Statistics Study Notes

Daniel Yang

2020 年 2 月 27 日

1 Time Series

ADF-Test: 检验结果为负，越负越拒绝原假设，即有单位根。单位根存在，即$y_t = a + by_{t-1} + e_t$，$|b| = 1$，为随机。

2 Maximum Likelihood

3 Kalman Filter

3.1 State Space Representation

$$\begin{aligned} \xi_{\mathbf{t+1}} &= \mathbf{F}\xi_{\mathbf{t}} + \mathbf{v_{t+1}} \\ \mathbf{y_t} &= \mathbf{A}'\mathbf{x_t} + \mathbf{H}'\xi_{\mathbf{t}} + \mathbf{w_t} \end{aligned}$$

3.2 Definition

$$\begin{aligned} \mathbf{P_{t|t-1}} &\equiv E[(\xi_t - \hat{\xi}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})'] \\ \mathbf{K_t} &= \mathbf{F}\mathbf{P_{t|t-1}}\mathbf{H}(\mathbf{H}'\mathbf{P_{t|t-1}}\mathbf{H} + \mathbf{R})^{-1} \end{aligned}$$

$$\begin{aligned} E[(\xi_t - \hat{\xi}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})'] &= \mathbf{P_{t|t-1}} \\ E[(\mathbf{y_t} - \hat{\mathbf{y}}_{t|t-1})(\mathbf{y_t} - \hat{\mathbf{y}}_{t|t-1})'] &= \mathbf{H}'\mathbf{P_{t|t-1}}\mathbf{H} + \mathbf{R} \\ E\{(\xi_t - \xi_{t|t-1})(\mathbf{y_t} - \hat{\mathbf{y}}_{t|t-1})'\} &= \mathbf{P_{t|t-1}}\mathbf{H} \end{aligned}$$

3.3 Steps

$$\hat{\xi}_{1|0} = E(\xi_1)$$

$$P_{1|0} = E\{[\xi_1 - E(\xi_1)][\xi_1 - E(\xi_1)]'\}$$

3.4 Iteration

3.5 Smoothing

$$\mathbf{J_t} \equiv \mathbf{P_{t|t}F'P^{-1}_{t+1|t}}$$

$$\hat{\xi_{t|T}} = \hat{\xi}_{t|t} + J(\hat{\xi}_{t+1|T} - \hat{\xi}_{t+1|t})$$