Mobile robot locomotion

Robótica Móvel e Inteligente

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Outline

- Introduction
  - Tracked locomotion, legged locomotion, wheeled locomotion
- Wheeled mobile robots
  - Wheel types and wheel configurations
  - Differential drive, Ackerman steering, Tricycle drive, Synchro drive, Omnidirectional drive
- Kinematic models of some popular wheeled configurations
  - Differential drive
  - Tricycle drive
  - Omnidirectional drive

Mobile robot

- A combination of various physical and computational units (hardware and software)
- Organized in a set of sub-systems:
  - **Sensing**: measures properties of the robot environment
  - **Reasoning**: maps measurements into high-level action commands
  - **Control**: transforms high-level action commands into low-level actions
  - **Actuation**: transforms low-level action commands into physical actions
  - **Locomotion**: **maps physical actions into movement, e.g, defines how the robot moves in its environment**
  - ***Communication**: provides communication with other robots, or with an external system*

Locomotion

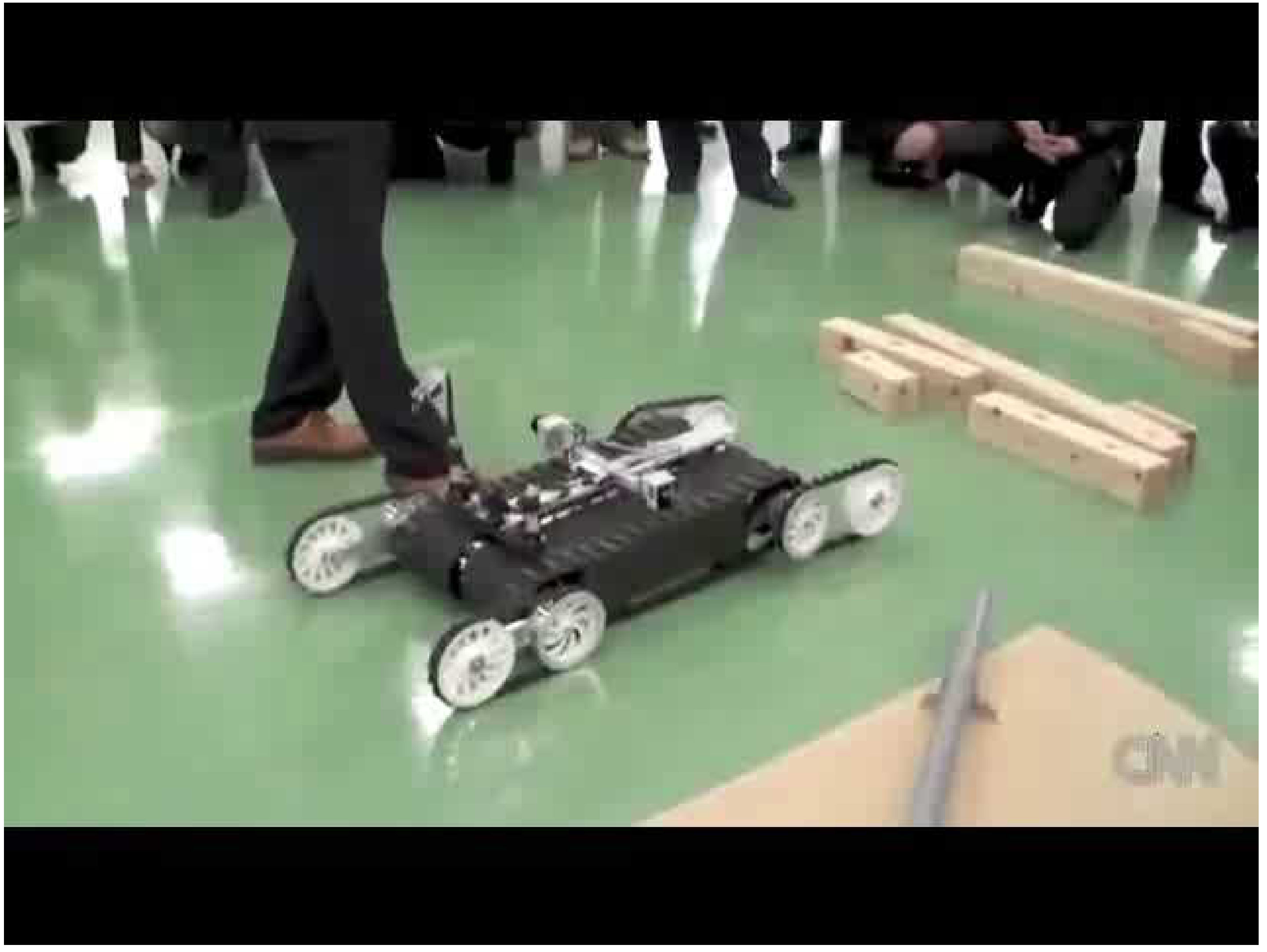
- The physical process that allows the robot to move in its environment

- Several solutions available:
  - Tracked locomotion
  - Legged locomotion
  - Wheeled locomotion

Tracked locomotion

- **Great traction power** - the track contact area with the ground is greater than the one provided by a wheel
- Locomotion system well suited for **robots that evolve in very rough terrain** (e.g., in natural disaster situations)
- **Change of direction is achieved by sliding the tracks**, which makes it very **difficult to use odometry** as a method of localization
- Requires a large amount of power to turn
- The robots that use this type of movement are **typically teleoperated**

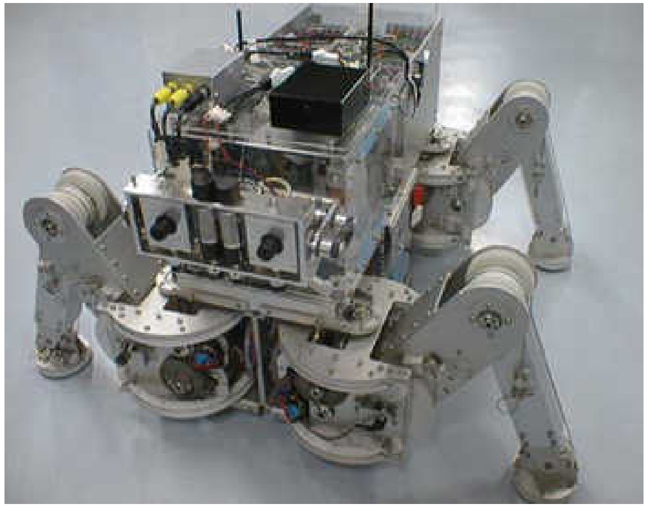
Tracked locomotion

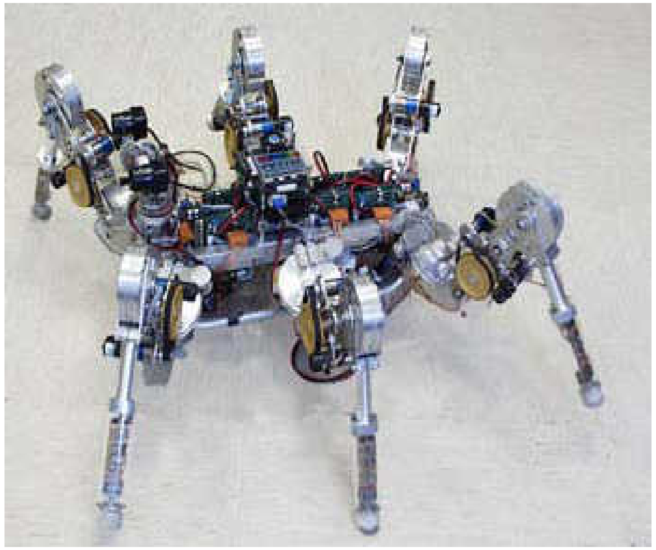


Legged locomotion

- Locomotion with legs is many times **based on living beings** (as those that move in difficult environments)
- The implementation of this type of locomotion system in robots is **complex**:
  - Mechanical complexity
  - Stability
  - Power consumption

Legged locomotion





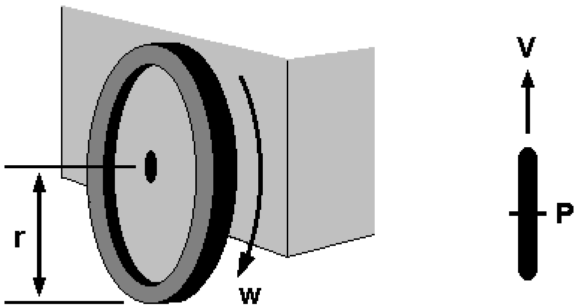
Legged locomotion (AlphaDog – Boston Dynamics)

Wheeled locomotion

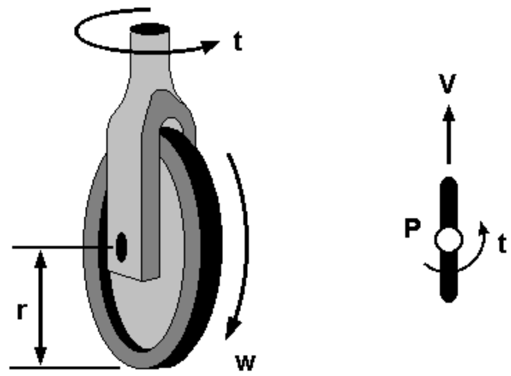
- The wheeled locomotion solution is the **most suitable for common applications**
  - rolling is very efficient!
- The **configuration and type of wheel** to use is **dependent on the application**
- Main constraint: **flat terrain** (or slightly irregular)
- Bigger wheels allow the robot to overcome bigger obstacles. However:
  - Motors with higher torque are needed (or gearboxes with higher reduction ratios, i.e., lower output speed for the same motor)

Wheel types

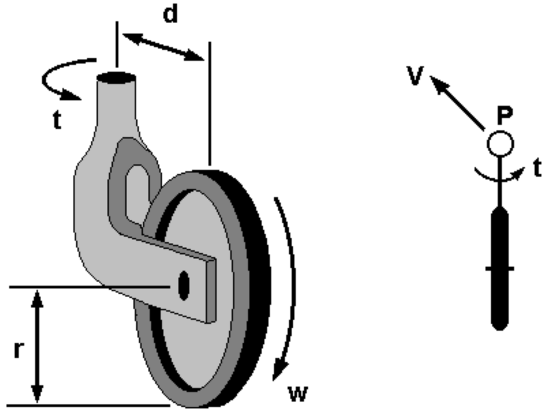
Standard wheel



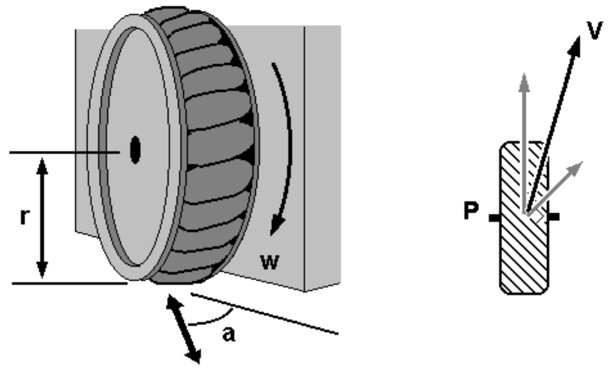
Steered standard wheel



Off-centered orientable wheel
(castor wheel)

