CS 4820, Spring 2017 Homework 10, Problem 3

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Solution:

1.) Prove m(s) tends to infinity as s tends to infinity

For s1<s2, suppose m(s1)>m(s2), this is a contradiction as if m(s1)>m(s2) according to the definition of m(s), s2 has to < s1. So m(s1)<=m(s2). Suppose m(s1)=m(s2), for Y={input y with length =m(s1)}, let s be the max halting step for any y in Y. If s2>s, m(s1)=m(s2) is a contradiction. So m(s1)<m(s2), since the length of y increases by an integer, as s goes to infinity, m(s) goes to infinity.

2.) Reduce halting problem to this problem

Suppose such a M exists, For a halting problem with input: N;x, denote the
binary representation of length of (N;x) by l, iteratively increase s from 1 and run M on x corresponding to s until the output of M(x) z>l, find m(s) for this z, now run universal turing machine U on (N;x) for s steps. If U halts within s steps, N halts on x. It is a YES instance. Otherwise N never halts on x, it is a NO instance. To prove this, suppose U halts at s3>s, according to the definition of m(s), z for this m(s)<=ssl which is a contradiction.

We now have a algorithm that decides the halting problem which is a contradiction as halting problem is undecidable. Therefore, there does not exist such a M.