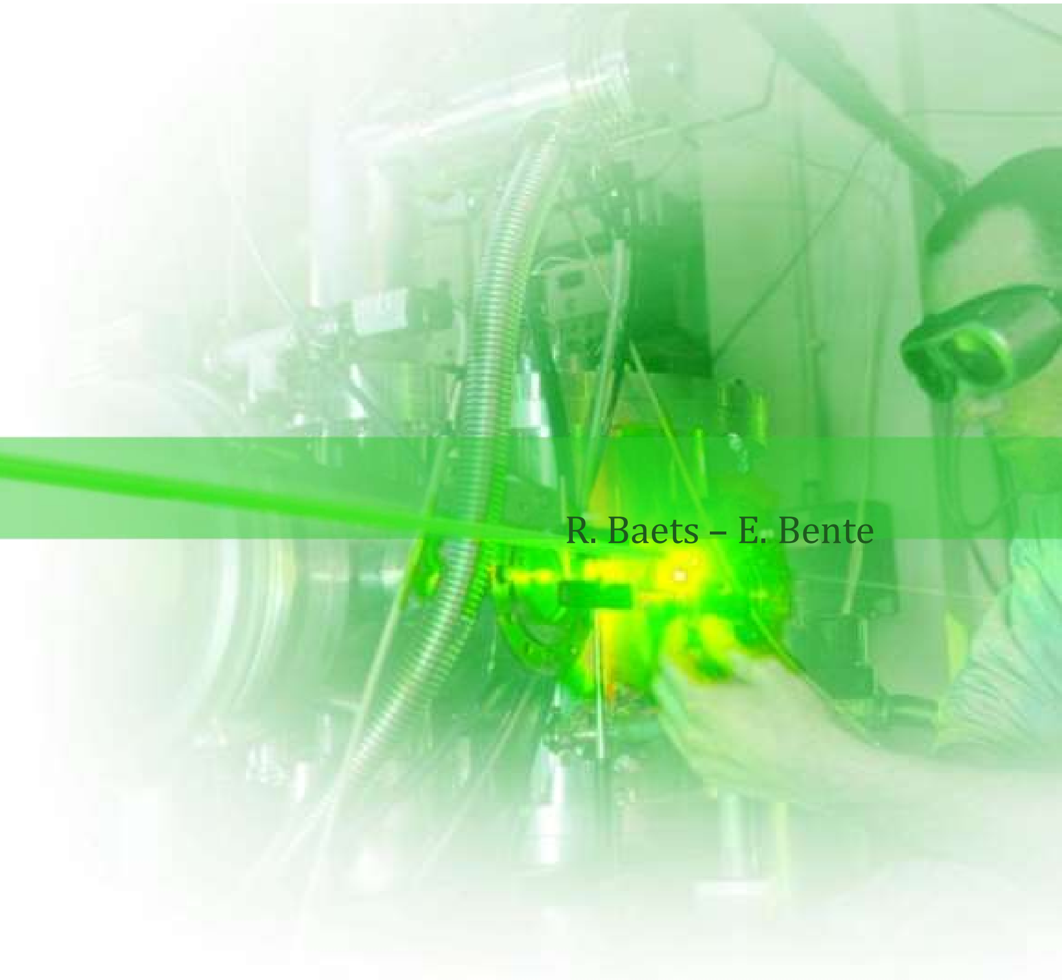
Photonics

R. Baets – E. Bente



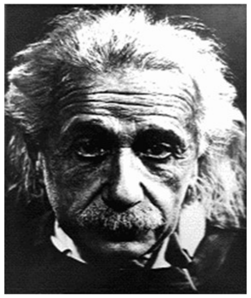
Lasers – Part A

Stimulated emission

Gain and amplification

The LASER

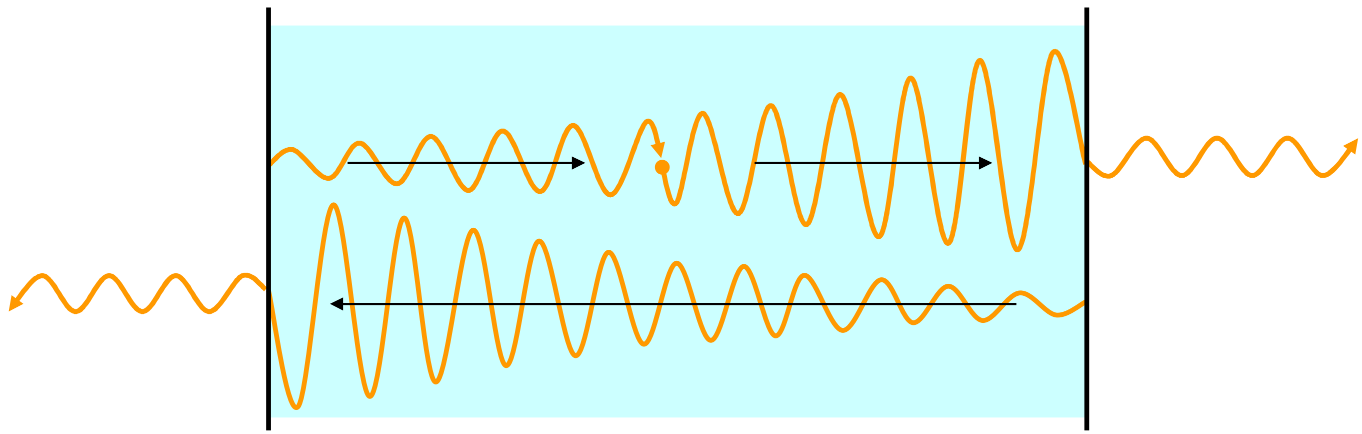
- **1917 :** Einstein postulates the concept of photons and stimulated emission
- **1954 :** First microwave laser (maser) by Charles Townes and Arthur Schawlow (Nobelprize in 1964)
- **1960 :** First optical laser (ruby laser) by Theodore Maiman
- **1962:** First semiconductor laser (3 independent teams in US and USSR)
- **Today:** From "a solution looking for a problem“ in 1960 to an irreplaceable tool in telecom and ICT, industry, medical world, measurement instruments, fundamental science



Laser cavity

Laser-oscillator = amplification+ feedback

- What is a laser oscillation?
- A harmonic (sinusoidal at optical frequencies) solution to Maxwell's equation (without external light source) taking into account the boundary conditions imposed by the laser cavity



This chapter

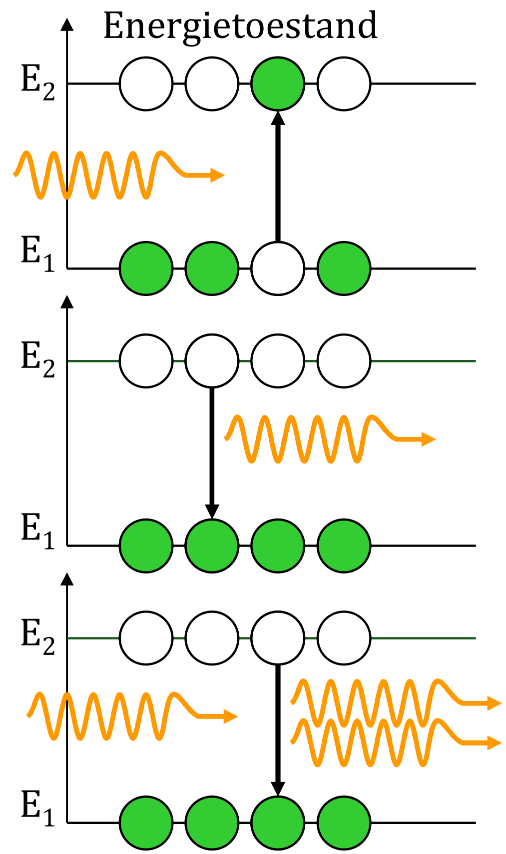
- Gain (amplification and attenuation)
  - Units of gain and loss
- Resonance in laser resonators
  - linear resonators – mode structure
- Properties of laser light
- Examples of lasers

- Gain (amplification and attenuation)

Emission and absorption (2 levels)

- Transition between atomic or molecular levels: emission or absorption of a photon

- Absorption:
  - Photon hits system
  - System in higher energy state
- Spontaneous emission
  - System falls back to ground state
  - Emission of photon
- Stimulated emission
  - Photon hits system
  - System falls back to ground state
  - Emission of identical photon



Einstein relations

- Absorption:
  - Proportional to N_1 (density of atoms in levels 1)
  - Proportional to photon energy density

$$\left.\frac{dN_2}{dt}\right|_{abs} = -\left.\frac{dN_1}{dt}\right|_{abs} = N_1 P_{abs} = B_{21}\rho(\nu_0)N_1$$

As defined (10.41)

- Spontaneous emission:
  - Proportional to N_2 (density of atoms in levels 2)

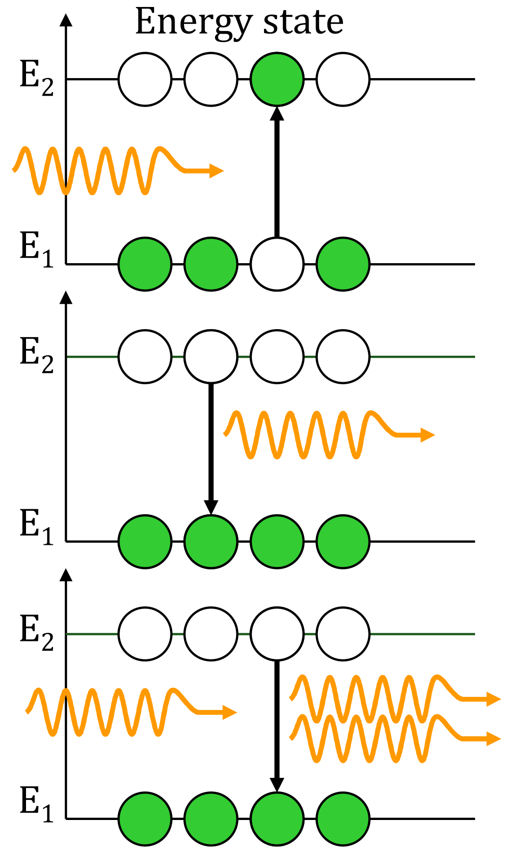
$$\left.\frac{dN_2}{dt}\right|_{sp} = -\left.\frac{dN_1}{dt}\right|_{sp} = -N_2 P_{sp} = -A_{21}N_2$$

As defined (10.29)

- Stimulated emission
  - Proportional to N_2
  - Proportional to photon energy dens.

$$\left.\frac{dN_2}{dt}\right|_{st} = -\left.\frac{dN_1}{dt}\right|_{st} = -N_2 P_{st} = -B_{21}\rho(\nu_0)N_2$$

As defined (10.36)



Solutions at thermal equilibrium

- Einstein relations $g_1 B_{12} = g_2 B_{21}$ $\quad A_{21}\frac{c^3}{8\pi h\nu^3} = B_{12}$

  (including degeneracy of levels 1 and 2)

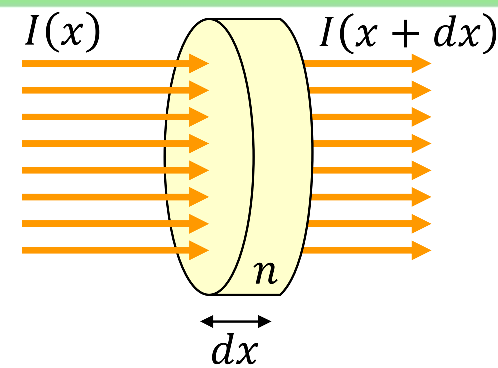
- Boltzmann distribution $$\frac{N_1}{N_2} = \frac{g_1}{g_2} exp\left(\frac{E_2 - E_1}{kT}\right) = \frac{g_1}{g_2} exp\left(\frac{h\nu}{kT}\right)$$

  Degeneracy

- At thermal equilibrium at room temperature
  - $k_B T \ll \hbar\omega$ at optical frequencies
  - $N_2 \ll N_1$: higher energy levels much less occupied
  - Absorption >> stimulated emission

- Amplification by stimulated emission?
  - Bring system out of thermal equilibrium – negative T

Absorption or amplification of a field



- Monochromatic EM field, intensity I(x) [W/m^2]

$$I(x) = \rho_\nu(x) v_g = N_{ph}(x)\ h\nu\ v_g \qquad v_g = \frac{c}{n}$$

  - Energy density ρ_ν [J/m^3]
  - Photon density N_{ph} [1/m^3] (all have freq ν)
  - Group velocity v_g (n = group index)

- Absorption:

$$\left.\frac{dN_{ph}}{dt}\right|_{abs} = \left.\frac{dN_1}{dt}\right|_{abs} = -B_{21}\rho_\nu N_1$$

Stimulated emission:

$$\left.\frac{dN_{ph}}{dt}\right|_{st} = -\left.\frac{dN_1}{dt}\right|_{st} = B_{21}\rho_\nu N_2$$

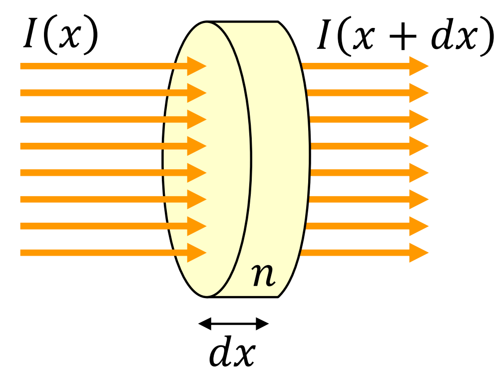
- Total: $$\frac{dN_{ph}}{dt} = \left.\frac{dN_{ph}}{dt}\right|_{st} + \left.\frac{dN_{ph}}{dt}\right|_{abs} = B_{21}\rho_\nu(N_2 - N_1) = B_{21}\frac{N_{ph}}{h\nu}(N_2 - N_1) \qquad B_{12} = B_{21}$$

- N_{ph} increases with time if $N_2 > N_1$, decreases if $N_2 < N_1$

Absorption or amplification of a field

$$\frac{dN_{ph}}{dt} = B_{21}\rho_\nu(N_2 - N_1) \qquad \rho_\nu(x) = \frac{I(x)}{v_g} \qquad N_{ph}(x) = \frac{\rho_\nu(x)}{h\nu} = I(x)\frac{1}{h\nu \cdot v_g}$$

- Change coordinate from t to x



$$\frac{dI}{dx} = \frac{dt}{dx}\frac{dI}{dt} = \frac{1}{v_g}\frac{dI}{dt} = h\nu\frac{dN_{ph}}{dt}$$

$$\frac{dI}{dx} = h\nu\rho_\nu(x)B_{21}(N_2 - N_1) = I(x)\frac{h\nu}{v_g}B_{21}(N_2 - N_1)$$

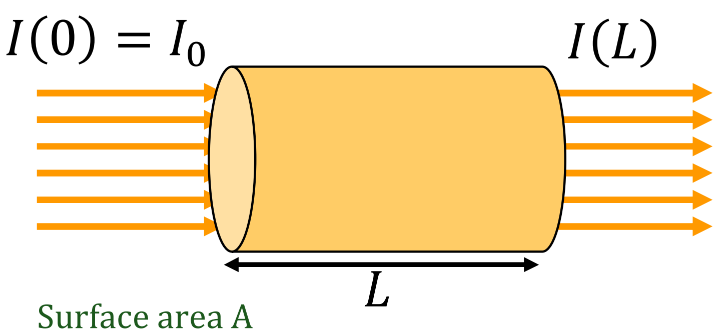
- Solution: $I(x) = I_0 e^{gx}$ where: $g \equiv \frac{h\nu}{v_g}B_{21}(N_2 - N_1)$

- g = gain= per unit distance [1/m]
- g > 0 intensity grows with x, g < 0 intensity decreases with x

Amplification and attenuation in uniform material

- Assume N_1 and N_2 are constant through the material. True if:
  - Intensity of the light is low

  and / or
  - For absorption: $\left.\frac{dN_2}{dt}\right|_{sp} \gg \left.\frac{dN_2}{dt}\right|_{abs}$ then N_2 will stay << N_1



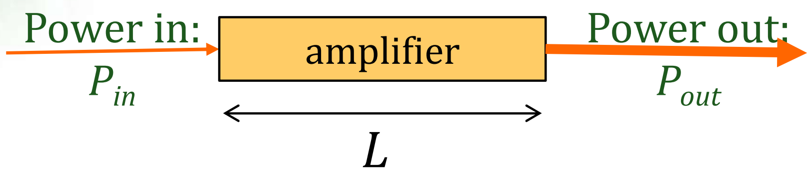
- Transmitted intensity

$$I(L) = I_0 e^{gL}$$

- amplification

$$G = \frac{I(L)}{I_0} = \frac{A \cdot I(L)}{A \cdot I_0} = \frac{P(L)}{P_0} = e^{gL}$$

The decibel (dB)



Amplification or gain

$$G = \frac{P_{out}}{P_{in}}$$

The dB unit is used to quantify power ratios, the amplification or gain can be expressed as:

$$G_{dB} \equiv 10 \cdot log \frac{P_{out}}{P_{in}}$$

- Power amplification in dB units

$$G = 10 \cdot log\left(\frac{I(L)}{I_0}\right) = 10 \cdot log(e^{gL}) = \frac{10}{ln10} \cdot gL = 4.343 \cdot g \cdot L$$

- Negative number for amplification in dB – intensity goes down (absorption)

Absorption and gain - effective cross-section

- Gain and loss properties are also often expressed as effective cross-sections for practical calculations for specific optical transitions and linewidth of the light source.

  - Gain $I(x) = I_0 e^{gx}$

- The effective gain cross-section σ_{eff} is defined as

$$g = (N_2 - N_1)\sigma_{eff} \qquad \text{or when } N_2 \gg N_1 \qquad g = N_2\sigma_{eff}$$

Units: $[\mathrm{m}^{-1}] = [\mathrm{m}^{-3}]\ [\mathrm{m}^2]$

- For **absorption**

$$\alpha = (N_1 - N_2)\sigma_{eff} \qquad \text{or when } N_1 \gg N_2 \qquad \alpha = N_1\sigma_{eff}$$

- Remember N_1 and N_2 are concentrations

Example 13.1

Example 13.1

Example 13.1 ctd.

How to achieve population inversion

Population inversion

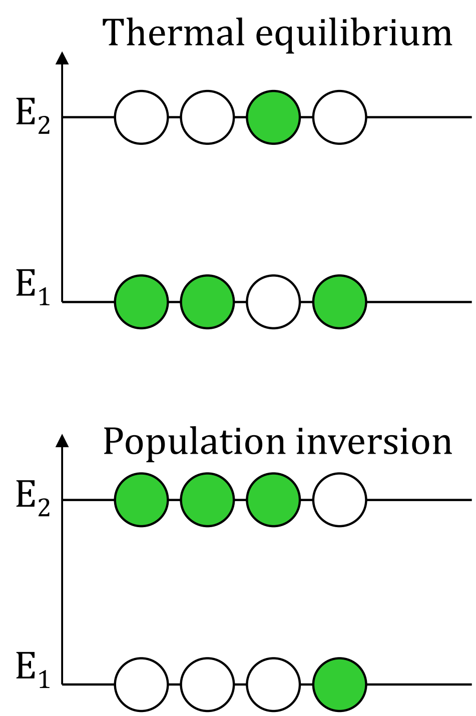
- Gain g: $$g = (N_2 - N_1)\sigma_{eff}$$

- Thermal equilibrium

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} exp\left(\frac{E_2 - E_1}{kT}\right) = \frac{g_1}{g_2} exp\left(\frac{h\nu}{kT}\right)$$

  - $N_1 > N_2$
  - $g < 0$: net absorption

- Population inversion
  - $N_1 < N_2$
  - $g > 0$: net gain
  - No thermal equilibrium



Pump

- Pump: bring system in population inversion through external energy source

- Pump mechanisms
  - Optical excitation
  - Gas discharge
  - Electron bombardment
  - Chemical energy
  - Current injection over a junction
    - Semiconductor lasers
    - Energy bands instead of energy levels

Pumping in a two-level system

- Pump with photons with energy

$$h\nu = E_2 - E_1$$

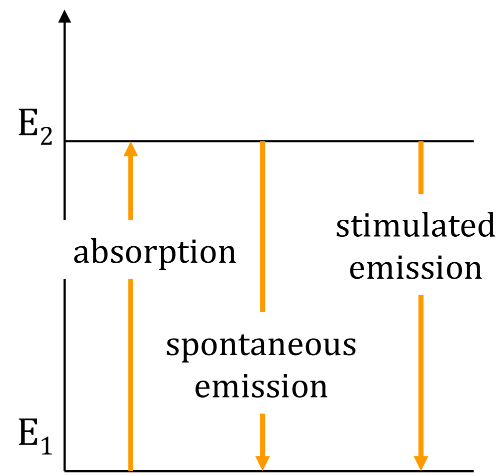
- Stationary regime
- absorption = emission

$$(N_2 - N_1)\sigma_{eff}\, N_{ph}\frac{c}{n} - \frac{N_2}{\tau_2} = 0$$

- and so

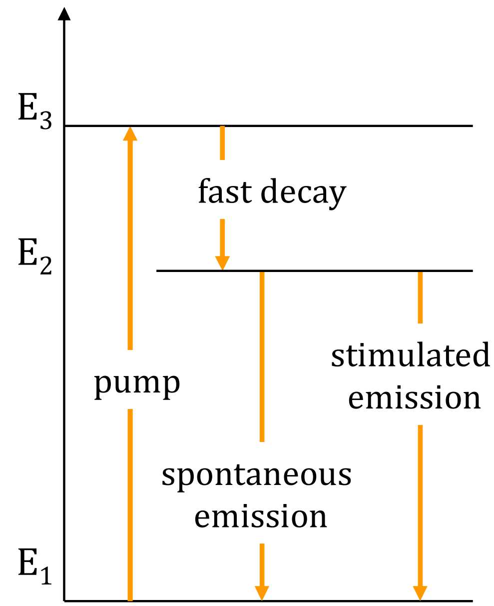
$$N_2 = \frac{N_1}{1 + \frac{n}{N_{ph}\tau_2\sigma_{eff}c}}$$

