Maxima's Christoffel

[Fwd: Maxima's christoffel]

computing christoffel symbols

Christoffel symbols of the first kind ill calculated

(%i2)
```
info:build_info()$info@version;
```

(%o2)

5.38.1

(%i2)
```
reset()$kill(all)$
```

(%i1)
```
load(linearalgebra)$
```

(%i2)
```
if get('itensor,'version)=false then load(itensor)$
```

(%i3)
```
imetric(g)$
```

(%i4)
```
if get('ctensor,'version)=false then load(ctensor)$
```

(%i10)
```
ctrgsimp:true$
ratchristof:true$
ratriemann:true$
rateinstein:true$
ratweyl:true$
ratfac:true$
```

(%i11)
```
derivabbrev:true$
```

(%i12)
```
declare(trigsimp,evfun)$
```

1 Thomas Widlar wrote:

I have compared Maxima's internal calculation (on the left) to our explicit calculation (on the right) which is attached. Our Gamma subscripts are in reverse order of Maxima's mcs which also have +1 added to each subcript. The values agree in all but five cases (marked XXX below) where Maxima mcs has zero and our Gamma has an expression. With Sean Carroll and Dr. Evans having checked them carefully by hand and our calculating them explicitly with Maxima, we have some confidence in the Gammas. However it could be that Viktor's solution is not the same problem as ours due to a difference in coordinate order or the metric specification. Does anything stand out to you? I have found ctensor.mac and the christof(dis) function. I will try to figure out what is going on. Viktor Toth's program.

```
(%i13) depends([a,b],[t,r])$
(%i17) assume(0≤r)$
       assume(0≤θ,θ≤π)$
       assume(0≤sin(θ))$
       assume(0≤ϕ,ϕ≤2*π)$
(%i18) ξ:ct_coords:[t,r,θ,ϕ]$
(%i19) dim:length(ct_coords)$
(%i24) lg:ident(4)$
       lg[1,1]:-exp(2*a)$
       lg[2,2]:exp(2*b)$
       lg[3,3]:r^2$
       lg[4,4]:r^2*sin(θ)^2$
(%i25) cmetric()$
```

Covariant Metric tensor

```
(%i26) ishow('g([μ,ν],[])=lg)$
```

$$g_{\mu\nu} = \begin{pmatrix} -e^{2a} & 0 & 0 & 0 \\ 0 & e^{2b} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin(\theta)^2 \end{pmatrix} \quad (\%t26)$$

Contravariant Metric tensor

```
(%i27) ishow('g([],[μ,ν])=ug)$
```

$$g^{\mu\nu} = \begin{pmatrix} -e^{-2a} & 0 & 0 & 0 \\ 0 & e^{-2b} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\sin(\theta)^2} \end{pmatrix} \quad (\%t27)$$

The determinant of the metric tensor

```
(%i28) expand(√(-gdet));
```

$$e^{b+a}\, r^2 \sin(\theta) \quad (\%o28)$$

Christoffel Symbol of the first kind

(%i29) `christof(lcs)$`

$$lcs_{1,1,1} = -e^{2a}\ (a_t) \tag{\%t29}$$

$$lcs_{1,1,2} = e^{2a}\ (a_r) \tag{\%t30}$$

$$lcs_{1,2,1} = -e^{2a}\ (a_r) \tag{\%t31}$$

$$lcs_{1,2,2} = e^{2b}\ (b_t) \tag{\%t32}$$

$$lcs_{2,2,1} = -e^{2b}\ (b_t) \tag{\%t33}$$

$$lcs_{2,2,2} = e^{2b}\ (b_r) \tag{\%t34}$$

$$lcs_{2,3,3} = r \tag{\%t35}$$

$$lcs_{2,4,4} = r\sin(\theta)^2 \tag{\%t36}$$

$$lcs_{3,3,2} = -r \tag{\%t37}$$

$$lcs_{3,4,4} = r^2\ \cos(\theta)\ \sin(\theta) \tag{\%t38}$$

$$lcs_{4,4,2} = -r\sin(\theta)^2 \tag{\%t39}$$

$$lcs_{4,4,3} = -r^2\ \cos(\theta)\ \sin(\theta) \tag{\%t40}$$

(%i41)
```
for i thru dim do for j:i thru dim do for k thru dim do
if lcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j],ξ[k]],[])=lcs[i,j,k])$
```

