

## Fast egg to chicken transformation - ICPC Library

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## 1 Data Structures

## 1.1 Disjoint Set Union

```
#include <bits/stdc++.h>
using namespace std;

const int maxn = 2e5+5;

namespace DSU {
```

```
int n, link[maxn], sz[maxn];
inline void init(int _n) {
    n = _n;
    for (int i = 0; i <= n; ++i) {
        link[i] = i;
        sz[i] = 1;
    }
}
int id(int x) { return link[x] == x ? x : id(link[x]); }
int same(int x, int y) { return (id(x) == id(y)); }
void unite(int x, int y) {
    x = id(x); y = id(y);
    if (x == y) return;
    if (sz[x] < sz[y]) swap(x,y);
    link[y] = x;
    sz[x] += sz[y];
}
int size(int x) { return sz[id(x)]; }
};
```

## 1.2 Fenwick Tree

```
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5+5;

namespace FenwickTree {

    int tree[maxn], a[maxn];
    int n;

    void build(int _n) {
        n = _n;
        for (int i = 0; i < maxn; ++i) {
            tree[i] = 0;
        }
    }

    void updateI(int idx, int x) { //increment a[idx] by x
        a[idx] = x;
        for (; idx <= n; idx += (idx & -idx)) {
            tree[idx] += x;
        }
    }

    void updateS(int idx, int x) { //set a[idx] = x
        int delta = x - a[idx];
        a[idx] = x;
        for (; idx <= n; idx += (idx & -idx)) {
            tree[idx] += delta;
        }
    }

    int query(int idx) { //return sum of a[1..idx]
        int rs = 0;
        for (; idx > 0; idx -= (idx & -idx)) {
            rs += tree[idx];
        }
        return rs;
    }
}
```

```

}

int query(int l, int r) {          //return sum of a[l..r]
    return query(r) - query(l - 1);
}

};

```

### 1.3 Merge Sort Tree

```

#include <bits/stdc++.h>
using namespace std;

#define all(x) (x).begin(), (x).end()

const int maxn = 1e5+5;

struct SegmentTree {

    struct Node {
        vector<int> v;
    };

    Node join(const Node &a, const Node &b) {
        Node ans;
        merge(all(a.v), all(b.v), back_inserter(ans.v));
        return ans;
    }

    Node tree[4*maxn];
    Node neutral;

    void build(int node, int l, int r, vector<int> &v) {
        if (l == r) {
            tree[node].v.push_back(v[l]);
            return;
        }
        int mid = (l+r)/2;
        int lc = (node << 1);
        build(lc, l, mid, v);
        build(lc+1, mid+1, r, v);
        tree[node] = join(tree[lc], tree[lc+1]);
    }

    int query(int node, int l, int r, int ql, int qr, int x) {
        if (r < l or qr < l or r < ql) return 0;
        if (ql <= l and r <= qr) return lower_bound(all(tree[node].v), x)
            - tree[node].v.begin(); //Count the numbers less than x in
            range [ql, qr]
        //if (ql <= l and r <= qr) return upper_bound(all(tree[node].v), x)
            - tree[node].v.begin(); //Count the numbers less or equal x
            in range [ql, qr]
        int mid = (l+r)/2;
        int lc = (node << 1);
        return query(lc, l, mid, ql, min(qr, mid), x) +
            query(lc+1, mid+1, r, max(ql, mid+1), qr, x);
    }
};

SegmentTree T;

```

```

int n, l, r, x, q;
vector<int> v;

int main() {
    ios_base::sync_with_stdio(0), cin.tie(0);

    cin >> n;

    v = vector<int>(n+1);

    for (int i = 1; i <= n; ++i) {
        cin >> v[i];
    }

    T.build(1, 1, n, v);

    cin >> q;

    while (q--) {
        cin >> l >> r >> x;
        cout << T.query(1, 1, n, l, r, x) << '\n';
    }

    return 0;
}

```

### 1.4 Segment Tree

```

#include <bits/stdc++.h>
using namespace std;
typedef long long int i64;
const int maxn = 1e5+5;
namespace SegmentTree {
    struct Node {
        i64 val;
        Node(i64 x) : val(x) {}
        Node () : val(0LL) {}
    };
    inline Node join(const Node &a, const Node &b) {
        return Node(a.val + b.val);
    }
    int n;
    Node tree[4*maxn];
    Node neutral;
    void build(int node, int l, int r, int *v) {
        if (l == r) { tree[node].val = v[l]; return; }
        int mid = l+(r-l)/2, lc = (node << 1);
        build(lc, l, mid, v);
        build(lc+1, mid+1, r, v);
        tree[node] = join(tree[lc], tree[lc+1]);
    }
    void update(int node, int l, int r, int idx, i64 val) {
        if (l == r) {
            tree[node].val = val; //to set value
            //tree[node].val += val; //to increment value
            return;
        }
        int mid = l+(r-l)/2, lc = (node << 1);
        if(idx <= mid) update(lc, l, mid, idx, val);
        else update(lc+1, mid+1, r, idx, val);
    }
}

```

```

    tree[node] = join(tree[lc], tree[lc+1]);
}
Node query(int node, int l, int r, int ql, int qr) {
    if (r < l or qr < l or r < ql) return neutral;
    if (ql <= l and r <= qr) return tree[node];
    int mid = l+(r-1)/2, lc = (node << 1);
    return join(query(lc, l, mid, ql, min(qr, mid)), query(lc+1, mid
+1, r, max(ql, mid+1), qr));
}
void build(int _n, int *v) {
    n = _n;
    build(1, 1, n, v);
}
i64 query(int l, int r) {
    return query(1, 1, n, l, r).val;
}
void update(int idx, int val) {
    update(1, 1, n, idx, val);
}
};

```

## 1.5 Segment Tree Kadane

```

#include <bits/stdc++.h>
using namespace std;
typedef long long int i64;
const int maxn = 1e5+5;
struct SegmentTree {
    struct Node {
        i64 ans, pref, suff, sum;
        Node(i64 x = 0LL) { //0 if can take empty set, -1inf if not
            ans = pref = suff = max(0LL, x); // max(0, x) if can take empty
                set, x if not
            sum = x; //always x
        }
    };
    inline Node join(const Node& a, const Node& b) {
        Node ans;
        ans.pref = max(a.pref, a.sum + b.pref);
        ans.suff = max(a.suff + b.sum, b.suff);
        ans.sum = a.sum + b.sum;
        ans.ans = max(a.suff + b.pref, max(a.ans, b.ans));
        return ans;
    }
    int n;
    Node tree[4*maxn];
    Node neutral;
    void build(int node, int l, int r, int *v) {
        if (l == r) { tree[node] = Node(v[l]); return; }
        int mid = l+(r-1)/2, lc = (node << 1);
        build(lc, l, mid, v);
        build(lc+1, mid+1, r, v);
        tree[node] = join(tree[lc], tree[lc+1]);
    }
    void update(int node, int l, int r, int idx, int val) {
        if (l == r) { tree[node] = Node(val); return; }
        int mid = l+(r-1)/2, lc = (node << 1);
        if(idx <= mid) update(lc, l, mid, idx, val);
        else update(lc+1, mid+1, r, idx, val);
        tree[node] = join(tree[lc], tree[lc+1]);
    }
};

```

```

}
Node query(int node, int l, int r, int ql, int qr) {
    if (r < l or qr < l or r < ql) return neutral;
    if (ql <= l and r <= qr) return tree[node];
    int mid = l+(r-1)/2, lc = (node << 1);
    return join(query(lc, l, mid, ql, min(qr, mid)), query(lc+1, mid
+1, r, max(ql, mid+1), qr));
}
void build(int _n, int *v) {
    n = _n;
    build(1, 1, n, v);
}
i64 query(int l, int r) {
    return query(1, 1, n, l, r).ans;
}
void update(int idx, int val) {
    update(1, 1, n, idx, val);
}
};

```

## 1.6 Segment Tree Lazy Propagation

```

#include <bits/stdc++.h>
using namespace std;

typedef long long int i64;
typedef vector<i64> vi64;
typedef long long ftype;

const int maxn = 1e5+5;

namespace SegmentTree {

    struct Node {
        ftype val;
        Node(ftype x) : val(x) {}
        Node () : val(0LL) {}
    };

    inline Node join(const Node &a, const Node &b) {
        return Node(a.val + b.val);
    }

    int n;
    ftype lazy[4*maxn];
    Node tree[4*maxn];
    Node neutral;
    ftype lazyNeutral = -1LL;

    inline void upLazy(int node, int l, int r) {
        if (lazy[node] == lazyNeutral) return;

        tree[node].val += lazy[node] * (r - l + 1LL); //To increment value
        // tree[node].val = lazy[node] * (r - l + 1LL); //To set value

        if (l != r) {
            int lc = (node << 1);
            // lazy[lc] = lazy[node]; //To set value
            // lazy[lc+1] = lazy[node]; //To set value
        }
    }
};

```

```

    lazy[lc] = (lazy[lc] == lazyNeutral ? lazy[node] : lazy[lc] +
        lazy[node]); //To increment value
    lazy[lc+1] = (lazy[lc+1] == lazyNeutral ? lazy[node] : lazy[lc
        +1] + lazy[node]); //To increment value
}

lazy[node] = lazyNeutral;
}

void build(int node, int l, int r, int *v) {
    lazy[node] = lazyNeutral;
    if (l == r) { tree[node].val = v[l]; return; }
    int mid = l+(r-l)/2, lc = (node << 1);
    build(lc, l, mid, v);
    build(lc+1, mid+1, r, v);
    tree[node] = join(tree[lc], tree[lc+1]);
}

void update(int node, int l, int r, int ul, int ur, ftype val) {
    upLazy(node, l, r);
    if (r < l or ur < ul or ur < l or r < ul) return;
    if (ul <= l and r <= ur) {
        // lazy[node] = val; // To set value
        lazy[node] = (lazy[node] == lazyNeutral ? val : lazy[node] + val
        ); //To increment value
        upLazy(node, l, r);
        return;
    }
    int mid = l+(r-l)/2, lc = (node << 1);
    update(lc, l, mid, ul, min(mid, ur), val);
    update(lc+1, mid+1, r, max(mid+1, ul), ur, val);
    tree[node] = join(tree[lc], tree[lc+1]);
}

Node query(int node, int l, int r, int ql, int qr) {
    upLazy(node, l, r);
    if (r < l or qr < ql or qr < l or r < ql) return neutral;
    if (ql <= l and r <= qr) return tree[node];
    int mid = l+(r-l)/2, lc = (node << 1);
    return join(query(lc, l, mid, ql, min(mid, qr)), query(lc+1, mid
        +1, r, max(mid+1, ql), qr));
}

void build(int _n, int *v) {
    n = _n;
    build(1, 1, n, v);
}

ftype query(int l, int r) {
    return query(1, 1, n, l, r).val;
}

void update(int l, int r, ftype val) {
    update(1, 1, n, l, r, val);
}
};

