Kerr Debney metric

[Fwd: Maxima's christoffel]

computing christoffel symbols

Christoffel symbols of the first kind ill calculated

galgebra_gr_metrics

(%i2) `info:build_info()$info@version;`

(%o2)

5.38.1

(%i2) `reset()$kill(all)$`

(%i1) `load(linearalgebra)$`

(%i2) `if get('itensor,'version)=false then load(itensor)$`

(%i3) `imetric(g)$`

(%i4) `if get('ctensor,'version)=false then load(ctensor)$`

(%i10)
```
ctrgsimp:true$
ratchristof:true$
ratriemann:true$
rateinstein:true$
ratweyl:true$
ratfac:true$
```

(%i11) `derivabbrev:true$`

(%i12) `declare(trigsimp,evfun)$`

1 Minkowski Spacetime Metric

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

(%i13) `ζ:ct_coords:[t,x,y,z]$`

(%i14) `dim:length(ct_coords)$`

(%i15) `lg:η:matrix([1,0,0,0],[0,-1,0,0],[0,0,-1,0],[0,0,0,-1])$`

Sets up the package for further calculations

(%i16) `cmetric()$`

Covariant Metric tensor

(%i17) `ishow('g([μ,ν],[])=lg)$`

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (\%t17)$$

(%i20)
```
remcomps(g([μ,ν],[]))$
components(g([μ,ν],[]),lg)$
showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (\%t20)$$

(%i22)
```
decsym(g,2,0,[sym(all)],[])$
dispsym(g,2,0);
```

$$[[sym, [[1,2]], []]] \quad (\%o22)$$

Contravariant Metric tensor

(%i23) `ishow('g([],[μ,ν])=ug)$`

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (\%t23)$$

(%i26)
```
remcomps(g([],[μ,ν]))$
components(g([],[μ,ν]),ug)$
showcomps(g([],[μ,ν]))$
```

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (\%t26)$$

(%i28) `decsym(g,0,2,[],[sym(all)])$`
`dispsym(g,0,2);`

$$[[sym,[],[[1,2]]]] \tag{\%o28}$$

The determinant of the metric tensor

(%i29) `gdet;`

$$-1 \tag{\%o29}$$

Physical components (coframe)

(%i30) `ishow(√(lg[1,1])*∂([ζ[1]],[]))$`

$$\partial_t \tag{\%t30}$$

(%i31) `ishow(√(-lg[2,2])*∂([ζ[2]],[]))$`

$$\partial_x \tag{\%t31}$$

(%i32) `ishow(√(-lg[3,3])*∂([ζ[3]],[]))$`

$$\partial_y \tag{\%t32}$$

(%i33) `ishow(√(-lg[4,4])*∂([ζ[4]],[]))$`

$$\partial_z \tag{\%t33}$$

Line element

(%i34) `ldisplay(ds^2=expand(transpose(diff(ζ)).lg.diff(ζ)))$`

$$ds^2 = -\mathrm{del}(z)^2 - \mathrm{del}(y)^2 - \mathrm{del}(x)^2 + \mathrm{del}(t)^2 \tag{\%t34}$$

Christoffel Symbol of the first kind

(%i35) `christof(lcs)$`

(%i36)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ζ[i],ζ[j],ζ[k]],[])=lcs[i,j,k])$
```

(%i37) `dispsym(ichr1,3,0);`

$$[[sym,[[1,2]],[]]] \tag{\%o37}$$

(%i38) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\Gamma_{\alpha\beta\mu} = \frac{g_{\beta\mu,\alpha} + g_{\alpha\mu,\beta} - g_{\alpha\beta,\mu}}{2} \tag{\%t38}$$

Christoffel Symbol of the second kind

(%i39)
```
christof(mcs)$
```

(%i40)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ζ[i],ζ[j]],[ζ[k]])=mcs[i,j,k])$
```

(%i41)
```
dispsym(ichr2,2,1);
```

$$[[sym,[[1,2]],[]]] \tag{\%o41}$$

(%i42)
```
ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$
```

$$\Gamma_{\alpha\beta}^{\mu}=\frac{\mathrm{g}^{\mu\nu}\ \left(\mathrm{g}_{\beta\nu,\alpha}+\mathrm{g}_{\alpha\nu,\beta}-\mathrm{g}_{\alpha\beta,\nu}\right)}{2} \tag{\%t42}$$

2 Kerr Debney metric

$$\begin{bmatrix} 0 & 0 & -e^{-z} & 0 \\ 0 & \frac{u^2 e^{4z}}{2} & 0 & 0 \\ -e^{-z} & 0 & 12e^{-2z} & ue^{-z} \\ 0 & 0 & ue^{-z} & \frac{u^2}{2} \end{bmatrix}$$

(%i43) `init_ctensor()$`

(%i44) `assume(0≤u)$`

(%i45) `ζ:ct_coords:[u,x,y,z]$`

(%i46) `dim:length(ct_coords)$`

(%i47) `lg:matrix([0,0,-exp(-z),0], [0,`$\frac{1}{2}$`*u**2*exp(4*z),0,0], [-exp(-z),0,12*exp(-2*z),u*exp(-z)], [0,0,u*exp(-z),`$\frac{1}{2}$`*u**2])$`

Sets up the package for further calculations

(%i48) `cmetric()$`

Covariant Metric tensor

(%i49) `ishow('g([μ,ν],[])=lg)$`

$$g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -e^{-z} & 0 \\ 0 & \frac{u^2 e^{4z}}{2} & 0 & 0 \\ -e^{-z} & 0 & 12e^{-2z} & u\,e^{-z} \\ 0 & 0 & u\,e^{-z} & \frac{u^2}{2} \end{pmatrix} \qquad (\%t49)$$

(%i52)
```
remcomps(g([μ,ν],[]))$
components(g([μ,ν],[]),lg)$
showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} 0 & 0 & -e^{-z} & 0 \\ 0 & \frac{u^2 e^{4z}}{2} & 0 & 0 \\ -e^{-z} & 0 & 12e^{-2z} & u\,e^{-z} \\ 0 & 0 & u\,e^{-z} & \frac{u^2}{2} \end{pmatrix} \qquad (\%t52)$$

(%i55)
```
remsym(g,2,0)$
decsym(g,2,0,[sym(all)],[])$
dispsym(g,2,0);
```

$$[[sym, [[1, 2]], []]] \qquad (\%o55)$$

Contravariant Metric tensor

(%i56) `ishow('g([],[μ,ν])=ug)$`

$$g^{\mu\nu} = \begin{pmatrix} -10 & 0 & -e^{z} & \frac{2}{u} \\ 0 & \frac{2e^{-4z}}{u^2} & 0 & 0 \\ -e^{z} & 0 & 0 & 0 \\ \frac{2}{u} & 0 & 0 & \frac{2}{u^2} \end{pmatrix} \qquad (\%t56)$$

(%i59) `remcomps(g([],[μ,ν]))$`
`components(g([],[μ,ν]),ug)$`
`showcomps(g([],[μ,ν]))$`

$$g^{\mu\nu} = \begin{pmatrix} -10 & 0 & -e^z & \frac{2}{u} \\ 0 & \frac{2e^{-4z}}{u^2} & 0 & 0 \\ -e^z & 0 & 0 & 0 \\ \frac{2}{u} & 0 & 0 & \frac{2}{u^2} \end{pmatrix} \tag{\%t59}$$

(%i62) `remsym(g,0,2)$`
`decsym(g,0,2,[],[sym(all)])$`
`dispsym(g,0,2);`

$$[[sym, [], [[1, 2]]]] \tag{\%o62}$$

The determinant of the metric tensor

(%i63) `gdet;`

$$-\frac{u^4\, e^{2z}}{4} \tag{\%o63}$$

Physical components (coframe)

(%i64) `ishow(√(lg[1,1])*∂([ζ[1]],[]))$`

$$0 \tag{\%t64}$$

(%i65) `ishow(√(lg[2,2])*∂([ζ[2]],[]))$`

$$\frac{u\, \partial_x\, e^{2z}}{\sqrt{2}} \tag{\%t65}$$

(%i66) `ishow(√(lg[3,3])*∂([ζ[3]],[]))$`

$$2\sqrt{3}\, \partial_y\, e^{-z} \tag{\%t66}$$

(%i67) `ishow(√(lg[4,4])*∂([ζ[4]],[]))$`

$$\frac{u\, \partial_z}{\sqrt{2}} \tag{\%t67}$$

Line element

(%i68) `ldisplay(ds²=expand(transpose(diff(ζ)).lg.diff(ζ)))$`

$$ds^2 = 2u\, e^{-z}\, \mathrm{del}(y)\, \mathrm{del}(z) - 2e^{-z}\, \mathrm{del}(u)\, \mathrm{del}(y) + \frac{u^2\, \mathrm{del}(z)^2}{2} + 12e^{-2z}\, \mathrm{del}(y)^2 + \frac{u^2\, e^{4z}\, \mathrm{del}(x)^2}{2} \tag{\%t68}$$

Christoffel Symbol of the first kind

(%i69) `christof(lcs)$`

$$lcs_{1,2,2} = \frac{u\,e^{4z}}{2} \tag{\%t69}$$

$$lcs_{1,4,3} = e^{-z} \tag{\%t70}$$

$$lcs_{1,4,4} = \frac{u}{2} \tag{\%t71}$$

$$lcs_{2,2,1} = -\frac{u\,e^{4z}}{2} \tag{\%t72}$$

$$lcs_{2,2,4} = -u^2\,e^{4z} \tag{\%t73}$$

$$lcs_{2,4,2} = u^2\,e^{4z} \tag{\%t74}$$

$$lcs_{3,3,4} = 12e^{-2z} \tag{\%t75}$$

$$lcs_{3,4,3} = -12e^{-2z} \tag{\%t76}$$

$$lcs_{4,4,1} = -\frac{u}{2} \tag{\%t77}$$

$$lcs_{4,4,3} = -u\,e^{-z} \tag{\%t78}$$

(%i79)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ζ[i],ζ[j],ζ[k]],[])=lcs[i,j,k])$
```

$$\Gamma_{uxx} = \frac{u\,e^{4z}}{2} \tag{\%t79}$$

$$\Gamma_{uzy} = e^{-z} \tag{\%t79}$$

$$\Gamma_{uzz} = \frac{u}{2} \tag{\%t79}$$

$$\Gamma_{xxu} = -\frac{u\,e^{4z}}{2} \tag{\%t79}$$

$$\Gamma_{xxz} = -u^2\,e^{4z} \tag{\%t79}$$

$$\Gamma_{xzx} = u^2\,e^{4z} \tag{\%t79}$$

$$\Gamma_{yyz} = 12e^{-2z} \tag{\%t79}$$

$$\Gamma_{yzy} = -12e^{-2z} \tag{\%t79}$$

$$\Gamma_{zzu} = -\frac{u}{2} \tag{\%t79}$$

$$\Gamma_{zzy} = -u\,e^{-z} \tag{\%t79}$$

(%i80) `dispsym(ichr1,3,0);`

$$[[sym, [[1,2]], []]] \tag{\%o80}$$

(%i81) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\Gamma_{\alpha\beta\mu} = \frac{g_{\beta\mu,\alpha} + g_{\alpha\mu,\beta} - g_{\alpha\beta,\mu}}{2} \qquad (\%t81)$$

(%i82) `ishow('Γ([α,β,1])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,1]),dim,dim)))$`

$$\Gamma_{\alpha\beta 1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{u\,e^{4z}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{u}{2} \end{pmatrix} \qquad (\%t82)$$

(%i83) `ishow('Γ([α,β,2])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,2]),dim,dim)))$`

$$\Gamma_{\alpha\beta 2} = \begin{pmatrix} 0 & \frac{u\,e^{4z}}{2} & 0 & 0 \\ \frac{u\,e^{4z}}{2} & 0 & 0 & u^2\,e^{4z} \\ 0 & 0 & 0 & 0 \\ 0 & u^2\,e^{4z} & 0 & 0 \end{pmatrix} \qquad (\%t83)$$

(%i84) `ishow('Γ([α,β,3])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,3]),dim,dim)))$`

$$\Gamma_{\alpha\beta 3} = \begin{pmatrix} 0 & 0 & 0 & e^{-z} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -12e^{-2z} \\ e^{-z} & 0 & -12e^{-2z} & -u\,e^{-z} \end{pmatrix} \qquad (\%t84)$$

(%i85) `ishow('Γ([α,β,4])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,4]),dim,dim)))$`

$$\Gamma_{\alpha\beta 4} = \begin{pmatrix} 0 & 0 & 0 & \frac{u}{2} \\ 0 & -u^2\,e^{4z} & 0 & 0 \\ 0 & 0 & 12e^{-2z} & 0 \\ \frac{u}{2} & 0 & 0 & 0 \end{pmatrix} \qquad (\%t85)$$

Christoffel Symbol of the second kind

(%i86) `christof(mcs)$`

$$mcs_{1,2,2} = \frac{1}{u} \qquad (\%t86)$$

$$mcs_{1,4,4} = \frac{1}{u} \qquad (\%t87)$$

$$mcs_{2,2,1} = 3u\,e^{4z} \qquad (\%t88)$$

$$mcs_{2,2,3} = \frac{u\,e^{5z}}{2} \qquad (\%t89)$$

$$mcs_{2,2,4} = -3e^{4z} \qquad (\%t90)$$

$$mcs_{2,4,2} = 2 \qquad (\%t91)$$

$$mcs_{3,3,1} = \frac{24e^{-2z}}{u} \qquad (\%t92)$$

$$mcs_{3,3,4} = \frac{24e^{-2z}}{u^2} \tag{\%t93}$$

$$mcs_{3,4,1} = 12e^{-z} \tag{\%t94}$$

$$mcs_{4,4,1} = 6u \tag{\%t95}$$

$$mcs_{4,4,3} = \frac{u\,e^{z}}{2} \tag{\%t96}$$

$$mcs_{4,4,4} = -1 \tag{\%t97}$$

(%i98)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ζ[i],ζ[j]],[ζ[k]])=mcs[i,j,k])$
```

$$\Gamma_{ux}^{x} = \frac{1}{u} \tag{\%t98}$$

$$\Gamma_{uz}^{z} = \frac{1}{u} \tag{\%t98}$$

$$\Gamma_{xx}^{u} = 3u\,e^{4z} \tag{\%t98}$$

$$\Gamma_{xx}^{y} = \frac{u\,e^{5z}}{2} \tag{\%t98}$$

$$\Gamma_{xx}^{z} = -3e^{4z} \tag{\%t98}$$

$$\Gamma_{xz}^{x} = 2 \tag{\%t98}$$

$$\Gamma_{yy}^{u} = \frac{24e^{-2z}}{u} \tag{\%t98}$$

$$\Gamma_{yy}^{z} = \frac{24e^{-2z}}{u^2} \tag{\%t98}$$

$$\Gamma_{yz}^{u} = 12e^{-z} \tag{\%t98}$$

$$\Gamma_{zz}^{u} = 6u \tag{\%t98}$$

$$\Gamma_{zz}^{y} = \frac{u\,e^{z}}{2} \tag{\%t98}$$

$$\Gamma_{zz}^{z} = -1 \tag{\%t98}$$

(%i99)
```
dispsym(ichr2,2,1);
```

$$[[sym, [[1, 2]], []]] \tag{\%o99}$$

(%i100)
```
ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$
```

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}\,(g_{\beta\nu,\alpha} + g_{\alpha\nu,\beta} - g_{\alpha\beta,\nu})}{2} \tag{\%t100}$$

(%i101)
```
ishow('Γ([α,β],[1])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,1]),dim,dim)))$
```

$$\Gamma_{\alpha\beta}^{1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3u\,e^{4z} & 0 & 0 \\ 0 & 0 & \frac{24e^{-2z}}{u} & 12e^{-z} \\ 0 & 0 & 12e^{-z} & 6u \end{pmatrix} \tag{\%t101}$$

(%i102) `ishow('Γ([α,β],[2])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,2]),dim,dim)))$`

$$\Gamma^{2}_{\alpha\beta} = \begin{pmatrix} 0 & \frac{1}{u} & 0 & 0 \\ \frac{1}{u} & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix} \quad (\%t102)$$

(%i103) `ishow('Γ([α,β],[3])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,3]),dim,dim)))$`

$$\Gamma^{3}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{u\,e^{5z}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{u\,e^{z}}{2} \end{pmatrix} \quad (\%t103)$$

(%i104) `ishow('Γ([α,β],[4])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,4]),dim,dim)))$`

$$\Gamma^{4}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{u} \\ 0 & -3e^{4z} & 0 & 0 \\ 0 & 0 & \frac{24e^{-2z}}{u^2} & 0 \\ \frac{1}{u} & 0 & 0 & -1 \end{pmatrix} \quad (\%t104)$$

