Photonics

Waveguides

Waveguide theory

Layered waveguides

~~Optical fibers~~

Step-index waveguide

- Core: refractive index n_1
- Cladding: refractive index n_2

$$n_1 > n_2$$

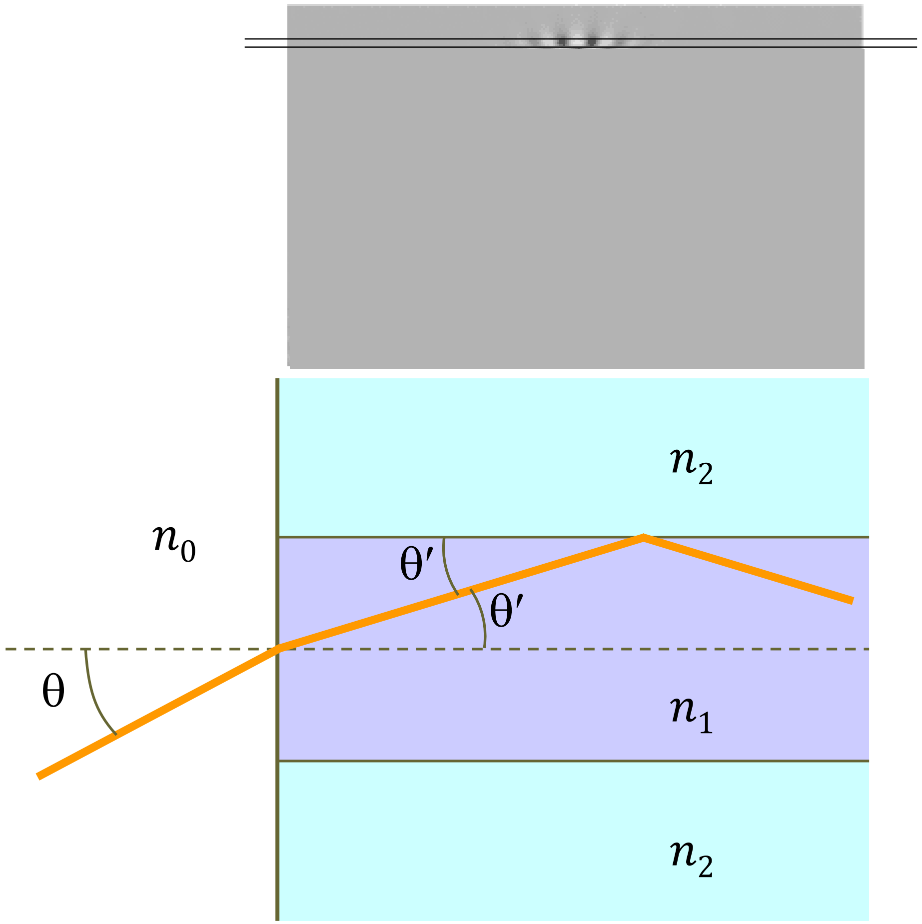
- Total internal reflection (TIR) angle θ' must be small enough

$$\theta'_{\max} = \arccos\frac{n_2}{n_1}$$

- Maximal incidence angle $\theta_{\max}$

$$\begin{aligned} n_0 \sin\theta_{\max} &= n_1 \sin\theta'_{\max} \\ &= n_1\sqrt{1-\cos^2\theta'_{\max}} \\ &= \sqrt{n_1^2 - n_2^2} \simeq \sqrt{2n\Delta n} \end{aligned}$$

- Numerical aperture $\mathrm{NA} = n_0 \sin\theta_{\max}$



Interference: fundamental mode

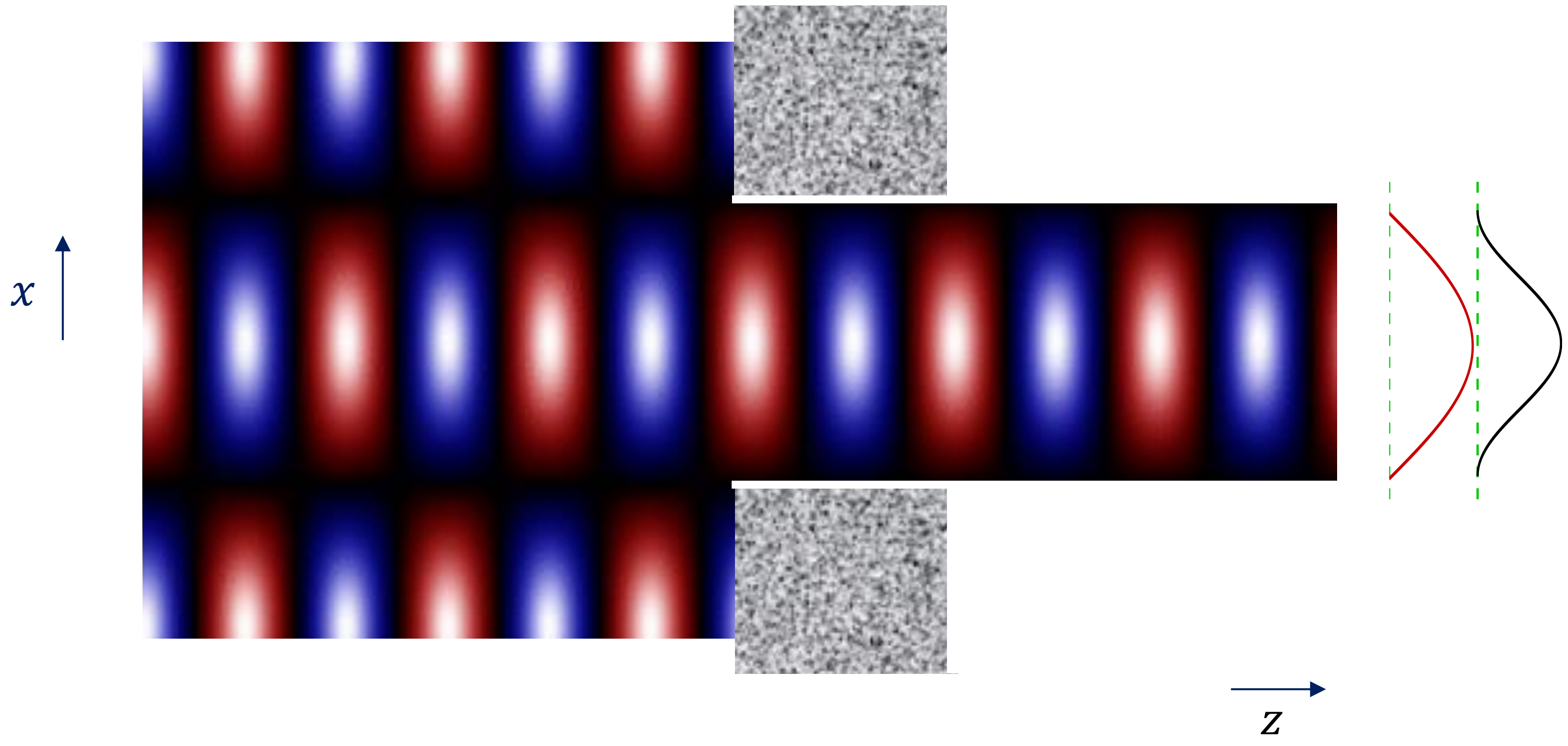
$$U(x,z) \propto e^{-j\vec{k}^{+}\cdot\vec{r}} = e^{-jk_x x} \cdot e^{-jk_z z}$$

