

CS 4820, Spring 2017 Homework 10, Problem 2

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Solution:

(1) Prove that INS is recursively enumerable (RE).

Suppose we have a TM that takes INS as input, let the machine enumerate strings in the alphabet, based on the definition, we know in finite time we can observe if the star is overwritten and if so, the TM accepts.

(2) Prove that INS is undecidable. First we show that we can reduce from Halting Problem to INS. Therefore if INS is decidable, then HP is decidable, which we know is not true. Then we raise the contradiction.

Say we have an instance of  $HP(M, x)$  where  $M$  runs on  $x$ . Construct a UTM  $M'$  that takes in  $M$  and  $x$ , append a star in front of  $x$  and make it  $x'$ , Assume we have a UTM  $M_{INS}$  that takes in  $M', x'$  and follows the rule:

Skip the first star, use  $M'$  to run  $M$  on the rest of the string. If it halts, then move the cursor to the star and overwrite it with 0 or 1; if it never halts,  $M_{INS}$  loops as well.

If  $M'$  halts, which is equivalent to  $M$  halts on  $x$ , then the star gets overwritten; on the other hand, if  $M'$  never halts, then  $M_{INS}$  will not overwrite  $x'$ . Thus  $M_{INS}$  overwrites  $x'$  if and only if  $M$  halts on  $x$ . If we have a way to decide on the language of INS then we can decide the halting problem. Thus we have the contradiction.