Photonics

Quantities and Units of Light

Basic concepts

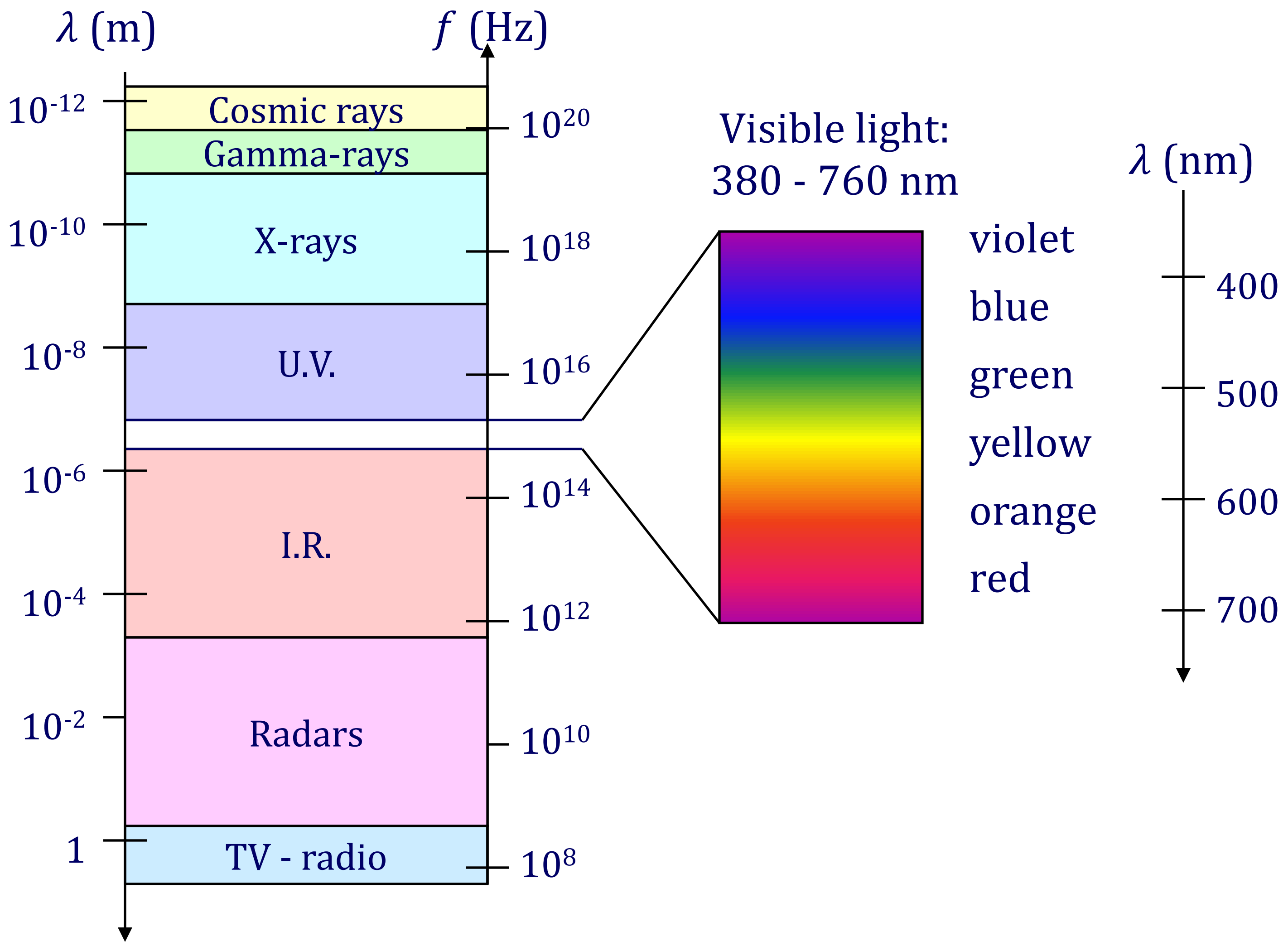
Energetic and photometric quantities

Human eye

Introduction

- Light is an EM-radiation, which is (almost) visible by the human eye:
  - visible light
  - Infrared (IR)
  - Ultraviolet (UV)

Electromagnetic spectrum



Basic quantities (1)

Quantity	Symbol	Unit	
Frequency	f, ν	THz	
Wavelength	λ	nm	$\lambda = \frac{c}{n \cdot f}$
Refractive index	n	-	
Speed	c/n	km/s	

Basic quantities (2)

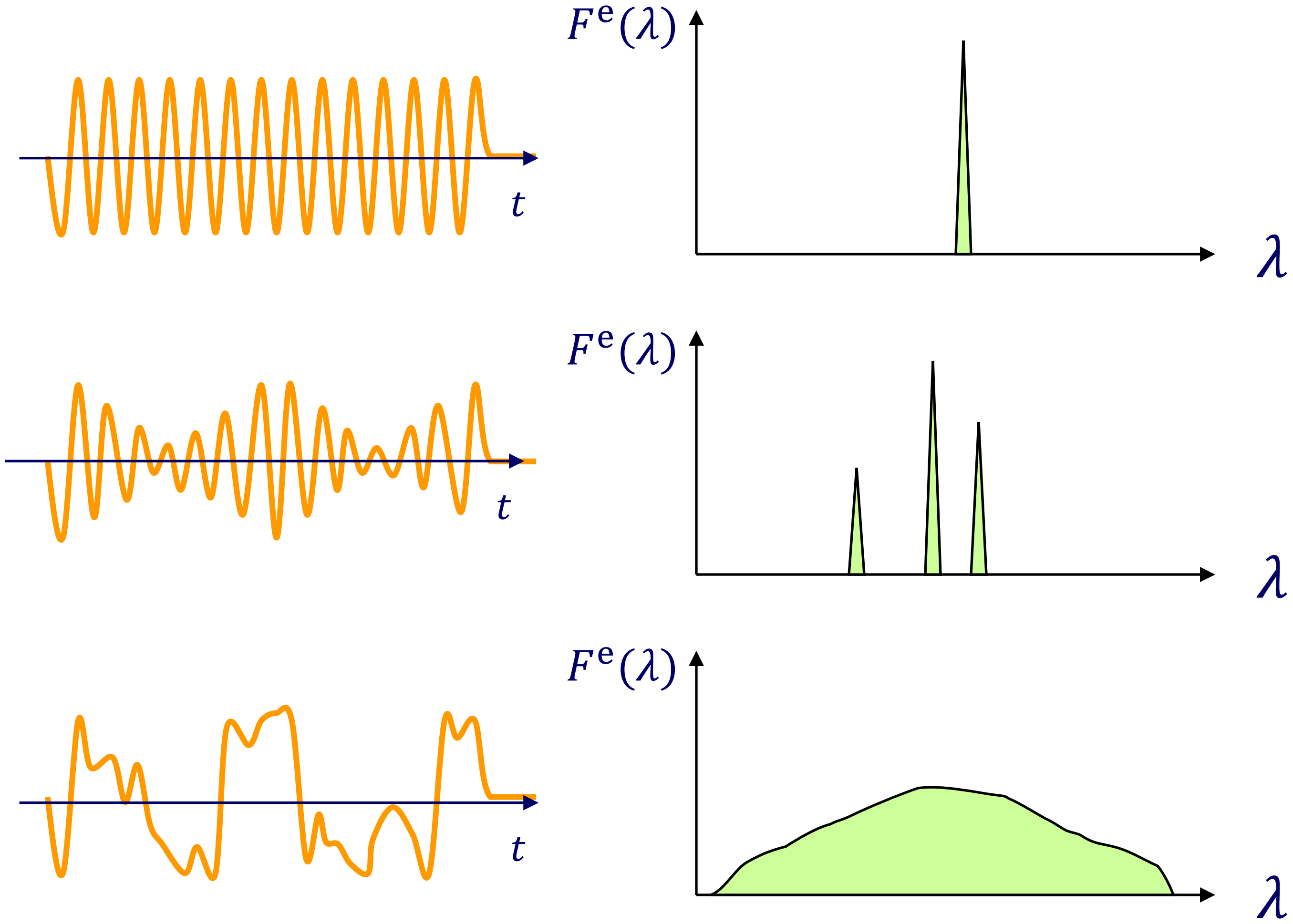
Quantity	Symbol	Unit	
Wave number	σ	1/cm	$\sigma = \frac{1}{\lambda}$
Wave number (EM)	k	1/cm	$k = \frac{2\pi}{\lambda}$
Photon energy	E	eV	$E = h \cdot \nu = h \cdot f$

