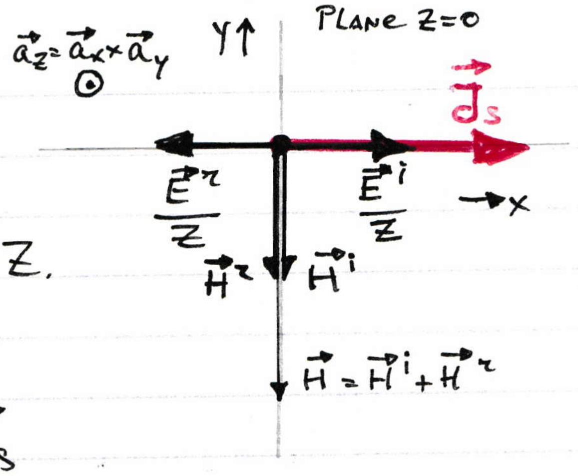
EM II Chapter 1

Question 1.1



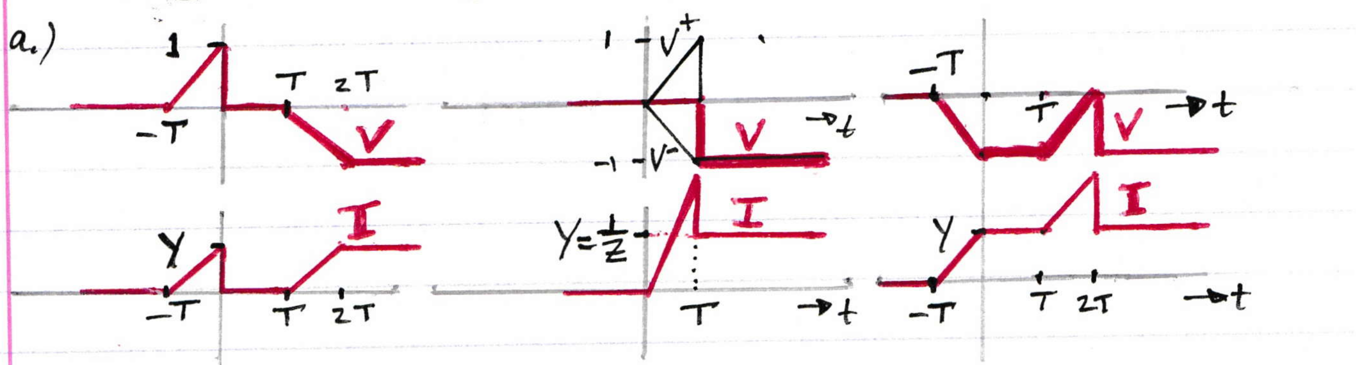
Plane wave $\vec{E}^i = V^i(z,t)\,\vec{a}_x$, travelling in the negative z-direction towards a PEC plate at z=0. Wave impedance Z. Reflected wave $\vec{E}^r = V^r(z,t)\,\vec{a}_x$

$$\vec{\nu} \times \vec{H}_1 = -\vec{J}_s \Rightarrow -\vec{a}_z \times (\vec{H}^i + \vec{H}^r) = -\vec{J}_s$$

Question 1.2

$v^+(t) = [u(t) - u(t-T)]\frac{t}{T}$ [V], $\quad V^-(t) = -u(t)\frac{t}{T} + u(t-T)(\frac{t}{T} - 1)$ [V]

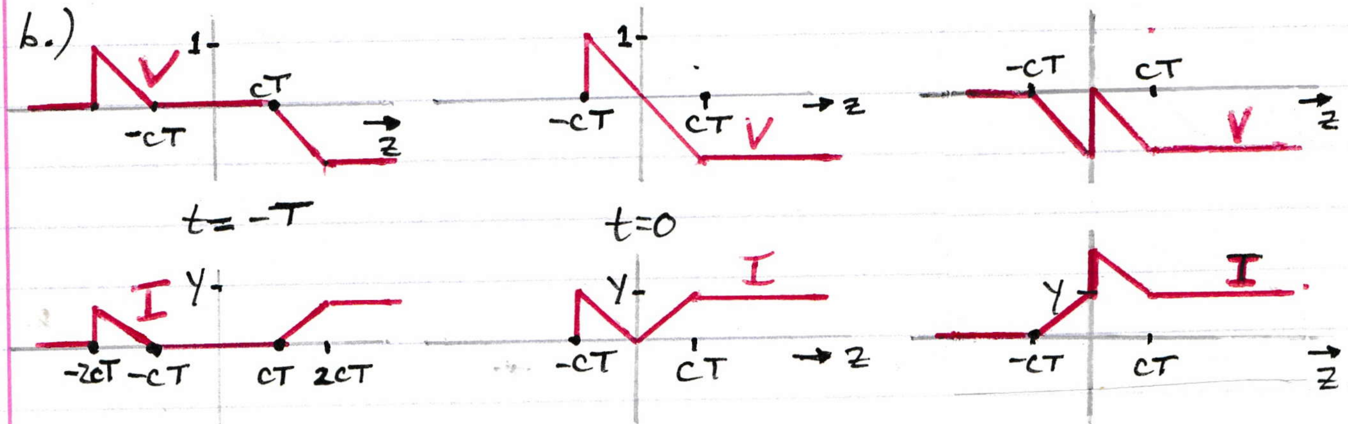
a.)



