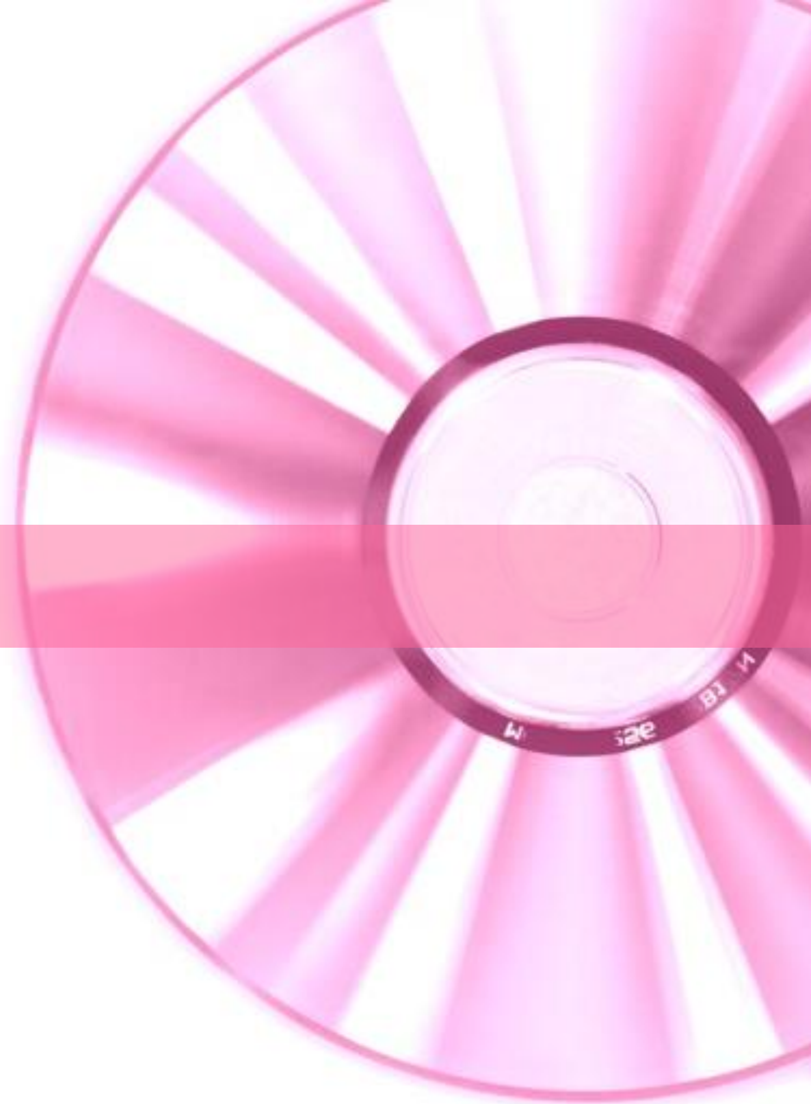
Photonics



Gaussian beam optics

Diffraction of a coherent beam

Gaussian beams and lens systems

Coherence and M^2 factor

Monochromatic light beam (in 2D)

- Helmholtz equation:

$$\nabla^2 U + k^2 U = 0$$

Solution: inhomogeneous plane wave

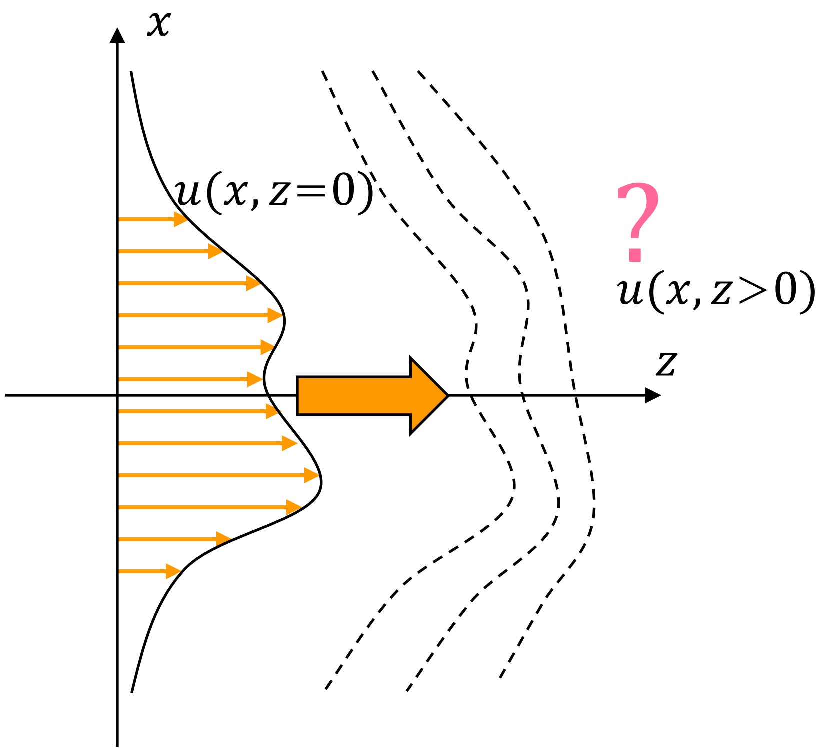
$$U(x,z) = A(x,z)\mathrm{e}^{-jkz}$$

- Paraxial approximation:

$$\left|\frac{\partial^2 A}{\mathrm{d}z^2}\right| \ll k \left|\frac{\partial A}{\mathrm{d}z}\right|$$

Paraxial Helmholtz equation:

$$\frac{\partial^2 A}{\mathrm{d}x^2} - 2jk\frac{\partial A}{\mathrm{d}z} = 0$$



Change of the amplitude profile during propagation = diffraction

Gaussian beam

- Paraxial Helmholtz equation:

$$\frac{\partial^2 A}{\mathrm{d}x^2} - 2jk\frac{\partial A}{\mathrm{d}z} = 0$$

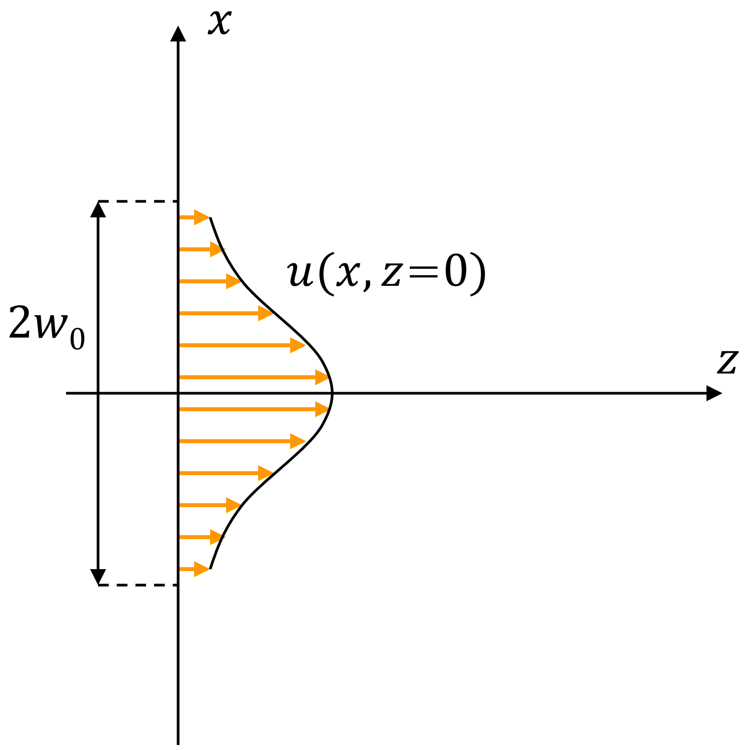
- Possible solution:
  Gaussian amplitude profile at $z = 0$

$$A(x, z = 0) = \mathrm{e}^{-\frac{x^2}{w_0^2}}$$

  - beam width $2w_0$
  - Perfect coherent beam
- Assumption for a solution at $z > 0$

$$A(x, z) = \mathrm{e}^{-j\left[p(z)+k\frac{x^2}{2q(z)}\right]}$$

  - beam remains Gaussian during propagation
  - $p(z)$: complex phase shift along the z-axis
  - $q(z)$: phase curvature and transversal amplitude profile



