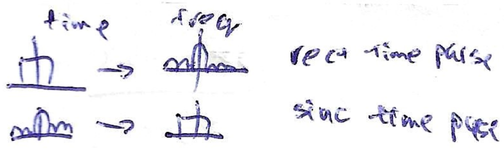
$10\log_{10}\left(\frac{P}{1mW}\right)$ dBm

$10\log_{10}\left(\frac{P_e}{P_i}\right)$ = dB



$\text{sinc}(f) = \frac{\sin(x)}{x}$

Equation sheet - Communications 1 (5ETC0)

PCM system

$M = 2^n$

$R = n \cdot f_s$

$$\left(\frac{S}{N}\right)_{out} = \frac{M^2}{1 + 4P_e(M^2 - 1)} \tag{1}$$

for pulse. 4.77 dB

$snr_{out} = 6.02n + \alpha$ [dB]

$L = 2^l \rightarrow l = \log_2(L)$

where

$$P_e = Q\left(\sqrt{\left(\frac{S}{N}\right)_{in}}\right) \tag{2}$$

Q-function plot ($P_e = \frac{1}{\sqrt{2\pi}z}e^{-z^2/2}$ for $z > 3$):

$snr_{in} = \frac{P_{sig}}{P_{noise}} = \frac{P}{N_0 \cdot n \cdot f_s}$

$N_0 R = N_0 B$

$= \frac{snr_{analog}}{\text{bandwidth ratio}}$

$snr_{old} \cdot N ratio \left(\frac{old}{new}\right)$

$snr_{in} = \frac{P_{sig}}{N} = \frac{P_{sig}}{N_0 B_{PCM}} = \frac{P_{sig}}{N_0 \cdot n \cdot f_s}$

$N = N_0 B_{PCM}$

ISI overlapping of signals

↓

bandwidth limit.

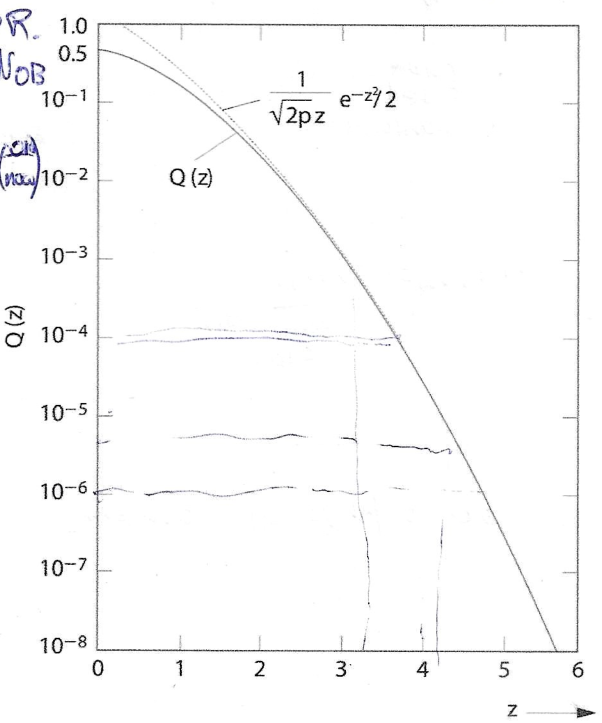


Figure 1: Q-function $Q(z)$ plot.

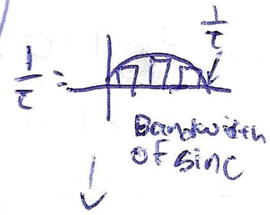
$5 = \sqrt{snr_{in}}$

$n\uparrow \rightarrow snr_{in}\downarrow \rightarrow P_e\uparrow$ $snr_{out}\uparrow$ until $\downarrow$ $snr_{out}\downarrow$

Power spectrum of a digital signal

Pulse shape $f(t) \Leftrightarrow F(f)$; Signal spectrum $P_s(f) = \frac{|F(f)|^2}{T_s}\sum_{k=-\infty}^{\infty} R(k)e^{j2\pi k f T_s}$

while signal bandwidth $\frac{1}{T}$



Raised cosine spectrum

$D_{max} = 2f_0$ max symbol rate

$$H_e(f) = \begin{cases} 1, & |f| \leq f_1 \\ \frac{1}{2}\left(1 + \cos\left(\frac{\pi(|f| - f_1)}{2f_\Delta}\right)\right), & f_1 \leq |f| \leq B \\ 0, & |f| \geq B \end{cases} \tag{3}$$