  - $N_2 < N_1$ Population inversion not possible



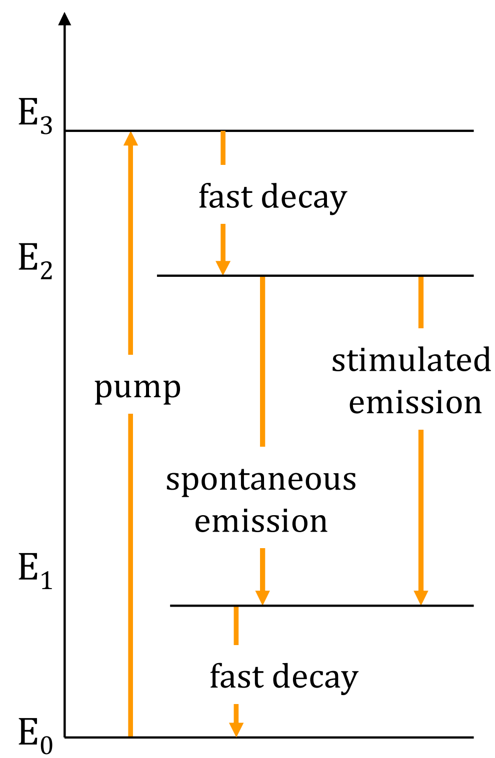
Pumping in a three level system

- Pumping to a higher energy level E_3
  - Short life time: little occupied
  - Spontaneous decay to E_2
  - No population inversion between E_3 en E_1: efficient pumping
- E_2: long lifetime
  - Little spontaneous emission
  - Population inversion between E_2 and E_1
- Drawback: need larger pump power to reduce N_1 below N_2
  - Pop. inversion requires low N_1
  - Possible absorption from N_1 to N_2

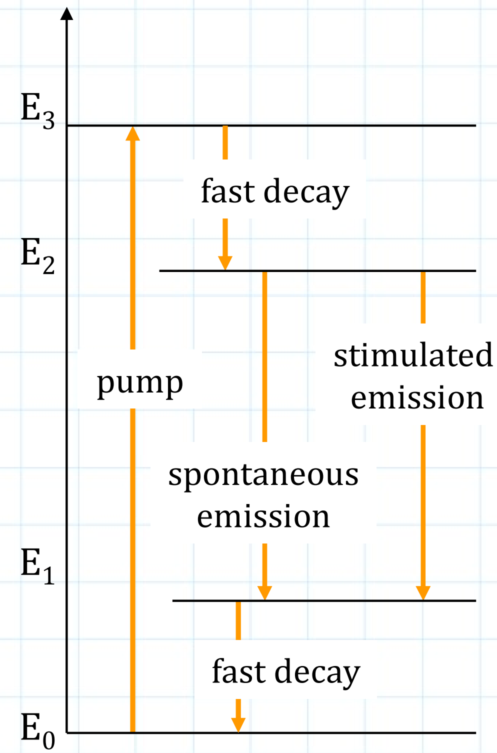


Pumping in a 4-level system

- Pumping from E_0 to E_3
  - Short lifetime: little occupied
  - Spontaneous decay to E_2
  - No population inversion between E_3 en E_0: efficient pumping
- E_2: long lifetime
  - Little spontaneous emission
  - Population inversion between E_2 en E_1
- E_1: short life time
  - Fast decay to E_0
- drawback: high pump energy (E_3-E_0): Energy difference $(E_3-E_0)-(E_2-E_1)$ is lost



Example 13.2


E$_3$
E$_2$
E$_1$
E$_0$
fast decay
pump
stimulated emission
spontaneous emission
fast decay

Example 13.2 ctd

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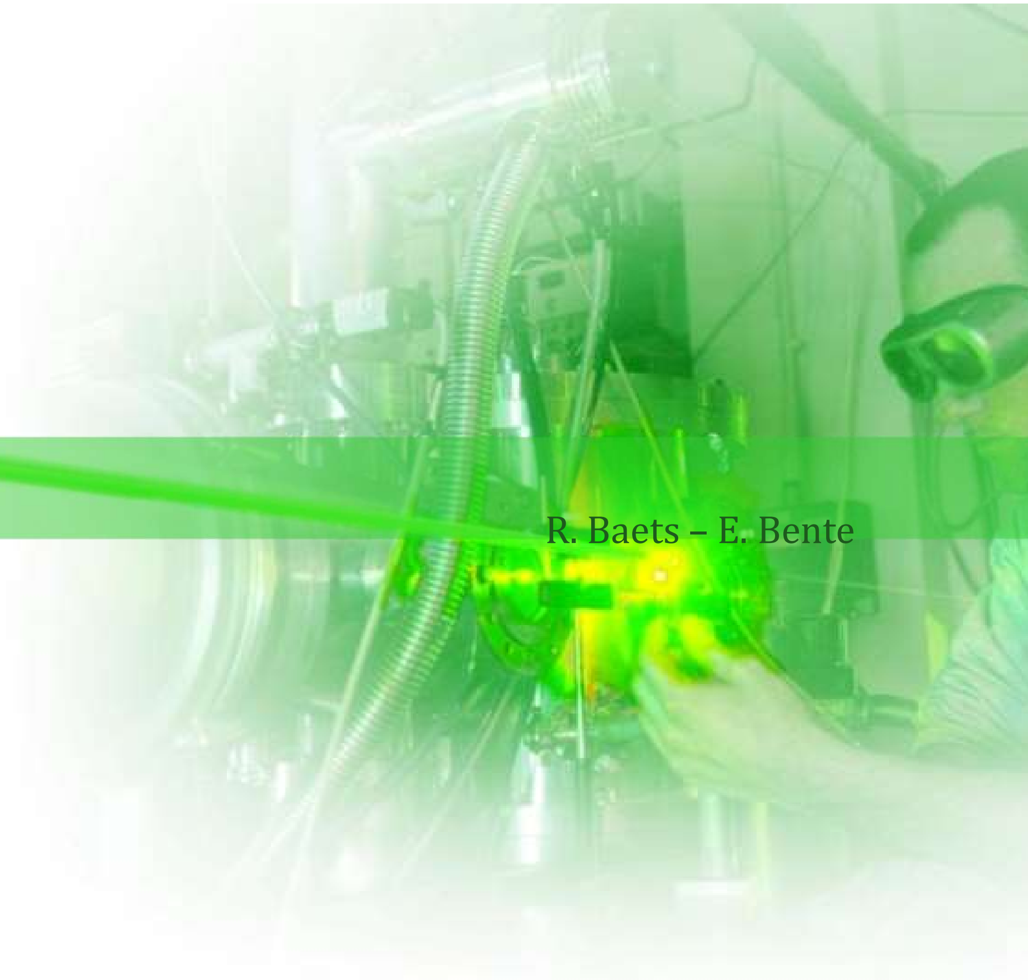
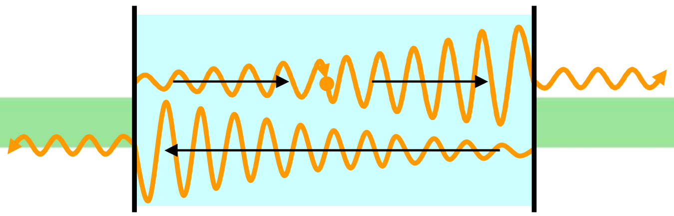


Lasers – Part B

Laser rate equations

Starting up a laser



1. Optical gain medium is pumped to population inversion
2. A fraction of the pumped particles give rise to spontaneous emission
3. The spontaneously emitting photons that propagate along the laser axis get amplified by stimulated emission
4. The mirrors result in a beam of light bouncing back and forth in the cavity
5. For certain wavelengths (frequencies), the spontaneous emission contributions (and their amplification) interfere constructively
6. This leads to stable laser operation with a loop gain of 1

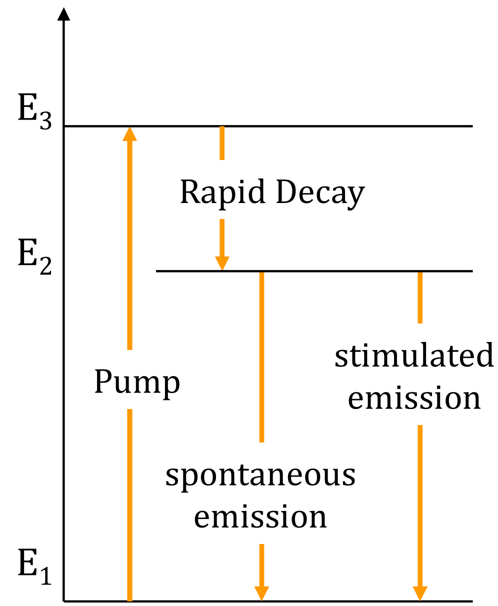
Rate equations (1)

- Rate equations:
  - Dynamics of the average particle density
  - No information about phase or frequency
- Pump rate R_p: from E_1 to E_2 over E_3
- Rate equations #photons, #systems in E_1 and E_2

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_2} + N_{ph} v_g \sigma (N_2 - N_1) - R_p \qquad v_g = \frac{c}{n}$$

$$\frac{dN_2}{dt} = R_p - \frac{N_2}{\tau_2} - N_{ph} v_g \sigma (N_2 - N_1)$$

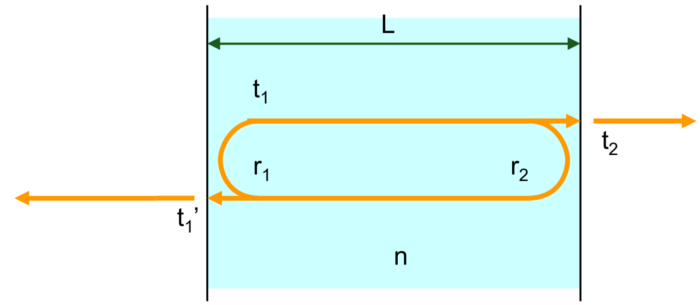
$$\frac{dN_{ph}}{dt} = N_{ph} v_g \sigma (N_2 - N_1) + \beta \frac{N_2}{\tau_2} - \frac{N_{ph}}{\tau_p}$$



  - β : the fraction of spontaneous emission that couples into the laser mode
  - τ_p : average lifetime of a photon in the cavity

Rate equations (2)

- Gain $g = (N_2 - N_1)\sigma$



- Given: total particle density: $N_1 + N_2 = N$
reduction to two equations:

$$\frac{dN_2}{dt} = R_p - \frac{N_2}{\tau_2} - v_g g N_{ph}$$

$$\frac{dN_{ph}}{dt} = v_g g N_{ph} + \beta \frac{N_2}{\tau_2} - \frac{N_{ph}}{\tau_p}$$

- Neglect spontaneous emission: $\frac{dN_{ph}}{dt} = N_{ph}\left(v_g g - \frac{1}{\tau_p}\right)$

- When $N_{ph} \ll N$, g can be considered constant

$$N_{ph}(t) = exp\left(\left(v_g g - \frac{1}{\tau_p}\right)t\right) \quad \text{using} \quad \frac{dN_{ph}}{dt} v_g = \frac{dN_{ph}}{dx} \quad N_{ph}(x) = exp\left(\left(g - \frac{1}{v_g \tau_p}\right)x\right)$$

- Passing through the cavity (length L): $loop\ gain = exp\left(\left(g - \frac{1}{v_g \tau_p}\right)2L\right)$

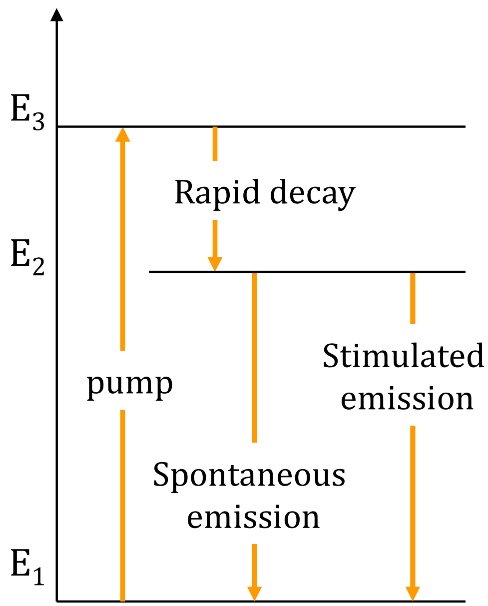
Stationary rate equations

- Stationary regime: d/dt = 0

$$0 = \frac{N_2}{\tau_2} + v_g g N_{ph} - R_p$$

$$0 = v_g g N_{ph} + \beta \frac{N_2}{\tau_2} - \frac{N_{ph}}{\tau_p}$$

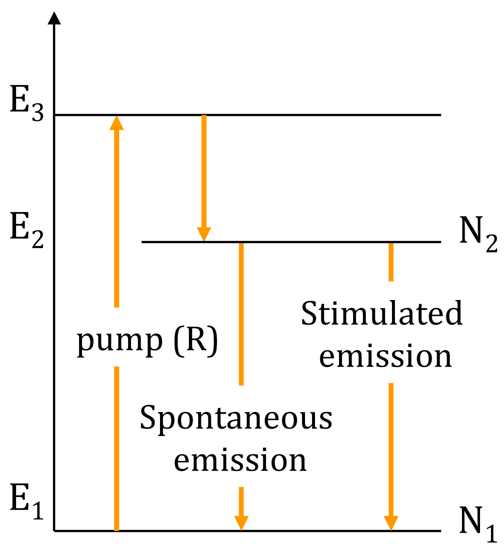
$$g = (2N_2 - N)\sigma$$

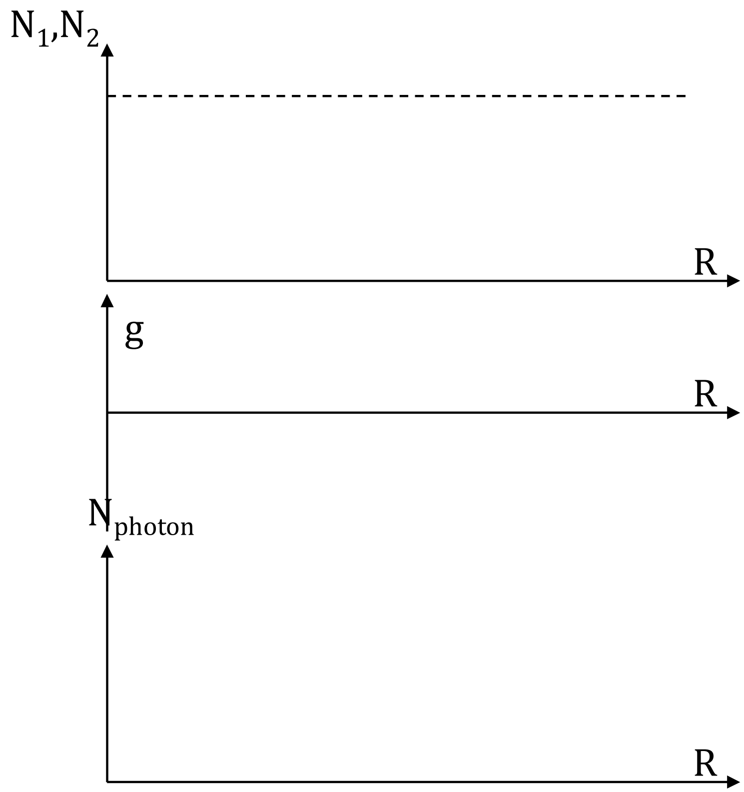


- Three regimes as a function of R
  - Low R: no population inversion
  - Transparency: population inversion starts
  - Oscillation threshold: loop gain = 1

Stationary rate equations

- How do the following quantities evolve when ramping up the pump rate (i.e. pump power, current etc) ?





Stationary rate equations

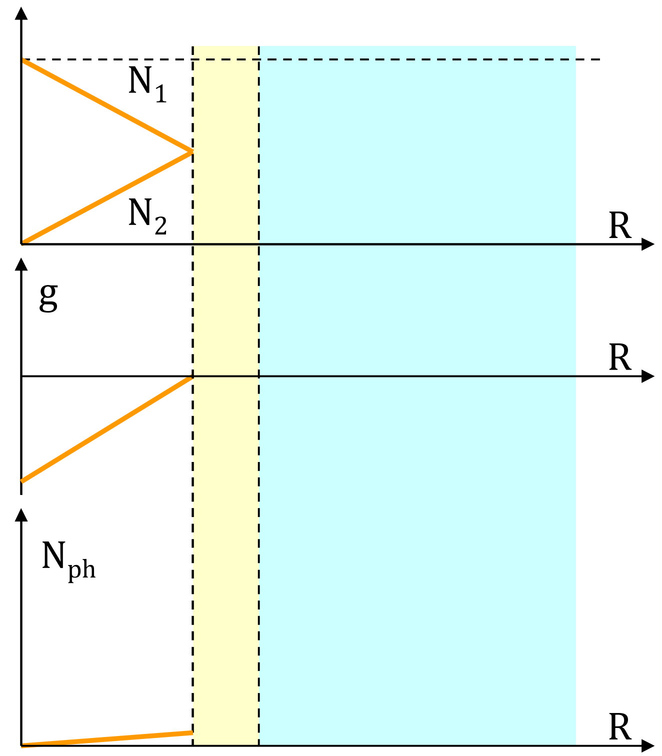
- R < oscillation threshold and transparency
  - No population inversion or gain < cavity loss
  - No amplification
  - $N_{ph} \sim 0$

$$N_2 = R_p \tau_2$$

$$N_2 - N_1 < 0$$

$$g < 0$$

$$N_{ph} \sim 0$$



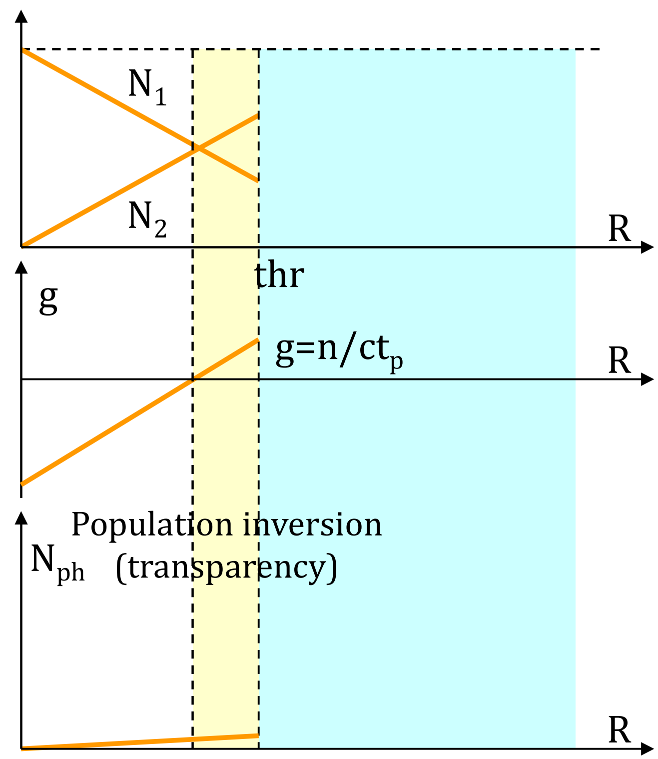
Stationary rate equations

- R = oscillation threshold
  - Population inversion
  - gain= cavity loss
  - loop gain < 1

$$N_{2thr} - N_{1thr} = \frac{1}{\sigma v_g \tau_p}$$

$$\boxed{g = \frac{1}{v_g \tau_p} = \frac{n}{c\tau_p}}$$

$$N_{ph} \sim 0$$



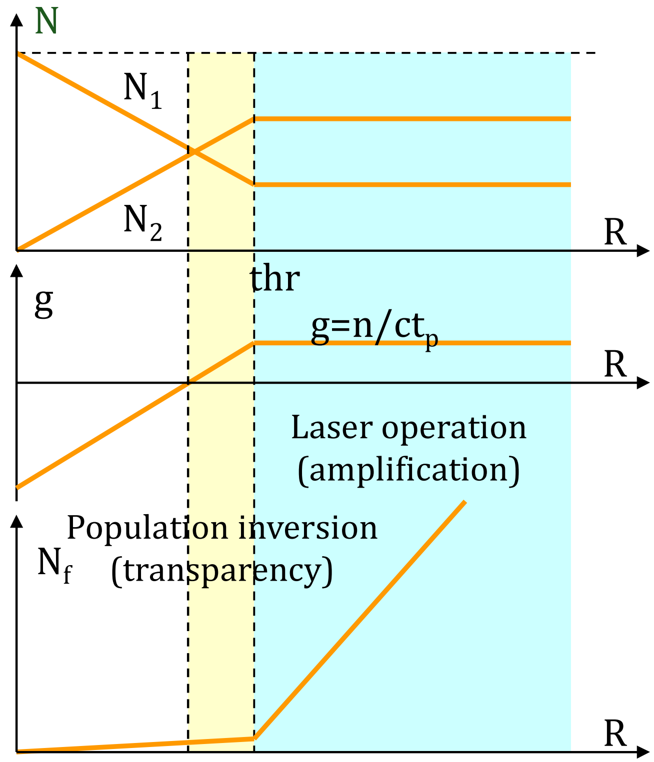
Stationary rate equations

- R > oscillation threshold
  - Population inversion
  - Loop gain= 1
  - N_2 en N_1 locked to threshold value

$$N_1 = N_{1thr} \qquad N_2 = N_{2thr} \qquad g = \frac{1}{v_g \tau_p}$$

$$N_{ph} = -\frac{R_p - R_{thr}}{\sigma v_g (N_{2thr} - N_{1th})} = \tau_p (R_p - R_{thr})$$

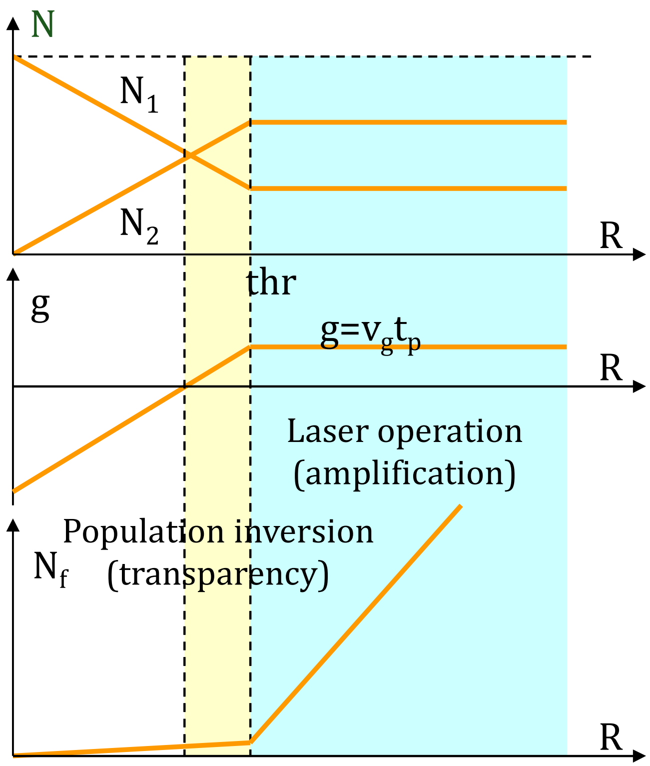
$$\frac{dN_2}{dt} = R_p - \frac{N_2}{\tau_2} - v_g g N_{ph} = 0$$

