5XCC0 Biopotential and Neural Interface Circuits

Amplifiers and Filters

Pieter Harpe

Outline

- Amplifier types
  - Transimpedance Amplifiers
  - Transconductance Amplifiers
  - Voltage Amplifiers
- Advanced Amplifier Techniques
- G_m-C Filters

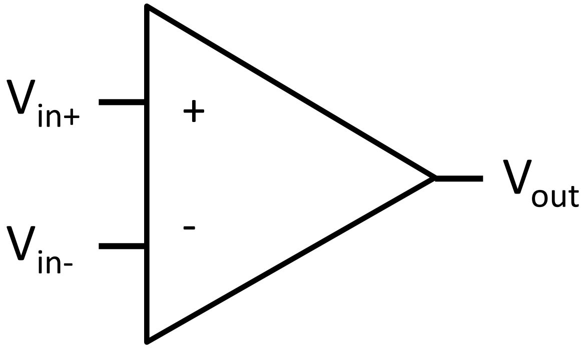
Basics

- You should be familiar with:
  - Differential pair
  - Current mirror
  - Common-mode, differential-mode
  - Common-mode feedback
  - Open-loop, closed-loop, stability, phase margin
- Recommended literature:
  - Razavi – “Design of Analog CMOS Integrated Circuits”
  - Sansen – “Analog Design Essentials”
  - Sarpeshkar – “Ultra Low Power Bioelectronics”

