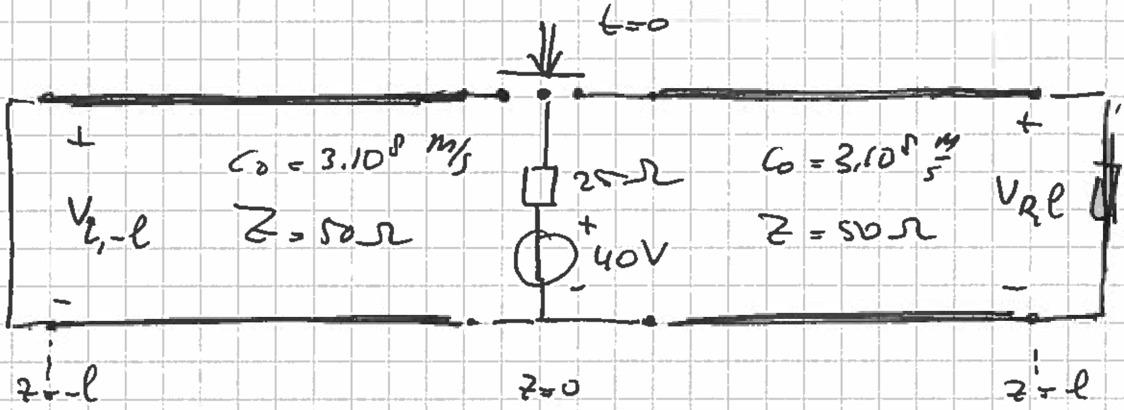
Solutions exam question made in class

1

Situation



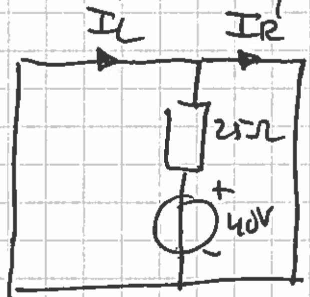
length $\ell = 0.9\ [m]$

(A) for $t \to \infty$ ⇒ steady state, so... no time varying voltage ?

→ TL is steady state

$V_{L,steady}(z,t)$?
$I_{L,steady}(z,t)$?

So, we get the following circuit representation:



note the directions (it is a choice !)

total current $= \frac{40}{25} = 1.6\ [A]$
will flow in both branches, therefore

$I_{L,steady}(z,t) = -0.8\ [A]$ (direction !)
$I_{R,steady}(z,t) = +0.8\ [A]$

$V_{L,steady}(z,t) = 0\ [V]$ (short circuit)
$V_{R,steady}(z,t) = 0\ [V]$

next step is the progressive and regressive waves

This is called wave-decomposition

So $V_{L,steady}$ $I_{L,steady}$ $\longrightarrow$ $V^{+/-}_{L,steady}$ $I^{+/-}_{L,steady}$

now we have

$$V^+_{L,steady} + V^-_{L,steady} = V_{L,steady} = \emptyset\ [V]$$
$$I^+_{L,steady} + I^-_{L,steady} = I_{L,steady} = -0.8\ [A]$$

$I^+_{L,st.} = V^+_{L,steady}/Z$ and $I^-_{L,steady} = - V^-_{L,steady}/Z$

So 4 equations, 4 unknowns --- must be possible

$$\frac{V^+_{L,steady}}{Z} - \frac{V^-_{L,steady}}{Z} = -0.8$$

$$\frac{V^+_{L,steady}}{Z} - \frac{-V^+_{L,steady}}{Z} = -0.8$$

$$\Rightarrow 2.V^+_{L,steady} = -0.8 . Z = -40$$

$$\Rightarrow V^+_{L}\ steady = -20\ [V]$$

$$\Rightarrow V^-_{L}\ steady = 20\ [V]$$

(check $\Sigma = 0$ ✓)

