Components in wireless technology, 5XTC0

Module 6
Exercise: Amplifier matching using distributed components

Vojkan Vidojkovic

TU/e Technische Universiteit Eindhoven University of Technology

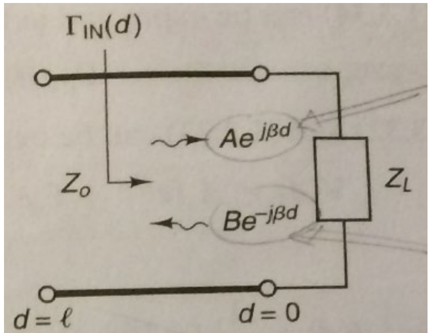
Outline

- Amplifier design
    - Solution for problem 1 from assignment
- Matching using distributed components
- Amplifier matching using distributed

Formula list available for exam

Power delivered to the load	$P_L = \frac{\lvert V_2^-\rvert^2}{2Z_0}\left(1-\lvert\Gamma_L\rvert^2\right)$
Input power to the network	$P_{in} = \frac{\lvert V_1^+\rvert^2}{2Z_0}\left(1-\lvert\Gamma_{in}\rvert^2\right)$
Input and output reflection coefficients of a transistor with a source and load: general case	$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$ $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$
Input and output reflection coefficients of a transistor with a source and load: unilateral case	$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$ $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22}$
Gain of the input matching network	$G_S = \frac{1-\lvert\Gamma_S\rvert^2}{\lvert 1-\Gamma_{\text{in}}\Gamma_S\rvert^2}$
Gain of the output matching network	$G_L = \frac{1-\lvert\Gamma_L\rvert^2}{\lvert 1-S_{22}\Gamma_L\rvert^2}$
Gain of the transistor (unilateral case)	$G_0 = \lvert S_{21}\rvert^2$
Transducer gain of the basic amplifier circuit (input matching, unilateral transistor, output matching)	$G_T = G_S G_0 G_L$ $G_{T,dB} = G_{S,dB} + G_{0,dB} + G_{L,dB}$
Maximum gain of the input and output matching networks	$G_{S_{\max}} = \frac{1}{1-\lvert S_{11}\rvert^2}$, $G_{L_{\max}} = \frac{1}{1-\lvert S_{22}\rvert^2}$.
Maximum transducer power gain, unilateral case	$G_{TU_{\max}} = \frac{1}{1-\lvert S_{11}\rvert^2}\lvert S_{21}\rvert^2\frac{1}{1-\lvert S_{22}\rvert^2}$
Normalized gain factors g_S and g_L	$g_S = \frac{G_S}{G_{S_{\max}}} = \frac{1-\lvert\Gamma_S\rvert^2}{\lvert 1-S_{11}\Gamma_S\rvert^2}(1-\lvert S_{11}\rvert^2)$, $g_L = \frac{G_L}{G_{L_{\max}}} = \frac{1-\lvert\Gamma_L\rvert^2}{\lvert 1-S_{22}\Gamma_L\rvert^2}(1-\lvert S_{22}\rvert^2)$.
Center and radius of the constant gain circle for the input matching network	$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)\lvert S_{11}\rvert^2}$, $R_S = \frac{\sqrt{1-g_S}\,(1-\lvert S_{11}\rvert^2)}{1-(1-g_S)\lvert S_{11}\rvert^2}$
Center and radius of the constant gain circle for the output matching network	$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)\lvert S_{22}\rvert^2}$, $R_L = \frac{\sqrt{1-g_L}\,(1-\lvert S_{22}\rvert^2)}{1-(1-g_L)\lvert S_{22}\rvert^2}$
Condition for "unconditionally stable" device, general case	for all $\lvert\Gamma_L\rvert<1$ and $\lvert\Gamma_S\rvert<1$ $\Rightarrow \begin{cases} \lvert\Gamma_{in}\rvert = \left\lvert S_{11}+\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right\rvert < 1 \\ \lvert\Gamma_{out}\rvert = \left\lvert S_{22}+\frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}\right\rvert < 1 \end{cases}$

Conditions for "unconditionally stable" device, unilateral case	$\lvert\Gamma_{in}\rvert = \lvert S_{11}\rvert < 1$ $\lvert\Gamma_{out}\rvert = \lvert S_{22}\rvert < 1$
Center and radius of the stability circles, load side	$C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{\lvert S_{22}\rvert^2-\lvert\Delta\rvert^2}$ (center), $R_L = \left\lvert\frac{S_{12}S_{21}}{\lvert S_{22}\rvert^2-\lvert\Delta\rvert^2}\right\rvert$ (radius). $\Delta = S_{11}S_{22} - S_{12}S_{21}$
Center and radius of the stability circles, source side	$C_S = \frac{(S_{11}-\Delta S_{22}^*)^*}{\lvert S_{11}\rvert^2-\lvert\Delta\rvert^2}$ (center), $R_S = \left\lvert\frac{S_{12}S_{21}}{\lvert S_{11}\rvert^2-\lvert\Delta\rvert^2}\right\rvert$ (radius) $\Delta = S_{11}S_{22} - S_{12}S_{21}$
Test for unconditional stability, general case	$\lvert\Delta\rvert = \lvert S_{11}S_{22}-S_{12}S_{21}\rvert < 1$ **and** $K = \frac{1-\lvert S_{11}\rvert^2-\lvert S_{22}\rvert^2+\Delta^2}{2\lvert S_{12}S_{21}\rvert} > 1$
Test for unconditional stability, unilateral case	$\lvert S_{11}\rvert < 1$ $\lvert S_{22}\rvert < 1$
Two-stage amplifier: Output noise and noise figure	$P_{N,total} = G_{A2}(G_{A1}P_{N,in}+P_{n1})+P_{n2}$ $\Rightarrow F_{total} = \frac{P_{N,total}}{P_{N,in}G_{A1}G_{A2}} = 1+\frac{P_{n1}}{P_{N,in}G_{A1}}+\frac{P_{n2}}{P_{N,in}G_{A1}G_{A2}}$ $F_{total} = F_1 + \frac{F_2-1}{G_{A1}}$, with $F_j = 1+\frac{P_{nj}}{P_{N,in}G_{Aj}}$, j=1,2
Noise figure of a 2-port amplifier	$F = F_{\min} + \frac{r_N}{g_S}\lvert \underline{y}_S - \underline{y}_{opt}\rvert^2$ $F = F_{\min} + 4r_N\frac{\lvert\underline{\Gamma}_S-\underline{\Gamma}_{opt}\rvert^2}{(1-\lvert\underline{\Gamma}_S\rvert^2)\cdot\lvert 1+\underline{\Gamma}_{opt}\rvert^2}$
Constant noise circles	$\underline{C}_F = \frac{\Gamma_{opt}}{1+N}$ $R_r = \frac{1}{1+N}\sqrt{N^2+N(1-\lvert\Gamma_{opt}\rvert^2)}$ $\Delta F_n' = N = (F-F_{\min})\frac{\lvert 1+\underline{\Gamma}_{opt}\rvert^2}{4r_n} = \frac{\lvert\underline{\Gamma}_S-\underline{\Gamma}_{opt}\rvert^2}{1-\lvert\underline{\Gamma}_S\rvert^2}$



$$V(d) = A_1 e^{j\beta d} + B_1 e^{-j\beta d}$$

$$I(d) = \frac{A_1}{Z_o}e^{j\beta d} - \frac{B_1}{Z_o}e^{-j\beta d}$$

$$\Gamma_0 = \Gamma_{IN}(0) = \frac{B_1}{A_1}$$

Problem 1

- Assignment, problem 1

A microwave amplifier is to be designed for $G_{TU,max}$ using a bipolar transistor with:

$$S_{11}=0.4, \measuredangle\ 130^{\circ}$$
$$S_{12} = 0$$
$$S_{21}=6, \measuredangle\ 40^{\circ}$$
$$S_{22}=0.7, \measuredangle\ -75^{\circ}$$