Amplifier types

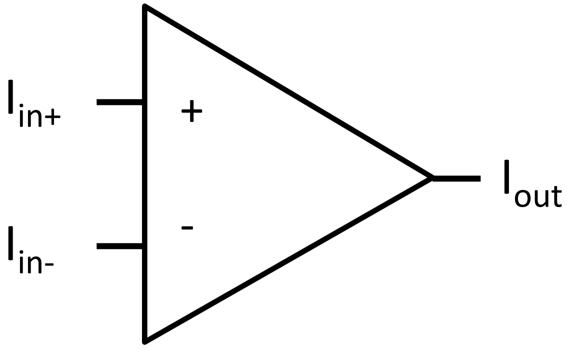
Voltage amplifier

$V_{out} = A \cdot V_{in}$



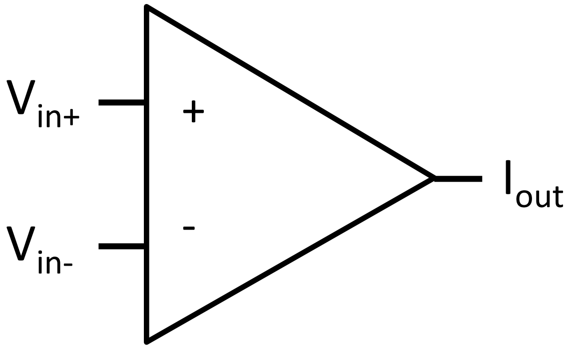
Current amplifier

$I_{out} = A \cdot I_{in}$



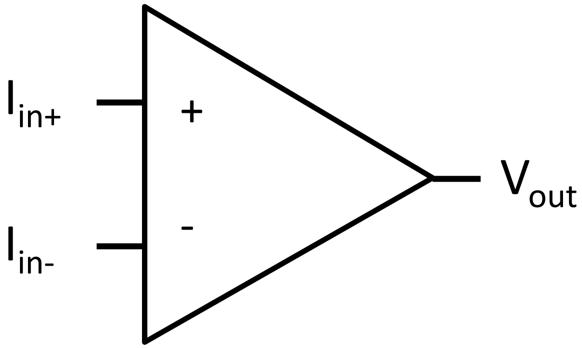
Transconductance amplifier

$I_{out} = G_m \cdot V_{in}$

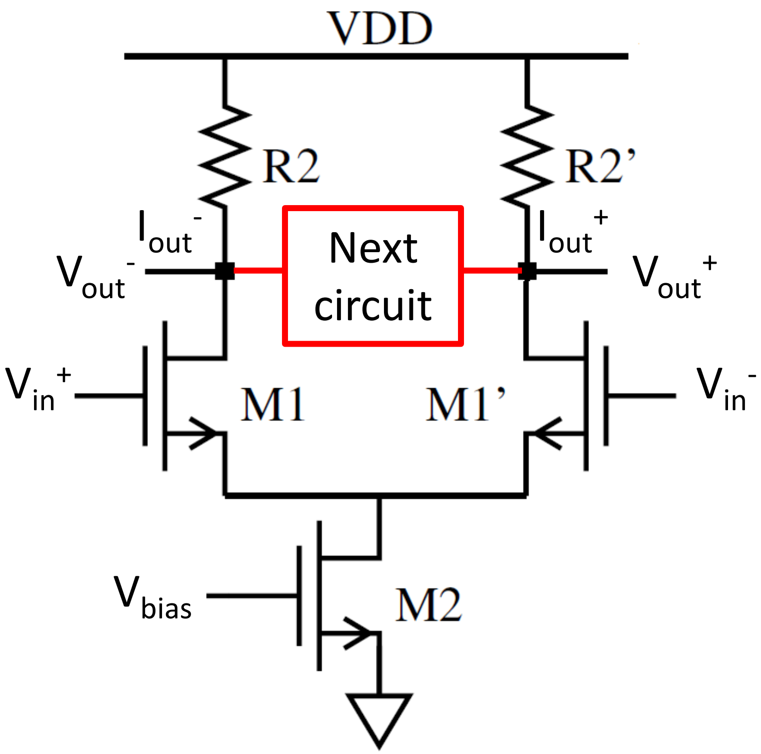


Transimpedance amplifier

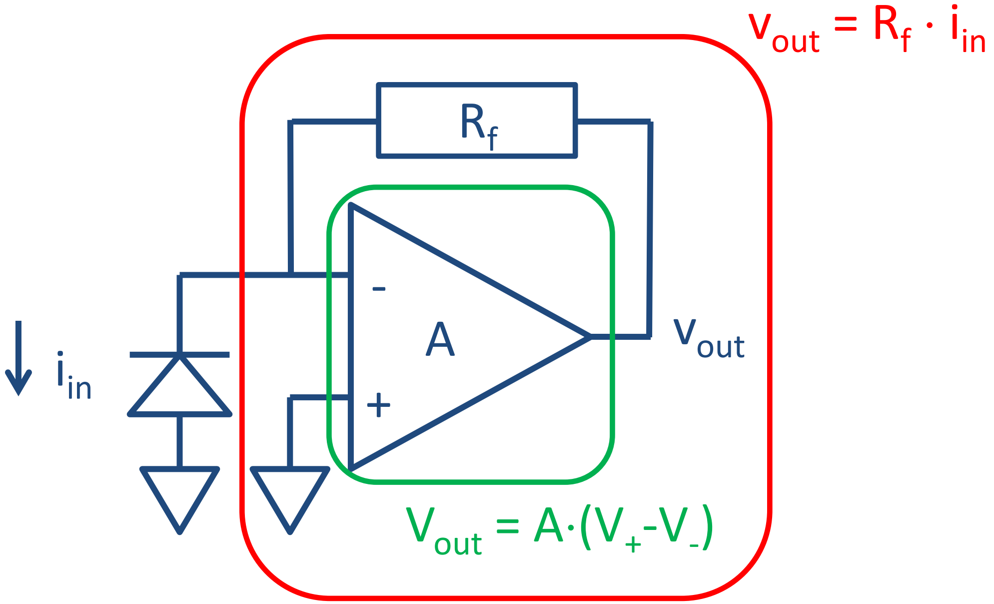
$V_{out} = Z \cdot I_{in}$



Amplifier types

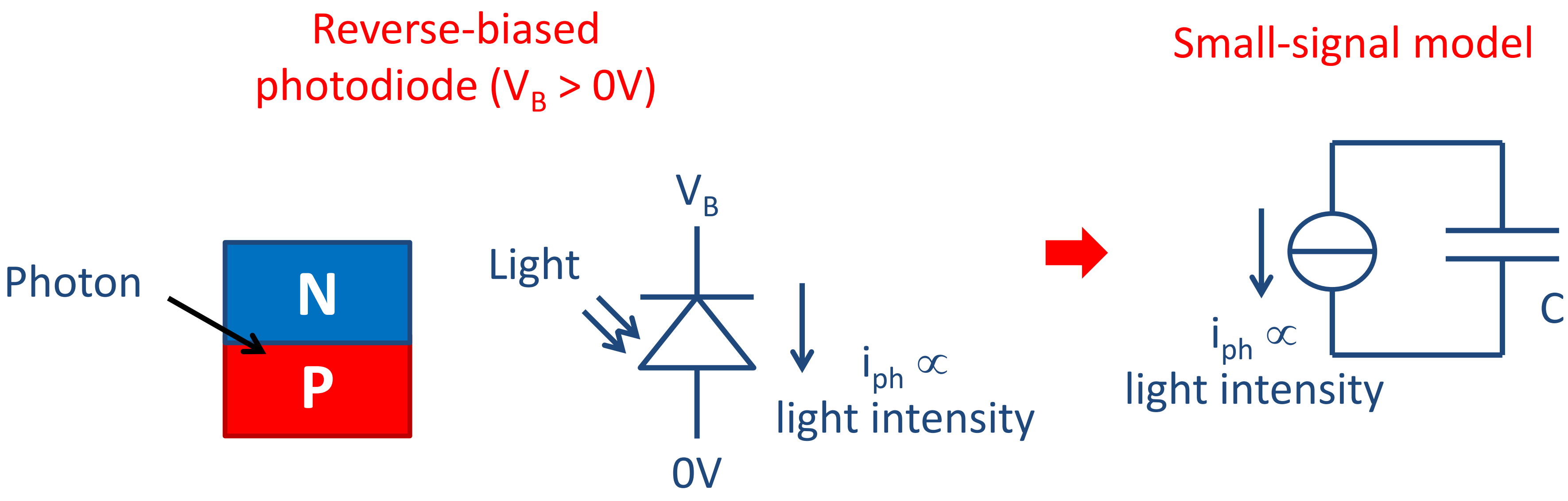


Amplifier type	Input	Output
Voltage	V	V
Transconductance	V	I
Transimpedance	I	V
Current	I	I



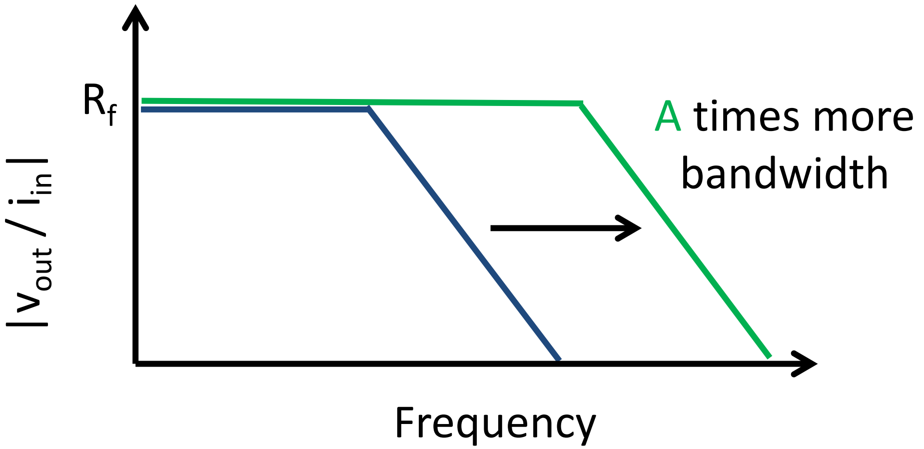
Photodiodes

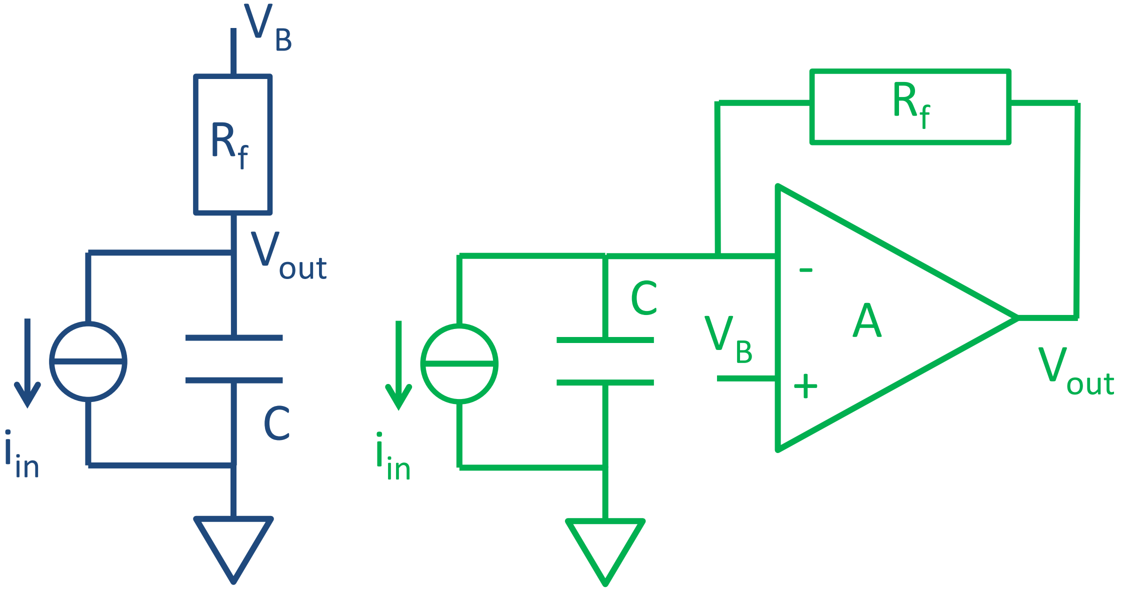
- Photon generates electron/hole pair in PN junction → Current created with the help of bias voltage V_B



Transimpedance Amplifier (TIA)

- Input I → Output V
- Many sensors have a current output
  - Photodiode (image sensors)
  - Microphone (cochlear implant)
  - (Some) ultrasound transducers





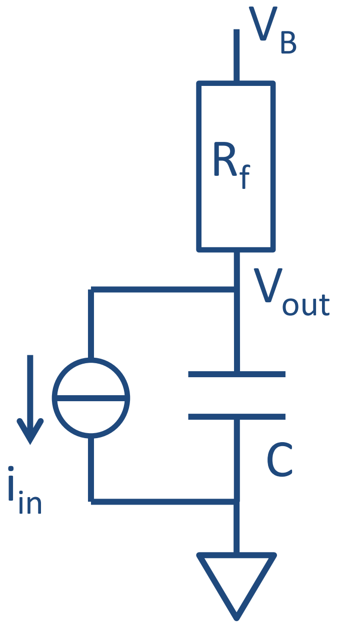
$v_{out}/i_{in} \approx -R_f / (1 + s\,C\,R_f)$

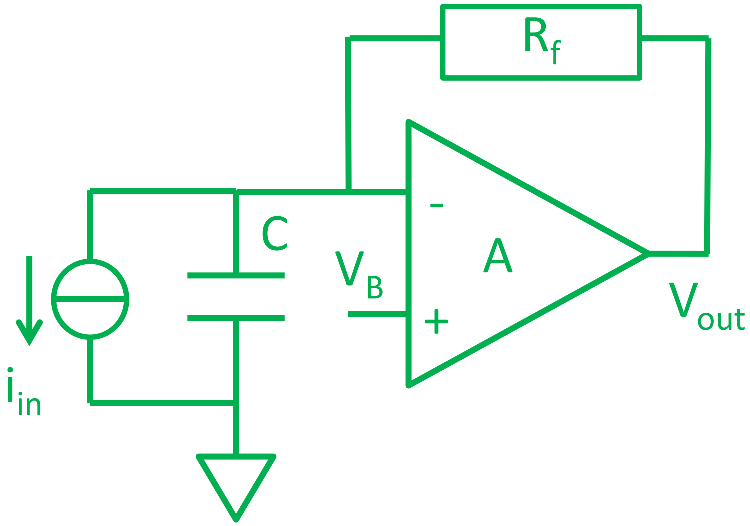
$v_{out}/i_{in} \approx R_f / (1 + s\,C\,R_f / A)$

TIA increases BW by a factor A,
and sets sensor bias voltage precisely

Exercise 1: TIA

a) Show that the two transfer functions as given on the previous slide (repeated here for convenience) are correct for the given circuits.



