Em2 - Part - 1 - ①Transmission lines - ②Scatter/Transfer Matrices (Matching and radio telescopes) - Slab waveguide - ③Antenna stab - Time-harmonic 1D waves - Power splitting/matrices.

$\varepsilon_0 = 8.85 \cdot 10^{-12}$
$\mu_0 = 4\pi \cdot 10^{-7}$

characteristic impedance

- propagation speed: $v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{LC}}$; • impedance $Z_0 = \sqrt{\frac{\varepsilon}{\mu}} = \sqrt{\frac{L}{C}} = 50\Omega$ usu
- matched impedance: $Z = 50\Omega$; load impedance: Z_L
- $\Gamma = V^-/V^+ = \frac{Z_L - Z_0}{Z_L + Z}$ • matched: $Z_L = Z \rightarrow \Gamma = 0$.
- open circuit: $Z_L = \infty \rightarrow \Gamma = 1$ • short circuit: $Z_L = 0 \rightarrow \Gamma = -1$
- $T = 1 + \Gamma$

Circuit analysis of $t=0$, t, $t=\infty$

- Z_g = source/generator impedance. → starts forward wave → no reflection if $Z_g = Z_0$ → at start of TL.
- Z_0 = characteristic impedance → wave prop → intrinsic → along TL.
- Z_L = load/output impedance → wave terminating → no reflection if $Z_L = Z_0$ → end of TL.
- before $t=0$: no signal; $V=0$; $I=0$. $I^+ = \frac{V_g}{Z_0 + Z_g}$
- immediate after on: V_g launch wave: $V^+ = \frac{Z_0}{Z_0 + Z_g} V_g$ forward propagation
- after reflection occurs: $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ → backward propagation: $V^- = \Gamma V^+$
- $V = V^+ + V^-$; $I = I^+ - I^- = \frac{V^+}{Z_0} - \frac{V^-}{Z_0}$ → traveling V, I.
- $t = \infty$ → steady state → no traveling wave → $V_{steady} = \frac{Z_L}{Z_L + Z_g} V_g$; $I_{steady} = \frac{V_g}{Z_L + Z_g}$
- Summary: $V(z,t)$ → total field from wave V^+, V^- superposition
  - $V^\pm$ → travelling wave components.
  - V_g → generator/supply voltage.
  - V_{steady} → final circuit value

! $\boxed{I_{steady} = \frac{V_g}{Z_L + Z_g}}$

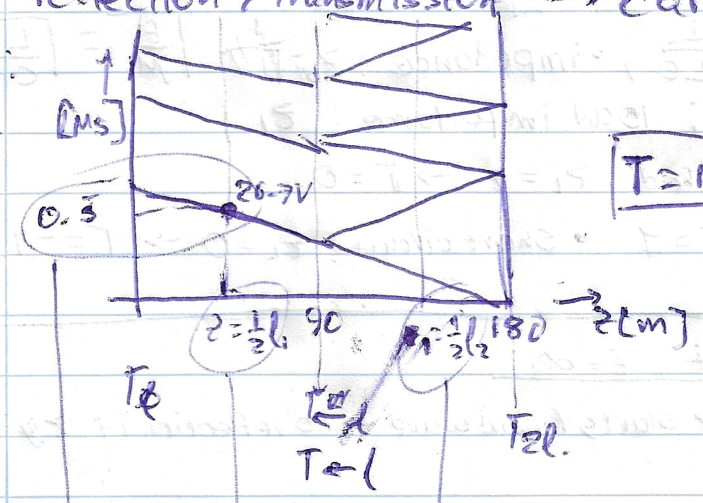
- Standing wave ratio: $V_{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$
- during the stage of reflections if we know V, I and want to decompose:
  $V^+ = 1/2 (V + Z_0 I)$; $V^- = 1/2 (V - Z_0 I)$
  ↳ no need to memorize its just derived from V, I eqs.
- $V = V^+ + V^-$, $I = \frac{1}{Z_0}(V^+ - V^-)$ → $V + Z_0 I = (V^+ + V^-) + (V^+ - V^-) = 2V^+$
  → $\frac{1}{2}(V + Z_0 I) = V^+$ //.
- note: Steady state V_{st}, I_{st} can be decomposed into V^+ and V^-
  - when there is multiple transmission lines means there is $V_1^+, V_1^-; V_2^+, V_2^-$

Bounce diagrams and time voltage plots

1. • time period of bounce propagation: $t = l/c$
2. • reflection / transmission → calculated.

