1. **Interpreting Bode diagrams:**
Given the bode diagram in Figure 1 representing a dynamical system, $G(s)$, sketch the output trajectory, $y(t)$, if this system is given a sinusoidal input with a frequency of 1 rad/s and amplitude of 1, e.g. $u(t) = \sin(t)$. Verify your sketch using MATLAB.

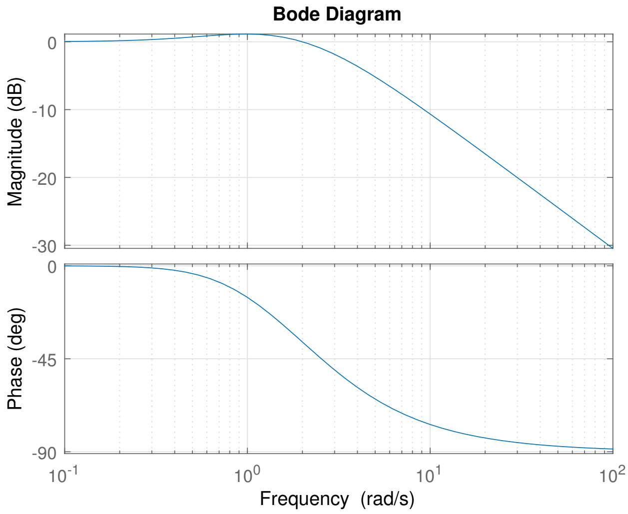

Figure 1: Bode diagram of $G(s)$ for problem 1.

Solution:
From the Bode diagram we have that for a frequency of 1 rad/s we have a slight amplification (magnitude over 0 dB) and a bit of phase lag (phase below 0 deg). Hence we expect our output, $y(t)$, to have an amplitude greater than 1 and it should be lagging behind our input, $u(t)$. Indeed, if we use `lsim` in MATLAB, we see that this is indeed the case, see Figure 2.

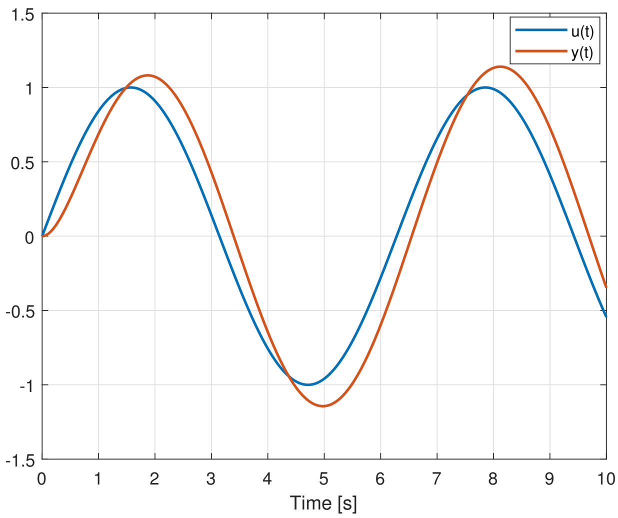

Figure 2: Input and output trajectory for exercise 1.

2. **Drawing Bode diagrams (Exam Level Question):**
Sketch Bode plots for the open-loop transfer functions $G(s)$ using the drawing rules. Then, verify your results using MATLAB.

   (a) $G(s) = \frac{100}{s(0.1s+1)(0.5s+1)}$,

(b) $G(s) = \frac{(s+2)}{s(s+1)(s+5)(s+10)}$,

(c) $G(s) = \frac{5000}{(s+7)(s+18)^3}$,

(d) $G(s) = \frac{s+5}{s^2+4s+5}$,

(e) $G(s) = \frac{s^2+2s+8}{s(s^2+2s+10)}$,

(f) $G(s) = \frac{s+2}{s(s-1)(s+6)^2}$,

(g) $G(s) = \frac{s^2+1}{s(s+1)}$,

(h) $G(s) = \frac{s+1}{s^2(s+5)(s+20)}$,

(i) $G(s) = \frac{1}{s(s^2+s+42.5)}$,

(j) $G(s) = \frac{s^2+s+12.5}{s^2(s+20)(s+10)}$.