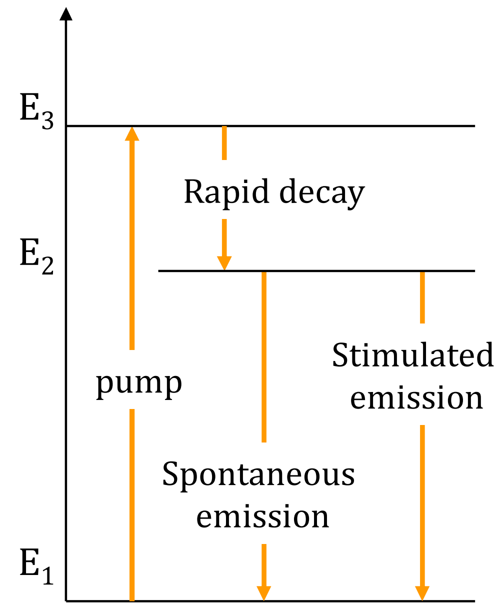


- Optical power linearly dependent on pump rate (above threshold) Note only part of N_{ph} is output

Stationary rate equations

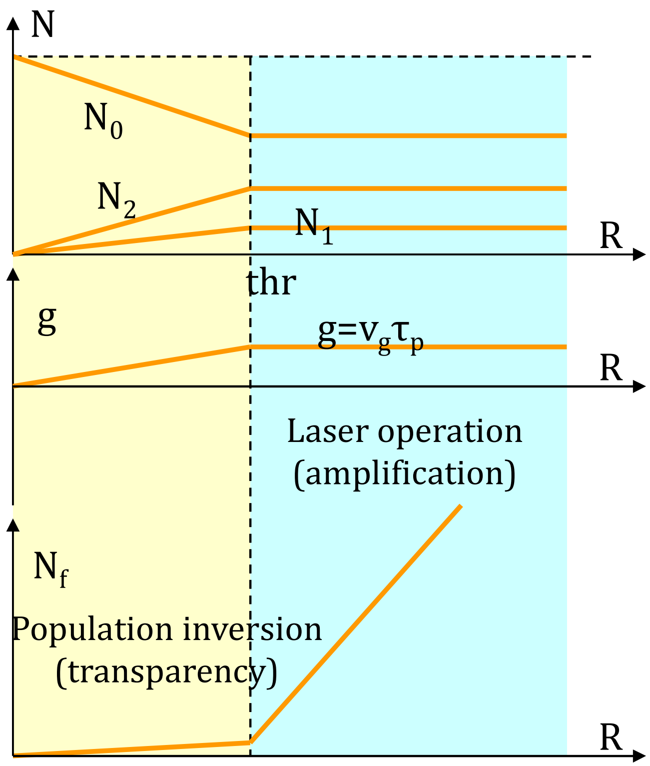
- Three level system

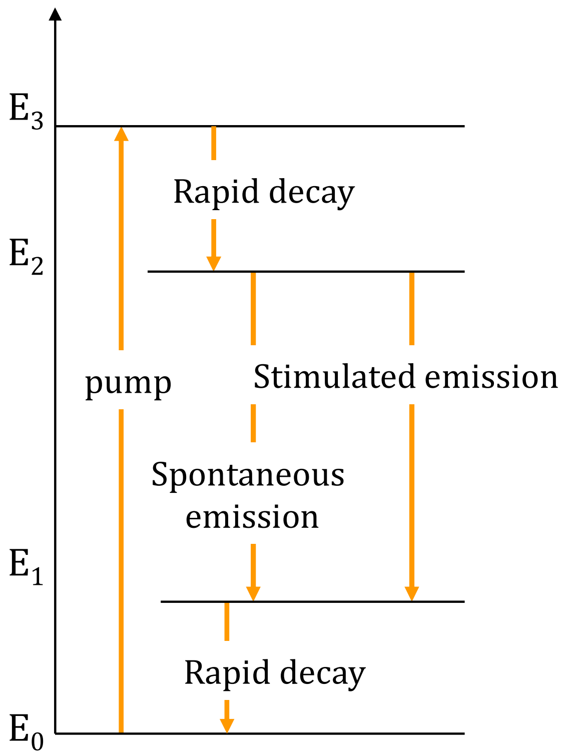




Stationary rate equations

- Four level system

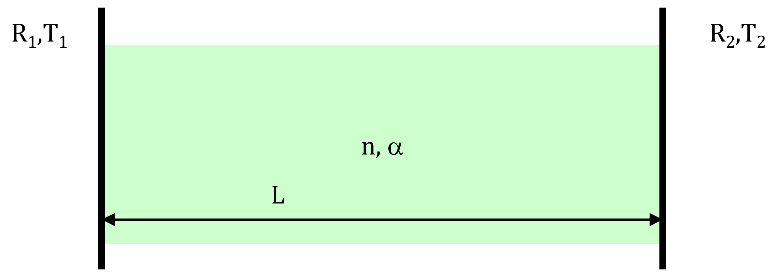




Example 13.3

Exercise: Basic laser

- Consider a cavity with plane mirrors with (power) reflectance R_1 and R_2, transmittance T_1 and T_2 and length L.
- The material has a refractive index n.
- The laser is well above the threshold so that spontaneous emission is negligible in the resonance condition.
- The cavity material has losses a [1/m] (due to scattering for example) next to the absorption and stimulated emission process.
- ➔ Derive an expression for the needed material gain g by stimulated emission for lasing as a function of the cavity parameters.
- ➔ Calculate the gain at threshold (in 1/cm) for the following parameters



Semiconductor laser with
$L = 0.3$ mm,
$R_1 = R_2 = 0.3$,
$\alpha = 30$ [dB/cm],
$n = 3.5$
He-Ne laser with
$L = 30$ cm,
$R_1 = 1$, $R_2 = 0.99$,
$\alpha = 0$, $n = 1$

Example 13.3

Example 13.3

Example 13.3

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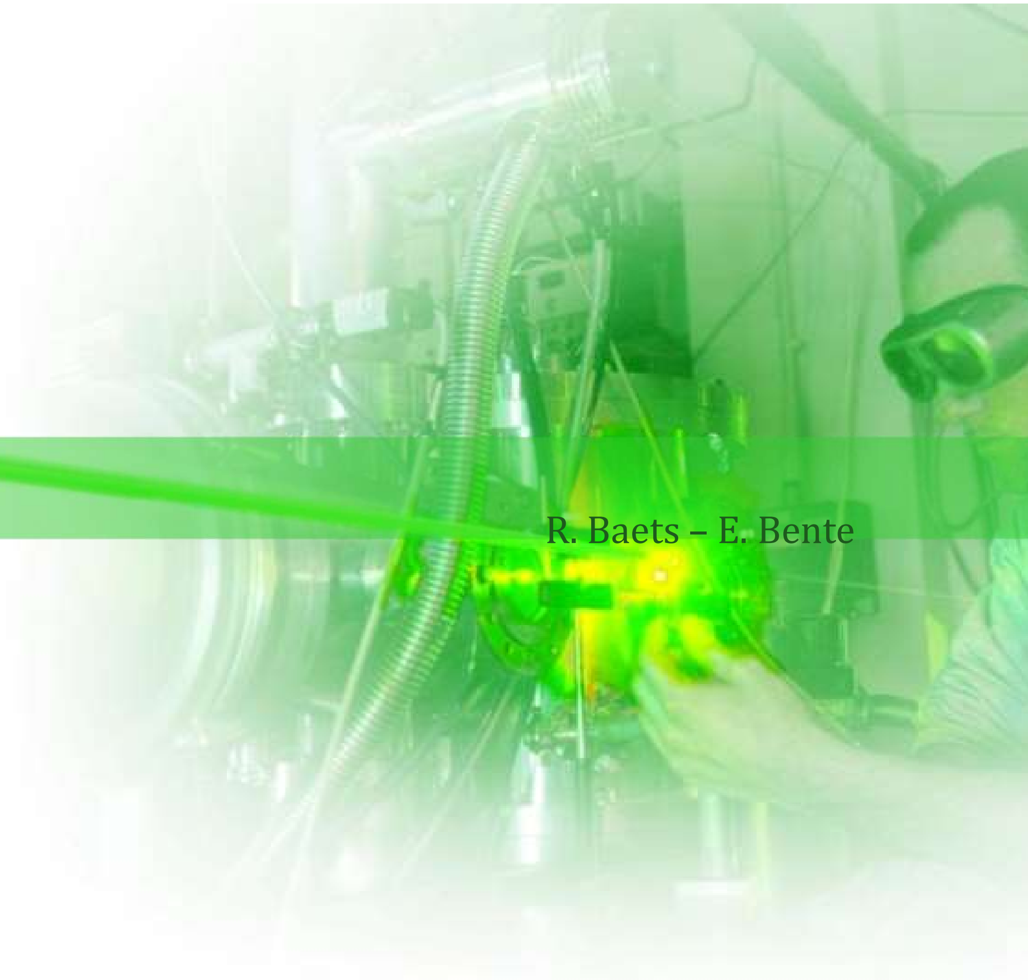
Lasers - Part C

Laser cavities

Passive Fabry-Perot resonator

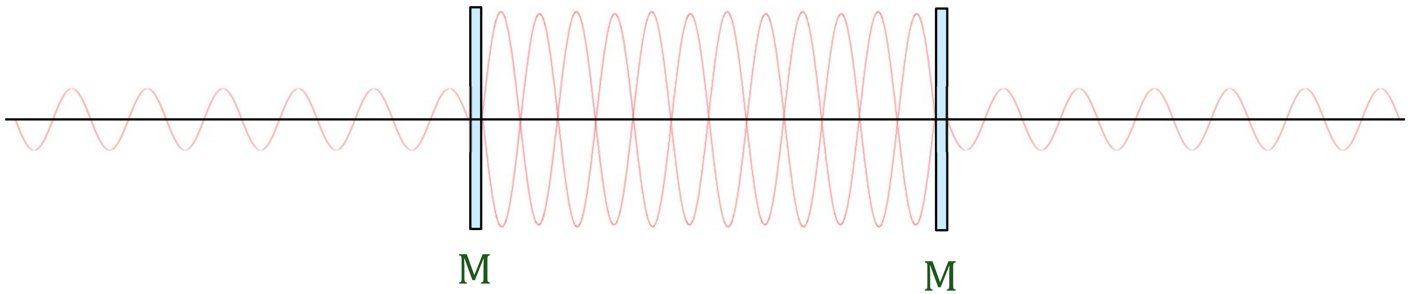
Resonator with optical amplifier



Laser resonator

- Up to this point only considered energy in the laser
- The resonator also determines the output frequency of the laser (together with the gain medium) and the quality of the beam of light
- Most simple two-mirror resonator:

Fabry-Pérot resonator

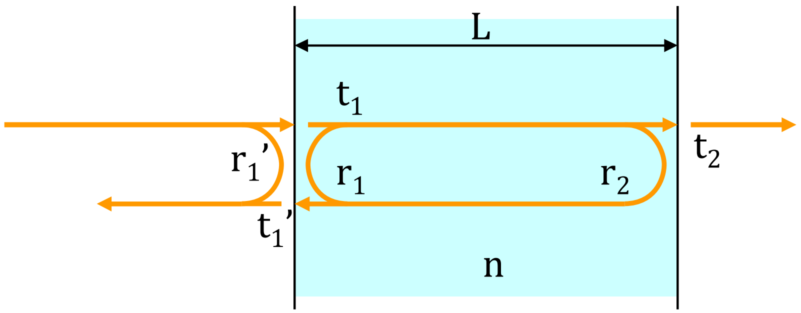


- Only frequencies of the standing waves are allowed: **Longitudinal Modes**

Resonance analysis with plane waves

(see also Chapter 6)

- 1D cavity = Fabry-Perot-etalon
  - walls = semi-transparent mirrors
  - Length L, refractive index n
  - Phase change $\phi = n \cdot L \cdot k_0$ $\quad k_0 = \frac{2\pi}{\lambda_0}$



- Electric field amplitude of transmission:

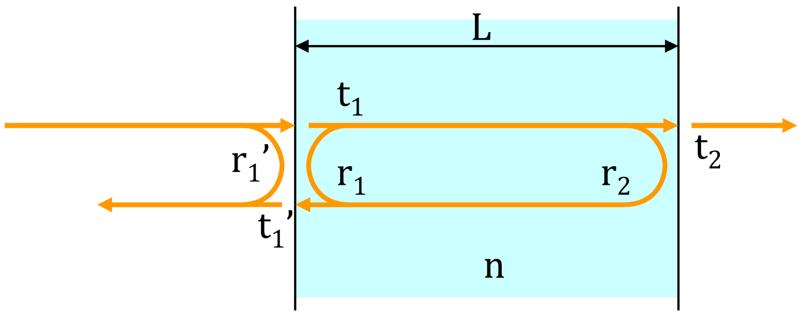
$$t(\phi) = t_1 t_2 e^{-j\phi} + t_1 t_2 r_1 r_2 e^{-j3\phi} + t_1 t_2 r_1^2 r_2^2 e^{-j5\phi} + \cdots \qquad \phi = \frac{2\pi \cdot n \cdot L}{\lambda_0}$$

$$t(\phi) = \frac{t_1 t_2 e^{-j\phi}}{1 - r_1 r_2 e^{-j2\phi}}$$

Transmission spectrum FP resonator

- Power transmission:

$$T = |t|^2 = \frac{t_1 t_2 \left(\frac{|t_1 t_2|}{1 - r_1 r_2}\right)^2}{1 + \frac{4 r_1 r_2}{(1 - r_1 r_2)^2} sin^2\phi} = \frac{T_{max}}{1 + F sin^2\phi}$$



$$F \equiv \frac{4 r_1 r_2}{(1 - r_1 r_2)^2} \qquad T_{max} \equiv t_1 t_2 \left(\frac{|t_1 t_2|}{1 - r_1 r_2}\right)^2$$

$$\phi = \frac{2\pi \cdot n \cdot L}{\lambda_0}$$

- $T_{max} = 1$ if $r_1 = r_2$: symmetrical structure

  and $t_1^2 t_2^2 = (1 - r_1^2)(1 - r_2^2)$: lossless mirrors

  and $sin\phi = 0 \qquad \phi = m \cdot \pi \quad m \in \mathbb{N}$

  $$\frac{2\pi nL}{\lambda} = m \cdot \pi \quad \Rightarrow \quad \lambda = \frac{2nL}{m}$$ Standing wave condition

Transmission spectrum FP etalon

- Power transmission: symmetric loss-less Fabry-Perot

$$T = \frac{1}{1 + F sin^2 \phi} \qquad F = \frac{4r^2}{(1 - r^2)^2}$$

- Maxima for $2\phi = 2 \cdot m \cdot \pi$, with m integer

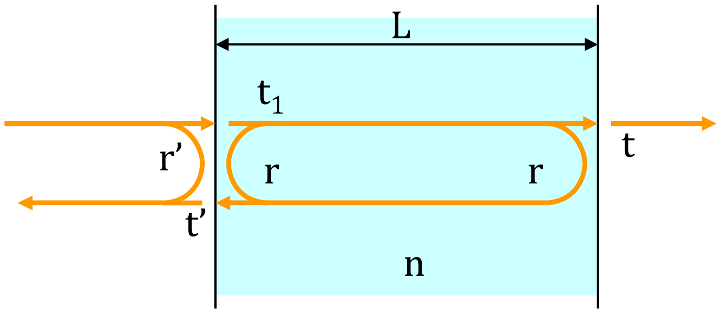
or $$L = m \frac{\lambda}{2n}$$

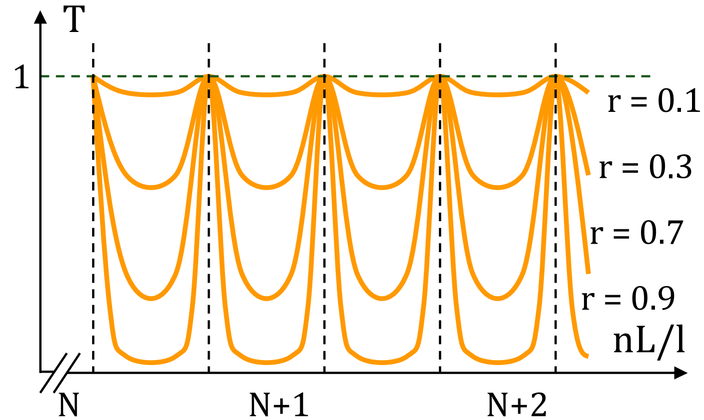
distance between transmission maxima

or $$\Delta\lambda \approx \frac{\lambda^2}{2nL} \qquad \Delta\nu = \frac{c}{2nL}$$

- Mode spacing = 1/round trip time in the cavity

$$T_{roundtrip} = \frac{1}{\Delta\nu} = \frac{2nL}{c}$$





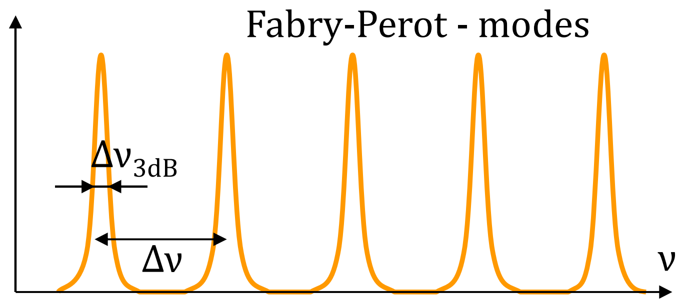
Finesse and quality factor of a cavity

- Finesse of a FP cavity

$$\text{finesse} \equiv \frac{\text{distance between maxima}}{\text{3dB width of a peak}} = \frac{\Delta\nu}{\Delta\nu_{3dB}}$$

- Q - factor of a cavity: 2π x the number of roundtrips the light has to make for the energy stored in the resonator to drop by a factor of $1/e$ (no other loss inside the cavity) (Chapter 6)

$$Q = \frac{2\pi}{ln\left(\frac{1}{r_1^2 r_2^2}\right)}$$



- Related to photon lifetime (Example 13.3)

$$\tau_p = \frac{Q}{2\pi} T_{roundtrip} = \frac{Q}{2\pi} \frac{2nL}{c}$$

Alternative quality factor of a resonator

- Q - factor of a cavity:
  2π x the number of light field oscillations the light field makes before the energy stored in the resonator has dropped by a factor of 1/e (only taking into account losses due to the mirror)

Example 13.4

Fabry-Perot etalon with gain

- With optical gain in the etalon

$$t = \frac{t_1 t_2 exp\left(\frac{g(\nu)L}{2}\right) exp(-j\phi)}{1 - r_1 r_2 exp(g(\nu)L) exp(-j2\phi)}$$

becomes infinite for: $r_1 r_2 exp(g(\nu)L) exp(-j2\phi) = 1$

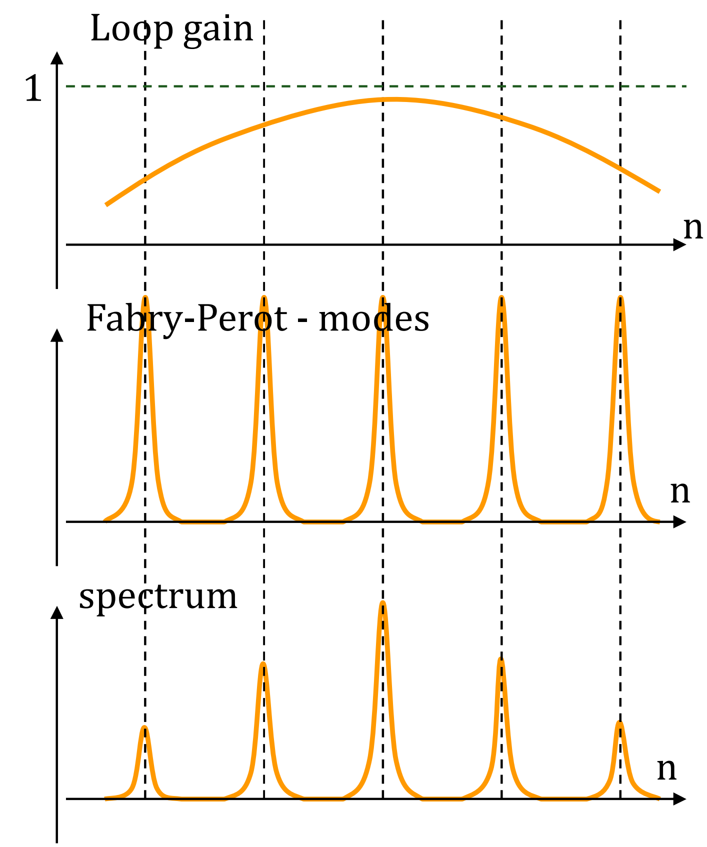
  - Equation for loop gain

$$r_1 r_2 exp(g(R,\nu)L) = 1$$

  - Equation for phase

$$\phi = \frac{2\pi n L}{\lambda} = m \cdot \pi \quad m \in \mathbb{N}$$

- Gain curve narrow compared to Δλ
  - Only 1 lasing frequency
- Gain curve broad compared to Δλ
  - Multiple longitudinal modes