Riemann tensor

(%i106)
```
riemann(false)$ for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ζ[a],ζ[b],ζ[c]],[ζ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}^{u}_{uzy} = \frac{12e^{-z}}{u} \quad (\%t106)$$

$$\mathbf{R}^{u}_{xyx} = 24e^{3z} \quad (\%t106)$$

$$\mathbf{R}^{u}_{xzx} = 6u\,e^{4z} \quad (\%t106)$$

$$\mathbf{R}^{u}_{yyu} = -\frac{24e^{-2z}}{u^2} \quad (\%t106)$$

$$\mathbf{R}^{x}_{yyx} = \frac{72e^{-2z}}{u^2} \quad (\%t106)$$

$$\mathbf{R}^{x}_{yzx} = \frac{12e^{-z}}{u} \quad (\%t106)$$

$$\mathbf{R}^{u}_{yzy} = -\frac{96e^{-2z}}{u} \quad (\%t106)$$

$$\mathbf{R}^{y}_{yzy} = -\frac{12e^{-z}}{u} \quad (\%t106)$$

$$\mathbf{R}^{u}_{zyu} = -\frac{12e^{-z}}{u} \quad (\%t106)$$

$$\mathbf{R}^{x}_{zyx} = \frac{12e^{-z}}{u} \quad (\%t106)$$

$$\mathbf{R}_{zzy}^{u} = 12e^{-z} \tag{\%t106}$$

(%i107)
```
dispsym(icurvature,3,1);
```

$$[[anti,[[2,3]],[]]] \tag{\%o107}$$

(%i109)
```
lriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if lriem[a,b,c,d]≠0 then
ishow('R([ζ[d],ζ[a],ζ[b],ζ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{xyyx} = 36e^{2z} \tag{\%t109}$$

$$\mathbf{R}_{xyzx} = 6u\, e^{3z} \tag{\%t109}$$

$$\mathbf{R}_{uyzy} = \frac{12e^{-2z}}{u} \tag{\%t109}$$

$$\mathbf{R}_{yzyu} = \frac{12e^{-2z}}{u} \tag{\%t109}$$

$$\mathbf{R}_{xzyx} = 6u\, e^{3z} \tag{\%t109}$$

$$\mathbf{R}_{yzzy} = -12e^{-2z} \tag{\%t109}$$

(%i111)
```
uriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if uriem[a,b,c,d]≠0 then
ishow('R([],[ζ[a],ζ[b],ζ[c],ζ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{xxuu} = \frac{48e^{-4z}}{u^4} \tag{\%t111}$$

$$\mathbf{R}^{xzxu} = \frac{48e^{-4z}}{u^5} \tag{\%t111}$$

$$\mathbf{R}^{yzuu} = \frac{24e^{z}}{u^3} \tag{\%t111}$$

$$\mathbf{R}^{zxux} = \frac{48e^{-4z}}{u^5} \tag{\%t111}$$

$$\mathbf{R}^{zyuu} = \frac{24e^{z}}{u^3} \tag{\%t111}$$

$$\mathbf{R}^{zzuu} = -\frac{144}{u^4} \tag{\%t111}$$

Ricci tensor

(%i114)
```
ric:zeromatrix(dim,dim)$
ricci(false)$
for i thru dim do for j:i thru dim do
if ric[i,j]≠0 then
ishow('R([ζ[i],ζ[j]])=ric[i,j])$
```

(%i117)
```
remcomps(R([μ,ν],[]))$
components(R([μ,ν],[]),ric)$
showcomps(R([μ,ν],[]))$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t117}$$

(%i119)
```
decsym(R,2,0,[sym(all)],[])$
dispsym(R,2,0);
```

$$[[sym, [[1,2]], []]] \tag{\%o119}$$

(%i122)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ζ[i],ζ[j]])=uric[i,j])$
```

(%i125)
```
remcomps(R([],[μ,ν]))$
components(R([],[μ,ν]),uric)$
showcomps(R([],[μ,ν]))$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t125}$$

(%i127)
```
decsym(R,0,2,[],[sym(all)])$
dispsym(R,0,2);
```

$$[[sym, [], [[1,2]]]] \tag{\%o127}$$

Scalar curvature

(%i128)
```
factor(radcan(scurvature()));
```

$$0 \tag{\%o128}$$

Kretschmann invariant

(%i129)
```
factor(radcan(rinvariant()));
```

$$0 \tag{\%o129}$$

Einstein tensor

(%i130) `kill(labels)$`

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ζ[i]],[ζ[j]])=ein[i,j])$
```

(%i4) `ishow('G([μ],[ν])=ein)$`

$$\mathbf{G}_{\mu}^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t4}$$

(%i7)
```
remcomps(G([μ],[ν]))$
components(G([μ],[ν]),ein)$
showcomps(G([μ],[ν]))$
```

$$\mathbf{G}_{\mu}^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t7}$$

(%i10)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ζ[i],ζ[j]],[])=lein[i,j])$
```

(%i11) `ishow('G([μ,ν],[])=lein)$`

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t11}$$

(%i14)
```
remcomps(G([μ,ν],[]))$
components(G([μ,ν],[]),lein)$
showcomps(G([μ,ν],[]))$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t14}$$

(%i16)
```
decsym(G,2,0,[sym(all)],[])$
dispsym(G,2,0);
```

$$[[sym, [[1, 2]], []]] \tag{\%o16}$$

Reduce Order

(%i18)
```
cv_coords:[U,X,Y,Z]$
depends(cv_coords,s)$
```

(%i22)
```
gradef(u,s,U)$
gradef(x,s,X)$
gradef(y,s,Y)$
gradef(z,s,Z)$
```

Geodesics

(%i23)
```
cgeodesic(false)$
```

Solve for second derivative of coordinates

(%i24)
```
geodsol:linsolve(listarray(geod),diff(ζ,s,2))$
```

(%i25)
```
map(ldisp,geodsol)$
```

$$U_s = -\frac{e^{-2z}\left(3X^2\,u^2\,e^{6z} + 6Z^2\,u^2\,e^{2z} + 24Y\,Z\,u\,e^z + 24Y^2\right)}{u} \qquad (\%t25)$$

$$X_s = -\frac{4X\,Z\,u + 2U\,X}{u} \qquad (\%t26)$$

$$Y_s = -\frac{X^2u\,e^{5z} + Z^2u\,e^z}{2} \qquad (\%t27)$$

$$Z_s = \frac{e^{-2z}\left(3X^2\,u^2\,e^{6z} + \left(Z^2\,u^2 - 2U\,Z\,u\right)\,e^{2z} - 24Y^2\right)}{u^2} \qquad (\%t28)$$

3 Schwarzschild Metric

The classic black hole solution. Uncharged and rotationally stationary.

$$\begin{bmatrix} -\frac{2GM}{c^2 r}+1 & 0 & 0 & 0 \\ 0 & -\frac{1}{-\frac{2GM}{c^2 r}+1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}$$

(%i29) `kill(labels,t,r,θ,ϕ)$`

(%i1) `init_ctensor()$`

(%i2) `unorder()$`

(%i3) `orderless(M)$`

(%i7)
```
assume(0≤r)$
assume(0≤θ,θ≤π)$
assume(0≤sin(θ))$
assume(0≤ϕ,ϕ≤2*π)$
```

(%i8) `ξ:ct_coords:[t,r,θ,ϕ]$`

(%i9) `dim:length(ct_coords)$`

(%i10) `assume(c>0,M>0,G>0)$`

(%i11) `S:1-(2*G*M)/(c^2*r)$`

(%i12) `lg:matrix([S,0,0,0], [0,-1/S,0,0], [0,0,-r^2,0], [0,0,0,-r^2*sin(θ)^2])$`

Sets up the package for further calculations

(%i13) `cmetric()$`

Covariant Metric tensor

(%i14) `ishow('g([μ,ν],[])=lg)$`

$$g_{\mu\nu} = \begin{pmatrix} 1-\frac{2MG}{c^2 r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2MG}{c^2 r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{pmatrix} \tag{\%t14}$$

(%i17)
```
remcomps(g([μ,ν],[]))$
components(g([μ,ν],[]),lg)$
showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} 1-\frac{2MG}{c^2 r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2MG}{c^2 r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{pmatrix} \tag{\%t17}$$

(%i20)
```
remsym(g,2,0)$
decsym(g,2,0,[sym(all)],[])$
dispsym(g,2,0);
```

$$[[sym,[[1,2]],[]]] \tag{\%o20}$$

Contravariant Metric tensor

(%i21) `ishow('g([],[μ,ν])=ug)$`

$$\mathrm{g}^{\mu\nu}=\begin{pmatrix}-\frac{c^2 r}{2MG-c^2 r} & 0 & 0 & 0\\ 0 & \frac{2MG-c^2 r}{c^2 r} & 0 & 0\\ 0 & 0 & -\frac{1}{r^2} & 0\\ 0 & 0 & 0 & -\frac{1}{r^2\sin(\theta)^2}\end{pmatrix} \quad (\%t21)$$

(%i24)
```
remcomps(g([],[μ,ν]))$
components(g([],[μ,ν]),ug)$
showcomps(g([],[μ,ν]))$
```

$$\mathrm{g}^{\mu\nu}=\begin{pmatrix}-\frac{c^2 r}{2MG-c^2 r} & 0 & 0 & 0\\ 0 & \frac{2MG-c^2 r}{c^2 r} & 0 & 0\\ 0 & 0 & -\frac{1}{r^2} & 0\\ 0 & 0 & 0 & -\frac{1}{r^2\sin(\theta)^2}\end{pmatrix} \quad (\%t24)$$

(%i27)
```
remsym(g,0,2)$
decsym(g,0,2,[],[sym(all)])$
dispsym(g,0,2);
```

$$[[sym,[],[[1,2]]]] \quad (\%o27)$$

The determinant of the metric tensor

(%i28) `gdet;`

$$-r^4\sin(\theta)^2 \quad (\%o28)$$

Physical components (coframe)

(%i29) `ishow(√(lg[1,1])*∂([ξ[1]],[]))$`

$$\sqrt{1-\frac{2MG}{c^2 r}}\,\partial_t \quad (\%t29)$$

(%i30) `ishow(√(-lg[2,2])*∂([ξ[2]],[]))$`

$$\frac{\partial_r}{\sqrt{1-\frac{2MG}{c^2 r}}} \quad (\%t30)$$

(%i31) `ishow(√(-lg[3,3])*∂([ξ[3]],[]))$`

$$r\,\partial_\theta \quad (\%t31)$$

(%i32) `ishow(√(-lg[4,4])*∂([ξ[4]],[]))$`

$$r\,\sin(\theta)\,\partial_\phi \quad (\%t32)$$

Line element

(%i33) `ldisplay(ds^2=expand(transpose(diff(ξ)).lg.diff(ξ)))$`

$$ds^2 = -r^2 \sin(\theta)^2 \operatorname{del}(\phi)^2 - r^2 \operatorname{del}(\theta)^2 - \frac{2MG \operatorname{del}(t)^2}{c^2 r} + \operatorname{del}(t)^2 - \frac{\operatorname{del}(r)^2}{1 - \frac{2MG}{c^2 r}} \quad (\%t33)$$

Christoffel Symbol of the first kind

(%i34) `christof(lcs)$`

$$lcs_{1,1,2} = -\frac{MG}{c^2 r^2} \quad (\%t34)$$

$$lcs_{1,2,1} = \frac{MG}{c^2 r^2} \quad (\%t35)$$

$$lcs_{2,2,2} = \frac{MG}{c^2 \left(1 - \frac{2MG}{c^2 r}\right)^2 r^2} \quad (\%t36)$$

$$lcs_{2,3,3} = -r \quad (\%t37)$$

$$lcs_{2,4,4} = -r \sin(\theta)^2 \quad (\%t38)$$

$$lcs_{3,3,2} = r \quad (\%t39)$$

$$lcs_{3,4,4} = -r^2 \cos(\theta) \sin(\theta) \quad (\%t40)$$

$$lcs_{4,4,2} = r \sin(\theta)^2 \quad (\%t41)$$

$$lcs_{4,4,3} = r^2 \cos(\theta) \sin(\theta) \quad (\%t42)$$

(%i43)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j],ξ[k]],[])=lcs[i,j,k])$
```

$$\Gamma_{ttr} = -\frac{MG}{c^2 r^2} \quad (\%t43)$$

$$\Gamma_{trt} = \frac{MG}{c^2 r^2} \quad (\%t43)$$

$$\Gamma_{rrr} = \frac{MG}{c^2 \left(1 - \frac{2MG}{c^2 r}\right)^2 r^2} \quad (\%t43)$$

$$\Gamma_{r\theta\theta} = -r \quad (\%t43)$$

$$\Gamma_{r\phi\phi} = -r \sin(\theta)^2 \quad (\%t43)$$

$$\Gamma_{\theta\theta r} = r \quad (\%t43)$$

$$\Gamma_{\theta\phi\phi} = -r^2 \cos(\theta) \sin(\theta) \quad (\%t43)$$

$$\Gamma_{\phi\phi r} = r \sin(\theta)^2 \quad (\%t43)$$

$$\Gamma_{\phi\phi\theta} = r^2 \cos(\theta) \sin(\theta) \quad (\%t43)$$

(%i44) `dispsym(ichr1,3,0);`

$$[[sym, [[1, 2]], []]] \tag{\%o44}$$

(%i45) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\mathit{\Gamma}_{\alpha\beta\mu} = \frac{g_{\beta\mu,\alpha} + g_{\alpha\mu,\beta} - g_{\alpha\beta,\mu}}{2} \tag{\%t45}$$

(%i46) `ishow('Γ([α,β,1])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,1]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 1} = \begin{pmatrix} 0 & \frac{MG}{c^2 r^2} & 0 & 0 \\ \frac{MG}{c^2 r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t46}$$

(%i47) `ishow('Γ([α,β,2])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,2]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 2} = \begin{pmatrix} -\frac{MG}{c^2 r^2} & 0 & 0 & 0 \\ 0 & \frac{MG\, c^2}{(2MG-c^2 r)^2} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r \sin(\theta)^2 \end{pmatrix} \tag{\%t47}$$

(%i48) `ishow('Γ([α,β,3])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,3]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & 0 & r^2 \cos(\theta) \sin(\theta) \end{pmatrix} \tag{\%t48}$$

(%i49) `ishow('Γ([α,β,4])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,4]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r \sin(\theta)^2 \\ 0 & 0 & 0 & -r^2 \cos(\theta) \sin(\theta) \\ 0 & -r \sin(\theta)^2 & -r^2 \cos(\theta) \sin(\theta) & 0 \end{pmatrix} \tag{\%t49}$$

Christoffel Symbol of the second kind

(%i50) `christof(mcs)$`

$$mcs_{1,1,2} = -\frac{MG\left(2MG - c^2 r\right)}{c^4 r^3} \tag{\%t50}$$

$$mcs_{1,2,1} = -\frac{MG}{r\,(2MG - c^2 r)} \tag{\%t51}$$

$$mcs_{2,2,2} = \frac{MG}{r\,(2MG - c^2 r)} \tag{\%t52}$$

$$mcs_{2,3,3} = \frac{1}{r} \tag{\%t53}$$

$$mcs_{2,4,4} = \frac{1}{r} \tag{\%t54}$$

$$mcs_{3,3,2} = \frac{2MG - c^2 r}{c^2} \tag{\%t55}$$

$$mcs_{3,4,4} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t56}$$

$$mcs_{4,4,2} = \frac{\left(2MG - c^2 r\right)\sin(\theta)^2}{c^2} \tag{\%t57}$$

$$mcs_{4,4,3} = -\cos(\theta)\sin(\theta) \tag{\%t58}$$

(%i59)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j]],[ξ[k]])=mcs[i,j,k])$
```

$$\Gamma_{tt}^{r} = -\frac{MG\left(2MG - c^2 r\right)}{c^4 r^3} \tag{\%t59}$$

$$\Gamma_{tr}^{t} = -\frac{MG}{r\left(2MG - c^2 r\right)} \tag{\%t59}$$

$$\Gamma_{rr}^{r} = \frac{MG}{r\left(2MG - c^2 r\right)} \tag{\%t59}$$

$$\Gamma_{r\theta}^{\theta} = \frac{1}{r} \tag{\%t59}$$

$$\Gamma_{r\phi}^{\phi} = \frac{1}{r} \tag{\%t59}$$

$$\Gamma_{\theta\theta}^{r} = \frac{2MG - c^2 r}{c^2} \tag{\%t59}$$

$$\Gamma_{\theta\phi}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t59}$$

$$\Gamma_{\phi\phi}^{r} = \frac{\left(2MG - c^2 r\right)\sin(\theta)^2}{c^2} \tag{\%t59}$$

$$\Gamma_{\phi\phi}^{\theta} = -\cos(\theta)\sin(\theta) \tag{\%t59}$$

(%i60)
```
dispsym(ichr2,2,1);
```

$$[[sym, [[1, 2]], []]] \tag{\%o60}$$

(%i61)
```
ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$
```

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}\left(g_{\beta\nu,\alpha} + g_{\alpha\nu,\beta} - g_{\alpha\beta,\nu}\right)}{2} \tag{\%t61}$$

(%i62)
```
ishow('Γ([α,β],[1])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,1]),dim,dim)))$
```

$$\Gamma_{\alpha\beta}^{1} = \begin{pmatrix} 0 & -\frac{MG}{r\left(2MG - c^2 r\right)} & 0 & 0 \\ -\frac{MG}{r\left(2MG - c^2 r\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t62}$$

(%i63) `ishow('Γ([α,β],[2])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,2]),dim,dim)))$`

$$\Gamma^{2}_{\alpha\beta} = \begin{pmatrix} -\frac{MG\left(2MG-c^2 r\right)}{c^4\, r^3} & 0 & 0 & 0 \\ 0 & \frac{MG}{r\,(2MG-c^2 r)} & 0 & 0 \\ 0 & 0 & \frac{2MG-c^2 r}{c^2} & 0 \\ 0 & 0 & 0 & \frac{\left(2MG-c^2 r\right)\sin(\theta)^2}{c^2} \end{pmatrix} \tag{\%t63}$$

