# Components in wireless technology, 5XTC0

## Module 5
## Exercise: Design of amplifiers and low-noise amplifiers

Vojkan Vidojkovic

TU/e Technische Universiteit Eindhoven University of Technology

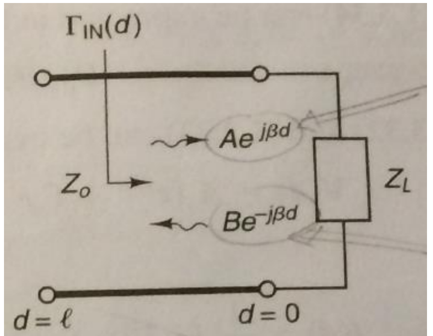
# Outline

- Matching networks
- Amplifier design
- Low-noise amplifier dsign

# Formula list available for exam

| | |
|---|---|
| **Power delivered to the load** | $P_L = \frac{\lvert V_2^-\rvert^2}{2Z_0}\left(1-\lvert\Gamma_L\rvert^2\right)$ |
| **Input power to the network** | $P_{in} = \frac{\lvert V_1^+\rvert^2}{2Z_0}\left(1-\lvert\Gamma_{in}\rvert^2\right)$ |
| **Input and output reflection coefficients of a transistor with a source and load: general case** | $\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$ <br> $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$ |
| **Input and output reflection coefficients of a transistor with a source and load: unilateral case** | $\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11}$ <br> $\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22}$ |
| **Gain of the input matching network** | $G_S = \frac{1-\lvert\Gamma_S\rvert^2}{\lvert 1-\Gamma_{in}\Gamma_S\rvert^2}$ |
| **Gain of the output matching network** | $G_L = \frac{1-\lvert\Gamma_L\rvert^2}{\lvert 1-S_{22}\Gamma_L\rvert^2}$ |
| **Gain of the transistor (unilateral case)** | $G_0 = \lvert S_{21}\rvert^2$ |
| **Transducer gain of the basic amplifier circuit (input matching, unilateral transistor, output matching)** | $G_T = G_S G_0 G_L$ <br> $G_{T,dB} = G_{S,dB} + G_{0,dB} + G_{L,dB}$ |
| **Maximum gain of the input and output matching networks** | $G_{S_{max}} = \frac{1}{1-\lvert S_{11}\rvert^2},$ <br> $G_{L_{max}} = \frac{1}{1-\lvert S_{22}\rvert^2}.$ |
| **Maximum transducer power gain, unilateral case** | $G_{TU_{max}} = \frac{1}{1-\lvert S_{11}\rvert^2}\lvert S_{21}\rvert^2\frac{1}{1-\lvert S_{22}\rvert^2}$ |
| **Normalized gain factors $g_s$ and $g_L$** | $g_S = \frac{G_S}{G_{S_{max}}} = \frac{1-\lvert\Gamma_S\rvert^2}{\lvert 1-S_{11}\Gamma_S\rvert^2}(1-\lvert S_{11}\rvert^2),$ <br> $g_L = \frac{G_L}{G_{L_{max}}} = \frac{1-\lvert\Gamma_L\rvert^2}{\lvert 1-S_{22}\Gamma_L\rvert^2}(1-\lvert S_{22}\rvert^2).$ |
| **Center and radius of the constant gain circle for the input matching network** | $C_S = \frac{g_S S_{11}^*}{1-(1-g_S)\lvert S_{11}\rvert^2},$ <br> $R_S = \frac{\sqrt{1-g_S}\,(1-\lvert S_{11}\rvert^2)}{1-(1-g_S)\lvert S_{11}\rvert^2}$ |
| **Center and radius of the constant gain circle for the output matching network** | $C_L = \frac{g_L S_{22}^*}{1-(1-g_L)\lvert S_{22}\rvert^2},$ <br> $R_L = \frac{\sqrt{1-g_L}\,(1-\lvert S_{22}\rvert^2)}{1-(1-g_L)\lvert S_{22}\rvert^2}$ |
| **Condition for "unconditionally stable" device, general case** | for all $\lvert\Gamma_L\rvert<1$ and $\lvert\Gamma_S\rvert<1$ <br> $\Rightarrow \begin{cases} \lvert\Gamma_{in}\rvert = \left\lvert S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}\right\rvert < 1 \\ \lvert\Gamma_{out}\rvert = \left\lvert S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}\right\rvert < 1 \end{cases}$ |

| | |
|---|---|
| **Conditions for "unconditionally stable" device, unilateral case** | $\lvert\Gamma_{in}\rvert = \lvert S_{11}\rvert < 1$ <br> $\lvert\Gamma_{out}\rvert = \lvert S_{22}\rvert < 1$ |
| **Center and radius of the stability circles, load side** | $C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{\lvert S_{22}\rvert^2-\lvert\Delta\rvert^2}$ (center), <br> $R_L = \left\lvert\frac{S_{12}S_{21}}{\lvert S_{22}\rvert^2-\lvert\Delta\rvert^2}\right\rvert$ (radius). <br> $\Delta = S_{11}S_{22} - S_{12}S_{21}$ |
| **Center and radius of the stability circles, source side** | $C_S = \frac{(S_{11}-\Delta S_{22}^*)^*}{\lvert S_{11}\rvert^2-\lvert\Delta\rvert^2}$ (center), <br> $R_S = \left\lvert\frac{S_{12}S_{21}}{\lvert S_{11}\rvert^2-\lvert\Delta\rvert^2}\right\rvert$ (radius) <br> $\Delta = S_{11}S_{22} - S_{12}S_{21}$ |
| **Test for unconditional stability, general case** | $\lvert\Delta\rvert = \lvert S_{11}S_{22} - S_{12}S_{21}\rvert < 1$ <br> and <br> $K = \frac{1-\lvert S_{11}\rvert^2-\lvert S_{22}\rvert^2+\Delta^2}{2\lvert S_{12}S_{21}\rvert} > 1$ |
| **Test for unconditional stability, unilateral case** | $\lvert S_{11}\rvert < 1$ <br> $\lvert S_{22}\rvert < 1$ |
| **Two-stage amplifier: Output noise and noise figure** | $P_{N,total} = G_{A2}(G_{A1}P_{N,in} + P_{n1}) + P_{n2}$ <br> $\Rightarrow F_{total} = \frac{P_{N,total}}{P_{N,in}G_{A1}G_{A2}} = 1 + \frac{P_{n1}}{P_{N,in}G_{A1}} + \frac{P_{n2}}{P_{N,in}G_{A1}G_{A2}}$ <br> $F_{total} = F_1 + \frac{F_2-1}{G_{A1}},$ <br> with $F_j = 1 + \frac{P_{nj}}{P_{N,in}G_{Aj}},\ j=1,2$ |
| **Noise figure of a 2-port amplifier** | $F = F_{min} + \frac{r_N}{g_S}\left\lvert \underline{y}_S - \underline{y}_{opt}\right\rvert^2$ <br> $F = F_{min} + 4r_N\frac{\lvert\underline{\Gamma}_S-\underline{\Gamma}_{opt}\rvert^2}{(1-\lvert\underline{\Gamma}_S\rvert^2)\cdot\lvert 1+\underline{\Gamma}_{opt}\rvert^2}$ |
| **Constant noise circles** | $\underline{C}_F = \frac{\underline{\Gamma}_{opt}}{1+N}$ <br> $R_r = \frac{1}{1+N}\sqrt{N^2+N(1-\lvert\underline{\Gamma}_{opt}\rvert^2)}$ <br> $\Delta F_n' = N = (F-F_{min})\frac{\lvert 1+\underline{\Gamma}_{opt}\rvert^2}{4r_n} = \frac{\lvert\underline{\Gamma}_S-\underline{\Gamma}_{opt}\rvert^2}{1-\lvert\underline{\Gamma}_S\rvert^2}$ |




$$V(d) = A_1 e^{j\beta d} + B_1 e^{-j\beta d}$$

$$I(d) = \frac{A_1}{Z_o}e^{j\beta d} - \frac{B_1}{Z_o}e^{-j\beta d}$$

$$\Gamma_0 = \Gamma_{IN}(0) = \frac{B_1}{A_1}$$

# Transmission lines: Problem 1

- Book of Gonzalez: Example 1.13
- Assignment, problem 2

(a) Calcuate the reflection coefficient $\Gamma_{IN}$ and the VSWR in the $\lambda/4$ line in Fig. 2.

(b) Calculate the voltage $V(\lambda/4)$, the power available from the source $P_{AVS}$, and the power delivered to the 100 Ω load.

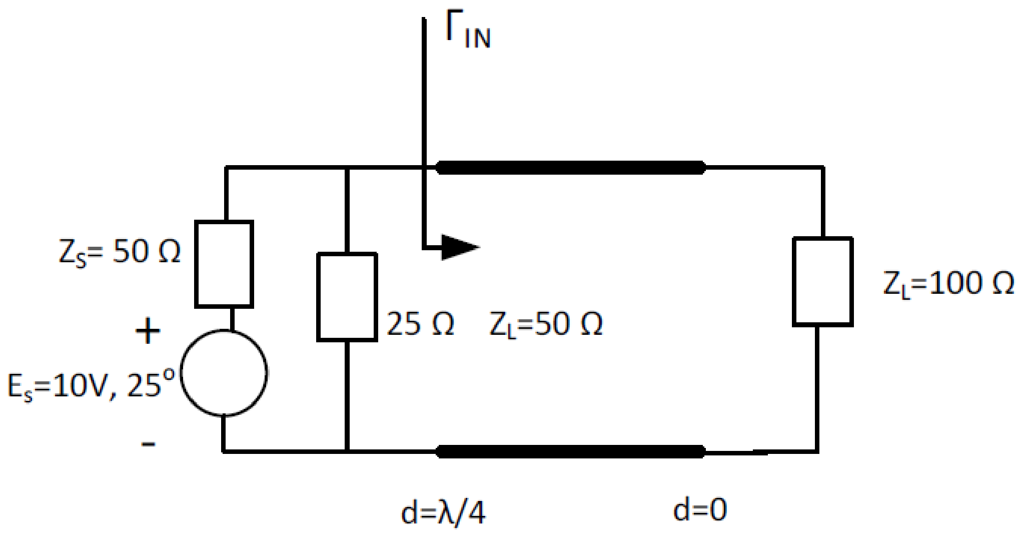
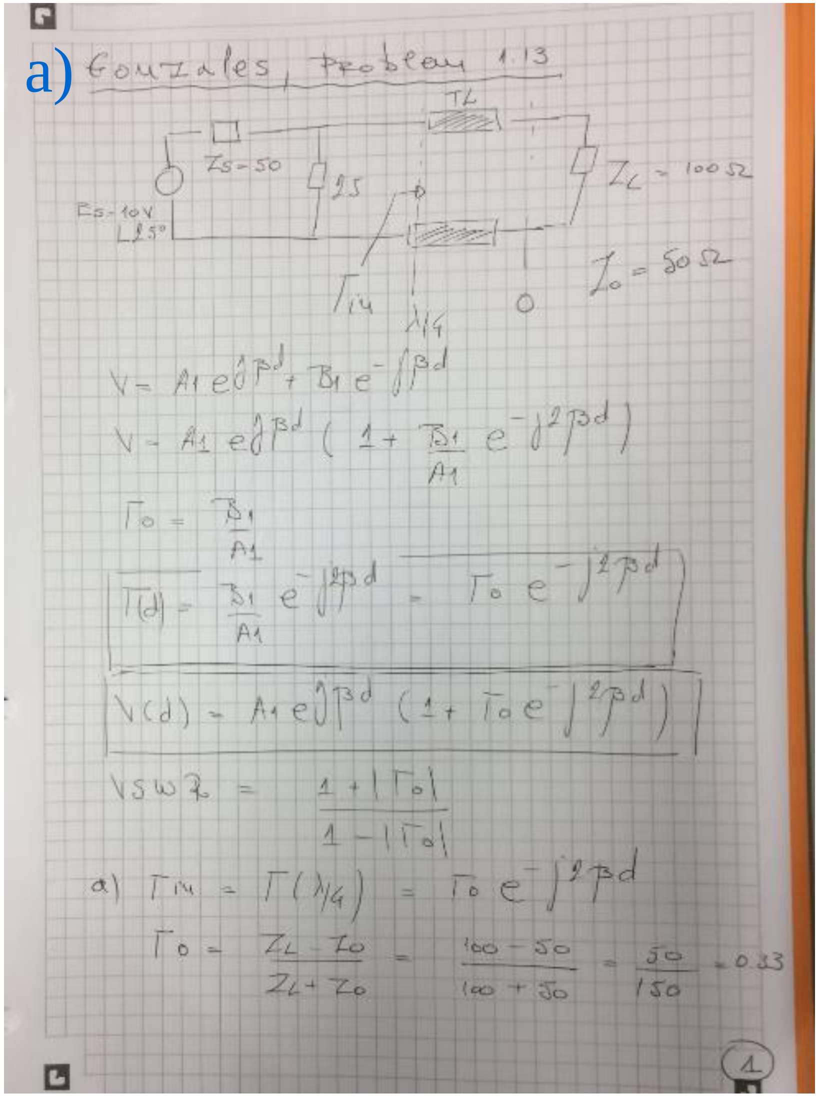