$$\Gamma_{ttt} = -e^{2a}\ (a_t) \qquad (\%t41)$$

$$\Gamma_{ttr} = e^{2a}\ (a_r) \qquad (\%t41)$$

$$\Gamma_{trt} = -e^{2a}\ (a_r) \qquad (\%t41)$$

$$\Gamma_{trr} = e^{2b}\ (b_t) \qquad (\%t41)$$

$$\Gamma_{rrt} = -e^{2b}\ (b_t) \qquad (\%t41)$$

$$\Gamma_{rrr} = e^{2b}\ (b_r) \qquad (\%t41)$$

$$\Gamma_{r\theta\theta} = r \qquad (\%t41)$$

$$\Gamma_{r\phi\phi} = r\sin(\theta)^2 \qquad (\%t41)$$

$$\Gamma_{\theta\theta r} = -r \qquad (\%t41)$$

$$\Gamma_{\theta\phi\phi} = r^2\ \cos(\theta)\ \sin(\theta) \qquad (\%t41)$$

$$\Gamma_{\phi\phi r} = -r\sin(\theta)^2 \qquad (\%t41)$$

$$\Gamma_{\phi\phi\theta} = -r^2\ \cos(\theta)\ \sin(\theta) \qquad (\%t41)$$

(%i42) `cdisplay(lcs,1)$`

$$lcs_1 = \begin{pmatrix} -e^{2a}\ (a_t) & e^{2a}\ (a_r) & 0 & 0 \\ -e^{2a}\ (a_r) & e^{2b}\ (b_t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(%i43) `cdisplay(lcs,2)$`

$$lcs_2 = \begin{pmatrix} -e^{2a}\,(a_r) & e^{2b}\,(b_t) & 0 & 0 \\ -e^{2b}\,(b_t) & e^{2b}\,(b_r) & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\sin(\theta)^2 \end{pmatrix}$$

(%i44) `cdisplay(lcs,3)$`

$$lcs_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & -r & 0 & 0 \\ 0 & 0 & 0 & r^2\,\cos(\theta)\,\sin(\theta) \end{pmatrix}$$

(%i45) `cdisplay(lcs,4)$`

$$lcs_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r\sin(\theta)^2 \\ 0 & 0 & 0 & r^2\,\cos(\theta)\,\sin(\theta) \\ 0 & -r\sin(\theta)^2 & -r^2\,\cos(\theta)\,\sin(\theta) & 0 \end{pmatrix}$$

(%i46) `ishow(Γ([μ,ν,1])=Γ_11:genmatrix(lambda([μ,ν],lcs[μ,ν,1]),dim,dim))$`

$$\mathit{\Gamma}_{\mu\nu 1} = \begin{pmatrix} -e^{2a}\,(a_t) & -e^{2a}\,(a_r) & 0 & 0 \\ -e^{2a}\,(a_r) & -e^{2b}\,(b_t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{\%t46}$$

(%i47) `ishow(Γ([μ,ν,2])=Γ_21:genmatrix(lambda([μ,ν],lcs[μ,ν,2]),dim,dim))$`

$$\mathit{\Gamma}_{\mu\nu 2} = \begin{pmatrix} e^{2a}\,(a_r) & e^{2b}\,(b_t) & 0 & 0 \\ e^{2b}\,(b_t) & e^{2b}\,(b_r) & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & 0 & 0 & -r\sin(\theta)^2 \end{pmatrix} \tag{\%t47}$$

(%i48) `ishow(Γ([μ,ν,3])=Γ_31:genmatrix(lambda([μ,ν],lcs[μ,ν,3]),dim,dim))$`

$$\mathit{\Gamma}_{\mu\nu 3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & 0 & -r^2\,\cos(\theta)\,\sin(\theta) \end{pmatrix} \tag{\%t48}$$

(%i49) `ishow(Γ([μ,ν,4])=Γ_41:genmatrix(lambda([μ,ν],lcs[μ,ν,4]),dim,dim))$`

$$\mathit{\Gamma}_{\mu\nu 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r\sin(\theta)^2 \\ 0 & 0 & 0 & r^2\,\cos(\theta)\,\sin(\theta) \\ 0 & r\sin(\theta)^2 & r^2\,\cos(\theta)\,\sin(\theta) & 0 \end{pmatrix} \tag{\%t49}$$

