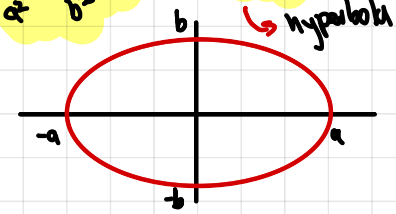
Week 1

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \leftrightarrow \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

hyperbola



Absolute Value

$|ax+b|=c \qquad |ax+b|\le 9$

$ax+b=\pm c \qquad -9\le ax+b\le 9$

$$\boxed{(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2\ldots\binom{n}{r}a^{n-r}b^r\ldots+b^n}$$

- Sum of even functions ⇒ even
- Sum of odd functions ⇒ odd

41) $|x+1|>|x-3|$

$x+1>-x+3$

$2x>2$

$x>1$

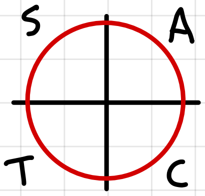
Reflections:

$a-x$ in place of x ⇒ reflect over $x=\frac{a}{2}$

$b-y$ in place of y ⇒ reflect over $y=\frac{b}{2}$

Switch x and y ⇒ Reflect over $y=x$

Week 2



$\sin(\frac{\pi}{2}-x)=\cos(x)$

$\cos(\frac{\pi}{2}-x)=\sin(x)$

$\tan(x)=\frac{\sin(x)}{\cos(x)}$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	DNE

Trig Identities-

$\sin^2(x)+\cos^2(x)=1$

$\sin(x\pm y)=\sin(x)\cos(y)\pm\sin(y)\cos(x)$

$\sin(2x)=2\sin(x)\cos(x)$

$\cos(x\pm y)=\cos(x)\cos(y)\mp\sin(x)\sin(y)$

$\cos(2x)=\cos^2(x)-\sin^2(x)$

$=2\cos^2(x)-1$

$=1-2\sin^2(x)$

Properties-

$f(x)=a\sin(b(x-c))+d$

a=amplitude

d= vertical shift

c= Horizontal Shift

period $=\frac{2\pi}{b}$

Vectors-

Lines ⇒ $L=\begin{pmatrix}a\\b\\c\end{pmatrix}+\lambda\begin{pmatrix}d\\e\\f\end{pmatrix}$

$$\frac{x-a}{d}=\frac{y-b}{e}=\frac{z-c}{f}$$

Dot Product:

$$u\cdot v=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}\cdot\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\begin{pmatrix}u_1\cdot v_1\\u_2\cdot v_2\\u_3\cdot v_3\end{pmatrix}=|u|\cdot|v|\cdot\cos\theta$$

Distances:

Point ⇒ line/plane:

- Line perpendicular that passes through P

Point ⇒ Line in $\mathbb{R}^3$:

- Plane perpendicular to line containing point and find point L that goes through plane

Intersection of 2 planes ⇒ Cross product of normals to plane for direction and find point

Cross Product:

$$u\times v=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}\times\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\begin{pmatrix}u_2v_3-u_3v_2\\u_3v_1-u_1v_3\\u_1v_2-u_2v_1\end{pmatrix}=|v||u|\cdot\sin\theta\cdot\hat{n}$$

Planes ⇒ $\Pi:\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}+\lambda\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}+\mu\begin{pmatrix}x_3\\y_3\\z_3\end{pmatrix}$

$ax+by+cz=d \qquad n=\begin{pmatrix}a\\b\\c\end{pmatrix}$

$r\cdot n=a\cdot n$

$n_1\cdot x+n_2\cdot y+n_3\cdot z=n_1x_0+n_2y_0+n_3z_0$

Give 2 planes with given intersection Line:

- Choose any point and find 2 vectors to 2 points on line
- Cross product gives normal