and therefore

$$I^+_{L,steady} = \frac{V^+_{L,steady}}{Z} = -0.4\ [A]$$

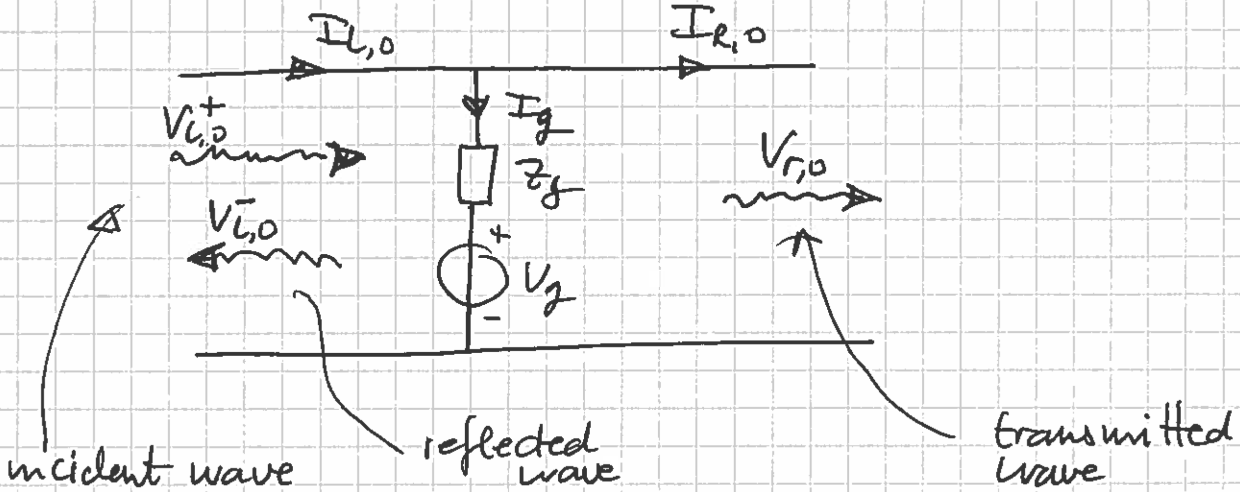
$$I^-_{L,steady} = \frac{-V^-_{L,steady}}{Z} = -0.4\ [A]$$

(check $\Sigma = -0.8\ A$. ✓)

B Reflection coëfficients

$$\Gamma_{L,L} = -1 \quad (\text{short}) \qquad \Gamma_{R,R} = -1 \quad (\text{short})$$

situation is



we know : $V_{L,0}^- = \Gamma V_{L,0}^+$

and $V_{R,0} = T \cdot V_{L,0}^+$

at this $z=0$:

<u>left side</u> : $V_{L,0} = V_{L,0}^+ + V_{L,0}^- = V_{L,0}^+(1+\Gamma)$

and $I_{L,0} = V_{L0}/50\,(1-\Gamma)$

Z_1 !!!

<u>right side</u> : $V_{r,0} = T V_{L,0}^+$

$I_{r,0} = T \cdot \dfrac{V_{L,0}^+}{50}$ ← Z_2

<u>now</u> <u>KVL</u> : $V_{L0} = V_{R,0}$

$$\Rightarrow V_{L0}^+(1+\Gamma) = V_{L0}^+(T)$$

$$\Rightarrow T = 1+\Gamma$$

<u>KCL</u> $I_{L0} = I_{R,0} + I_G$

$$\frac{V_{L0}^+}{50}(1-\Gamma) = \frac{V_{L0}^+}{50}\cdot T + I_g$$

$$I_g \rightarrow \frac{\overbrace{V_{L0}^+ \cdot T}^{V_{r,0}}}{25}$$

<u>So</u>

$$\frac{V_{L0}^+}{50}(1-\Gamma) = \frac{V_{L0}^+}{50}T + \frac{V_{L0}^+}{25}\cdot T$$