Solution:

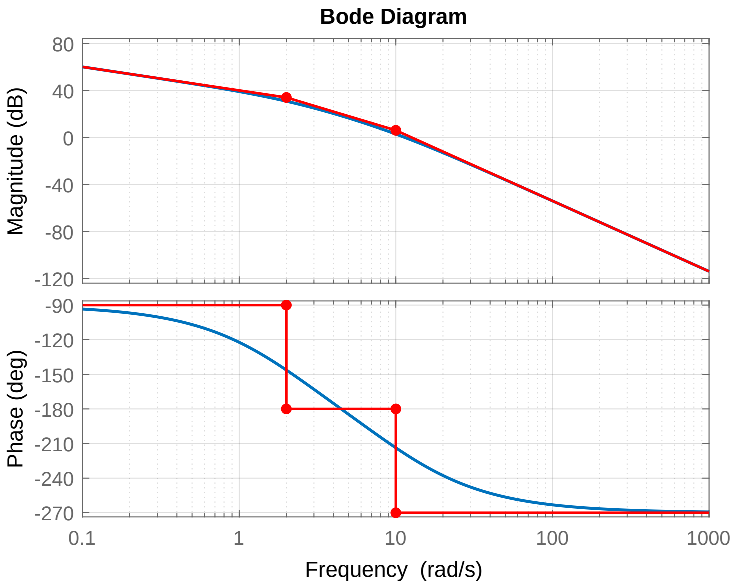

Figure 3: $G(s) = \frac{100}{s(0.1s+1)(0.5s+1)}$

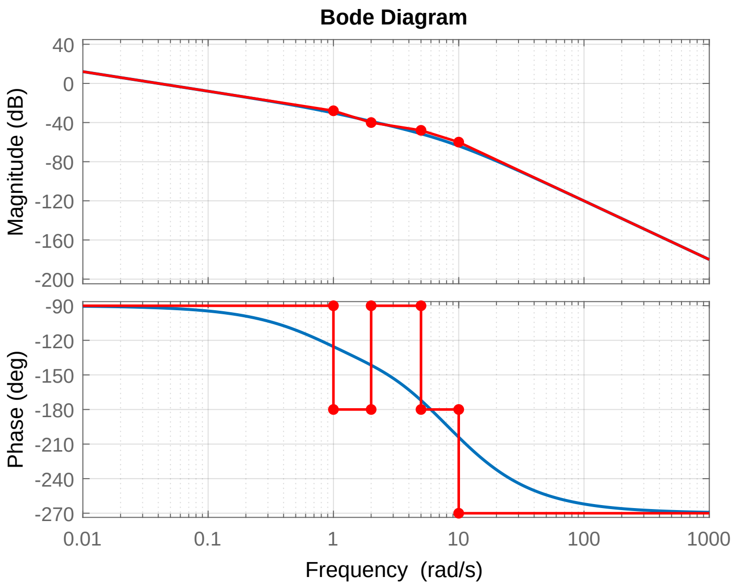

Figure 4: $G(s) = \frac{(s+2)}{s(s+1)(s+5)(s+10)}$; Bode form: $G(s) = \frac{\frac{1}{25}(\frac{s}{2}+1)}{s(s+1)(\frac{s}{5}+1)(\frac{s}{10}+1)}$.