Spectrum of a “light” source

monochromatic

line spectrum

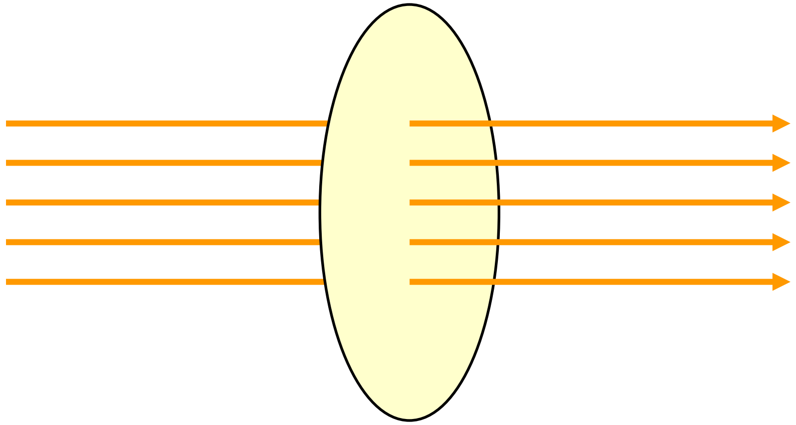
continuous spectrum



Energetic quantities (1)

- Radiant energy
  - symbol: Q^{e}
  - unit: Joule
- Radiant Flux
  - Amount of radiation through a surface per time unit
  - symbol: F^{e}
  - unit: Watt

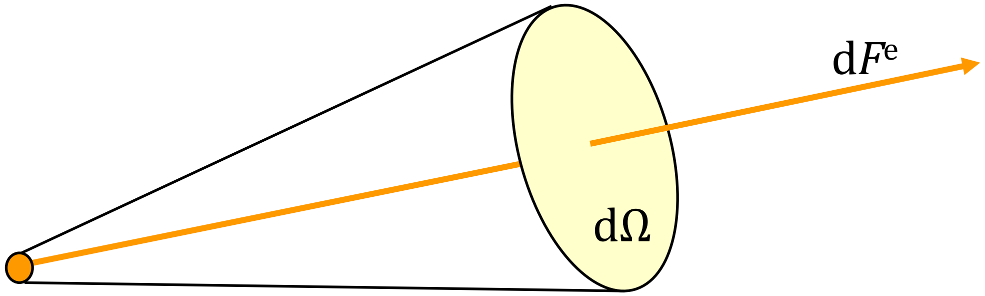
$$F^{e} = \frac{\mathrm{d}Q^{e}}{\mathrm{d}t}$$



Energetic quantities (2)

- Radiant Intensity
  - Radiant flux in a given direction per unit solid angle (for a point source)
  - symbol: I^{e}
  - unit: Watt / sr

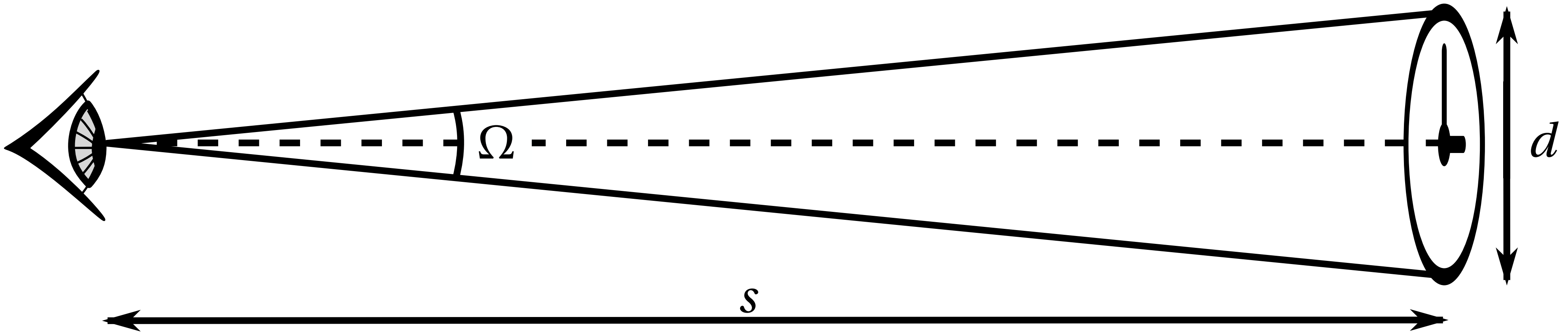
$$I^{e} = \frac{\mathrm{d}F^{e}}{\mathrm{d}\Omega}$$



Exercise

You look straight at a clock on the wall, which is at a distance $s = 3\ \mathrm{m}$. The circular clock has a diameter of $d = 25\ \mathrm{cm}$.

- Calculate the solid angle that corresponds to your view of the clock.

