INTEC

Photonics

Electromagnetism

Maxwell equations, dielectric media, polarization, reflection and transmission, layered structures

Maxwell EM wave equations

- **Maxwell**:
  (in a medium
  with no free charges)

$$\begin{cases} \nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{D} = 0 \\ \nabla \cdot \mathbf{B} = 0 \end{cases}$$

- **Constitutive laws**:

$$\begin{cases} \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} \end{cases}$$

- **P and M describe the response of a material to the incident field**
- **Poynting vector**: $\mathbb{P} = \mathbf{E} \times \mathbf{H}$

Dielectric media

- **Linear media**: linear relations between $\mathbf{P}(\mathbf{r},t)$ and $\mathbf{E}(\mathbf{r},t)$
- **Homogeneous media**: relation between $\mathbf{P}(\mathbf{r},t)$ and $\mathbf{E}(\mathbf{r},t)$ is not dependent on the position $\mathbf{r}$
- **Isotropic media**: relation between $\mathbf{P}(\mathbf{r},t)$ and $\mathbf{E}(\mathbf{r},t)$ is not dependent on the direction of $\mathbf{E}(\mathbf{r},t)$
- **Non-dispersive media**: material response is instantaneous; $\mathbf{P}(\mathbf{r},t)$ at a time t is determined by $\mathbf{E}(\mathbf{r},t)$ at the same time t, and not by the values of $\mathbf{E}(\mathbf{r},t)$ at previous times
- **Spatially non-dispersive media**: $\mathbf{P}(\mathbf{r},t)$ at a location $\mathbf{r}$ is determined by $\mathbf{E}(\mathbf{r},t)$ at the same location $\mathbf{r}$

birefringence in calcite crystal:
non-isotropic medium

Homogeneous, linear, non-dispersive, non-magnetic and isotropic media

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$
$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

- $\mathbf{P}(\mathbf{r}, t)$ and $\mathbf{D}(\mathbf{r}, t)$ can be written as:
  $\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$
  $\mathbf{D} = \varepsilon \mathbf{E}$ with $\varepsilon = \varepsilon_0 (1 + \chi)$
- Maxwell equations become:

$$\begin{cases} \nabla \times \mathbf{H} = \varepsilon \dfrac{\partial \mathbf{E}}{\partial t} \\ \nabla \times \mathbf{E} = -\mu_0 \dfrac{\partial \mathbf{H}}{\partial t} \\ \nabla \cdot \mathbf{E} = 0 \\ \nabla \cdot \mathbf{H} = 0 \end{cases}$$

- Scalar wave equation holds for every field component u:

$$\nabla^2 u - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0, \quad \text{with} \quad v^2 = \frac{1}{\varepsilon \mu_0} \quad \text{and} \quad n = \frac{c}{v}$$

TEM wave

- The transversal electromagnetic plane wave (TEM) is expressed by complex amplitudes:

$$\begin{cases} \mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{E}(\mathbf{r})e^{j\omega t}\right] \\ \mathbf{H}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{H}(\mathbf{r})e^{j\omega t}\right] \end{cases}$$

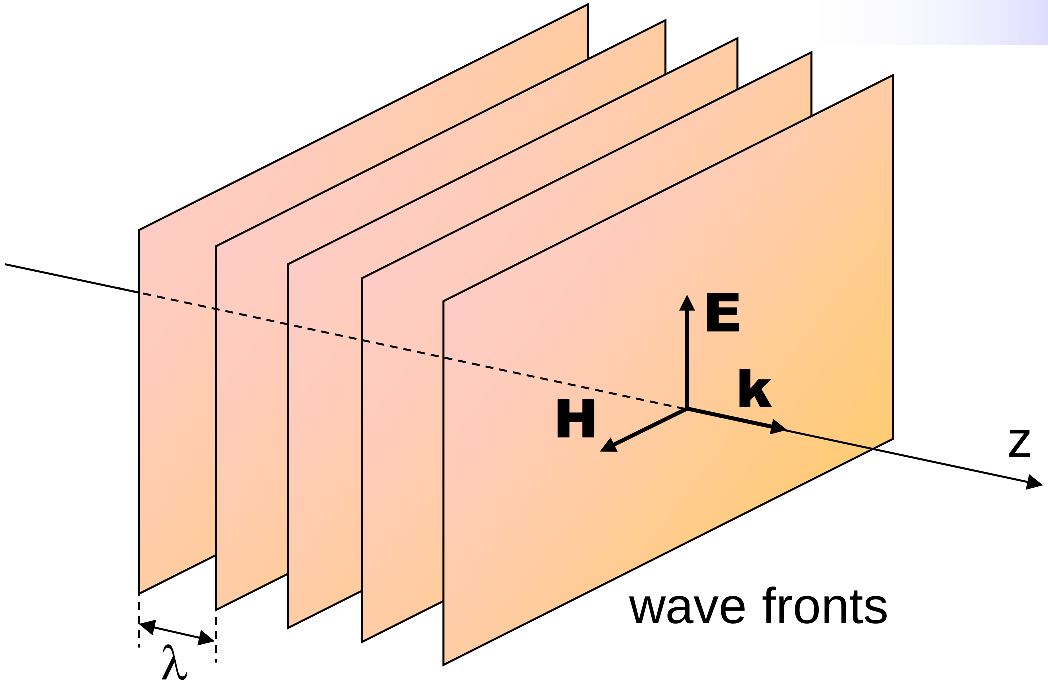
$$\begin{cases} \mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}} \\ \mathbf{H}(\mathbf{r}) = \mathbf{H}_0 e^{-j\mathbf{k}\cdot\mathbf{r}} \end{cases}$$

$$\frac{E_0}{H_0} = Z = \frac{Z_0}{n} = \frac{\omega\mu_0}{k}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

- To satisfy the Maxwell equations we have:

$$k = \omega\sqrt{\varepsilon\mu_0} = \frac{\omega}{v} = \frac{n\omega}{c} = nk_0$$



Polarization of EM waves

- **Polarization:** orientation of the field vector in a particular location in space changes in time
- Polarization is **important for interaction with matter**:
  - Reflection and transmission coefficients are dependent on the polarization

Polarization of EM waves

- **Polarization:** orientation of the field vector in a particular location in space changes in time
- Polarization is **important for interaction with matter**:
  - Reflection and transmission coefficients are dependent on the polarization
  - Absorption is polarization dependent
  - n of anisotropic materials depends on the polarization
- In general, a monochromatic plane wave with frequency ν propagating in the z-direction with the electric field in the xy-plane is described by:

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\mathbf{A}\mathrm{e}^{j2\pi\nu\left(t-\frac{z}{c}\right)}\right] \quad \text{or} \quad \mathbf{E}(z,t) = \mathrm{Re}\left[\mathbf{A}\mathrm{e}^{j\omega t}\mathrm{e}^{-jkz}\right]$$

$$\mathbf{A} = A_x\mathbf{e}_x + A_y\mathbf{e}_y$$

Elliptical polarization

- Substitution with $A_x = a_x \mathrm{e}^{j\phi_x}$ and $A_y = a_y \mathrm{e}^{j\phi_y}$ gives

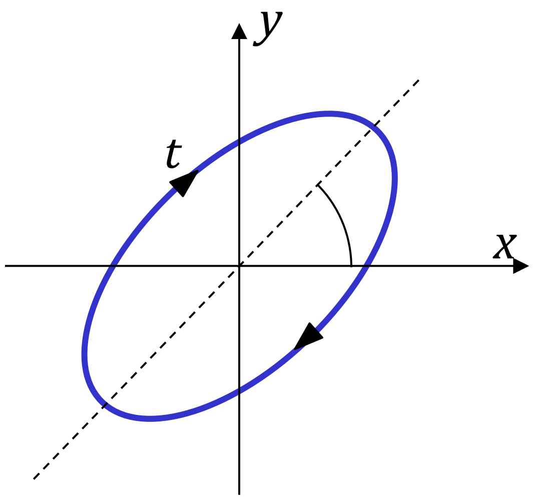
$$\mathbf{E}(z,t) = E_x \mathbf{e}_x + E_y \mathbf{e}_y$$

