Filtering a Continuous-Time Signal Digitally

5ESE0 - January 2023

Daniel Tyukov - 1819283

I. Abstract

This final lab examines a signal processing mechanism that utilizes a combination of analog-to-digital and digital-to-analog conversion, along with two distinct filtering stages. The investigation employed the use of MATLAB for data analysis and calculations. The results revealed that the system effectively eliminated one signal and significantly reduced another while leaving the last one unchanged.

II. Introduction

The aim of this report is to gain a comprehensive understanding of the Signals course by applying the acquired knowledge to various systems. The report will proceed by outlining the steps involved in the process and subsequently drawing conclusions. Specific components such as a C/D convertor, FIR filters, a Bandpass filter, and a D/C converter will be discussed and evaluated. Additionally, the concept of aliasing and its impact on the system will be explored as a crucial aspect of the report.

III. C/D Converter

1.1 Exercise 1

	x(t)	Amp	θ	φ	Alias	x(n)
x1	$2cos(2\pi 200t + \frac{\pi}{3})$	2	$\frac{2}{5}\pi$	$\frac{\pi}{3}$	False	$2cos(\frac{2}{5}\pi n + \frac{\pi}{3})$
x2	$5\cos(2\pi 350t + \frac{\pi}{2})$	5	$\frac{7}{10}\pi$	$\frac{\pi}{2}$	False	$5\cos(\frac{7}{10}\pi n + \frac{\pi}{2})$
x3	$5cos(2\pi 600t + \frac{\pi}{4})$	5	$\frac{4}{5}\pi$	$-\frac{\pi}{4}$	True	$5cos(\frac{4}{5}\pi n - \frac{\pi}{4})$

Figure 1.1.1: x(t) & x(n) properties

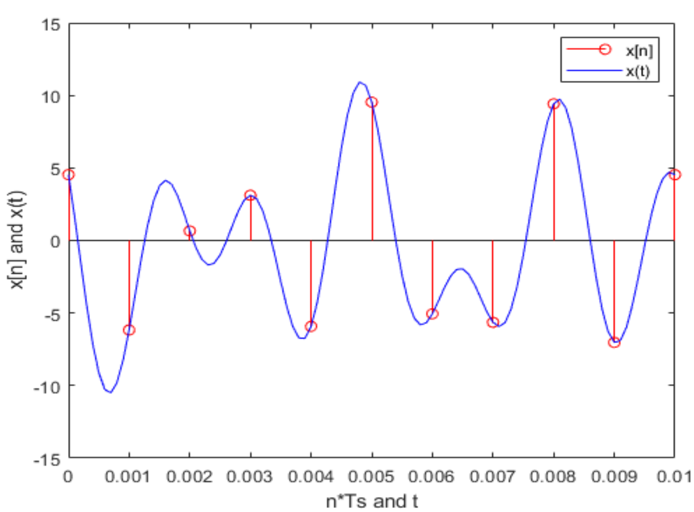

Figure 1.1.2: x(t) and x(n) plot

Upon hearing the signal, one can make various observations and justify them through analysis. The order of the pitches can be identified as x1(t) being the lowest, followed by x2(t), and then x3(t). This is because x1(t) has the lowest frequency out of the three components, while x3(t) has the highest. Additionally, the difference in volume is noticeable, with x2(t) and x3(t) being significantly louder than x1(t). This can be attributed to the fact that the amplitude of x2(t) and x3(t) is 5, while x1(t) has an amplitude of 2.

Conversion to discrete time domain is performed:

$x(n) = Acos(\theta n + \varphi)$	$\theta = 2\pi \frac{f0}{fs}$

The process of converting continuous-time signals to discrete-time signals usually preserves the amplitude and phase shift unless aliasing takes place. Aliasing causes a change in the sign of the phase shift, from positive to negative or vice versa. In this particular scenario, aliasing is observed in the case of x3(t), as the normalized radian frequency for x3(t) falls outside the given domain. As a result, the new phase shift for x3(t) is $-\frac{\pi}{4}$, instead of $\frac{\pi}{4}$. A summary of these values can be found in the accompanying **Figure 1.1.1**.

The frequency of the discrete signals x1(n) and x2(n) align with their corresponding continuous-time signals x1(t) and x2(t) respectively. However, the frequency of x3(n) is perceived as lower than x3(t) due to the effect of aliasing during the sampling process. As a result, the combined frequency of x(n) is also lower than that of x(t). Refer to **Figure 1.1.2** for a visual representation of x(t) and its sampled points x(n).