$r = \frac{f_\Delta}{f_0}$

$\frac{T}{T} \cdot \text{sinc}(f \cdot \tau)$

with $f_0 = f_1 + f_\Delta = B - f_\Delta$; Roll-off factor $r = \frac{f_\Delta}{f_0}$

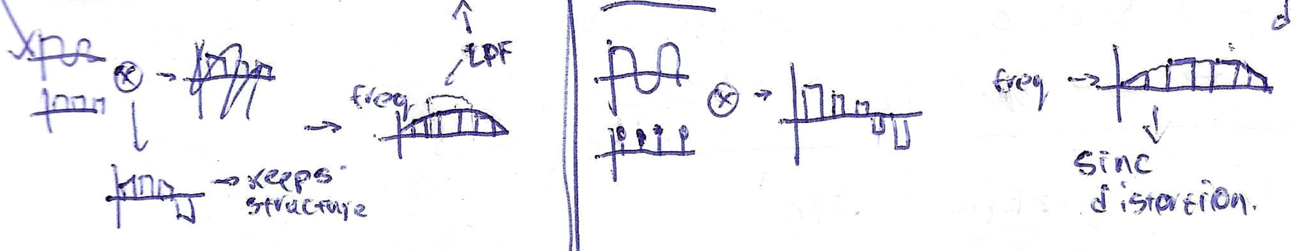
$B = f_0(1 + r)$

absolute 0 bandwidth at $f = \frac{1}{\tau}$

$f_s = \frac{1}{T_s}$

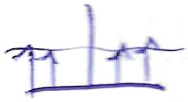
$d = \frac{\tau}{T_s}$

sampling: natural



$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$



→ magnitude plots

- num of symbols: $N = 2BT$.
- total info: $I =$ ~~...~~ # symbols · bits/symb
- aliasing → overlap.
- noise power level: $N = \left(\frac{N_0}{2}\right) B_{PCM} = \left(\frac{N_0}{2}\right)(n f_s \cdot n) = [W] = P_{noise}$

double sided bandwidth

noise spectral density

$P_{noise} = \frac{N_0}{2} \cdot 2B$.

- $SNR_{new\,in} = \frac{SNR_{in\,old}}{\frac{new\ bandwidth}{old\ bandwidth}}$

$\hat{n} = n f_s$

B_{PCM} ~~RX~~ $n = \frac{R}{B}$

$n = \frac{B}{R}$?

- $B_{PCM} = n \cdot B_{analog}$.
- $SNR_{in} = \frac{P_{sig}}{P_{noise}}$.
- $SNR_{in\,new} = \frac{SNR_{old}}{\left[\frac{new\ bits}{old}\right]}$.
- $P_{in} = SNR_{in} \cdot N_0 \cdot B$.

- line coding: convert digital data to physical waveforms

1010011 Hamming (7,4)

msg: 1011

$P_1\ P_2\ 1\ P_3\ 0\ 1\ 1$

1. ~~P_1 101 → even → $P_1 = 0$.~~

1. P_1 1 0 1 → even ⇒ $P_1 = 0$. (every 1)
2. P_2 1 1 1 → odd = $P_2 = 1$ (every 2)
3. P_3 0 1 1 → even = $P_3 = 0$. (every 3)

0110011 // XOR

1010011 ⊕ 0110011

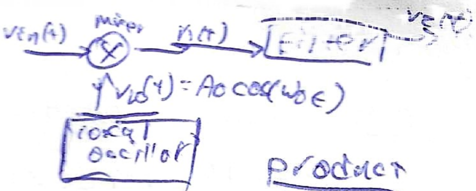
= 11 0000 → $2^0 + 2^1 = 1 + 2 = 3$.

→ 3rd bit flipped → 1000011

- $P_{noise} = \frac{N_0}{2} \cdot 2B_{PCM} = N_0 B_{PCM}$

via mixer.

upconversion → baseband to bandpass.
down conversion → ~~band~~ bandpass to baseband.



product detector

• oscillator produces carrier

euler funcs for magnitude spectrum drawing.