where

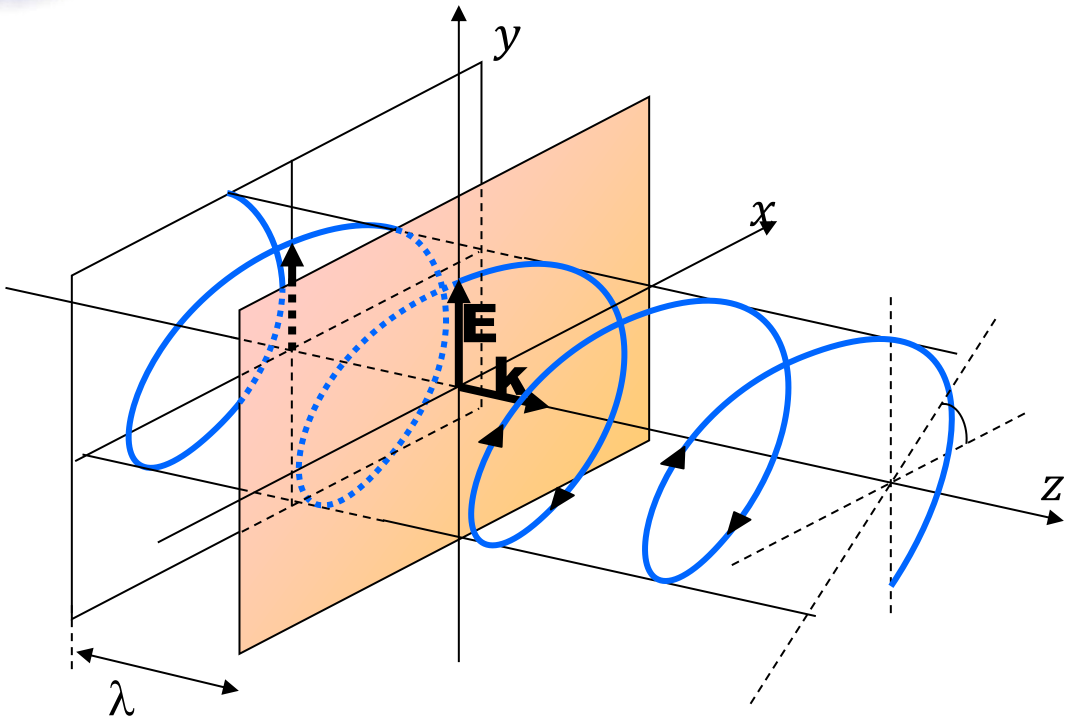
$$\begin{cases} E_x = a_x \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \phi_x\right] \\ E_y = a_y \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \phi_y\right] \end{cases}$$

$$\Rightarrow \quad \frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\phi \frac{E_x E_y}{a_x a_y} = \sin^2\phi$$

$$\mathbf{E}(z,t) = \mathrm{Re}\left[\mathbf{A}\mathrm{e}^{j\pi\nu(t-z/c)}\right]$$
$$\mathbf{A} = A_x \mathbf{e}_x + A_y \mathbf{e}_y$$

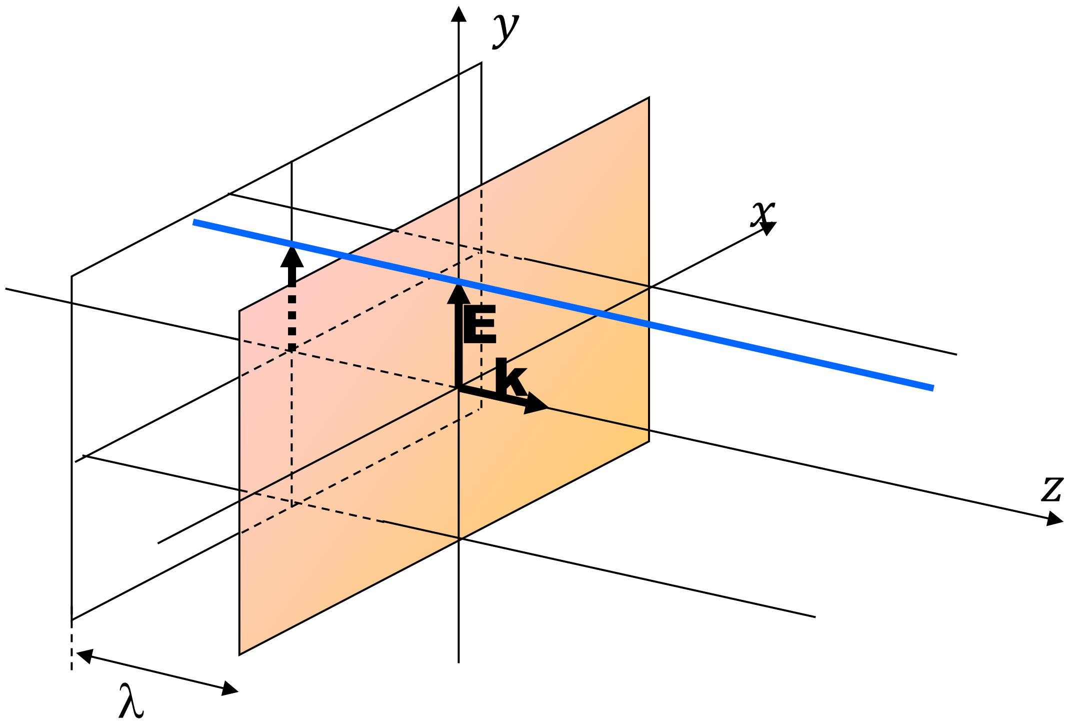


Elliptical polarization



Snapshot at <u>one</u> moment

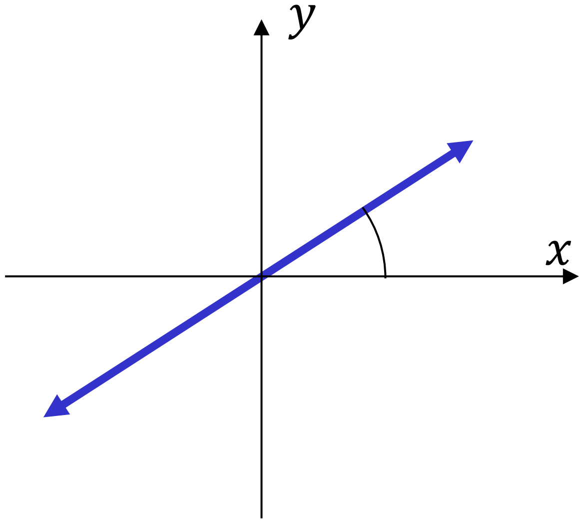
Travelling with the beam
as a function of time

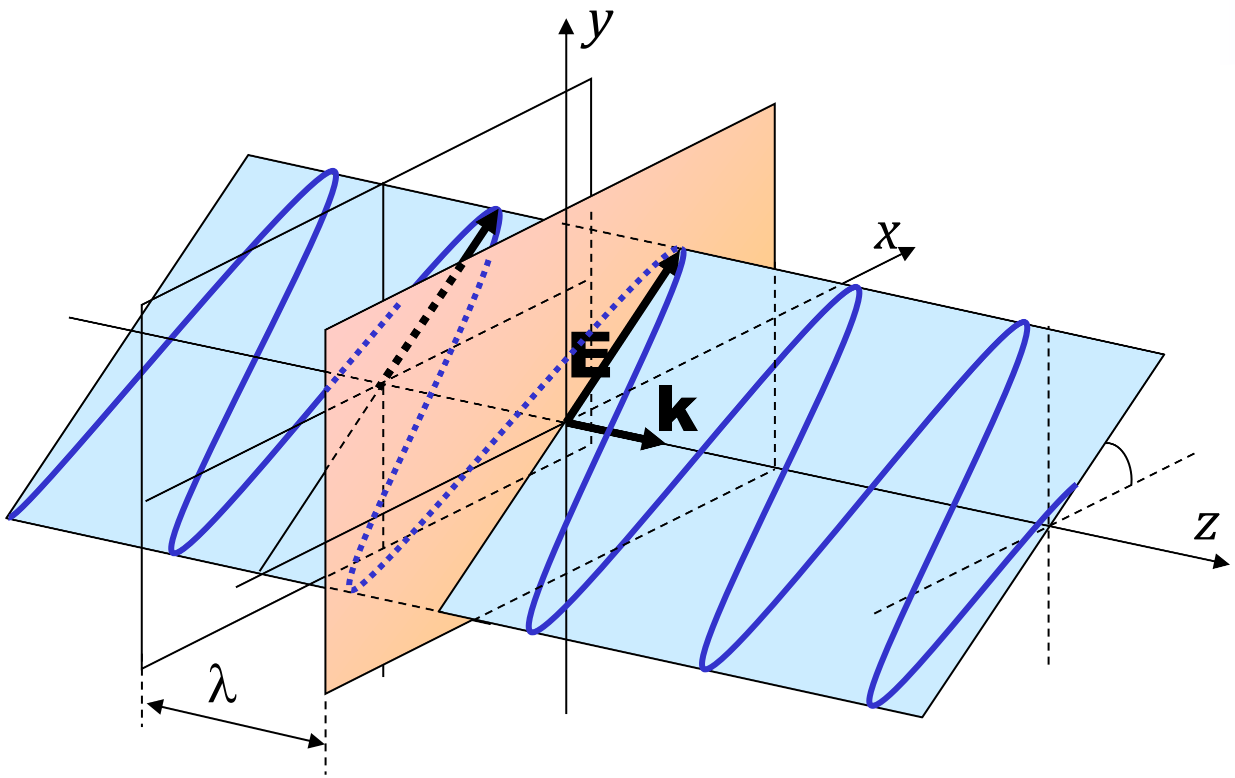


Linear polarization

- A special case of elliptical polarization
  - One of the components is dropped, e.g.: $a_x = 0$
  - If $\phi = 0$ or $\phi = \pi$

