Proposed solutions $\boxed{8a}$

1) $$\nabla^2 A_z + k^2 A_z = 0$$

$$\nabla^2 A_z = \nabla\cdot(\nabla A_z) = \nabla\cdot(\vec{a}_r \partial_r A_z) = \frac{1}{r^2}\partial_r(r^2 \partial_r A_z)$$

$$= \frac{1}{r^2}\partial_r\left(r^2 \partial_r \frac{(A_z r)}{r}\right) = \frac{1}{r^2}\partial_r\left(\cancel{r^2}\frac{(\partial_r(A_z r))r - A_z r}{\cancel{r^2}}\right) =$$

$$= \frac{1}{r^2}\left(\partial_r^2(A_z r)r + \cancel{\partial_r(A_z r)} - \cancel{\partial_r(A_z r)}\right) = \frac{1}{r}\partial_r^2(A_z r)$$

$$\Rightarrow \nabla^2 A_z + k^2 A_z = \frac{1}{r}\partial_r^2(rA_z) + k^2 A_z = 0$$

$$A_z = C^+ \frac{e^{-jkr}}{r} + C^- \frac{e^{jkr}}{r}$$

Sommerfeld radiation condition $\rightarrow$ $C^- = 0$

We integrate $\nabla^2 A_z + k^2 A_z = -\mu I_0 d\, \delta(\bar{r})$ over a volume determined by a small sphere, centered at the origin, with radius Δr, and then we tend $\Delta r \to 0$

$$\iiint_{V_{\Delta r}} \nabla^2 A_z + k^2 A_z \, dV = \iiint_{V_{\Delta r}} -\mu I_0 d\, \delta(\bar{r})\, dV$$

$$\iiint_{V_{\Delta r}} \nabla\cdot\nabla A_z \, dV + k^2 \underbrace{\iiint_{V_{\Delta r}} A_z \, dV}_{\text{this integral vanishes at the limit } \Delta r \to 0} = -\mu I_0 d$$

$$\oiint_{S_{\Delta r}} \nabla A_z \cdot \vec{a}_r \, dS = \oiint_{S_{\Delta r}} \partial_r A_z \, r^2 \sin\theta \, d\theta\, d\varphi =$$

(Gauss theorem)

$$= 4\pi (\Delta r)^2 \partial_r A_z(\Delta r) = 4\pi (\Delta r)^2 C^+ \left(-jk - \frac{1}{\Delta r}\right) \frac{e^{-jk\Delta r}}{\Delta r}$$

$$= 4\pi C^+ (-jk\Delta r - 1)\, e^{-jk\Delta r} \xrightarrow[\Delta r \to 0]{} -4\pi C^+$$

$$\Rightarrow \boxed{C^+ = \frac{\mu\, I_o d}{4\pi}}$$

2) $$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \iiint_{V'} \vec{J}(\vec{r}') \frac{e^{-j\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\, dV'$$

$$\vec{J}(\vec{r}') = I_o d\, \delta(x')\, \delta(y')\, \delta(z')\, \vec{a}_z$$

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} I_o d \frac{e^{-j\beta|\vec{r}|}}{|\vec{r}|} \vec{a}_z$$ (sampling property of δ)

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} I_o d \frac{e^{-j\beta r}}{r} \vec{a}_z$$

3) $$\vec{E} = \frac{1}{j\omega\varepsilon\mu}\left[\nabla \nabla\cdot\vec{A} + k^2 \vec{A}\right]$$

$$\nabla\cdot\vec{A} = \partial_z A_z(r) = \partial_r A_z(r)\, \partial_z r = \partial_r A_z(r)\, \partial_z \sqrt{x^2+y^2+z^2}$$

$$= I_o d\mu \frac{z}{r} \frac{d}{dr}\left[\frac{e^{-jkr}}{4\pi r}\right] = I_o d\mu \left(-jk - \frac{1}{r}\right) \frac{e^{-jkr}}{4\pi r} \cos\theta$$