Interference: fundamental mode

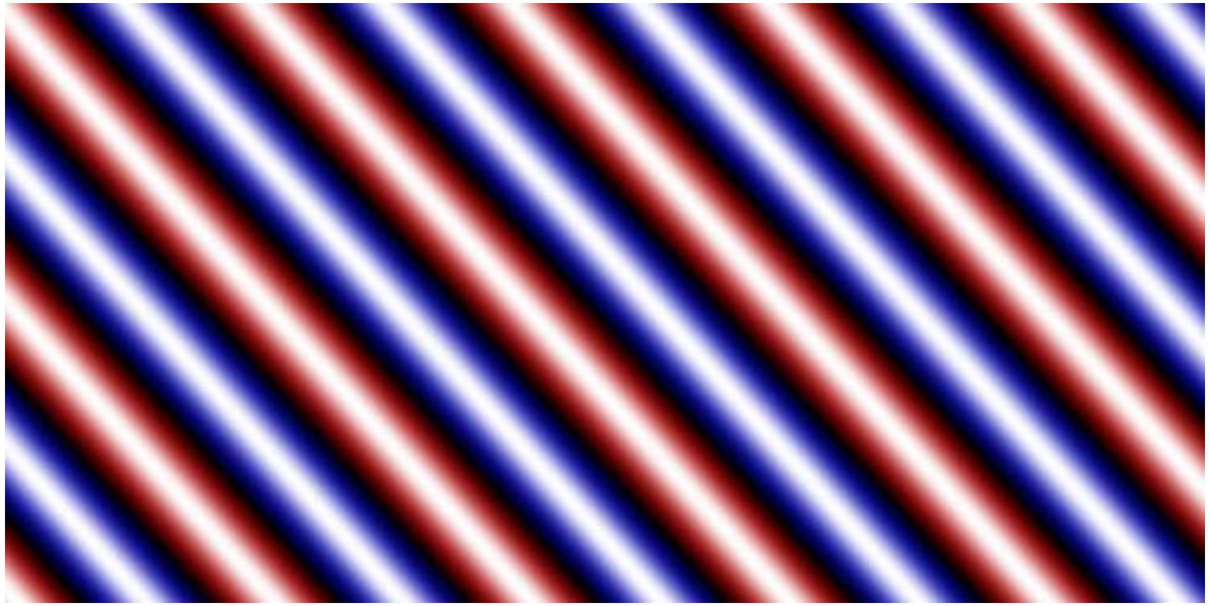
$$U(x,z) \propto e^{-j\vec{k}^{-}\cdot\vec{r}} = e^{-jk_x^{-}x} \cdot e^{-jk_z z} = e^{jk_x x} \cdot e^{-jk_z z}$$

Interference: fundamental mode

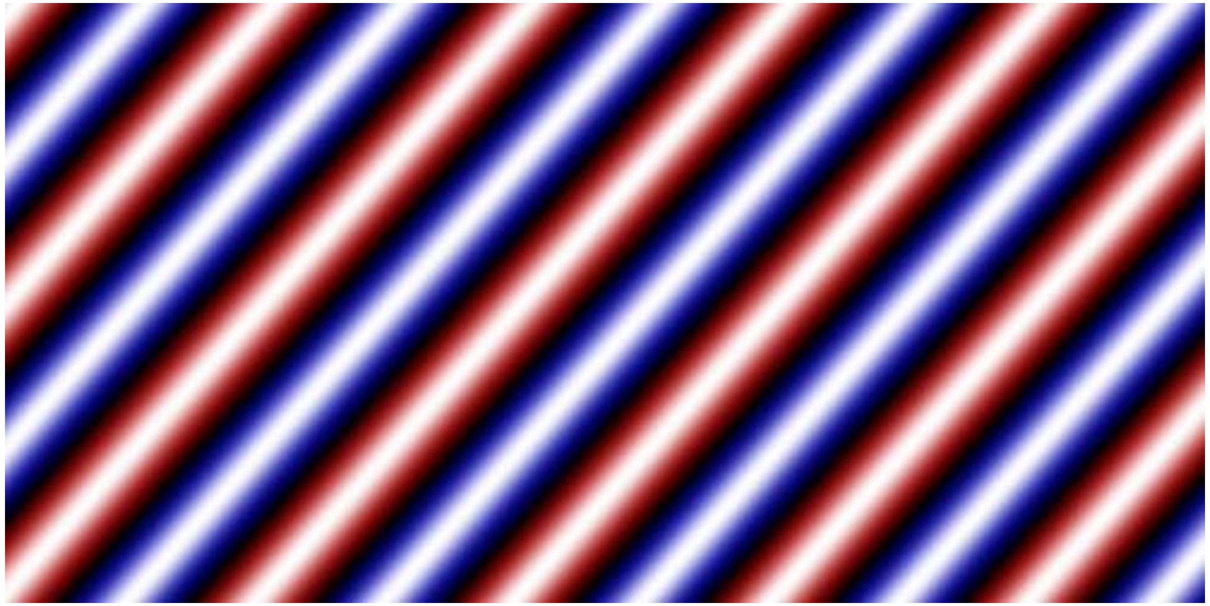
$$U(x,z) \propto e^{-j\vec{k}^{-}\cdot\vec{r}} + e^{-j\vec{k}^{+}\cdot\vec{r}} =$$
$$\left(e^{jk_x x} + e^{-jk_x x}\right)\cdot e^{-jk_z z} = 2\cos(k_x x)\cdot e^{-jk_z z}$$