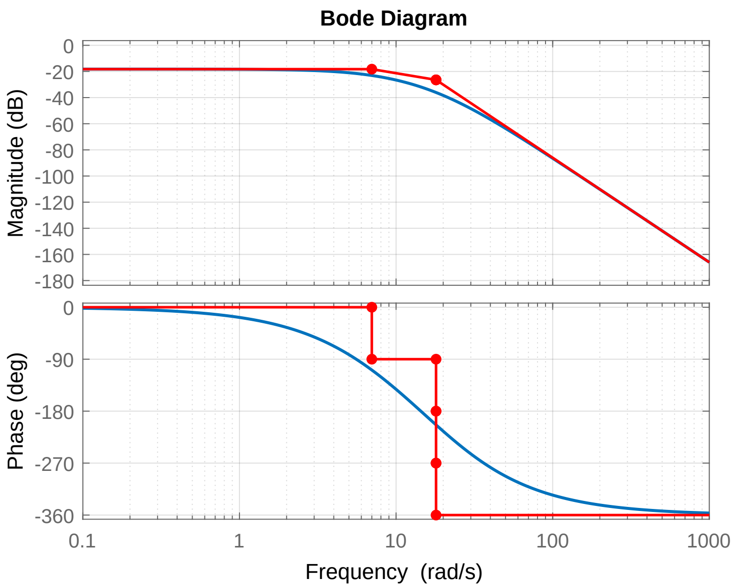

Figure 5: $G(s) = \frac{5000}{(s+7)(s+18)^3}$; Bode form: $G(s) = \frac{0.12248}{(1+s/18)^3(1+s/7)}$.

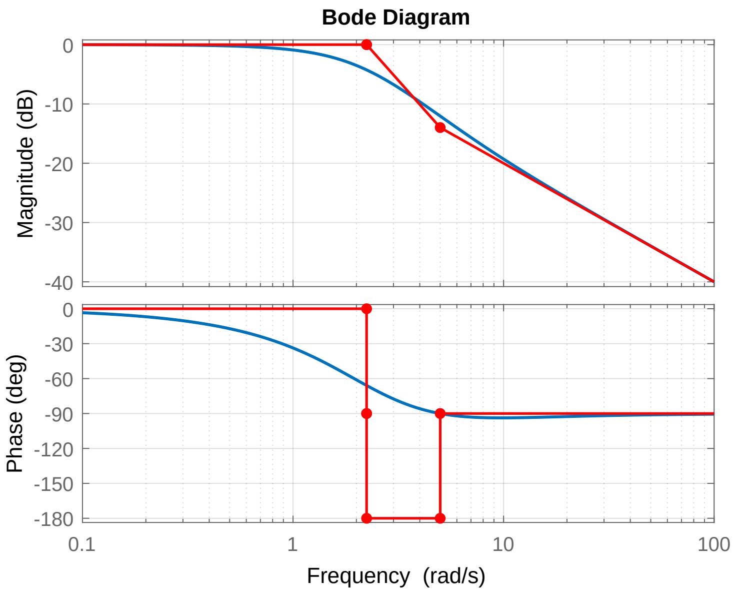

Figure 6: $G(s) = \frac{s+5}{s^2+4s+5}$; Bode form: $G(s) = \frac{(1+s/5)}{1+1.789(s/2.236)+(s/2.236)^2}$.

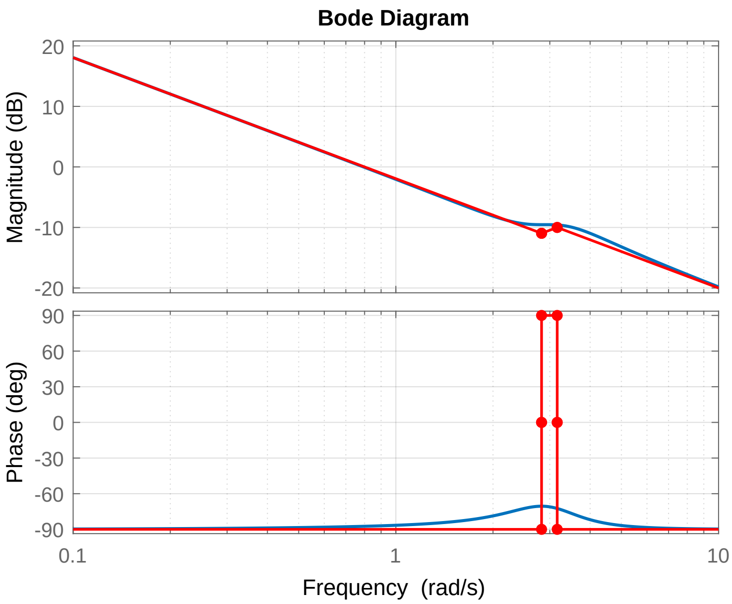

Figure 7: $G(s) = \frac{s^2+2s+8}{s(s^2+2s+10)}$; Bode form: $G(s) = \frac{4}{5}(s^{-1})\frac{1+s/4+s^2/8}{1+s/5+s^2/10}$

