

CS 4820, Spring 2017 Homework 8, Problem 1

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(a) There may be some problems with the reduction from Test Network Design to Subset Sum problem because the given edges may not be manual set and the requirement 4, which is for each i , there are edges connecting..., may not take full consideration for the explanation of both L and T shall be connected, which means exceptions exist.

(b) Firstly, we need to prove that Test Network Design is NP. We need prove the following things, v_1 is a vertex of L , v_2 is a vertex of T , T has exactly k vertices, and that L and T are connected graphs. All these things can be finished in $O(m)$ by applying such as DFS for checking the connection of L and T where m is the number of the edges in the graph G .

Next, we need prove that Test Network Design is NP-hard and we find Independent Set \leq p Test Network Design.

The reduction. Given an instance of Independent Set $G^* = (V^*, E^*)$ and a parameter k , we construct an instance of Test Network Design $G = (V, E)$ as follows:

- Two vertices $\{v_1, v_2\}$ belong to the live network and test network
- $\{v_1, v_2\}$ have edges to every v belongs to V^* to make the whole graph connected graph
- For every edge e belongs to E^* , a set $W(e) = \{W_0(e), \dots, W_{k-1}(e)\}$ of k vertices, which might be the possible solution for test network, should satisfy the connection requirements which is for every W in this set, there is an edge from this edge to $W_0(e)$

Running time of the reduction:

According to the Independent Set problem, the running time for creating the adjacency list representation of G requires $O(mk)$, where m is the number of the edges in G^*

If G^* has an independent set of size k , then the Test Network Design has a solution. We can create a test network T consisting of v_2 and $\{S - v_2\}$ whose size is $k-1$ and it is with all the edges between them in G^* . Note that T is connected from the previous requirements. The live network L is also connected and every vertex belongs to L is connected to v_1 .

If the Test Network Design has a solution, then G^* has an independent set of size k . Let L, T be the solutions of the Test Network Design instance. Firstly, both L and T are connected graphs and can be

connected by margin edge to form an overall connected graph G' . Next is that L and T are separated because L got the remaining vertices in the graph, which means no two vertices from both L and T are connected. L and T are exactly two sub independent sets. It shows G^* has an independent set of size k .