Diffraction of a Gaussian beam (1)

- Substitution of a Gaussian beam in the Helmholtz equation:

$$2k\left(\frac{\mathrm{d}p}{\mathrm{d}z}+\frac{j}{2q}\right)+\left(\frac{kx}{q}\right)^2\left(1-\frac{\mathrm{d}q}{\mathrm{d}z}\right)=0$$

- Should hold for all x and all z:

$$\begin{cases}\dfrac{\mathrm{d}q}{\mathrm{d}z}=1\\ \dfrac{\mathrm{d}q}{\mathrm{d}z}=-\dfrac{j}{2q}\end{cases}$$

with boundary conditions ($z=0$):

$$\begin{cases}q(0)=j\dfrac{kw_0^2}{2}\\ p(0)=0\end{cases}$$

- Integration gives:

$$q(z)=z+j\frac{kw_0^2}{2}$$

$$jp(z)=-\ln\left(\sqrt{\frac{w_0}{w(z)}}\right)-\frac{j}{2}\arctan\left(\frac{z}{b_0}\right)$$

$$A(x,z=0)=\mathrm{e}^{-\frac{x^2}{w_0^2}}$$

$$A(x,z)=\mathrm{e}^{-j\left[p(z)+k\frac{x^2}{2q(z)}\right]}$$

Diffraction of a Gaussian beam (2)

- Solution for $q(z)$ gives:

$$q(z) = z + j\frac{kw_0^2}{2}$$

- We can split $\dfrac{1}{q(z)}$ into a real and imaginary part:

$$\frac{1}{q(z)} \triangleq \frac{1}{R(z)} - j\frac{2}{kw^2(z)}$$

  - $R(z)$: radius of curvature of the phase front (paraxial approximation)
  - $w(z)$: half $1/\mathrm{e}^2$ width of a Gaussian beam at z

$$R(z) = z\left(1 + \frac{b_0^2}{z^2}\right)$$

$$w(z) = w_0\sqrt{1 + \frac{z^2}{b_0^2}}$$

where $b_0 = \dfrac{kw_0^2}{2} = \dfrac{\pi w_0^2}{\lambda}$: the Rayleigh range

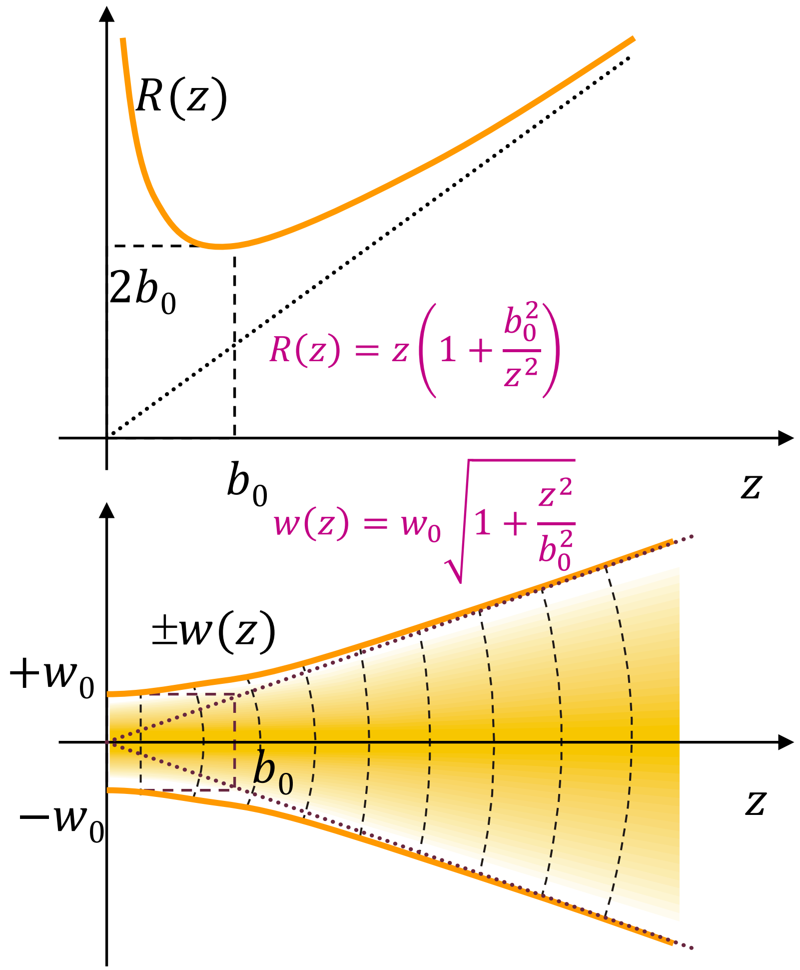
Diffraction of a Gaussian beam (3)