Christoffel Symbol of the second kind

(%i50) `christof(mcs)$`

$$mcs_{1,1,1} = a_t \tag{\%t50}$$

$$mcs_{1,1,2} = (a_r)\ e^{2a-2b} \tag{\%t51}$$

$$mcs_{1,2,1} = a_r \tag{\%t52}$$

$$mcs_{1,2,2} = b_t \tag{\%t53}$$

$$mcs_{2,2,1} = e^{2b-2a}\ (b_t) \tag{\%t54}$$

$$mcs_{2,2,2} = b_r \tag{\%t55}$$

$$mcs_{2,3,3} = \frac{1}{r} \tag{\%t56}$$

$$mcs_{2,4,4} = \frac{1}{r} \tag{\%t57}$$

$$mcs_{3,3,2} = -e^{-2b} r \tag{\%t58}$$

$$mcs_{3,4,4} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t59}$$

$$mcs_{4,4,2} = -e^{-2b} r \sin(\theta)^2 \tag{\%t60}$$

$$mcs_{4,4,3} = -\cos(\theta)\ \sin(\theta) \tag{\%t61}$$

(%i62)
```
for i thru dim do for j:i thru dim do for k thru dim do
if mcs[i,j,k]≠0 then
ishow('Γ([ξ[i],ξ[j]],[ξ[k]])=mcs[i,j,k])$
```

$$\mathit{\Gamma}^{t}_{tt} = a_t \tag{\%t62}$$

$$\mathit{\Gamma}^{r}_{tt} = (a_r)\ e^{2a-2b} \tag{\%t62}$$

$$\mathit{\Gamma}^{t}_{tr} = a_r \tag{\%t62}$$

$$\mathit{\Gamma}^{r}_{tr} = b_t \tag{\%t62}$$

$$\mathit{\Gamma}^{t}_{rr} = e^{2b-2a}\ (b_t) \tag{\%t62}$$

$$\mathit{\Gamma}^{r}_{rr} = b_r \tag{\%t62}$$

$$\mathit{\Gamma}^{\theta}_{r\theta} = \frac{1}{r} \tag{\%t62}$$

$$\mathit{\Gamma}^{\phi}_{r\phi} = \frac{1}{r} \tag{\%t62}$$

$$\mathit{\Gamma}^{r}_{\theta\theta} = -e^{-2b} r \tag{\%t62}$$

$$\mathit{\Gamma}^{\phi}_{\theta\phi} = \frac{\cos(\theta)}{\sin(\theta)} \tag{\%t62}$$

$$\mathit{\Gamma}^{r}_{\phi\phi} = -e^{-2b} r \sin(\theta)^2 \tag{\%t62}$$

$$\mathit{\Gamma}^{\theta}_{\phi\phi} = -\cos(\theta)\ \sin(\theta) \tag{\%t62}$$

(%i63) cdisplay(mcs,1)$

$$mcs_1 = \begin{pmatrix} a_t & (a_r)\ e^{2a-2b} & 0 & 0 \\ a_r & b_t & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(%i64) cdisplay(mcs,2)$

$$mcs_2 = \begin{pmatrix} a_r & b_t & 0 & 0 \\ e^{2b-2a}\ (b_t) & b_r & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r} \end{pmatrix}$$

(%i65) cdisplay(mcs,3)$

$$mcs_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & -e^{-2b} r & 0 & 0 \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \end{pmatrix}$$

(%i66) cdisplay(mcs,4)$

$$mcs_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & -e^{-2b} r \sin(\theta)^2 & -\cos(\theta)\ \sin(\theta) & 0 \end{pmatrix}$$

(%i67) ishow(Γ([μ,ν],[1])=Γ_12:genmatrix(lambda([μ,ν],mcs[μ,ν,1]),dim,dim))$

$$\Gamma^1_{\mu\nu} = \begin{pmatrix} a_t & a_r & 0 & 0 \\ a_r & e^{2b-2a}\ (b_t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (\%t67)$$

(%i68) ishow(Γ([μ,ν],[2])=Γ_22:genmatrix(lambda([μ,ν],mcs[μ,ν,2]),dim,dim))$

$$\Gamma^2_{\mu\nu} = \begin{pmatrix} (a_r)\ e^{2a-2b} & b_t & 0 & 0 \\ b_t & b_r & 0 & 0 \\ 0 & 0 & -e^{-2b} r & 0 \\ 0 & 0 & 0 & -e^{-2b} r \sin(\theta)^2 \end{pmatrix} \quad (\%t68)$$

(%i69) ishow(Γ([μ,ν],[3])=Γ_32:genmatrix(lambda([μ,ν],mcs[μ,ν,3]),dim,dim))$

$$\Gamma^3_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & -\cos(\theta)\ \sin(\theta) \end{pmatrix} \quad (\%t69)$$

