5XTCO - June 2020

Q1

a) $z_L = (15 + j39)\,\Omega$

$z_1 = 150\,\Omega$

$\frac{z_L}{z_1} = \frac{15}{150} + j\frac{39}{150} = 0.1 + j\,0.26 \longrightarrow$ find in SC

$\underline{|\Gamma_L| = 0.83}$

$\underline{\angle = 151^\circ}$

b)

$l_1 = 0.321\,m$, $\lambda = \frac{c}{f} = \frac{3\cdot 10^8\,m/s}{400\cdot 10^6\,1/s} = 0.75\,m$

$\frac{l_1}{\lambda} = 0.428 \Longleftrightarrow l_1 = 0.428\lambda \longrightarrow$ Turn 0.428λ towards generator

$\frac{z_L'}{z_1} = 0.1 - j\,0.2 \leftarrow$ in SC

$\underline{z_L' = (15 - j\,30)\,\Omega}$

c)

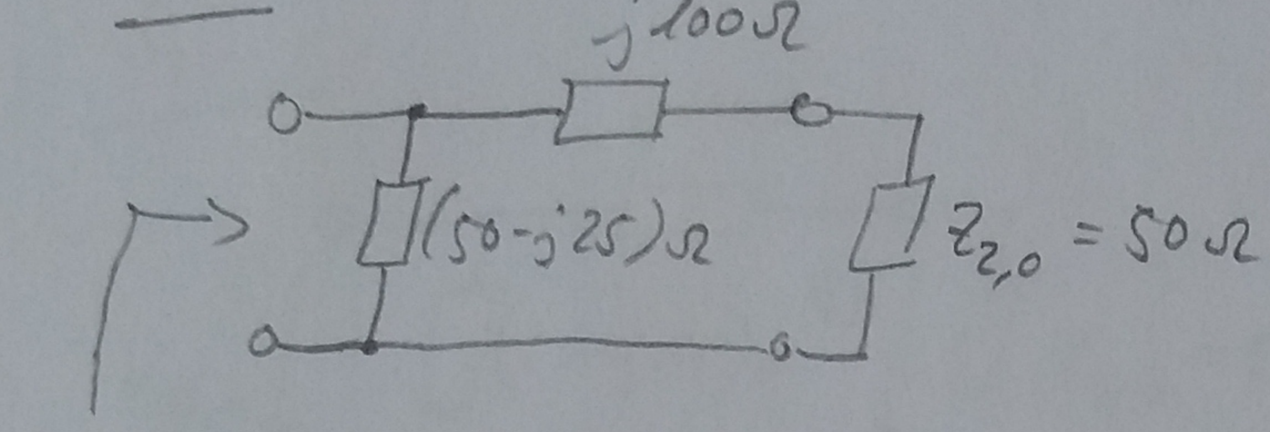
$\frac{z_L}{z_0} = \frac{15}{75} - j\frac{30}{75} = 0.2 - j\,0.4 \longrightarrow$ find in SC

from SC : $\frac{y_L'}{y_0} = 1 + j\,2 \Longrightarrow \underline{z_S = 0\,\Omega}$

$y_p = (-j\,2)\,y_0 = -j\frac{2}{75}\frac{1}{\Omega}$

$\underline{z_p = j\,37.5\,\Omega}$

d) S_{11}: Ref. impedance $\underline{Z_0 = 50\,\Omega}$



$\Gamma_1 = S_{11}$

$Z_{in,1} = \underbrace{(50 - j25)\,\Omega}_{\text{find in SC}} \parallel \underbrace{(50 + j100)\,\Omega}_{\text{find in SC}} \rightarrow \frac{Z_2}{Z_0} = \frac{50}{50} + j\frac{100}{50} = \underbrace{1 + j2}_{\text{from SC}}$

$\frac{Z_1}{Z_0} = \frac{50}{50} - j\frac{25}{50}$

$\underbrace{\frac{Z_1}{Z_0} = 1 - j0.5}_{\text{from SC}}$

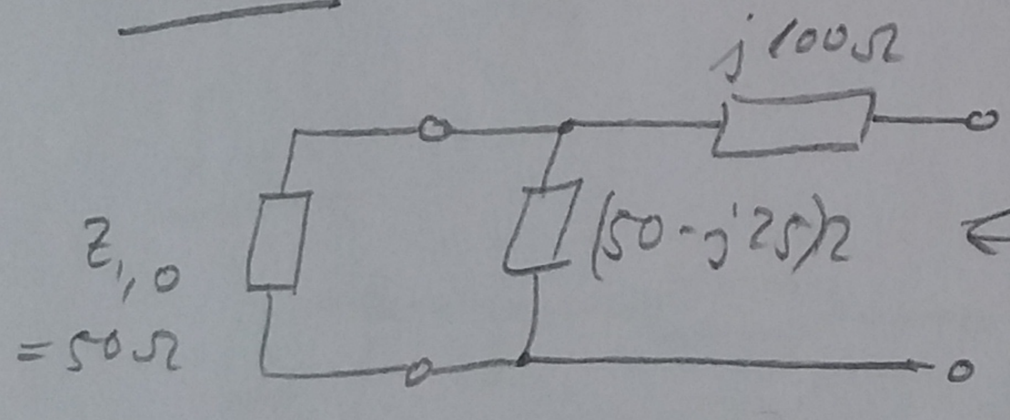
$\frac{Y_1}{Y_0} = 0.8 + j0.4$ → $\boxed{\frac{Y_1 + Y_2}{Y_0} = 1}$ condition of SC!