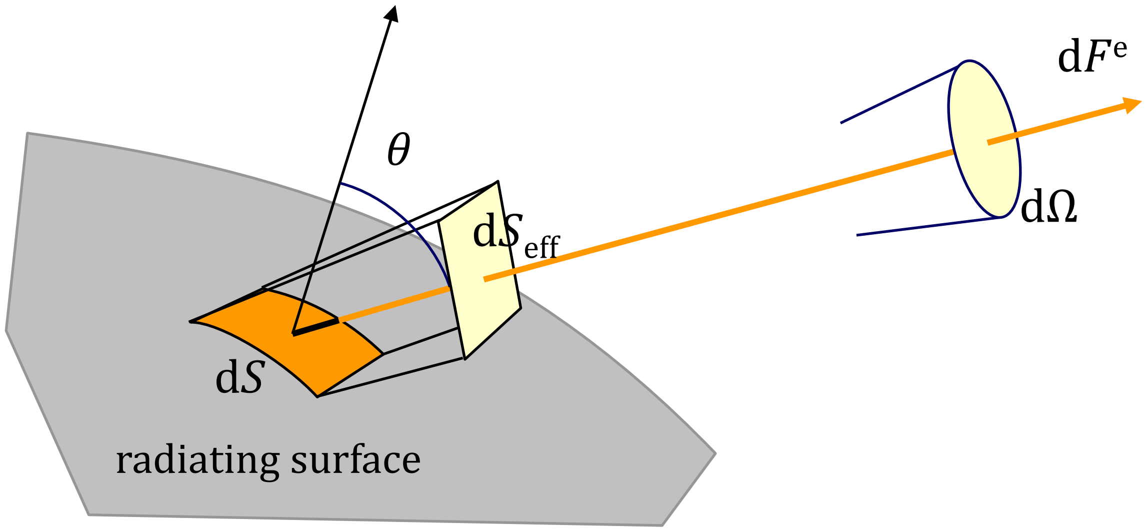


Energetic quantities (3)

- Radiance
  - Radiant intensity of a surface around a given point
    in a given direction
    per unit of effective area of the surface
  - symbol: L^e
  - unit: Watt / sr / m^2

$$L^e = \frac{dI^e}{dS_{eff}}$$

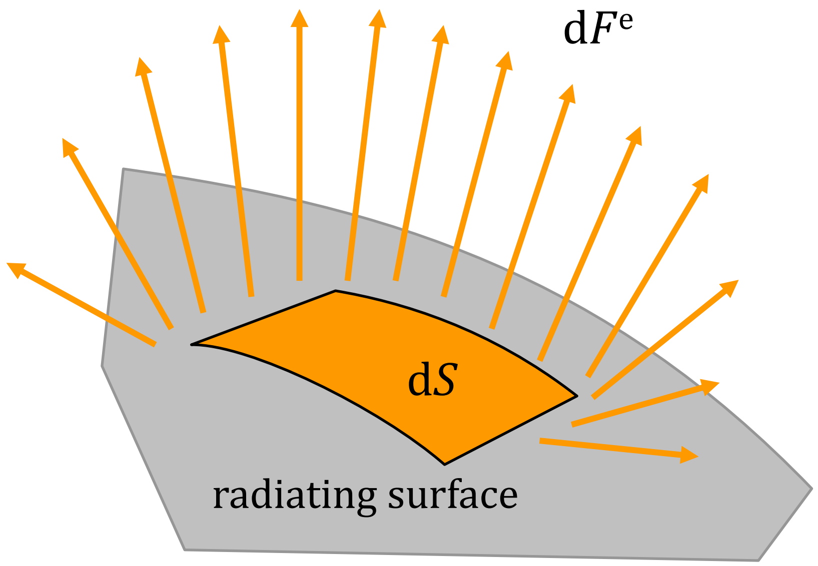
$$dS_{eff} = dS\cos\theta$$



Energetic quantities (4)

- Radiant exitance
  - Radiant flux emitted per unit area
  - symbol: M^{e}
  - unit: Watt / m^2

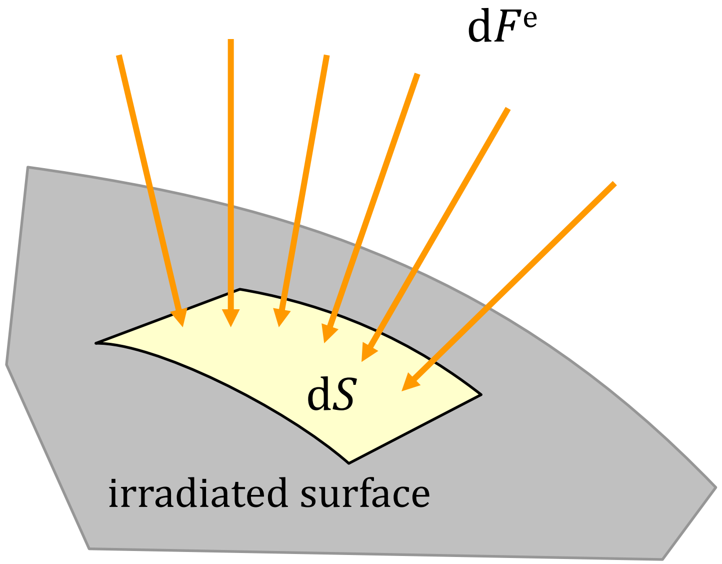
$$M^{e} = \frac{dF^{e}}{dS}$$



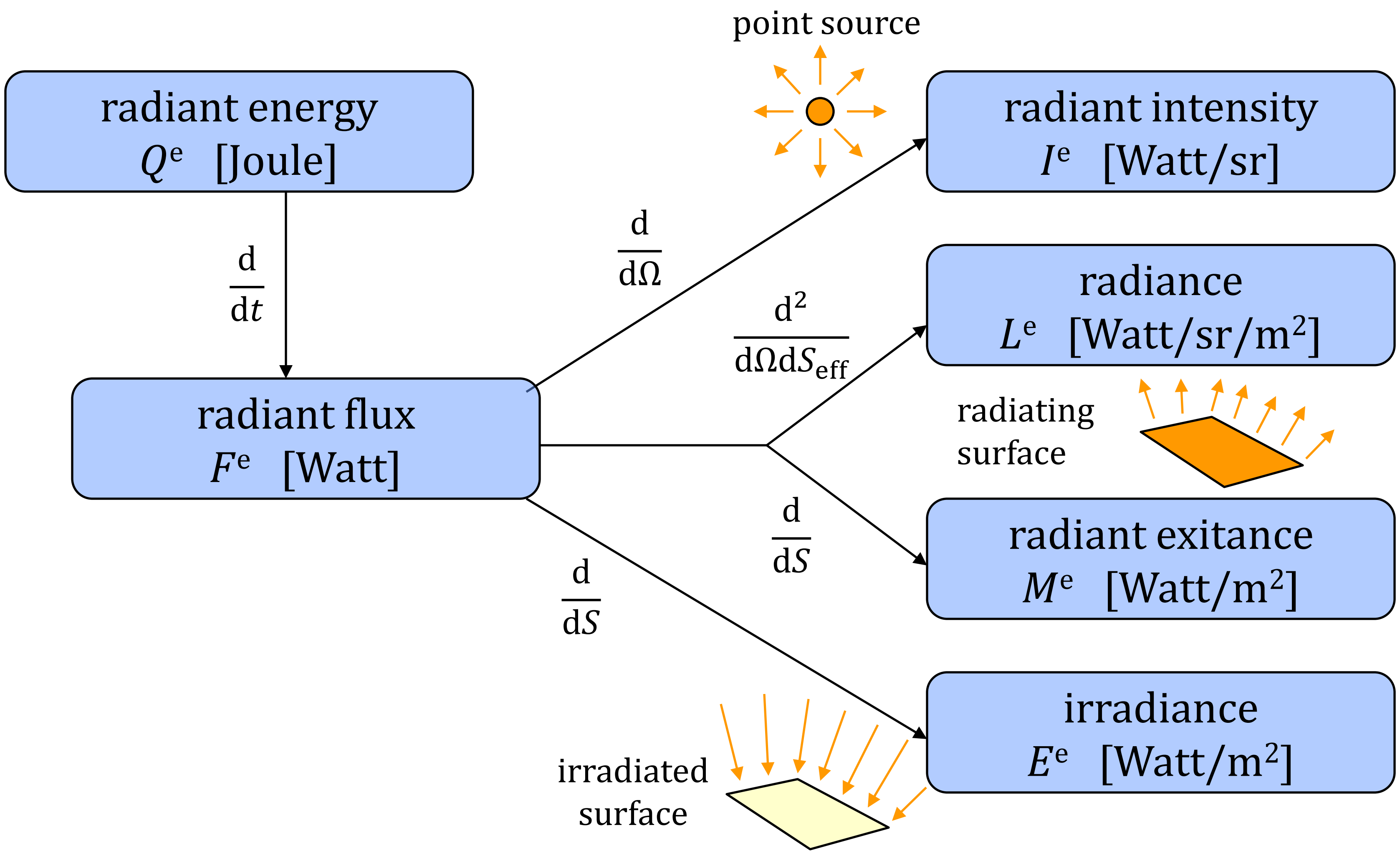
Energetic quantities (5)

