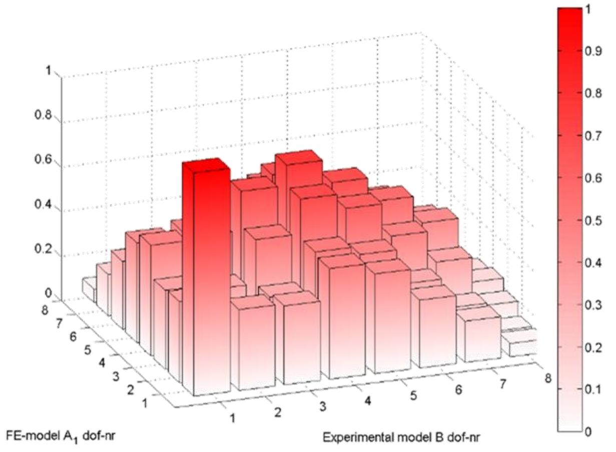
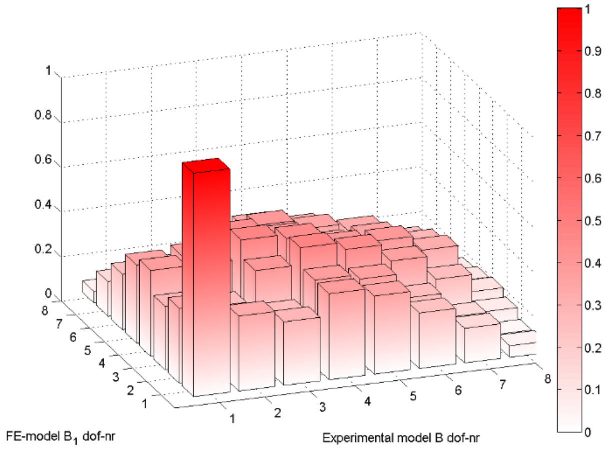


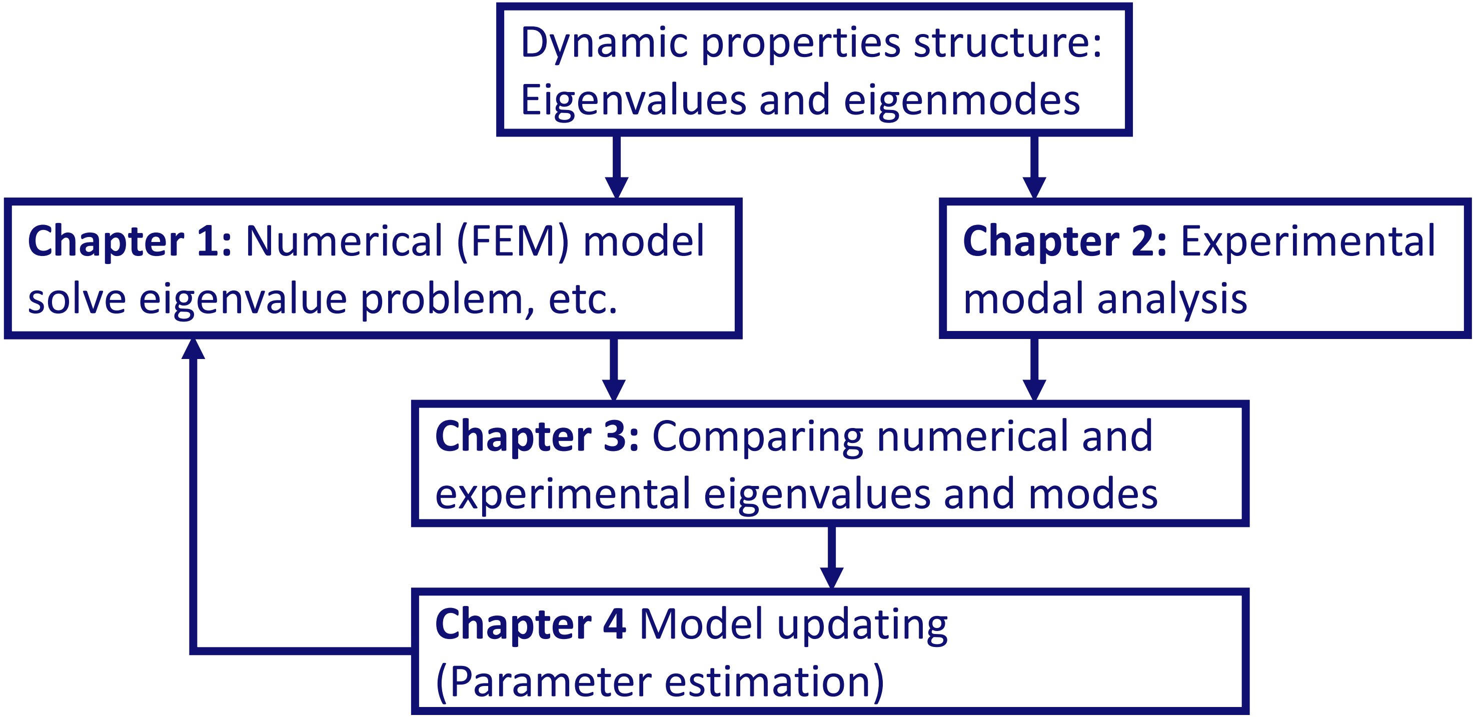


13. Model updating

Structural Dynamics part of 4DM00

dr.ir. R.H.B. (Rob) Fey, ir. D.W.M. (Daniël) Veldman

Structure of the course (SD part)



System error matrices

Find the parts of the numerical model which could be responsible for differences between numerical and experimental data.

Undamped orthogonality properties:

$$U^\top M U = I \quad \Rightarrow \quad M = (U^\top)^{-1} U^{-1} \quad \Rightarrow \quad M^{-1} = U U^\top$$

$$U^\top K U = \Omega^2 \quad \Rightarrow \quad K = (U^\top)^{-1} \Omega^2 U^{-1} \quad \Rightarrow \quad K^{-1} = U \Omega^{-2} U^\top$$

In practice we only have e measured modes U_{ne} and measured eigenfrequencies: Ω_{ee}^2.

