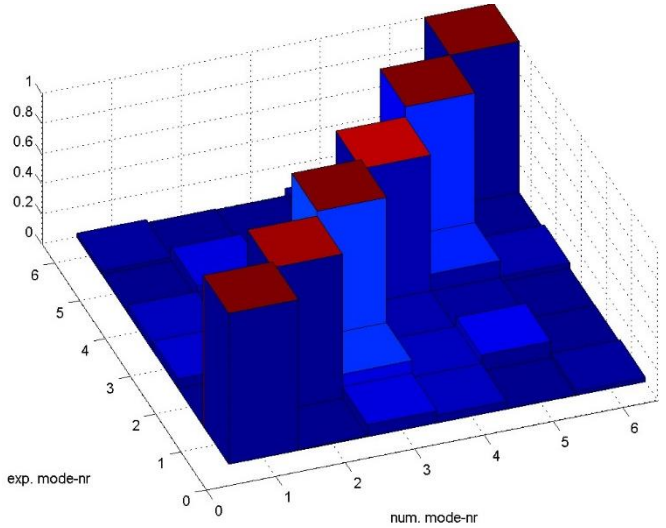

EINDHOVEN UNIVERSITY OF TECHNOLOGY

12. Comparing numerical and experimental data

Structural Dynamics part of 4DM00

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Introduction

If FE-model is used to predict and/or improve the dynamic behaviour,
it should reflect the real (measured) dynamic behaviour!

Is it necessary to update a model based on experimental data?

First **compare** experimental results from prototype structures
with numerical results e.g. from Finite Element Analyses

Comparison of:

1) eigenvalues,
2) eigenmodes,
3) FRF's.

In this course: focus on comparison of experimental and numerical eigenmodes

General problems when comparing:

1) modelling of damping is difficult,
2) differences in number of dof.

Comparing experimental and numerical eigenvalues

Problems:

- FEM calculations usually result in undamped eigenfrequencies ω_{Oi}^{n} (sorting: $\omega_{O,i}^{n} \leq \omega_{O,i+1}^{n}$), i.e. imaginary eigenvalues $\lambda_i^n = j\omega_{Oi}^{n}$
  Experiments usually lead to measured, complex eigenvalues $\lambda_i^e = \mu_i^e + j\nu_i^e$ (sorting: $|\nu_i^e| \leq |\nu_{i+1}^e|$ or $|\lambda_i^e| \leq |\lambda_{i+1}^e|$)
- Number of numerical eigenvalues may differ from number of experimental eigenvalues
- Complex situations occur when comparing
  clusters of experimental eigenvalues to clusters of numerical eigenvalues.
  Mode shapes are needed to pair the correct numerical and experimental eigenvalues!

Summarizing:
Just numbering eigenvalues in increasing order may lead to wrong conclusions

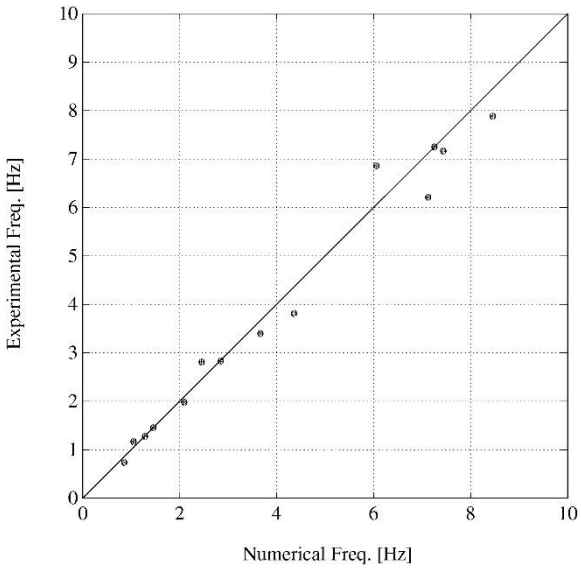
Comparing experimental and numerical eigenvalues

Visualization: Plot points (ω_{Oi}^n, ν_i^e) or $(\omega_{Oi}^n, |\lambda_i^e| = |\mu_i^e + j\nu_i^e|)$.