- Irradiance
  - Radiant flux received per unit area
  - symbol: E^e
  - unit: Watt / m^2

$$E^e = \frac{dF^e}{dS}$$



Energetic quantities (6)



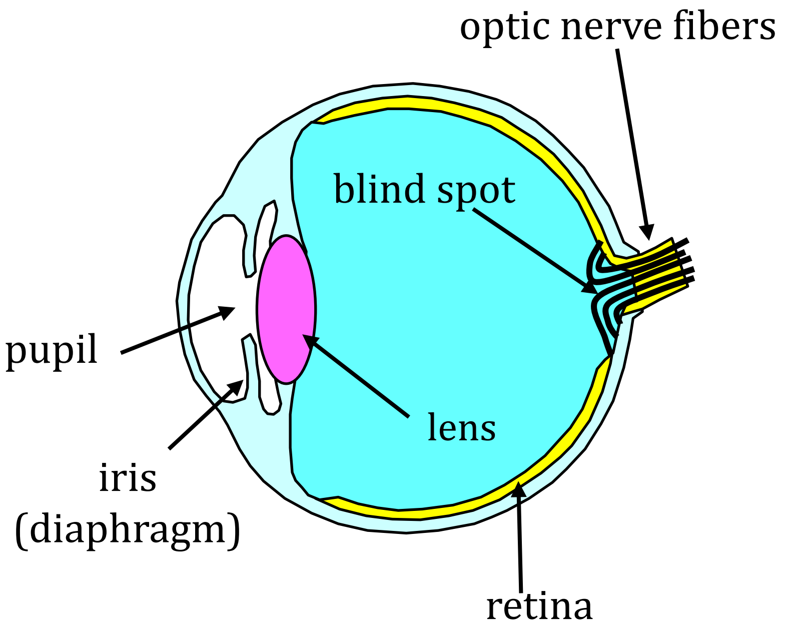
Spectral density

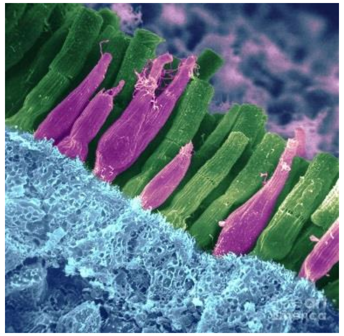
- Quantity per wavelength interval
  - symbol: subscript S
  - e.g. spectral density of the radiant flux

$$F_S^e(\lambda) = \frac{dF^e}{d\lambda}$$

Human eye

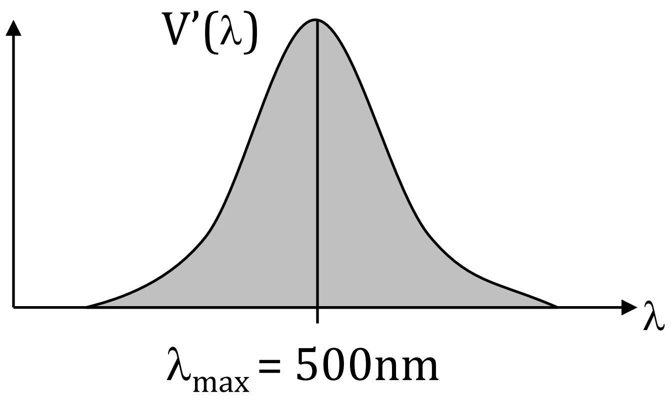
- 2 types of light sensitive neurons in the retina
  - cones
    - 3 types (red, green, blue)
    - good at normal illumination
      = photopic sight, color vision
  - rods
    - 1 type
    - important at low light intensity
      = scotopic sight, night vision

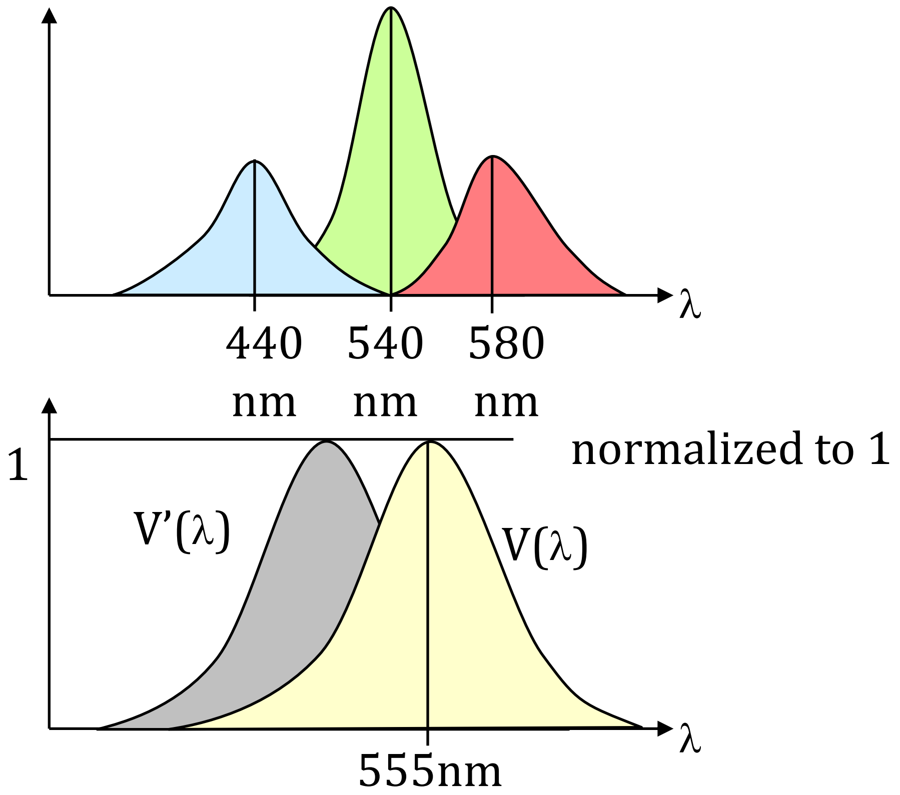




Eye sensitivity

- rods
  (scotopic sight)
- cones
  (photopic sight)

