EINDHOVEN UNIVERSITY OF TECHNOLOGY

Communication Theory (5ETB0)
Module 8.1

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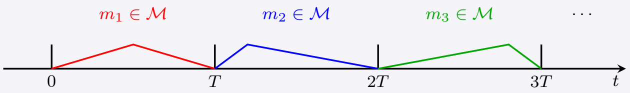
Module 8.1

Presentation Outline

Motivation: A Stream of Messages

Preliminaries

- Previously: considered transmission of a **single randomly-chosen message** $m \in \mathcal{M}$ over a waveform channel
- Now: Transmission of a **stream** of messages over the AWGN waveform channel
- Assumption 1: The signals $s_m(t)$ are only non-zero inside the time-interval $0 \leq t < T$
- Assumption 2: Equally likely messages (i.e., $\Pr\{M = m\} = 1/|\mathcal{M}|$ for all $m \in \mathcal{M}$)



Definitions and Problem Statement

Definitions

- **Transmit Power** is P_s ([Joule/sec] or [Watt])
- **Average Energy** is $E_s = P_s T$ [Joule]
- **Transmission rate** R is defined as

$$R \triangleq \frac{\log_2 |\mathcal{M}|}{T} \quad \left[\frac{\text{bits}}{\text{second}}\right]$$

- **Energy per transmitted bit** is

$$E_b \triangleq \frac{E_s}{\log_2 |\mathcal{M}|} = \frac{E_s}{T} \frac{T}{\log_2 |\mathcal{M}|} = \frac{P_s}{R} \quad \left[\frac{\text{Joule}}{\text{bit}}\right]$$

Questions to be Answered

- What is the **maximum rate at which we can communicate reliably** over a waveform channel when the available power is P_s?
- What are the signals that are to be used to achieve this maximum rate?
- Two systems considered: bit-by-bit and block-orthogonal signaling

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Presentation Outline

Bit-by-Bit Signaling: Definitions

Rate and Transmitted Waveform

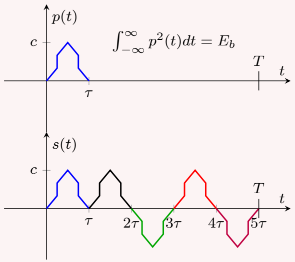
Transmit K binary digits $b_1 b_2 \cdots b_K$ in T seconds. Then

$$|\mathcal{M}| = 2^K, \quad R = \frac{\log_2 |\mathcal{M}|}{T} = \frac{K}{T}$$

Transmit signal $s(t)$, composed of K pulses $p(t)$ that are time shifted:

$$s(t) = \sum_{i=1}^{K} (-1)^{b_i+1} p(t-(i-1)\tau)$$

Signal set is: $\mathcal{S} = \{s_1(t), s_2(t), \ldots, s_{2^K}(t)\}$



Message to be transmitted: 11010

Bit-by-Bit Signaling: Building-block Waveform

Building-block Waveforms

The building-block waveforms are time-shifts over multiples of τ of the normalized pulse $p(t)/\sqrt{E_b}$

$$\varphi_i(t) \triangleq \frac{p(t-(i-1)\tau)}{\sqrt{E_b}}, \qquad i = 1, 2, \ldots, K$$

Questions...

Q1: Can the messages $s_m(t)$ be written as a linear combination of $\varphi_i(t)$? Yes!

$$s(t) = \sum_{i=1}^{K} (-1)^{b_i+1} p(t-(i-1)\tau) = \sum_{i=1}^{K} (-1)^{b_i+1} \sqrt{E_b} \varphi_i(t)$$

Q2: Are $\varphi_i(t)$ orthonormal? Yes!

$$\int_{-\infty}^{\infty} \varphi_i(t)\varphi_j(t)dt = \frac{1}{E_b}\int_{-\infty}^{\infty} p(t-(i-1)\tau)p(t-(j-1)\tau)dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Q3: What is the dimensionality of the signal space? $N = K$