The S-parameters were measured in a 50 Ω system at f=1 GHz, $V_{CE} = 15$ V, and $I_C = 15$ mA.

a) Determine whether this amplifier is unconditionally stable.

b) Determine $G_{TU,max}$. Express your answer in dB.

c) Determine the optimum source impedance and the optimum load impedance for maximum gain. Illustrate your answer in the Smith chart.

d) Design a 2-element matching network at the input and a 2-element matching network at the output of the amplifier to reach conjugate matching to a 50 Ω source and load impedance. Illustrate your answer on the Smith chart.

e) Calculate the constant gain circle for G_L=1.5 dB and indicate it in the Smith chart.

Problem 1: Solution

Problem

$S_{11} = 0.4\, e^{j130}$

$S_{12} = 0$

$S_{21} = 6\, e^{j40}$

$S_{22} = 0.7\, e^{j-75}$

a) $\Delta = S_{11} S_{22} - S_{12} S_{21} < 1$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + \Delta^2}{2\,|S_{12}\, S_{21}|} > 1$$

→ conditions for unconditional stability

unilateral case: $|S_{11}| = 0.4 < 1$

$|S_{22}| = 0.7 < 1$

amplifier unconditionally stable

b) $$G_{TU\,max} = \frac{1}{1-|S_{11}|^2}\,|S_{21}|^2\,\frac{1}{1-|S_{22}|^2}$$

unilateral case

$G_{TU\,max} = 1{,}2 \cdot 36 \cdot 2 = 86{,}4$

$G_{TU\,max}[dB] = 10 \log G_{TU\,max}$

$G_{TU\,max}[dB] = 19{,}4$ dB

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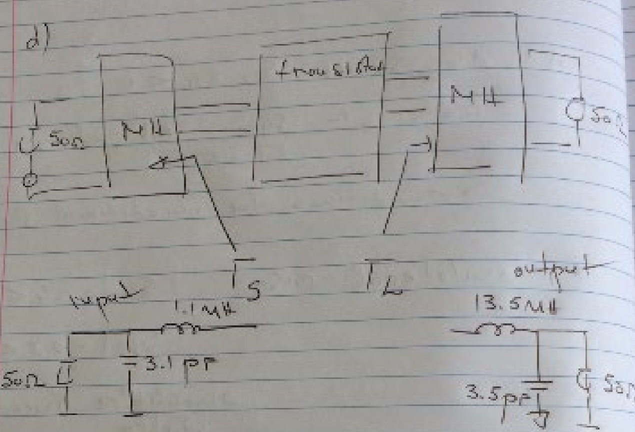
c) $\Gamma_S = S_{11}^* = 0.4\, e^{-j130}$

$\Gamma_L = S_{22}^* = 0.7\, e^{j75}$

d)



e) $G_L = 1.5$ dB $\Rightarrow$ $G_L = 1.41$

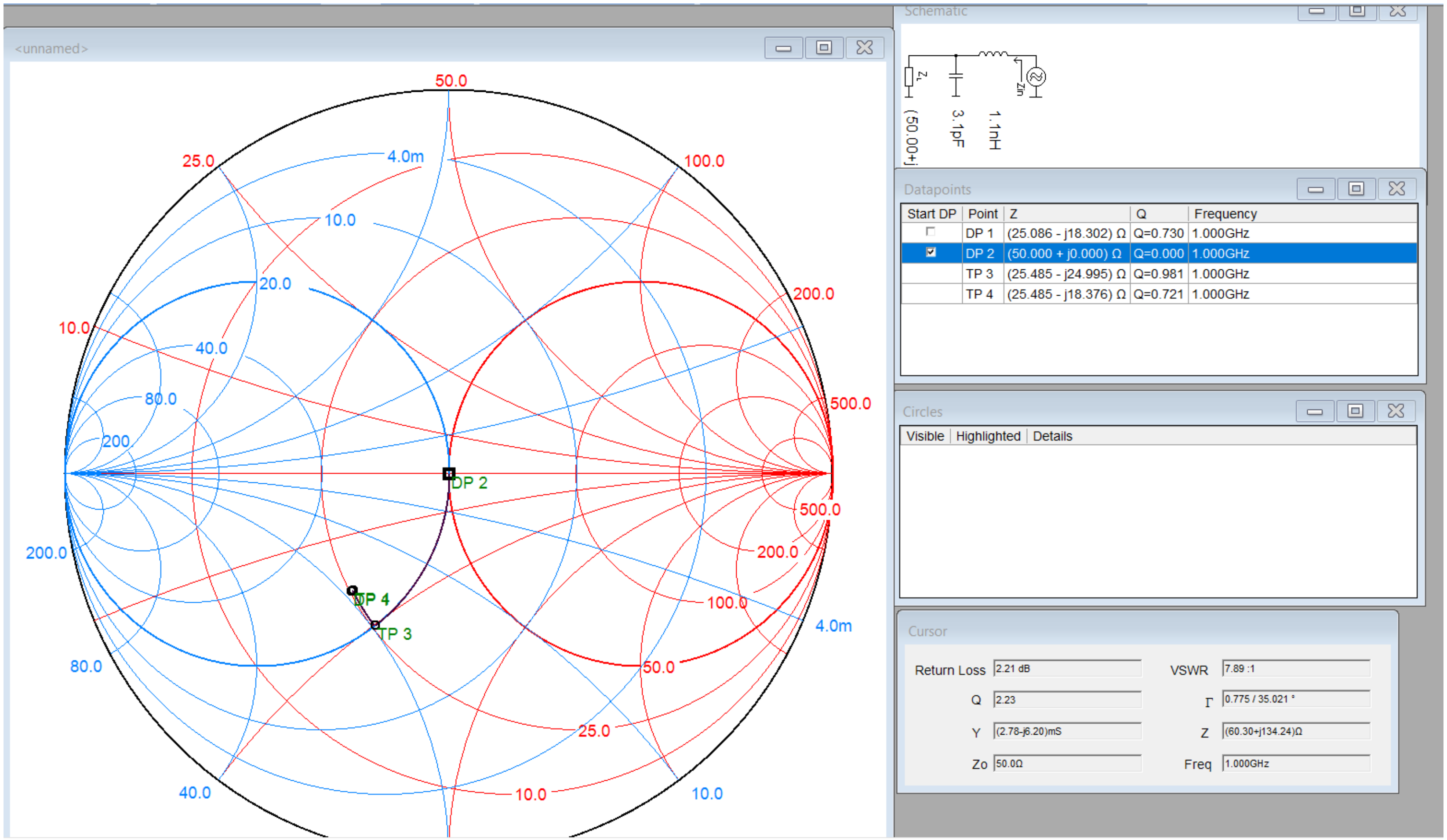
$$G_{L,max} = \frac{1}{1-|S_{22}|^2} = 2$$

