✓ $P_1, P_2 \dots P_n \sim$ std. $\quad P_j = |e_j\rangle\langle e_j| \quad \tilde{e}_j := \mu_j e_j$

✓ $Q_1, Q_2 \dots Q_n \quad Q_k = |f_k\rangle\langle f_k| \quad \tilde{f}_k := \lambda_k f_k$

$R_1, \dots R_n \quad R_\ell = |g_\ell\rangle\langle g_\ell|$

$$c_{jk,\ell} = \mathrm{Tr}(P_j Q_k R_\ell) = \langle e_j, f_k\rangle\langle f_k, g_\ell\rangle\langle g_\ell, e_j\rangle$$

$$\ell = 1 : \quad c_{jk,\ell} = c_{jk,1} = \langle e_j, f_k\rangle \underbrace{\langle f_k, g_1\rangle}_{\lambda_k \frac{1}{\sqrt{n}}} \underbrace{\langle g_1, e_j\rangle}_{\bar{\mu}_j \frac{1}{\sqrt{n}}} = \frac{1}{n}\langle \mu_j e_j, \lambda_k f_k\rangle$$

$$X_{k,\ell} := \sqrt{n}\, P_k Q_\ell$$

$$\langle X_{k,\ell} \mid X_{k',\ell'} \rangle_{H\text{-Sch}} = n \cdot \mathrm{Tr}\left(P_k \underbrace{P_{k'}}\, Q_{\ell'} \underbrace{Q_\ell} \right) = n\, \delta_{k,k'}\, \delta_{\ell,\ell'} \underbrace{\mathrm{Tr}(P_k Q_\ell)}_{\frac{1}{n}\ |\langle e_k, f_\ell \rangle|^2}$$

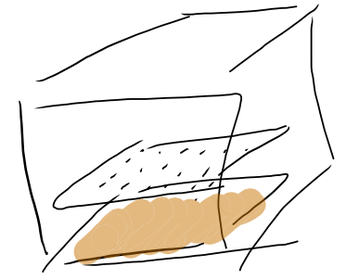
$$= \delta_{k,k'}\, \delta_{\ell,\ell'}$$

$\rightsquigarrow$ $X_{k,\ell}$ $(k,\ell = 1,\dots,n)$ egy ONB - t alkotnak $M_n(\mathbb{C})$ - ben

$\rightsquigarrow$ $\forall Z \in M_n(\mathbb{C}) \quad Z = \sum_{k,\ell=1}^{n} \langle X_{k,\ell} \mid Z \rangle_{H\text{-Sch}}\, X_{k,\ell} = \sum_{k,\ell=1}^{n} n\, \mathrm{Tr}(Q_\ell P_k Z)\, P_k Q_\ell$

$$\rightsquigarrow \forall Z \in M_n(\mathbb{C}) \quad Z = \sum_{k,l=1}^{n} \langle X_{k,l}, Z \rangle_{H\text{-}S} \, X_{k,l} = \sum_{k,l=1}^{n} n \, \mathrm{Tr}(Q_l P_k Z) \, P_k Q_l$$

ha $Z = R_j$



$$R_j = n \sum_{k,l=1}^{n} \mathrm{Tr}\left(\underbrace{Q_l P_k R_j}_{(R_j P_k Q_l)^*} \right) P_k Q_l =$$

$$= n \sum_{k,l=1}^{n} \overline{\mathrm{Tr}(P_k Q_l R_j)} \, P_k Q_l = n \sum_{k,l=1}^{n} \overline{C_{k,l,j}} \, P_k Q_l$$

Áll.: ha $Z \geq 0$ akkor

Z egy valódi projekció $\Longleftrightarrow$ $\mathrm{Tr}(Z) = \mathrm{Tr}(Z^* Z) = \underline{1}$

$\mathrm{Tr}(R_j) = ? = \underline{1}$ ✓ (átdobásos megfelelőség)

$\mathrm{Tr}(R_j^* R_j) = ? = \underline{1}$ ✓ (j-edik lapon az elemek abszolút négyzetösszege $= \underline{1}$)