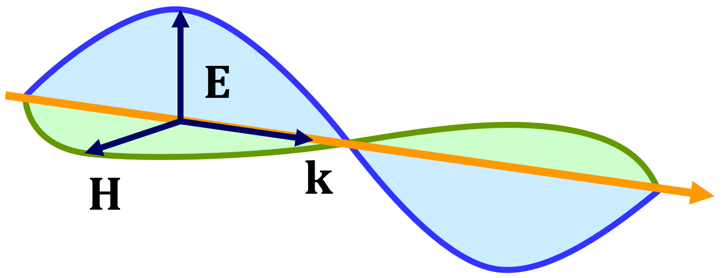
$$E_y = \pm \frac{a_y}{a_x} E_x$$



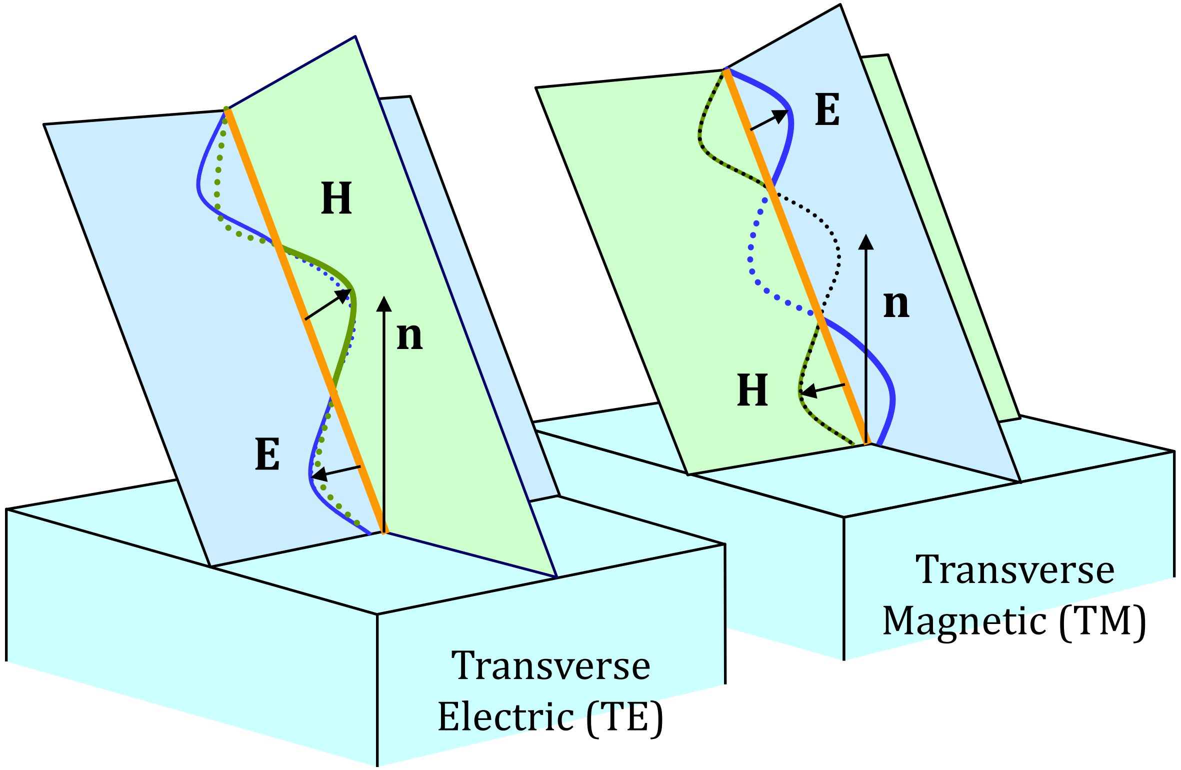


Reflection and transmission: TE and TM waves

- TEM wave
  - electrical component **E**
  - magnetic component **H**
  - **E ⊥ H ⊥ k**



- Orientation to the surface
  - TE wave: **E ⊥ n**
    “s-polarization”
  - TM wave: **H ⊥ n**
    “p-polarization”

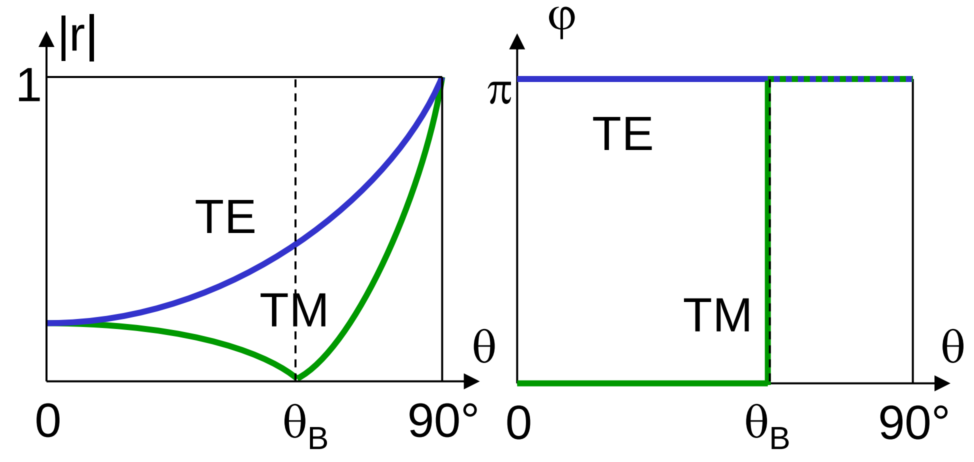


External reflection and transmission

- From lower to higher n: $(n' > n)$

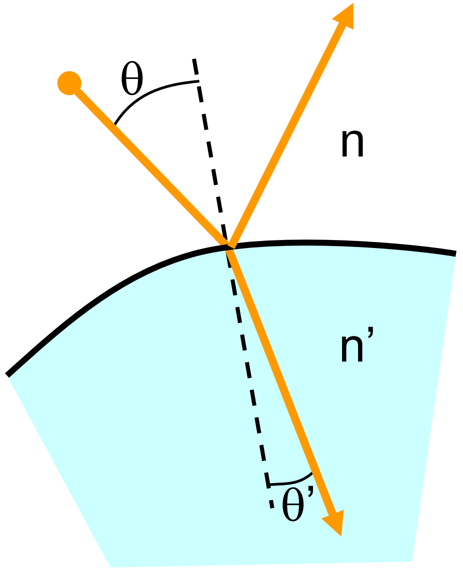
$$r_{\mathrm{TE}} = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'}$$

$$r_{\mathrm{TM}} = \frac{n'\cos\theta - n\cos\theta'}{n'\cos\theta + n\cos\theta'}$$



