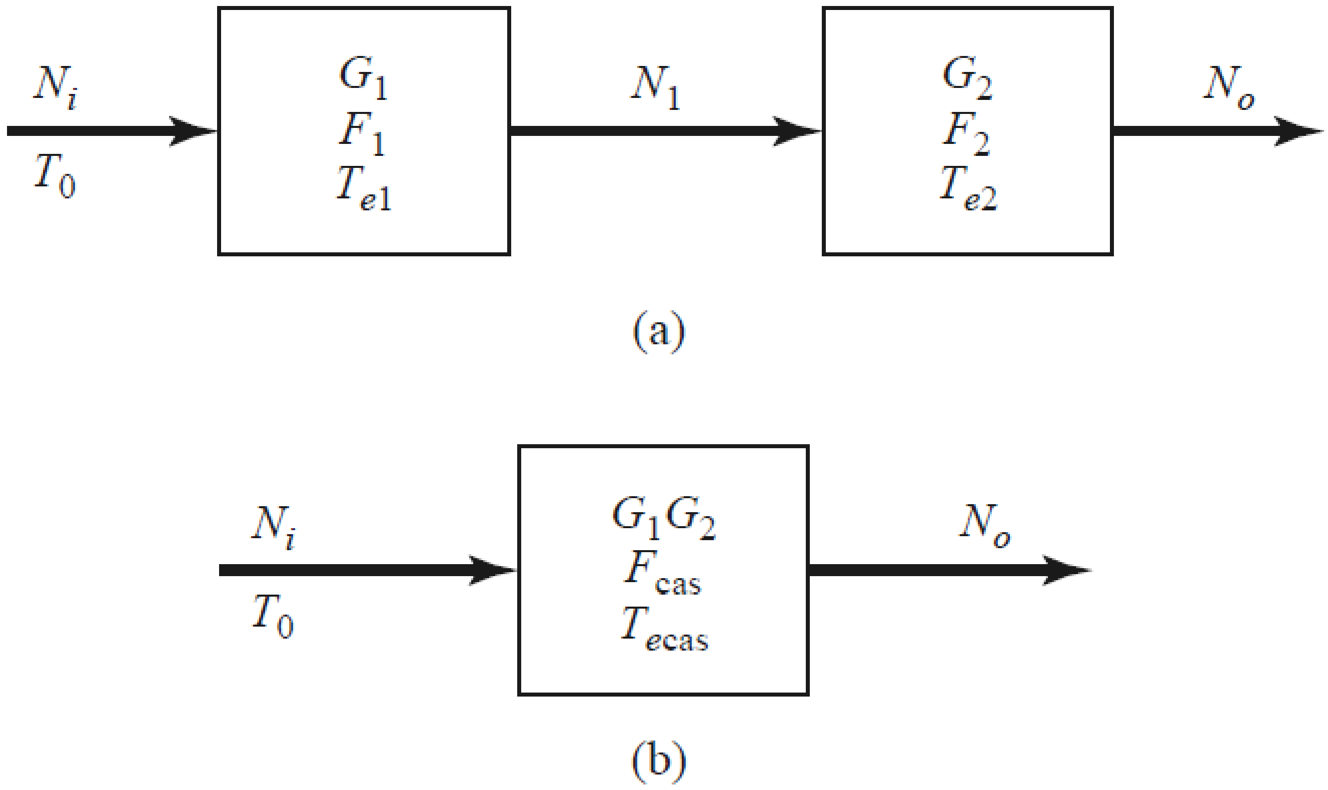
$$\Rightarrow F_{total} = \frac{P_{N,total}}{P_{N,in}G_{A1}G_{A2}} = 1 + \frac{P_{n1}}{P_{N,in}G_{A1}} + \frac{P_{n2}}{P_{N,in}G_{A1}G_{A2}}$$