(%i64) `ishow('Γ([α,β],[3])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,3]),dim,dim)))$`

$$\Gamma^{3}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\,\sin(\theta) \end{pmatrix} \tag{\%t64}$$

(%i65) `ishow('Γ([α,β],[4])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,4]),dim,dim)))$`

$$\Gamma^{4}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{pmatrix} \tag{\%t65}$$

Riemann tensor

(%i67)
```
riemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ξ[a],ξ[b],ξ[c]],[ξ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}^{t}_{rrt} = -\frac{2MG}{r^2\,\left(2MG-c^2 r\right)} \tag{\%t67}$$

$$\mathbf{R}^{t}_{\theta\theta t} = -\frac{MG}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{r}_{\theta\theta r} = -\frac{MG}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{t}_{\phi\phi t} = -\frac{MG\sin(\theta)^2}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{r}_{\phi\phi r} = -\frac{MG\sin(\theta)^2}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{\theta}_{\phi\phi\theta} = \frac{2MG\sin(\theta)^2}{c^2 r} \tag{\%t67}$$

(%i68) `dispsym(icurvature,3,1);`

$$[[anti,[[2,3]],[]]] \tag{\%o68}$$

(%i70)
```
lriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if lriem[a,b,c,d]≠0 then
ishow('R([ξ[d],ξ[a],ξ[b],ξ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{trrt} = \frac{2MG}{c^2\,r^3} \tag{\%t70}$$

$$\mathbf{R}_{t\theta\theta t} = \frac{MG\,\left(2MG - c^2 r\right)}{c^4\,r^2} \tag{\%t70}$$

$$\mathbf{R}_{r\theta\theta r} = -\frac{MG}{2MG - c^2 r} \tag{\%t70}$$

$$\mathbf{R}_{t\phi\phi t} = \frac{MG\,\left(2MG - c^2 r\right)\,\sin\left(\theta\right)^2}{c^4\,r^2} \tag{\%t70}$$

$$\mathbf{R}_{r\phi\phi r} = -\frac{MG\sin\left(\theta\right)^2}{2MG - c^2 r} \tag{\%t70}$$

$$\mathbf{R}_{\theta\phi\phi\theta} = -\frac{2MGr\sin\left(\theta\right)^2}{c^2} \tag{\%t70}$$

(%i72)
```
uriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if uriem[a,b,c,d]≠0 then
ishow('R([],[ξ[a],ξ[b],ξ[c],ξ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{rrtt} = \frac{2MG}{c^2\,r^3} \tag{\%t72}$$

$$\mathbf{R}^{\theta\theta tt} = \frac{MG}{r^4\,\left(2MG - c^2 r\right)} \tag{\%t72}$$

$$\mathbf{R}^{\theta\theta rr} = -\frac{MG\,\left(2MG - c^2 r\right)}{c^4\,r^6} \tag{\%t72}$$

$$\mathbf{R}^{\phi\phi tt} = \frac{MG}{r^4\,\left(2MG - c^2 r\right)\,\sin\left(\theta\right)^2} \tag{\%t72}$$

$$\mathbf{R}^{\phi\phi rr} = -\frac{MG\,\left(2MG - c^2 r\right)}{c^4\,r^6\sin\left(\theta\right)^2} \tag{\%t72}$$

$$\mathbf{R}^{\phi\phi\theta\theta} = -\frac{2MG}{c^2\,r^7\sin\left(\theta\right)^2} \tag{\%t72}$$

Ricci tensor

(%i75)
```
ric:zeromatrix(dim,dim)$
ricci(false)$
for i thru dim do for j:i thru dim do
if ric[i,j]≠0 then
ishow('R([ξ[i],ξ[j]])=ric[i,j])$
```

(%i78)
```
remcomps(R([μ,ν],[]))$
components(R([μ,ν],[]),ric)$
showcomps(R([μ,ν],[]))$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t78}$$

(%i80)
```
decsym(R,2,0,[sym(all)],[])$
dispsym(R,2,0);
```

$$[[sym, [[1, 2], [1, 2]], []]] \tag{\%o80}$$

(%i83)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ξ[i],ξ[j]])=uric[i,j])$
```

(%i86)
```
remcomps(R([],[μ,ν]))$
components(R([],[μ,ν]),uric)$
showcomps(R([],[μ,ν]))$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t86}$$

(%i88)
```
decsym(R,0,2,[],[sym(all)])$
dispsym(R,0,2);
```

$$[[sym, [], [[1, 2], [1, 2]]]] \tag{\%o88}$$

Scalar curvature

(%i89)
```
factor(radcan(scurvature()));
```

$$0 \tag{\%o89}$$

Kretschmann invariant

(%i90)
```
factor(radcan(rinvariant()));
```

$$\frac{48M^2\,G^2}{c^4\,r^6} \tag{\%o90}$$

Einstein tensor

(%i91)
```
kill(labels)$
```

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ξ[i]],[ξ[j]])=ein[i,j])$
```

(%i4)
```
ishow('G([μ],[ν])=ein)$
```

$$\mathbf{G}_{\mu}^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t4}$$

(%i7)
```
remcomps(G([μ],[ν]))$
components(G([μ],[ν]),ein)$
showcomps(G([μ],[ν]))$
```

$$\mathbf{G}_{\mu}^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t7}$$

(%i10)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ξ[i],ξ[j]],[])=lein[i,j])$
```

(%i11)
```
ishow('G([μ,ν],[])=lein)$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t11}$$

(%i14)
```
remcomps(G([μ,ν],[]))$
components(G([μ,ν],[]),lein)$
showcomps(G([μ,ν],[]))$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t14}$$

(%i16)
```
decsym(G,2,0,[sym(all)],[])$
dispsym(G,2,0);
```

$$[[sym, [[1, 2], [1, 2]], []]] \tag{\%o16}$$

Reduce Order

(%i18)
```
cv_coords:[T,R,Θ,Φ]$
depends(cv_coords,s)$
```

(%i22)
```
gradef(t,s,T)$
gradef(r,s,R)$
gradef(θ,s,Θ)$
gradef(φ,s,Φ)$
```

Geodesics

(%i23)
```
cgeodesic(false)$
```

Solve for second derivative of coordinates

(%i24)
```
geodsol:linsolve(listarray(geod),diff(ξ,s,2))$
```

(%i25)
```
map(ldisp,geodsol)$
```

$$T_s = -\frac{2MGRT}{c^2\,r^2 - 2MGr} \tag{\%t25}$$

$$R_s = (M^2\,G^2\,c^2\,r^3\,\left(4\Phi^2\sin(\theta)^2 + 4\Theta^2\right) + c^6\,r^5\,\left(\Phi^2\sin(\theta)^2 + \Theta^2\right) + MG\,c^4\,r^4\,\left(-4\Phi^2\sin(\theta)^2 - 4\Theta^2\right) + \left(MG\,R^2 - MG\,T^2\right)\,c \tag{\%t26}$$

$$\Theta_s = \frac{r\,\Phi^2\,\cos(\theta)\,\sin(\theta) - 2R\Theta}{r} \tag{\%t27}$$

$$\Phi_s = -\frac{2R\Phi\,\sin(\theta) + 2r\Theta\Phi\,\cos(\theta)}{r\,\sin(\theta)} \tag{\%t28}$$

4 Einstein-Rosen Bridge Metric

The most famous wormhole solution.

$$\begin{bmatrix} \frac{-2m+r}{r} & 0 & 0 & 0 \\ 0 & -\frac{4r}{-4m+2r} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin(\theta)^2 \end{bmatrix}$$

(%i29) `kill(labels,t,r,θ,ϕ)$`

(%i1) `init_ctensor()$`

(%i2) `unorder()$`

(%i3) `orderless(m)$`

(%i7)
```
assume(0≤r)$
assume(0≤θ,θ≤π)$
assume(0≤sin(θ))$
assume(0≤ϕ,ϕ≤2*π)$
```

(%i8) `ξ:ct_coords:[t,r,θ,ϕ]$`

(%i9) `dim:length(ct_coords)$`

(%i10) `assume(m>0)$`

(%i11)
```
lg:matrix([(-2*m+r)/r,0,0,0], [0,-(4*r)/(-4*m+2*r),0,0], [0,0,-r^2,0],
[0,0,0,-r^2*sin(θ)^2])$
```

Sets up the package for further calculations

(%i12) `cmetric()$`

Covariant Metric tensor

(%i13) `ishow('g([μ,ν],[])=lg)$`

$$g_{\mu\nu} = \begin{pmatrix} \frac{r-2m}{r} & 0 & 0 & 0 \\ 0 & -\frac{4r}{2r-4m} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin(\theta)^2 \end{pmatrix} \qquad (\%t13)$$

(%i16)
```
remcomps(g([μ,ν],[]))$
components(g([μ,ν],[]),lg)$
showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} \frac{r-2m}{r} & 0 & 0 & 0 \\ 0 & -\frac{4r}{2r-4m} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin(\theta)^2 \end{pmatrix} \qquad (\%t16)$$

(%i19)
```
remsym(g,2,0)$
decsym(g,2,0,[sym(all)],[])$
dispsym(g,2,0);
```

$$[[sym,[[1,2]],[]]] \qquad (\%o19)$$

Contravariant Metric tensor

(%i20) `ishow('g([],[μ,ν])=ug)$`

$$g^{\mu\nu} = \begin{pmatrix} -\frac{r}{2m-r} & 0 & 0 & 0 \\ 0 & \frac{2m-r}{2r} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2 \sin(\theta)^2} \end{pmatrix} \quad (\%t20)$$

(%i23)
```
remcomps(g([],[μ,ν]))$
components(g([],[μ,ν]),ug)$
showcomps(g([],[μ,ν]))$
```

$$g^{\mu\nu} = \begin{pmatrix} -\frac{r}{2m-r} & 0 & 0 & 0 \\ 0 & \frac{2m-r}{2r} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2 \sin(\theta)^2} \end{pmatrix} \quad (\%t23)$$

(%i26)
```
remsym(g,0,2)$
decsym(g,0,2,[],[sym(all)])$
dispsym(g,0,2);
```

$$[[sym, [], [[1, 2]]]] \quad (\%o26)$$

The determinant of the metric tensor

(%i27) `gdet;`

$$-2r^4 \sin(\theta)^2 \quad (\%o27)$$

Physical components (coframe)

(%i28) `ishow(√(lg[1,1])*∂([ξ[1]],[]))$`

$$\frac{\sqrt{r-2m}\,\partial_t}{\sqrt{r}} \quad (\%t28)$$

(%i29) `ishow(√(-lg[2,2])*∂([ξ[2]],[]))$`

$$\frac{2\sqrt{r}\,\partial_r}{\sqrt{2r-4m}} \quad (\%t29)$$

(%i30) `ishow(√(-lg[3,3])*∂([ξ[3]],[]))$`

$$r\,\partial_\theta \quad (\%t30)$$

(%i31) `ishow(√(-lg[4,4])*∂([ξ[4]],[]))$`

$$r\,\sin(\theta)\,\partial_\phi \quad (\%t31)$$

Line element

(%i32) `ldisplay(ds`^{2}`=expand(transpose(diff(ξ)).lg.diff(ξ)))$`

$$ds^2 = -r^2 \sin(\theta)^2 \operatorname{del}(\phi)^2 - r^2 \operatorname{del}(\theta)^2 - \frac{2m \operatorname{del}(t)^2}{r} + \operatorname{del}(t)^2 - \frac{4r \operatorname{del}(r)^2}{2r - 4m} \tag{\%t32}$$

Christoffel Symbol of the first kind

(%i33) `christof(lcs)$`

$$lcs_{1,1,2} = \frac{\frac{r-2m}{r^2} - \frac{1}{r}}{2} \tag{\%t33}$$

$$lcs_{1,2,1} = \frac{\frac{1}{r} - \frac{r-2m}{r^2}}{2} \tag{\%t34}$$

$$lcs_{2,2,2} = \frac{\frac{8r}{(2r-4m)^2} - \frac{4}{2r-4m}}{2} \tag{\%t35}$$

$$lcs_{2,3,3} = -r \tag{\%t36}$$

$$lcs_{2,4,4} = -r \sin(\theta)^2 \tag{\%t37}$$

$$lcs_{3,3,2} = r \tag{\%t38}$$

$$lcs_{3,4,4} = -r^2 \cos(\theta) \sin(\theta) \tag{\%t39}$$

$$lcs_{4,4,2} = r \sin(\theta)^2 \tag{\%t40}$$

$$lcs_{4,4,3} = r^2 \cos(\theta) \sin(\theta) \tag{\%t41}$$

(%i42)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j],ξ[k]],[])=lcs[i,j,k])$
```

$$\mathit{\Gamma}_{ttr} = \frac{\frac{r-2m}{r^2} - \frac{1}{r}}{2} \tag{\%t42}$$

$$\mathit{\Gamma}_{trt} = \frac{\frac{1}{r} - \frac{r-2m}{r^2}}{2} \tag{\%t42}$$

$$\mathit{\Gamma}_{rrr} = \frac{\frac{8r}{(2r-4m)^2} - \frac{4}{2r-4m}}{2} \tag{\%t42}$$

$$\mathit{\Gamma}_{r\theta\theta} = -r \tag{\%t42}$$

$$\mathit{\Gamma}_{r\phi\phi} = -r \sin(\theta)^2 \tag{\%t42}$$

$$\mathit{\Gamma}_{\theta\theta r} = r \tag{\%t42}$$

$$\mathit{\Gamma}_{\theta\phi\phi} = -r^2 \cos(\theta) \sin(\theta) \tag{\%t42}$$

$$\mathit{\Gamma}_{\phi\phi r} = r \sin(\theta)^2 \tag{\%t42}$$

$$\mathit{\Gamma}_{\phi\phi\theta} = r^2 \cos(\theta) \sin(\theta) \tag{\%t42}$$

(%i43) `dispsym(ichr1,3,0);`

$$[[sym,[[1,2]],[]]] \quad (\%o43)$$

(%i44) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\mathit{\Gamma}_{\alpha\beta\mu}=\frac{g_{\beta\mu,\alpha}+g_{\alpha\mu,\beta}-g_{\alpha\beta,\mu}}{2} \quad (\%t44)$$

(%i45) `ishow('Γ([α,β,1])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,1]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 1}=\begin{pmatrix} 0 & \frac{m}{r^2} & 0 & 0 \\ \frac{m}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (\%t45)$$

(%i46) `ishow('Γ([α,β,2])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,2]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 2}=\begin{pmatrix} -\frac{m}{r^2} & 0 & 0 & 0 \\ 0 & \frac{2m}{(2m-r)^2} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\sin(\theta)^2 \end{pmatrix} \quad (\%t46)$$

(%i47) `ishow('Γ([α,β,3])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,3]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 3}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & 0 & r^2\cos(\theta)\sin(\theta) \end{pmatrix} \quad (\%t47)$$

(%i48) `ishow('Γ([α,β,4])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,4]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 4}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r\sin(\theta)^2 \\ 0 & 0 & 0 & -r^2\cos(\theta)\sin(\theta) \\ 0 & -r\sin(\theta)^2 & -r^2\cos(\theta)\sin(\theta) & 0 \end{pmatrix} \quad (\%t48)$$

Christoffel Symbol of the second kind

(%i49) `christof(mcs)$`

$$mcs_{1,1,2}=-\frac{m\,(2m-r)}{2r^3} \quad (\%t49)$$

$$mcs_{1,2,1}=-\frac{m}{(2m-r)\,r} \quad (\%t50)$$

$$mcs_{2,2,2}=\frac{m}{(2m-r)\,r} \quad (\%t51)$$

$$mcs_{2,3,3}=\frac{1}{r} \quad (\%t52)$$

$$mcs_{2,4,4} = \frac{1}{r} \tag{\%t53}$$

$$mcs_{3,3,2} = \frac{2m - r}{2} \tag{\%t54}$$

$$mcs_{3,4,4} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t55}$$

$$mcs_{4,4,2} = \frac{(2m - r)\sin(\theta)^2}{2} \tag{\%t56}$$

$$mcs_{4,4,3} = -\cos(\theta)\sin(\theta) \tag{\%t57}$$

(%i58)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j]],[ξ[k]])=mcs[i,j,k])$
```

$$\Gamma_{tt}^{r} = -\frac{m(2m - r)}{2r^3} \tag{\%t58}$$

$$\Gamma_{tr}^{t} = -\frac{m}{(2m - r)r} \tag{\%t58}$$

$$\Gamma_{rr}^{r} = \frac{m}{(2m - r)r} \tag{\%t58}$$

$$\Gamma_{r\theta}^{\theta} = \frac{1}{r} \tag{\%t58}$$

$$\Gamma_{r\phi}^{\phi} = \frac{1}{r} \tag{\%t58}$$

$$\Gamma_{\theta\theta}^{r} = \frac{2m - r}{2} \tag{\%t58}$$

$$\Gamma_{\theta\phi}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t58}$$

$$\Gamma_{\phi\phi}^{r} = \frac{(2m - r)\sin(\theta)^2}{2} \tag{\%t58}$$

$$\Gamma_{\phi\phi}^{\theta} = -\cos(\theta)\sin(\theta) \tag{\%t58}$$

(%i59)
```
dispsym(ichr2,2,1);
```

$$[[sym, [[1, 2]], []]] \tag{\%o59}$$

(%i60)
```
ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$
```

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}\left(g_{\beta\nu,\alpha} + g_{\alpha\nu,\beta} - g_{\alpha\beta,\nu}\right)}{2} \tag{\%t60}$$

(%i61)
```
ishow('Γ([α,β],[1])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,1]),dim,dim)))$
```

$$\Gamma_{\alpha\beta}^{1} = \begin{pmatrix} 0 & -\frac{m}{(2m-r)r} & 0 & 0 \\ -\frac{m}{(2m-r)r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t61}$$

(%i62) `ishow('Γ([α,β],[2])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,2]),dim,dim)))$`

$$\Gamma^{2}_{\alpha\beta} = \begin{pmatrix} -\frac{m\,(2m-r)}{2r^3} & 0 & 0 & 0 \\ 0 & \frac{m}{(2m-r)r} & 0 & 0 \\ 0 & 0 & \frac{2m-r}{2} & 0 \\ 0 & 0 & 0 & \frac{(2m-r)\sin(\theta)^2}{2} \end{pmatrix} \tag{\%t62}$$

(%i63) `ishow('Γ([α,β],[3])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,3]),dim,dim)))$`

$$\Gamma^{3}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\,\sin(\theta) \end{pmatrix} \tag{\%t63}$$

(%i64) `ishow('Γ([α,β],[4])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,4]),dim,dim)))$`

$$\Gamma^{4}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{pmatrix} \tag{\%t64}$$

Riemann tensor

(%i66)
```
riemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ξ[a],ξ[b],ξ[c]],[ξ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}^{t}_{rrt} = -\frac{2m}{(2m-r)\,r^2} \tag{\%t66}$$

$$\mathbf{R}^{t}_{\theta\theta t} = -\frac{m}{2r} \tag{\%t66}$$

$$\mathbf{R}^{r}_{\theta\theta r} = -\frac{m}{2r} \tag{\%t66}$$

$$\mathbf{R}^{t}_{\phi\phi t} = -\frac{m\sin(\theta)^2}{2r} \tag{\%t66}$$

$$\mathbf{R}^{r}_{\phi\phi r} = -\frac{m\sin(\theta)^2}{2r} \tag{\%t66}$$

$$\mathbf{R}^{\theta}_{\phi\phi\theta} = \frac{(r+2m)\,\sin(\theta)^2}{2r} \tag{\%t66}$$

