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**(1)** (5 points)

For any positive integer  $n$ , let  $L_n$  denote an L-shaped region in the plane obtained by starting with a square of side length  $2^n$  and deleting its upper right quadrant. For example,  $L_1$  is the “L-shaped tromino tile” discussed in class on Wednesday.

Prove that for every positive integer  $n$ , it is possible to tile  $L_n$  using copies of  $L_1$ . In other words, you should prove that  $L_n$  can be partitioned into regions, each congruent to  $L_1$ .

Try to make your proof as clear and precise as possible. You do not need to describe an algorithm to compute the tiling, nor to analyze its running time. You only need to prove that such a tiling exists.

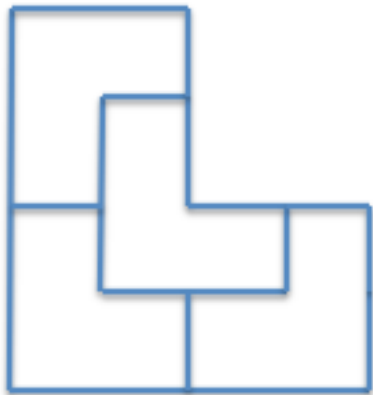
*Hint: Use mathematical induction.*

Basis: Show that the statement holds for  $n=1$ .

For  $n=1$ ,  $L_1$  is tiled by 1  $L_1$ . Thus it has been shown that  $L_1$  holds.

Inductive step: show that if  $L_n$  holds, then  $L_{n+1}$  also holds.

According to the construction of the L shape,  $L_{n+1}$  could always be tiled by 4  $L_n$  in the pattern shown in the figure below.



Since  $L_n$  could be tiled by  $L_1$  according to the induction hypothesis.  $L_{n+1}$  could therefore also be tiled by  $L_1$ .

Since both the basis and the inductive step have been performed, by mathematical induction, we have shown that  $L_n$  can be partitioned into regions, each congruent to  $L_1$ .