(%i70) ishow(Γ([μ,ν],[4])=Γ_42:genmatrix(lambda([μ,ν],mcs[μ,ν,4]),dim,dim))$

$$\Gamma^4_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & \frac{\cos(\theta)}{\sin(\theta)} \\ 0 & \frac{1}{r} & \frac{\cos(\theta)}{\sin(\theta)} & 0 \end{pmatrix} \quad (\%t70)$$

Riemann Tensor

(%i72)
```
riemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([" ",ξ[a],ξ[b],ξ[c]],[ξ[d]])=riem[a,b,c,d])$
```

$$\mathbf{R}^{t}{}_{rrt} = e^{-2a}(e^{2b}\ (b_{tt}) + e^{2b}\,(b_t)^2 - (a_t)\ e^{2b}\ (b_t) + e^{2a}\ (a_r)\ (b_r) - e^{2a}\ (a_{rr}) - e^{2a}\,(a_r)^2) \quad (\%t72)$$

$$\mathbf{R}^{t}{}_{\theta\theta t} = -\,(a_r)\ e^{-2b} r \quad (\%t72)$$

$$\mathbf{R}^{r}{}_{\theta\theta t} = e^{-2b}\ (b_t)\, r \quad (\%t72)$$

$$\mathbf{R}^{t}{}_{\theta\theta r} = -e^{-2a}\ (b_t)\, r \quad (\%t72)$$

$$\mathbf{R}^{r}{}_{\theta\theta r} = e^{-2b}\ (b_r)\, r \quad (\%t72)$$

$$\mathbf{R}^{t}{}_{\phi\phi t} = -\,(a_r)\ e^{-2b} r \sin(\theta)^2 \quad (\%t72)$$

$$\mathbf{R}^{r}{}_{\phi\phi t} = e^{-2b}\ (b_t)\, r \sin(\theta)^2 \quad (\%t72)$$

$$\mathbf{R}^{t}{}_{\phi\phi r} = -e^{-2a}\ (b_t)\, r \sin(\theta)^2 \quad (\%t72)$$

$$\mathbf{R}^{r}{}_{\phi\phi r} = e^{-2b}\ (b_r)\, r \sin(\theta)^2 \quad (\%t72)$$

$$\mathbf{R}^{\theta}{}_{\phi\phi\theta} = e^{-2b}\ \left(e^{b} - 1\right)\ \left(e^{b} + 1\right)\ \sin(\theta)^2 \quad (\%t72)$$

(%i74)
```
lriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if lriem[a,b,c,d]≠0 then
ishow('R([ξ[d],ξ[a],ξ[b],ξ[c]],[])=lriem[a,b,c,d])$
```

$$\mathbf{R}_{trrt} = -e^{2b}\ (b_{tt}) - e^{2b}\,(b_t)^2 + (a_t)\ e^{2b}\ (b_t) - e^{2a}\ (a_r)\ (b_r) + e^{2a}\ (a_{rr}) + e^{2a}\,(a_r)^2 \quad (\%t74)$$

$$\mathbf{R}_{t\theta\theta t} = (a_r)\ e^{2a-2b} r \quad (\%t74)$$

$$\mathbf{R}_{r\theta\theta t} = (b_t)\, r \quad (\%t74)$$

$$\mathbf{R}_{t\theta\theta r} = (b_t)\, r \quad (\%t74)$$

$$\mathbf{R}_{r\theta\theta r} = (b_r)\, r \quad (\%t74)$$

$$\mathbf{R}_{t\phi\phi t} = (a_r)\ e^{2a-2b} r \sin(\theta)^2 \quad (\%t74)$$

$$\mathbf{R}_{r\phi\phi t} = (b_t)\, r \sin(\theta)^2 \quad (\%t74)$$

$$\mathbf{R}_{t\phi\phi r} = (b_t)\, r \sin(\theta)^2 \quad (\%t74)$$

$$\mathbf{R}_{r\phi\phi r} = (b_r)\, r \sin(\theta)^2 \quad (\%t74)$$

$$\mathbf{R}_{\theta\phi\phi\theta} = e^{-2b}\ \left(e^{b} - 1\right)\ \left(e^{b} + 1\right)\ r^2 \sin(\theta)^2 \quad (\%t74)$$