$$g_L = \frac{1.41}{2} = 0.71$$

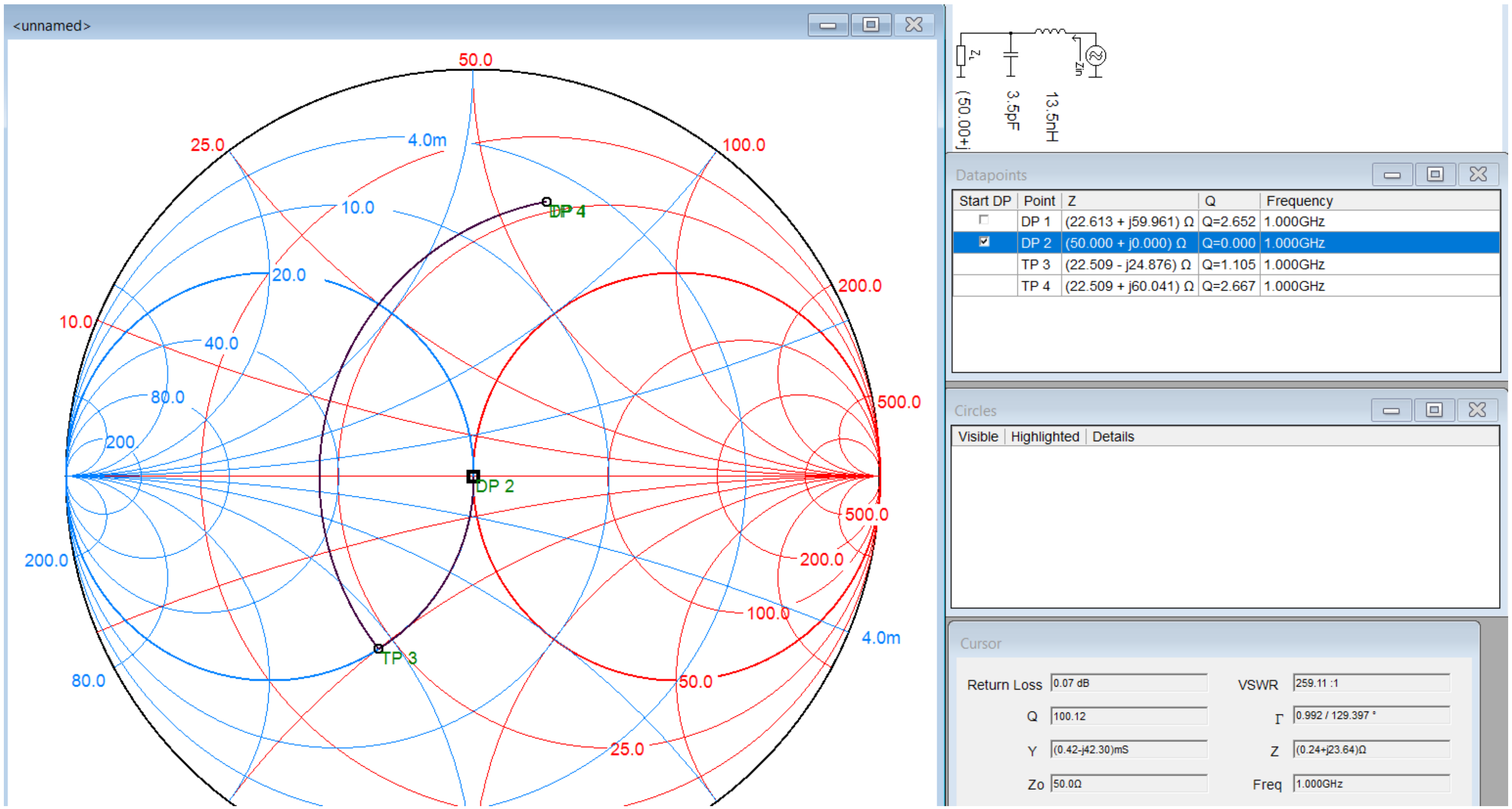
$$C_L = \frac{g_L\, S_{22}^*}{1-(1-g_L)|S_{22}|^2}$$

$$R_L = \frac{\sqrt{1-g_L}\,(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$