Pseudo-inverses:

$$(M_{nn}^N)^{-1} \approx U_{ne}^N (U_{ne}^N)^\top \qquad (K_{nn}^N)^{-1} \approx U_{ne}^N (\Omega_{ee}^N)^{-2} (U_{ne}^N)^\top$$

$$(M_{nn}^E)^{-1} \approx U_{ne}^E (U_{ne}^E)^\top \qquad (K_{nn}^E)^{-1} \approx U_{ne}^E (\Omega_{ee}^E)^{-2} (U_{ne}^E)^\top$$

Intuition: there is a good match between numerical and experimental data when $(M^N)^{-1} \approx (M_{nn}^E)^{-1}$ and $(K^N)^{-1} \approx (K_{nn}^E)^{-1}$.

System error matrices

Define the error stiffness matrix:

$$\Delta K = K^E - K^N$$

Note: we cannot compute ΔK because only the pseudo inverse of K^E is available.

We can also write:

$$K^E = K^N(I + (K^N)^{-1}\Delta K), \qquad \Rightarrow \qquad (K^E)^{-1} = (I + (K^N)^{-1}\Delta K)^{-1}(K^N)^{-1}.$$

We use a Taylor series expansion for $(I + (K^N)^{-1}\Delta K)^{-1}$:

$$(K^E)^{-1} = (K^N)^{-1} - (K^N)^{-1}\Delta K(K^N)^{-1} + \left((K^N)^{-1}\Delta K\right)^2(K^N)^{-1}$$

If all eigenvalue of $(K^N)^{-1}\Delta K$ have sufficiently small magnitudes (ΔK sufficiently small), the higher order terms can be neglected:

$$\boxed{\Delta K \approx K^N\left((K^N)^{-1} - (K^E)^{-1}\right)K^N}$$

System error matrices

$$\Delta K \approx K^N\left((K^N)^{-1} - (K^E)^{-1}\right)K^N$$

- $(K^E)^{-1}$ is calculated by $U_{ne}^E(\Omega_{ee}^E)^{-2}\,(U_{ne}^E)^\top$
- $(K^N)^{-1}$ is calculated by $U_{ne}^N(\Omega_{ee}^N)^{-2}\,(U_{ne}^N)^\top$

Note: when the number of measured dof m is smaller than n,
the matrix K^N of dimension (n, n) should be reduced to dimension (m, m)

System error matrices

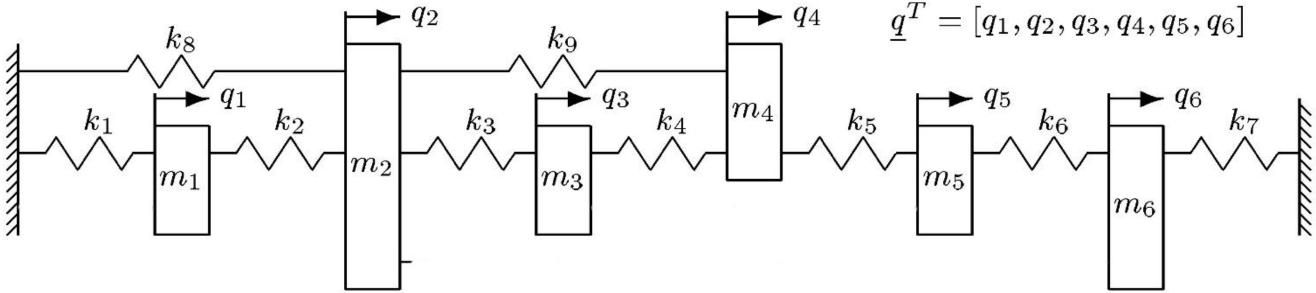
A similar procedure can be used to find the error mass matrix ΔM

Often, errors in K^N are larger than errors in M^N:
modelling of stiffness usually is more complicated than modelling of mass

ΔK and ΔM:

- could be seen as correction matrices to directly update K^N and M^N,
  but due to limited accuracy and interpretations problems they rather….
- give indications of DOFs in the FE model where modelling should be improved
  (identifying candidate parameters for model updating)

Example 1: 6 dof model



Data: $m_1 = m_2 = m_5 = 2\ [kg], \quad m_3 = m_4 = m_6 = 1\ [kg]$
$k_1 = k_2 = k_3 = k_4 = k_6 = k_7 = 10^6\ [N/m], \quad k_5 = k_8 = k_9 = 2\ 10^6\ [N/m]$

'Experimental' modes are obtained by setting $k_1 = 2 \cdot 10^6$ N/m, $k_7 = 0.5 \cdot 10^6$ N/m.

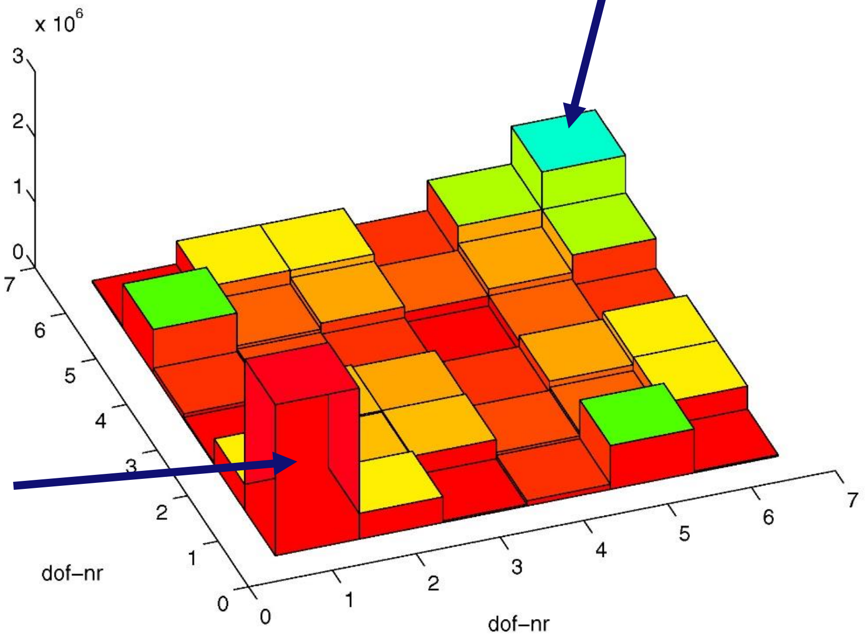
All DOFs are 'measured': $m = n = 6$

$e = 2$ (the first 2 eigenfrequencies and eigenmodes are both calculated and measured)

Example 1: 6 dof model

Numerical eigenfrequencies	Experimental eigenfrequencies
$\omega_1^n = 585.5$ rad/s	$\omega_1^e = 552.1$ rad/s
$\omega_2^n = 940.1$ rad/s	$\omega_2^e = 1031.5$ rad/s