(%i76)
```
uriemann(false)$
for a thru dim do for b thru dim do
for c thru (if symmetricp(lg,dim) then b else dim) do
for d thru (if symmetricp(lg,dim) then a else dim) do
if riem[a,b,c,d]≠0 then
ishow('R([],[ξ[a],ξ[b],ξ[c],ξ[d]])=uriem[a,b,c,d])$
```

$$\mathbf{R}^{rrtt} = -e^{-4b-4a}\left(e^{2b}\,(b_{tt}) + e^{2b}\,(b_t)^2 - (a_t)\,e^{2b}\,(b_t) + e^{2a}\,(a_r)\,(b_r) - e^{2a}\,(a_{rr}) - e^{2a}\,(a_r)^2\right) \quad (\%t76)$$

$$\mathbf{R}^{\theta\theta tt} = \frac{(a_r)\,e^{-2b-2a}}{r^3} \quad (\%t76)$$

$$\mathbf{R}^{\theta\theta tr} = -\frac{e^{-2b-2a}\,(b_t)}{r^3} \quad (\%t76)$$

$$\mathbf{R}^{\theta\theta rt} = -\frac{e^{-2b-2a}\,(b_t)}{r^3} \quad (\%t76)$$

$$\mathbf{R}^{\theta\theta rr} = \frac{e^{-4b}\,(b_r)}{r^3} \quad (\%t76)$$

$$\mathbf{R}^{\phi\phi tt} = \frac{(a_r)\,e^{-2b-2a}}{r^3\sin(\theta)^2} \quad (\%t76)$$

$$\mathbf{R}^{\phi\phi tr} = -\frac{e^{-2b-2a}\,(b_t)}{r^3\sin(\theta)^2} \quad (\%t76)$$

$$\mathbf{R}^{\phi\phi rt} = -\frac{e^{-2b-2a}\,(b_t)}{r^3\sin(\theta)^2} \quad (\%t76)$$

$$\mathbf{R}^{\phi\phi rr} = \frac{e^{-4b}\,(b_r)}{r^3\sin(\theta)^2} \quad (\%t76)$$

$$\mathbf{R}^{\phi\phi\theta\theta} = \frac{e^{-2b}\,\left(e^b-1\right)\,\left(e^b+1\right)}{r^6\sin(\theta)^2} \quad (\%t76)$$

Ricci Tensor

(%i79)
```
ric:zeromatrix(dim,dim)$
ricci(false)$
for i thru dim do for j:i thru dim do
if ric[i,j]≠0 then
ishow('R([ξ[i],ξ[j]])=ric[i,j])$
```

$$\mathbf{R}_{tt} = -(e^{-2b}(e^{2b}\,(b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\,e^{2b}\,(b_t)\,r + e^{2a}\,(a_r)\,(b_r)\,r - e^{2a}\,(a_{rr})\,r - e^{2a}\,(a_r)^2 r - 2e^{2a}\,(a_r)))/r \tag{\%t79}$$

$$\mathbf{R}_{tr} = \frac{2\,(b_t)}{r} \tag{\%t79}$$

$$\mathbf{R}_{rr} = (e^{-2a}(e^{2b}\,(b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\,e^{2b}\,(b_t)\,r + e^{2a}\,(a_r)\,(b_r)\,r - e^{2a}\,(a_{rr})\,r - e^{2a}\,(a_r)^2 r + 2e^{2a}\,(b_r)))/r \tag{\%t79}$$

$$\mathbf{R}_{\theta\theta} = e^{-2b}\,\left((b_r)\,r - (a_r)\,r + e^{2b} - 1\right) \tag{\%t79}$$

$$\mathbf{R}_{\phi\phi} = e^{-2b}\,\left((b_r)\,r - (a_r)\,r + e^{2b} - 1\right)\,\sin(\theta)^2 \tag{\%t79}$$

Returns a list of the unique differential equations

(%i80)
```
map(ldisp,efe:findde(ric,2))$
```

$$e^{2b}\,(b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\,e^{2b}\,(b_t)\,r + e^{2a}\,(a_r)\,(b_r)\,r - e^{2a}\,(a_{rr})\,r - e^{2a}\,(a_r)^2 r - 2e^{2a}\,(a_r) \tag{\%t80}$$

$$b_t \tag{\%t81}$$

$$e^{2b}\,(b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\,e^{2b}\,(b_t)\,r + e^{2a}\,(a_r)\,(b_r)\,r - e^{2a}\,(a_{rr})\,r - e^{2a}\,(a_r)^2 r + 2e^{2a}\,(b_r) \tag{\%t82}$$

$$(b_r)\,r - (a_r)\,r + e^{2b} - 1 \tag{\%t83}$$

(%i84)
```
factor(efe[1]-efe[3]);
```

$$-2e^{2a}\,(b_r + a_r) \tag{\%o84}$$

(%i87)
```
uric:zeromatrix(dim,dim)$
uricci(false)$
for i thru dim do for j:i thru dim do
if uric[i,j]≠0 then
ishow('R([],[ξ[i],ξ[j]])=uric[i,j])$
```

