ESTIMATING BOTH SUPPLY AND DEMAND ELASTICITIES USING VARIATION IN A SINGLE TAX RATE

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We show how an insight from taxation theory allows identification of both the supply and demand elasticities using only one instrument. Most models of taxation since Ramsey (1927) assume that a tax levied on the demand side only affects demand through the price after taxation. Econometrically, we show that this assumption acts as an exclusion restriction. Under the Ramsey Exclusion Restriction (RER), a single tax reform can serve to simultaneously identify the demand and supply elasticity. We develop an estimation method, which includes 2SLS estimators for the elasticities, and a test for strength of the instrument. We discuss possible applications.

KEYWORDS: Tax reform, instrumental variable, supply and demand elasticities.

> "*The econometric challenge in implementing any of these structural methods is simultaneity: identification of the slope of the supply and demand curves requires 2J instruments.*"
>
> Chetty (2009)

1. INTRODUCTION

ESTIMATING THE SLOPE OF BOTH THE DEMAND AND THE SUPPLY CURVE generally requires at least two instruments (Wright (1928)). Tracing out the slope of the demand curve requires variation in the supply curve, and vice versa. However, in specific cases, economic theory enables us to restrict the demand and supply equations, such that the two slopes are identified with a single instrument (Koopmans (1949)).

In this note, we show that a relatively simple insight from taxation theory provides a plausible restriction, allowing variation in a single tax rate to identify both the supply and

demand elasticities. In the context of a demand-side tax, a standard assumption in models of taxation since Ramsey (1927) is that the supply of a good depends on the before-tax price, whereas demand depends on the price after taxation. The implication is that the tax can be excluded from the demand equation, when demand is expressed in the after-tax prices. In honor of Frank Ramsey, we name this exclusion restriction the Ramsey Exclusion Restriction (RER).

The RER and the Standard Exclusion Restriction (SER), stating that a demand-side tax does not directly appear in the supply equation, jointly make the tax rate a valid instrument for estimating the structural supply and demand elasticities.

In this note, we derive 2SLS estimators for the structural demand and supply elasticities using the tax rate as an instrument. In addition, we show how to test whether the instrument has sufficient strength.

Our approach provides several advantages over the existing empirical tax literature, which mainly focuses on estimating reduced-form, rather than structural, elasticities. First, structural demand and supply elasticities are useful for welfare analysis of a large range of policies. Second, structural elasticities allow for comparison between estimates obtained through variation in the tax rate and estimates obtained using alternative sources of variation. Third, our method can be used to decompose heterogeneity in reduced-form elasticities into heterogeneity in structural demand and supply elasticities.

The note is organized as follows. In the next section, we show that the SER and the RER jointly allow for identification of the supply and demand elasticities. Section 3 provides an estimation framework. Section 4 lists possible applications, advantages, and limitations to our approach. In the Supplemental Material (Zoutman, Gavrilova, and Hopland (2018)), we extend our result by considering supply-side taxes, nonlinear taxes, and a setting with multiple goods and taxes. In addition, we provide a test for the RER which relies on the existence of a second valid instrument, and link the RER to the sufficient-statistics literature.

2. THE MODEL

Assume that we observe panel data for a good on the equilibrium quantity, Y_{it}, and the before-tax price, P_{it}. The cross-sectional indicator i may indicate specific regions, firms, or individuals, and t denotes the time index. The good is taxed with an ad valorem tax rate τ_{it} that is levied on the demand side. Suppose τ_{it} is exogenous, possibly after controlling for a vector of control variables x_{it}. Assuming log-linearity, the supply-demand system is given by

$$y_{it} = \varepsilon^S p_{it} + \eta z_{it} + \Gamma^S x_{it} + \nu^S_{it}, \tag{1}$$

$$y_{it} = \varepsilon^D p_{it} + \gamma z_{it} + \Gamma^D x_{it} + \nu^D_{it}, \tag{2}$$

where lowercase letters y_{it} and p_{it} denote logged quantities and prices, $z_{it} \equiv f(\tau_{it})$ is a pre-specified function of the tax rate chosen such that y_{it} is linear in z_{it}, and ν^S_{it} (ν^D_{it}) is the disturbance in the supply (demand) equation. We initially allow the instrument to enter both the demand and the supply equations.