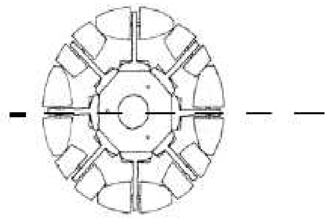


Swedish wheel (omnidirectional)

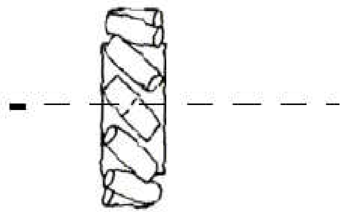


Wheel types – Swedish wheel

- Small rollers around the wheel circumference, with axes antiparallel to the main axis
- The wheel can be driven with full force, but will also slide laterally with very low friction
- Omnidirectional property
- Three degrees of freedom:
  - Rotation around the wheel axle (motorized)
  - Around the rollers
  - Around the contact point with the ground



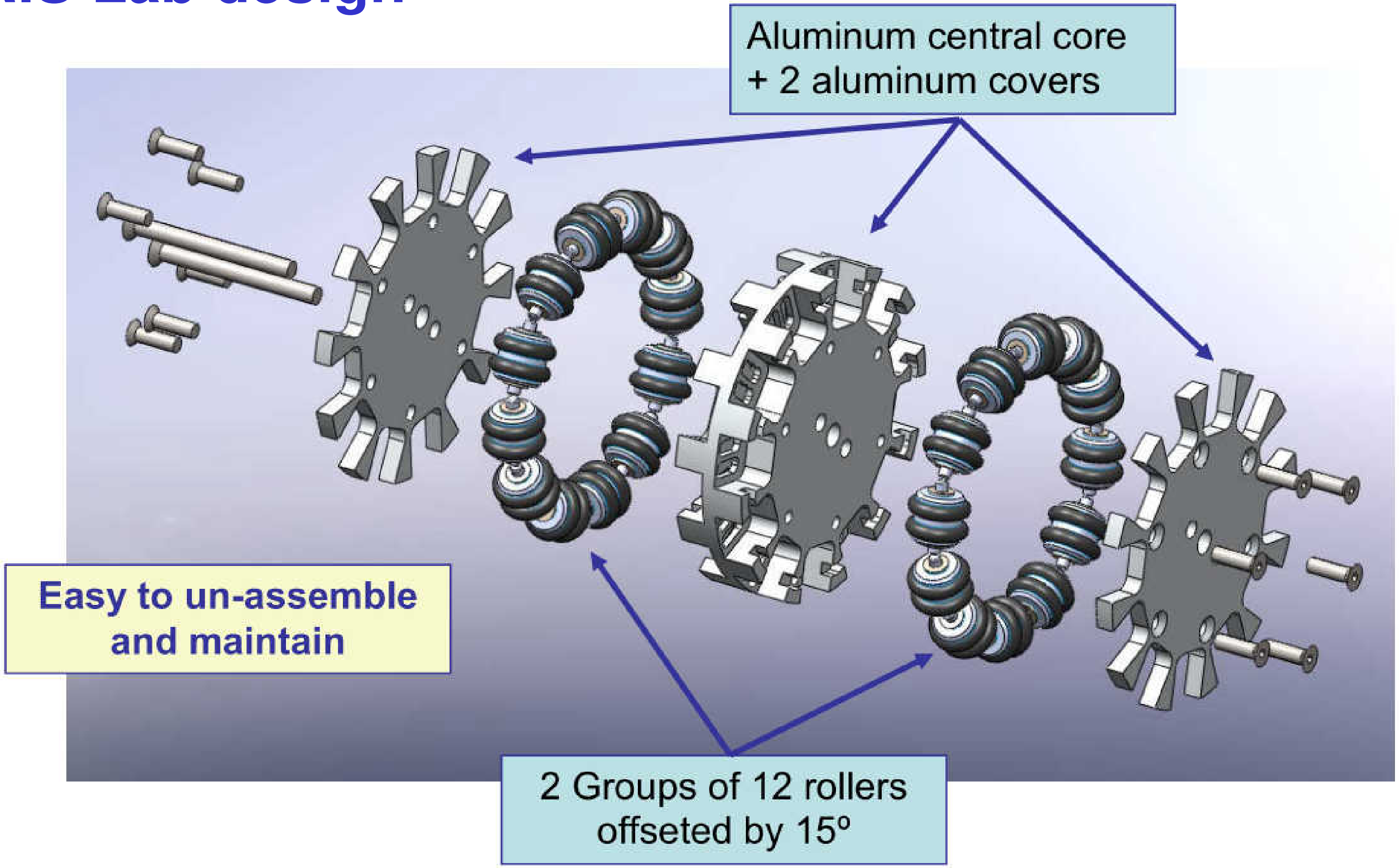
swedish 90°



swedish 45°

Wheel types – Swedish wheel

- **IRIS Lab design**



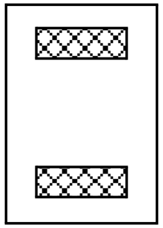
Wheeled locomotion – static stability

- Two wheels
  - Minimum number of wheels to achieve stability
  - **Center of mass must be below the axle** that links the wheels
- Three wheels
  - Stable configuration
  - **Center of mass must be inside the triangle formed by the ground contact points of the wheels**
- Four wheels
  - Stable configuration
  - **Requires a suspension system** to compensate for irregularities in the environment where the robot has to move
- More than four wheels
  - Configuration dependent

Wheel configurations[1]

- 2 wheels

One steering wheel in the front and one traction wheel in the rear

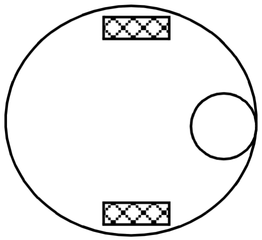


Two-wheel differential drive with the center of mass below the axle

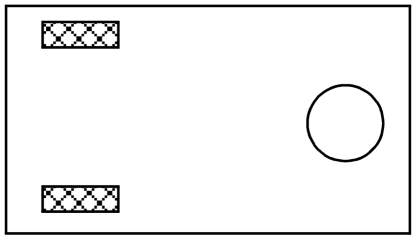
1) From: *R. Siegwart, I. Nourbakhsh*

Wheel configurations

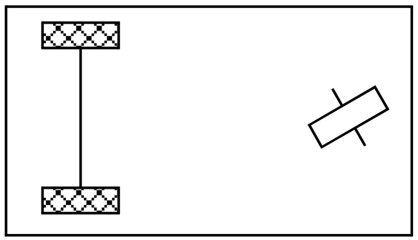
- 3 wheels



Two-wheel centered differential drive with a third point of contact



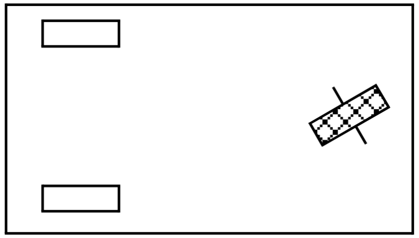
Two independently driven wheels in the rear/front, one steered free wheel (unpowered) in the front/rear



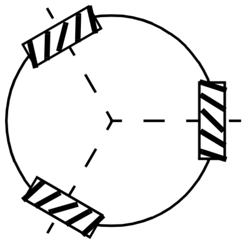
Two connected traction wheels (differential gear) in rear, one steered free wheel in front

Wheel configurations

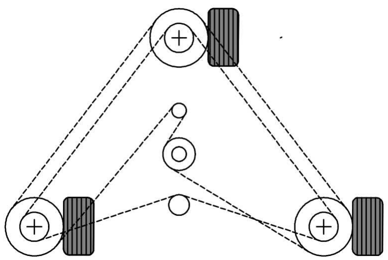
- 3 wheels



Two free wheels in rear, one steered traction wheel in front



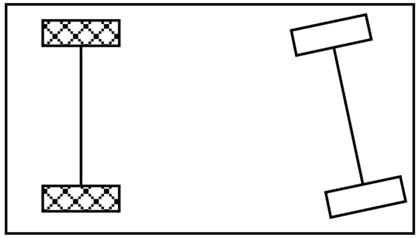
Three motorized Swedish or spherical wheels arranged in a triangle; omnidirectional movement is possible



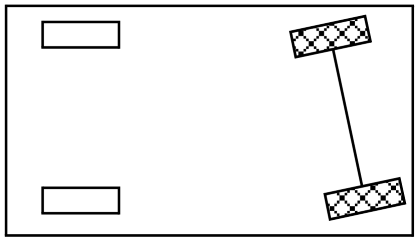
Three synchronously motorized and steered wheels; the chassis orientation is not controllable

Wheel configurations

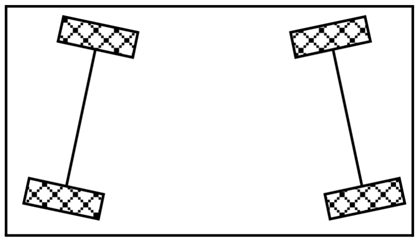
- 4 wheels



Two motorized wheels in the rear, two steered wheels in the front; steering has to be different for the two wheels to avoid slipping/skidding.



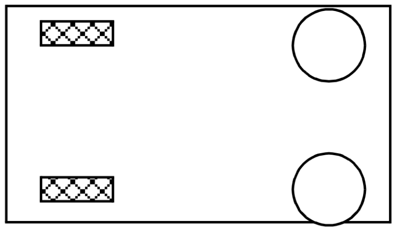
Two motorized and steered wheels in the front, two free wheels in the rear; steering has to be different for the two wheels to avoid slipping/skidding.