*Figure 2 Problem 2.*

# Transmission lines: Solution for Problem 1a



a) Gonzales, Problem 1.13

TL

$Z_S = 50$

25

$Z_L = 100\,\Omega$

$E_S = 10V \angle 25°$

$\Gamma_{in}$

$\lambda/4$

$Z_0 = 50\,\Omega$

$V = A_1 e^{j\beta d} + B_1 e^{-j\beta d}$

$V = A_1 e^{j\beta d}\left(1 + \frac{B_1}{A_1} e^{-j2\beta d}\right)$

$\Gamma_0 = \frac{B_1}{A_1}$

$\boxed{\Gamma(d) = \frac{B_1}{A_1} e^{-j2\beta d} = \Gamma_0 e^{-j2\beta d}}$

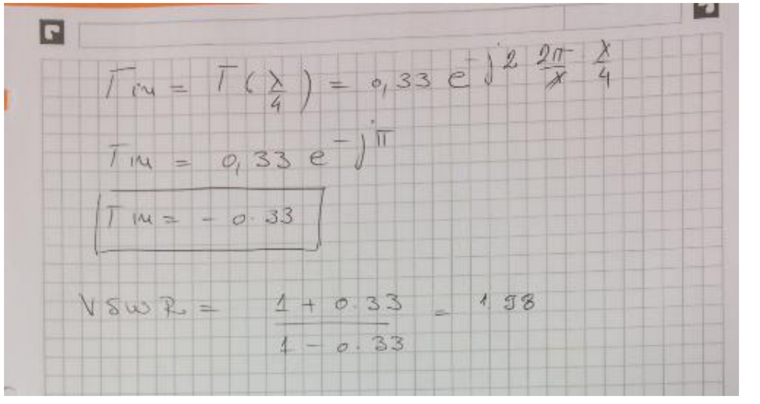
$\boxed{V(d) = A_1 e^{j\beta d}\left(1 + \Gamma_0 e^{-j2\beta d}\right)}$

$VSWR = \frac{1+|\Gamma_0|}{1-|\Gamma_0|}$

a) $\Gamma_{in} = \Gamma(\lambda/4) = \Gamma_0 e^{-j2\beta d}$

$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50} = \frac{50}{150} = 0.33$

1

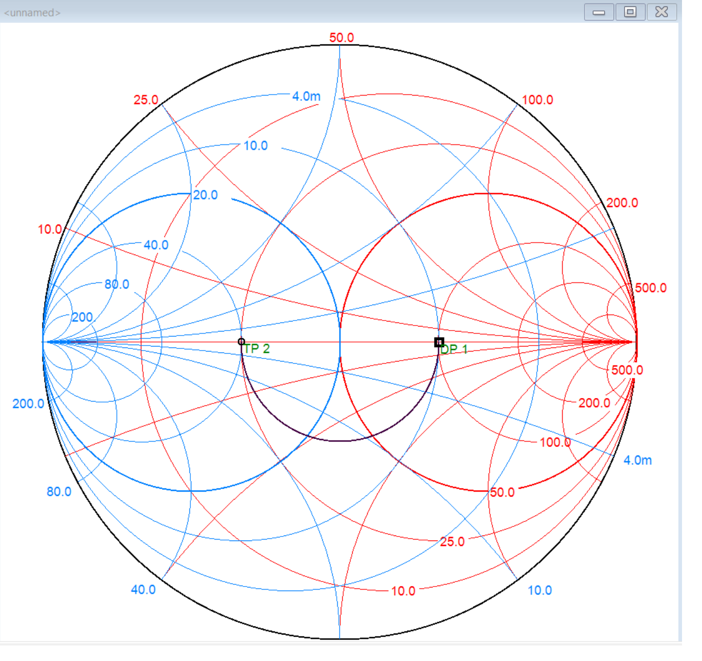


$\Gamma_{in} = \Gamma\left(\frac{\lambda}{4}\right) = 0{,}33\, e^{-j2\frac{2\pi}{\lambda}\frac{\lambda}{4}}$

$\Gamma_{in} = 0{,}33\, e^{-j\pi}$

$\boxed{\Gamma_{in} = -0.33}$

$VSWR = \frac{1+0.33}{1-0.33} = 1.98$



# Transmission lines: Solution for Problem 1 b (1/2)

b) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d}$

$d = \frac{\lambda}{4}$

$\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi/2$

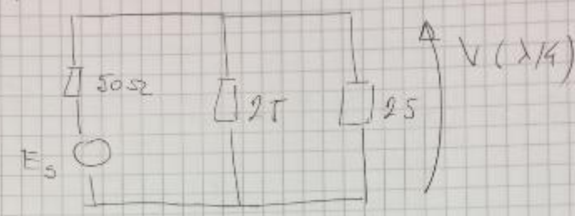
$\tan \pi/2 = \infty$

$Z_{in} = \dfrac{Z_0^2}{Z_L}$

$Z_{in} = \dfrac{50^2}{100} = 25\,\Omega$

(2)

Equivalent circuit:




$V(\frac{\lambda}{4}) = E_s \cdot \dfrac{12,5}{50+12,5}$

$V(\frac{\lambda}{4}) = 10\, e^{j25} \cdot 0.2$

$\boxed{V(\frac{\lambda}{4}) = 2\, e^{j25}}$

$V(\frac{\lambda}{4}) = A_1 e^{j\beta d}\left(1 + \Gamma_0 e^{-j2\beta d}\right)$

$\frac{\lambda}{4} = d \Rightarrow \beta d = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2}$

$2\beta d = \pi$

$2 \cdot e^{j25} = A_1 e^{j\frac{\pi}{2}}\left(1 + 0.33\, e^{-j\pi}\right)$

$2\, e^{-j65} = A_1 (1 - 0.33)$

$\boxed{A_1 = 3\, e^{-j65}}$

$e^{-j\pi} = \cos(-\pi) + j\sin(-\pi)$ (3)

$e^{-j\pi} = -1$

# Transmission lines: Solution for Problem 1 b (2/2)


$P_{avs}$ @ $Z_L = Z_S^* = \mathrm{Re}\{V_L \cdot I_L^*\}$

$Z_S = R_S + jX_S$

$Z_L = Z_S^* = R_S - jX_S$

$V_L = \frac{E_S}{2}$

$I_L = \frac{E_S}{2R_S}$

$$\boxed{P_{avs} = \frac{|E_S|^2}{8R_S} = \frac{100}{8\cdot 50} = 0{,}25\,W}$$

$V(0) = A_1(1+\Gamma_0)$

$V(0) = 3e^{-j65}(1+0{,}33) = \cancel{3.99}$

$$\boxed{V(0) = 3.99\,e^{-j65}}$$

$I(0) = \frac{A_1(1-\Gamma_0)}{Z_0} = \frac{3e^{-j65}\cdot 0.67}{50}$

$I(0) = 0.04\,e^{-j65}$

4

$P_0 = \frac{\mathrm{Re}}{2}\{V(0)\cdot I(0)^*\}$

max → $P_{avs}$

$P_0 = \frac{\mathrm{Re}}{2}\{3.99e^{-j65}\cdot 0.04e^{j65}\}$

$$\boxed{P_0 = 0.08\,W}$$

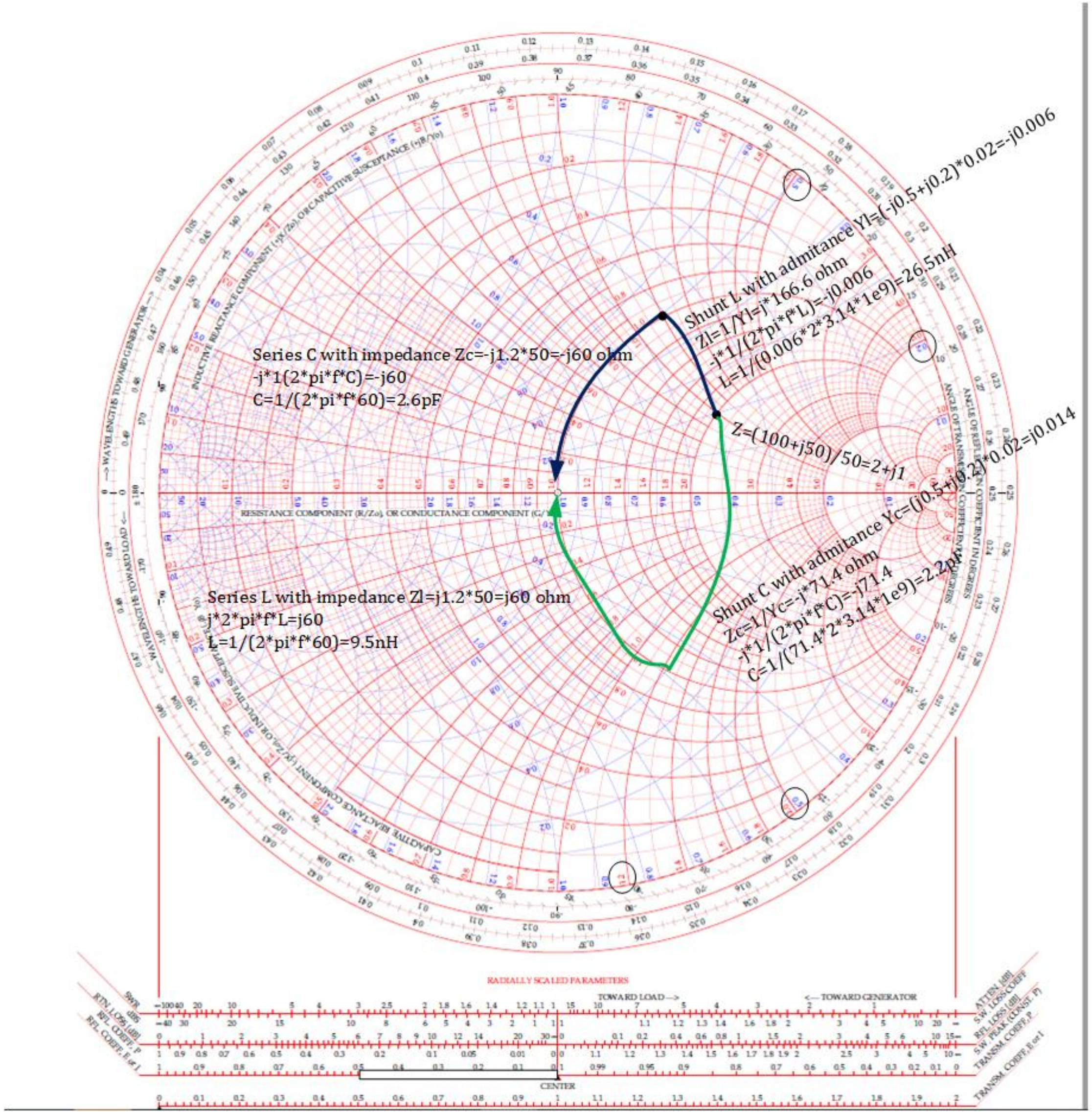
# Matching: Problem 2

- Assignment, problem 3

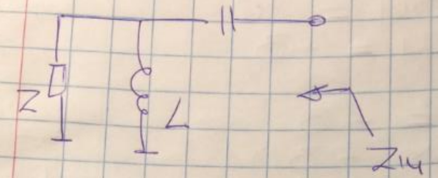
Design an LC matching network that transforms impedance Z=100+j50 to 50ohm. Operating frequency is 1GHz.