- Perpendicular incidence:

$$R = \left(\frac{n-n'}{n+n'}\right)^2 \qquad T = \frac{4nn'}{(n+n')^2}$$



- Brewster's angle:
  No reflection for TM-waves

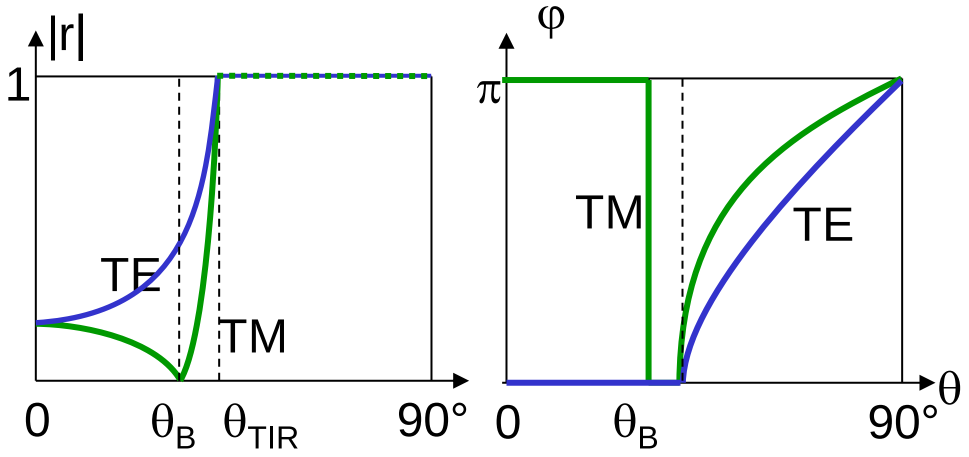
$$\tan\theta_{\mathrm{B}} = \frac{n'}{n}$$

Internal reflection and transmission

- From higher to lower n ($n > n'$), same equations:

$$r_{\mathrm{TE}} = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'}$$

$$r_{\mathrm{TM}} = \frac{n'\cos\theta - n\cos\theta'}{n'\cos\theta + n\cos\theta'}$$

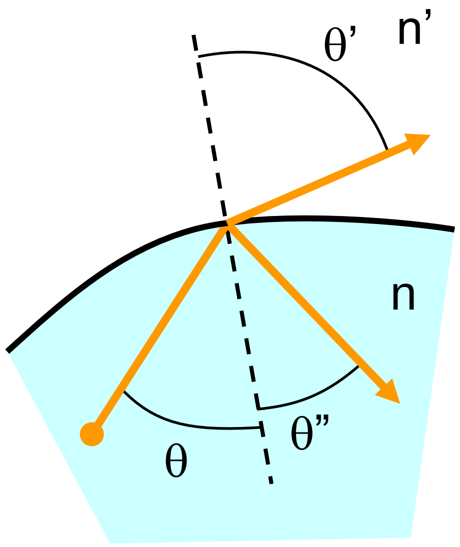


- Brewster's angle:

$$\tan\theta_{\mathrm{B}} = \frac{n'}{n}$$

- Total internal reflection (**TIR**):
  No more transmission as the incidence angle becomes too large

$$\sin\theta_{\mathrm{TIR}} = \frac{n'}{n}$$



Exercise: Brewster angle

Consider a glass plate ($n = 1.5$) in air.

For external reflection:

- Calculate θ_{B}
- Calculate θ_{TIR}

For internal reflection:

- Calculate θ_{B}
- Calculate θ_{TIR}

Power reflection and transmission

- Reflection and transmission of power:

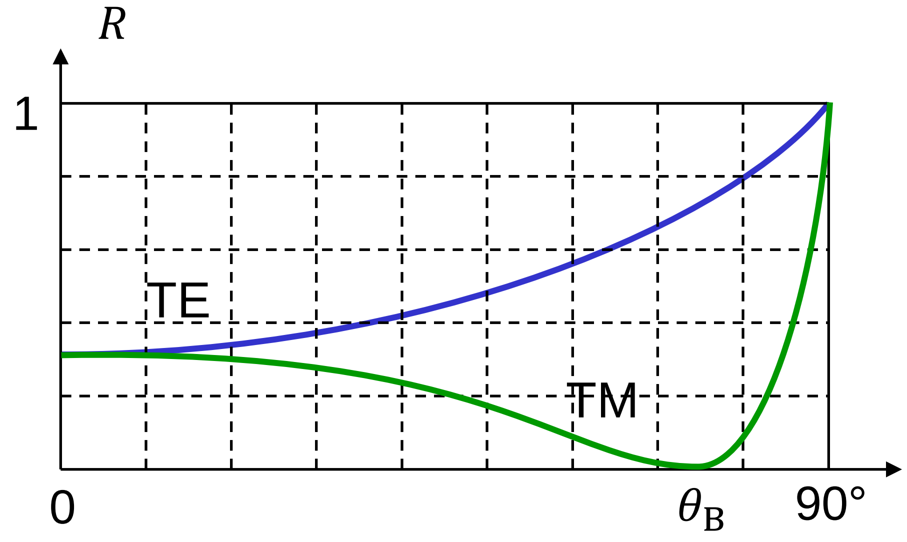
$$R_{\mathrm{TE}} = |r_{\mathrm{TE}}|^2$$

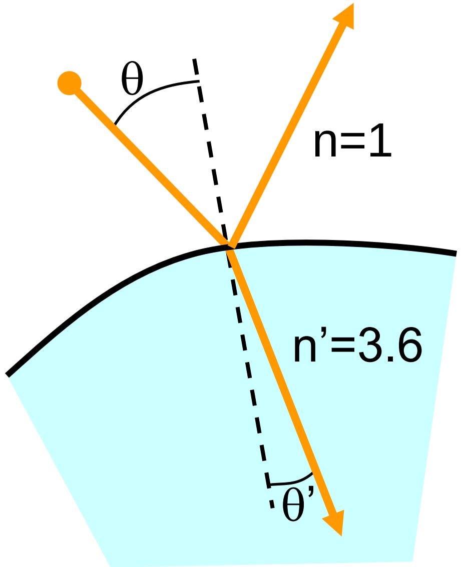
$$T_{\mathrm{TE}} = 1 - R_{\mathrm{TE}} = \frac{n' \cos\theta'}{n \cos\theta} |t_{\mathrm{TE}}|^2$$

$$R_{\mathrm{TM}} = |r_{\mathrm{TM}}|^2$$

$$T_{\mathrm{TM}} = 1 - R_{\mathrm{TM}} = \frac{n' \cos\theta'}{n \cos\theta} |t_{\mathrm{TM}}|^2$$

- Example: GaAs: $n' = 3.6$





Absorption

- Dielectric materials which absorb light:
  - described by a complex susceptibility:

$$\chi = \chi_{\mathrm{R}} + j\chi_{\mathrm{I}}, \quad \varepsilon = \varepsilon_0(1 + \chi_{\mathrm{R}} + j\chi_{\mathrm{I}})$$

$$\varepsilon = \varepsilon_0 \varepsilon_{\mathrm{r}}$$

$$n = \sqrt{\varepsilon_{\mathrm{r}}}$$

  - It means:

$$k = k_0\sqrt{1 + \chi_{\mathrm{R}} + j\chi_{\mathrm{I}}} = k_0(n_{\mathrm{R}} + jn_{\mathrm{I}}) = \beta - \frac{j}{2}\alpha$$

  - Therefore:

$$\mathrm{e}^{-jkz} = \mathrm{e}^{-\frac{1}{2}\alpha z}\mathrm{e}^{-j\beta z}$$

Where α: attenuation or absorption coefficient

and β: propagation constant

The power will decrease exponentially with distance: $P(z) = P_0\mathrm{e}^{-\alpha z}$

Dispersion

- Dispersive materials are characterized by:
  - $\chi(\nu)$, $n(\nu)$ and $v(\nu)$
- Phase velocity:

$$v_\mathrm{p} = \frac{\omega}{\beta} \qquad n = \frac{c}{v_\mathrm{p}} \qquad \text{refractive index}$$

- Group velocity:

$$v_\mathrm{g} = \frac{\mathrm{d}\omega}{\mathrm{d}\beta} \qquad N = \frac{c}{v_\mathrm{g}} \qquad \text{group index}$$