```

## 2 Graph Algorithms

### 2.1 LCA

```

#include <bits/stdc++.h>
using namespace std;
using ii = pair<int, int>;

const int maxn = 1e4+5;
const int L = 21;

namespace LCA {
    int anc[maxn][L], tin[maxn], tout[maxn], deep[maxn], h[maxn];
    vector<ii> gr[maxn];
    int n, timer;

    inline void init(int _n) {
        n = _n;
        timer = 0;
        for (int i = 0; i < n; ++i) {
            deep[i] = 0;
            h[i] = 0;
            tin[i] = tout[i] = 0;
            gr[i].clear();
            for (int j = 0; j < L; ++j) anc[i][j] = 0;
        }
    }

    void dfs(int u, int p) {
        tin[u] = ++timer;
        anc[u][0] = p;
        for (int i = 1; i < L; ++i) anc[u][i] = anc[ anc[u][i-1] ][i-1];
        for (auto [to, w] : gr[u]) if (to != p) {
            deep[to] = deep[u] + 1;
            h[to] = h[u] + w;
            dfs(to, u);
        }
        tout[u] = ++timer;
    }

    inline void addEdge(int u, int v, int w) {
        gr[u].emplace_back(v, w);
    }

    inline bool is_anc(int u, int v) {
        return (tin[u] <= tin[v] and tout[v] <= tout[u]);
    }

    inline int lca(int u, int v) {
        if (is_anc(u, v)) return u;
        if (is_anc(v, u)) return v;
        for (int i = L-1; i >= 0; --i) if (!is_anc(anc[u][i], v)) u = anc[
            u][i];
        return anc[u][0];
    }

    inline int kth_anc(int u, int k) {
        if (--k == 0) return u;
    }
}

```

```

    for (int i = L-1; i >= 0; --i) {
        if (k - (1 << i) >= 0) {
            u = anc[u][i];
            k -= (1 << i);
        }
    }
    return u;
}

inline int dist_w(int a, int b) {
    return h[a] + h[b] - 2 * h[lca(a, b)];
}

inline int dist(int a, int b) {
    return deep[a] + deep[b] - 2 * deep[lca(a, b)];
}

inline void build() {
    dfs(0, 0);
}
};

```

## 2.2 Articulation Point

```

#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5+5;

vector<int> gr[maxn];
int used[maxn], tin[maxn], low[maxn];
int n, timer;

void is_cutpoint(int u) {
    return;
}

void dfs(int u, int p = -1) {
    used[u] = true;
    tin[u] = low[u] = timer++;
    int children = 0;
    for (int to : gr[u]) if (to != p) {
        if (used[to]) { //Is a back edge
            low[u] = min(low[u], tin[to]);
        } else {
            dfs(to, u);
            low[u] = min(low[u], low[to]);
            if (low[to] >= tin[u] and p != -1) {
                is_cutpoint(u);
            }
            ++children;
        }
    }
    if (p == -1 and children > 1)
        is_cutpoint(u);
}

void find_cutpoints() {
    timer = 0;
    for (int i = 0; i < n; ++i) {

```

```

        used[i] = false;
        tin[i] = -1;
        low[i] = -1;
    }
    for (int i = 0; i < n; ++i)
        if (!used[i]) dfs(i);
}

```

## 2.3 Bridges

```

#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5+5;

vector<int> gr[maxn];
int used[maxn], tin[maxn], low[maxn];
int n, timer;

void is_bridge(int u, int v) {
    return;
}

void dfs(int u, int p = -1) {
    used[u] = true;
    tin[u] = low[u] = timer++;
    for (int to : gr[u]) if (to != p) {
        if (used[to]) { //Is a back edge
            low[u] = min(low[u], tin[to]);
        } else {
            dfs(to, u);
            low[u] = min(low[u], low[to]);
            if (low[to] > tin[u]) {
                is_bridge(u, to);
            }
        }
    }
}

void find_bridges() {
    timer = 0;
    for (int i = 0; i < n; ++i) {
        used[i] = false;
        tin[i] = -1;
        low[i] = -1;
    }
    for (int i = 0; i < n; ++i)
        if (!used[i]) dfs(i);
}

```

## 2.4 HLD Edges

```

#include <bits/stdc++.h>
using namespace std;
using i64 = long long int;
using vi64 = vector<i64>;
using ii = pair<int, int>;

```

```

#define fi first
#define se second

const int maxn = 1e5+5;

namespace SegmentTree {

    struct Node {
        i64 val;
        Node(i64 x) : val(x) {}
        Node () : val(0LL) {}
    };

    inline Node join(const Node &a, const Node &b) {
        return Node(a.val + b.val);
    }

    int n;
    i64 lazy[4*maxn];
    Node tree[4*maxn];
    Node neutral;
    i64 lazyNeutral = -1LL;

    inline void upLazy(int node, int l, int r) {
        if (lazy[node] == lazyNeutral) return;

        tree[node].val += lazy[node] * (r - l + 1LL); //To increment value
        // tree[node].val = lazy[node] * (r - l + 1LL); //To set value

        if (l != r) {
            int lc = (node << 1);
            // lazy[lc] = lazy[node]; //To set value
            // lazy[lc+1] = lazy[node]; //To set value
            lazy[lc] = (lazy[lc] == lazyNeutral ? lazy[node] : lazy[lc] +
                lazy[node]); //To increment value
            lazy[lc+1] = (lazy[lc+1] == lazyNeutral ? lazy[node] : lazy[lc]
                +1 + lazy[node]); //To increment value
        }

        lazy[node] = lazyNeutral;
    }

    void build(int node, int l, int r, int *v) {
        lazy[node] = lazyNeutral;
        if (l == r) { tree[node].val = v[l]; return; }
        int mid = l+(r-l)/2, lc = (node << 1);
        build(lc, l, mid, v);
        build(lc+1, mid+1, r, v);
        tree[node] = join(tree[lc], tree[lc+1]);
    }

    void update(int node, int l, int r, int ul, int ur, i64 val) {
        upLazy(node, l, r);
        if (r < l or ur < ul or ur < l or r < ul) return;
        if (ul <= l and r <= ur) {
            lazy[node] = val; // To set value
            lazy[node] = (lazy[node] == lazyNeutral ? val : lazy[node] + val
                ); // To increment value
            upLazy(node, l, r);
            return;
        }

        int mid = l+(r-l)/2, lc = (node << 1);
        update(lc, l, mid, ul, min(mid, ur), val);
        update(lc+1, mid+1, r, max(mid+1, ul), ur, val);
        tree[node] = join(tree[lc], tree[lc+1]);
    }

    Node query(int node, int l, int r, int ql, int qr) {
        upLazy(node, l, r);
        if (r < l or qr < ql or qr < l or r < ql) return neutral;
        if (ql <= l and r <= qr) return tree[node];
        int mid = l+(r-l)/2, lc = (node << 1);
        return join(query(lc, l, mid, ql, min(mid, qr)), query(lc+1, mid
            +1, r, max(mid+1, ql), qr));
    }

    void build(int _n, int *v) {
        n = _n;
        build(1, 1, n, v);
    }

    i64 query(int l, int r) {
        return query(1, 1, n, l, r).val;
    }

    void update(int l, int r, i64 val) {
        update(1, 1, n, l, r, val);
    }
};

namespace HLD {
    struct edge {
        int a; i64 w;
        edge () {}
        edge (int to, i64 ww) : a(to), w(ww) {}
    };

    vector<edge> gr[maxn];
    int pos[maxn], st[maxn], pai[maxn];
    int sobe[maxn], h[maxn], v[maxn], timer;
    int hei[maxn], deep[maxn];

    inline void addEdge(int a, int b, i64 w = 1LL) {
        gr[a].push_back(edge(b, w));
    }

    //O(n)
    void dfs(int u, int p = -1) {
        st[u] = 1;
        for (auto &e : gr[u]) if (e.a != p) {
            sobe[e.a] = e.w;
            dfs(e.a, u);
            st[u] += st[e.a];
            if (st[e.a] > st[gr[u][0].a] or gr[u][0].a == p) swap(e, gr[u]
                [0]);
        }
    }

    //O(n)
    void build_hld(int u, int p = -1) {
        pos[u] = ++timer;

```

```

v[pos[u]] = sobe[u];
for (auto e : gr[u]) if (e.a != p) {
    pai[e.a] = u;
    h[e.a] = (e.a == gr[u][0].a ? h[u] : e.a);
    build_hld(e.a, u);
}
}

inline void build(int root = 0) {
    timer = 0;
    h[root] = 0;
    hei[root] = 0;
    deep[root] = 0;
    dfs(root);
    build_hld(root);
    SegmentTree::build(timer, v);
}

//O(log^2 (n))
i64 query_path(int a, int b) {
    if (a == b) return 0LL;
    if (pos[a] < pos[b]) swap(a, b);

    if (h[a] == h[b]) return SegmentTree::query(1+pos[b], pos[a]);
    return SegmentTree::query(pos[h[a]], pos[a]) +
        query_path(pai[h[a]], b);
}

//O(log^2 (n))
void update_path(int a, int b, int x) {
    if (a == b) return;
    if (pos[a] < pos[b]) swap(a, b);
    if (h[a] == h[b]) return (void) SegmentTree::update(1+pos[b], pos[
        a], x);
    SegmentTree::update(1, 1, timer, pos[h[a]], pos[a], x);
    update_path(pai[h[a]], b, x);
}

//O(log(n))
inline i64 query_subtree(int a) {
    if (st[a] == 1) return 0LL;
    return SegmentTree::query(1+pos[a], pos[a]+st[a]-1);
}

//O(log(n))
inline void update_subtree(int a, int x) {
    if (st[a] == 1) return;
    SegmentTree::update(1+pos[a], pos[a]+st[a]-1, x);
}

//O(log(n))
int lca(int a, int b) {
    if (pos[a] < pos[b]) swap(a, b);
    return (h[a] == h[b] ? b : lca(pai[h[a]], b));
}

//O(log(n))
i64 distw(int a, int b) {
    return hei[a] + hei[b] - 2 * hei[lca(a, b)];
}

