Equation sheet - Intro Telecomm (5ETA0)

PCM system

$$\left(\frac{S}{N}\right)_{out} = \frac{M^2}{1 + 4P_e(M^2 - 1)} \tag{1}$$

where

$$P_e = Q\left(\sqrt{\left(\frac{S}{N}\right)_{in}}\right) \tag{2}$$

Q-function plot ($P_e = \frac{1}{\sqrt{2\pi}z}e^{-z^2/2}$ for $z > 3$):

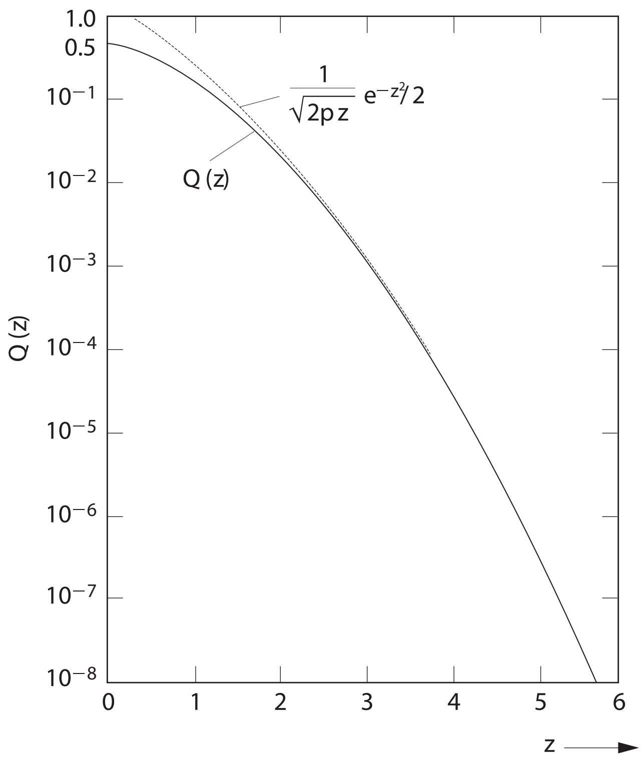

Figure 1: Q-function $Q(z)$ plot.

Power spectrum of a digital signal

Pulse shape $f(t) \Leftrightarrow F(f)$; Signal spectrum $P_s(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k)e^{j2\pi kfT_s}$

Raised cosine spectrum

$$H_e(f) = \begin{cases} 1, & |f| \leq f_1 \\ \frac{1}{2}\left(1 + \cos(\frac{\pi(|f|-f_1)}{2f_\Delta})\right), & f_1 \leq |f| \leq B \\ 0, & |f| \geq B \end{cases} \tag{3}$$

with $f_0 = f_1 + f_\Delta = B - f_\Delta$; Roll-off factor $r = \frac{f_\Delta}{f_0}$

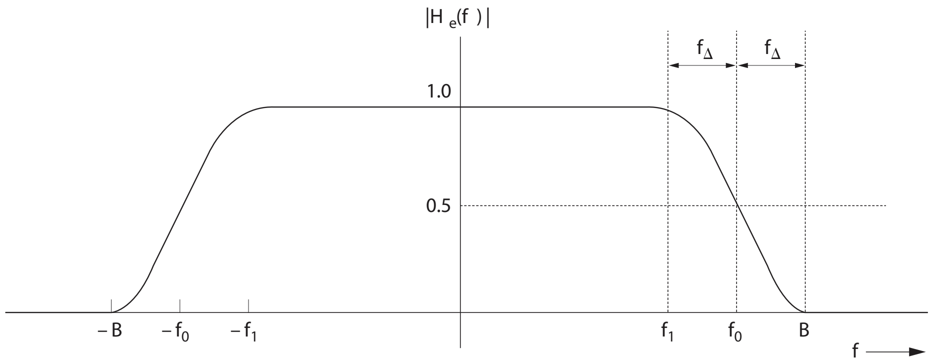

Figure 2: Raised cosine spectrum

The impulse response of the root-raised cosine filter:

$$h_e(t) = 2f_0 \left(\frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \right) \frac{\cos(2\pi f_\Delta t)}{1-(4f_\Delta t)^2} \tag{4}$$

Information theory

Channel capacity:

$$C = B\log_2\left(1+\frac{S}{N}\right) \tag{5}$$

Analog modulation

Frequency modulation (FM):

$$s(t) = A_c \cos\left(\omega_c t + D_f \int m(\tau)d\tau\right); \qquad \beta_f = \frac{\Delta F}{B} \tag{6}$$

$$\text{Carsons rule:} \quad B_T = 2(\beta+1)B; \qquad \Delta F = \frac{1}{2\pi}\cdot D_f \cdot \max[m(t)] \tag{7}$$

