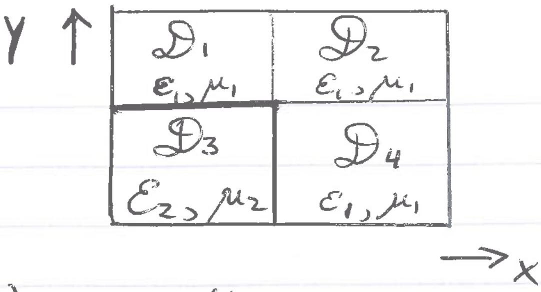
EM II Chapter 2

Question 2.1



a.) $H_y^{(2)} = H_y^{(1)}$

b.) $\mu_2 H_y^{(3)} = \mu_1 H_y^{(1)}$
$\Rightarrow H_y^{(3)} = \frac{\mu_1}{\mu_2} H_y^{(1)}$

c.) $H_y^{(4)} = H_y^{(2)} = H_y^{(1)}$

d.) $H_y^{(4)} = H_y^{(3)} = \frac{\mu_1}{\mu_2} H_y^{(1)}$ contradiction

At the right angle where the two line segments meet, (A corner) the interface is <u>Not smooth</u>. The field has singular components at a corner, and the piecewise constant approximation fails.

Question 2.2

a.) $V_{steady} = \frac{450}{450+50} \times 120\ V = 108\ V_s$ $\quad I_{steady} = \frac{120}{450+50} = 0.24 A$

b.) $V^+_{steady} = \frac{1}{2}(V_{steady} + Z I_{steady}) = \frac{1}{2}(108+36) = 72\ V$
$V^+_{steady}/Z = I^+_{steady} = 0.48\ A$
$V^-_{steady} = \frac{1}{2}(V_{steady} - Z I_{steady}) = \frac{1}{2}(108-36) = 36\ V$
$-V^-_{steady}/Z = I^-_{steady} = -0.24 A$

c.) At $z=0$, after $t=0$, the total current must be 0, because of the open switch.

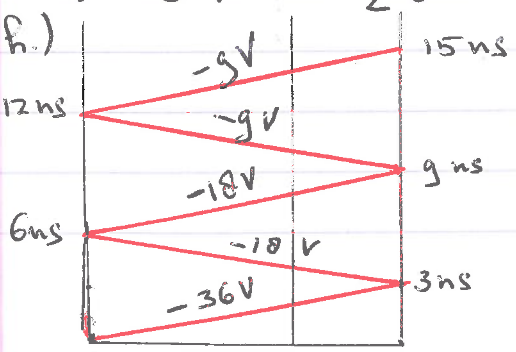
d.) $I_{total}\big|_{z=0,t>0} = 0 \Rightarrow \frac{V_{comp}}{Z} + I_{steady} = 0 \Rightarrow V_{comp} = -36V$

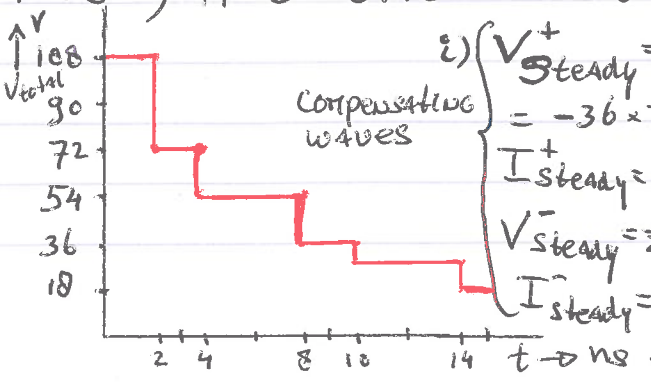
e.) $\frac{Z_L - Z}{Z_L + Z} = \Gamma_L = \frac{1}{2}$ $\quad \Gamma_g = \frac{\infty - Z}{\infty + Z} = 1$ (open switch).

