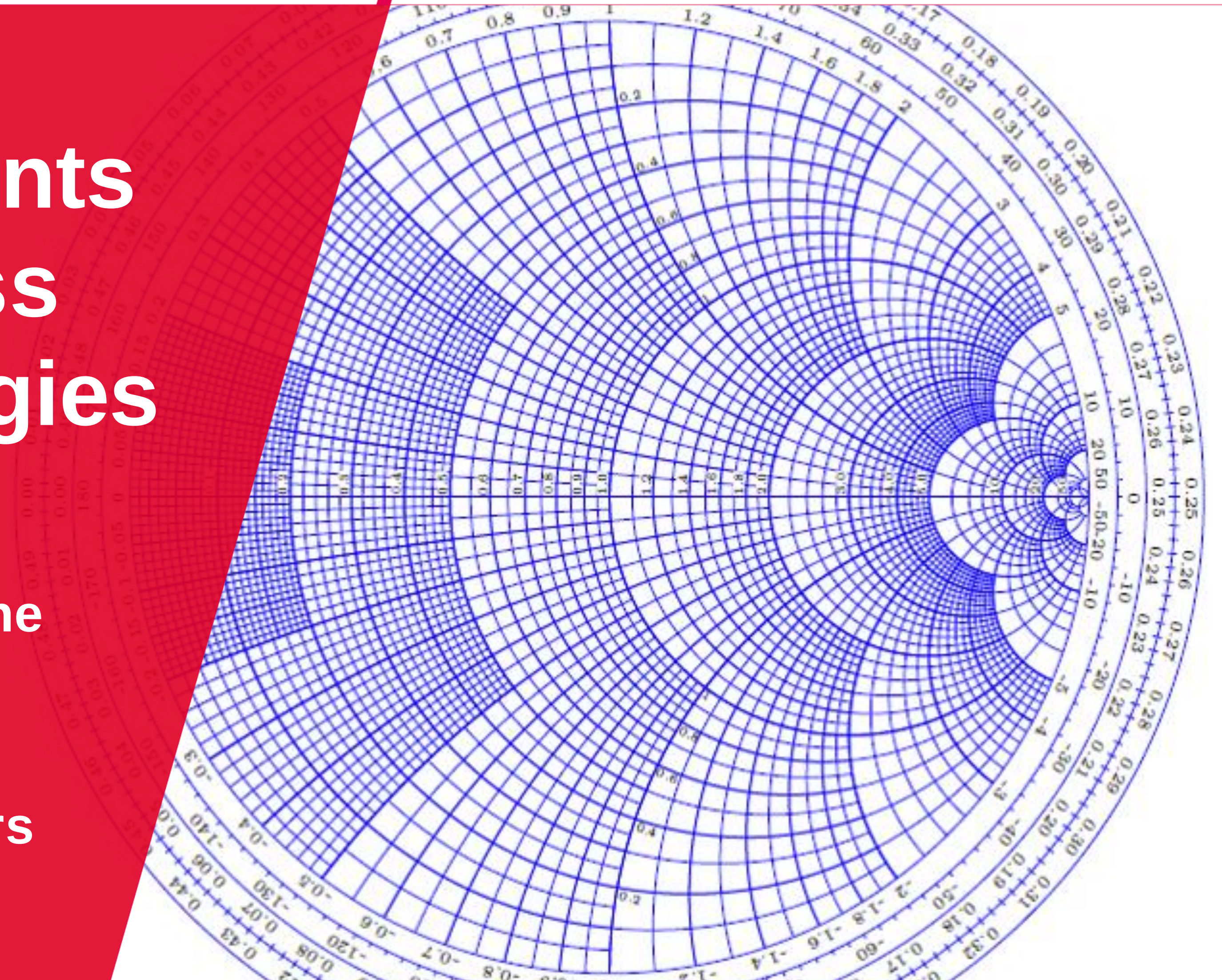
Technische Universiteit Eindhoven University of Technology

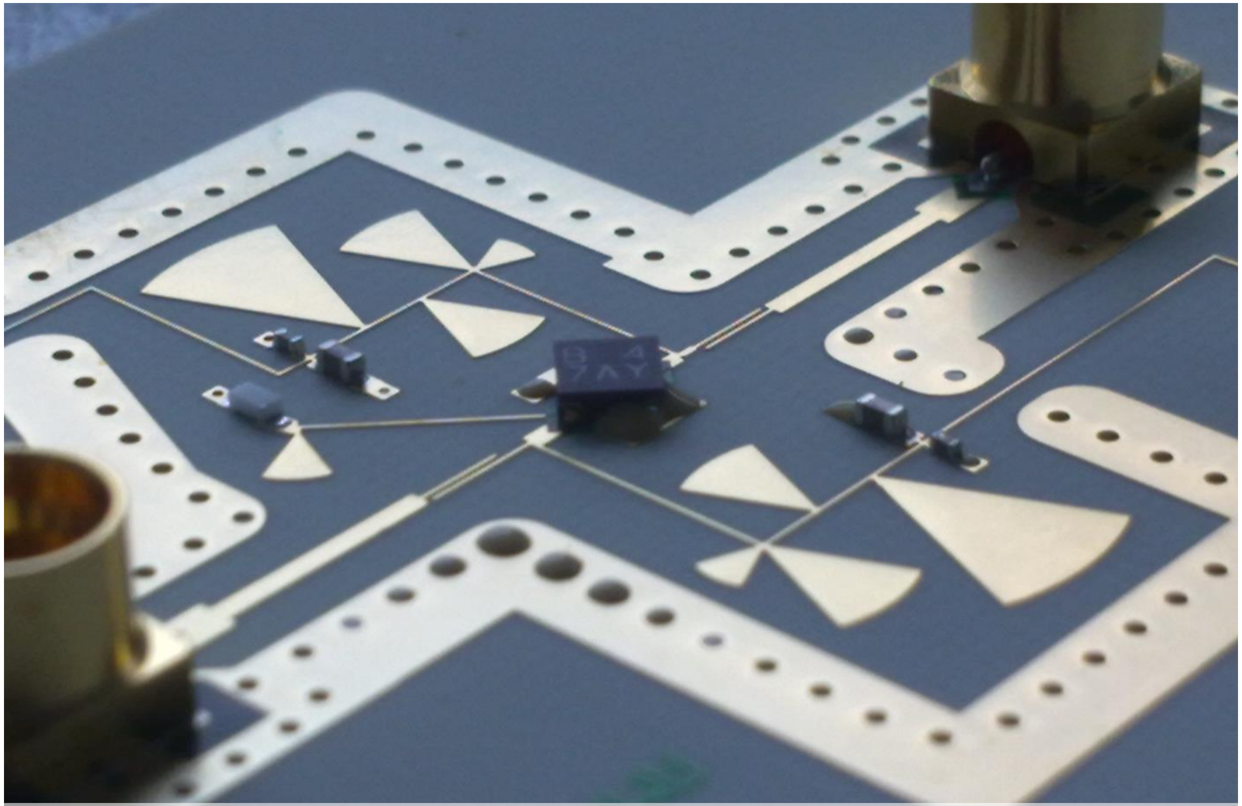


Components in Wireless Technologies

Module 1: Transmission Line Theory

Sander Bronckers

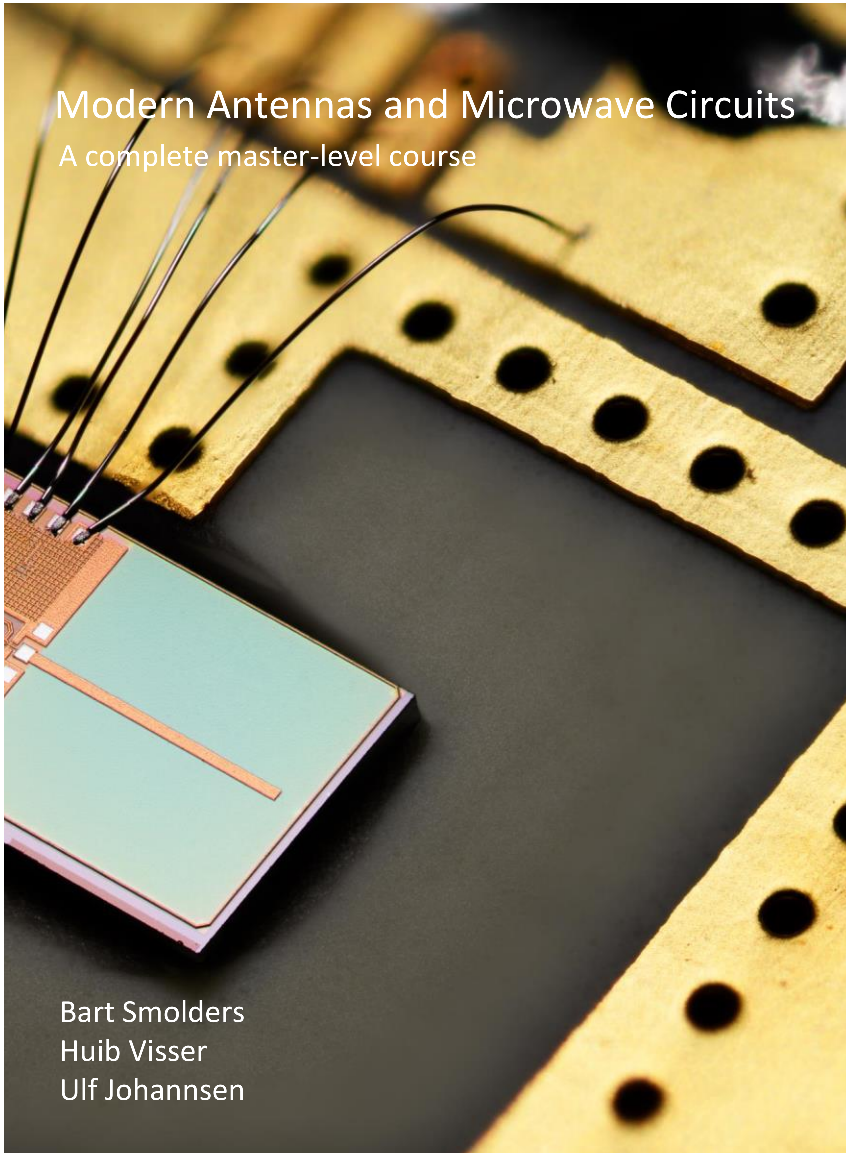
Transmission Lines of a 24 GHz Low-Noise Amplifier



