Tuning parameters and Evaluation of a PID controller

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***Abstract*—This practice has been worked with a circuit developed with amplifier operations that works as a second order system to use it as the plant of the complete system. To work with a PID controller using tuning parameters and evaluations through different methods. In a specified way the Ziegler and Nichols Methods.**

I. Introduction

In any system, being electrical, mechanical, thermal or their combinations, four different responses can be observed depending on the damping of those systems, since they are susceptible to environmental disturbances, which can lead to a system unwanted response causing it to destabilize. Given this situation is gotten the need,despite the destabilization in the system response, to correct it and make them stable, for this it is necessary to have a controller which will keep the system response at a stable value according to its input regardless of any disturbances it may have.

II. Theoretical Framework

A PID (Proportional Integral Derivative) is a controller that allows to maintain certain values for a required action, the importance of this devices is it does the controlling process better than a analog sensor could do, those for the fact that it works with a feedback that gives information of the controlled system.

In specific therms, a PID is a configuration of OP-AMP that brings its qualities, this because the action of the Proportional, Integral and Derivative in common affects the input signal, causing what is seen in the section of Result Analysis. It means, that depending if its used a P (Proportional) or I (Integral) or D (Derivative) in arrangement, the result will not be the same, some example of common organization in the industry are:

- P
- I
- PI
- PD

The selection of with use, depends in the features of each component

A. Proportional

As the name indicates, P is a gain for the system, represented with

$$K_p$$

It's used to reduce the stationary state error and perturbations, nevertheless it will never make them equal to zero (0). This gain cannot be to big, because it could unstabilize the system, on the other side K_p may bring a faster response, this doesn't happens in all the cases.

B. Integral

This configuration of OP AMP integrates the signal respect time, giving some properties to the resulting wave, as:

- Cancel the stationary state error.
- Cancel the perturbation in the stable state in the system.
- Speeds the response of the system.

Also, it presents a respective gain, the integrative constant:

$$K_i$$

C. Derivative

As the P and I, the D has a respective gain for the system

$$K_d$$

The function of a derivative configuration is seen when the state of the system es transitional, because here it brings an anticipative response. In other hand, this arrangement present some problems:

- Doesn't do any correction to the stationary state error.
- Increments the noise in the system, this by amplifying it.
- Doesn't cancel perturbations in the system.

For the practice, it was used the the Ziegler & Nichols method, for the reason that the principal requirement was to obtain a 25% of overshoot, to accomplish this, it was evaluated both variants for the mentioned method:

D. Ziegler & Nichols, open-loop dynamics

The open-Loop dynamics was thought for systems with the behavior of a sigmoide function and present instability.

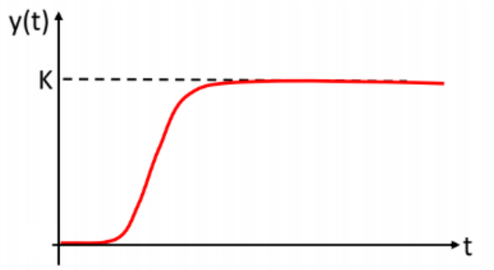

Figure 1: Open-Loop curve

Its name comes from the fact that to obtain a the values of the gain of P, I and D for a PID, the feedback, integrative and derivative sections, must be disconnected, creating a ”open-loop”.
Following the procedure, it is necessary to find the the inflexion point, and trace a line over it, this will give values that will be used with some equations to finally find K_p, K_i and K_d. Using as reference Figure (1)

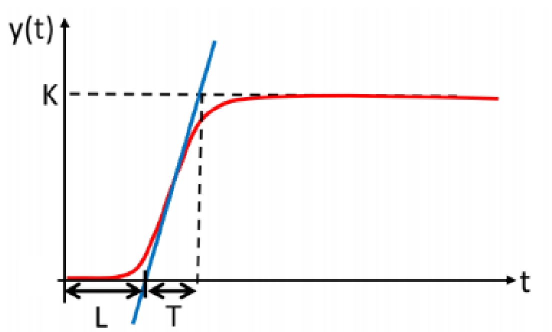

Figure 2: Inflexion point

Where:

- L is the retard in the system.
- T is the time constant.

$$G(s) \approx \frac{Ke^{-Ls}}{Ts+1} \tag{1}$$

The equation (1) is the approximation we obtain for the curve of Figure (1).
The value of L and T from the Figure (2) are important because they belong to the constants that define the constants of the PID