$$= \frac{V_{L0}^+}{50}(1-\Gamma) = \frac{V_{L0}^+}{50}(T+2T)$$

$$\Rightarrow \quad 1-\Gamma = 3T$$

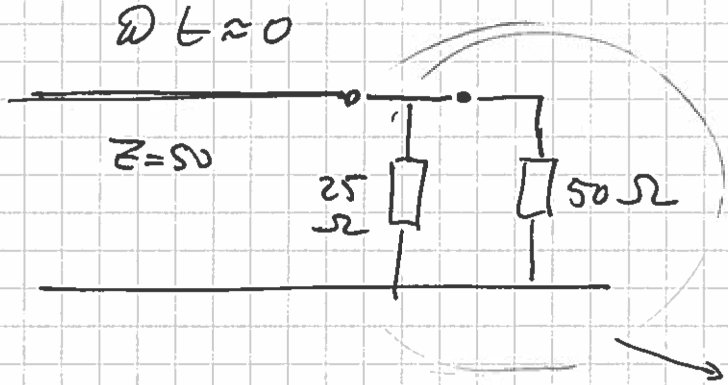
now

$$\begin{array}{r} T = 1+\Gamma \\ 3T = 1-\Gamma \\ \hline 4T = 2 \end{array} \longrightarrow \boxed{T = 1/2}$$

$$\Rightarrow \boxed{\Gamma = -\frac{1}{2}}$$

<u>Alternative solution</u>

@ $t \approx 0$



$\longrightarrow 16\frac{2}{3}\,\Omega$

that means

$$\Gamma = \frac{Z_L - Z}{Z_L + Z} = \frac{-50+16\frac{2}{3}}{50+16\frac{2}{3}} = -\frac{1}{2}$$

$$T = 1+\Gamma \Rightarrow \underline{T = 1/2}$$

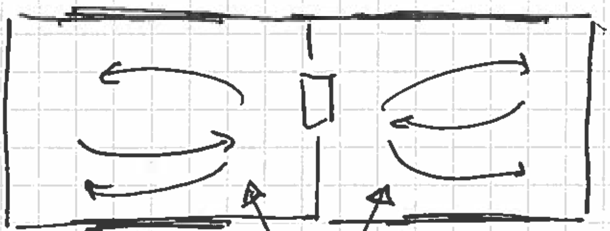
last part of the question

-- the other way around.

$\Rightarrow$ same answers $\quad \Gamma = -\frac{1}{2} \quad T = \frac{1}{2}$

(c) Now we need to sketch the bounce diagram of the total voltage at $z = 0.6$ m for $t \in [0, 18\,ns]$

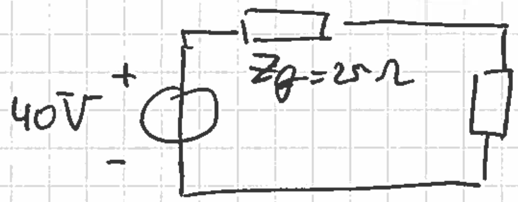
So, what happends is.



same situation at both sides!

length = 0.9 m $\longrightarrow$ $c = 3 \cdot 10^8$ m/s $\longrightarrow$ single trip = 3 ns

at $t = \phi$ $\longrightarrow$ voltage divider:



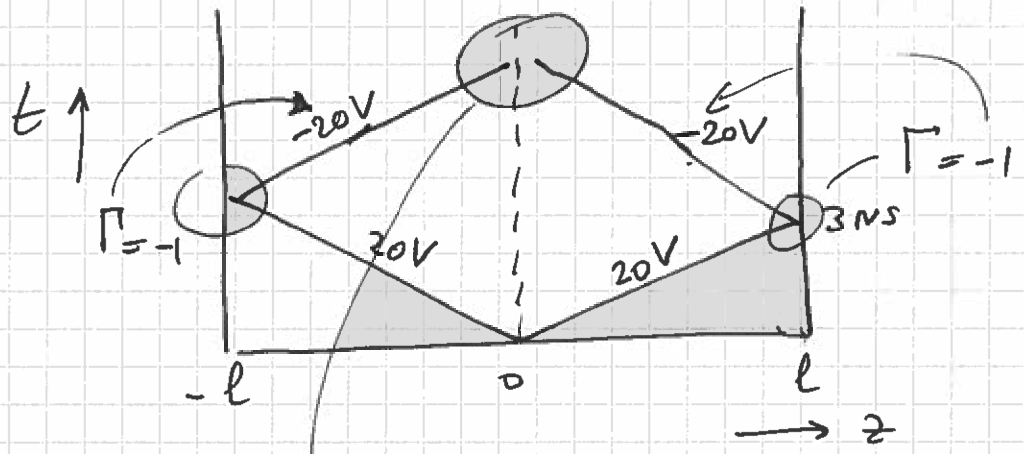
$\Leftarrow$ two transmission lines in parallel