$$\nabla \nabla \cdot \vec{A} = \partial_r \nabla \cdot \vec{A}\, \vec{a}_r + \frac{1}{r} \partial_\theta \nabla \cdot \vec{A}\, \vec{a}_\theta =$$

$$= I_0 d\mu \left[\frac{1}{r^2} \frac{e^{-jkr}}{4\pi r} + \left(-jk - \frac{1}{r}\right) \left(\frac{-jk e^{-jkr} r}{4\pi r^2} - \frac{e^{-jkr}}{4\pi r^2} \right) \right] \cos\theta\, \vec{a}_r +$$

$$+ \frac{I_0 d\mu}{r} \left(jk + \frac{1}{r}\right) \frac{e^{-jkr}}{4\pi r} \sin\theta\, \vec{a}_\theta =$$

$$= I_0 d\mu \left(\frac{1}{r} + \left(-jk - \frac{1}{r}\right)(-jkr - 1) \right) \frac{e^{-jkr}}{4\pi r^2} \cos\theta\, \vec{a}_r +$$

$$+ \frac{I_0 d\mu}{r} \left(jk + \frac{1}{r}\right) \frac{e^{-jkr}}{4\pi r} \sin\theta\, \vec{a}_\theta$$

$$k^2 \vec{A} = k^2 A_z \vec{a}_z = k^2 A_z \cos\theta\, \vec{a}_r - k^2 A_z \sin\theta\, \vec{a}_\theta$$

$$E_r = \frac{1}{j\omega\varepsilon\mu} \left[I_0 \mu d \left(\frac{1}{r} - k^2 r + jk + jk + \frac{1}{r} \right) \frac{e^{-jkr}}{4\pi r^2} \cos\theta + \right.$$

$$\left. + \frac{k^2 I_0 d\mu}{4\pi} \frac{e^{-jkr}}{r} \cos\theta \right]$$

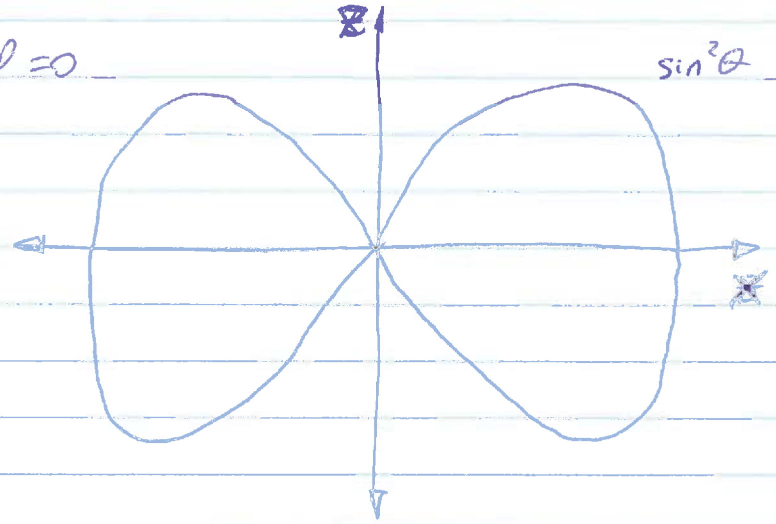
$$\boxed{E_r = \frac{1}{j\omega\varepsilon} \frac{I_0 d}{2\pi} \left(jk + \frac{1}{r}\right) \frac{e^{-jkr}}{r^2} \cos\theta}$$

$$E_\theta = \frac{1}{j\omega\varepsilon\mu} I_0 d\mu \left(\frac{1}{r}\left(jk + \frac{1}{r}\right) - k^2 \right) \frac{e^{-jkr}}{4\pi r} \sin\theta$$