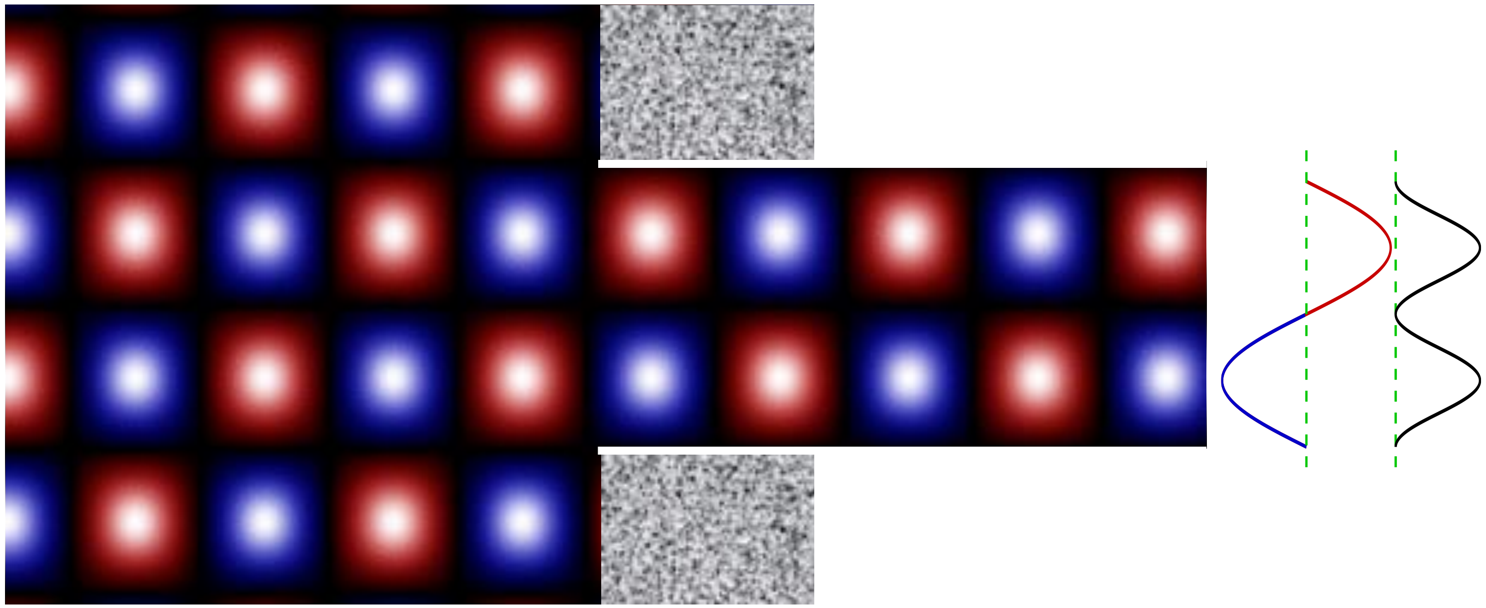
Interference: first order mode



Interference: first order mode



Interference: first order mode



Interfering waves

WebTOP

Department of Physics and Astronomy
Mississippi State University

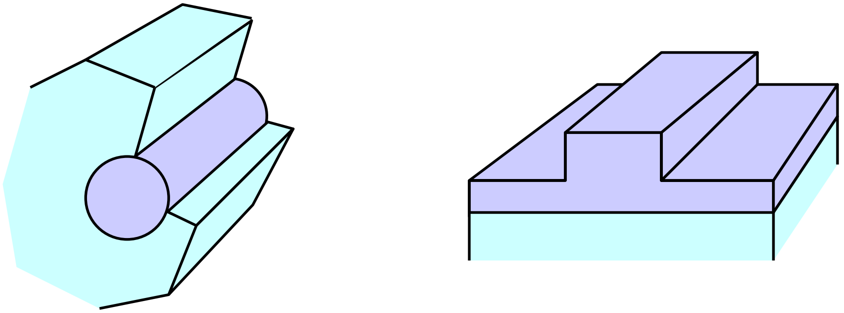
http://webtop.org

WebTOP is a 3D interactive computer graphics system that simulates and visualizes optical phenomena.

Start

Three-dimensional waveguides

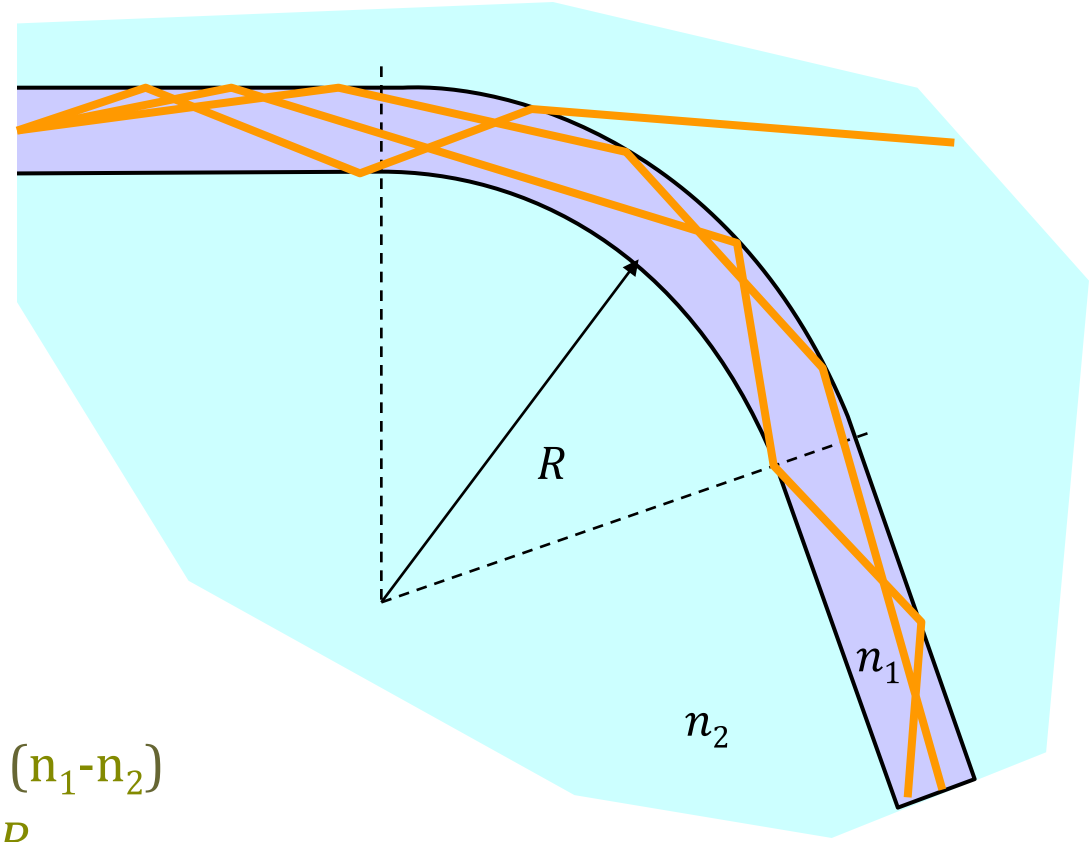
- Rectangular core
- Cylindrical core
    - step-index
    - graded-index



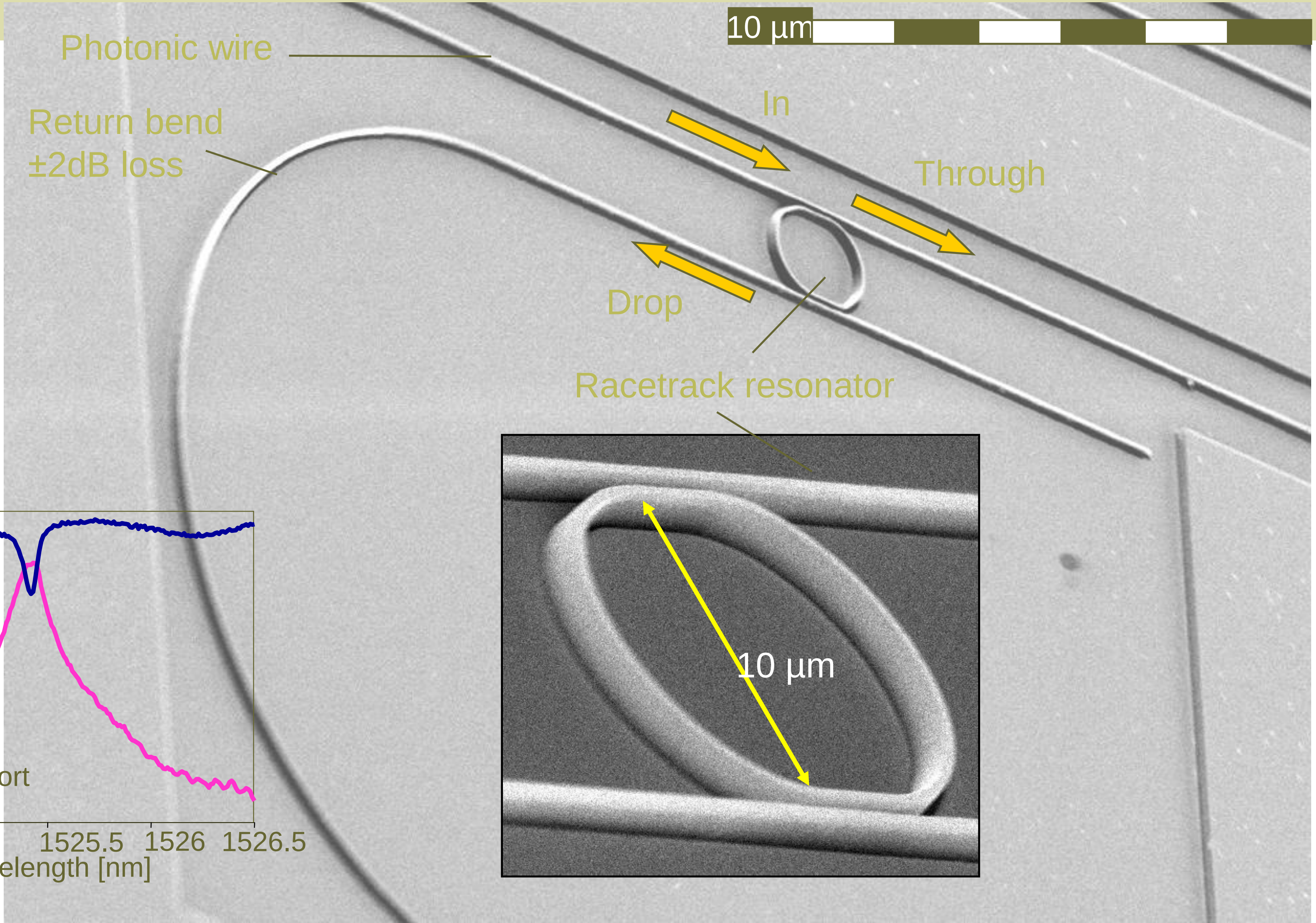
- e.g. optical fibers
    - multi-mode: $D_{core} = 50\ \mu m$
    - single-mode: $D_{core} = 10\ \mu m$
      (geometrical optics is not applicable anymore)

