Final Project - Bayesian Analysis

```r
library(readxl)
library(dplyr)
library(tidyr)
library(lubridate)
library(stringr)
library(lme4)
```

```r
load("beatspy.RData")
```

Frequentist

```r
#logistic regression models fitted within each sector
library(purrr)
library(modelsummary)

sector_models = m3_df |>
  group_split(gics_sector_name) |>
  setNames(unique(m3_df$gics_sector_name)) |>
  map(~ glm(
    beat_spy ~ log_pe + div_yield,
    data = .x,
    family = binomial(link = "logit")
  ))

modelsummary(sector_models)
```

```r
#mixed effects w/ firm-level random slopes
mixed_random_slopes = glmer(
  beat_spy ~ log_pe + div_yield +
    (1 + log_pe + div_yield | Ticker),
  data = m3_df,
  family = binomial(link = "logit"),
  control = glmerControl(optimizer = "bobyqa")
)
```

```
## boundary (singular) fit: see help('isSingular')
```

```r
mixed_random_slopes |> summary()
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
```

	Health Care	Information Technology	Consumer Staples	Industrials	Utilities	Financials	Materials
(Intercept)	−0.846	1.524	0.684	−0.559	0.512	−2.651	0.231
	(1.388)	(0.741)	(1.268)	(0.571)	(0.566)	(1.001)	(0.726)
log_pe	0.215	−0.304	−0.262	0.049	0.022	0.791	0.000
	(0.403)	(0.212)	(0.373)	(0.153)	(0.168)	(0.293)	(0.214)
div_yield	−0.044	−0.279	−0.181	−0.012	−0.184	−0.038	−0.074
	(0.097)	(0.089)	(0.090)	(0.052)	(0.060)	(0.096)	(0.081)
Num.Obs.	94	283	306	189	603	321	577
AIC	131.3	387.2	395.5	257.9	826.0	427.6	803.8
BIC	138.9	398.2	406.7	267.7	839.2	438.9	816.9
Log.Lik.	−62.641	−190.622	−194.752	−125.971	−410.016	−210.811	−398.899
RMSE	0.49	0.49	0.47	0.49	0.49	0.48	0.50

```
##   Approximation) [glmerMod]
##  Family: binomial  ( logit )
## Formula: beat_spy ~ log_pe + div_yield + (1 + log_pe + div_yield | Ticker)
##    Data: m3_df
## Control: glmerControl(optimizer = "bobyqa")
##
##       AIC       BIC    logLik -2*log(L)  df.resid
##    4632.9    4688.0   -2307.4    4614.9      3391
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.1874 -0.9363 -0.6846  1.0067  2.9505
##
## Random effects:
##  Groups Name        Variance Std.Dev. Corr
##  Ticker (Intercept) 0.289481 0.53803
##         log_pe      0.012842 0.11332  -1.00
##         div_yield   0.007972 0.08928  -1.00  1.00
## Number of obs: 3400, groups:  Ticker, 406
##
## Fixed effects:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.24602    0.20147   1.221    0.222
## log_pe       0.02221    0.05797   0.383    0.702
## div_yield   -0.17073    0.02119  -8.057 7.84e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) log_pe
## log_pe    -0.949
## div_yield -0.565  0.335
## optimizer (bobyqa) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

```r
#pooled logistic model w sector fixed effects
pooled_fe = glm(
  beat_spy ~ log_pe + div_yield + factor(gics_sector_name),
  data = m3_df,
  family = binomial(link = "logit")
)