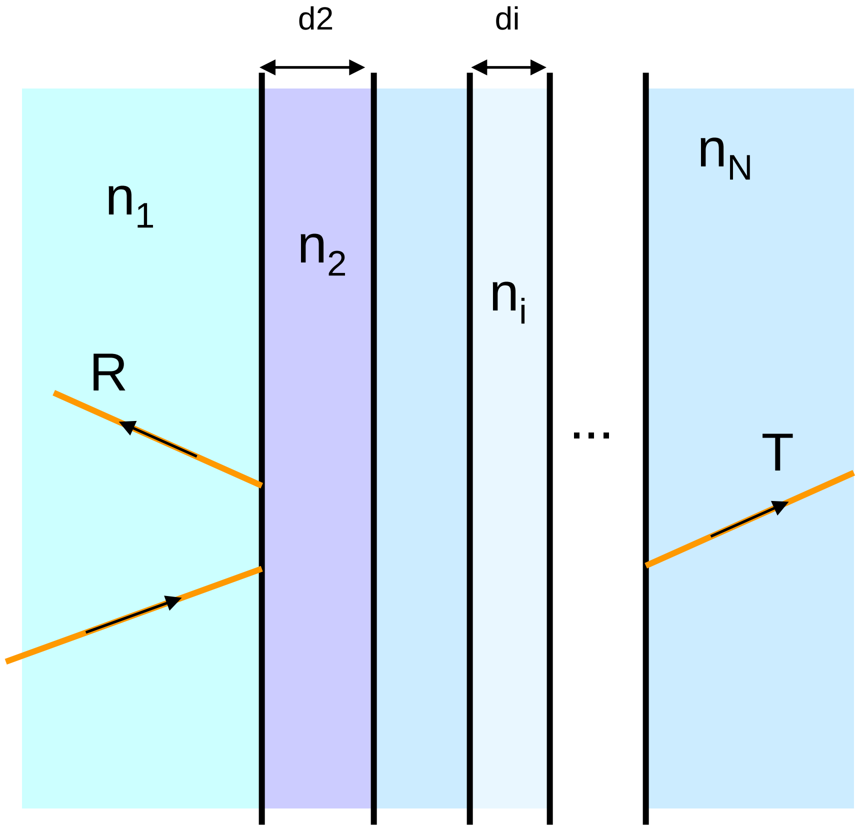
- Material dispersion:

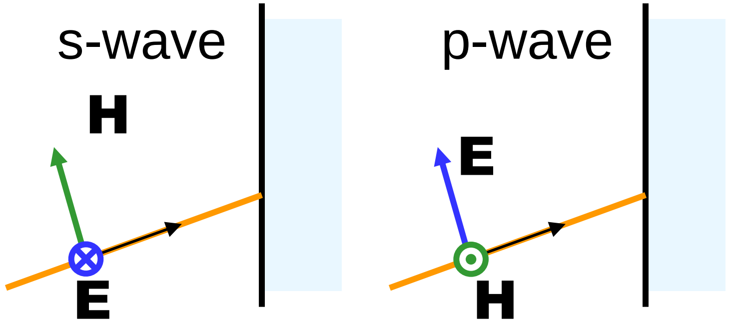
$$\frac{\mathrm{d}n}{\mathrm{d}\lambda} \qquad N = n - \lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}$$

Multilayered structure – layered media

- Layered structure:
  refractive index n_i, thickness d_i
- To determine:

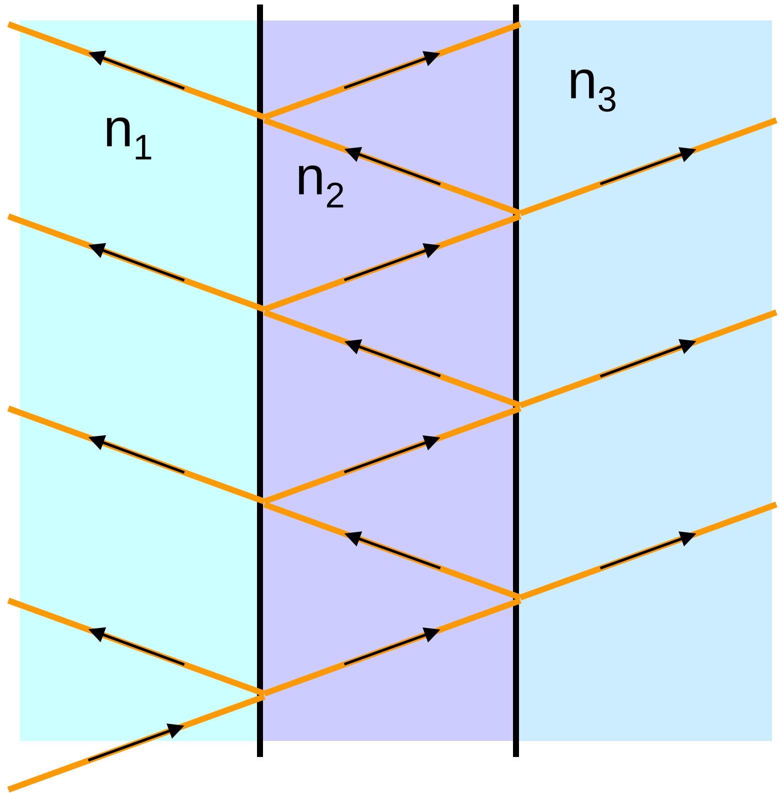
  what is the reflection and transmission for an incident plane wave with a given incidence direction, wavelength and polarization
- Polarization: 2 independent cases
  - **E**-field parallel to the surface: s-wave
  - **H**-field parallel to the surface: p-wave



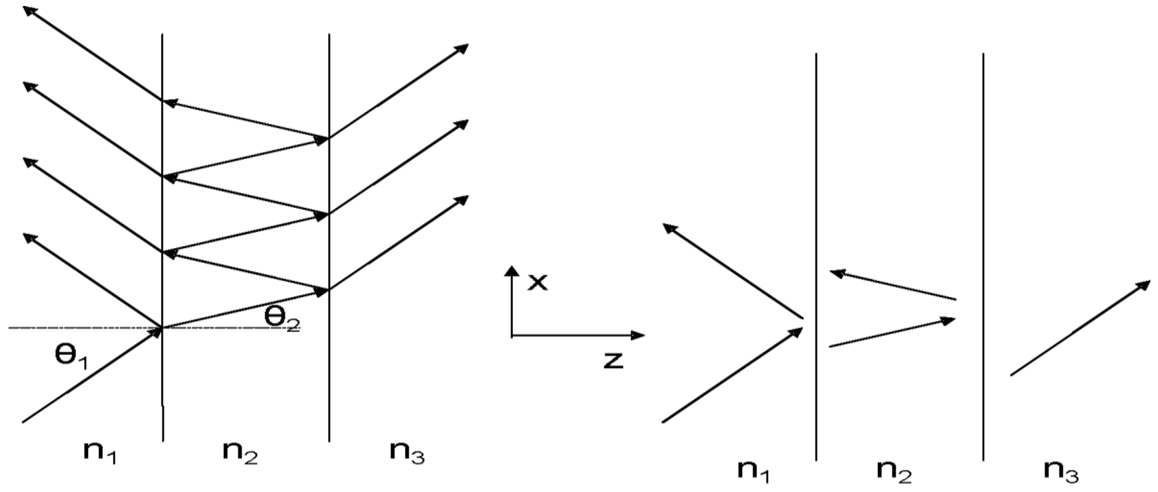


Three-layer structure

- Simplest layered medium
- = 2 semi-transparent mirrors
- = “Fabry-Perot interferometer”
- = “Fabry-Perot etalon”



Three-layer structure: 2 analysis methods



(a) (b)

Method 1

- Wave impinges
- Calculate transmission and reflection (Fresnel)
- Propagation in the layer
- Calculate transmission and reflection (Fresnel)
- Etc. Etc. Etc.
- Sum up all contributions

Method 2

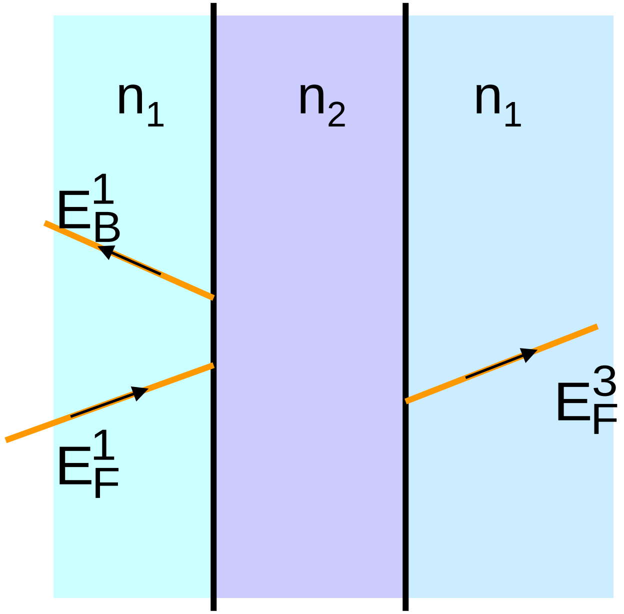
- An incident plane wave from 1 results in:
- 1 forward and 1 backward wave in layers 1 and 2
- 1 forward wave in layer 3
- Apply boundary conditions on the both interfaces
- Solve a linear system

The final result of both methods is the same!