(%i67) `dispsym(icurvature,3,1);`

$$[[anti,[[2,3]],[]]] \tag{\%o67}$$

(%i69)
```
lriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if lriem[a,b,c,d]≠0 then
ishow('R([ξ[d],ξ[a],ξ[b],ξ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{trrt} = \frac{2m}{r^3} \tag{\%t69}$$

$$\mathbf{R}_{t\theta\theta t} = \frac{m\,(2m-r)}{2r^2} \tag{\%t69}$$

$$\mathbf{R}_{r\theta\theta r} = -\frac{m}{2m-r} \tag{\%t69}$$

$$\mathbf{R}_{t\phi\phi t} = \frac{m\,(2m-r)\,\sin(\theta)^2}{2r^2} \tag{\%t69}$$

$$\mathbf{R}_{r\phi\phi r} = -\frac{m\sin(\theta)^2}{2m-r} \tag{\%t69}$$

$$\mathbf{R}_{\theta\phi\phi\theta} = -\frac{r\,(r+2m)\,\sin(\theta)^2}{2} \tag{\%t69}$$

(%i71)
```
uriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if uriem[a,b,c,d]≠0 then
ishow('R([],[ξ[a],ξ[b],ξ[c],ξ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{rrtt} = \frac{m}{2r^3} \tag{\%t71}$$

$$\mathbf{R}^{\theta\theta tt} = \frac{m}{2\,(2m-r)\,r^4} \tag{\%t71}$$

$$\mathbf{R}^{\theta\theta rr} = -\frac{m\,(2m-r)}{4r^6} \tag{\%t71}$$

$$\mathbf{R}^{\phi\phi tt} = \frac{m}{2\,(2m-r)\,r^4\sin(\theta)^2} \tag{\%t71}$$

$$\mathbf{R}^{\phi\phi rr} = -\frac{m\,(2m-r)}{4r^6\sin(\theta)^2} \tag{\%t71}$$

$$\mathbf{R}^{\phi\phi\theta\theta} = -\frac{r+2m}{2r^7\sin(\theta)^2} \tag{\%t71}$$

Ricci tensor

(%i74)
```
ric:zeromatrix(dim,dim)$
ricci(false)$
for i thru dim do for j:i thru dim do
if ric[i,j]≠0 then
ishow('R([ξ[i],ξ[j]])=ric[i,j])$
```

$$\mathbf{R}_{\theta\theta} = \frac{1}{2} \tag{\%t74}$$

$$\mathbf{R}_{\phi\phi} = \frac{\sin(\theta)^2}{2} \tag{\%t74}$$

(%i75) `ishow('R([μ,ν],[])=ric)$`

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{\sin(\theta)^2}{2} \end{pmatrix} \tag{\%t75}$$

(%i78)
```
remcomps(R([μ,ν],[]))$
components(R([μ,ν],[]),ric)$
showcomps(R([μ,ν],[]))$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{\sin(\theta)^2}{2} \end{pmatrix} \tag{\%t78}$$

(%i81)
```
remsym(R,2,0)$
decsym(R,2,0,[sym(all)],[])$
dispsym(R,2,0);
```

$$[[sym, [[1, 2]], []]] \tag{\%o81}$$

(%i84)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ξ[i],ξ[j]])=uric[i,j])$
```

$$\mathbf{R}^{\theta\theta} = -\frac{1}{2r^2} \tag{\%t84}$$

$$\mathbf{R}^{\phi\phi} = -\frac{1}{2r^2} \tag{\%t84}$$

(%i85) `ishow('R([],[μ,ν])=uric)$`

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2r^2} \end{pmatrix} \tag{\%t85}$$

(%i88)
```
remcomps(R([],[μ,ν]))$
components(R([],[μ,ν]),uric)$
showcomps(R([],[μ,ν]))$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2r^2} \end{pmatrix} \tag{\%t88}$$

(%i91)
```
remsym(R,0,2)$
decsym(R,0,2,[],[sym(all)])$
dispsym(R,0,2);
```

$$[[sym, [], [[1, 2]]]] \tag{\%o91}$$

Scalar curvature

(%i92)
```
factor(radcan(scurvature()));
```

$$-\frac{1}{r^2} \tag{\%o92}$$

Kretschmann invariant

(%i93)
```
factor(radcan(rinvariant()));
```

$$\frac{r^2+4mr+12m^2}{r^6} \tag{\%o93}$$

Einstein tensor

(%i94)
```
kill(labels)$
```

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ξ[i]],[ξ[j]])=ein[i,j])$
```

$$\mathbf{G}_t^t = \frac{1}{2r^2} \tag{\%t3}$$

$$\mathbf{G}_r^r = \frac{1}{2r^2} \tag{\%t3}$$

(%i4)
```
ishow('G([μ],[ν])=ein)$
```

$$\mathbf{G}_\mu^\nu = \begin{pmatrix} \frac{1}{2r^2} & 0 & 0 & 0 \\ 0 & \frac{1}{2r^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t4}$$

(%i7)
```
remcomps(G([μ],[ν]))$
components(G([μ],[ν]),ein)$
showcomps(G([μ],[ν]))$
```

$$\mathbf{G}_\mu^\nu = \begin{pmatrix} \frac{1}{2r^2} & 0 & 0 & 0 \\ 0 & \frac{1}{2r^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t7}$$

(%i10)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ξ[i],ξ[j]],[])=lein[i,j])$
```

$$\mathbf{G}_{tt} = \frac{r-2m}{2r^3} \tag{\%t10}$$

$$\mathbf{G}_{rr} = -\frac{2}{r\,(2r-4m)} \tag{\%t10}$$

(%i11)
```
ishow('G([μ,ν],[])=lein)$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} \frac{r-2m}{2r^3} & 0 & 0 & 0 \\ 0 & -\frac{2}{r\,(2r-4m)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t11}$$

(%i14)
```
remcomps(G([μ,ν],[]))$
components(G([μ,ν],[]),lein)$
showcomps(G([μ,ν],[]))$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} \frac{r-2m}{2r^3} & 0 & 0 & 0 \\ 0 & -\frac{2}{r\,(2r-4m)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t14}$$

(%i17)
```
remsym(G,2,0)$
decsym(G,2,0,[sym(all)],[])$
dispsym(G,2,0);
```

$$[[sym, [[1,2]], []]] \tag{\%o17}$$

Reduce Order

(%i19)
```
cv_coords:[T,R,Θ,Φ]$
depends(cv_coords,s)$
```

(%i23)
```
gradef(t,s,T)$
gradef(r,s,R)$
gradef(θ,s,Θ)$
gradef(φ,s,Φ)$
```

Geodesics

(%i24)
```
cgeodesic(false)$
```

Solve for second derivative of coordinates

(%i25)
```
geodsol:linsolve(listarray(geod),diff(ξ,s,2))$
```

(%i26)
```
map(ldisp,geodsol)$
```

$$T_s = -\frac{2mRT}{r^2-2mr} \tag{\%t26}$$

$$R_s = (m^2\,r^3\,\left(4\,\Phi^2\sin(\theta)^2+4\,\Theta^2\right)+r^5\,\left(\Phi^2\sin(\theta)^2+\Theta^2\right)+m\,r^4\,\left(-4\,\Phi^2\sin(\theta)^2-4\,\Theta^2\right)+\left(2m\,R^2-m\,T^2\right)\,r^2+4m^2\,T^2r-4m^3 \tag{\%t27}$$

$$\Theta_s = \frac{r\,\Phi^2\,\cos(\theta)\,\sin(\theta)-2R\Theta}{r} \tag{\%t28}$$

$$\Phi_s = -\frac{2R\Phi\,\sin(\theta)+2r\,\Theta\Phi\,\cos(\theta)}{r\,\sin(\theta)} \tag{\%t29}$$

5 Weak Field Approximation

$$\begin{bmatrix} c^2\left(-\frac{2GM}{c^2 r}+1\right) & 0 & 0 & 0 \\ 0 & -\frac{1}{-\frac{2GM}{c^2 r}+1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{bmatrix}$$

```
(%i30) kill(labels,t,r,θ,φ)$
(%i1)  init_ctensor()$
(%i2)  unorder()$
(%i3)  orderless(M)$
(%i7)  assume(0≤r)$
       assume(0≤θ,θ≤π)$
       assume(0≤sin(θ))$
       assume(0≤φ,φ≤2*π)$
(%i8)  ξ:ct_coords:[t,r,θ,φ]$
(%i9)  dim:length(ct_coords)$
(%i10) assume(c>0,M>0,G>0)$
(%i11) S:1-(2*G*M)/(c^2*r)$
(%i12) lg:matrix([c^2*S,0,0,0], [0,-1/S,0,0], [0,0,-r^2,0], [0,0,0,-r^2*sin(θ)^2])$
```

Sets up the package for further calculations

```
(%i13) cmetric()$
```

Covariant Metric tensor

```
(%i14) ishow('g([μ,ν],[])=lg)$
```

$$g_{\mu\nu} = \begin{pmatrix} c^2\left(1-\frac{2MG}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2MG}{c^2 r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{pmatrix} \quad (\%t14)$$

```
(%i17) remcomps(g([μ,ν],[]))$
       components(g([μ,ν],[]),lg)$
       showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} c^2\left(1-\frac{2MG}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2MG}{c^2 r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin(\theta)^2 \end{pmatrix} \quad (\%t17)$$

```
(%i20) remsym(g,2,0)$
       decsym(g,2,0,[sym(all)],[])$
       dispsym(g,2,0);
```

$$[[sym,[[1,2]],[]]] \quad (\%o20)$$

Contravariant Metric tensor

(%i21) `ishow('g([],[μ,ν])=ug)$`

$$g^{\mu\nu} = \begin{pmatrix} -\frac{r}{2MG-c^2r} & 0 & 0 & 0 \\ 0 & \frac{2MG-c^2r}{c^2r} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin(\theta)^2} \end{pmatrix} \quad (\%t21)$$

(%i24)
```
remcomps(g([],[μ,ν]))$
components(g([],[μ,ν]),ug)$
showcomps(g([],[μ,ν]))$
```

$$g^{\mu\nu} = \begin{pmatrix} -\frac{r}{2MG-c^2r} & 0 & 0 & 0 \\ 0 & \frac{2MG-c^2r}{c^2r} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin(\theta)^2} \end{pmatrix} \quad (\%t24)$$

(%i27)
```
remsym(g,0,2)$
decsym(g,0,2,[],[sym(all)])$
dispsym(g,0,2);
```

$$[[sym, [], [[1,2]]]] \quad (\%o27)$$

The determinant of the metric tensor

(%i28) `gdet;`

$$-c^2\, r^4 \sin(\theta)^2 \quad (\%o28)$$

Physical components (coframe)

(%i29) `ishow(√(lg[1,1])*∂([ξ[1]],[]))$`

$$c\sqrt{1-\frac{2MG}{c^2r}}\,\partial_t \quad (\%t29)$$

(%i30) `ishow(√(-lg[2,2])*∂([ξ[2]],[]))$`

$$\frac{\partial_r}{\sqrt{1-\frac{2MG}{c^2r}}} \quad (\%t30)$$

(%i31) `ishow(√(-lg[3,3])*∂([ξ[3]],[]))$`

$$r\,\partial_\theta \quad (\%t31)$$

(%i32) `ishow(√(-lg[4,4])*∂([ξ[4]],[]))$`

$$r\,\sin(\theta)\,\partial_\phi \quad (\%t32)$$

Line element

(%i33) `ldisplay(ds²=expand(transpose(diff(ξ)).lg.diff(ξ)))$`

$$ds^2 = -r^2 \sin(\theta)^2 \operatorname{del}(\phi)^2 - r^2 \operatorname{del}(\theta)^2 - \frac{2MG \operatorname{del}(t)^2}{r} + c^2 \operatorname{del}(t)^2 - \frac{\operatorname{del}(r)^2}{1 - \frac{2MG}{c^2 r}} \tag{\%t33}$$

Christoffel Symbol of the first kind

(%i34) `christof(lcs)$`

$$lcs_{1,1,2} = -\frac{MG}{r^2} \tag{\%t34}$$

$$lcs_{1,2,1} = \frac{MG}{r^2} \tag{\%t35}$$

$$lcs_{2,2,2} = \frac{MG}{c^2 \left(1 - \frac{2MG}{c^2 r}\right)^2 r^2} \tag{\%t36}$$

$$lcs_{2,3,3} = -r \tag{\%t37}$$

$$lcs_{2,4,4} = -r \sin(\theta)^2 \tag{\%t38}$$

$$lcs_{3,3,2} = r \tag{\%t39}$$

$$lcs_{3,4,4} = -r^2 \cos(\theta) \sin(\theta) \tag{\%t40}$$

$$lcs_{4,4,2} = r \sin(\theta)^2 \tag{\%t41}$$

$$lcs_{4,4,3} = r^2 \cos(\theta) \sin(\theta) \tag{\%t42}$$

(%i43)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j],ξ[k]],[])=lcs[i,j,k])$
```

$$\Gamma_{ttr} = -\frac{MG}{r^2} \qquad (\%t43)$$

$$\Gamma_{trt} = \frac{MG}{r^2} \qquad (\%t43)$$

$$\Gamma_{rrr} = \frac{MG}{c^2 \left(1 - \frac{2MG}{c^2 r}\right)^2 r^2} \qquad (\%t43)$$

$$\Gamma_{r\theta\theta} = -r \qquad (\%t43)$$

$$\Gamma_{r\phi\phi} = -r \sin(\theta)^2 \qquad (\%t43)$$

$$\Gamma_{\theta\theta r} = r \qquad (\%t43)$$

$$\Gamma_{\theta\phi\phi} = -r^2 \cos(\theta) \sin(\theta) \qquad (\%t43)$$

$$\Gamma_{\phi\phi r} = r \sin(\theta)^2 \qquad (\%t43)$$

$$\Gamma_{\phi\phi\theta} = r^2 \cos(\theta) \sin(\theta) \qquad (\%t43)$$

(%i44) `dispsym(ichr1,3,0);`

$$[[sym, [[1,2]], []]] \tag{\%o44}$$

(%i45) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\Gamma_{\alpha\beta\mu} = \frac{g_{\beta\mu,\alpha} + g_{\alpha\mu,\beta} - g_{\alpha\beta,\mu}}{2} \tag{\%t45}$$

(%i46) `ishow('Γ([α,β,1])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,1]),dim,dim)))$`

$$\Gamma_{\alpha\beta 1} = \begin{pmatrix} 0 & \frac{MG}{r^2} & 0 & 0 \\ \frac{MG}{r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t46}$$

(%i47) `ishow('Γ([α,β,2])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,2]),dim,dim)))$`

$$\Gamma_{\alpha\beta 2} = \begin{pmatrix} -\frac{MG}{r^2} & 0 & 0 & 0 \\ 0 & \frac{MG\,c^2}{(2MG-c^2r)^2} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\sin(\theta)^2 \end{pmatrix} \tag{\%t47}$$

(%i48) `ishow('Γ([α,β,3])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,3]),dim,dim)))$`

$$\Gamma_{\alpha\beta 3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & 0 & r^2\cos(\theta)\sin(\theta) \end{pmatrix} \tag{\%t48}$$

(%i49) `ishow('Γ([α,β,4])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,4]),dim,dim)))$`

$$\Gamma_{\alpha\beta 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r\sin(\theta)^2 \\ 0 & 0 & 0 & -r^2\cos(\theta)\sin(\theta) \\ 0 & -r\sin(\theta)^2 & -r^2\cos(\theta)\sin(\theta) & 0 \end{pmatrix} \tag{\%t49}$$

Christoffel Symbol of the second kind

(%i50) `christof(mcs)$`

$$mcs_{1,1,2} = -\frac{MG\left(2MG - c^2 r\right)}{c^2\, r^3} \tag{\%t50}$$

$$mcs_{1,2,1} = -\frac{MG}{r\,(2MG - c^2 r)} \tag{\%t51}$$

$$mcs_{2,2,2} = \frac{MG}{r\,(2MG - c^2 r)} \tag{\%t52}$$

$$mcs_{2,3,3} = \frac{1}{r} \tag{\%t53}$$

$$mcs_{2,4,4} = \frac{1}{r} \tag{\%t54}$$

$$mcs_{3,3,2} = \frac{2MG - c^2 r}{c^2} \tag{\%t55}$$

$$mcs_{3,4,4} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t56}$$

$$mcs_{4,4,2} = \frac{\left(2MG - c^2 r\right)\,\sin(\theta)^2}{c^2} \tag{\%t57}$$

$$mcs_{4,4,3} = -\cos(\theta)\,\sin(\theta) \tag{\%t58}$$

(%i59)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j]],[ξ[k]])=mcs[i,j,k])$
```