```

```

//O(log(n))
int dist(int a, int b) {
    return deep[a] + deep[b] - 2 * deep[lca(a, b)];
}
};

```

## 2.5 HLD Nodes

```

#include <bits/stdc++.h>
using namespace std;
using i64 = long long int;
using vi64 = vector<i64>;

const int maxn = 1e5+5;

namespace SegmentTree {

    struct Node {
        i64 val;
        Node(i64 x) : val(x) {}
        Node () : val(0LL) {}
    };

    inline Node join(const Node &a, const Node &b) {
        return Node(a.val + b.val);
    }

    int n;
    i64 lazy[4*maxn];
    Node tree[4*maxn];
    Node neutral;
    i64 lazyNeutral = -1LL;

    inline void upLazy(int node, int l, int r) {
        if (lazy[node] == lazyNeutral) return;

        tree[node].val += lazy[node] * (r - l + 1LL); //To increment value
        // tree[node].val = lazy[node] * (r - l + 1LL); //To set value

        if (l != r) {
            int lc = (node << 1);
            // lazy[lc] = lazy[node]; //To set value
            // lazy[lc+1] = lazy[node]; //To set value
            lazy[lc] = (lazy[lc] == lazyNeutral ? lazy[node] : lazy[lc] +
                lazy[node]); //To increment value
            lazy[lc+1] = (lazy[lc+1] == lazyNeutral ? lazy[node] : lazy[lc
                +1] + lazy[node]); //To increment value
        }

        lazy[node] = lazyNeutral;
    }

    void build(int node, int l, int r, int *v) {
        lazy[node] = lazyNeutral;
        if (l == r) { tree[node].val = v[l]; return; }
        int mid = l+(r-l)/2, lc = (node << 1);
        build(lc, l, mid, v);
        build(lc+1, mid+1, r, v);
        tree[node] = join(tree[lc], tree[lc+1]);
    }
}

```

```

void update(int node, int l, int r, int ul, int ur, i64 val) {
    upLazy(node, l, r);
    if (r < l or ur < ul or ur < l or r < ul) return;
    if (ul <= l and r <= ur) {
        // lazy[node] = val; // To set value
        lazy[node] = (lazy[node] == lazyNeutral ? val : lazy[node] + val);
        // To increment value
        upLazy(node, l, r);
        return;
    }
    int mid = l+(r-l)/2, lc = (node << 1);
    update(lc, l, mid, ul, min(mid, ur), val);
    update(lc+1, mid+1, r, max(mid+1, ul), ur, val);
    tree[node] = join(tree[lc], tree[lc+1]);
}

Node query(int node, int l, int r, int ql, int qr) {
    upLazy(node, l, r);
    if (r < l or qr < ql or qr < l or r < ql) return neutral;
    if (ql <= l and r <= qr) return tree[node];
    int mid = l+(r-l)/2, lc = (node << 1);
    return join(query(lc, l, mid, ql, min(mid, qr)), query(lc+1, mid+1, r, max(mid+1, ql), qr));
}

void build(int _n, int *v) {
    n = _n;
    build(1, 1, n, v);
}

i64 query(int l, int r) {
    return query(1, 1, n, l, r).val;
}

void update(int l, int r, i64 val) {
    update(1, 1, n, l, r, val);
}
};

namespace HLD {
    struct edge {
        int a; i64 w;
        edge () {}
        edge (int to, i64 ww) : a(to), w(ww) {}
    };

    vector<edge> gr[maxn];
    int pos[maxn], st[maxn], pai[maxn];
    int h[maxn], v[maxn], val[maxn], timer;
    int deep[maxn], hei[maxn];

    inline void addEdge(int a, int b, i64 w = 1LL) {
        gr[a].push_back(edge(b, w));
    }

    //O(n)
    void dfs(int u, int p = -1) {
        st[u] = 1;
        for (auto &e : gr[u]) if (e.a != p) {

```

```

            pai[e.a] = u;
            deep[e.a] = deep[u] + 1;
            hei[e.a] = hei[u] + e.w;
            dfs(e.a, u);
            st[u] += st[e.a];
            if (st[e.a] > st[gr[u][0].a] or gr[u][0].a == p) swap(e, gr[u][0]);
        }
    }

    //O(n)
    void build_hld(int u, int p = -1) {
        pos[u] = ++timer;
        v[pos[u]] = val[u];
        for (auto e : gr[u]) if (e.a != p) {
            h[e.a] = (e.a == gr[u][0].a ? h[u] : e.a);
            build_hld(e.a, u);
        }
    }

    void build(int root = 0) {
        timer = 0;
        h[root] = 0;
        hei[root] = 0;
        deep[root] = 0;
        dfs(root);
        build_hld(root);
        SegmentTree::build(timer, v);
    }

    //O(log^2 (n))
    i64 query_path(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);
        if (h[a] == h[b]) return SegmentTree::query(pos[b], pos[a]);
        return SegmentTree::query(pos[h[a]], pos[a]) + query_path(pai[h[a]], b);
    }

    //O(log^2 (n))
    void update_path(int a, int b, i64 x) {
        if (pos[a] < pos[b]) swap(a, b);
        if (h[a] == h[b]) return (void) SegmentTree::update(pos[b], pos[a], x);
        SegmentTree::update(pos[h[a]], pos[a], x);
        update_path(pai[h[a]], b, x);
    }

    //O(log(n))
    inline i64 query_subtree(int a) {
        return SegmentTree::query(pos[a], pos[a]+st[a]-1);
    }

    //O(log(n))
    inline void update_subtree(int a, i64 x) {
        SegmentTree::update(pos[a], pos[a]+st[a]-1, x);
    }

    //O(log(n))
    int lca(int a, int b) {
        if (pos[a] < pos[b]) swap(a, b);
        return (h[a] == h[b] ? b : lca(pai[h[a]], b));
    }
}

```



```

}

//O(log(n))
i64 distw(int a, int b) {
    return hei[a] + hei[b] - 2 * hei[lca(a, b)];
}

//O(log(n))
int dist(int a, int b) {
    return deep[a] + deep[b] - 2 * deep[lca(a, b)];
}
};

```

## 2.6 Strongly Connected Component

```

#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5+5;

namespace SCC {
    vector<int> gr[maxn], gt[maxn];
    vector<int> order;
    int comp[maxn], used[maxn];
    int n, timer, scc;

    void init(int _n) {
        n = _n;
        scc = 0;
        order.clear();
        for (int i = 0; i < n; ++i) {
            used[i] = false;
            comp[i] = 0;
            gr[i].clear();
            gt[i].clear();
        }
    }

    void addEdge(int u, int v) {
        gr[u].push_back(v);
        gt[v].push_back(u);
    }

    void dfs1(int u) {
        used[u] = timer;
        for (int to : gr[u]) if (used[to] != timer) {
            dfs1(to);
        }
        order.push_back(u);
    }

    void dfs2(int u) {
        used[u] = timer;
        comp[u] = scc;
        for (int to : gt[u]) if (used[to] != timer) {
            dfs2(to);
        }
    }

    int get_scc() {

```

```

        ++timer;
        for (int u = 0; u < n; ++u)
            if (used[u] != timer) dfs1(u);

        ++timer;
        for (int i = n-1; i >= 0; --i)
            if (used[order[i]] != timer) {
                dfs2(order[i]);
                ++scc;
            }
        return scc;
    }
};

```

## 3 Dynamic Programming

### 3.1 Prefix Sum Array

```

#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5+5;

/*
Answer queries:
Q(L, R) = 1*A[L] + 2*A[L+1] + 3*A[L+2] + ... (R-L+1)*A[R]
*/

int n, a[maxn];
int psa[maxn], ips[maxn];
int q, l, r, ans;

void computePSA() {
    for (int i = 1; i <= n; ++i) {
        psa[i] = psa[i-1] + a[i];
        ips[i] = ips[i-1] + i * a[i];
    }
    while (q--) {
        cin >> l >> r;
        ans = ips[r] - ips[l-1] - (l - 1) * (psa[r] - psa[l-1]);
        cout << ans << '\n';
    }
}

```

## 4 Math

### 4.1 Basic Math

```

#include <bits/stdc++.h>
using namespace std;

namespace ModHash{
    const uint64_t MOD = (1ll<<61) - 1;
    uint64_t modmul(uint64_t a, uint64_t b){

```

```

uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b
    >>32;
uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) + ((m <<
    35) >> 3) + 1;
ret = (ret & MOD) + (ret>>61);
ret = (ret & MOD) + (ret>>61);
return ret-1;
}
};

uint64_t modMul(uint64_t a, uint64_t b, uint64_t MOD) {
    return (__uint128_t)a*b%MOD;
}

uint64_t binpow(uint64_t base, uint64_t exp, uint64_t MOD) {
    base %= MOD;
    uint64_t res = 1;
    while (exp > 0) {
        if (exp & 1) res = modMul(res, base, MOD);
        base = modMul(base, base, MOD);
        exp >>= 1;
    }
    return res;
}

uint64_t bigExp(uint64_t base, string exp, uint64_t MOD) {
    base %= MOD;
    uint64_t ans = 1LL;
    for (char c : exp) {
        ans = binpow(ans, 10LL, MOD);
        ans = modMul(ans, binpow(base, c-'0', MOD));
    }
    return ans;
}

uint64_t gcd(uint64_t a, uint64_t b) { return (b == 0 ? a : gcd(b, a%b)); }
uint64_t binary_gcd(uint64_t a, uint64_t b) {
    if (a == 0 or b == 0)
        return a ^ b;
    int shift = __builtin_ctzll(a | b);
    a >>= __builtin_ctzll(a);
    do {
        b >>= __builtin_ctzll(b);
        if (a > b)
            swap(a, b);
        b -= a;
    } while (b);
    return a << shift;
}

uint64_t lcm(uint64_t a, uint64_t b) { return a / binary_gcd(a, b) * b; }
};

```

## 4.2 Fatorial

```

#include <bits/stdc++.h>
#include "modular_inverse.h"
using namespace std;

```

```

const int64_t MOD = 1000000007LL;
const int MAXV = 1e6+5;

int64_t fat[MAXV], ifat[MAXV];

void init() {
    fat[0] = 1LL;
    for (int64_t i = 1; i < MAXV; ++i) {
        fat[i] = (i * fat[i-1]) % MOD;
    }
    ifat[MAXV-1] = inv_mod(fat[MAXV-1], MOD);
    for (int64_t i = MAXV-1; i >= 1; --i) {
        ifat[i-1] = (ifat[i] * i) % MOD;
    }
}