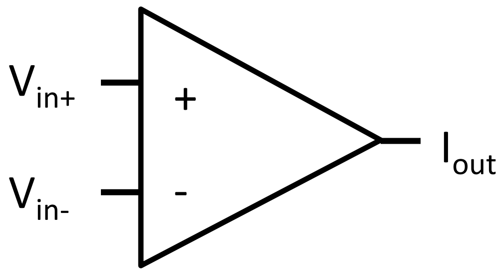


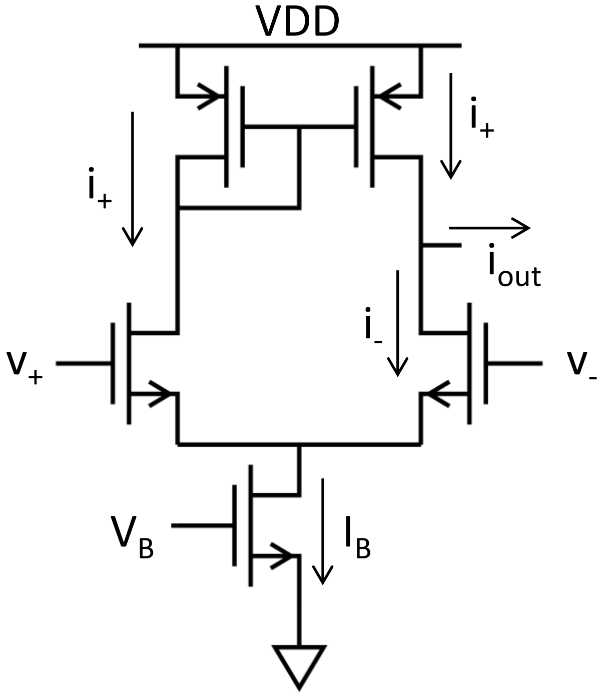
$v_{out}/i_{in} \approx -R_f\,/\,(1 + s\,C\,R_f)$

$v_{out}/i_{in} \approx R_f\,/\,(1 + s\,C\,R_f\,/\,A)$

Transconductance Amplifier

- OTA: Operational Transconductance Amplifier
- Input V → Output I
- G_m: $i_{out} = G_m\, v_{in}$





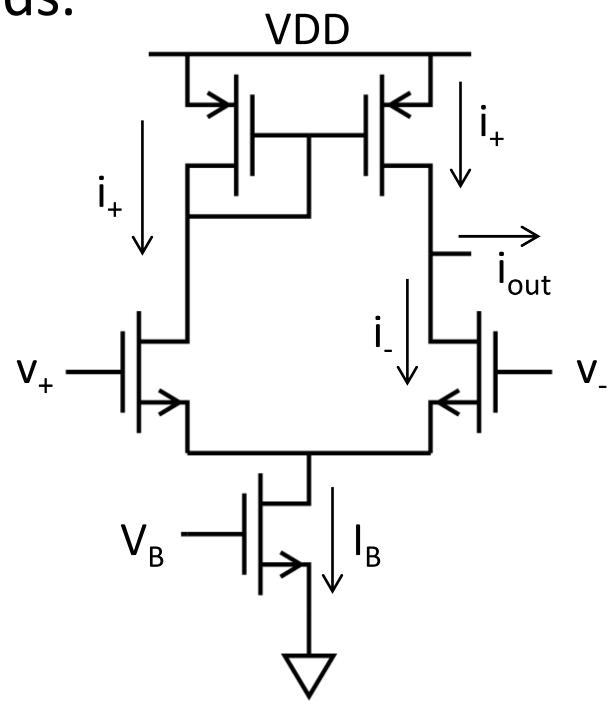
$$i_{out} = i_+ - i_-;\ v_{in} = v_+ - v_-$$
$$i_+ = g_m \cdot v_+;\ i_- = g_m \cdot v_-$$
$$i_{out} = g_m \cdot v_+ - g_m \cdot v_- = g_m \cdot v_{in}$$
$$g_m = K_s/\Phi_t \cdot \tfrac{1}{2}\, I_B$$
$$i_{out}/v_{in} = G_m = K_s/\Phi_t \cdot \tfrac{1}{2}\, I_B$$

Exercise 2: OTA Noise

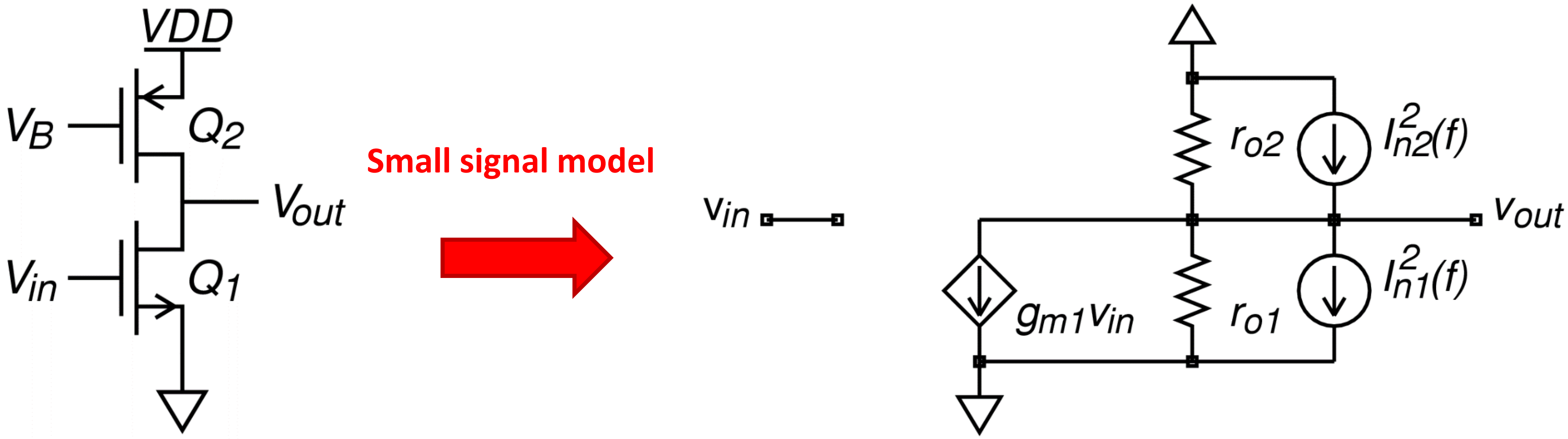
Consider the OTA also discussed on the previous slide.

a) Express the input-referred noise power spectral density as function of the bias current I_B (and other parameters). You may assume that only the two input transistors are critical for the overall noise, that those transistors are biased in sub-threshold, and that for each transistor $V_{gn}^2(f) = kT / 9I_{DS}$ holds.

b) Assume that we need an OTA which has a total input-referred noise power of $2\mu V_{rms}$ in a bandwidth of 10kHz. How should we set I_B?



Common-Source (CS) Voltage Amplifier (VA)



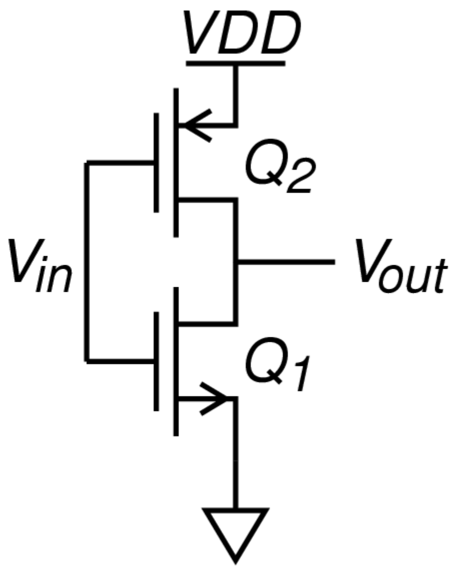
- Gain:

$$A = -g_{m1} \cdot (r_{o1} \mathbin{//} r_{o2})$$

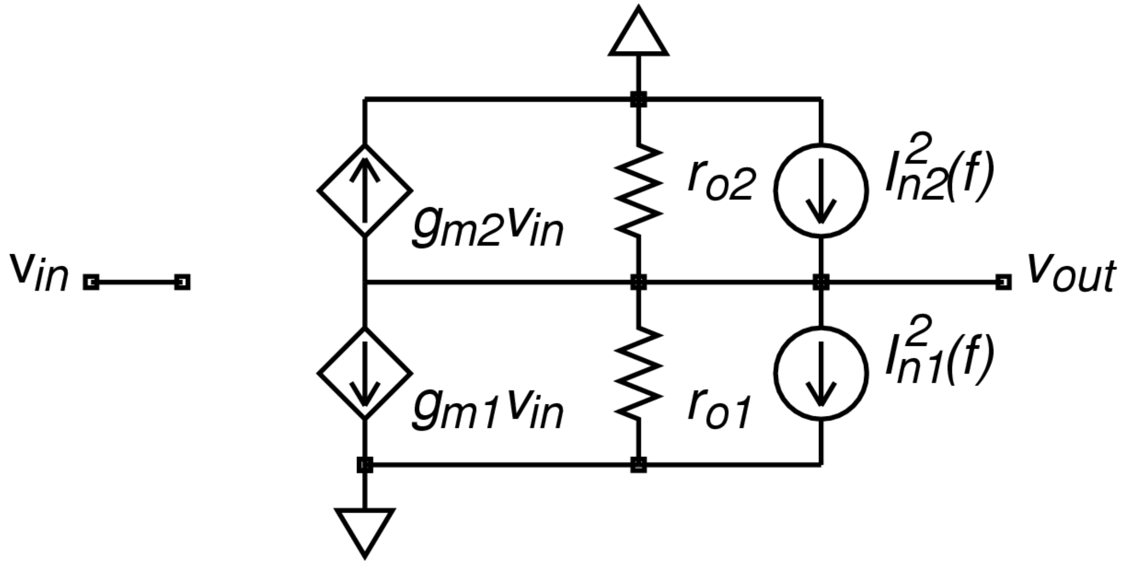
- Input-referred noise:

$$V_n^2(f) = \{I_{n1}^2(f) + I_{n2}^2(f)\} / g_{m1}^2$$

Inverter-Based Voltage Amplifier



Small signal model



- Gain:

$$A = -(g_{m1} + g_{m2}) \cdot (r_{o1} \mathbin{//} r_{o2})$$

- Input-referred noise:

$$V_n^2(f) = \{I_{n1}^2(f) + I_{n2}^2(f)\} / (g_{m1} + g_{m2})^2$$

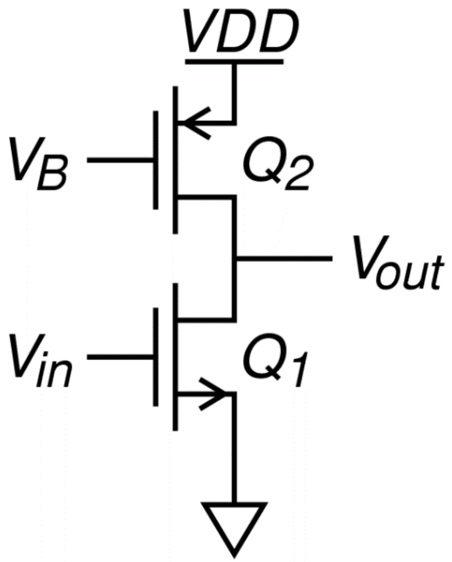
More gain (about 2x) and lower input-referred noise power spectral density (about 4x) compared to CS VA

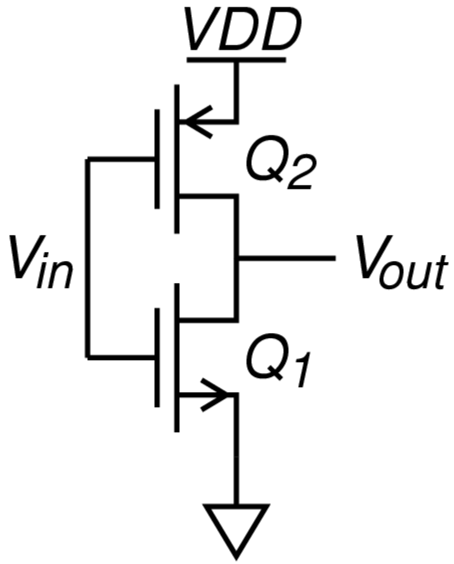
Exercise 3: CS VA versus INV VA

Assume that the bias current for the circuits below is set to 1µA and assume that all transistors are biased in sub-threshold.

a) For the CS VA, what will be the input-referred noise power spectral density?

b) For the INV VA, what will be the input-referred noise power spectral density?

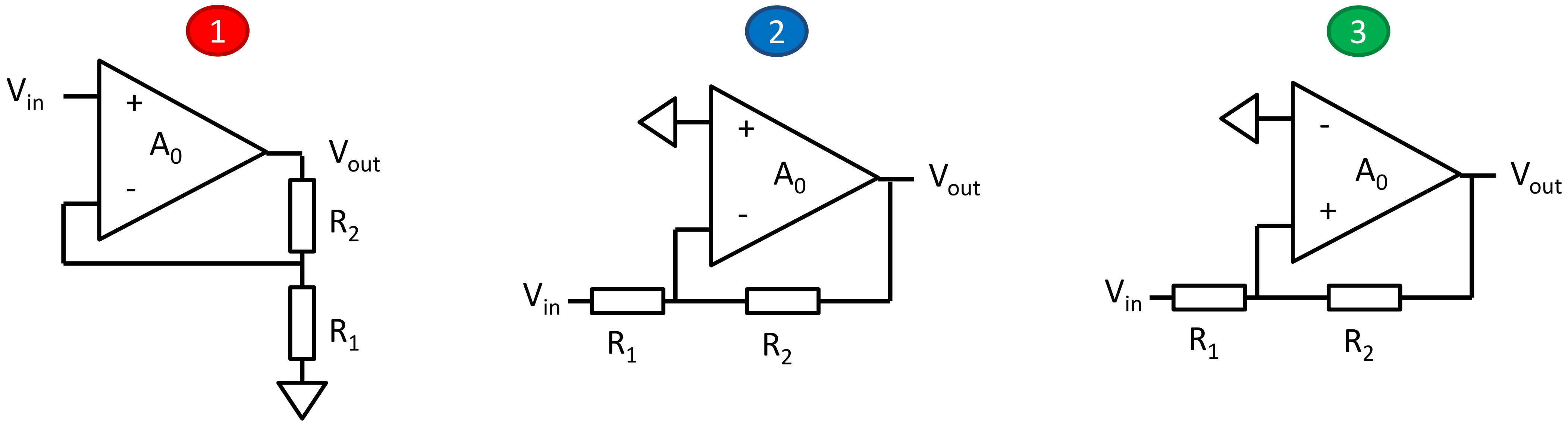




Low-Voltage Analog Design

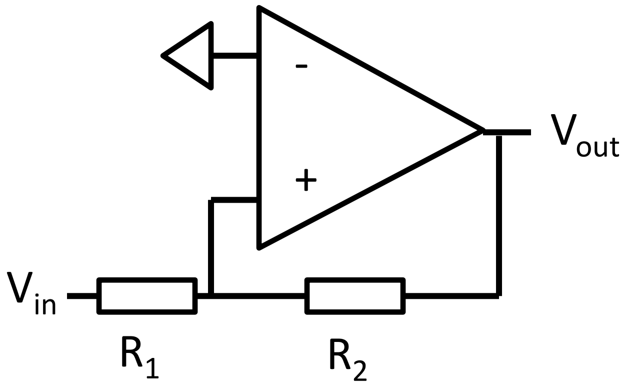
- Each transistor needs a certain V_{DSAT} →
  - Minimize number of stacked transistors
  - Use sub-threshold biasing (lower V_{DSAT})
- Use cascaded stages rather than cascoded transistors to increase gain
- Increase DC gain by positive feedback