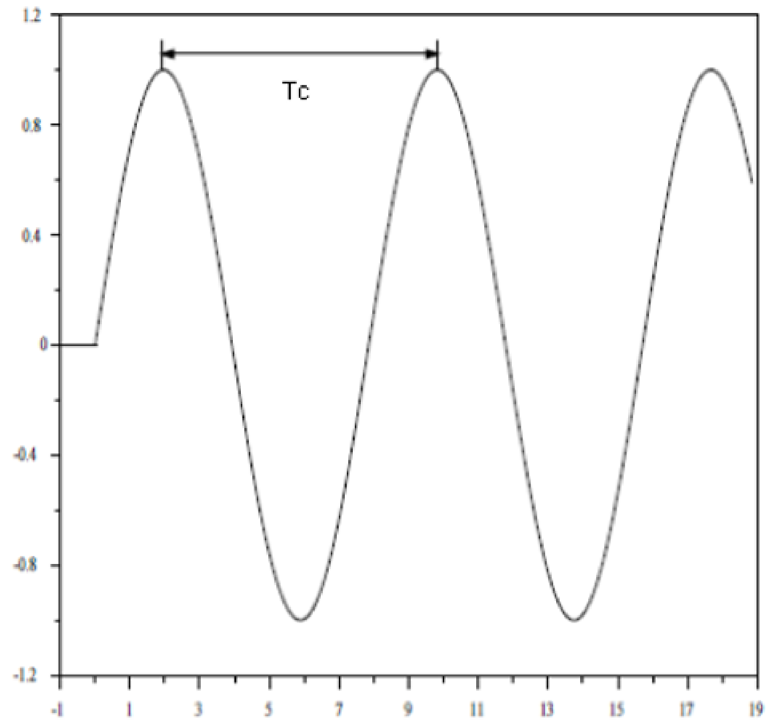

Figure 3: Open-loop dynamics

Type of controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\left(\frac{T}{L}\right)$	$\frac{L}{0.3}$	0
PID	$1.2\left(\frac{T}{L}\right)$	$2L$	$0.5L$

Table I: —-NOMBRE—-

E. Ziegler & Nichols, closed-loop dynamics

This method is similar to the Open-loop dynamics, the difference es that in this case the presence of feedback.
The oscillation method (other way to name it) has its features because the resulting wave form is like

III. Results Analysis

In this practice it was made a circuit of a PID controller, by doing the circuit shown on figure AA.

This controller was conditioned with the second order plant made for the first practice shown on Figure 4

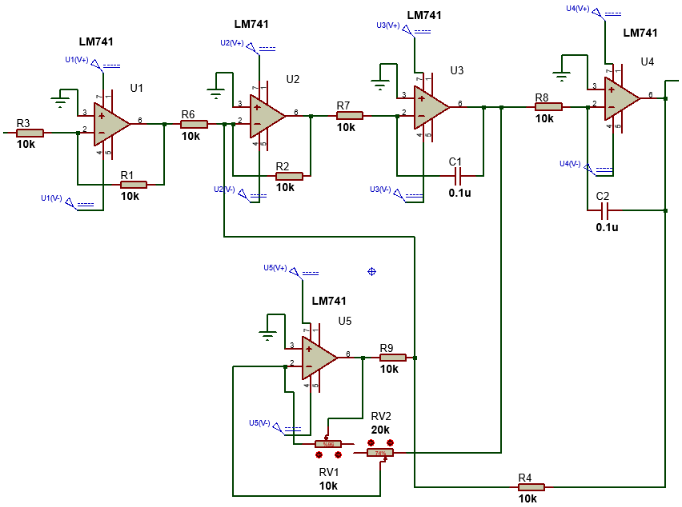

Figure 4: PROTEUS schematic plant circuit

Because of that, transfer function becomes like.

$$H(s) = \frac{10^6}{s^2 + \left(\frac{R_2}{R_1}\right)(10^3)s + 10^6} \tag{2}$$

Analysing the damping coefficient as the same as the last practice, it can be seen that this damping coefficient belongs to a underdamping system.

$$H(s) = \frac{10^6}{s^2 + \left(\frac{14000}{37}\right)s + 10^6}$$

Because as it can be seen, the damping coefficient is less than 1. To do the PID controller system, it must be analyzed the way it works, shown on Figure 5

