

# Regression Discontinuity Design

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# Plan

- Overview of RDD
- Meaning and validity of RDD
- Several examples from the literature
- Estimation (where most decisions are made)
- Discussion of Almond et al (low birth weight)
- Stata code and data for *all* examples will be available on Chalk. Email me if you have questions: mcoca@uchicago.edu

# Basics

- Method developed to estimate treatment effects in non-experimental settings
- Provides causal estimates of treatment effects
- Good internal validity; some assumptions can be empirically verified
- Treatment effects are local (LATE)
- Limits external validity
- Relatively easy to estimate (like RCT)
- First application: Thistlethwaite and Campbell (1960)

# Thistlethwaite and Campbell

- They studied the impact of merit awards on future academic outcomes
- Awards allocated based on test scores
- If a person had a score greater than  $c$ , the cutoff point, then she received the award
- Simple way of analyzing: compare those who received the award to those who didn't. (Why is this the wrong approach?)
- Confounding: factors that influence the test score are also related to future academic outcomes (income, parents' education, motivation)
- Thistlethwaite and Campbell realized they could compare individuals just above and below the cutoff point.

# Validity

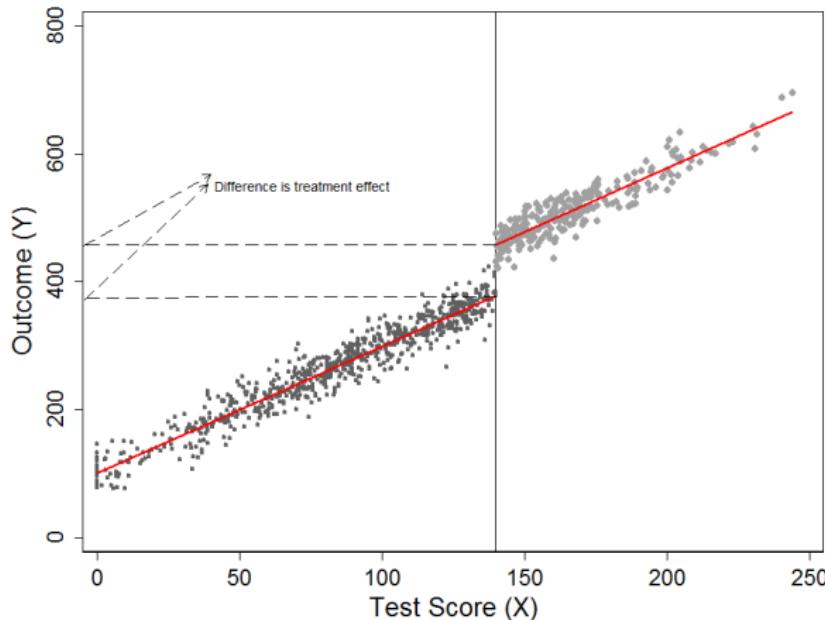
- Simple idea: assignment mechanism is known
- We know that the probability of treatment jumps to 1 if test score  $> c$
- Assumption is that individuals cannot manipulate with precision their assignment variable (think about the SAT)
- Key word: *precision*. Consequence: comparable individuals near cutoff point
- If treated and untreated individuals are similar near the cutoff point then data can be analyzed as if it were a (conditionally) randomized experiment
- If this is true, then background characteristics should be similar near  $c$  (can be checked empirically)
- The estimated treatment effect applies to those near the cutoff point (limits external validity)

# Validity

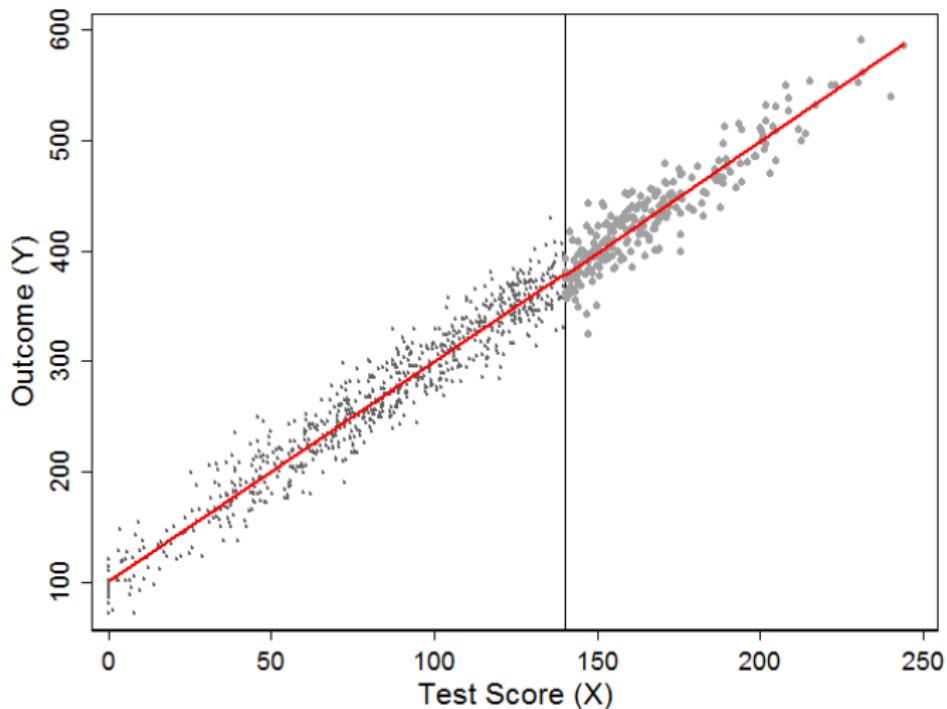
- Careful when you read that the validity depends on rule being “arbitrary” or assignment variable measured with error (e.g. Moscoe et al. 2015)
- Validity hinges on assignment mechanism being known and free of manipulation with precision or cutoff point in some way related to outcome of interest
- Manipulation example 1: Test with few questions and plenty of time
- Manipulation example 2: DMV test to get a driving license
- Example 3: Some mechanism makes cutoff point related to outcome (think biology: blood pressure). What if measured with error?
- Example 4: Eligibility criteria to obtain some benefit (say, below income of 28K). Why? How could you verify assumptions?
- A comment on continuity
- Again: some manipulation is fine (you can always study harder, for example). Precision and lack of relation to outcome is the key to identify causal effects

# Graphical Example

- Simulated data with  $c = 140$
- $\text{gen } y = 100 + 80*T + 2*x + \text{rnormal}(0, 20)$