We can represent this demand-supply system in reduced form as follows:

$$\begin{bmatrix} y_{it} \\ p_{it} \end{bmatrix} = \begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} z_{it} + \begin{bmatrix} \Pi_{xy} \\ \Pi_{xp} \end{bmatrix} x_{it} + \xi_{it}, \tag{3}$$

where π_{zy} is the coefficient between the instrument and the traded quantity, π_{zp} is the coefficient between the instrument and the price, and Π_{xy} and Π_{xp} are vectors of reduced-form coefficients for the control variables. By assumption, z_{it} and x_{it} are jointly exogenous, allowing the coefficients π and Π to be estimated through OLS. Hence, the structural demand and supply coefficients in (1), (2) are identified if they can be expressed in terms of the reduced-form coefficients in (3).

If we do not impose any restrictions, the relationship between reduced-form and structural coefficients can be found by solving equations (1), (2) for y_{it} and p_{it}:

$$\begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} = \begin{bmatrix} \dfrac{\varepsilon^S \gamma - \varepsilon^D \eta}{\varepsilon^S - \varepsilon^D} \\ \dfrac{\gamma - \eta}{\varepsilon^S - \varepsilon^D} \end{bmatrix}. \tag{4}$$

The left-hand side of this expression has two reduced-form coefficients, while the right-hand side has four structural coefficients. Therefore, the structural demand and supply elasticities are not identified unless we impose additional restrictions on (1), (2).

In this note, we suggest two restrictions. The standard way to restrict (1), (2) in the instrumental variable literature since Wright (1928) is to specify the way in which the instrument z_{it} enters (1). Because the tax is levied on the demand side, it is logical to assume that the instrument z_{it} does not directly enter the supply equation. This is explicitly stated as follows:

STANDARD EXCLUSION RESTRICTION: *If the tax τ_{it} is levied on the demand side, then $\eta = 0$.*

Our main insight is that a second restriction follows from taxation theory. The budget constraint of consumers is not a general function of the before-tax price P_{it} and the tax rate τ_{it}. Instead, the variable that enters the consumers' budget constraint is the after-tax price $P_{it}^\tau \equiv (1 + \tau_{it}) P_{it}$. Therefore, if consumers are rational, demand depends on taxation only through P_{it}^τ. The assumption that a tax only affects demand through the after-tax price is implicit in most economic models of taxation dating back to Ramsey (1927), and we therefore label it as the RER.

Under the RER, the demand equation (2) can be written as

$$\begin{aligned} y_{it} &= \varepsilon^D p_{it}^\tau + \Gamma^D x_{it} + \nu_{it}^D \\ &= \varepsilon^D p_{it} + \varepsilon^D \log(1 + \tau_{it}) + \Gamma^D x_{it} + \nu_{it}^D. \end{aligned} \tag{5}$$

By comparing (5) to (2), we can see the effect of the RER. When we choose the instrument z_{it} as the logarithm of the gross-of-tax rate $z_{it} \equiv \log(1 + \tau_{it})$, the coefficient on the instrument γ equals the demand elasticity, $\gamma = \varepsilon^D$. Intuitively, an increase in P_{it} and an increase in the gross-of-tax rate $1 + \tau_{it}$ affect the consumers' budget in an identical manner. As a consequence, a one-percentage change in either variable should have the exact same impact on demand. Formally, this is stated as follows:

RAMSEY EXCLUSION RESTRICTION: *Demand depends only on the price after taxation. It follows that $z_{it} \equiv \log(1 + \tau_{it})$ and $\gamma = \varepsilon^D$.*

When we impose SER and RER on (4), we arrive at:[1]

$$\begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} = \begin{bmatrix} \dfrac{\varepsilon^S \varepsilon^D}{\varepsilon^S - \varepsilon^D} \\ \dfrac{\varepsilon^D}{\varepsilon^S - \varepsilon^D} \end{bmatrix}. \tag{6}$$

The right-hand side of equation (6) depends on only two structural coefficients. Hence, it is possible to express both the demand and the supply elasticities in terms of reduced-form coefficients:

$$\varepsilon^S = \frac{\pi_{zy}}{\pi_{zp}}, \tag{7}$$

$$\varepsilon^D = \frac{\pi_{zy}}{1 + \pi_{zp}}. \tag{8}$$

Therefore, if the SER and RER hold, the instrument is valid for estimating both the demand and supply elasticities.