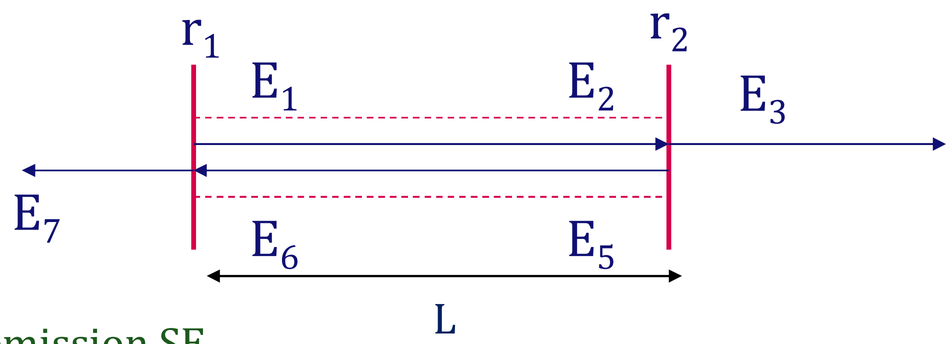


Fabry-Perot laser – threshold spectrum

steady state electric field A described by the complex amplitude E of the envelope:

$$A = E e^{i\omega t} \qquad \omega = 2\pi \frac{c}{\lambda}$$

optical amplifier between two mirrors, gain g, index n



- Single transverse mode – plane wave
- Optical amplifier gain g and adds spontaneous emission SE
- Relations between amplitudes:

$$E_2 = E_1 exp\left(\frac{g}{2}L + i\frac{2\pi n}{\lambda}L\right) + SE$$

$$E_5 = E_2 \cdot r_2$$

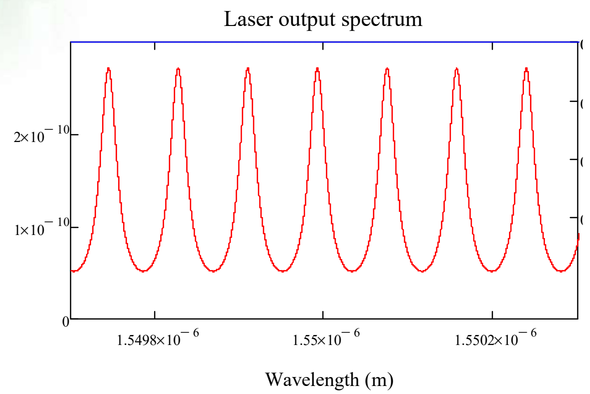
$$E_6 = E_5 exp\left(\frac{g}{2}L + i\frac{2\pi n}{\lambda}L\right)$$

$$E_1 = E_6 \cdot r_1$$

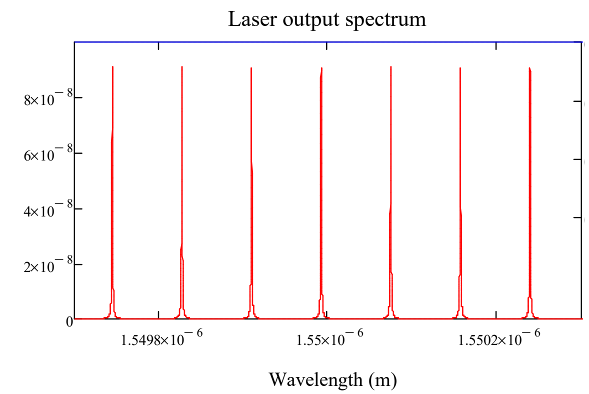
- Electric field gain is ½ the power gain
- Solution (calculate E_3 from E_2)

$$\boxed{E_2 = \frac{SE}{r_1 r_2 exp\left(gL + i\frac{2\pi n}{\lambda}2L\right) - 1}}$$

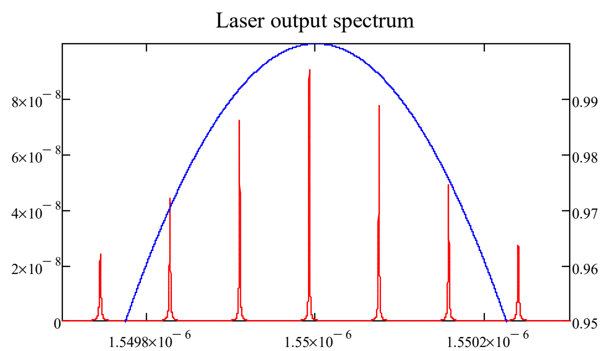
FP laser cavity output spectra



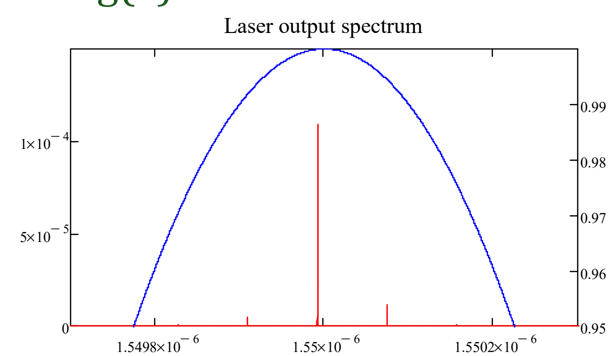
- Low gain
- g(λ) = constant



- Gain close to threshold
- g(λ) = constant



- Gain close to threshold
- g(λ) variable

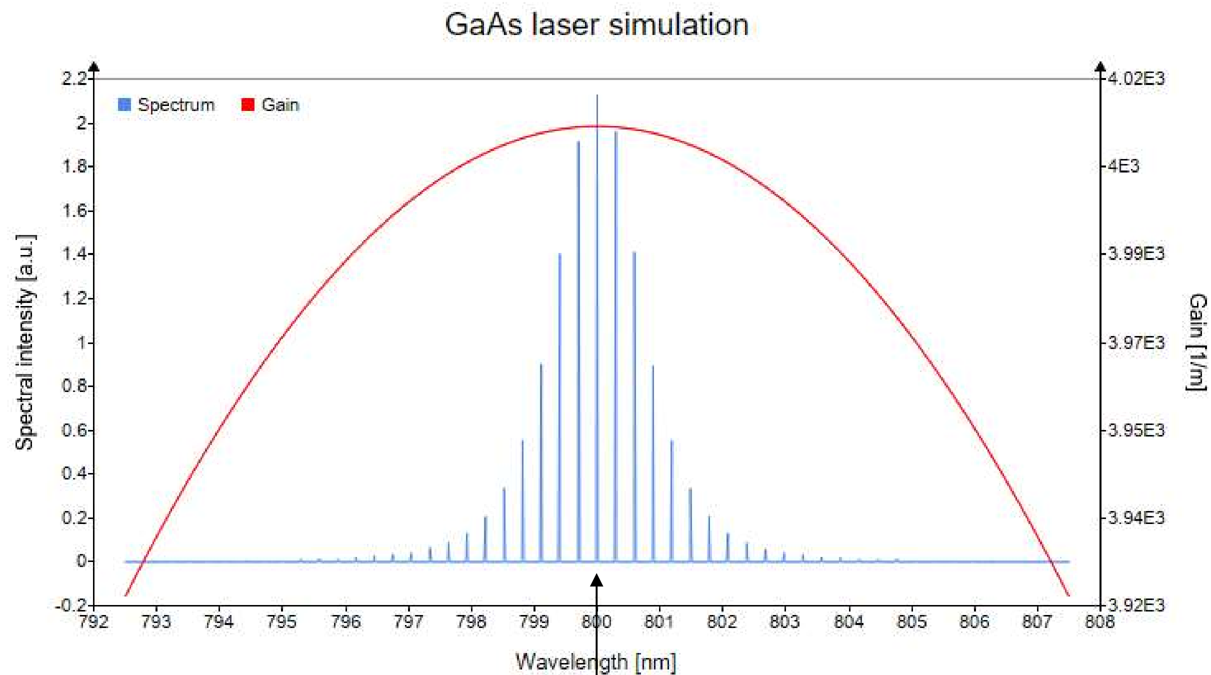


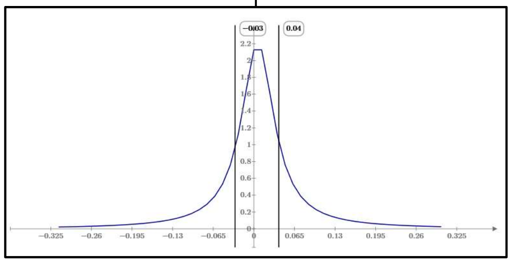
- Gain at threshold (gain + SE = 1)
- g(λ) variable

Mode spacing
Example 13.5 - GaAs laser

- Example: GaAs semiconductor laser
  - Length $L=0.3$ mm
    The mode spacing becomes 140 GHz or 0.4 nm
  - Very broad gain curve (can be 50 nm)
  - Multiple longitudinal modes

Example 13.5 – GaAs laser spectrum



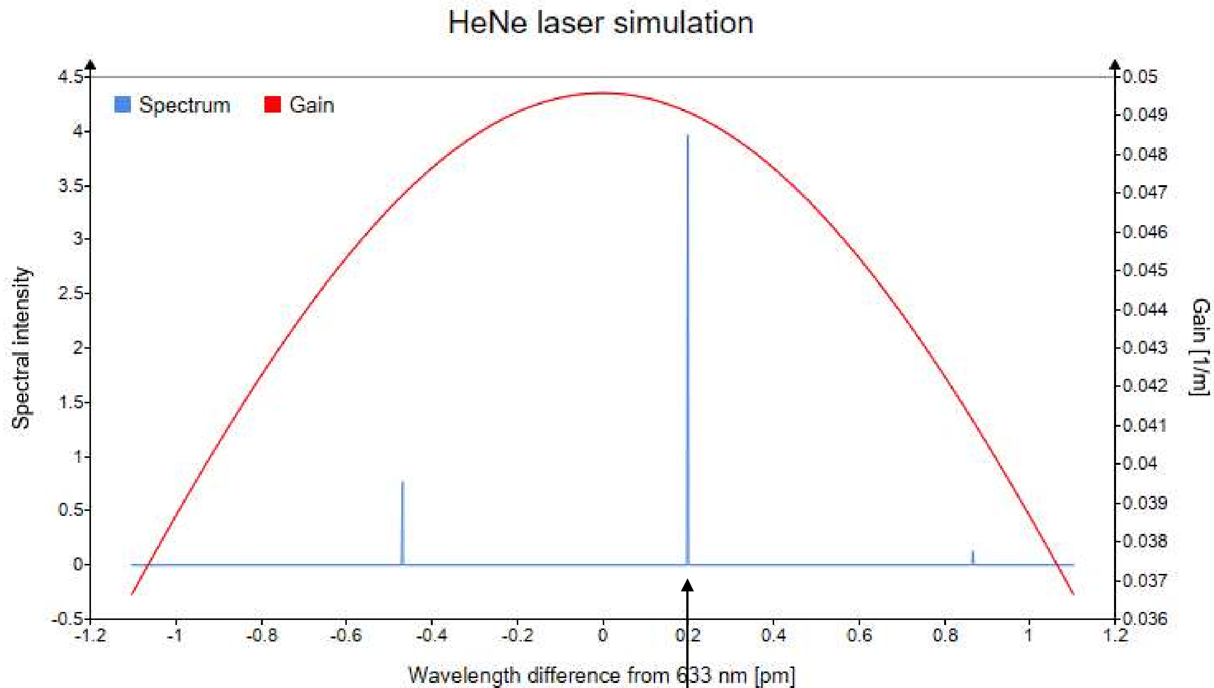


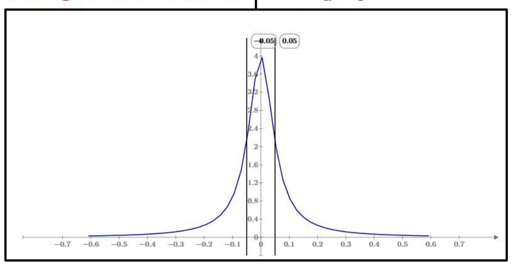
width of the strongest resonance
is 0.07 pm = 33 MHz

Mode spacing
Example 13.5 – He-Ne laser

- Example: He-Ne laser
  - wavelength: l=633 nm
  - Length L= 30 cm
    The mode spacing becomes 500 MHz or 0.0007 nm
  - Gain curve: Doppler broadening ~ 1.5 GHz
  - Few longitudinal modes

Example 13.5 He-Ne laser spectrum



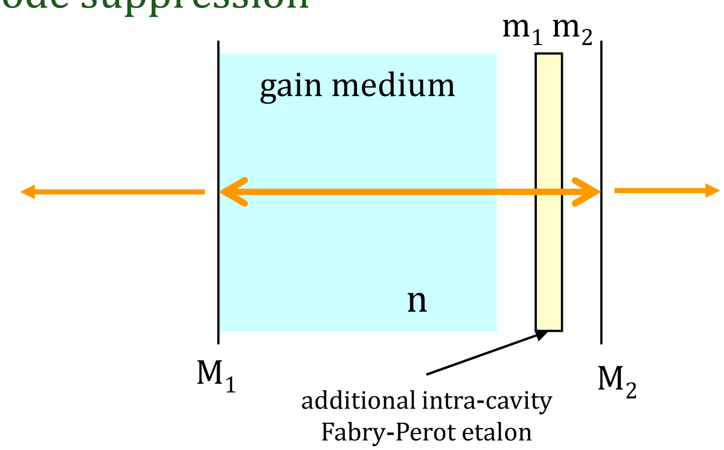


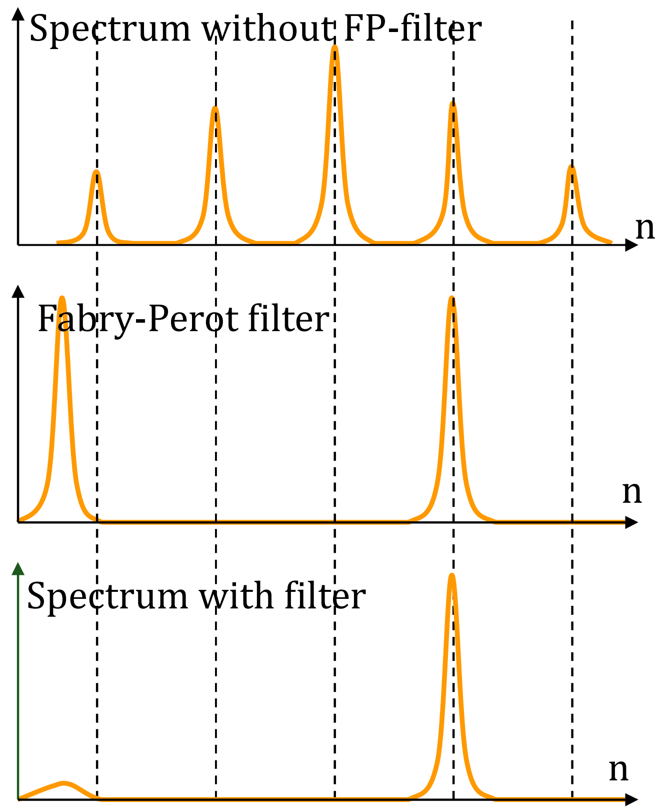
width of the strongest resonance is 1 fm = 75 kHz

Longitudinal mode selection

- Laser with broad gain spectrum: multiple longitudinal modes

- Monomode spectrum can obtained
  - Using an additional spectral filter (short FP etalon) inside
  - Sharp filter – high finesse
  - Side mode suppression





Photonics

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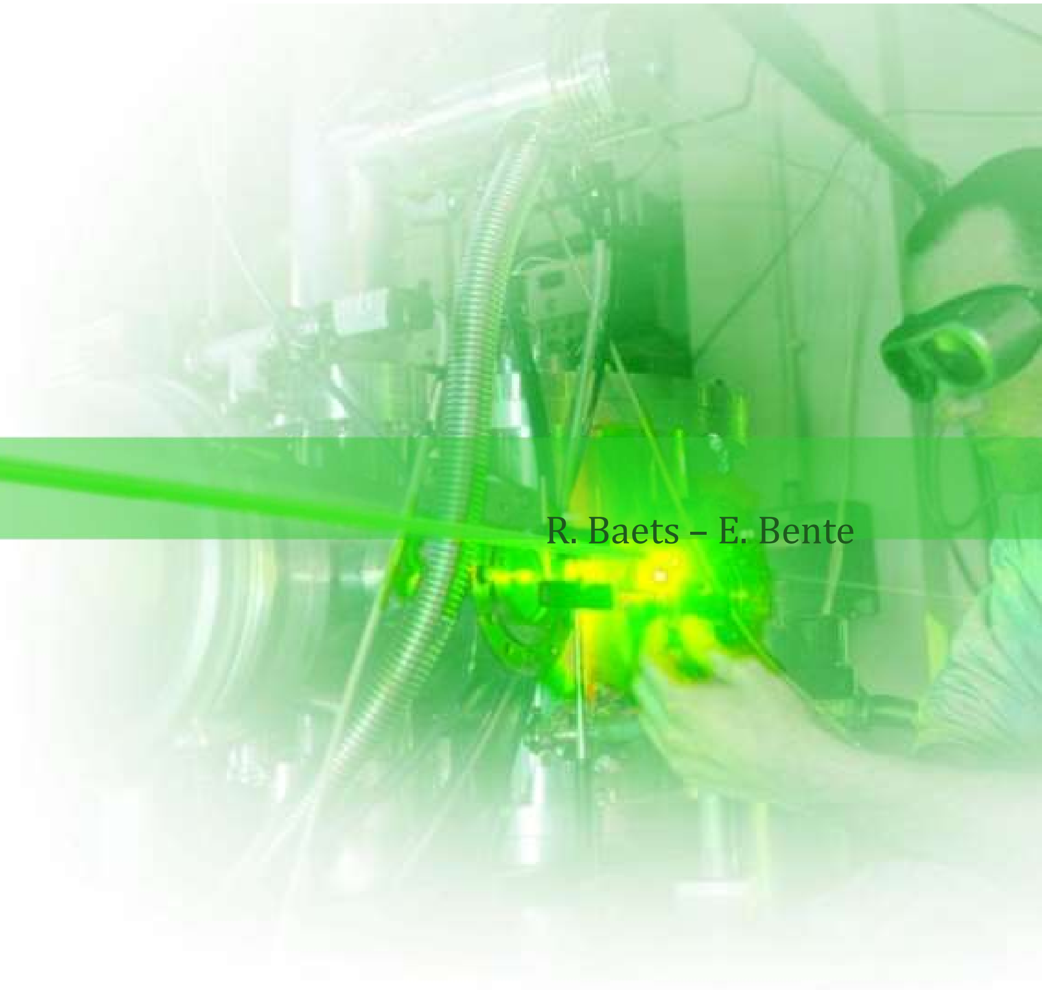


Lasers

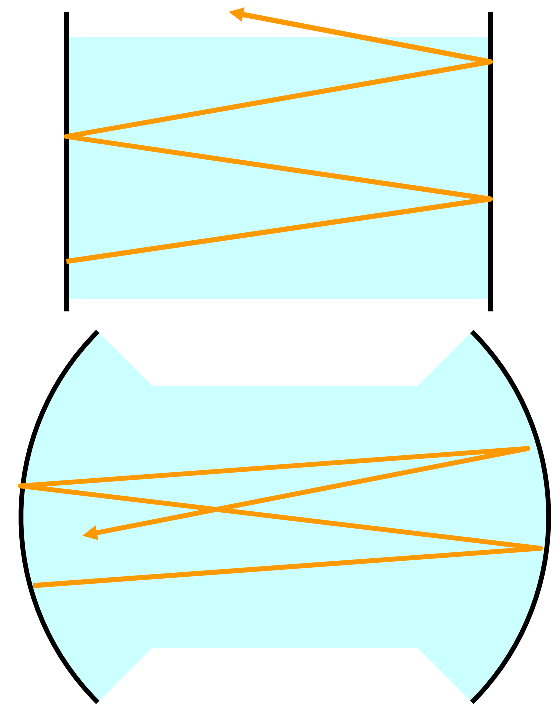
Cavity design using mirrors

Transverse modes

Cavity with finite dimensions

- 1-dimensional laser: plane waves in the cavity
  => No diffraction losses in the cavity

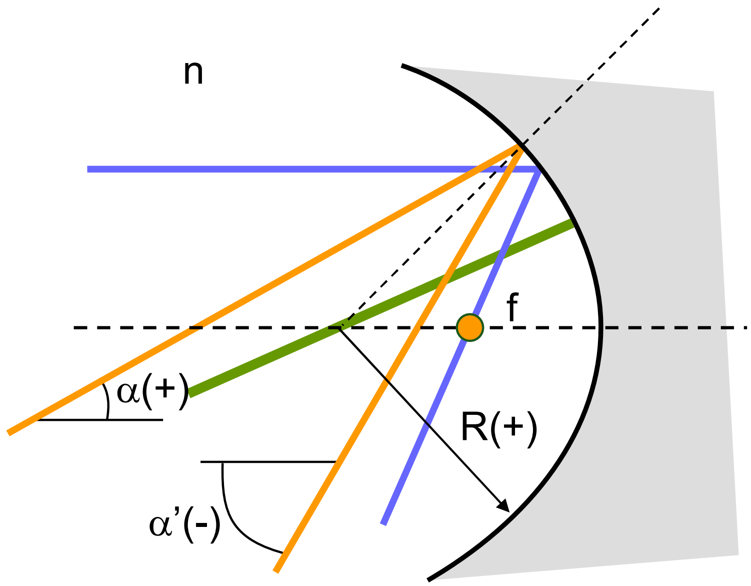
- Real optical cavities 3D: finite dimensions
  - Plane mirrors:
    - Diffraction loss
    - Lasing only possible when gain compensates loss
  - Spherically curved mirror
    - Light converging inside cavity
    - Deduce radius of curvature with **ray optics**



Spherical mirrors

- Paraxial approximation:

$$\begin{bmatrix} x' \\ n\alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix} \begin{bmatrix} x \\ n\alpha \end{bmatrix} \qquad P = \frac{2n}{R}$$

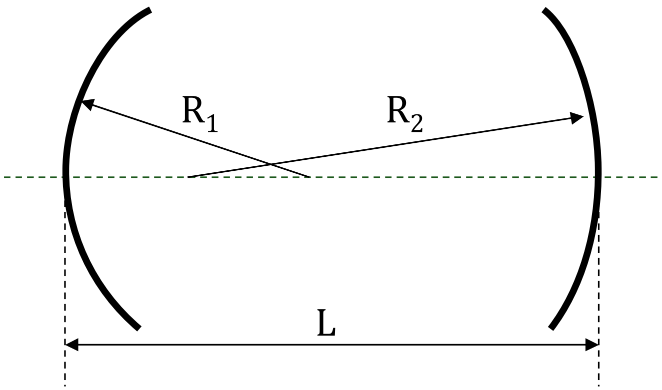


- Expansion of the sign convention
  - concave mirror: R>0
  - take into account the propagation direction