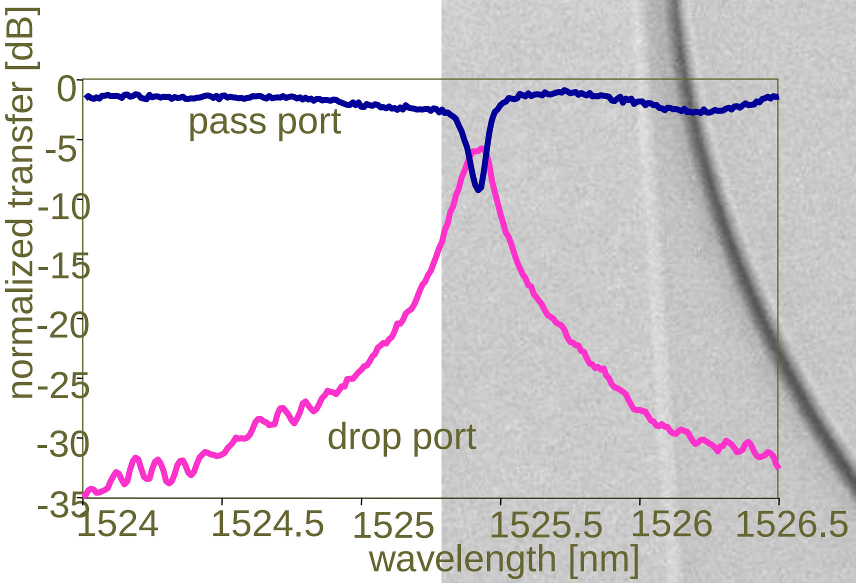
Bends in waveguides

- radius of curvature R
  - should be large
  - the larger $(n_1 - n_2)$ is, the smaller R can be
- e.g. optical fiber: R should be larger than 1cm
- Always losses
  - lower losses with higher $(n_1\text{-}n_2)$
  - lower losses with larger R



Ring resonators in Silicon-on-Insulator





Wave equations

- Combining Maxwell equations
  → wave equations for **E**- and **H**-fields

- Piecewise constant refractive index n
  → Scalar Helmholtz equation for every
  E- and **H**-component

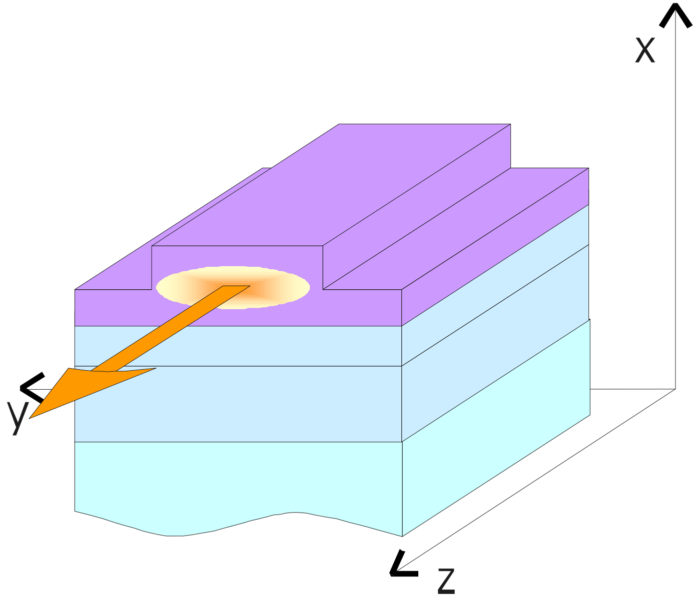
$$\nabla^2 U(r) + k_0^2 n^2(r) U(r) = 0$$

Longitudinally invariant waveguides (1)

- Invariant in the propagation direction

$$n(r) = n(x, y)$$

  $\rightarrow$ eigenmodes:
  - transverse shape invariant
  - guided or not-guided



- Forward propagating eigenmode:

$$\mathbf{E}(x, y, z) = \mathbf{e}(x, y)\mathrm{e}^{-j\beta z}$$
$$\mathbf{H}(x, y, z) = \mathbf{h}(x, y)\mathrm{e}^{-j\beta z}$$

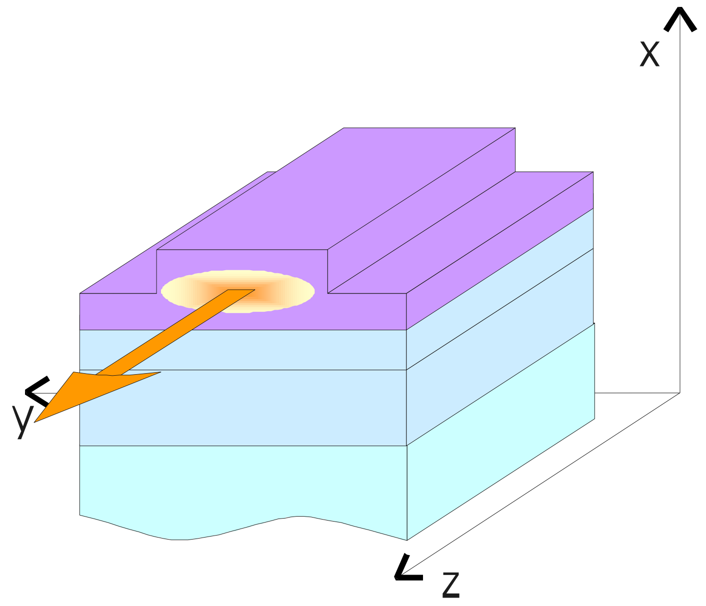
Longitudinally invariant waveguides (2)