Positive Feedback Loops

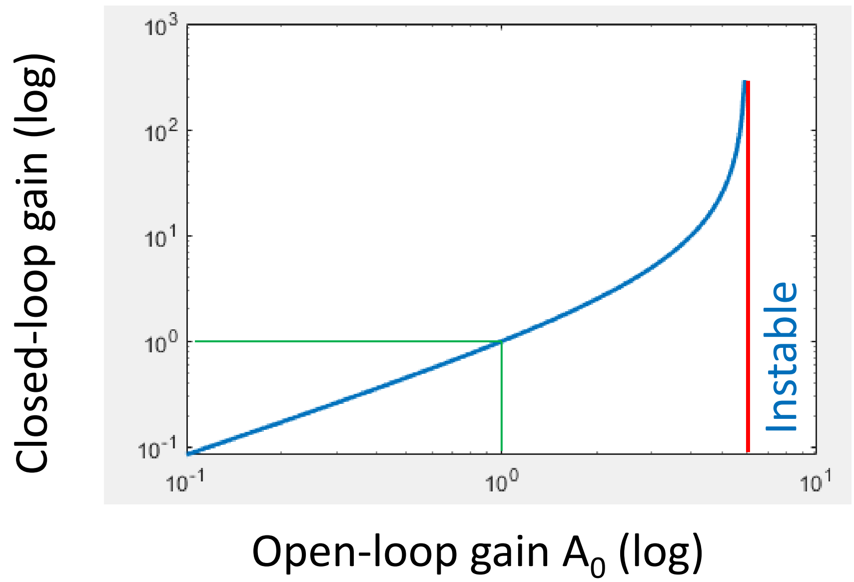


- $V_{out} = A_0 \cdot (V_+ - V_-)$; what is the closed-loop gain $A_{cl} = V_{out}/V_{in}$?
  - (1): negative feedback, non-inverting amplifier, $|A_{cl}| < A_0$
  - (2): negative feedback, inverting amplifier, $|A_{cl}| < A_0$
  - (3): positive feedback, non-inverting amplifier, $|A_{cl}| > A_0$

Positive Feedback to Enhance Gain



Example for R_2 = $5R_1$

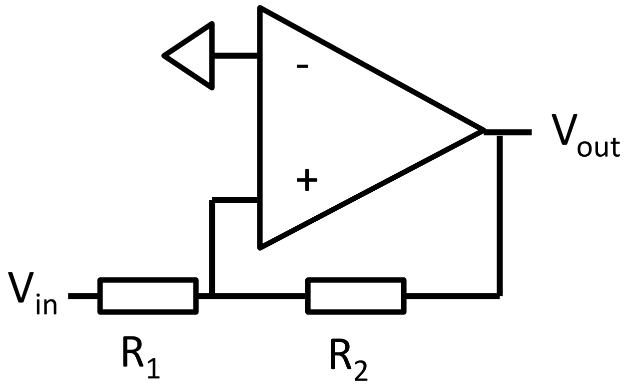


- $A_{cl} = V_{out}/V_{in} = A_0\, R_2 \,/\, [R_1 + R_2 - A_0\, R_1]$
- Only stable when $R_1 + R_2 - A_0\, R_1 > 0$
- Gain can be $> A_0$!

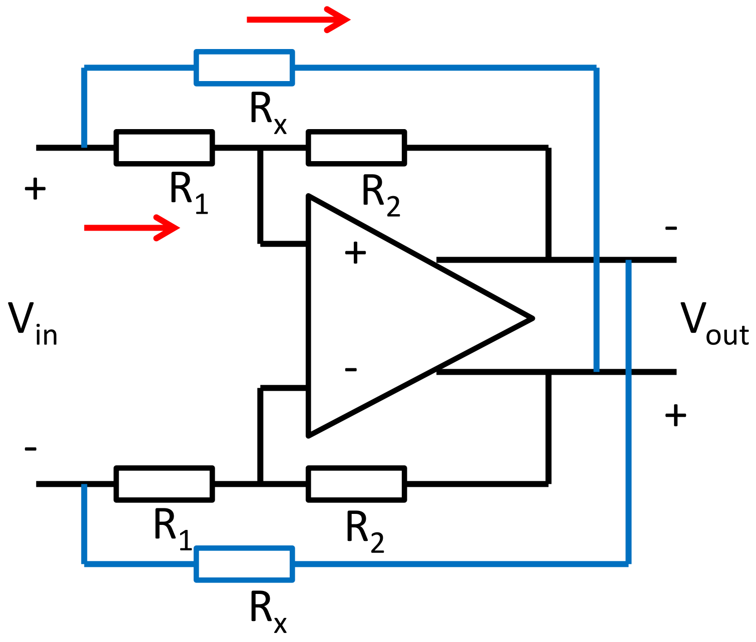
A_0	A_{cl}
$A_0 < 1$	$A_{cl} < 1$
$A_0 = 1$	$A_{cl} = 1$
$A_0 > 1$	$A_{cl} > A_0$

Exercise 4: Positive Feedback to Enhance Gain

a) For the circuit below, assume that the amplifier's open-loop gain is 20x and that R_2 = 1MΩ. What should the value of R_1 be to reach a closed-loop gain of 100x?



Positive Feedback to Enhance Z_{in}



$I_{in} = I_{R1} + I_{Rx}$

$I_{R1} = 0.5V_{in} / R_1$

$I_{Rx} = (0.5V_{in} – 0.5V_{out}) / R_x$

For $Z_{in} = \infty$, $I_{in} = 0$:

$0.5V_{in} / R_1 + (0.5V_{in} – 0.5V_{out}) / R_x = 0$

Since $V_{out} = R_2 / R_1\ V_{in}$, this leads to:

$R_x = R_2 – R_1$

- $V_{out} = R_2 / R_1 \cdot V_{in}$
- What is the input impedance?
- $Z_{in} \approx 2R_1$ (differential)
- With positive feedback, Z_{in} can be increased to infinity (theory)

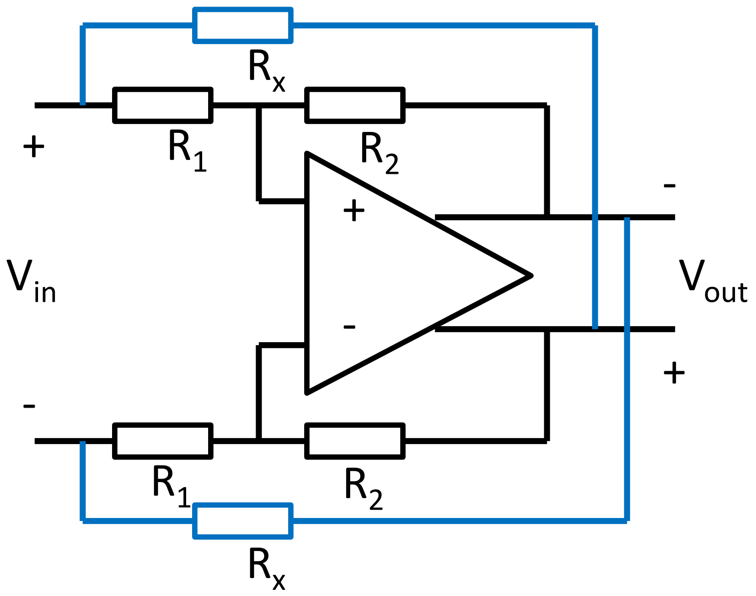
Exercise 5: Positive Feedback to Enhance Z_{in}

For the circuit below, assume that the amplifier's open-loop gain is infinite and that R_1 = 1MΩ. For questions a) and b), you may assume R_x is not yet present.

a) What should the value of R_2 be, to get a closed-loop gain of 100x?

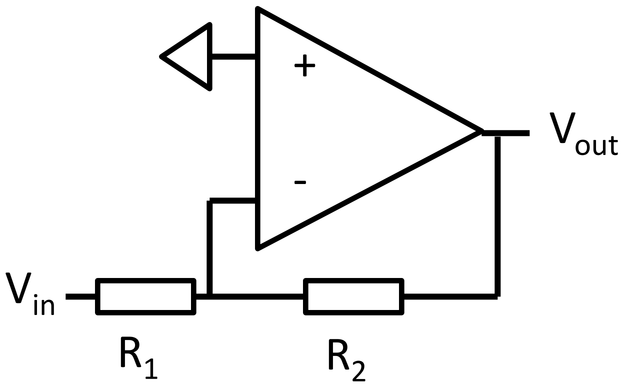
b) What is the differential input impedance of the circuit?