f.) $\begin{cases} V^+_{2n} = (\Gamma_L \Gamma_g)^n V_{comp} = 2^{-n} \cdot V_{comp} = -36 \times 2^{-n}\ V. \\ I^+_{2n} = Y\, V^+_{2n} = -0.24 \times 2^{-n}\ A \end{cases}$

g.) $\begin{cases} V^-_{2n+1} = \Gamma_L (\Gamma_L \Gamma_g)^n V_{comp} = -18 \times 2^{-n}\ V \\ I^-_{2n+1} = -\frac{1}{2}(-0.24 \times 2^{-n})\ A = 0.12 \times 2^{-n}\ A \end{cases}$

h.)





i) $\begin{cases} V^+_{steady} = \sum_{n=0}^{\infty} -36 \times 2^{-n} = -36 \times \frac{1}{1-\frac{1}{2}} = -72V \\ I^+_{steady} = -0.48 A \\ V^-_{steady} = \frac{1}{2}(-72) = -36V \\ I^-_{steady} = 0.24 A \end{cases}$

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Question 2.3

a.) Because of the circular symmetry of the configuration, the scalar potential Φ must be independent of φ

$$\Rightarrow \frac{\partial^2}{\partial\varphi^2}\Phi = 0$$

b.) $$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\Phi}{d\rho}\right) = 0 \Rightarrow \frac{d}{d\rho}\left(\rho\frac{d\Phi}{d\rho}\right) = 0 \Rightarrow \rho\frac{d\Phi}{d\rho} = A \text{ (constant)}$$

c.) $$\frac{d\Phi}{d\rho} = \frac{A}{\rho} \Rightarrow \Phi(\rho) = \int_{\rho'=a}^{\rho}\frac{A}{\rho'}d\rho' + \Phi(a) = A(\mathrm{Log}\,\rho - \mathrm{Log}\,a) + \Phi(a) = A\,\mathrm{Log}\frac{\rho}{a} + \Phi(a)$$

(LN)

d.) $$\Phi(a) = 1 \qquad \Phi(b) = \Phi(a) + A\,\mathrm{Log}\frac{b}{a} = 1 + A\,\mathrm{Log}\frac{b}{a} = 0.$$

$$\Rightarrow A = \frac{-1}{\mathrm{Log}\frac{b}{a}} \Rightarrow \Phi(\rho) = 1 - \frac{\mathrm{Log}\frac{\rho}{a}}{\mathrm{Log}\frac{b}{a}} = \frac{\mathrm{Log}\frac{b}{\rho}}{\mathrm{Log}\frac{b}{a}}$$

e.) $$C = \varepsilon\oint_{C_R}\underbrace{-\underline{\nu}\cdot\nabla_t\Phi}_{-\frac{d\Phi}{d\rho}}\,dl = \frac{\varepsilon}{\mathrm{Log}\frac{b}{a}}\oint_{C_R}\frac{1}{\rho}\underbrace{dl}_{\rho\,d\varphi} = \frac{2\pi\varepsilon}{\mathrm{Log}\frac{b}{a}}$$

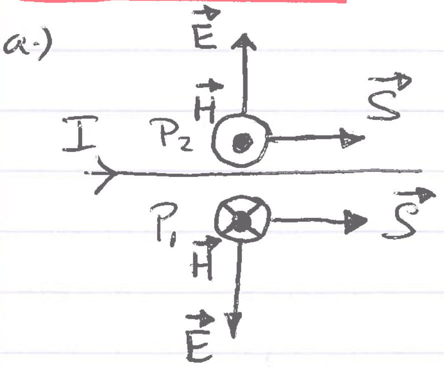
$$CL = \varepsilon\mu \Rightarrow L = \frac{\mu}{2\pi}\mathrm{Log}\frac{b}{a}.$$

f.) $$Z = \sqrt{L/C} = \sqrt{\frac{\mu}{\varepsilon}}\cdot\frac{1}{2\pi}\mathrm{Log}\frac{b}{a}.$$

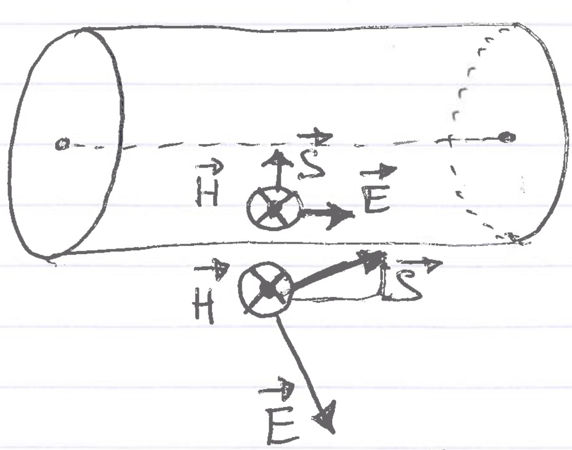
Note: Log is a more common notation for Ln

Question 2.4

a.)



b.)



