CS 4820, Spring 2017 Homework 7, Problem 2

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2a

In the circulations with demands and lower bounds, we can make the reduction to the max flow problem by introducing super source S^* and sink T^*. S^* connects to all positive demand nodes in graph and T^*connects to all negative demand nodes. It is obvious that all demands turn in 0 at the same time for the equivalent graph transformation. If $l_e = 0$, it means we need not change the vertex capacity for the s-t. At this time, for min cut, the sum demand comes from the source. According to the max flow min cut theorem, because the flow of cut {S,T} equals f(S,T), however, the max flow in the graph G equals to the net flow of a certain cut which is f(S,T) – f(T,S), which means if f(T,S) = 0, the max flow in the graph equals to the min cut flow, we can conclude for this equivalently new s-t, the graph has a feasible circulation with demands if and only if $\sum_{v\epsilon B} d_v \le c(A,B)$. The sum of demands equals to net flow in the s-t. The contradiction is that the sum of demands is not 0 in the previous circulations which means unknown external flow and not feasible. So the proof is correct.

2b

In the circulations with demands and lower bounds, if the demands are 0. If we need to satisfied the capacity constraints, but not the demand constraints, l_v should equal to $f^{in}(v) - f^{out}(v)$. Recall the question 2a, d equals to $f^{in}(v) - f^{out}(v)$ (without the lower bound). So it is just the same with the reduction of changing each capacity from c to $c\text{-}l_i$ for circulations transforming into max flow graph. So we can draw the proof that the circulation problem is feasible if and only if every partition of the vertices of G into two sets A, B satisfies the inequality $\sum_{u\epsilon B}\sum_{v\epsilon A} l_{uv} \le \sum_{v\epsilon B}\sum_{v\epsilon A} c_{uv}$.

2c

$$\sum_{u\epsilon B}\sum_{v\epsilon A} l_{uv} + \sum_{v\epsilon B} d_v \le \sum_{u\epsilon A}\sum_{v\epsilon B} c_{uv}$$

We can transform it into the problem with only demands. We define the new demand for node v in Graph G’ as d’v = dv – Lv where Lv = $f^{in}(v) - f^{out}(v)$ And the new capacity of edge becomes C’uv = Cuv – luv

The feasibility of the problem without lower bound is:

$$\sum_{v\epsilon B} d'_v \le \sum_{u\epsilon A}\sum_{v\epsilon B} c'_{uv}$$

Note that for all edges with both nodes u, v belong to B, their contribution to Lv cancel out. Therefore, only the edges that cross the boundary of A, B remains in Lv.

$$\sum_{v \epsilon B} d_v - (\sum_{u \epsilon A} \sum_{v \epsilon B} l_{uv} - \sum_{u \epsilon B} \sum_{v \epsilon A} l_{uv}) \leq \sum_{u \epsilon A} \sum_{v \epsilon B} c_{uv} - \sum_{u \epsilon A} \sum_{v \epsilon B} l_{uv}$$

Therefore,

$$\sum_{v \epsilon B} d_v + \sum_{u \epsilon B} \sum_{v \epsilon A} l_{uv} \leq \sum_{u \epsilon A} \sum_{v \epsilon B} c_{uv}$$

If le = 0 the formula becomes that in (2a), and if dv = 0 it becomes that in (2b).