$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$

magnitude plot you just math the euler stuff and plot

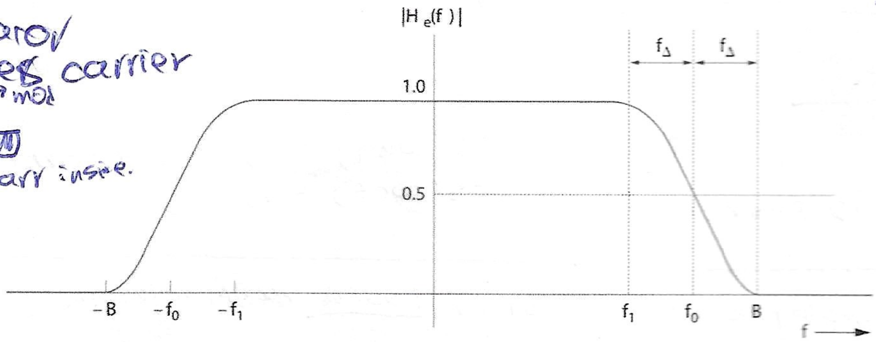
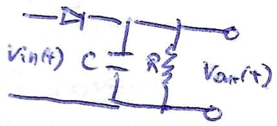


Figure 2: Raised cosine spectrum



The impulse response of the root-raised cosine filter:

$$h_e(t) = 2f_0 \left(\frac{\sin(2\pi f_0 t)}{2\pi f_0 t}\right) \frac{\cos(2\pi f_\Delta t)}{1-(4f_\Delta t)^2} \tag{4}$$

$\frac{R}{B} = \eta$

Information theory

Channel capacity:

Mbits/sec

$$C = B\log_2\left(1+\frac{S}{N}\right) \tag{5}$$

$\frac{C}{B} = \eta$

Analog modulation

Frequency modulation (FM):

$P_{avg} = \frac{A_c^2}{2R_b}$

for PM: $\Delta\theta$.

$$s(t) = A_c \cos\left(\omega_c t + D_f \int m(\tau)d\tau\right); \qquad \beta_f = \frac{\Delta F}{B} \tag{6}$$

$\beta = A_m$

$P_{avg} = \frac{A_c^2}{2R_b} \langle m^2(t) \rangle$

$$\text{Carsons rule:} \quad B_T = 2(\beta+1)\overset{f_m}{B}; \qquad \Delta F = \frac{1}{2\pi} \cdot D_f \cdot \max[m(t)] \tag{7}$$

Amplitude modulation (AM): $s(t) = A_c[1+m(t)]\cos(\omega_c t)$. $m(t) = A_m\cos(\omega_m t)$.

modulation signal

pros of AM modulation with carrier → envelope detector, ↳ but wasting energy.

when this > 100% → product detector no this

$$\%modulation = \frac{A_{max} - A_{min}}{2A_c} \cdot 100\% \tag{8}$$

$B = 2 f_m$

AM

$PEP = \frac{A_c^2}{2R_p}\{1+\max[m(t)]\}^2$

$$\%positive\ Modulation = \frac{A_{max} - A_c}{A_c} \cdot 100\% = max(m(t)) \cdot 100\% \tag{9}$$

$$\%negative\ Modulation = \frac{A_c - A_{min}}{A_c} \cdot 100\% = min(m(t)) \cdot 100\% \tag{10}$$

DSB

$PEP = \frac{A_c^2}{2R_p}\{\max[m(t)]\}^2$ efficiency

sinusoidal DSB $\langle m^2(t)\rangle = \frac{1}{2}$

$$E = \frac{<m^2(t)>}{1+<m^2(t)>} \cdot 100\% \tag{11}$$

$\rightarrow \frac{A_m^2}{2} = \langle m^2(t) \rangle$

Trigonometric identities: %mod $= \frac{A_m}{A_c} \cdot 100\%$ → ~~for~~ DSB-SC → $\infty\%$ → since no carrier.

DSB-SC → no carrier

$A\cos(\omega_m t)\cos(\omega_c t)$.

$A\, m(t)\cos(\omega_c t)$.

$$2\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta) + \sin(\alpha-\beta) \tag{12}$$

$$2\cos(\alpha)\sin(\beta) = \sin(\alpha+\beta) - \sin(\alpha-\beta) \tag{13}$$

$$2\cos(\alpha)\cos(\beta) = \cos(\alpha+\beta) + \cos(\alpha-\beta) \tag{14}$$

for DSB-SC → $E = \frac{\langle m^2(t)\rangle}{\langle m^2(t)\rangle} \cdot 100\% = 100\%$.

how to find Am.

$A_c(1 \pm A_m) = A_{max/min}$

A_c → ~~quadrature~~ when ~~?~~ appears.

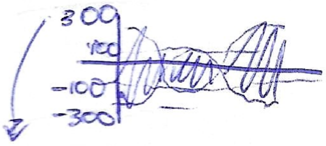
• Am modulation → carrier signal amplitude to encode info; extracted by envelope detector. for mod % < 100%

• DSB-SC. lacks carrier. → requires product detector for demodulation.

↳ when efficiency equal or over 100% → product detector.

e.g amplitude modulation.