$Z_L = 50\,\Omega // 50\,\Omega$
$= 25\,\Omega$

that means $V_0 = 20\,V$

we also know $\Gamma_{L,L} = \Gamma_{L,R} = -1$

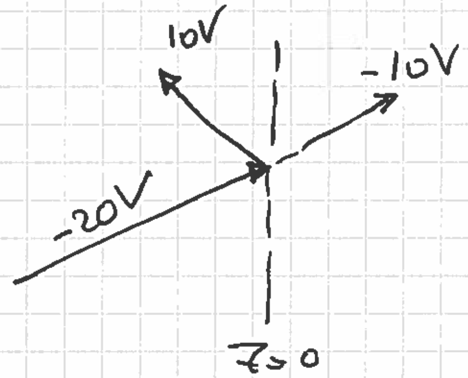
So, let's sketch the beginning of the bounce diagram.



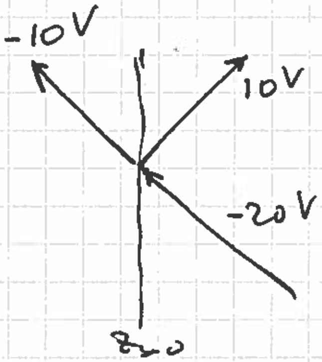
Here it becomes interesting!
we first have to determine the reflection coefficients.

$\Gamma = \frac{Z_g - Z}{Z_g + Z} = \ldots$ see before $\Gamma = -\frac{1}{2}$, $T = \frac{1}{2}$

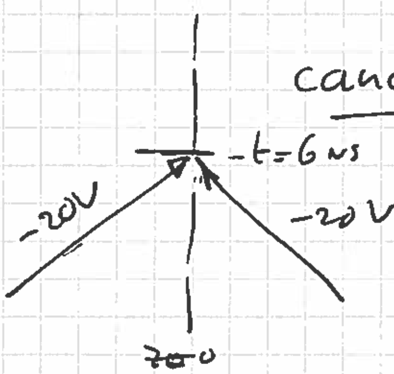
We have



and at the same time ($t = 6ns$)