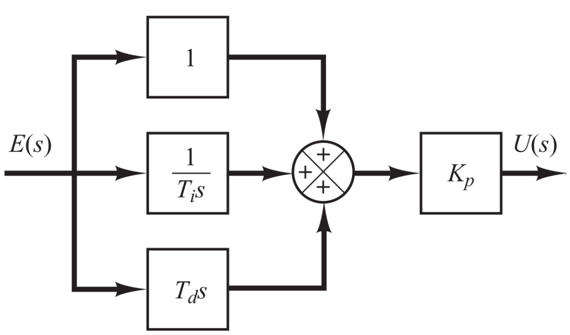

Figure 5: PID Controller (Obtained of Ogata)

This initially has a differential adder that calculates the error of the E (s) system by subtracting the signal from SetPoint (S.P.) with the output of the Y plant (s). This is subsequently carried to the integrator, the shunt and an unit gain cable, which outputs are subsequently added and passed through an inverting amplifier that determines the proportional gain and produces the control stress signal.
Finally, it's carried to another adder where a signal of perturbation is introduced. Its exit goes to the entrance of the plant.

The controller has the transfer function shown in equation.

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{3}$$

To know the parameters, it was done the two different Ziegler Methods, by Matlab.

A. Closed-loop method system

The first Ziegler method it was done by analysing the plant with a proportional controler which is which is a gain received with an unit step input signal.
The damping coefficient for this method must be 0 or the closest to 0. In the case of this practice, it was decided to work with a damping coefficient of 0.1, because in real terms it is almost impossible to reach a damping coefficient of 0 with the said plant.

The oscillations method, looks for a system which would be a perfect case without any damping coefficient. However for this system it's not possible to reach instability.
Developing the Rough Hurwitz method for knowing the system's stability can be proved that the proportional gain must be less than 1 to be unstable.
Because of that, Rough Hurwitz analysis couldn't be used to find the proportional gain for this practice. Due to that, the named proportional gain had to be chosen at personal discretion.

It was simulated the system on Simulink doing the following block diagram shown on Figure 6

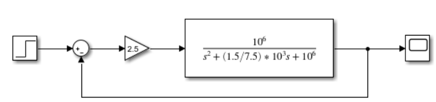

Figure 6: Simulink block diagram oscillation method

According to this, the proportional gain was being changed until finding the response desired.
The proportional gain chosen as personal discretion was 2,5.
For this method it can also be shown the simulink's graph on figure 7

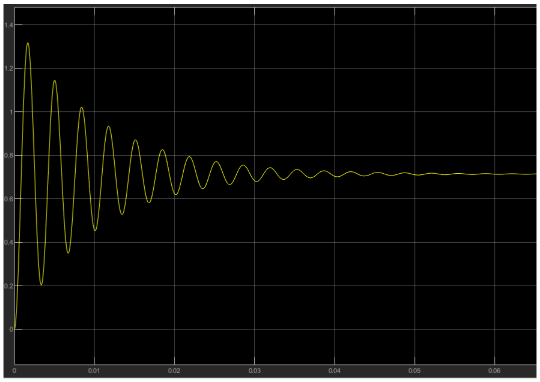

Figure 7: Simulink step response for oscillation method

The Matlab graph for the same system can be seen on Figure II

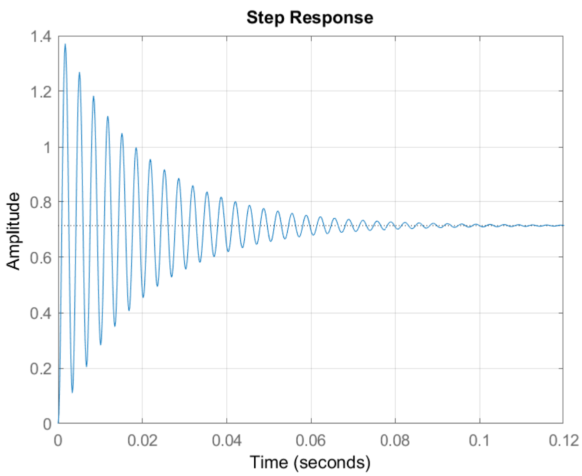

Figure 8: Simulink's oscillation method response

According to this, it was found that with a proportional gain of 2,5 and with the kind of plant own to this system, it's gotten a critical period:

$$P_c = (3,33)10^{-3} \tag{4}$$

$$K_c = 2,5 \tag{5}$$

Now having this information, it can be found Kp for a proportional controller, Ti for a integrator controller and Td for a shunt controller.