ΔK

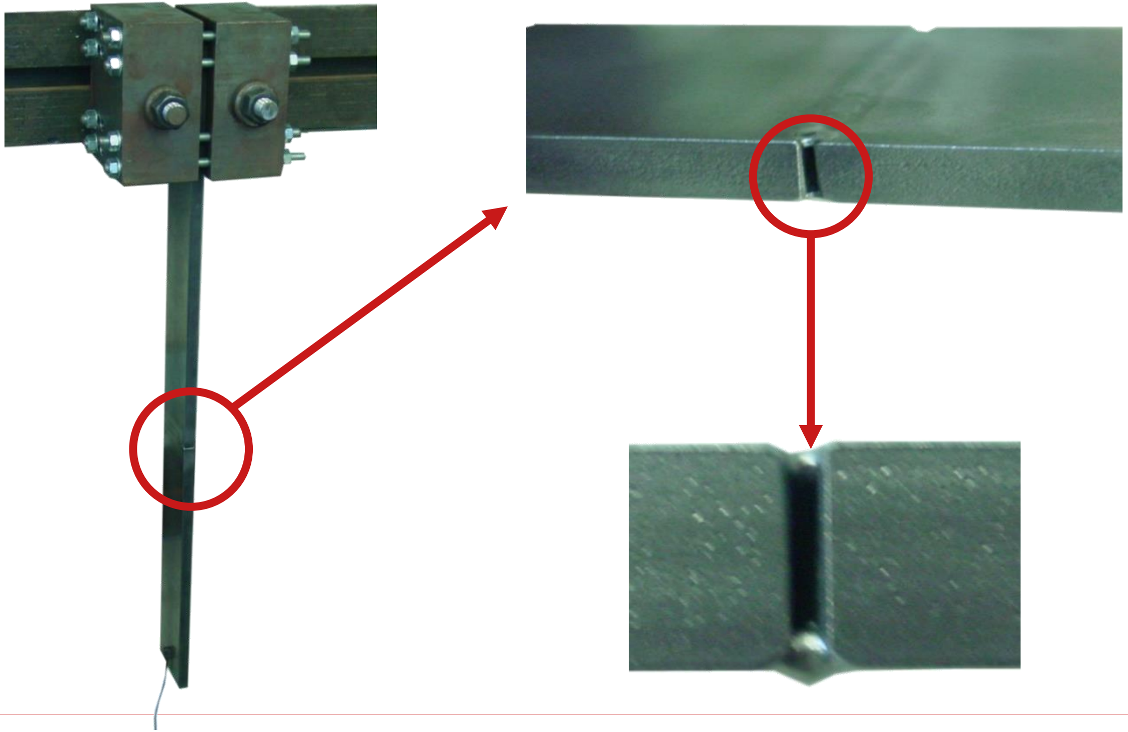


The largest elements of ΔK are related to DOFs 1 and 6.

These are also DOFs at which stiffness was modified for the ‘experimental’ eigenmodes.

Example 2: Clamped-Free beam with weld

Experimental setup



Example 2: Clamped-Free beam with weld

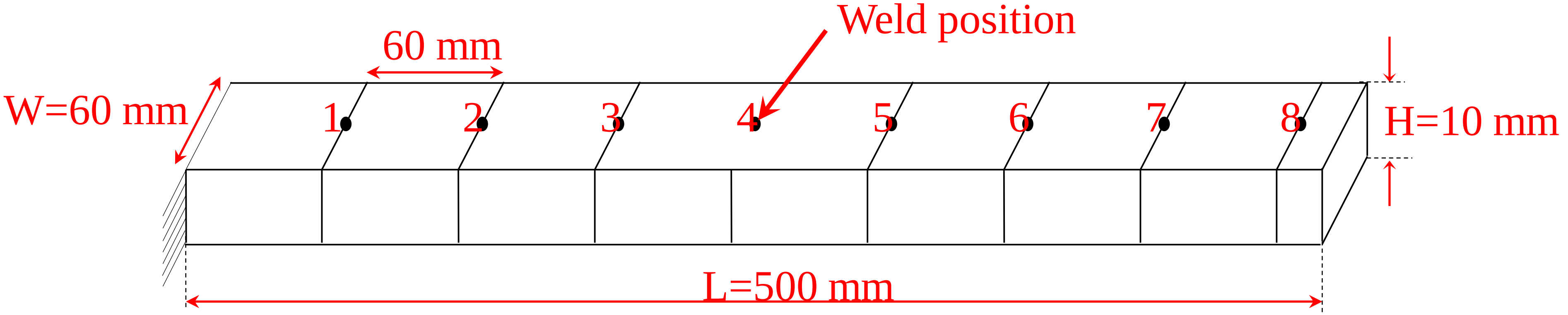
<u>Experimental Modal Analysis:</u>

Number of sensor positions: $m = 8$

Lowest 5 eigenfrequences/eigenmodes (bending) measured: $e = 5$ $(e \leq m)$

Experimental eigenmodes: U^E_{me} $(8,5)$, each mode mass-normalized $m^E_k = 1$

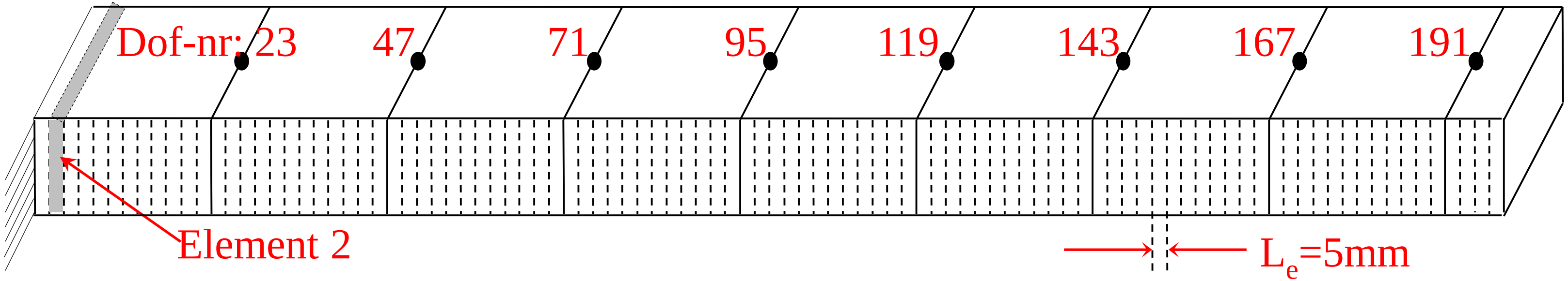
Experimental eigenfrequencies: Ω^E_{ee} $(5,5)$



Example 2: Clamped-Free beam with weld

Finite Element model

Beam is assumed to be homogeneous!
100 Euler beam elements (only bending, no shear): 200 DOFs



Original FE-model: $M^N \ddot{x}(t) + K^N x(t) = 0$

Eigenvalue problem $\left(-\omega_{O,i}^2 M^N + K^N\right) u_{O,i} = 0$ is solved for the lowest $m = 8$ eigenfrequencies

Corresponding eigenmodes: U_{nm}^N: (200,8)   Mass-normalized: $(U_{nm}^N)^T M^N U_{nm}^N = I_{mm}$.