By inspecting equations (7), (8), we can verify which conditions need to be satisfied in order to strongly identify the demand and supply elasticities. If $\pi_{zp} = 0$, the right-hand side of equation (7) is not defined. Hence, strong identification of the supply elasticity requires that the null hypothesis $\pi_{zp} = 0$ can be rejected. By a similar argument, the demand elasticity is strongly identified if the null hypothesis $\pi_{zp} = -1$ can be rejected.

These conditions have a straightforward economic interpretation. If $\pi_{zp} = 0$, variation in the tax rate does not affect the price prior to taxation. In this case, the entire incidence of the tax falls on the demand side. As such, the instrument does not provide variation in the price that is relevant for the supply side and, hence, the supply elasticity is not identified. By considering equation (6), we can see that $\pi_{zp} = 0$ if $\varepsilon^D = 0$ and/or $\varepsilon^S = \infty$.

If $\pi_{zp} = -1$, the price after taxation is independent of the tax rate. Hence, the incidence of the tax falls entirely on the supply side. Therefore, there is no variation in the price consumers pay for the good, and it becomes impossible to estimate the demand elasticity. By considering equation (6), we can see that $\pi_{zp} = -1$ if $\varepsilon^D = -\infty$ and/or $\varepsilon^S = 0$.

In the market for a single good, the SER and RER jointly allow for identification of both the demand and supply elasticities, as long as the incidence of the tax is shared between both sides of the market.

Figure 1 provides the intuition behind the proof. The figure shows the effect of an increase in the tax rate on equilibrium supply and demand when we impose both restrictions. The left panel of the figure considers the effect of the tax reform expressed in p_{it}. The right panel considers the effect of the exact same tax reform but expressed in p_{it}^{τ}. By the SER, the tax reform does not directly affect the supply curve when the model is expressed in before-tax prices. In the left panel, this implies that the reform shifts demand along the supply curve, without simultaneously shifting the supply curve. As such, this reform identifies the slope of the supply curve: the supply elasticity. Conversely, by the RER, the tax rate does not directly appear in the demand equation when the same model

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[1]Equation (6) represents the standard tax incidence equations that are taught in undergraduate textbooks (e.g., Gruber (2016), Chapter 20).
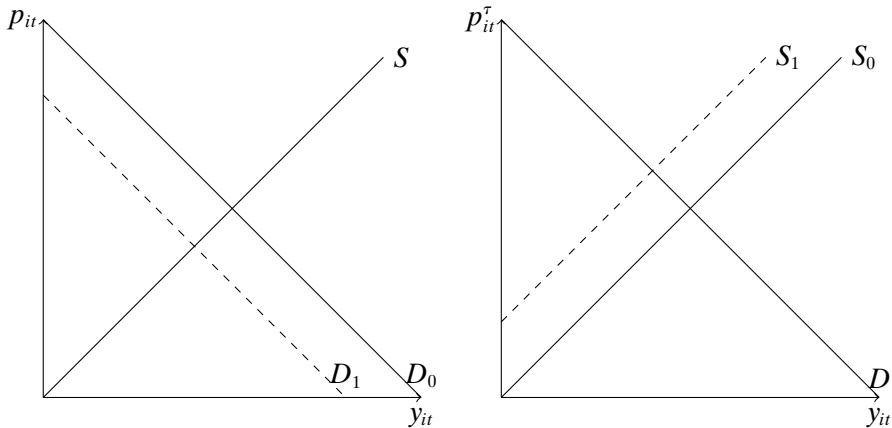
FIGURE 1.—The effect of a tax rate increase.

is expressed in after-tax prices p_{it}^τ.[2] Hence, in the right panel, the tax reform shifts the supply curve along the demand curve, without simultaneously shifting the demand curve. Therefore, the same reform also identifies the slope of the demand curve.

