CS 4820, Spring 2017 Homework 8, Problem 2

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2.For each state i, we could check if the candidate spends at least ki days campaigning in state i during the interval from si to di -1 in polynomial time by counting the days candidate spends campaigning in state i during the interval from si to di -1. We could also check if the candidate has already won at least mi of the states in Ai by counting the states the candidate has won. So we could check if a candidate wins a state in polynomial time, therefore we could check how many states the candidate wins in poly-time and determines if the candidate wins at least p states. Thus the problem belongs to NP.

We reduce vertex cover problem to this problem. For a vertex cover problem with n vertices, m edges, find if k vertices could cover all edges. There are k+2 days in the campaign. Each vertex corresponds to a state whose vote day is k+1th day and the candidate spends at least 1 day campaigning for it to win it, mi for it is 1, si for it is 1. Each edge corresponds to a state whose vote day is k+2th day, the candidate spends at least k+1 days campaigning for it to win it, mi for it is 1, Ai is the vertex states connected to this edge state, si for it is 1. Find if the candidate could win at least p=k+m states.

In order to win p states, the candidate has to win at least k vertex states. The candidate could win at most k vertex states since there are only k days to campaign for all vertex states, so the candidate must win k vertex states and m edge states to win p states. In order to win a vertex state, the candidate has to campaign one day for that state since no state is voted before the vertex state. In order to win an edge state, it has to connect to at least one of the selected k vertex state meaning at least one influential state of it wins. An edge state can't win by campaigning for k+1 days as we could only win 1 state this way, k>=1.

If we have a feasible solution for the vertex cover problem, it means k vertices could cover all m edges. We also have a feasible solution for the campaign problem for p=k+m as each of k vertex state could campaign one day and each of m edge state has at least one vertex state connected to it which is influential and has won. Thus we win k+m=p states according to state winning criteria described by the problem reduction.

If we have a feasible solution for the campaign problem, we have p=k+m winning states. K of them must be vertex states as there are k campaign days for the vertex states and each vertex state needs 1 day to win, we can't campaign for edge state as they require k+1 campaign, we could win 1 edge state if we campaign k+1 for it, and k>=1, so campaigning for edges states does not work. So we should all campaign for vertex states and therefore we have exactly k vertex state win and m edge state wins. According to the winning criteria for the edge states in the problem reduction, each edge state must connect to at least 1 winning vertex state, thus all edges are covered by the k vertices which is a feasible solution to the vertex cover problem.

For the problem reduction, the description for each vertex state and edge state is a constant(di,si,ki,Ai,mi), there are k+2 campaign days so the problem could be reduced from the vertex cover problem in polynomial time..