Amplitude modulation (AM):

$$\%modulation = \frac{A_{max}-A_{min}}{2A_c}\cdot 100\% \tag{8}$$

$$\%positive\ Modulation = \frac{A_{max}-A_c}{A_c}\cdot 100\% = max(m(t))\cdot 100\% \tag{9}$$

$$\%negative\ Modulation = \frac{A_c-A_{min}}{A_c}\cdot 100\% = min(m(t))\cdot 100\% \tag{10}$$

$$E = \frac{< m^2(t) >}{1+< m^2(t) >}\cdot 100\% \tag{11}$$

Trigonometric identities:

$$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta)+\sin(\alpha-\beta) \tag{12}$$

$$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta)-\sin(\alpha-\beta) \tag{13}$$

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha+\beta)+\cos(\alpha-\beta) \tag{14}$$

$$2\sin(\alpha)\sin(\beta) = -\cos(\alpha+\beta) + \cos(\alpha-\beta) \tag{15}$$

$$2\sin(\alpha)\cos(\alpha) = \sin(2\alpha) \tag{16}$$

$$2\cos^2(\alpha) = 1 + \cos(2\alpha) \tag{17}$$

$$2\sin^2(\alpha) = 1 - \cos(2\alpha) \tag{18}$$

order\betta	0.5	1	2	3	4	5	6	7	8	9	10
0	0.93847	0.765198	0.223891	-0.26005	-0.39715	-0.1776	0.150645	0.300079	0.171651	-0.09033	-0.24594
1	**0.242268**	0.440051	0.576725	0.339059	-0.06604	-0.32758	-0.27668	-0.00468	0.234628	0.245307	0.043472
2	0.030604	**0.114903**	0.352834	0.486091	0.364128	0.046565	-0.24287	-0.30142	-0.11299	0.144846	0.25463
3	0.002564	0.019563	**0.128943**	0.309063	0.430171	0.364831	0.114768	-0.16756	-0.29113	-0.18093	0.05838
4	0.000161	0.002477	0.033996	**0.132034**	0.281129	0.391232	0.357642	0.157798	-0.10535	-0.26547	-0.2196
5		0.00025	0.00704	0.043028	**0.132087**	0.261141	0.362087	0.347896	0.185775	-0.05504	-0.23406
6			0.001202	0.011394	0.049088	**0.131049**	0.245837	0.339197	0.337569	0.204312	-0.01446
7			0.000175	0.002547	0.015176	0.053376	**0.129587**	0.233584	0.320578	0.327456	0.21671
8				0.000493	0.004029	0.018405	0.056532	**0.127971**	0.223455	0.305063	0.317854
9					0.000939	0.00552	0.021165	0.058921	**0.126321**	0.214881	0.291856
10					0.000195	0.001468	0.006964	0.023539	0.060767	**0.124694**	0.207486
11						0.000351	0.002048	0.008335	0.025597	0.062217	**0.123117**
12							0.000545	0.002656	0.009624	0.027393	0.06337
13							0.000133	0.00077	0.003275	0.01083	0.028972
14								0.000205	0.001019	0.003895	0.011957
15									0.000293	0.001286	0.004508
16										0.000393	0.001567

Figure 3: Bessel function table

Line codings

Line coding (1 bit per symbol)	First Null Bandwidth (R is bit rate)	SE (Bits/Hz)
Unipolar/Polar NRZ	R	1
Unipolar RZ (50%)	$2R$	0.5
Sinc Pulses	$0.5R$	2
Raised cosine pulses	$(1+r)\frac{R}{2}$	$\frac{2}{1+r}$