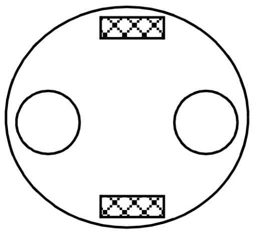
Four steered and motorized wheels

Wheel configurations

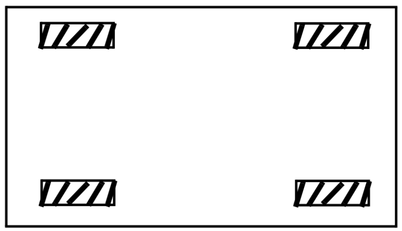
- 4 wheels



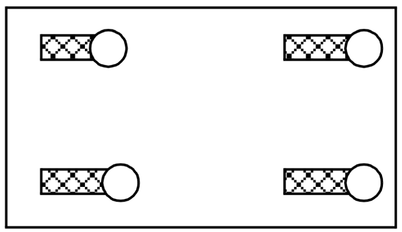
Two traction wheels (differential) in rear/front, two omnidirectional wheels in the front/rear



Two-wheel differential drive with two additional points of contact



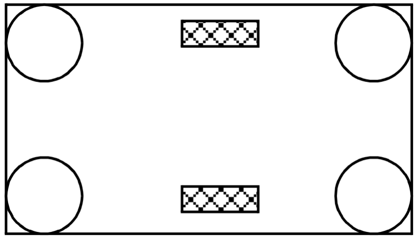
Four omnidirectional wheels



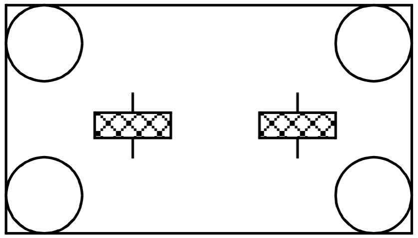
Four motorized and steered castor wheels

Wheel configurations

- 6 wheels



Two traction wheels (differential) in center, one omnidirectional wheel at each corner



Two motorized and steered wheels aligned in center, one omnidirectional wheel at each corner

Non-standard configurations

SHRIMP (EPFL)

Kinematics

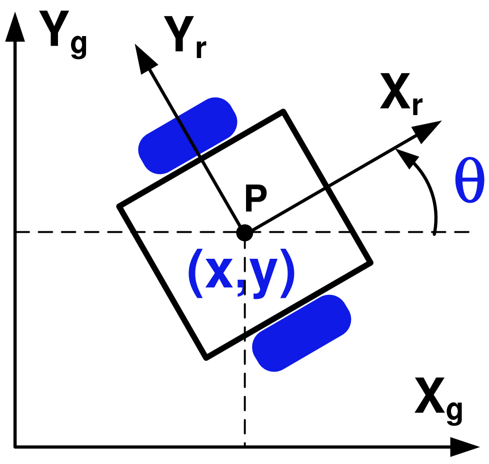
- **Locomotion**
  - The process that causes the movement of the robot
  - In order to produce a motion, forces must be applied to the robot
- **Dynamics**
  - The study of motion, in which forces are modeled
- **Kinematics**
  - Modeling the motion without considering the forces that cause the object to move

Local and global reference frames

- **global** reference frame: {Xg, Yg}
- **local** (robot) reference frame: {Xr, Yr}

- Orthogonal rotation matrix:

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\xi R = [x\ y\ \theta]^T$$

- Mapping velocities **from global reference frame** to robot reference frame:

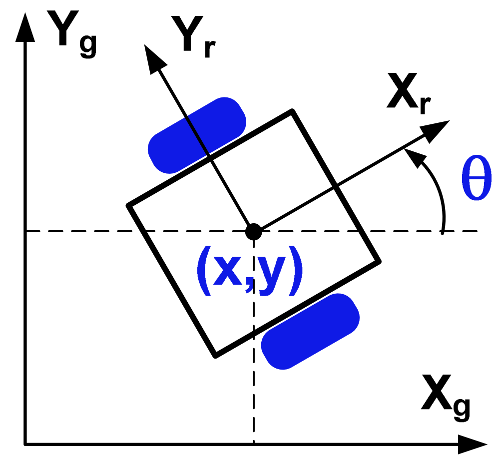
$$\dot{\xi} R = R(\theta)\, \dot{\xi} G = R(\theta) \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T \qquad \dot{\xi} R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

($\dot{x}$ – linear velocity along X_g, $\dot{y}$ – linear velocity along Y_g, $\dot{\theta}$ – angular velocity)

Local and global reference frames

- Inverse of the orthogonal rotation matrix:

$$R(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



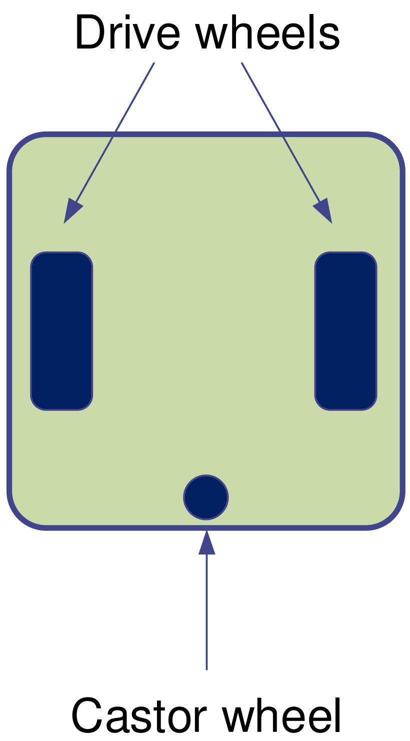
- Mapping velocities from robot reference frame to global reference frame:

$$\dot{\xi}G = R(\theta)^{-1}\dot{\xi}R = R(\theta)^{-1}\begin{bmatrix} Vx & Vy & \dot{\theta} \end{bmatrix}^T \qquad \dot{\xi}G = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Vx & Vy & \dot{\theta} \end{bmatrix}^T$$

(Vx – linear velocity along X_r, Vy – linear velocity along Y_r, $\dot{\theta}$ – angular velocity)

Differential drive

- **Common configuration**:
  - 2 active independent drive wheels
  - 1 or 2 passive castor wheels
- Robot follows a **trajectory** which is **defined by the speed of each wheel**
- Trajectory is **sensitive to differences in the relative velocity of the two wheels**
  - caused by asymmetries in motors and/or wheels
  - a small error results in a path different from that intended
- **Easy mechanical implementation**



Differential drive – kinematics

W_R – angular velocity, right wheel

W_L – angular velocity, left wheel

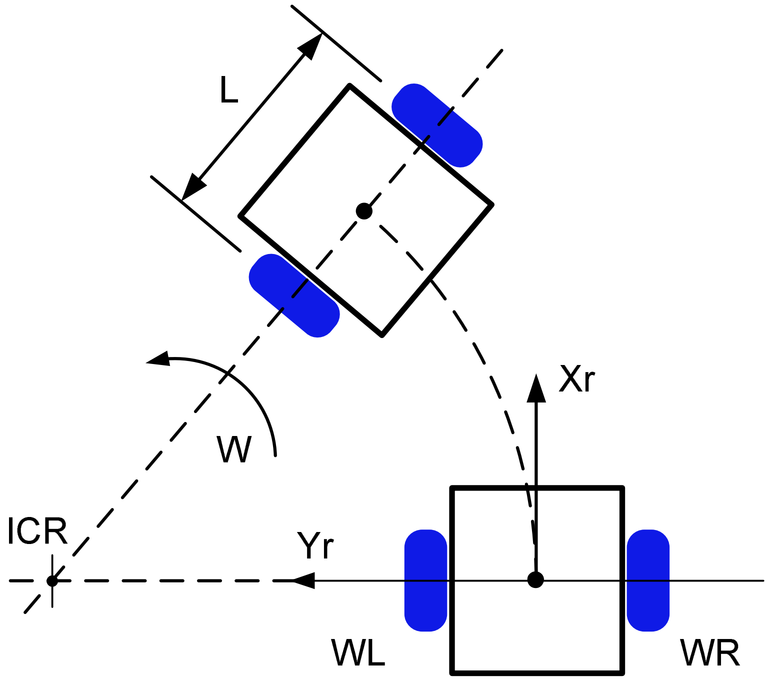
V_R – linear velocity, right wheel

V_L – linear velocity, left wheel

W – angular velocity of the robot about ICR

r – wheel radius

L – distance between wheels



Differential drive – kinematics

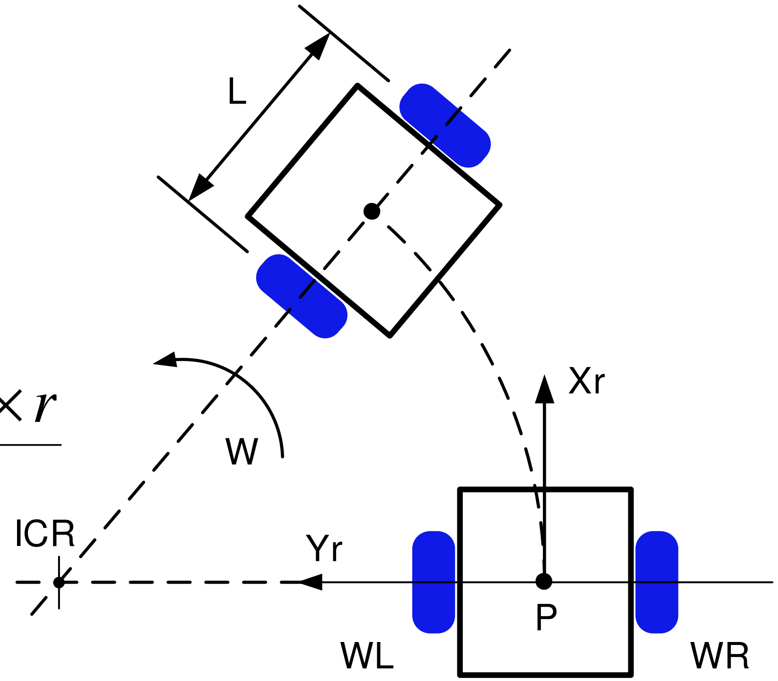
$$V_R(t) = W_R(t) \times r$$

$$V_L(t) = W_L(t) \times r$$

$$V_X(t) = \frac{V_R(t)}{2} + \frac{V_L(t)}{2} = W_R(t)\frac{r}{2} + W_L(t)\frac{r}{2}$$

$$V_Y(t) = 0$$

$$W(t) = \frac{V_R(t)}{L} - \frac{V_L(t)}{L} = \frac{W_R(t)\times r - W_L(t)\times r}{L}$$



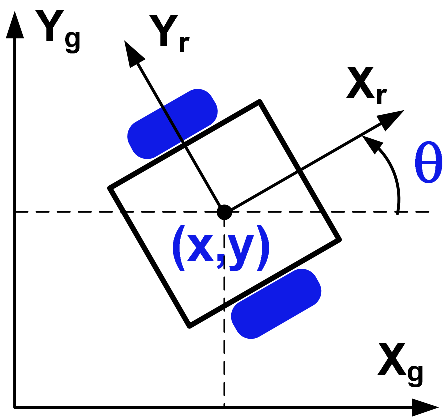
Kinematic model in local frame

$$\begin{bmatrix} V_X(t) \\ V_Y(t) \\ W(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} W_L(t) \\ W_R(t) \end{bmatrix}$$