Ideally: all points on straight line $y = x$

Typical deviations with specific reasons:

- All points approx. on $y = \alpha x$, but $\alpha \neq 1$
  Cause: E-modulus or density in numerical model wrong?
- First points on $y = x$, then sudden deviation to one side
  Cause: some num. or exp. eigenvalues may be missing
- Randomly distributed points in high frequency range
  Cause: higher eigenfrequencies are less accurate
  Experimental eigenvalues: measurement or fit errors (EMA)
  Numerical eigenvalues: modelling errors, discretisation errors



Comparing experimental and numerical eigenmodes

u_i^e: experimental mode i

u_j^n: numerical mode j

Tools for scaling:

- MSF: Modal Scale Factor

Tools for comparing:

- Visual inspection (plot real and imaginary parts)
- MAC: Modal Assurance Criterion
- COMAC: Coordinate Modal Assurance Criterion

General problem in comparing eigenmodes

The number of DOFs in u_i^e (experimental mode i)
is usually much lower than the number of DOFs in u_j^n (numerical mode j)

Solutions:

1. Reduce u_j^n by deleting non-measured DOFs
2. Expand u_{im}^e by solving non-measured u_{io}^e from second row of

$$\left\{-\omega_{mi}^2 \begin{bmatrix} M_{mm} & M_{mo} \\ M_{om} & M_{oo} \end{bmatrix} + \begin{bmatrix} K_{mm} & K_{mo} \\ K_{om} & K_{oo} \end{bmatrix}\right\} \begin{bmatrix} u_{im}^e \\ u_{io}^e \end{bmatrix} = \begin{bmatrix} 0_m \\ 0_o \end{bmatrix}$$

$$u_{io}^e = \left(-\omega_{mi}^2 M_{oo} + K_{oo}\right)^{-1} \left(-\omega_{mi}^2 M_{om} + K_{om}\right) u_{im}^e.$$

Modal Scale Factor (MSF)

Assume: u_i^e and u_j^n have same number of DOFs.

To account for the difference in normalization, multiply u_i^e by

$$\mathrm{MSF}_e[i,j] = \frac{u_i^{eH} u_j^n}{u_i^{eH} u_i^e}.$$

Alternatively, multiply u_j^n by

$$\mathrm{MSF}_n[i,j] = \frac{u_i^{nH} u_j^e}{u_i^{nH} u_i^n}.$$

Note: $x^H = \bar{x}^\top$ denotes the Hermitian transpose.

Example 1: $u_1^e = [1, 1+j]^\top, u_1^n = [j, j-1]^\top, \;\; MSF_e[1,1] = (MSF_n[1,1])^{-1} = j.$

Example 2: $u_2^e = [1, 2-j]^\top, u_2^n = [2, 4-2j]^\top, MSF_e[2,2] = (MSF_n[2,2])^{-1} = 2$

Visual inspection

Generally viscously damped system, complex eigenmodes!

Right eigenvalue problem: $(\lambda_k C + D)v_k = 0$ $\qquad v_k = \begin{bmatrix} u_k \\ \lambda_k u_k \end{bmatrix}$

Process the numerical eigenmode as follows:

1. Only consider the 'displacement' part u_k
2. Delete non-measured DOFs in u_k
3. Search for (complex) entry in u_k with highest modulus
4. Divide u_k by this (complex) entry
5. Plot real and imaginary part of u_k

Rescale the experimental mode by **MSF$_e$** and plot the corresponding DOFs.

Modal Assurance Criterion (MAC)

$$\mathrm{MAC}[i,j] = \frac{\left|u_i^{eH} u_j^n\right|^2}{\left(u_i^{eH} u_i^e\right)\left(u_j^{nH} u_j^n\right)}$$

- $0 \leq \mathrm{MAC}[i,j] \leq 1$
- $\mathrm{MAC}[i,j]$ gives the degree of correlation between u_i^e and u_j^n
- $MAC[i,j] = 1$ if and only if u_i^e is a multiple of u_j^n
- Normalisation/scaling does not influence the MAC number!
- u_i^e and u_j^n should have the same number of DOFs

Modal Assurance Criterion (MAC)

$$\mathrm{MAC}[i,j] = \frac{\left|u_i^{eH} u_j^{n}\right|^2}{\left(u_i^{eH} u_i^{e}\right)\left(u_j^{nH} u_j^{n}\right)}$$

In general we have n experimental modes and m numerical modes: MAC matrix is $n \times m$.

If $n = m$, the MAC-matrix indicates a good correspondence between the experimental and numerical eigenmodes if

- $\mathrm{MAC}[i,i] \approx 1$ for $i = 1, \ldots, n$
- $\mathrm{MAC}[i,j]$ is small (say < 0.1) for $i \neq j$.
  However, even when experimental and numerical eigenmodes are equal, $\mathrm{MAC}[i,j]$ for $i \neq j$ will generally not be zero.

Literature

Allemang, R.J., The Modal Assurance Criterion, Twenty years of use and abuse, Sound and Vibration, pp. 14-21, August 2003.