Symmetrical three-layer structure (1)

- Reflection and transmission of a plate:

$$R = \frac{4|r_{12}|^2 \sin^2\phi}{|1 - r_{12}^2 e^{-j2\phi}|^2} \qquad T = \frac{|t_{12}t_{21}|^2}{|1 - r_{12}^2 e^{-j2\phi}|^2}$$



- where $\phi = k_{2,z}d = k_0 n_2 d\cos\theta_2 = \frac{2\pi}{\lambda_0} n_2 d\cos\theta_2$
- Reflection and transmission of 1 interface

$R_1 = |r_{12}|^2$, $T_1 = |t_{12}t_{21}|$ and $T_1 = 1 - R_1$

- Transmission of a plate:

$$T = \frac{T_1^2}{1 + R_1^2 - 2R_1\cos 2\phi} = \frac{(1-R_1)^2}{(1-R_1)^2 + 2R_1 - 2R_1\cos 2\phi}$$

$\cos 2\phi = 1 - 2\sin^2\phi$

$$= \frac{1}{1 + F\sin^2\phi} \qquad \text{where} \qquad F = \frac{4R_1}{(1-R_1)^2}$$

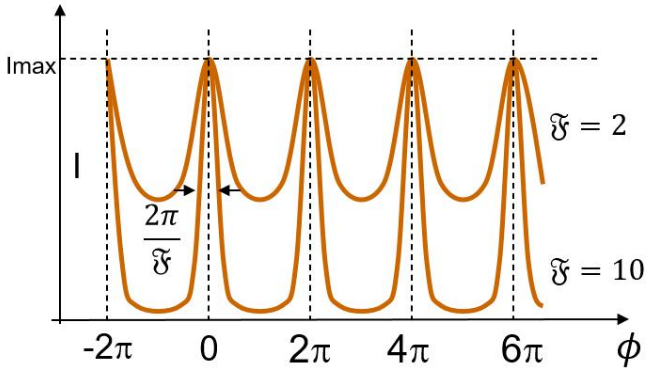
Airy equation

Symm

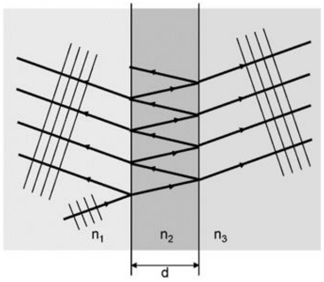
- Reflect

Interference between multiple waves (3)

$$\boxed{I = \frac{I_{max}}{1 + (2\mathfrak{F}/\pi)^2 \sin^2(\phi/2)}}$$



$$\begin{cases} I_{max} = \dfrac{I_0}{(1-|h|)^2} \\ \mathfrak{F} = \dfrac{\pi\sqrt{|h|}}{1-|h|} \quad \text{finesse} \end{cases}$$



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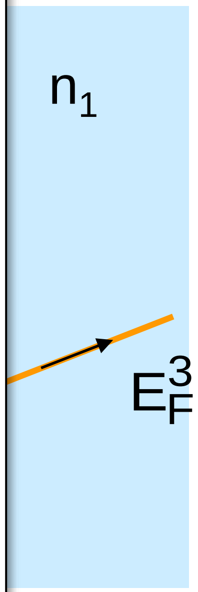


- where
- Reflect

$R_1 = |$

- Transm

$$1 + R_1 - 2R_1 \cos 2\phi \quad (1 - R_1)^2 + 2R_1 - 2R_1 \cos 2\phi$$

$$= \frac{1}{1 + F\sin^2\phi} \quad \text{where} \quad F = \frac{4R_1}{(1-R_1)^2}$$

Airy equation

Symmetrical three-layer structure (2)

- Transmission of a plate: $$T = \frac{1}{1 + F\sin^2\phi}$$
  where $$F = \frac{4R_1}{(1-R_1)^2}$$
- Perpendicular incidence and maximum transmission:

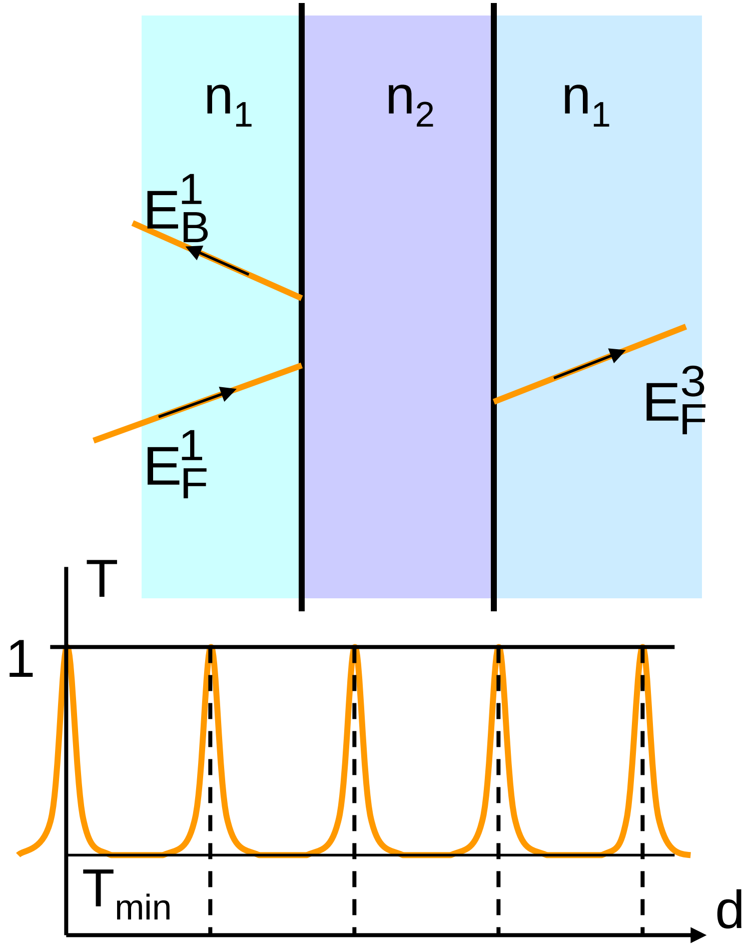
  i.e. $\phi = m\pi \quad d = m\frac{\lambda}{2n_2}$

  minimal transmission: $$T_{\min} = \frac{1}{1+F}$$

- Desirable: $T_{\min}$ as small as possible

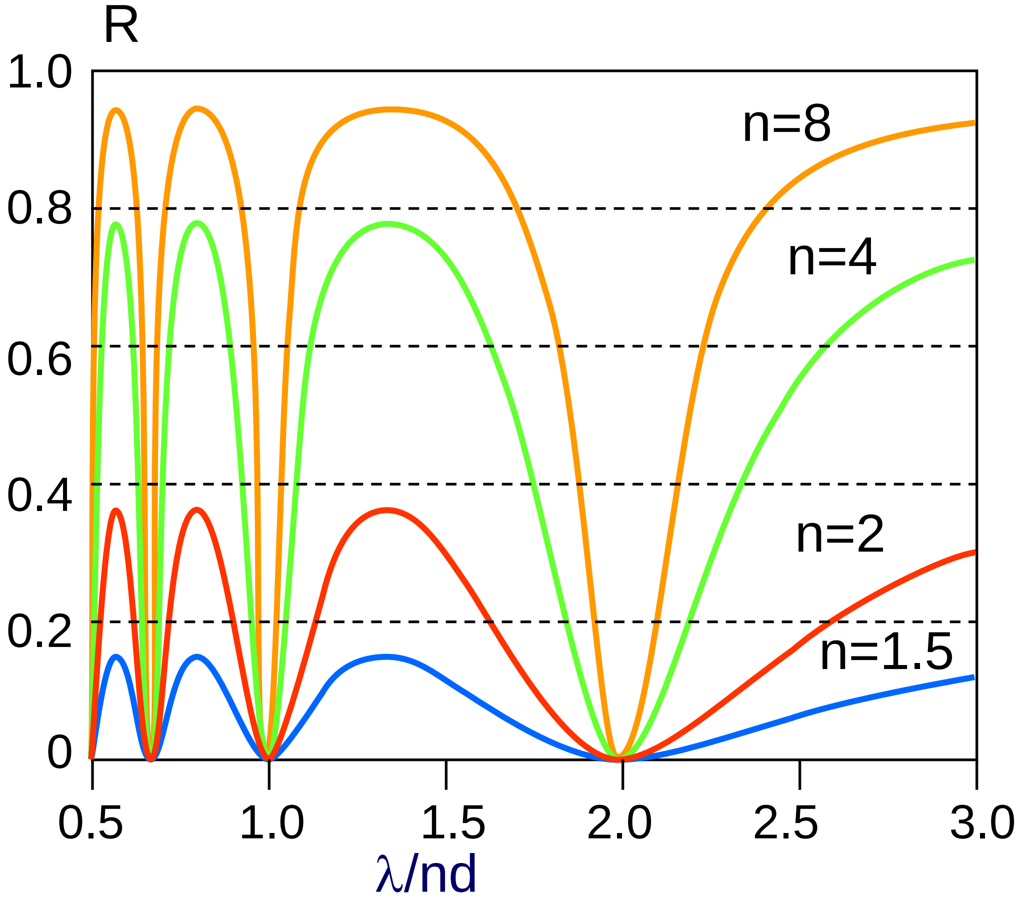
  $\rightarrow$ large $F \rightarrow R_1$ close to 1