No effect



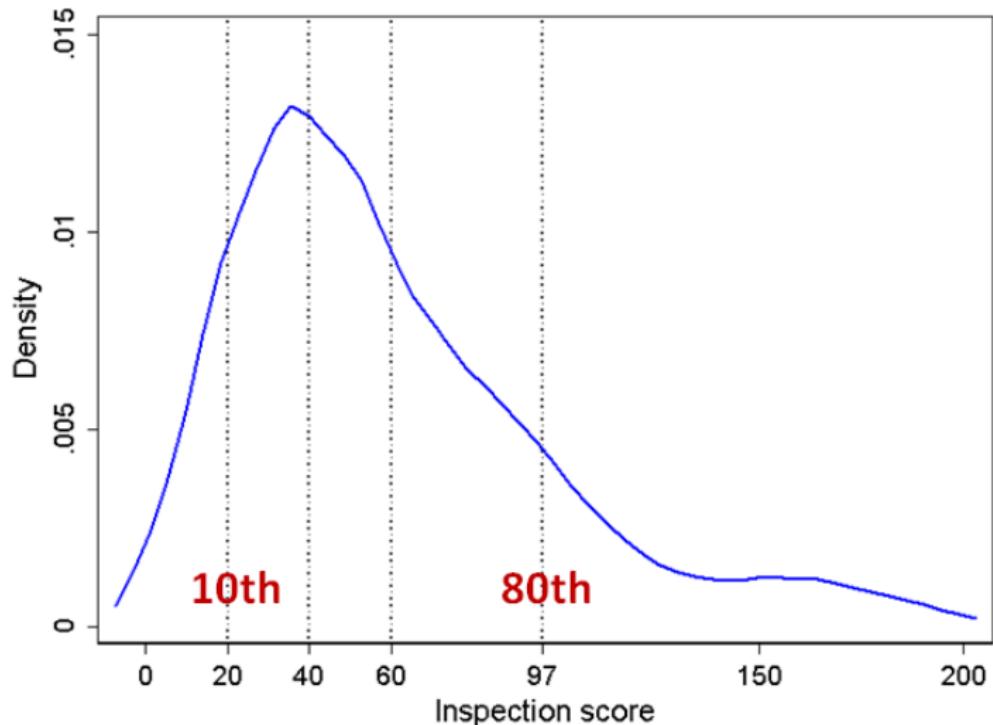
# Sharp and fuzzy RDD

- Sharp RDD: Assignment or running variable completely determines treatment. A jump in the probability of treatment before and after cutoff point.
- Fuzzy RDD: Cutoff point increases the *probability* of treatment but doesn't completely determine treatment.
- Which brings us back to the world of instrumental variables...
- Not used often but has a lot of potential
- Think of encouragement designs or imperfect compliance (like the Oregon study)

## Examples from literature

- Almond et al. (2010): Assignment variable is birth weight. Infants with low birth weight (< 1,500 grams or about 3 pounds) receive more medical treatment.
- We'll talk more about this paper next class. Don't forget to read it!
- Lee, Moretti, Buttler (2004): The vote share (0 to 100 percent) for a candidate is a continuous variable. A candidate is elected if he or she obtains more than 50% of the votes. They evaluated voting record of candidates in close elections.
- CMS rates nursing homes using 1 to 5 stars. Overall stars are assigned based on deficiency data transformed into a points system. Outcome: new admissions six months after the release of ratings.

## Assignment of stars based on scores



## Examples from literature

- Anderson and Magruder (2012) and Lucas (2012): Yelp.com ratings have an underlying continuous score. Distribution determines cutoff points for 1 to 5 stars. Effect of an extra star on future reservations and revenue.
- Anderson et al. (2012): Young adults lose their health insurance as they age (older than 18 and in college but different after ACA). Age changes the probability of having health insurance (fuzzy design).

# Estimation: Parametric

- Simplest case is linear relationship between  $Y$  and  $X$

$$Y_i = \beta_0 + \beta_1 T_i + \beta_3 X_i + \epsilon_i$$

- $T_i = 1$  if subject  $i$  received treatment and  $T_i = 0$  otherwise. You can also write this as  $T_i = \mathbf{1}(X_i > c)$  or  $T_i = \mathbb{1}_{[X_i > c]}$
- $X$  is the assignment variable (sometimes called “forcing” or “running” variable)
- Usually centered at cutoff point
- $Y_i = \beta_0 + \beta_1 T_i + \beta_3(X_i - c) + \epsilon_i$ . Treatment effect is given by  $\beta_1$ .
- $E[Y|T = 1, X = c] = \beta_0 + \beta_1$  and  $E[Y|T = 0, X = c] = \beta_0$ .
- $E[Y|T = 1, X = c] - E[Y|T = 0, X = c] = \beta_1$ .

## Reminder on centering

- Centering changes the interpretation of the intercept:

$$\begin{aligned} Y &= \beta_0 + \beta_1(Age - 65) + \beta_2Edu \\ &= \beta_0 + \beta_1Age - \beta_165 + \beta_2Edu \\ &= (\beta_0 - \beta_165) + \beta_1Age + \beta_2Edu \end{aligned}$$

- Compare to:

$$Y = \alpha_0 + \alpha_1Age + \alpha_2Edu$$

- $\beta_1 = \alpha_1$ ,  $\beta_2 = \alpha_2$ , but  $\alpha_0 \neq (\beta_0 - \beta_165)$
- Useful with interactions:

$$Y = \alpha_0 + \alpha_1Age + \alpha_2Edu + \alpha_3Age \times Edu$$

- Compare to:

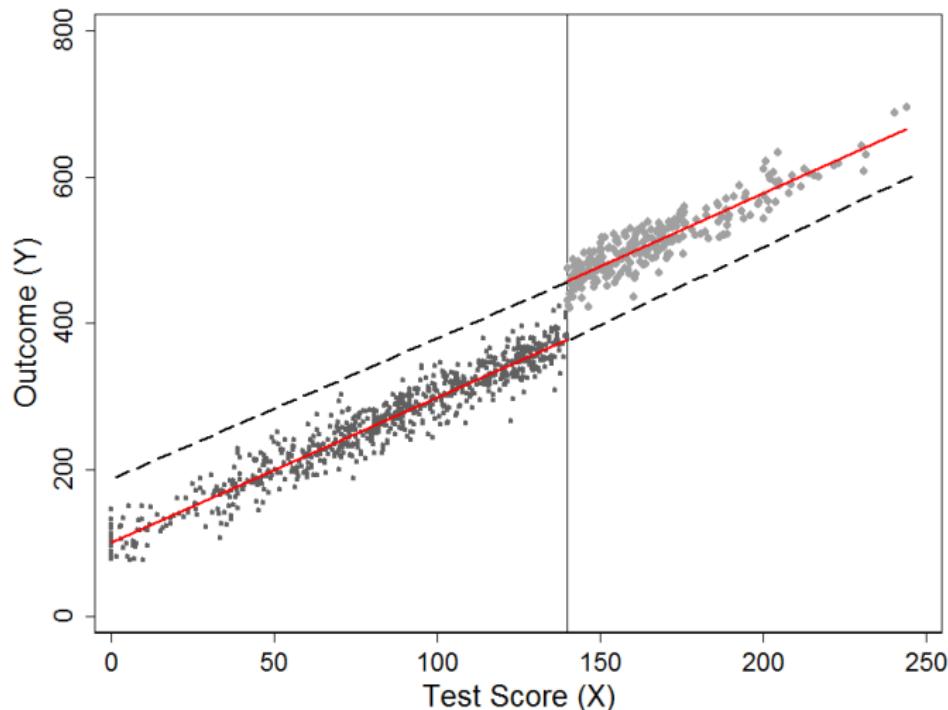
$$Y = \beta_0 + \beta_1(Age - 65) + \beta_2(Edu - 12) + \beta_3(Age - 65) \times (Edu - 12)$$