Example 2: Clamped-Free beam with weld

Reduction of numerical mass and stiffness matrices

To compute ΔM and ΔK, the $(200{,}200)$-matrices M^N and K^N are reduced to $(8{,}8)$-matrices.

1. Approximate the state $x(t)$ in terms of the 8 lowest eigenmodes:
   $x(t) = U_{nm}^N p(t)$ $(8{,}1)$-column $p(t)$ contains generalized dof.
2. Obtained matrices based on the measured DOFs $x_m(t)$:
   $$x_m(t) = U_{mm}^N p(t) \qquad p(t) = (U_{mm}^N)^{-1} x_m(t)$$

So $x_m(t)$ contains the dof's:
23,47,71,95,119,143,167,191

Resulting transformation matrix T is $(200{,}8)$

$$x(t) = U_{nm}^N (U_{mm}^N)^{-1} x_m(t) = T x_m(t)$$

Reduced $(8{,}8)$-matrices:

$$M_{mm}^N = T^T M^N T, \qquad K_{mm}^N = T^T K^N T$$

Example 2: Clamped-Free beam with weld

Error stiffness and mass matrices

Error stiffness matrix:

$$\Delta K = K^E - K^N \approx K_{mm}^N[(K_{mm}^N)^{-1} - (K_{mm}^E)^{-1}]K_{mm}^N$$

where:

$$(K_{mm}^E)^{-1} \approx U_{me}^E(\Omega_{ee}^E)^{-2}\,(U_{me}^E)^\top$$

$$(K_{mm}^N)^{-1} \approx U_{me}^N(\Omega_{ee}^N)^{-2}\,(U_{me}^N)^\top$$

Error mass matrix:

$$\Delta M = M^E - M^N \approx M_{mm}^N[(M_{mm}^N)^{-1} - (M_{mm}^E)^{-1}]M_{mm}^N$$

where:

$$(M_{mm}^E)^{-1} \approx U_{me}^E\,(U_{me}^E)^\top$$

$$(M_{mm}^N)^{-1} \approx U_{me}^N\,(U_{me}^N)^\top$$

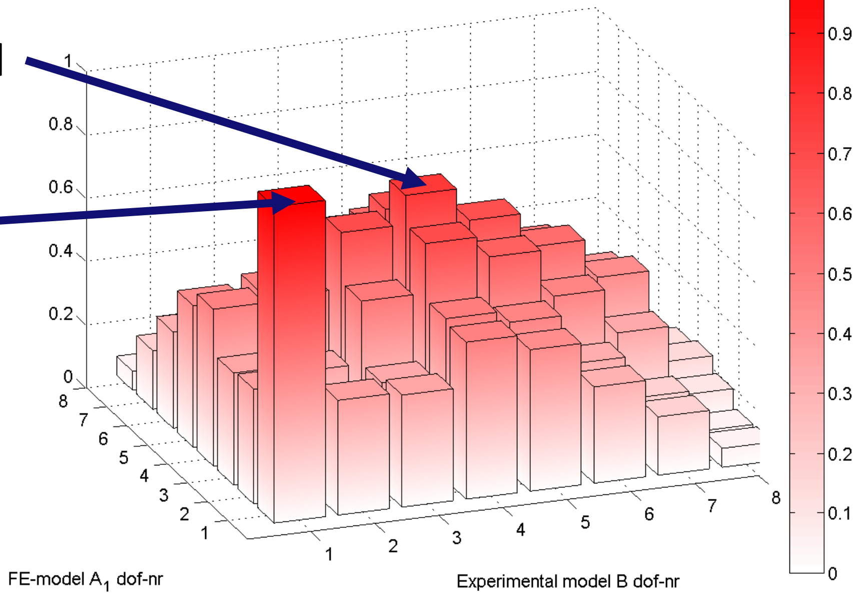
Example 2: Clamped-Free beam with weld

Error stiffness matrix

$$\Delta K = |K^E - K^N_{mm}|$$

weld

clamping

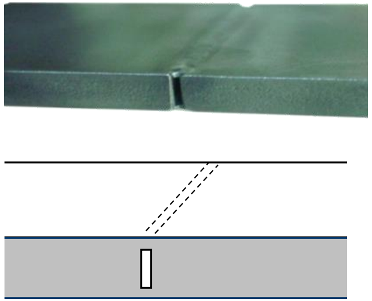


Error of 1 corresponds to $7.90\ 10^7\ N/m^2$

Example 2: Clamped-Free beam with weld

Improvement of the FE model

1. Better modelling of the weld



Use a beam element to model the weld

This element has different properties than the other elements,
e.g. lower secondary moment of area