```

## 4.3 Prime Number

```

#include <bits/stdc++.h>
#include "math.h"

using namespace std;

bool check_composite(uint64_t n, uint64_t a, uint64_t d, int s) {
    uint64_t x = binpow(a, d, n);
    if (x == 1 or x == n - 1)
        return false;
    for (int r = 1; r < s; ++r) {
        x = modMul(x, x, n);
        if (x == n - 1)
            return false;
    }
    return true;
}

bool MillerRabin(uint64_t n) {
    if (n < 4)
        return (n == 2 or n == 3);

    uint64_t d = n - 1;
    int s = __builtin_ctzll(d);
    d >>= s;

    for (uint64_t a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        // int a = 2 + rand() % (n - 3); (nondeterministic version)
        if (n == a)
            return true;
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

uint64_t pollard(uint64_t n) {
    auto f = [n](uint64_t x) { return (modMul(x, x, n) + 1) % n; };
    uint64_t x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    while (t++ % 40 or binary_gcd(prd, n) == 1) {
        if (x == y)
            x = ++i, y = f(x);
        if ((q = modMul(prd, max(x, y) - min(x, y), n)))

```

```

    prd = q;
    x = f(x), y = f(f(y));
}
return binary_gcd(prd, n);
}

vector<uint64_t> factor(uint64_t n) {
    if (n == 1)
        return {};
    if (MillerRabin(n))
        return {n};
    uint64_t x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), r.begin(), r.end());
    return l;
}

```

## 4.4 Matrix Exponentiation

```

#include <bits/stdc++.h>
using namespace std;

typedef long long int i64;

const int mod = 1e9+7;
const int D = 3;

int d = D;

struct M {
    i64 m[D][D];

    i64* operator[](int i) {
        return m[i];
    }

    M operator-(M oth) {
        M res;
        for (int i = 0; i < d; ++i) {
            for (int j = 0; j < d; ++j) {
                res[i][j] = m[i][j] - oth[i][j];
            }
        }
        return res;
    }

    M operator+(M oth) {
        M res;
        for (int i = 0; i < d; ++i) {
            for (int j = 0; j < d; ++j) {
                res[i][j] = m[i][j] + oth[i][j];
            }
        }
        return res;
    }

    M operator*(M oth) {
        M res;
        for (int i = 0; i < d; ++i) {
            for (int j = 0; j < d; ++j) {

```

```

                res[i][j] = 0;
                for (int k = 0; k < d; ++k) {
                    res[i][j] = (res[i][j] + m[i][k] * oth[k][j] % mod + mod) %
                        mod;
                }
            }
        }
        return res;
    }

    M exp(i64 e) {
        M res;
        for (int i = 0; i < d; ++i)
            for (int j = 0; j < d; ++j)
                res[i][j] = (i==j);
        M base = *this;
        while (e > 0) {
            if (e & 1LL) res = res * base;
            base = base * base;
            e >>= 1LL;
        }
        return res;
    }
};

```

## 4.5 Array Permutation

```

#include <bits/stdc++.h>
using namespace std;
using i64 = long long int;

const int mod = 1e9+7;
const int D = 15;

int d = D;

struct M {
    i64 m[D][D];

    i64* operator[](int i) {
        return m[i];
    }

    M operator-(M oth) {
        M res;
        for (int i = 0; i < d; ++i) {
            for (int j = 0; j < d; ++j) {
                res[i][j] = m[i][j] - oth[i][j];
            }
        }
        return res;
    }

    M operator+(M oth) {
        M res;
        for (int i = 0; i < d; ++i) {
            for (int j = 0; j < d; ++j) {
                res[i][j] = m[i][j] + oth[i][j];
            }
        }
        return res;
    }

    M operator*(M oth) {
        M res;
        for (int i = 0; i < d; ++i) {
            for (int j = 0; j < d; ++j) {
                res[i][j] = m[i][j] + oth[i][j];
            }
        }
    }
};

```

```

    return res;
}

M operator*(M oth) {
    M res;
    for (int i = 0; i < d; ++i) {
        for (int j = 0; j < d; ++j) {
            res[i][j] = 0;
            for (int k = 0; k < d; ++k) {
                res[i][j] = (res[i][j] + m[i][k] * oth[k][j] % mod + mod) %
                    mod;
            }
        }
    }
    return res;
}

M exp(i64 e) {
    M res;
    for (int i = 0; i < d; ++i)
        for (int j = 0; j < d; ++j)
            res[i][j] = (i==j);
    M base = *this;
    while (e > 0) {
        if (e & 1LL) res = res * base;
        base = base * base;
        e >>= 1LL;
    }
    return res;
}
};

// O(n^3)*log(k)
void apply_permutation(vector<int> &seq, vector<int> &perm, int k) {
    d = perm.size();
    M base;
    for (int i = 0; i < d; ++i) {
        for (int j = 0; j < d; ++j) {
            base[i][j] = 0;
        }
    }
    for (int i = 0; i < d; ++i) {
        base[i][perm[i]-1] = 1;
    }
    base = base.exp(k);
    vector<int> ans(d, 0);
    for (int i = 0; i < d; ++i) {
        for (int j = 0; j < d; ++j) {
            ans[i] += seq[j] * base[i][j];
        }
    }
    for (int i = 0; i < d; ++i) {
        seq[i] = ans[i];
    }
}

// O(n)
int dfs(int u, vector<vector<int>> &gr, vector<bool> &used, vector<int>
    > &order) {
    int rs = 1;
    order.push_back(u);

```

```

    used[u] = true;
    if (!used[gr[u][0]]) {
        rs += dfs(gr[u][0], gr, used, order);
    }
    return rs;
}

// O(n)
void apply_permutation_with_graph(vector<int> &seq, vector<int> &perm,
    int k) {
    int n = seq.size();
    vector<vector<int>> gr(n+1);
    for (int i = 0; i < n; ++i) {
        gr[perm[i]].push_back(i+1);
    }

    vector<bool> used(n+1, false);
    vector<int> ans(n+1);
    vector<int> order;

    for (int i = 1; i <= n; ++i) {
        if (!used[i]) {
            order.clear();
            int sz = dfs(i, gr, used, order);
            int pos = k % sz;
            for (int j = 0; j < sz; ++j) {
                int u = order[j];
                int to = order[(j+pos)%sz];
                ans[to-1] = seq[u-1];
            }
        }
    }

    for (int i = 0; i < n; ++i) {
        seq[i] = ans[i];
    }
}

```

## 4.6 Modular Arithmetic

```

#include <bits/stdc++.h>
#include "extended_euclidean.h"
using namespace std;

const int64_t MOD = 1e9+7;

inline int64_t modSum(int64_t a, int64_t b) {
    return (a+b >= MOD ? a+b-MOD : a+b);
}

inline int64_t modSub(int64_t a, int64_t b) {
    return (a+b < 0 ? a-b+MOD : a-b);
}

inline int64_t modMul(int64_t a, int64_t b) {
    return (a*1LL*b)%MOD;
}

int64_t inv_mod(int64_t a, int64_t mod = MOD) {
    int64_t x, y;
    extended_gcd(a, mod, x, y);
    return (x%mod + mod)%mod;
}

```

```

int64_t modDiv(int64_t a, int64_t b) {
    return modMul(a, inv_mod(b, MOD));
}
/*
    O(log(a))
*/
int64_t bigModMul(int64_t a, int64_t b) {
    int64_t ans = 0LL;
    b %= MOD;
    while (a > 0) {
        if (a & 1) ans = modAdd(ans, b, MOD);
        b = modMul(b, 2LL, MOD);
        a >>= 1;
    }
    return ans;
}
uint64_t bigModMul_2(uint64_t a, uint64_t b) {
    long double x;
    uint64_t c;
    int64_t r;
    if (a >= MOD) a %= MOD;
    if (b >= MOD) b %= MOD;
    x = a;
    c = (x * b) / MOD;
    r = (int64_t) (a * b - c * MOD) % (int64_t)MOD;
    return (r < 0 ? r+MOD : r);
}

```

## 4.7 Extended Euclidean

```

#include <bits/stdc++.h>
using namespace std;

int64_t extended_gcd(int64_t a, int64_t b, int64_t &x, int64_t &y) {
    if (b == 0) {
        x = 1; y = 0;
        return a;
    }
    int64_t g = extended_gcd(b, a%b, y, x);
    y -= x*(a/b);
    return g;
}

```

## 4.8 Geometric Operations

```

#include <bits/stdc++.h>
using namespace std;
typedef long long int i64;
typedef double ftype;

const int mod = 1e9+7;
const int D = 4;

int d = D;

struct M {
    ftype m[D][D];

```

```

ftype* operator[](int i) {
    return m[i];
}

M operator-(M oth) {
    M res;
    for (int i = 0; i < d; ++i) {
        for (int j = 0; j < d; ++j) {
            res[i][j] = m[i][j] - oth[i][j];
        }
    }
    return res;
}

M operator+(M oth) {
    M res;
    for (int i = 0; i < d; ++i) {
        for (int j = 0; j < d; ++j) {
            res[i][j] = m[i][j] + oth[i][j];
        }
    }
    return res;
}

M operator*(M oth) {
    M res;
    for (int i = 0; i < d; ++i) {
        for (int j = 0; j < d; ++j) {
            res[i][j] = 0;
            for (int k = 0; k < d; ++k) {
                // res[i][j] = (res[i][j] + m[i][k] * oth[k][j] % mod + mod)
                // % mod;
                res[i][j] += m[i][k] * oth[k][j];
            }
        }
    }
    return res;
}

M exp(i64 e) {
    M res;
    for (int i = 0; i < d; ++i)
        for (int j = 0; j < d; ++j)
            res[i][j] = (i==j);
    M base = *this;
    while (e > 0) {
        if (e & 1LL) res = res * base;
        base = base * base;
        e >>= 1LL;
    }
    return res;
}

};

struct operation {
    int op, a, b, c;
    double ang;
};

void geometric_operations(int &x, int &y, int &z, vector<pair<
    operation, int>> &v) {