Problem 1: Solution, matching at input

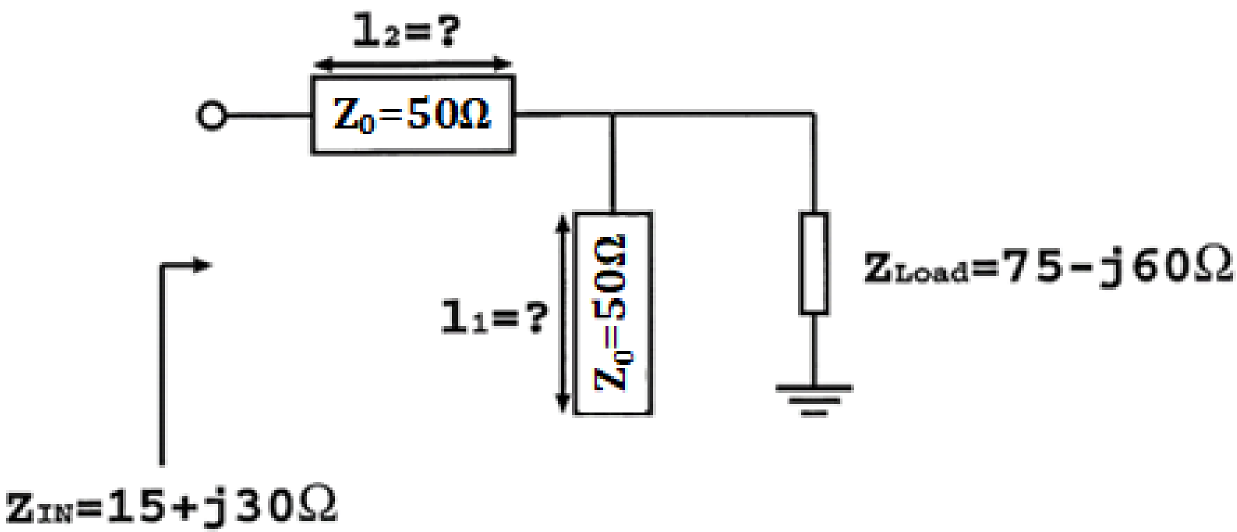


Problem 1: Solution, matching at output

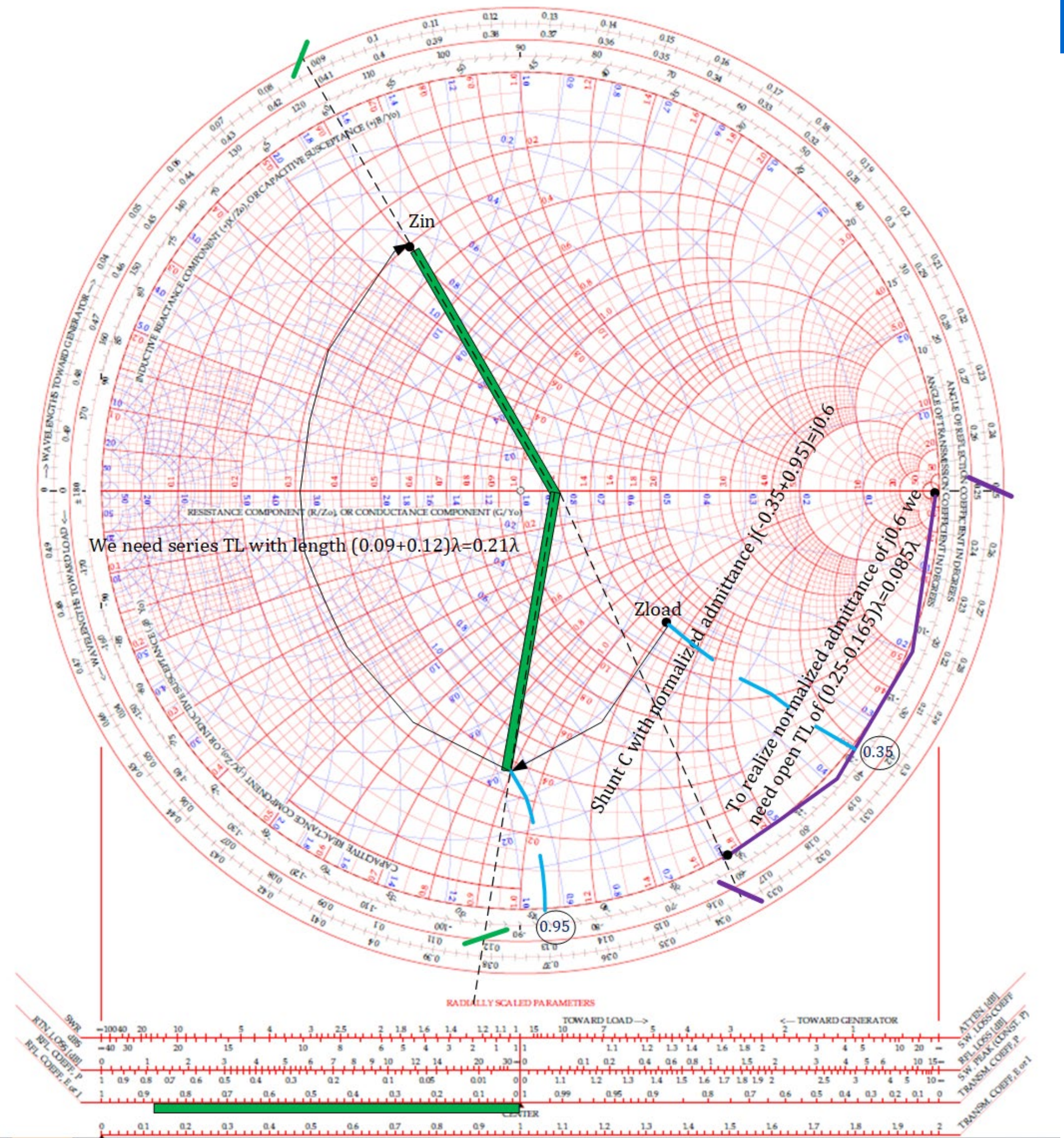


Problem 2 - Matching using TL

Using the Smith chart, design a matching network with microstrip lines (open-circuited stub and series line) which transforms the load impedance Z_{Load} into the input impedance Z_{IN}. Choose shortest possible line length.



Problem 2, Matching using TL - Solution



Step 1

Shunt C with normalized admitance j(-0.35+0.95)=j0.6
To realize normalized admittance of j0.6 we need open TL of (0.25-0.165)λ=0.085λ

l1=0.085λ

Step 2

We need series TL with length (0.09+0.12)λ=0.21λ

l2= 0.21λ

Example 12.4 book of Pozar – solution (1/4)

Next, we use (11.59) and (11.61) to compute the center and radius of the 2 dB noise figure circle:

$$N = \frac{F - F_{min}}{4R_N/Z_0}|1+\Gamma_{opt}|^2 = \frac{1.58 - 1.445}{4(20/50)}|1 + 0.62\angle 100^\circ|^2$$

$$= 0.0986,$$

$$C_F = \frac{\Gamma_{opt}}{N+1} = 0.56\angle 100^\circ$$

$$R_F = \frac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{N+1} = 0.24.$$

This noise figure circle is plotted in Figure 11.15a. Minimum noise figure ($F_{min} = 1.6$ dB) occurs for $\Gamma_S = \Gamma_{opt} = 0.62\angle 100^\circ$.

Next we calculate data for several input section constant gain circles. From (11.52),

G_S(dB)	g_S	C_S	R_S
1.0	0.805	$0.52\angle 60^\circ$	0.300
1.5	0.904	$0.56\angle 60^\circ$	0.205
1.7	0.946	$0.58\angle 60^\circ$	0.150

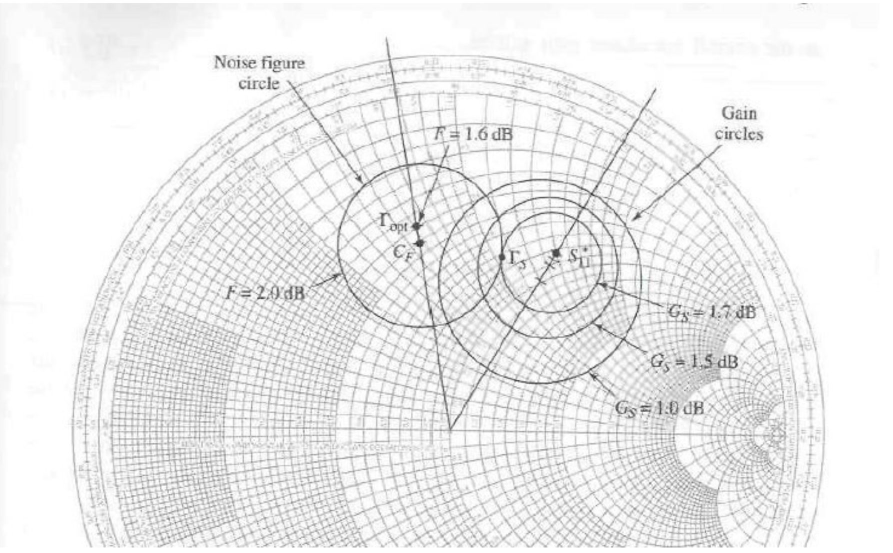
These circles are also plotted in Figure 11.15a. We see that the $G_S = 1.7$ dB gain circle just intersects the $F = 2$ dB noise figure circle, and that any higher gain will result in a worse noise figure. From the Smith chart the optimum solution is then $\Gamma_S = 0.53\angle 75^\circ$, yielding $G_S = 1.7$ dB and $F = 2.0$ dB.