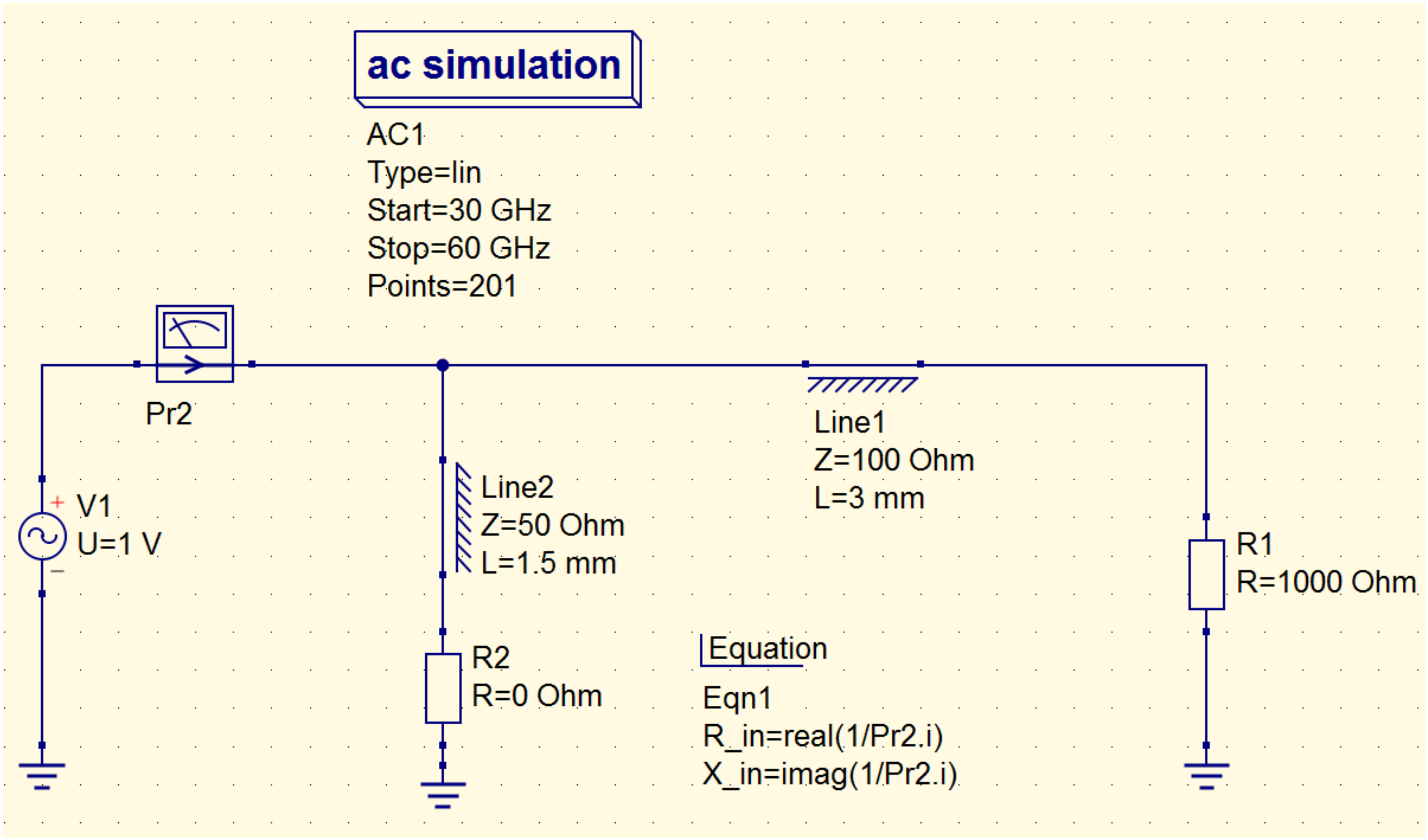
Let’s start investigating transmission lines, using Qucs

Free online book:

https://arxiv.org/abs/1911.08484



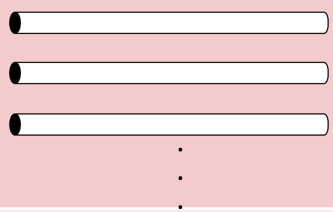
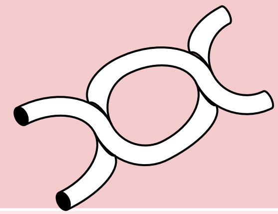
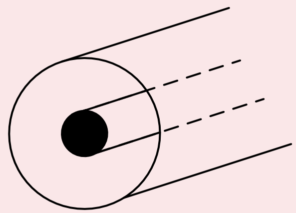
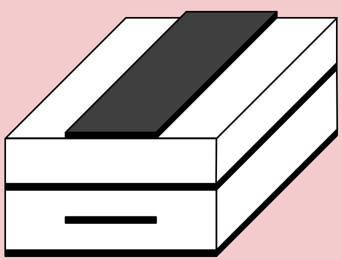
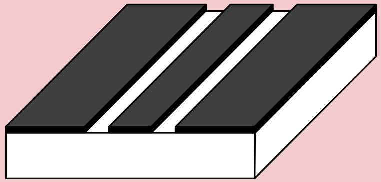
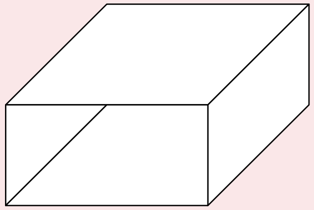
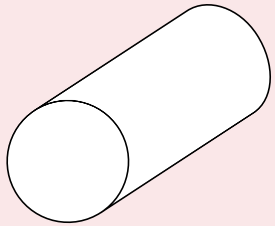
One of the goals of today is to understand this!



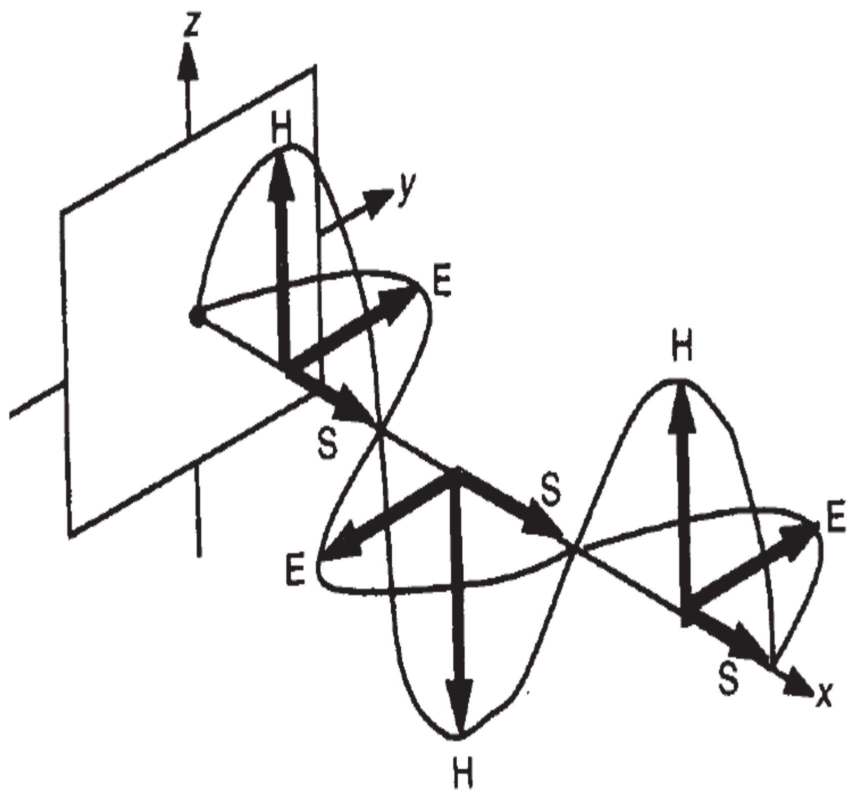
Transmission Line Theory

Content

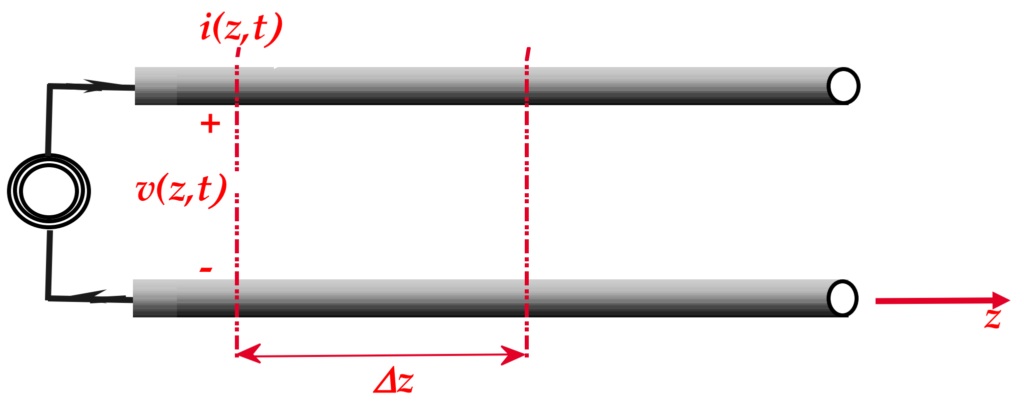
- Telegrapher Equations
- Wave Propagation on a Transmission Line
- The Lossless Transmission Line