Type of controller	K_p	T_i	T_d
P	$0.5k_c$	∞	0
PI	$0.45k_c$	$\frac{T_c}{1.2}$	0
PID	$0.6k_c$	$0.5T_c$	$0.125T_c$

Table II: Parameter's table for oscillations method

According to this table, is gotten the following values:

$$Kp = 1,5 \tag{6}$$

$$Ti = 1,665 * 10^{-}3 seconds \tag{7}$$

$$Td = 4,1625 * 10^{(} - 4) seconds \tag{8}$$

With the prevously said gains of each controller (proportional, integrator, shunt) it was done the PID controller for the system.
The plant with its respective PID controller can be seen on Figure 9

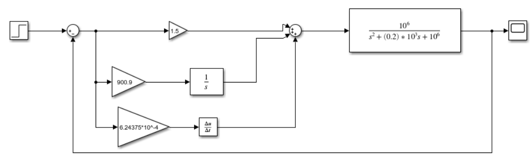

Figure 9: PID controller of oscillations method

And the system's response having the PID controller can be seen on the following figure 10

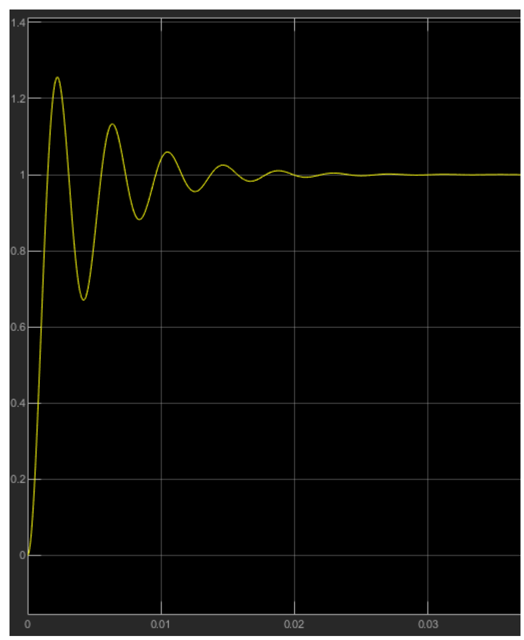

Figure 10: Response of oscillations method

It can be evidenced that the response overshoot reaches approximately a 25Ďue to this, this system with the previously said values for each gain was set on Proteus, setting the gains with the amplifier's resistors ratio.

EXPLICAR RELACIONES DE RESISTENCIAS POR METODO DE OSCILACIONES

B. Open loop method system

For this method, it was clear that the plant with an open loop system must be connected directly to the unit step entrance. The plant must be with a damping coefficient to make it an overdamped system, it means, greater than 1.
For the worked system, it was decided to use a damping coefficient of 2.
First on matlab with this damping coefficient it was gotten the following graph shown on Figure 13

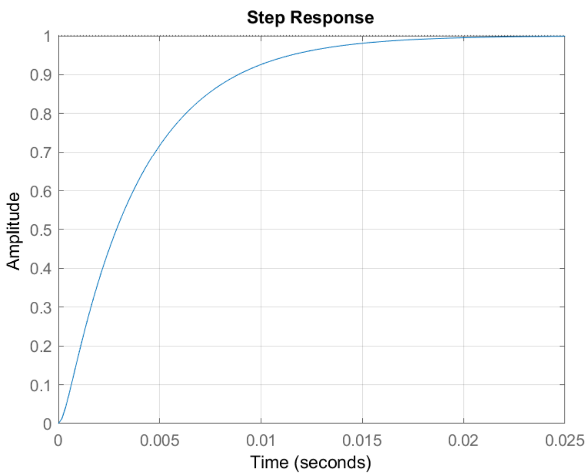

Figure 11: Response of overdamped system

In order to achieve the same graph, the following formula had to be used, in order to find the values of the variables sought according to the previously explained open-loop system method.

$$H(s) = \frac{Ke^{-Ls}}{s^2 + Ts + 1}$$

It was generated on Matlab a comparison between the step response of the overdamping plant shown on Figure
refbsab and the induced transfer function according to the previous formula to find the parameters L and T, that make the transfer function as similar as possible.