```

```

int m = v.size();
M bases[m];
for (int i = 0; i < m; ++i) {
    operation op = v[i].first;
    int k = v[i].second;
    if (op.op == 1) { //Shift operation
        M base;
        for (int j = 0; j < 4; ++j)
            for (int k = 0; k < 4; ++k)
                base[j][k] = (j == k);

        base[3][0] = op.a;
        base[3][1] = op.b;
        base[3][2] = op.c;
        bases[i] = base.exp(k);
    } else if (op.op == 2) { //Scaling operation
        M base;
        for (int j = 0; j < 4; ++j)
            for (int k = 0; k < 4; ++k)
                base[j][k] = 0;

        base[0][0] = op.a;
        base[1][1] = op.b;
        base[2][2] = op.c;
        base[3][3] = 1;
        bases[i] = base.exp(k);
    } else if (op.op == 3) { //Rotation operation around x
        M base;
        for (int j = 0; j < 4; ++j)
            for (int k = 0; k < 4; ++k)
                base[j][k] = 0;

        base[0][0] = 1.0;
        base[1][1] = +cos(op.ang); //cos(theta)
        base[1][2] = -sin(op.ang); //sin(theta)
        base[2][2] = +cos(op.ang); //cos(theta)
        base[2][1] = +sin(op.ang); //sin(theta)
        base[3][3] = 1.0;
        bases[i] = base.exp(k);
    }
}
for (int i = 1; i < m; ++i) {
    bases[i] = bases[i-1] * bases[i];
}
M ans = bases[m-1];
int _x = x, _y = y, _z = z;
x = _x * ans[0][0] + _y * ans[1][0] + _z * ans[2][0] + 1 * ans[3][0];
y = _x * ans[0][1] + _y * ans[1][1] + _z * ans[2][1] + 1 * ans[3][1];
z = _x * ans[0][2] + _y * ans[1][2] + _z * ans[2][2] + 1 * ans[3][2];
}

```

## 5 Geometry

### 5.1 Basic Geometry

```

using namespace std;

#define eps 1e-9
#define eq(a, b) (abs(a - b) < eps)
#define lt(a, b) (a < b - eps)
#define gt(a, b) (a > b + eps)
#define le(a, b) (a < b + eps)
#define ge(a, b) (a > b - eps)
#define ftype long double
/*#define eq(a, b) (a == b)
#define lt(a, b) (a < b)
#define gt(a, b) (a > b)
#define le(a, b) (a <= b)
#define ge(a, b) (a >= b)
#define ftype long long*/

// Begin Point 2D
struct point2d {
    ftype x, y;

    point2d() : x(0.0), y(0.0) {}
    point2d(const ftype& x, const ftype& y) : x(x), y(y) {}

    point2d& operator=(const point2d& oth) {
        x = oth.x; y = oth.y;
        return (*this);
    }
    point2d& operator+=(const point2d& oth) {
        x += oth.x; y += oth.y;
        return (*this);
    }
    point2d& operator-=(const point2d& oth) {
        x -= oth.x; y -= oth.y;
        return (*this);
    }
    point2d& operator*=(const ftype& factor) {
        x *= factor; y *= factor;
        return (*this);
    }
    point2d& operator/=(const ftype& factor) {
        x /= factor; y /= factor;
        return (*this);
    }
};

point2d operator+(const point2d& a, const point2d& b) {
    return point2d(a.x + b.x, a.y + b.y);
}
point2d operator-(const point2d& a, const point2d& b) {
    return point2d(a.x - b.x, a.y - b.y);
}
point2d operator*(const point2d& a, const ftype& factor) {
    return point2d(a.x * factor, a.y * factor);
}
point2d operator*(const ftype& factor, const point2d& a) {
    return point2d(factor * a.x, factor * a.y);
}
point2d operator/(const point2d& a, const ftype& factor) {
    return point2d(a.x / factor, a.y / factor);
}

bool operator==(const point2d& a, const point2d& b) {
    return (eq(a.x, b.x) and eq(a.y, b.y));
}

```

```
#include <bits/stdc++.h>
```

```

}
bool operator!=(const point2d& a, const point2d& b) {
    return !(a==b);
}
bool operator < (const point2d& a, const point2d& b) {
    return (lt(a.x, b.x) or (eq(a.x, b.x) and lt(a.y, b.y)));
}
bool operator > (const point2d& a, const point2d& b) {
    return (b < a);
}
bool operator <= (const point2d& a, const point2d& b) {
    return !(a > b);
}
bool operator >= (const point2d& a, const point2d& b) {
    return !(a < b);
}
// > 0 if |angle| < pi/2
// = 0 if |angle| = pi
// < 0 if |angle| > pi/2
ftype operator*(const point2d& a, const point2d& b) {
    return (a.x * b.x + a.y * b.y);
}
// < 0 if a comes before b in ccw
// = 0 if a is collinear to b
// > 0 if a comes after b in ccw
ftype operator^(const point2d& a, const point2d& b) {
    return (a.x * b.y - a.y * b.x);
}
ftype ccw(const point2d& a, const point2d& b) {
    return (a ^ b);
}
// ccw(a, b, c) : > 0 if a comes before b counterclockwise in origin
// ccw(a, b, c) : < 0 if a comes after b counterclockwise in origin
ftype ccw(const point2d& a, const point2d& b, const point2d& origin) {
    return ccw(a - origin, b - origin);
}
ftype abs(const point2d& a) {
    return (a * a);
}
ftype norm(const point2d& a) {
    return sqrt(abs(a));
}
ftype dist(const point2d& a, const point2d& b) {
    return norm(a - b);
}
ftype dist2(const point2d& a, const point2d& b) {
    return abs(a - b);
}
ftype dist_point_to_line(const point2d& a, const point2d& p1, const
    point2d& p2) {
    return (a-p1)^(p2-p1)/norm(p2-p1);
}
ftype distance_segment_to_point(const point2d& p, const point2d& q,
    const point2d& a) {
    ftype l2 = dist2(p, q);
    if (eq(l2, 0)) return dist(p, a);
    ftype t = max((ftype)0, min((ftype)1, (a-p)*(q-p)/l2));
    point2d proj = p + t * (q-p);
    return dist(a, proj);
}
ftype proj(const point2d& a, const point2d& b) {

```

```

    return (a*b)/(b*b);
}
point2d pointProj(const point2d& a, const point2d& b) {
    return proj(a, b)*b;
}
ftype angle(const point2d& a) {
    return atan2(a.y, a.x);
}
ftype angle(const point2d& a, const point2d& b) {
    return atan2(a ^ b, a * b);
}
ftype angle(const point2d& a, const point2d& b, const point2d& origin)
{
    return angle(a - origin, b - origin);
}
// Left rotation. Angle (rad)
point2d rotate(const point2d& a, const ftype& angleSin, const ftype&
    angleCos) {
    return point2d(a.x * angleCos - a.y * angleSin, a.x * angleSin + a
        .y * angleCos);
}
point2d rotate(const point2d& a, const ftype& angle) {
    return rotate(a, sin(angle), cos(angle));
}
// Pi/2 left rotation
point2d perp(const point2d& a) {
    return point2d(-a.y, a.x);
}
// 0 to 1 and 2 quadrant. 1 to 3 and 4
int half(const point2d& p) {
    if (gt(p.y, 0) or (eq(p.y, 0) and ge(p.x, 0))) return 0;
    return 1;
}
// angle(a) < angle(b)
bool cmpByAngle(const point2d& a, const point2d& b) {
    int ha = half(a), hb = half(b);
    if (ha != hb) return ha < hb;
    ftype c = a^b;
    if (eq(c, 0)) return lt(norm(a), norm(b));
    return gt(c, 0);
}
inline int sgn(ftype x) {
    return (ge(x, 0) ? (eq(x, 0) ? 0 : 1) : -1);
}
// Intersection of lines r : a + d1 * t
point2d intersect(const point2d& a1, const point2d& d1, const point2d&
    a2, const point2d& d2) {
    return a1 + ((a2-a1)^d2)/(d1^d2) * d1;
}
ftype area(vector<point2d> &pts){
    ftype ret = 0.0;
    for (int i = 2; i < (int)pts.size(); i++) {
        ret += ccw(pts[i] - pts[0], pts[i - 1] - pts[0]);
    }
    return abs(ret * 0.5);
}
ftype signed_area_parallelogram(const point2d& a, const point2d& b,
    const point2d& c) {
    return ccw(a, b, c);
}
ftype triangle_area(const point2d& a, const point2d& b, const point2d&

```

```

    c) {
        return abs(signed_area_parallelogram(a, b, c) * 0.5);
    }
    bool point_in_triangle(const point2d& a, const point2d& b, const
        point2d& c, const point2d& p) {
        ftype s1 = abs(ccw(b, c, a));
        ftype s2 = abs(ccw(a, b, p)) + abs(ccw(b, c, p)) + abs(ccw(c, a, p
            ));
        return eq(s1, s2);
    }
    bool pointInSquare(const point2d& A, const point2d& B, const point2d&
        C, const point2d& D, const point2d& P) {
        ftype s1 = 2*abs((B-A)^(D-A));
        ftype s2 = abs((B-P)^(A-P)) + abs((C-P)^(B-P)) + abs((D-P)^(C-P))
            + abs((A-P)^(D-P));
        return eq(s1, s2);
    }
    bool between(ftype l, ftype r, ftype x) {
        return (le(min(l, r), x) and ge(max(l, r), x));
    }
    bool pointInSegment(const point2d& a, const point2d& b, const point2d&
        p) {
        if (!eq(ccw(a, b, p), 0.0)) return false;
        return between(a.x, b.x, p.x) and between(a.y, b.y, p.y);
    }
    ftype up2(ftype a) {
        return (ftype)a * a;
    }
    // End Point 2D

    // Begin Line
    ftype det(ftype a, ftype b, ftype c, ftype d){
        return a * d - b * c;
    }
    struct Line {
        ftype a, b, c;
        Line () {}
        Line (ftype a1, ftype b1, ftype c1) : a(a1), b(b1), c(c1) {
            normalize();
        }
        Line (const point2d& p1, const point2d& p2) {
            a = p1.y - p2.y;
            b = p2.x - p1.x;
            c = -a * p1.x - b * p1.y;
            normalize();
        }
        void normalize() {
            ftype z = sqrt(up2(a) + up2(b));
            if (!eq(z, 0)) { a /= z, b /= z, c /= z; }
            if (lt(a, 0.0) or (eq(a, 0.0) and lt(b, 0.0))) {
                a = -a;
                b = -b;
                c = -c;
            }
        }
    };
    bool intersection_point_of_lines(const Line& m, const Line& n, point2d
        &res) {
        ftype zn = det(m.a, m.b, n.a, n.b);
        if (eq(zn, 0.0)) return false;
        res.x = -det(m.c, m.b, n.c, n.b) / zn;