# Extrapolation

- Note that the estimation of treatment effect in RDD depends on extrapolation
- To the left of cutoff point only non-treated observations
- To the right of cutoff point only treated observations
- What is the treatment effect at  $X = 130$ ? Just plug in:
- $E[Y|T, X = 130] = \beta_0 + \beta_1 T + \beta_3(130 - 140)$

# Extrapolation...

Dashed lines are extrapolations



# Counterfactuals

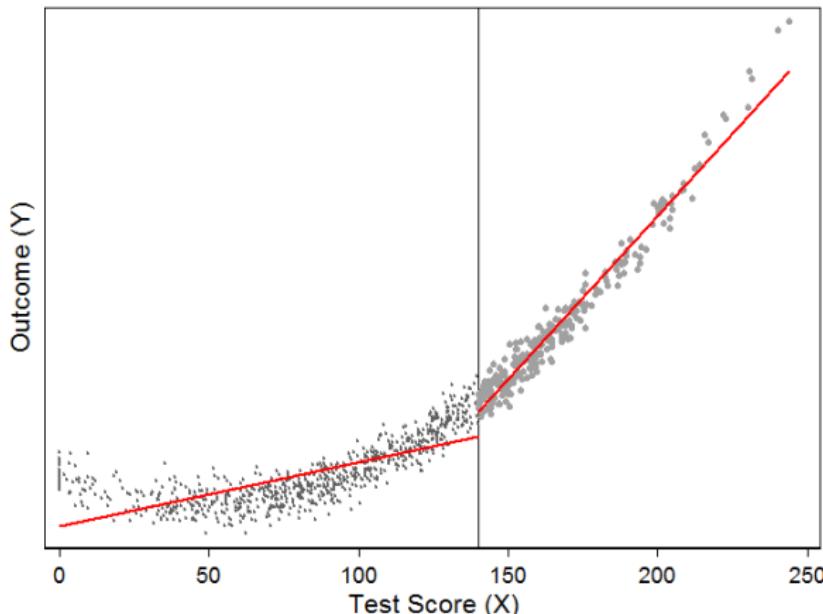
- The extrapolation is a counterfactual or potential outcome
- Each person  $i$  has two potential outcomes (Rubin's causal framework).
- $Y_i(1)$  denotes the outcome of person  $i$  if in the treated group
- $Y_i(0)$  denotes the outcome of person  $i$  if in the non-treated group
- Causal effect of treatment for person  $i$  is  $Y_i(1) - Y_i(0)$
- Average treatment effect is  $E[Y_i(1) - Y_i(0)]$
- Only one potential outcome is observed. In randomized experiments, one group provides the counterfactual for the other because they are comparable (exchangeable)
- Exchangeability (epi). Also called “selection on observables” or “no unmeasured confounders”

## Counterfactuals, II

- In RDD the counterfactuals are conditional on  $X$  as in a conditionally randomized trial (think severity)
- We are interested in the treatment effect at  $X = c$ :  
 $E[Y_i(1) - Y_i(0)|X_i = c]$
- Treatment effect is  $\lim_{x \rightarrow c} E[Y_i|X_i = x] - \lim_{x \leftarrow c} E[Y_i|X_i = x]$
- Estimation possible because of the continuity of  $E[Y_i(1)|X]$  and  $E[Y_i(0)|X]$
- See Hahn, Todd, and Van der Klaauw (2001) for details
- The estimation of the treatment effect is based on extrapolation because of lack of overlap. Therefore, the functional relationship between  $X$  and  $Y$  must be correctly specified

# Need to model relationship between $X$ and $Y$ correctly

- What if nonlinear? Could result in a biased treatment effect if one assumes a linear model.



# Other specifications

- More general:  $Y_i = \beta_0 + \beta_1 T_i + \beta_3 f(X_i - c) + \epsilon_i$
- If  $(X_i - c) = \tilde{X}_i$  then  $Y_i = \beta_0 + \beta_1 T_i + \beta_3 f(\tilde{X}_i) + \epsilon_i$
- Most common form for  $f(\tilde{X}_i)$  are polynomials

- Polynomials of order  $p$ :

$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 \tilde{X}_i + \beta_3 \tilde{X}_i^2 + \beta_4 \tilde{X}_i^3 + \cdots + \beta_{p+1} \tilde{X}_i^p + \epsilon_i$$

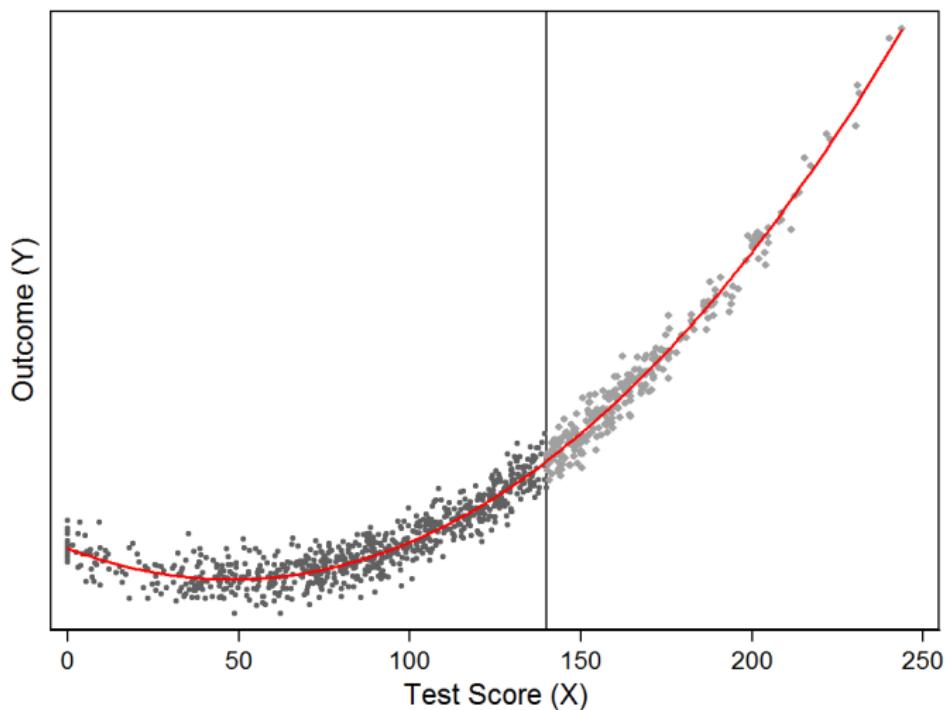
- More flexibility with interactions

- 2nd degree with interactions:

$$Y_i = \beta_0 + \beta_1 T_i + \beta_3 \tilde{X}_i + \beta_4 \tilde{X}_i^2 + \beta_5 \tilde{X}_i \times T_i + \beta_6 \tilde{X}_i^2 \times T_i + \epsilon_i$$

- Question: Why not controlling for other covariates?

