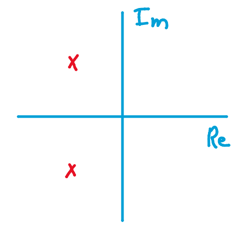
3 - Frequency response - Type 3 building blocks, nonminimum-phase systems & sketching procedure

Frequency response analysis

- 3rd class / Class 3: Second order complex conjugate pair of poles and zeros
- Frequency response of RHP zero
- Drawing rules bode plot



Class 3

$$\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^{\pm 1}$$

Damping coefficient (ζ)

natural Frequency (ω_n)

Break point $\frac{1}{\omega_n}$

Behaviour:

- $\omega/\omega_n \gg 1$, ± 40dB/decade
- $\omega/\omega_n \cong 1$, resonant peak M_r
- Phase lead / lag 180°, 90° at ω_n

$$\frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}$$ (Complex conjugate pole pair)

Re: $\dfrac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}$

Im: $\dfrac{-2\zeta\left(\frac{\omega}{\omega_n}\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}$

Three ranges:

1) $\omega \ll \omega_n$
2) $\omega \gg \omega_n$
3) $\omega = \omega_n$

$\omega \ll \omega_n$

$\frac{\omega}{\omega_n} \to 0$ is very small

Re: $\dfrac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2} \cong \dfrac{1}{1+0}$

Im: $\dfrac{-2\zeta\left(\frac{\omega}{\omega_n}\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2} \cong \dfrac{0}{1+0}$

Gain: $\sqrt{1^2+0^2} = 1 = 0$dB

phase: $\arctan(0/1) = 0^\circ$

$\underline{\omega \gg \omega_n}$

$$1-\left(\frac{\omega}{\omega_n}\right)^2 \cong -\left(\frac{\omega}{\omega_n}\right)^2$$

Re: $$\frac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2} \cong \frac{-\left(\frac{\omega}{\omega_n}\right)^2}{\left(-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\underbrace{\left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}} \cong \frac{-\left(\frac{\omega}{\omega_n}\right)^2}{\left(\left(\frac{\omega}{\omega_n}\right)^2\right)^2} = -\left(\frac{\omega}{\omega_n}\right)^{-2}$$

small compared to

Im: $$\frac{-2\zeta\left(\frac{\omega}{\omega_n}\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2} \cong \frac{-2\zeta}{\left(\frac{\omega}{\omega_n}\right)^3}$$

Gain: $$\left[\left(1/(\omega/\omega_n)\right)^2+\underbrace{\left(-2\zeta/(\omega/\omega_n)^3\right)^2}_{\text{small compared to}}\right]^{1/2} \approx \left(\left(\frac{\omega}{\omega_n}\right)^{-4}\right)^{1/2} = \left(\frac{\omega}{\omega_n}\right)^{-2}$$

In decibels: $20\log_{10}\left(\frac{\omega}{\omega_n}\right)^{\pm 2} = \pm 2 \cdot 20\log_{10}(\omega/\omega_n)$

Phase: $$\arctan\left(\frac{-2\zeta\left(\frac{\omega}{\omega_n}\right)^{-3}}{\left(\frac{\omega}{\omega_n}\right)^{-2}}\right) = \arctan\left(-2\zeta\left(\frac{\omega}{\omega_n}\right)^{-1}\right)$$

$= 0°$ sign information ignored, $\underline{\pm 180°}$

$\underline{\omega = \omega_n}$

$$\frac{\omega_0}{\omega_n} = 1$$

	Zeros	Poles
$\zeta < 1/2$	$-$	$+$
$\zeta > 1/2$	$+$	$-$

$+$: amplification

$-$: attenuation

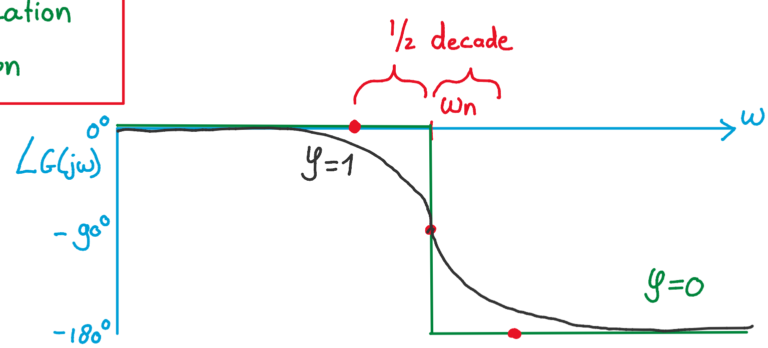
Phase: $\arctan\left(\frac{Im}{\underbrace{Re}_{\text{Very small}}}\right) = \infty$