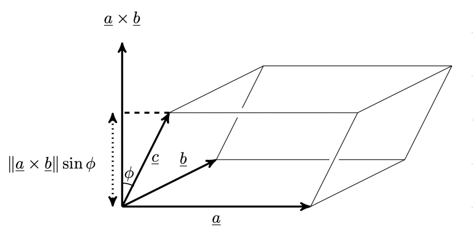
or

- $L=\begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix}+\lambda\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$

$x=x_0+\lambda x_1 \qquad \lambda=\frac{x-x_0}{x_1}$

$y=y_0+\lambda y_1$ sub into

$z=z_0+\lambda z_1$



$V=|a\times b\cdot c|$

Week 3

eg/ $f(x)=\sqrt{x}$ not differentiable at $x=0$ as can't approach from left

Continuity:

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = \lim_{x\to c} f(x)$$

Differentiability:

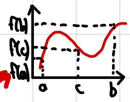
$$\underbrace{\lim_{h\to 0^-} \frac{f(x+h)-f(x)}{h}}_{\text{slope from left}} = \underbrace{\lim_{x\to 0^+} \frac{f(x+h)+f(x)}{h}}_{\text{slope from right}}$$

• Differentiability implies continuity but not vice versa

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\pm 0}{0} \rightarrow$$ evaluate both sides which will go to $\pm\infty$

$= \pm\infty$ or DNE if different

eg/ $\lim_{x\to 0} \frac{1}{x} =$ DNE



Squeeze Theorem ⇒

$f(x) \leq g(x) \leq h(x)$ around $x=a$

then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x) \leq \lim_{x\to a} h(x)$

$$\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad \lim_{x\to \infty} \frac{\sin(x)}{x} = 0}$$

Difficult Limits:

- $\frac{\pm 0}{0} \rightarrow$ Consider sides
- $\frac{0}{0} \rightarrow$ Factor / L'Hopital / Taylor
- $\frac{\infty}{\infty} \rightarrow$ Divide by highest power
- $\infty - \infty \rightarrow$ Write 1 term
- $0 \cdot \infty \rightarrow$ Which goes to $0/\infty$ fastest or write as $\frac{0}{0}$ / $\frac{\infty}{\infty}$
- 1^∞, ∞^0, 0^0 → $e^{\ln}$ trick

Intermediate Value Theorem:

• If $f(a) \leq s \leq f(b)$ there exists value $a \leq c \leq b$ such that $f(c)=s$

• To show there is a solution on interval

$$\boxed{f(x)=|x| \qquad f'(x)=\frac{x}{|x|}}$$

$$\boxed{f(x)=a^x \quad f'(x)=\ln a \cdot a^x}$$

$$f(x)^{g(x)} \Rightarrow e^{\ln(f(x)^{g(x)})} = e^{g(x)\ln(f(x))}$$

Tangent:

$$y = f(a) + f'(a)(x-a)$$

Differentiation Rules:

$$\frac{d}{dx} u\cdot v = u\cdot\frac{dv}{dx} + v\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$$

Trig Differentiation:

$f(x)=\sin(x)$	$f'(x)=\cos(x)$
$f(x)=\cos(x)$	$f'(x)=-\sin(x)$
$f(x)=\tan(x)$	$f'(x)=\frac{1}{\cos^2 x}=1+\tan^2 x$
$f(x)=\arctan x$	$f'(x)=\frac{1}{1+x^2}$
$f(x)=\arcsin x$	$f'(x)=\frac{1}{\sqrt{1-x^2}}$
$f(x)=\arccos x$	$f'(x)=\frac{-1}{\sqrt{1-x^2}}$

Inverse Functions:

• Reflect over $y=x$

• Prove that function is one to one → Show derivative is always increasing/decreasing on interval

$$(f^{-1}(y))' = \frac{1}{f'(x)}$$

Important

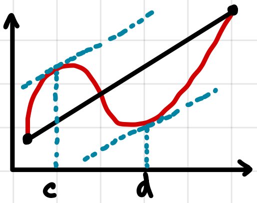
Week 4

$$(f^{-1})'(y) = \frac{1}{f'(x)}$$

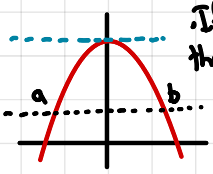
$$\boxed{\arccos x = \frac{\pi}{2} - \arcsin x}$$