Coordinate Modal Assurance Criterion (COMAC)

Warning:

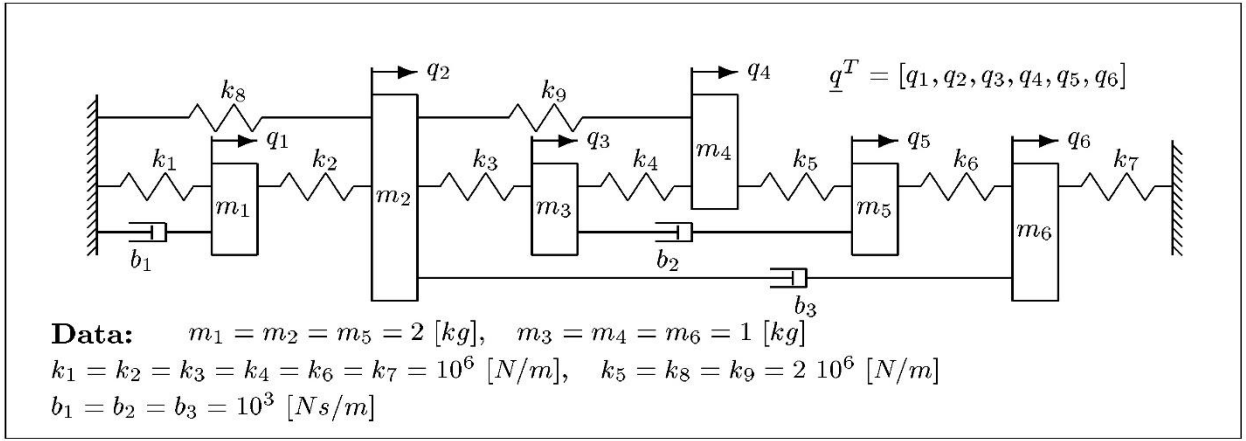
- u_i^e now denotes the i-th **row** of the matrix with experimental eigenmodes.
- u_j^n now denotes the i-th **row** of the matrix with numerical eigenmodes.

u_i^e and u_j^n thus contains the mode shape values of all modes at DOF i

$$\mathrm{COMAC}[i,j] = \frac{\left|u_i^e u_j^{nH}\right|^2}{\left(u_i^e u_i^{eH}\right)\left(u_j^n u_j^{nH}\right)}$$

- $0 \leq \mathrm{COMAC}[i,j] \leq 1$
- COMAC can be used for identifying a badly working sensor

Example: 6 dof model



General viscous damping, underdamped.

- Numerical modes u_j^n derived according to above model
- 'Experimental' modes derived according to above model,
  but with $k_1 = 1.2 \cdot 10^6$ N/m, $b_1 = 1200$ Ns/m, $b_2 = 800$ Ns/m, $b_3 = 1100$ Ns/m.

Example: 6 dof model

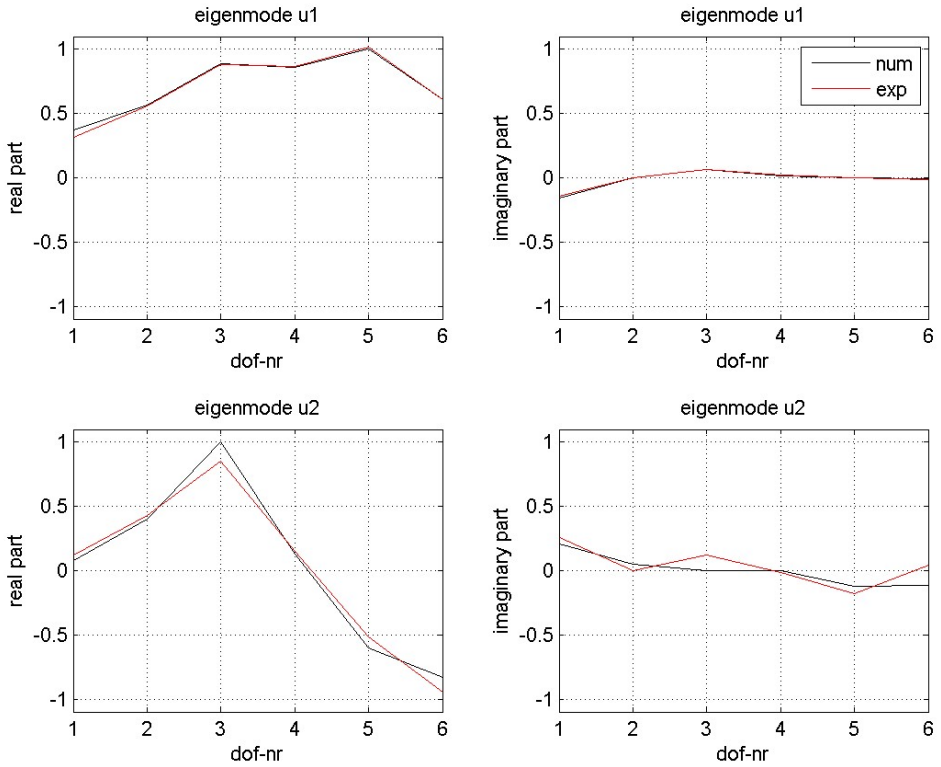
Eigenvalue number	Experimental eigenvalues (*1000)	Numerical eigenvalues (*1000)
1	-0.0169 + 0.5984i	-0.0188 + 0.5931i
2	-0.6724 + 0.9086i	-0.7203 + 0.8570i
3	-0.3049 + 0.9927i	-0.2520 + 0.9529i
4	-0.6779 + 1.4367i	-0.7087 + 1.4477i
5	-0.0280 + 1.7467i	-0.0255 + 1.7460i
6	-0.0248 + 2.4619i	-0.0247 + 2.4643i

Experimental and numerical eigenvalues match quite well.