$s(t) = 100(1 + 2\cos(\omega_m t))\cos(\omega_c t)$.



→ 300, -300, 100, -100

→ when $\cos(\omega_m t) = 1, -1$] 4 combos
when $\cos(\omega_c t) = 1, -1$]

in general PEP is peak of envelope

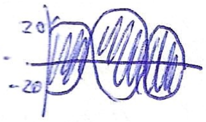
- envelope detector: simple passive device, but, for positive AM modulation under 100% efficiency.
- product detector: for AM and PM → uses local oscillator for down conversion allows when %mod > 100%, works for negative modulation.

---

e.g DSB-SC modulation.

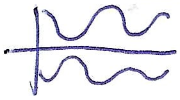
$s(t) = 20\cos(2\pi 10^3 t) \cdot \cos(2\pi 3 \cdot 10^5 t)$.

when $\cos(\omega_m t) = 1, -1$] 2 combos →
$\cos(\omega_c t) = 1, -1$]



- PM, FM → same one is derivative of other

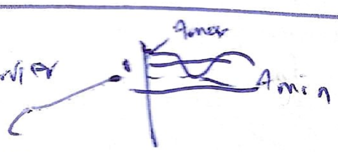
- implement envelope detection for under 100% %mod →



100% ≥ →

you get these distortions need product detector.

- $\frac{A_{max}(t)}{A(t)_{min}} = 2$. → DSB + carrier



- $\frac{1+m_0}{1-m_0} = 2$.
↳ $m_0 = \frac{1}{3}$ → 33.3% → envelope.

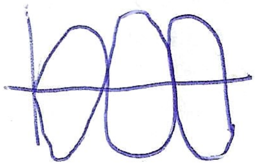
- see notes for IQ demodulator.
- DSB-SC → $g(t) = m(t)$ → to be able to detect with envelope detector we have to have carrier → $g(t) = 1 + m(t)$

- $A \% mod = \frac{A_{max} - A_{min}}{2A_c} = \frac{1.4A_c - 0.6A_c}{2}$ for example ✓

- $\beta = \frac{D_f \cdot A_m}{\omega_m}$, $D_f = \frac{\beta \omega_m}{A_m}$, $\Delta F = \beta \cdot f_m$.

° carsons rule we need 98% of power for good signal reconstruction.

DSB-SC
%= ∞%
↓
needs product detector



- if lower sideband negative → narrowband FM.

$A_c m(t) \cdot \cos(w_c t) \cos(w_c t + \varphi) = \frac{1}{2}[\cos(2w_c t + \varphi) + \cos(\varphi)]$

$R = D \cdot l$

$\frac{R}{l} = D$

$R = lD$

$R = 5 \cdot 10^6$

$$2\sin(\alpha)\sin(\beta) = -\cos(\alpha+\beta) + \cos(\alpha-\beta) \tag{15}$$

$$2\sin(\alpha)\cos(\alpha) = \sin(2\alpha) \tag{16}$$

$$2\cos^2(\alpha) = 1 + \cos(2\alpha) \tag{17}$$

$$2\sin^2(\alpha) = 1 - \cos(2\alpha) \tag{18}$$

4 level
unipolar RZ 20%
$L = \log_2(4) = 2 = l$
multilevel
$R/l = 2.5 \cdot 10^3$
$\frac{2.5}{0.2} = 12.5$
$B = R/l \cdot \%$
$\eta = l = 2$
~~$\eta = B \cdot l = 2 \cdot 0.2$~~
$= 0.4$

order\betta	0.5	1	2	3	4	5	6	7	8	9	10
0	0.93847	0.765198	0.223891	-0.26005	-0.39715	-0.1776	0.150645	0.300079	0.171651	-0.09033	-0.24594
1	**0.242268**	0.440051	0.576725	0.339059	-0.06604	-0.32758	-0.27668	-0.00468	0.234628	0.245307	0.043472
2	0.030604	**0.114903**	0.352834	0.486091	0.364128	0.046565	-0.24287	-0.30142	-0.11299	0.144846	0.25463
3	0.002564	0.019563	**0.128943**	0.309063	0.430171	0.364831	0.114768	-0.16756	-0.29113	-0.18093	0.05838
4	0.000161	0.002477	0.033996	**0.132034**	0.281129	0.391232	0.357642	0.157798	-0.10535	-0.26547	-0.2196
5		0.00025	0.00704	0.043028	**0.132087**	0.261141	0.362087	0.347896	0.185775	-0.05504	-0.23406
6			0.001202	0.011394	0.049088	**0.131049**	0.245837	0.339197	0.337569	0.204312	-0.01446
7			0.000175	0.002547	0.015176	0.053376	**0.129587**	0.233584	0.320578	0.327456	0.21671
8				0.000493	0.004029	0.018405	0.056532	**0.127971**	0.223455	0.305063	0.317854
9					0.000939	0.00552	0.021165	0.058921	**0.126321**	0.214881	0.291856
10					0.000195	0.001468	0.006964	0.023539	0.060767	**0.124694**	0.207486
11						0.000351	0.002048	0.008335	0.025597	0.062217	**0.123117**
12							0.000545	0.002656	0.009624	0.027393	0.06337
13							0.000133	0.00077	0.003275	0.01083	0.028972
14								0.000205	0.001019	0.003895	0.011957
15									0.000293	0.001286	0.004508
16										0.000393	0.001567

