Student-led tutorials 5EPB0 ‘EM II’

SLT-A

April 29th 2025

Use of AI for solving SLT questions:

You are allowed to use generative AI, specifically to create content (like images, (source) code, video, text or any other kind). You are allowed to use AI tools as a source for inspiration, e.g. to enhance design and/or research activities, or to improve/enhance the quality of products, but you are not allowed to copy-paste pieces of writing from the AI tool into your assignment. You should be able to explain why you opted to use specific tools and evaluate their usefulness.

Q1 Philosophy

1. (a) Plane waves propagate along a certain medium/material with a certain characteristic impedance. What kind of mediums exist for plane waves? Do they need to contain metal? Can they consist of multiple materials?

   (b) A metal transmission line has a certain impedance, usually denoted by Z_0 measured in Ohms. If we consider the same metal line and place a voltage over it and measure the current, we get a resistance $R = \frac{I}{V}$ measured in Ohms due to the finite conductivity of the material. Yet the impedance and resistance are not the same, why?

Q2 Layered Media

Consider the setup shown in the Fig. 1. It shows a plane wave propagating through Medium 1 in the $+z$-direction and impinging at $z = 0$ under normal incidence on a layered dielectric stack consisting of Medium 2-4. The properties of the media are given by:

$$
\begin{array}{lcl|lcl}
\epsilon_{r,1} & = & 4 & \mu_{r,1} & = & 1 \\
\epsilon_{r,2} & = & 25 & \mu_{r,2} & = & 4 \\
\epsilon_{r,3} & = & 8 & \mu_{r,3} & = & 2 \\
\epsilon_{r,4} & = & 4 & \mu_{r,4} & = & 9 \\
l_1 & = & l_1 & l_2 & = & l_1
\end{array}
$$

2. (a) Calculate the phase velocity and wave impedance of the four media and explain what phase velocity and wave impedance is.

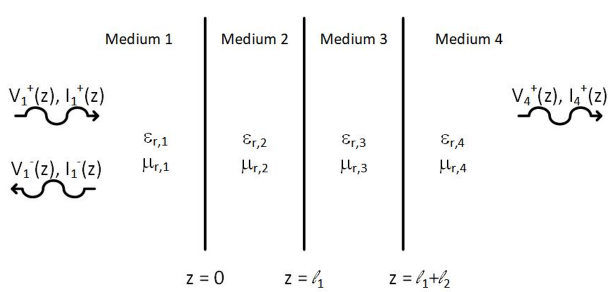

Figure 1: Layered dielectric stack of the second question

(b) Give a description of what will happen with the waves (reflections, transmission). For a description of transmission, please read Chapter 2 of the reader. Here only a description is needed. In Chapter 2 also values will be introduced.

(c) What happens to the wavelength in the different media?

Q3 Bouncing waves

3. Figure 2 shows a pulse at $t_0 = 0T$ travelling with speed $c = L/T$ along a transmission line with characteristic impedance Z_0=50 Ω. At $z = 10L$ the transmission line is terminated.

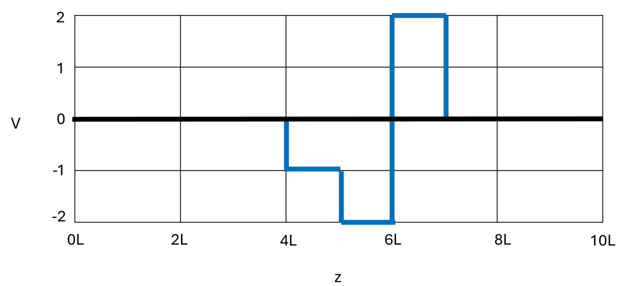

Figure 2: A pulse travelling in a transmission line in positive z direction.

(a) Sketch the voltage distribution at $t_1 = 10T$ if the line is terminated in an open circuit.

(b) Sketch the voltage distribution at $t_2 = 6T$ if the line is terminated in a short circuit.

(c) Sketch the voltage distribution at $t_1 = 10T$ if the line is terminated with a 25 ohm load.

(d) Sketch the voltage distribution at $t_1 = 10T$ if the line is terminated with a 50 ohm load.

Q4 Equivalent circuit

In the reader it is explained that the voltage and current along a transmission line can be described using the following equations:

$$-\frac{\partial v}{\partial z} = L\frac{\partial i}{\partial t}, \tag{1}$$

and

$$-\frac{\partial i}{\partial z} = C\frac{\partial v}{\partial t}. \tag{2}$$

Note that on the left side of the equations, the voltage (or current) is differentiated with respect to location and on the right side with respect to time. As we know from circuit theory, these relations can be represented by lumped elements.

4. (a) Derive an equivalent circuit model that fulfils the two equations given above. For this, consider a finite length of transmission line Δz such that you can replace $-\frac{\partial}{\partial z}$ by a finite difference. Please use a sketch as shown in Fig. 3, fill in the missing components and update the equations.

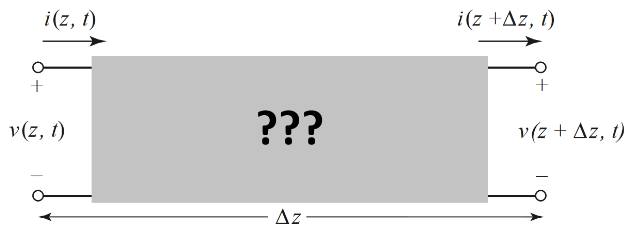

Figure 3: An equivalent circuit model of a transmission line.

(b) How would the equivalent circuit change, if losses are present in the transmission line? Update all equations as well as your sketch. What are the units of the lumped elements?