$$\mathbf{R}^{tt} = (e^{-2b-2a}(e^{2b}\,(b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\,e^{2b}\,(b_t)\,r + e^{2a}\,(a_r)\,(b_r)\,r - e^{2a}\,(a_{rr})\,r - e^{2a}\,(a_r)^2 r - 2e^{2a}\,(a_r)))/r \tag{\%t87}$$

$$\mathbf{R}^{tr} = \frac{2e^{-2b}\,(b_t)}{r} \tag{\%t87}$$

$$\mathbf{R}^{rr} = (e^{-2b-2a}(e^{2b}\,(b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\,e^{2b}\,(b_t)\,r + e^{2a}\,(a_r)\,(b_r)\,r - e^{2a}\,(a_{rr})\,r - e^{2a}\,(a_r)^2 r + 2e^{2a}\,(b_r)))/r \tag{\%t87}$$

$$\mathbf{R}^{\theta\theta} = \frac{e^{-2b}\,\left((b_r)\,r - (a_r)\,r + e^{2b} - 1\right)}{r^2} \tag{\%t87}$$

$$\mathbf{R}^{\phi\phi} = \frac{e^{-2b}\,\left((b_r)\,r - (a_r)\,r + e^{2b} - 1\right)}{r^2} \tag{\%t87}$$

Returns a list of the unique differential equations

(%i88) `map(ldisp,efe:findde(uric,2))$`

$$e^{2b}\left(b_{tt}\right)r+e^{2b}\left(b_t\right)^2r-\left(a_t\right)e^{2b}\left(b_t\right)r+e^{2a}\left(a_r\right)\left(b_r\right)r-e^{2a}\left(a_{rr}\right)r-e^{2a}\left(a_r\right)^2r-2e^{2a}\left(a_r\right) \quad (\%t88)$$

$$b_t \quad (\%t89)$$

$$e^{2b}\left(b_{tt}\right)r+e^{2b}\left(b_t\right)^2r-\left(a_t\right)e^{2b}\left(b_t\right)r+e^{2a}\left(a_r\right)\left(b_r\right)r-e^{2a}\left(a_{rr}\right)r-e^{2a}\left(a_r\right)^2r+2e^{2a}\left(b_r\right) \quad (\%t90)$$

$$\left(b_r\right)r-\left(a_r\right)r+e^{2b}-1 \quad (\%t91)$$

(%i92) `factor(efe[1]-efe[3]);`

$$-2e^{2a}\left(b_r+a_r\right) \quad (\%o92)$$

Scalar curvature

(%i93) `factor(radcan(scurvature()));`

$$(2e^{-2b-2a}(e^{2b}\left(b_{tt}\right)r^2+e^{2b}\left(b_t\right)^2r^2-\left(a_t\right)e^{2b}\left(b_t\right)r^2+e^{2a}\left(a_r\right)\left(b_r\right)r^2-e^{2a}\left(a_{rr}\right)r^2-e^{2a}\left(a_r\right)^2r^2+2e^{2a}\left(b_r\right)r-2e^{2a}\left(a_r\right)r+e^{2b+2} \quad (\%o93)$$

Kretschmann invariant

(%i94) `factor(radcan(rinvariant()));`

$$(4e^{-4b-4a}(e^{4b}\left(b_{tt}\right)^2r^4+2e^{4b}\left(b_t\right)^2\left(b_{tt}\right)r^4-2\left(a_t\right)e^{4b}\left(b_t\right)\left(b_{tt}\right)r^4+2\left(a_r\right)e^{2b+2a}\left(b_r\right)\left(b_{tt}\right)r^4-2\left(a_{rr}\right)e^{2b+2a}\left(b_{tt}\right)r^4-2(a_r)^2e^{2b} \quad (\%o94)$$

Einstein Tensor

(%i95) kill(labels)$

(%i3)
```
ein:zeromatrix(dim,dim)$
einstein(false)$
for i thru dim do for j:i thru dim do
if ein[i,j]≠0 then
ishow('G([ξ[i]],[ξ[j]])=fullratsimp(ein[i,j]))$
```

$$\mathbf{G}_t^{t} = -\frac{e^{-2b}\left(2\left(b_r\right)r+e^{2b}-1\right)}{r^2} \tag{\%t3}$$

$$\mathbf{G}_t^{r} = \frac{2e^{-2b}\left(b_t\right)}{r} \tag{\%t3}$$

$$\mathbf{G}_r^{r} = \frac{e^{-2b}\left(2\left(a_r\right)r-e^{2b}+1\right)}{r^2} \tag{\%t3}$$