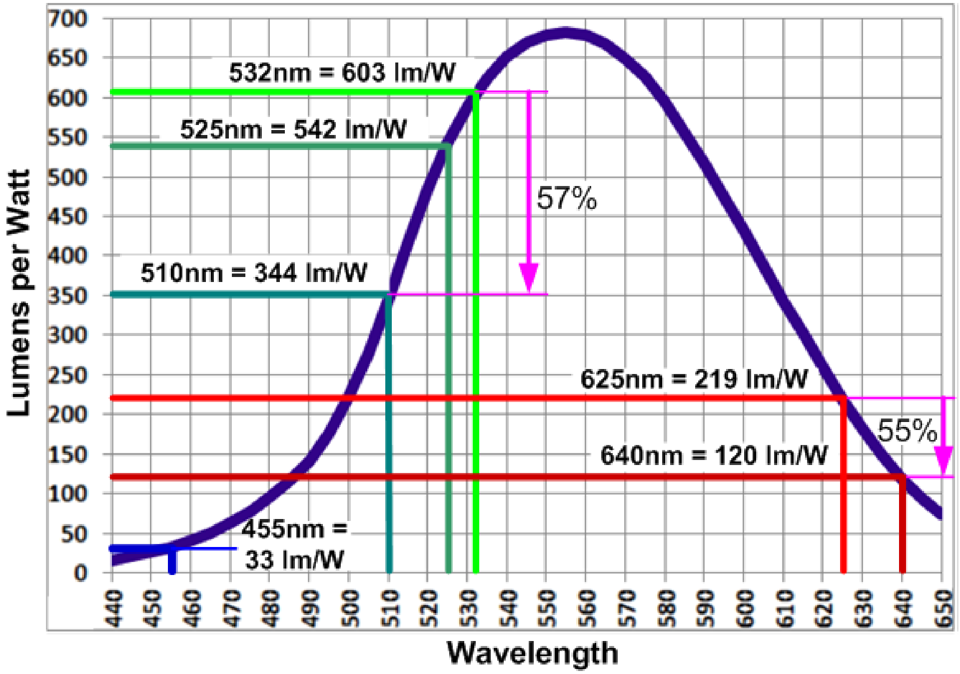




Photometric quantities (1)

- Photometric:
  take into account the response of the human eye

- Luminous flux
  - symbol: F
  - unit: lumen [lm]

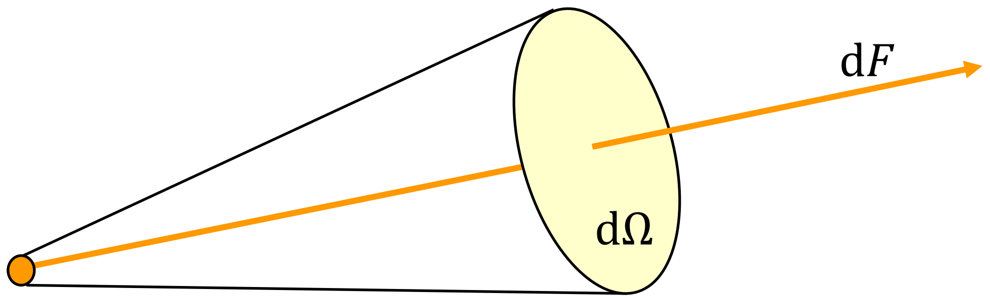


$$F = K \int F_S^e \cdot V(\lambda) \mathrm{d}\lambda \text{ with } K = 683 \text{ lumen/Watt}$$

Photometric quantities (2)

- Luminous intensity
  - Luminous flux in a given direction per unit solid angle (for a point source)
  - symbol: I
  - unit: candela = lumen / sr, [cd] or [lm/sr]

$$I = \frac{\mathrm{d}F}{\mathrm{d}\Omega}$$

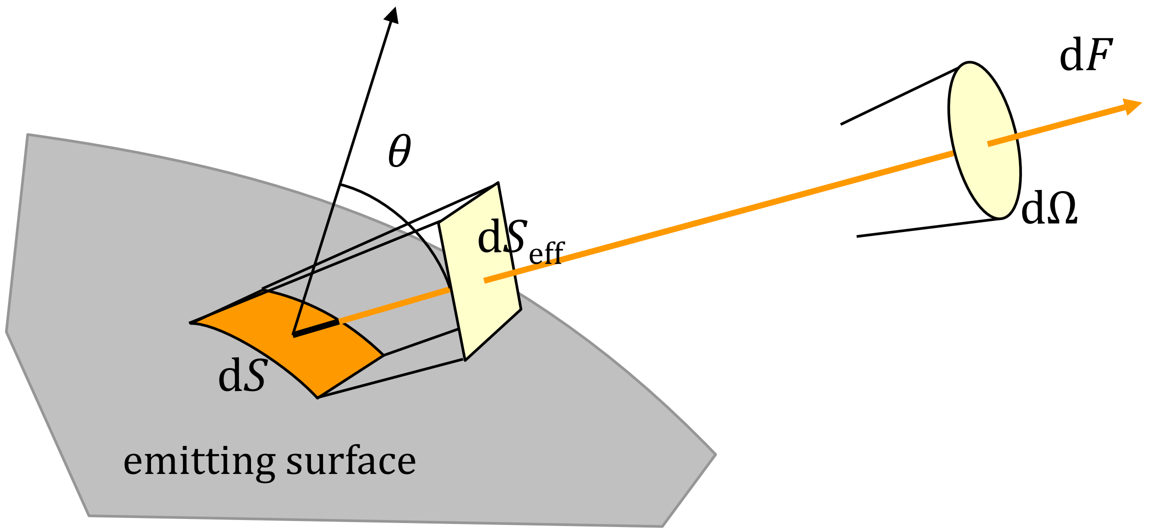


Photometric quantities (3)

- Luminance; brightness
  - luminous intensity of a surface around a given point in a given direction per unit of effective area of the surface
  - symbol: L
  - unit: candela / m^2, [cd/m^2] (or nit)

$$L = \frac{\mathrm{d}I}{\mathrm{d}S_{\mathrm{eff}}}$$

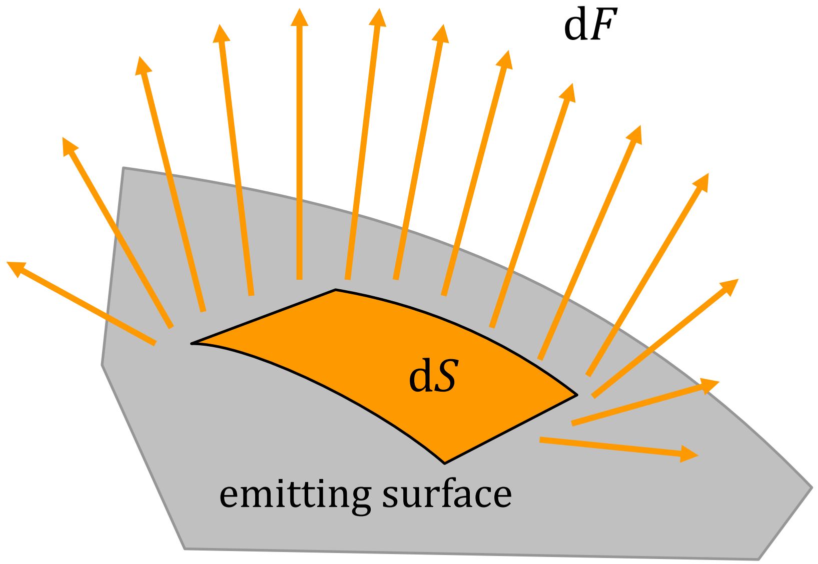
$$\mathrm{d}S_{\mathrm{eff}} = \mathrm{d}S\cos\theta$$