- Solution of the wave equation:

$$U(x,z) = A(x,z)\mathrm{e}^{-jkz}$$

$$A(x,z) = \sqrt{\frac{w_0}{w(z)}}\ \mathrm{e}^{-\frac{x^2}{w^2(z)}}\ \mathrm{e}^{-j\frac{kx^2}{2R(z)}}\ \mathrm{e}^{\frac{j}{2}\arctan\frac{z}{b_0}}$$

- Radius of curvature $R(z)$:
  - For $z = 0$: $R(0) = \infty$
  - Minimum: $R(b_0) = 2b_0$
  - For $z \to \infty$: $R(z) = z$
- Width $w(z)$:
  - Always increases: $w(z) > w_0$
  - For $z < b_0$: $w(z) \simeq w_0$
  - For $z > b_0$: $w(z) \simeq z\frac{w_0}{b_0}$



The Rayleigh range

- b_0 is the *Rayleigh range*
- $z < b_0$
  - Quasi constant width

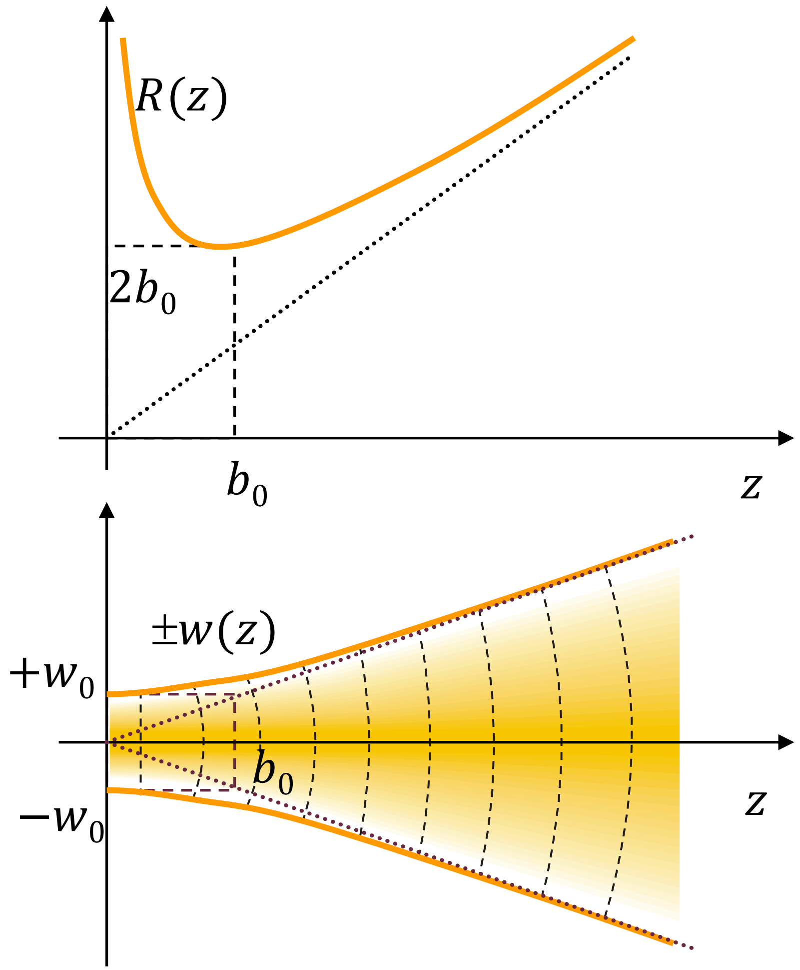
$$w(b_0) = \sqrt{2}w_0$$

  - Wave front begins to bend
- $z > b_0$
  - Spherical fan out
    θ = half divergence angle

$$\theta = \pm \arctan \frac{w_0}{b_0}$$

$$\simeq \pm \frac{w_0}{b_0} = \pm \frac{2}{kw_0} = \pm \frac{\lambda}{\pi w_0}$$