2. Including shear
   (Timoshenko beam elements instead of Euler beam elements)

Clamping is still modelled as infinitely stiff… ☹

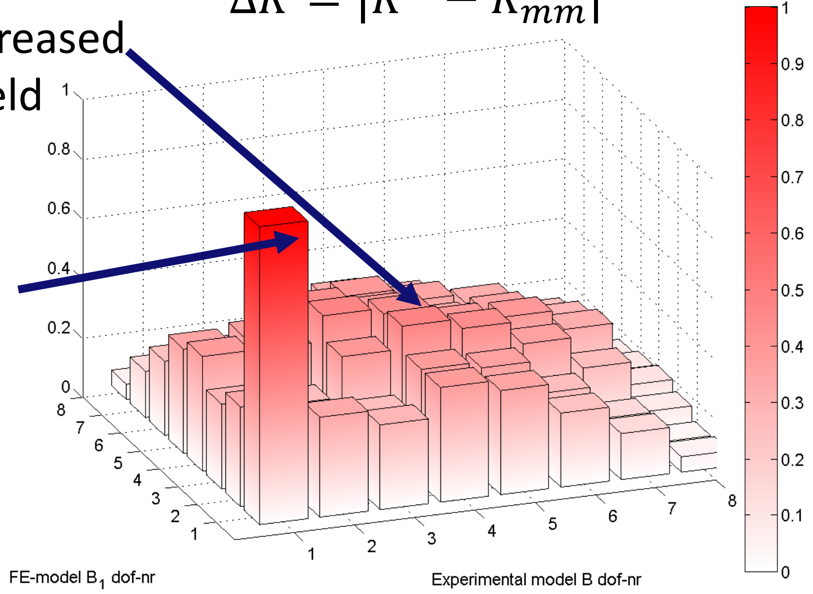
Example 2: Clamped-Free beam with weld

Error stiffness matrix

$$\Delta K = |K^E - K^N_{mm}|$$

ΔK has decreased near the weld

What about this element?



Error of 1 corresponds to $7.81\ 10^7$ N/m^2

Model updating using eigenvalue sensitivity

Given: λ_e, a column of e experimentally measured eigenvalues.

Given: a numerical model $(\lambda_k C(p) + D(p))v_k = 0$
depending on a vector of q parameters p.
Note: typically, $q < 2e$.

Problem: find/adapt the parameter values p such that $\lambda_n = \lambda_n(p)$, a vector containing e eigenvalues of the numerical model, match with λ_e.
Note: Corresponding eigenmodes should be similar.

Model updating using eigenvalue sensitivity

Iterative approach: $p_{(i)}$ denotes the parameter values at iteration i

For small parameter variations $\Delta p_{(i)}$ around $p_{(i)}$

$$\lambda_n\big(p_{(i)} + \Delta p_{(i)}\big) \approx \lambda_n\big(p_{(i)}\big) + \left.\frac{\partial \lambda_n}{\partial p}\right|_{p=p_{(i)}} \Delta p_{(i)} =: \lambda_{n(i)} + S_{(i)}\Delta p_{(i)},$$

where

$$\lambda_{n(i)} \coloneqq \lambda_n\big(p_{(i)}\big), \qquad S_{(i)} = \left.\frac{\partial \lambda_n}{\partial p}\right|_{p=p_{(i)}}.$$

Note: $S_{(i)}$ is computed using sensitivity analysis.

Introduce

$$\Delta\lambda_{(i)} \coloneqq \lambda_e - \lambda_{n(i)} - S_{(i)}\Delta p_{(i)}$$

Note: $\Delta\lambda_{(i)} \neq \lambda_e - \lambda_n(p)$, but $\Delta\lambda_{(i)} \approx \lambda_e - \lambda_n(p)$ for small $\Delta p_{(i)}$.

Model updating using eigenvalue sensitivity

The number of parameters q is typically small (i.e. $q < 2e$).
$\Rightarrow$ it is typically impossible to achieve $\lambda_n(p) = \lambda_e$.

Least squares approach: Minimize

$$\varepsilon_{(i)} = \Delta\lambda_{(i)}^H W \, \Delta\lambda_{(i)}$$

- $\varepsilon_{(i)}$ is a scalar-valued cost function
- $\Delta\lambda_{(i)} = \lambda_e - \lambda_{n(i)} - S_{(i)}\Delta p_{(i)}$
- W is a real, positive definite, symmetric (often diagonal) weighting matrix.

Optimal $\boldsymbol{\Delta p_{(i)}}$: $$\frac{\partial \varepsilon}{\partial \Delta p_{(i)}} = 0, \qquad \mathrm{Re}\left(S_{(i)}^H W S_{(i)}\right) \Delta p_{(i)} = \mathrm{Re}\left(S_{(i)}^H W\left(\lambda_e - \lambda_{n(i)}\right)\right)$$

Update: Parameter update $p_{(i+1)} = p_{(i)} + \Delta p_{(i)}$ and model update $\lambda_{n(i+1)}$ and $S_{(i+1)}$.

Stop when $\varepsilon_{(i)}$ and/or the relative changes $\Delta p_{\alpha(i)}/p_{\alpha(i)}$ $(\alpha = 1, \dots, q)$ are small enough.

FEMTOOLS

Developed by Dynamic Design Solutions (DDS), Leuven (B)

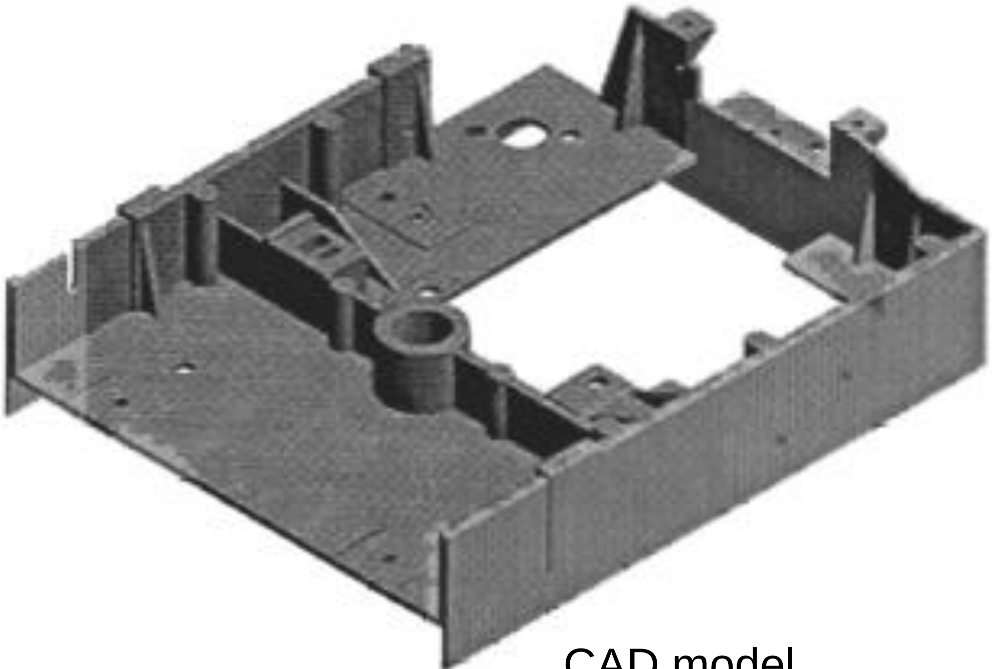
Finite Element Package → FEMTOOLS

Experimental Modal Analysis Package → FEMTOOLS

FEMTOOLS (http://www.femtools.com/) offers among others:

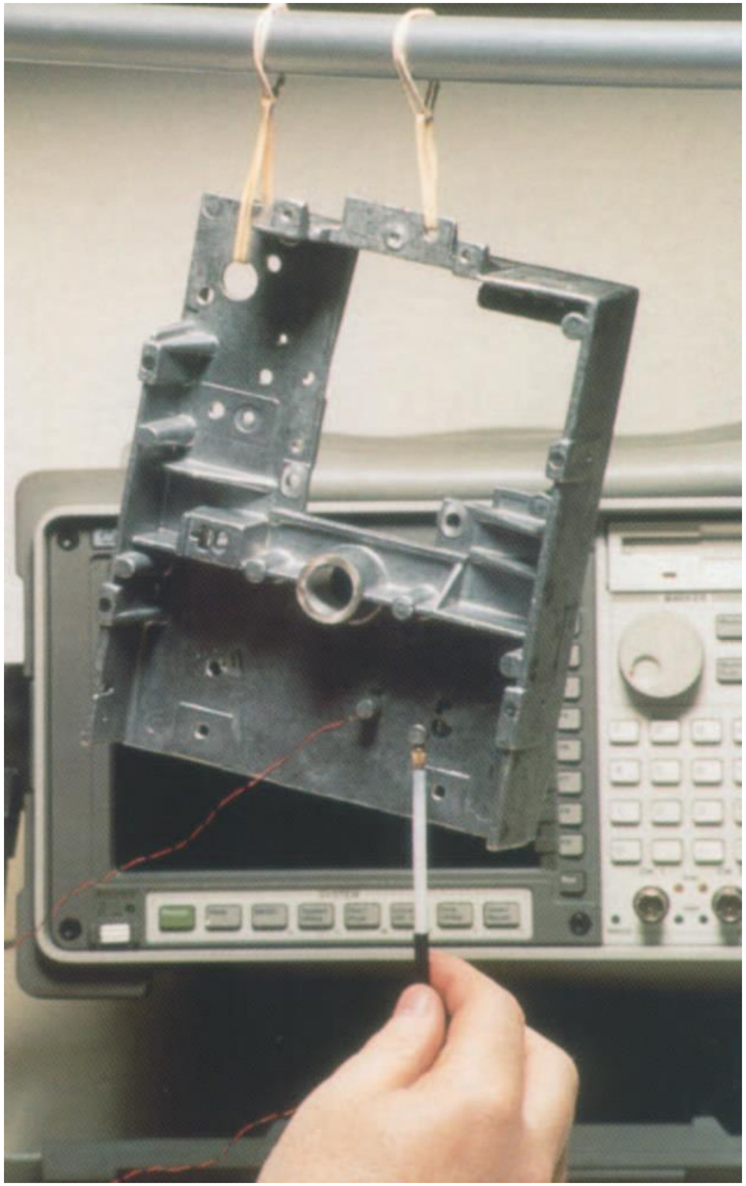
- Sensor position selection
- FE-EMA correlation (e.g. MAC)
- Finite Element Model Updating

Example Model Updating: Housing of Hard Disk



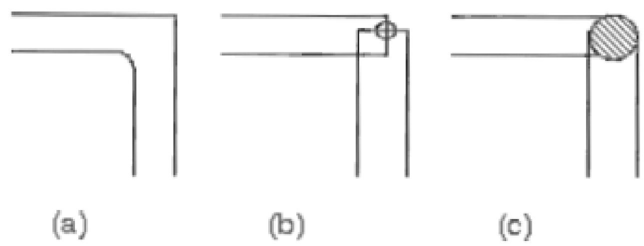
CAD model

Modal test setup

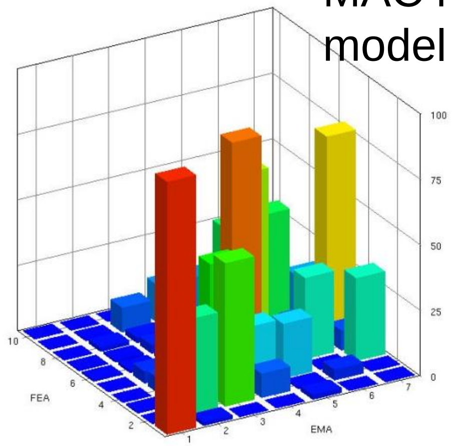


Housing of Hard Disk - FE model uncertainties/errors

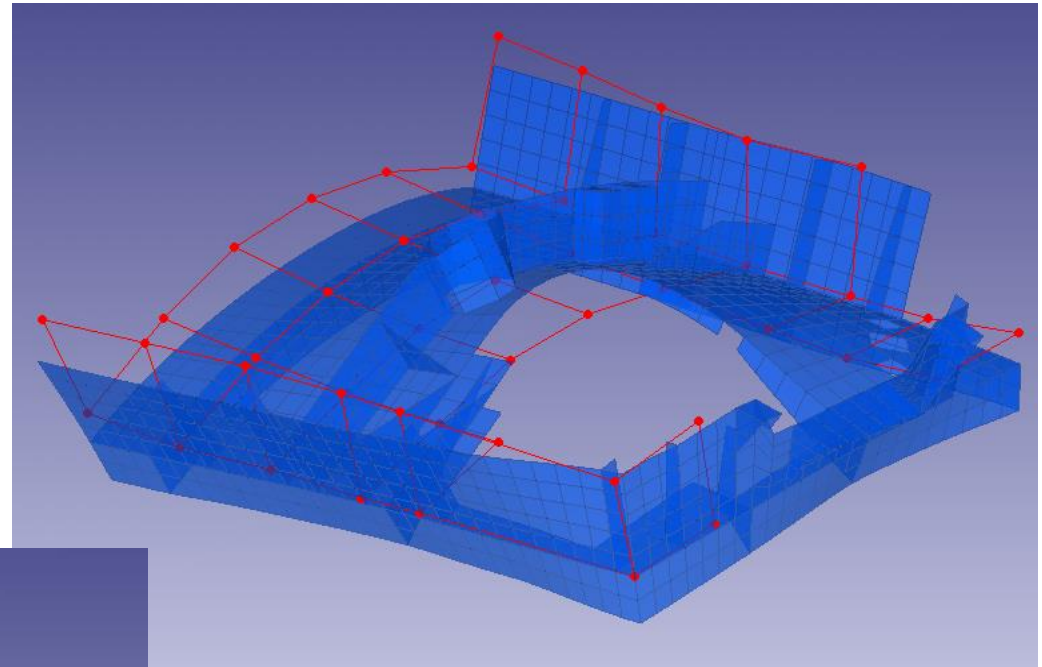
- Use of plate and beam elements (actual structure is 3D)
- Uncertainty in plate thickness
  - Coarse measurements
  - Variation of thickness
- Errors in modeling of fillets (see figure below)
- Uncertainty about mass density (injection moulding)
- Errors due to geometrical simplifications (small holes not modeled)