summary(pooled_fe)
```

```
##
## Call:
## glm(formula = beat_spy ~ log_pe + div_yield + factor(gics_sector_name),
##     family = binomial(link = "logit"), data = m3_df)
##
## Coefficients:
##                                                Estimate Std. Error z value
## (Intercept)                                    -0.51439    0.31086  -1.655
## log_pe                                          0.16173    0.06488   2.493
## div_yield                                      -0.10129    0.02318  -4.370
## factor(gics_sector_name)Consumer Discretionary  0.25790    0.24538   1.051
## factor(gics_sector_name)Consumer Staples       -0.26459    0.24560  -1.077
## factor(gics_sector_name)Energy                 -0.02855    0.26369  -0.108
## factor(gics_sector_name)Financials              0.42514    0.22947   1.853
## factor(gics_sector_name)Health Care            -0.12594    0.24359  -0.517
## factor(gics_sector_name)Industrials             0.31252    0.23072   1.355
## factor(gics_sector_name)Information Technology  0.67038    0.24352   2.753
## factor(gics_sector_name)Materials              -0.01109    0.25882  -0.043
## factor(gics_sector_name)Real Estate            -0.39848    0.26779  -1.488
## factor(gics_sector_name)Utilities               0.12672    0.24909   0.509
##                                                Pr(>|z|)
## (Intercept)                                     0.09798 .
## log_pe                                          0.01267 *
## div_yield                                      1.24e-05 ***
## factor(gics_sector_name)Consumer Discretionary  0.29324
## factor(gics_sector_name)Consumer Staples        0.28134
## factor(gics_sector_name)Energy                  0.91379
## factor(gics_sector_name)Financials              0.06392 .
## factor(gics_sector_name)Health Care             0.60513
## factor(gics_sector_name)Industrials             0.17555
## factor(gics_sector_name)Information Technology  0.00591 **
## factor(gics_sector_name)Materials               0.96582
## factor(gics_sector_name)Real Estate             0.13675
## factor(gics_sector_name)Utilities               0.61092
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 4692.6  on 3399  degrees of freedom
## Residual deviance: 4553.0  on 3387  degrees of freedom
## AIC: 4579
##
## Number of Fisher Scoring iterations: 4
```

```r
#pooled logistic model w sector fe + interactions with DivYield
pooled_fe_interact = glm(
  beat_spy ~
    log_pe +
    div_yield * factor(gics_sector_name),
  data = m3_df,
  family = binomial(link = "logit")
)

summary(pooled_fe_interact)
```

```
##
## Call:
## glm(formula = beat_spy ~ log_pe + div_yield * factor(gics_sector_name),
##     family = binomial(link = "logit"), data = m3_df)
##
## Coefficients:
##                                                        Estimate Std. Error
## (Intercept)                                           -0.598294   0.382658
## log_pe                                                 0.141407   0.065402
## div_yield                                             -0.056334   0.072561
## factor(gics_sector_name)Consumer Discretionary         0.651732   0.386631
## factor(gics_sector_name)Consumer Staples              -0.061950   0.397369
## factor(gics_sector_name)Energy                        -0.269860   0.406570
## factor(gics_sector_name)Financials                     0.724126   0.350106
## factor(gics_sector_name)Health Care                    0.132709   0.362233
## factor(gics_sector_name)Industrials                    0.364062   0.355297
## factor(gics_sector_name)Information Technology         1.078496   0.379784
## factor(gics_sector_name)Materials                      0.046934   0.386036
## factor(gics_sector_name)Real Estate                   -0.436358   0.490961
## factor(gics_sector_name)Utilities                      0.190843   0.517561
## div_yield:factor(gics_sector_name)Consumer Discretionary -0.149613   0.107978
## div_yield:factor(gics_sector_name)Consumer Staples    -0.064631   0.100157
## div_yield:factor(gics_sector_name)Energy               0.050386   0.088010
## div_yield:factor(gics_sector_name)Financials          -0.105227   0.087464
## div_yield:factor(gics_sector_name)Health Care         -0.107264   0.105975
## div_yield:factor(gics_sector_name)Industrials          0.002932   0.103243
## div_yield:factor(gics_sector_name)Information Technology -0.161282   0.107027
## div_yield:factor(gics_sector_name)Materials           -0.012621   0.096039
## div_yield:factor(gics_sector_name)Real Estate         -0.000487   0.103716
## div_yield:factor(gics_sector_name)Utilities           -0.026034   0.120654
##                                                       z value Pr(>|z|)
## (Intercept)                                            -1.564  0.11793
## log_pe                                                  2.162  0.03061 *
## div_yield                                              -0.776  0.43754
## factor(gics_sector_name)Consumer Discretionary          1.686  0.09186 .
## factor(gics_sector_name)Consumer Staples               -0.156  0.87611
## factor(gics_sector_name)Energy                         -0.664  0.50685
## factor(gics_sector_name)Financials                      2.068  0.03861 *
## factor(gics_sector_name)Health Care                     0.366  0.71409
## factor(gics_sector_name)Industrials                     1.025  0.30552
## factor(gics_sector_name)Information Technology          2.840  0.00451 **
## factor(gics_sector_name)Materials                       0.122  0.90323
```