plot:

eg 2TL

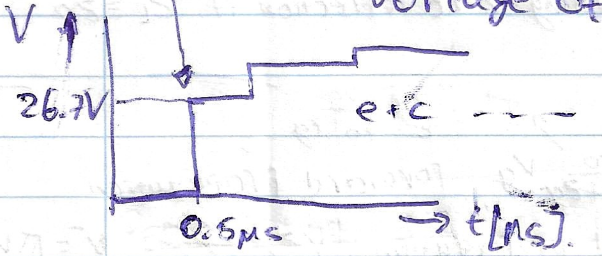


$\Gamma_{1l}, \Gamma_{2l}, \Gamma_{12l}, \Gamma_{\to l}, \Gamma_{\leftarrow l}$

$\downarrow T_{\to l}$ $\quad \downarrow T_{\leftarrow l}$

$\boxed{T = 1 + \Gamma}$

voltage of half way plots.



~~plot & same~~

1 plot per TL.

• [remember bounce diagram starts from where switch closes].

~~$|\Gamma|^2 \to$ Power~~

~~$|\Gamma|^2 \cdot 100 \to$ power % loss~~

$P_{loss} = |\Gamma|^2 \cdot 100\%$

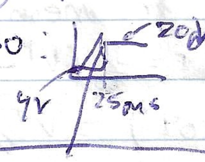
$P_{gain} = |1 - |\Gamma|^2| \cdot 100\%$

• bounce diagram start with V_{comp}.
↓
but when drawing graph

you start with whats before

eg: 4V before → $V_{comp} = 16V$ at 25ns

so:



• Compensating voltage is [after toggle there is new voltage → ~~Vcom~~ V_{comp} is $V_{before} \pm V_{comp} = V_{after}$.

eg: $V_{before} = 4V \to V_{after} = 20V$ $\quad V_{comp} = +16V$

Part I

Scatter and Transfer Matrices - Matching

- Impedance Matching (quarter-wave transformer).
- $Z_2 = \sqrt{Z_1 Z_3}$ $\quad l = \frac{\lambda}{4}$ $\rightarrow c = \lambda \cdot f \rightarrow \lambda = \frac{c}{f}$

Z match of stub. $\quad$ Z generator $\quad$ Z load $\quad$ → only quarter wave stubs.

A-matrix $\rightarrow A = D_1 T C_2 = \frac{1}{2}\begin{pmatrix} 1 & Z_1 \\ 1 & -Z_1 \end{pmatrix}\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ Y_2 & -Y_2 \end{pmatrix}$

(T ↓ can be multiple)

↳ • Start with decomposition matrix

$A = \begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & Z_1 \\ 1 & -Z_1 \end{pmatrix}\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$.

• decomposition matrix

$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = T_2 T_3 \begin{pmatrix} V_4 \\ I_4 \end{pmatrix}$ → eg.

↳ $A = \begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & Z_1 \\ 1 & -Z_1 \end{pmatrix} T_2 T_3 \begin{pmatrix} V_4 \\ I_4 \end{pmatrix}$.

• composition matrix

$\begin{pmatrix} V_4 \\ I_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ Y_2 & -Y_2 \end{pmatrix}\begin{pmatrix} V_4^+ \\ V_4^- \end{pmatrix}$

↳ $A = \begin{pmatrix} V^+ \\ V^- \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & Z_1 \\ 1 & -Z_1 \end{pmatrix} T_2 T_3 \begin{pmatrix} 1 & 1 \\ Y_2 & -Y_2 \end{pmatrix}\begin{pmatrix} V_4^+ \\ V_4^- \end{pmatrix}$.

↳ so now we described 1 parts of propagating waves using the other.

• $T = \begin{pmatrix} \cos(\kappa l) & jZ \sin(\kappa l) \\ jY \sin(\kappa l) & \cos(\kappa l) \end{pmatrix}$ $\rightarrow$ $\kappa = \frac{2\pi}{\lambda}$, l = length or $\frac{\lambda}{4}$ for quarter stub.