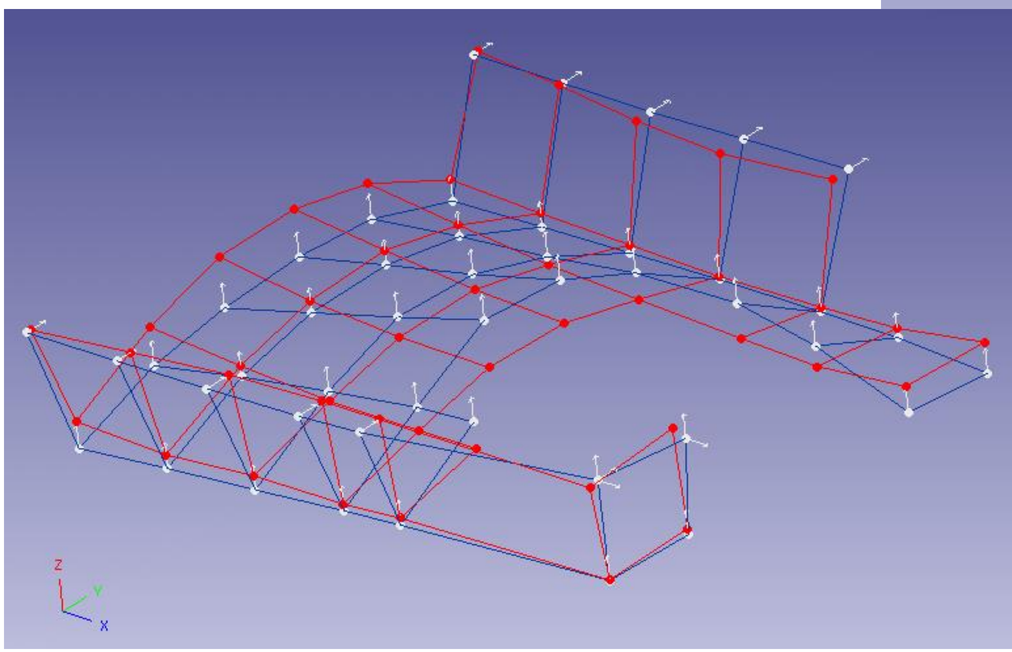
Housing of Hard Disk – Correlation of theor. and exp. modes



MAC matrix prior to model updating



Mode shapes (FEA + EMA)



Mode shapes (FEA + EMA) reduced to measured dof's

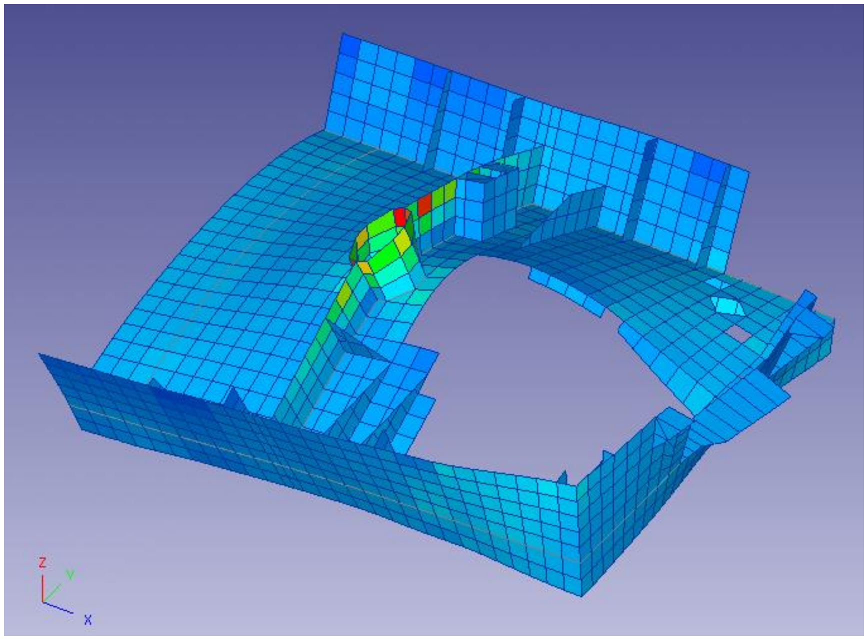
Housing of Hard Disk – Correlation prior to model updating

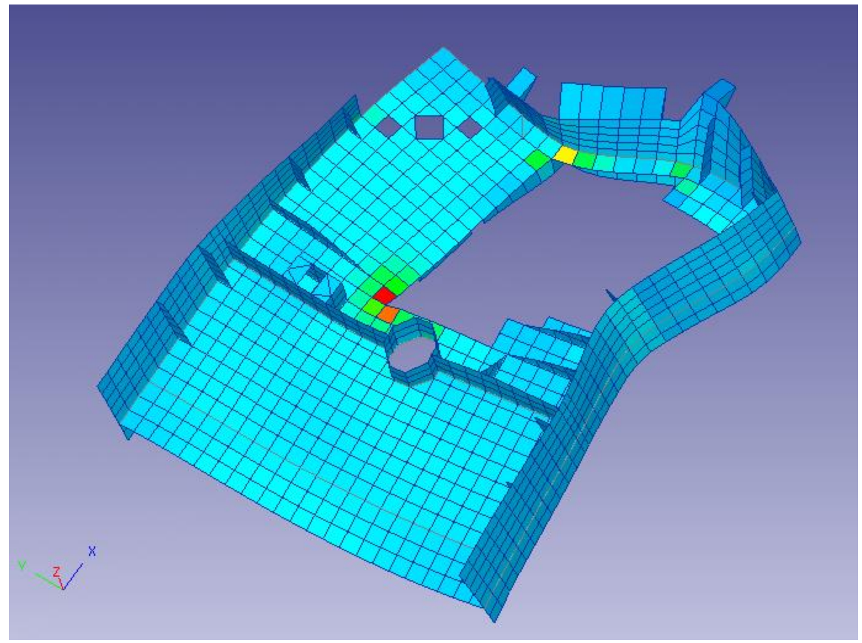
FEA-EMA correlation of eigenfrequencies and mode shapes
Start values prior to model updating

