Proposed Solutions EM II (5EPB0) - Reflection and Refraction

1) Reflection and Refraction A p-polarized wave $\mathbf{E} = \mathbf{A}\exp(-jk_0(x\cos\theta + z\sin\theta))$ is incident at angle of $\theta = \pi/6$ to the normal upon a perfectly conducting medium that is separated from free space by an interface $x = 0$. The frequency $f = 3.183$ GHz. It is also the case that a sum wave of incident plus reflected waves appears to propagate parallel to the interface.

1. What is the wavelength associated with this parallel motion?

$$\vec{E}^{+} = \vec{A}e^{-jk_0(x\cos\theta + z\sin\theta)}$$

p-polarized $\Rightarrow \vec{A} = A_x\vec{a_x} + A_z\vec{a_z}$

$$\vec{E}^{-} = (A_x\vec{a_x} - A_z\vec{a_z})\, e^{-jk_0(-x\cos\theta + z\sin\theta)}$$
$$\vec{E}_1 = A_x\vec{a_x}e^{-jk_0 z\sin\theta}\left[e^{-jk_0 x\cos\theta} + e^{jk_0 x\cos\theta}\right] + A_z\vec{a_z}e^{-jk_0 z\sin\theta}\left[e^{-jk_0 x\cos\theta} - e^{jk_0 x\cos\theta}\right]$$

"Apparent" wave motion propagating // to $x = 0 \propto e^{-jk_0 z\sin\theta}$

$$\lambda_{app} = \frac{2\pi}{k_0\sin\theta} = 2\lambda_0 = \frac{2c_0}{f} = 18.85\ cm$$

2. What is the velocity associated with this motion?

$$c_{app} = \lambda_{app}f = 2\lambda_0 f = 2c_0 = 6e8\ m/s \quad (!?)$$

3. Where can a metal plate be placed so that the wave structure between the two interfaces is unchanged?

z-component must be zero (B.C. @ metal)

$\sin(k_0 x\cos\theta) = 0 \quad \Rightarrow \quad k_0 x\cos\theta = m\pi \quad m = 0, 1, 2, ...$

$$x = \frac{m\pi}{k_0\cos\theta}$$

x-component must be maximum in amplitude (B.C. @ metal)

$|\cos(k_0 x\cos\theta)| = 1 \quad \Rightarrow \quad k_0 x\cos\theta = m\pi \quad m = 0, 1, 2, ...$

$$x = \frac{m\pi}{k_0\cos\theta}$$

2) Reflection and Refraction A uniform time-harmonic plane wave $\mathbf{E}^+ = 10\mathbf{a}_y \exp(-j(6x+8z))$ in V/m is incident from vacuum upon a perfect conductor beyond the interface $x = 0$.

1. What is the polarization of such a wave?

s-polarized

2. Find the frequency f and wavelength λ of the wave.

$k = \omega\sqrt{\varepsilon\mu} = \vec{k} \cdot \vec{k} \quad \Rightarrow \quad f = \frac{kc_0}{w\pi} = 477.46\ MHz; \quad \lambda = \frac{2\pi}{k} = 62.83\ cm$

3. Write down expressions for the actual fields $\mathbf{E}^+(\mathbf{r}, t)$ and $\mathbf{H}^+(\mathbf{r}, t)$.

$$\vec{E}^+(\vec{r}, t) = 10\vec{a_y}e^{-j(6x+8z+\omega t)} \quad [V/m]$$

$$\vec{H}^+(\vec{r}, t) = \frac{\vec{k} \times \vec{E}}{\omega\mu} = \frac{(-80\vec{a_x} + 60\vec{a_z})}{\omega\mu}e^{-j(6x+8z+\omega t)} \quad [A/m]$$

4. What is the angle of incidence θ_1?

$\theta_1 = \arccos\frac{k_x}{k} = \arcsin\frac{k_z}{k} = 53.13°$

5. Find the reflected waves $\mathbf{E}^-$ and $\mathbf{H}^-$.

$$\vec{E}_s^- = -10\vec{a_y}e^{-j(-6x+8z)} \quad \text{[V/m]}$$

$$\vec{H}_s^- = \frac{\vec{k} \times \vec{E}_s^-}{\omega\mu} = \frac{(80\vec{a_x} + 60\vec{a_z})}{\omega\mu}e^{-j(-6x+8z)} \quad \text{[A/m]}$$

6. Find the *total fields* $\mathbf{E}_1$ and $\mathbf{H}_1$

$$\vec{E}_1 = \vec{E}^+ + \vec{E}^- = -20j\vec{a_y}\sin(6x)e^{-j(8z+\omega t)} \quad \text{[V/m]}$$

$$\vec{H}_1 = \vec{H}^+ + \vec{H}^- \quad \Leftrightarrow \quad \vec{H}_1 = \frac{\vec{k} \times \vec{E}_1}{\omega\mu} \qquad \text{(check!)}$$

$$\vec{H}_1 = \frac{10}{\omega\mu}\left[16j\sin(6x)\vec{a_x} + 12\cos(6x)\vec{a_z}\right]e^{-j(8z+\omega t)} \quad \text{[A/m]}$$