- Focal length f=n/P $\quad f = \dfrac{R}{2}$

Cavity system matrix

- Spherical mirror ⇨ $\begin{bmatrix} 1 & 0 \\ -P & 1 \end{bmatrix}$, P = 2/R
- Translation ⇨ $\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$



- Matrix formalism for 1 roundtrip (length 2L)

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -P_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -P_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - P_1 L & L(2 - P_1 L) \\ P_1 P_2 L - P_1 P_2 & 1 - P_1 L - 2P_2 L + P_1 P_2 L^2 \end{bmatrix}$$

with $P_i = \frac{1}{f_i} = \frac{2}{R_i}$

Cavity system matrix

- Transformation for two consecutive periods

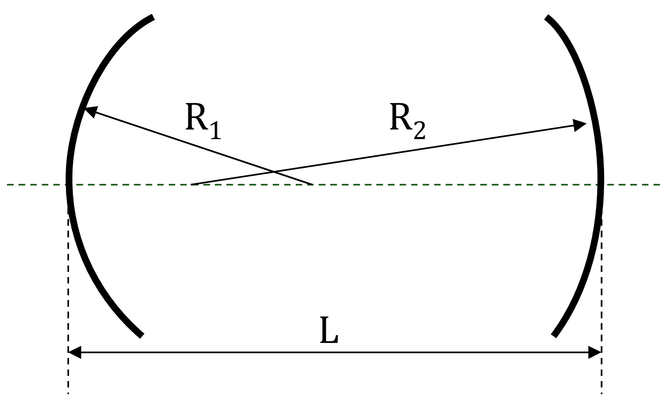
$$\begin{bmatrix} x_{n+1} \\ \alpha_{n+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_n \\ \alpha_n \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x_{n+2} \\ \alpha_{n+2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_{n+1} \\ \alpha_{n+1} \end{bmatrix}$$

- Eliminate α_i: $x_{n+2} - (A+D)\, x_{n+1} + (AD - BC)\, x_n = 0$ and (AD-BC)=1

det(**M**)=n/n'

with

$$A + D = 2\left[1 - P_1 L - P_2 L + \frac{P_1 P_2 L^2}{2}\right] = 2\left[2\left(1 - \frac{P_1 L}{2}\right)\left(1 - \frac{P_2 L}{2}\right) - 1\right]$$



Cavity system matrix

- Recursive relation for x_i: $$x_{n+2} - (A + D)\, x_{n+1} + x_n = 0$$
- Proposed solution: $$x_n = \lambda \cdot x_{n-1} = \lambda^n \cdot x_0 = e^{\pm jn\theta} x_0$$
- Equation becomes: $$\cos\theta = \frac{1}{2}(A + D)$$ $$A + D = 2\left[2\left(1 - \frac{P_1 L}{2}\right)\left(1 - \frac{P_2 L}{2}\right) - 1\right]$$

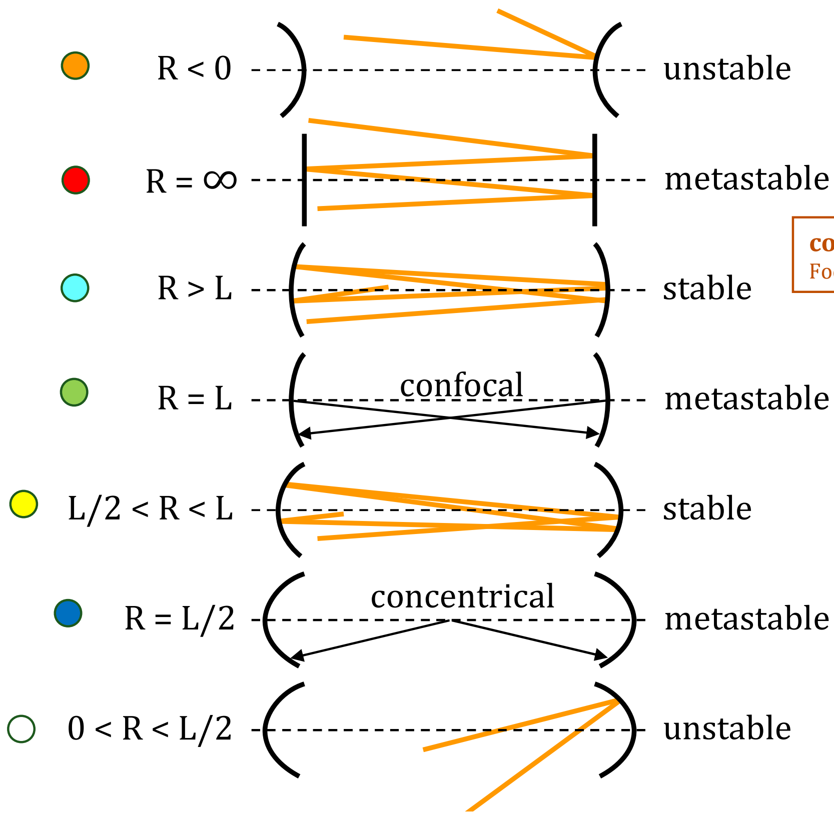
- Solution is non-divergent if q is real: -1 < cos q < 1

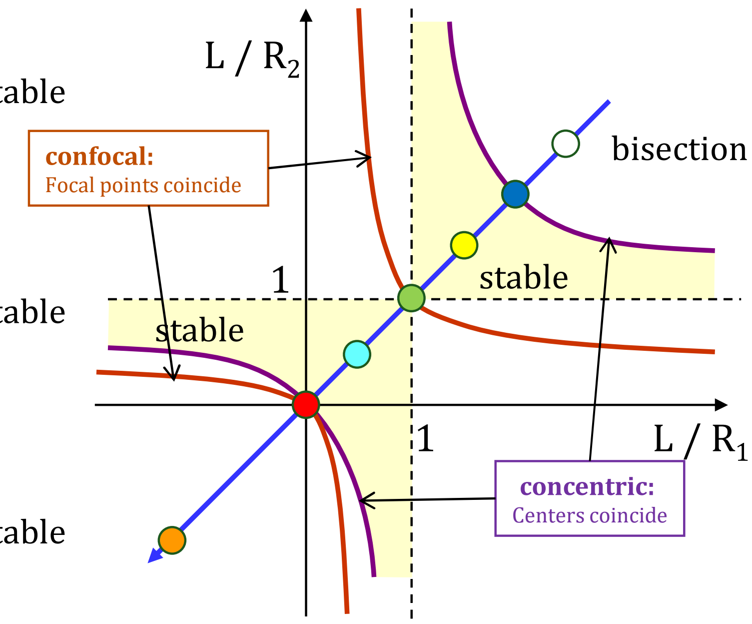
or: $$\boxed{0 \le \left(1 - \frac{P_1 L}{2}\right)\left(1 - \frac{P_2 L}{2}\right) \le 1 \qquad \text{Stability condition}}$$

Cavity stability

- Stability condition: $$0 \le \left(1 - \frac{P_1 L}{2}\right)\left(1 - \frac{P_2 L}{2}\right) \le 1$$
- For a symmetrical cavity: $R_1=R_2=R$

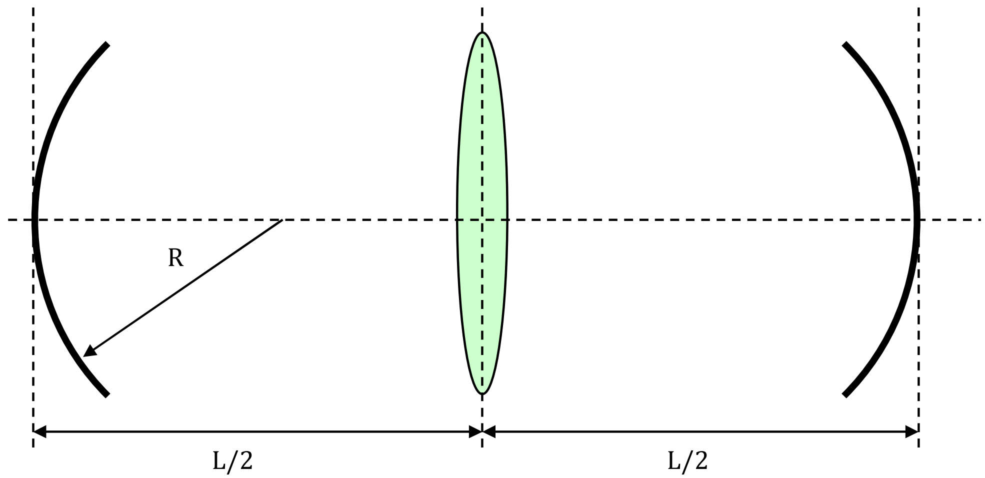
$$f = \frac{R}{2} \qquad P = \frac{2}{R} \qquad 0 \le \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \le 1$$





Exercise: Laser stability: problem

- Suppose we have a symmetrical cavity with spherical mirrors. When a lens (focal length f) is now placed in the middle of this cavity, which condition does f have to fulfill for the cavity to remain stable?



- Tip: set-up the matrix M' for half the roundtrip. The full roundtrip is then M'·M'. Derive the stability criterium for the A' and D' in M' . It may also help to use a symbolic calculation tool.

Gaussian beam in a cavity

- Gaussian beam: spherical phase front https://en.wikipedia.org/wiki/Gaussian_beam
- Resonance: beam must be the same after one round trip

$$q_2 = q_1 \qquad q_2 = \frac{Aq_1 + B}{Cq_1 + D} \qquad \frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi \cdot n \cdot w^2(z)}$$

- Curvature spherical phase front = mirror curvature

$R_1 = z_1 + \frac{b_0^2}{z_1}$ and $R_2 = z_2 + \frac{b_0^2}{z_2}$

rayleigh range b_0

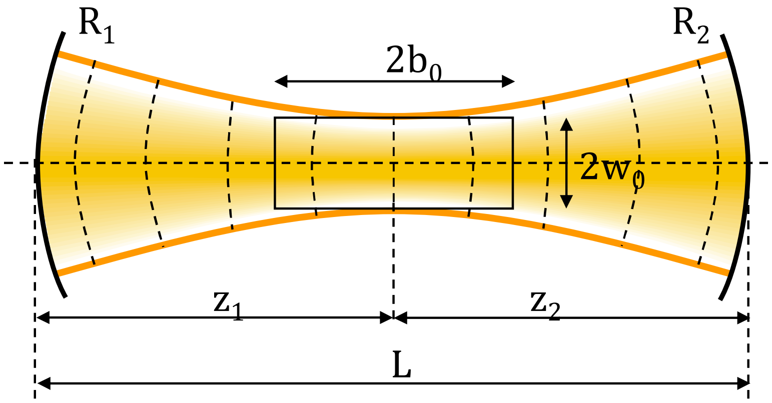
- If $R_1 = R_2 = R$:

$$z_1 = z_2 = \frac{L}{2}$$

$$b_0^2 = \frac{L}{2}\left(R - \frac{L}{2}\right)$$

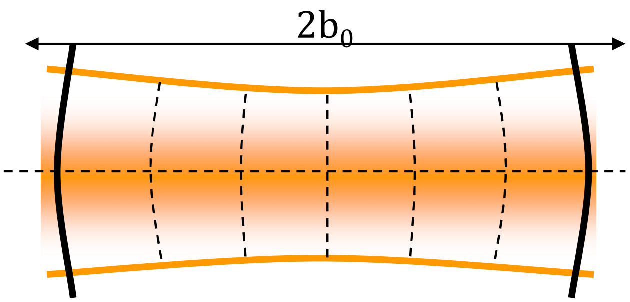
$$w_0 = \sqrt{\frac{2}{k}\sqrt{\frac{L}{2}\left(R - \frac{L}{2}\right)}}$$

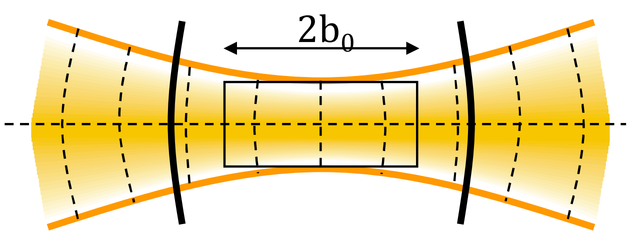
beam waist



Gaussian beam in a cavity

- $z_1 = z_2 = \frac{L}{2}$, $b_0^2 = \frac{L}{2}\left(R - \frac{L}{2}\right)$ and $w_0 = \sqrt{\frac{2}{k}\sqrt{\frac{L}{2}\left(R - \frac{L}{2}\right)}}$
- Only real solution when $R \geq \frac{L}{2}$ (same condition as for ray theory)
  - If $R > L$, then $b_0 > L/2$:
    - Beam is parallel in the laser and diffracts outside the mirrors
  - If $R = L$, then $b_0 = L/2$:
    - Rayleigh range of the Gaussian beam equals the cavity length
  - When $R < L$, then $b_0 < L/2$:
    - beam already diffracts within the laser cavity,
      on the outside you see a diverging beam





Transversal modes

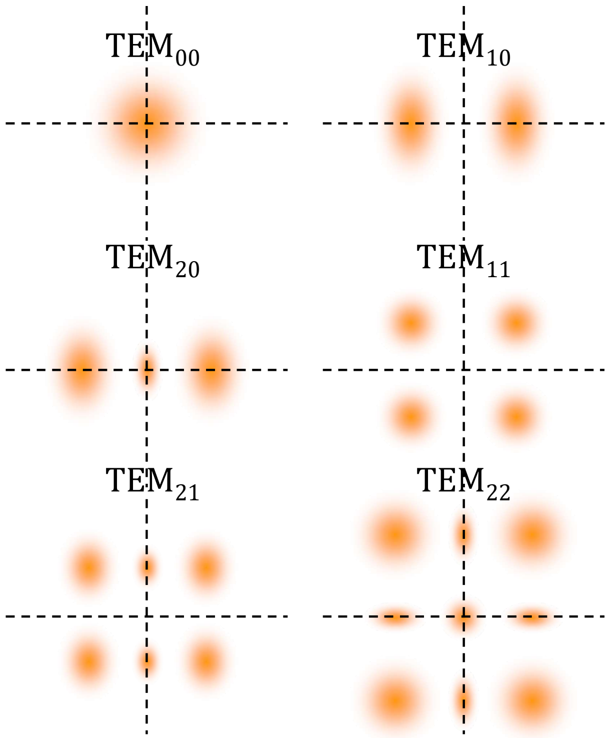
- Wave equation $$\frac{\partial^2 A}{dx^2} - 2jk\frac{\partial A}{dz} = 0$$
- General solution (2D-cavity):

$$\Psi(x,z) = \sqrt{\frac{w_0}{w(z)}} H_m\left(\frac{\sqrt{2}x}{w(z)}\right)$$

$$e^{-\frac{x^2}{w^2(z)}} e^{-jkz} e^{j\left(m+\frac{1}{2}\right)\arctan\frac{z}{b_0} - j\frac{kx^2}{2R(z)}}$$

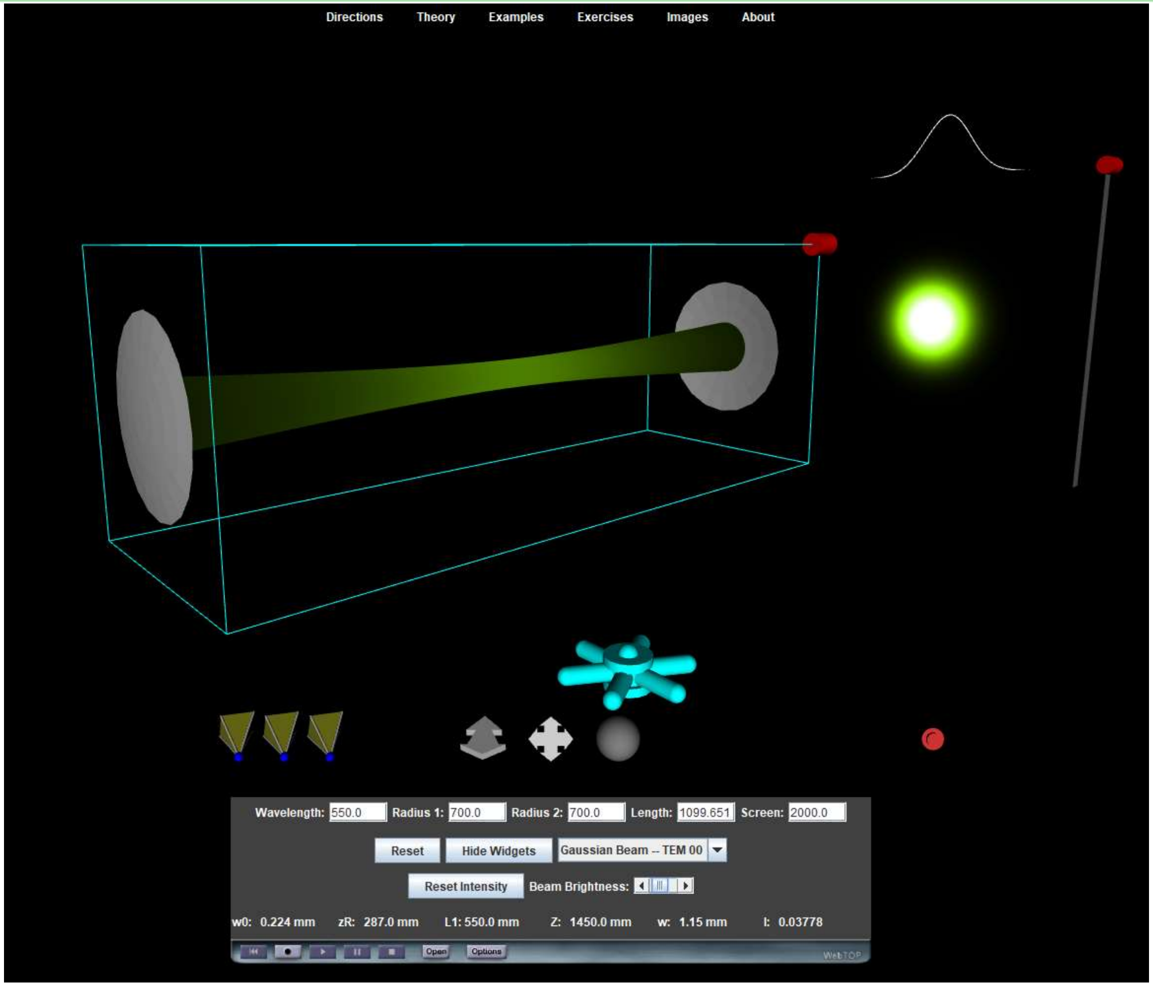
with H_m Hermite-polynomials

- General solution (3D-cavity):
  - product of 2 2D-solutions
  - 2 transversal mode numbers



WebTop simulation – Lasers

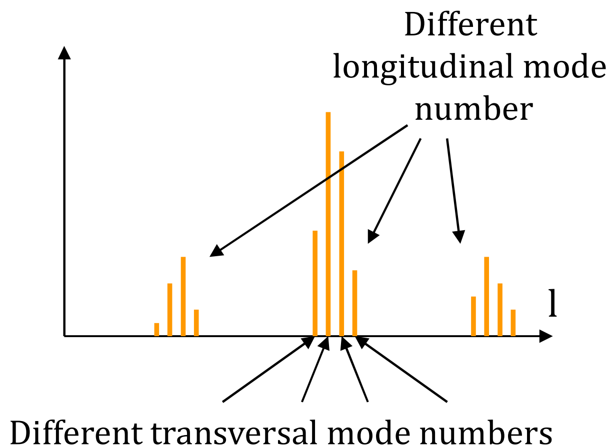
https://sourceforge.net/projects/webtop-optics/



Laser modes

- Lasermode =
  Harmonic solution of Maxwell's equations that satisfy the resonance condition
  (an EM field that is the same after one roundtrip)

- Discrete solutions: characterized by three mode numbers
  - 1 mode number due to $2k_0nL = 2m\pi$
    "Longitudinal mode number"
  - 2 mode numbers due to the different
    Gauss-Hermite-solutions
    "Transversal mode numbers"

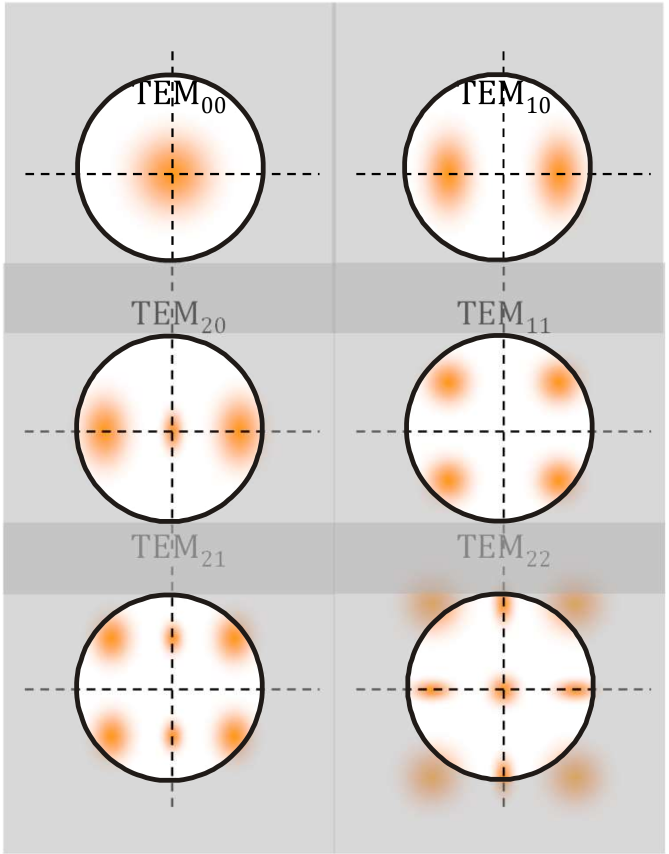


Lateral mode filter

- Applications that require strong spatial coherence:
  - high radiance
  - Diffraction limited beam