#	FEA	Hz	EMA	Hz	Diff.(%)	MAC(%)
1	1	391.13	1	372.93	4.88	96.4
2	2	1284.48	3	1569.81	-18.18	54.7
3	4	1961.58	4	1772.64	10.66	89.3
4	5	2209.86	7	2224.44	-0.66	77.5
5	6	2414.55	5	1978.92	22.01	65.6
6	7	2532.22	6	2119.41	19.48	42.7

Housing of Hard Disk – Sensitivity analysis

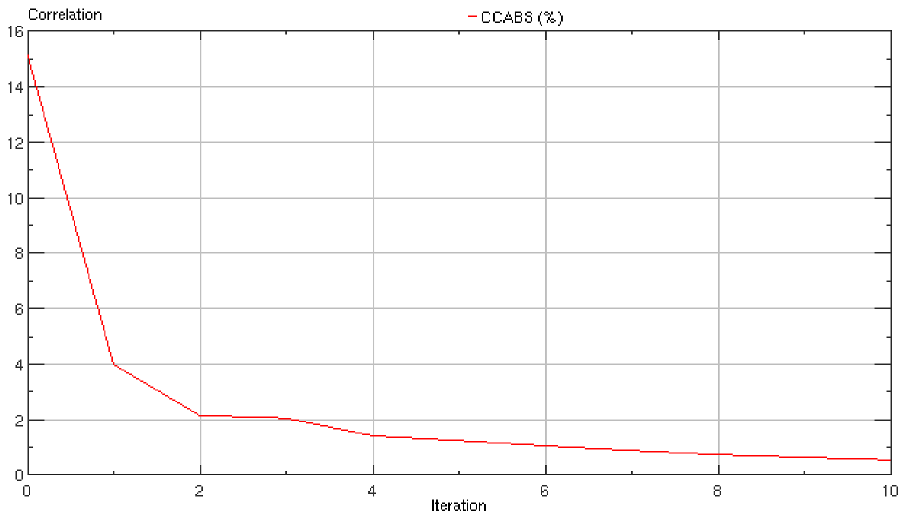
- Local plate thickness as updating parameters
- Eigenfrequencies as target quantities





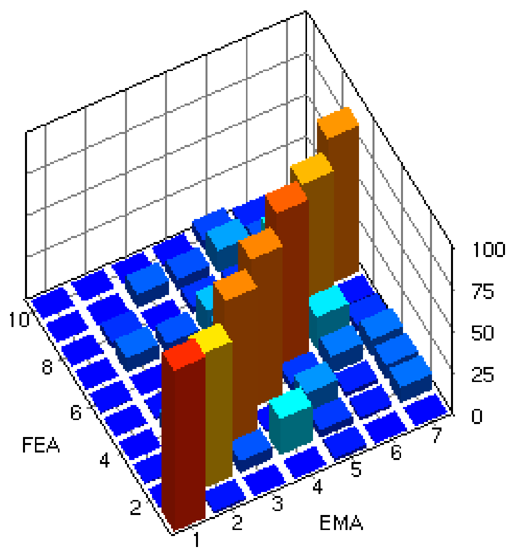
Colors indicate sensitivities of eigenfrequencies w.r.t. local plate thickness

Housing of Hard Disk – Model updating

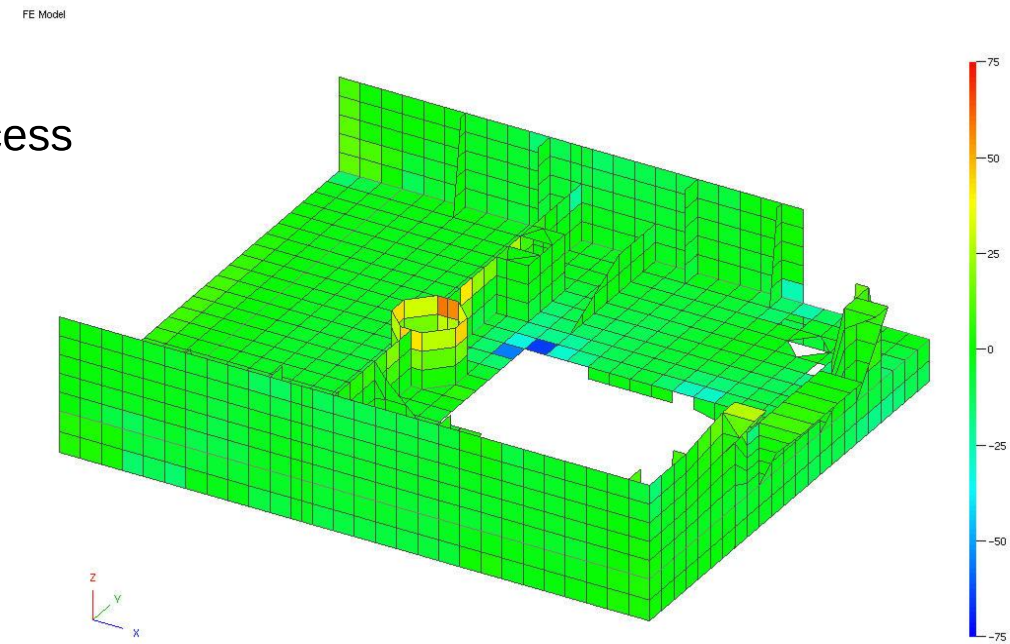


Convergence in model updating iteration process

Model updating by using measured eigenfrequencies



MAC after model updating



Required model modifications

Housing of Hard Disk – Correlation after model updating

FEA-EMA correlation of eigenfrequencies and mode shapes after model updating

#	FEA	Hz	EMA	Hz	Diff.(%)	MAC(%)
1	1	376.26	1	372.93	0.89	96.1
2	2	1379.30	2	1373.77	0.40	80.1
3	3	1566.80	3	1569.81	-0.19	89.8
4	4	1772.40	4	1772.64	-0.01	83.8
5	5	1984.82	5	1978.92	0.30	90.9
6	6	2146.99	6	2119.41	1.30	82.9
7	7	2195.90	7	2224.44	-1.28	87.7