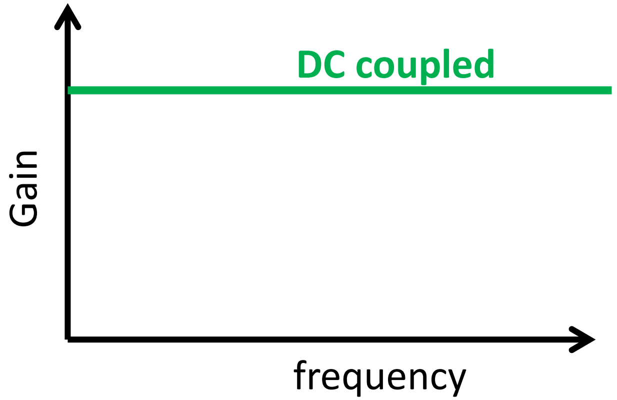
c) If we add R_x, and if R_x is chosen equal to R_2, what will the differential input impedance then be?



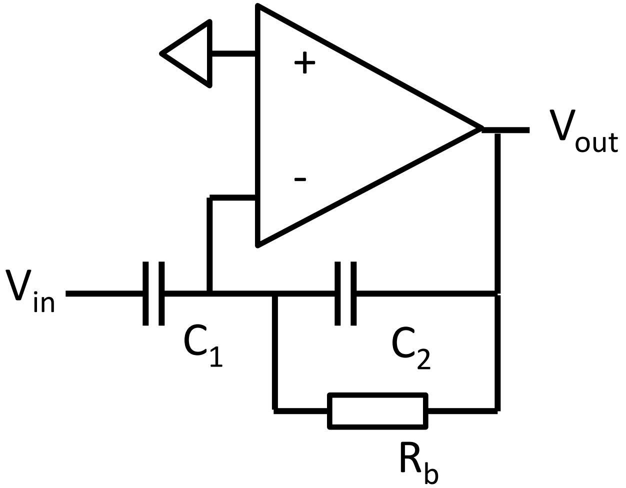
Capacitively-Coupled Amplifiers

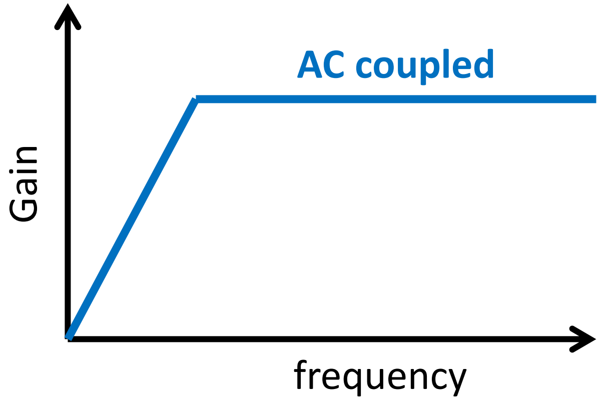
- Gain: R_2 / R_1
  - R_2 and R_1 contribute in-band noise (4kTR)





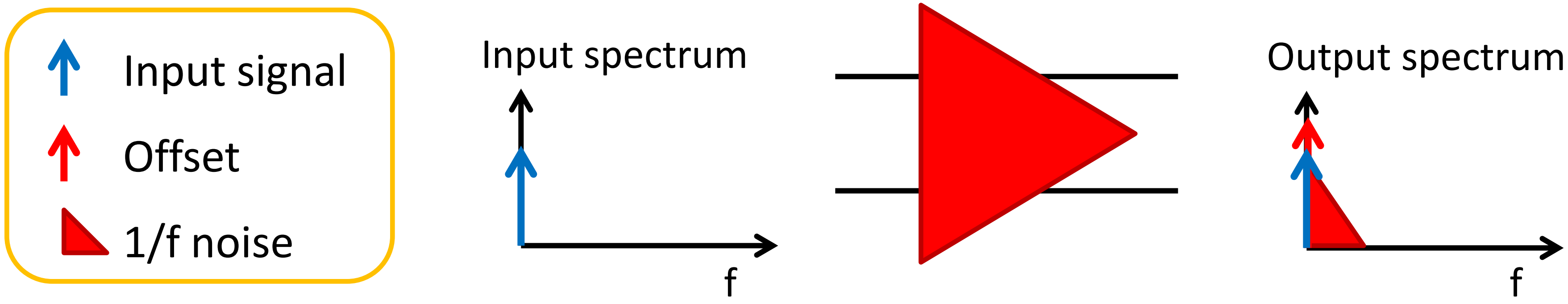
- Gain: C_1 / C_2
  - Does not work for DC signals
  - Bias resistor (R_b) needed
  - Most of the noise of R_b is out of band

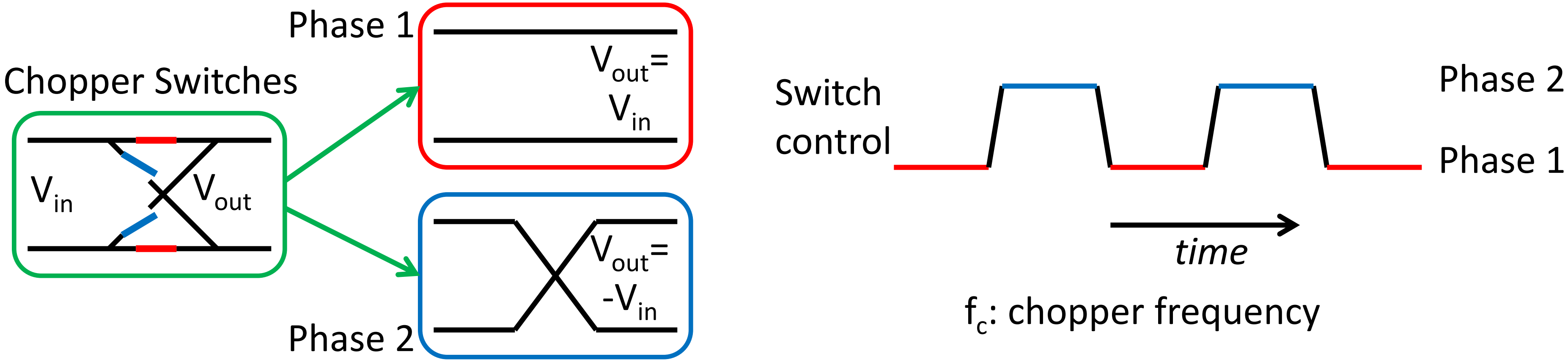




Chopping Amplifier (1)

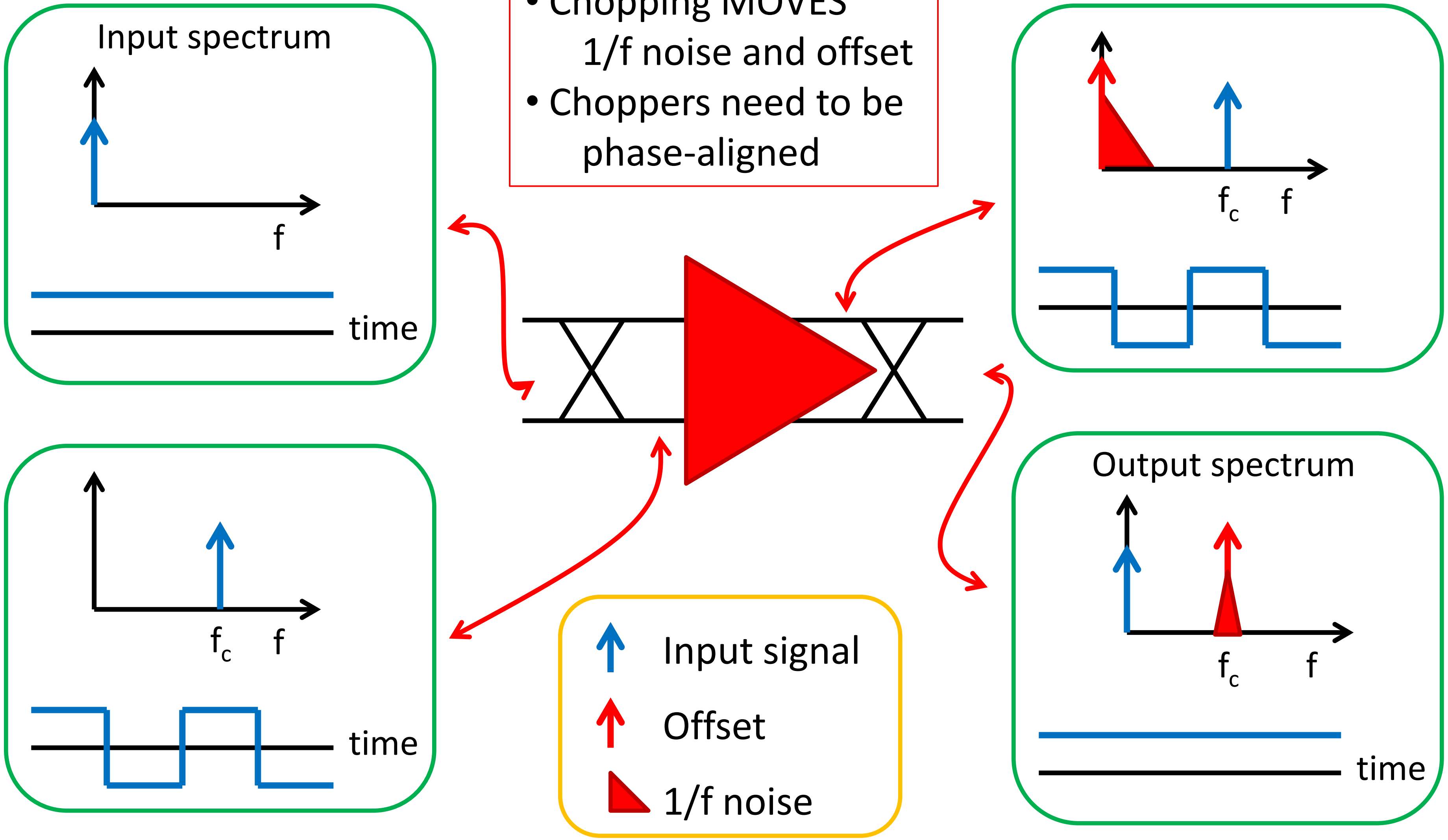
- Input is a DC (or low-frequency) signal
- Amplifier with 1/f noise and offset