Examples of transmission line	Schematic view	Field mode
Parallel wires and twisted pair	 	TEM
Coaxial		TEM
Micro-/Strip and coplanar waveguide	 	Quasi-TEM
Hollow waveguides	 	Non-TEM

- Following theory is derived for transmission lines with TEM field modes!

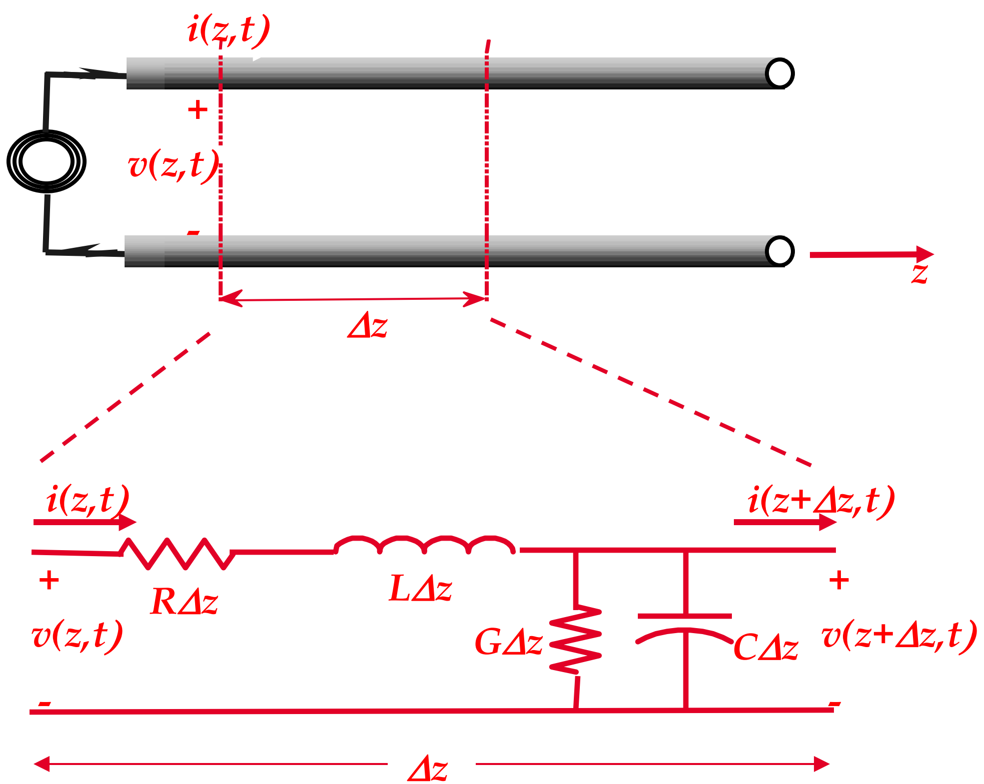


Basic Transmission line along z-axis



- Two-wire line representation
- Tline with TEM wave propagation has at least two conductors
- A short piece of length is Δz ($<< \lambda$) and can be modelled as a lumped-element circuit
- Question: How???

Lumped-element circuit model



R: series resistance per unit length (Ohm/m)

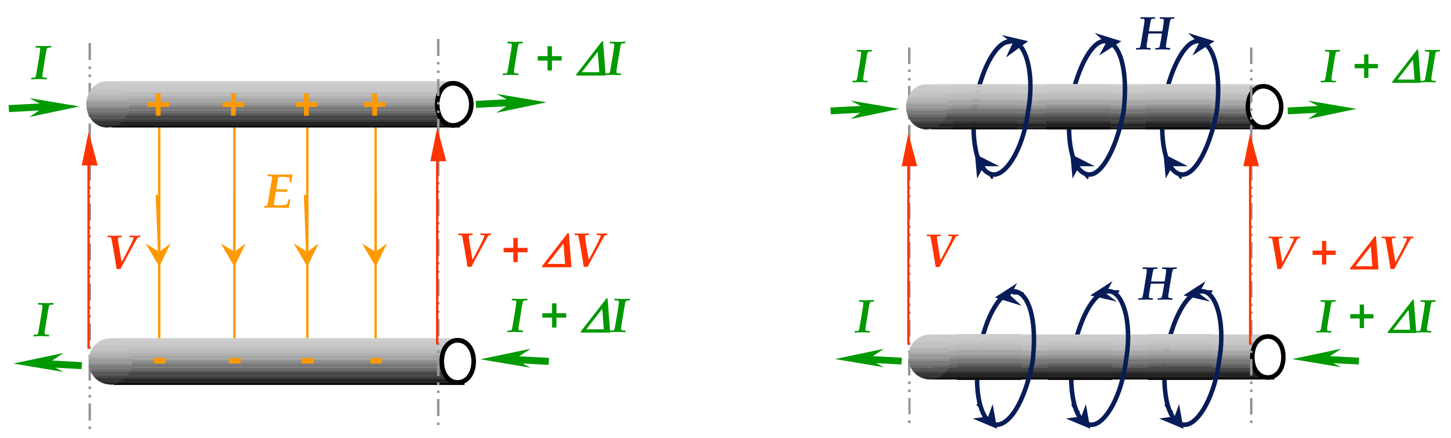
L: series inductance per unit length (H/m)

G: shunt conductance per unit length (S/m)

C: shunt capacitance per unit length (F/m)

Physical interpretation

“Simplified view”



Both Electric and Magnetic fields are present in the transmission lines

- These fields are perpendicular to each other and to the direction of wave propagation for TEM mode waves

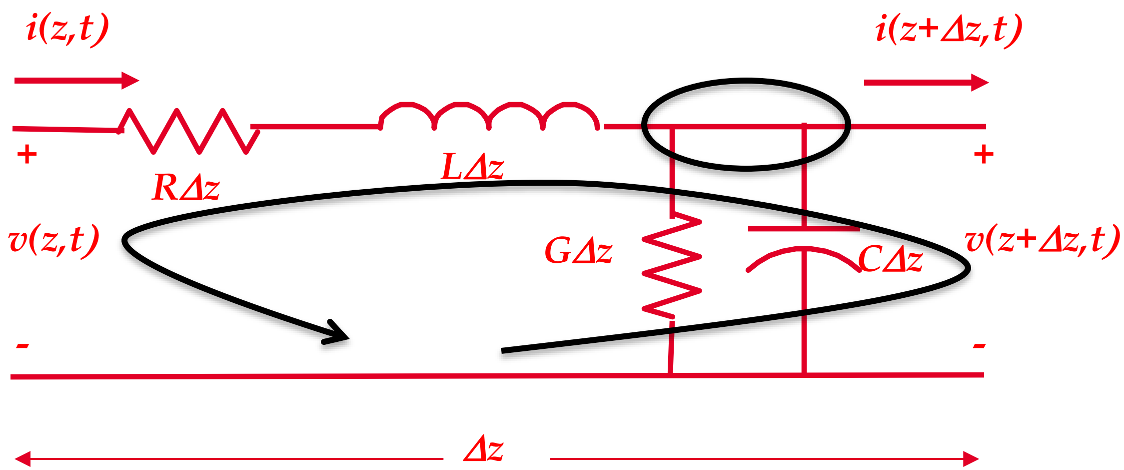
Electric field is established by a potential difference between two conductors.

- Implies equivalent circuit model must contain capacitor.

Magnetic field induced by current flowing on the line

- Implies equivalent circuit model must contain inductor.