The Poynting vector still points mostly from the battery ~~from~~ to the lightbulb. But it has a small component into the wire, where power is dissipated.

Electromagnetics II, Assignment 1, 2016-2017

1a) 2

$$V_{L,\text{steady}}(z,t)\big|_{z<0} \overset{\frac{1}{2}}{=} 0$$ (due to short circuit at $z=\pm\ell$)

$$I_{L,\text{steady}}(z,t)\big|_{z=0} \overset{\frac{1}{2}}{=} -\frac{1}{2}\,\frac{40}{25}\,A = -0.8\,A.$$

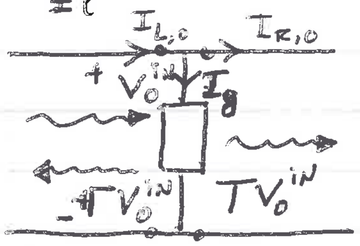
direction of current; symmetry of configuration

$$V^{\pm}_{L,\text{steady}}\Big|_{z<0} = \frac{1}{2}\Big(V_{L,\text{steady}} \pm \underset{\underset{50\Omega}{\parallel}}{Z}\, I_{L,\text{steady}}\Big)\Big|_{z<0} \overset{\frac{1}{2}}{=} \mp 20\,V.$$

$$I^{\pm}_{L,\text{steady}}\Big|_{z<0} = \pm\frac{V^{\pm}_{L,\text{steady}}}{Z} \overset{\frac{1}{2}}{=} -0.4\,A$$

1b) 2

$$\Gamma_{\pm\ell} = -1$$ (short circuit) $\frac{1}{2}$



$\frac{1}{2}$

$$V_0^{in} + \Gamma V_0^{in} = T\,V_0^{in}$$ (voltage amplitude continuity).

$\frac{1}{2}$

$$\underbrace{\frac{V_0^{in}}{Z} - \frac{\Gamma V_0^{in}}{Z}}_{I_{L,0}} = \underbrace{\frac{(1+\Gamma)V_0^{in}}{25(=Z_g)}}_{I_g} + \frac{T\,V_0^{in}}{Z}$$

$Z = 50\,\Omega.$

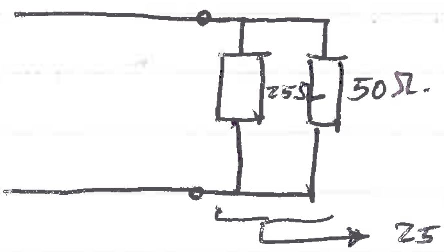
$$\Rightarrow \left.\begin{aligned} &1+\Gamma = T \\ &\frac{1-\Gamma}{50} = (1+\Gamma)\left(\frac{1}{25}+\frac{1}{50}\right)\end{aligned}\right\}\ 1-\Gamma = 3(1+\Gamma) \Rightarrow \Gamma = -\frac{1}{2}$$