A. How many solution are existing?
   Draw the solutions in the Smith Chart
B. Calculate impedance of L and C for all solutions
C. Calculate values of L and C for all solutions
D. Take one of the solutions and check solution based on Smith Chart with analytical calculation

# Matching: Problem 2 – Solution (1/2)



# Matching: Problem 2 – Solution (2/2)




$$Z_{in} = \frac{1}{j\omega C} + \frac{1}{\frac{1}{j\omega L} + \frac{1}{Z}}$$

$$Z_{in} = -j\frac{1}{\omega C} + \frac{1}{-j\frac{1}{\omega L} + \frac{1}{Z}}$$

$$\frac{1}{\omega C} = A, \quad \frac{1}{\omega L} = B \quad \dots (1)$$

$$Z_{in} = -j\cdot A + \frac{1}{-jB + R + jI_m}$$

$\frac{1}{Z} = R + jI_m \to$ known $\begin{cases} Z = 100 + j50 \\ R = 8\cdot 10^{-3},\ I_m = 4\cdot 10^{-3} \end{cases}$

$$Z_{in} = -j\cdot A + \frac{R - j(I_m - B)}{R^2 + (I_m - B)^2}$$

$Z_{in} = 50 \to$ known

$$50 = \frac{R}{R^2 + (I_m - B)^2} \quad \dots (3)$$

$$A + \frac{I_m - B}{R^2 + (I_m - B)^2} = 0 \quad \dots (4)$$

$$(I_m - B)^2 = \frac{R}{50} - R^2$$

$$B = \pm\sqrt{\frac{R}{50} - R^2} + I_m$$

$$\boxed{B = 5{,}8\cdot 10^{-3} \Rightarrow L = 27{,}45\ nH}$$

from (1) $\Rightarrow L = \frac{1}{2\cdot pi\cdot f\, B}$

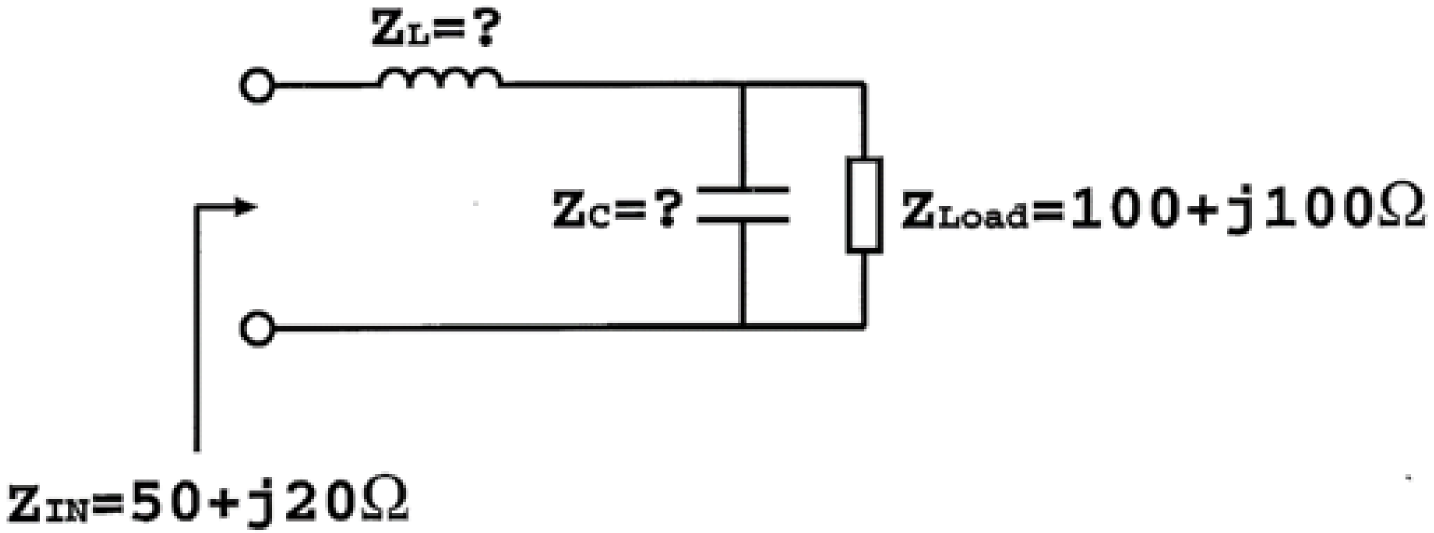
$$A = \frac{B - I_m}{R^2 + (I_m - B)^2}$$

$A = 61{,}23 \Rightarrow C = \frac{1}{2\cdot pi\cdot f\cdot A} =$

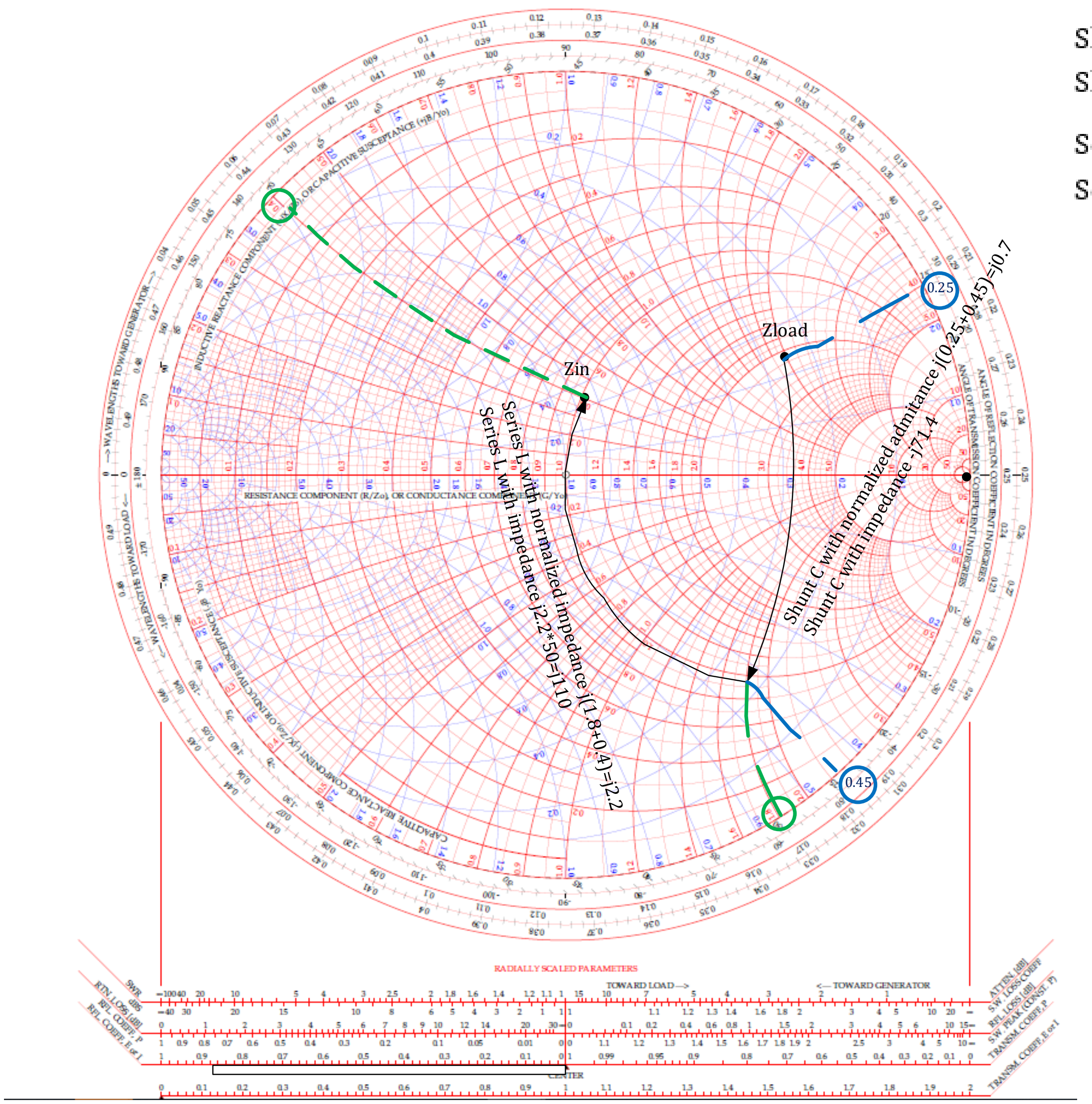
$C = 2.6\ pF$

# Matching: Problem 3

Using the Smith chart, design a matching network with lumped elements (L and C) as shown below which transforms the load impedance $Z_{Load}$ into the input impedance $Z_{IN}$.



# Probklem 3: solution




Shunt C with normalized admitance j(0.25+0.45)=j0.7

Shunt C with impedance (1/j0.7)*50=-j71.4

Series L with normalized impedance j(1.8+0.4)=j2.2

Series L with impedance j2.2*50=j110

# Amplifier design:
# Similar as example 12.1 book of Pozar

A microwave transistor has the following $S$ parameters at 10 GHz, with a 50 $\Omega$ reference impedance:

$$S_{11} = 0.45\angle 150^\circ$$

$$S_{12} = 0.01\angle -10^\circ$$

$$S_{21} = 2.05\angle 10^\circ$$

$$S_{22} = 0.40\angle -150^\circ$$

The source impedance is $Z_S = 20\ \Omega$ and the load impedance is $Z_L = 30\ \Omega$. Compute the power gain, the available gain, and the transducer power gain.

