Communication Theory (5ETB0)
Module 6.2

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Module 6.2

Presentation Outline

Motivation for Correlation Receiver

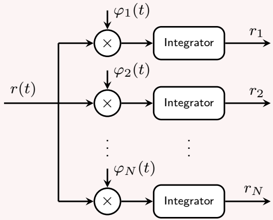
Recovery of Signal Vectors

Gram-Schmidt: $s_m(t) = \sum_{i=1}^{N} s_{mi}\varphi_i(t)$

Components r_i are determined as follows:

$$r_i \triangleq \int_{-\infty}^{\infty} r(t)\varphi_i(t)dt$$

for $i = 1, 2, \ldots, N$



Optimum Receiver

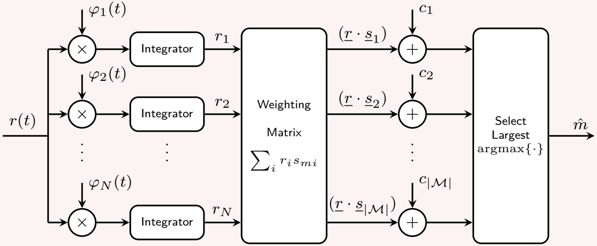
The optimum receiver applies the rule

$$\hat{m}^{\text{MAP}} = \operatorname*{argmax}_{m\in\mathcal{M}} \{(\underline{r}\cdot\underline{s}_m) + c_m\}, \qquad c_m = \frac{N_0}{2}\ln \Pr\{M=m\} - \frac{E_m}{2}$$

Correlation Receiver

Correlation Receiver: Implementation with Matrix Multiplications

Structure:

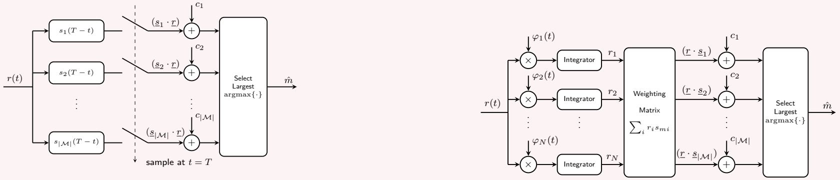


Dot products:

$$\begin{pmatrix} (\underline{r} \cdot \underline{s}_1) \\ (\underline{r} \cdot \underline{s}_2) \\ \vdots \\ (\underline{r} \cdot \underline{s}_{|\mathcal{M}|}) \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{|\mathcal{M}|1} & s_{|\mathcal{M}|2} & \cdots & s_{|\mathcal{M}|N} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{pmatrix}$$

Direct Receiver vs. Correlation Receiver

Two Receivers Side-by-side



Three Questions

- Q1: Which receiver is simpler in terms of $\mathrm{argmax}\{\cdot\}$?
- Q2: Which receiver is simpler in terms of filters?
- Q3: Can we always guarantee this?

Example: $s_m(t) = m \cdot p(t)$, $m = 1, 2, 3, \ldots, 128$ and $0 < T < 1$ ps (transmission rate is 7 Gbps). In this case, $N = 1$ and $|\mathcal{M}| = 128 \Rightarrow$ x100 simpler

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Presentation Outline

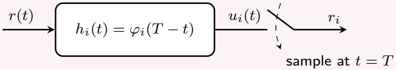
Matched-Filter Receiver

Two Important Concepts

Integral of a multiplication = dot product = Filter+sampling (M6.1)

r-values: Integral of multiplication ($r(t)$ and $\varphi_i(t)$)

Matched Filter Receiver

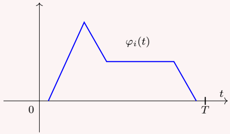


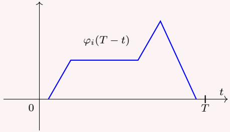
$$u_i(t) = \int_{-\infty}^{\infty} r(\alpha) h_i(t-\alpha) d\alpha$$

$$= \int_{-\infty}^{\infty} r(\alpha) \varphi_i(T-t+\alpha) d\alpha$$

$$\stackrel{t=T}{=} \int_{-\infty}^{\infty} r(\alpha) \varphi_i(\alpha) d\alpha = r_i$$

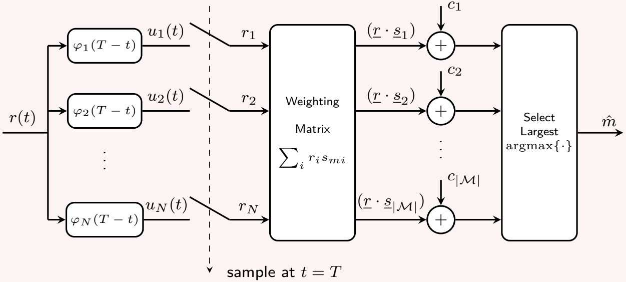
Building-block $\varphi_i(t)$ and impulse response $\varphi_i(T-t)$:





Matched-Filter Receiver

Matched-filter Receiver Structure



Two Questions

- Q1: Is the matched-filter receiver simpler than the direct receiver?
- Q2: What are differences between the matched-filter and correlation receiver?