$\frac{Y_2}{Y_0} = 0.2 - j0.4$ ←

$\Rightarrow Z_{in,1} = 50\,\Omega$

$\Rightarrow \underline{S_{11} = 0}$

$\underline{S_{22}}$:



$\Gamma_2 = S_{22}$

$Z_{in,2} = j100\,\Omega + \{50\,\Omega \,//\, (50 - j25)\,\Omega\}$

$\frac{Y_1}{Y_0} = 1 + 0.8 + j0.4$

$\frac{Y_1}{Y_0} = 1.8 + j0.4$

$\frac{Z_1}{Z_0} = 0.53 - j0.12$

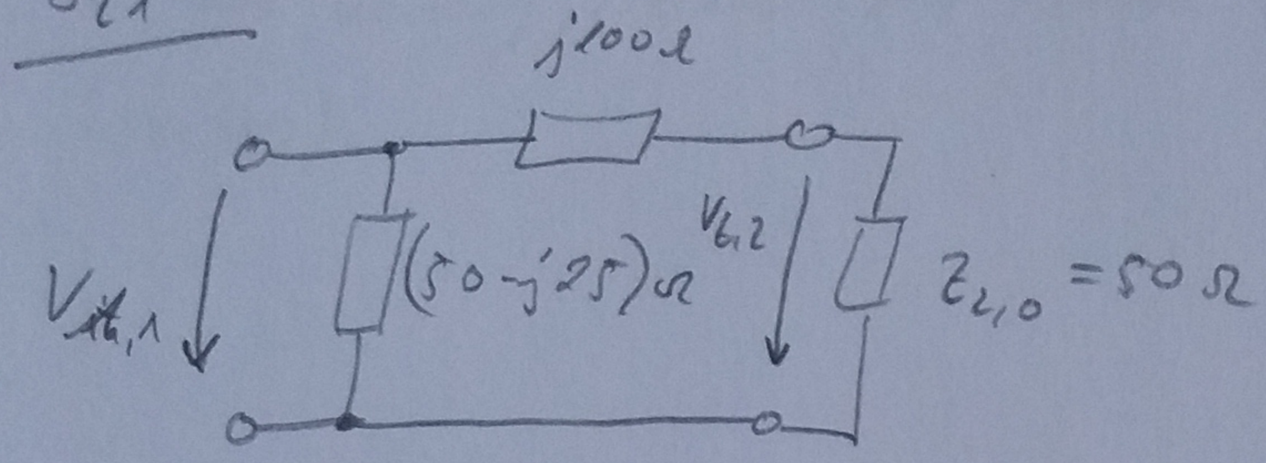
~~$j100\,\Omega$~~ + $Z_{in} = 26.5 - j6$

$Z_{in,2} = (26.5 - j9)\,\Omega$

$S_{22} = \frac{Z_{in,2} - Z_0}{Z_{in,2} + Z_0} = 0.48 + j0.66$

d) cont'd

S_{21}:



$V_{t,1} = (1+S_{11})\,V_{in,1}$; $\underline{V_{in,1}}$: incident voltage wave on port 1 (from the left)

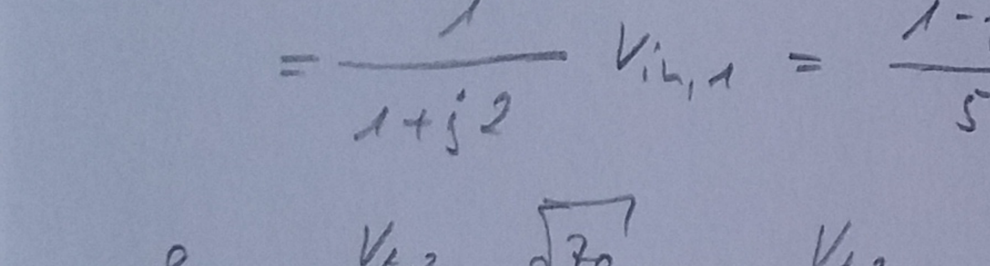
$V_{t,2} = \frac{Z_{2,0}}{Z_{2,0}+j100\Omega}\,V_{t,1} = \frac{Z_{2,0}}{Z_{2,0}+j100\Omega}\,(1+S_{11})\,V_{in,1}$; $\underline{V_{t,2}}$: accepted/dissipated power in $Z_{2,0}$ (i.e. port 2)

$= \frac{50\Omega}{(50+j100)\Omega}\,(1+0)\,V_{in,1}$

$= \frac{1}{1+j2}\,V_{in,1} = \frac{1-j2}{5}\,V_{in,1} = (0.2-j0.4)\,V_{in,1}$

$\underline{\underline{S_{21}}} = \underbrace{\frac{V_{t,2}}{\sqrt{Z_0}}}_{b_2} \cdot \underbrace{\frac{\sqrt{Z_0}}{V_{in,1}}}_{\frac{1}{a_1}} = \frac{V_{t,2}}{V_{in,1}} = \underline{\underline{0.2-j0.4}}$

S_{12}:



$V_{in,2}$: incident voltage wave on port 2 (from the right)

$V_{t,1}$: accepted/dissipated power in $Z_{1,0}$ (i.e. port 1)

$V_{t,1} = \frac{\overbrace{50\Omega \,||\, (50-j25)\Omega}^{=Z_1}}{j100+\{\underbrace{50\Omega \,||\, (50-j25)\Omega}_{=Z_1=26.5-j6}\}}\,(1+\underbrace{S_{22}}_{=0.48+j0.64})\,V_{in,2}$

$\underline{\underline{S_{12}}} = \frac{V_{t,1}}{V_{in,2}} = \underline{\underline{0.2-j0.4}} = S_{21}$! → This result could have been known without calculations. The network is passive and must be reciprocal ⇒ S-matrix must be symmetrical!

c) The scattering matrix of the setup shown in Fig. 2 is not unitary, because the network contains lossy components (resistors).