  - Evolution to spherical wave front with center in $z = 0$



Transition from 2D to 3D

- Solution of the wave equation in 2D:

$$U(x,z) = A(x,z)\mathrm{e}^{-jkz}$$

$$A(x,z) = \sqrt{\frac{w_0}{w(z)}}\,\mathrm{e}^{-\frac{x^2}{w^2(z)}}\;\mathrm{e}^{-j\frac{kx^2}{2R(z)}}\;\mathrm{e}^{\frac{j}{2}\arctan\frac{z}{b_0}}$$

- Solution in 3D (for a circular beam)

$$U(x,y,z) = A(x,y,z)\mathrm{e}^{-jkz}$$

$$A(x,y,z) = \sqrt{\frac{w_0}{w(z)}}\,\mathrm{e}^{-\frac{x^2}{w^2(z)}}\;\mathrm{e}^{-j\frac{kx^2}{2R(z)}}\sqrt{\frac{w_0}{w(z)}}\,\mathrm{e}^{-\frac{y^2}{w^2(z)}}\;\mathrm{e}^{-j\frac{ky^2}{2R(z)}}\;\mathrm{e}^{\frac{j}{2}\arctan\frac{z}{b_0}}$$

$$= \frac{w_0}{w(z)}\mathrm{e}^{-\frac{x^2+y^2}{w^2(z)}}\;\mathrm{e}^{-jk\frac{x^2+y^2}{2R(z)}}\;\mathrm{e}^{\frac{j}{2}\arctan\frac{z}{b_0}}$$

Gaussian beams and lens systems

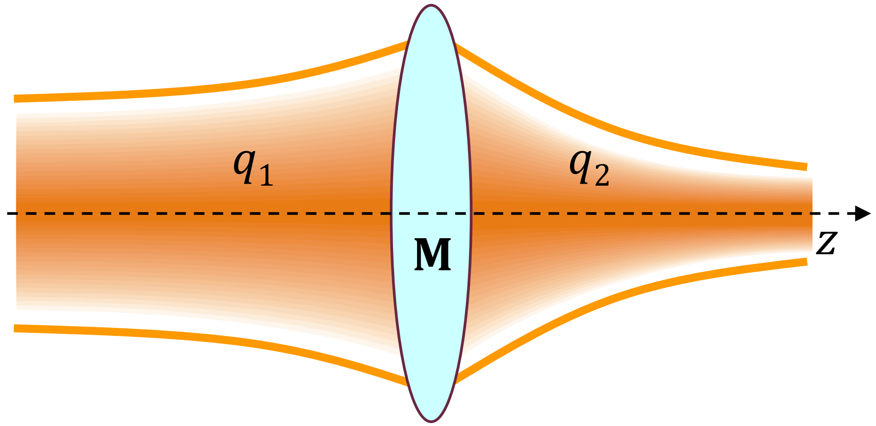
- Gaussian beam through a thin lens:
  - again Gaussian beam
  - different phase curvature

- Gaussian beam after an arbitrary lens system
  - Incident beam: q_1
  - Lens system matrix $\mathbf{M}$