3) Radio communication over a lake. Consider a ground-to-air communication system as shown in Figure 1. The receiver antenna is on an aircraft over a huge lake circling at a horizontal distance of ~8 km from a transmitter antenna as it waits for a landing time. The transmitter antenna is located right at the shore mounted on top of a 50-m tower above the lake surface overlooking the lake and transmits a p-polarized signal. The transmitter operates in the VHF band (30 - 300 MHz). The pilot of the aircraft experiences noise (sometimes called *ghosting* effect) in his receiver due to the destructive interference between the direct wave and the ground-reflected wave and needs to adjust his altitude to minimize this interference. Assuming the lake to be flat and lossless with $\varepsilon_r \simeq 79$, calculate the critical height of the aircraft in order to achieve clear transmission between the transmitter and the receiver.

We need to look for Brewster's angle θ_B

$$\theta_B = \arctan\sqrt{\varepsilon_r} \approx 1.4588 \text{ rad} \approx 83.6°$$
$$\tan\theta_B = \sqrt{\varepsilon_r} = \frac{L_1}{50} = \frac{L_2}{h} \qquad L_1 + L_2 = 8 \text{ km}$$
$$\Rightarrow h \approx 850 \text{ m}$$

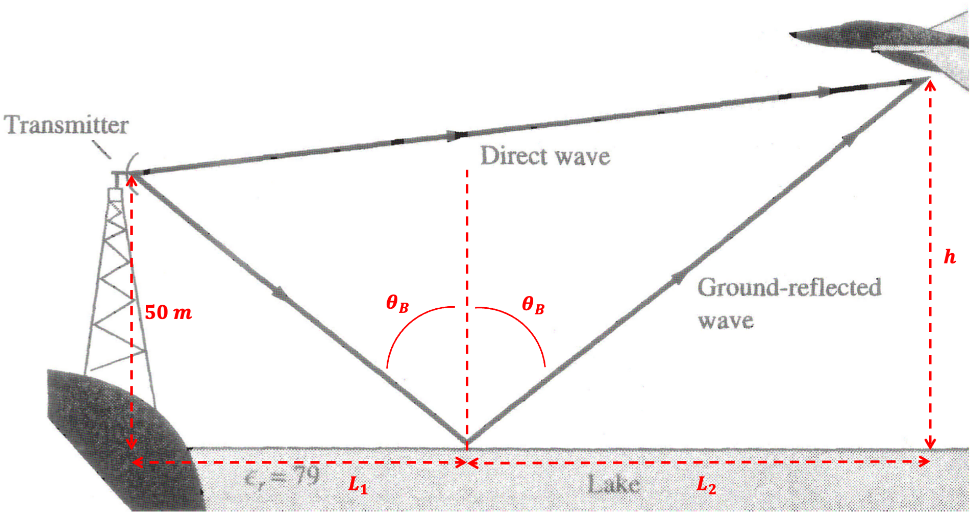

Figure 1: Communication over a lake.

Book. Solve problems 12.10, 12.13, 12.14, 12.21, 12.24 and 12.25.

12.10

a) $\varepsilon'_{r1} = 9$; b) $\varepsilon'_{r1} = 34$; c) $\varepsilon'_{r1} = 4$

12.14

a)

$$\vec{E}_1 = \vec{E}^+ + \vec{E}^- = E_0(\vec{a_x} + j\vec{a_y})e^{-j\beta z} + (-E_0)(\vec{a_x} + j\vec{a_y})e^{j\beta z}$$
$$\vec{E}_1 = 2E_0 \sin(\beta z)(\vec{a_y} - j\vec{a_x})$$

b)

$$\mathrm{Re}\left\{\vec{E}_1 e^{j\omega t}\right\} = 2E_0 \sin(\beta z)\left[\cos(\omega t)\vec{a_y} + \sin(\omega t)\vec{a_x}\right]$$

c) It is a LHCP standing wave of amplitude $2E_0 \sin(\beta z)$. Since standing waves do not propagate, handedness is defined by using the (common) +z direction as a reference...

12.24

"Entry" angle and "exit" angle must be Brewster (no reflection loss).
From the geometry we see that $\theta_B^2 = \frac{\alpha}{2}$

$$\Rightarrow \alpha = 2 \arcsin\left(\frac{1}{\sqrt{1+n^2}}\right) \approx 1.21 \text{ rad} \approx 69.2°$$

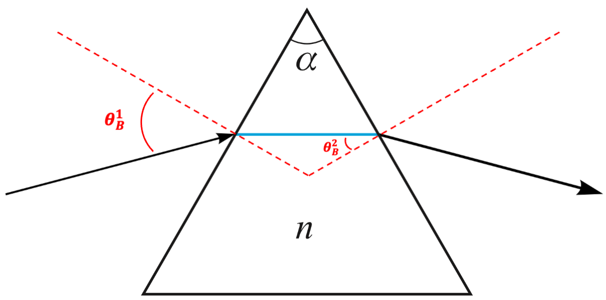

Figure 2: Problem 12.24