IV. Nulling Filter FIR1

2.1 Exercise 2

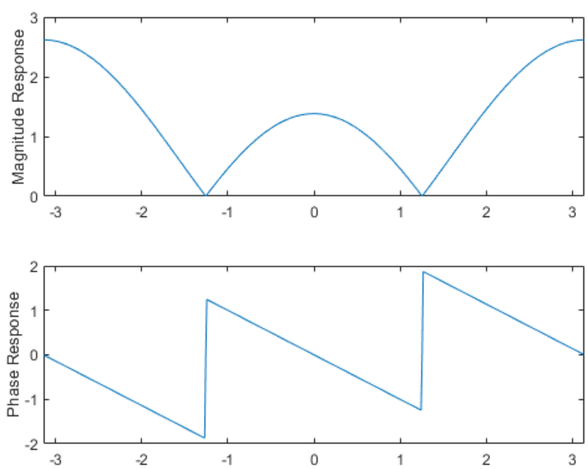

Figure 2.1.1: Magnitude/Phase of FIR1

The initial filter is a 3-level nulling filter:

$v(n) = x(n) - 2cos(\theta null)x(n - 1) + x(n - 2)$

0.4π was provided as θnull and so therefor we can derive the impulse and the frequency in the following way:

$h1(n) = \delta(n) - 2cos(0.4\pi)\delta(n - 1) + \delta(n - 2)$
$H1(e^{j\theta}) = 1 - 2cos(0.4\pi)e^{-j\theta} + e^{-2j\theta}$
$H1(e^{j\theta}) = e^{-j\theta}(e^{j\theta} - 2cos(0.4\pi) + e^{-j\theta})$
$H1(e^{j\theta}) = e^{-j\theta}(2cos(\theta) - 2cos(0.4\pi))$

And so therefor we conclude that the magnitude and phase response are the following:

$\|H1(e^{j\theta})\| = 2cos(\theta) - 2cos(0.4\pi)$	$\Phi H(e^{j\theta}) = e^{-j\theta}$

It can be observed that when $\theta = 0.4\pi$, the amplitude of the signal is cancelled out by the two cosines. This is also depicted in **Figure 2.1.1**, where there is a dip in amplitude at the points of $- 0.4\pi$ and 0.4π.

2.2 Exercise 3

Nulling filter only affects signals with a specific frequency, in this case θnull = 0.4π, so only x1(n) is nullified. When tested in MATLAB v(n) sounds very similar to vth(n), indicating that x1(n) has nullified. But if observing **Figure 2.1.1** there is an affect that the filter caused on other signals that is not noticeable to the human ear.

