

DEE3504:
Homework #1 (due in class October 23, 2017)

Instructions: Submit your solution **BEFORE** the class begins. No late submission allowed. Please list your collaborator and/or references (if any) for each problem.

Reading:

1. Chapters 1 and 2 including solved exercises.
2. The tips on page 3.

Exercises:

1. (10 pt) Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample. (Explanation is required to get the full points.)

True or false? Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m . Then in every stable matching S for this instance, the pair (m, w) belongs to S .

2. (25 pt) Peripatetic Shipping Lines, Inc. is a shipping company that owns n ships and provides service to n ports. Each of its ships has a *schedule* which says, for each day of the month, which of the ports it's currently visiting, or whether it's out at sea. (You can assume the "month" here has m days, for some $m > n$.) Each ship visits each port for exactly one day during the month. For safety reasons, PSL Inc. has the following strict requirement:

(*) *No two ships can be in the same port on the same day.*

The company wants to perform maintenance on all the ships this month, via the following scheme. They want to *truncate* each ship's schedule: for each ship s_i , there will be some day when it arrives in its scheduled port and simply remains there for the rest of the month (for maintenance). This means that s_i will not visit the remaining ports on its schedule (if any) that month, but this is okay. So the *truncation* of s_i 's schedule will simply consist of its original up to a certain specified day on which it is in a port p_j ; the remainder of the truncated schedule simply has it remain in port p_j .

Now the company's question to you is the following: Given the schedule for each ship, find a truncation for each so that condition (*) continues to hold: no two ships are ever in the same port on the same day.

Show that such a set of truncations can always be found, and give an efficient algorithm to find them. [Hint: Stable Matching]

3. (20 pt) Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$.
 - (a) $f_1(n) = n^{2.5}$
 - (b) $f_2(n) = \sqrt{2n}$
 - (c) $f_3(n) = n + 10$
 - (d) $f_4(n) = 10^n$
 - (e) $f_5(n) = 100^n$
 - (f) $f_6(n) = n^2 \log n$

4. (25 pt) There's a class of folk songs and holiday songs in which each verse consists of the previous verse, with one extra line added on. "The Twelve Days of Christmas" has this property; for example, when you get to the fifth verse, you sing about the five golden rings and then, reprising the lines from the fourth verse, also cover the four calling birds, the three French hens, the two turtle doves, and of course the partridge in the pear tree. [see page 4]
These songs tend to last a long time, despite having relatively short scripts. In particular, you can convey the words plus instructions for one of these songs by specifying just the new line that is added in each verse without having to write out all the previous lines each time. (So the phrase "five golden rings" only has to be written once, even though it will appear in verses five and onward.)
There's something asymptotic that can be analyzed here. Suppose, for concreteness, that each line has a length that is bounded by a constant c , and suppose that the song, when sung out loud, runs for n words total. Show how to encode such a song using a script that has length $f(n)$, for a function $f(n)$ that grows as slowly as possible.
5. (20 pt) Suppose that you run the program below (brute-force solution to the 4-sum problem) for $N = 1,000$ and observe that it takes 1,000 seconds.

```
int brute(int a[], int N)
{
    int i, j, k, m;
    for (i = 0; i < N; i++)
        for (j = i+1; j < N; j++)
            for (k = j+1; k < N; k++)
                for (m = k+1; m < N; m++)
                    if (a[i] + a[j] + a[k] + a[m] == 0) return 1;
    return 0;
}
```

- (a) Predict its running time (in seconds) for $N = 10,000$.
(b) Give a formula that estimates the running time (in seconds) as a function of N . (Please specify the coefficient.)

Study and, in general, the pursuit of truth and beauty is a sphere of activity in which we are permitted to remain children all our lives.

-- A. Einstein

Survival Kits for Homework

- Homework will be assigned after 1~2 topics; it should be handed in lecture, at the beginning of class, on the day it is due. Late homeworks will not receive credit. (If a genuine emergency situation prevents you from handing in an assignment on time, come talk to me and we can work something out.)
- **Do not try to start the assignment the night before it's due.** It is often helpful to look at the problems early; even if you don't spend a lot of time on them right away, it will help to have the problems “stewing” in your head for a few days.
- You are expected to support the answers to the homework with **proofs**. Much of the homework will consist of questions asking you to design algorithms for various problems. A complete answer consists of a clear description of an algorithm (an English or Chinese description is fine), followed by an analysis of its running time and a proof that it works correctly; you do not need to implement the algorithm. You should try to **make your algorithms as efficient as possible**. We reserve the right to **deduct** a significant number of points for solutions that consist only of pseudo-code with no explanation, even if they turn out to be correct.
- You may discuss the homework problems with other members of the class, but you must write up the assignment separately and **list the names of the people with whom you discussed the assignment**. There is no penalty for working in groups.
- You should **cite any sources** that you use in working on a homework problem.
- **Regrading:** If you believe your solution to a question was correct, and it was marked incorrect, you should write up an explanation of the grading error, attach it to your graded homework, and bring it to the TA **within one week** after the homework has been returned. Note that we're talking here about correct algorithms that were treated as incorrect; in general, we will not look at regrading requests that are simply arguing about the amount of partial credit assigned. We will process regrading requests periodically in batch mode, rather than on the spot.

The Twelve Days of Christmas

On the first day of Christmas
my true love sent to me:
A Partridge in a Pear Tree

On the second day of Christmas
my true love sent to me:
2 Turtle Doves
and a Partridge in a Pear Tree

On the third day of Christmas
my true love sent to me:
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the fourth day of Christmas
my true love sent to me:
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the fifth day of Christmas
my true love sent to me:
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the sixth day of Christmas
my true love sent to me:
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the seventh day of Christmas
my true love sent to me:
7 Swans a Swimming
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the eighth day of Christmas
my true love sent to me:
8 Maids a Milking
7 Swans a Swimming
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens

2 Turtle Doves
and a Partridge in a Pear Tree

On the ninth day of Christmas
my true love sent to me:
9 Ladies Dancing
8 Maids a Milking
7 Swans a Swimming
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the tenth day of Christmas
my true love sent to me:
10 Lords a Leaping
9 Ladies Dancing
8 Maids a Milking
7 Swans a Swimming
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the eleventh day of Christmas
my true love sent to me:
11 Pipers Piping
10 Lords a Leaping
9 Ladies Dancing
8 Maids a Milking
7 Swans a Swimming
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

On the twelfth day of Christmas
my true love sent to me:
12 Drummers Drumming
11 Pipers Piping
10 Lords a Leaping
9 Ladies Dancing
8 Maids a Milking
7 Swans a Swimming
6 Geese a Laying
5 Golden Rings
4 Calling Birds
3 French Hens
2 Turtle Doves
and a Partridge in a Pear Tree

<http://www.youtube.com/watch?v=KQEOBZLx-Z8>