For the output section we choose $\Gamma_L = S_{22}^* = 0.5\angle 60^\circ$ for a maximum G_L of

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB}.$$

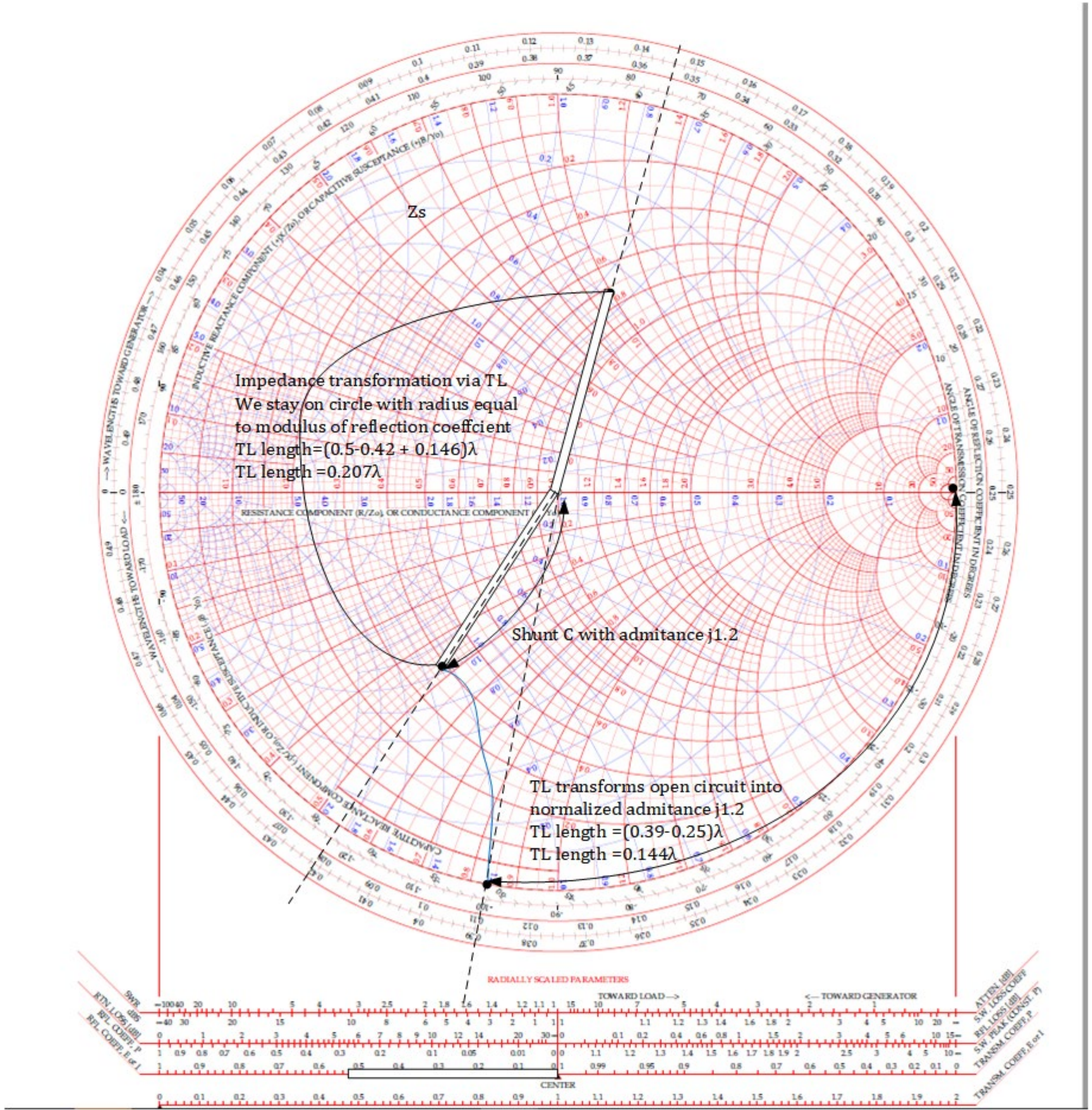
The transistor gain is

$$G_0 = |S_{21}|^2 = 3.61 = 5.58 \text{ dB},$$

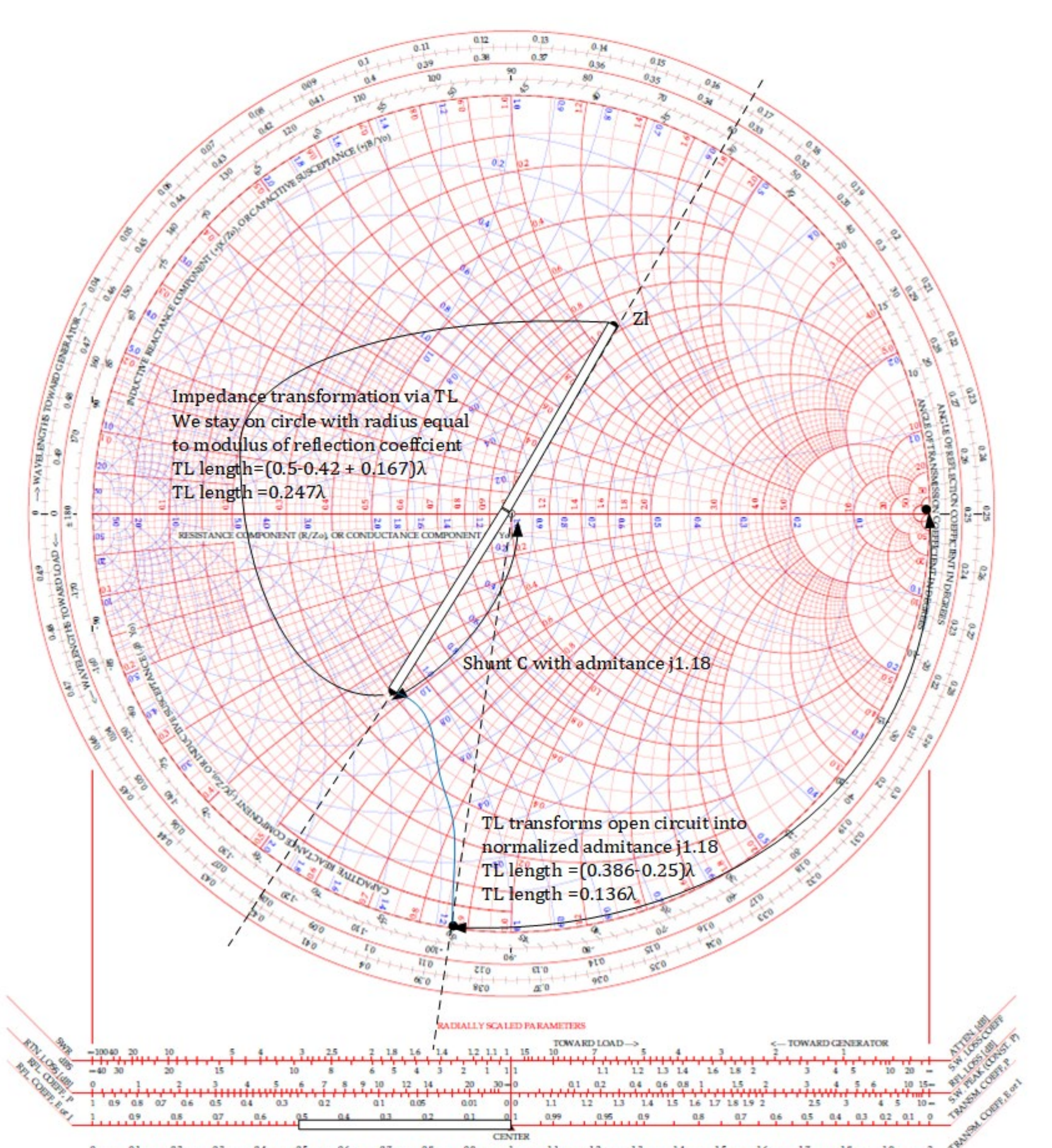


$G_T = G_s + G_0 + G_L$ [dB] = 8.5 dB

Example 12.4 book of Pozar – solution (2/4) Input matching network

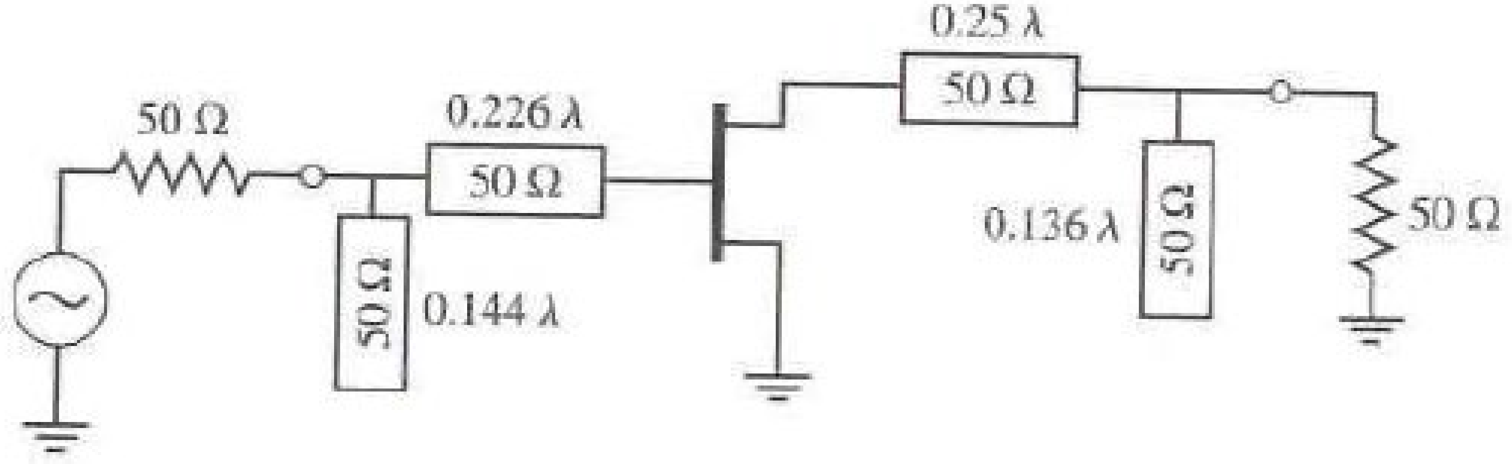


Example 12.4 book of Pozar – solution (3/4) Output matching network



Example 12.4 book of Pozar – solution (4/4) Amplifier schematic

(a)



Amplifier design – matching using TL Example 12.3 book of Pozar

EXAMPLE 11.3 Conjugately Matched Amplifier Design

Design an amplifier for maximum gain at 4.0 GHz using single-stub matching sections. ~~Calculate and plot the input return loss and the gain from 3 to 5 GHz.~~ The GaAs FET has the following S parameters ($Z_0 = 50\,\Omega$):

f (GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3.0	$0.80\angle{-89^\circ}$	$2.86\angle{99^\circ}$	$0.03\angle{56^\circ}$	$0.76\angle{-41^\circ}$
4.0	$0.72\angle{-116^\circ}$	$2.60\angle{76^\circ}$	$0.03\angle{57^\circ}$	$0.73\angle{-54^\circ}$
5.0	$0.66\angle{-142^\circ}$	$2.39\angle{54^\circ}$	$0.03\angle{62^\circ}$	$0.72\angle{-68^\circ}$

Amplifier design - matching using TL
Example 12.3 book of Pozar – Solution (1/4)

Example 11.3, book of Pozar, page 620