$vth = x2(n) + x3(n)$

V. Bandpass Filter FIR2

3.1 Exercise 4

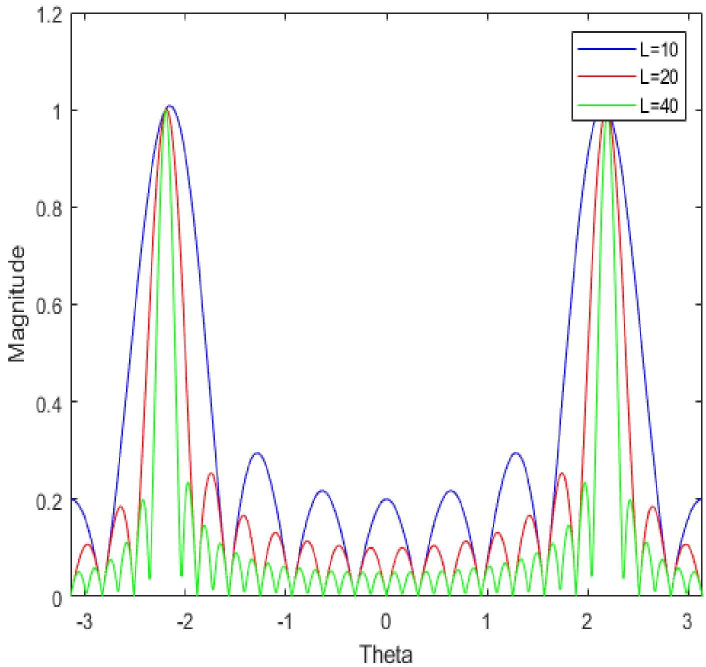

Figure 3.1.1: BPF for L's

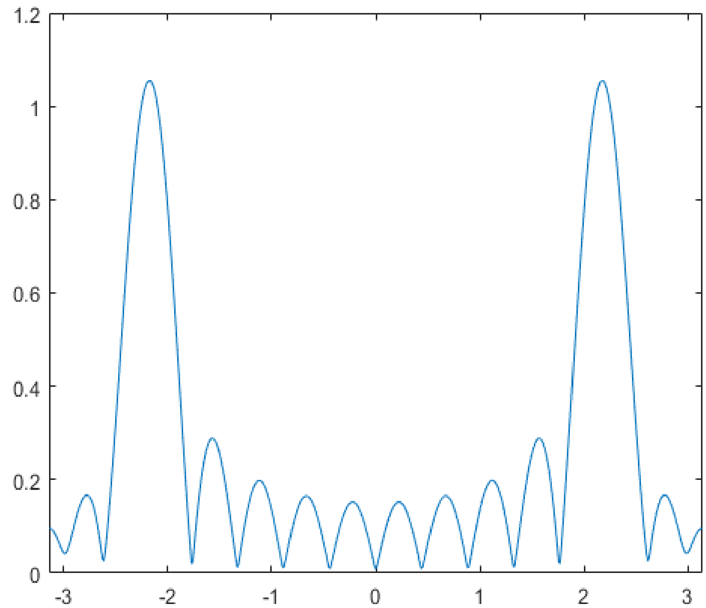

Figure 3.1.2: for L = 14

L	Wpb
10	0.527787565803085
20	0.263893782901543
40	0.125663706143592

Figure 3.1.3: Length and PB Width

Bandpass filter definition:

$$h2(n) = \frac{2}{L} cos(n\theta c)\ for\ n = 0, 1, (L - 1)$$

The parameter θc has been fixed at 0.7π and the task is to examine the plots when L is 10, 20, and 40. By analyzing these plots, it becomes simple to determine the width of the passband and thus, establish a relationship between L and Wpb.

Figure 3.1.1 illustrates the three plots with L values of 10, 20, and 40. From the figure, it is clear that as the filter length decreases, the width of the passband increases. To gain a more precise understanding, **Figure 3.1.3** displays the specific values of the width of the passband and the corresponding filter length. By analyzing these values, it can be deduced that as the filter length is doubled, the width of the passband is halved.

If more samples were sampled the results would showcase such a relationship even more clearly.

The value of L that results in a reduction by a factor of 10 is L = 14, as depicted in **Figure 3.1.2**. The plot indicates that any signals with a magnitude close to zero will be diminished, while signals with a magnitude close to 1 will remain unchanged. In the case of $\theta = 0.82\pi$, the signal is resulting in suppression.

The output signal is denoted with y(n), after testing the sounds inside MATLAB there is now a clear difference between v(n) and y(n). Such difference in sound is due to the following facts: x2(n) does not get suppressed since if looked at the previous figures lays somewhere in the middle. While x3(n) does get suppressed but not completely as the magnitude is not 0. If we could increase the length more then full suppression of x3(n) is theoretically possible.

VI. D/C Converter

4.1 Exercise 5

fs	f0	y(t)
800Hz	280Hz	$5cos(2\pi 280t\ + \frac{\pi}{2})$
1000Hz	350Hz	$5cos(2\pi 350t\ + \frac{\pi}{2})$
1200Hz	420Hz	$5cos(2\pi 420t\ + \frac{\pi}{2})$

Figure 4.1.1: Final Signal

We assume even though not completely true that now our final signal in the discrete-time domain is represented as shown below (no longer considering x3) alongside the formula to convert it to the continuous time domain adjusted per sampling frequency used.

$y(n) = x2(n)$	$f0 = \frac{\theta}{2\pi} fs$

As shown in **Figure 4.1.1**

If the sampling rate is 1000Hz, the output signal y(t) will be identical to x2(t). However, if the sampling rate is decreased to 800Hz, y(t) will have a lower pitch compared to x2(t). On the other hand, if the sampling rate is increased to 1200Hz, y(t) will have a higher pitch in comparison to x2(t). It is worth noting that the amplitude and phase of y(t) will remain unchanged in all the above scenarios.

VII. Overall System

5.1 Exercise 6

The continuous-time signal x(t) was transformed into discrete-time samples x(n) by a C/D converter. However, during this process, x3(t) was affected by aliasing. Then x(n) went through a nulling filter, which nullified x1(n). The output of this filter was the samples v(n). These samples were then filtered by a bandpass filter, which only partially suppressed x3(n) due to the filter length L being insufficient. The output of this filter was the samples y(n). These samples were then converted back to a continuous-time signal, y(t), by a D/C converter. It was determined that the only correct sampling frequency was fs = 1000Hz.

To simplify the system, FIR1 could be removed as FIR2 would be able to filter out x1(n). To improve the system, FIR2 could be made more efficient by increasing its filter length L which would result in a shorter passband width Wpb and complete suppression of x1(n) and x3(n).

VIII. Conclusion

In conclusion, a thorough analysis can lead to better solutions, as the desired outcome of completely removing the signals x1(t) and x2(t) was not achieved. But as explained before, a simpler system may be more efficient. The design should be chosen based on the desired outcome.