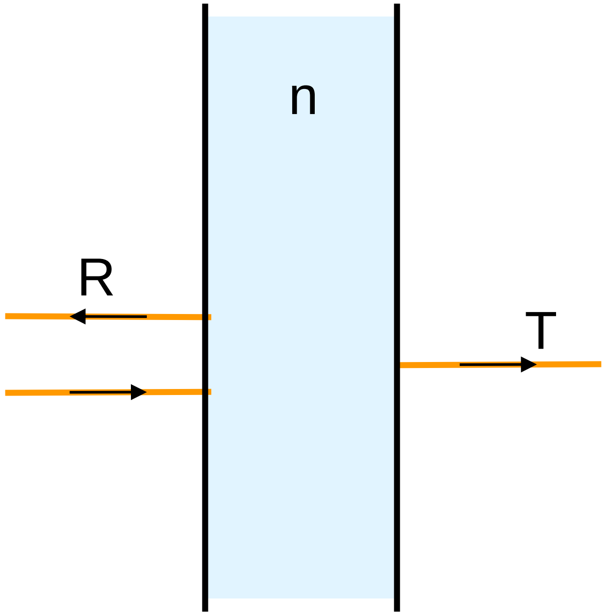
  $\Rightarrow$ difficult with available materials



Symmetrical three-layer structure (3)

- Reflection of parallel plate with different refractive indices



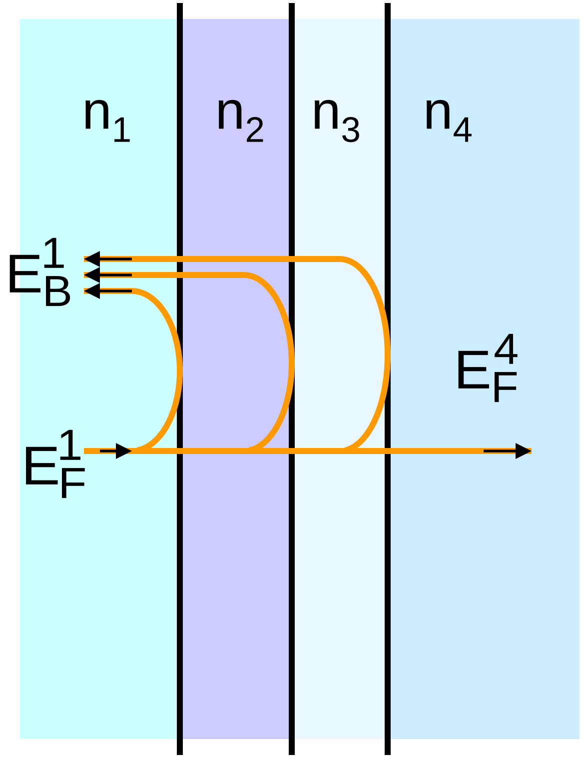


Coatings

- Multiple layer structures
  - Multiple reflections at the interfaces
  - Interference of the reflections

    destructive: anti-reflection coating

    constructive: high-reflection coating
  - Usually designed for perpendicular incidence

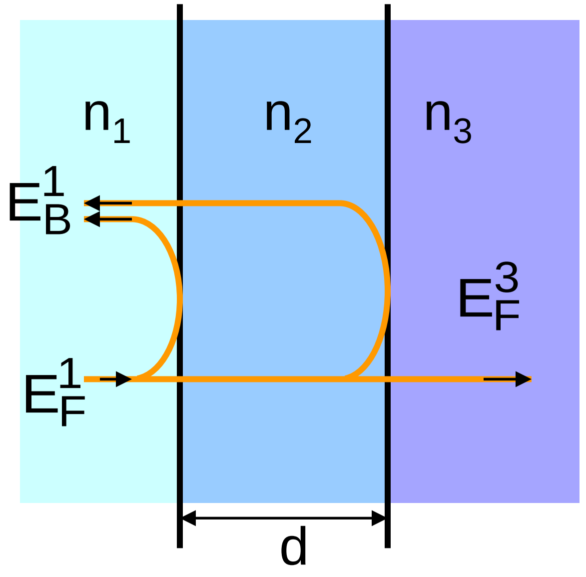


- Application:
  - Dielectric mirrors (HR-coatings)
  - AR-coatings for lenses

Anti-reflection coating

- Reflected beams must interfere **destructively**
- if $n_1 < n_2 < n_3$ (phase shift π)

$$\boxed{d = \frac{1}{4}\frac{\lambda_0}{n_2} = \frac{\lambda_2}{4}}$$ (quarter wave plate)

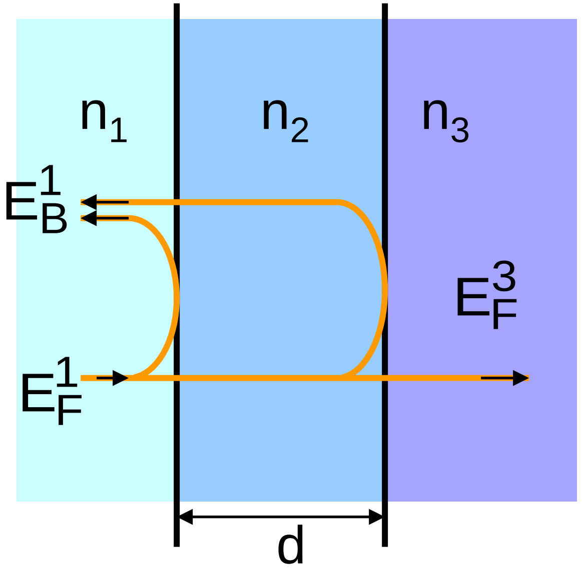


- From Fresnel equations: $r_{12} = r_{23}$
- Perpendicular incidence: $r_{ij} = \dfrac{n_i - n_j}{n_i + n_j}$