$$\Gamma^{r}_{tt} = -\frac{MG\left(2MG - c^2 r\right)}{c^2\,r^3} \tag{\%t59}$$

$$\Gamma^{t}_{tr} = -\frac{MG}{r\,(2MG - c^2 r)} \tag{\%t59}$$

$$\Gamma^{r}_{rr} = \frac{MG}{r\,(2MG - c^2 r)} \tag{\%t59}$$

$$\Gamma^{\theta}_{r\theta} = \frac{1}{r} \tag{\%t59}$$

$$\Gamma^{\phi}_{r\phi} = \frac{1}{r} \tag{\%t59}$$

$$\Gamma^{r}_{\theta\theta} = \frac{2MG - c^2 r}{c^2} \tag{\%t59}$$

$$\Gamma^{\phi}_{\theta\phi} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t59}$$

$$\Gamma^{r}_{\phi\phi} = \frac{\left(2MG - c^2 r\right)\,\sin(\theta)^2}{c^2} \tag{\%t59}$$

$$\Gamma^{\theta}_{\phi\phi} = -\cos(\theta)\,\sin(\theta) \tag{\%t59}$$

(%i60)
```
dispsym(ichr2,2,1);
```

$$[[sym,[[1,2]],[]]] \tag{\%o60}$$

(%i61)
```
ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$
```

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}\,\left(g_{\beta\nu,\alpha} + g_{\alpha\nu,\beta} - g_{\alpha\beta,\nu}\right)}{2} \tag{\%t61}$$

(%i62)
```
ishow('Γ([α,β],[1])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,1]),dim,dim)))$
```

$$\Gamma^{1}_{\alpha\beta} = \begin{pmatrix} 0 & -\frac{MG}{r\,(2MG - c^2 r)} & 0 & 0 \\ -\frac{MG}{r\,(2MG - c^2 r)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t62}$$

(%i63) `ishow('Γ([α,β],[2])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,2]),dim,dim)))$`

$$\Gamma^{2}_{\alpha\beta} = \begin{pmatrix} -\frac{MG\left(2MG-c^2 r\right)}{c^2\, r^3} & 0 & 0 & 0 \\ 0 & \frac{MG}{r\,(2MG-c^2 r)} & 0 & 0 \\ 0 & 0 & \frac{2MG-c^2 r}{c^2} & 0 \\ 0 & 0 & 0 & \frac{\left(2MG-c^2 r\right)\sin(\theta)^2}{c^2} \end{pmatrix} \tag{\%t63}$$

(%i64) `ishow('Γ([α,β],[3])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,3]),dim,dim)))$`

$$\Gamma^{3}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\,\sin(\theta) \end{pmatrix} \tag{\%t64}$$

(%i65) `ishow('Γ([α,β],[4])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,4]),dim,dim)))$`

$$\Gamma^{4}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{pmatrix} \tag{\%t65}$$

Riemann tensor

(%i67)
```
riemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ξ[a],ξ[b],ξ[c]],[ξ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}^{t}_{rrt} = -\frac{2MG}{r^2\,\left(2MG-c^2 r\right)} \tag{\%t67}$$

$$\mathbf{R}^{t}_{\theta\theta t} = -\frac{MG}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{r}_{\theta\theta r} = -\frac{MG}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{t}_{\phi\phi t} = -\frac{MG\sin(\theta)^2}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{r}_{\phi\phi r} = -\frac{MG\sin(\theta)^2}{c^2 r} \tag{\%t67}$$

$$\mathbf{R}^{\theta}_{\phi\phi\theta} = \frac{2MG\sin(\theta)^2}{c^2 r} \tag{\%t67}$$

(%i68) `dispsym(icurvature,3,1);`

$$[[anti,[[2,3]],[]]] \tag{\%o68}$$

(%i70)
```
lriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if lriem[a,b,c,d]≠0 then
ishow('R([ξ[d],ξ[a],ξ[b],ξ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{trrt} = \frac{2MG}{r^3} \tag{\%t70}$$

$$\mathbf{R}_{t\theta\theta t} = \frac{MG\,\left(2MG - c^2 r\right)}{c^2\,r^2} \tag{\%t70}$$

$$\mathbf{R}_{r\theta\theta r} = -\frac{MG}{2MG - c^2 r} \tag{\%t70}$$

$$\mathbf{R}_{t\phi\phi t} = \frac{MG\,\left(2MG - c^2 r\right)\,\sin\left(\theta\right)^2}{c^2\,r^2} \tag{\%t70}$$

$$\mathbf{R}_{r\phi\phi r} = -\frac{MG\sin\left(\theta\right)^2}{2MG - c^2 r} \tag{\%t70}$$

$$\mathbf{R}_{\theta\phi\phi\theta} = -\frac{2MGr\sin\left(\theta\right)^2}{c^2} \tag{\%t70}$$

(%i72)
```
uriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if uriem[a,b,c,d]≠0 then
ishow('R([],[ξ[a],ξ[b],ξ[c],ξ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{rrtt} = \frac{2MG}{c^4\,r^3} \tag{\%t72}$$

$$\mathbf{R}^{\theta\theta tt} = \frac{MG}{c^2\,r^4\,\left(2MG - c^2 r\right)} \tag{\%t72}$$

$$\mathbf{R}^{\theta\theta rr} = -\frac{MG\,\left(2MG - c^2 r\right)}{c^4\,r^6} \tag{\%t72}$$

$$\mathbf{R}^{\phi\phi tt} = \frac{MG}{c^2\,r^4\,\left(2MG - c^2 r\right)\,\sin\left(\theta\right)^2} \tag{\%t72}$$

$$\mathbf{R}^{\phi\phi rr} = -\frac{MG\,\left(2MG - c^2 r\right)}{c^4\,r^6\sin\left(\theta\right)^2} \tag{\%t72}$$

$$\mathbf{R}^{\phi\phi\theta\theta} = -\frac{2MG}{c^2\,r^7\sin\left(\theta\right)^2} \tag{\%t72}$$

Ricci tensor

(%i75)
```
ric:zeromatrix(dim,dim)$
ricci(false)$
for i thru dim do for j:i thru dim do
if ric[i,j]≠0 then
ishow('R([ξ[i],ξ[j]])=ric[i,j])$
```

(%i76) `ishow('R([μ,ν],[])=ric)$`

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t76}$$

(%i79)
```
remcomps(R([μ,ν],[]))$
components(R([μ,ν],[]),ric)$
showcomps(R([μ,ν],[]))$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t79}$$

(%i82)
```
remsym(R,2,0)$
decsym(R,2,0,[sym(all)],[])$
dispsym(R,2,0);
```

$$[[sym,[[1,2]],[]]] \tag{\%o82}$$

(%i85)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ξ[i],ξ[j]])=uric[i,j])$
```

(%i86) `ishow('R([],[μ,ν])=uric)$`

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t86}$$

(%i89)
```
remcomps(R([],[μ,ν]))$
components(R([],[μ,ν]),uric)$
showcomps(R([],[μ,ν]))$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t89}$$

(%i92)
```
remsym(R,0,2)$
decsym(R,0,2,[],[sym(all)])$
dispsym(R,0,2);
```

$$[[sym,[],[[1,2]]]] \tag{\%o92}$$

Scalar curvature

(%i93) `factor(radcan(scurvature()));`

$$0 \tag{\%o93}$$

Kretschmann invariant

(%i94) `factor(radcan(rinvariant()));`

$$\frac{48M^2\,G^2}{c^4\,r^6} \tag{\%o94}$$

Einstein tensor

(%i95) `kill(labels)$`

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ξ[i]],[ξ[j]])=ein[i,j])$
```

(%i4) `ishow('G([μ],[ν])=ein)$`

$$\mathbf{G}_{\mu}^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t4}$$

(%i7)
```
remcomps(G([μ],[ν]))$
components(G([μ],[ν]),ein)$
showcomps(G([μ],[ν]))$
```

$$\mathbf{G}_{\mu}^{\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t7}$$

(%i10)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ξ[i],ξ[j]],[])=lein[i,j])$
```

(%i11) `ishow('G([μ,ν],[])=lein)$`

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t11}$$

(%i14) `remcomps(G([`μ`,`ν`],[]))$`
`components(G([`μ`,`ν`],[]),lein)$`
`showcomps(G([`μ`,`ν`],[]))$`

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t14}$$

(%i17) `remsym(G,2,0)$`
`decsym(G,2,0,[sym(all)],[])$`
`dispsym(G,2,0);`

$$[[sym, [[1, 2]], []]] \tag{\%o17}$$

Reduce Order

(%i19) `cv_coords:[T,R,`Θ`,`Φ`]$`
`depends(cv_coords,s)$`

(%i23) `gradef(t,s,T)$`
`gradef(r,s,R)$`
`gradef(`θ`,s,`Θ`)$`
`gradef(`ϕ`,s,`Φ`)$`

Geodesics

(%i24) `cgeodesic(false)$`

Solve for second derivative of coordinates

(%i25) `geodsol:linsolve(listarray(geod),diff(`ξ`,s,2))$`

(%i26) `map(ldisp,geodsol)$`

$$T_s = -\frac{2MGRT}{c^2\,r^2 - 2MGr} \tag{\%t26}$$

$$R_s = (M^2\,G^2\,r^3\,\left(4\Phi^2\sin(\theta)^2 + 4\Theta^2\right) + c^4\,r^5\,\left(\Phi^2\sin(\theta)^2 + \Theta^2\right) + MG\,c^2\,r^4\,\left(-4\Phi^2\sin(\theta)^2 - 4\Theta^2\right) + \left(MG\,R^2\,c^2 - MG\,T^2\,c^4\right. \tag{\%t27}$$

$$\Theta_s = \frac{r\,\Phi^2\,\cos(\theta)\,\sin(\theta) - 2R\Theta}{r} \tag{\%t28}$$

$$\Phi_s = -\frac{2R\Phi\,\sin(\theta) + 2r\,\Theta\Phi\,\cos(\theta)}{r\,\sin(\theta)} \tag{\%t29}$$

6 Friedmann Lemaitre Robertson Walker metric

The spacetime for an expanding universe.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a^2}{-kr^2+1} & 0 & 0 \\ 0 & 0 & -r^2a^2 & 0 \\ 0 & 0 & 0 & -r^2a^2\sin(\theta)^2 \end{bmatrix}$$

(%i30) `kill(labels,t,r,θ,φ)$`

(%i1) `init_ctensor()$`

(%i5)
```
assume(0≤r)$
assume(0≤θ,θ≤π)$
assume(0≤sin(θ))$
assume(0≤φ,φ≤2*π)$
```

(%i6) `ξ:ct_coords:[t,r,θ,φ]$`

(%i7) `dim:length(ct_coords)$`

(%i8) `assume(a>0)$`

(%i9) `depends(a,t)$`

(%i10) `lg:matrix([1,0,0,0], [0,-a^2/(-k*r^2+1),0,0], [0,0,-a^2*r^2,0], [0,0,0,-a^2*r^2*sin(θ)^2])$`

Sets up the package for further calculations

(%i11) `cmetric()$`

Covariant Metric tensor

(%i12) `ishow('g([μ,ν],[])=lg)$`

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a^2}{1-k\,r^2} & 0 & 0 \\ 0 & 0 & -a^2\,r^2 & 0 \\ 0 & 0 & 0 & -a^2\,r^2\sin(\theta)^2 \end{pmatrix} \tag{\%t12}$$

(%i15)
```
remcomps(g([μ,ν],[]))$
components(g([μ,ν],[]),lg)$
showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a^2}{1-k\,r^2} & 0 & 0 \\ 0 & 0 & -a^2\,r^2 & 0 \\ 0 & 0 & 0 & -a^2\,r^2\sin(\theta)^2 \end{pmatrix} \tag{\%t15}$$

(%i18)
```
remsym(g,2,0)$
decsym(g,2,0,[sym(all)],[])$
dispsym(g,2,0);
```

$$[[sym,[[1,2]],[]]] \tag{\%o18}$$

Contravariant Metric tensor

(%i19) `ishow('g([],[μ,ν])=ug)$`

$$g^{\mu\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \frac{k\,r^2-1}{a^2} & 0 & 0\\ 0 & 0 & -\frac{1}{a^2\,r^2} & 0\\ 0 & 0 & 0 & -\frac{1}{a^2\,r^2\,\sin(\theta)^2}\end{pmatrix} \tag{\%t19}$$

(%i22) `remcomps(g([],[μ,ν]))$`
`components(g([],[μ,ν]),ug)$`
`showcomps(g([],[μ,ν]))$`

$$g^{\mu\nu}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \frac{k\,r^2-1}{a^2} & 0 & 0\\ 0 & 0 & -\frac{1}{a^2\,r^2} & 0\\ 0 & 0 & 0 & -\frac{1}{a^2\,r^2\,\sin(\theta)^2}\end{pmatrix} \tag{\%t22}$$

(%i25) `remsym(g,0,2)$`
`decsym(g,0,2,[],[sym(all)])$`
`dispsym(g,0,2);`

$$[[sym,[],[[1,2]]]] \tag{\%o25}$$

The determinant of the metric tensor

(%i26) `gdet;`

$$\frac{a^6\,r^4\,\sin(\theta)^2}{k\,r^2-1} \tag{\%o26}$$

Physical components (coframe)

(%i27) `ishow(√(lg[1,1])*∂([ξ[1]],[]))$`

$$\partial_t \tag{\%t27}$$

(%i28) `ishow(√(-lg[2,2])*∂([ξ[2]],[]))$`

$$\frac{a\,\partial_r}{\sqrt{1-k\,r^2}} \tag{\%t28}$$

(%i29) `ishow(√(-lg[3,3])*∂([ξ[3]],[]))$`

$$ar\,\partial_\theta \tag{\%t29}$$

(%i30) `ishow(√(-lg[4,4])*∂([ξ[4]],[]))$`

$$ar\,\sin(\theta)\,\partial_\phi \tag{\%t30}$$

Line element

(%i31) `ldisplay(ds^2=expand(transpose(diff(ξ)).lg.diff(ξ)))$`

$$ds^2 = -a^2\, r^2 \sin(\theta)^2 \operatorname{del}(\phi)^2 - a^2\, r^2 \operatorname{del}(\theta)^2 + \operatorname{del}(t)^2 - \frac{a^2 \operatorname{del}(r)^2}{1-k\, r^2} \tag{\%t31}$$

Christoffel Symbol of the first kind

(%i32) `christof(lcs)$`

$$lcs_{1,2,2} = -\frac{a\ (\dot{a})}{1-k\, r^2} \tag{\%t32}$$

$$lcs_{1,3,3} = -a\ (\dot{a})\ r^2 \tag{\%t33}$$

$$lcs_{1,4,4} = -a\ (\dot{a})\ r^2 \sin(\theta)^2 \tag{\%t34}$$

$$lcs_{2,2,1} = \frac{a\ (\dot{a})}{1-k\, r^2} \tag{\%t35}$$

$$lcs_{2,2,2} = -\frac{a^2 k r}{(1-k\, r^2)^2} \tag{\%t36}$$

$$lcs_{2,3,3} = -a^2 r \tag{\%t37}$$

$$lcs_{2,4,4} = -a^2 r \sin(\theta)^2 \tag{\%t38}$$

$$lcs_{3,3,1} = a\ (\dot{a})\ r^2 \tag{\%t39}$$

$$lcs_{3,3,2} = a^2 r \tag{\%t40}$$

$$lcs_{3,4,4} = -a^2\, r^2\ \cos(\theta)\ \sin(\theta) \tag{\%t41}$$

$$lcs_{4,4,1} = a\ (\dot{a})\ r^2 \sin(\theta)^2 \tag{\%t42}$$

$$lcs_{4,4,2} = a^2 r \sin(\theta)^2 \tag{\%t43}$$

$$lcs_{4,4,3} = a^2\, r^2\ \cos(\theta)\ \sin(\theta) \tag{\%t44}$$

(%i45)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j],ξ[k]],[])=lcs[i,j,k])$
```

$$\Gamma_{trr} = -\frac{a\ (\dot{a})}{1-k\, r^2} \tag{\%t45}$$

$$\Gamma_{t\theta\theta} = -a\ (\dot{a})\ r^2 \tag{\%t45}$$

$$\Gamma_{t\phi\phi} = -a\ (\dot{a})\ r^2 \sin(\theta)^2 \tag{\%t45}$$

$$\Gamma_{rrt} = \frac{a\ (\dot{a})}{1-k\, r^2} \tag{\%t45}$$

$$\Gamma_{rrr} = -\frac{a^2 k r}{(1-k\, r^2)^2} \tag{\%t45}$$

$$\Gamma_{r\theta\theta} = -a^2 r \tag{\%t45}$$

$$\Gamma_{r\phi\phi} = -a^2 r \sin(\theta)^2 \tag{\%t45}$$

$$\mathit{\Gamma}_{\theta\theta t} = a\ (\dot{a})\ r^2 \tag{\%t45}$$

$$\mathit{\Gamma}_{\theta\theta r} = a^2 r \tag{\%t45}$$

$$\mathit{\Gamma}_{\theta\phi\phi} = -a^2\, r^2\, \cos(\theta)\, \sin(\theta) \tag{\%t45}$$

$$\mathit{\Gamma}_{\phi\phi t} = a\ (\dot{a})\ r^2 \sin(\theta)^2 \tag{\%t45}$$

$$\mathit{\Gamma}_{\phi\phi r} = a^2 r \sin(\theta)^2 \tag{\%t45}$$

$$\mathit{\Gamma}_{\phi\phi\theta} = a^2\, r^2\, \cos(\theta)\, \sin(\theta) \tag{\%t45}$$