Photometric quantities (4)

- Luminous exitance
  - Luminous flux emitted per unit area
  - symbol: M
  - unit: lumen / m^2

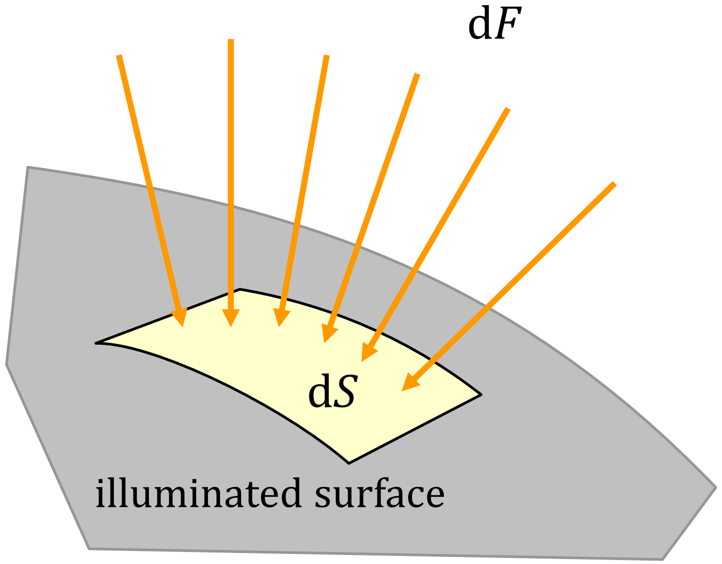
$$M = \frac{\mathrm{d}F}{\mathrm{d}S}$$



Photometric quantities (5)

- Illuminance
  - Luminous flux received per unit area
  - symbol: E
  - unit: lux = lumen / m^2

$$E = \frac{\mathrm{d}F}{\mathrm{d}S}$$



Energetic vs. photometric

- Energetic
  - Radiant flux F^{e}
- Photometric
  - Luminous flux F

$$F = K \int F_S^{e}(\lambda) \cdot V(\lambda) \mathrm{d}\lambda$$

with $K = 683$ lumen/Watt

- Radiant intensity I^{e} ⟷ Luminous intensity I
- Radiance L^{e} ⟷ Luminance L
- Radiant exitance M^{e} ⟷ Luminous exitance M
- Irradiance E^{e} ⟷ Illuminance E

Calculation of the illuminance

- Point source
  - solid angle

$$\mathrm{d}\Omega = \frac{\mathrm{d}S\cos\theta}{D^2}$$

  - luminous flux $\mathrm{d}F$ on $\mathrm{d}S$

$$\mathrm{d}F = I(\theta,\phi)\frac{\mathrm{d}S\cos\theta}{D^2}$$

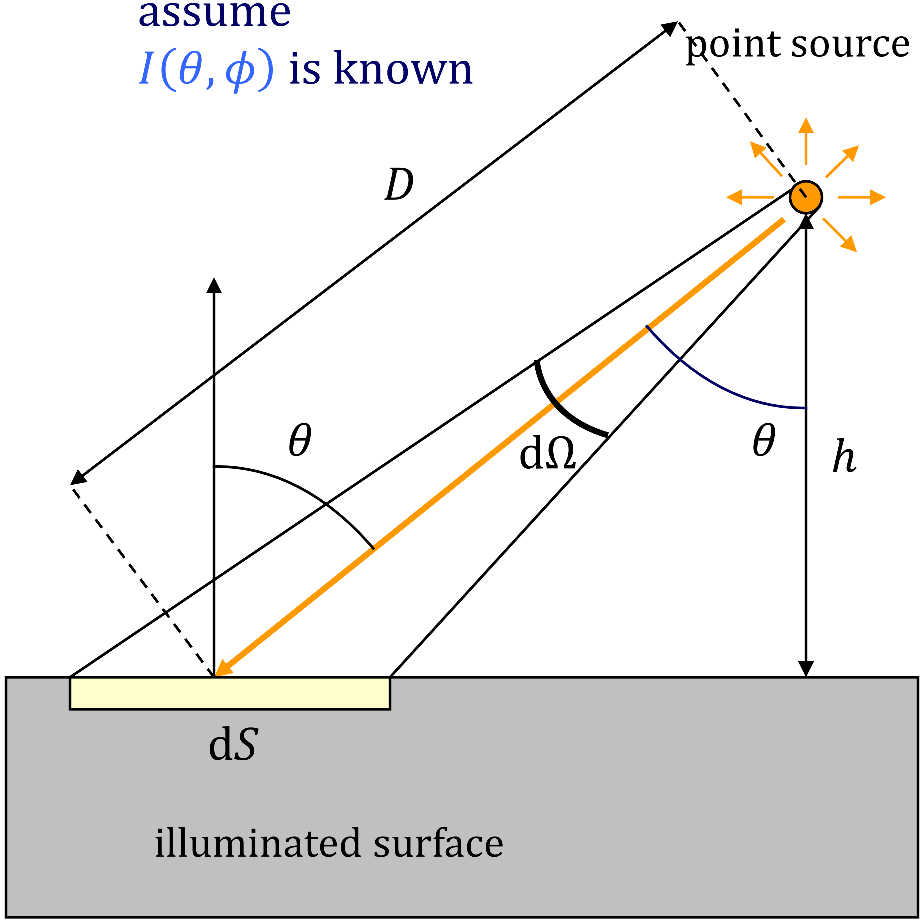
  - Illuminance E of the surface

$$E = \frac{\mathrm{d}F}{\mathrm{d}S} = \frac{I\cos\theta}{D^2} = \frac{I\cos^3\theta}{h^2}$$

  - Perpendicular incidence ($\theta = 0$)