Differential drive – kinematics

Kinematic model in world frame

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Vx(t) \\ 0 \\ W(t) \end{bmatrix}$$

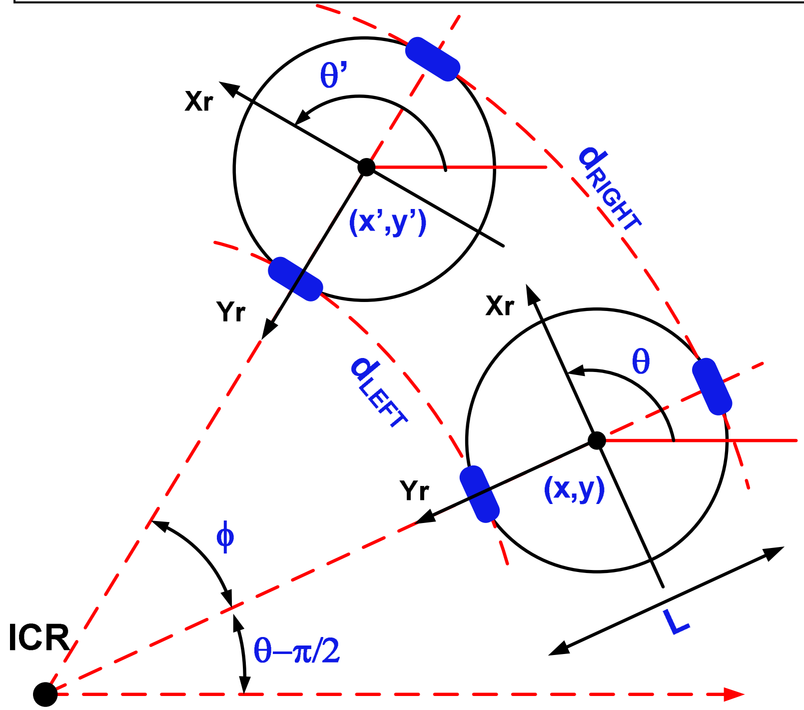


$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V(t) \\ W(t) \end{bmatrix}$$

Differential drive – position estimation

- **(x, y, θ)** – pose (position and orientation) of the robot in the **world frame**
- Supposing the robot's pose is **(x, y, θ)**, the **position estimation** consists in finding **(x', y', θ')** given:
  - $\mathbf{d_{RIGHT}}$ - distance travelled by the right wheel
  - $\mathbf{d_{LEFT}}$ - distance travelled by the left wheel
- $\mathbf{d_{RIGHT}}$ and $\mathbf{d_{LEFT}}$ measured by wheel encoders

Robot is moving counter-clockwise



L - distance between robot wheels

Differential drive – position estimation

- Over a small time period, the robot's motion can be approximated by an arc

$$d_{CENTER} = \frac{d_{RIGHT} + d_{LEFT}}{2}$$

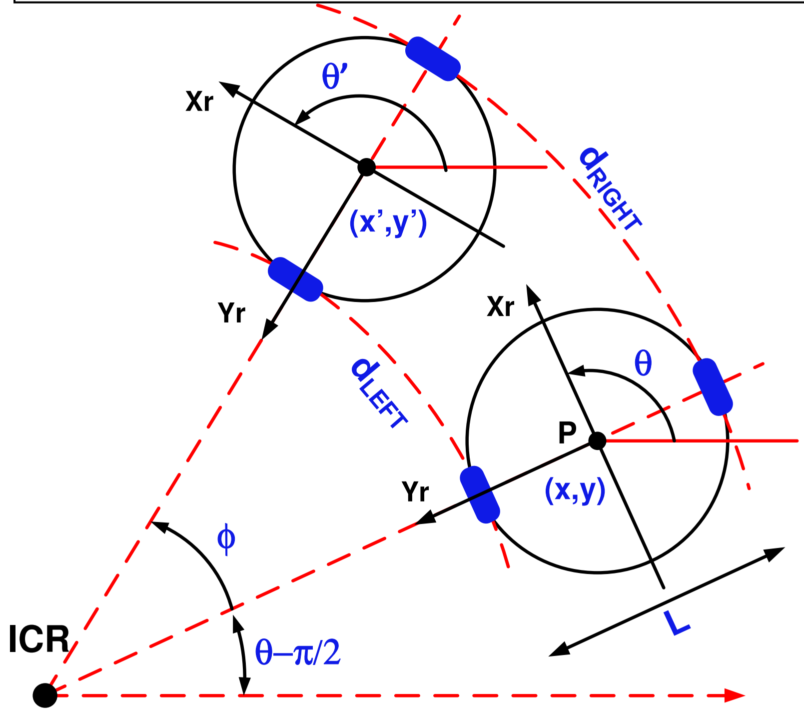
$$\phi = \frac{dist}{R}$$

$$\phi \times R_{RIGHT} = d_{RIGHT}$$
$$\phi \times R_{LEFT} = d_{LEFT}$$

$$\phi = \frac{d_{RIGHT} - d_{LEFT}}{L}$$

$$(R_{RIGHT} - R_{LEFT} = L)$$

Robot is moving counter-clockwise



ICR: Instantaneous Center of Rotation

Differential drive – position estimation

$$d_{CENTER} = \frac{d_{RIGHT} + d_{LEFT}}{2}$$

$$\phi = \frac{d_{RIGHT} - d_{LEFT}}{L}$$

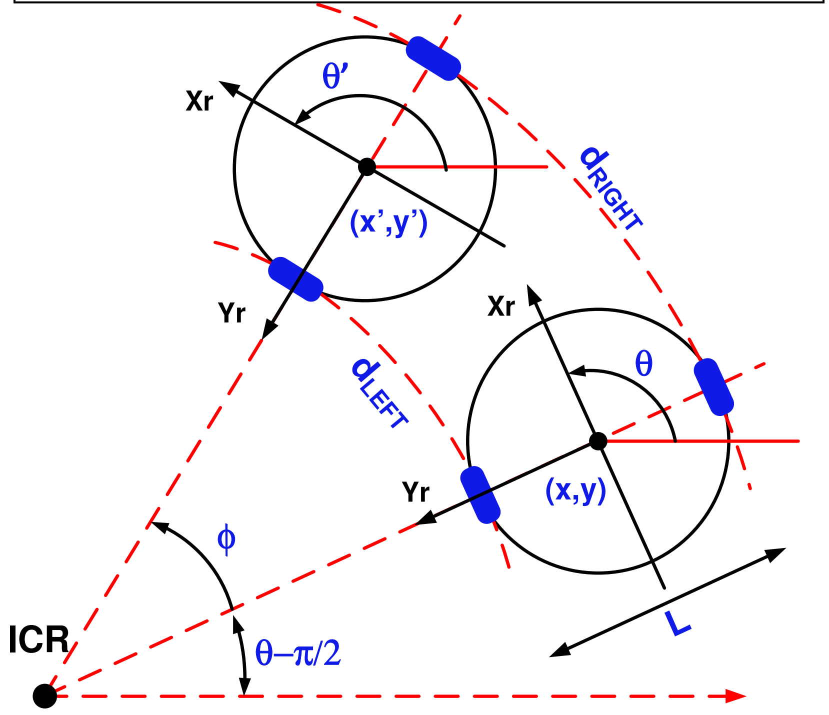
- For small displacements, such that sin(ϕ)≅ϕ and cos(ϕ)≅1):

$$x' = x + d_{CENTER} \times \cos(\theta)$$

$$y' = y + d_{CENTER} \times \sin(\theta)$$

$$\theta' = \theta + \phi$$

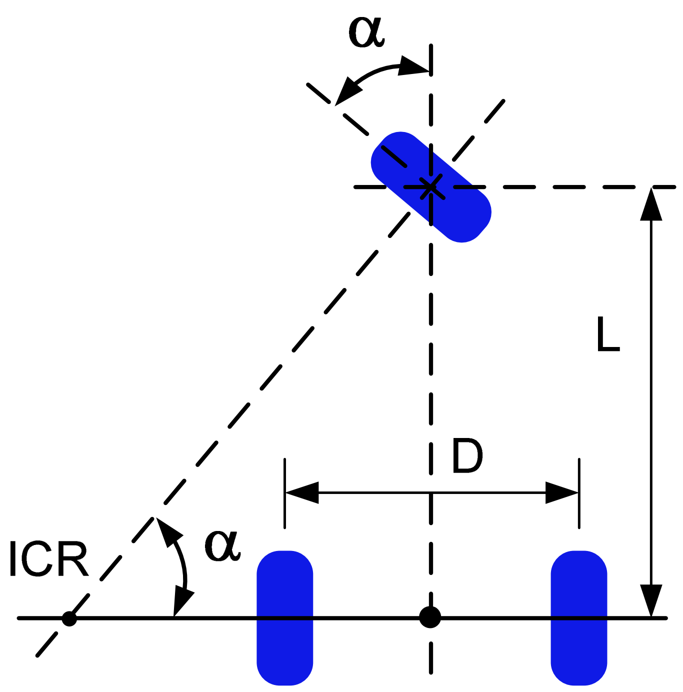
Robot is moving counter-clockwise



ICR: Instantaneous Center of Rotation

Tricycle drive

- **Three wheels**: two rear wheels and one (steering) front wheel
- Two possible configurations of traction:
  - **Front wheel is passive** - the two rear wheels are driving wheels (must use differential gear)
  - **Driving wheel on the front** (rear wheels are passive) – easier to implement
- **Main problems** of the front wheel drive configuration:
  - When going uphill, the driving wheel may **loose traction** due to the displacement of the center of mass
  - The traction **contact area** with the ground is half of the rear wheel drive configuration