Maximum transducer power gain → Review

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S \Gamma_{in}|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_{T\,max} \quad \Gamma_{in} = \Gamma_S^*, \quad \Gamma_{out} = \Gamma_L^*$$

$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

page 620 @ Pozar

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$\cancel{\Delta = S_{11}S_{22} - S_{12}S_{21}}$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^* \qquad \Delta = S_{11}S_{22} - S_{12}S_{21}$$

1

We first check the stability of the transistor by calculating Δ and K at 4.0 GHz:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.488\angle{-162^\circ},$$

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.195.$$

Since $|\Delta| < 1$ and $K > 1$, the transistor is unconditionally stable at 4.0 GHz. There is no need to plot the stability circles.

For maximum gain, we should design the matching sections for a conjugate match to the transistor. Thus, $\Gamma_S = \Gamma_{in}^*$ and $\Gamma_L = \Gamma_{out}^*$, and Γ_S, Γ_L can be determined from (11.43):

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} = 0.872\angle{123^\circ}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876\angle{61^\circ}.$$

Then the effective gain factors of (11.19) can be calculated as

$$G_S = \frac{1}{1-|\Gamma_S|^2} = 4.17 = 6.20 \text{ dB},$$

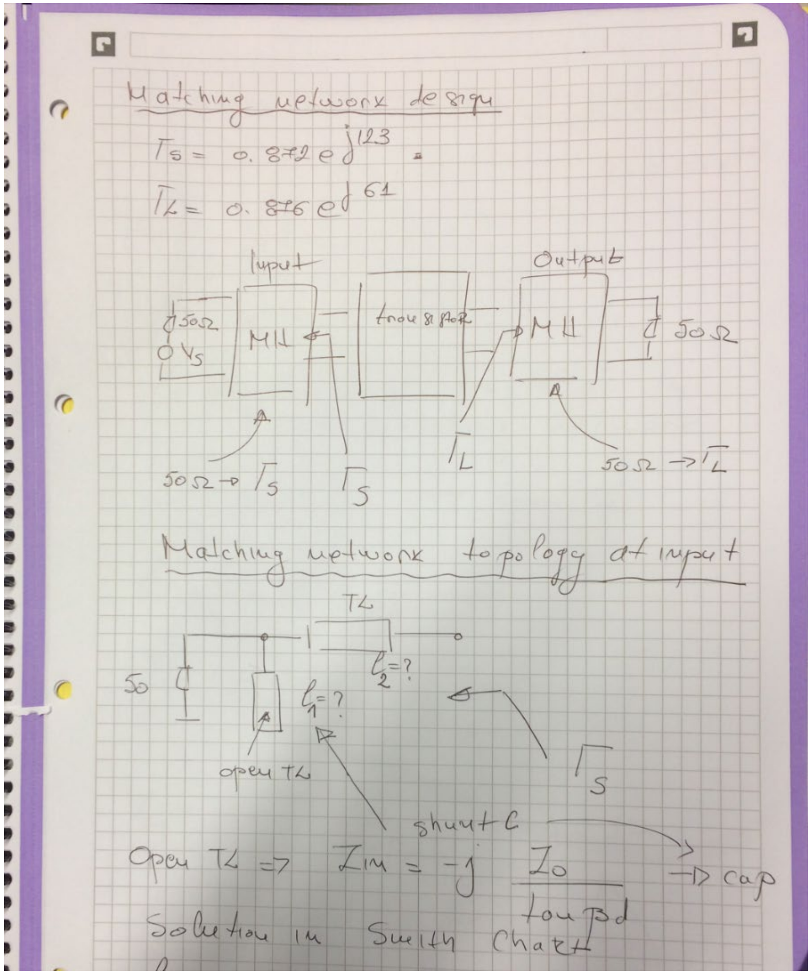
$$G_0 = |S_{21}|^2 = 6.76 = 8.30 \text{ dB},$$