Chopping Amplifier (2)

- Chopping MOVES 1/f noise and offset
- Choppers need to be phase-aligned

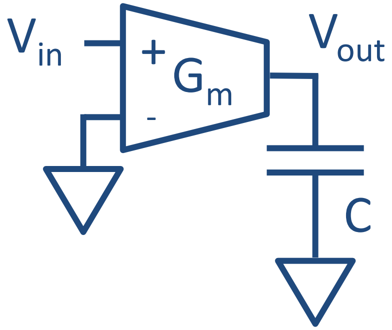


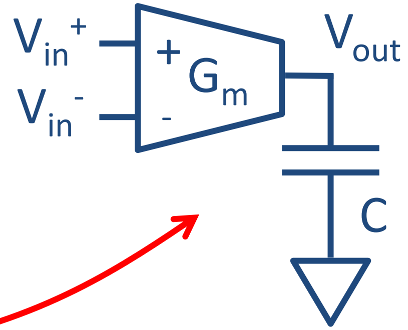
Filters

- Passive RLC filters
  - – Large L and C values for low frequencies → Cannot be integrated in an IC
- Opamp-RC
  - – Overdesigned GBW needed → Power hungry
- Switched capacitor
  - – Opamps and f_{switch} far beyond bandwidth → Power hungry
- Mosfet-RC
  - – Transistors are non-linear → Filter will have poor linearity
- G_m-C filters
  - – Open-loop G_m stages → Power-efficient, modest linearity

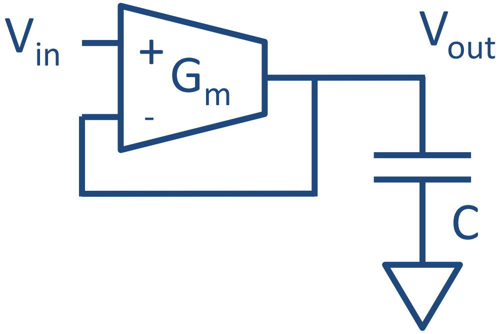
G_m-C Filters

- Basic stage: G_m + C,
  acts as integrator:
  - $H(s) = 1/s\tau$, with $\tau = C / G_m$
  - $V_{out}(s) = 1 / s\tau \cdot V_{in}(s)$
  - $V_{out}(s) = 1 / s\tau \cdot (V_{in}^{+}(s) - V_{in}^{-}(s))$



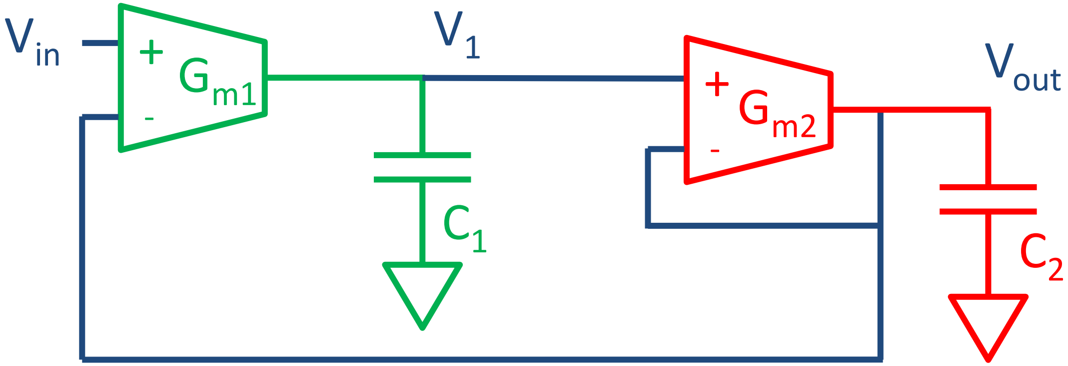


- How to synthesize: 1st order filter example
  - $V_{out}(s) / V_{in}(s) = 1 / (s\tau + 1)$
  - $V_{out}(s)(s\tau + 1) = V_{in}(s)$
  - $V_{out}(s)\, s\tau = V_{in}(s) - V_{out}(s)$
  - $V_{out}(s) = 1 / s\tau \cdot (V_{in}(s) - V_{out}(s))$



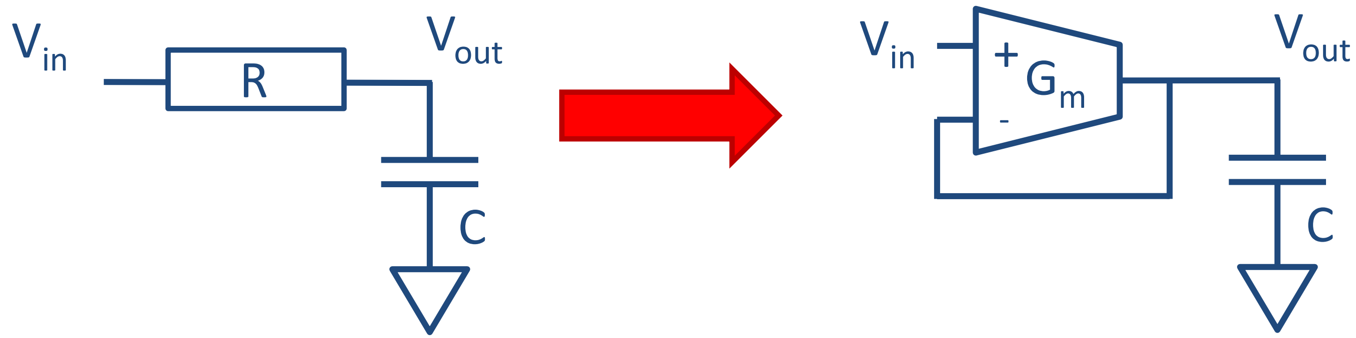
2^{nd} Order G_m-C Filter

- $V_{out}(s) / V_{in}(s) = 1 / (\tau_1\tau_2 s^2 + \tau_1 s + 1)$
- $V_{out}(s) \cdot (\tau_1\tau_2 s^2 + \tau_1 s + 1) = V_{in}(s)$
- $V_{out}(s) \cdot \tau_1 s(\tau_2 s + 1) = V_{in}(s) - V_{out}(s)$
- $V_{out}(s) \cdot (\tau_2 s + 1) = 1 / s\tau_1 (V_{in}(s) - V_{out}(s))$
- $V_{out}(s) \cdot \tau_2 s = 1 / s\tau_1 (V_{in}(s) - V_{out}(s)) - V_{out}(s)$
- $V_{out}(s) = 1 / s\tau_2 \{1 / s\tau_1 (V_{in}(s) - V_{out}(s)) - V_{out}(s)\}$