Telegrapher Equations (Time Domain)

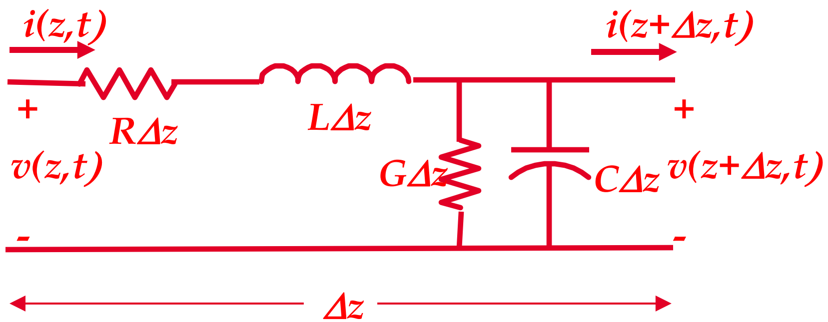


Apply Kirchhoff's voltage and current laws:

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Telegrapher Equations (Time Domain)



$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

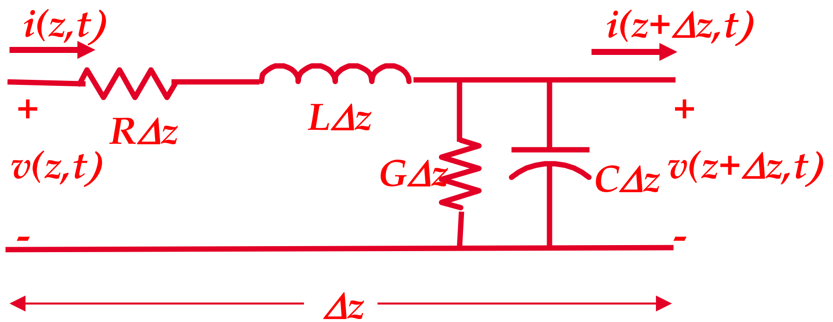
$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Divide by Δz and take limit Δz -> 0:

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

Telegrapher equations (Frequency Domain)



$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

$$\boxed{\frac{\partial v(z,t)}{\partial t} \leftrightarrow j\omega V(z)}$$

$$\frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)$$

$$\frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

- Telegrapher Equations

- Wave Propagation on a Transmission Line
  - Propagation constant
  - Characteristic impedance

- The Lossless Transmission Line

Wave propagation on a transmission line

Solving the Telegrapher equations:

$$I. \quad \frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z)$$

$$II. \quad \frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z)$$

Telegrapher equations (see previous slide)

$$\overset{II.}{\Leftrightarrow} V(z) = -\frac{1}{(G + j\omega C)}\frac{\partial I(z)}{\partial z}$$

$$\overset{I.}{\rightarrow} \frac{\partial V(z)}{\partial z} = -\frac{1}{(G + j\omega C)}\frac{\partial^2 I(z)}{\partial z^2} = -(R + j\omega L)I(z)$$

$$\frac{\partial^2 I(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)I(z) = \gamma^2 I(z)$$

Wave propagation on a transmission line

Solving the Telegrapher equations gives:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

Where γ is the **complex propagation constant**:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Wave propagation on a Tline

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

The relation between Voltage and Current amplitudes is:

$$I_0^+ = \frac{\gamma}{R + j\omega L} V_0^+, I_0^- = \frac{-\gamma}{R + j\omega L} V_0^-$$
$$Z_0 = \frac{R + j\omega L}{\gamma}$$

Why…?
(next slide)

Z_0 is the **characteristic impedance** of the Tline.

Wave propagation on a Tline

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

I. $$\frac{\partial V(z)}{\partial z} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z} = -(R + j\omega L) I(z)$$

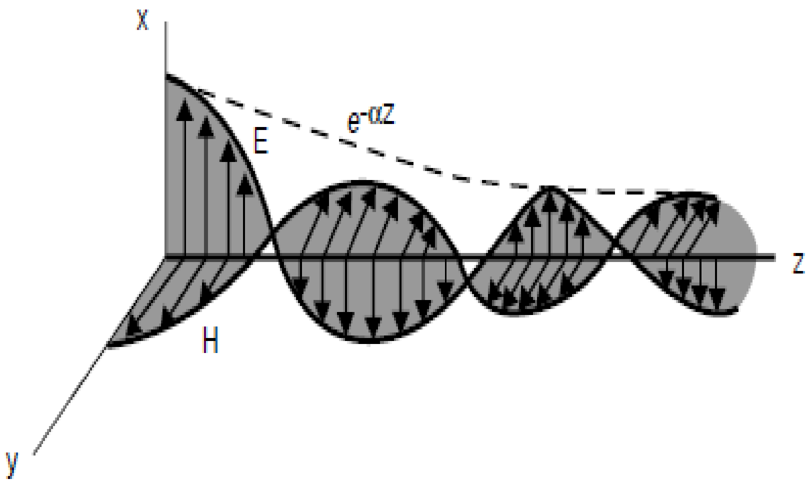
$$= -(R + j\omega L) I_0^+ e^{-\gamma z} - (R + j\omega L) I_0^- e^{\gamma z}$$

$$I_0^+ = \frac{\gamma}{R + j\omega L} V_0^+ \qquad I_0^- = \frac{-\gamma}{R + j\omega L} V_0^-$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

Z_0 is the **characteristic impedance** of the Tline.

Complex propagation constant



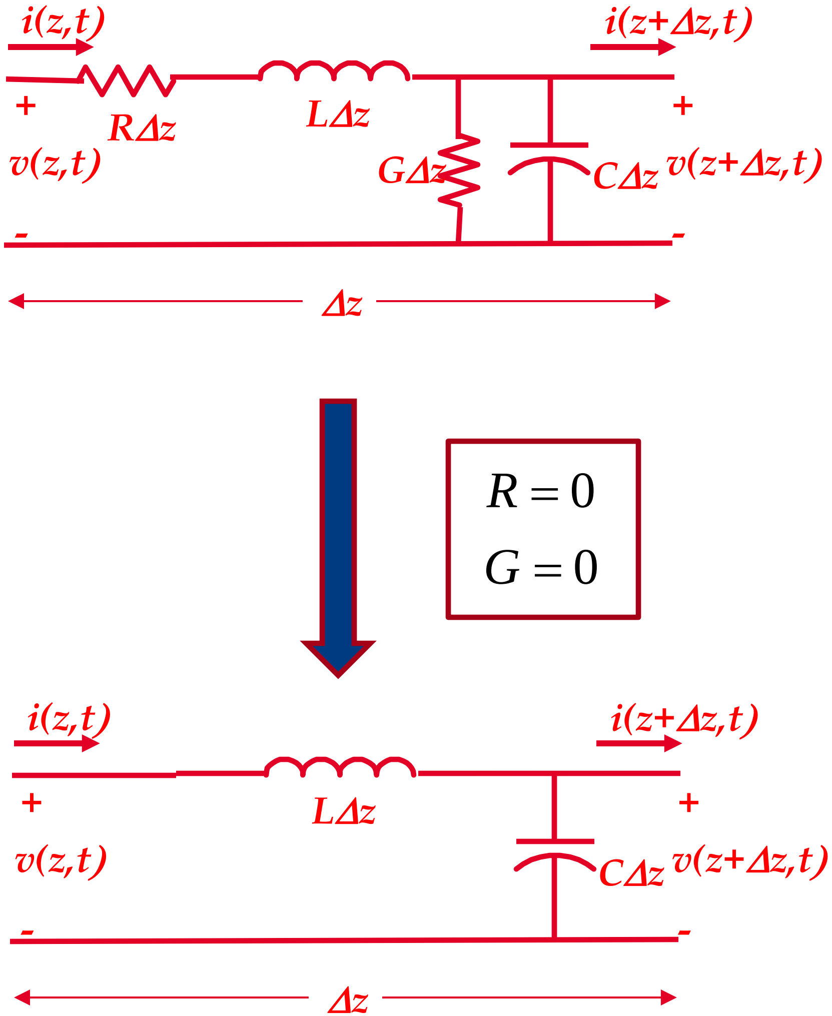
$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

α is called **attenuation constant**.

β is called **phase constant**.

- Telegrapher Equations
- Wave Propagation on a Transmission Line
- The Lossless Transmission Line
  - Reflection coefficient
  - Impedance transformation

The lossless line



The lossless line

Since R = 0 and G = 0:

$$\gamma = \alpha + j\beta = j\beta = j\omega\sqrt{LC},$$

$$\beta = \omega\sqrt{LC},$$

$$\alpha = 0,$$

$$Z_0 = \sqrt{\frac{L}{C}},$$

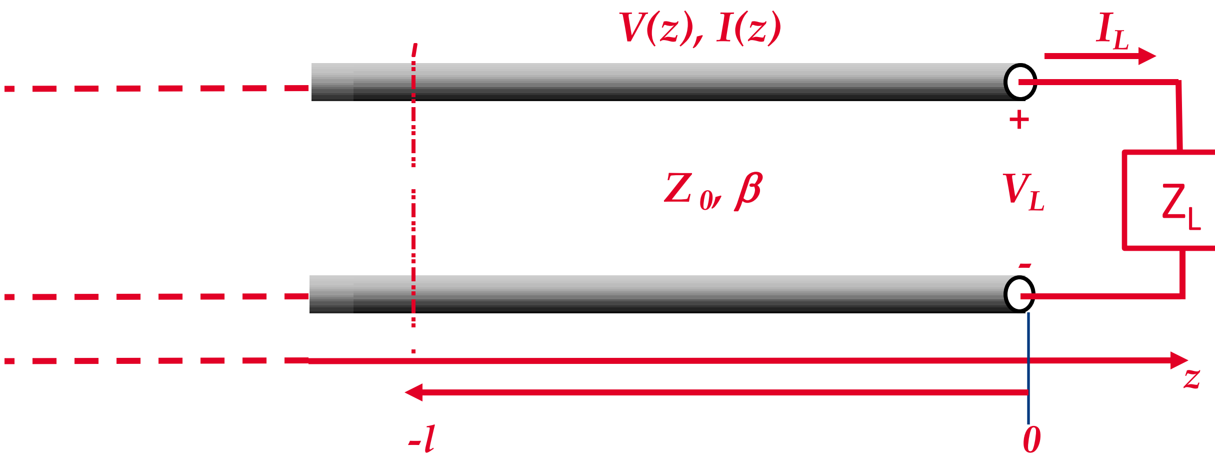
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}},$$