# Amplifier design
# Similar as example 12.1 book of Pozar - Solution

Example 11.1, book of Pozar, page 539

$S_{11} = 0.45\angle 150° = 0.45 e^{j150°}$

$S_{12} = 0.01 e^{j-10}$

$S_{21} = 2{,}05\, e^{j10}$

$S_{22} = 0.4\, e^{-j150}$

$Z_S = 20\,\Omega$

$Z_L = 30\,\Omega$

$Z_0 = 50\,\Omega$

Power gain $= G = \dfrac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)\,|1 - S_{22}\Gamma_L|^2}$

Available power gain $= G_A = \dfrac{|S_{21}|^2 (1 - |\Gamma_S|^2)}{|1 - S_{11}\Gamma_S|^2 (1 - |\Gamma_{out}|^2)}$

Transducer power gain $= G_T = \dfrac{|S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_S\Gamma_{in}|^2\,|1 - S_{22}\Gamma_L|^2}$

[S] parameters are known

$\Gamma_L = ?$, $\Gamma_S = ?$, $\Gamma_{out} = ?$, $\Gamma_{in} = ?$

1

$\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = -0{,}250$

$\Gamma_S = \dfrac{Z_S - Z_0}{Z_S + Z_0} = -0.429$

$\Gamma_{out} = S_{22} + \dfrac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$

$\Gamma_{in} = S_{11} + \dfrac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$

$\Gamma_{out} = 0.408\, e^{-j151}$

$\Gamma_{in} = 0.455\, e^{j150}$

$G = 5.94$

$G_A = 5.85$

$G_T = 5.49$

2

# Amplifier design: Example 3.4.1 book of Gonzalez

The $S$ parameters of a BJT measured at $V_{CE} = 10$ V, $I_C = 30$ mA, and $f = 1$ GHz, in a 50-Ω system are

$$S_{11} = 0.73\angle 175^\circ$$

$$S_{12} = 0$$

$$S_{21} = 4.45\angle 65^\circ$$

$$S_{22} = 0.21\angle -80^\circ$$

(a) Calculate the optimum terminations.
(b) Calculate $G_{s,\max}$, $G_{L,\max}$, and $G_{TU,\max}$ in decibels.
(c) Draw several $G_s$ constant-gain circles.
(d) Design the input matching network for $G_s = 2$ dB.

# Example 3.4.1 book of Gonzalez – Solution (1/3)

Example 3.4.1, book of Gonzalez, page 232

$f = 1\ \text{GHz}$

$S_{11} = 0.73\, e^{j175}$

$S_{12} = 0 \rightarrow$ unilateral case

$S_{21} = 4.45\, e^{j65}$

$S_{22} = 0.21\, e^{j-80}$

$$G_T = \underbrace{\frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}}_{G_S}\ \underbrace{|S_{21}|^2}_{G_0}\ \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{G_L}$$

1) $G_{T\max}$ / Conjugate match at input, output

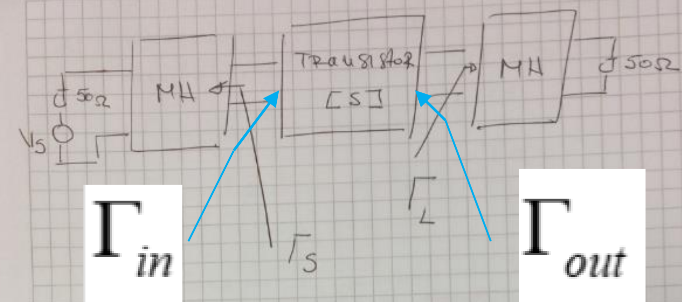
$\Gamma_{in} = \Gamma_S^*$, $\Gamma_{out} = \Gamma_L^*$

$$G_{S\max} = \frac{1-|\Gamma_S|}{(1-|\Gamma_S|^2)^2} = \frac{1}{1-|\Gamma_S|^2}$$

$$G_{T\max} = \frac{1}{1-|\Gamma_S|^2}\,|S_{21}|^2\,\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$




2) $G_{TU\max}$ / Unilateral case $S_{12}=0$

Conjugate match at input and output

$\Gamma_{in} = \Gamma_S^*$, $\Gamma_{out} = \Gamma_L^*$

$\Gamma_S^* = S_{11} \Rightarrow \Gamma_S = S_{11}^*$

$\Gamma_L^* = S_{22} \Rightarrow \Gamma_L = S_{22}^*$

$$G_{TU\max} = \frac{1}{1-|S_{11}|^2}\,|S_{21}|^2\,\frac{1}{1-|S_{22}|^2}$$

$$G_{SU\max} = \frac{1}{1-|S_{11}|^2}$$

$$G_{LU\max} = \frac{1}{1-|S_{22}|^2}$$

2

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

# Example 3.4.1 book of Gonzalez – Solution (2/3)

a) optimum terminations →
unilateral case $S_{12} = 0$ →

$$\Gamma_S = S_{11}^* = 0.73\, e^{-j175}$$

$$\Gamma_L = S_{22}^* = 0.21\, e^{+j80}$$

b)

$$G_{S\,max} = \frac{1}{1-|S_{11}|^2} = 2.141$$

$$G_{S\,max}[dB] = 10 \log G_{S\,max} = 3.31\, dB$$

$$G_{L\,max} = \frac{1}{1-|S_{22}|^2} = 1.046$$

$$G_{L\,max}[dB] = 0.195$$

$$G_0 = |S_{21}|^2 = 19.8$$

$$G_0[dB] = 12.97\, dB$$

$$G_{TU,max} = 16.47\, dB = G_{S\,max} + G_0 + G_{L\,max}$$ → all in dB

c) $G_S = 2\, dB$ → in dB

$G_S = 1{,}59$

$$g_S = \frac{G_S}{G_{S\,max}} = \frac{1.59}{2.141} = 0.74$$

(3)

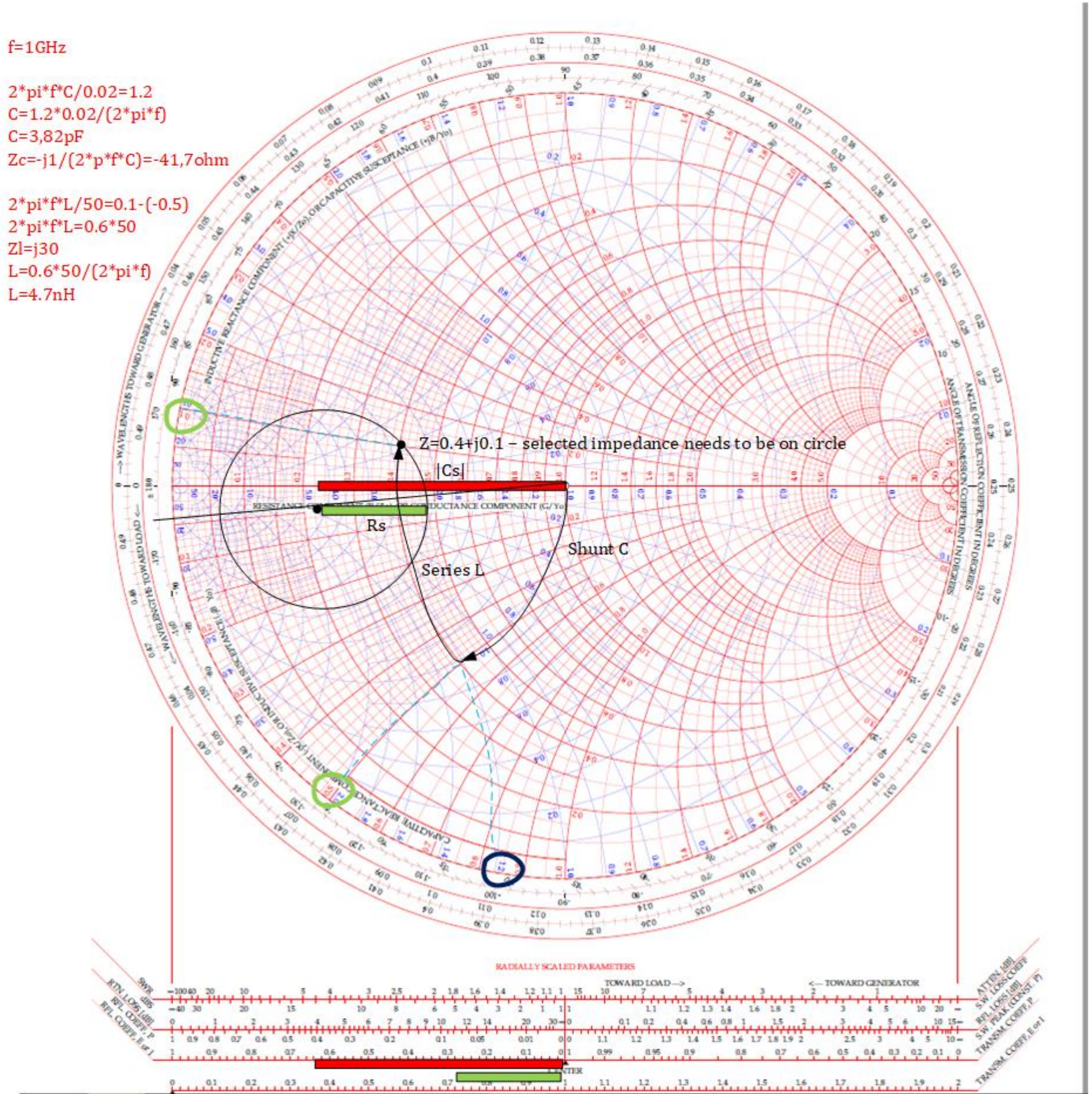
$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)|S_{11}|^2}$$

$$R_S = \frac{\sqrt{1-g_S}\,(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}$$

$$C_S = 0.629\, e^{j-175}$$

$$R_S = 0.274$$

# Example 3.4.1 book of Gonzalez – Solution (3/3)



# Amplifier design: problem 4

A microwave amplifier design is to be designed for $G_{TU,max}$ using a transistor with

$$S_{11}=0.5, \measuredangle 140^{\circ}$$
$$S_{12}=0$$
$$S_{21}=5, \measuredangle 45^{\circ}$$
$$S_{22}=0.6, \measuredangle -95^{\circ}$$

The S-parameters were measured in a 50 Ω system at f=900 MHz, $V_{CE}$ = 15 V, and $I_C$ = 15 mA.

a) Determine whether this amplifier is unconditionally stable.

b) Determine $G_{TU,max}$ . Express your answer in dB.

c) Determine the optimum source impedance and the optimum load impedance for maximum gain. Illustrate your answer in the Smith chart.

d) Design a 2-element matching network at the input and a 2-element matching network at the output of the amplifier to reach conjugate matching to a 50 Ω source and load impedance. Illustrate your answer on the Smith chart.

e) Calculate the constant gain circle for $G_L$=1dB and indicate it in the Smith chart.

