希腊字母

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1 Pre

1.1 Normal Distribution

$N(x)$为标准正态分布（Standard Normal）的累积分布函数（CDF，Cumulative Distribution Function），由正态分布可知$N(-x) = 1 - N(x)$。

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dz$$

$N'(x)$为标准正态分布的概率密度函数（PDF，Probability Density Function），由正态分布的对称性，可知$N'(x) = N'(-x)$。

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

1.2 Greeks

Greeks	
Delta	$\delta = \partial V / \partial S$
Gamma	$\gamma = \partial^2 V / \partial S^2$
Theta	$\theta = \partial V / \partial t$
Rho	$\rho = \partial V / \partial r$
Vega	$\nu = \partial V / \partial \sigma$

2 Black formula

$$c = e^{-r(T-t)}[FN(d_1) - KN(d_2)] \qquad p = e^{-r(T-t)}[KN(-d_2) - FN(-d_1)]$$

2.1 d_1,d_2

$$
\begin{aligned}
d_1 &= \frac{\log(F/K) + \sigma^2/2(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= \frac{\log(F/K) + \sigma^2/2(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= d_1 - \sigma\sqrt{T-t} \\
\frac{\partial d_1}{\partial F} = \frac{\partial d_2}{\partial F} &= \frac{\frac{\partial \log(F/K)}{\partial F}\sigma\sqrt{T-t}}{(\sigma\sqrt{T-t})^2} = \frac{\partial(\log F - \log K)/\partial F}{\sigma\sqrt{T-t}} = \frac{1}{F\sigma\sqrt{T-t}}
\end{aligned}
$$

2.2 Delta

$$
\begin{aligned}
FN'(d_1) &= \frac{F}{\sqrt{2\pi}}e^{-d_1^2/2} \\
KN'(d_2) &= \frac{K}{\sqrt{2\pi}}e^{-d_2^2/2} = \frac{K}{\sqrt{2\pi}}e^{-d_1^2/2 + d_1\sigma\sqrt{T-t} - \sigma^2(T-t)/2} \\
&= \frac{K}{\sqrt{2\pi}}e^{-d_1^2/2 + \ln(F/K)} \qquad \left(d_1\sigma\sqrt{T-t} = \ln(F/K) + \sigma^2(T-t)\right) \\
&= \frac{F}{\sqrt{2\pi}}e^{-d_1^2/2} \\
&= FN'(d_1) \\
\delta = \frac{\partial V_c}{\partial F} &= e^{-r(T-t)}[N(d_1) + FN'(d_1)\frac{\partial d_1}{\partial F} - KN'(d_2)\frac{\partial d_2}{\partial F}] \\
&= e^{-r(T-t)}N(d_1) \\
\delta = \frac{\partial V_p}{\partial F} &= e^{-r(T-t)}[KN'(-d_2)\frac{\partial d_2}{\partial F} - N(-d_1) - FN'(-d_1)\frac{\partial d_1}{\partial F}] \\
&= e^{-r(T-t)}(-N(-d_1)) \qquad \left(KN'(-d_2) = KN'(d_2),\, FN'(-d_1) = FN'(d_1)\right) \\
&= e^{-r(T-t)}(N(d_1) - 1)
\end{aligned}
$$

3 BSM formula

$$
c = Se^{-q(T-t)}N(d_1) - Ke^{-r(T-t)}N(d_2) \qquad p = Ke^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1)
$$

3.1 d_1,d_2

$$
\begin{aligned}
d_1 &= \frac{\log(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= \frac{\log(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= d_1 - \sigma\sqrt{T-t} \\
\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} &= \frac{\frac{\partial \log(S/K)}{\partial S}\sigma\sqrt{T-t}}{(\sigma\sqrt{T-t})^2} = \frac{\partial(\log S - \log K)/\partial S}{\sigma\sqrt{T-t}} = \frac{1}{S\sigma\sqrt{T-t}}
\end{aligned}
$$

3.2 Delta

$$
\begin{aligned}
\delta = \frac{\partial V_c}{\partial S} &= N(d_1) + \frac{1}{S\sigma\sqrt{T-t}}\left[SN'(d_1) - Ke^{-r(T-t)}N'(d_2)\right] \\
&= N(d_1) + \frac{1}{S\sigma\sqrt{T-t}}\left[S\frac{1}{\sqrt{2\pi}}e^{-d_1^2/2} - Ke^{-r(T-t)}\frac{1}{\sqrt{2\pi}}e^{-d_2^2/2}\right] \\
&= N(d_1) + \frac{1}{S\sigma\sqrt{2\pi(T-t)}}\left[Se^{-d_1^2/2} - Ke^{-r(T-t)}e^{-d_2^2/2}\right] \\
&= N(d_1) + \frac{1}{S\sigma\sqrt{2\pi(T-t)}}\left[Se^{-d_1^2/2} - Ke^{-r(T-t)}e^{-(d_1-\sigma\sqrt{T-t})^2/2}\right] \\
&= N(d_1) + \frac{1}{S\sigma\sqrt{2\pi(T-t)}}\left[Se^{-d_1^2/2} - Ke^{-r(T-t)}e^{-d_1^2/2+d_1\sigma(T-t)-\sigma^2(T-t)/2}\right] \\
&= N(d_1) + \frac{1}{S\sigma\sqrt{2\pi(T-t)}}\left[Se^{-d_1^2/2} - Ke^{-r(T-t)}e^{-d_1^2/2+ln(S/K)+(r+\sigma^2/2)(T-t)-\sigma^2(T-t)/2}\right] \\
&= N(d_1) + \frac{1}{S\sigma\sqrt{2\pi(T-t)}}\left[Se^{-d_1^2/2} - Ke^{-(d_1^2)/2+ln(S/K)}\right] \\
&= N(d_1) \\
\delta = \frac{\partial V_p}{\partial S} &= N(d_1) + \left[SN'(d_1) - Ke^{-r(T-t)}N'(d_2)\right]
\end{aligned}
$$