Third degree polynomial. Actual model second degree polynomial (see Stata do file). However...



## A note on higher order polynomials

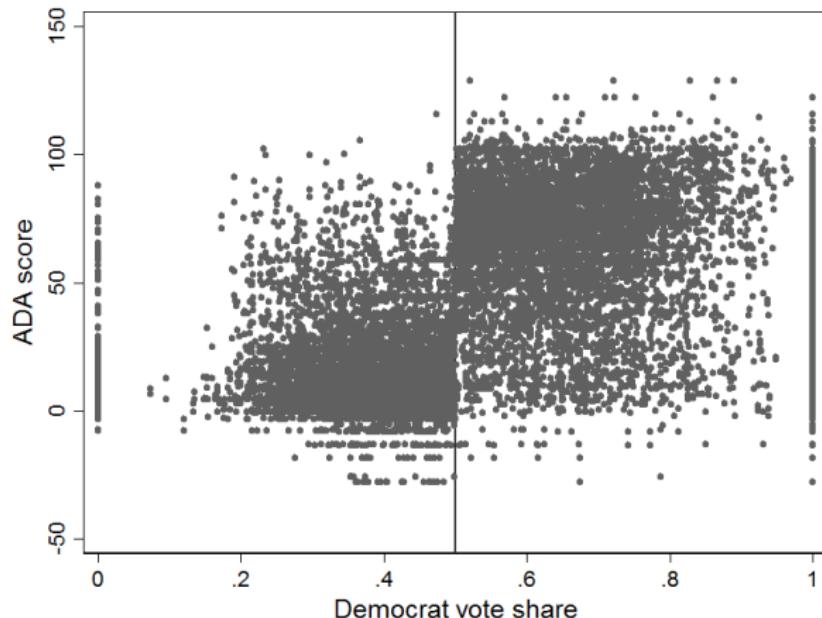
- We will see an example in which using higher order polynomials does not influence results
- In some cases, however, it may matter
- Gelman and Inbems (2014) subtle paper: “Why High-order Polynomials Should not be Used in Regression Discontinuity Designs”
- “We argue that estimators for causal effects based on [higher order polynomials] can be misleading, and we recommend researchers do not use them, and instead use estimators based on local linear or quadratic polynomials...”

## Real dataset

- Data from Lee, Moretti, Buttler (2004)
- U.S. House elections (1946-1995)
- Forcing variable is Democratic vote share. If share > 50 then Democratic candidate is elected
- Outcome is a liberal voting score from the Americans for Democratic Action (ADA)
- Do candidates who are elected in close elections tend to moderate their congressional voting?
- “We find that the degree of electoral strength has no effect on a legislator’s voting behavior”
- Data and code are on Chalk

# Graph a bit messy (about 13,500 obs)

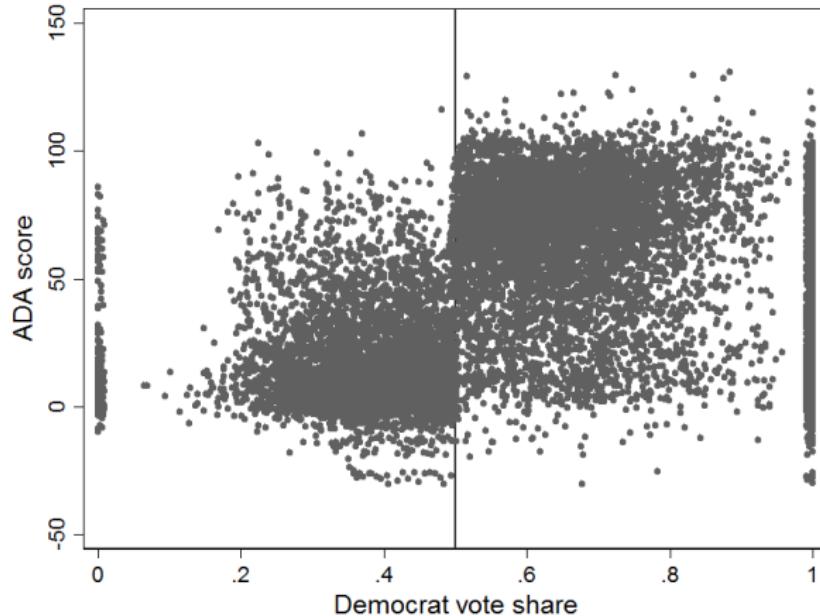
```
scatter score demvoteshare, msize(tiny) xline(0.5) ///
xtitle("Democrat vote share") ytitle("ADA score")
```



## Good idea to add some “jittering”

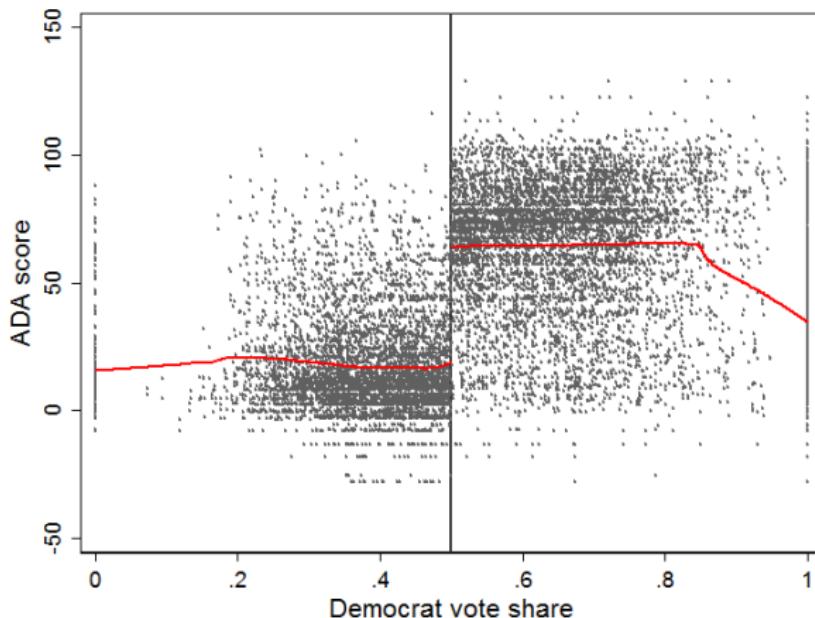
With the jitter option, it is easier to see where is the mass

```
scatter score demvoteshare, msize(tiny) xline(0.5) ///
xtitle("Democrat vote share") ytitle("ADA score") jitter(5)
```



# Useful to “smooth” data with LOWESS

```
lowess score demvoteshare if democrat ==1, gen (lowess_y_d1) nograph bw(0.5)  
lowess score demvoteshare if democrat ==0, gen (lowess_y_d0) nograph bw(0.5)  
....  
....
```

