Note of Demand Elasticities

Recall that we have the following share equations from AIDS

$$\omega_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln(\frac{X}{P}), \forall i \tag{1}$$

$$lnP = \alpha_0 + \sum_j \alpha_j lnp_j + \frac{1}{2}\sum_i \sum_j \gamma_{ij} lnp_i lnp_j \tag{2}$$

and if we use Stone approximation

$$ln\tilde{P} = \sum_k \omega_k lnp_k \tag{3}$$

We first show that $\frac{d\omega_i}{dlnp_j}$ is connected to demand elasticities $\frac{dlnq_i}{dlnp_j}$.

If $i = j$, i.e., we are interested in own elasticity, then

$$\frac{d\omega_i}{dlnp_j} = \frac{d(p_i q_i)}{dlnp_i} \cdot \frac{1}{X} = [p_i q_i + \frac{dq_i}{dlnp_i} p_i]/X = \frac{p_i q_i}{X}[1 + \frac{dlnq_i}{dlnp_i}] \equiv \omega_i [1 + \eta_{ii}] \tag{4}$$

where η_{ii} is the own demand elasticity.

If $i \neq j$, i.e., we are interested in cross elasticity, then

$$\frac{d\omega_i}{dlnp_j} = \frac{d(p_i q_i)}{dlnp_j} \cdot \frac{1}{X} = [0 + \frac{dq_i}{dlnp_j} p_i]/X = \frac{p_i q_i}{X}\frac{dlnq_i}{dlnp_j} \equiv \omega_i \eta_{ij} \tag{5}$$

where η_{ij} is the cross demand elasticity.

Making use of the AIDS demand system, we know that

$$\frac{d\omega_i}{dlnp_j} = \gamma_{ij} - \beta_i \frac{dlnP}{dlnp_j} \tag{6}$$

Thus the bottom-line is that we have

$$\eta_{ii} = -1 + [\gamma_{ii} - \beta_i \frac{dlnP}{dlnp_i}]/\omega_i \tag{7}$$

$$\eta_{ij} = [\gamma_{ij} - \beta_i \frac{dlnP}{dlnp_j}]/\omega_i \tag{8}$$

Finally, the calculation of $\frac{dlnP}{dlnp_j}$ can either use AIDS exact price aggregation

$$\frac{dlnP}{dlnp_j} = \alpha_j + \sum_k \gamma_{kj} lnp_k \tag{9}$$

or use Stone approximation formula (by assuming a constant expenditure share)[1]

$$\frac{dlnP}{dlnp_j} = \omega_j \tag{10}$$

[1] Alston et al (94) shows this is a pretty good approximation with Monte Carlo Simulations.