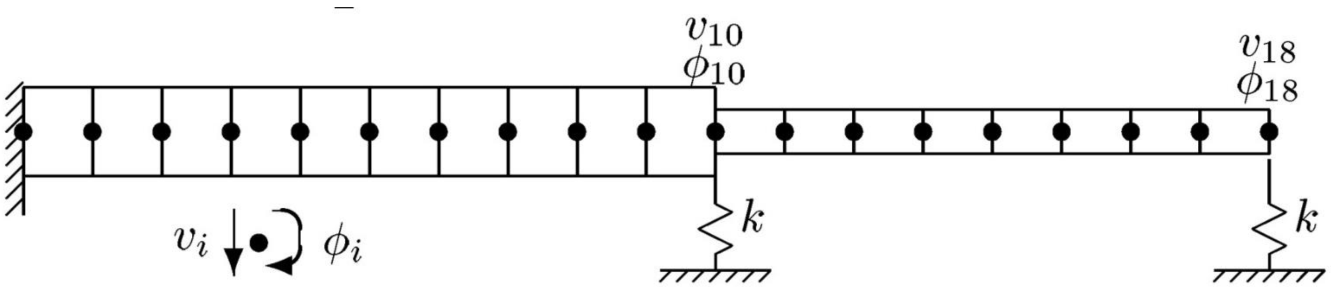


10. Sensor placement

Structural Dynamics part of 4DM00

dr.ir. R.H.B. (Rob) Fey, ir. D.W.M. (Daniël) Veldman

Sensor placement

Based on the **principle of modal independence:**
place sensors such that the mode shapes are as linearly independent as possible.

s: number of sensors.
e: number of modes of interest.

Asssume $s \geq e$.

Sensor placement (preperation)

1. Start from a FEM model with n DOFs.
   Compute the (mass-normalized) eigenmodes:
$$\left(-\omega_i^2 M + K\right)u_i = 0, \qquad u_i^T M u_i = 1.$$

2. Collect all eigenmodes in a matrix $U_{nn} = [u_1 \quad u_2 \quad \cdots \quad u_n]$.

3. Select the e relevant eigenmodes
   Remove the columns of U_{nn} corresponding to other eigenmodes
   Result: matrix $U_{ne} = [u_{k_1} \quad u_{k_2} \quad \cdots \quad u_{k_e}]$.

4. Select the m potential sensor locations ($m > s$)
   Remove all rotational, internal, inaccessible dof from U_{ne}
   Result: matrix U_{me}.

Sensor placement (criterion)

Using U_{me} form the **Fisher Information Matrix**

$$F_{ee} = U_{me}^{\top} U_{me}$$

Define the matrix

$$G_{mm} = U_{me} F_{ee}^{-1} U_{me}^{\top}.$$

Note that $G_{mm}^2 = G_{mm}$ (G_{mm} is idempotent). It therefore holds that

$$\mathrm{trace}(G_{mm}) = \mathrm{rank}(G_{mm}) = e.$$

Diagonal elements of G_{mm} represent partial contributions of each dof to the rank of G_{mm}.

Idea of selection procedure (on next slide):
Remove of the dof with the smallest diagonal element of G_{mm} and recompute G_{mm}.

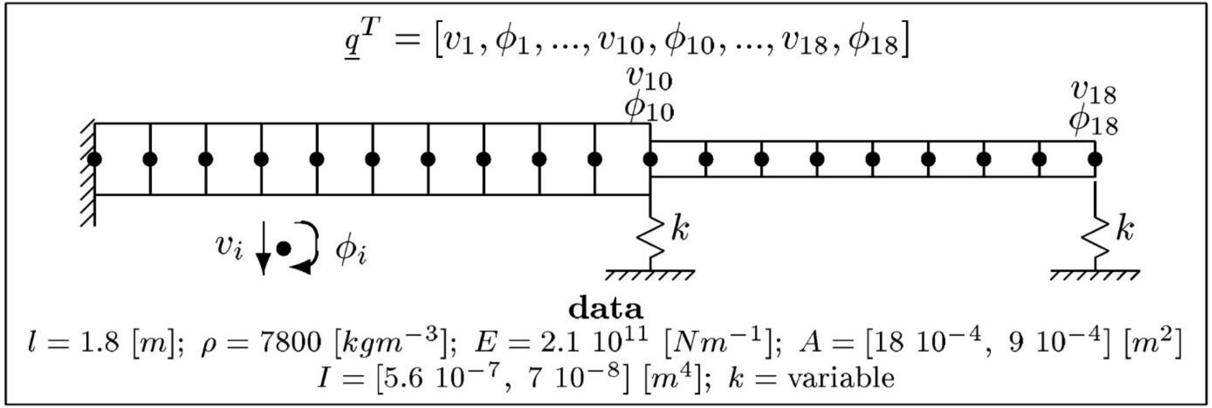
Sensor placement (procedure)

While $m > s$ (i.e. there are more potential sensor locations than sensors)

1. Form the matrices F_{ee} and G_{mm}
2. Find the smallest diagonal element of G_{mm}
3. Remove the DOF corresponding to this element and set $m := m - 1$.

End while

Example: clamped-free beam

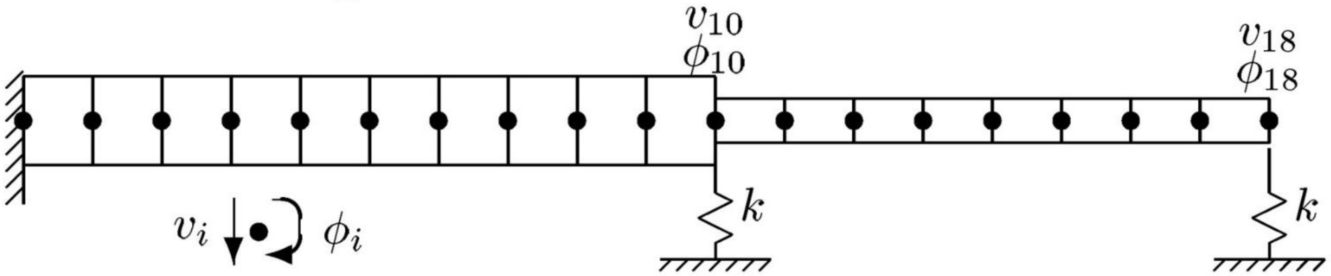


Non-uniform cross-section and 2 discrete springs, number of DOFs: $n = 36$

Assume: only lowest 4 modes important ($e = 4$)
Candidate positions: only translational dof v_i, $i = 1, \dots, 18$ ($m = 18$)
Search for 4 resp. 6 optimal sensor positions ($s = 4{,}6$)

Repeat procedure for 5 different values of the spring stiffness k.

Example: clamped-free beam



Stiffness k	4 Sensors	6 Sensors
0	6 11 15 18	5 6 11 14 15 18
$5.0\ 10^6$	6 11 15 18	5 6 11 14 15 18
$1.0\ 10^7$	5 10 13 16	5 6 10 13 16 17
$2.0\ 10^7$	5 9 13 16	5 9 10 13 14 16
$5.0\ 10^7$	4 8 13 16	4 5 8 12 13 16