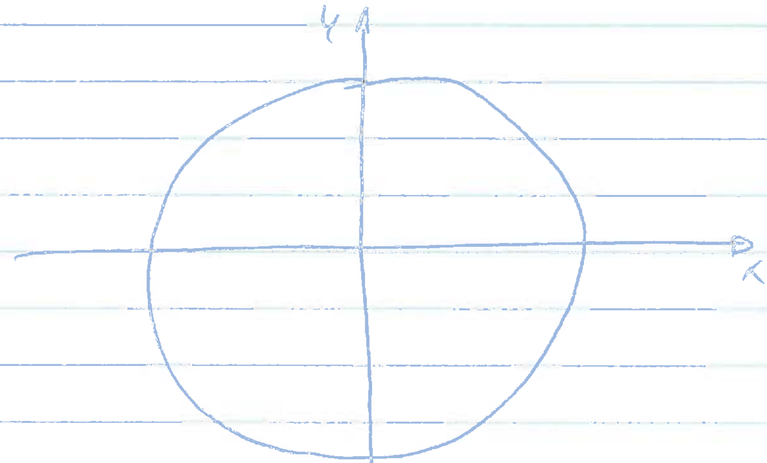
$$\boxed{E_\theta = \frac{1}{j\omega\varepsilon} \frac{I_0 d}{4\pi} \left(-k^2 + \frac{jk}{r} + \frac{1}{r^2} \right) \frac{e^{-jkr}}{r} \sin\theta}$$

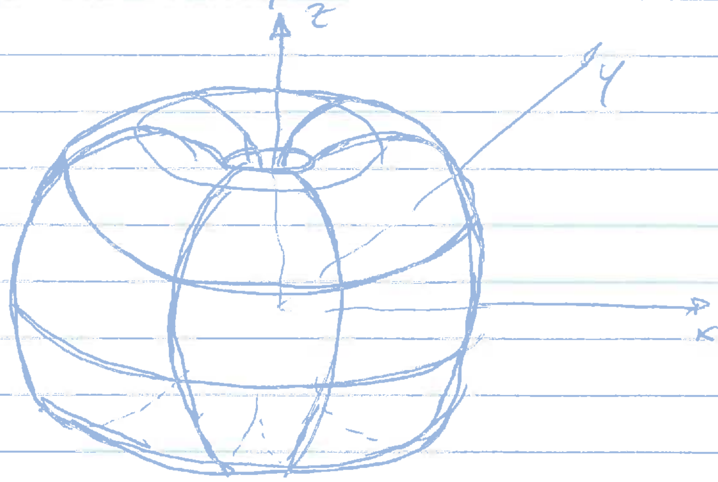
4)

$\varphi = 0$ $\sin^2\theta$



$\theta = 0$





(sorry...)

$\boxed{\text{Book}}$ (even problems)

14.4) $E_r = \dfrac{-j I_0 Z d}{2\pi k r^3} \cos\theta$

$E_\theta = \dfrac{-j I_0 Z d}{4\pi k r^3} \sin\theta$

This solution is equivalent to the static dipole provided that

$I_0 = j\omega Q \Rightarrow I_0 = \partial_t Q$

14.6) $\langle \vec{S} \rangle = \frac{1}{2} \mathbb{R}\{\vec{E} \times \vec{H}^*\} = \frac{1}{2}\left(\frac{I_0 d k}{4\pi r}\right)^2 Z \sin^2\theta \, \vec{a}_r$ W/m²

14.8) $\langle \vec{S} \rangle_{md} = \frac{1}{2}\left(\frac{I_0 k}{4\pi r}\right)^2 (\omega\mu \pi a^2)^2 \frac{1}{Z} \sin^2\theta \, \vec{a}_r$

$\langle \vec{S} \rangle_{ed} = \frac{1}{2}\left(\frac{I_0 k}{4\pi r}\right)^2 d^2 Z \sin^2\theta \, \vec{a}_r$

For equal power $\rightarrow$ $\boxed{d = \dfrac{(2\pi a)^2}{2\lambda}}$