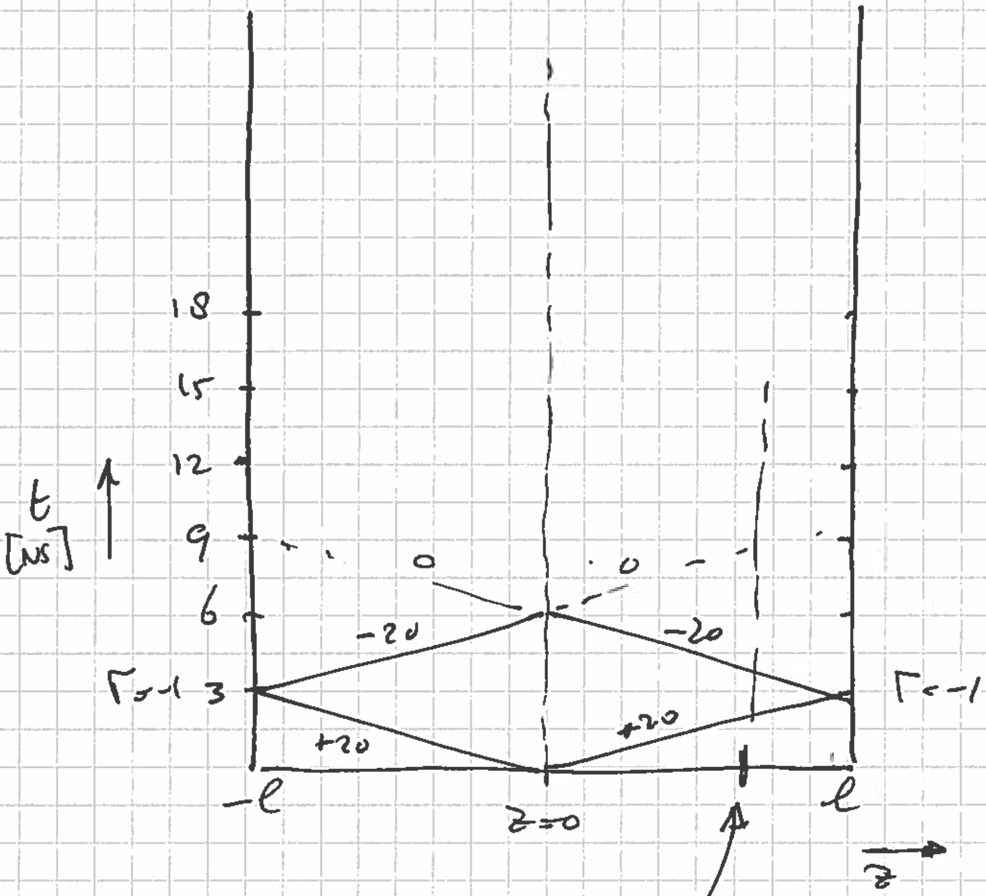


Superposition gives



cancel out!

So, total bounce diagram

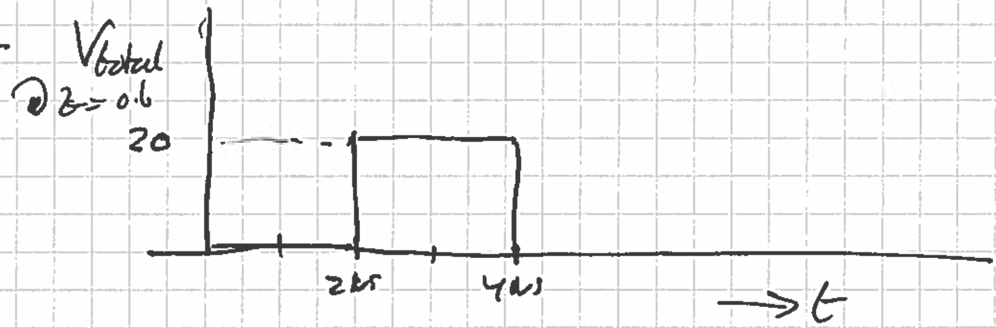


now at z = 0.6

0.6 meter ⟶ $c = 3 \cdot 10^8$ m/s ⟶ t = 2ns

regressive wave at t = 4ns

So



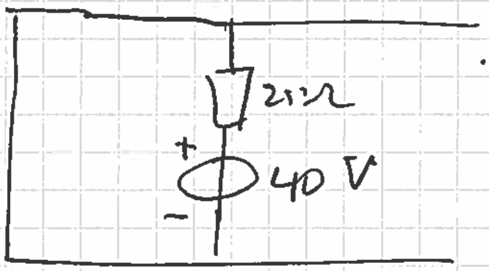
D Now we make an "open" at the right branche.

$\underline{Q}$: What is $V_{R,steady}(z,t)$ and $I_{R,steady}(z,t)$

+ with wave decomposition

$V^{+/-}_{R,steady}(z,t)$ and $I^{+/-}_{R,steady}(z,t)$

now situation is



$\left.\right\}$ $t \to \infty$
$V = 0$ (short at $z=\ell$.)

$V_{R,steady}(z,t) = 0$ [V]

Since it is open at the right side $\Rightarrow$ $I_{R,steady}(z,t) = 0$ [A]

no current at an open end.