# Amplifier design: Problem 4 - solution (1/5)

Problem 1

$S_{11} = 0.5\, e^{j140}$
$S_{12} = 0$
$S_{21} = 5\, e^{j45}$
$S_{22} = 0.6\, e^{j-95}$

a) $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21} < 1$

$K = \dfrac{1 - |S_{11}|^2 - |S_{22}|^2 + \Delta^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1$ } general case

~~For~~ $|S_{11}| < 1$, $|S_{22}| < 1$ } unilateral case

Conditions for unconditional stability

$S_{12} = 0$, unilateral case

$|S_{11}| = 0.5 < 1$, $|S_{22}| = 0.6 < 1 \rightarrow$ amplifier ~~[illegible]~~ unconditionally stable

b) ~~$G_{T\,max} = \frac{1}{1-|\Gamma|}$~~

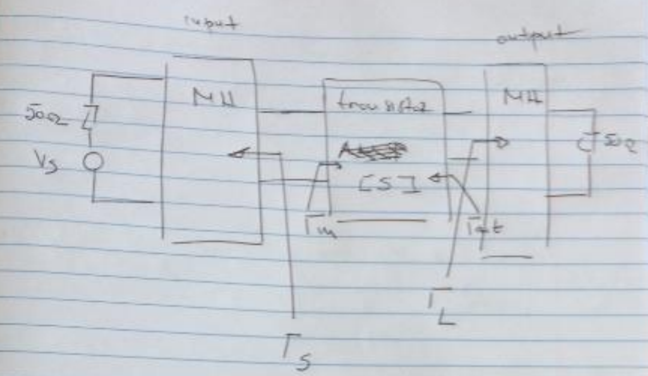
$G_{TU\,max}$ / unilateral case $S_{12} = 0$

$$G_{TU\,max} = \frac{1}{1 - |S_{11}|^2} \, |S_{21}|^2 \, \frac{1}{1 - |S_{22}|^2}$$

$\Gamma_S = S_{11}^*$

$\Gamma_L = S_{22}^*$

(1)




$$G_{TU\,max} = \frac{1}{1 - |0.5|^2} \, |5|^2 \, \frac{1}{1 - |0.6|^2}$$

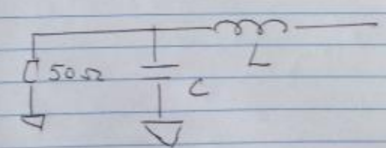
$G_{TU\,max} = 1.33 \cdot 25 \cdot 1.56 = 52$

$G_{TU\,max}\,[dB] = 10 \log G_{TU\,max} = 17.16\ dB$

c) $\Gamma_S = S_{11}^* = 0.5\, e^{-j140}$

$\Gamma_L = S_{22}^* = 0.6\, e^{j95}$

d) MN @ input



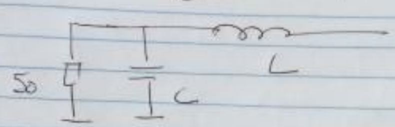

$C = 6.7\ pF$

$L = 1.44\ nH$

(2)

# Amplifier design: Problem 4 - solution (2/5)

Matching at output




$C = 4\ pF$

$L = 11.7\ nH$

e) $G_{L[dB]} = 1\ dB \Rightarrow G_L = 1.26$

$$G_{Lmax} = \frac{1}{1-|S_{22}|^2} = 1.56$$

$$g_L = \frac{G_L}{G_{Lmax}} = 0.8$$

$$C_L = \frac{g_L \cdot S_{22}^*}{1-(1-g_L)|S_{22}^*|^2}$$

$$R_L = \frac{\sqrt{1-g_L}\,(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$