Mean Value Theorem -

$$\underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{Average slope}} = f'(c)$$

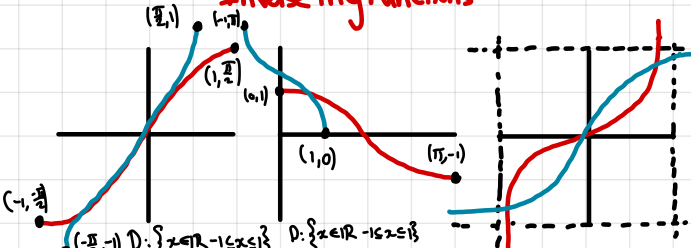


Rolle's Theorem



If $f(a)=f(b)$, then there exists c on $[a,b]$ so that $f'(c)=0$

Inverse Trig Functions -



arcsinx: D: $\{x\in\mathbb{R} \mid -1\leq x\leq 1\}$, R: $\{y\in\mathbb{R} \mid -\frac{\pi}{2}\leq y\leq\frac{\pi}{2}\}$

arccosx: D: $\{x\in\mathbb{R} \mid -1\leq x\leq 1\}$, R: $\{y\in\mathbb{R} \mid 0\leq y\leq\pi\}$

arctanx: D: $\{x\in\mathbb{R}\}$, R: $\{y\in\mathbb{R} \mid -\frac{\pi}{2}< y<\frac{\pi}{2}\}$

$$\boxed{\begin{aligned} \arcsin'(x) &= \frac{1}{\sqrt{1-x^2}} \\ \arccos'(x) &= \frac{-1}{\sqrt{1-x^2}} \\ \arctan'(x) &= \frac{1}{1+x^2} \end{aligned}}$$

$$\boxed{f(x)^{g(x)} = e^{\ln(f(x)^{g(x)})} = e^{g(x)\cdot\ln(f(x))}}$$

Taylor Series -

$$\boxed{f(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + f^n(a)\frac{(x-a)^n}{n!}}$$

$O(x^n) \rightarrow$ Something with x^n

$x\to 0 \Rightarrow$ O highest power

$x\to\infty \Rightarrow$ Omit smallest power

Important MacLaurin Series -

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots$$

$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3 \ldots$$

$$\arctan(x) \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

Week 5

Applying Taylor Series:

eg/ $\frac{1}{8-x} = \frac{1}{8}\cdot\frac{1}{1-\frac{x}{8}} \approx 1+\left(\frac{x}{8}\right)+\left(\frac{x}{8}\right)^2 \ldots$

eg/ $e^{x+3} = e^{x}\cdot e^{3} \approx e^{3}\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\ldots\right)$

eg/ e^{x}, $a=1$, $y=x-1$ $x=y+1$

↳ $e^{3(y+1)} = e^{3y}\cdot e^{3} \approx e^{3}\left(1+(3y)+\frac{(3y)^2}{2}\ldots\right)$

$= e^{3}\left(1+3(x-1)+\frac{9(x-1)^2}{2}\ldots\right)$

L'Hopital - $\left(\lim \frac{0}{0} / \frac{\infty}{\infty}\right)$

$$\boxed{\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}}$$

Limit - $\infty^0, 1^\infty$ etc.