Same situation for the progressive and regressive wave in the $t \to \infty$ situation

So $V^{+/-}_{R,steady}(z,t) = \phi$

$I^{+/-}_{R,steady}(z,t) = \phi$

E What is the total voltage at $z = \phi.6$ for $t \in [0, 18]$ ns

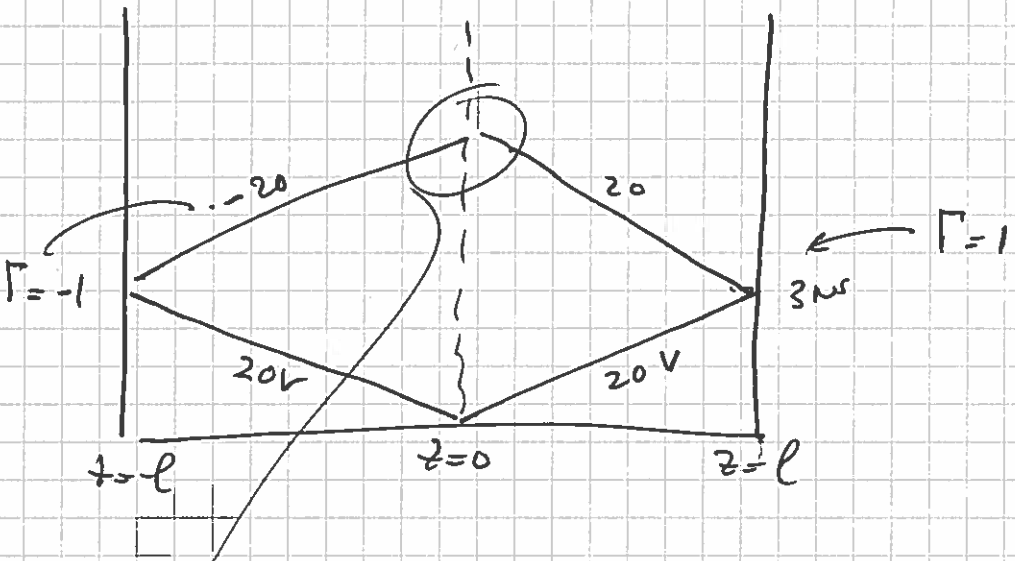
At the beginning: $V_0 = 40 \cdot \dfrac{\cancel{80}\ 25}{\cancel{80+}\ 25+25} = \underline{20\,[V]}$

is 2 50Ω TL in parallel.

same situation as before.

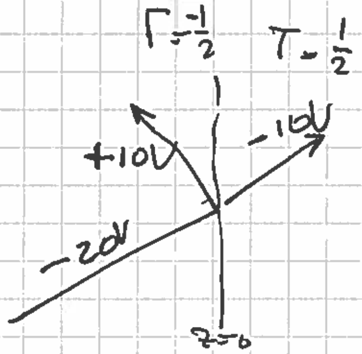
now $\Gamma_{L,L} = -1$ (short)
$\Gamma_{L,R} = 1$ (open).