with $F_j = 1 + \frac{P_{nj}}{P_{N,in}G_{Aj}}, \quad j = 1,2$

$$F_{total} = F_1 + \frac{F_2 - 1}{G_{A1}}$$

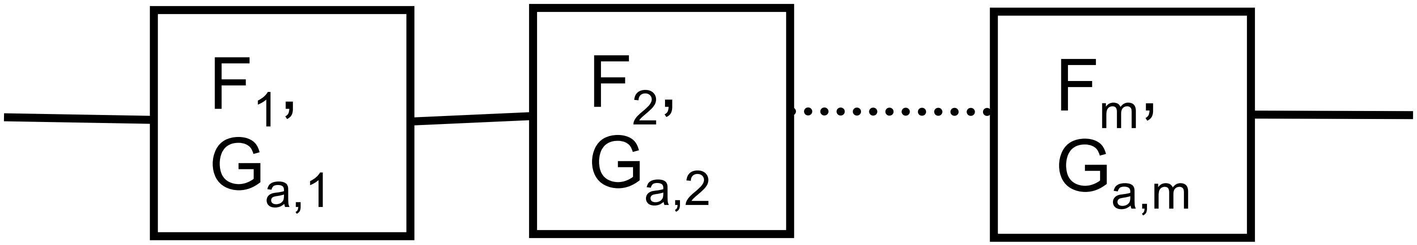
F_1 has the strongest impact on the overall noise figure!

Gain, F and T_e of a cascaded system



$$G_{cas} = G_1 G_2 \qquad T_{cas} = T_{e1} + \frac{1}{G_1} T_{e2} \qquad F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

Cascaded NF: Friis' formula



System with cascaded sub-systems with noise figure F_m and available gain $G_{a,m}$

$$F_{total} = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_{a,1}} + \ldots + \frac{F_m - 1}{G_{a,1} G_{a,2} \ldots G_{a,(m-1)}}$$

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Noise figure of an amplifier (1)

- Noise figure of a 2-port amplifier:

$$F = F_{\min} + \frac{r_N}{g_S}\left|\underline{y}_S - \underline{y}_{opt}\right|^2$$

Normalized equivalent noise resistor: $r_n = \frac{R_n}{Z_0}$

Source admittance: $\underline{Y}_S = g_S + jb_S$

Minimum noise figure for the chosen bias point: $F_{\min} = \min(F)$

- Expression with the reflection coefficients Γ_S and Γ_{opt}

Scaling factor "sensitivity to offest"

$$F = F_{\min} + 4r_N \underbrace{\frac{\left|\underline{\Gamma}_S - \underline{\Gamma}_{opt}\right|^2}{\left(1-\left|\underline{\Gamma}_S\right|^2\right)\cdot\left|1+\underline{\Gamma}_{opt}\right|^2}}_{\text{Offset to optimum value}}$$

Noise figure of an amplifier (2)

$$F = F_{\min} + 4r_N \frac{\left|\underline{\Gamma}_S - \underline{\Gamma}_{opt}\right|^2}{\left(1-\left|\underline{\Gamma}_S\right|^2\right)\cdot\left|1+\underline{\Gamma}_{opt}\right|^2}$$

Which values of $\underline{\Gamma}_S$ lead to a constant value of F?

The values are on circles and we can calculate the center and the radius. The circles are called constant noise circles.