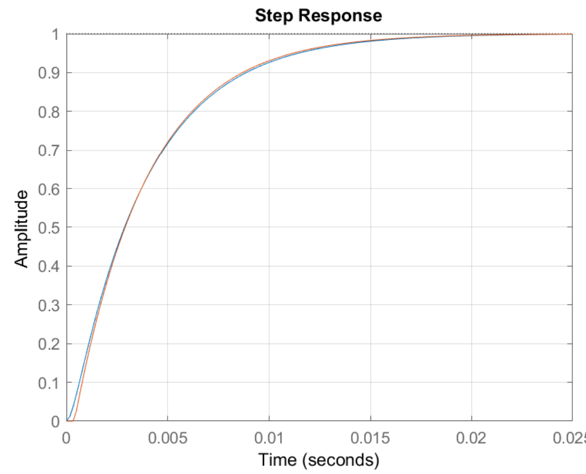

Figure 12: Comparison with the plant vs the induced transfer function

According to this, it was gotten the following parameters.

$$T = 0,0036 \tag{9}$$

$$L = 0,0004 \tag{10}$$

Those parameters are necessary to find the gains of each of the amplifiers that correspond to a controller. The following table is used to find these gains.

Tipo de controlador	Kp	Ti	Td
P	T/L	∞	0
PI	0.9 T/L	L/0.3	0
PID	1.2 T/L	2 L	0.5 L

Figure 13: Table for knowing the PID gains of the open loop method

For the system worked, it was gotten the following parameters.

$$K_p = 10,8 \tag{11}$$

$$T_i = 0,0008 \tag{12}$$

$$T_d = 0,0002 \tag{13}$$

Because of that, it was gotten the following gains.

$$K_i = 13500 \tag{14}$$

$$K_d = 0,00216 \tag{15}$$

With the previously named gains for this system, it was simulated with simulink tho following block diagram for the PID controller of the system.

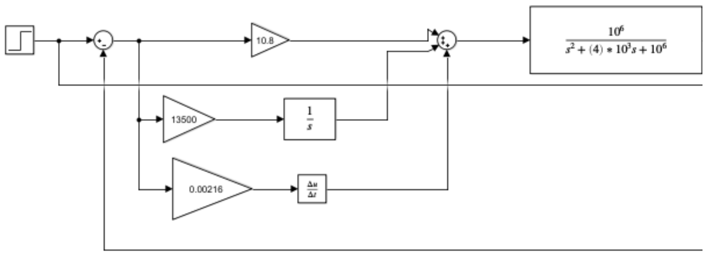

Figure 14: Block diagram of open loop method

And the obtained response for the complete system on Simulink is shown on Figure 15

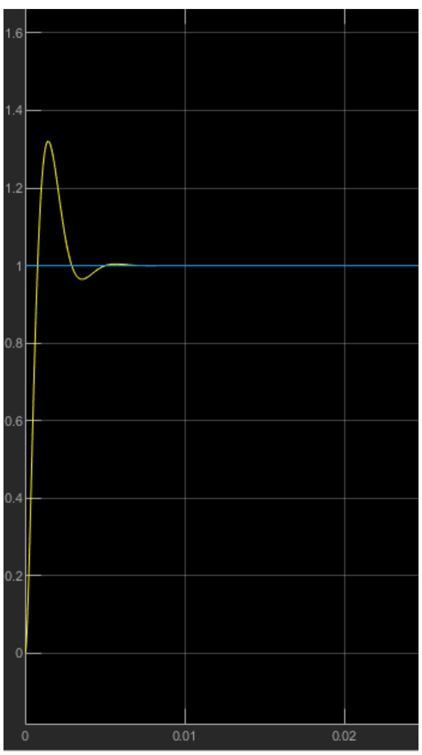

Figure 15: Response for complete system on Simulink open loop method

It was done the same simulation but on Matlab code, and that simulation can be evidenced on Figure 17

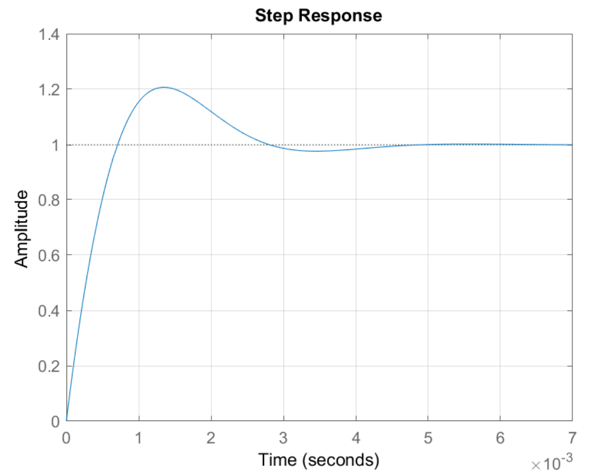

Figure 16: Response for complete system on Matlab open loop method

As it can also be seen, the response has an overshoot approximately of 25
For the complete analysis of the practice it was decided to also perform the PI method which can be shown below.