$$C_L = \frac{0.8 \cdot 0.6 e^{j95}}{1-(1-0.8)|0.6|^2}$$

$$C_L = \frac{0.48 e^{j95}}{0.928}$$

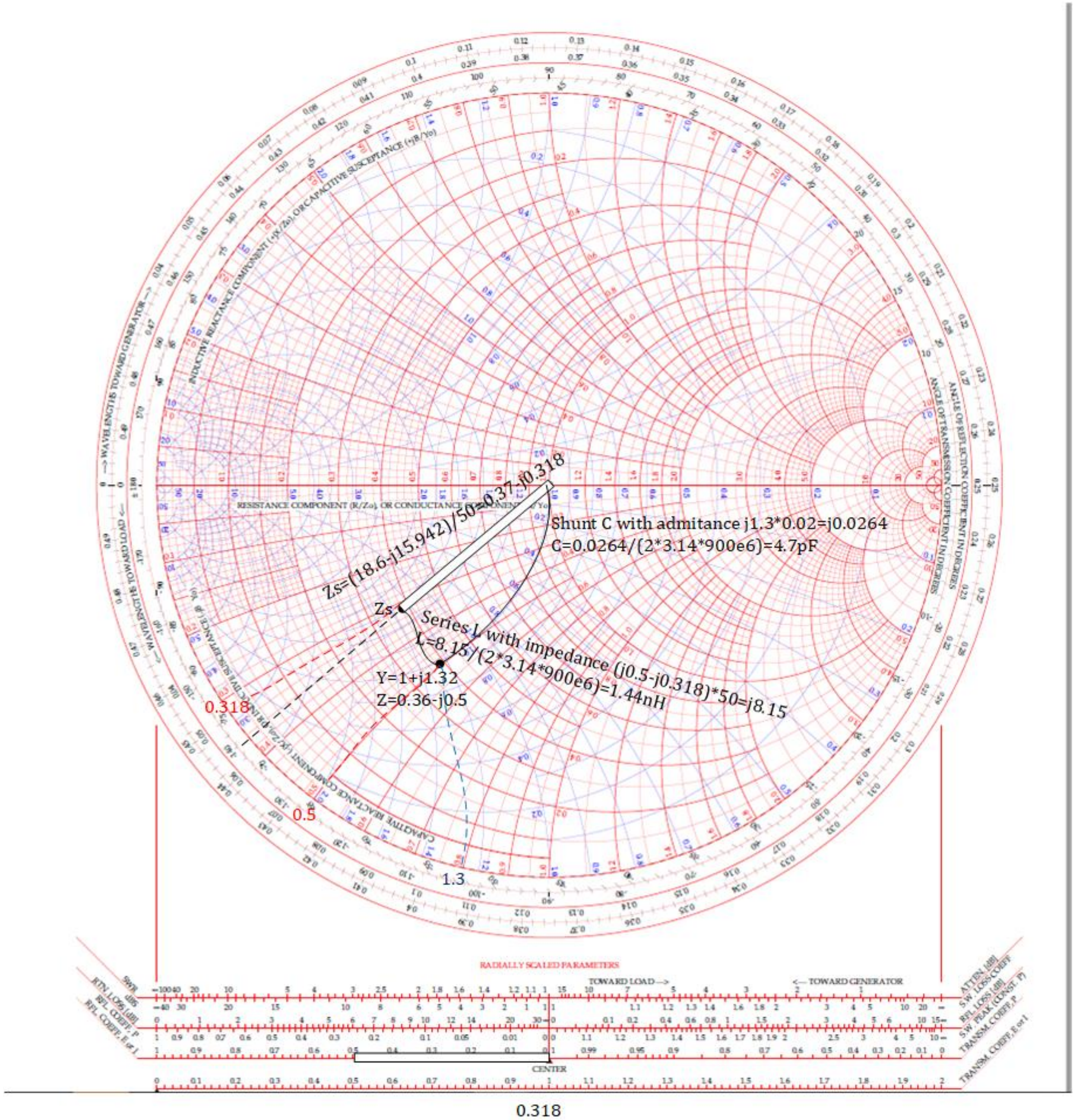
$$\boxed{C_L = 0.52 e^{j95}}$$

$$R_L = \frac{\sqrt{1-0.8}\,(1-0.6^2)}{1-(1-0.8)(0.6)^2}$$

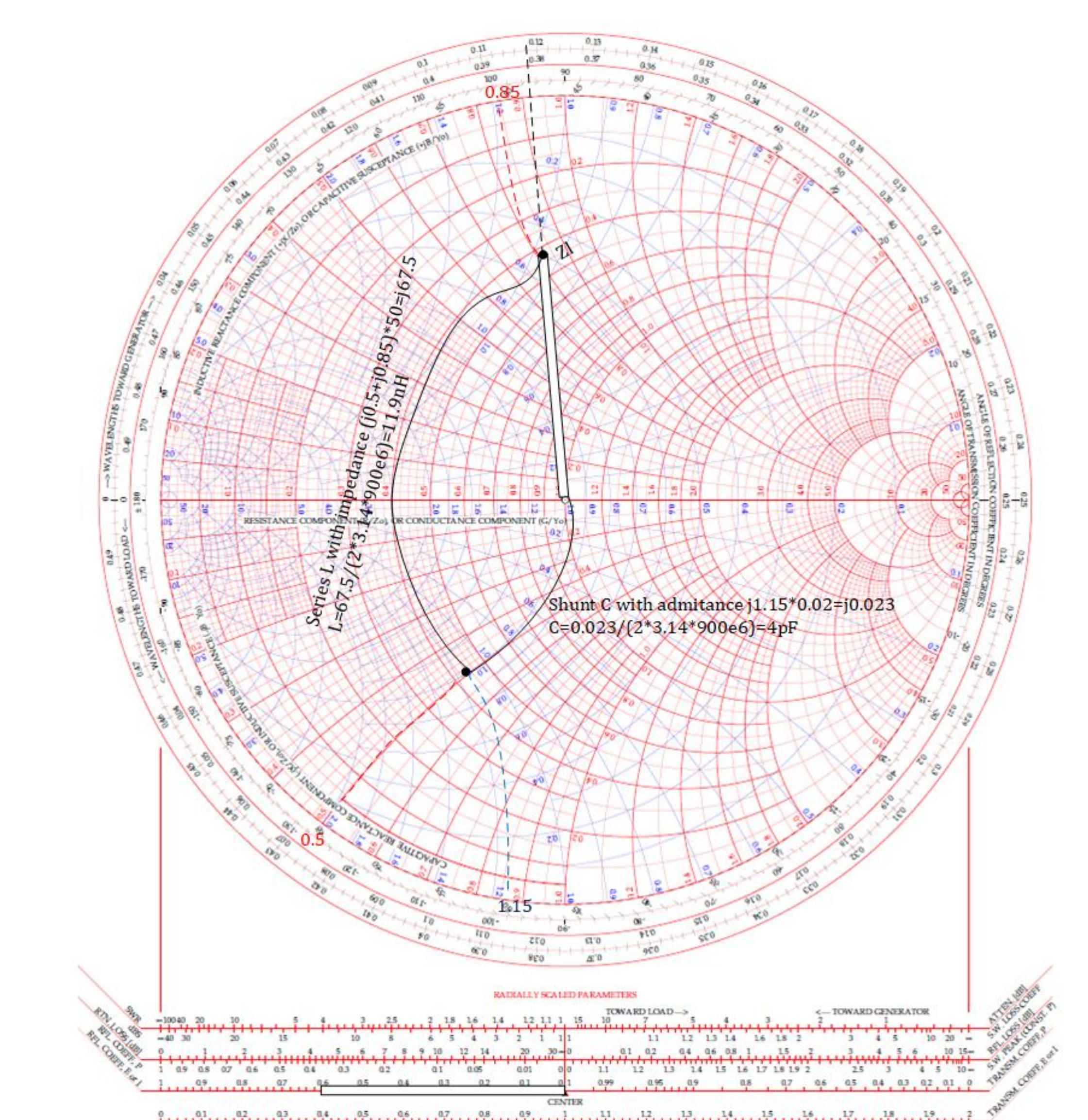
$$R_L = \frac{0.286}{0.928}$$

$$\boxed{R_L = 0.31}$$

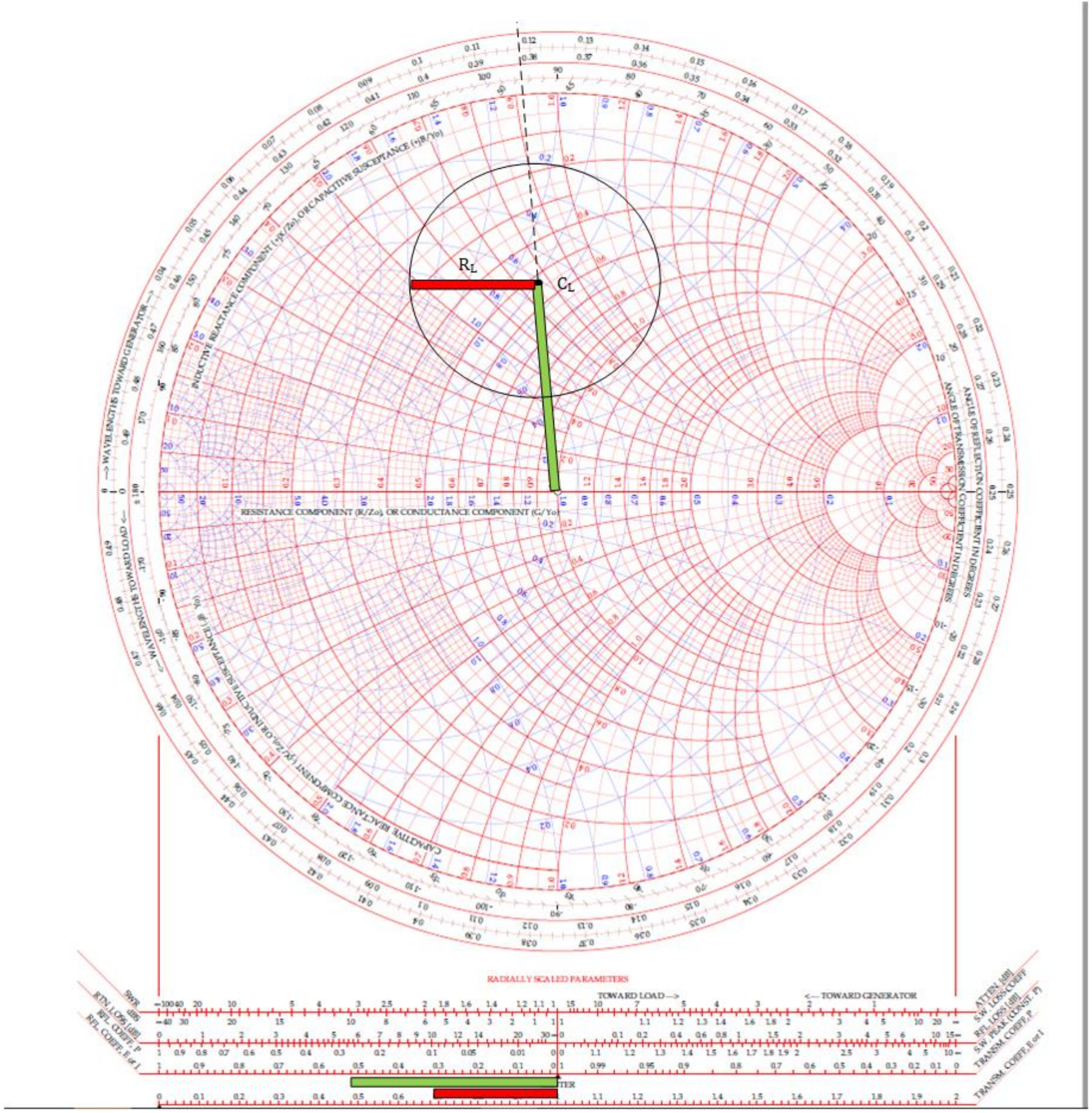
# Amplifier design: Problem 4 - solution, input matching (3/5)



# Amplifier design: problem 4 - solution, output matching (4/5)



# Amplifier design: problem 4 - solution, output matching (5/5)



# Problem 5 – will be in assignment

A microwave amplifier is to be designed for $G_{TU,max}$ using a bipolar transistor with:

$$S_{11}=0.4, \measuredangle\ 130^\circ$$
$$S_{12} = 0$$
$$S_{21}=6, \measuredangle\ 40^\circ$$
$$S_{22}=0.7, \measuredangle\ -75^\circ$$

The S-parameters were measured in a 50 Ω system at f=1 GHz, $V_{CE}$ = 15 V, and $I_C$ = 15 mA.

a) Determine whether this amplifier is unconditionally stable.

b) Determine $G_{TU,max}$ . Express your answer in dB.

c) Determine the optimum source impedance and the optimum load impedance for maximum gain. Illustrate your answer in the Smith chart.

d) Design a 2-element matching network at the input and a 2-element matching network at the output of the amplifier to reach conjugate matching to a 50 Ω source and load impedance. Illustrate your answer on the Smith chart.

e) Calculate the constant gain circle for $G_L$=1.5 dB and indicate it in the Smith chart.

# Amplifier design
# Similar as example 12.2 book of Pozar

The $S$ parameters for the HP HFET-102 GaAs FET at 2 GHz with a bias voltage $V_{gs} = 0$ are given as follows ($Z_0 = 50\,\Omega$):

$$S_{11} = 0.894\angle{-60.6^\circ},$$
$$S_{21} = 3.122\angle{123.6^\circ},$$
$$S_{12} = 0.020\angle{62.4^\circ},$$
$$S_{22} = 0.781\angle{-27.6^\circ}.$$

Determine the stability of this transistor by calculating $K$ and $|\Delta|$, and plot the stability circles.

# Amplifier design
# Example 11.2 book of Pozar - Solution

From (11.31) and (11.24) we compute $K$ and $|\Delta|$ as

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.696\angle{-83^\circ},$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} = 0.607.$$

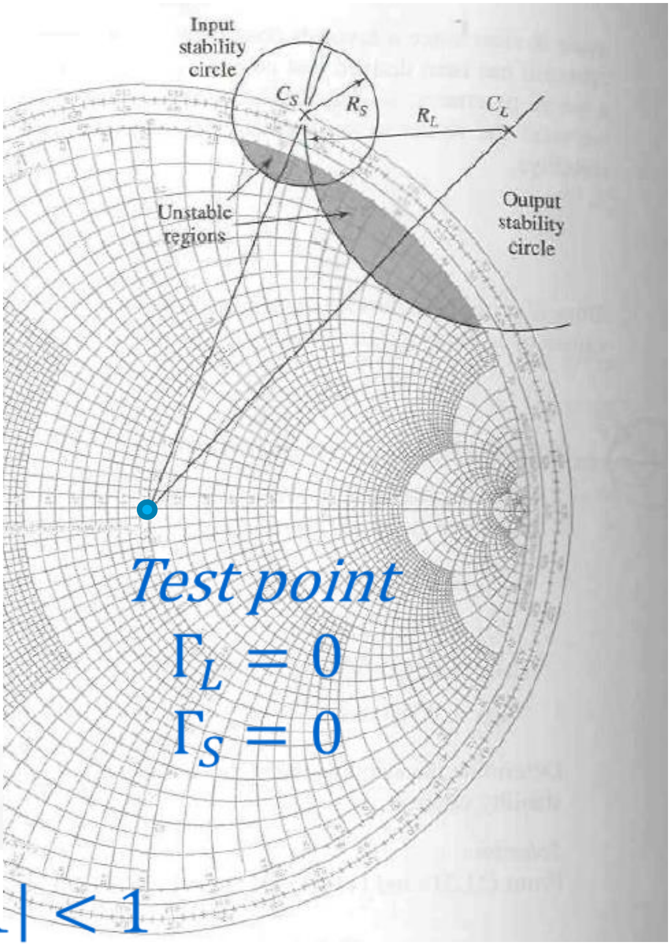
We have $|\Delta| = 0.696 < 1$, but $K < 1$, so the device is potentially unstable. The centers and radii of the stability circles are given by (11.28) and (11.29):

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 1.361\angle 47^\circ,$$

$$R_L = \frac{|S_{12}S_{21}|}{|S_{22}|^2 - |\Delta|^2} = 0.50,$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = 1.132\angle 68^\circ,$$

$$R_S = \frac{|S_{12}S_{21}|}{|S_{11}|^2 - |\Delta|^2} = 0.199.$$




$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{in} = S11 \; for \; \Gamma_L = 0, |S11| < 1$$

$$\Gamma_{out} = S22 \; for \; \Gamma_S = 0, |S22| < 1$$

# Amplifier design: problem 5

The S-parameters and the noise parameters of a transistor at 1 GHz are

$S_{11} = 0.76 \angle -160°$
$S_{12} = 0$
$S_{21} = 5.01 \angle 85°$
$S_{22} = 0.508 \angle -20°$

$F_{min} = 2.2$ dB
$\Gamma_{opt} = 0.5 \angle -150°$
$r_n = 6.6\ \Omega$

a) Proof that the transistor is unconditionally stable.
b) How must the source reflection coefficient $\Gamma_S$ and load reflection coefficient $\Gamma_L$ be chosen to achieve maximum tranducer power gain $G_{TU,max}$. Calculate the required $\Gamma_S$ and $\Gamma_L$ and indicate them on the Smith chart. What are the corresponding values of the source impedance $Z_S$ and the load impedance $Z_L$?
c) For the case of conjugate matching at the input side of the transistor, design an input matching network consisting of an inductor and a capacitor. Calculate the value of the inductance L and the capacitance C required for this network. Illustrate your answer on the Smith chart.
d) Determine the maximum transducer power gain $G_{TU,max}$. Give your answer in dB.
e) For the case that the input matching network is not conjugately matched, calculate the constant gain circle that will lead to a 3 dB lower $G_{TU,max}$ Draw this constant gain circle on the Smith chart.
f) Calculate the center and the radius of the 3 dB constant noise figure circle. Draw the 3 dB constant noise figure circle on the Smith chart.

# Amplifier design: problem 5 – solution a)

a) $|\Delta| = |S_{11} S_{22} - S_{12} \cdot S_{21}| < 1$

$|\Delta| = |0.76\, e^{-j160} \cdot 0.508\, e^{-j20}|$

$|\Delta| = 0.386 < 1 \rightarrow$ OK

$K = \infty > 1 \rightarrow$ OK

generic case

$|S_{11}| < 1 \Rightarrow 0.76 < 1 \rightarrow$ OK

$|S_{22}| < 1 \Rightarrow 0.508 < 1 \rightarrow$ ok

unilateral case

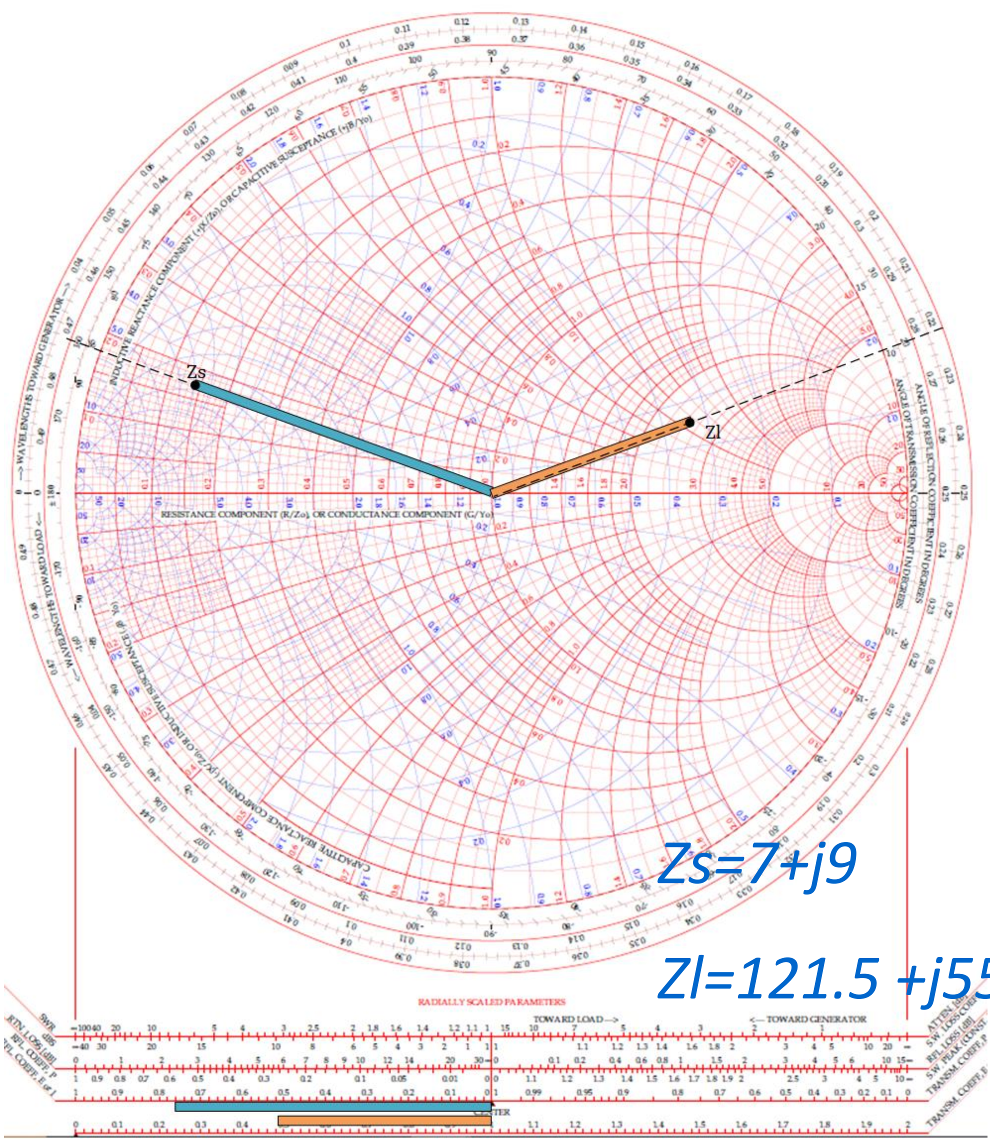
b) $G_{TU\,max}$ / unilateral case $S_{12} = 0$

