Note of Linear GMM

We warm up by stating two simple propositions of derivatives with Matrix algebra:

(1) For vector x, $\frac{\partial A'x}{\partial x} = \frac{\partial x'A}{\partial x} = A'$

(2). If A is not a function of x and A is symmetric, then $\frac{\partial x'Ax}{\partial x} = 2x'A$.

Now we visit our linear GMM objective function, which is $(Z'Y)'W(Z'Y)+b'(Z'X)'W(Z'X)b-(Z'Y)'W(Z'X)b - b'(Z'X)'W(Z'Y)$.

Apply propositions (1) and (2), we have the first order condition as

$$2b'[(Z'X)'W(Z'X)] - 2[(Z'Y)'W(Z'X)] = 0$$

Thus $b' = [(Z'X)'W(Z'X)]^{-1}[(Z'Y)'W(Z'X)]$, which implies that $b^* = [(Z'X)'W(Z'X)]^{-1}[(Z'X)'W(Z'Y)]$