Higher-Order G_m-C Filters

- Either use:
  - Concatenation of 1st and 2nd order stages
  - Apply element replacement synthesis method: {R, L, C} is replaced by its equivalent in G_m-C
    e.g.: $V_{out}(s) / V_{in}(s) = 1 / (s\tau + 1)$



Noise in G_m-C Filters

- Before:
  - RC filter, S&H: $P_{noise} = kT / C$
- G_m/C filter: $P_{noise} \propto kT / C$
- Approach:
  - Size the capacitors based on the noise requirement
  - Determine G_m's to reach the proper $\tau = C / G_m$
  - Calculate I_{BIAS} currents for each G_m stage

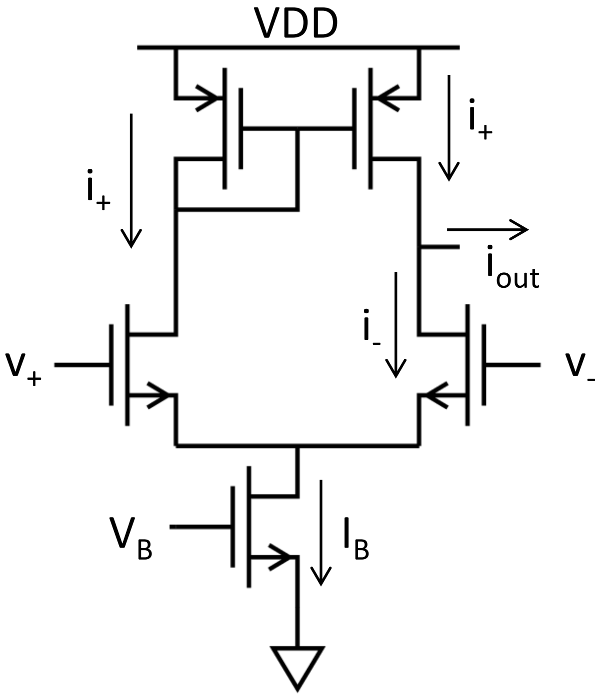
Exercise 6: First-Order G_m-C filter

We would like to design a first-order G_m-C low-pass filter with a cut-off frequency of 2kHz. It is already given that C = 100fF.

a) What is the circuit topology that we need for this filter?

b) What is the required value of G_m?

c) Assuming we use the OTA given below, and assuming it is biased in sub-threshold, what is thus the required bias current I_B?



Summary

- Amplifiers
- Positive feedback to increase gain or input impedance
- Chopping amplifiers
- G_m-C filters

Solution 1: TIA

a)

- First circuit:
  - KCL: $i_{in} + V_{out}\,sC + (V_{out} – V_B) / R_f = 0$
  - When determining the transfer function from source i_{in} to output V_{out}, you may set the other sources to 0, so $V_B = 0V$.
  - $i_{in} + v_{out}\,sC + v_{out} / R_f = 0$
  - $v_{out} / i_{in} = -R_f / (1 + s\,C\,R_f)$
- Second circuit:
  - Amplifier equation: $V_{out} = A\,(V_+ - V_-) = A\,(V_B – V_-)$
  - KCL: $i_{in} + V_-\,sC + (V_- – V_{out}) / R_f = 0$
  - Combining the above equations and setting V_B to 0V as before:
  - $v_- = -v_{out} / A$
  - $i_{in} – v_{out}\,sC / A – v_{out}\,(1 / A + 1) / R_f = 0$
  - $v_{out} / i_{in} \approx R_f / (1 + s\,C\,R_f / A)$

Solution 2: OTA Noise

a) Each of the two input transistors has a gate noise of $V_{gn}^2(f) = kT / 9I_{DS}$.
Together, that gives a total IRN of $V_{in,n}^2(f) = 2kT / 9I_{DS}$.
$I_{DS} = ½ I_B$, so the overall IRN as function of I_B is $V_{in,n}^2(f) = 4kT / 9I_B$.

b) $2\mu V_{rms}$ means a total noise power of $4pV^2$ in the 10kHz BW.
This is equivalent to a PSD of $V_{in,n}^2(f) = 4pV^2 / 10kHz = 0.4fV^2/Hz$.
So: $0.4fV^2/Hz = 4kT / 9I_B$, which results in an I_B of 4.6µA.

Solution 3: CS VA versus INV VA

Assume that the bias current for the circuits below is set to 1µA and assume that all transistors are biased in sub-threshold.

a) $V_n^2(f) = \{I_{n1}^2(f) + I_{n2}^2(f)\} / g_{m1}^2$.
   $I_{n1}^2(f) = I_{n2}^2(f) = 2qI_D$, with $I_D = 1\mu A$.
   We can estimate g_{m1} by e.g.: $g_{m1} \approx 25 I_D = 25\mu A/V$.
   Solving this gives: $V_n^2(f) \approx 1 fV^2/Hz$.

b) $V_n^2(f) = \{I_{n1}^2(f) + I_{n2}^2(f)\} / (g_{m1} + g_{m2})^2$
   The values are the same as before and $g_{m2} = g_{m1}$.
   Solving this gives: $V_n^2(f) \approx 0.26 fV^2/Hz$.

Solution 4: Positive Feedback to Enhance Gain

a) Amplifier equation: $V_{out} = AV_+$, where $A = 20$.
KCL: $(V_+ - V_{in}) / R_1 + (V_+ - V_{out}) / R_2 = 0$, where $R_2 = 1M\Omega$.
Final goal: $V_{out} = 100V_{in}$.

Combining the equations gives:
$V_+ = 0.05V_{out}$
$V_{in} = 0.01V_{out}$
$(0.05V_{out} - 0.01V_{out}) / R_1 + (0.05V_{out} - V_{out}) / 1M\Omega = 0$
$0.04 / R_1 = 0.95 / 1M\Omega \rightarrow R_1 = 42k\Omega$.

Solution 5: Positive Feedback to Enhance Z_{in}

a) $R_2 = 100M\Omega$, because the closed-loop gain is R_2/R_1.

b) $2R_1 = 2M\Omega$.

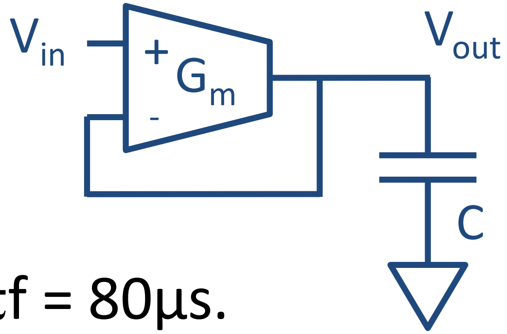
c) Analyzing single-ended:
$i_{in} = i_{R1} + i_{Rx} = 0.5v_{in} / R_1 + (0.5v_{in} – 0.5v_{out}) / R_x$.
$v_{out} = 100\ v_{in}$.
$R_x = R_2 = 100R_1$.
$Z_{in,diff} = 2Z_{in,single\text{-}ended} = v_{in} / i_{in}$.

Combining the equations gives:
$i_{in} = 0.5v_{in} / R_1 – 49.5v_{in} / R_x = v_{in} (0.5 / R_1 – 0.495 / R_1) = 0.005\ v_{in} / R_1$
$Z_{in,diff} = v_{in} / i_{in} = 1 / (0.005 / R_1) = 200R_1 = 200M\Omega$.

Solution 6: First-Order G_m-C filter

We would like to design a first-order G_m-C low-pass filter with a cut-off frequency of 2kHz. It is already given that C = 100fF.

a) Topology as shown in figure
The equation for it is
$V_{out}(s) / V_{in}(s) = 1 / (s\tau + 1)$
where $\tau = C / G_m$



b) 2kHz cut-off frequency, so $\tau = 1 / 2\pi f = 80\mu s$.
Since C = 100fF, that implies G_m must be 100fF/80µs = 1.26nA/V.

c) For this OTA, the G_m is approximately $12.5 I_B$. So $I_B = 0.1$nA.