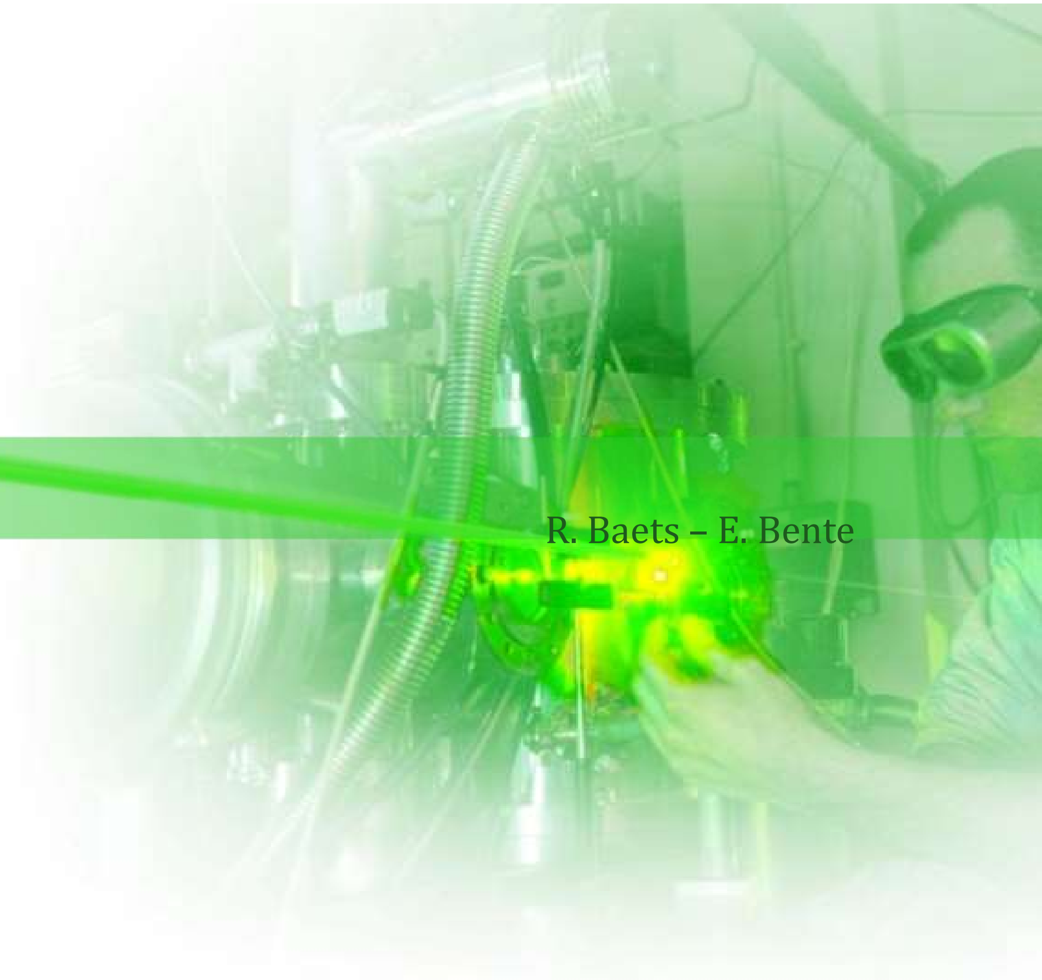
  => Suppress all transversal modes except TEM_{00}

- Higher transversal modes:
  - Wider than TEM_{00}
  - Suppress by small lateral dimension in cavity
    $Loss_{HO} > Loss_{TEM00}$
  - TEM_{00} will lase first



INTEC

Photonics



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Lasers

Proporties of laser beams

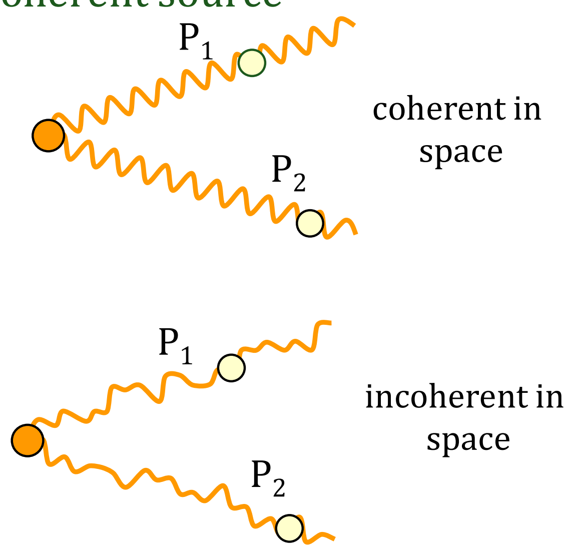
Pulsed lasers

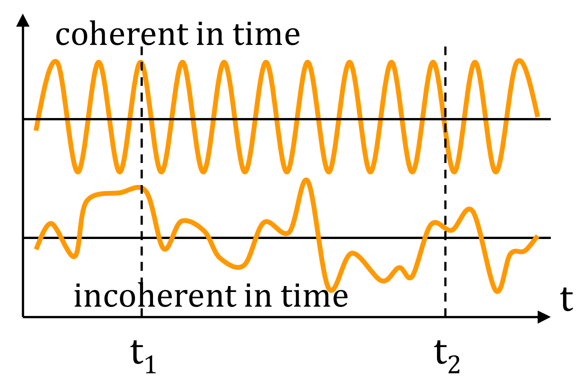
Properties of laser beams

- Monochromatic
  - Light constrained to very small frequency interval
  - Because of line shape and cavity
- Coherent
- Directional
  - Diffraction limited beam (minimal divergence for give beam width)
- Intense
  - Very high radiance
  - Very high intensity by focusing beam
  - Dangerous for the eye

Coherence

- Coherence means: fixed phase relation between fields
- Case 1: Perfect monochromatic source
  - pure sinusoidal
  - perfect coherent
  - <u>fixed phase phase-relation in time and space</u>
- Case 2: incoherent source





Coherence

$$E(x,y,t) = A\sin(k\cdot y - \omega\cdot t + \phi)$$

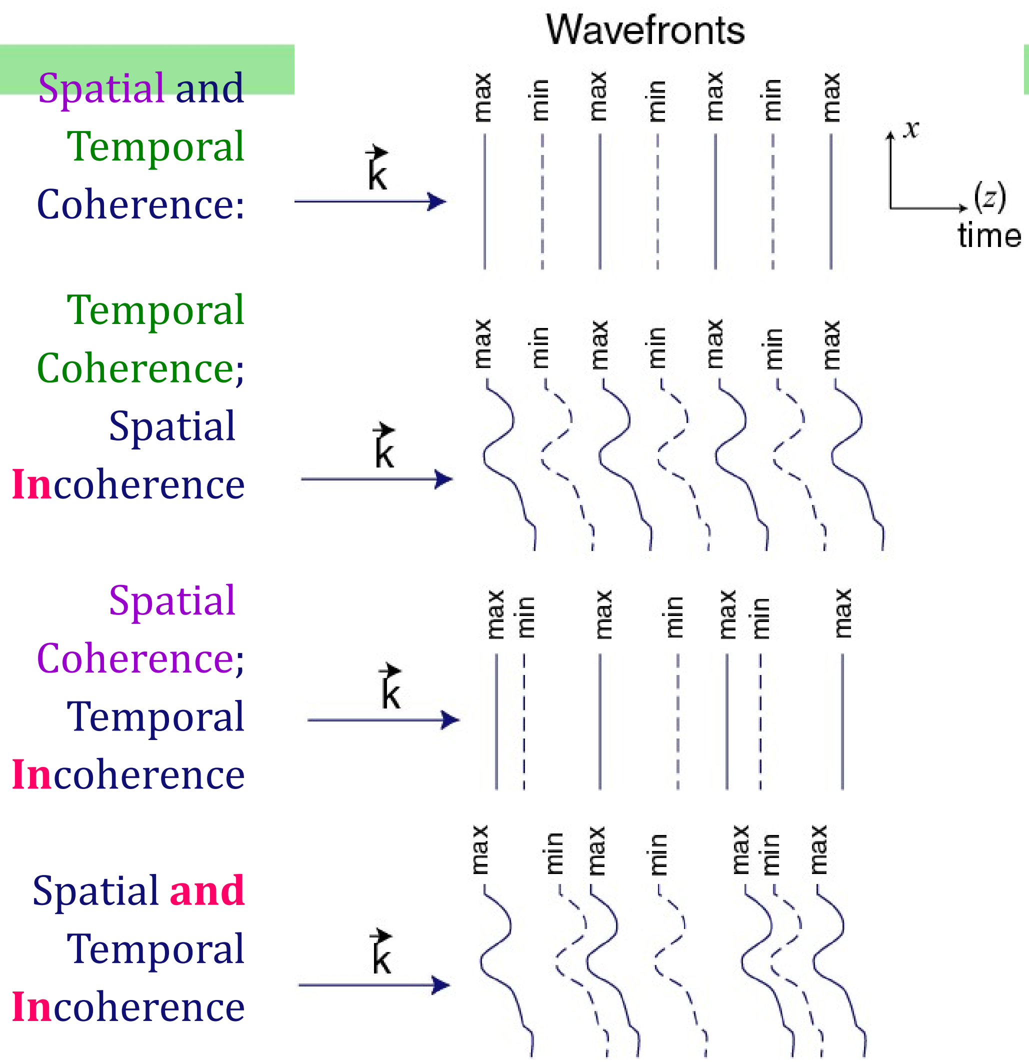
Beams can be coherent

or only partially coherent

(indeed, even incoherent)

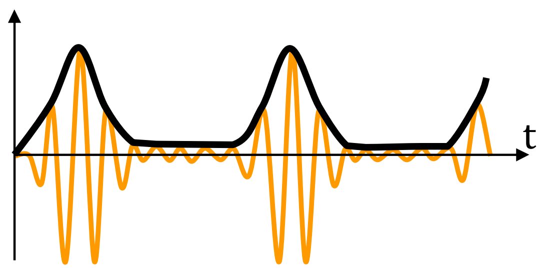
in both space and time.

Source: prof R. Trebino
Optics course

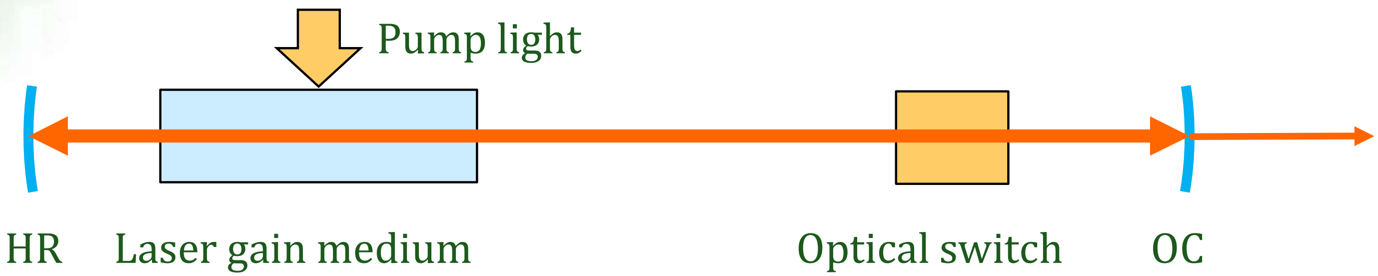


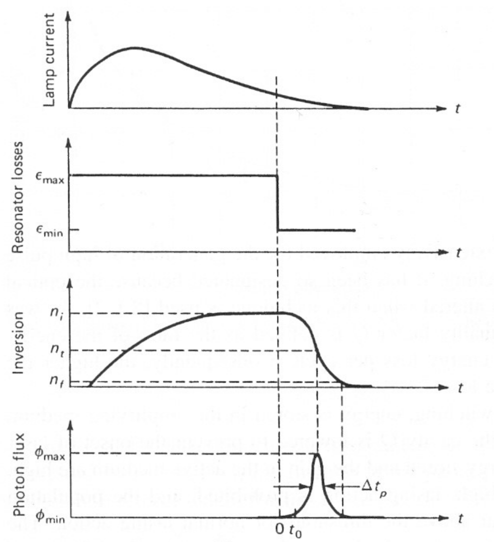
Pulsed lasers

- 2 types of lasers
  - Continuous Wave (CW):constant output power
    - continuous pump
  - Pulsed lasers: light in short pulses (ns, ps, fs)
    - continuous or pulsed pump
- What are pulsed lasers good for?
  - high peak powers
  - ultra fast optics (ps, fs)
  - broad band spectrum
  - Non-linear optics
  - Optical clocks
  - Tele/data-communication
  - (Bio)-imaging



Q-switching



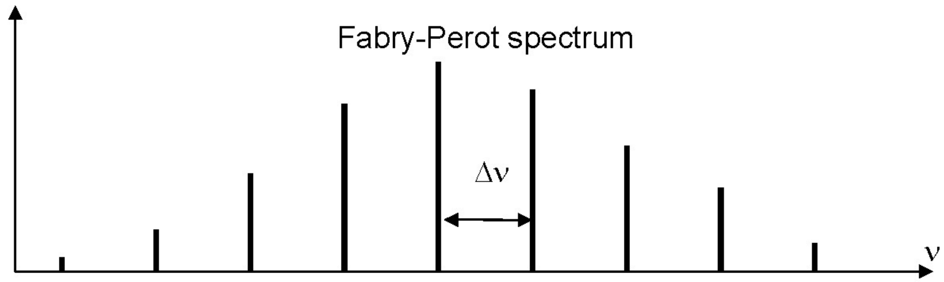


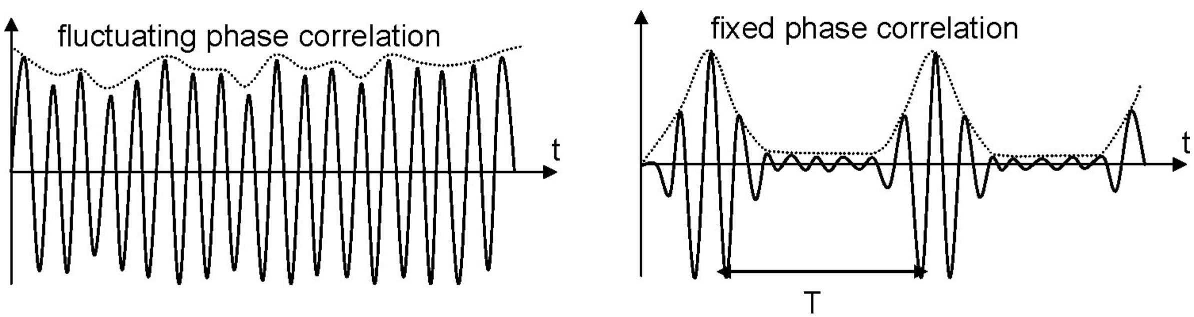
W. Koechner, Solid-State Laser Engineering 5th ed.

- Optical switch in the laser cavity
- Initially closed
- Upper lasing level occupation keeps on increasing due to pumping
- Switch opens – large optical gain short pulse can form (several to tens of nanoseconds typ. Energy can be J

https://www.rp-photonics.com/video/q_switching/q_switching.mp4

Modelocking





- Laser spectrum has equally spaced laser modes

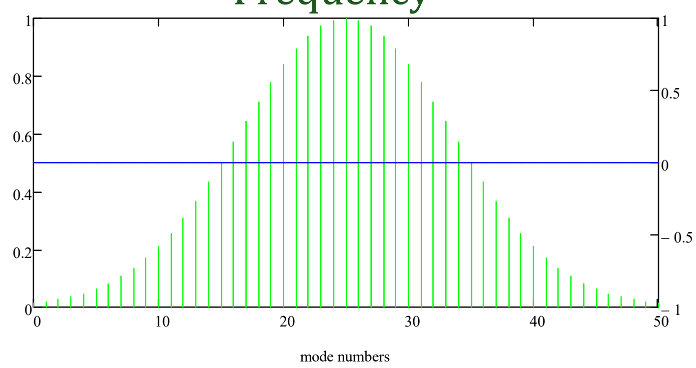
  e.g. in a linear cavity $\Delta\nu = \frac{c_0}{2nL}$ $T = \frac{1}{\Delta\nu}$

- If the phases of all the modes are equal -> pulse train output

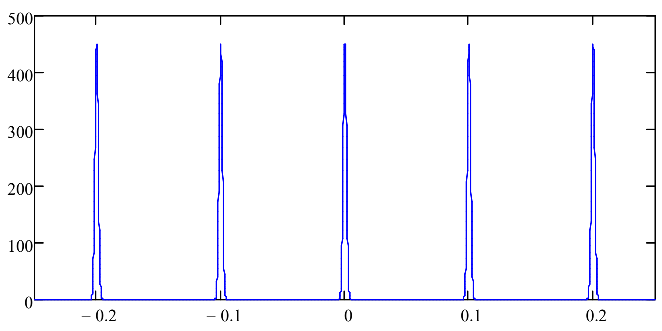
Modelocked lasers

- The phase locking can be achieved with modulators or saturable absorbers inside the cavity
- The wider the laser spectrum, the shorter the pulses
- The pulse train can be very regular (stable) -> clocks
  - These clocks are now the most stable in the world (10^{-20} stability)

Frequency



Time



Applications of pulsed lasers

- Laser machining
  The laser pulse heats the material with ns or even fs
  Material evaporates
    Laser drilling, cutting
    Industrial and medical (e.g. Femtosecond laser cateract surgery)

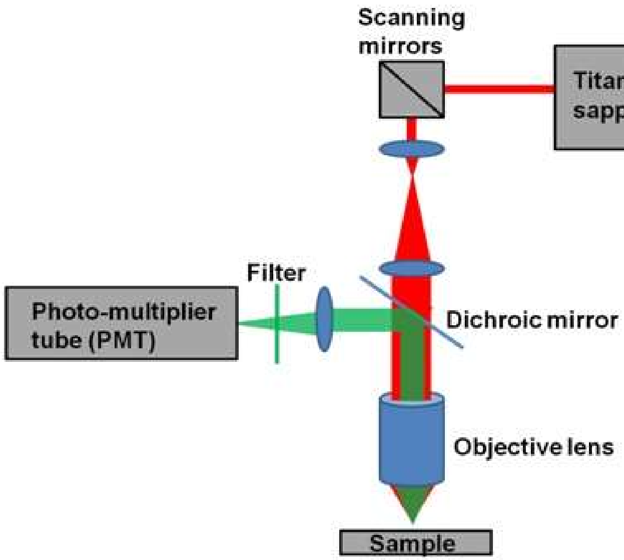
      Many examples at e.g. http://www.laser-community.com/en/

- Telecommunication
  - Directly modulated lasers (on-off keying)
  - Free space optical communication systems

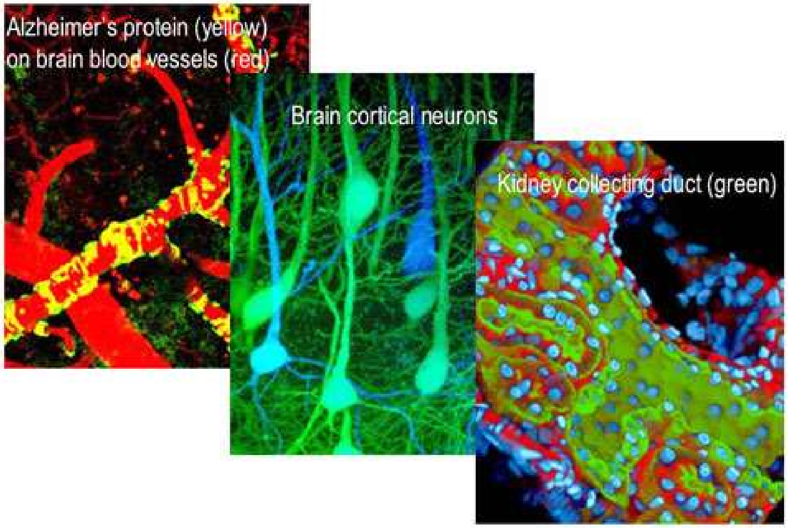
- Rapid prototyping – 3D laser printing

Applications of pulsed lasers

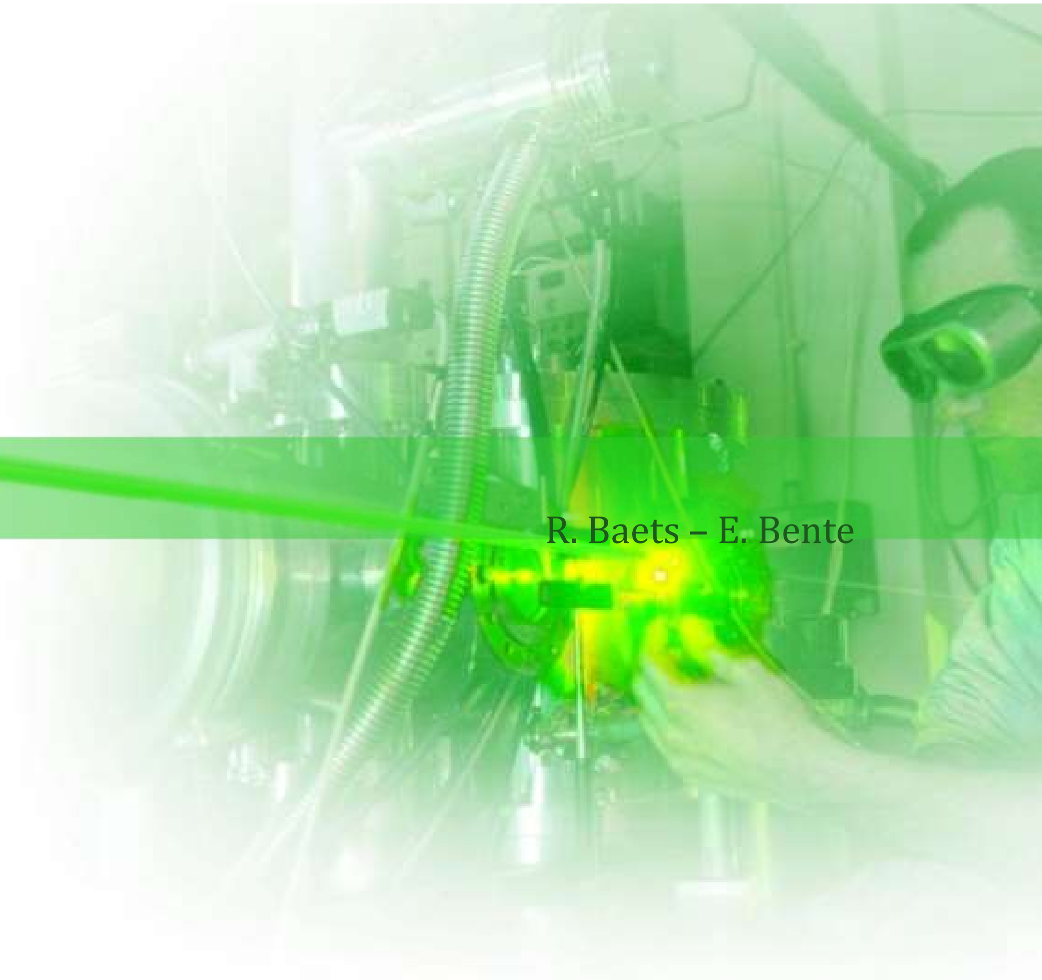
- Imaging
  - Use of pico-second or femtosecond lasers pulses in microscopy



http://biophotonics.illinois.edu/imaging-technology/imaging-techniques/multiphoton-microscopy



http://www.utoledo.edu/corelabs/amic/mutli_photon.html



Photonics

R. Baets – E. Bente

Lasers – Part F

Laser types

Lasertypes

- Laser variety due to:
  - choice of gain material
  - cavity design for high Q-factors
  - tailored to applications

