

Sustainability Aware Asset Management

Project: Asset Allocation with a Carbon Objective

Due Friday May 27, 2025, at midnight

The goal of this project is to implement some climate aware asset management concepts seen in class. The first part of the project consists in building a portfolio based on the mean-variance criterion. The second part focuses on the impact of the portfolio on climate. You are given a certain freedom within the framework, provided you explain the methodology that seems the most relevant to you.

Please submit a PDF-report with the details of your methodology and an analysis of the results, and a folder containing your code files such that I can replicate your results by running the code. You are graded based on your report and, to a lesser extent, on your code.

You must use the strategy assigned to your group, i.e., the region and the climate strategy. Projects provided after the deadline will not be graded.

Data

In the "Static.xlsx" file, you are given the following information for 2565 firms with MSCI with carbon data:

- ISIN code
- Name of the company
- Country and region

Then you are given additional files containing time series on carbon data and market values:

- CO₂ emissions (annual) for Scope 1 and Scope 2 (in tonnes CO₂) ($E_{i,Y}$) (Scope_1.xlsx and Scope_2.xlsx)
- Revenues (annual) (in million USD) ($Rev_{i,Y}$) (DS_REV_USD_Y.xlsx)
- Market capitalization (end of year) (in million USD) ($Cap_{i,Y}$) (DS_MV_T_USD_Y.xlsx)
- Market capitalization (end of month) (in million USD) ($Cap_{i,t}$) (DS_MV_T_USD_M.xlsx)
- Total return index (price index, including dividend payments) (end of year) (in USD) ($P_{i,Y}$) (DS_RI_T_USD_Y.xlsx)
- Total return index (price index, including dividend payments) (end of month) (in USD) ($P_{i,t}$) (DS_RI_T_USD_M.xlsx)

Additional important information:

- The data is collected from end 1999 to end 2024 for market values and from end 2013 to end 2023 (last year available) for carbon data.
- Price data are collected at the monthly frequency. Prices should be converted into returns using the simple return definition. Returns at date t are collected in the vector $R_t = \{R_{1,t}, \dots, R_{N,t}\}$, where N denotes the number of firms. The 10 years of data from 2004 to 2013 are used to initialize the calculation of expected returns and the covariance matrix. See point 1.1 below.
- Revenues and CO₂ emissions are updated annually. You should compute the carbon intensity by dividing CO₂ emissions by the Revenues of the corresponding year. Some data might be missing. When the missing value is between 2 available years or at the end of the sample, just use the number from the previous year. When the missing value is at the beginning of the sample, you cannot invest in this firm until numbers are made available. The allocation is based on information available at the end of the year (carbon and return data) for the next year. The portfolio is rebalanced only once per year. However, the performance of the portfolio is calculated month by month.
- It is important to distinguish the frequency of returns (monthly) and the frequency of rebalancing (annual). The investment decision is made at the end of a given year for the next year. However, expected returns and the covariance must be calculated based on monthly returns. So, to avoid confusion, I use the index t when the variable is monthly and the index Y when the variable is annual. The reason for computing monthly return is that this allows us to compute some important characteristics of the portfolio, such as its volatility or Sharpe ratio.

Part I - Standard Asset Allocation (Discussion: April 8)

1 Standard Asset Allocation

We start with the construction of a minimum variance portfolio. The region defining the investment set is given in the strategy assigned to the group.

- 1.1 We compute the minimum variance portfolio out of sample. "Out of sample" means that we use only past data to compute the optimal portfolio for the next period. For instance, we use the 10 years of monthly returns (from Jan. 2004 to Dec. 2013) to compute the vector of expected returns and the covariance matrix. Therefore, we have $\tau = 10 \times 12 = 120$ months of data available to compute the expected returns and the covariance matrix valid for the allocation at the end of Dec. 2013 for the allocation for Dec. 2014. Dec. 2013 corresponds to year $Y_0 = 2013$ for annual data and to $t_0 = \tau = 120$ for monthly data. Dec. 2014 corresponds to $Y_0 + 1 = 2014$ and $t_0 + 12 = 132$. We associate year Y to month t (Dec. of the year).