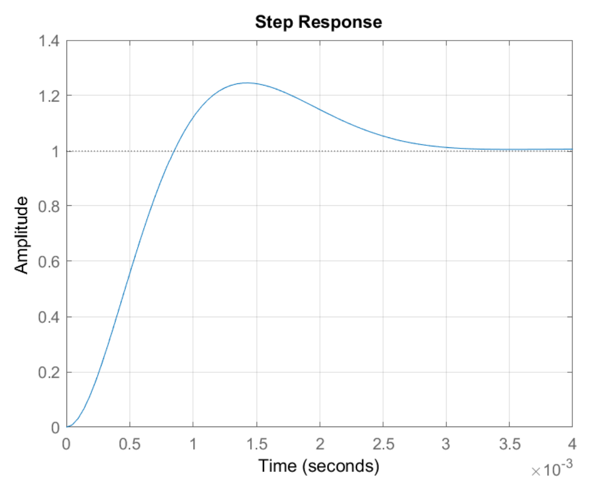

Figure 17: Response for PI on Matlab open loop method

With those two responses, it can be made a comparison between the PID controller and the PI controller gotten from this open loop method.

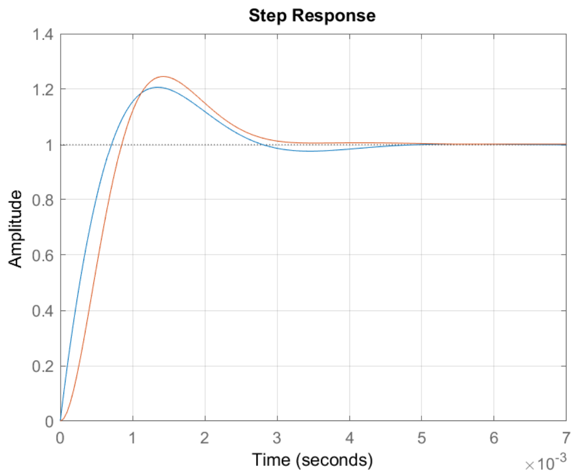

Figure 18: PI vs PID open-loop method

When entering the disturbance in the case of the system with PID controller in the step in which it is introduced, a peak appears whose magnitude is proportional to the magnitude of the disturbance but less than the magnitude of the peak in the PI controller.
Below is shown the poles and zeros diagram of the system with the PID controller.

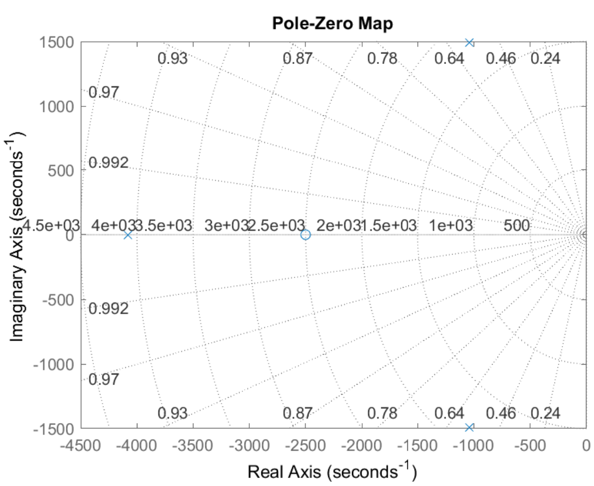

Figure 19: PI vs PID open-loop method

According to these results, it was done the simulation circuit on Proteus taking into account the respective gains for each controller in the amplifiers resistors ratio.
For the analysis of possible configurations with P, I or D, in the circuit of Figure (21)