so, beginning of the bounce diagram

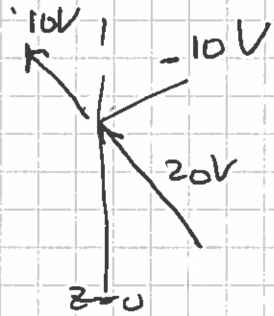


Here it is interesting again

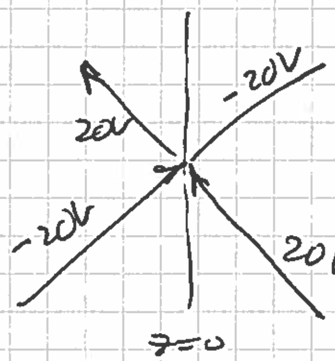
we have



+

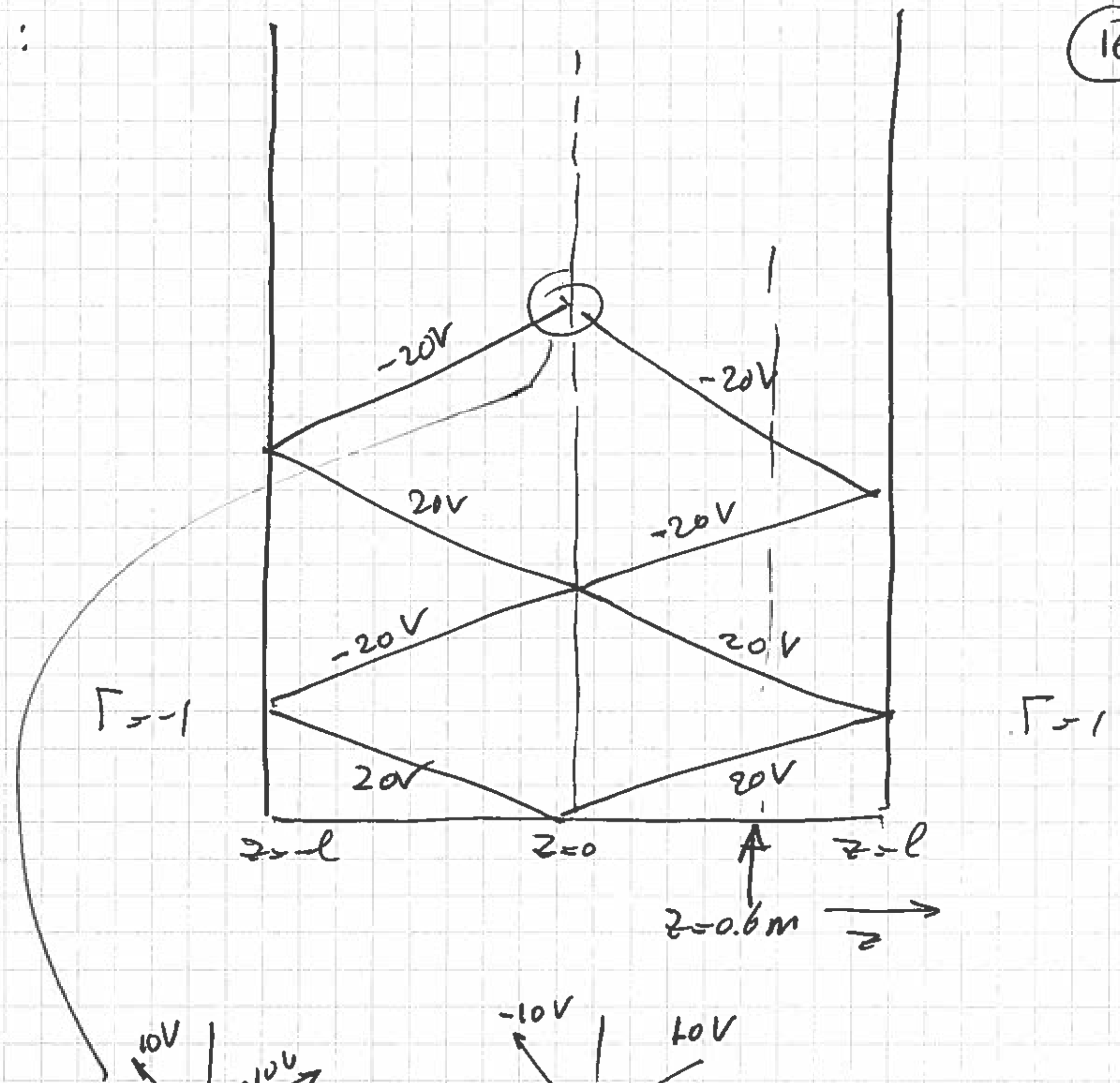


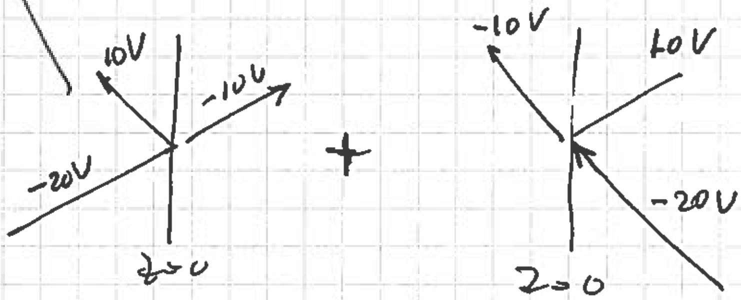
=



(superposition)

So:





= cancel out again!

So, now at z=0.6 m