Re: $$\frac{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2} = 0$$

Im: $$\frac{-2\zeta\left(\frac{\omega}{\omega_n}\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2} = \frac{-1}{2\zeta}$$

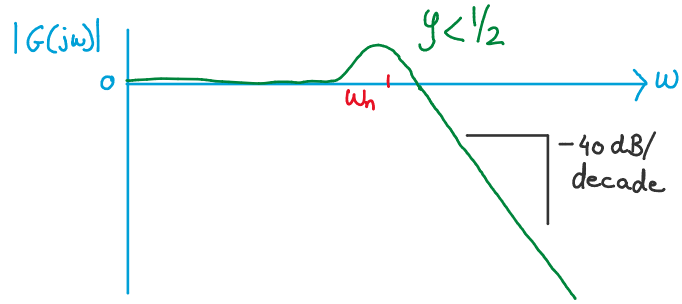
Gain: $\sqrt{\left(\frac{-1}{2\zeta}\right)^2} = \frac{1}{2\zeta}$, In decibels $\pm 20\log_{10}(2\zeta)$

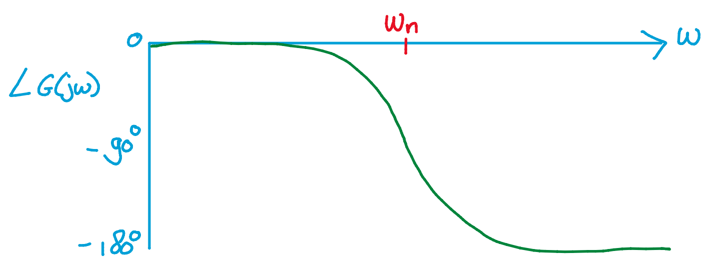
$$\begin{cases} \zeta = 1/2 & \log_{10}(2\zeta) = 0 \\ \zeta < 1/2 & \log_{10}(2\zeta) \text{ negative} \\ \zeta > 1/2 & \log_{10}(2\zeta) \text{ positive} \end{cases}$$



Class 3 bode plot

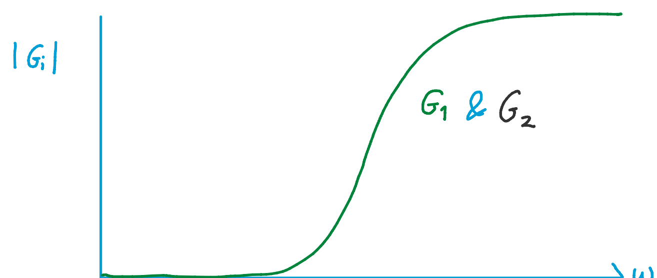
Complex conjugate pole pair





RHP zeros

$$G_1 = 10\,\frac{s+1}{s+10}$$
$$G_2 = 10\,\frac{s-1}{s+10}$$

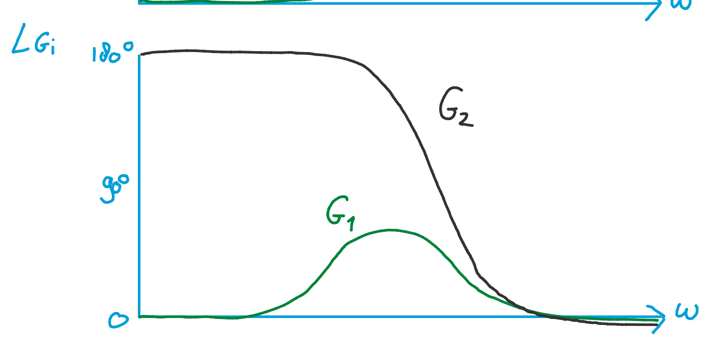


RHP zero

Gain: 20 dB/decade

Phase: −90°

$$|G_1(j\omega)| = |G_2(j\omega)|$$



Design procedure bode plots

1.) Write TF in bode form, identify building blocks and their break point frequencies
2.) Determine n in $K_0 (j\omega_0)^n$, plot low frequency asymptote through K_0
    Bode gain-phase: gain $n \times 20$ dB/decade phase $n \times 90^\circ$
3.) Extend asymptote untill first break point. Change slope based on pole/zero at break points
    Continue in ascending order
4.) At the break points, magnitude deviates 3 dB (1st order) or ζ-dependent (2nd order)
5.) Plot low frequency asymptote $n \times 90^\circ$
6.) Phase change of $\pm 90^\circ$ or $\pm 180^\circ$ at break points
7.) Draw asymptotes for phase and intersect them
8.) Account for 11° phase difference at asymptote intersection

Summary

- Frequency response of 2nd order system as function of ζ

$$\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1 \right]^{\pm 1}$$

- Zero in LHP vs RHP
- Overall design procedure
    Transfer function → Bode plot → System properties