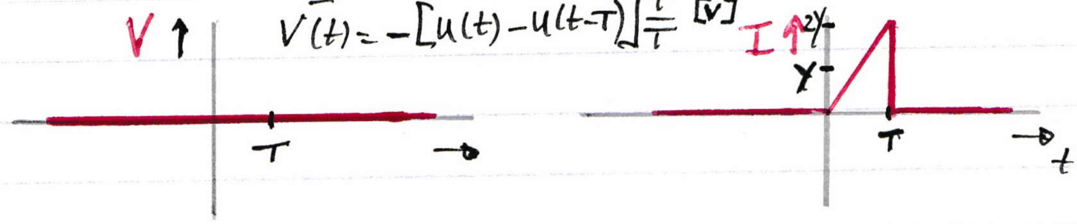
z=-cT $\quad$ z=0 $\quad$ z=cT

$$I^{\pm}(t \mp \tfrac{z}{c}) = \pm Y\, V^{\pm}(t \mp \tfrac{z}{c}) \qquad Y = Z^{-1}$$

b.)



c.)



$V^-(t) = -[u(t) - u(t-T)]\frac{t}{T}$ [V]

d.) $Z\Big|_{z=0} = \frac{V}{I}\Big|_{z=0} = 0\ \Omega$

The voltage constituents V^+ and V^- cancel at z=0, indicative of a perfect electric conductor at z=0

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Question 1.3

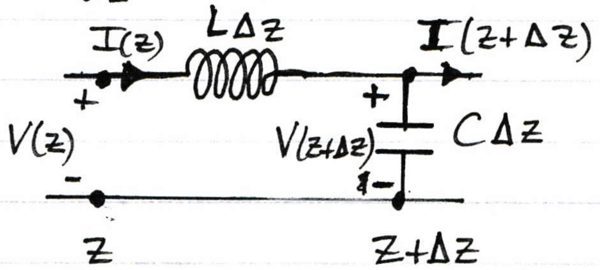
a.) $$-\frac{\partial V}{\partial z} = L\frac{\partial I}{\partial t} \Rightarrow \frac{V(z,t)-V(z+\Delta z,t)}{\Delta z} = L\frac{\partial I}{\partial t}$$

$$\Rightarrow V(z,t) - V(z+\Delta z,t) = L\Delta z\,\frac{\partial I}{\partial t}(z,t)$$

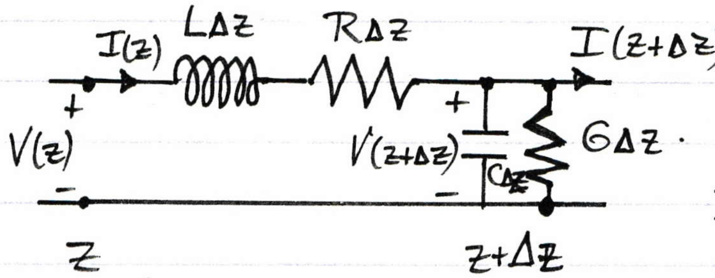
Likewise:

$$-\frac{\partial I}{\partial z} = C\frac{\partial V}{\partial t} \Rightarrow I(z,t) - I(z+\Delta z,t) = C\Delta z\,\frac{\partial V}{\partial t}(z,t)$$

b.)



c.)



$$V(z,t)-V(z+\Delta z,t) = L\Delta z\frac{\partial I}{\partial t} + R\Delta z\, I$$

$$I(z,t)-I(z+\Delta z,t) = C\Delta z\frac{\partial V}{\partial t} + G\Delta z\, V$$

(R: resistance per unit length, G: conductante per unit length)

Question 1.4

a.) $V^-(t) = \Gamma V^+(t-T) \Rightarrow V^-(t+\frac{\ell}{c}) = \Gamma V^+(t+\frac{\ell}{c}-T)$

$$= \Gamma V^+(t - \frac{cT-\ell}{c})$$

$$\Rightarrow \frac{V}{I} = Z\,\frac{V^+(t-\frac{\ell}{c}) + \Gamma V^+(t-\frac{cT-\ell}{c})}{V^+(t-\frac{\ell}{c}) - \Gamma V^+(t-\frac{cT-\ell}{c})} = Z\,\frac{1+\Gamma}{1-\Gamma} = Z_L$$