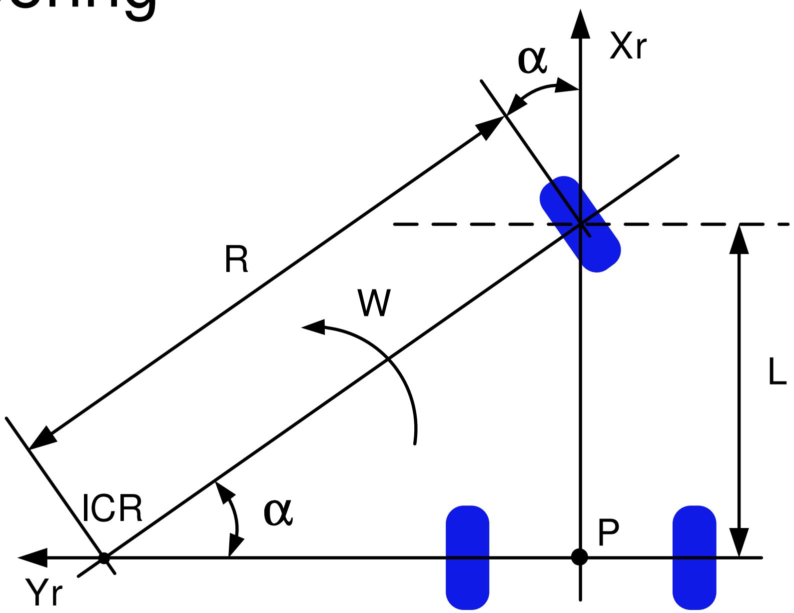


Tricycle drive – kinematics

V_S – linear velocity of the steering wheel

W_S – angular velocity of the steering wheel

r – steering wheel radius



W – angular velocity of the robot about ICR

α – steering angle

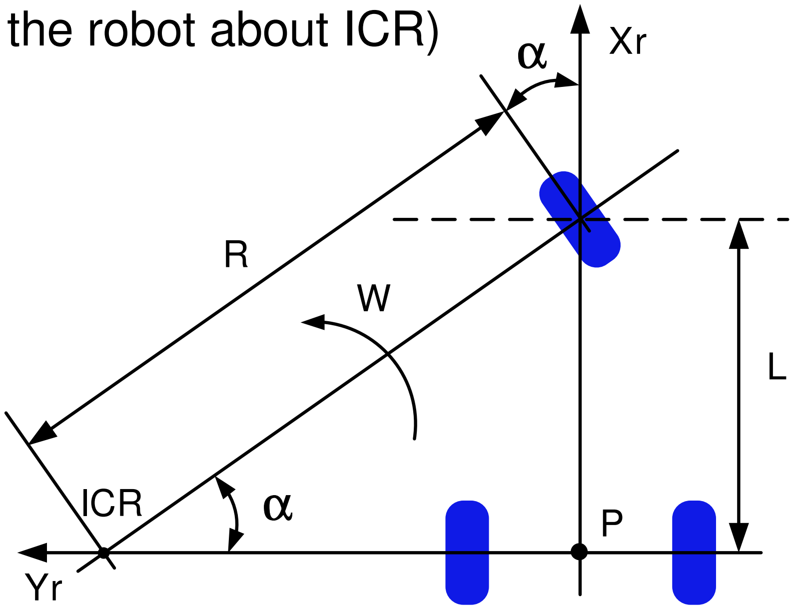
Tricycle drive – kinematics

$$Vs = Ws \times r$$ (linear velocity of the steering wheel)

$$Ws = \frac{Vs}{r}$$ (angular velocity of the steering wheel)

$$R = \frac{L}{\sin\alpha}$$

$$W = \frac{Vs}{R} = \frac{Vs \times \sin\alpha}{L}$$ (angular velocity of the robot about ICR)



Kinematic model in local frame

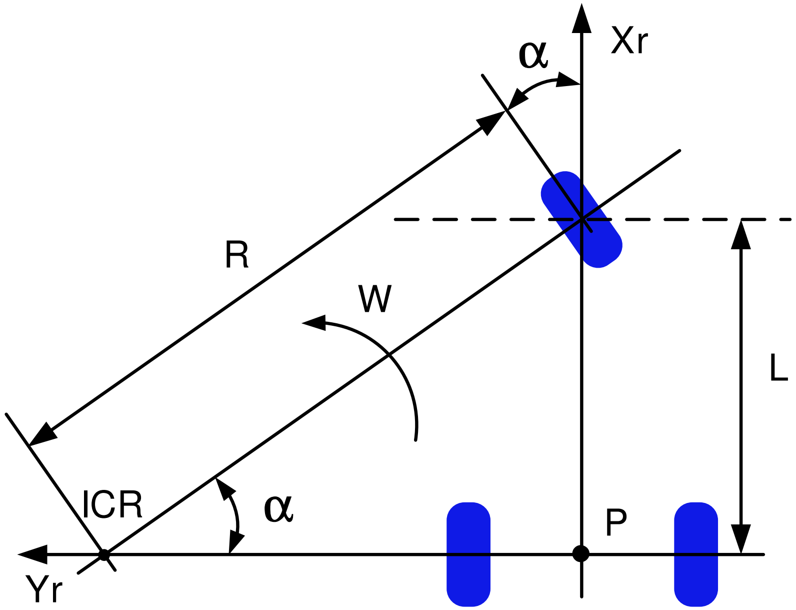
$$VX(t) = VS(t) \times \cos\alpha\,(t)$$

$$VY(t) = 0$$

$$W(t) = \frac{VS(t)}{L} \times \sin\alpha\,(t)$$

Tricycle drive – kinematics

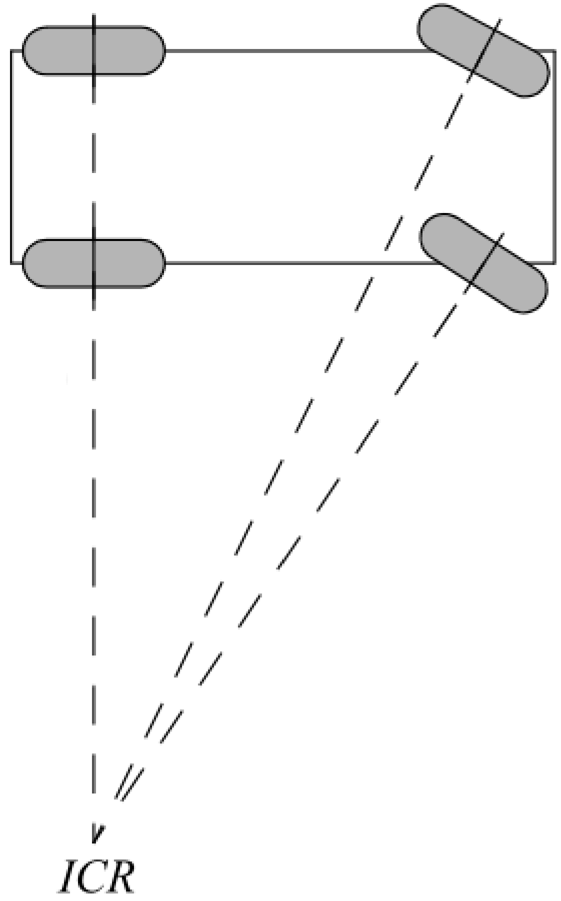
Kinematic model in world frame



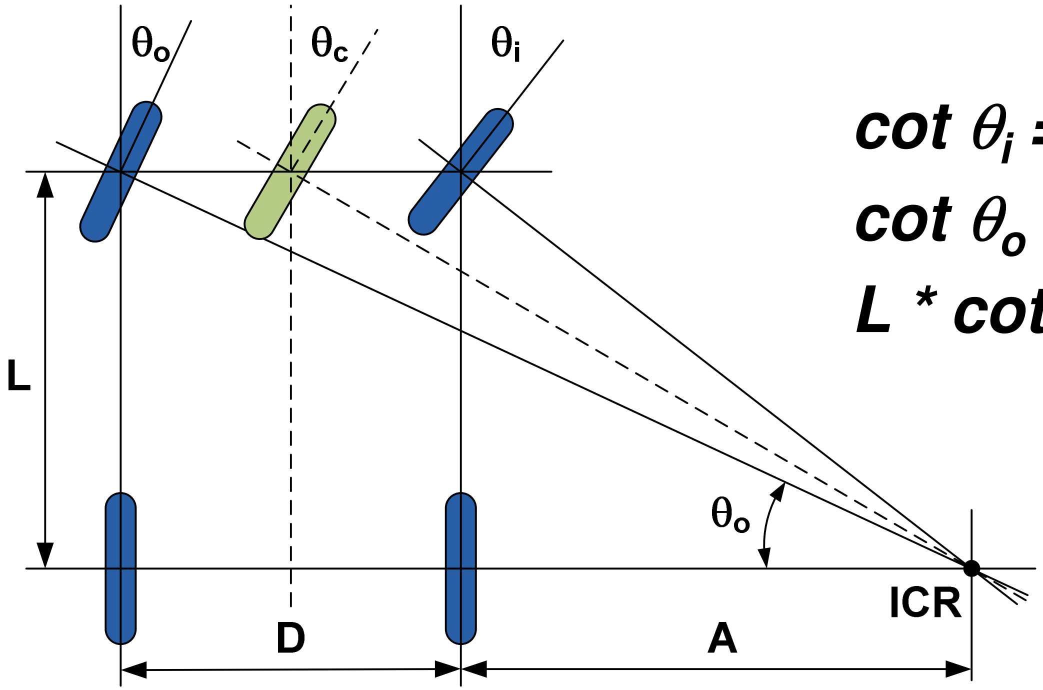
$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Vs(t)\times\cos\alpha(t) \\ 0 \\ \dfrac{Vs(t)}{L}\times\sin\alpha(t) \end{bmatrix}$$

Ackerman steering

- Generally the **method of choice for outdoor autonomous robots**
- The **inside front wheel is turned slightly more than the outside wheel** (reduces tire slippage)
- The extension of the axis of all four wheels intersects a common point ICR
- 4 or 3 wheel system support rear and/or front traction
- A **differential gear must be used in the traction axel** (unless a single motorized wheel is used in that axel)