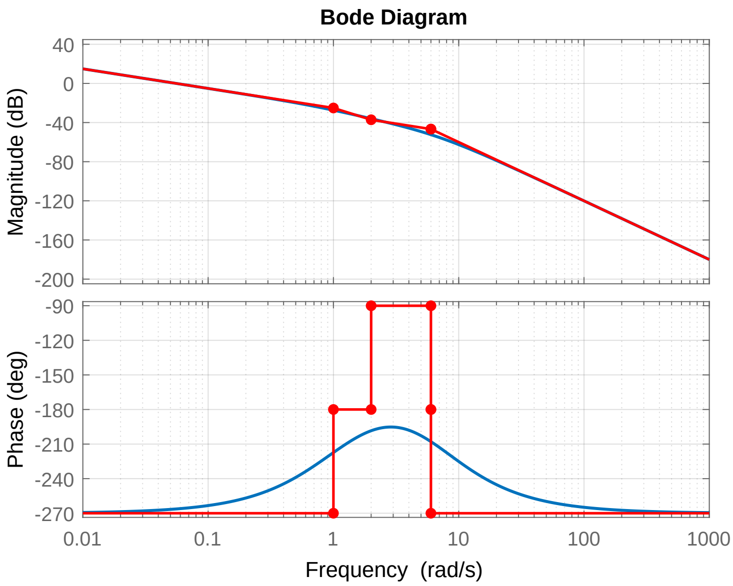

Figure 8: $G(s) = \frac{s+2}{s(s-1)(s+6)^2}$; Bode form: $G(s) = -\frac{1}{18}\frac{1+s/2}{s(1-s)(1+s/6)^2}$.

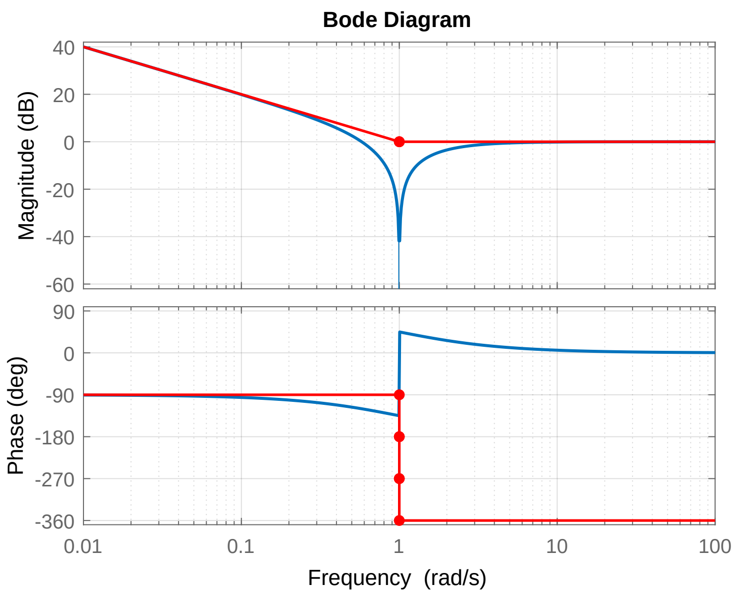

Figure 9: $G(s) = \frac{s^2+1}{s(s+1)}$.