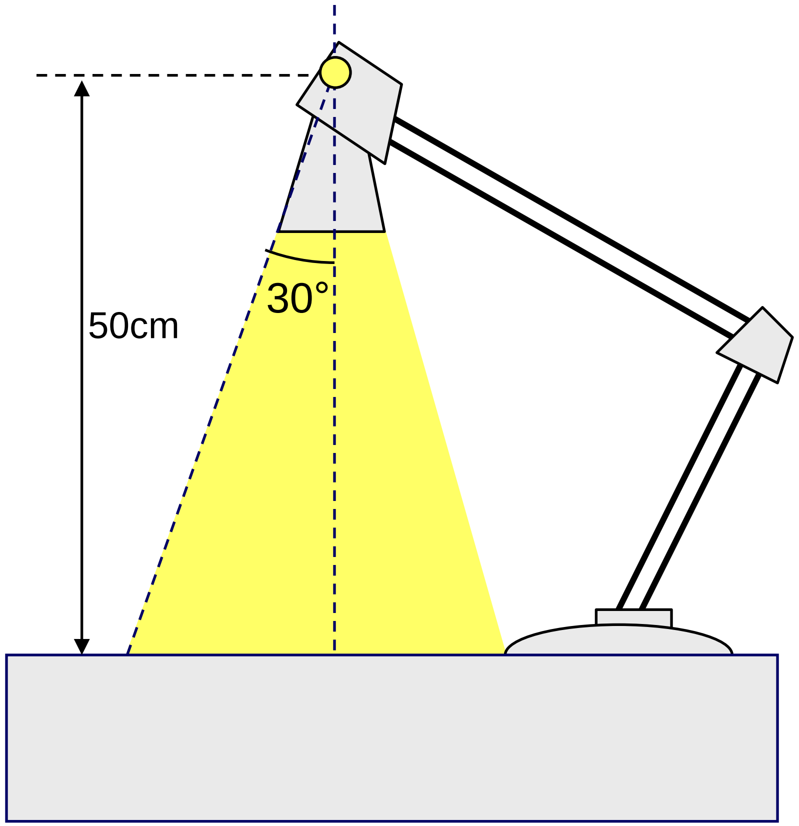
$$E = \frac{I}{h^2}$$

assume $I(\theta,\phi)$ is known



Exercise: desk lamp

- A desk lamp contains a light bulb with an electrical power of 60 Watt. The conversion efficiency is 15 lumen per (electric) Watt. Due to the fitting the luminous flux is divided uniformly over a cone with a half opening angle of 30 degrees. The lamp is 50 cm above the desk surface.
- Calculate the average illuminance on the desk surface.
- Calculate the illuminance on the desk surface in the center and at the edge of the light cone.



Calculation of the illuminance

- Not a point source
  - Luminance L
  - Luminous flux $\mathrm{d}F$ in $\mathrm{d}\Omega$

$$\mathrm{d}F = L(\theta, \phi)(\mathrm{d}S\cos\theta)\mathrm{d}\Omega$$

$$\mathrm{d}\Omega = \frac{\mathrm{d}S'\cos\theta'}{D^2}$$

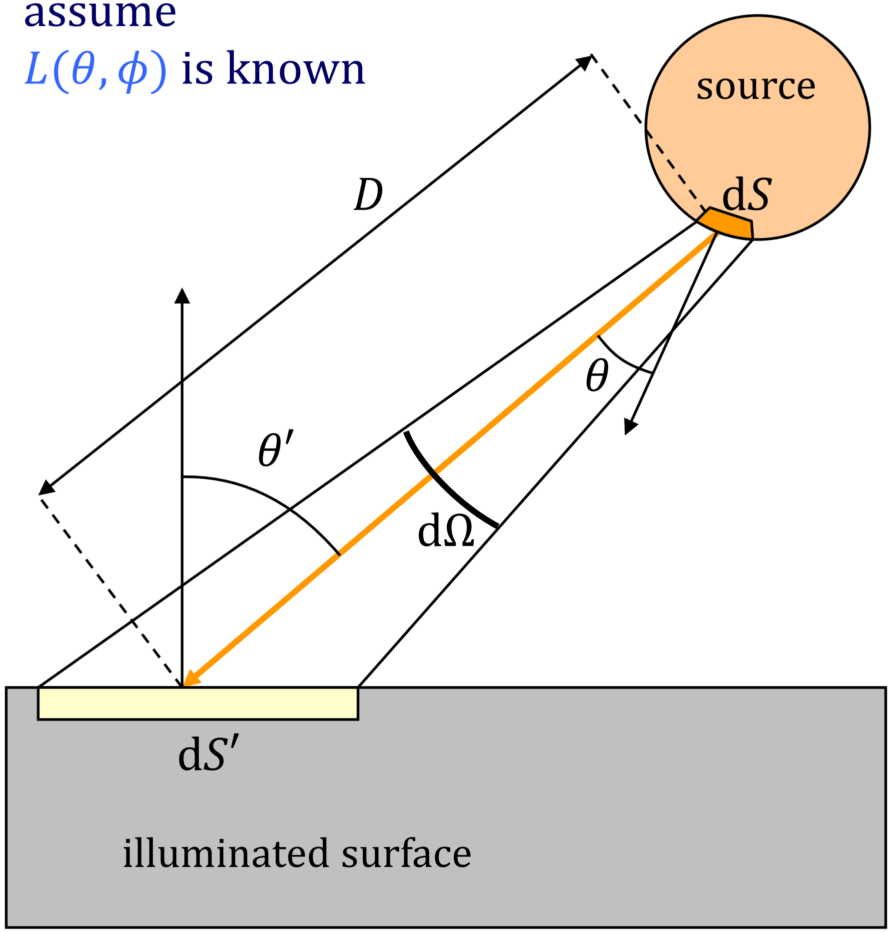
  - Illuminance $\mathrm{d}E$ on $\mathrm{d}S'$

$$\mathrm{d}E = L\frac{\cos\theta\cos\theta'}{D^2}\mathrm{d}S$$

  - Total illuminance E

$$E = \iint_{\text{source}} L\frac{\cos\theta\cos\theta'}{D^2}\mathrm{d}S$$

assume
$L(\theta, \phi)$ is known



Retina illuminance as function of a light source

- Luminous flux from $\mathrm{d}S$ to S'

$$\mathrm{d}F = L\mathrm{d}S_{\mathrm{eff}}\mathrm{d}\Omega$$

and $\mathrm{d}\Omega = \dfrac{S'}{D^2}$, so $\mathrm{d}F = L\mathrm{d}S_{\mathrm{eff}}\dfrac{S'}{D^2}$