We compute the expected returns as the average over the 10 years before year $Y + 1$:

$$\hat{\mu}_{Y+1} = \frac{1}{\tau} \sum_{k=0}^{\tau-1} R_{t-k}$$

The covariance matrix is

$$\Sigma_{Y+1} = \frac{1}{\tau} \sum_{k=0}^{\tau-1} (R_{t-k} - \hat{\mu}_{Y+1})'(R_{t-k} - \hat{\mu}_{Y+1})$$

The allocation is determined at the end of year Y for year $Y + 1$. We compute the optimal allocation of the portfolio using Markowitz's approach. In general, the covariance matrix Σ_{Y+1} is not invertible, so using the optimal weight formula will not work. Instead, we use the following maximization problem, while restricting the optimal weights to be positive:

$$\begin{aligned} \min_{\{\alpha_Y\}} \quad & \sigma_{p,Y+1}^2 = \alpha_Y' \Sigma_{Y+1} \alpha_Y \\ \text{s.t.} \quad & \alpha_Y' e = 1 \\ \text{s.t.} \quad & \alpha_{i,Y} \geq 0 \quad \text{for all } i \end{aligned}$$

Then, we roll the window by one year and iterate until the end of the sample, so that our portfolio is rebalanced every year from Dec. 2013 to Dec. 2023.

We compute the ex-post performance of the portfolio. The portfolio return can be computed every month of year $Y + 1$ using monthly stock returns of year $Y + 1$ and optimal weights calculated at the end of year Y . This gives: $R_{p,t+k} = \alpha_{t+k-1}' R_{t+k}$, for $k = 1, \dots, 12$, where $\alpha_{i,t+k-1} = \alpha_{i,t+k-2} \times (1 + R_{i,t+k-1}) / (1 + R_{p,t+k-1})$, with $\alpha_t = \alpha_Y$. We now have a time series of ex post portfolio returns: $\{R_{p,\tau+1}, \dots, R_{p,T}\}$, where $T = 240$ denotes the total number of months in the sample (from 2004 to 2023).

Compute the characteristics of this portfolio (denoted " $P_{oos}^{(mv)}$ ") over the sample: annualized average return ($\bar{\mu}_p$), annualized volatility (σ_p), Sharpe ratio (SR_p), minimum, and maximum.

Remark: The reason for considering a long-only portfolio is to help interpreting the carbon footprint of the portfolio.

- 1.2 Compare these properties to those of the value-weighted portfolio (denoted " $P^{(vw)}$ ") ("benchmark"). The performance of the value-weighted portfolio is

$$R_{t+1}^{(vw)} = \sum_{i=1}^N w_{i,t} R_{i,t+1}$$

where $w_{i,t} = \text{Cap}_{i,t} / \sum_{j=1}^N \text{Cap}_{j,t}$ denotes the relative market capitalization of firm i at the end of month t .

In particular, plot the cumulative return series of both strategies and compare their summary statistics.

Part II - Asset Allocation with a Carbon Emissions Reduction (Deadline: May 27)

2 Allocation with a 50% Reduction in Carbon Emissions

We now add a layer in the portfolio construction by taking into account the CO₂ emissions associated to the portfolio. The scope of the CO₂ emissions is given in the strategy assigned to the group.

- 2.1 We consider the carbon intensity ($CI_{i,Y}$) of all firms in our investment set. It is computed in tons of CO₂ equivalent per million U.S. dollars of revenue. We compute the weighted-average carbon intensity and the carbon footprint of the portfolio as the amount of annual carbon emissions that can be allocated to the investor per million U.S. dollars invested:

$$WACI_Y^{(p)} = \sum_{i=1}^N \alpha_{i,Y} CI_{i,Y} \quad CF_Y^{(p)} = \frac{1}{V_Y} \sum_{i=1}^N o_{i,Y} E_{i,Y}$$

where $o_{i,Y} = V_{i,Y}/Cap_{i,Y}$ measures the fraction of the equity of the firm owned by the portfolio, with $V_{i,Y} = \alpha_{i,Y} V_Y$ the dollar value invested in firm i and $V_Y = \sum_{i=1}^N V_{i,Y}$ the dollar value of the portfolio. Compute the carbon footprint of $P_{oos}^{(+)}$ for every year in the sample, assuming a starting wealth equal to $V_{2013} = 1$ million U.S. dollars.

