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9/19/22 **Write Up 1** Advanced Topics in Math, Foil

1. Prove that an integer is odd if and only if it is the sum of two consecutive integers.

   Proof. Let x be the integer
   ($\Rightarrow$) Suppose x is odd, so there is an integer a such that $x = 2a + 1$. $x = 2a + 1 = a + a + 1 = a + (a + 1)$, and since a is integer, $a + 1$ must also be a integer. Hence, x is the sum of two consecutive integers a and $a + 1$.
   ($\Leftarrow$) Suppose x is the sum of two consecutive integers. Let the two consecutive integer be a and $a + 1$. Then, we have $x = a + (a + 1) = 2a + 1$. By defination, x is an odd number. ∎

2. Let x and y be integers. Prove that $x \leq y - 1$ if and only if $x < y$.

   Proof. Let x and y be integers.
   ($\Rightarrow$) Suppose $x \leq y - 1$. It indicates $y - x \geq 1$ after organize it according to the property of inequalities. Since $1 > 0$, we have $y - x \geq 1 > 0$ and hence $y - x > 0$. Move x to the opposite side, we can conclude that $x < y$.
   ($\Leftarrow$) Suppose $x < y$. This indicates that $y - x > 0$. We know that $y - x$ is integer since both y and x are integers. Sinice there difference is greater than zero and is a integer, it needs to be at least 1. Hence, we have $y - x \geq 1$

   $$\begin{aligned} y - x &\geq 1 \\ 1 &\leq y - x \\ x + 1 &\leq y \\ x &\leq y - 1 \end{aligned}$$

   By organizing the inequality, we can conclude that $x \leq y - 1$. ∎

3. Let a be an integer. Prove that a is odd if and only if there is an integer x such that $a = 2x - 1$.

   Proof. Let a be an integer.
   ($\Rightarrow$) Suppose a is odd. By defination, there exist an integer b such that $a = 2b + 1$. Let integer $x = b + 1$. We can rewrite $a = 2b + 1 = 2(b + 1) - 1 = 2x - 1$. Hence, we can conclude $a = 2x - 1$.
   ($\Leftarrow$) Suppose there is an integer x such that $a = 2x - 1$. We can rewrite it as $a = 2(x - 1) + 1$. Let integer $y = x - 1$. Then, we will have $a = 2y + 1$, and a is odd clearly. ∎

4. Prove that the difference between consecutive perfect squares is odd.

Proof. Let x be an integer. Then x^2 and $(x+1)^2$ must be consecutive perfect square. Let a be there difference and $a = (x+1)^2 - x^2$.

$$\begin{aligned} &= (x+1)^2 - x^2 \\ &= x^2 + 2x + 1 - x^2 \\ &= 2x + 1 \end{aligned}$$

Therefore, $a = 2x + 1$, by defination, a is odd and hence the difference is odd. ∎

5. Disprove the proposition: Two right triangles have the same area is and only if the lengths of their hypotenuses are the same.

Proof. Let the first triangle has legs of 4 and 6 and the second triangle has legs of 3 and 8. They have the same area $4 * 6 * \frac{1}{2} = 3 * 8 * \frac{1}{2} = 12$. However, the hypotenuse of the first triangle is $\sqrt{4^2 + 6^2} = \sqrt{52}$ and the hypotenuse of the second triangle is $\sqrt{3^2 + 8^2} = \sqrt{73}$. Obviously, $\sqrt{52} \neq \sqrt{73}$ according the Pythagorean theorem . Hence, the length of their hypotenuses are not the same, the statement is disproved. ∎