- Propagation characteristics of an eigenmode:
  - propagation constant β
  - effective refractive index n_{eff}

$$n_{\text{eff}} = \beta / k_0$$

  - effective dielectric constant

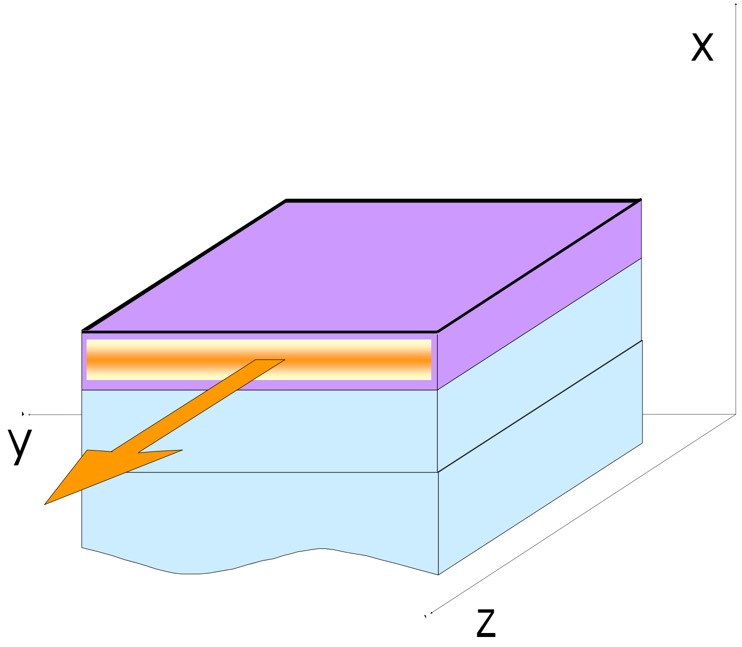
$$\varepsilon_{\text{eff}} = n_{\text{eff}}^2$$

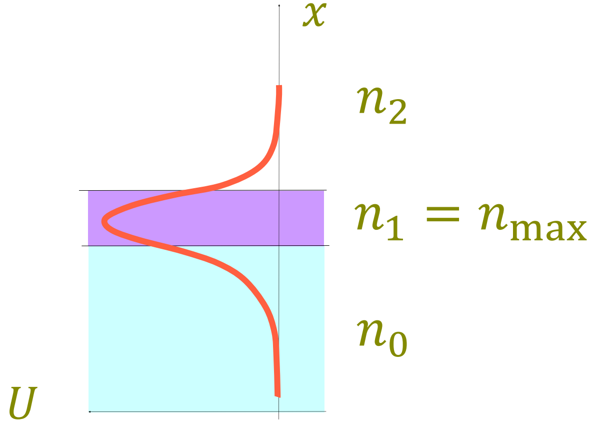


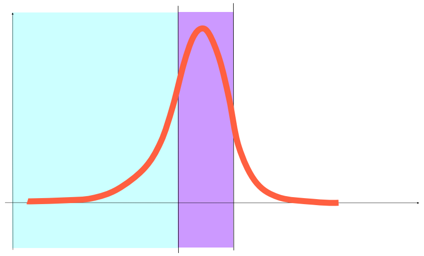
Simplest waveguide: slab

- Only confined in transverse direction (x), propagating in z-direction

$$\mathbf{E}(x,y,z) = \mathbf{e}(x)\mathrm{e}^{-j\beta z}$$
$$\mathbf{H}(x,y,z) = \mathbf{h}(x)\mathrm{e}^{-j\beta z}$$

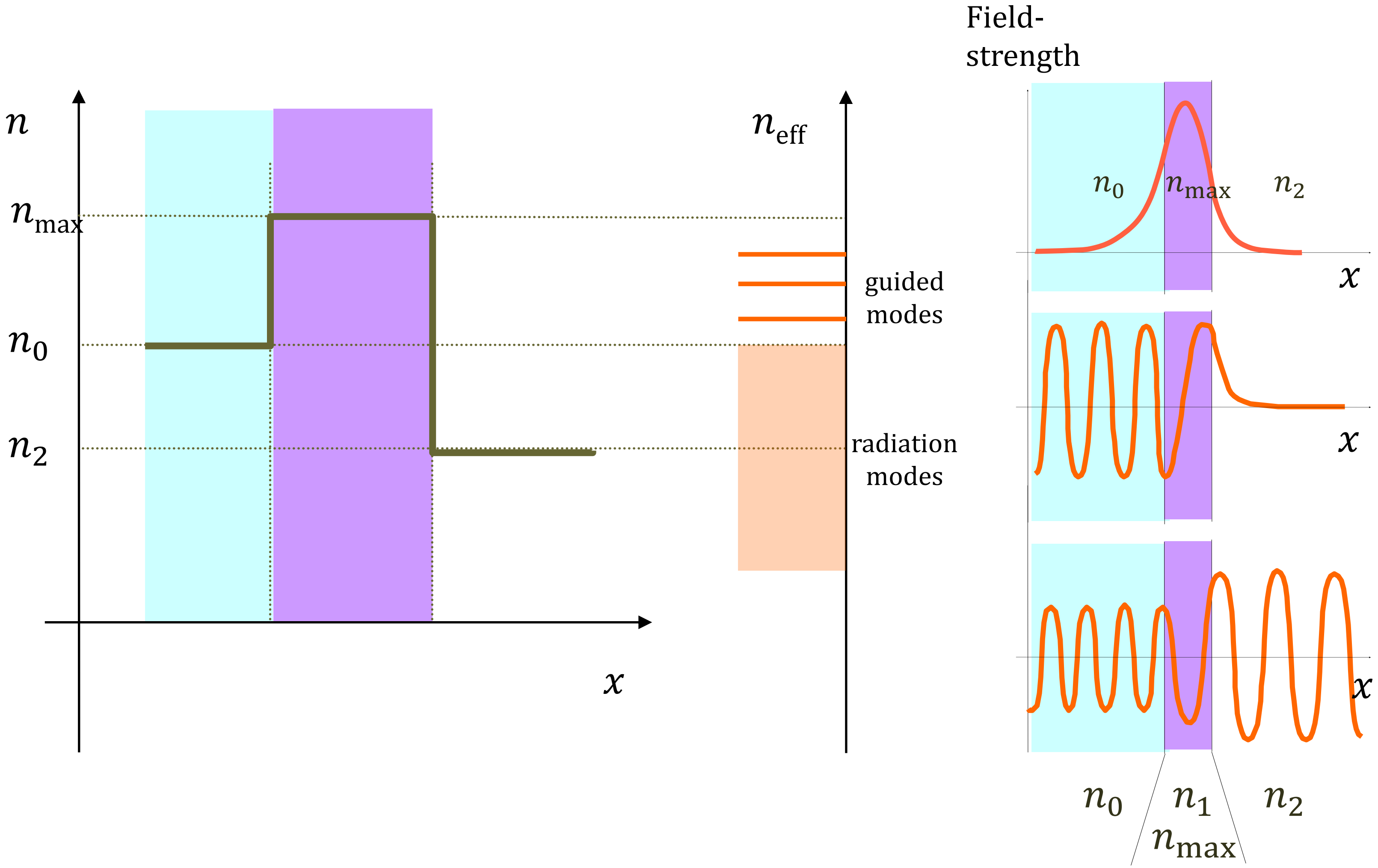






Lossless waveguide (1)

- Refractive index profile:



Lossless waveguide (2)

