# HOMEWORK 10

**Problem 1.** *Both the dihedral group $D_8$ and the quaternion group $Q_8$ have cardinality 8. Are they isomorphic to one another? If yes, exhibit an isomorphism. If not, explain why.*

*Proof.* □

**Problem 2.** *Both the dihedral group $D_6$ and the permutation group $S_3$ have cardinality 6. Are they isomorphic to one another. If yes, exhibit an isomorphism. If not, explain why. (Hint: Label the vertices of the triangle using numbers $1, 2, 3...$)*

For the following problem, you might want to review the cartesian products of two groups.

**Problem 3.** *Consider the abelian group*

$$G = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z},$$

*and the element $g = (\bar{1}, \bar{2}) \in G$. (I use the bar notation to denote an integer modulo n for some n which should be understood from the context...)*

   (1) *What is $\mathrm{Ord}(g)$?*
   (2) *What is $H = \langle g \rangle$?*
   (3) *Enumerate all the elements in $G/H$.*
   (4) *What is $(G : H)$?*

*Solution.* □

**Problem 4.** *Consider the group morphism*

$$\pi_2 : \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \longrightarrow \mathbb{Z}/4\mathbb{Z}$$

*given by*

$$(\bar{a}, \bar{b}) \mapsto \pi_2(\bar{a}, \bar{b}) = \bar{b}.$$

   (1) *What is $\ker(\pi_2)$?*
   (2) *Let $H = \ker(\pi_2)$. Enumerate all the elements in $G/H$, where $G = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$.*

*Solution.* □

**Problem 5.** *Consider $S_4$ and let $\sigma = (2\,3\,4\,1) \in S_4$. Let $H = \langle (2\,3\,4\,1) \rangle$.*

   (1) *What is $\mathrm{Ord}(g)$?*
   (2) *What is $|H|$?*
   (3) *What is $(G : H)$?*
   (4) *Find a representative for each left coset.*

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