  - Gain state of matter
    - gas, liquid, solid
  - Wavelength: deep UV to IR and THz
  - Mechanisms for pumping
    - optical: flash-lamps, pump-lasers
    - electrical
  - length scale: from nm to > 1m

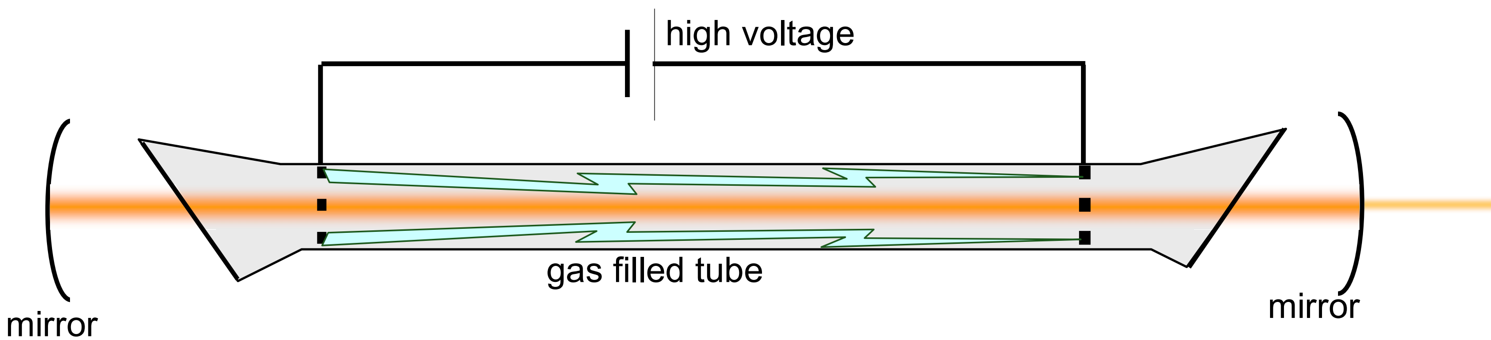
Gaslasers (1)

- Lasing levels: Atomic or Molecular electron transitions
- Pumping: electric plasma discharge causes excitation
  - high voltage!
- Typical gases
  - He-Ne (632.8nm)
    - research, alignement
  - Argon (488nm, 512nm)
    - popular pump lasers
  - CO_2 (10.6µm)
    - material processing

Gaslasers (2)

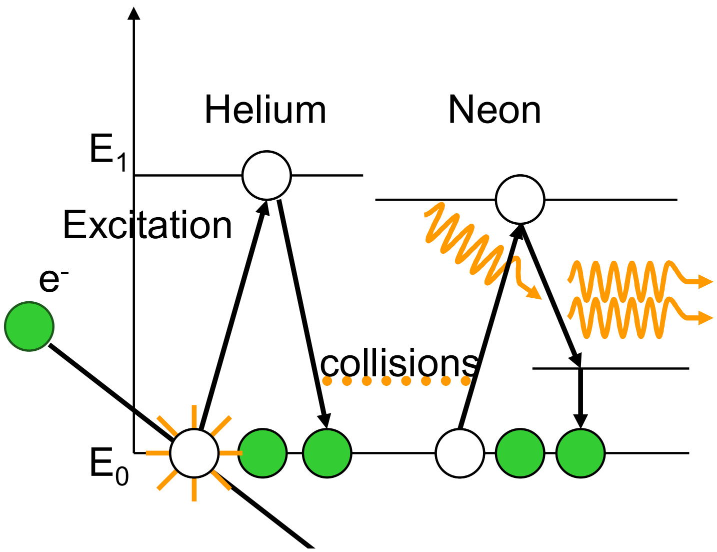
- Well defined lasing wavelength
  - not tunable

- low material gain
  - Long cavities and high Q (L = 30 cm - 3 m)
  - need for excellent mirrors (>99%)



He-Ne laser

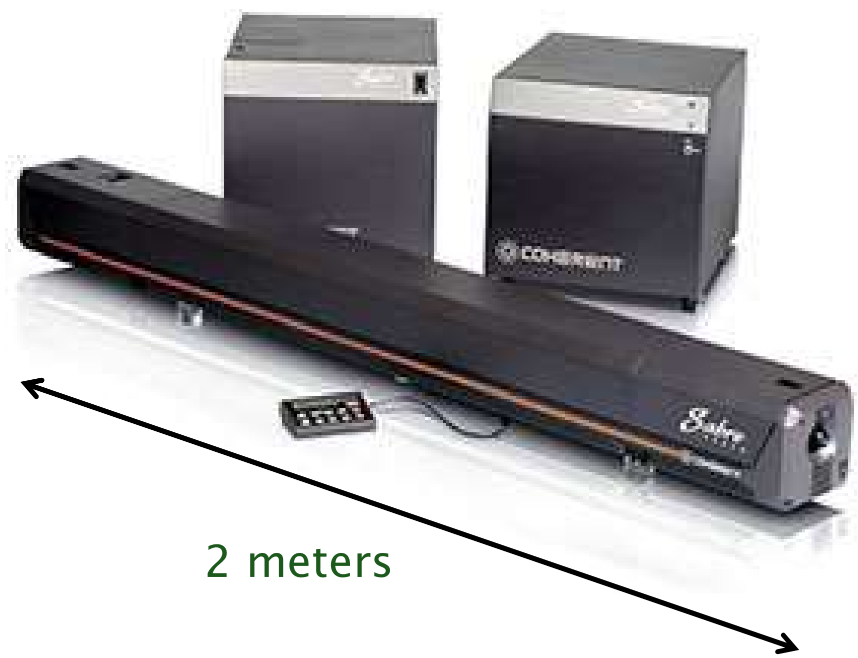
- mostly red emission (632.8 nm), some green or IR
- low-pressure gas mixture:
  90% He + 10% Ne
- pump gas: Helium
  - excitation through electron scattering
  - energy transfer to neon
- Laser gas: Neon
  - several sharp atomic transitions
  - pick lasing linewith cavity
- low output power 1 - 10 mW
- low efficiency



Ion laser

- Gas: Argon, Krypton, Copper vapour
- Ionized gas in magnetic field contained plasma
  - Argon: sharply defined lasing lines λ = 350 - 520 nm
  - Krypton: sharp defined lines in the visible
- Wavelength selection
  - prism in cavity
  - mirror reflex coating
- high output: >20 Watt
  - low efficiency
    (Ar^+: 400V at 50A for 2.5W 350nm)
  - (lots of) water cooling necessary
- Largely replaced by solid state lasers but still in use for specific applications

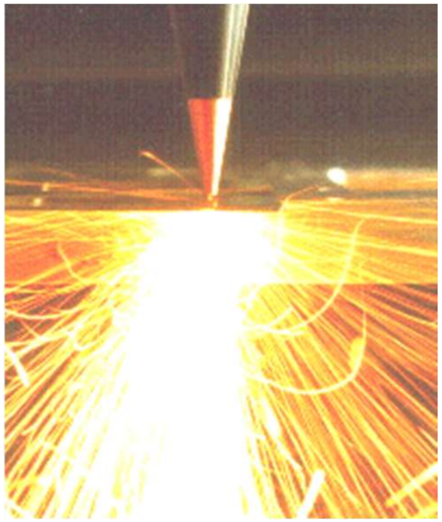
Argon ion laser 25 Watt visible – 7 W UV



www.Coherent.com

Molecular lasers

- Molecular transitions for lasing
  - CO_2, N_2, excimer: ArF KrF
- CO_2-laser
  - Pump gas: Nitrogen-Helium mix
  - Laser gas: CO_2 (vibrational and rotational transitions)
  - $\lambda = 10.6\ \mu m$ (Far IR)
  - Optics for IR: Ge, GaAs, ZnSe, diamond
  - High efficiency: up to 30%
  - Output: upto over 20kWatt
    - → material processing, **EUV source**
  - Important industrial laser



- Nitrogen-laser
  - Laser gas: N_2
  - $\lambda = 337.1$ nm

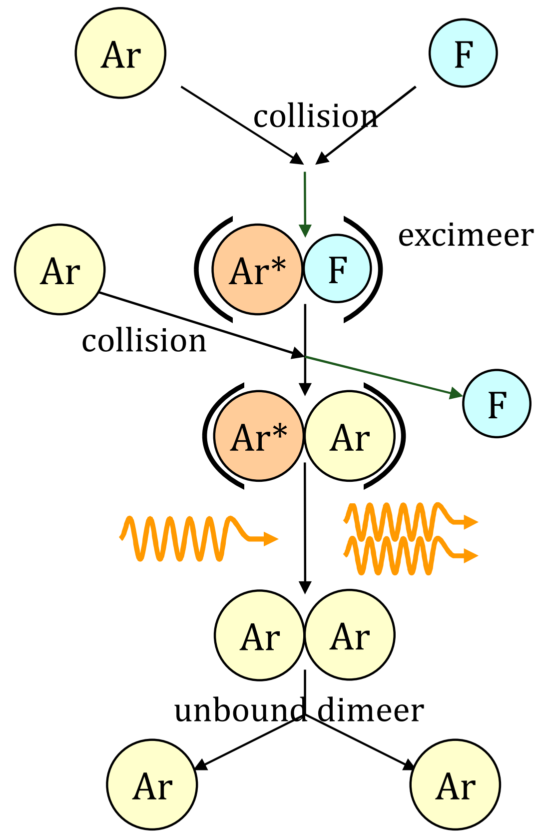
20kW CO_2 laser system for EUV source

- http://www.laser-community.com/en/euv-lithography-laser-trumpf/

Excimeerlasers

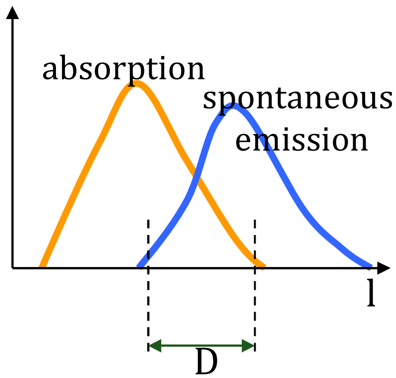
- short wavelengths (deep-UV)
  - l = 125 - 500 nm
  - high resolution (DUV) lithography
  - Medical (LASIK eye correction)
  - 100W – 1kW maximum power
  - high photon-energy:
    → photo chemistry
- Excimeer = excited dimeer
  - molecule with halogen (F, Cl,...) and noble gas (Ar, Kr, Xe,...)
  - meta-stable molecule
  - only one atom excited
- only pulsed operation – tens of ns

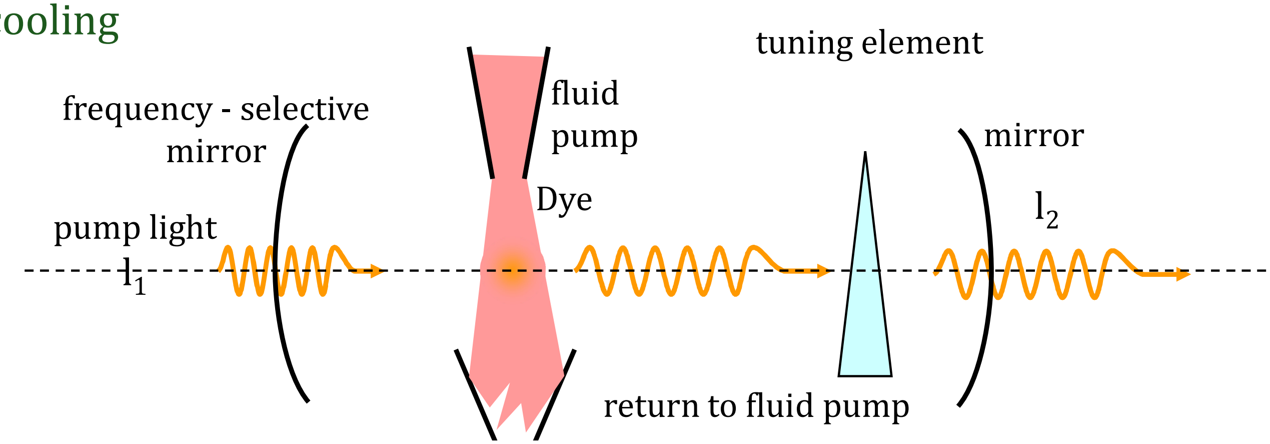
- ArF system for 193nm

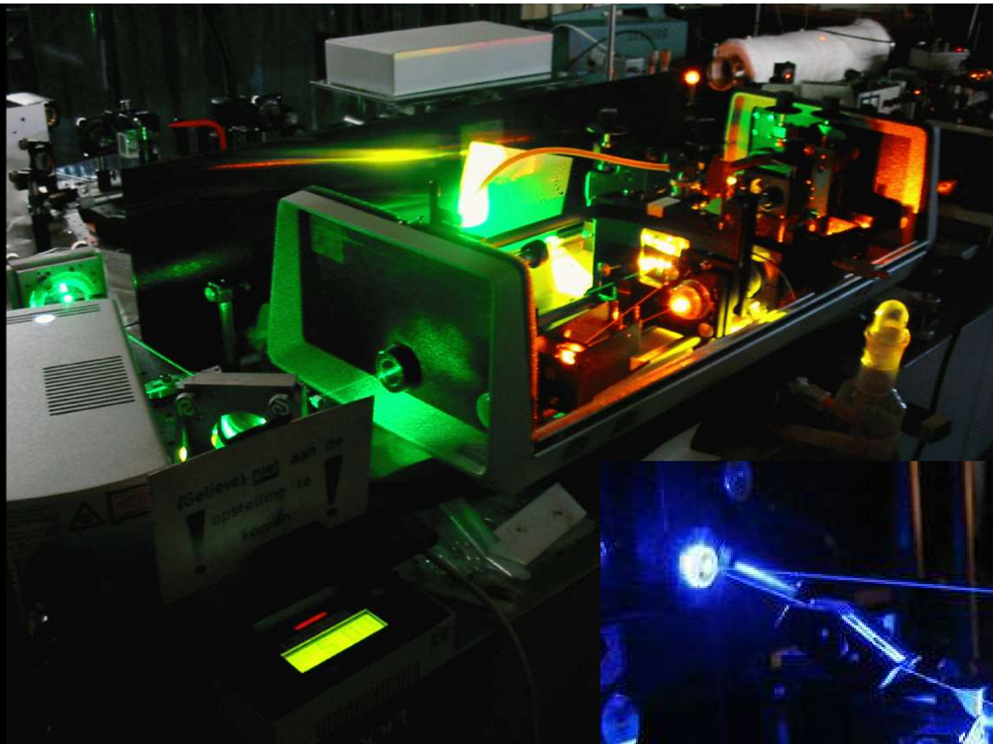


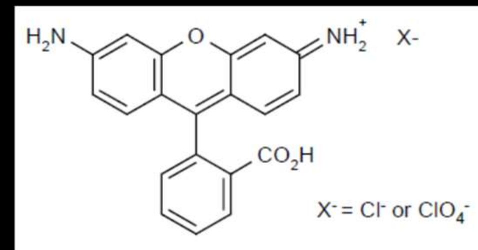
Dye-laser (liquid)

- Organic (Dyes)
  - Strong light absorption and efficient emission
  - broad spectrum: D ≈ 50 nm → tunable
  - available in VIS and IR
- Cavity
  - optical pumped (by eg. argon laser)
  - wavelength tuning with tuning element e.g. prism
- Dye: dissolved in water or alcohol
  - closed cycle fluid pump
  - allows for cooling

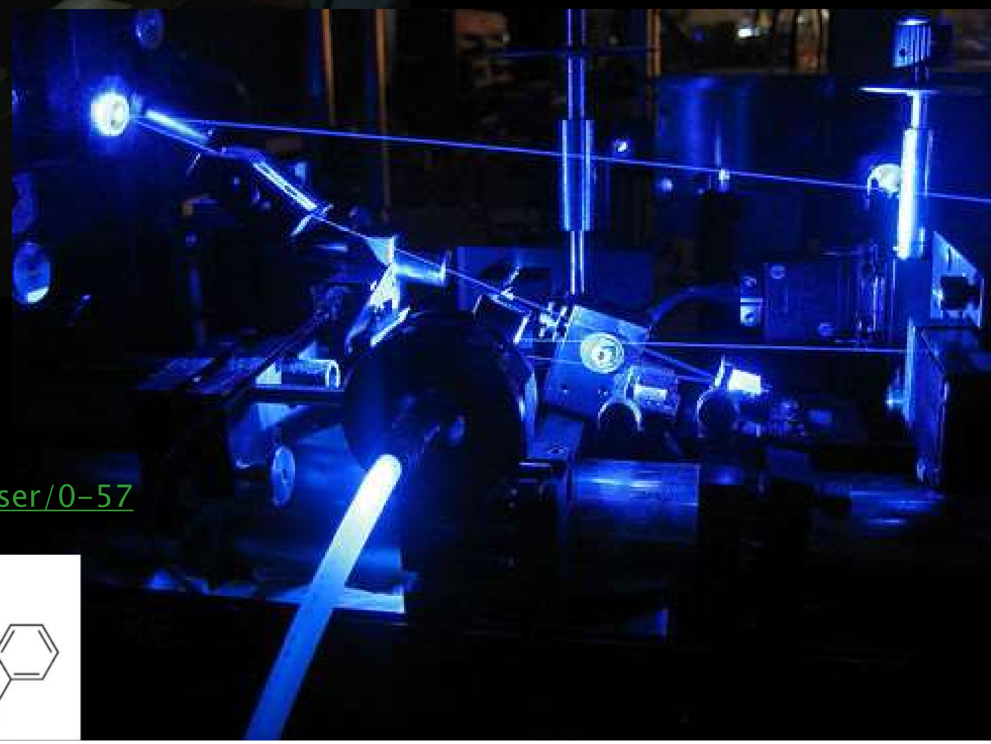




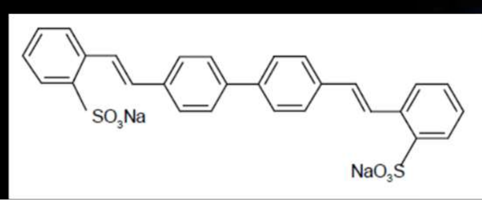




Ar-ion laser
pumped dye lasers

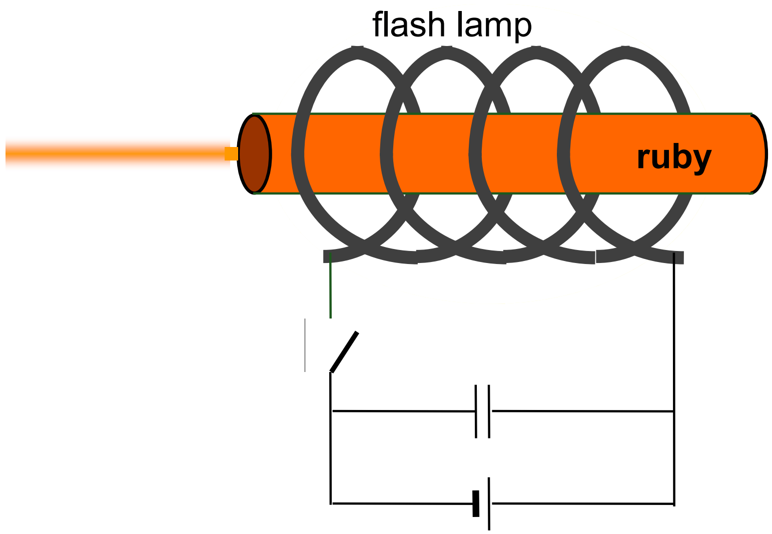


http://drhart.ucoz.com/index/dye_laser/0-57



Doped solid state lasers

- Gain medium
  - Crystalline / glass isolator (host material)
  - Doping with **metal ions**
  - **Optical** pumping
    - Using flashlamps
    - Using **laser diodes (diode pumped solid state lasers)**
- Ruby-laser
  - first laser (Maiman, 1960)
  - Al_2O_3 met $0.05\%_{vol}$ Cr^{3+}
  - Three-level system
    → hard pump
  - L=10cm en ∅=1cm
  - flashlight pumped
  - λ = 694.3 nm