$$\mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

  - Outgoing beam q_2

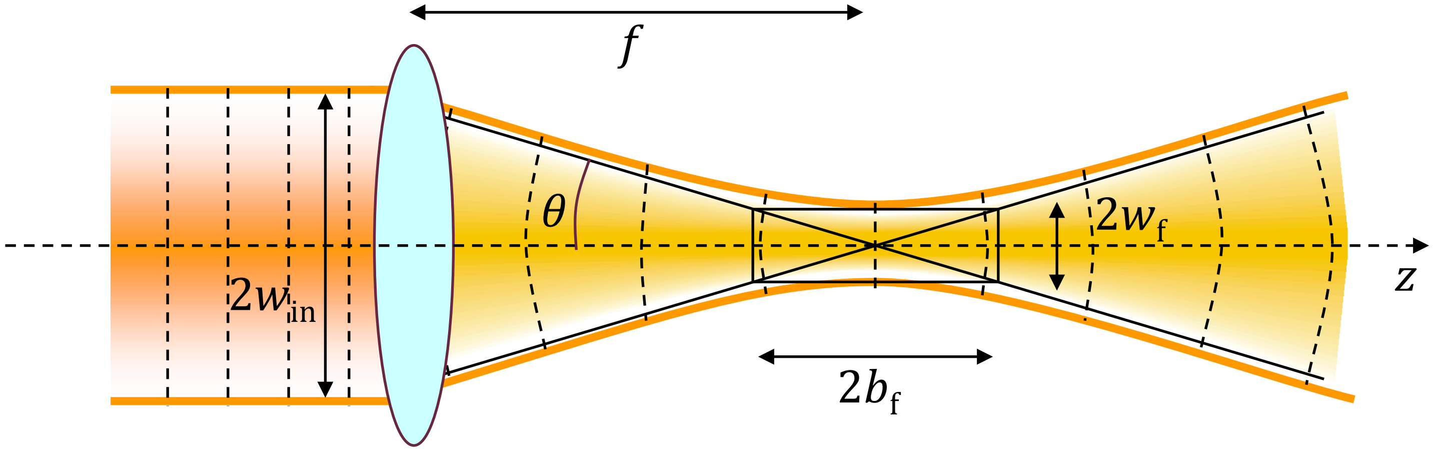
$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$



Focusing a Gaussian beam (1)

- Incident laser beam:
  - plane phase front
  - width w_{in}
- Outgoing beam
  - converges with an angle $\theta = \dfrac{w_{\mathrm{in}}}{f}$
  - θ is correlated with the w_{f} in the focus plane $\theta = \dfrac{\lambda}{\pi w_{\mathrm{f}}}$

$$\theta = \frac{\lambda}{\pi w_0}$$



Focusing a Gaussian beam (2)

- The width in the focus plane: $$2w_\mathrm{f} = \frac{2\lambda f}{\pi w_\mathrm{in}}$$
  - small spot: small focal length $\rightarrow$ strong refraction
  - If the beam is as wide as the lens:

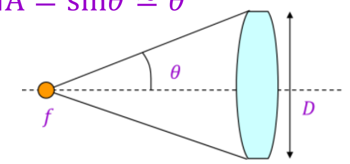
$$\theta = \mathrm{NA}_\mathrm{lens}$$

$$2w_\mathrm{f} = \frac{2\lambda}{\pi \mathrm{NA}_\mathrm{lens}} = \frac{0.64\lambda}{\mathrm{NA}}$$

- Depth of field $2b_\mathrm{f}$ (the Rayleigh range): $2b_\mathrm{f} = k w_\mathrm{f}^2$