treat this as normal table

$\eta = \frac{R}{B} = \frac{D}{B}$

if multi level

$\eta = \frac{R}{B} = \frac{lD}{B}$

Figure 3: Bessel function table

%

$\eta = B \cdot l$

$\eta\%$

$B = \frac{D}{d}$

duty cycle

~~R~~ $R = l \cdot D$

Line codings

D/B or R/B or $\frac{R}{B}$

Line coding (1 bit per symbol)	First Null Bandwidth (R is bit rate)	SE (Bits/Hz)
Unipolar/Polar NRZ	R	1
Unipolar RZ (50%)	$2R$	0.5
Sinc Pulses	$0.5R$	2
Raised cosine pulses	$(1+r)D$ $(1+r)\frac{R}{2}$ → D.K. → D. $l = 2$	$\frac{2}{1+r}$ → $\frac{l}{1+r}$

$\frac{R}{D} = \frac{\text{bits}}{\text{per symbol}} = l$

Table 1: Bandwidth and Spectral Efficiency of Various Line Coding Schemes

multilevel polar NRZ? R/l $r[0,1]$ l

$D_{max} = 2f_0$

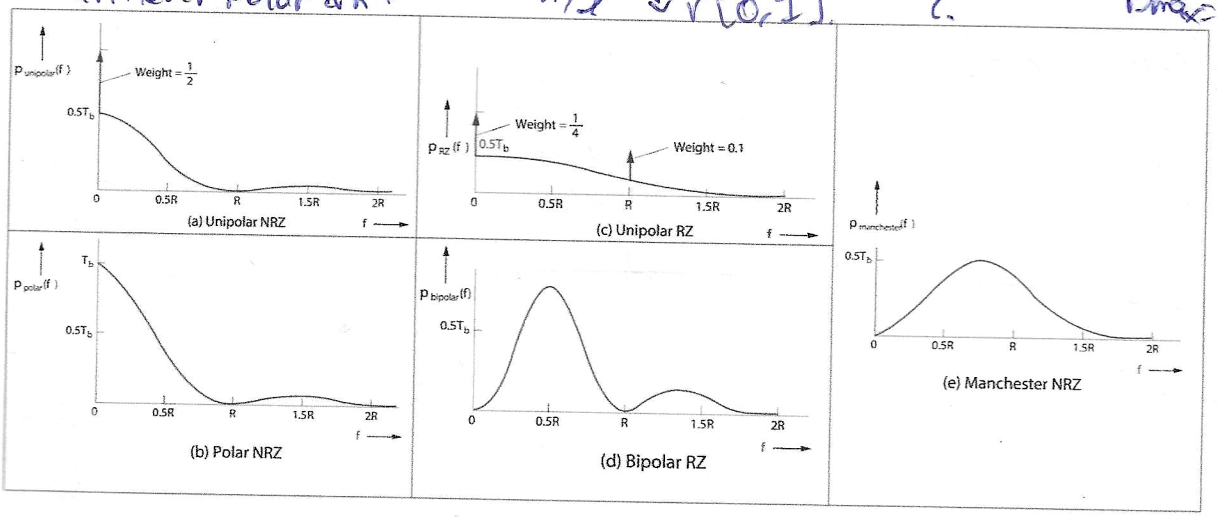
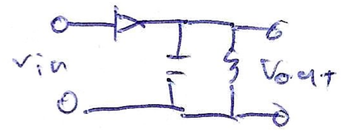


Figure 4: Line codings and their spectras

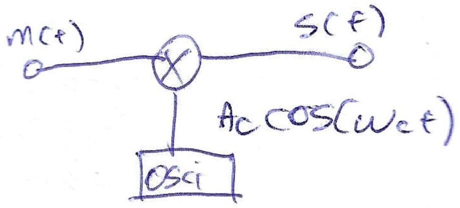
- $\eta_{max} = C/B = \log_2(1 + SNR)$

- Power limited region $SNR \ll 1$, capacity inc with snr.
- bandwidth limited region $SNR \gg 1$, capacity constraint by bandwidth