$$G_{TU\,max} = \frac{1}{1-|S_{11}|^2}\,|S_{21}|^2\,\frac{1}{1-|S_{22}|^2}$$

$$\Gamma_S = S_{11}^* = 0.76\, e^{j160}$$

$$\Gamma_L = S_{22}^* = 0.508\, e^{j20}$$

# Amplifier design: problem 5 – solution b)




*Zs=7+j9*

*Zl=121.5 +j55*

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$\Gamma_S (Z_S + Z_0) = Z_S - Z_0$$

$$\Gamma_S Z_S - Z_S = -Z_0 - \Gamma_S Z_0$$

$$Z_S(\Gamma_S - 1) = -Z_0(1+\Gamma_S)$$

$$Z_S = Z_0 \cdot \frac{1+\Gamma_S}{1-\Gamma_S}$$

$$Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

---

$$Z_S = Z_0 \frac{1+0.76 e^{j160}}{1-0.76 e^{+j160}} = Z_0 \frac{1-0.714+j0.26}{1+0.714-j0.26}$$

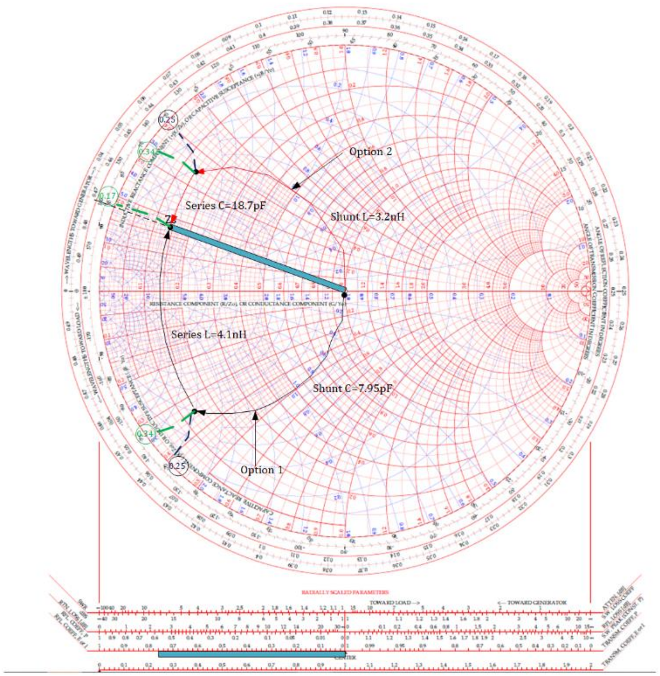
$$Z_S = Z_0 \frac{0.286+j0.26}{1.71-j0.26} \cdot \frac{1.71+j0.26}{1.71+j0.26}$$

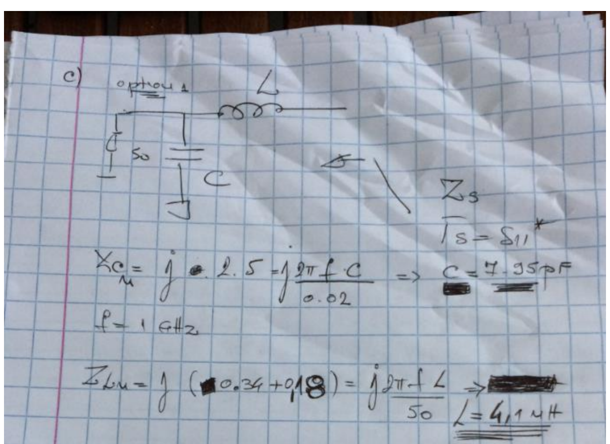
$$\boxed{Z_S = Z_0 \frac{0.42+j0.52}{3} = Z_0 (0.14+j0.18)}$$

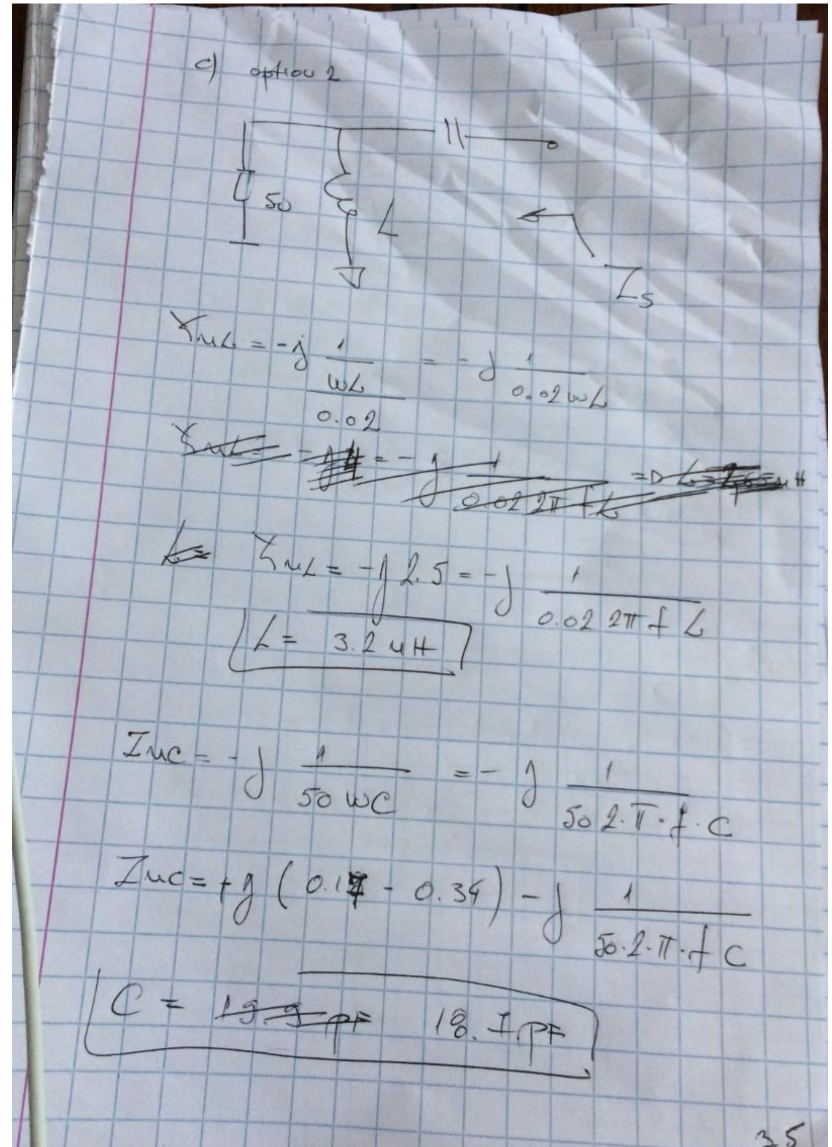
$$Z_L = Z_0 \frac{1+0.508 e^{j20}}{1-0.508 e^{j20}} = Z_0 \frac{1.47+j0.17}{0.52-j0.17}$$

$$Z_L = Z_0 \frac{0.73+j0.33}{0.3} = Z_0 \cdot 2.43+j1.1$$

# Amplifier design: problem 5 – solution c)



# Amplifier design: problem 5 – solution d)

$$G_{TU\,max} = \frac{1}{1-|S_{11}|^2}\,|S_{21}|^2\,\frac{1}{1-|S_{22}|^2}$$

$$G_{TU\,max} = 2{,}36 \cdot 25{,}1 \cdot 1{,}34$$

$$G_{TU\,max} = 79{,}4$$

$$\boxed{G_{TU\,max}[dB] = 19\ dB}$$

# Amplifier design: problem 5 – solution e)

e) $G_{s,max} = \dfrac{1}{1-|S_{11}|^2} = 2.36$

$G_{s,max} = 3.73\text{dB}$

$G_{s\,required} = G_{s\,max} - 3\ \text{dB}$ [dB]

$G_{s\,required} = 0.73\ \text{dB}$ [dB]

$G_{s\,required} = 1.18$

$g_s = \dfrac{1.18}{2.36} = 0.5$

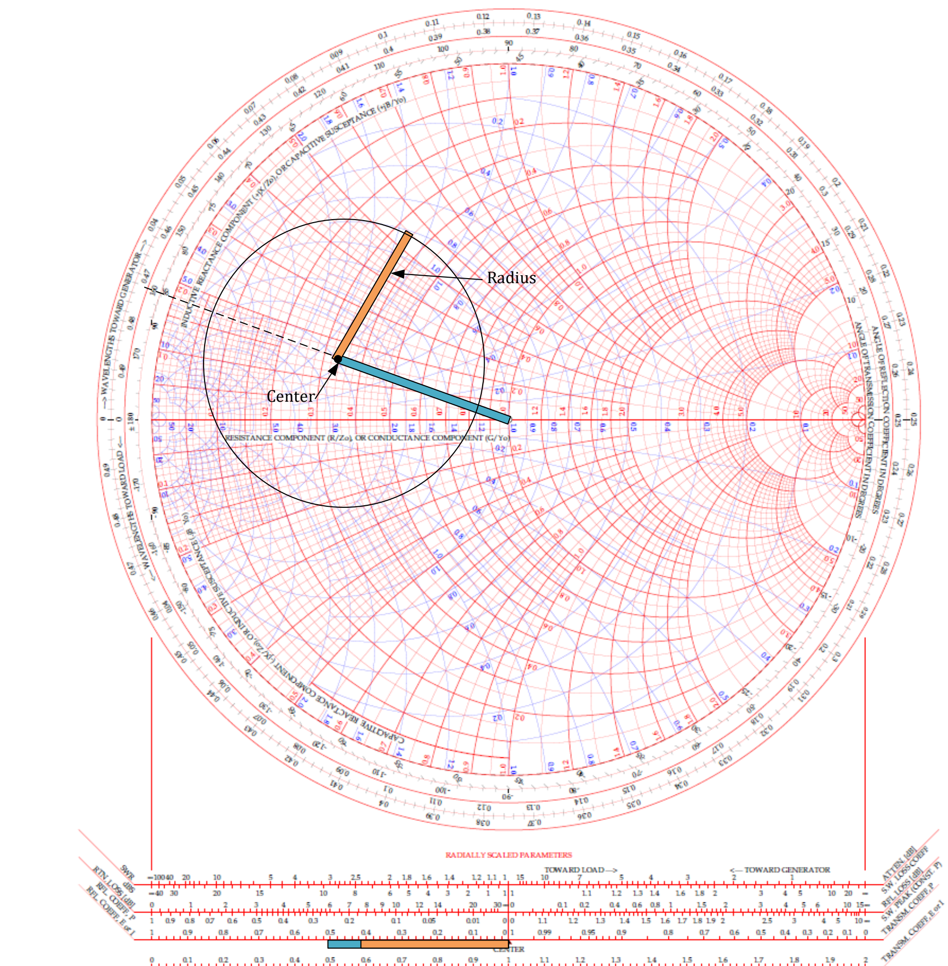
$C_s = \dfrac{0.5 \cdot S_{11}^*}{1-(1-g_s)|S_{11}|^2}$