Table 1: Bandwidth and Spectral Efficiency of Various Line Coding Schemes

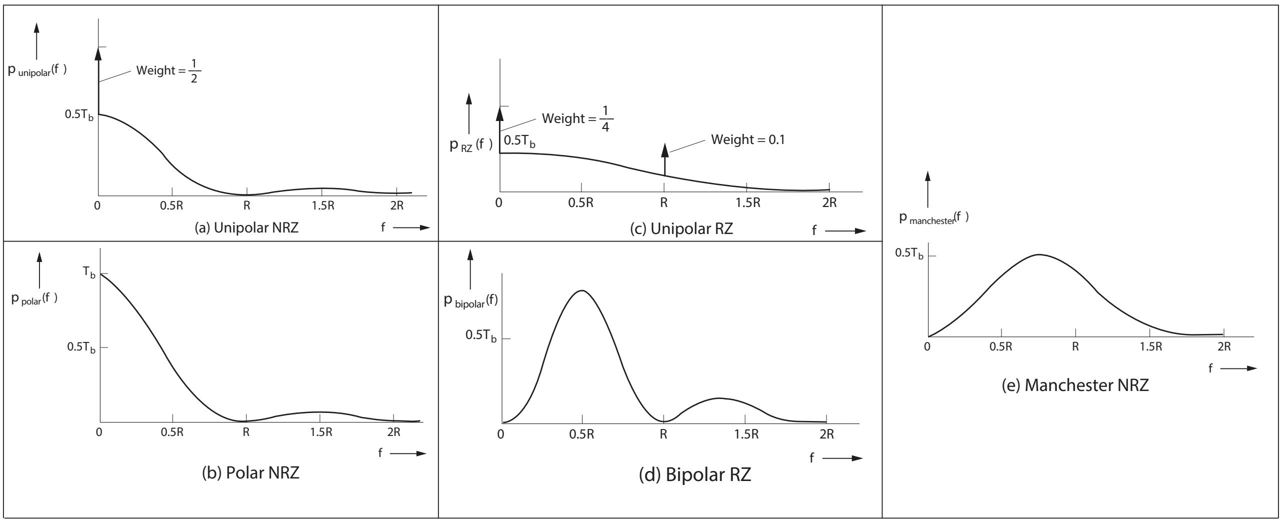

Figure 4: Line codings and their spectras

Received signal power in a radio communication link

$$P_{Rx} = P_{Tx} \cdot G_{AT} \cdot G_{AR} \cdot \left[\frac{\lambda}{4\pi d}\right]^2 \tag{19}$$

$$P_{Rx} \approx P_{TX} G_{TX} G_{RX} \left(\frac{h_{TX} h_{RX}}{d^2}\right)^2 \tag{20}$$

$$d_{break} = \frac{4\pi h_{Tx} h_{Rx}}{\lambda} \tag{21}$$

Knife edge

$$v = \alpha \sqrt{\frac{2 d_1 d_2}{\lambda (d_1 + d_2)}} \tag{22}$$

$$\alpha = \beta + \gamma \tag{23}$$

$$\beta = \tan^{-1}\left(\frac{h_{obs} - h_{TX}}{d_1}\right) \tag{24}$$

$$\gamma = \tan^{-1}\left(\frac{h_{obs} - h_{RX}}{d_2}\right) \tag{25}$$

The equation below should only be used for $v > 0$!

$$A(v) = 6.9 + 20 \log_{10}\left(\sqrt{v^2+1} + v - 0.1\right) \quad [dB] \tag{26}$$

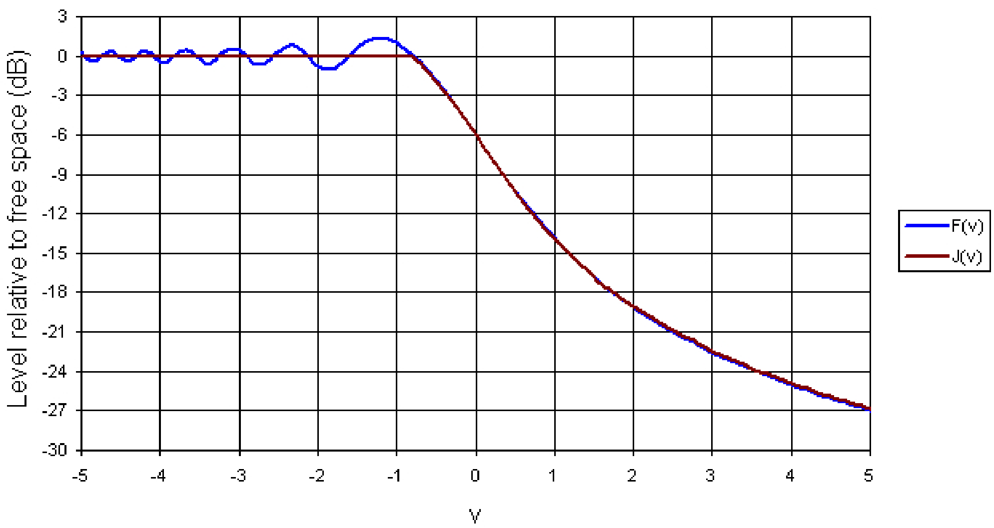

Figure 5: Diffraction loss. This figure can be used for all values of v!