from which: $\boxed{n_2 = \sqrt{n_1 n_3}}$

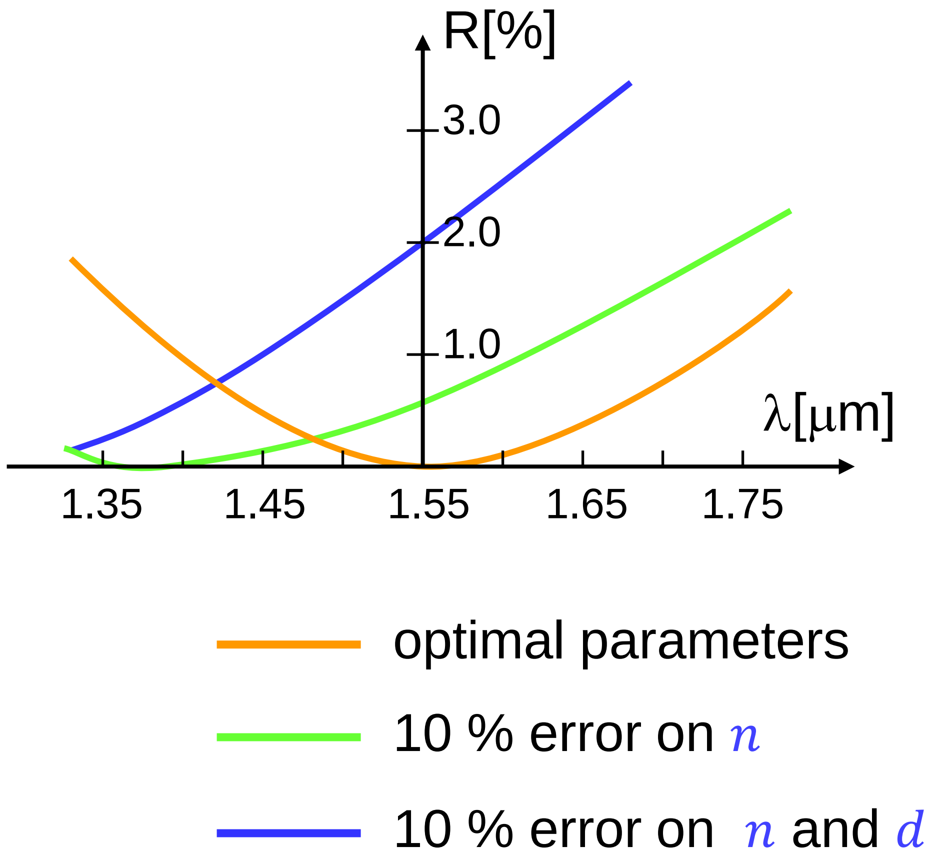
Exercise: anti-reflection coating

- GaAs: $n_3 = 3.2$ for telecom wavelength, 1550 nm
  - Calculate n_2 for minimum reflection
  - Calculate d for minimum reflection
  - What is then the total reflection of the coated semiconductor?



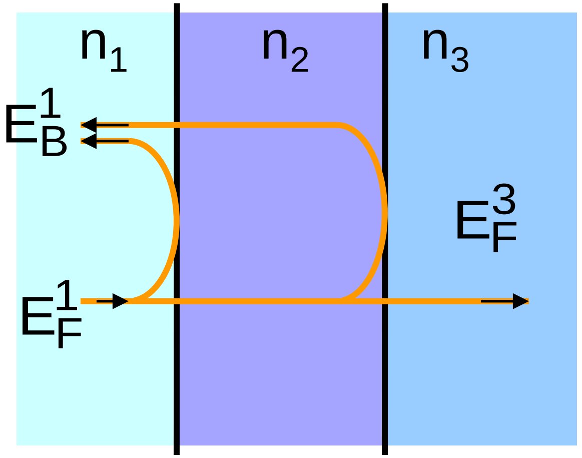
Exercise: anti-reflection coating

- GaAs: $n_3 = 3.2$ for telecom wavelength
  - n_2
  - d
  - Reflection < 0.5% between $1450\ \mathrm{nm}$ and $1650\ \mathrm{nm}$
- Difficult to fabricate in practice
  - Material with exact refraction index
  - Thickness d is critical

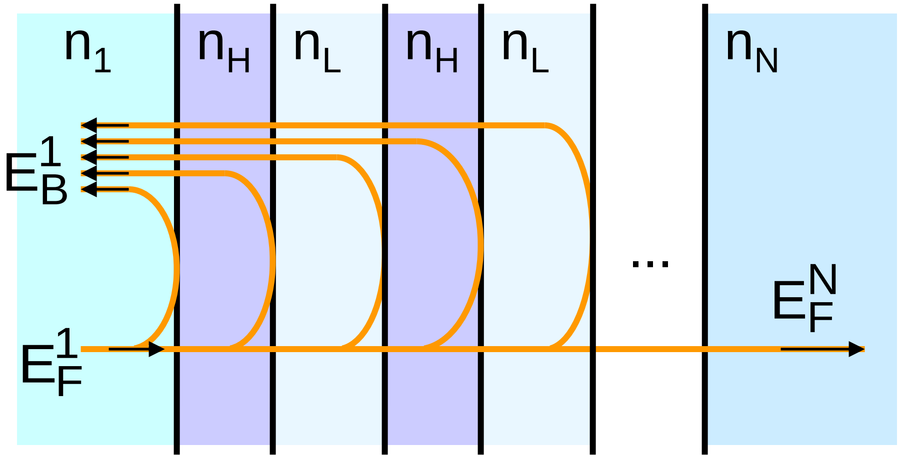


Highly reflective coating (1)

- Two possibilities:
  - quarter-wave plate with $n_2 > n_1$ and $n_2 > n_3$

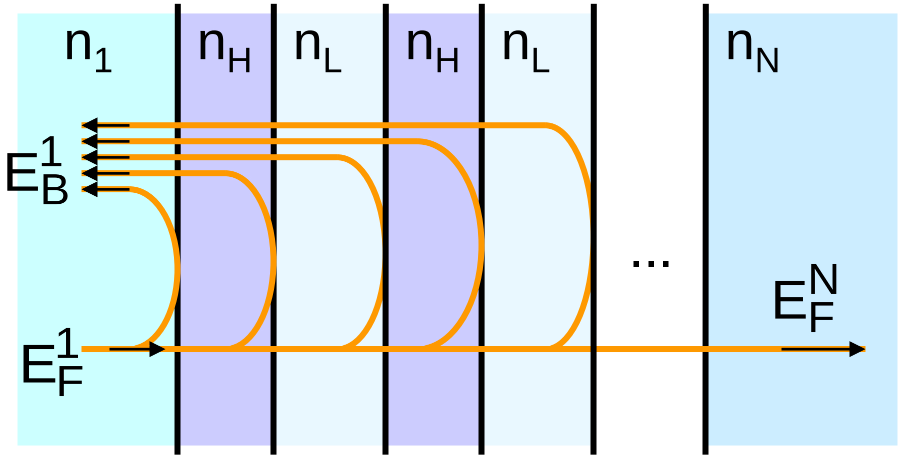


  - multilayered structure of quarter wave plates with high and low refraction index: acts as a Bragg-reflector



Highly reflective coating (2)

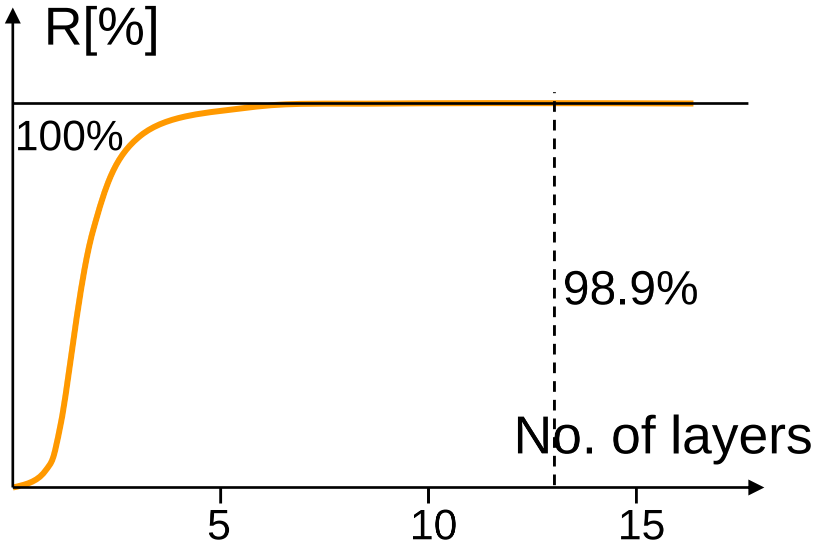
- Constructive interference if $n_\mathrm{H} d_\mathrm{H} = n_\mathrm{L} d_\mathrm{L} = \frac{\lambda_0}{4}$



- $$R = \left(\frac{1 - \left(\frac{n_\mathrm{H}}{n_\mathrm{L}}\right)^{2N}}{1 + \left(\frac{n_\mathrm{H}}{n_\mathrm{L}}\right)^{2N}} \right)^2$$

$\rightarrow$ converges to 1 for large N

$\rightarrow$ better converges for higher $\frac{n_\mathrm{H}}{n_\mathrm{L}}$



- HR-coating for He-Ne laser
  - $\lambda = 633\ \mathrm{nm}$
  - $n_\mathrm{H} = 2.32$ (zinc sulfide)
  - $n_\mathrm{L} = 1.38$ (magnesium fluoride)

Complex coatings

- Applications
  - filters (wide and narrow band)
  - power splitters
  - polarization splitters
- Design: with special CAD-tools
- Example: sunglasses
  - $T < 1\%$ between 400nm and 500nm
  - $15\% < T < 25\%$ between 510nm and 790nm
  - $T < 1\%$ between 800nm and 900nm

  - 29 layers of TiO_2 and SiO_2 with thicknesses between 20nm and 200nm