(%i46) `dispsym(ichr1,3,0);`

$$[[sym, [[1,2]], []]] \tag{\%o46}$$

(%i47) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\mathit{\Gamma}_{\alpha\beta\mu} = \frac{g_{\beta\mu,\alpha} + g_{\alpha\mu,\beta} - g_{\alpha\beta,\mu}}{2} \tag{\%t47}$$

(%i48) `ishow('Γ([α,β,1])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,1]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{a\,(\dot{a})}{k\,r^2-1} & 0 & 0 \\ 0 & 0 & a\ (\dot{a})\ r^2 & 0 \\ 0 & 0 & 0 & a\ (\dot{a})\ r^2 \sin(\theta)^2 \end{pmatrix} \tag{\%t48}$$

(%i49) `ishow('Γ([α,β,2])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,2]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 2} = \begin{pmatrix} 0 & \frac{a\,(\dot{a})}{k\,r^2-1} & 0 & 0 \\ \frac{a\,(\dot{a})}{k\,r^2-1} & -\frac{a^2 k r}{(k\,r^2-1)^2} & 0 & 0 \\ 0 & 0 & a^2 r & 0 \\ 0 & 0 & 0 & a^2 r \sin(\theta)^2 \end{pmatrix} \tag{\%t49}$$

(%i50) `ishow('Γ([α,β,3])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,3]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 3} = \begin{pmatrix} 0 & 0 & -a\ (\dot{a})\ r^2 & 0 \\ 0 & 0 & -a^2 r & 0 \\ -a\ (\dot{a})\ r^2 & -a^2 r & 0 & 0 \\ 0 & 0 & 0 & a^2\, r^2\, \cos(\theta)\, \sin(\theta) \end{pmatrix} \tag{\%t50}$$

(%i51) `ishow('Γ([α,β,4])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,4]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 4} = \begin{pmatrix} 0 & 0 & 0 & -a\ (\dot{a})\ r^2 \sin(\theta)^2 \\ 0 & 0 & 0 & -a^2 r \sin(\theta)^2 \\ 0 & 0 & 0 & -a^2\, r^2\, \cos(\theta)\, \sin(\theta) \\ -a\ (\dot{a})\ r^2 \sin(\theta)^2 & -a^2 r \sin(\theta)^2 & -a^2\, r^2\, \cos(\theta)\, \sin(\theta) & 0 \end{pmatrix} \tag{\%t51}$$

Christoffel Symbol of the second kind

(%i52) `christof(mcs)$`

$$mcs_{1,2,2} = \frac{\dot{a}}{a} \tag{\%t52}$$

$$mcs_{1,3,3} = \frac{\dot{a}}{a} \tag{\%t53}$$

$$mcs_{1,4,4} = \frac{\dot{a}}{a} \tag{\%t54}$$

$$mcs_{2,2,1} = -\frac{a\ (\dot{a})}{k\,r^2 - 1} \tag{\%t55}$$

$$mcs_{2,2,2} = -\frac{kr}{k\,r^2 - 1} \tag{\%t56}$$

$$mcs_{2,3,3} = \frac{1}{r} \tag{\%t57}$$

$$mcs_{2,4,4} = \frac{1}{r} \tag{\%t58}$$

$$mcs_{3,3,1} = a\ (\dot{a})\ r^2 \tag{\%t59}$$

$$mcs_{3,3,2} = r\ \left(k\,r^2 - 1\right) \tag{\%t60}$$

$$mcs_{3,4,4} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t61}$$

$$mcs_{4,4,1} = a\ (\dot{a})\ r^2 \sin(\theta)^2 \tag{\%t62}$$

$$mcs_{4,4,2} = r\ \left(k\,r^2 - 1\right)\ \sin(\theta)^2 \tag{\%t63}$$

$$mcs_{4,4,3} = -\cos(\theta)\ \sin(\theta) \tag{\%t64}$$

(%i65)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j]],[ξ[k]])=mcs[i,j,k])$
```

$$\Gamma_{tr}^{r} = \frac{\dot{a}}{a} \tag{\%t65}$$

$$\Gamma_{t\theta}^{\theta} = \frac{\dot{a}}{a} \tag{\%t65}$$

$$\Gamma_{t\phi}^{\phi} = \frac{\dot{a}}{a} \tag{\%t65}$$

$$\Gamma_{rr}^{t} = -\frac{a\ (\dot{a})}{k\,r^2 - 1} \tag{\%t65}$$

$$\Gamma_{rr}^{r} = -\frac{kr}{k\,r^2 - 1} \tag{\%t65}$$

$$\Gamma_{r\theta}^{\theta} = \frac{1}{r} \tag{\%t65}$$

$$\Gamma_{r\phi}^{\phi} = \frac{1}{r} \tag{\%t65}$$

$$\mathit{\Gamma}^{t}_{\theta\theta} = a\,\left(\dot{a}\right)\,r^2 \tag{\%t65}$$

$$\mathit{\Gamma}^{r}_{\theta\theta} = r\,\left(k\,r^2-1\right) \tag{\%t65}$$

$$\mathit{\Gamma}^{\phi}_{\theta\phi} = \frac{\cos\left(\theta\right)}{\sin\left(\theta\right)} \tag{\%t65}$$

$$\mathit{\Gamma}^{t}_{\phi\phi} = a\,\left(\dot{a}\right)\,r^2\sin\left(\theta\right)^2 \tag{\%t65}$$

$$\mathit{\Gamma}^{r}_{\phi\phi} = r\,\left(k\,r^2-1\right)\,\sin\left(\theta\right)^2 \tag{\%t65}$$

$$\mathit{\Gamma}^{\theta}_{\phi\phi} = -\cos\left(\theta\right)\,\sin\left(\theta\right) \tag{\%t65}$$

(%i66) `dispsym(ichr2,2,1);`

$$[[sym,[[1,2]],[]]] \tag{\%o66}$$

(%i67) `ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$`

$$\mathit{\Gamma}^{\mu}_{\alpha\beta} = \frac{\mathrm{g}^{\mu\nu}\,\left(\mathrm{g}_{\beta\nu,\alpha}+\mathrm{g}_{\alpha\nu,\beta}-\mathrm{g}_{\alpha\beta,\nu}\right)}{2} \tag{\%t67}$$

(%i68) `ishow('Γ([α,β],[1])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,1]),dim,dim)))$`

$$\mathit{\Gamma}^{1}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{a\,\left(\dot{a}\right)}{k\,r^2-1} & 0 & 0 \\ 0 & 0 & a\,\left(\dot{a}\right)\,r^2 & 0 \\ 0 & 0 & 0 & a\,\left(\dot{a}\right)\,r^2\sin\left(\theta\right)^2 \end{pmatrix} \tag{\%t68}$$

(%i69) `ishow('Γ([α,β],[2])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,2]),dim,dim)))$`

$$\mathit{\Gamma}^{2}_{\alpha\beta} = \begin{pmatrix} 0 & \frac{\dot{a}}{a} & 0 & 0 \\ \frac{\dot{a}}{a} & -\frac{k\,r}{k\,r^2-1} & 0 & 0 \\ 0 & 0 & r\,\left(k\,r^2-1\right) & 0 \\ 0 & 0 & 0 & r\,\left(k\,r^2-1\right)\,\sin\left(\theta\right)^2 \end{pmatrix} \tag{\%t69}$$

(%i70) `ishow('Γ([α,β],[3])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,3]),dim,dim)))$`

$$\mathit{\Gamma}^{3}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & \frac{\dot{a}}{a} & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ \frac{\dot{a}}{a} & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos\left(\theta\right)\,\sin\left(\theta\right) \end{pmatrix} \tag{\%t70}$$

(%i71) `ishow('Γ([α,β],[4])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,4]),dim,dim)))$`

$$\mathit{\Gamma}^{4}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & \frac{\dot{a}}{a} \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos\left(\theta\right)}{\sin\left(\theta\right)} \\ \frac{\dot{a}}{a} & \frac{1}{r} & \frac{\cos\left(\theta\right)}{\sin\left(\theta\right)} & 0 \end{pmatrix} \tag{\%t71}$$

Riemann tensor

(%i73)
```
riemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ξ[a],ξ[b],ξ[c]],[ξ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}_{rrt}^{t} = -\frac{a\ (\ddot{a})}{k\,r^2-1} \tag{\%t73}$$

$$\mathbf{R}_{\theta\theta t}^{t} = a\ (\ddot{a})\ r^2 \tag{\%t73}$$

$$\mathbf{R}_{\theta\theta r}^{r} = \left(k+(\dot{a})^2\right)\ r^2 \tag{\%t73}$$

$$\mathbf{R}_{\phi\phi t}^{t} = a\ (\ddot{a})\ r^2\sin(\theta)^2 \tag{\%t73}$$

$$\mathbf{R}_{\phi\phi r}^{r} = \left(k+(\dot{a})^2\right)\ r^2\sin(\theta)^2 \tag{\%t73}$$

$$\mathbf{R}_{\phi\phi\theta}^{\theta} = \left(k+(\dot{a})^2\right)\ r^2\sin(\theta)^2 \tag{\%t73}$$

(%i74)
```
dispsym(icurvature,3,1);
```

$$[[anti,[[2,3]],[]]] \tag{\%o74}$$

(%i76)
```
lriemann(false)$ for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if lriem[a,b,c,d]≠0 then
ishow('R([ξ[d],ξ[a],ξ[b],ξ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{trrt} = -\frac{a\ (\ddot{a})}{k\,r^2-1} \tag{\%t76}$$

$$\mathbf{R}_{t\theta\theta t} = a\ (\ddot{a})\ r^2 \tag{\%t76}$$

$$\mathbf{R}_{r\theta\theta r} = \frac{a^2\ \left(k+(\dot{a})^2\right)\ r^2}{k\,r^2-1} \tag{\%t76}$$

$$\mathbf{R}_{t\phi\phi t} = a\ (\ddot{a})\ r^2\sin(\theta)^2 \tag{\%t76}$$

$$\mathbf{R}_{r\phi\phi r} = \frac{a^2\ \left(k+(\dot{a})^2\right)\ r^2\sin(\theta)^2}{k\,r^2-1} \tag{\%t76}$$

$$\mathbf{R}_{\theta\phi\phi\theta} = -a^2\ \left(k+(\dot{a})^2\right)\ r^4\sin(\theta)^2 \tag{\%t76}$$

(%i78)
```
uriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if uriem[a,b,c,d]≠0 then
ishow('R([],[ξ[a],ξ[b],ξ[c],ξ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{rrtt} = -\frac{(\ddot{a})\ \left(k\,r^2-1\right)}{a^3} \tag{\%t78}$$

$$\mathbf{R}^{\theta\theta tt} = \frac{\ddot{a}}{a^3\,r^2} \tag{\%t78}$$

$$\mathbf{R}^{\theta\theta rr} = \frac{\left(k+(\dot{a})^2\right)\,\left(k\,r^2-1\right)}{a^6\,r^2} \tag{\%t78}$$

$$\mathbf{R}^{\phi\phi tt} = \frac{\ddot{a}}{a^3\,r^2\,\sin(\theta)^2} \tag{\%t78}$$

$$\mathbf{R}^{\phi\phi rr} = \frac{\left(k+(\dot{a})^2\right)\,\left(k\,r^2-1\right)}{a^6\,r^2\,\sin(\theta)^2} \tag{\%t78}$$

$$\mathbf{R}^{\phi\phi\theta\theta} = -\frac{k+(\dot{a})^2}{a^6\,r^4\,\sin(\theta)^2} \tag{\%t78}$$

Ricci tensor

(%i81)
```
ric:zeromatrix(dim,dim)$
ricci(false)$
for i thru dim do for j:i thru dim do
if ric[i,j]≠0 then
ishow('R([ξ[i],ξ[j]])=ric[i,j])$
```

$$\mathbf{R}_{tt} = -\frac{3\,(\ddot{a})}{a} \tag{\%t81}$$

$$\mathbf{R}_{rr} = -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{k\,r^2-1} \tag{\%t81}$$

$$\mathbf{R}_{\theta\theta} = \left(2k+a\,(\ddot{a})+2(\dot{a})^2\right)\,r^2 \tag{\%t81}$$

$$\mathbf{R}_{\phi\phi} = \left(2k+a\,(\ddot{a})+2(\dot{a})^2\right)\,r^2\sin(\theta)^2 \tag{\%t81}$$

(%i82)
```
matrixp(ric);
```

$$\text{true} \tag{\%o82}$$

(%i83)
```
diagmatrixp(ric,dim);
```

$$\text{true} \tag{\%o83}$$

(%i84)
```
symmetricp(ric,dim);
```

$$\text{true} \tag{\%o84}$$

(%i85)
```
ishow('R([μ,ν],[])=ric)$
```

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3(\ddot{a})}{a} & 0 & 0 & 0 \\ 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{k\,r^2-1} & 0 & 0 \\ 0 & 0 & \left(2k+a\,(\ddot{a})+2(\dot{a})^2\right)\,r^2 & 0 \\ 0 & 0 & 0 & \left(2k+a\,(\ddot{a})+2(\dot{a})^2\right)\,r^2\sin(\theta)^2 \end{pmatrix} \tag{\%t85}$$

(%i88)
```
remcomps(R([μ,ν],[]))$
components(R([μ,ν],[]),ric)$
showcomps(R([μ,ν],[]))$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} -\frac{3(\ddot{a})}{a} & 0 & 0 & 0 \\ 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{k\,r^2-1} & 0 & 0 \\ 0 & 0 & \left(2k+a\,(\ddot{a})+2(\dot{a})^2\right)\,r^2 & 0 \\ 0 & 0 & 0 & \left(2k+a\,(\ddot{a})+2(\dot{a})^2\right)\,r^2\sin\left(\theta\right)^2 \end{pmatrix} \tag{\%t88}$$

(%i91)
```
remsym(R,2,0)$
decsym(R,2,0,[sym(all)],[])$
dispsym(R,2,0);
```

$$[[sym,[[1,2]],[]]] \tag{\%o91}$$

(%i92)
```
map(ldisp,efe:findde(ric,2))$
```

$$\ddot{a} \tag{\%t92}$$

$$2k+a\,(\ddot{a})+2(\dot{a})^2 \tag{\%t93}$$

(%i94)
```
eliminate(efe,[diff(a,t,2)]);
```

$$[2\left(k+(\dot{a})^2\right)] \tag{\%o94}$$

(%i95)
```
solve(%,diff(a,t));
```

$$[\dot{a}=-\sqrt{-k},\dot{a}=\sqrt{-k}] \tag{\%o95}$$

(%i98)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ξ[i],ξ[j]])=uric[i,j])$
```

$$\mathbf{R}^{tt} = -\frac{3\,(\ddot{a})}{a} \tag{\%t98}$$

$$\mathbf{R}^{rr} = -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} \tag{\%t98}$$

$$\mathbf{R}^{\theta\theta} = -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} \tag{\%t98}$$

$$\mathbf{R}^{\phi\phi} = -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} \tag{\%t98}$$

(%i99)
```
matrixp(uric);
```

$$\text{true} \tag{\%o99}$$

(%i100) `diagmatrixp(uric,dim);`

$$\text{true} \tag{\%o100}$$

(%i101) `symmetricp(uric,dim);`

$$\text{true} \tag{\%o101}$$

(%i102) `ishow('R([],[μ,ν])=uric)$`

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} -\frac{3(\ddot{a})}{a} & 0 & 0 & 0 \\ 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} & 0 & 0 \\ 0 & 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} & 0 \\ 0 & 0 & 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} \end{pmatrix} \tag{\%t102}$$