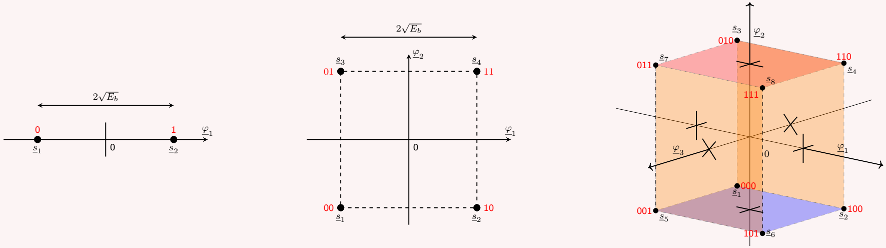
Bit-by-Bit Signaling: Geometry

Geometric Representation

The signals are

$$s(t) = \sum_{i=1}^{K} (-1)^{b_i+1} \sqrt{E_b} \varphi_i(t)$$

The vectorial representation is $\underline{s}_m = \sqrt{E_b}((-1)^{b_1+1}, (-1)^{b_2+1}, \ldots, (-1)^{b_N+1})$



$K = N = 1, |\mathcal{M}| = 2$ $\qquad$ $K = N = 2, |\mathcal{M}| = 4$ $\qquad$ $K = N = 3, |\mathcal{M}| = 8$

Bit-by-Bit Signaling: Reception

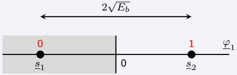
Optimum Receiver

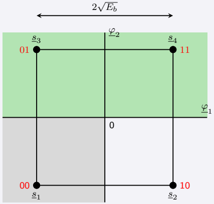
The optimum receiver decides $\hat{m} = 1$ if

$$r_i < 0, \text{ for all } i = 1, ..., K$$

Geometric Interpretation

If $m = 1$ is transmitted: $\underline{s}_1 = \left(-\sqrt{E_b}, -\sqrt{E_b} \dots, -\sqrt{E_b}\right)$





Note: To estimate b_i with $i = 1, 2, \dots, K$, only r_i in dimension i is needed.

Bit-by-Bit Signaling: Error Probability (1/2)

Correct and Error Probabilities

- The signal hypercube is symmetrical
- Assume that $\underline{s}_1$ was transmitted
- No error occurs if $r_i = -\sqrt{E_b} + n_i < 0$ for all $i = 1, \ldots, K$:

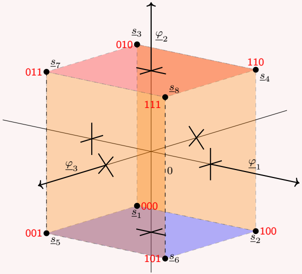
$$n_i < \sqrt{E_b} \text{ for all } i = 1, \ldots, K$$

- Correct Probability

$$P_{\mathsf{c}} = \left(1 - Q(\sqrt{2E_b/N_0})\right)^K$$

- Error Probability

$$P_{\mathsf{e}} = 1 - \left(1 - Q(\sqrt{2E_b/N_0})\right)^K$$



Bit-by-Bit Signaling: Error Probability (2/2)

Error Probabilities Considerations

- Using $K = RT$ and $E_b = P_s/R$

$$P_\mathrm{e} = 1 - \left(1 - Q\left(\sqrt{\frac{2P_s}{RN_0}}\right)\right)^{RT}$$

- Fix P_s and R and consider two extreme cases for T:
  - $T = 1/R \Rightarrow K = 1 \Rightarrow$

$$P_\mathrm{e} = Q\left(\sqrt{\frac{2P_s}{RN_0}}\right)$$

    Conclusion: P_e can be decreased by increasing P_s or by decreasing R
  - $T \to \infty \Rightarrow P_\mathrm{e} \to 1$
    Conclusion: Reliability **cannot** be increased by increasing T

Is this the end of the story?

Can we increase reliability by increasing T? Yes! With block-orthogonal signaling

Summary Module 8.1

Take Home Messages

- Introduced the problem of serial transmission
- Bit-by-bit signalling model and analysis
- Increasing dimensionality in bit-by-bit signalling does not help

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