```
## factor(gics_sector_name)Real Estate                              -0.889  0.37412
## factor(gics_sector_name)Utilities                                 0.369  0.71232
## div_yield:factor(gics_sector_name)Consumer Discretionary  -1.386  0.16587
## div_yield:factor(gics_sector_name)Consumer Staples        -0.645  0.51874
## div_yield:factor(gics_sector_name)Energy                   0.573  0.56698
## div_yield:factor(gics_sector_name)Financials              -1.203  0.22894
## div_yield:factor(gics_sector_name)Health Care             -1.012  0.31146
## div_yield:factor(gics_sector_name)Industrials              0.028  0.97734
## div_yield:factor(gics_sector_name)Information Technology  -1.507  0.13183
## div_yield:factor(gics_sector_name)Materials               -0.131  0.89544
## div_yield:factor(gics_sector_name)Real Estate             -0.005  0.99625
## div_yield:factor(gics_sector_name)Utilities               -0.216  0.82916
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 4692.6  on 3399  degrees of freedom
## Residual deviance: 4542.1  on 3377  degrees of freedom
## AIC: 4588.1
##
## Number of Fisher Scoring iterations: 4
```

Model Comparison

```r
# manually back out AIC + BIC
# AIC = 2l + 2k
# BIC = 2l + log(n) * k

sector_logLik = sum(sapply(sector_models, logLik))
sector_k = sum(sapply(sector_models, function(m) attr(logLik(m), "df")))

sector_AIC = -2 * as.numeric(sector_logLik) + 2 * sector_k
sector_BIC = -2 * as.numeric(sector_logLik) +
              log(nrow(m3_df)) * sector_k

# AIC
aic_comp = AIC(pooled_fe,
               pooled_fe_interact,
               mixed_random_slopes)
aic_comp = data.frame(
  Model = rownames(aic_comp),
  df = aic_comp$df,
  AIC = aic_comp$AIC,
  row.names = NULL
) |>
  rbind(data.frame(Model = "sector_models",
                   df = sector_k,
                   AIC = sector_AIC))

#BIC
bic_comp = BIC(pooled_fe,
```

```r
                pooled_fe_interact,
                mixed_random_slopes)
bic_comp = data.frame(
  Model = rownames(bic_comp),
  df = bic_comp$df,
  BIC = bic_comp$BIC,
  row.names = NULL
) |>
  rbind(data.frame(Model = "sector_models",
                   df = sector_k,
                   BIC = sector_BIC))
```

```r
aic_comp |> arrange(AIC)
```

```
##                 Model df      AIC
## 1           pooled_fe 13 4579.022
## 2  pooled_fe_interact 23 4588.136
## 3       sector_models 33 4590.064
## 4 mixed_random_slopes  9 4632.865
```

```r
bic_comp |> arrange(BIC)
```

```
##                 Model df      BIC
## 1           pooled_fe 13 4658.732
## 2 mixed_random_slopes  9 4688.048
## 3  pooled_fe_interact 23 4729.162
## 4       sector_models 33 4792.405
```

(to-do – LOOCV)

Bayesian

```r
library(brms)
library(tidybayes)
library(bayesplot)
library(posterior)
```

Model 1 (Logistic)

$$Y_{i,t} \sim Bernoulli(p_{i,t})$$
$$logit(p_{i,t}) = \beta_0 + \beta_1 + log(PE_{i,t}) + \beta_2 DivYield_{i,t}$$

```r
bayes_model1 <- brm(
  beat_spy ~ log_pe + div_yield,
  data = m3_df,
  family = bernoulli(link = "logit"),
  seed = 123
)
```

```
## Compiling Stan program...