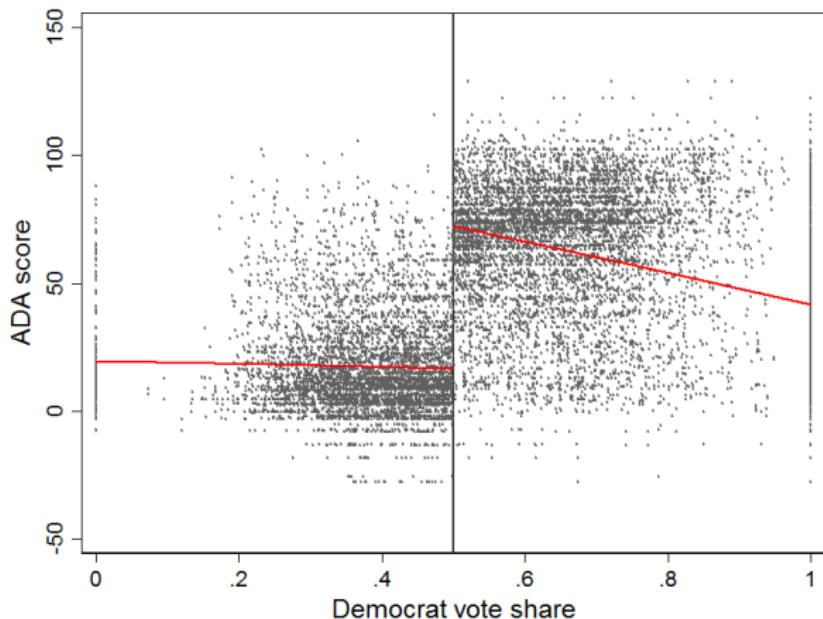


# LOWESS

- LOcally WEighted Scatterplot Smoothing
- Non-parametric graphical method
- Computationally intensive (one regression per data point)
- For each data point, run a *weighted* linear regression (linear or polynomials on  $X$ ) using all the observations within a window.  
Weights give more importances to observations close to data point
- Predicted  $y$ ,  $\hat{y}$ , is then the “smoothed”  $(y_i, x_i)$  point

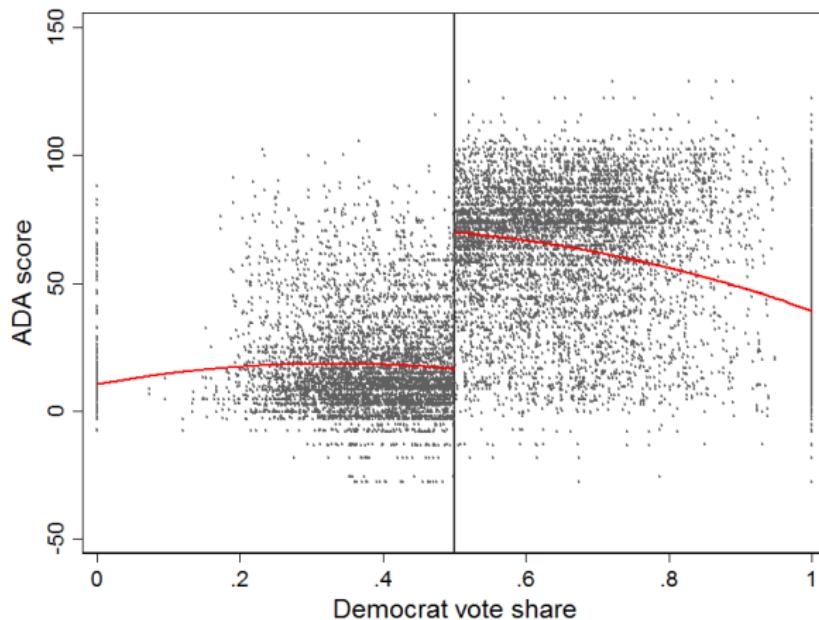
## Parametric: Linear relationship

```
scatter score demvoteshare, msize(tiny) xline(0.5) xtitle("Democrat vote share") ///
ytitle("ADA score") || lfit score demvoteshare if democrat ==1, color(red) || ///
lfit score demvoteshare if democrat ==0, color(red) legend(off)
```



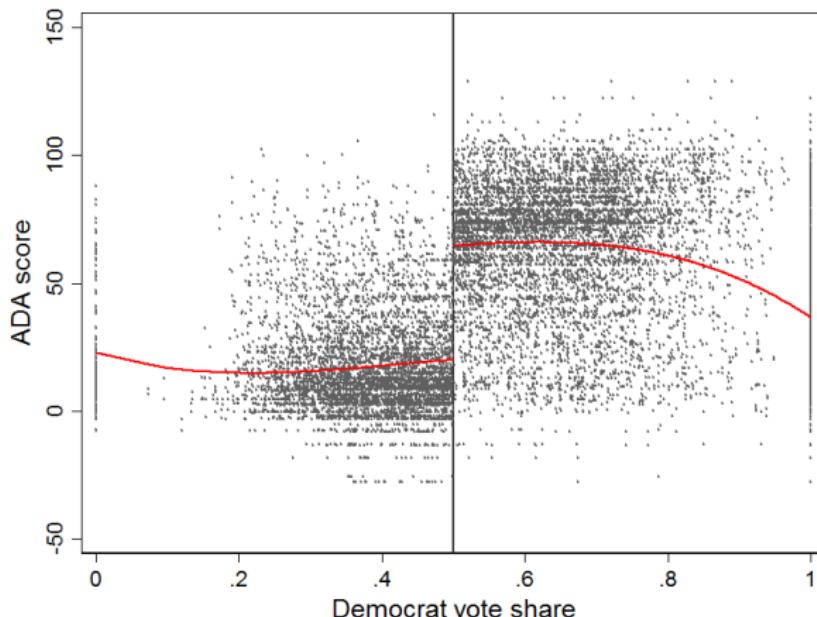
# Quadratic

```
gen demvoteshare2 = demvoteshare^2  
reg score demvoteshare demvoteshare2 democrat  
predict scorehat0
```



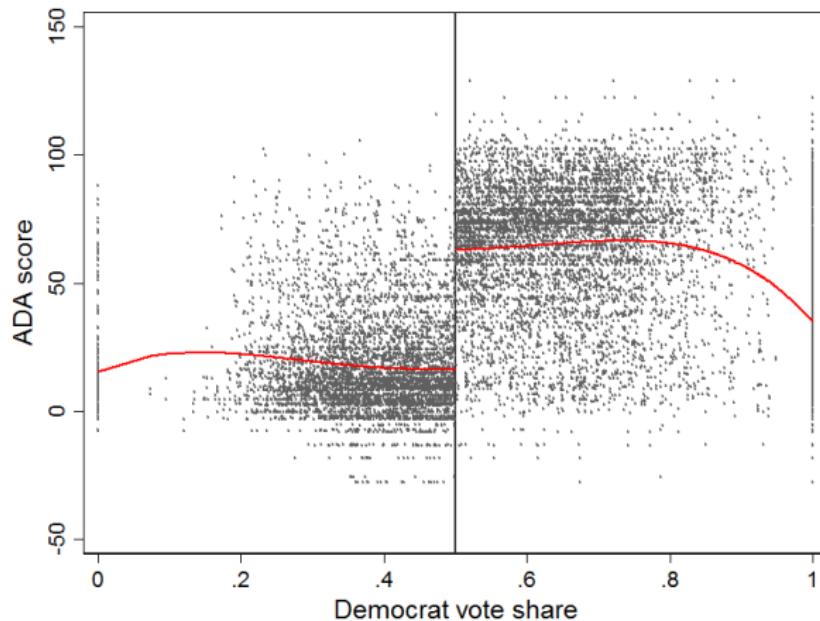
# Third degree polynomial

```
gen demvoteshare3 = demvoteshare^3  
reg score demvoteshare demvoteshare2 demvoteshare3 democrat  
predict scorehat01
```