3. ESTIMATION

In this section, we propose a simple estimation framework that allows researchers to arrive at consistent estimates for the supply and demand elasticities. We divide the estimation of the two elasticities into two two-stage regressions. The first two-stage regression directly corresponds to the left panel of Figure 1. In this regression, the price before taxation p_{it} is the endogenous variable, y_{it} serves as the dependent variable, and $z_{it} = \log(1 + \tau_{it})$ is the instrument. As such, the first- and second-stage equations are given by

$$p_{it} = \pi_{zp} \log(1 + \tau_{it}) + \Pi_{xp} x_{it} + \xi_{it}, \tag{9}$$

$$y_{it} = \varepsilon^S \hat{p}_{it} + \Gamma^S x_{it} + \nu_{it}^S. \tag{10}$$

We use the first-stage equation to instrument the price before taxation, $\hat{p}_{it}$. The second-stage equation regresses traded quantities on the instrumented price, thus estimating the supply elasticity ε^S.

By virtue of the results obtained in the previous section, a standard 2SLS estimator of equations (9), (10) provides consistent estimates of the supply elasticity if the SER holds, and the null hypothesis that $\pi_{zp} = 0$ can be rejected. π_{zp} is the coefficient on the instrument in the first stage. Therefore, the strength of the instrument for the supply

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[2]To see this, note that the SER and RER jointly allow us to rewrite (1), (2) in terms of after-tax prices as follows:

$$y_{it} = \varepsilon^S p_{it}^\tau - \varepsilon^S \log(1 + \tau_{it}) + \Gamma^S x_{it} + \nu_{it}^S,$$

$$y_{it} = \varepsilon^S p_{it}^D + \Gamma^S x_{it} + \nu_{it}^S.$$

Written in this form, the tax rate only appears in the supply equation.

elasticity can be tested using a standard F-test that is built into most statistical software (e.g., Stock and Yogo (2005)).

To estimate the demand elasticities, we run a similar regression but use the price after taxation, rather than the price before taxation, as the endogenous variable, corresponding to the right-hand panel of Figure 1. The first-stage equation is given by

$$p_{it}^{\tau} = \pi_{zp}^{\tau} \log(1 + \tau_{it}) + \Pi_{xp} x_{it} + \xi_{it}^{p}. \tag{11}$$

We then use the instrumented price $\hat{p}_{it}^{\tau}$ to estimate the demand elasticities with the following regression equation:

$$y_{it} = \varepsilon^{D} \hat{p}_{it}^{\tau} + \Gamma^{D} x_{it} + \nu_{it}^{D}. \tag{12}$$

This system of equations can again be estimated using a standard 2SLS estimator. By the results obtained in the previous section, the demand elasticity is strongly identified if the RER holds and the null hypothesis that $\pi_{zp}^{\tau} = 0$ can be rejected.[3] π_{zp}^{τ} is the coefficient on the instrument in the first stage, and hence, standard F-tests can be used to test whether the instrument has sufficient strength to identify the demand elasticity.

There are two alternative approaches to the estimation framework we discuss here. First, for simplicity we estimate the two elasticities one-by-one using two 2SLS equations. Alternatively, the two 2SLS equations can be combined using a system 2SLS estimator, as detailed in Chapter 8.3.2 in Wooldridge (2002). The two approaches yield identical estimates. However, the advantage of the system 2SLS estimator is that it provides a sample covariance between the demand and the supply elasticities.

Second, Saez, Matsaganis, and Tsakloglou (2012) estimate reduced-form elasticities by estimating (3) and then back out the structural demand and supply elasticities through equations (7), (8). This approach yields identical central estimates, but our approach is more convenient for (i) estimating 2SLS-corrected standard errors, (ii) testing the strength of the instrument, and (iii) testing the RER by means of an additional instrument, as we show in the Supplemental Material.

4. APPLICATIONS, ADVANTAGES, AND LIMITATIONS

In the previous two sections, we have shown that the RER allows for simultaneous identification and estimation of the supply and the demand elasticities using variation in only one tax rate. In this section, we discuss possible applications of our method and list advantages and limitations to our approach.

4.1. *Applications*

Our approach is useful in settings with exogenous variation in the tax rate, when the RER is plausible. In economic models, the RER follows from the assumption that individuals rationally base their demand on the price after taxation.

The most obvious application is variation in a consumption tax such as a value-added, sales, or excise tax. In this setting, the RER is particularly plausible when the after-tax price is listed on the price tag, because this ensures that consumers respond to the correct price.

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[3]Note that the following relationship must hold mechanically: $\pi_{zp}^{\tau} = \pi_{zp} + 1$. Hence, the null hypothesis that $\pi_{zp} = -1$ is equivalent to the null hypothesis that $\pi_{zp}^{\tau} = 0$.