- $\mathrm{d}S_{\mathrm{eff}}$ is projected on $\mathrm{d}S''$

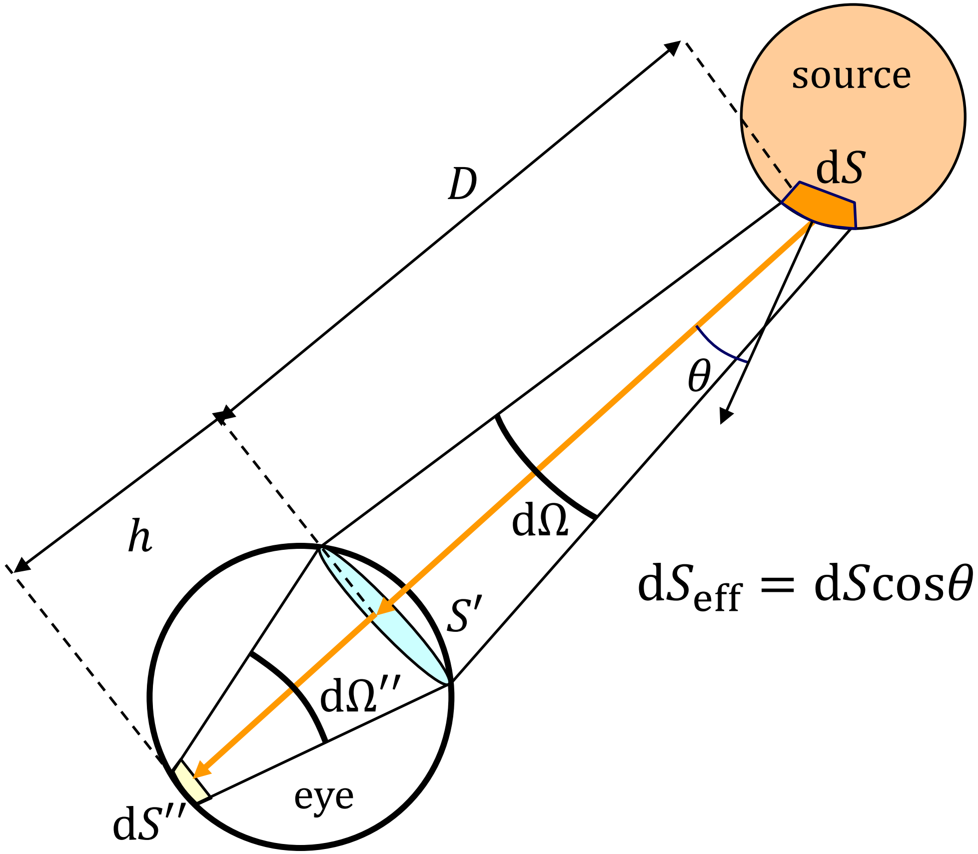
$$\frac{\mathrm{d}S''}{\mathrm{d}S_{\mathrm{eff}}} = \frac{h^2}{D^2}, \text{ or } \frac{\mathrm{d}S_{\mathrm{eff}}}{D^2} = \frac{\mathrm{d}S''}{h^2}$$

- Luminous flux to $\mathrm{d}S''$

$$\mathrm{d}F = L\frac{\mathrm{d}S''S'}{h^2}$$

- Illuminance at retina

$$E = \frac{\mathrm{d}F}{\mathrm{d}S''} = L\frac{S'}{h^2} = L\mathrm{d}\Omega''$$



$\mathrm{d}S_{\mathrm{eff}} = \mathrm{d}S\cos\theta$

$\boxed{\text{Independent of distance } D}$

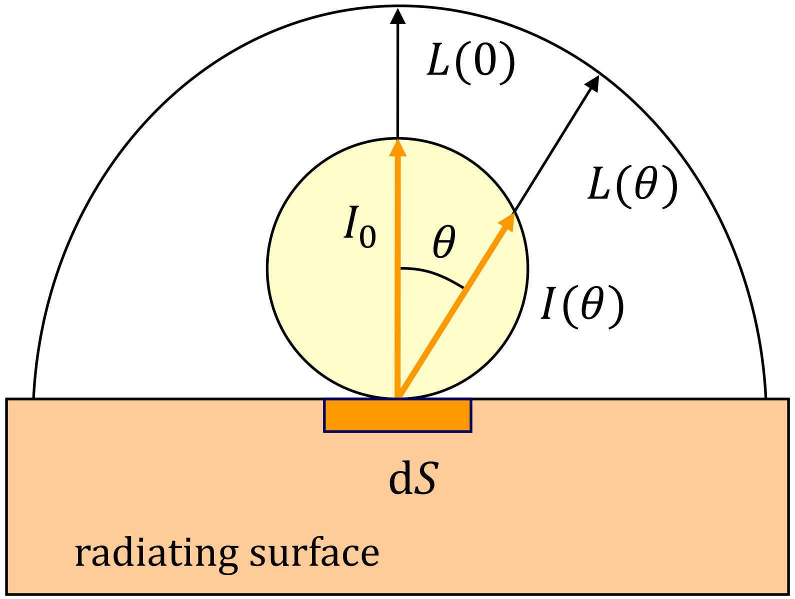
Lambert's law

Lambert surface:

$$L(\theta, \phi) = \text{constant}$$

Examples:

- Many incoherent sources, like the sun, incandescent lamp…
- Rough, diffusely reflecting surfaces



- Luminous intensity of $\mathrm{d}S$

$$\begin{aligned}\mathrm{d}I(\theta) &= L \cdot \mathrm{d}S_{\text{eff}}(\theta) \\ &= L \cdot \mathrm{d}S \cdot \cos\theta \\ &= \mathrm{d}I_0 \cos\theta\end{aligned}$$

- Luminous flux from $\mathrm{d}S$

$$\mathrm{d}F = \int \mathrm{d}I \mathrm{d}\Omega$$

$$\boxed{\mathrm{d}\Omega = \sin\theta \, \mathrm{d}\theta \mathrm{d}\phi}$$

$$\begin{aligned}&= \mathrm{d}I_0 \int_0^{\pi/2} \cos\theta \sin\theta \, \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi \\ &= \pi \mathrm{d}I_0 = \pi L \mathrm{d}S\end{aligned}$$

- Luminous exitance of the surface

$$M = \frac{\mathrm{d}F}{\mathrm{d}S} = \pi L$$

Exercise: lambertian emitter

A lambertian emitter with a surface of $S = 4\ \mathrm{m}^2$ has a luminance of $L = 1\ \mathrm{cd/m}^2$.

- What is the luminous exitance M of the surface?
- What is the total luminous flux F emitted by the surface?

Illuminances

Summer sun	100000 lux
Winter sun	10000 lux
Sunrise	500 lux
Full moon	0.25 lux
Retina sensitivity	10^{-9} lux
ISO-400 film sensitivity (1 s exposure)	10^{-2} lux

Recommended illuminances

- Eye functions optimally from 10000 lux

- Daylight 1000-100000 lux
- Artificial light
  - Offices: 500-1000 lux
  - Very precise work: 1000-5000 lux
  - Living area (local): 500-1000 lux
  - Living area (general): 50-100 lux

Luminance (brightness)

Sun	$1.65\ 10^{9}$ cd/m^2
Moon	$2.5\ 10^{3}$ cd/m^2
Filament of an incandescent lamp	$7\ 10^{6}$ cd/m^2
Fluorescent lamp	$8\ 10^{3}$ cd/m^2
LED	10^{4} - 10^{6} cd/m^2
Laser (1W - green)	10^{15} cd/m^2
Computer monitor	300 cd/m^2
White paper (80% reflection - 400 lux)	10^{2} cd/m^2
needed for photopic sight (cones)	> 1 - 10 cd/m^2
needed for scotopic sight (rods)	> 0.01 - 0.1 cd/m^2

Examples

- Projectors
  - light output in lumen
  - www.barco.com
- Lamps
  - luminous intensity in cd
  - www.osram.com
- Monitor / TV
  - brightness in cd/m^2 (nit)
  - tweakers.net