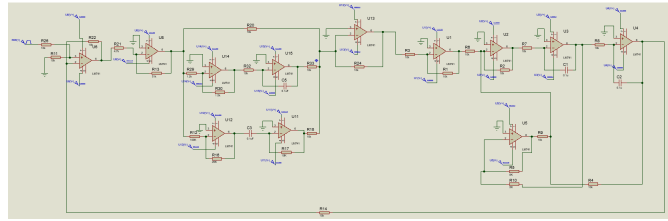

Figure 20: Proteus diagram open-loop method

It es necessary to see closer to the controller of this system

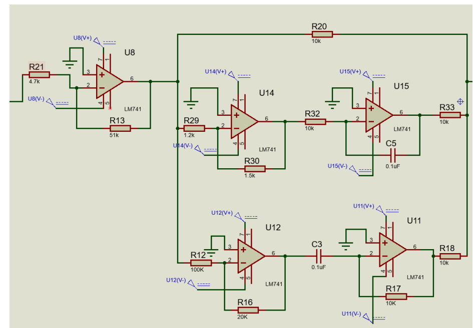

Figure 21: Controller

Observing the last image, it is possible to prove what in the theoretical framework was said about the controller P, that it's unable to cancel the stationary error, bur certainly it can give a approximation of the response that is required. For this, it was simulated en Proteus the output of the whole system, without the connection of the Integrative and Derivative amplificators.

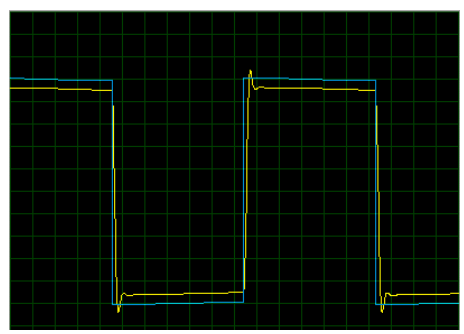

Figure 22: Proportional controller

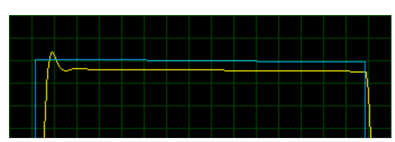

Figure 23: Proportional controller

Other configuration that can be seen is a PD controller, this doing the same process as in the P controller, instead, this case integrative amplificator is disconnected.

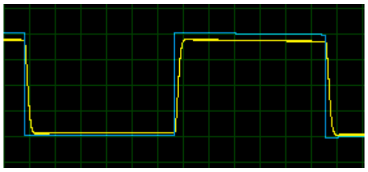

Figure 24: PD controller

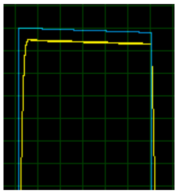

Figure 25: PD controller

The response shown in the Figure (24) and (25) is due to the derivative controller approaching the response for what is required, nonetheless it can't because it doesn't cancel the stationary state error, neither the proportional con do it, this is the reason it never reach the excepted response.

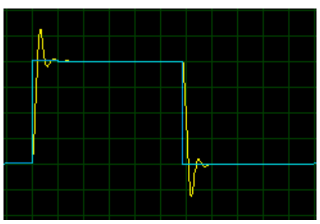

Figure 26: PI controller

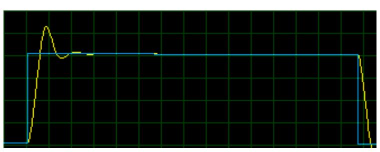

Figure 27: PI controller

Figures (26) and (27), correspond to a PI controller, as seen its behavior is congruent with the theory, because the P reduces the stationary state error, the I takes this value and cancels the error, both configuration the increase the speed of the response. The result is a behavior with a fast response and no stationary state error.

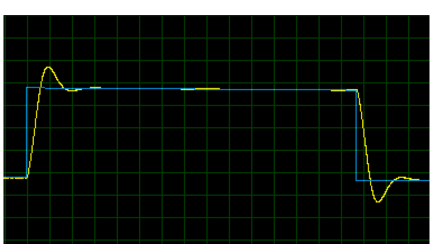

Figure 28: PID controller

The union of P, I and D is seen in Figure (28), the final result and excepted response for the system.

C. Tests at the university electronics lab

Both methods were tested at the university with a circuit made on protoboard. Next, the open-loop method, taken from the oscilloscope of the electronic laboratory, can be evidenced.