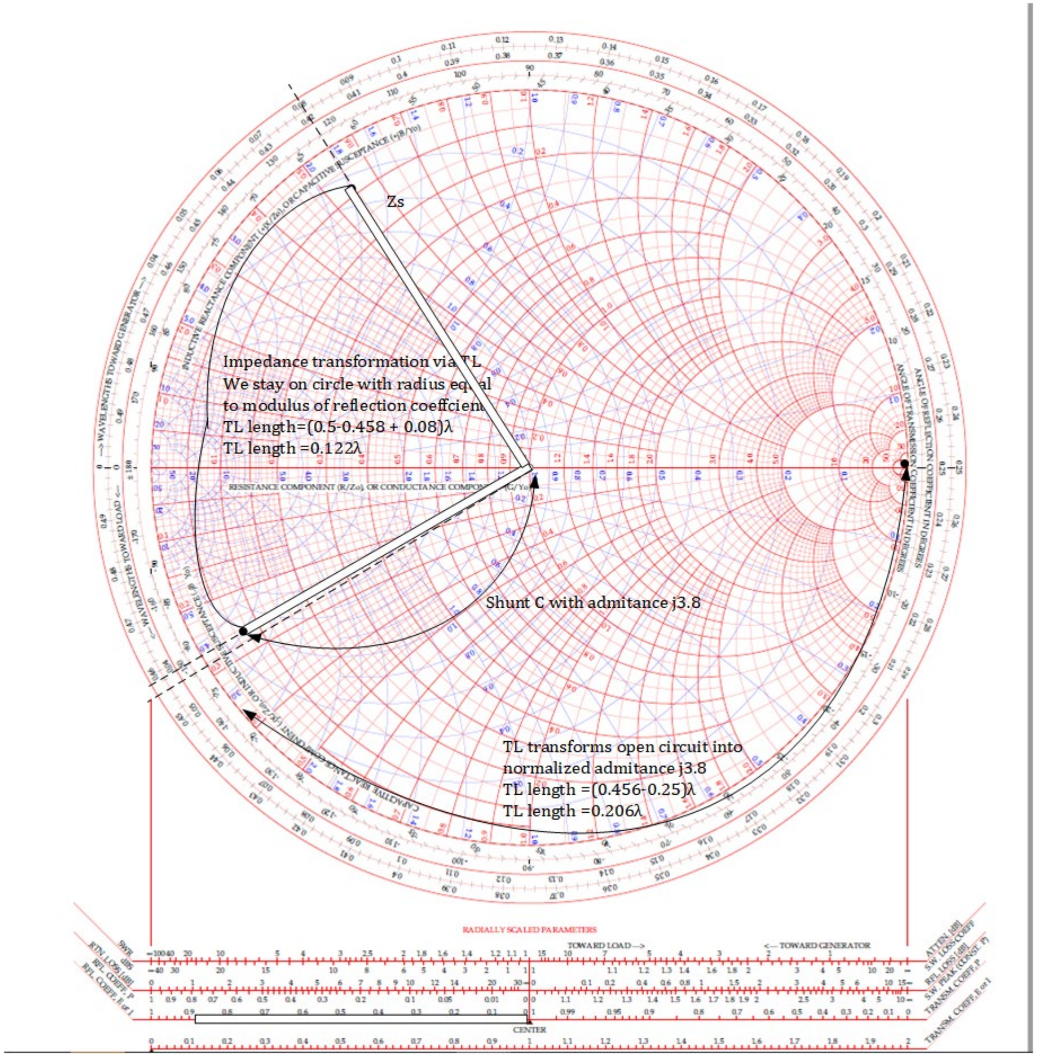
$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = 1.67 = 2.22 \text{ dB}.$$

So the overall transducer gain will be

$$G_{T_{max}} = 6.20 + 8.30 + 2.22 = 16.7 \text{ dB}.$$

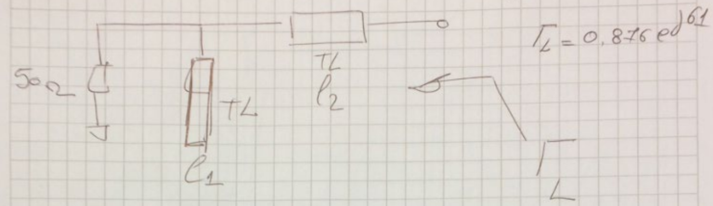
Amplifier design - matching using TL
Example 12.3 book of Pozar – Solution (2/4)





Amplifier design - matching using TL
Example 12.3 book of Pozar – Solution (3/4)

