

List of Equations (only some of these equations are required to solve question 3 and 4)

Power delivered to the load	$P_L = \frac{ V_2^- ^2}{2Z_0} (1 - \Gamma_L ^2)$
Input power to the network	$P_{in} = \frac{ V_1^+ ^2}{2Z_0} (1 - \Gamma_{in} ^2)$
Input and output reflection coefficients of a transistor with a source and load: general case	$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$ $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$
Input and output reflection coefficients of a transistor with a source and load: unilateral case	$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$ $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22}$
Gain of the input matching network	$G_S = \frac{1 - \Gamma_S ^2}{ 1 - \Gamma_{in}\Gamma_S ^2}$
Gain of the output matching network	$G_L = \frac{1 - \Gamma_L ^2}{ 1 - S_{22}\Gamma_L ^2}$
Gain of the transistor (unilateral case)	$G_0 = S_{21} ^2$
Transducer gain of the basic amplifier circuit (input matching, unilateral transistor, output matching)	$G_T = G_S G_0 G_L$ $G_{T,dB} = G_{S,dB} + G_{0,dB} + G_{L,dB}$
Maximum gain of the input and output matching networks	$G_{S_{max}} = \frac{1}{1 - S_{11} ^2},$ $G_{L_{max}} = \frac{1}{1 - S_{22} ^2}.$
Maximum transducer power gain, unilateral case	$G_{TU_{max}} = \frac{1}{1 - S_{11} ^2} S_{21} ^2 \frac{1}{1 - S_{22} ^2}$
Normalized gain factors g_s and g_L	$g_s = \frac{G_S}{G_{S_{max}}} = \frac{1 - \Gamma_S ^2}{ 1 - S_{11}\Gamma_S ^2} (1 - S_{11} ^2),$ $g_L = \frac{G_L}{G_{L_{max}}} = \frac{1 - \Gamma_L ^2}{ 1 - S_{22}\Gamma_L ^2} (1 - S_{22} ^2).$
Center and radius of the constant gain circle for the input matching network	$C_S = \frac{g_s S_{11}^*}{1 - (1 - g_s) S_{11} ^2},$ $R_S = \frac{\sqrt{1 - g_s} (1 - S_{11} ^2)}{1 - (1 - g_s) S_{11} ^2}$
Center and radius of the constant gain circle for the output matching network	$C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L) S_{22} ^2},$ $R_L = \frac{\sqrt{1 - g_L} (1 - S_{22} ^2)}{1 - (1 - g_L) S_{22} ^2}$

Condition for “unconditionally stable” device, general case	<p>for all $\Gamma_L < 1$ and $\Gamma_S < 1$</p> $\Rightarrow \begin{cases} \Gamma_{in} = \left S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right < 1 \\ \Gamma_{out} = \left S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right < 1 \end{cases}$
Conditions for “unconditionally stable” device, unilateral case	$ \Gamma_{in} = S_{11} < 1$ $ \Gamma_{out} = S_{22} < 1$
Center and radius of the stability circles, load side	$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{ S_{22} ^2 - \Delta ^2} \quad (\text{center}),$ $R_L = \left \frac{S_{12}S_{21}}{ S_{22} ^2 - \Delta ^2} \right \quad (\text{radius}).$ $\Delta = S_{11}S_{22} - S_{12}S_{21}$
Center and radius of the stability circles, source side	$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{ S_{11} ^2 - \Delta ^2} \quad (\text{center}),$ $R_S = \left \frac{S_{12}S_{21}}{ S_{11} ^2 - \Delta ^2} \right \quad (\text{radius}).$ $\Delta = S_{11}S_{22} - S_{12}S_{21}$
Test for unconditional stability, general case	$ \Delta = S_{11}S_{22} - S_{12}S_{21} < 1$ <p>and</p> $K = \frac{1 - S_{11} ^2 - S_{22} ^2 + \Delta^2}{2 S_{12}S_{21} } > 1$
Test for unconditional stability, unilateral case	$ S_{11} < 1$ $ S_{22} < 1$
Noise figure of a 2-port amplifier	$F = F_{\min} + \frac{r_N}{g_S} \left y_S - y_{opt} \right ^2$ $F = F_{\min} + 4r_N \frac{ \Gamma_S - \Gamma_{opt} ^2}{(1 - \Gamma_S ^2)(1 + \Gamma_{opt} ^2)}$
Constant noise circles	$C_F = \frac{\Gamma_{opt}}{1 + N}$ $R_F = \frac{1}{1 + N} \sqrt{N^2 + N(1 - \Gamma_{opt} ^2)}$ $F_{\min} = 10^{\frac{NF_{\min}}{10}}$ $\Delta F_n' = N = (F - F_{\min}) \frac{1 + \Gamma_{opt} ^2}{4r_n} = \frac{ \Gamma_S - \Gamma_{opt} ^2}{1 - \Gamma_S ^2}$

Output noise, input noise, equivalent noise temperature, noise factor and noise figure	$N_o = Gk_B(T_0 + T_e)B$ $N_i = k_B T_0 B$ $F = \frac{\frac{S_i}{N_i}}{\frac{S_o}{N_o}} = \frac{S_i}{S_o} \frac{N_o}{N_i} = \frac{1}{G} \frac{Gk_B(T_0 + T_e)B}{k_B T_0 B} = 1 + \frac{T_e}{T_0}$ $NF = 10 \log_{10} F$
Three-stage amplifier: Output noise ($P_{n,total}$), noise factor (F_{total}) noise figure (NF_{total})	$P_{n,total} = G_{A3}G_{A2}G_{A1}P_{n,in} + G_{A3}G_{A2}P_{n1} + G_{A3}P_{n2} + P_{n3}$ $F_{total} = \frac{P_{n,total}}{G_{A3}G_{A2}G_{A1}P_{n,in}}$ $F_{total} = 1 + \frac{P_{n1}}{G_{A1}P_{n,in}} + \frac{P_{n2}}{G_{A1}G_{A2}P_{n,in}} + \frac{P_{n3}}{G_{A3}G_{A2}G_{A1}P_{n,in}}$ $F_{total} = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1}G_{A2}}$ $NF_{total} = 10 \log_{10} F_{total}$ <p>Noise factor of single stage $F_j = 1 + \frac{P_{nj}}{G_{Aj}P_{n,in}}, j = 1, 2, 3$</p> <p>Noise figure of single stage $NF_j = 10 \log_{10} F_j, j = 1, 2, 3$</p>
Receiver sensitivity	$P_{sens} [dBm] = k_B T_0 B [dBm] + NF_{total} [dB] + SNR [dB]$ $P_{sens} [dBm] = -174 + NF_{total} + 10 \log_{10} B + SNR$ <p>k_B – Boltzman constant, $k_B = 1.38 \cdot 10^{-23} \frac{Watt \cdot s}{K}$</p>
Conversion Watt to dBm	$P_{sens} [dBm] = 10 \log_{10} \frac{P_{sens} [Watt]}{1mWatt}$ $P_{sens} [dBm] = 10 \log_{10} \frac{P_{sens} [Watt]}{10^{-3} Watt}$
Gain conversion from linear to dB	$G [dB] = 10 \log_{10} G$
Friis radio link formula	$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi R} \right)^2$ <p>P_{RX} – power at RX input P_{TX} – power at TX output G_{RX} – RX antenna gain G_{TX} – TX antenna gain λ – wave length $\lambda = \frac{c}{f}$ c – speed of light, $c = 3 \cdot 10^8 \frac{m}{s}$ f – frequency</p>