- No eigenmodes with $n_{\mathrm{eff}} > n_{\max}$

- Guided modes: Discrete set of eigenvalues n_{eff}

$$n_{\max} > n_{\mathrm{eff}} > \max(n_{\mathrm{cladding}})$$

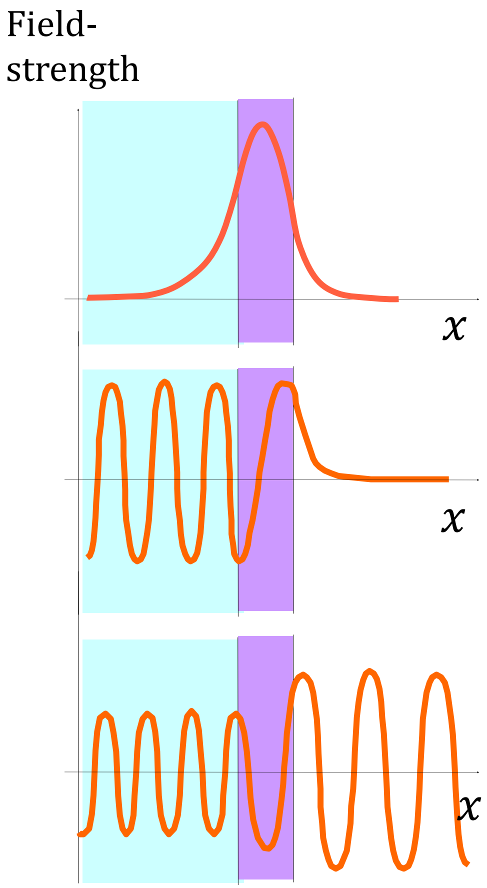
  field strength decays in the cladding (transversal direction, r_{t}):

$$\lim_{|r_{\mathrm{t}}| \to \infty} U(r_{\mathrm{t}}) = 0$$

There is not necessarily a guided mode

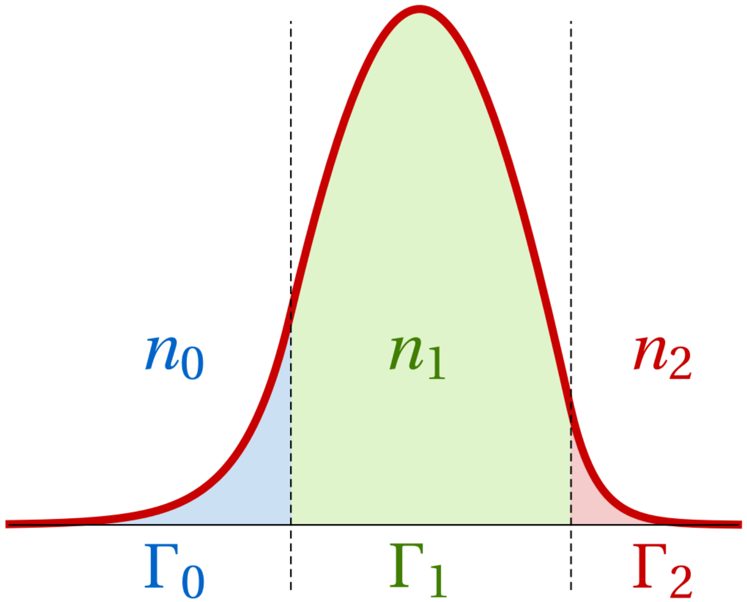
- Radiation modes: continuous set of eigenvalues

$$n_{\mathrm{eff}} < \max(n_{\mathrm{cladding}})$$



Effective index

$$n_{\text{eff}} = \frac{\beta}{k_0} \qquad \beta = n_{\text{eff}} k_0$$



$$n_{\text{eff}} \simeq \sum_i \Gamma_i n_i$$

$$\Gamma_i = \frac{P_i}{P_{\text{tot}}}$$

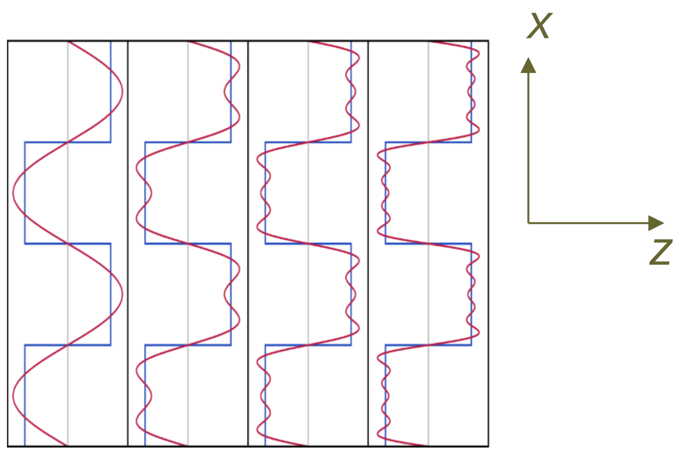
Lossless waveguide (3)

- Guided and radiation modes: form a complete set of functions
  $\rightarrow$ any field is a weighted sum of these modes

$$E(x,y,z) = \sum_m a_m \mathbf{e}_m(x,y) + \int a(k)\mathbf{e}_k(x,y)\mathrm{d}k$$

$$E(x) = \frac{\cos x}{1} - \frac{\cos 3x}{3} + \cdots$$

$$E(x) = \sum a_i \mathrm{e}^{-jk_{x,i}x}$$



Slab waveguide

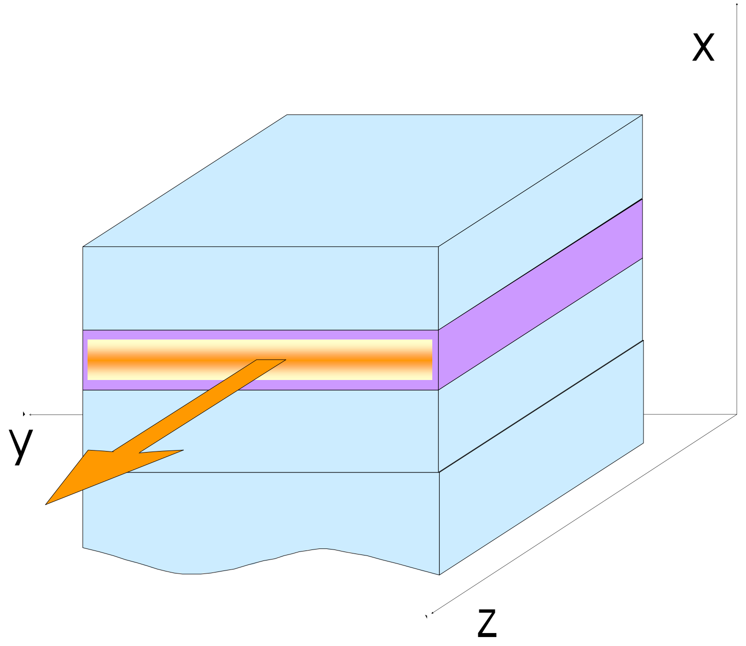
- Slab waveguide
  - 1D-structure
  - 2D-problem

  $\rightarrow$ decouples into
  Transverse Electric (TE) problem and Transverse Magnetic (TM) problem

- All fields are independent of y

- **E**- and **H**-field:

$$\mathbf{E}(x,z) = \mathbf{e}(x)\mathrm{e}^{-j\beta z}$$
$$\mathbf{H}(x,z) = \mathbf{h}(x)\mathrm{e}^{-j\beta z}$$



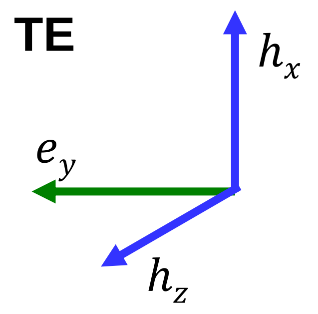
$U(x,z) = u(x)\mathrm{e}^{-j\beta z}$ for all field components

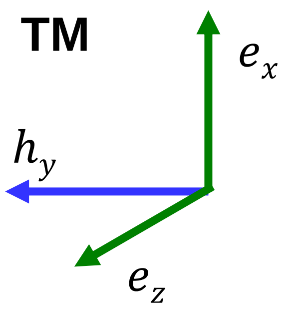
TE modes (TM analogous)