Another potential application concerns the housing market. Many countries levy demand- and/or supply-side taxes on property transactions. In the housing market, consumers likely base their demand on the price after taxation, because optimization errors where individuals do not rationally take into account the effect of taxation on the after-tax price when they make a transaction will be rather costly.

Another possible application is in the labor market. In the labor market, most countries levy a supply-side tax in the form of a labor-income tax, and a demand-side tax in the form of a payroll tax. Similarly to the housing market, labor income for workers and labor costs for firms are a large part of their respective budgets, making the RER plausible in this setting as well.

4.2. *Advantages*

Most of the recent empirical literature on taxation focuses on estimating reduced-form elasticities between the traded quantity or price, and the gross-of-tax rate. As can be seen from (6), reduced-form elasticities are a function of both the demand and the supply elasticities. On the contrary, our method allows for direct identification of the structural demand and supply elasticities. This provides several advantages. The first advantage lies in welfare analysis. It is well known that both structural elasticities and reduced-form elasticities can be used to calculate the excess burden of a tax (Harberger (1964a, 1964b), Chetty (2009)). However, structural demand and supply elasticities also allow researchers to decompose the excess burden into the share borne by the demand side and the supply side. Additionally, structural demand and supply elasticities can be used to analyze the welfare effects of policies that are not related to taxation, such as price floors and ceilings, whereas reduced-form elasticities obtained from variation in the tax rate can only be used to analyze the welfare effects of tax policy.

A second advantage of our method is that it allows researchers to compare estimates between different strands of literature. There is a large literature that studies the effect of variations in price floors and ceilings.[4] In standard models of microeconomics, the estimated effect of equilibrium sales on a price floor (ceiling) depends on the structural demand (supply) elasticity. By identifying structural demand and supply elasticities, our method enhances comparability between these estimates and estimates obtained through variation in the tax rate.

A third advantage is that our method allows for decomposition of heterogeneity in the reduced-form elasticity into heterogeneity in the demand and supply elasticities. For instance, it is well known that female labor supply responds more to changes in the income tax rate than male labor supply (see, e.g., Meghir and Phillips (2010)). What is less clear is whether the heterogeneity is driven by differences in the demand or the supply elasticity. Our approach allows the researcher to split the sample, and estimate the demand and supply elasticity for different groups, thereby decomposing the source of the heterogeneity.[5]

A final advantage is that we devise a test for the RER in the Supplemental Material, which works when a second instrument is available. The reduced-form literature does

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[4]See, for instance, Card and Krueger (2015) for an overview of the minimum wage literature and Autor, Palmer, and Pathak (2014) for a recent study on rent controls.

[5]If different groups face independent variation in the tax rate, as is, for instance, the case for males and females in Gelber (2014), the multiple-good extension of our model in the Supplemental Material allows estimation of supply and demand cross-elasticities as well.

not require the RER to identify reduced-form elasticities, but the RER often plays an important role in the interpretation of the estimates (see Supplemental Material). Our test allows researchers to verify whether the RER is justified in their setting.

4.3. *Limitations*

The most important limitation to our approach is that it relies on the RER. The RER can only be tested if a second instrument is available, and may not always be plausible. For instance, Chetty, Looney, and Kroft (2009) show that sales taxes in the United States are not salient because they are not included on the price tag. As such, consumers do not act rationally to changes in the tax rate, and, hence, the RER fails to hold. Another example is tax avoidance. In settings with tax avoidance, the RER may be violated because a change in the tax rate does not elicit the same change in demand as a change in the before-tax price because the tax can be avoided, whereas the price cannot (see, e.g., Slemrod (2001), Kopczuk et al. (2016)). Our approach is therefore not appropriate in settings where taxes are not salient or are easily avoidable.

In addition, our approach requires that incidence is shared between the demand and the supply side. If all incidence falls on the supply (demand) side, the demand (supply) elasticity is not identified. This could pose an issue in settings where prices are rigid due to price floors, ceilings, or unionization (see, e.g., Saez, Matsaganis, and Tsakloglou (2012), Lehmann, Marical, and Rioux (2013)). However, the assumption that incidence is shared can be tested with our approach. If incidence is not shared, this results in a low F-statistic for the instrument when estimating either the demand or the supply equation.

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