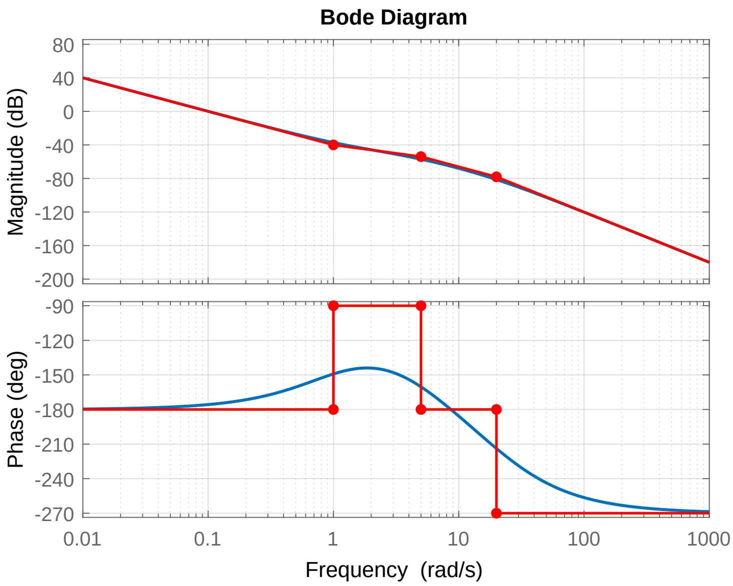

Figure 10: $G(s) = \frac{s+1}{s^2(s+5)(s+20)}$; Bode form: $G(s) = \frac{0.01(1+s)}{s^2(1+s/20)(1+s/5)}$

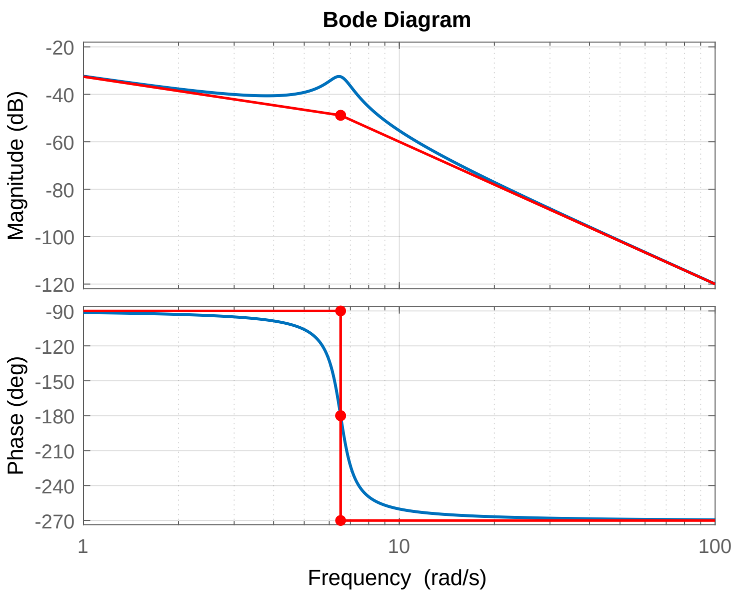

Figure 11: $G(s) = \frac{1}{s(s^2+s+42.5)}$; Bode form: $G(s) = \frac{0.023529}{s(1+0.1534(s/6.519)+(s/6.519)^2)}$