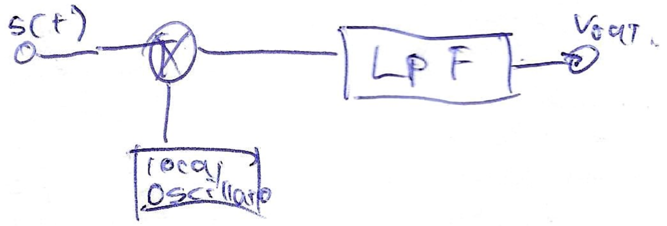
envelope detector



product modulator



product detector



$\frac{R}{B} = 2.5 \rightarrow$ multilevel. $R = lD.$

$l = 2 \rightarrow \frac{lD}{B} = 2.5 \rightarrow \frac{2D}{B} = 2.5 \rightarrow \frac{D}{B} = 1.25$

raised cosine rolloff

$\eta = \frac{l}{1+r} \rightarrow 1.25 = \frac{2}{1+r} \rightarrow r = 0.6$

$l = 4$

$n = 6.$

polar nrz

$\eta = 1 = \frac{R}{B} \rightarrow$

since multilevel.

$B = R = lD$

$B = \frac{R}{l} = D.$

$$\boxed{B = \frac{nf_s}{l}}$$

in milliwatt
dBm → $10\log_{10}\left(\frac{P_{measure}}{1mW}\right)$ → −(if FSPL)

Friis eq.

Received signal power in a radio communication link

free space path loss

$\boxed{FSPL = 20 \left(\frac{4\pi d}{\lambda}\right)^2}$

dBm $$P_{Rx} = P_{Tx} \cdot G_{AT} \cdot G_{AR} \cdot \left[\frac{\lambda}{4\pi d}\right]^2 [mW] \quad \text{single path} \qquad (19)$$

$$P_{Rx} \approx P_{TX} G_{TX} G_{RX} \left(\frac{h_{TX} h_{RX}}{d^2}\right)^2 [mW] \quad \text{multi path.} \qquad (20)$$

when attenuation its FSPL + KE

$$d_{break} = \frac{4\pi h_{Tx} h_{Rx}}{\lambda} \qquad (21)$$

if $d > d_{break}$

Knife edge

if want to calc knowing A(v). want h_{obs}

→ $\frac{v}{K} = \beta + \gamma$.

$$v = \alpha \sqrt{\frac{2 d_1 d_2}{\lambda (d_1 + d_2)}} \qquad (22)$$

$$\alpha = \beta + \gamma \qquad (23)$$

$$\beta = \tan^{-1}\left(\frac{h_{obs} - h_{TX}}{d_1}\right) \qquad (24)$$

$$\gamma = \tan^{-1}\left(\frac{h_{obs} - h_{RX}}{d_2}\right) \qquad (25)$$

The equation below should only be used for $v > 0$!

$K = \sqrt{\frac{2 d_1 d_2}{\lambda (d_1 + d_2)}}$

$$A(v) = 6.9 + 20\log_{10}\left(\sqrt{v^2 + 1} + v - 0.1\right) \quad [dB] \qquad (26)$$

while we obtain from

Physics

$P_R = P_{Tx} - L \cdot d$ dB/km

$\left(\frac{1}{d}\right)$ → direct line of sight.

$\left(\frac{1}{d^4}\right)$ → M.P.

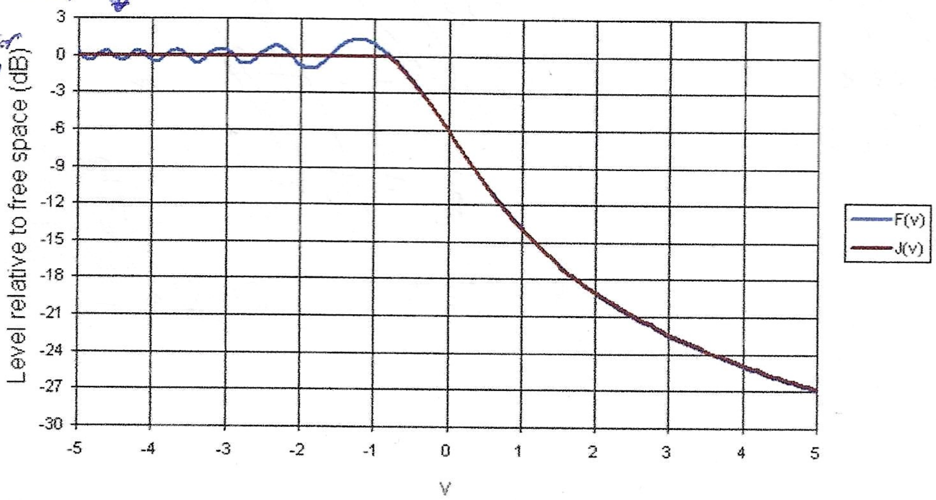


Figure 5: Diffraction loss. This figure can be used for all values of v!