$C_s = \dfrac{0.5 \quad 0.76\, e^{j160}}{1-(1-0.5)\,0.76^2} = 0.53\, e^{j160}$

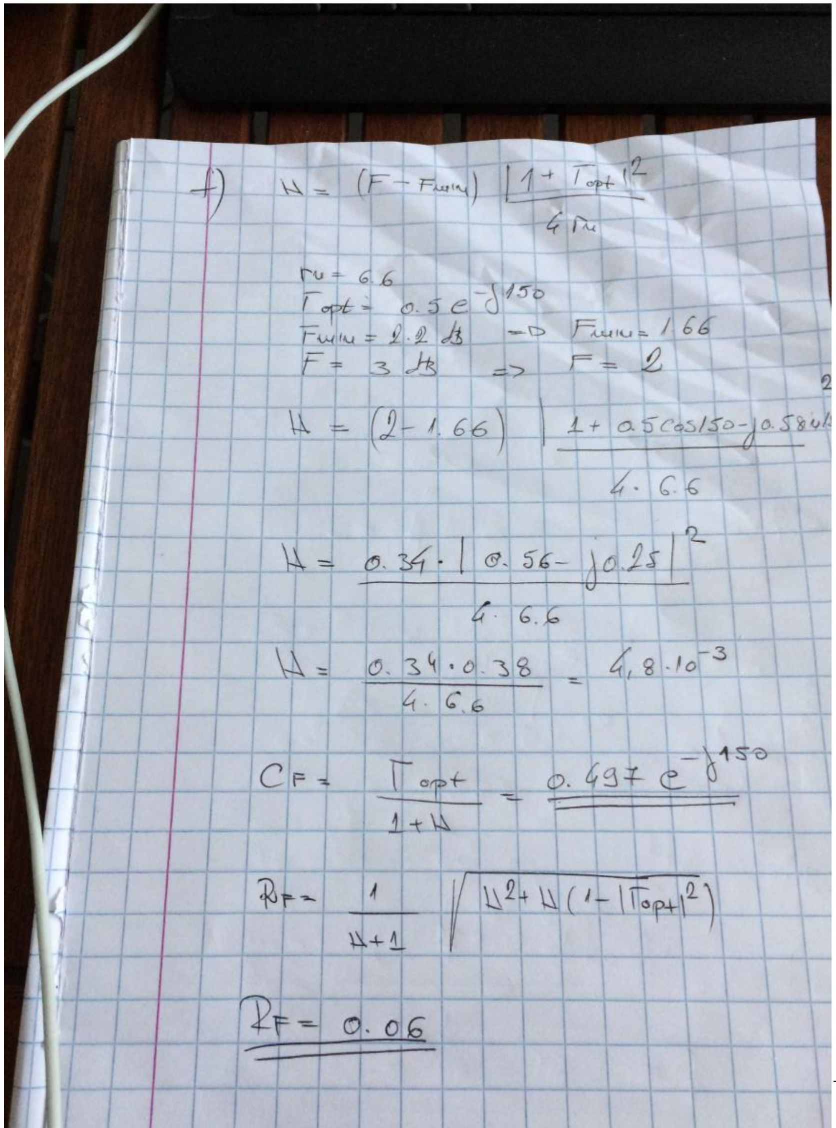
$R_s = \dfrac{\sqrt{1-g_s}\,(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2} = 0.62$

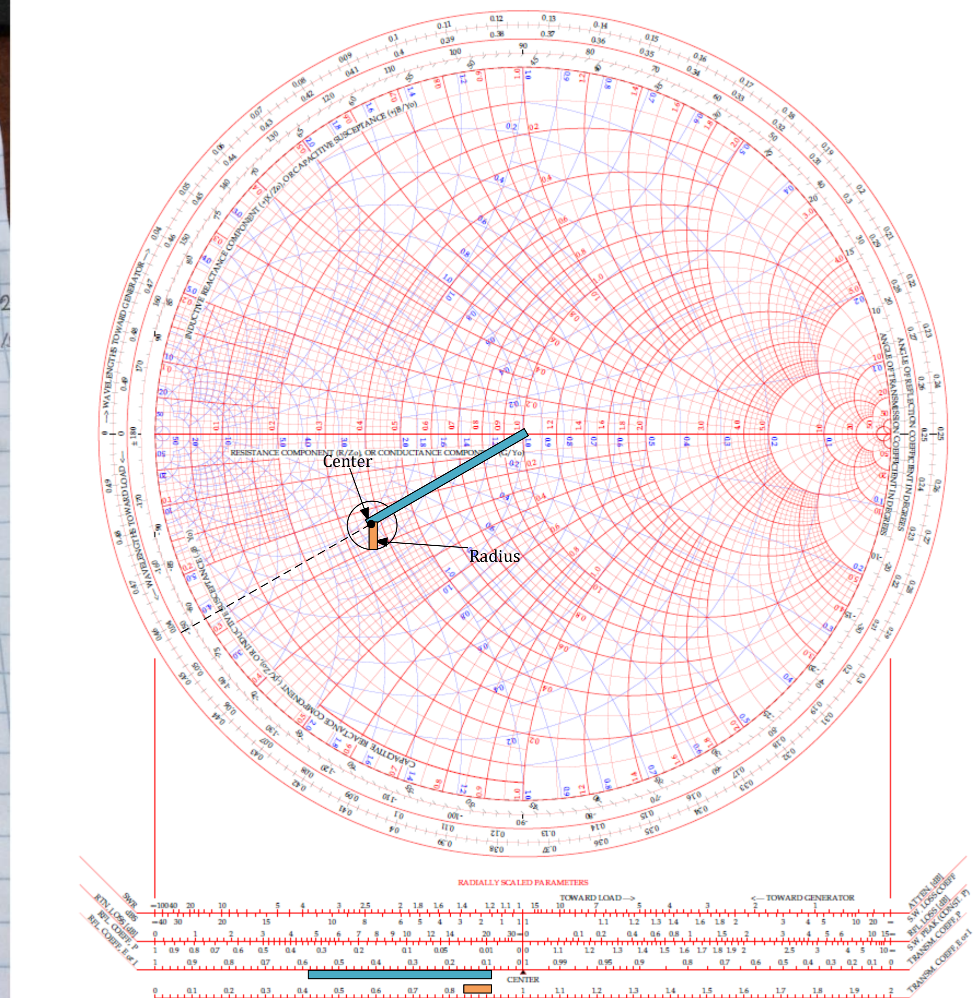
~~$R_s = \dfrac{}{0.71}$~~

4



# Amplifier design: problem 5 – solution f)



# Example 11.5 book of Pozar

A GaAs FET is biased for minimum noise figure, and has the following $S$ parameters and noise parameters at 4 GHz ($Z_0 = 50\,\Omega$): $S_{11} = 0.6\angle{-60^\circ}$, $S_{21} = 1.9\angle{81^\circ}$, $S_{12} = 0.05\angle{26^\circ}$, $S_{22} = 0.5\angle{-60^\circ}$; $F_{min} = 1.6$ dB, $\Gamma_{opt} = 0.62\angle{100^\circ}$, $R_N = 20\,\Omega$. For design purposes, assume the device is unilateral, ~~and calculate the maximum error in $G_T$ resulting from this assumption~~. Then design an amplifier having a 2.0 dB noise figure with the maximum gain that is compatible with this noise figure.

# Example 11.4 book of Pozar – solution (1/2)

Next, we use (11.59) and (11.61) to compute the center and radius of the 2 dB noise figure circle:

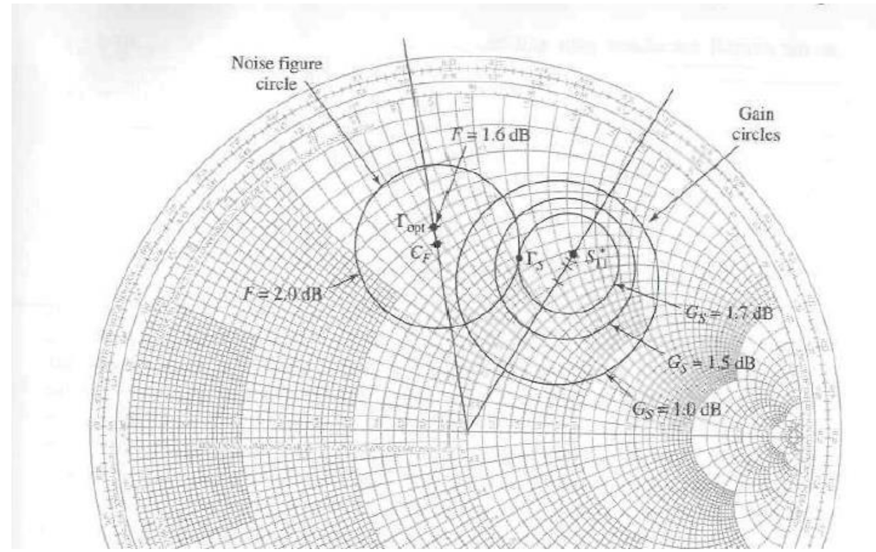
$$N = \frac{F - F_{min}}{4R_N/Z_0}|1+\Gamma_{opt}|^2 = \frac{1.58 - 1.445}{4(20/50)}|1 + 0.62\angle 100^\circ|^2$$

$$= 0.0986,$$

$$C_F = \frac{\Gamma_{opt}}{N+1} = 0.56\angle 100^\circ$$

$$R_F = \frac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{N+1} = 0.24.$$

This noise figure circle is plotted in Figure 11.15a. Minimum noise figure ($F_{min} = 1.6$ dB) occurs for $\Gamma_S = \Gamma_{opt} = 0.62\angle 100^\circ$.




Next we calculate data for several input section constant gain circles. From (11.52),

| $G_S$(dB) | $g_S$ | $C_S$ | $R_S$ |
|---|---|---|---|
| 1.0 | 0.805 | $0.52\angle 60^\circ$ | 0.300 |
| 1.5 | 0.904 | $0.56\angle 60^\circ$ | 0.205 |
| 1.7 | 0.946 | $0.58\angle 60^\circ$ | 0.150 |

These circles are also plotted in Figure 11.15a. We see that the $G_S = 1.7$ dB gain circle just intersects the $F = 2$ dB noise figure circle, and that any higher gain will result in a worse noise figure. From the Smith chart the optimum solution is then $\Gamma_S = 0.53\angle 75^\circ$, yielding $G_S = 1.7$ dB and $F = 2.0$ dB.

For the output section we choose $\Gamma_L = S_{22}^* = 0.5\angle 60^\circ$ for a maximum $G_L$ of

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB}.$$

The transistor gain is

$$G_0 = |S_{21}|^2 = 3.61 = 5.58 \text{ dB},$$