Matching network topology at output

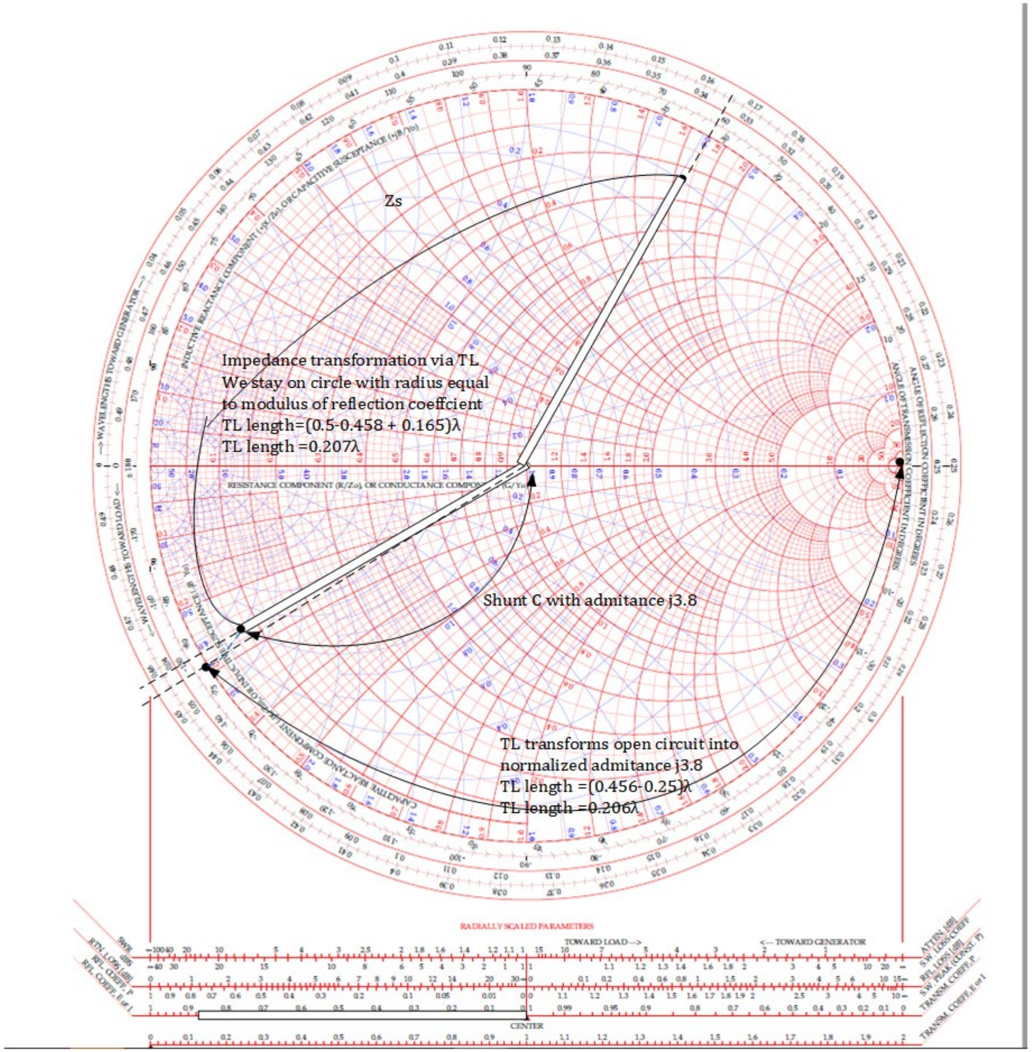


l_1 → the same length as in input matching network

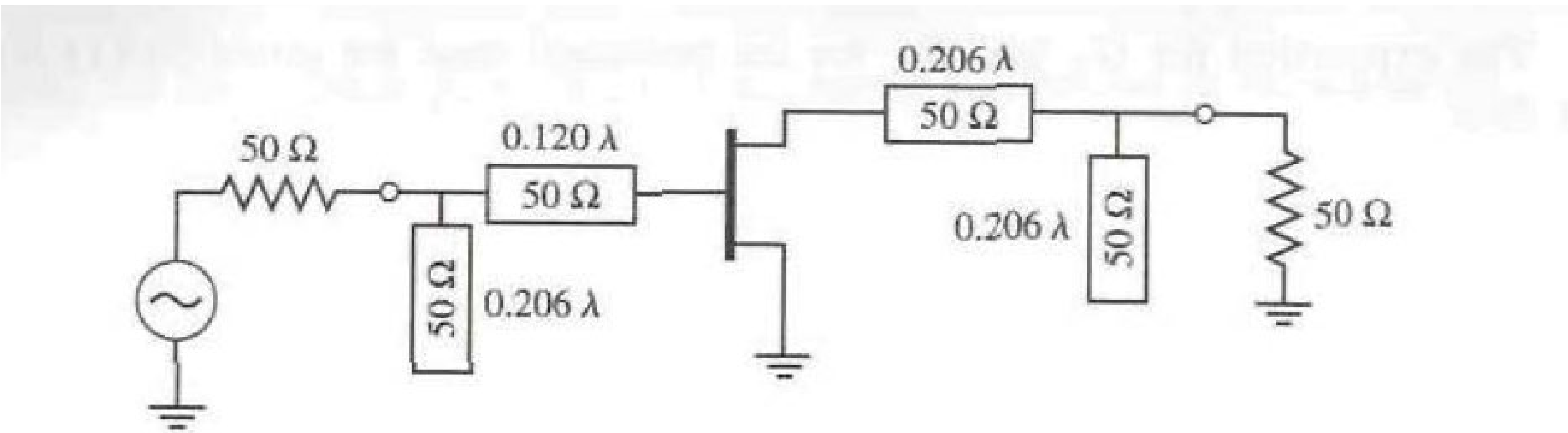
$l_1 = 0{,}208\ \lambda$ → open TL

$l_2 = 0{,}207\ \lambda$

$Z_{in,\,open\ TL} = -j\,\dfrac{Z_0}{\tan\beta d}$



Amplifier design
Example 12.3 book of Pozar – Solution (4/4)



Example 11.5 book of Pozar

A GaAs FET is biased for minimum noise figure, and has the following S parameters and noise parameters at 4 GHz ($Z_0 = 50\,\Omega$): $S_{11} = 0.6\angle{-60^\circ}$, $S_{21} = 1.9\angle{81^\circ}$, $S_{12} = 0.05\angle{26^\circ}$, $S_{22} = 0.5\angle{-60^\circ}$; $F_{min} = 1.6$ dB, $\Gamma_{opt} = 0.62\angle{100^\circ}$, $R_N = 20\,\Omega$. For design purposes, assume the device is unilateral, ~~and calculate the maximum error in G_T resulting from this assumption~~. Then design an amplifier having a 2.0 dB noise figure with the maximum gain that is compatible with this noise figure.

Example 11.4 book of Pozar

Design an amplifier to have a gain of 11 dB at 4.0 GHz. Plot constant gain circles for $G_S = 2$ dB and 3 dB, and $G_L = 0$ dB and 1 dB. Calculate and plot the input return loss and overall amplifier gain from 3 to 5 GHz. The FET has the following S parameters ($Z_0 = 50\,\Omega$):

f (GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3	$0.80\angle{-90^\circ}$	$2.8\angle{100^\circ}$	0	$0.66\angle{-50^\circ}$
4	$0.75\angle{-120^\circ}$	$2.5\angle{80^\circ}$	0	$0.60\angle{-70^\circ}$
5	$0.71\angle{-140^\circ}$	$2.3\angle{60^\circ}$	0	$0.58\angle{-85^\circ}$

Example 11.4 book of Pozar - Solution

Since $S_{12} = 0$ and $|S_{11}| < 1$ and $|S_{22}| < 1$, the transistor is unilateral and unconditionally stable. From (11.48) we calculate the maximum matching section gains as

$$G_{S_{max}} = \frac{1}{1 - |S_{11}|^2} = 2.29 = 3.6 \text{ dB},$$

$$G_{L_{max}} = \frac{1}{1 - |S_{22}|^2} = 1.56 = 1.9 \text{ dB}.$$

The gain of the mismatched transistor is

$$G_o = |S_{21}|^2 = 6.25 = 8.0 \text{ dB},$$

Chapter 11: Design of Microwave Amplifiers and Oscillators

so the maximum unilateral transducer gain is

$$G_{TU_{max}} = 3.6 + 1.9 + 8.0 = 13.5 \text{ dB}.$$

Thus we have 2.5 dB more gain than is required by the specifications.

We use (11.49), (11.52), and (11.53) to calculate the following data for the constant gain circles:

$G_S = 3$ dB	$g_S = 0.875$	$C_S = 0.706\angle 120^\circ$	$R_S = 0.166$
$G_S = 2$ dB	$g_S = 0.691$	$C_S = 0.627\angle 120^\circ$	$R_S = 0.294$
$G_L = 1$ dB	$g_L = 0.806$	$C_L = 0.520\angle 70^\circ$	$R_L = 0.303$
$G_L = 0$ dB	$g_L = 0.640$	$C_L = 0.440\angle 70^\circ$	$R_S = 0.440$