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2/15/22 **Write Up 7** Advanced Topics in Math, Foil

For problems 1-3, let $H, K \leq G$.

1. Prove that $H \subseteq K \implies H \leq K$.

   Suppose $H \subseteq K$. We want to show that H is a subgroup of K, that is, we want to show that H satisfies the three axioms of a group.

   First, since H is a subset of K, any element of H is also an element of K. So H inherits the associative property of the binary operation from K.

   Second, since $H \subseteq K$, the identity element of K must also be an element of H. Therefore, H has an identity element.

   Third, for any element h in H, its inverse in K is also in H because $H \subseteq K$. Therefore, H has inverses for all its elements.

   Since H satisfies the three axioms of a group, we can conclude that H is a subgroup of K.

2. Show that $H \cap K \leq G$.

3. Let G be an abelian group, and define HK as follows:

$$HK = \{hk \mid h \subseteq H \text{ and } k \in K\}$$

   Prove that $HK \leq G$.

4. Suppose a group G is generated by two elements a and b.
   Prove that $ab = ba \implies G$ is abelian.

   Proof. Let us assume that $ab = ba$. We know that G is a group and hence we only need to prove G is communitive for the group to be abelian. ■

5. Define the center of a group to be

$$C = \{g \in G \mid gx = xg, \forall x \in G\},$$

   that is, the set of all elements of G that commute with every element of G.
   Prove $C \leq G$.