Q5 Liberty of Internet

Underwater internet cables connect countries worldwide to the internet. One such cable is drawn between Brandon Bay in Ireland and the Statue of Liberty in New York, which is approximately 5100 km and luckily lossless. Since last week this cable is no longer working, and due to rising world tensions, a sabotage action is assumed. It is your job to find out where the sabotage action happened such that the cable can be repaired. To find this out, we can make use of time domain reflectometry (TDR), where at $t = 0\ s$ a continuous 1 V (Heavyside step function) pulse is sent over the cable ($\epsilon_r = 6$).

5. (a) You've just made your way to Brandon Bay with the necessary equipment which you need to set up. What is the maximum time we need to wait to determine the faulty location?

(b) To test if the equipment works properly, you can make use of a similar cable next to the broken one, with the same characteristics. Your colleague in New York has placed an open connection at his side of the working cable. What type of signal to you expect to receive back at your side?

(c) The more senior engineer in New York wants to make sure that you understand what you are doing, and is therefore terminating his side of the working cable with a for you unknown impedance. The signal that you measure looks like Figure 4. Determine what load the more senior engineer has placed at the other side.

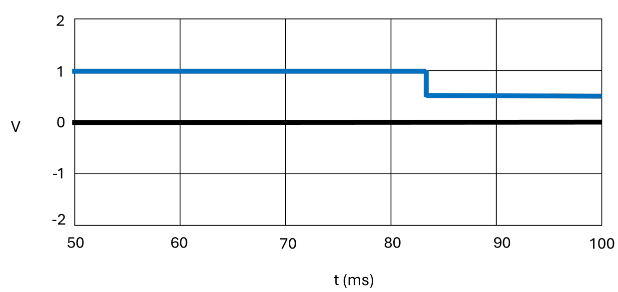

Figure 4: The measured signal for question 5c.

(d) Now you can start doing your work on the broken cable. The signal you receive is shown in Fig. 5. Can you determine at what distance the sabotage action is located from Brandon Bay and what type of termination the broken cable is.

(e) Now it is your time to school the more senior engineer. Can you predict what type of signal his TDR device should give? Draw the predicted signal that he/she should see. Think about the cable termination. But be careful, since his TDR device is a bit weaker and can only transmit a 0.5 V signal.

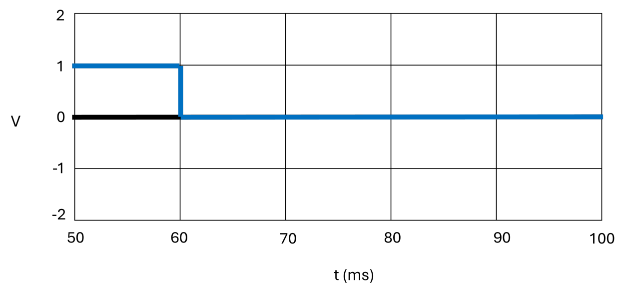

Figure 5: The measured signal in your TDR device when connecting to the broken cable for question 5d.

Q6 Transmission Line

A lossless transmission line with a characteristic impedance of $Z = 50\ \Omega$ and a length of $l = 5$ m is connected to a load resistor $Z_L = 100\ \Omega$. The line is driven by a voltage source of 500 V with an (internal) impedance of 50 Ω.

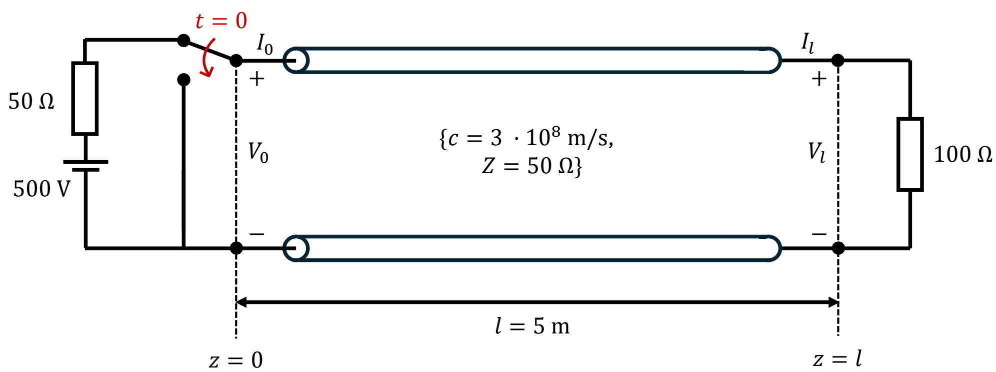

Figure 6: Transmission line problem 6.

6. (a) A steady state has been reached for $t = 0^-$. What are the steady state total voltage amplitude $V_{\text{steady}}(z, 0^-)$ and the associated steady state total current amplitude $I_{\text{steady}}(z, 0^-)$ (anywhere along the transmission line), before the switch is closed?

   (b) Despite the stationary nature of the steady state, it may be decomposed into forward and backward propagating waves. Determine the respective steady state TEM-wave voltage amplitude $V^{\pm}_{\text{steady}}(z, 0^-)$ of these forward and backward propagating waves.

   (c) Determine the associated steady state TEM-wave current amplitude $I^{\pm}_{\text{steady}}(z, t)$ of these forward and backward propagating waves.

   (d) Now the switch is closed. This will generate a wave on the transmission line. The details of this will be treated in the following weeks of the course. What will happen in the process of $t \to \infty$? What are the steady state total voltage amplitude $V_{\text{steady}}(z, \infty)$ and the associated steady state total current amplitude $I_{\text{steady}}(z, \infty)$ (anywhere along the transmission line)?