```

```

        res.y = -det(m.a, m.c, n.a, n.c) / zn;
        return true;
    }
    bool parallel(const Line& m, const Line& n) {
        return eq(det(m.a, m.b, n.a, n.b), 0.0);
    }
    bool equivalent(const Line& m, const Line& n) {
        return eq(det(m.a, m.b, n.a, n.b), 0.0) and
            eq(det(m.a, m.c, n.a, n.c), 0.0) and
            eq(det(m.b, m.c, n.b, n.c), 0.0);
    }
    ftype dist(const Line& m, const point2d& p) {
        return abs(m.a * p.x + m.b * p.y + m.c) /
            sqrt(up2(m.a) + up2(m.b));
    }
    // End Line
    // Begin Segment
    struct Segment {
        point2d a, b;
        Segment () {}
        Segment (const point2d& a1, const point2d b1) : a(a1), b(b1) {}
    };
    bool interl(ftype a, ftype b, ftype c, ftype d) {
        if (a > b) swap(a, b);
        if (c > d) swap(c, d);
        return le(max(a, c), min(b, d));
    }
    bool check_intersection(const Segment& s1, const Segment& s2) {
        point2d a = s1.a, b = s1.b, c = s2.a, d = s2.b;
        if (ccw(a, d, c) == 0 and ccw(b, d, c) == 0)
            return (interl(a.x, b.x, c.x, d.x) and
                interl(a.y, b.y, c.y, d.y));
        return sgn(ccw(b, c, a) != ccw(b, d, a) and
            ccw(d, a, c) != ccw(d, b, c));
    }
    bool intersection_point_of_segments(const Segment& s1, const Segment&
        s2, Segment &ans) {
        point2d a = s1.a, b = s1.b, c = s2.a, d = s2.b;
        if (!interl(a.x, b.x, c.x, d.x) or
            !interl(a.y, b.y, c.y, d.y)) return false;
        Line m(a, b);
        Line n(c, d);
        if (parallel(m, n)) {
            if (!equivalent(m, n)) return false;
            if (b < a) swap(a, b);
            if (d < c) swap(c, d);
            ans = Segment(max(a, c), min(b, d));
            return true;
        } else {
            point2d p(0, 0);
            intersection_point_of_lines(m, n, p);
            ans = Segment(p, p);
            return between(a.x, b.x, p.x) and between(a.y, b.y, p.y) and
                between(c.x, d.x, p.x) and between(c.y, d.y, p.y);
        }
    }
    // End Segment

    // Begin Circle
    struct Circle {
        ftype x, y, r;

```



```

    Circle () {}
    Circle(ftype x1, ftype y1, ftype r1) : x(x1), y(y1), r(r1) {}
};
bool pointInCircle(const Circle& c, const point2d& p) {
    return ge(c.r, dist(point2d(c.x, c.y), p));
}
Circle circumCircle(const point2d& a, const point2d& b, const point2d&
    c) {
    point2d u((b-a).y, -((b-a).x));
    point2d v((c-a).y, -((c-a).x));
    point2d n = (c-b) * 0.5;
    ftype t = (u^n) / (v^u);
    point2d ct = ((a+c) * 0.5) + (v * t);
    ftype r = dist(ct, a);
    return Circle(ct.x, ct.y, r);
}
Circle inCircle(const point2d& a, const point2d& b, const point2d& c)
{
    ftype m1 = dist(a, b);
    ftype m2 = dist(a, c);
    ftype m3 = dist(b, c);
    point2d ct = ((c * m1) + (b * m2) + a * m3) / (m1 + m2 + m3);
    ftype sp = 0.5 * (m1 + m2 + m3);
    ftype r = sqrt(sp * (sp - m1) * (sp - m2) * (sp - m3)) / sp;
    return Circle(ct.x, ct.y, r);
}
// Minimum enclosing circle
Circle minimumCircle(vector<point2d> p) {
    random_shuffle(p.begin(), p.end());
    Circle c = Circle(p[0].x, p[0].y, 0.0);
    for (int i = 0; i < (int)p.size(); ++i) {
        if (pointInCircle(c, p[i])) continue;
        c = Circle(p[i].x, p[i].y, 0.0);
        for (int j = 0; j < i; ++j) {
            if (pointInCircle(c, p[j])) continue;
            c = Circle((p[j].x + p[i].x)*0.5, (p[j].y + p[i].y)*0.5,
                0.5*dist(p[j], p[i]));
            for (int k = 0; k < j; ++k) {
                if (pointInCircle(c, p[k])) continue;
                c = circumCircle(p[j], p[i], p[k]);
            }
        }
    }
    return c;
}
int circle_line_intersection(const Circle& circ, const Line& line,
    point2d& p1, point2d& p2) {
    ftype r = circ.r;
    ftype a = line.a, b = line.b, c = line.c + line.a * circ.x + line.
        b * circ.y; //take a circle to the (0, 0)
    ftype x0 = -a * c / (up2(a) + up2(b)), y0 = -b * c / (up2(a) + up2
        (b)); // (x0, y0) is the shortest distance point of the
        line for (0, 0)
    if (gt(up2(c), up2(r) * (up2(a) + up2(b)))) return 0;
    if (eq(up2(c), up2(r) * (up2(a) + up2(b)))) {
        p1.x = p2.x = x0 + circ.x;
        p1.y = p2.y = y0 + circ.y;
        return 1;
    } else {
        ftype d_2 = up2(r) - up2(c) / (up2(a) + up2(b));
        ftype mult = sqrt(d_2 / (up2(a) + up2(b)));

```

```

        p1.x = x0 + b * mult + circ.x;
        p2.x = x0 - b * mult + circ.x;
        p1.y = y0 - a * mult + circ.y;
        p2.y = y0 + a * mult + circ.y;
        return 2;
    }
}
int circle_intersection(const Circle& c1, const Circle& c2, point2d&
    p1, point2d& p2) {
    if (eq(c1.x, c2.x) and eq(c1.y, c2.y)){
        if (eq(c1.r, c2.r)) return -1; //INF
        else return 0;
    } else {
        Circle circ(0, 0, c1.r);
        Line line;
        line.a = -2 * (c2.x - c1.x);
        line.b = -2 * (c2.y - c1.y);
        line.c = up2(c2.x - c1.x) + up2(c2.y - c1.y) + up2(c1.r) - up2
            (c2.r);
        int sz = circle_line_intersection(circ, line, p1, p2);
        p1.x += c1.x;
        p2.x += c1.x;
        p1.y += c1.y;
        p2.y += c1.y;
        return sz;
    }
}
bool check_segment_covered_by_circles(const vector<Circle> &vc, const
    Segment& s) {
    vector<point2d> v = {s.a, s.b};
    Line l(s.a, s.b);
    for (Circle c : vc) {
        point2d p1, p2;
        int inter = circle_line_intersection(c, l, p1, p2);
        if (inter >= 1 and between(s.a.x, s.b.x, p1.x) and between(s.a
            .y, s.b.y, p1.y))
            v.push_back(p1);
        if (inter == 2 and between(s.a.x, s.b.x, p2.x) and between(s.a
            .y, s.b.y, p2.y))
            v.push_back(p2);
    }
    sort(v.begin(), v.end());
    bool ans = true;
    for (int i = 1; i < (int)v.size(); i++){
        bool has = false;
        for (Circle c : vc) {
            if (pointInCircle(c, v[i - 1]) and pointInCircle(c, v[i]))
                {
                    has = true;
                    break;
                }
        }
        ans &= has;
    }
    return ans;
}
void tangents(const point2d& c, double r1, double r2, vector<Line> &
    ans) {
    double r = r2 - r1;
    double z = up2(c.x) + up2(c.y);
    double d = z - up2(r);

```

```

    if (lt(d, 0)) return;
    d = sqrt(abs(d));
    Line l;
    l.a = (c.x * r + c.y * d) / z;
    l.a = (c.y * r + c.x * d) / z;
    l.c = r1;
    ans.push_back(l);
}
vector<Line> tangents(const Circle& a, const Circle& b) {
    vector<Line> ans;
    for (int i = -1; i <= 1; i += 2)
        for (int j = -1; j <= 1; j += 2)
            tangents(point2d(b.x - a.x, b.y - a.y), a.r * i, b.r * j,
                      ans);
    for (int i = 0; i < (int)ans.size(); ++i) {
        ans[i].c -= ans[i].a * a.x + ans[i].b * a.y;
        ans[i].normalize();
    }
    return ans;
}
// End Circle

```

## 5.2 Convex Hull

```

#include <bits/stdc++.h>
using namespace std;

vector<point2d> convex_hull(vector<point2d> a){
    if (a.size() == 1) return a;
    sort(a.begin(), a.end());
    a.erase(unique(a.begin(), a.end()), a.end());

    vector<point2d> up, down;
    point2d p1 = a[0], p2 = a.back();
    up.push_back(p1);
    down.push_back(p1);

    for (int i = 1; i < (int)a.size(); i++) {
        if ((i == (int)a.size() - 1) or ge(ccw(p2, a[i], p1), 0)) { //
            Accept collinear points
            // if ((i == (int)a.size() - 1) or gt(ccw(p2, a[i], p1), 0)) {
            // Don't accept collinear points
            while (up.size() >= 2 and lt(ccw(a[i], up.back(), up[up.
                size()-2]), 0)) up.pop_back(); // Accept collinear
                points
            // while (up.size() >= 2 and le(ccw(a[i], up.back(), up[up.
                size()-2]), 0)) up.pop_back(); // Don't accept
                collinear points
            up.push_back(a[i]);
        }
        if ((i == (int)a.size() - 1) or ge(ccw(a[i], p2, p1), 0)) { //
            Accept collinear points
            // if ((i == (int)a.size() - 1) or gt(ccw(a[i], p2, p1), 0)) {
            // Don't accept collinear points
            while (down.size() >= 2 and gt(ccw(a[i], down.back(), down.
                size()-2]), 0)) down.pop_back(); // Accept
                collinear points
            // while (down.size() >= 2 and ge(ccw(a[i], down.back(),
                down[down.size()-2]), 0)) down.pop_back(); // Don't
                accept collinear points
        }
    }
}

```

```

        down.push_back(a[i]);
    }
}

a.clear();
for (int i = 0; i < (int)up.size(); i++) a.push_back(up[i]);
for (int i = (int)down.size()-2; i >= 1; i--) a.push_back(down[i]);
return a;
}