Wavelength

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Phase velocity

The terminated lossless Transmission Line

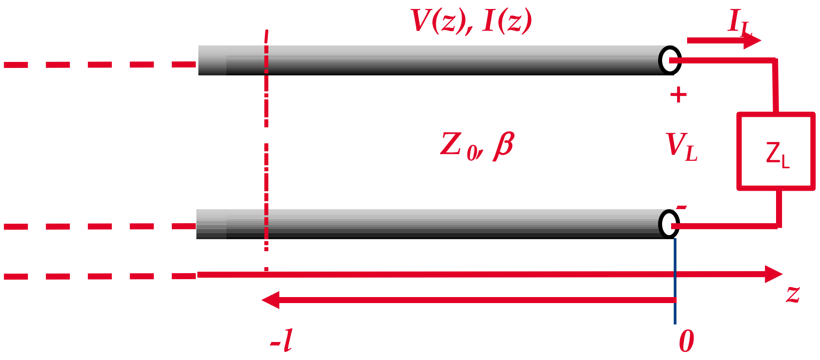


Total voltage and current are now:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

The terminated lossless Transmission Line



Total voltage and current at the load at $z=0$:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

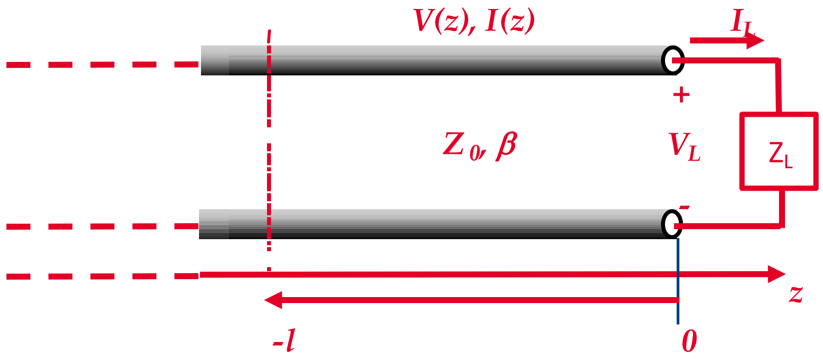
Rewriting gives:

$$V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection coefficient

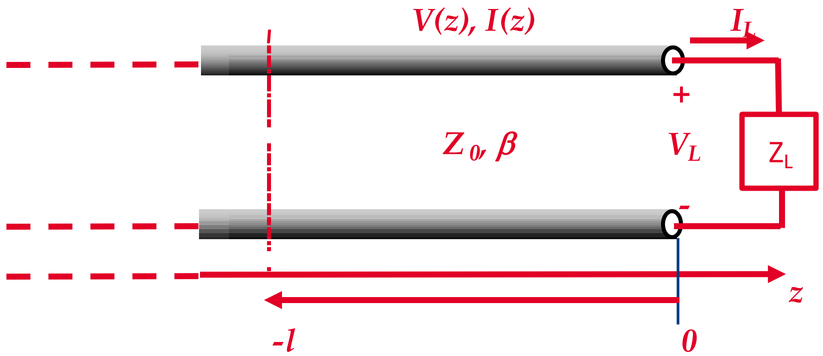
The terminated lossless Transmission Line



Special cases:

$$Z_L = Z_0 => \Gamma = 0,$$
$$Z_L = 0 => \Gamma = -1,$$
$$Z_L = \infty => \Gamma = 1.$$

The terminated lossless Transmission Line



Γ can be generalised to any point l on the transmission line:

$$\Gamma(z=-l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \Gamma e^{-2j\beta l},$$
$$\Gamma = \Gamma(z=0).$$

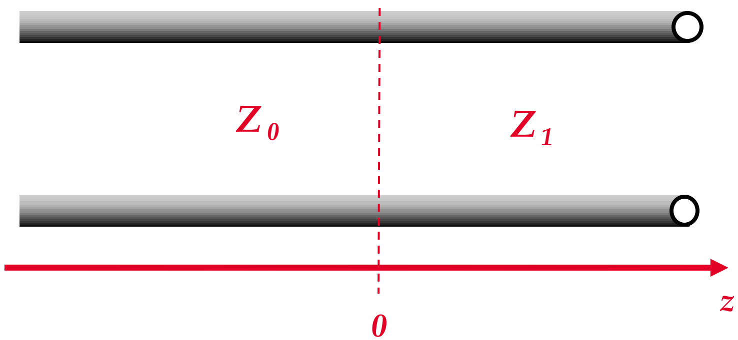
The **impedance** Z_{in} at z=**-**l is now:

$$Z_{in} = \frac{V(z=-l)}{I(z=-l)} = \frac{V_0^+\left[e^{j\beta l} + \Gamma e^{-j\beta l}\right]}{V_0^+\left[e^{j\beta l} - \Gamma e^{-j\beta l}\right]} Z_0 = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

Insertion Loss

Consider two transmission lines connected to each other.

Transmitted wave for *z>0:*



$$V(z) = V_0^+ T e^{-j\beta z}$$

T is the transmission coefficient:

$$T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

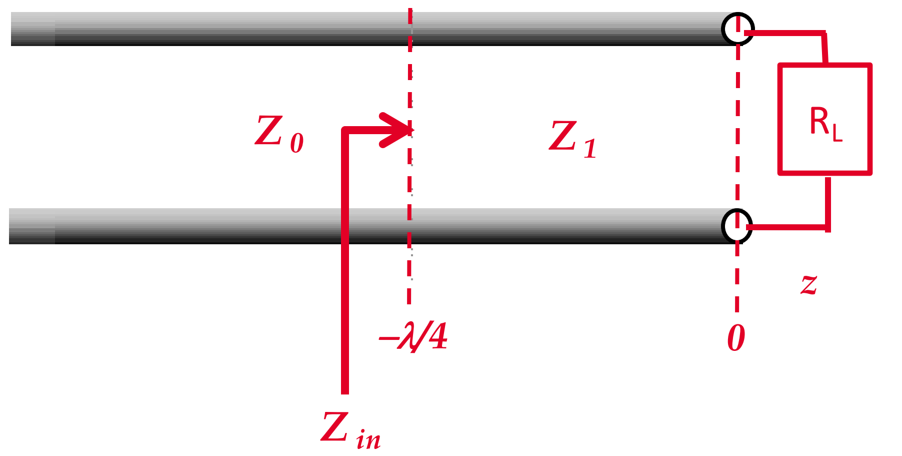
The insertion loss (*IL*) is now:

$$IL = -10\log\left(\frac{P_1}{P_0}\right) = -10\log\left(\frac{|V_1|^2 / Z_1}{|V_0|^2 / Z_0}\right)$$

$$= -20\log|T| + 10\log\left(Z_1 / Z_0\right)$$

Minus sign because we are calculating the loss

Quarter-wave transformer
"The impedance viewpoint"



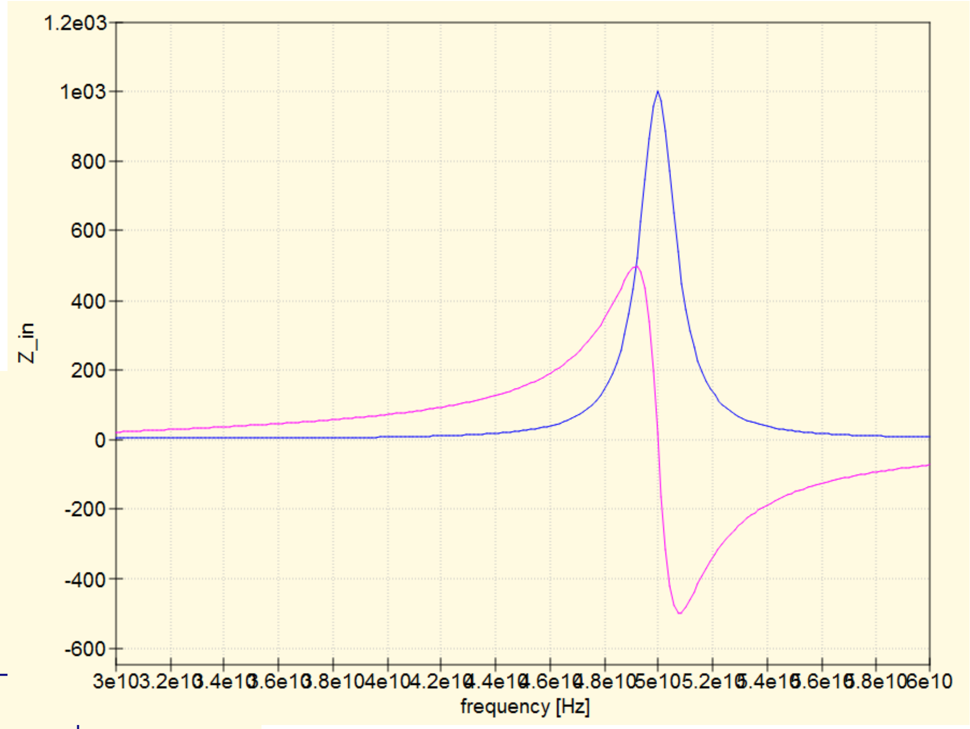
Input impedance at $z=-\lambda/4$:

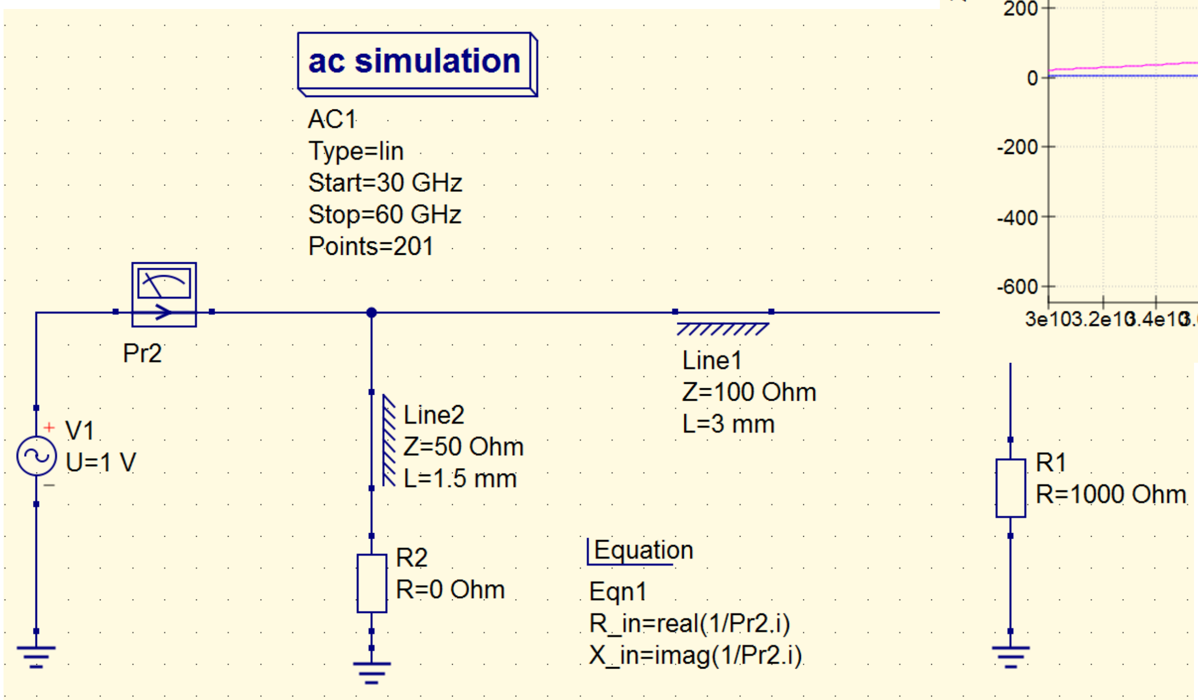
$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan \beta l}{Z_1 + jR_L \tan \beta l}$$

Now $$\beta l = (2\pi / \lambda)(\lambda / 4) = \pi / 2$$

$$Z_{in} = \frac{Z_1^2}{R_L} \quad \Rightarrow \quad \Gamma = 0 \quad \text{if} \quad Z_1 = \sqrt{Z_0 R_L}$$

Can we now understand this?

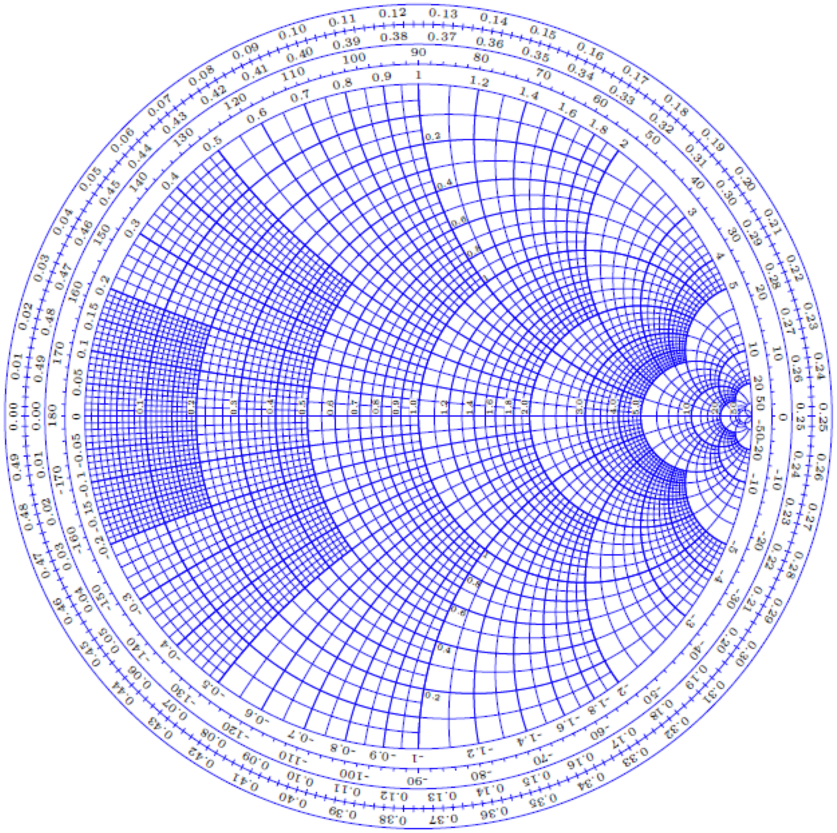




The Smith Chart

Smith Chart

Phillip H. Smith (1905-1987)

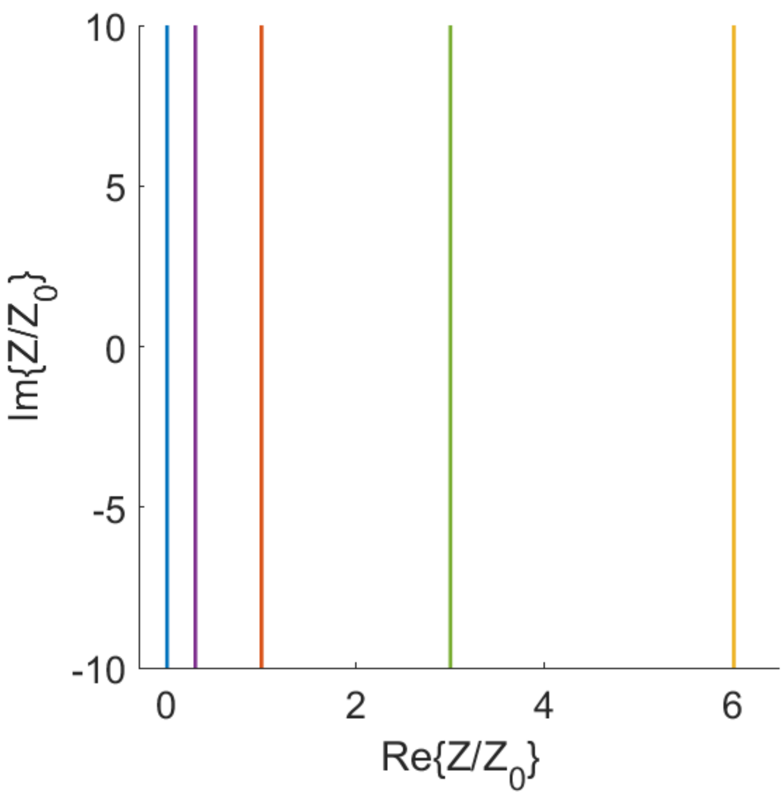


- Invented by Phillip H. Smith in 1939
- Easily usable graphical representation of the complex reflection coefficient Γ
- Easily reading the associated complex terminating impedance

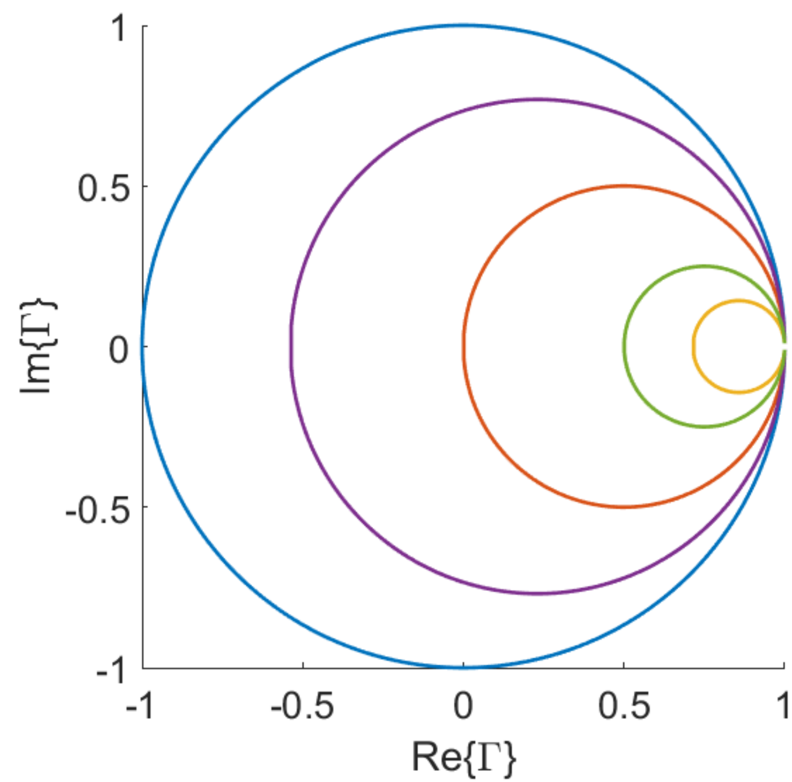
“Derivation” of the Smith Chart

Re{Z} = const.