Example: 6 dof model

- u_1^e and u_1^n match well

- u_2^e and u_2^n match well



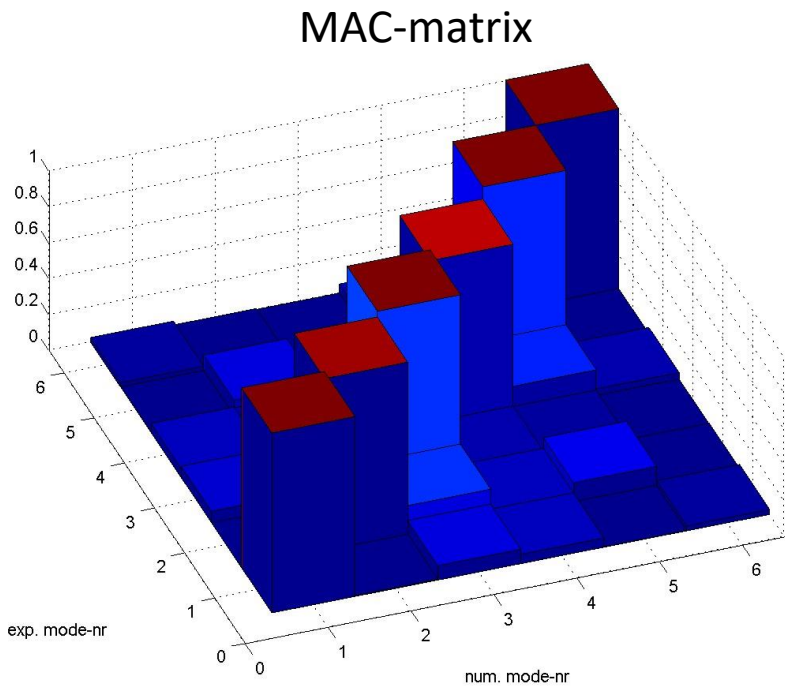
Example: 6 dof model

MAC	num 1	num 2	num 3	num 4	num 5	num 6
exp 1	0.9988	0.0047	0.0801	0.0521	0.0023	0.0271
exp 2	0.0003	0.9594	0.1631	0.0488	0.1074	0.0056
exp 3	0.0632	0.1847	0.9917	0.0332	0.0305	0.0001
exp 4	0.0502	0.0453	0.0168	0.9313	0.1428	0.0365
exp 5	0.0026	0.0882	0.0245	0.1069	0.9991	0.0021
exp 6	0.0264	0.0051	0.0002	0.0506	0.0021	0.9999

Inspection of the MAC matrix:

- Diagonal elements of MAC matrix is showing values close to 1:
  Good match between experimental mode i and numerical mode i
- Off-diagonal terms almost all have values < 0.1

Example: 6 dof model



Example: COMAC

COMAC:
all sensors are working perfectly

COMAC	n DOF 1	n DOF 2	n DOF 3	n DOF 4	n DOF 5	n DOF 6
e DOF 1	0.9453	0.2658	0.0851	0.1935	0.1934	0.1752
e DOF 2	0.2725	0.2850	0.8499	0.6288	0.5495	0.1115
e DOF 3	0.1185	0.8693	0.6711	0.7907	0.4880	0.1716
e DOF 4	0.2534	0.6485	0.7558	0.7201	0.8031	0.4241
e DOF 5	0.2824	0.5776	0.4801	0.8222	0.8465	0.7931
e DOF 6	0.2869	0.1133	0.1441	0.4141	0.7630	0.2975

COMAC:
sensor number 3 is giving low random numbers

COMAC	n dof 1	n dof 2	n dof 3	n dof 4	n dof 5	N dof 6
e dof 1	0.9453	0.2658	0.0851	0.1935	0.1934	0.1752
e dof 2	0.2725	0.2850	0.8499	0.6288	0.5495	0.1115
e dof 3	0.1065	0.0238	0.1638	0.0469	0.0140	0.1409
e dof 4	0.2534	0.6485	0.7558	0.7201	0.8031	0.4241
e dof 5	0.2824	0.5776	0.4801	0.8222	0.8465	0.7931
e dof 6	0.2869	0.1133	0.1441	0.4141	0.7630	0.2975