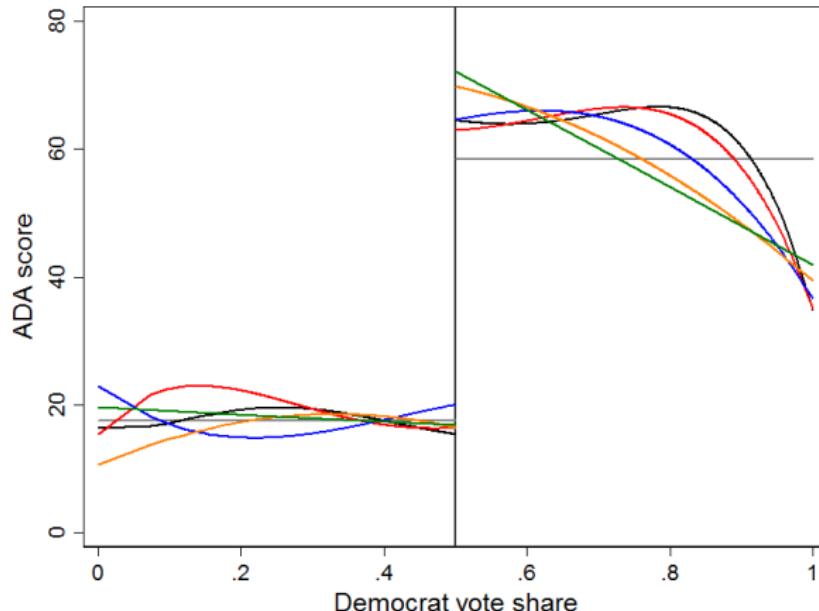
# Fourth degree polynomial

```
gen demvoteshare4 = demvoteshare^4  
reg score demvoteshare demvoteshare2 demvoteshare3 demvoteshare4 ///  
    democrat  
predict scorehat02
```



# Mean (null model) to fifth degree polynomial

```
line scorehat04 demvoteshare if democrat ==1, sort color(gray) || ///
line scorehat04 demvoteshare if democrat ==0, sort color(gray) legend(off) ....
```



# Parametric

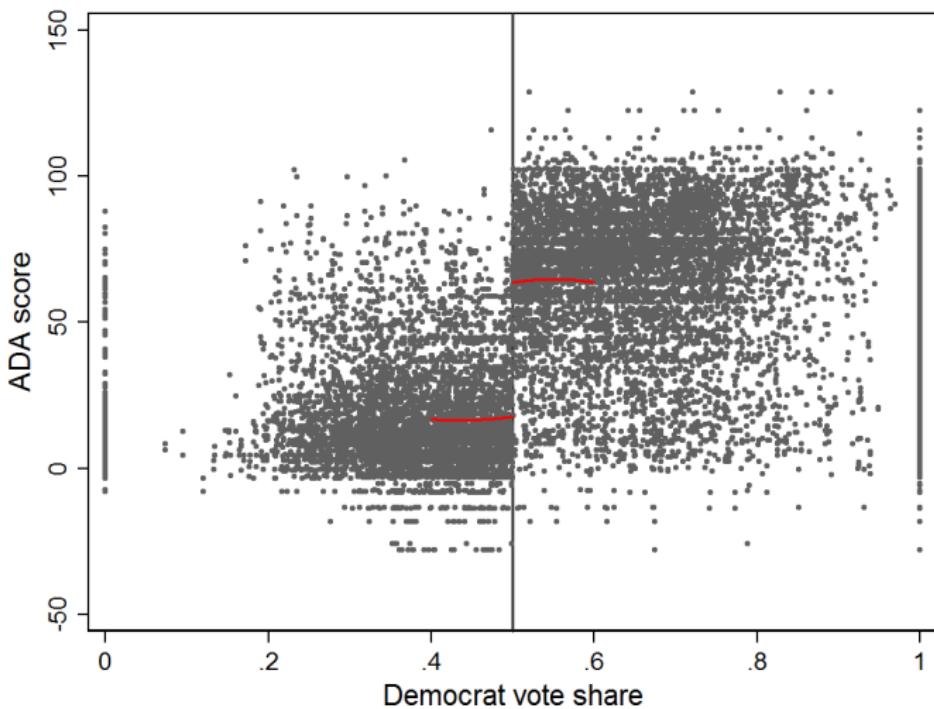
- Note that polynomials “smooth” the data (like LOWESS)
- We used *all* the data even though we want treatment effect at  $c$
- But polynomials give weight to points away from  $c$  and tend to provide smaller SEs
- In other datasets, the choice of polynomial degree will matter (see Gelman and Inbems, 2014)
- Why not only use data close to  $c$ ? Bias and variance trade-off

## Restrict to a window

- Run a flexible regression like a polynomial with interactions (stratified) but don't use observations away from the cutoff. Choose a bandwidth around  $X = 0.5$ . Lee et al (2004) used 0.4 to 0.6.

```
reg score demvoteshare demvoteshare2 if democrat ==1 & ///
(demvoteshare>.40 & demvoteshare<.60)
predict scorehat1 if e(sample)
reg score demvoteshare demvoteshare2 if democrat ==0 & ///
(demvoteshare>.40 & demvoteshare<.60)
predict scorehat0 if e(sample)

scatter score demvoteshare, mszie(tiny) xline(0.5) xtitle("Democrat vote share") ///
ytitle("ADA score") || ///
line scorehat1 demvoteshare if democrat ==1, sort color(red) || ///
line scorehat0 demvoteshare if democrat ==0, sort color(red) legend(off)
graph export lee3_1.png, replace
```



# Limit to window, 2nd degree polynomial

```
gen x_c = demvoteshare - 0.5
gen x2_c = x_c^2
reg score i.democrat##(c.x_c c.x2_c) if (demvoteshare>.40 & demvoteshare<.60)
```