- 2.2 We now construct an optimal long-only portfolio with a carbon footprint 50% below the carbon footprint of the optimal long-only portfolio $P_{oos}^{(mv)}$ determined in point 1.2 (*minimum variance active investor*). For every year in the sample, compute the optimal weights of the long-only portfolio with a carbon footprint lower or equal to $0.5 \times$ the carbon footprint of $P_{oos}^{(mv)}$. This can be done by solving the problem:

$$\begin{aligned} \min_{\{\alpha_Y\}} \quad & \sigma_{p,Y}^2 = \alpha_Y' \Sigma_{Y+1} \alpha_Y \\ \text{s.t.} \quad & CF_Y^{(p)} \leq 0.5 \times CF_Y^{(P_{oos}^{(mv)})} \\ \text{s.t.} \quad & \alpha_{i,Y} \geq 0 \quad \text{for all } i \end{aligned}$$

We call this portfolio “ $P_{oos}^{(mv)}(0.5)$ ”. Compute the characteristics of this out-of-sample portfolio over the sample. As before, plot the cumulative return series of both strategies and compare summary statistics.

- 2.3 Another interesting decarbonization strategy consists in designing the portfolio that is as close as possible to the benchmark, while reducing the carbon footprint by 25% (*otherwise passive investor*). This is done by solving the minimum variance criterion for the tracking error every year:

$$\begin{aligned} \min_{\{\alpha_Y\}} \quad & (TE_{p,Y})^2 = (\alpha_Y - \alpha_Y^{(vw)})' \Sigma_{Y+1} (\alpha_Y - \alpha_Y^{(vw)}) \\ \text{s.t.} \quad & CF_Y^{(p)} \leq 0.5 \times CF_Y^{(P^{(vw)})} \\ \text{s.t.} \quad & \alpha_{i,Y} \geq 0 \quad \text{for all } i \end{aligned}$$

where $CF_Y^{(P^{(vw)})} = \frac{1}{Cap_Y} \sum_{i=1}^N E_{i,Y}$ denotes the carbon footprint of the value-weighted portfolio, with $Cap_Y = \sum_{i=1}^N Cap_{i,Y}$ the total market value of the investment set.

We call this portfolio “ $P_{oos}^{(vw)}(0.5)$ ”. Compute the characteristics of the portfolio over the sample. Again, plot the cumulative return series of both strategies and compare summary statistics.

- 2.4 Comment on the trade-off between the financial performance of the portfolio and the reduction in its carbon footprint. Elaborate on the difference between portfolios “ $P_{oos}^{(mv)}$ ” and “ $P_{oos}^{(mv)}(0.5)$ ” and between portfolios “ $P_{oos}^{(vw)}$ ” and “ $P_{oos}^{(vw)}(0.5)$ ”.

3 Allocation with a Net Zero Objective

Finally, we want to construct a minimum variance portfolio, while cumulatively reducing its carbon emissions.

- 3.1 Finally, we implement a decarbonization strategy in which the carbon footprint of the portfolio is reduced by $\theta = 10\%$ per year every year from Dec. 2013 to Dec. 2023.

We adopt the point of view of the otherwise passive investor. The optimization problem is the same as in point 2.3 except that the carbon emissions reduction constraint is now defined as

$$CF_Y^{(p)} \leq (1 - \theta)^{Y - Y_0 + 1} \times CF_{Y_0}^{(P^{(vw)})} \quad \text{for } Y = 2013, \dots, 2023$$

with $Y_0 = 2013$.

We call this portfolio “ $P_{oos}^{(vw)}(NZ)$ ”. Compute the characteristics of this portfolio over the sample. Again, plot the cumulative return series of both strategies and compare summary statistics.

- 3.2 Compare the cumulative performance of the three portfolios “ $P_{oos}^{(vw)}$ ”, “ $P_{oos}^{(vw)}(0.5)$ ”, and “ $P_{oos}^{(vw)}(NZ)$ ”. Comment on the possible cost of constructing a net zero portfolio.