

CS 4820, Spring 2017 Homework 6, Problem 3

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Description:

- For the edges that have nonzero flows in the given maximum flow, they are not useless.
- For the edges left, iterate through each edge whose tail and head nodes are  $u, v$ :

Do DFS following the direction of each edge from  $v$ :

If the search hits  $u$  again, we find a cycle containing edge. Then all edges in the cycle are not useless.

If no cycle containing the edge is found, then record the edge as useless, and continue to the next available edge.

- When the iteration is done, output all the useless edges.

Correction:

We know that each flow can be represented as a linear combination of paths and cycles. So the difference between different maximum flows can be paths and cycles. As all maximum flows in a graph has the same flow value, consider the fact that cycles result in no flow value change while paths do change the flow value, thus different maximum flows cannot differ in paths but only in cycles.

From above we know by adding or subtracting cycles we can transform a given maximum flow to another one. Therefore we desire to find all the cycles in the graph. For a given edge, we can use DFS to test if the search hits back to the edge; if yes, then we find a cycle and all edges constituting the cycle is not useless. After all iterations, we know which edges do not participate in any of the cycle or the original given maximum flow and those are the output.

Running time :

Iterating the given maximum flow costs  $O(m)$

Number of iteration steps the edges is  $O(m)$ , each iterative step costs  $O(m+n)$  for the DFS.

Overall it is  $O((m+n)*m) = O(m^2)$ .