$$\mathbf{G}_\theta^{\theta} = -(e^{-2b-2a}((e^{2b}\left(b_{tt}\right)+e^{2b}\left(b_t\right)^2-\left(a_t\right)e^{2b}\left(b_t\right)+e^{2a}\left(a_r\right)\left(b_r\right)-e^{2a}\left(a_{rr}\right)-e^{2a}\left(a_r\right)^2)r+e^{2a}\left(b_r\right)-e^{2a}\left(a_r\right)))/r \tag{\%t3}$$

$$\mathbf{G}_\phi^{\phi} = -(e^{-2b-2a}((e^{2b}\left(b_{tt}\right)+e^{2b}\left(b_t\right)^2-\left(a_t\right)e^{2b}\left(b_t\right)+e^{2a}\left(a_r\right)\left(b_r\right)-e^{2a}\left(a_{rr}\right)-e^{2a}\left(a_r\right)^2)r+e^{2a}\left(b_r\right)-e^{2a}\left(a_r\right)))/r \tag{\%t3}$$

Returns a list of the unique differential equations

(%i4) map(ldisp,efe:findde(ein,2))$

$$2\left(b_r\right)r+e^{2b}-1 \tag{\%t4}$$

$$b_t \tag{\%t5}$$

$$2\left(a_r\right)r-e^{2b}+1 \tag{\%t6}$$

$$e^{2b}\left(b_{tt}\right)r+e^{2b}\left(b_t\right)^2r-\left(a_t\right)e^{2b}\left(b_t\right)r+e^{2a}\left(a_r\right)\left(b_r\right)r-e^{2a}\left(a_{rr}\right)r-e^{2a}\left(a_r\right)^2r+e^{2a}\left(b_r\right)-e^{2a}\left(a_r\right) \tag{\%t7}$$

(%i8) factor(efe[1]-efe[3]);

$$2\left(\left(b_r\right)r-\left(a_r\right)r+e^{2b}-1\right) \tag{\%o8}$$

(%i11)
```
lein:zeromatrix(dim,dim)$
leinstein(false)$
for i thru dim do for j:i thru dim do
if lein[i,j]≠0 then
ishow('G([ξ[i],ξ[j]],[])=fullratsimp(lein[i,j]))$
```

$$\mathbf{G}_{tt} = \frac{e^{2a-2b}\left(2\left(b_r\right)r+e^{2b}-1\right)}{r^2} \tag{\%t11}$$

$$\mathbf{G}_{tr} = \frac{2\left(b_t\right)}{r} \tag{\%t11}$$

$$\mathbf{G}_{rr} = \frac{2\left(a_r\right)r-e^{2b}+1}{r^2} \tag{\%t11}$$

$$\mathbf{G}_{\theta\theta} = -e^{-2b-2a} r ((e^{2b}\ (b_{tt}) + e^{2b}\,(b_t)^2 - (a_t)\ e^{2b}\ (b_t) + e^{2a}\ (a_r)\ (b_r) - e^{2a}\ (a_{rr}) - e^{2a}\,(a_r)^2) r + e^{2a}\ (b_r) - e^{2a}\ (a_r)) \tag{\%t11}$$

$$\mathbf{G}_{\phi\phi} = -e^{-2b-2a} r ((e^{2b}\ (b_{tt}) + e^{2b}\,(b_t)^2 - (a_t)\ e^{2b}\ (b_t) + e^{2a}\ (a_r)\ (b_r) - e^{2a}\ (a_{rr}) - e^{2a}\,(a_r)^2) r + e^{2a}\ (b_r) - e^{2a}\ (a_r)) \sin(\theta)^2 \tag{\%t11}$$

Returns a list of the unique differential equations

(%i12) `map(ldisp,efe:findde(lein,2))$`

$$2\,(b_r)\,r + e^{2b} - 1 \tag{\%t12}$$

$$b_t \tag{\%t13}$$

$$2\,(a_r)\,r - e^{2b} + 1 \tag{\%t14}$$

$$e^{2b}\ (b_{tt})\,r + e^{2b}\,(b_t)^2 r - (a_t)\ e^{2b}\ (b_t)\,r + e^{2a}\ (a_r)\ (b_r)\,r - e^{2a}\ (a_{rr})\,r - e^{2a}\,(a_r)^2 r + e^{2a}\ (b_r) - e^{2a}\ (a_r) \tag{\%t15}$$