(%i105)
```
remcomps(R([],[μ,ν]))$
components(R([],[μ,ν]),uric)$
showcomps(R([],[μ,ν]))$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} -\frac{3(\ddot{a})}{a} & 0 & 0 & 0 \\ 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} & 0 & 0 \\ 0 & 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} & 0 \\ 0 & 0 & 0 & -\frac{2k+a\,(\ddot{a})+2(\dot{a})^2}{a^2} \end{pmatrix} \tag{\%t105}$$

(%i108)
```
remsym(R,0,2)$
decsym(R,0,2,[],[sym(all)])$
dispsym(R,0,2);
```

$$[[sym, [], [[1, 2]]]] \tag{\%o108}$$

Scalar curvature

(%i109) `factor(radcan(scurvature()));`

$$-\frac{6\left(k + a\,(\ddot{a}) + (\dot{a})^2\right)}{a^2} \tag{\%o109}$$

Kretschmann invariant

(%i110) `factor(radcan(rinvariant()));`

$$\frac{12\left(k^2 + 2(\dot{a})^2 k + a^2\,(\ddot{a})^2 + (\dot{a})^4\right)}{a^4} \tag{\%o110}$$

Einstein tensor

(%i111) `kill(labels)$`

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ξ[i]],[ξ[j]])=ein[i,j])$
```

$$\mathbf{G}_t^t = \frac{3\left(k + (\dot{a})^2\right)}{a^2} \quad (\%t3)$$

$$\mathbf{G}_r^r = \frac{k + 2a\,(\ddot{a}) + (\dot{a})^2}{a^2} \quad (\%t3)$$

$$\mathbf{G}_\theta^\theta = \frac{k + 2a\,(\ddot{a}) + (\dot{a})^2}{a^2} \quad (\%t3)$$

$$\mathbf{G}_\phi^\phi = \frac{k + 2a\,(\ddot{a}) + (\dot{a})^2}{a^2} \quad (\%t3)$$

(%i4)
```
matrixp(ein);
```

true (%o4)

(%i5)
```
diagmatrixp(ein,dim);
```

true (%o5)

(%i6)
```
symmetricp(ein,dim);
```

true (%o6)

(%i7)
```
ishow('G([μ],[ν])=ein)$
```

$$\mathbf{G}_\mu^\nu = \begin{pmatrix} \frac{3\left(k+(\dot{a})^2\right)}{a^2} & 0 & 0 & 0 \\ 0 & \frac{k+2a\,(\ddot{a})+(\dot{a})^2}{a^2} & 0 & 0 \\ 0 & 0 & \frac{k+2a\,(\ddot{a})+(\dot{a})^2}{a^2} & 0 \\ 0 & 0 & 0 & \frac{k+2a\,(\ddot{a})+(\dot{a})^2}{a^2} \end{pmatrix} \quad (\%t7)$$

(%i10)
```
remcomps(G([μ],[ν]))$
components(G([μ],[ν]),ein)$
showcomps(G([μ],[ν]))$
```

$$\mathbf{G}_\mu^\nu = \begin{pmatrix} \frac{3\left(k+(\dot{a})^2\right)}{a^2} & 0 & 0 & 0 \\ 0 & \frac{k+2a\,(\ddot{a})+(\dot{a})^2}{a^2} & 0 & 0 \\ 0 & 0 & \frac{k+2a\,(\ddot{a})+(\dot{a})^2}{a^2} & 0 \\ 0 & 0 & 0 & \frac{k+2a\,(\ddot{a})+(\dot{a})^2}{a^2} \end{pmatrix} \quad (\%t10)$$

(%i11)
```
map(ldisp,efe:findde(ein,2))$
```

$$k + (\dot{a})^2 \quad (\%t11)$$

$$k+2a\,(\ddot{a})+(\dot{a})^2 \tag{\%t12}$$

(%i13) `eliminate(efe,[diff(a,t)]);`

$$[4a^2\,(\ddot{a})^2] \tag{\%o13}$$

(%i14) `solve(%,diff(a,t,2));`

$$[\ddot{a}=0] \tag{\%o14}$$

(%i17)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ξ[i],ξ[j]],[])=lein[i,j])$
```

$$\mathbf{G}_{tt}=\frac{3\left(k+(\dot{a})^2\right)}{a^2} \tag{\%t17}$$

$$\mathbf{G}_{rr}=-\frac{k+2a\,(\ddot{a})+(\dot{a})^2}{1-k\,r^2} \tag{\%t17}$$

$$\mathbf{G}_{\theta\theta}=-\left(k+2a\,(\ddot{a})+(\dot{a})^2\right)\,r^2 \tag{\%t17}$$

$$\mathbf{G}_{\phi\phi}=-\left(k+2a\,(\ddot{a})+(\dot{a})^2\right)\,r^2\sin(\theta)^2 \tag{\%t17}$$

(%i18) `matrixp(lein);`

$$\text{true} \tag{\%o18}$$

(%i19) `diagmatrixp(lein,dim);`

$$\text{true} \tag{\%o19}$$

(%i20) `symmetricp(lein,dim);`

$$\text{true} \tag{\%o20}$$

(%i21) `ishow('G([μ,ν],[])=lein)$`

$$\mathbf{G}_{\mu\nu}=\begin{pmatrix} \frac{3\left(k+(\dot{a})^2\right)}{a^2} & 0 & 0 & 0 \\ 0 & -\frac{k+2a\,(\ddot{a})+(\dot{a})^2}{1-k\,r^2} & 0 & 0 \\ 0 & 0 & -\left(k+2a\,(\ddot{a})+(\dot{a})^2\right)\,r^2 & 0 \\ 0 & 0 & 0 & -\left(k+2a\,(\ddot{a})+(\dot{a})^2\right)\,r^2\sin(\theta)^2 \end{pmatrix} \tag{\%t21}$$

(%i24) remcomps(G([μ,ν],[]))$
components(G([μ,ν],[]),lein)$
showcomps(G([μ,ν],[]))$

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} \frac{3\left(k+(\dot{a})^2\right)}{a^2} & 0 & 0 & 0 \\ 0 & -\frac{k+2a\,(\ddot{a})+(\dot{a})^2}{1-k\,r^2} & 0 & 0 \\ 0 & 0 & -\left(k+2a\,(\ddot{a})+(\dot{a})^2\right)\,r^2 & 0 \\ 0 & 0 & 0 & -\left(k+2a\,(\ddot{a})+(\dot{a})^2\right)\,r^2\sin(\theta)^2 \end{pmatrix} \tag{\%t24}$$

(%i27) remsym(G,2,0)$
decsym(G,2,0,[sym(all)],[])$
dispsym(G,2,0);

$$[[sym,[[1,2]],[]]] \tag{\%o27}$$

Reduce Order

(%i29) cv_coords:[T,R,Θ,Φ]$
depends(cv_coords,s)$

(%i33) gradef(t,s,T)$
gradef(r,s,R)$
gradef(θ,s,Θ)$
gradef(ϕ,s,Φ)$

Geodesics

(%i34) cgeodesic(false)$

Solve for second derivative of coordinates

(%i35) geodsol:linsolve(listarray(geod),diff(ξ,s,2))$

(%i36) map(ldisp,geodsol)$

$$T_s = -\frac{\left(a\,(\dot{a})\,k\,r^4 - a\,(\dot{a})\,r^2\right)\,\Phi^2\sin(\theta)^2 + \left(a\,(\dot{a})\,k\,r^4 - a\,(\dot{a})\,r^2\right)\,\Theta^2 - R^2a\,(\dot{a})}{k\,r^2-1} \tag{\%t36}$$

$$R_s = -((a\,k^2\,r^5 - 2ak\,r^3 + ar)\,\Phi^2\sin(\theta)^2 + (a\,k^2\,r^5 - 2ak\,r^3 + ar)\,\Theta^2 + 2RT\,(\dot{a})\,k\,r^2 - R^2akr - 2RT\,(\dot{a}))/(ak\,r^2 - a) \tag{\%t37}$$

$$\Theta_s = \frac{ar\,\Phi^2\cos(\theta)\sin(\theta) + (-2T\,(\dot{a})\,r - 2Ra)\,\Theta}{ar} \tag{\%t38}$$

$$\Phi_s = -\frac{(2T\,(\dot{a})\,r + 2Ra)\,\Phi\sin(\theta) + 2ar\,\Theta\Phi\cos(\theta)}{ar\,\sin(\theta)} \tag{\%t39}$$

7 Hypersphere Metric (Glome)

A metric which describes a spacetime where the cosmic time is assigned to the meaning of a 4D hypersphere radius. The essential idea behind this spacetime is that the "3+1" dimensionality commonly referenced in physics can be meaningfully mapped to the "3+1" dimensionality associated with a hypersphere; by the "3" angular coordinates and the "1" radial coordinate.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -t^2 & 0 & 0 \\ 0 & 0 & -t^2 \sin(\psi)^2 & 0 \\ 0 & 0 & 0 & -t^2 \sin(\psi)^2 \sin(\theta)^2 \end{bmatrix}$$

(%i40) `kill(labels,t,ψ,θ,φ)$`

(%i1) `init_ctensor()$`

(%i7)
```
assume(0≤t)$
assume(0≤ψ,ψ≤π)$
assume(0≤sin(ψ))$
assume(0≤θ,θ≤π)$
assume(0≤sin(θ))$
assume(0≤φ,φ≤2*π)$
```

(%i8) `ξ:ct_coords:[t,ψ,θ,φ]$`

(%i9) `dim:length(ct_coords)$`

(%i10) `lg:matrix([1,0,0,0], [0,-t^2,0,0], [0,0,-t^2*sin(ψ)^2,0], [0,0,0,-t^2*sin(ψ)^2*sin(θ)^2])$`

Sets up the package for further calculations

(%i11) `cmetric()$`

Covariant Metric tensor

(%i12) `ishow('g([μ,ν],[])=lg)$`

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -t^2 & 0 & 0 \\ 0 & 0 & -t^2 \sin(\psi)^2 & 0 \\ 0 & 0 & 0 & -t^2 \sin(\theta)^2 \sin(\psi)^2 \end{pmatrix} \quad (\%t12)$$

(%i15)
```
remcomps(g([μ,ν],[]))$
components(g([μ,ν],[]),lg)$
showcomps(g([μ,ν],[]))$
```

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -t^2 & 0 & 0 \\ 0 & 0 & -t^2 \sin(\psi)^2 & 0 \\ 0 & 0 & 0 & -t^2 \sin(\theta)^2 \sin(\psi)^2 \end{pmatrix} \quad (\%t15)$$

(%i18)
```
remsym(g,2,0)$
decsym(g,2,0,[sym(all)],[])$
dispsym(g,2,0);
```

$$[[sym, [[1, 2]], []]] \quad (\%o18)$$

Contravariant Metric tensor

(%i19) `ishow('g([],[μ,ν])=ug)$`

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{t^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{t^2 \sin(\psi)^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{t^2 \sin(\theta)^2 \sin(\psi)^2} \end{pmatrix} \tag{\%t19}$$

(%i22) `remcomps(g([],[μ,ν]))$`
`components(g([],[μ,ν]),ug)$`
`showcomps(g([],[μ,ν]))$`

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{t^2} & 0 & 0 \\ 0 & 0 & -\frac{1}{t^2 \sin(\psi)^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{t^2 \sin(\theta)^2 \sin(\psi)^2} \end{pmatrix} \tag{\%t22}$$

(%i25) `remsym(g,0,2)$`
`decsym(g,0,2,[],[sym(all)])$`
`dispsym(g,0,2);`

$$[[sym, [], [[1,2]]]] \tag{\%o25}$$

The determinant of the metric tensor

(%i26) `gdet;`

$$-t^6 \sin(\theta)^2 \sin(\psi)^4 \tag{\%o26}$$

Physical components (coframe)

(%i27) `ishow(√(lg[1,1])*∂([ξ[1]],[]))$`

$$\partial_t \tag{\%t27}$$

(%i28) `ishow(√(-lg[2,2])*∂([ξ[2]],[]))$`

$$t\,\partial_\psi \tag{\%t28}$$

(%i29) `ishow(√(-lg[3,3])*∂([ξ[3]],[]))$`

$$t\,\partial_\theta\, \sin(\psi) \tag{\%t29}$$

(%i30) `ishow(√(-lg[4,4])*∂([ξ[4]],[]))$`

$$t\, \sin(\theta)\, \partial_\phi\, \sin(\psi) \tag{\%t30}$$

Line element

(%i31) `ldisplay(ds^2=expand(transpose(diff(ξ)).lg.diff(ξ)))$`

$$ds^2 = -t^2 \operatorname{del}(\psi)^2 - t^2 \sin(\theta)^2 \sin(\psi)^2 \operatorname{del}(\phi)^2 - t^2 \sin(\psi)^2 \operatorname{del}(\theta)^2 + \operatorname{del}(t)^2 \quad (\%t31)$$

Christoffel Symbol of the first kind

(%i32) `christof(lcs)$`

$$lcs_{1,2,2} = -t \quad (\%t32)$$

$$lcs_{1,3,3} = -t \sin(\psi)^2 \quad (\%t33)$$

$$lcs_{1,4,4} = -t \sin(\theta)^2 \sin(\psi)^2 \quad (\%t34)$$

$$lcs_{2,2,1} = t \quad (\%t35)$$

$$lcs_{2,3,3} = -t^2 \cos(\psi) \sin(\psi) \quad (\%t36)$$

$$lcs_{2,4,4} = -t^2 \sin(\theta)^2 \cos(\psi) \sin(\psi) \quad (\%t37)$$

$$lcs_{3,3,1} = t \sin(\psi)^2 \quad (\%t38)$$

$$lcs_{3,3,2} = t^2 \cos(\psi) \sin(\psi) \quad (\%t39)$$

$$lcs_{3,4,4} = -t^2 \cos(\theta) \sin(\theta) \sin(\psi)^2 \quad (\%t40)$$

$$lcs_{4,4,1} = t \sin(\theta)^2 \sin(\psi)^2 \quad (\%t41)$$

$$lcs_{4,4,2} = t^2 \sin(\theta)^2 \cos(\psi) \sin(\psi) \quad (\%t42)$$

$$lcs_{4,4,3} = t^2 \cos(\theta) \sin(\theta) \sin(\psi)^2 \quad (\%t43)$$

(%i44)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j],ξ[k]],[])=lcs[i,j,k])$
```

$$\Gamma_{t\psi\psi} = -t \quad (\%t44)$$

$$\Gamma_{t\theta\theta} = -t \sin(\psi)^2 \quad (\%t44)$$

$$\Gamma_{t\phi\phi} = -t \sin(\theta)^2 \sin(\psi)^2 \quad (\%t44)$$

$$\Gamma_{\psi\psi t} = t \quad (\%t44)$$

$$\Gamma_{\psi\theta\theta} = -t^2 \cos(\psi) \sin(\psi) \quad (\%t44)$$

$$\Gamma_{\psi\phi\phi} = -t^2 \sin(\theta)^2 \cos(\psi) \sin(\psi) \quad (\%t44)$$

$$\Gamma_{\theta\theta t} = t \sin(\psi)^2 \quad (\%t44)$$

$$\Gamma_{\theta\theta\psi} = t^2 \cos(\psi) \sin(\psi) \quad (\%t44)$$

$$\Gamma_{\theta\phi\phi} = -t^2 \cos(\theta) \sin(\theta) \sin(\psi)^2 \quad (\%t44)$$

$$\Gamma_{\phi\phi t} = t \sin(\theta)^2 \sin(\psi)^2 \quad (\%t44)$$

$$\Gamma_{\phi\phi\psi} = t^2 \sin(\theta)^2 \cos(\psi) \sin(\psi) \quad (\%t44)$$

$$\Gamma_{\phi\phi\theta} = t^2 \cos(\theta) \sin(\theta) \sin(\psi)^2 \quad (\%t44)$$

(%i45) `dispsym(ichr1,3,0);`

$$[[sym,[[1,2]],[]]] \tag{\%o45}$$

(%i46) `ishow('Γ([α,β,μ])=subst([%1=ν],rename(ev(ichr1([α,β,μ]),ichr1))))$`

$$\mathit{\Gamma}_{\alpha\beta\mu} = \frac{g_{\beta\mu,\alpha} + g_{\alpha\mu,\beta} - g_{\alpha\beta,\mu}}{2} \tag{\%t46}$$

(%i47) `ishow('Γ([α,β,1])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,1]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t\sin(\psi)^2 & 0 \\ 0 & 0 & 0 & t\sin(\theta)^2\sin(\psi)^2 \end{pmatrix} \tag{\%t47}$$

(%i48) `ishow('Γ([α,β,2])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,2]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 2} = \begin{pmatrix} 0 & -t & 0 & 0 \\ -t & 0 & 0 & 0 \\ 0 & 0 & t^2\cos(\psi)\sin(\psi) & 0 \\ 0 & 0 & 0 & t^2\sin(\theta)^2\cos(\psi)\sin(\psi) \end{pmatrix} \tag{\%t48}$$

(%i49) `ishow('Γ([α,β,3])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,3]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 3} = \begin{pmatrix} 0 & 0 & -t\sin(\psi)^2 & 0 \\ 0 & 0 & -t^2\cos(\psi)\sin(\psi) & 0 \\ -t\sin(\psi)^2 & -t^2\cos(\psi)\sin(\psi) & 0 & 0 \\ 0 & 0 & 0 & t^2\cos(\theta)\sin(\theta)\sin(\psi)^2 \end{pmatrix} \tag{\%t49}$$

(%i50) `ishow('Γ([α,β,4])=fullratsimp(genmatrix(lambda([α,β],lcs[α,β,4]),dim,dim)))$`

$$\mathit{\Gamma}_{\alpha\beta 4} = \begin{pmatrix} 0 & 0 & 0 & -t\sin(\theta)^2\sin(\psi)^2 \\ 0 & 0 & 0 & -t^2\sin(\theta)^2\cos(\psi)\sin(\psi) \\ 0 & 0 & 0 & -t^2\cos(\theta)\sin(\theta)\sin(\psi)^2 \\ -t\sin(\theta)^2\sin(\psi)^2 & -t^2\sin(\theta)^2\cos(\psi)\sin(\psi) & -t^2\cos(\theta)\sin(\theta)\sin(\psi)^2 & 0 \end{pmatrix} \tag{\%t50}$$