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Constant noise circles

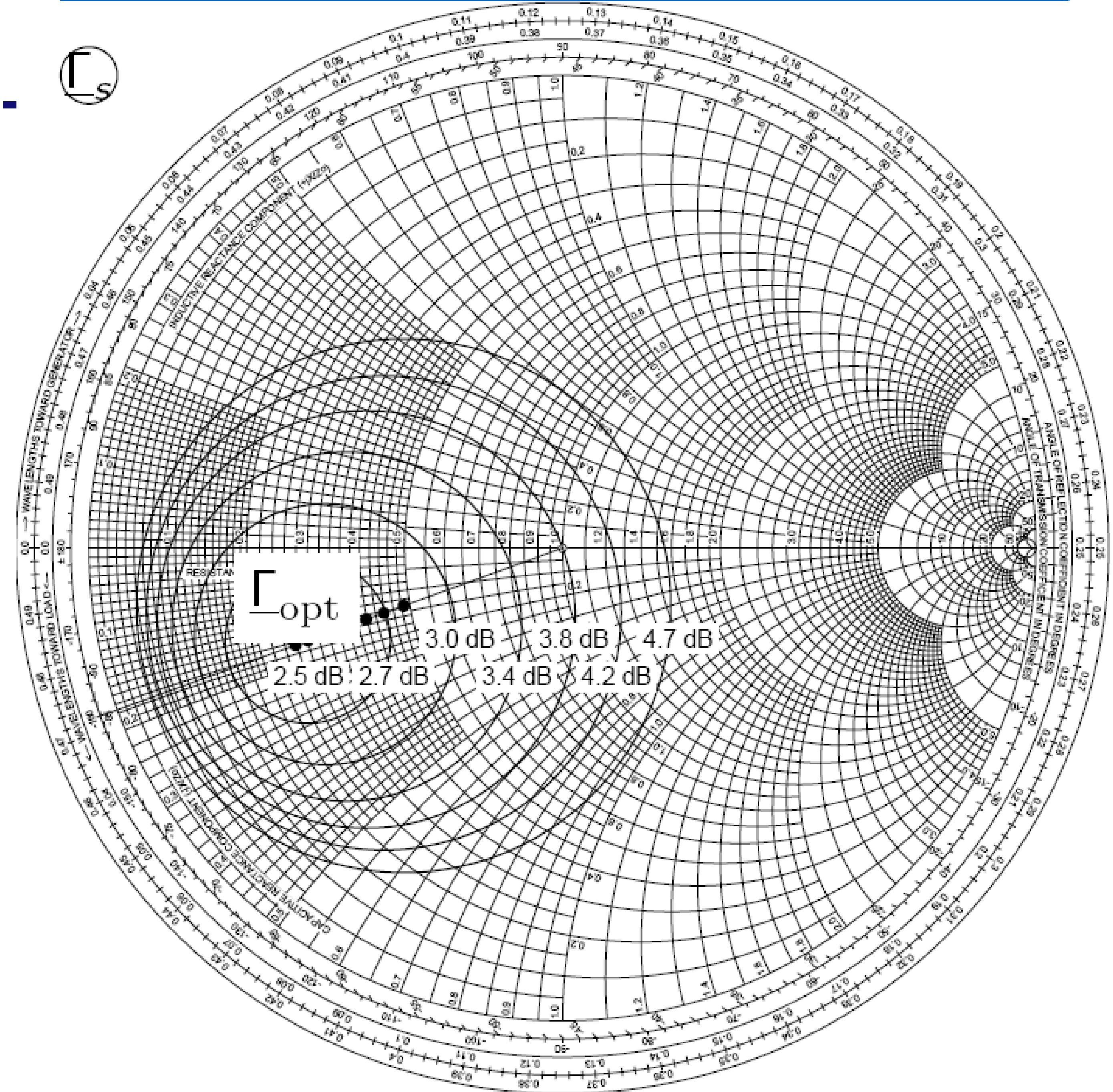
Centers: $\underline{C}_F = \dfrac{\Gamma_{opt}}{1+N}$

Radii: $R_F = \dfrac{1}{1+N}\sqrt{N^2 + N(1-\left|\Gamma_{opt}\right|^2)}$

With the “Noise figure parameter N” defined as:

$$\Delta F_n' = N = \left(F - F_{\min}\right)\frac{\left|1+\underline{\Gamma}_{opt}\right|^2}{4r_n} = \frac{\left|\underline{\Gamma}_S - \underline{\Gamma}_{opt}\right|^2}{1-\left|\underline{\Gamma}_S\right|^2}$$

Constant noise-circles in the source-reflection-coefficient Smith Chart



Design for specific noise figure

Typically the values of Γ_{opt}, r_n and F_{min} are known for the transistor.

The amplifier specification requires a noise figure F and a gain G.

Procedure:

1. Calculate N
2. Calculate C_F and R_F
3. Draw the constant noise circle for the required F in the Smith chart as well as the input section constant gain circle for several G_S
4. Choose a value for Γ_S that is on the desired noise circle and a certain gain circle
5. The remaining gain must come from the transistor and the output matching stage

Study guide amplifier design

- Study the slides

- Pozar: Read Paragraphs 12.1, 12.2, 12.3

- Be able to calculate the exercises from the book of Pozar: Example 12.1, 12.2, 12.3, 12.4 and 12.5

- Extra training:
  Book: G. Gonzalez, Microwave transistor amplifiers
  Various exercises in Chapter 2, 3, 4