- TE: $e_x = e_z = h_y = 0,\ \partial_y = 0$
- Maxwell:

$$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$

Homogeneous media



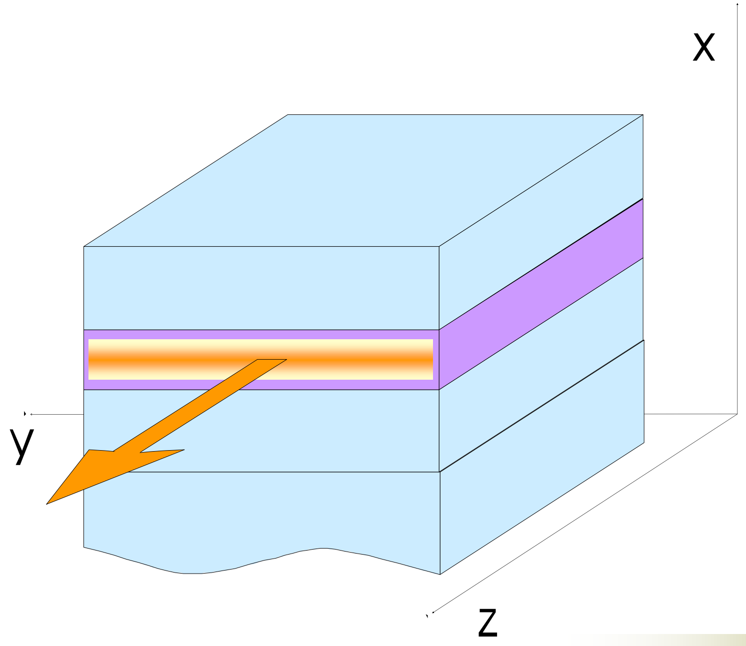


$$\nabla \times \begin{pmatrix} 0 \\ e_y \\ 0 \end{pmatrix} = \begin{pmatrix} -\partial_z e_y \\ 0 \\ \partial_x e_y \end{pmatrix} = -j\omega\mu_0 \begin{pmatrix} h_x \\ 0 \\ h_z \end{pmatrix}$$

$$u(x,z) = u(x)\mathrm{e}^{-j\beta z}$$

two equations

$$j\beta e_y = -j\omega\mu_0 h_x \Rightarrow \boxed{\beta e_y = -\omega\mu_0 h_x}$$
$$\partial_x e_y = -j\omega\mu_0 h_z \Rightarrow \boxed{\partial_x e_y = -j\omega\mu_0 h_z}$$



$$\nabla \times \begin{pmatrix} h_x \\ 0 \\ h_z \end{pmatrix} = \begin{pmatrix} 0 \\ \partial_z h_x - \partial_x h_z \\ 0 \end{pmatrix} = j\omega\varepsilon \begin{pmatrix} 0 \\ e_y \\ 0 \end{pmatrix}$$

$$\varepsilon = \varepsilon_0\varepsilon_\mathrm{r} = \varepsilon_0 n^2$$

one equation

$$-j\beta h_x - \partial_x h_z = j\omega\varepsilon_0 n^2 e_y \Rightarrow \boxed{\omega\varepsilon_0 n^2 e_y = -\beta h_x + j\partial_x h_z}$$

TE modes (2)

$$\beta e_y = -\omega\mu_0 h_x$$
$$\partial_x e_y = -j\omega\mu_0 h_z$$
$$\omega\varepsilon_0 n^2 e_y = -\beta h_x + j\partial_x h_z$$

$$\omega^2\varepsilon_0\mu_0 = k_0^2$$

- h_x and h_z are easily found from e_y

$$h_x = \frac{-\beta}{\omega\mu_0} e_y$$
$$h_z = \frac{j}{\omega\mu_0} \partial_x e_y$$

and therefore

$$\omega\varepsilon_0 n^2 e_y = \frac{\beta^2}{\omega\mu_0} e_y - \frac{1}{\omega\mu_0}\partial_x^2 e_y \Rightarrow \partial_x^2 e_y + k_0^2 n^2 e_y = \beta^2 e_y$$

TE- and TM-problem

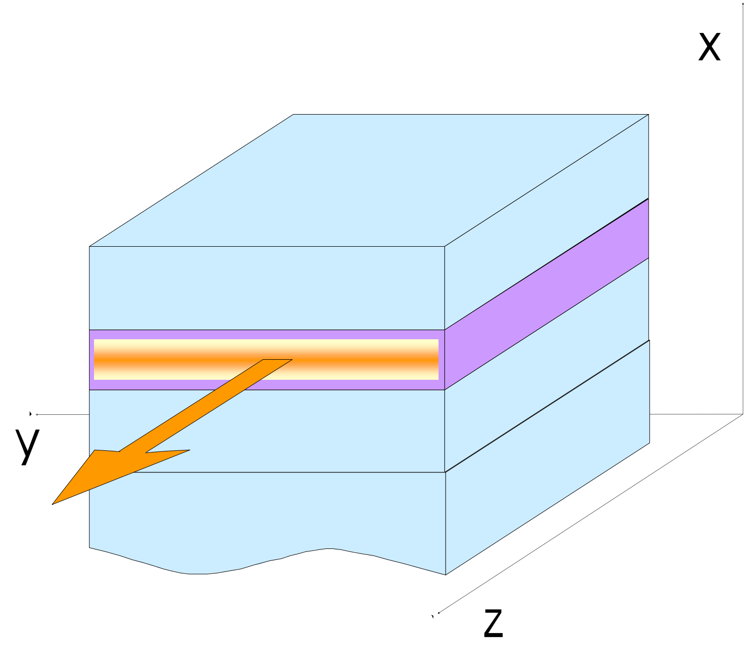
- Eliminating x- and z-component

TE: $\frac{\mathrm{d}^2 e_y(x)}{\mathrm{d}x^2} + k_0^2 n^2(x) e_y(x) = \beta^2 e_y(x)$

TM: $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{k_0^2 n^2(x)} \frac{\mathrm{d}h_y(x)}{\mathrm{d}x}\right) + h_y(x) = \frac{1}{k_0^2 n^2(x)} \beta^2 h_y(x)$

TE- and TM-equations are identical
(with different boundary conditions)

- Calculate x, z components from u_y:



TE	TM
$h_x = \frac{-\beta}{\omega\mu_0} e_y$	$e_x = \frac{\beta}{\omega\varepsilon_0 n^2} h_y$
$h_z = \frac{j}{\omega\mu_0} \partial_x e_y$	$e_z = \frac{-j}{\omega\varepsilon_0 n^2} \partial_x h_y$

Three-layer slab waveguide (1)

- Solutions for e_y

$$e_{y,i} = \begin{cases} A\mathrm{e}^{-\delta x} & x \geq 0 \\ A\cos\kappa x + B\sin\kappa x & -d \leq x \leq 0 \\ (A\cos\kappa d - B\sin\kappa d)\mathrm{e}^{\gamma(x+d)} & x \leq -d \end{cases}$$

where:

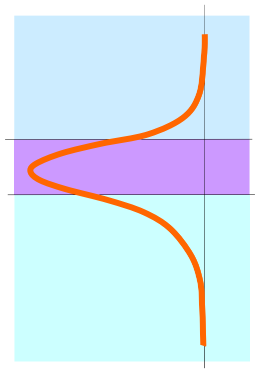
$$\delta = \sqrt{\beta^2 - n_2^2 k_0^2}$$