$\frac{1}{4}$

$$T = \frac{1}{2}$$

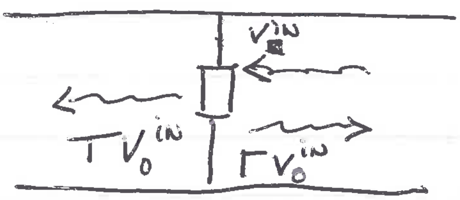
$\frac{1}{4}$

(Equivalent to



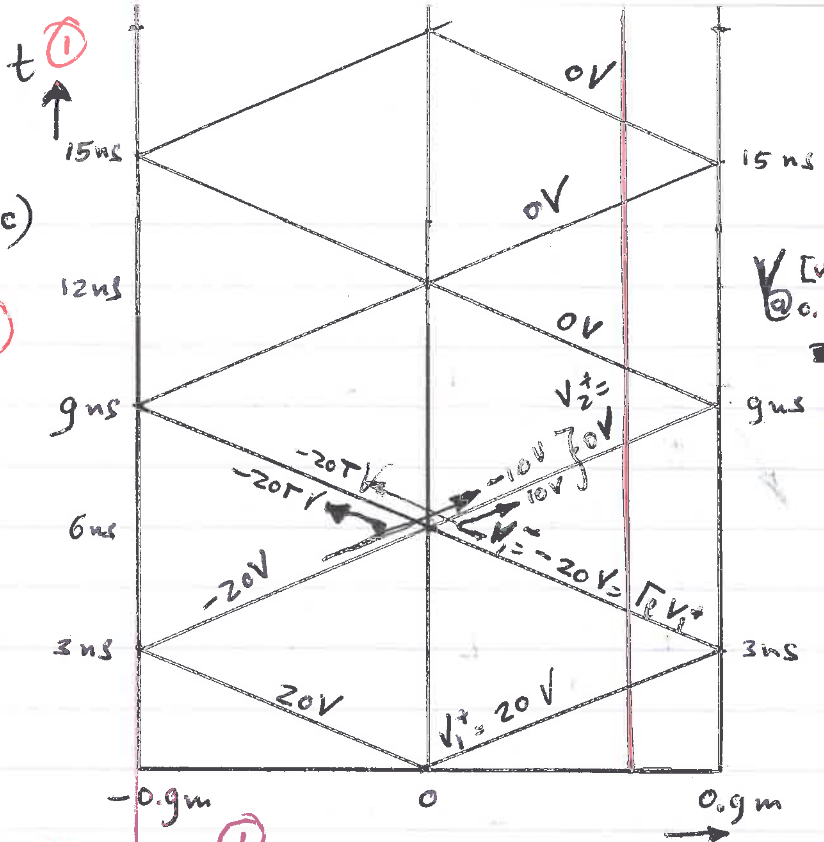
$$\Gamma = \frac{\frac{50}{3} - 50}{\frac{50}{3} + 50} = -\frac{1}{2}$$

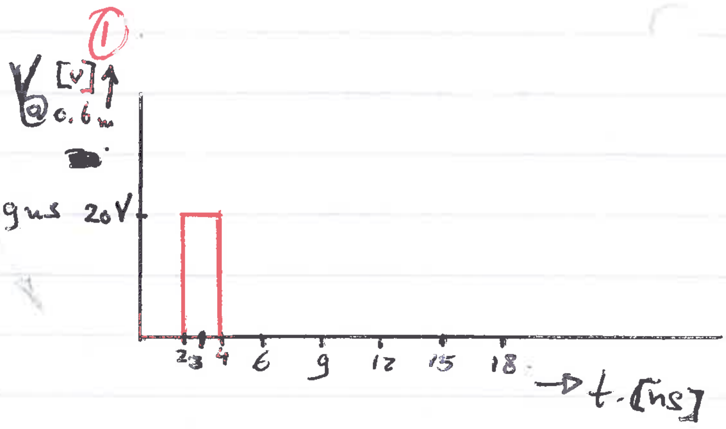
$$25\,\Omega \,//\, 50\,\Omega = \frac{50}{3}\,\Omega$$)

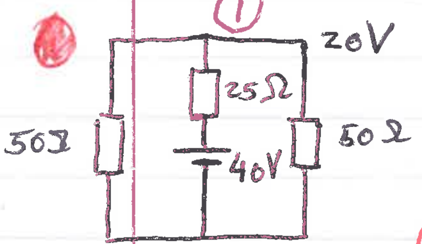


$$\Rightarrow \Gamma = -\frac{1}{2} \quad T = \frac{1}{2}$$ (the same of course)

1c)







$(50\,\Omega // 50\,\Omega = 25\,\Omega)$.

1d) $V_{R,steady}\big|_{z>0} = 0$ (due to short circuit at $z=-\ell$).

$I_{R,steady}\big|_{z>0} = 0$ (due to open end at $z=\ell$).

$V^{\pm}_{R,steady} = 0\,V$

$I^{\pm}_{R,steady} = 0\,A$.

1e)