Christoffel Symbol of the second kind

(%i51) `christof(mcs)$`

$$mcs_{1,2,2} = \frac{1}{t} \tag{\%t51}$$

$$mcs_{1,3,3} = \frac{1}{t} \tag{\%t52}$$

$$mcs_{1,4,4} = \frac{1}{t} \tag{\%t53}$$

$$mcs_{2,2,1} = t \tag{\%t54}$$

$$mcs_{2,3,3} = \frac{\cos(\psi)}{\sin(\psi)} \tag{\%t55}$$

$$mcs_{2,4,4} = \frac{\cos(\psi)}{\sin(\psi)} \tag{\%t56}$$

$$mcs_{3,3,1} = t\sin(\psi)^2 \tag{\%t57}$$

$$mcs_{3,3,2} = -\cos(\psi)\,\sin(\psi) \tag{\%t58}$$

$$mcs_{3,4,4} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t59}$$

$$mcs_{4,4,1} = t\sin(\theta)^2\sin(\psi)^2 \tag{\%t60}$$

$$mcs_{4,4,2} = -\sin(\theta)^2\,\cos(\psi)\,\sin(\psi) \tag{\%t61}$$

$$mcs_{4,4,3} = -\cos(\theta)\,\sin(\theta) \tag{\%t62}$$

(%i63)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j]],[ξ[k]])=mcs[i,j,k])$
```

$$\Gamma_{t\psi}^{\psi} = \frac{1}{t} \tag{\%t63}$$

$$\Gamma_{t\theta}^{\theta} = \frac{1}{t} \tag{\%t63}$$

$$\Gamma_{t\phi}^{\phi} = \frac{1}{t} \tag{\%t63}$$

$$\Gamma_{\psi\psi}^{t} = t \tag{\%t63}$$

$$\Gamma_{\psi\theta}^{\theta} = \frac{\cos(\psi)}{\sin(\psi)} \tag{\%t63}$$

$$\Gamma_{\psi\phi}^{\phi} = \frac{\cos(\psi)}{\sin(\psi)} \tag{\%t63}$$

$$\Gamma_{\theta\theta}^{t} = t\sin(\psi)^2 \tag{\%t63}$$

$$\Gamma_{\theta\theta}^{\psi} = -\cos(\psi)\,\sin(\psi) \tag{\%t63}$$

$$\Gamma_{\theta\phi}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t63}$$

$$\Gamma_{\phi\phi}^{t} = t\sin(\theta)^2\sin(\psi)^2 \tag{\%t63}$$

$$\Gamma_{\phi\phi}^{\psi} = -\sin(\theta)^2\,\cos(\psi)\,\sin(\psi) \tag{\%t63}$$

$$\Gamma_{\phi\phi}^{\theta} = -\cos(\theta)\,\sin(\theta) \tag{\%t63}$$

(%i64)
```
dispsym(ichr2,2,1);
```

$$[[sym,[[1,2]],[]]] \tag{\%o64}$$

(%i65)
```
ishow('Γ([α,β],[μ])=subst([%1=ν],rename(ev(ichr2([α,β],[μ]),ichr2))))$
```

$$\Gamma_{\alpha\beta}^{\mu} = \frac{g^{\mu\nu}\,(g_{\beta\nu,\alpha} + g_{\alpha\nu,\beta} - g_{\alpha\beta,\nu})}{2} \tag{\%t65}$$

(%i66) `ishow('Γ([α,β],[1])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,1]),dim,dim)))$`

$$\Gamma^{1}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t\sin(\psi)^2 & 0 \\ 0 & 0 & 0 & t\sin(\theta)^2\sin(\psi)^2 \end{pmatrix} \tag{\%t66}$$

(%i67) `ishow('Γ([α,β],[2])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,2]),dim,dim)))$`

$$\Gamma^{2}_{\alpha\beta} = \begin{pmatrix} 0 & \frac{1}{t} & 0 & 0 \\ \frac{1}{t} & 0 & 0 & 0 \\ 0 & 0 & -\cos(\psi)\sin(\psi) & 0 \\ 0 & 0 & 0 & -\sin(\theta)^2\cos(\psi)\sin(\psi) \end{pmatrix} \tag{\%t67}$$

(%i68) `ishow('Γ([α,β],[3])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,3]),dim,dim)))$`

$$\Gamma^{3}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & \frac{1}{t} & 0 \\ 0 & 0 & \frac{\cos(\psi)}{\sin(\psi)} & 0 \\ \frac{1}{t} & \frac{\cos(\psi)}{\sin(\psi)} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\sin(\theta) \end{pmatrix} \tag{\%t68}$$

(%i69) `ishow('Γ([α,β],[4])=fullratsimp(genmatrix(lambda([α,β],mcs[α,β,4]),dim,dim)))$`

$$\Gamma^{4}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{t} \\ 0 & 0 & 0 & \frac{\cos(\psi)}{\sin(\psi)} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ \frac{1}{t} & \frac{\cos(\psi)}{\sin(\psi)} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{pmatrix} \tag{\%t69}$$

Riemann tensor

(%i71)
```
riemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ξ[a],ξ[b],ξ[c]],[ξ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}^{\psi}_{\theta\theta\psi} = 2\sin(\psi)^2 \tag{\%t71}$$

$$\mathbf{R}^{\psi}_{\phi\phi\psi} = 2\sin(\theta)^2\sin(\psi)^2 \tag{\%t71}$$

$$\mathbf{R}^{\theta}_{\phi\phi\theta} = \sin(\theta)^2\left(\sin(\psi)^2 - \cos(\psi)^2 + 1\right) \tag{\%t71}$$

(%i72) `dispsym(icurvature,3,1);`

$$[[anti, [[2,3]], []]] \tag{\%o72}$$

```
(%i74) lriemann(false)$
       for a thru dim do for b thru dim do
       for c thru (if symmetricp(lg,dim) then b else dim) do
       for d thru (if symmetricp(lg,dim) then a else dim) do
       if lriem[a,b,c,d]≠0 then
       ishow('R([ξ[d],ξ[a],ξ[b],ξ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{\psi\theta\theta\psi} = -2t^2 \sin(\psi)^2 \tag{\%t74}$$

$$\mathbf{R}_{\psi\phi\phi\psi} = -2t^2 \sin(\theta)^2 \sin(\psi)^2 \tag{\%t74}$$

$$\mathbf{R}_{\theta\phi\phi\theta} = -t^2 \sin(\theta)^2 \sin(\psi)^2 \left(\sin(\psi)^2 - \cos(\psi)^2 + 1\right) \tag{\%t74}$$

```
(%i76) uriemann(false)$
       for a thru dim do for b thru dim do
       for c thru (if symmetricp(lg,dim) then b else dim) do
       for d thru (if symmetricp(lg,dim) then a else dim) do
       if uriem[a,b,c,d]≠0 then
       ishow('R([],[ξ[a],ξ[b],ξ[c],ξ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{\theta\theta\psi\psi} = -\frac{2}{t^6 \sin(\psi)^2} \tag{\%t76}$$

$$\mathbf{R}^{\phi\phi\psi\psi} = -\frac{2}{t^6 \sin(\theta)^2 \sin(\psi)^2} \tag{\%t76}$$

$$\mathbf{R}^{\phi\phi\theta\theta} = -\frac{\sin(\psi)^2 - \cos(\psi)^2 + 1}{t^6 \sin(\theta)^2 \sin(\psi)^6} \tag{\%t76}$$

Ricci tensor

```
(%i79) ric:zeromatrix(dim,dim)$
       ricci(false)$
       for i thru dim do for j:i thru dim do
       if ric[i,j]≠0 then
       ishow('R([ξ[i],ξ[j]])=ric[i,j])$
```

$$\mathbf{R}_{\psi\psi} = 2^2 \tag{\%t79}$$

$$\mathbf{R}_{\theta\theta} = 3\sin(\psi)^2 - \cos(\psi)^2 + 1 \tag{\%t79}$$

$$\mathbf{R}_{\phi\phi} = \sin(\theta)^2 \left(3\sin(\psi)^2 - \cos(\psi)^2 + 1\right) \tag{\%t79}$$

```
(%i80) ishow('R([μ,ν],[])=ric)$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2^2 & 0 & 0 \\ 0 & 0 & 3\sin(\psi)^2 - \cos(\psi)^2 + 1 & 0 \\ 0 & 0 & 0 & \sin(\theta)^2 \left(3\sin(\psi)^2 - \cos(\psi)^2 + 1\right) \end{pmatrix} \tag{\%t80}$$

(%i83)
```
remcomps(R([μ,ν],[]))$
components(R([μ,ν],[]),ric)$
showcomps(R([μ,ν],[]))$
```

$$\mathbf{R}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2^2 & 0 & 0 \\ 0 & 0 & 3\sin(\psi)^2 - \cos(\psi)^2 + 1 & 0 \\ 0 & 0 & 0 & \sin(\theta)^2 \left(3\sin(\psi)^2 - \cos(\psi)^2 + 1\right) \end{pmatrix} \tag{\%t83}$$

(%i86)
```
remsym(R,2,0)$
decsym(R,2,0,[sym(all)],[])$
dispsym(R,2,0);
```

$$[[sym, [[1,2]], []]] \tag{\%o86}$$

(%i89)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ξ[i],ξ[j]])=uric[i,j])$
```

$$\mathbf{R}^{\psi\psi} = -\frac{4}{t^2} \tag{\%t89}$$

$$\mathbf{R}^{\theta\theta} = -\frac{3\sin(\psi)^2 - \cos(\psi)^2 + 1}{t^2 \sin(\psi)^2} \tag{\%t89}$$

$$\mathbf{R}^{\phi\phi} = -\frac{3\sin(\psi)^2 - \cos(\psi)^2 + 1}{t^2 \sin(\psi)^2} \tag{\%t89}$$

(%i90)
```
ishow('R([],[μ,ν])=uric)$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{4}{t^2} & 0 & 0 \\ 0 & 0 & -\frac{3\sin(\psi)^2-\cos(\psi)^2+1}{t^2\sin(\psi)^2} & 0 \\ 0 & 0 & 0 & -\frac{3\sin(\psi)^2-\cos(\psi)^2+1}{t^2\sin(\psi)^2} \end{pmatrix} \tag{\%t90}$$

(%i93)
```
remcomps(R([],[μ,ν]))$
components(R([],[μ,ν]),uric)$
showcomps(R([],[μ,ν]))$
```

$$\mathbf{R}^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{4}{t^2} & 0 & 0 \\ 0 & 0 & -\frac{3\sin(\psi)^2-\cos(\psi)^2+1}{t^2\sin(\psi)^2} & 0 \\ 0 & 0 & 0 & -\frac{3\sin(\psi)^2-\cos(\psi)^2+1}{t^2\sin(\psi)^2} \end{pmatrix} \tag{\%t93}$$

(%i96)
```
remsym(R,0,2)$
decsym(R,0,2,[],[sym(all)])$
dispsym(R,0,2);
```

$$[[sym, [], [[1,2]]]] \tag{\%o96}$$

Scalar curvature

(%i97) `factor(radcan(scurvature()));`

$$-\frac{2\left(5\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1\right)}{t^{2}\sin\left(\psi\right)^{2}} \tag{\%o97}$$

Kretschmann invariant

(%i98) `factor(radcan(rinvariant()));`

$$\frac{4\left(9\sin\left(\psi\right)^{4}-2\cos\left(\psi\right)^{2}\sin\left(\psi\right)^{2}+2\sin\left(\psi\right)^{2}+\cos\left(\psi\right)^{4}-2\cos\left(\psi\right)^{2}+1\right)}{t^{4}\sin\left(\psi\right)^{4}} \tag{\%o98}$$

Einstein tensor

(%i99) `kill(labels)$`

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ξ[i]],[ξ[j]])=ein[i,j])$
```

$$\mathbf{G}_{t}^{t}=\frac{5\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1}{t^{2}\sin\left(\psi\right)^{2}} \tag{\%t3}$$

$$\mathbf{G}_{\psi}^{\psi}=\frac{\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1}{t^{2}\sin\left(\psi\right)^{2}} \tag{\%t3}$$

$$\mathbf{G}_{\theta}^{\theta}=\frac{2}{t^{2}} \tag{\%t3}$$

$$\mathbf{G}_{\phi}^{\phi}=\frac{2}{t^{2}} \tag{\%t3}$$

(%i4) `ishow('G([μ],[ν])=ein)$`

$$\mathbf{G}_{\mu}^{\nu}=\begin{pmatrix}\frac{5\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1}{t^{2}\sin\left(\psi\right)^{2}} & 0 & 0 & 0\\ 0 & \frac{\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1}{t^{2}\sin\left(\psi\right)^{2}} & 0 & 0\\ 0 & 0 & \frac{2}{t^{2}} & 0\\ 0 & 0 & 0 & \frac{2}{t^{2}}\end{pmatrix} \tag{\%t4}$$

(%i7)
```
remcomps(G([μ],[ν]))$
components(G([μ],[ν]),ein)$
showcomps(G([μ],[ν]))$
```

$$\mathbf{G}_{\mu}^{\nu}=\begin{pmatrix}\frac{5\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1}{t^{2}\sin\left(\psi\right)^{2}} & 0 & 0 & 0\\ 0 & \frac{\sin\left(\psi\right)^{2}-\cos\left(\psi\right)^{2}+1}{t^{2}\sin\left(\psi\right)^{2}} & 0 & 0\\ 0 & 0 & \frac{2}{t^{2}} & 0\\ 0 & 0 & 0 & \frac{2}{t^{2}}\end{pmatrix} \tag{\%t7}$$

(%i10)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ξ[i],ξ[j]],[])=lein[i,j])$
```

$$\mathbf{G}_{tt} = \frac{5\sin(\psi)^2 - \cos(\psi)^2 + 1}{t^2 \sin(\psi)^2} \tag{\%t10}$$

$$\mathbf{G}_{\psi\psi} = -\frac{\sin(\psi)^2 - \cos(\psi)^2 + 1}{\sin(\psi)^2} \tag{\%t10}$$

$$\mathbf{G}_{\theta\theta} = -2\sin(\psi)^2 \tag{\%t10}$$

$$\mathbf{G}_{\phi\phi} = -2\sin(\theta)^2 \sin(\psi)^2 \tag{\%t10}$$

(%i11)
```
ishow('G([μ,ν],[])=lein)$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} \frac{5\sin(\psi)^2-\cos(\psi)^2+1}{t^2\sin(\psi)^2} & 0 & 0 & 0 \\ 0 & -\frac{\sin(\psi)^2-\cos(\psi)^2+1}{\sin(\psi)^2} & 0 & 0 \\ 0 & 0 & -2\sin(\psi)^2 & 0 \\ 0 & 0 & 0 & -2\sin(\theta)^2\sin(\psi)^2 \end{pmatrix} \tag{\%t11}$$

(%i14)
```
remcomps(G([μ,ν],[]))$
components(G([μ,ν],[]),lein)$
showcomps(G([μ,ν],[]))$
```

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} \frac{5\sin(\psi)^2-\cos(\psi)^2+1}{t^2\sin(\psi)^2} & 0 & 0 & 0 \\ 0 & -\frac{\sin(\psi)^2-\cos(\psi)^2+1}{\sin(\psi)^2} & 0 & 0 \\ 0 & 0 & -2\sin(\psi)^2 & 0 \\ 0 & 0 & 0 & -2\sin(\theta)^2\sin(\psi)^2 \end{pmatrix} \tag{\%t14}$$

(%i17)
```
remsym(G,2,0)$
decsym(G,2,0,[sym(all)],[])$
dispsym(G,2,0);
```

$$[[sym, [[1,2]], []]] \tag{\%o17}$$

Reduce Order

(%i19)
```
cv_coords:[T,Ψ,Θ,Φ]$
depends(cv_coords,s)$
```

(%i23)
```
gradef(t,s,T)$ gradef(ψ,s,Ψ)$
gradef(θ,s,Θ)$
gradef(ϕ,s,Φ)$
```

Geodesics

(%i24)
```
cgeodesic(false)$
```

Solve for second derivative of coordinates

(%i25) `geodsol:linsolve(listarray(geod),diff(ξ,s,2))$`

(%i26) `map(ldisp,geodsol)$`

$$T_s = \left(-t\,\Phi^2 \sin(\theta)^2 - t\,\Theta^2\right) \sin(\psi)^2 - t\,\Psi^2 \tag{\%t26}$$

$$\Psi_s = \frac{\left(t\,\Phi^2 \sin(\theta)^2\,\cos(\psi) + t\,\Theta^2\,\cos(\psi)\right)\,\sin(\psi) - 2T\,\Psi}{t} \tag{\%t27}$$

$$\Theta_s = \frac{\left(t\,\Phi^2\,\cos(\theta)\,\sin(\theta) - 2T\,\Theta\right)\,\sin(\psi) - 2t\,\Theta\,\Psi\,\cos(\psi)}{t\,\sin(\psi)} \tag{\%t28}$$

$$\Phi_s = -\frac{(2T\,\Phi\,\sin(\theta) + 2t\,\Theta\,\Phi\,\cos(\theta))\,\sin(\psi) + 2t\,\Phi\,\Psi\,\sin(\theta)\,\cos(\psi)}{t\,\sin(\theta)\,\sin(\psi)} \tag{\%t29}$$