## Trying to compile a simple C file

## Start sampling
```

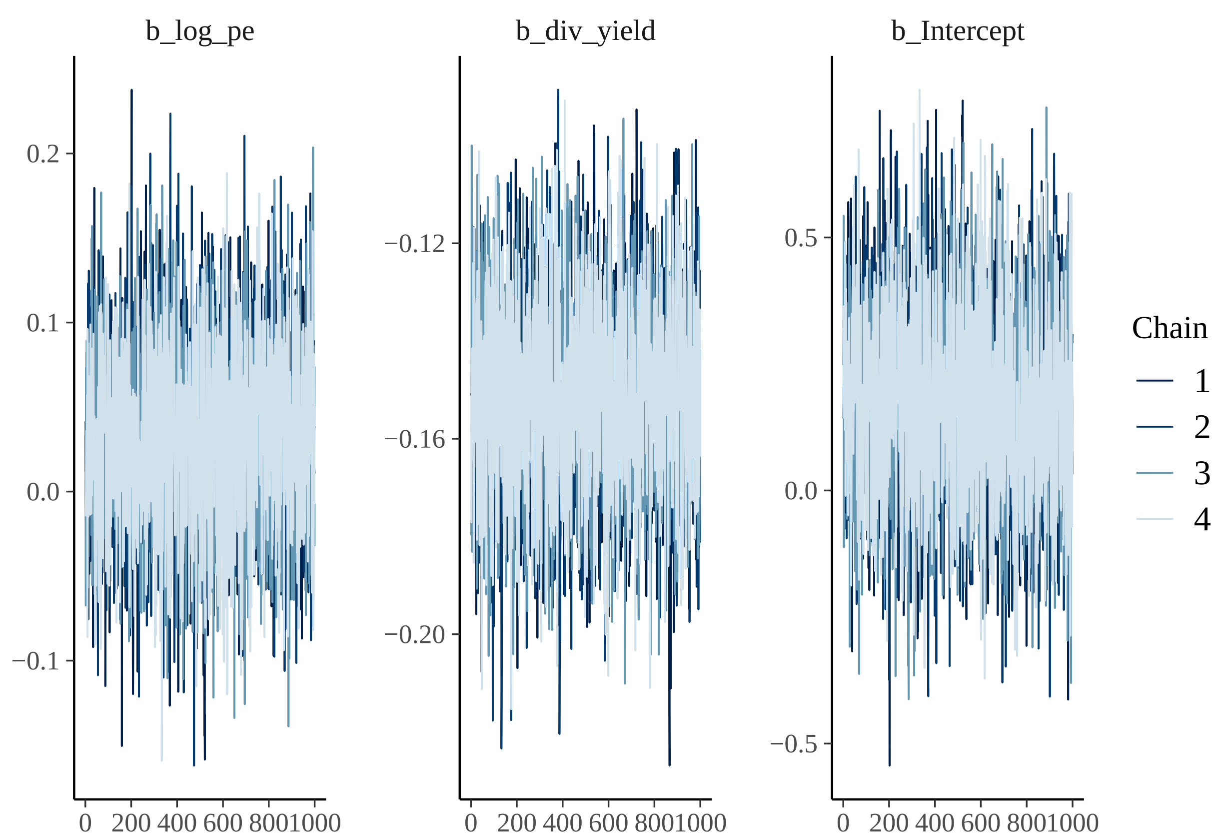
```
summary(bayes_model1)$fixed
```

```
##               Estimate  Est.Error   l-95% CI   u-95% CI     Rhat Bulk_ESS
## Intercept  0.17273278 0.18791917 -0.1830719  0.5376506 1.001122 3007.931
## log_pe     0.03149262 0.05432937 -0.0760189  0.1343352 1.001685 3368.702
## div_yield -0.15189077 0.01951898 -0.1906835 -0.1138507 1.000259 2819.099
##           Tail_ESS
## Intercept 2906.451
## log_pe    3373.531
## div_yield 2796.484
```

Rhats are ~1 and effective sample sizes » 100

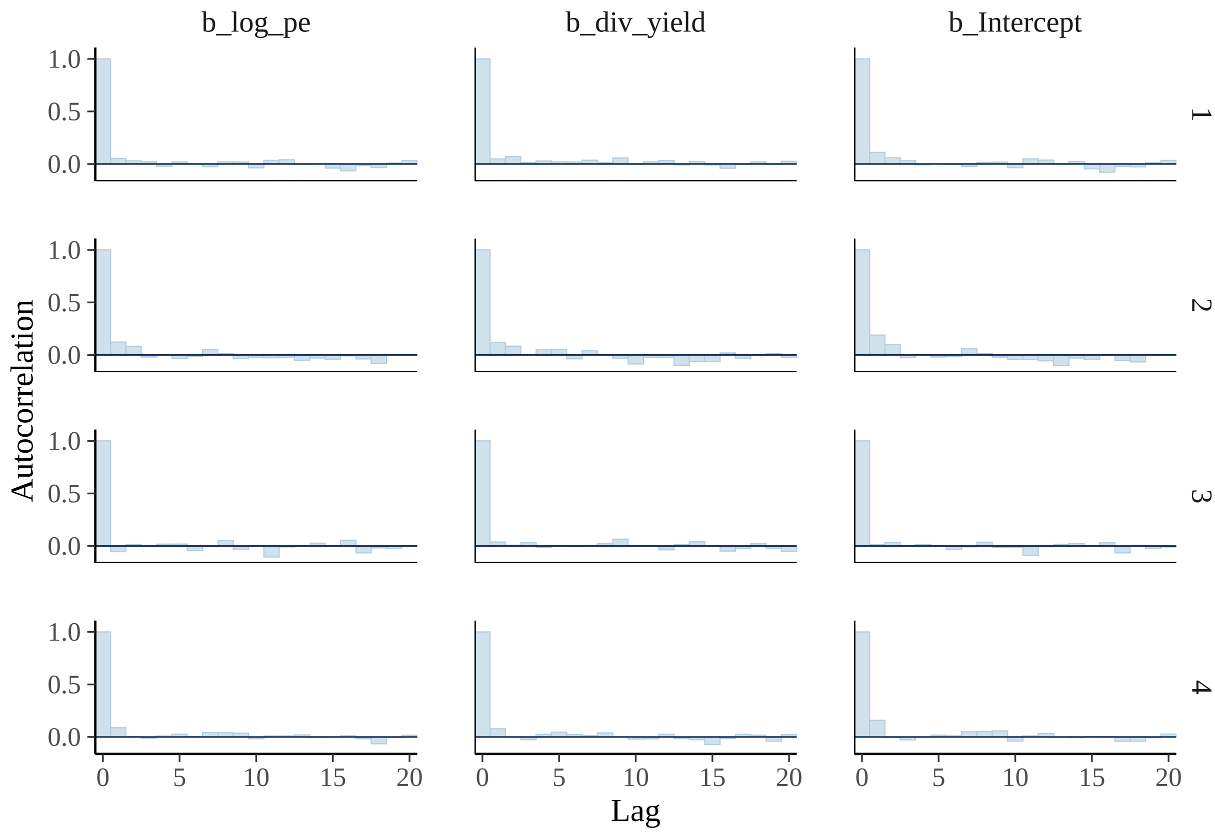
```
draws_model1 = bayes_model1 |>
  as_draws_array()

mcmc_trace(draws_model1,
           pars = c("b_log_pe", "b_div_yield", "b_Intercept"))
```



No discernable pattern from trace plots

```r
mcmc_acf_bar(
  draws_model1,
  pars = c("b_log_pe", "b_div_yield", "b_Intercept")
)
```



acfs fall off quickly

Model 2 (Nested random intercepts)

$$logit(p_{i,t}) = \beta_0 + \beta_1 log(PE_{i,t}) + \beta_2 DivYield_{i,t} + u_j + v_i$$

```r
bayes_model2 = brm(
  beat_spy ~ log_pe + div_yield + (1 | gics_sector_name/Ticker),
  data = m3_df,
  family = bernoulli(link = "logit"),
  seed = 123
)
```