• how is matching achieved → even though diff impedances → its not 1 wave its multiple waves → add em up → matches

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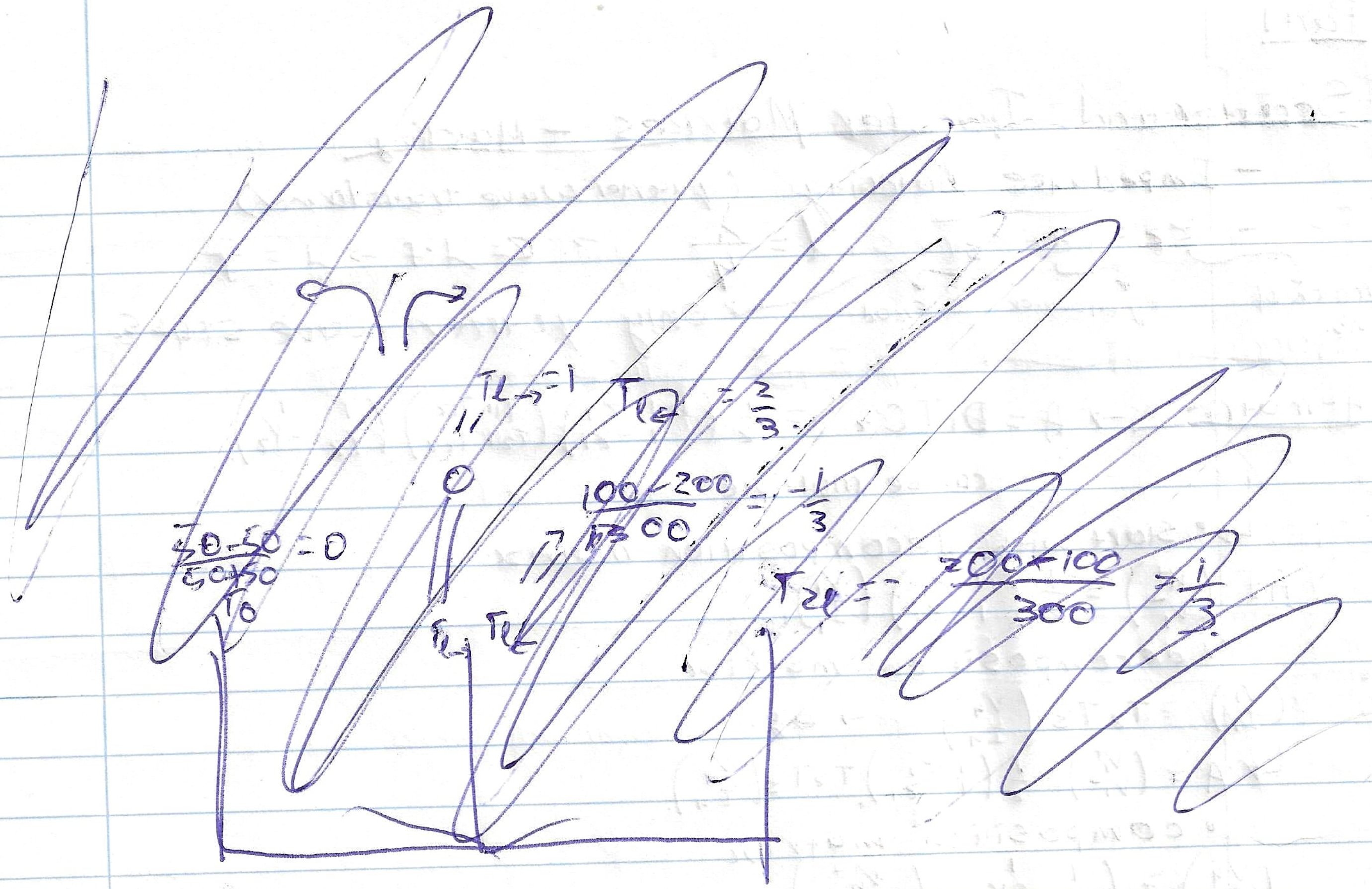
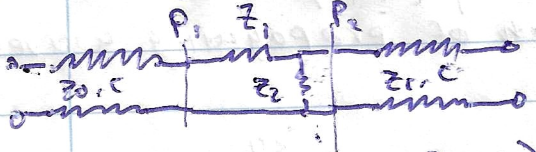


<u>Resist Aug 2022</u> $Z_0, Z_1 = 50\Omega$



$T = \begin{bmatrix} 1.5 & 100 \\ 0.005 & 1 \end{bmatrix}$. Find Z_1, Z_2

Tseries: $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$.

Tshunt: $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$.

Tseries · Tshunt $= \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$

↳ $Z_1 = 100\Omega$, $Z_2 = 200\Omega$.