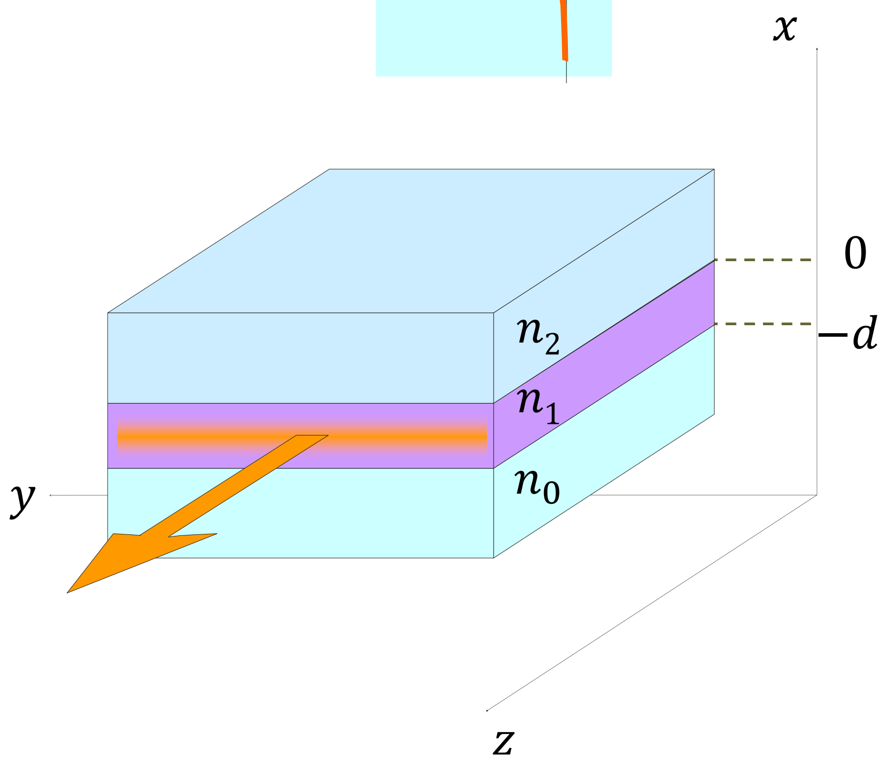
$$\kappa = \sqrt{n_1^2 k_0^2 - \beta^2}$$

$$\gamma = \sqrt{\beta^2 - n_0^2 k_0^2}$$

- Boundary conditions give:

$$\tan\kappa d = \frac{\kappa(\gamma + \delta)}{\kappa^2 - \gamma\delta}$$





Three-layer slab waveguide (2)

- eigenvalue equation for TE modes

$$\tan \kappa d = \frac{\kappa(\gamma + \delta)}{\kappa^2 - \gamma\delta}$$

$\rightarrow$ Discrete solutions for β

where:

$$\delta = \sqrt{\beta^2 - n_2^2 k_0^2}$$

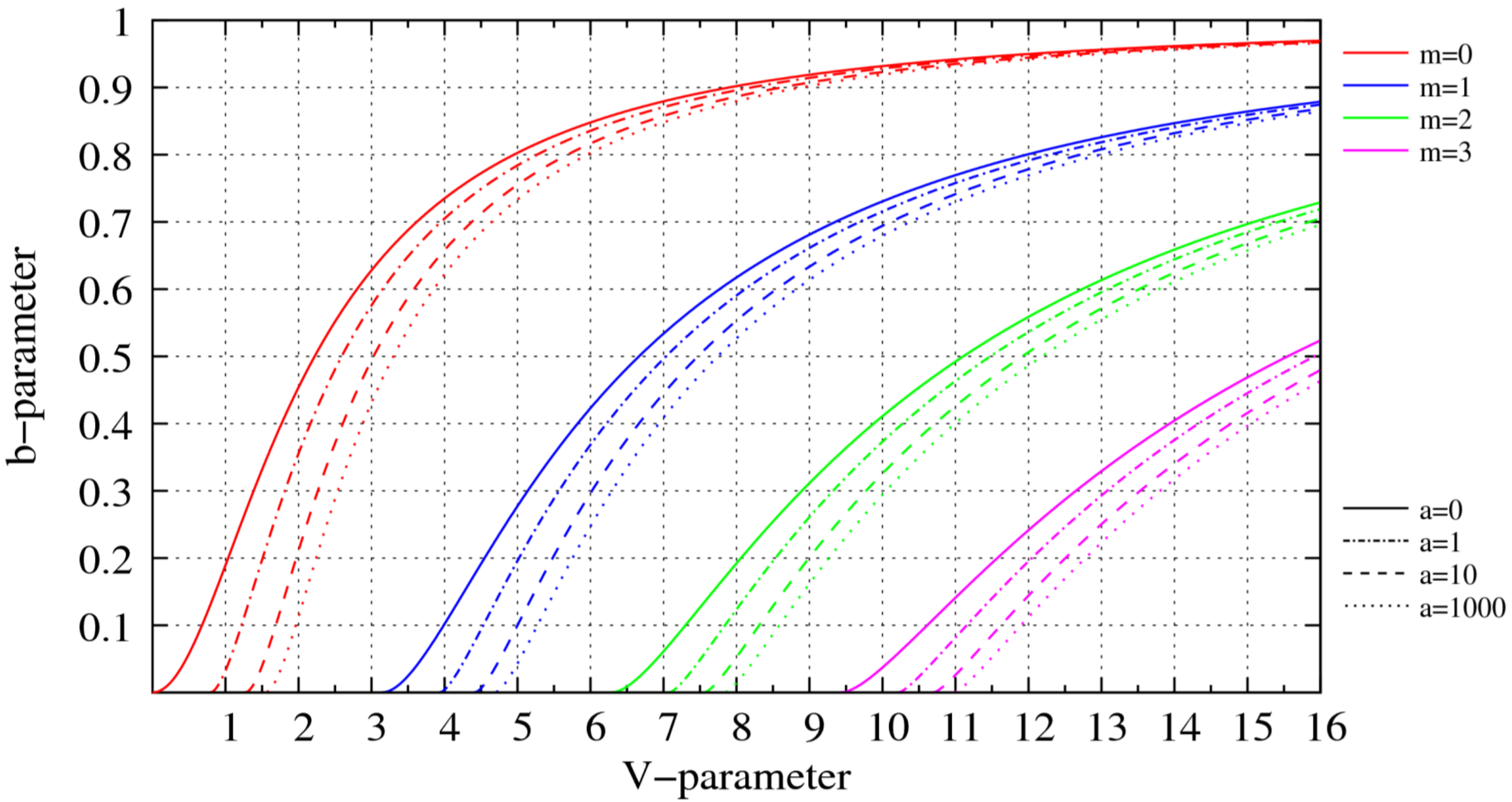
$$\kappa = \sqrt{n_1^2 k_0^2 - \beta^2}$$

$$\gamma = \sqrt{\beta^2 - n_0^2 k_0^2}$$

- Normalized quantities:
  - normalized frequency $V = k_0 d \sqrt{n_1^2 - n_0^2}$
  - relative effective index $b = \dfrac{n_{\text{eff}}^2 - n_0^2}{n_1^2 - n_0^2}$
  - asymmetry parameter $a_{\text{TE}} = \dfrac{n_0^2 - n_2^2}{n_1^2 - n_0^2}$

b-V diagram

- Normalized parameters: $V = k_0 d\sqrt{n_1^2 - n_0^2}$ $\quad b = \dfrac{n_{\mathrm{eff}}^2 - n_0^2}{n_1^2 - n_0^2}$ $\quad a_{\mathrm{TE}} = \dfrac{n_0^2 - n_2^2}{n_1^2 - n_0^2}$



Field distribution for Ey