(%i16) `factor(efe[1]-efe[3]);`

$$2\left((b_r)\,r - (a_r)\,r + e^{2b} - 1\right) \tag{\%o16}$$

(%i17) `expand(solve(%,diff(a,r)));`

$$\left[a_r = \frac{e^{2b}}{r} - \frac{1}{r} + b_r\right] \tag{\%o17}$$

Weyl Conformal tensor

(%i19)
```
weyl(false)$
for i thru dim do
for j from (if symmetricp(lg,dim) then i+1 else 1) thru dim do
for k from (if symmetricp(lg,dim) then i else 1) thru dim do
for l from (if symmetricp(lg,dim) then k+1 else 1) thru (if (symmetricp(lg,dim) and k=i)
then j else dim) do
if weyl[i,j,k,l]≠0 then
ishow('W([ξ[i],ξ[j],ξ[k],ξ[l]],[])=weyl[i,j,k,l])$
```

$$\mathbf{W}_{trtr} = (e^{2b}\,(b_{tt})\,r^2+e^{2b}\,(b_t)^2\,r^2-(a_t)\,e^{2b}\,(b_t)\,r^2+e^{2a}\,(a_r)\,(b_r)\,r^2-e^{2a}\,(a_{rr})\,r^2-e^{2a}\,(a_r)^2\,r^2-e^{2a}\,(b_r)\,r+e^{2a}\,(a_r)\,r+e^{2b+2a}- \tag{\%t19}$$

$$\mathbf{W}_{t\theta t\theta} = -(e^{-2b}(e^{2b}\,(b_{tt})\,r^2+e^{2b}\,(b_t)^2\,r^2-(a_t)\,e^{2b}\,(b_t)\,r^2+e^{2a}\,(a_r)\,(b_r)\,r^2-e^{2a}\,(a_{rr})\,r^2-e^{2a}\,(a_r)^2\,r^2-e^{2a}\,(b_r)\,r+e^{2a}\,(a_r)\,r+e \tag{\%t19}$$

$$\mathbf{W}_{t\phi t\phi} = -(e^{-2b}(e^{2b}\,(b_{tt})\,r^2+e^{2b}\,(b_t)^2\,r^2-(a_t)\,e^{2b}\,(b_t)\,r^2+e^{2a}\,(a_r)\,(b_r)\,r^2-e^{2a}\,(a_{rr})\,r^2-e^{2a}\,(a_r)^2\,r^2-e^{2a}\,(b_r)\,r+e^{2a}\,(a_r)\,r+ \tag{\%t19}$$

$$\mathbf{W}_{r\theta r\theta} = (e^{-2a}(e^{2b}\,(b_{tt})\,r^2+e^{2b}\,(b_t)^2\,r^2-(a_t)\,e^{2b}\,(b_t)\,r^2+e^{2a}\,(a_r)\,(b_r)\,r^2-e^{2a}\,(a_{rr})\,r^2-e^{2a}\,(a_r)^2\,r^2-e^{2a}\,(b_r)\,r+e^{2a}\,(a_r)\,r+e^{2} \tag{\%t19}$$

$$\mathbf{W}_{r\phi r\phi} = (e^{-2a}(e^{2b}\,(b_{tt})\,r^2+e^{2b}\,(b_t)^2\,r^2-(a_t)\,e^{2b}\,(b_t)\,r^2+e^{2a}\,(a_r)\,(b_r)\,r^2-e^{2a}\,(a_{rr})\,r^2-e^{2a}\,(a_r)^2\,r^2-e^{2a}\,(b_r)\,r+e^{2a}\,(a_r)\,r+e^{2} \tag{\%t19}$$

$$\mathbf{W}_{\theta\phi\theta\phi} = -(e^{-2b-2a}\,r^2(e^{2b}\,(b_{tt})\,r^2+e^{2b}\,(b_t)^2\,r^2-(a_t)\,e^{2b}\,(b_t)\,r^2+e^{2a}\,(a_r)\,(b_r)\,r^2-e^{2a}\,(a_{rr})\,r^2-e^{2a}\,(a_r)^2\,r^2-e^{2a}\,(b_r)\,r+e^{2a}\,(\tag{\%t19}$$

2 Viktor T. Toth wrote:

The routine christof() only displays UNIQUE values of the Christoffel symbols, which are defined to be symmetric in the first two indices in Maxima. So when, for instance, mcs[4,2,4] is not displayed, it's not because it's zero, but because it has the same value as mcs[2,4,4], which was already displayed. You can verify this by explicitly checking the value of mcs[4,2,4], and so on. I verified and in this particular case, apart from the differences in index ordering, Maxima with ctensor yields the same results as Maple 11's built-in tensor package. Viktor