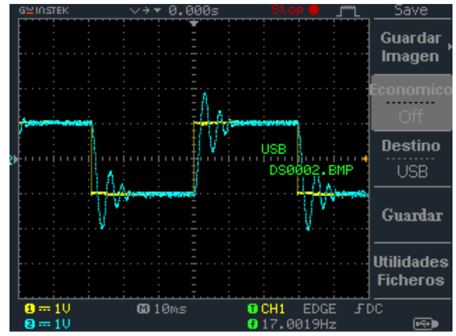

Figure 29: Real life response open loop system at electronics lab

For this Kp = 10,8 the actual poles have become conjugated complexes, so the system resembles an underdamped second order one. Additionally, the stabilization time decreases with increasing Kp, since in general the dominant poles move away from the imaginary axis.
It's also concluded that when entering the disturbance with a PI controller the peak's amplitude is greater, however it also reaches stability.

As Kp increases, one pole gains dominance while two other poles lose it, so the system should have an overdamped second order one (since it has two dominant real poles). However, for large values of K real poles are separated from the real axis and also become conjugate complexes, so the system should tend to a second order underdamped with a shorter establishment time and that's what can be seen on the Figure 29.

IV. Conclusions

The Ziegler Nichols and Cohen Coon Methods are very useful for tuning the controller in systems with different damping

The stabilization time tends to decrease as Td increases, however for very large values it increases again. This can happen because having such a high lead time rates small changes in error cause the controller tends to saturate and send a maximum magnitude control effort that moves the system away from the Set Point again.

The errors obtained at the time of measurement are due to miscalculation when approximating the asymptote since it is at the discretion of the person.
If the appropriate type of controller is implemented the closed loop method, allows to eliminate the error in a stable state

and keep the variables of a process at the desired value, even in the presence of disturbances, and if these could not be compensated by the controller, the processes would have to be completely upset and any small obligation errors to perform maintenance and stop production.
However, Finally, the PID controller, is very ideal to achieve all the benefits previously exposed, since having an integrator eliminates the error in stable state; by changing the three K parameters, you can also modify all the characteristics of the transient response of the system according to the requirements. This allows the plant to be approximated to a first-order system or to a second-order damped system.

V. APPENDIX

Here is shown the Matlab code used for the correct development of this practice.
It is clear that the Simulink block diagram was shown previously on the results analysis report.

```
%%% The following process is to show the transfer function for the
%%% developed system.
s=tf('s');
a=10^6/(s^2+(4*10^3)*s + 10^6); %%%as can be seen,
%%the damping coefficient for the system is 2.

%%Here is shown the way to find the correct parameters L and T
%%with the following formula getting transfer function as similar as possible
step(a)
grid on
hold on
K=1;
L=0.0004;
T=0.0036;
a1=K*exp(-L*s)/(T*s+1);
hold on
step(a1)


%%Being for this case and method a plant with an overdamping system.
step(a)


%%
hold on
grid on
K=1;
L=0.0004;
T=0.0036;
a1=K*exp(-L*s)/(T*s+1);
Kp=1.2*T/L
Ti=2*L;
Td=0.5*L;
Ki=Kp/Ti
Kd=Kp*Td
PID= Kp+Kd*s+Ki/s;
v= PID*a;
pid=feedback(v,1);
step(pid)


%%Here is the poles and zeros diagram for the complete PID system
pzmap(pid)
grid on
hold on
```

```
%%%Here is done the P controller alone.
KPP= T/L;
Z=a*KPP;
PP=feedback(Z,1);
step(PP)
grid on
hold on


%%Here is done the PI controller for the system
Kp1=0.9*T/L;
Ti1=L/0.3;
Ki1=Kp1/Ti1;
PI=Kp1+Ki1/s;
w=a*PI;
pi=feedback(w,1);
step(pi)
grid on
hold on
%step(a1)

%%OTHER METHOD
%%% Here is shown the oscillations method
s=tf('s');
num=[10^6];
den=[1 (0.2)*10^3 10^6];
%%As can be seen the damping coefficient for this system is 0.1, almost 0.
a=tf(num,den);
Kc=2;
Kp1=0.5*Kc;
b=a*Kp1;
h=feedback(b,1);
step(h) %%%Here is shown the transfer function with the Kc implemented
hold on
grid on



%R=10000;%% ohms
%C= 0.1*10^-6; %% farad
Kp= 0.6*Kc;
Tc=3.33*10^-3;
Ti=0.5*Tc;
Td=Tc/8;
Ki=Kp/Ti;
Kd=Kp*Td;

%%Here is developed the complete PID controller for the method
PID= Kp+Kd*s+Ki/s;
o=PID*a;
pid=feedback(o,1);

%%Here is shown the complete PID controller for the oscillations method
step(pid)
hold on
grid on
```