```

## 5.3 Convex Polygon

```

#include <bits/stdc++.h>
using namespace std;

namespace ConvexPolygon {
    vector<point2d> vp;
    void init(const vector<point2d>& aux) {
        vp = convex_hull(aux);
    }
    bool pointInPolygon(const point2d& point) {
        if (vp.size() < 3) return pointInSegment(vp[0], vp[1], point);

        if (!eq(ccw(vp[1], point, vp[0]), 0.0) and
            sgn(ccw(vp[1], point, vp[0])) != sgn(ccw(vp[1], vp.back(),
                vp[0]))) return false;

        if (!eq(ccw(vp.back(), point, vp[0]), 0.0) and
            sgn(ccw(vp.back(), point, vp[0])) != sgn(ccw(vp.back(),
                vp[1], vp[0]))) return false;

        if (eq(ccw(vp[1], point, vp[0]), 0.0)) return ge(norm(vp[1]-vp
            [0]), norm(point-vp[0]));

        int pos = 1, l = 1, r = vp.size() - 2;
        while (l <= r) {
            int mid = (l + r) / 2;
            if (le(ccw(vp[mid], point, vp[0]), 0.0)) {
                pos = mid;
                l = mid + 1;
            } else {
                r = mid - 1;
            }
        }

        return point_in_triangle(vp[0], vp[pos], vp[pos+1], point);
    }
};

```

## 6 String Algorithms

### 6.1 Aho Corasick

```

#include <bits/stdc++.h>
using namespace std;
#define fi first

```

```

#define se second

typedef pair<int, int> ii;

const int K = 26;
inline int getId(char c) {
    return c - 'a';
}

namespace Aho {
    struct Vertex {
        int next[K], go[K];
        int suff_link = -1, end_link = -1;
        int leaf = -1, p = -1, sz, match = -1;
        char pch;
        Vertex(int p1 = -1, char ch = '$', int sz1 = 0) : p(p1), pch(ch),
            sz(sz1) {
            fill(begin(next), end(next), -1);
            fill(begin(go), end(go), -1);
        }
    };
    vector<Vertex> trie;
    inline void init() {
        trie.clear();
        trie.emplace_back();
    }
    int add_string(const string &s, int id = 1) {
        int v = 0;
        for (char ch : s) {
            int c = getId(ch);
            if (trie[v].next[c] == -1) {
                trie[v].next[c] = trie.size();
                trie.emplace_back(v, ch, trie[v].sz + 1);
            }
            v = trie[v].next[c];
        }
        trie[v].leaf = id;
        return v;
    }
    int go(int v, char ch);
    int get_suff_link(int v) {
        if (trie[v].suff_link == -1) {
            if (v == 0 or trie[v].p == 0) {
                trie[v].suff_link = 0;
            } else {
                trie[v].suff_link = go(get_suff_link(trie[v].p), trie[v].pch);
            }
        }
        return trie[v].suff_link;
    }
    int get_end_link(int v) {
        if (trie[v].end_link == -1) {
            if (v == 0 or trie[v].p == 0) {
                trie[v].end_link = 0;
            } else {
                int suff_link = get_suff_link(v);
                if (trie[suff_link].leaf != -1) {
                    trie[v].end_link = suff_link;
                } else {
                    trie[v].end_link = get_end_link(suff_link);
                }
            }
        }
    }
}

```

```

    }
    return trie[v].end_link;
}
int go(int v, char ch) {
    int c = getId(ch);
    if (trie[v].go[c] == -1) {
        if (trie[v].next[c] != -1) {
            trie[v].go[c] = trie[v].next[c];
        } else {
            trie[v].go[c] = (v == 0 ? 0 : go(get_suff_link(v), ch));
        }
    }
    return trie[v].go[c];
}
};

/*
    Get match positions in  $O(|t| * \sqrt{|t|})$ 
    Answer: {i, j} -> Range of match
*/
vector<ii> getMatch(const string &t) {
    auto addMatch = [](vector<ii> &ans, int v, int i) {
        while (v != 0) {
            ans.emplace_back(i - Aho::trie[v].sz + 1, i);
            v = Aho::get_end_link(v);
        }
    };
    int v = 0;
    vector<ii> ans;
    for (int i = 0; i < (int)t.size(); ++i) {
        v = Aho::go(v, t[i]);
        if (Aho::trie[v].leaf != -1) {
            addMatch(ans, v, i);
        } else {
            addMatch(ans, Aho::get_end_link(v), i);
        }
    }
    sort(ans.begin(), ans.end());
    return ans;
}

int countMatch(int v) {
    if (Aho::trie[v].match == -1) {
        if (v == 0 or Aho::trie[v].p == 0) {
            Aho::trie[v].match = (Aho::trie[v].leaf != -1 ? 1 : 0);
        } else {
            Aho::trie[v].match = (Aho::trie[v].leaf != -1 ? 1 : 0) +
                countMatch(Aho::get_end_link(v));
        }
    }
    return Aho::trie[v].match;
}

/*
    Get match amount in  $O(|t|)$ 
    Answer: Amount of matches
*/
int64_t matchAmount(const string &t) {
    int v = 0;
    int64_t ans = 0;
    for (char ch : t) {

```

```

    v = Aho::go(v, ch);
    ans += countMatch(v);
}
return ans;
}

```

## 6.2 String Hashing

```

#include <bits/stdc++.h>
using namespace std;

/*
    Small Primes:
    31, 53
    Large Primes:
    (1e6+3), (1e8+7), 100003621, (1e9+7), (1e9+9), (1LL<<61)-1
*/

struct StringHashing {
    const uint64_t MOD = (1LL<<61)-1;
    const int base = 31;
    uint64_t modMul(uint64_t a, uint64_t b){
        uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b
        >>32;
        uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
        uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) + ((m <<
        35) >> 3) + 1;
        ret = (ret & MOD) + (ret>>61);
        ret = (ret & MOD) + (ret>>61);
        return ret-1;
    }
    inline int getInt(char c) {
        return (c-'a'+1);
    }
    /*
        hs[i] = s[0]*p^(i) + s[1]*p(i-1) + ... + s[i-1]*p + s[i]
    */
    vector<uint64_t> hs, p;
    StringHashing(const string &s) {
        int n = s.size();
        hs.resize(n); p.resize(n);
        p[0] = 1;
        hs[0] = getInt(s[0]);
        for (int i = 1; i < n; ++i) {
            p[i] = modMul(p[i-1], base);
            hs[i] = (modMul(hs[i-1], base) + getInt(s[i]))%MOD;
        }
    }
    /*
        hs[i..j] = hs[j] - hs[i-1] * p^(j-i+1)
    */
    uint64_t getValue(int l, int r) {
        if (l > r) return -1;
        uint64_t res = hs[r];
        if (l > 0) res = (res + MOD - modMul(p[r-l+1], hs[l-1]))%MOD;
        return res;
    }
};

struct StringHashingDoubleMod {

```

```

    const uint64_t MOD1 = 1e6+3;
    const uint64_t MOD2 = 1e8+7;
    const int base = 31;
    uint64_t modMul(uint64_t a, uint64_t b){
        uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2 = b
        >>32;
        uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
        uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) + ((m <<
        35) >> 3) + 1;
        ret = (ret & MOD) + (ret>>61);
        ret = (ret & MOD) + (ret>>61);
        return ret-1;
    }
    int getInt(char c) {
        return (c-'a'+1);
    }
    vector<uint64_t> h1, h2, p1, p2;
    StringHashing(const string &s) {
        int n = s.size();
        h1.resize(n); h2.resize(n); p1.resize(n); p2.resize(n);
        p1[0] = 1;
        p2[0] = 1;
        h1[0] = getInt(s[0]);
        h2[0] = getInt(s[0]);
        for (int i = 1; i < n; ++i) {
            p1[i] = modMul(p1[i-1], base, MOD1);
            p2[i] = modMul(p2[i-1], base, MOD2);
            h1[i] = (modMul(h1[i-1], base, MOD1) + getInt(s[i]))%MOD1;
            h2[i] = (modMul(h2[i-1], base, MOD2) + getInt(s[i]))%MOD2;
        }
    }
    pair<uint64_t, uint64_t> getValue(int l, int r) {
        if (l > r) return {-1, -1};
        pair<uint64_t, uint64_t> res;
        res.first = h1[r];
        if (l > 0) res.first = (res.first + MOD1 - modMul(p1[r-l+1], h1[l-1], MOD1))%MOD1;
        res.second = h2[r];
        if (l > 0) res.second = (res.second + MOD2 - modMul(p2[r-l+1], h2[l-1], MOD2))%MOD2;
        return res;
    }
};

```

## 6.3 Prefix Function

```

#include <bits/stdc++.h>
using namespace std;

/*
    p[i] is the length of the longest proper prefix of s[0..i]
    which is also a suffix of this string
    Run in O(|s|)
*/
vector<int> prefix_function(const string &s) {
    int n = s.size();
    vector<int> pi(n);
    for (int i = 1, j = 0; i < n; ++i) {
        while (j > 0 and s[i] != s[j]) j = pi[j-1];
        if (s[i] == s[j]) ++j;
    }
}

```

```

    pi[i] = j;
}
return pi;
}

/*
Returns a vector with the initial positions of
all occurrences of s in t
Using O(|s|) memory
Run in O(|s|+|t|)
*/
vector<int> kmp(const string &s, const string &t) {
    vector<int> p = prefix_function(s+'$'), match;
    for (int i = 0, j = 0; i < (int)t.size(); ++i) {
        while (j > 0 and s[j] != t[i]) j = p[j-1];
        if (s[j] == t[i]) ++j;
        if (j == (int)s.size()) match.push_back(i-j+1);
    }
    return match;
}

/*
ans[i] is the amount of occurrences of the prefix s[0..i] in s
*/
vector<int> prefix_occurrences(const string &s) {
    vector<int> pi = prefix_function(s);
    int n = pi.size();
    vector<int> ans(n+1);
    for (int i = 0; i < n; i++)
        ans[pi[i]]++;
    for (int i = n-1; i > 0; i--)
        ans[pi[i-1]] += ans[i];
    for (int i = 0; i <= n; i++)
        ans[i]++;
    return ans;
}

inline int getInt(char c) {
    return c-'a';
}

/*
Run in O(26*|s|)
*/
struct autKMP {
    vector<vector<int>> nxt;
    autKMP (const string &s) : nxt(26, vector<int>(s.size()+1)) {
        vector<int> p = prefix_function(s);
        nxt[getId(d[0])][0] = 1;
        for (char c = 0; c < 26; ++c) {
            for (int i = 1; i <= (int)s.size(); ++i) {
                nxt[c][i] = (getId(s[i-1]) == c ? i+1 : nxt[c][p[i-1]]);
            }
        }
    }
};

/*
Returns a vector with the initial positions of
all occurrences of s in t
Run in O(|t|)
*/

