Voss-Weyl formula

Based on Tensor Calculus Video Video 49 - Voss Weyl Formula

Based on Tensor Calculus Video Video 50 - Voss Weyl Examples

Based on Pavel Grinfeld Lecture Tensor Calculus Lecture 7d: The Voss-Weyl Formula

Based on WikiPedia Article Divergence

Based on WikiPedia Article Laplacian

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(%i2) `info:build_info()$info@version;`

(%o2)

5.38.1

(%i2) `reset()$kill(all)$`

(%i1) `derivabbrev:true$`

1 Using tensor packages

(%i3)
```
if get('itensor,'version)=false then load(itensor)$
if get('ctensor,'version)=false then load(ctensor)$
```

(%i4)
```
imetric(Z)$
```

$$\frac{1}{\sqrt{Z}}\frac{\partial}{\partial\,Z^i}\left(\sqrt{Z}\,A^i\right)$$

(%i5)
```
ishow(sublis([%1=m,%2=n],rename(expand(idiff(√(determinant(Z))*A([],[i]),i)/√(determinant(Z)))
```

$$A^m\,\Gamma^n_{nm} + A^m_{,m} \tag{\%t5}$$

$$\nabla_i A^i$$

(%i6)
```
ishow(sublis([%1=n,%2=m],rename(covdiff(A([],[i]),i))))$
```

$$A^m\,\Gamma^n_{mn} + A^n_{,n} \tag{\%t6}$$

(%i7)
```
Eq1:ic_convert(Q1=%)$
```

$$\frac{1}{\sqrt{Z}}\frac{\partial}{\partial Z^i}\left(\sqrt{Z}Z^{ij}\frac{\partial f}{\partial Z^j}\right)$$

(%i8)
```
ishow(sublis([%1=a,%2=b,%3=c],rename(expand(idiff(√(determinant(Z))*Z([],[i,j])*idiff(f([],[]),j
```

$$f_{,b}\,Z^{ab}\,\Gamma^c_{ca} + f_{,b}\,Z^{ab}_{,a} + f_{,ab}\,Z^{ab} \tag{\%t8}$$

$$\nabla_i\nabla^i f = \nabla_i Z^{ij}\nabla_j f$$

(%i9)
```
ishow(sublis([%1=a,%2=b,%3=c],rename(expand(covdiff(Z([],[i,j])*covdiff(f([],[]),j),i)))))$
```

$$f_{,b}\,Z^{ab}\,\Gamma^c_{ac} + f_{,b}\,Z^{ab}_{,a} + f_{,ab}\,Z^{ab} \tag{\%t9}$$

(%i10)
```
Eq2:ic_convert(Q2=%)$
```

```
(%i14) assume(0≤r)$
       assume(0≤θ,θ≤π)$
       assume(0≤sin(θ))$
       assume(0≤φ,φ≤2*π)$
(%i15) ξ:ct_coords:[r,θ,φ]$
(%i16) dim:length(ξ)$
(%i17) R:[r*sin(θ)*cos(φ),r*sin(θ)*sin(φ),r*cos(θ)]$
(%i18) ct_coordsys(append(R,[ξ]),all)$
(%i21) lg:trigsimp(lg)$
       ug:trigsimp(ug)$
       gdet:trigsimp(gdet)$
(%i22) sf:makelist(√(lg[i,i]),i,1,dim);
```

$$[1, r, r\ \sin(\theta)] \tag{sf}$$

```
(%i23) √(gdet);
```

$$r^2\ \sin(\theta) \tag{\%o23}$$

Divergence

```
(%i26) A:[A_1,A_2,A_3]$
       declare(A,scalar)$
       depends(A,ξ)$
(%i27) Q3:ev(Eq1,expand);
```

$$\frac{A_2 \cos(\theta)}{\sin(\theta)} + \frac{2 A_1}{r} + A_{3\phi} + A_{2\theta} + A_{1r} \tag{Q3}$$

Normalized

```
(%i31) A:[A_r,A_θ,A_φ]$
       declare(A,scalar)$
       depends(A,ξ)$
       ldisplay(A:A/sf)$
```

$$A = \left[A_r, \frac{A_\theta}{r}, \frac{A_\phi}{r\ \sin(\theta)}\right] \tag{\%t31}$$

```
(%i32) ev(Eq1,expand);
```

$$\frac{A_\theta \cos(\theta)}{r\ \sin(\theta)} + \frac{A_{\phi\phi}}{r\ \sin(\theta)} + \frac{A_{\theta\theta}}{r} + \frac{2 A_r}{r} + A_{rr} \tag{\%o32}$$