Who Cares? We do!

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Optimum Detection in Presence of Nonlinear Distortions with Memory

Gabriele Liga[(1)], Alex Alvarado[(1)], Erik Agrell[(2)], Marco Secondini[(3)], Robert I. Killey[(1)] and Polina Bayvel[(1)]

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Abstract *The performance of nonlinearity-tailored detection for single channel, single span optical fibre systems is studied. Monotonically decreasing bit error rate with transmitted power can be achieved without any nonlinearity compensation.*

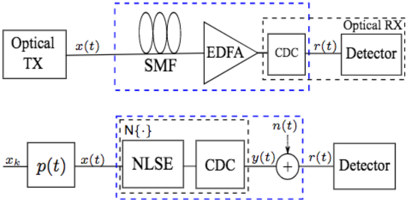

Fig. 1: Single span system and equivalent channel schematic.

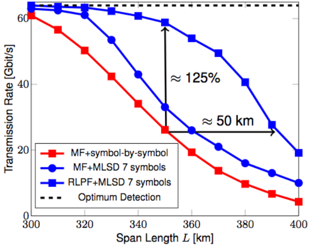

Fig. 4: Achievable transmission rates vs. L for different detection strategies.

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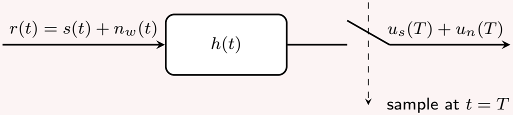
Presentation Outline

Signal and Noise Components

AWGN Through Linear Filter



$$u(T) = \int_{-\infty}^{\infty} r(T-\alpha)h(\alpha)d\alpha = u_s(T) + u_n(T)$$

with

$$u_s(T) \triangleq \int_{-\infty}^{\infty} s(T-\alpha)h(\alpha)d\alpha$$

$$u_n(T) \triangleq \int_{-\infty}^{\infty} n_w(T-\alpha)h(\alpha)d\alpha$$

Signal-to-Noise Ratio (SNR)

SNR Definition

We can now define the *signal-to-noise ratio* as

$$\text{SNR} \triangleq \frac{u_s^2(T)}{E[U_n^2(T)]} \tag{1}$$

Noise variance:

$$\begin{aligned}
E[U_n^2(T)] &= E\left[\int_{-\infty}^{\infty} N_w(T-\alpha)h(\alpha)d\alpha \int_{-\infty}^{\infty} N_w(T-\beta)h(\beta)d\beta\right] \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[N_w(T-\alpha)N_w(T-\beta)]h(\alpha)h(\beta)d\alpha d\beta \\
&= \frac{N_0}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\beta-\alpha)h(\alpha)h(\beta)d\alpha d\beta \\
&= \frac{N_0}{2}\int_{-\infty}^{\infty} h^2(\alpha)d\alpha.
\end{aligned}$$

Matched Filter Maximizes the SNR

Schwarz Inequality

For two finite-energy waveforms $a(t)$ and $b(t)$ the inequality

$$\left(\int_{-\infty}^{\infty} a(t)b(t)dt\right)^2 \leq \int_{-\infty}^{\infty} a^2(t)dt \int_{-\infty}^{\infty} b^2(t)dt \tag{2}$$

holds. Equality is obtained only if $b(t) \equiv Ca(t)$ for some constant C.

Result: Maximum Attainable SNR

$$\text{SNR} = \frac{\left[\int_{-\infty}^{\infty} s(T-\alpha)h(\alpha)d\alpha\right]^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} h^2(\alpha)d\alpha} \leq \frac{\int_{-\infty}^{\infty} s^2(T-\alpha)d\alpha \int_{-\infty}^{\infty} h^2(\alpha)d\alpha}{\frac{N_0}{2}\int_{-\infty}^{\infty} h^2(\alpha)d\alpha}$$

$$= \frac{\int_{-\infty}^{\infty} s^2(T-\alpha)d\alpha}{\frac{N_0}{2}} = \frac{E_s}{\frac{N_0}{2}}$$

The Matched-filter Receiver not only minimizes P_e but it also maximizes the SNR!

Summary Module 6.2

Take Home Messages

- Correlation receiver
- Matched-filter receiver (correlation receiver using filters)
- Comparison of the three receivers: direct, correlation and matched-filer
- SNR and optimality of Matched-filter receiver

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