```

```

*/
vector<int> matching_aut(const string& s, const string& t) {
    auto aut = autKMP(s);
    vector<int> match;
    int at = 0;
    for (int i = 0; i < (int)t.size(); ++i) {
        at = aut.nxt[getId(t[i])][at];
        if (at == (int)s.size()) match.push_back(i-at+1);
    }
    return match;
}

```

## 6.4 Suffix Array

```

#include <bits/stdc++.h>
using namespace std;

vector<int> sort_cyclic_shifts(const string &s) {
    int n = s.size();
    const int alphabet = 256;

    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; ++i)
        ++cnt[s[i]];
    for (int i = 1; i < alphabet; ++i)
        cnt[i] += cnt[i-1];
    for (int i = 0; i < n; ++i)
        p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; ++i) {
        if (s[p[i]] != s[p[i-1]])
            ++classes;
        c[p[i]] = classes - 1;
    }

    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        int h2 = (1 << h);
        for (int i = 0; i < n; ++i) {
            pn[i] = p[i] - h2;
            if (pn[i] < 0) pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin()+classes, 0);
        for (int i = 0; i < n; ++i)
            ++cnt[c[pn[i]]];
        for (int i = 1; i < classes; ++i)
            cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; --i)
            p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; ++i) {
            pair<int, int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
            pair<int, int> prev = {c[p[i-1]], c[(p[i-1] + (1 << h)) % n]};
            if (cur != prev)
                ++classes;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
}

```

```

    }
    return p;
}

/*
    Run in  $O(|s| \cdot \log(|s|))$ 
*/
vector<int> suffix_array_construction(string s) {
    s += '$';
    vector<int> sorted_shifts = sort_cyclic_shifts(s);
    sorted_shifts.erase(sorted_shifts.begin());
    return sorted_shifts;
}

/*
    Kasai's algorithm run in  $O(n)$ 
*/
vector<int> lcp_construction(const string &s, vector<int> &p) {
    int n = (int)s.size();
    vector<int> rank(n, 0);
    for (int i = 0; i < n; ++i) {
        rank[p[i]] = i;
    }
    int k = 0;
    vector<int> lcp(n-1, 0);
    for (int i = 0; i < n; ++i) {
        if (rank[i] == n-1) {
            k = 0;
            continue;
        }
        int j = p[rank[i] + 1];
        while (i + k < n and j + k < n and s[i+k] != s[j+k]) {
            ++k;
        }
        lcp[rank[i]] = k;
        if (k) --k;
    }
    return lcp;
}

/*
    Number of different substrings:
     $(n^2 + n) / 2 - \sum_{i=0}^{n-2} lcp[i]$ 
*/

```

## 6.5 Z Function

```

#include <bits/stdc++.h>
using namespace std;

/*
     $z[i]$  is the length of the largest common prefix
    between  $s[0..n-1]$  and  $s[i..n-1]$ 
*/
vector<int> z_function(const string &s) {
    int n = (int)s.size();
    vector<int> z(n);
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while (i + z[i] < n and s[z[i]] == s[i + z[i]])

```

```

            ++z[i];
        if (i + z[i] - 1 > r)
            l = i, r = i + z[i] - 1;
    }
    return z;
}

```

## 6.6 Trie

```

#include <bits/stdc++.h>
using namespace std;
const int K = 26;
inline int getId(char c) {
    return c - 'a';
}

namespace Trie {
    struct Vertex {
        int next[K];
        int leaf, count;
        Vertex () {
            fill(begin(next), end(next), -1);
            leaf = count = 0;
        }
    };
    vector<Vertex> trie;
    void init() {
        trie.clear();
        trie.emplace_back();
    }
    /*
        Insert a string in  $O(|s|)$ 
    */
    void add(const string &s) {
        int v = 0;
        ++trie[v].count;
        for (char ch : s) {
            int c = getId(ch);
            if (trie[v].next[c] == -1) {
                trie[v].next[c] = trie.size();
                trie.emplace_back();
            }
            v = trie[v].next[c];
            ++trie[v].count;
        }
        ++trie[v].leaf;
    }
    /*
        Get amount of occurrences of  $s$  in  $O(|s|)$ 
    */
    int countStr(const string &s) {
        int v = 0;
        for (char ch : s) {
            int c = getId(ch);
            if (trie[v].next[c] == -1) return 0;
            v = trie[v].next[c];
        }
        return trie[v].leaf;
    }
    /*
        Get amount of occurentes of prefix  $s$  in  $O(|s|)$ 
    */

```

```

*/
int countPre(const string &s) {
    int v = 0;
    for (char ch : s) {
        int c = getId(ch);
        if (trie[v].next[c] == -1) return 0;
        v = trie[v].next[c];
    }
    return trie[v].count;
}
/*
Remove a string s in O(|s|) and returns true if it's removed
*/
bool remove(const string &s) {
    vector<int> rm;
    int v = 0;
    rm.push_back(v);
    for (char ch : s) {
        int c = getId(ch);
        if (trie[v].next[c] == -1) return false;
        v = trie[v].next[c];
        rm.push_back(v);
    }
    if (trie[v].leaf > 0) {
        --trie[v].leaf;
        for (int x : rm) --trie[x].count;
        return true;
    }
    return false;
}
};

```

```

    }
    v = trie[v].next[b];
    ++trie[v].pre;
    }
    trie[v].val = val;
    }
    int min_xor(int val) {
        int v = 0;
        for (int i = SZ-1; i >= 0; --i) {
            bool b = val & (1 << i);
            if (trie[v].next[b] != -1) {
                v = trie[v].next[b];
            } else {
                v = trie[v].next[b^1];
            }
        }
        return val ^ trie[v].val;
    }
    int max_xor(int val) {
        int v = 0;
        for (int i = SZ-1; i >= 0; --i) {
            bool b = val & (1 << i);
            if (trie[v].next[b^1] != -1) {
                v = trie[v].next[b^1];
            } else {
                v = trie[v].next[b];
            }
        }
        return val ^ trie[v].val;
    }
};

```

## 6.7 Trie Int

```

#include <bits/stdc++.h>
using namespace std;
const int K = 2;
const int SZ = 32;
namespace Trie {
    struct Vertex {
        int next[K];
        int val, pre;
        Vertex () {
            fill(begin(next), end(next), -1);
            pre = val = 0;
        }
    };
    vector<Vertex> trie;
    void build() {
        trie.clear();
        trie.emplace_back();
    }
    void add(int val) {
        int v = 0;
        ++trie[v].pre;
        for (int i = SZ-1; i >= 0; --i) {
            bool b = val & (1 << i);
            if (trie[v].next[b] == -1) {
                trie[v].next[b] = trie.size();
                trie.emplace_back();
            }
        }
    }
}

```

## 7 Miscellaneous

### 7.1 Longest Increasing Subsequence

```

#include <bits/stdc++.h>
using namespace std;

int lis(vector<int> &aux) {
    vector<int> d;
    for (int &x : aux) {
        auto it = upper_bound(d.begin(), d.end(), x); //for repeated
        values
        //auto it = lower_bound(d.begin(), d.end(), x); //for non repeated
        values
        if (it == d.end()) d.pb(x);
        else *it = x;
    }
    return (int)d.size();
}

```

### 7.2 Mo Algorithm

```

#include <bits/stdc++.h>
using namespace std;

```

```

typedef vector<int> vi;

const int BLOCK_SIZE = 800;
const int maxn = 5e5+5;

int v[maxn], f[maxn];
int ans;

void remove(int idx) {
    --f[v[idx]];
    if (f[v[idx]] == 0) --ans;
}

void add(int idx) {
    ++f[v[idx]];
    if (f[v[idx]] == 1) ++ans;
}

int get_answer() {
    return ans;
}

struct Query {
    int l, r, idx;
    bool operator < (const Query oth) const {
        if (l / BLOCK_SIZE != oth.l / BLOCK_SIZE) return l < oth.l;
        return (l / BLOCK_SIZE & 1) ? (r < oth.r) : (r > oth.r);
    }
};

vi mo_s_algorithm(vector<Query> queries) {
    vi answers(queries.size());
    sort(queries.begin(), queries.end());
    int l = 0, r = 0;
    for (Query q : queries) {
        while (q.l < l) add(--l);
        while (r < q.r) add(++r);
        while (l < q.l) remove(l++);
        while (q.r < r) remove(r--);
        answers[q.idx] = get_answer();
    }
    return answers;
}

```

## 7.3 Ternary Search

```

#include <bits/stdc++.h>
using namespace std;

#define ftype long double;
#define f(x) x

const int inf = 0x3f3f3f3f;

void ternary_search_of_min(ftype top) {
    ftype lo = 0.0, hi = top;
    for (int i = 0; i < 100; ++i) {
        m1 = (lo * 2 + hi) / 3.0;
        m2 = (lo + 2 * hi) / 3.0;

```

```

        if (f(m1) > f(m2)) {
            lo = m1;
            ans = m2;
        } else {
            hi = m2;
            ans = m1;
        }
    }
    cout << f(ans) << " = " << ans << '\n';
}

void ternary_search_of_max(ftype top) {
    ftype lo = 0.0, hi = top;
    for (int i = 0; i < 100; ++i) {
        m1 = (lo * 2 + hi) / 3.0;
        m2 = (lo + 2 * hi) / 3.0;
        if (f(m1) < f(m2)) {
            lo = m1;
            ans = m2;
        } else {
            hi = m2;
            ans = m1;
        }
    }
    cout << f(ans) << " = " << ans << '\n';
}

void ternary_search_of_min_on_integers(int top) {
    int lo = 0, hi = top, ans = inf;
    while (hi - lo > 4) {
        int m1 = (lo + hi) / 2;
        int m2 = m1 + 1;
        if (f(m1) > f(m2)) {
            lo = m1;
        } else {
            hi = m2;
        }
    }
    for (int i = lo; i <= hi; ++i) {
        ans = min(ans, f(i));
    }
}

void ternary_search_of_max_on_integers(int top) {
    int lo = 0, hi = top, ans = -inf;
    while (hi - lo > 4) {
        int m1 = (lo + hi) / 2;
        int m2 = m1 + 1;
        if (f(m1) < f(m2)) {
            lo = m1;
        } else {
            hi = m2;
        }
    }
    for (int i = lo; i <= hi; ++i) {
        ans = max(ans, f(i));
    }
}

```