Source	SS	df	MS	Number of obs	=	4632
Model	2622762.02	5	524552.404	F( 5, 4626)	=	1153.29
Residual	2104043.2	4626	454.829918	Prob > F	=	0.0000
Total	4726805.22	4631	1020.6878	R-squared	=	0.5549
				Adj R-squared	=	0.5544
				Root MSE	=	21.327

score	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
1.democrat	45.9283	1.892566	24.27	0.000	42.21797 49.63863
x_c	38.63988	60.77525	0.64	0.525	-80.5086 157.7884
x2_c	295.1723	594.3159	0.50	0.619	-869.9704 1460.315
democrat#c.x_c					
1	6.507415	88.51418	0.07	0.941	-167.0226 180.0374
democrat#c.x2_c					
1	-744.0247	862.0435	-0.86	0.388	-2434.041 945.9916
_cons	17.71198	1.310861	13.51	0.000	15.14207 20.28189

## So what should you do?

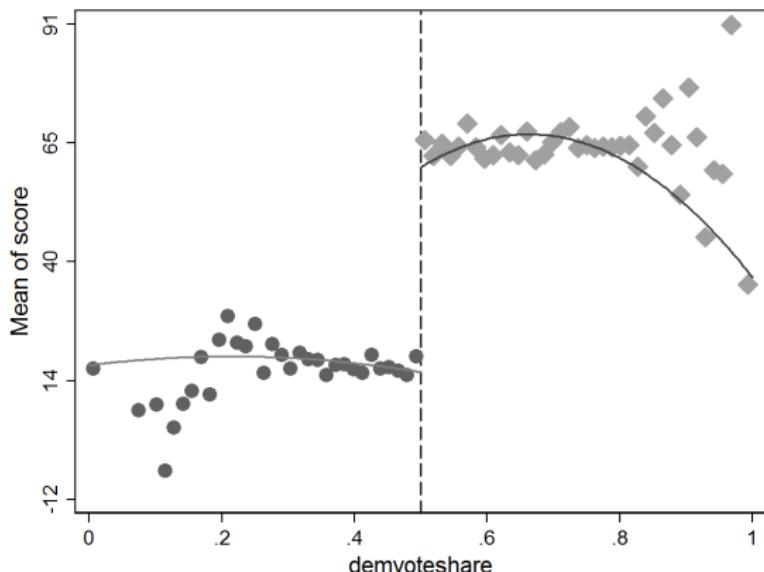
- Best case: Whatever you do gives you similar results (like in this example)
- Most common strategy is to restrict estimation to a window adjusting for covariates
- It used to be popular to use higher order polynomials
- Try different windows and present sensitivity analyses
- Balance should determine the size of window
- Try non-parametric methods

# Nonparametric methods

- Paper by Hahn, Todd, and Van der Klaauw (2001) clarified assumptions about RDD and framed estimation as a nonparametric problem
- Emphasized using local polynomial regression instead of something like LOWESS
- “Nonparametric methods” means a lot of things in statistics
- In the context of RDD, the idea is to estimate a model that does not assume a functional form for the relationship between  $Y$  and  $X$ . The model is something like  $Y_i = f(X_i) + \epsilon_i$
- A very basic method: calculate  $E[Y]$  for each bin on  $X$  (think of a histogram)

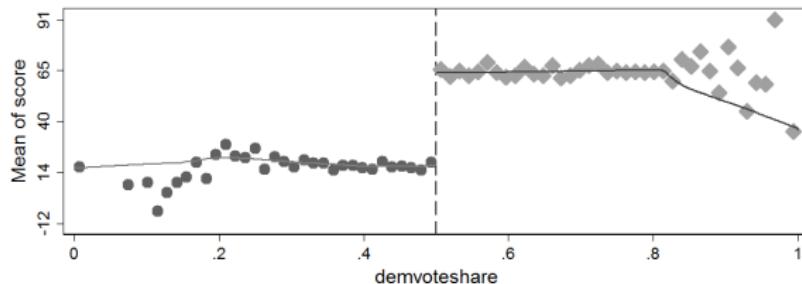
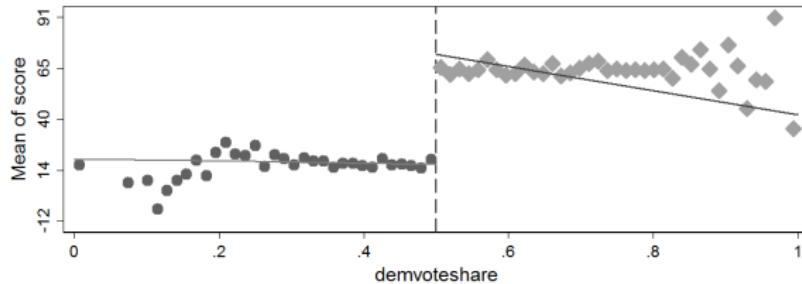
- Stata has a command to do just that: `cmogram`
- After installing the command (`ssc install cmogram`) type `help cmogram`. Lots of useful options
- Common way to show RDD data. See for example Figure II of Almond et al. (2010). To recreate something like Figure 1 of Lee et al (2004):

```
cmogram score demvoteshare, cut(.5) scatter line(.5) qfit
```



# Compare to linear and LOWESS fits

```
cmogram score demvoteshare, cut(.5) scatter line(.5) lfit  
cmogram score demvoteshare, cut(.5) scatter line(.5) lowess
```



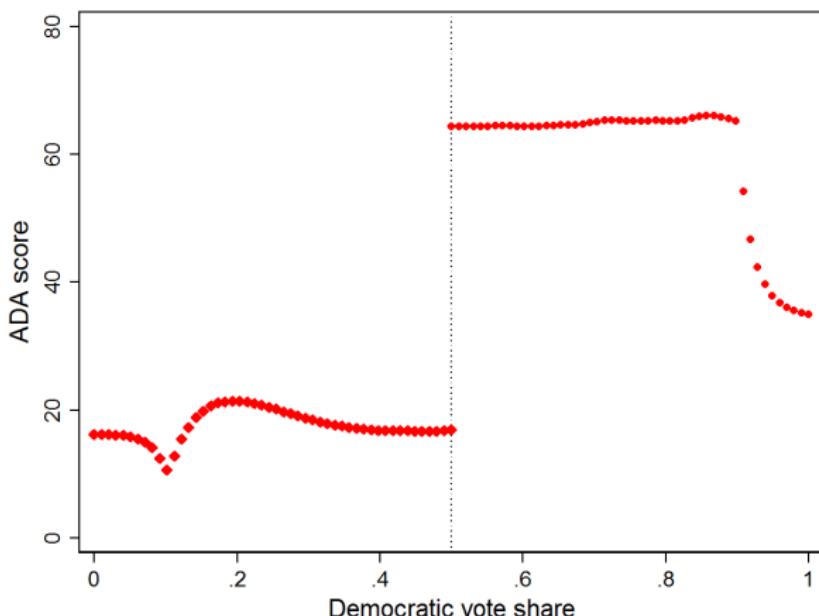
# Local polynomial regression

- Hahn, Todd, and Van der Klaauw (2001) showed that one-side Kernel estimation (like LOWESS) may have poor properties because the point of interest is at a boundary
- Proposed to use instead a local linear nonparametric regression
- Stata's `lpoly` command estimates kernel-weighted local polynomial regression
- Think of it as a weighted regression restricted to a window (hence "local"). The Kernel provides the weights
- A rectangular Kernel would give the same result as taking  $E[Y]$  at a given bin on  $X$ . The triangular Kernel gives more importance to observations close to the center
- Method sensitive to choice of bandwidth (window)