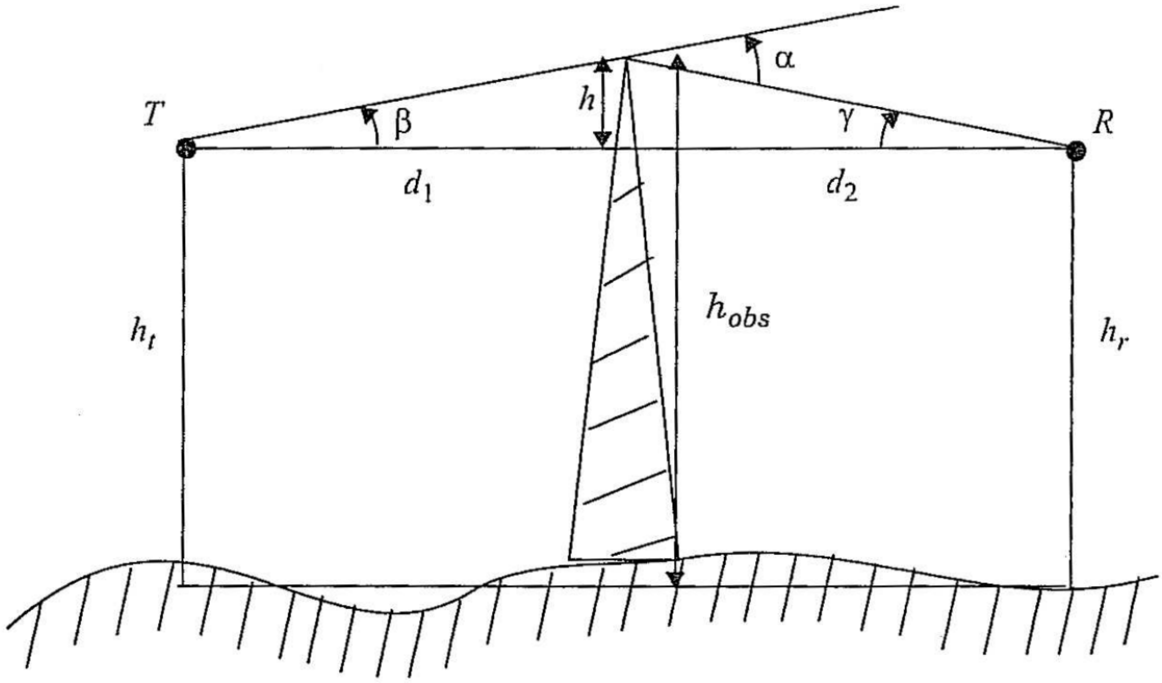

Figure 6: Knife-edge diffraction geometry. The point T denotes the transmitter and R the receiver, with an infinite knife-edge obstruction blocking the life-of-sight path.

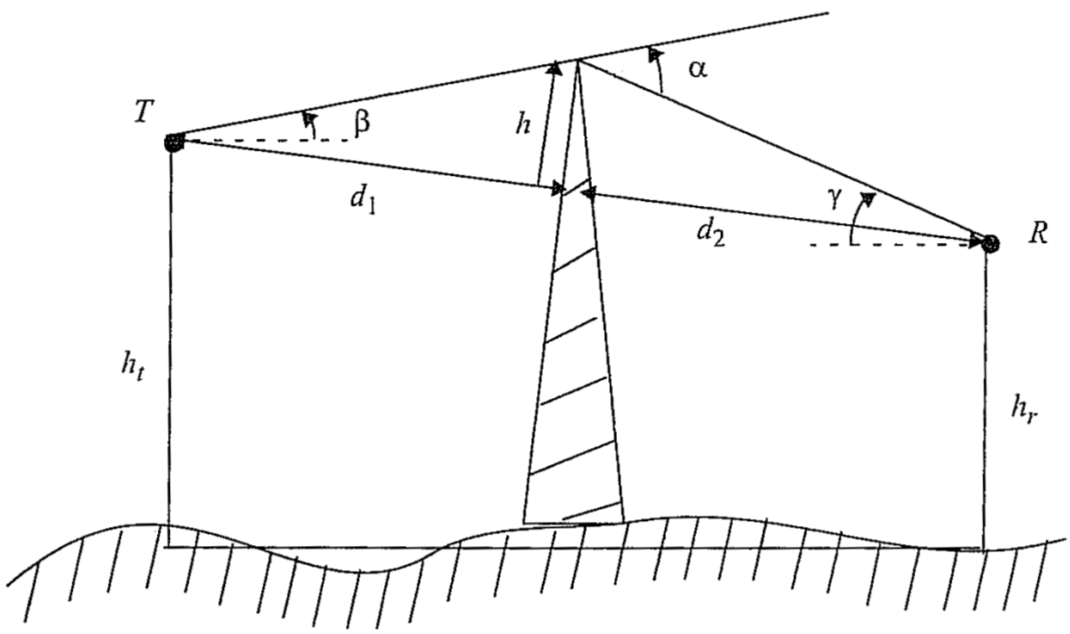

Figure 7: Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if α and β are small and $h << d_1$ and d_2, then h and h' are virtually identical.