Communication Theory (5ETB0)
Module 5.1

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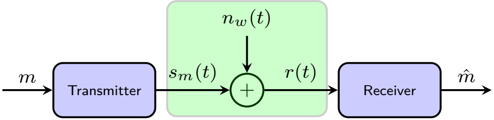
Module 5.1

Presentation Outline

System Description and AWG Noise



Definitions

- Transmitter: Chooses waveform $s_m(t)$ when message m is to be transmitted. Set of used waveforms: $s_1(t), s_2(t), \ldots, s_{|\mathcal{M}|}(t)$.
- Waveform Channel: Accepts input $s_m(t)$ and adds Gausian noise $n_w(t)$ such that

$$r(t) = s_m(t) + n_w(t)$$

  Autocorrelation function of noise process:

$$R_{N_w}(t,s) \stackrel{\Delta}{=} E[N_w(t)N_w(s)] = \frac{N_0}{2}\delta(t-s),$$

- Receiver: Forms an *estimate* $\hat{m}$ based on received waveform $r(t)$.

Module 5.1

Presentation Outline

Waveforms: Energy, Orthogonality, and Orthonormality

Energy of a waveform

The energy of a waveform $x(t)$ is defined as

$$E_x \triangleq \int_{-\infty}^{\infty} x^2(t)dt.$$

Orthogonality and orthonormality

The waveforms $\varphi_i(t), i = 1, \ldots, N$ are said to be **orthogonal** if

$$\int_{-\infty}^{\infty} \varphi_i(t)\varphi_j(t)dt = \begin{cases} E_i & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

If $E_i = 1$ the waveforms are said to be **orthonormal**

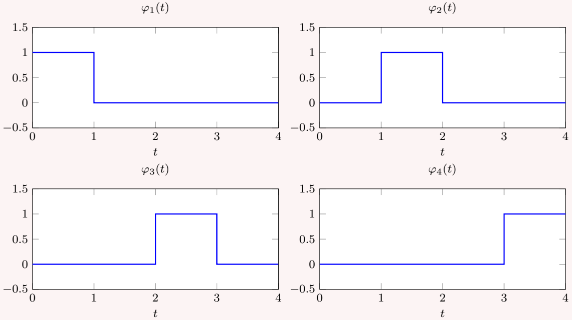
Example 5.1

Time-translated Orthogonal Pulses

Four building-block waveform, time-translated orthogonal pulses:

Question: Are the pulses orthonormal?

$$\int_{-\infty}^{\infty} \varphi_i(t)\varphi_j(t)dt = ?$$



Module 5.1

Presentation Outline

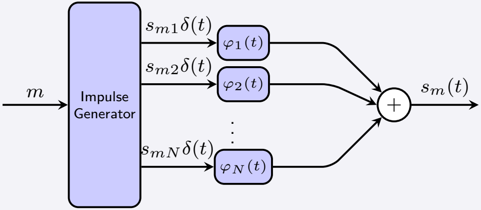
Waveform Synthesis

Waveform Synthesis: From Vectors to Signals

Assume the signal waveform $s_m(t)$ can be expressed as

$$s_m(t) = \sum_{i=1}^{N} s_{mi}\varphi_i(t), \text{ for } m \in \mathcal{M} = \{1, 2, \ldots, |\mathcal{M}|\}.$$

where $\varphi_i(t)$ are called **building-block waveforms**, which are assumed to be *orthonormal*. Signals $s_m(t)$ can be synthesized as:



Canonical transmitter is based on N **building-block waveforms**

Example 5.2

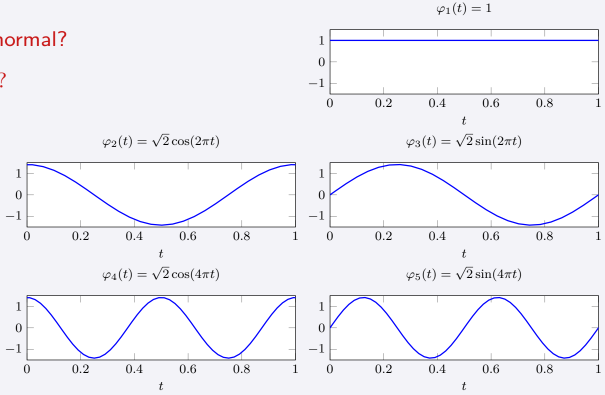
Sine and Cosine Waves

Five building-block waveforms: a pulse with amplitude 1 and four sine and cosine waves. Waveforms are zero outside this time interval.

Question:

Are the pulses orthonormal?

$$\int_{-\infty}^{\infty} \varphi_i(t)\varphi_j(t)dt = ?$$



Summary Module 5.1

Take Home Messages

- Waveform channels are of great practical importance
- AWGN Channel
- Two signal properties: Energy and Orthogonality/Orthonormality
- From vectors to signals using building-block waveforms

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