Z domain



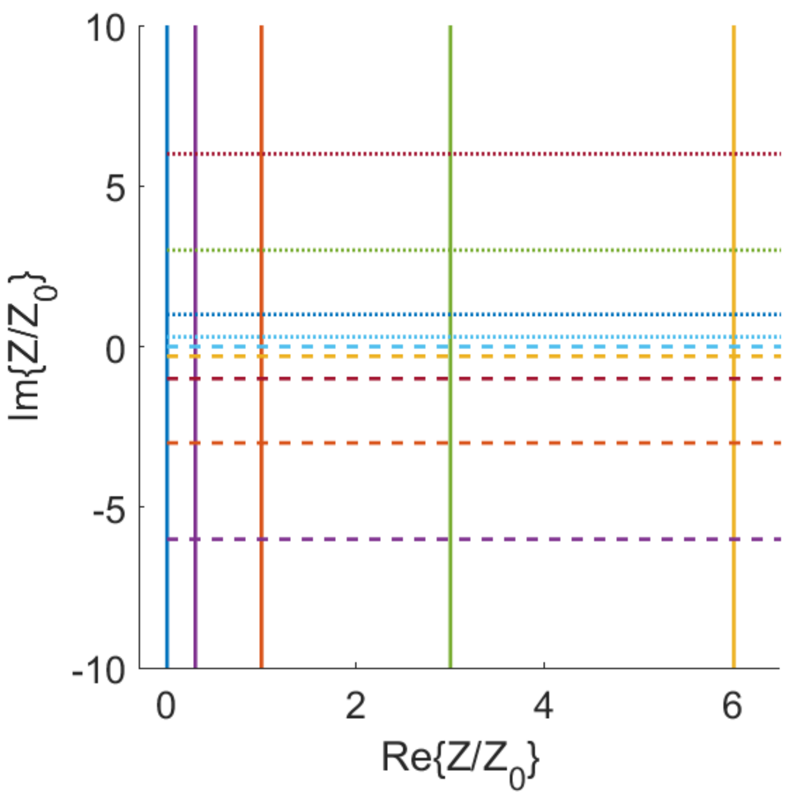
Γ domain



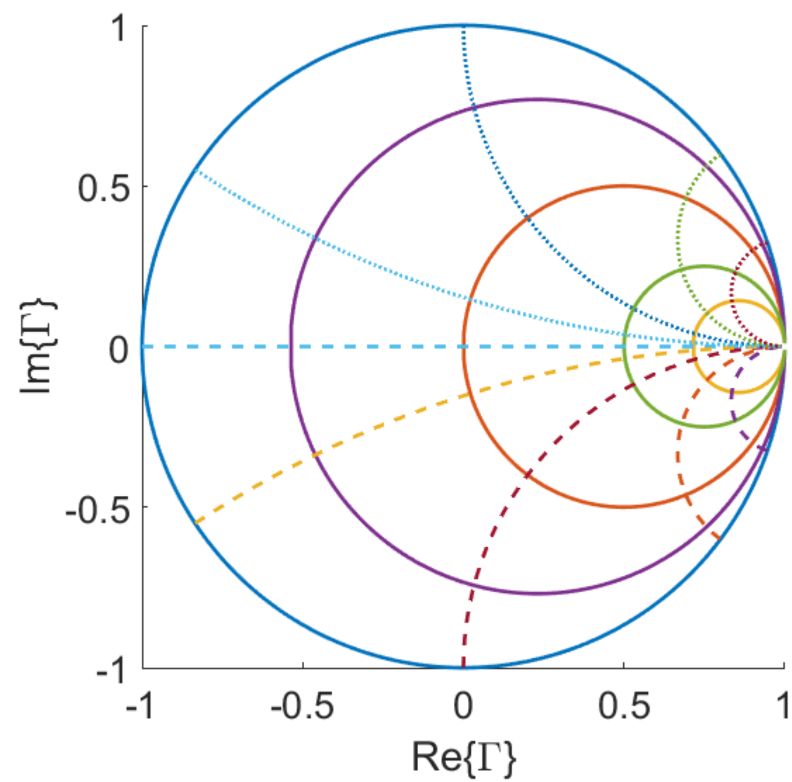
“Derivation” of the Smith Chart

Im{Z} = const.

Z domain



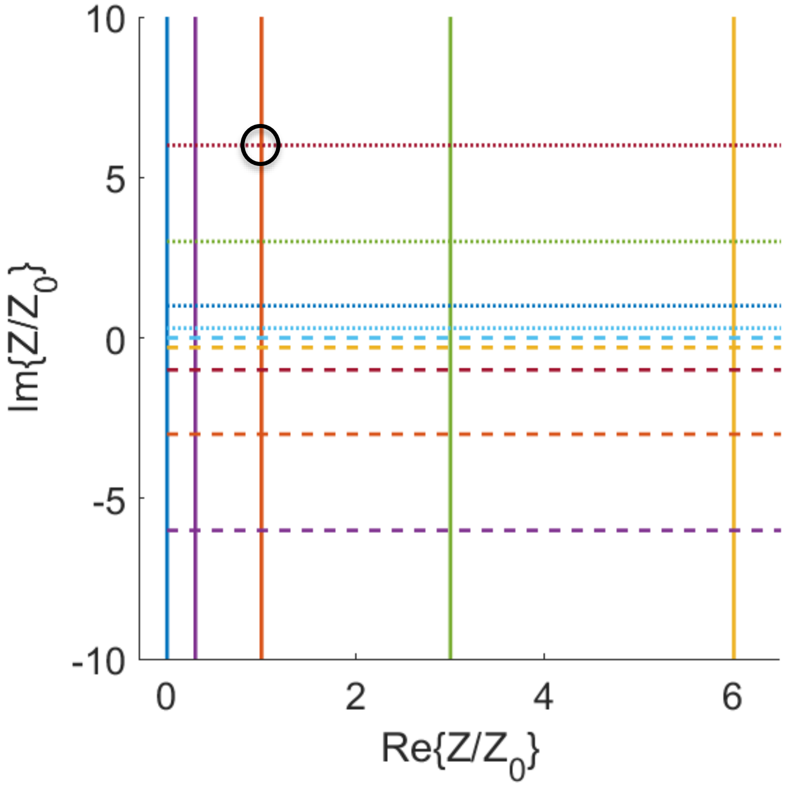
Γ domain



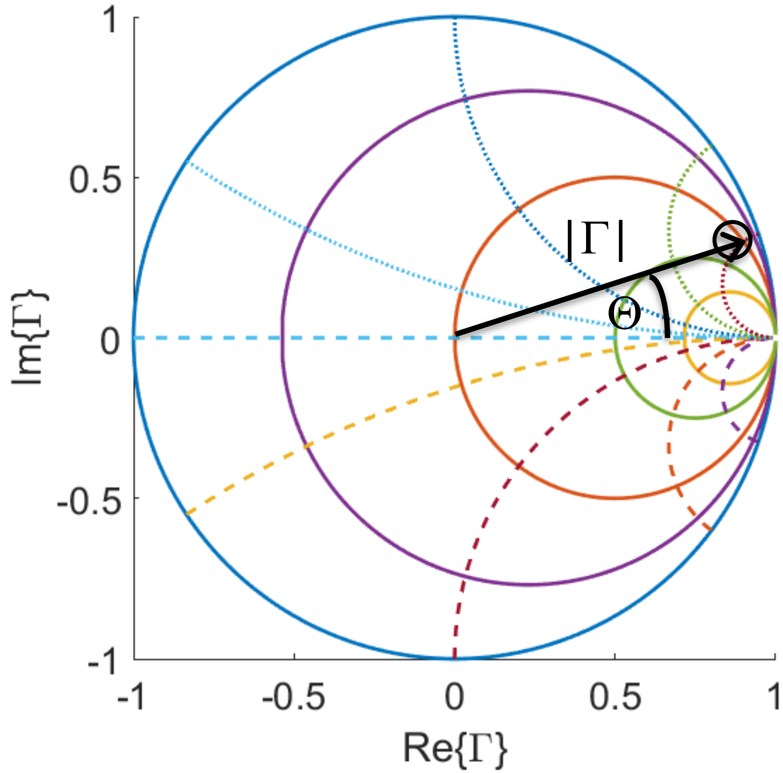
“Derivation” of the Smith Chart

Im{Z} = const.