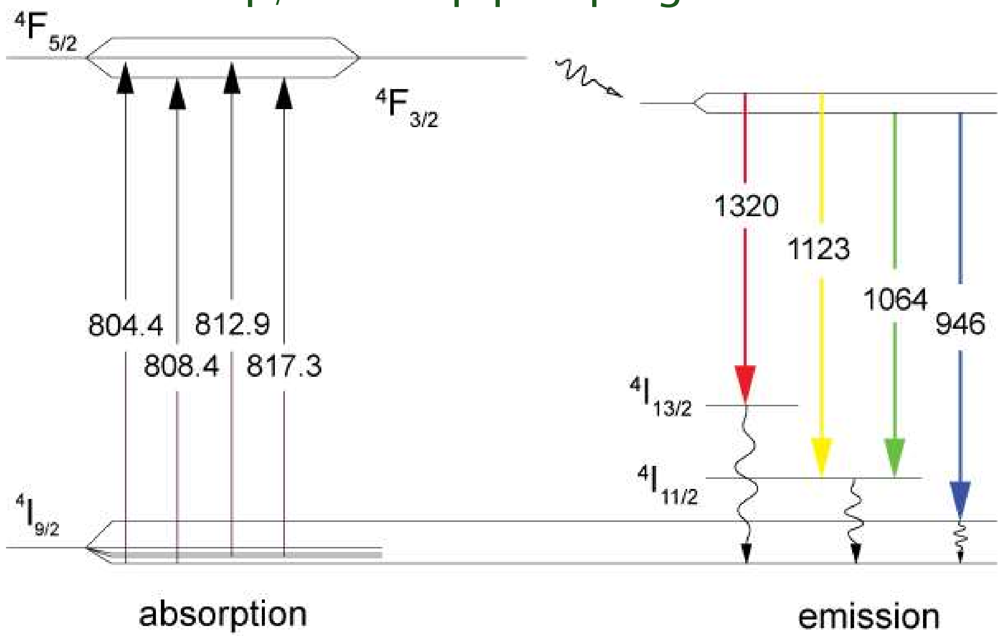
Overview of ions and host materials

Ion	host	λ	τ upper	Effective emission σ	Broadening
Nd^{3+}	YAG	1064nm	230 μs	$2.8 \cdot 10^{-19}$ cm²	homogeneous
	glass	1050–1060nm	300 μs	$\pm 4 \cdot 10^{-20}$ cm²	inhomogeneous
	YVO_4	1064nm	100 μs	$1.56 \cdot 10^{-18}$ cm²	homogeneous
Yt^{3+}	YAG	1030nm	950 μs	$2.1 \cdot 10^{-20}$ cm²	homogeneous
Er^{3+}	glass	1520–1560nm	8 ms	$5 \cdot 10^{-21}$ cm²	inhomogeneous
Ti^{3+}	Al_2O_3	700–1000nm	3.2 μs	$4 \cdot 10^{-19}$ cm² (peak λ)	homogeneous
Tm^{3+}	YAG	1870–2160nm	10 ms	$2 \cdot 10^{-21}$ cm²	inhomogeneous

Thermal and optical properties of the host material are important
Ion and host determine wavelength/tuning range, lifetime and cross-section

Nd:YAG levels

- Absorption and lasing wavelengths in nm.
- Diode pumping or flashlamp/arc lamp pumping



- Absorption lines for diode laser pumping

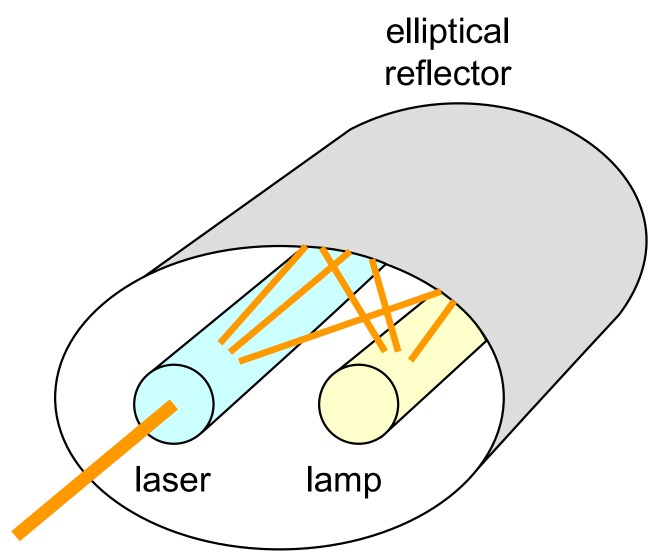
http://www.photonics.ld-didactic.com/Educational%20Kits/P5853.html

https://www.rp-photonics.com/yag_lasers.html

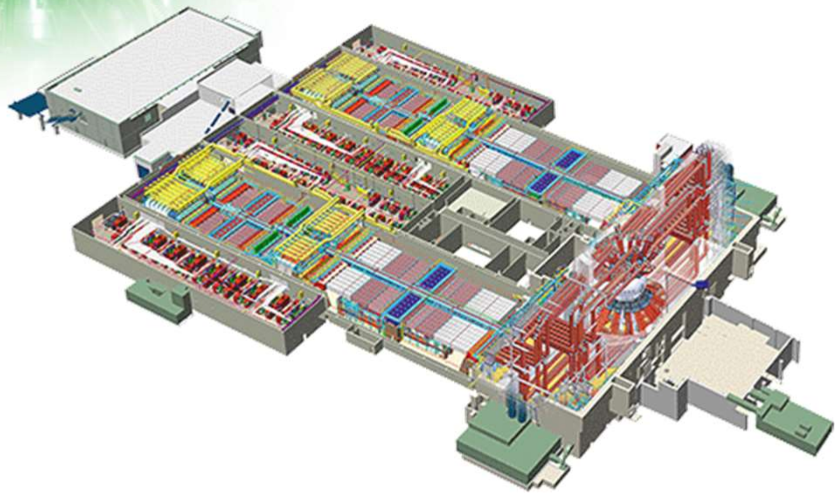
Neodymium-YAG-laser

- Gain medium
  - Yttrium-Aluminum garnet ($Y_3Al_5O_{12}$) doped with Nd^{3+}
  - four-level-system
  - λ = 1.06um -1.3 um

- Optically pumped
  - Pulsed or continuous
  - Flash lamps – laser diode arrays (808nm)
  - Elliptical reflector with lamp and laser in focus
  - high power (>100 Watt)
  - efficiency : few %
  - material processing



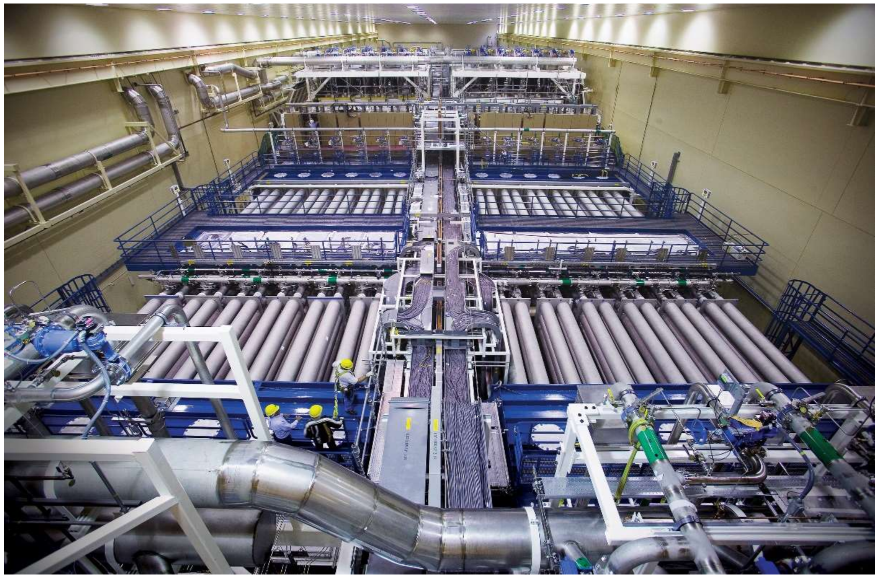
National Ignition Facility



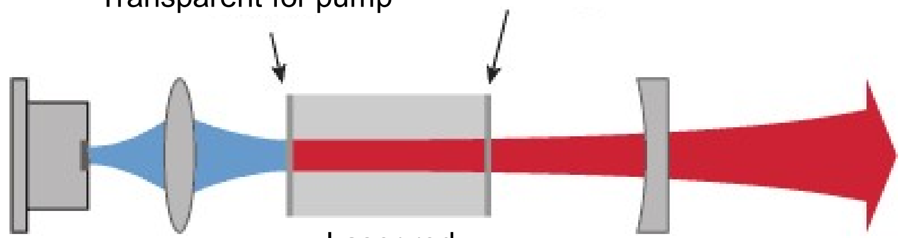
192 laser beams
Nd:glass laser system
Frequency tripling to 350 nm

https://lasers.llnl.gov/about/what-is-nif

https://lasers.llnl.gov/content/assets/images/media/photo-gallery



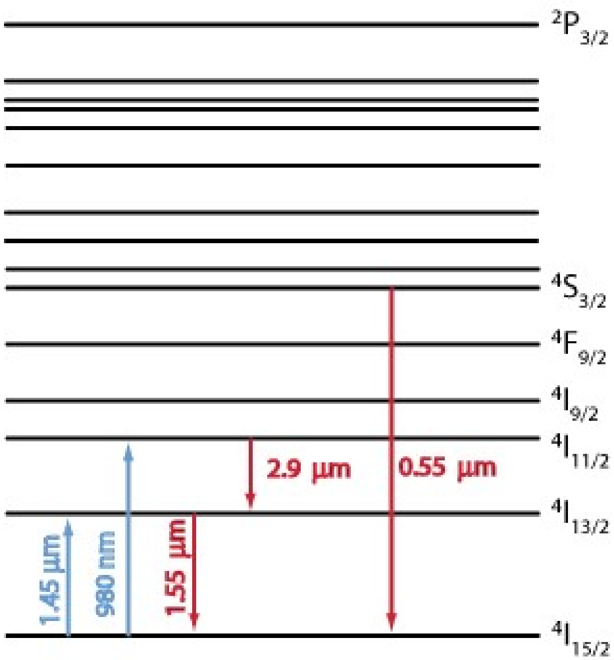
Diode pumped solid state laser



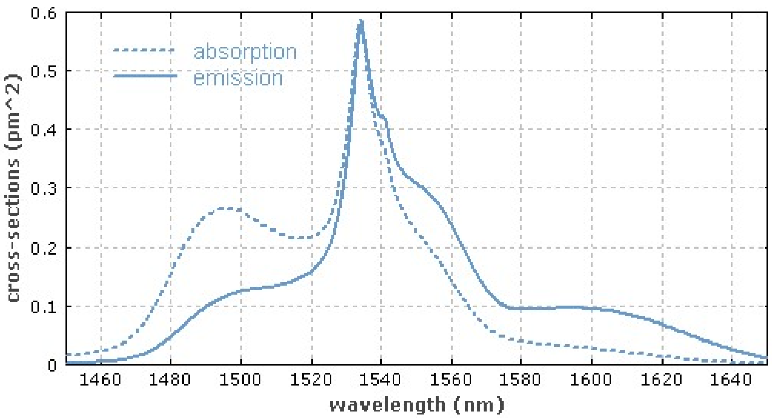
- Pump laser diodes – efficient (e.g. 60%)
  - Low beam quality, bandwidth (1–2 nm)
  - Can be fibre coupled using multimode fibre (large core 50 – 200 μm)
- Efficient absorption in laser material
- High quantum efficiency

https://s7d1.scene7.com/is/image/coherentinc/Verdi_V2?$vlarge$

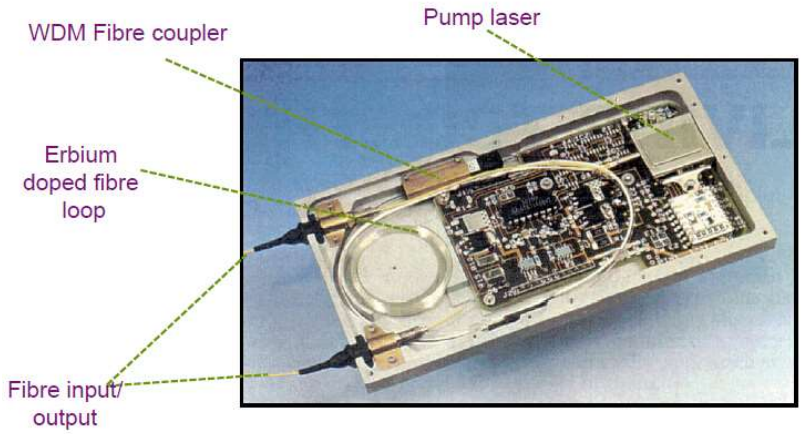
Er:glass levels – basis of EDFA



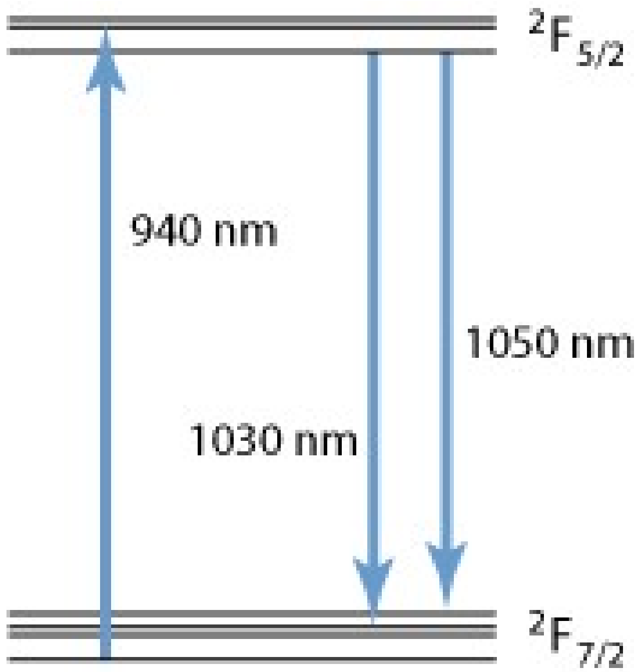
https://www.rp-photonics.com/erbium_doped_gain_media.html



- Erbium doped fibre amplifier



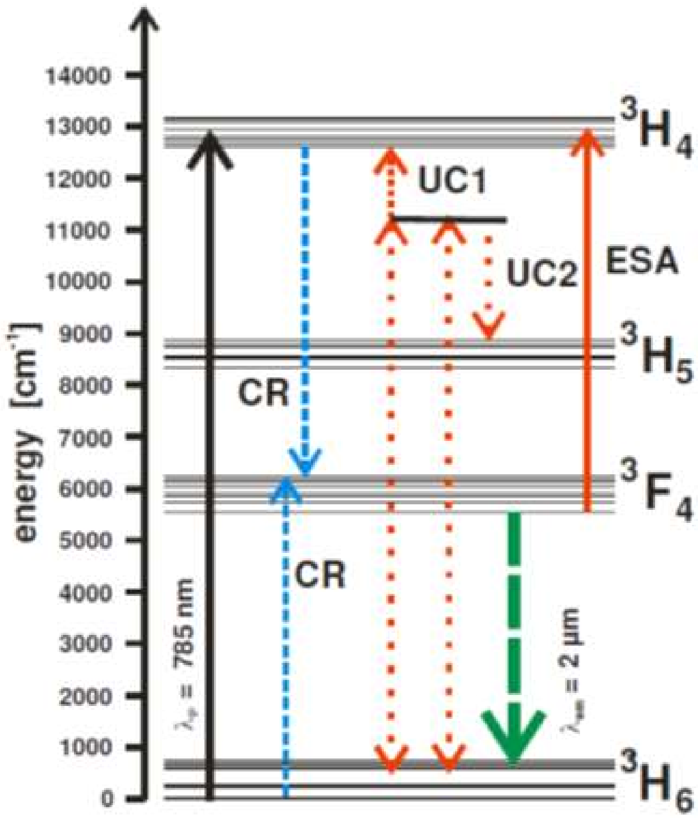
Yt:YAG levels



- Very high quantum efficiency,
- Low heat load
- Quasi three level laser
  - Lower laser level populated at room temperature
- Yt: glass material much used in high power fibre lasers

https://www.rp-photonics.com/ytterbium_doped_gain_media.html

Tm:YAG laser



http://cdn.intechopen.com/pdfs/8446/InTech-2_m_laser_sources_and_their_possible_applications.pdf

Output at 2 micron

Quantum efficiency can be close to 2 !
Decay from highest level to upper lasing level is by emission of a photon that is absorbed by ions in the ground state to the upper lasing level!

Medical applications

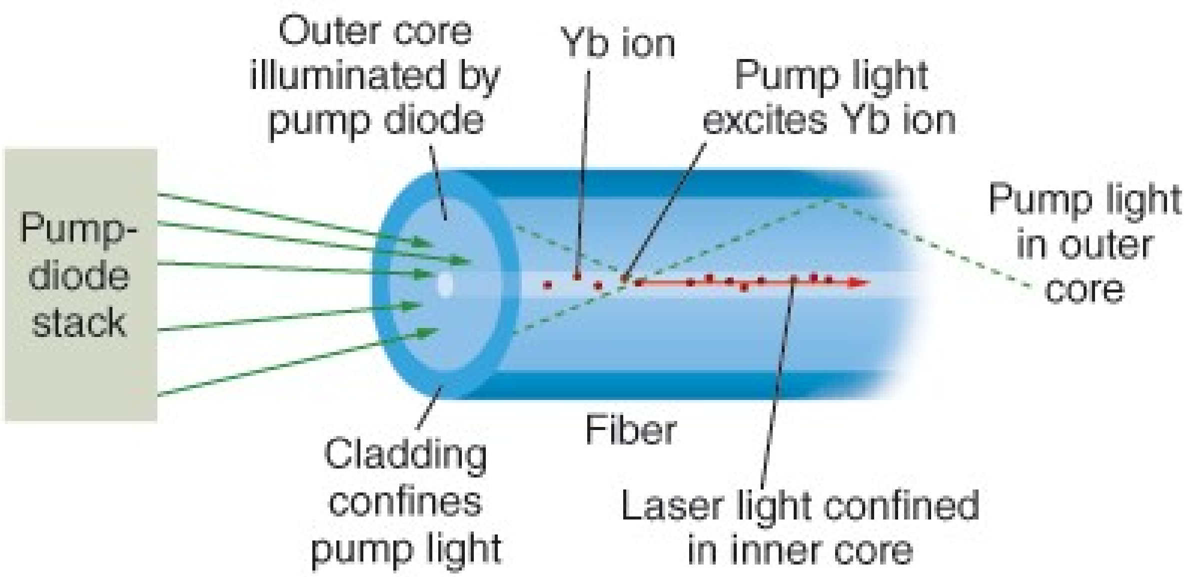
Tunable solid-state-lasers

- gain medium
  - broad gain for tuneability
- Wavelength selection with
  - intra-cavity filter and mirror coating

- Titanium-sapphire-laser
  - Sapphire doped with Ti^{3+} ions
  - Optically pumped with gas or diode laser
  - tunable between $\lambda=0.7\mu m$ and $\lambda=0.95\ \mu m$
  - Modelocked operation:
    extremely short pulses downto 5 fs.

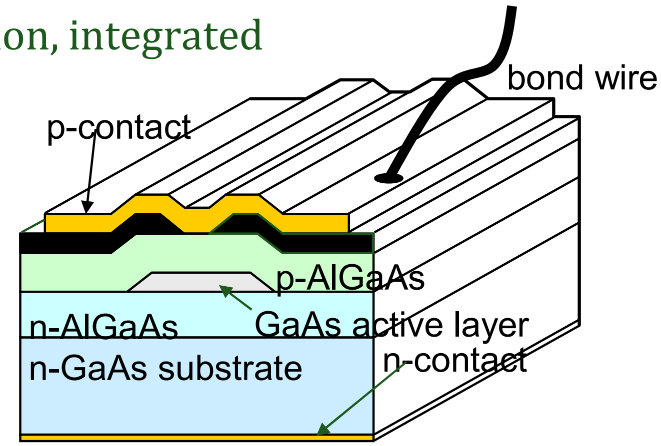
Fiber lasers

- Doping of the fiber core using rare earths (Ytterbium, Erbium, Thulium,…) to address different wavelength ranges
- Cavity can be a ring-type cavity, can have external mirrors,…
- Has started to replace many other type of lasers – upto 100 kW

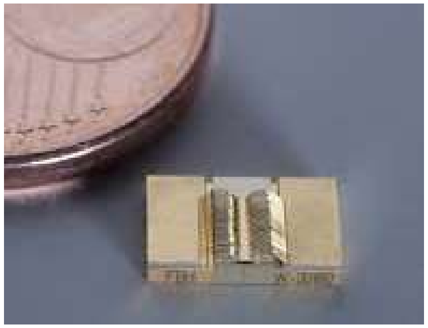


Semiconductor laser diodes

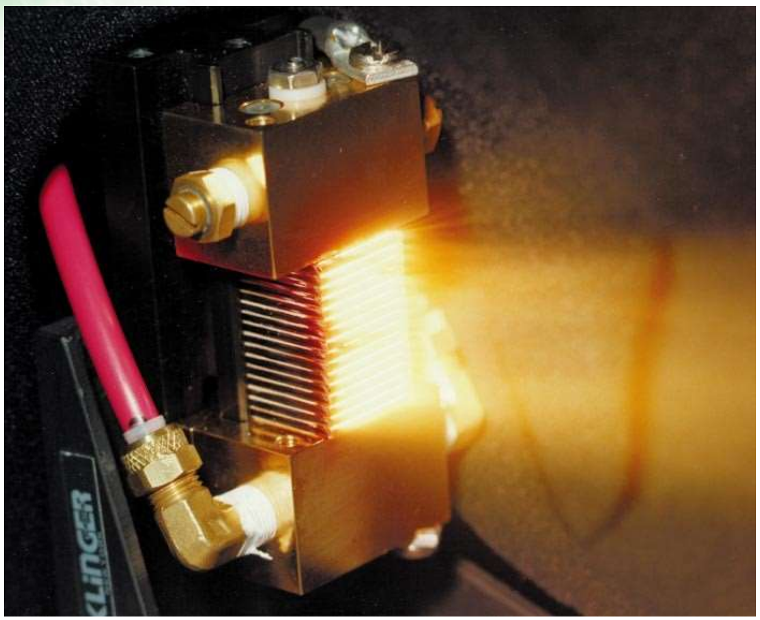
- Pump mechanism: electron hole recombination
- Cavity= reflection cleaved facets, distributed Bragg reflection, integrated circuits



- very small (μm → mm)
- broad gain media → tuneable
- available in wide λ range
  UV to FIR
- Cheap (mass production)
- disadvantages:
  - beam quality
  - line width
  - lasing wavelength variation
  - limited power <100W



High power 2D diode laser array



A high-power (1.45-kW CW) semiconductor laser diode array using a microchannel cooler (electrical-optical eff. 60%) 1cm x 2.5cm

- High power CW or long pulsed operation (e.g. 0.5 ms), low beam quality
- Each stripe is a linear array of laser diodes
- These arrays are stacked
- Solid state laser pumping
- Heating – e.g. soldering