```
## Compiling Stan program...
```

```
## Trying to compile a simple C file
```

```
## Start sampling
```

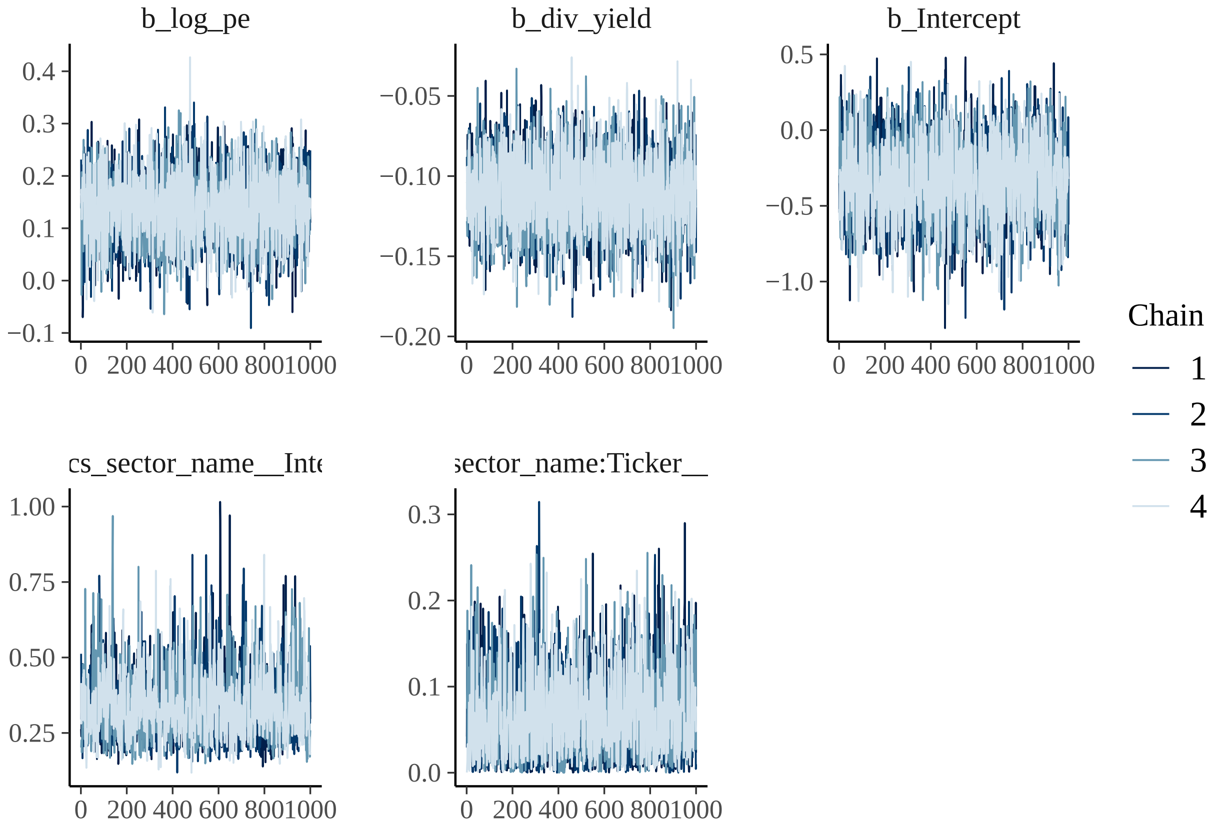
```r
summary(bayes_model2)
```

```
##  Family: bernoulli
##   Links: mu = logit
## Formula: beat_spy ~ log_pe + div_yield + (1 | gics_sector_name/Ticker)
##    Data: m3_df (Number of observations: 3400)
##   Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##          total post-warmup draws = 4000
##
## Multilevel Hyperparameters:
## ~gics_sector_name (Number of levels: 11)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.34      0.10     0.19     0.59 1.00     1348     2025
##
## ~gics_sector_name:Ticker (Number of levels: 406)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.06      0.05     0.00     0.17 1.00     1607     1590
##
## Regression Coefficients:
##           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept    -0.32      0.26    -0.82     0.17 1.00     2110     2593
## log_pe        0.14      0.06     0.02     0.26 1.00     3616     2987
## div_yield    -0.11      0.02    -0.15    -0.07 1.00     3638     2948
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