Laplacian

```
(%i33) depends(f,ξ)$
(%i34) ev(Eq2);
```

$$\frac{(f_\theta)\ \cos(\theta)}{r^2\ \sin(\theta)} + \frac{f_{\phi\phi}}{r^2 \sin(\theta)^2} + \frac{2\,(f_r)}{r} + \frac{f_{\theta\theta}}{r^2} + f_{rr} \tag{\%o34}$$

2 Using vect and diff_form packages

(%i2) `reset()$kill(allbut(ξ,dim,R,Q1,Q2,Q3))$`

(%i1) `derivabbrev:true$`

(%i3)
```
if get('vect,'version)=false then load(vect)$
if get('diff_form,'version)=false then load(diff_form)$
```

(%i4)
```
inv_i1(_pform):=block([a_],a_:makelist(coeff(_pform,basis[i]),i,1,dim),
list_matrix_entries(a_ .  sqrt(diag(norm_table))))$
```

(%i8)
```
assume(0≤r)$
assume(0≤θ,θ≤π)$
assume(0≤sin(θ))$
assume(0≤ϕ,ϕ≤2*π)$
```

(%i9) `scalefactors(append([R],ξ))$`

(%i10) `sf;`

$$[1, r, r\ \sin(\theta)] \qquad (\%o10)$$

(%i11) `sfprod;`

$$r^2\ \sin(\theta) \qquad (\%o11)$$

(%i12) `dimension;`

$$3 \qquad (\%o12)$$

Divergence, Normalized

(%i15)
```
A:[A_r,A_θ,A_ϕ]$
declare(A,scalar)$
depends(A,ξ)$
```

(%i16) `ev(express(div(A)),diff,expand);`

$$\frac{A_\theta \cos(\theta)}{r\ \sin(\theta)} + \frac{A_{\phi\,\phi}}{r\ \sin(\theta)} + \frac{A_{\theta\,\theta}}{r} + \frac{2 A_r}{r} + A_{r\,r} \qquad (\%o16)$$

(%i17) `is(%=Q1);`

$$\text{true} \qquad (\%o17)$$

(%i18) `fstar_with_clf(ξ,R,nest2([h_st,d,h_st,vtof1],A));`

$$\frac{A_\theta \cos(\theta)}{r\ \sin(\theta)} + \frac{A_{\phi\,\phi}}{r\ \sin(\theta)} + \frac{A_{\theta\,\theta}}{r} + \frac{2 A_r}{r} + A_{r\,r} \qquad (\%o18)$$

(%i19) `is(%=Q1);`

$$\text{true} \qquad (\%o19)$$

Divergence, non-Normalized

(%i23)
```
A:[A_1,A_2,A_3]$
declare(A,scalar)$
depends(A,ξ)$
ldisplay(A:A*sf)$
```

$$A = [A_1, A_2 r, A_3 r \sin(\theta)] \tag{\%t23}$$

(%i24) `ev(express(div(A)),diff,expand);`

$$\frac{A_2 \cos(\theta)}{\sin(\theta)} + \frac{2A_1}{r} + A_{3\phi} + A_{2\theta} + A_{1r} \tag{\%o24}$$

(%i25) `is(%=Q3);`

$$\text{true} \tag{\%o25}$$

(%i26) `fstar_with_clf(ξ,R,nest2([h_st,d,h_st,vtof1],A));`

$$\frac{A_2 \cos(\theta)}{\sin(\theta)} + \frac{2A_1}{r} + A_{3\phi} + A_{2\theta} + A_{1r} \tag{\%o26}$$

(%i27) `is(%=Q3);`

$$\text{true} \tag{\%o27}$$

Laplacian

(%i28) `depends(f,ξ)$`

(%i29) `ev(express(laplacian(f)),diff,expand);`

$$\frac{(f_\theta)\cos(\theta)}{r^2 \sin(\theta)} + \frac{f_{\phi\phi}}{r^2 \sin(\theta)^2} + \frac{2(f_r)}{r} + \frac{f_{\theta\theta}}{r^2} + f_{rr} \tag{\%o29}$$

(%i30) `is(%=Q2);`

$$\text{true} \tag{\%o30}$$

(%i31) `fstar_with_clf(ξ,R,nest2([h_st,d,h_st,d],f));`

$$\frac{(f_\theta)\cos(\theta)}{r^2 \sin(\theta)} + \frac{f_{\phi\phi}}{r^2 \sin(\theta)^2} + \frac{2(f_r)}{r} + \frac{f_{\theta\theta}}{r^2} + f_{rr} \tag{\%o31}$$

(%i32) `is(%=Q2);`

$$\text{true} \tag{\%o32}$$