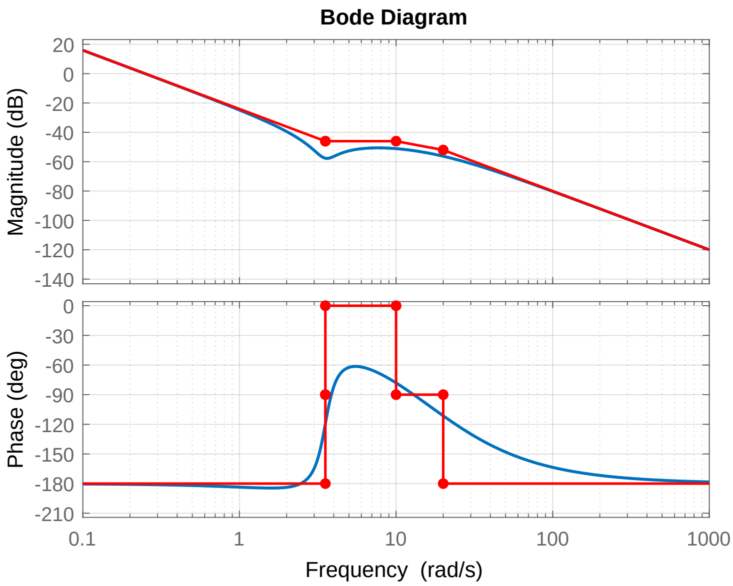

Figure 12: $G(s) = \frac{s^2+s+12.5}{s^2(s+20)(s+10)}$; Bode form: $G(s) = \frac{0.0625(1+0.2828(s/3.536)+(s/3.536)^2)}{s^2(1+s/20)(1+s/10)}$

3. **System type and steady state error from Bode plots:**
Consider the closed-loop system (as shown in Figure 13), generated by the open-loop transfer function:

$$K\,G(s) = 10\,\frac{s+1}{s^2(s+2)},$$

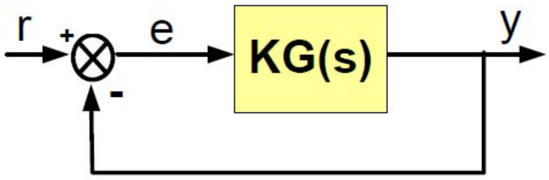

Figure 13: Block diagram for Problems 2 and 3.

- Sketch the bode plot of $K\,G(s)$
- Make a low frequency approximation of $K\,G(s)$
- Sketch the bode plot of the low frequency approximation of $K\,G(s)$ and determine the system type
- Give the steady state error for the closed-loop system for a step, ramp and parabolic input

Solution:
Writing $K\,G(s)$ in Bode form gives

$$K\,G(s) = 5\frac{1+s}{s^2\left(1+\frac{1}{2}s\right)}$$

resulting in the Bode diagram given by Figure 14.

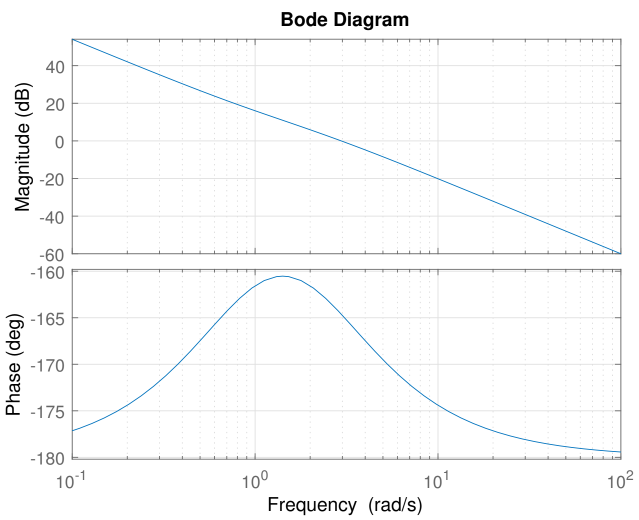

Figure 14: Bode plot of $K\,G(s)$

Using the Bode form we can obtain a low frequency approximation of the form

$$K\,G(j\omega) = \frac{5}{(j\omega)^2}$$

It follows then that for low frequency the slope of magnitude is -40 dB/dec, hence, the system type is 2.

From the block diagram we have that

$$\begin{aligned} E(s) &= R(s) - Y(s) \\ &= R(s) - K\,G(s)E(s) \\ &= \frac{1}{1 + K\,G(s)} R(s) \end{aligned}$$

Pluggin in the low frequency approximation and applying the FVT we get

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1+\frac{5}{s^2}} R(s) = \lim_{s\to 0} \frac{s^3}{s^2+5} R(s)$$

Note that for $R(s) = 1/s$ (step) and $R(s) = 1/s^2$ (ramp) we get $e_{ss} = 0$. Instead, for $R(s) = 1/s^3$ (parabola), we have that

$$e_{ss} = \lim_{s\to 0} \frac{s^3}{s^2+5} \frac{1}{s^3} = \frac{1}{5} \neq 0$$