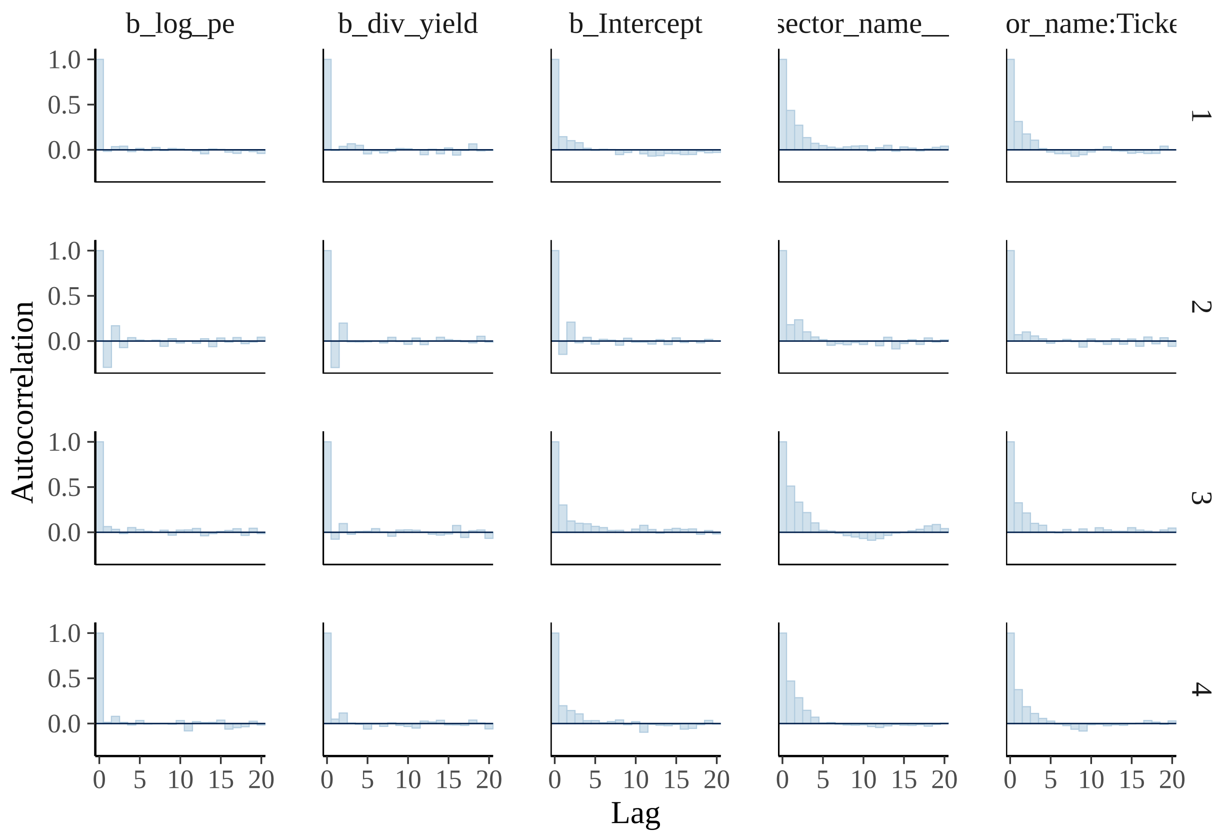
Rhats are all ~1, effective sample sizes » 100

```r
draws_model2 = bayes_model2 |>
  as_draws_array()

mcmc_trace(
  draws_model2,
  pars = c("b_log_pe",
           "b_div_yield",
           "b_Intercept",
           "sd_gics_sector_name__Intercept",
           "sd_gics_sector_name:Ticker__Intercept")
)
```




```
mcmc_acf_bar(
  draws_model2,
  pars = c("b_log_pe", "b_div_yield", "b_Intercept", "sd_gics_sector_name__Intercept", "sd_gics_sector_r
)
```



acfs fall off quickly

Model 3 (Mixed Effects + firm-level random slopes/intercepts)

Covariates are centered

```
bayes_model3 = brm(
  beat_spy ~ log_pe + div_yield + (1 + log_pe + div_yield | Ticker),
  data = m3_df |> mutate(log_pe = scale(log_pe),
                         div_yield = scale(div_yield)),
  family = bernoulli(link = "logit"),
  seed = 123
)
```

```
## Compiling Stan program...
```

```
## Trying to compile a simple C file
```

```
## Start sampling
```

```
summary(bayes_model3)
```