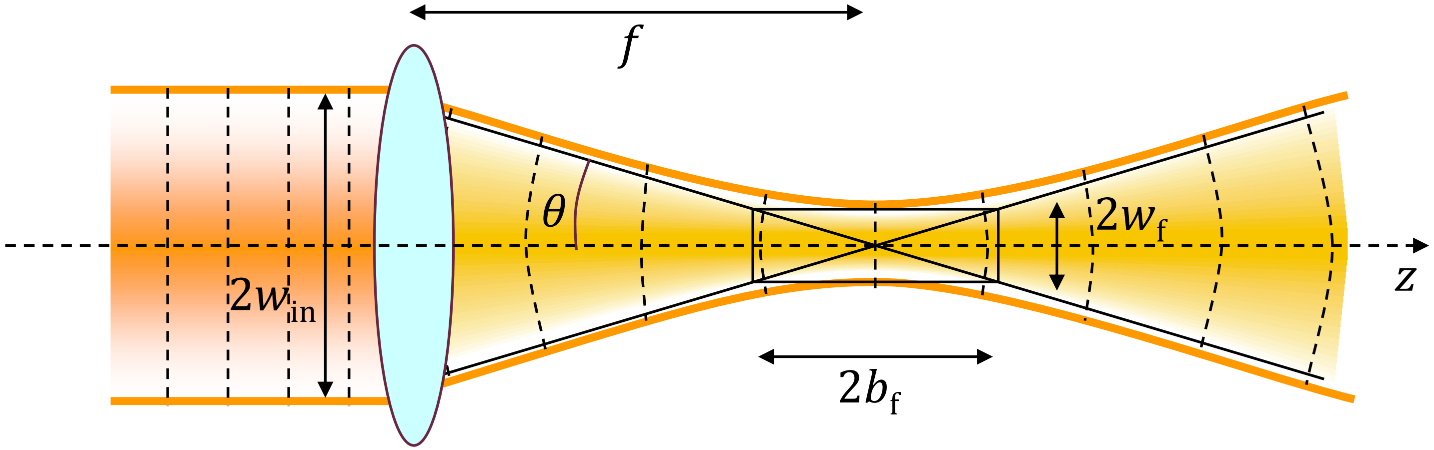
- Numerical aperture

$\mathrm{NA} = \sin\theta \simeq \theta$



$$\theta = \frac{w_\mathrm{in}}{f}$$

$$b_0 = \frac{\pi w_0^2}{\lambda}$$



Focusing a Gaussian beam (3)

$$\boxed{2w_{\mathrm{f}} = \frac{0.64\lambda}{\mathrm{NA}} \simeq \lambda}$$

- It is impossible to focus a beam by a focusing lens system into a spot that is (substantially) smaller than the wavelength.
- To obtain smallest possible spot a lens with largest possible NA should be used
- It holds not only for Gaussian beams, but is valid in general

Focusing a Gaussian beam (3)

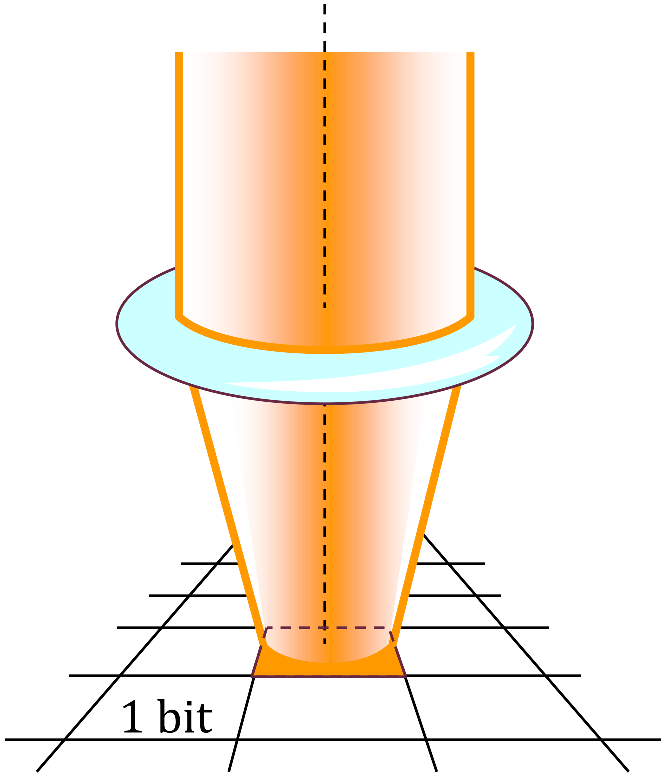
15 mm inner diameter
12 cm outer diameter
Beethoven's 9th

- Application: Bit density on a CD
- 1 CD: capacity $= 750\text{ MB} \times 17/8 \times 36/24$ (error correction)
  $= 1.9 \cdot 10^{10}$ bits
- 1 CD: area $= \pi \times (5.8^2 - 2.5^2)\text{ cm}^2$
  $= 86\text{ cm}^2 = 86 \cdot 10^8\ \mu\text{m}^2$

Bit density:

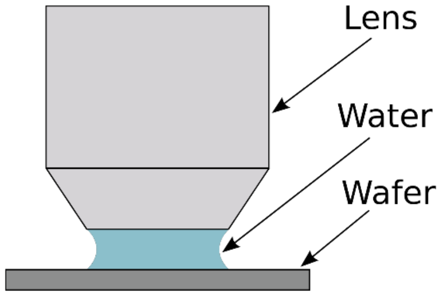
$$\frac{1.9 \cdot 10^{10}\text{ bits}}{86 \cdot 10^8\ \mu\text{m}^2} = 2.2\,\frac{\text{bits}}{\mu\text{m}^2}$$

- Physical limit:
  - wavelength $\lambda = 0.78\ \mu\text{m}$
  - 1 bit $\simeq$ circle with diameter λ
  - Bit density: ≈ 2.1 bit / μm^2
  - The only way to increase the density: decrease the wavelength

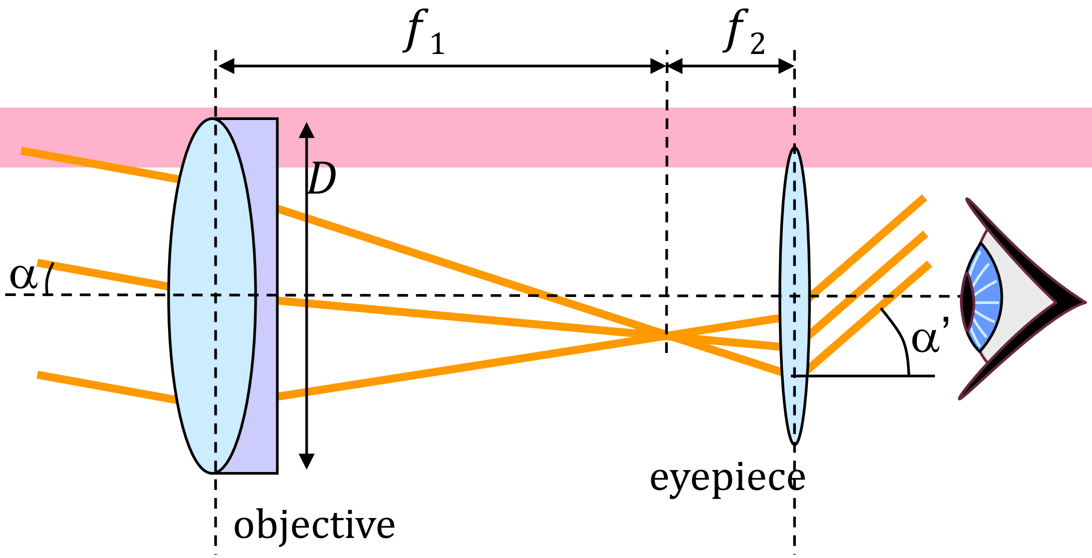


Pattern definition at IC production

- Imaging of a mask on a wafer with 4 × reduction
- Source: 193 nm excimer laser (deep UV)
- NA close to 1
- Smallest feature: 0.64 λ=120 nm
- In practice 90 nm achieved!!!
- Improvements:
  - Immersion lithography: water between the lens and a wafer: smallest feature: $0.64\ \lambda/n$
  - Source with a shorter wavelength: EUV (13.5 nm)



Telescopes



- Resolution at the image plane after objective: $\boxed{2w_\mathrm{f} \approx 0.64\lambda/\mathrm{NA}}$
- It corresponds to the angular resolution in the object plane
- Diameter of the objective lens is the determining factor for the resolution (and for the light intensity as well)

$$\boxed{\Delta\alpha \approx 0.64\frac{\lambda/\mathrm{NA}}{f_1} \approx 1.28\frac{\lambda}{D}}$$

M² factor

- Gaussian beams: $\theta = \dfrac{\lambda}{\pi w_0}$
- Therefore: $\pi\theta \dfrac{w_0}{\lambda} = 1$
- Non-Gaussian beams
  - Amplitude profile is not Gaussian
  - And/or phase profile is not parabolic: $\pi\theta \dfrac{w_0}{\lambda} \geq 1$

- Definition: $M^2 = \pi\theta \dfrac{w_0}{\lambda} \geq 1$
  - M^2 is a measure for a beam quality
  - $M^2 = 1$: “diffraction-limited beam” (it means: θ is minimal for a given beam width)