$\Downarrow$

if $\frac{\ell}{c} = \frac{cT-\ell}{c} \Rightarrow \ell = \frac{1}{2}cT$

b. $|\Gamma|>1$ Firstly, let us assume that V^+ is the incident wave, and that $V^- = \Gamma V^+$ at $z=\ell=\frac{1}{2}cT$

$\Rightarrow Z_L = Z\,\frac{1+\Gamma}{1-\Gamma} < 0$ if $|\Gamma|>1 \Rightarrow$ Negative impedance (Active Load)

Secondly, Let us assume that V^- is the incident wave, and that $V^- = \Gamma V^+$ at $z=\ell=\frac{1}{2}cT \Rightarrow V^+ = \frac{1}{\Gamma}V^-$ at $z=\ell=\frac{1}{2}cT$

$\Rightarrow Z_L = Z\,\frac{1+\Gamma^{-1}}{1-\Gamma^{-1}} = -Z\,\frac{1+\Gamma}{1-\Gamma} > 0$ if $|\Gamma|>1 \Rightarrow$ positive impedance

($\Rightarrow$ passive Load)

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Question 1.5

a.) $I = I^+ + I^-$, $V = V^+ + V^-$; $I^{\pm} = \pm \frac{1}{Z} V^{\pm}$

b.) At $z=0$, where the load is located, we have

$$I = I_G + I_C = GV + C\frac{dV}{dt}$$

$$= (I^+ + I^-)\Big|_{z=0} = \frac{1}{Z}(V^+ - V^-) = \left(C\frac{d}{dt} + G\right)V^- + \left(C\frac{d}{dt} + G\right)V^+$$

Multiply by $Z \Rightarrow$

$$\left(CZ\frac{d}{dt} + GZ + 1\right)V^- = -\left(CZ\frac{d}{dt} + GZ - 1\right)\underbrace{V^+}_{u(t)\,:\ \text{known}}$$

c.) A homogeneous, first-order differential equation for a function $f(t)$ can always be cast in the form

$$\left(\tau\frac{d}{dt} + 1\right)f(t) = 0, \quad f(0) = f_0 \Rightarrow f(t) = f_0 e^{-\frac{t}{\tau}}$$

$\Rightarrow$ divide the equation for V^- found above by $(GZ+1) \Rightarrow$

$$\left(\tau\frac{d}{dt} + 1\right)V^- = -\left(\tau\frac{d}{dt} + \frac{GZ-1}{GZ+1}\right)u(t) = -\tau\delta(t) + \frac{1-GZ}{1+GZ}u(t)$$

$\Rightarrow \tau = \frac{CZ}{1+GZ}$ is the time constant.

d.) $\tau = \frac{CZ}{1+GZ} \Rightarrow \left(\tau\frac{d}{dt} + 1\right)V^- = \left(\tau\frac{d}{dt} + 1\right)\left[\left(A + Be^{-\frac{t}{\tau}}\right)u(t)\right]$

$$= \tau\underbrace{\left(A + Be^{-\frac{t}{\tau}}\right)}_{\to 1 \text{ for } t=0}\delta(t) \cancel{- Be^{-\frac{t}{\tau}}u(t)} + Au(t) \cancel{+ Be^{-\frac{t}{\tau}}u(t)}$$

$$= -\tau\delta(t) + \frac{1-GZ}{1+GZ}u(t) \Rightarrow A + B = -1 \ \& \ A = \frac{1-GZ}{1+GZ}$$

$$\Rightarrow B = \frac{-2}{1+GZ}$$

e.) $\lim\limits_{t\downarrow 0} V(t) = \lim\limits_{t\downarrow 0}\left(V^+(t) + V^-(t)\right) = 1 + A + B = 0$

$\lim\limits_{t\downarrow 0} I(t) = \frac{1}{Z}(1 - A - B) = \frac{2}{Z}$ For $t\downarrow 0$, the empty capacitor acts as a short circuit

f.) $\lim\limits_{t\to\infty} V(t) = 1 + A = -B = \frac{2}{1+GZ}$; $\lim\limits_{t\to\infty} I(t) = \frac{1}{Z}(1-A) = \frac{2GZ}{1+GZ}$

$\Rightarrow \lim\limits_{t\to\infty} \frac{V}{I} = \frac{1}{G}$ For $t\to\infty$, the charged capacitor acts as an open circuit

g.) Time shift: $V^-\left(t + \frac{z}{c}\right) = \left(A + Be^{-\frac{t}{\tau} - \frac{z}{c\tau}}\right)u\left(t + \frac{z}{c}\right)$ for $z<0$.