$e^{\ln(f(x)^{g(x)})} = e^{g(x)\ln(f(x))}$

Week 6

U substitution -

Try find a u, who's derivative is equal to something in integral

eg/ $\int \tan x\,dx = \int \frac{\sin x}{\cos x}$ $u=\cos x$

$= \int -\frac{1}{u}\,du = -\ln|\cos x| + c$ $du = -\sin x\,dx$

eg/ $\int x^3\sqrt{x^2-2}\,dx$ $u=x^2-2,\ x^2=u+2$

$du = 2x$

$= \frac{1}{2}\int (u+2)\sqrt{u}\,du$

$= \frac{1}{2}\int u^{\frac{3}{2}} + 2u^{\frac{1}{2}}\,du = \frac{1}{2}\left[\frac{2}{5}(x^2+2)^{\frac{5}{2}} - \frac{4}{3}(x^2+2)^{\frac{3}{2}}\right] + c$

eg/ $\int \ln(x)\,dx = x\ln(x) - \int \frac{x}{x}\,dx$

$= x\ln(x) - x$

Integration by Parts:

$$\boxed{\int u\cdot dv = uv - \int v\,du}$$

↳ get rid of u and dv in integral

Differentiating Integrals

$$\frac{d}{dx}\left(\int_{g(x)}^{f(x)} h(t)\,dt\right) = f'(x)\cdot h(f(x)) - g'(x)h(g(x))$$

$$\boxed{\begin{array}{l}\int_{-a}^{a} f(x) = 0 \Rightarrow \text{If } f(x) \text{ is odd} \\ \int_{-a}^{a} f(x) = 2\int_{0}^{a} f(x) \Rightarrow \text{If } f(x) \text{ is even}\end{array}}$$

Taylor Series Sum Notations:

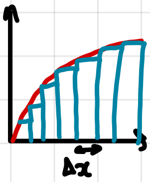
$e^x \approx \sum_{k=0}^{n} \frac{x^k}{k!}$

$\sin x \approx \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$

$\cos x \approx \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$

Rieman's Sums:

$A \approx \sum_{i=1}^{n} f\left(x_i + \frac{\Delta x}{2}\right)\cdot \Delta x$



Average Value Function:

$$\boxed{\frac{1}{b-a}\int_a^b f(x)\,dx}$$

Integrals $\sin^n(x), \cos^m(x)$:

m odd → u substitution

m even → trig identity

Integrals to infinity

$\int_a^\infty f(x) = \lim_{b\to\infty}\int_a^b f(x)$

↳ same goes for asymptotes

If integral infinity $\in \mathbb{R} \Rightarrow$ converges

If integral infinity $\in \pm\infty \Rightarrow$ diverges

$0 \le f(x) \le g(x)$

$f(x)$ diverges $\Rightarrow g(x)$ diverges

$g(x)$ converges $\Rightarrow f(x)$ diverges

Standard Integrals:

$\int x^r = \frac{x^{r+1}}{r+1}$ $\int \frac{1}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$

$\int \frac{1}{x} = \ln(x)$ $\int \frac{1}{\sqrt{a^2-x^2}} = \frac{1}{a}\arcsin\left(\frac{x}{a}\right)$

$\int \sin(ax) = \frac{-1}{a}\cos(ax)$ $\int \frac{-1}{\sqrt{a^2-x^2}} = \frac{1}{a}\arccos\left(\frac{x}{a}\right)$

$\int \cos(ax) = \frac{1}{a}\sin(x)$ $\int \tan x = -\ln|\cos(x)|$

$\int a^x = \frac{1}{\ln a}\cdot a^x$ $\int \frac{1}{\cos^2(x)} = \tan(x)$

$\int e^{f(x)} = \frac{1}{f(x)}\cdot e^{f(x)}$

Rational Fraction Expansion

$\frac{ax+b}{cx^2+dx+e} = \frac{A}{x+f} + \frac{B}{x+g}$

power denominator > power numerator

eg/ $\frac{1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

eg/ $\frac{x^2+x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

• Sometimes have to complete square

↳ then u substitution

Week 7

Seperable differential equations:

$\frac{dy}{dx} = f(x)\cdot g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)\,dx$

Linear differential equations

$\frac{dy}{dx} = p(x)y + q(x)$

Solve $\frac{dy}{dx} = p(x)y$ then $c \Rightarrow c(x)$

or

Multiply all by integration constant

$I = e^{\int p(x)dx}$

Easier