Ollscoil na Gaillimhe
University of Galway

Semester II Examinations 2022/2023

Exam Codes	1EM1, 1OA1, 2BA1, 2BCW1, 2BCT1, 2BPT1, 2BS1, 2EH1, 3BS9
Exam	Second Year Arts and Science Third Year Science
Module	LINEAR ALGEBRA
Module Code	MA203
External Examiner	Prof. Colva Roney-Dougal
Internal Examiners	Prof. Goetz Pfeiffer Dr Kevin Jennings ⋆

Instructions	**Answer all questions.**
Duration	2 hours
No. of Pages	4 pages including this page
School	Mathematical and Statistical Sciences

Requirements:

Release in Exam venue	Yes ☑	No ☐
MCQ	Yes ☐	No ☑
Statistical / Log Tables	Yes ☑	No ☐

Q1. (a) [13 marks] Consider the following system of equations

$$\begin{array}{rcrcrcrcl} 4x_1 & - & 3x_2 & + & 4x_3 & - & 2x_4 & = & 11 \\ x_1 & - & x_2 & + & x_3 & - & x_4 & = & 3 \\ x_1 & & & + & x_3 & + & x_4 & = & 2 \end{array}$$

(i) Write down the augmented matrix for this system of equations.

(ii) Using elementary row operations, convert the augmented matrix to reduced row echelon form.

(iii) Write down the general solution of the system of equations.

(b) [12 marks] For each of the following statements, declare with justification whether the statement is true or false.

(i) A system of four linear equations in three unknowns cannot have a solution.

(ii) $4x + 2y - 2z = 0$ is the equation of a plane through the origin in $\mathbb{R}^3$, with normal vector $(2, 1, -1)$

(iii) It is possible to determine if two lines in $\mathbb{R}^3$ intersect by solving an appropriate system of linear equations.

Q2. (a) [8 marks] Let

$$A = \begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 5 & 6 \end{pmatrix}.$$

Of the products AB, BA, A^2 and B^2, compute all those that are defined.

(b) [11 marks] Use elementary row operations to find the inverse (if it exists) of the matrix A below. Determine the rank of A and the kernel of A.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 4 & 1 & 5 \\ 4 & 1 & 4 \end{pmatrix}.$$

(c) [6 marks] For each of the following statements, declare with justification whether the statement is true or false.

(i) Elementary row operations do not change the determinant of a matrix.

(ii) Any set of three linearly independent vectors in $\mathbb{R}^3$ is a basis for $\mathbb{R}^3$.

(iii) If the rows of a square matrix A are linearly dependent, then A is not invertible.

Q3. (a) [10 marks] Express the vector $(-1, 4, 7)$ as a linear combination of the vectors $(1, -1, 0), (1, 0, 2)$ and $(0, 1, 1)$. What is the span of the set of vectors $\{(1, -1, 0), (1, 0, 2), (0, 1, 1)\}$?

(b) [6 marks] Let $e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)$ be the standard basis vectors in $\mathbb{R}^3$ and consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^3$ which describes an anticlockwise rotation by an angle θ in the $e_1 - e_3$ plane about the e_2-axis. Find the matrix for L (w.r.t. the standard basis for $\mathbb{R}^3$).

(c) [9 marks] For each of the following statements, declare with justification whether the statement is true or false.

(i) If 0 is an eigenvalue of a square matrix A, then A has no inverse matrix.

(ii) If the characteristic polynomial of a 3×3 matrix A is $P(\lambda) = \lambda^3 - 6\lambda^2 + 11\lambda - 6$, then 2 is an eigenvalue of A.

(iii) A 3×3 matrix can have four distinct eigenvalues.

Q4. The town of Ballymarcove has three mobile phone providers ONE, TWO and THREE and every resident of Ballymarcove is a customer of exactly one provider.

- Every year 10% of ONE customers switch to TWO and 20% switch to THREE (with 70% remaining with ONE).
- Every year 30% of TWO customers switch to ONE and 20% switch to THREE (with 50% remaining with TWO).
- Every year 30% of THREE customers switch to ONE and 30% switch to TWO (with 40% remaining with THREE).

(a) [9 marks] Write down the transition matrix for this Markov process.

(b) [8 marks] Explain why the transition matrix has 1 as an eigenvalue.

(c) [8 marks] TWO is a relative newcomer to Ballymarcove and in the long term it aims to have a third of the market. Currently ONE has 60% of the market, TWO has 10% and THREE has 30%. If current trends continue, can TWO expect to achieve their aim?