Ackerman steering



$$\cot \theta_i = A / L$$

$$\cot \theta_o = (A + D) / L$$

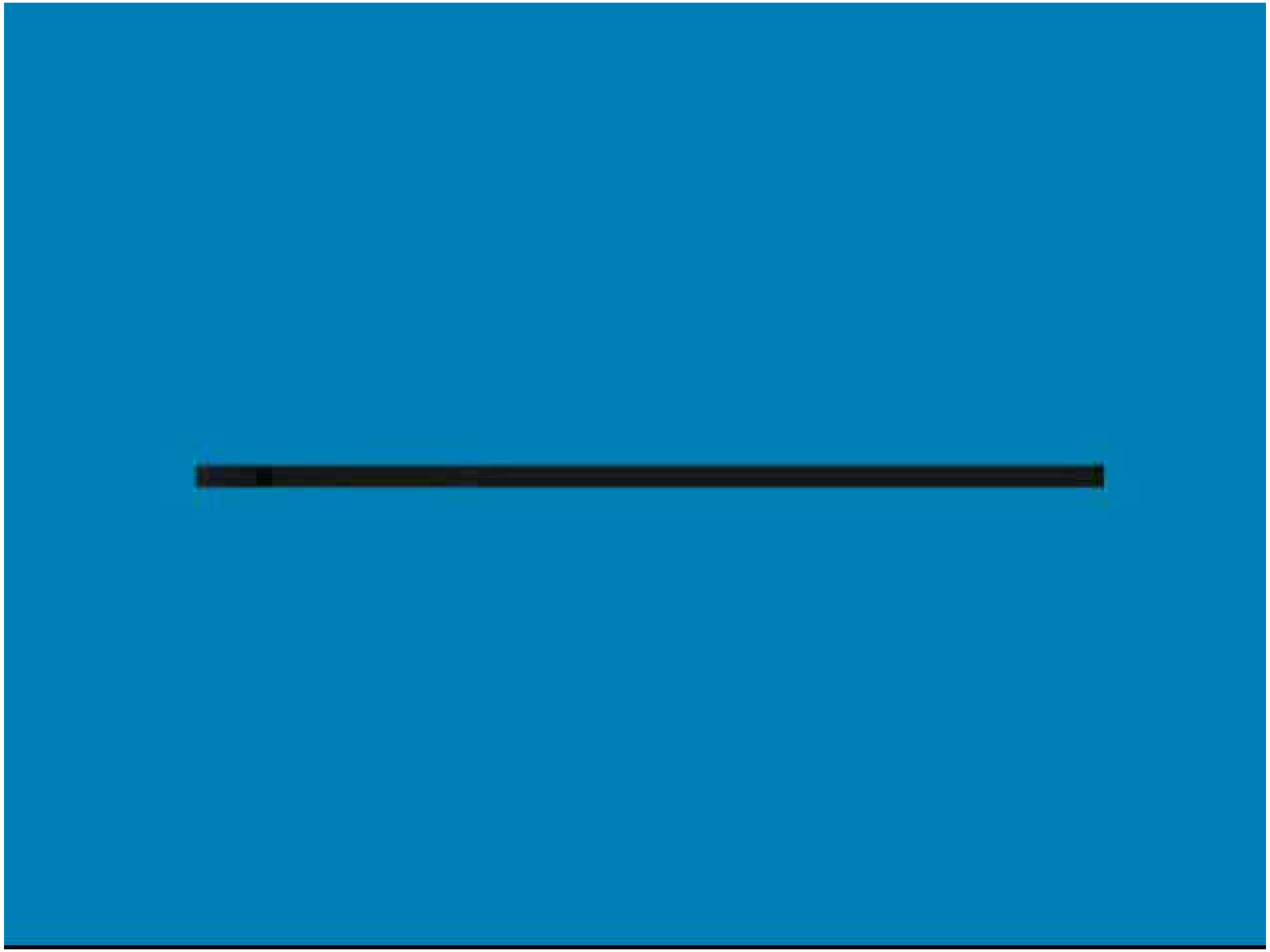
$$L * \cot \theta_o = L * \cot \theta_i + D$$

$$\boxed{\cot \theta_c = \cot \theta_o - D / (2 * L)}$$

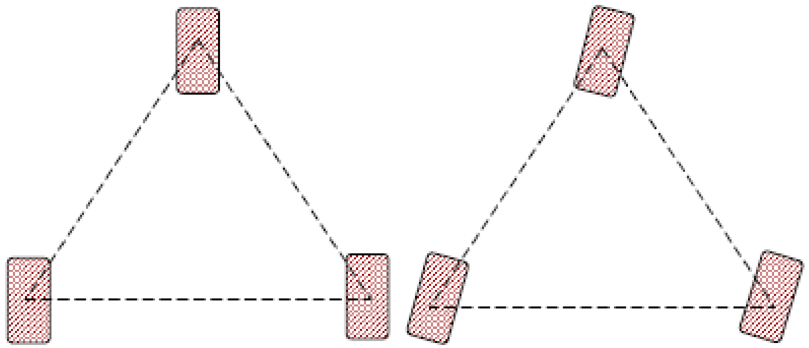
$$\boxed{\cot \theta_o - \cot \theta_i = D / L}$$

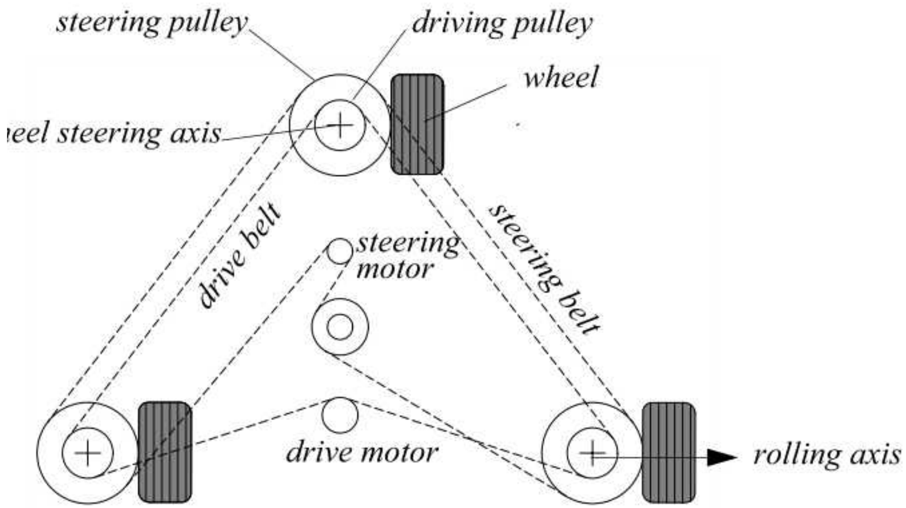
- Axis of all wheels intersects a common point ICR
- Kinematic model: tricycle drive

Ackerman steering (ROTA robot)

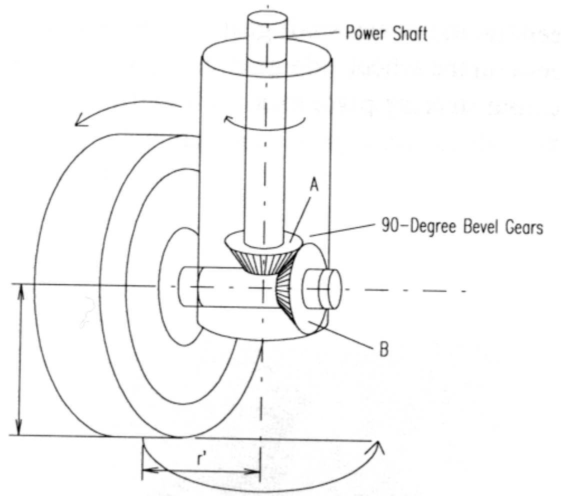


Synchro drive

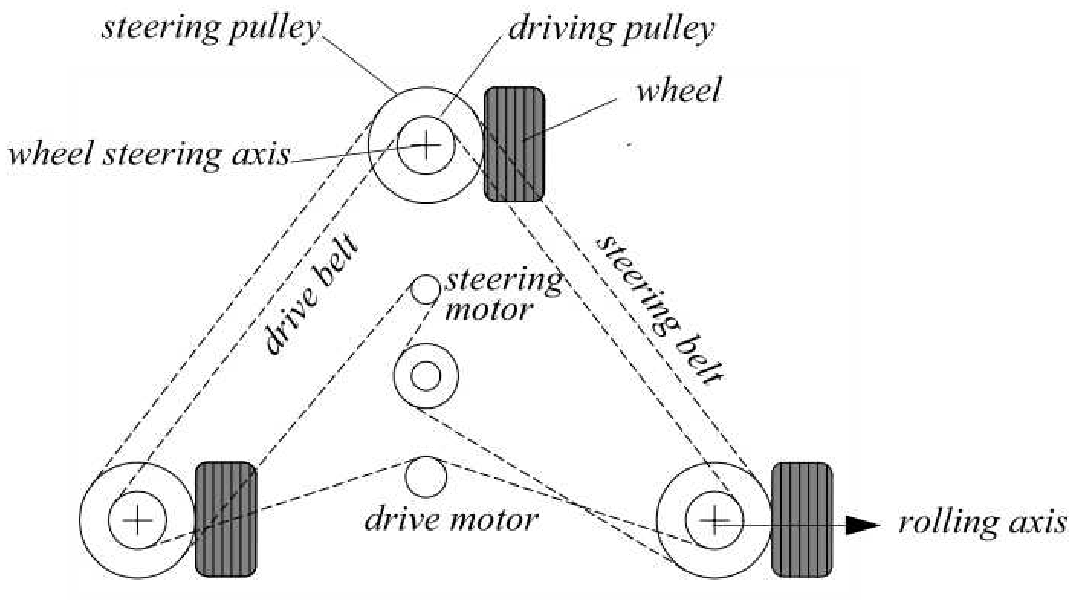




- Three (or more) wheels
- Two motors:
  - Translation motor sets the speed of all three wheels together
  - Steering motor turns all the wheels together about each of their individual vertical steering axes