G = dB → ratio of power → $10\log_{10}\left(\frac{P_o}{P_i}\right)$ [dB].

P_{Tx}, P_{Rx} = dBm → power [mW] → $10\log_{10}\left(\frac{P}{1mW}\right)$ [dBm] shorter pulses.

[high bandwidth → $\lambda \downarrow$ → low dispersion.
low bandwidth → $\lambda \uparrow$ → low attenuation. → power not concentrated in shorter pulses.

things to consider for.

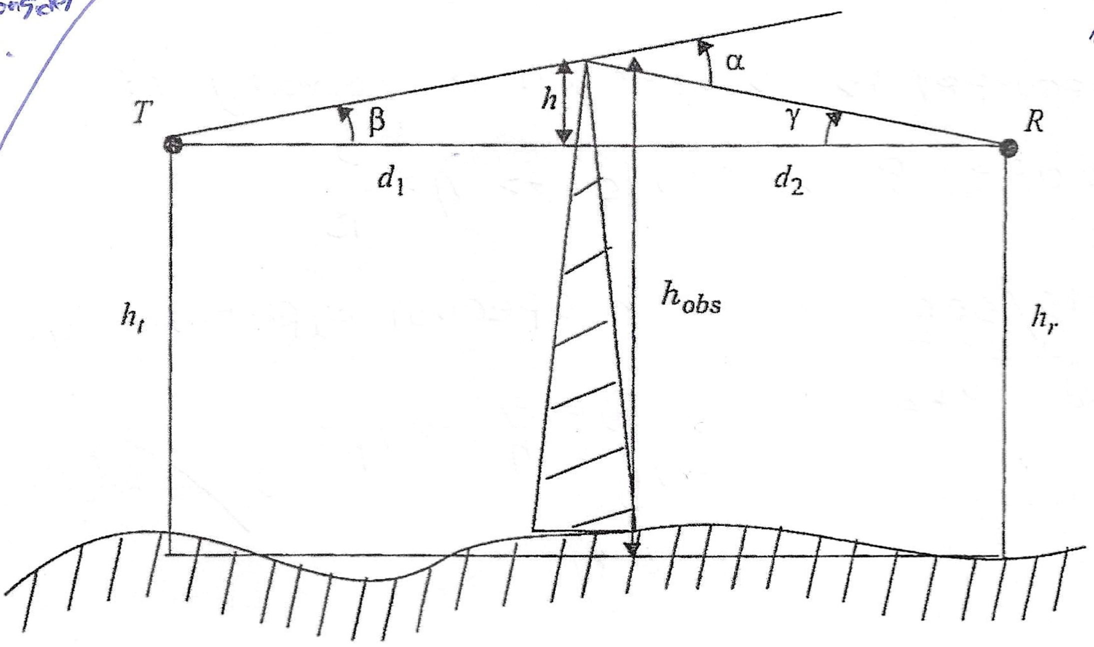
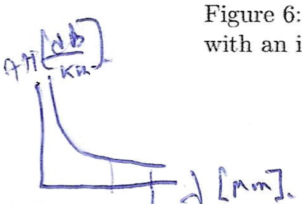
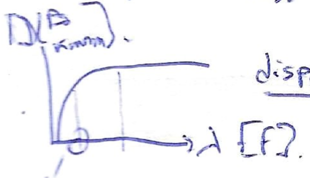


Figure 6: Knife-edge diffraction geometry. The point T denotes the transmitter and R the receiver, with an infinite knife-edge obstruction blocking the life-of-sight path.



higher wavelength less losss.



dispersion

work at lower wavelength for less dispersion
~~Longer dispersion~~ phase.