# Local regression is a smoothing method

- Kernel-weighted local polynomial regression is a smoothing method

```
lpoly score demvoteshare if democrat == 0, nograph kernel(triangle) gen(x0 sdem0) bwidth(0.1)  
lpoly score demvoteshare if democrat == 1, nograph kernel(triangle) gen(x1 sdem1) bwidth(0.1)  
<omitted>
```



# Treatment effect

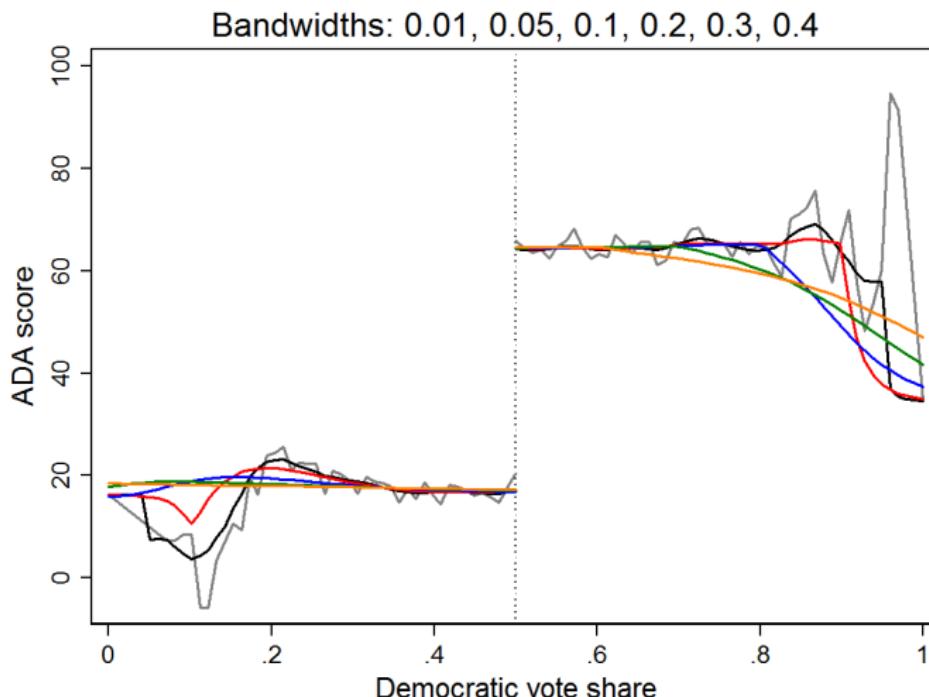
- We're interested in getting the treatment at  $X = 0.5$

```
gen forat = 0.5 in 1
lpoly score demvoteshare if democrat == 0, nograph kernel(triangle) gen(sdem0) ///
    at(forat) bwidth(0.1)
lpoly score demvoteshare if democrat == 1, nograph kernel(triangle) gen(sdem1) ///
    at(forat) bwidth(0.1)
gen dif = sdem1 - sdem0
list sdem1 sdem0 dif in 1/1
```

	sdem1	sdem0	dif
1.	64.395204	16.908821	47.48639

# Different windows

- What happens when we change the bandwidth?



# Nonparametric

- With non-parametric methods in RDD came several methods to choose “optimal windows”
- In practical applications, you may want to check balance around that window
- Standard error of treatment effect can be bootstrapped
- Could add other variables to nonparametric methods but more complicated
- See Stata do file for examples using command `rdrobust`

# Using rdrobust

```
. rdrobust score demvoteshare, c(0.5) all bwselect(IK)
```

Sharp RD Estimates using Local Polynomial Regression

Cutoff c = .5   Left of c Right of c			Number of obs = 13577
<hr/>			Rho (h/b) = 0.770
Number of obs   3535	3318		NN Matches = 3
Order Loc. Poly. (p)   1	1		BW Type = IK
Order Bias (q)   2	2		Kernel Type = Triangular
BW Loc. Poly. (h)   0.152	0.152		
BW Bias (b)   0.197	0.197		

score	Loc. Poly.	Robust [Robust				
		Coef.	Std. Err.	z	P> z	
demvoteshare	47.171	1.262	36.9043	0.000	44.1	49.047108

All Estimates. Outcome: score. Running Variable: demvoteshare.

Method	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
Conventional	47.171	.98131	48.0692	0.000	45.247 49.093991
Bias-Corrected	46.574	.98131	47.4608	0.000	44.65 48.496943
Robust	46.574	1.262	36.9043	0.000	44.1 49.047108

# Parametric or non-parametric?

- When would parametric or non-parametric or window size matter?
  - Small effect
  - Relationship between  $Y$  and  $X$  different away from cutoff
  - Functional form not well captured by polynomials (or other functional form)
- Parametric: can add random effects, clustering SEs,...
- But more important: What about if the outcome cannot be assumed to distribute normal?
- The curse and blessing of so many good RDD guides...
- With counts, for example, need to use Poisson or Negative Binomial models
- If conclusions are different, do worry

# Marginal returns to medical care

- Big picture: is spending more money on health care worth it (in terms of health gained)?
- Actual research: is spending more money on low-weight newborns worth it in terms of mortality reductions? Compare marginal costs (dollars) to marginal benefits (mortality transformed into dollars).
- On jargon: In economics marginal = additional. So compare additional spending to additional benefit
- In IV language, the “marginal” patient is the “complier”
- RDD part used to estimate marginal benefits. Data from U.S Census birth 1983 to 2002
- Forcing variable is newborn weight. Cutoff point  $c = 1,500$  grams (almost 3 lbs)

# Data

- Did they use a fuzzy or sharp RDD?
- Related question: What is the “treatment”?
- What models did they use? And what was the outcome?

## Estimating equation

- Their model is:

$$Y_i = \alpha_0 + \alpha_1 VLBW_i + \alpha_2 VLBW_i \times (g_i - 1500) + \\ \alpha_3 (1 - VLBW_i)(g_i - 1500) + \alpha_t + \alpha_s + \delta X'_i + \epsilon_i \quad (1)$$

- Change notation so  $VLBW = T$  and  $(g_i - 1500) = \tilde{X}$  and after doing some algebra the model is:

$$Y = \alpha_0 + \alpha_1 T + \alpha_3 \tilde{X} + (\alpha_2 - \alpha_3) T \times \tilde{X} + (\alpha_t + \alpha_s + \delta X') + \epsilon$$

- $(\alpha_t + \alpha_s + \delta X')$  are covariates

# Covariates

- They compared means of covariates above and beyond cutoff point
- They found some differences (large sample) so they include covariates in the model
- They did a RDD-type analysis on covariates to see if they were “smooth” (no jump at VLBW cutoff)