Z domain

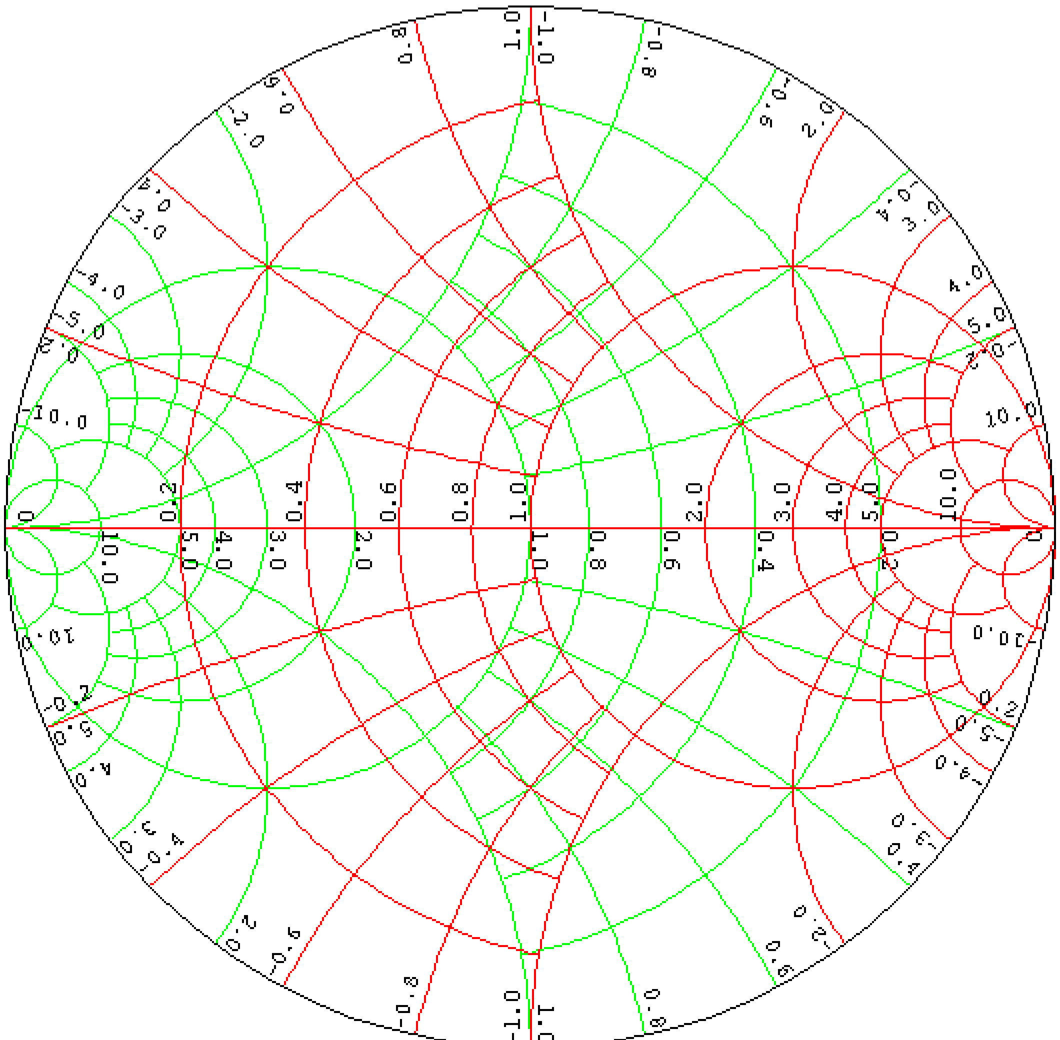


Γ domain



“Derivation” of the Smith Chart

The same can be done for admittances!



$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z - Z_0}{Z + Z_0}$$

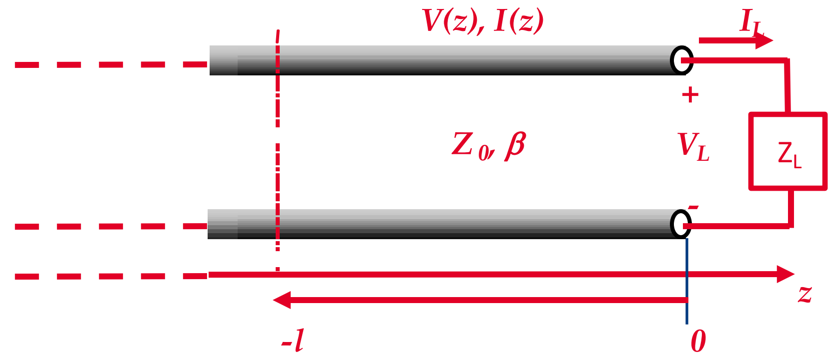
$$\Gamma = \frac{\frac{Z}{Z_0} - 1}{\frac{Z}{Z_0} + 1} = \frac{1 - \frac{Y}{Y_0}}{1 + \frac{Y}{Y_0}}$$

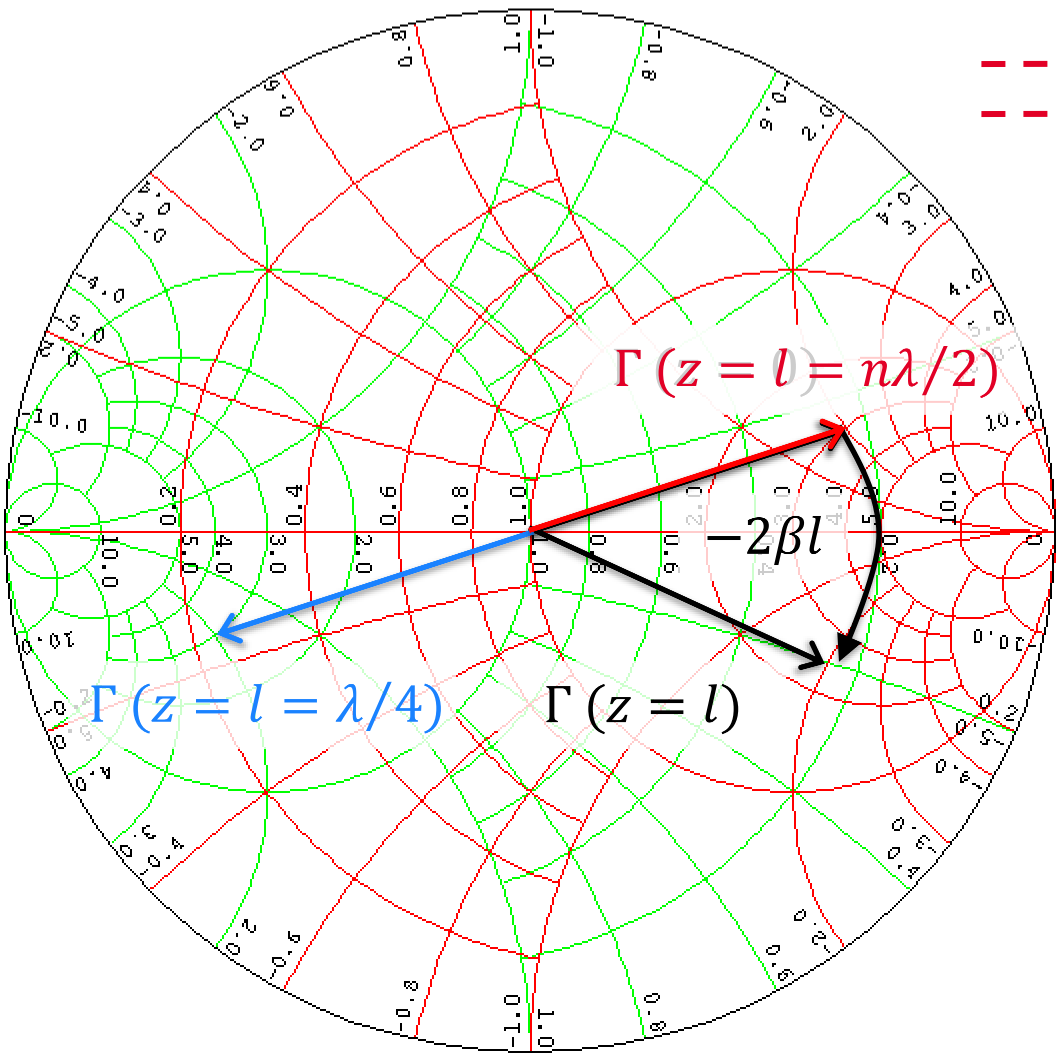
Red lines: $\frac{Z}{Z_0}$

Green lines: $\frac{Y}{Y_0}$

Picture source: www.microwaves101.com

The terminated lossless Transmission Line





From previously:

Γ at any point l on the transmission line:

$$\Gamma(z=-l)=\frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}}=\Gamma e^{-2j\beta l}$$

$$\beta=\frac{2\pi}{\lambda}$$

$$\Rightarrow -2\beta l=-2\pi \quad \text{for } l=\frac{\lambda}{2}$$

Smith Chart - Summary

- The Smith Chart contains
  - Magnitude, $|\Gamma|$, of the reflection coefficient
  - Phase, Θ, of the reflection coefficient
  - Real and imaginary part of the reflection coefficient
  - Real and imaginary part of the load impedance and admittance (normalised to the reference impedance Z_0 and admittance Y_0, respectively)