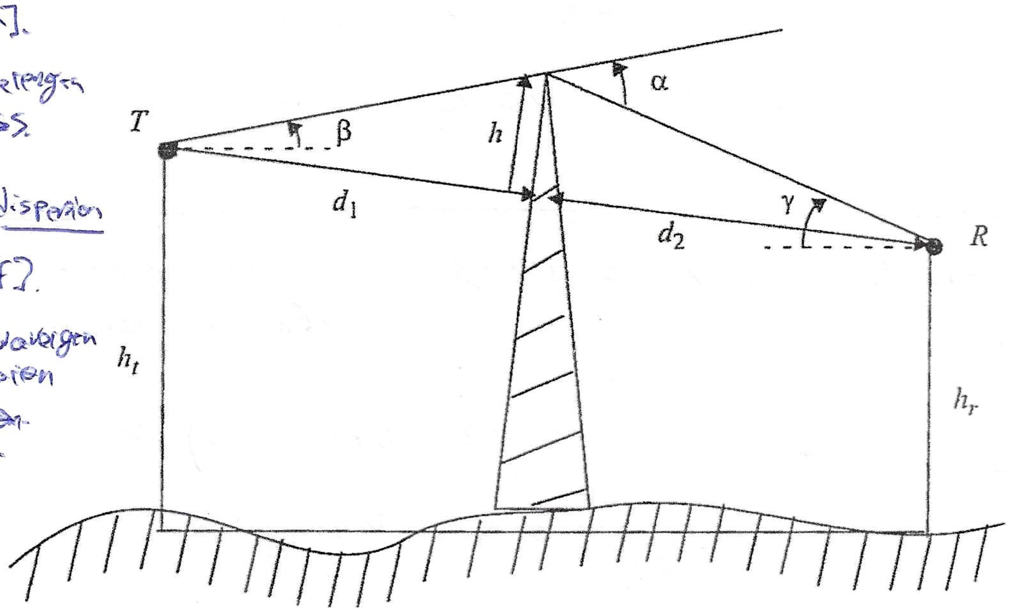


Figure 7: Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if α and β are small and $h << d_1$ and d_2, then h and h' are virtually identical.

Physical Layer

• $P_{Rx} = P_{Tx} - L\,[\mathrm{dB/km}] \cdot d\,[\mathrm{km}]$.

$\left(\frac{1}{d^2}\right)$ → direct line of sight.

$\left(\frac{1}{d^4}\right)$ → M.P.

• speed ~~fast~~ in medium
$v = \frac{c}{n} \left[\frac{m}{s}\right]$

~~Calc~~ • $\theta_1 = 0°$ → $L = 10km$ direct line of sight

• $\theta_2 = 2°$ → $L = \frac{10km}{\cos(\theta_2)} = 10006.1\ m$

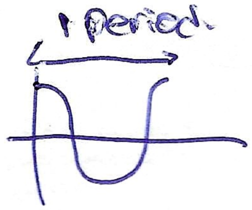
• $T_1 - T_0$ = delay

$\frac{L_1}{v} - \frac{L_0}{v}$

• Problem caused for multimode fibers.
→ delay causes interference.

• problem for fast modulation.

• Most fibers are single mode



- bits/sec per hz → spectral efficiency of bit rate R → $R = nf_s$ → $\eta = \frac{R}{B}$
- symbols/sec per hz → spectral efficiency of baud rate ~~R = ηB → D = R/η~~ → $\eta = \frac{D}{B}$

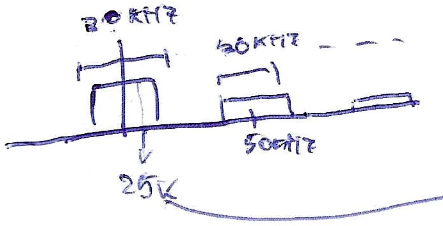
$R = lD$

$\boxed{l = \%_0 \cdot l}$

! doublesided ($N_0/2$) $P_{noise} = \frac{N_0}{2} \cdot 2 \cdot f_{null}$
singlesided (N_0) $P_{noise} = N_0 \cdot f_{null}$] → ~~No~~ fan.

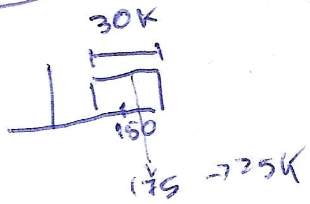
↳ allways treat as single sid

- sampling circuit $f_s = 50KHz$ → gating → filter 30KHz
↳ produce 25KHz exact



$\sin(2\pi \cdot 25K \cdot t + \pi/3)$ ✓

$\sin(2\pi \cdot 175Kt\ \frac{\pi}{8})$ ✓



$\sin(\pi + 12 \cdot 50 \cdot t + \frac{\pi}{2})$

↓

$2\pi \cdot 625K$ ✓

$\sin(\pi \cdot 25K \cdot t)$ ✗

↓

12.5K