NUI Galway
OÉ Gaillimh

Semester II Examinations 2021/2022

Exam Codes	1EM1, 1OA1, 2BA1, 2BCW1, 2BCT1, 2BPT1, 2BS1, 2EH1, 3BS9
Exam	Second Year Arts and Science Third Year Science
Module	LINEAR ALGEBRA
Module Code	MA203
External Examiner	Prof. Colva Roney-Dougal
Internal Examiners	Dr. Niall Madden Dr. John Burns

Instructions **Answer all questions.**

Duration	2 hours
No. of Pages	4 pages including this page
School	Mathematics, Statistics & Applied Mathematics

Requirements:

Release in Exam venue	Yes ☑	No ☐
MCQ	Yes ☐	No ☑
Statistical / Log Tables	Yes ☑	No ☐

Q1. (a) [13 marks] Consider the following system of equations

$$\begin{array}{rcrcrcrcr} 2x_1 & - & x_2 & - & x_3 & + & x_4 & = & 0 \\ x_1 & - & x_2 & + & x_3 & - & x_4 & = & 3 \\ x_1 & & & + & x_3 & + & x_4 & = & 2 \end{array}$$

(i) Write down the augmented matrix for this system of equations.

(ii) Using elementary row operations, convert the augmented matrix to reduced row echelon form.

(iii) Write down the general solution of the system of equations.

(b) [12 marks] For each of the following statements, declare whether the statement is true or false and justify your answer.

(i) A system of four linear equations in three unknowns cannot have a solution.

(ii) $3x + 3y - 2z = 0$ is the equation of a plane through the origin in $\mathbb{R}^3$, with normal vector $(3, 3, -2)$

(iii) It is possible to determine if two lines in $\mathbb{R}^3$ intersect by solving an appropriate system of linear equations.

Q2. (a) [8 marks] Let

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \end{pmatrix}.$$

Of the products AB, BA, A^2 and B^2, compute all those that are defined.

(b) [11 marks] Use elementary row operations to find the inverse of the matrix A below, if it exists. Determine the rank of A and the dimension of the kernel of A.

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & -4 \\ 2 & 1 & -11 \end{pmatrix}.$$

(c) [6 marks] For each of the following statements, declare whether the statement is true or false and justify your answer.

(i) Elementary row operations do not change the determinant of a matrix.

(ii) Any set of three linearly independent vectors in $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$.

(iii) If a square matrix A is invertible, then the rows of A are linearly independent.

Q3. (a) [10 marks] Express the vector $(2, 1, 3)$ as a linear combination of the vectors $(1, -1, 0), (1, 0, 2)$ and $(0, 1, 1)$. What is the span of the set of vectors $\{(1, -1, 0), (1, 0, 2), (0, 1, 1)\}$?

(b) [6 marks] Let $n = (n_1, n_2, n_3) \in \mathbb{R}^3$ and consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $L(v) := n \times v$ (the cross product of n and v). Find the second column of the matrix for L (w.r.t. the standard basis for $\mathbb{R}^3$).

(c) [9 marks] For each of the following statements, declare whether the statement is true or false and justify your answer.

(i) If 0 is an eigenvalue of a square matrix A, then A has no inverse matrix.

(ii) If the characteristic polynomial of a 3×3 matrix A is $P(\lambda) = \lambda^3 - 6\lambda^2 + 11\lambda - 6$, then 2 is an eigenvalue of A.

(iii) Any non-zero multiple of an eigenvector of an $n \times n$ matrix A is also an eigenvector of A.

Q4. A marketing company analysed customer switching behaviour between mobile phone providers ONE, TWO and THREE and observed that:

- Every year 10% of ONE customers switch to TWO and 10% switch to THREE (with 80% remaining with ONE).
- Every year 3% of TWO customers switch to ONE and 2% switch to THREE (with 95% remaining with TWO).
- Every year 20% of THREE customers switch to ONE and 5% switch to TWO (with 75% remaining with THREE).

(a) [9 marks] Write down the transition matrix for this Markov process.

(b) [6 marks] Explain why the transition matrix has 1 as an eigenvalue.

(c) [10 marks] The marketing company predicts that TWO will (in the long term) have more that 60% of the market. Is this realistic if current trends continue? Currently ONE has 45% of the market, TWO has 25% and THREE has 30%.