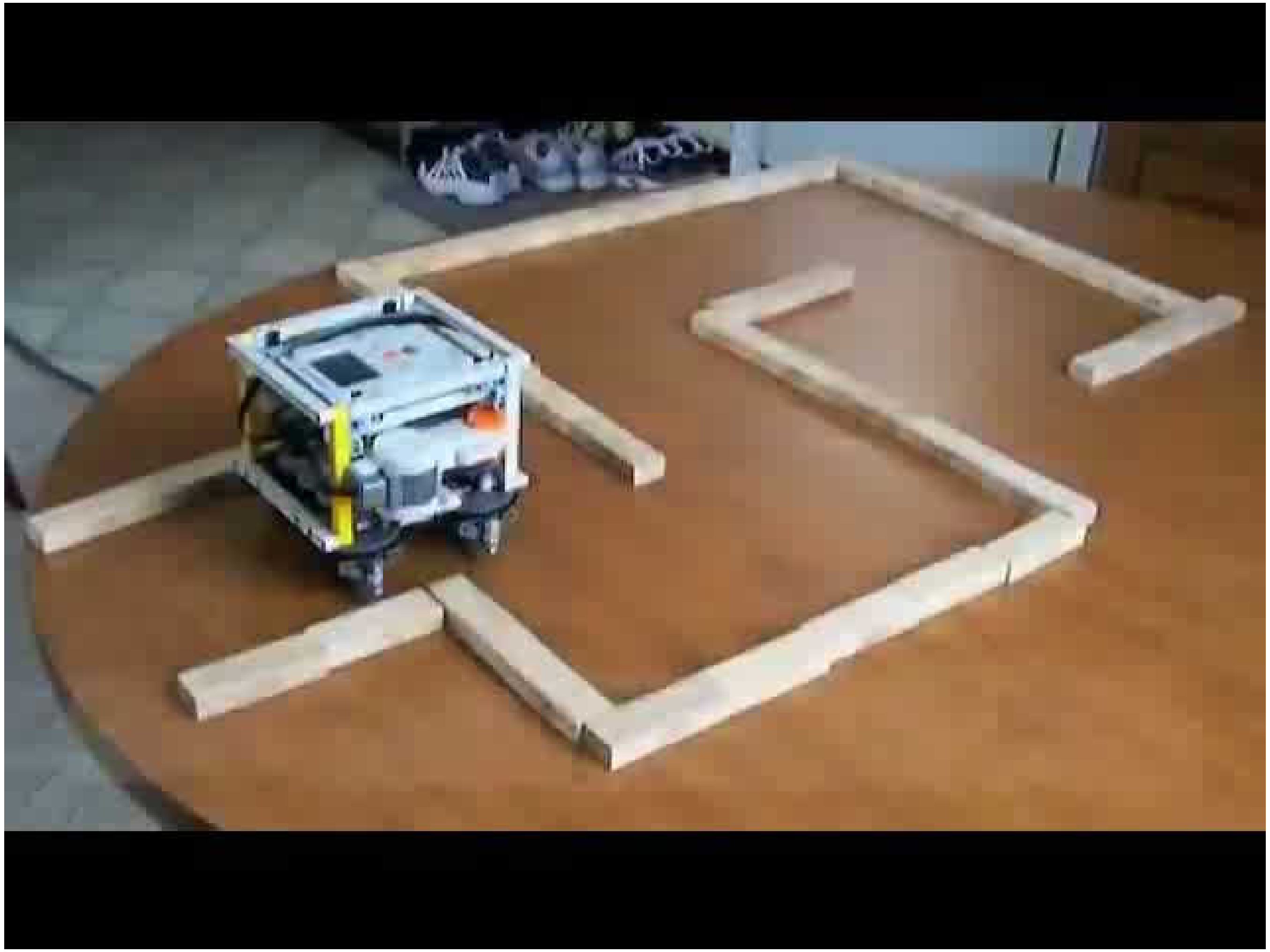


Synchro drive



- The robot **can move in any direction**
- The robot can always reorient its wheels and **move along a new trajectory without changing its footprint**
- However, the **orientation of the chassis is not controllable** (since the wheels are being steered with respect to the robot chassis)

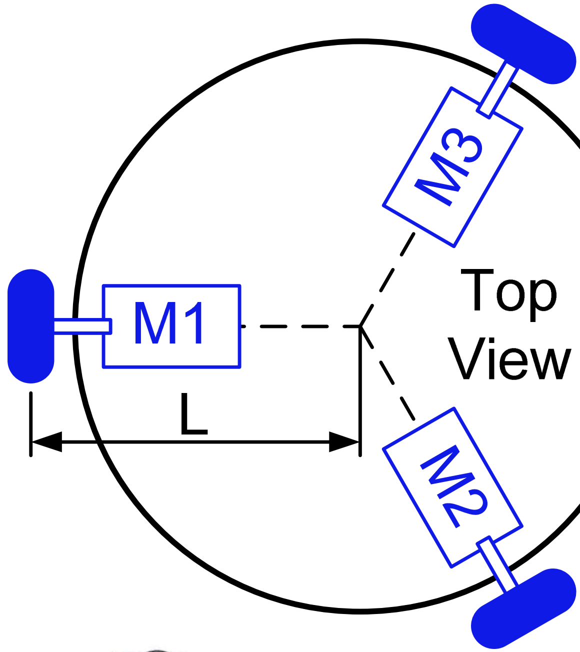
Synchro drive (LEGO)



http://y2u.be/MFxjIthqXVs

Omnidirectional drive

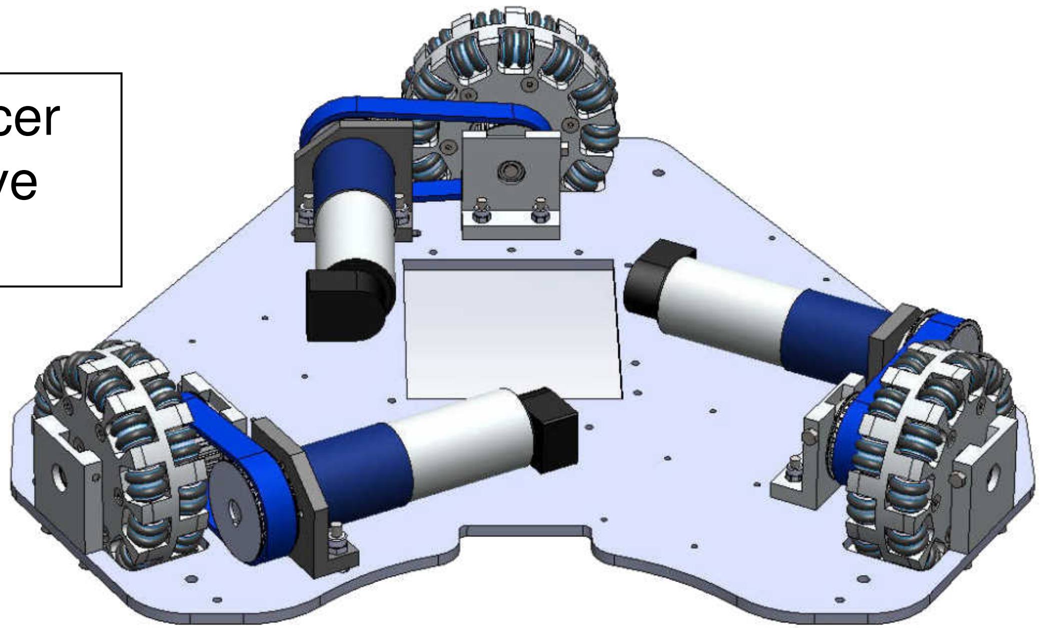
- Uses **Swedish wheels**
- **Each wheel has one independent drive motor**
- Allows **movement in any direction** by setting appropriate speeds in each of the three motors
- Allows complex movements (for instance translation combined with rotation)
- Three wheels configuration:
  - the wheels are spaced 120º



Omnidirectional drive

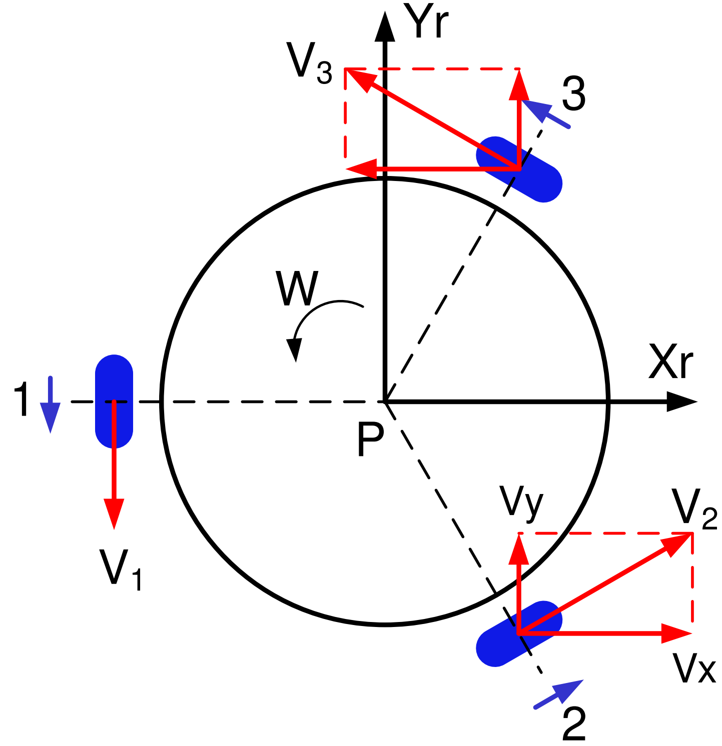
- Allows the generation of complex movements, such as to go straight changing, at the same time, the robot orientation
- **Excellent maneuverability**
- 4-wheel configuration: greater traction but more sensitive to uneven floors

CAMBADA (IRIS Lab) soccer robot – omnidirectional drive structure



Omnidirectional drive – kinematics

- The translation velocities of the wheels, V1, V2 and V3, determine the global velocity of the robot on the environment
- The translation velocity of the wheel hub "i" (Vi) can be divided in two parts:
  - pure translation of the robot
  - pure rotation of the robot



