Department of Electronics Engineering
National Chiao Tung University
Algorithms, Fall 2017

December 20, 2017
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DEE3504:
Homework #4 (due 11:59AM, January 2, 2018)

Instructions: Submit your solution to TAs at ED413. No late submission allowed. Please list your collaborator and/or references (if any) for each problem.

Reading:

1. Chapters 7 and 8 including solved exercises, Chapter 17 in Cormen book.

Exercises:

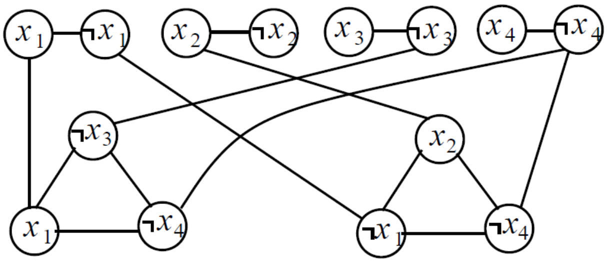
1. (25 pt) Decide whether you think each of the following statements is or false. If it is true, give a short explanation. If it is false, give a counter example.
   a. Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e)=c_e$).
   b. Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; let (A, B) be a minimum s-t cut with respect to these capacities $\{c_e: e\in E\}$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum s-t cut with respect to new capacities $\{1+c_e: e\in E\}$.

2. (25 pt) In this year of great depression, a realtor needs to maximize the number of apartments sold; otherwise, she will soon go into bankruptcy. She has p apartments to sell and q potential customers for these apartments. She has m salesmen working for her. Each salesman is assigned a list of apartments and clients interested in these apartments. A salesman can sell an apartment to any of his customers. Salesman i can sell at most b_i apartments. Also, any apartment cannot be owned by more than one person. For $m = 2$, $p = 4$, $q = 5$, $b_1 = b_2 = 2$, and the following assignments of customers and apartments to the salesmen, construct the flow network for the underlying problem. **How to find** the maximum number of apartments that can be sold?

Salesman	Customers	Apartments
1	1, 2, 3, 4	1, 2, 3
2	3, 4, 5	3, 4

3. (25 pt) A vertex cover of an undirected graph $G = (V, E)$ is a subset $V' \subseteq V$ such that if $(u, v) \in E$ then u or $v \in V'$ (or both). The Vertex-Cover Problem (VC) is to find a vertex cover of minimum size in G.
   a. Formally describe the decision problem of VC. Let it be VC_D.
   b. The 3-CNF-SAT problem (3SAT) is defined as follows:
      Definition: 3SAT = {<ϕ>: Boolean formula ϕ in 3-conjunctive normal form is satisfiable}.
      Use the reduction from 3SAT shown below to prove that VC_D is NP-complete. No partial credit for reduction from any other NP-complete problem. (Hint: What is the size of the

vertex cover for the reduced graph?)



$\phi = (x_1 \vee \neg x_3 \vee \neg x_4) \wedge (\neg x_1 \vee x_2 \vee \neg x_4)$

4. (25 pt) Assume you are creating an array data structure that has a fixed size of *n*. You want to backup this array after every so many insertion operations. Unfortunately, the backup operation is quite expensive, it takes *n* time to do the backup. Insertions without a backup just take 1 time unit.
   a. How frequently can you do a backup and still guarantee that the amortized cost of insertion is O(1)?
   b. Prove that you can do backups in O(1) amortized time by using the potential method. (Hint: Let $\phi_i = i \bmod n$.)