$$V_i = V_{transl,i} + V_{rot}$$

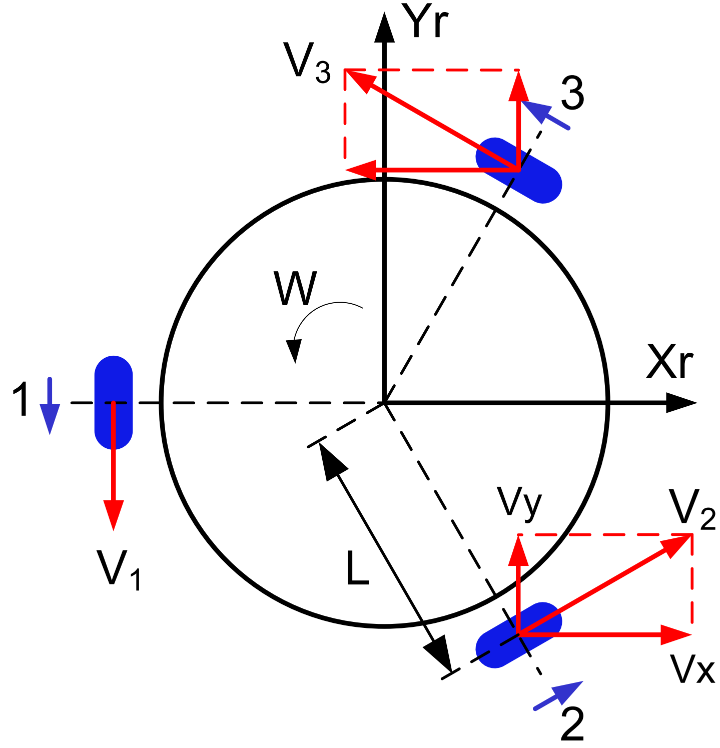
Omnidirectional drive – kinematics

- When the robot performs a pure rotation, the hub "i" velocity becomes

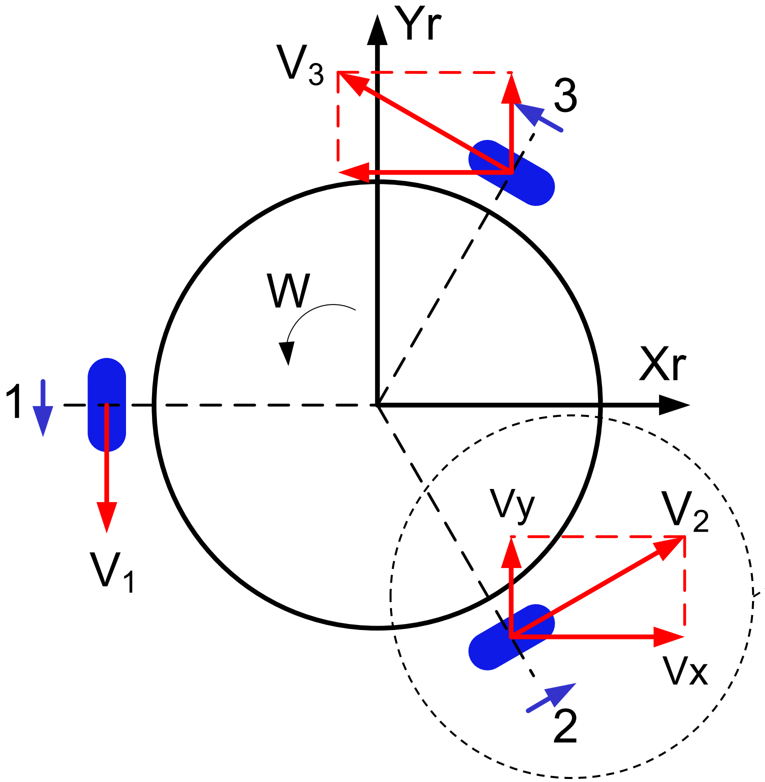
$$V_i = LW$$

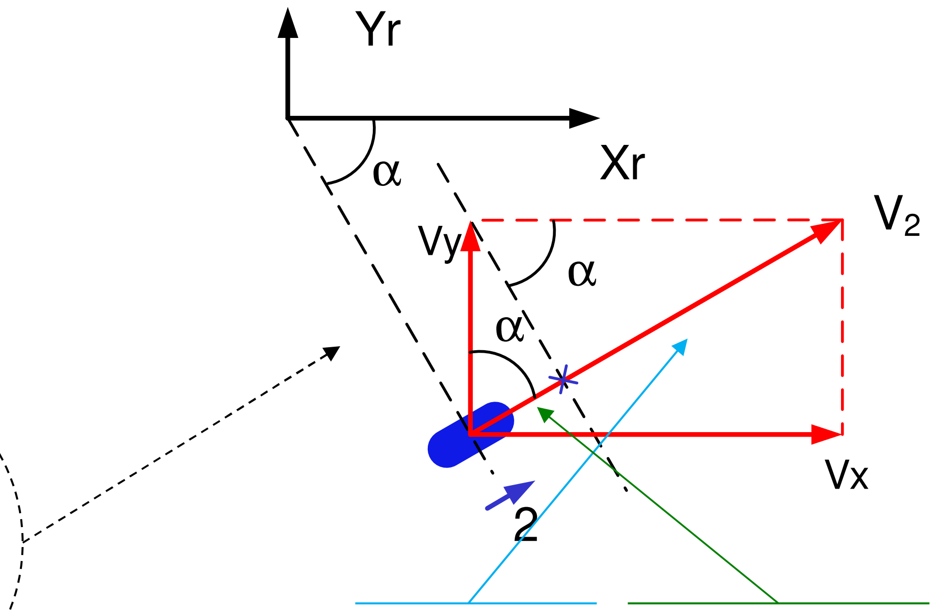
where:

- L: is the distance from the geometric center of the robot to the wheel
- W: angular velocity of the robot



Omnidirectional drive – kinematics



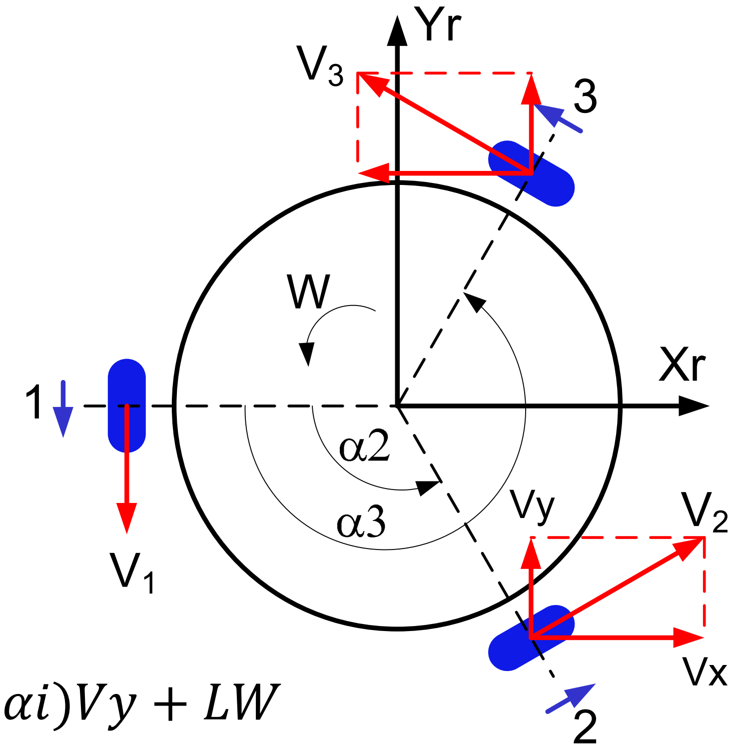


$$V_{transl,\,2} = \sin(\alpha)Vx + \cos(\alpha) \times Vy$$

$$V_{transl,\,3} = -\sin(\alpha)Vx + \cos(\alpha)Vy$$

$$V_{transl,\,1} = \sin(0)Vx - \cos(0)Vy = -Vy$$

Omnidirectional drive – kinematics



- Taking hub 1 as reference, the hub angles are:

$$\alpha 1 = 0º, \alpha 2 = 120º, \alpha 3 = 240º$$

- The pure translation velocity at wheel hub "i" can then be generalized as:

$$V_{transl, i} = \sin(\alpha i)Vx - \cos(\alpha i)Vy$$

- And Vi becomes:

$$Vi = \mathrm{V}_{\mathrm{transl, i}} + \mathrm{Vrot} = \sin(\alpha i)Vx - \cos(\alpha i)Vy + LW$$

- But, $V_i = r \,.\, W_i$, (*r is* the wheel radius and W_i the wheel angular velocity)

$$\mathrm{rW}i = \sin(\alpha i)Vx - \cos(\alpha i)Vy + LW$$

$$\mathrm{W}i = (\sin(\alpha i)Vx - \cos(\alpha i)Vy + LW) \,/\, r$$

Omnidirectional drive – kinematics

- We can then write:

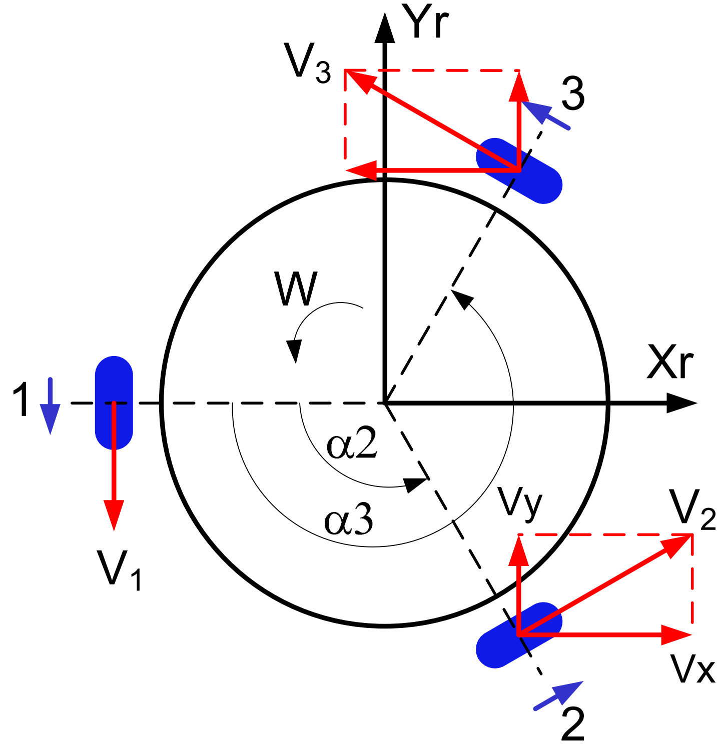
$$W_1 = \frac{1}{r}(-Vy + LW)$$

$$W_2 = \frac{1}{r}(\frac{\sqrt{3}}{2}Vx + 0.5Vy + LW)$$

$$W_3 = \frac{1}{r}(-\frac{\sqrt{3}}{2}Vx + 0.5Vy + LW)$$

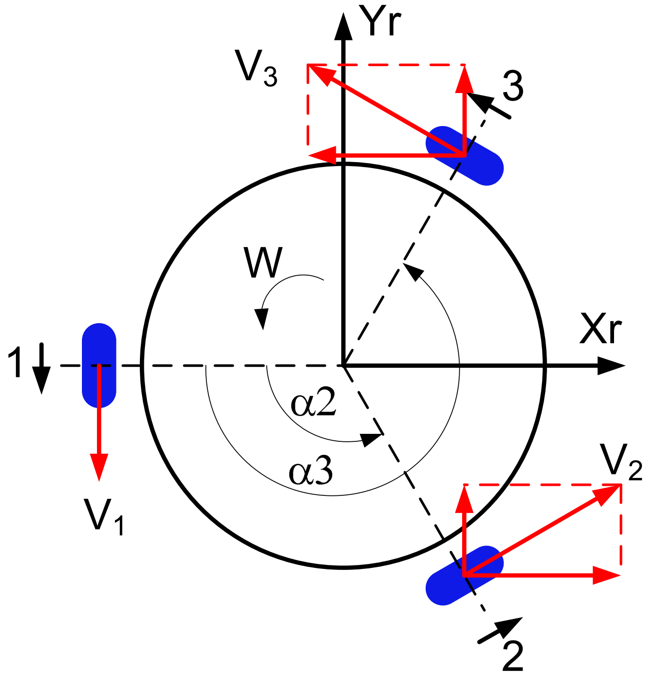
- Solving for Vx, Vy and W:

$$\begin{bmatrix} Vx \\ Vy \\ W \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{3}} \times r & -\frac{1}{\sqrt{3}} \times r \\ -\frac{2}{3}r & \frac{1}{3}r & \frac{1}{3}r \\ \frac{r}{3L} & \frac{r}{3L} & \frac{r}{3L} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$$



Kinematic model in local frame

Omnidirectional drive - kinematics



Kinematic model in global frame

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Vx \\ Vy \\ W \end{bmatrix}$$

Omnidirectional drive (CAMBADA)

http://y2u.be/PXq89EONEz0