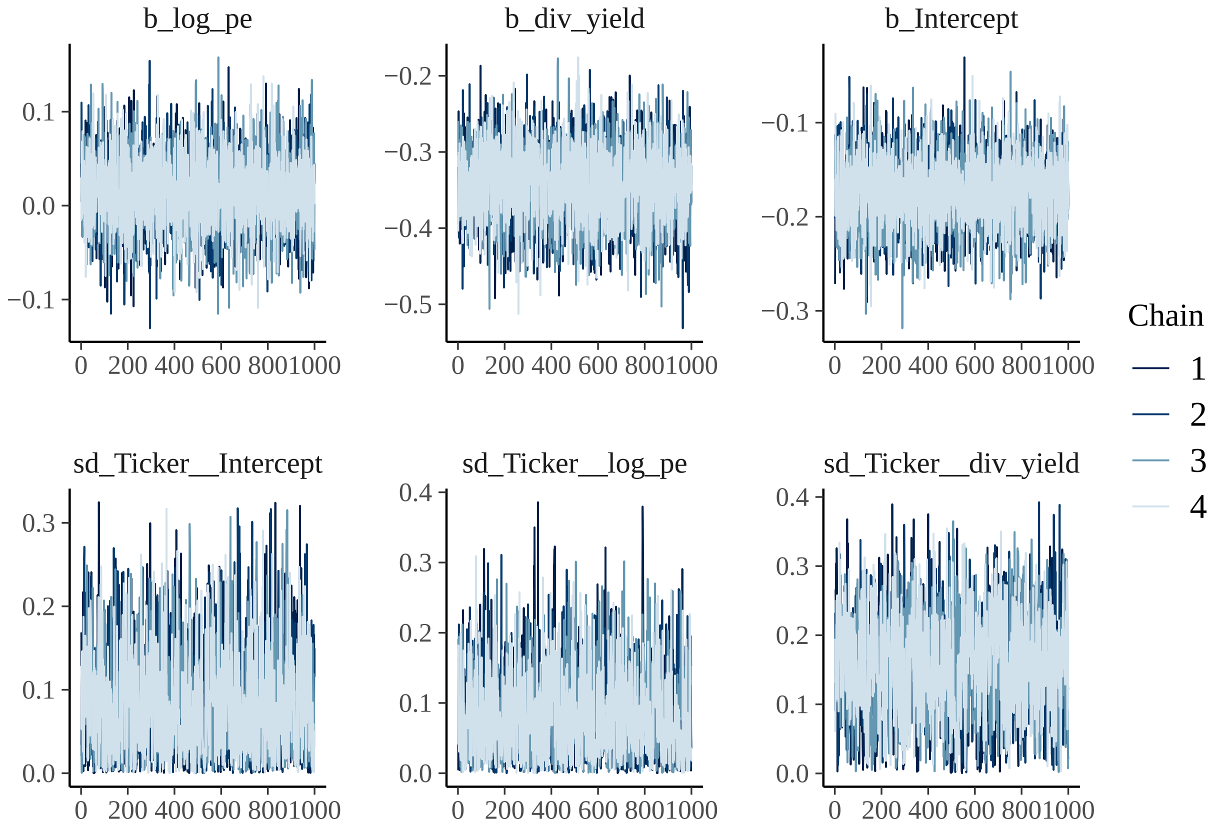
```
##  Family: bernoulli
##   Links: mu = logit
```

```
## Formula: beat_spy ~ log_pe + div_yield + (1 + log_pe + div_yield | Ticker)
##    Data: mutate(m3_df, log_pe = scale(log_pe), div_yield =  (Number of observations: 3400)
##   Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
##          total post-warmup draws = 4000
##
## Multilevel Hyperparameters:
## ~Ticker (Number of levels: 406)
##                          Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## sd(Intercept)                0.09      0.06     0.00     0.23 1.00     1265
## sd(log_pe)                   0.08      0.06     0.00     0.23 1.00     1412
## sd(div_yield)                0.16      0.07     0.02     0.30 1.00     1070
## cor(Intercept,log_pe)        0.03      0.50    -0.87     0.89 1.00     2685
## cor(Intercept,div_yield)     0.10      0.48    -0.83     0.90 1.00     1388
## cor(log_pe,div_yield)        0.20      0.48    -0.78     0.92 1.00     1513
##                          Tail_ESS
## sd(Intercept)                1754
## sd(log_pe)                   2025
## sd(div_yield)                1397
## cor(Intercept,log_pe)        2449
## cor(Intercept,div_yield)     2368
## cor(log_pe,div_yield)        2618
##
## Regression Coefficients:
##           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept    -0.17      0.04    -0.24    -0.10 1.00     6658     3278
## log_pe        0.01      0.04    -0.06     0.09 1.00     6592     3033
## div_yield    -0.34      0.05    -0.44    -0.25 1.00     3832     3085
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Rhats are ~1 and effective sample sizes » 100

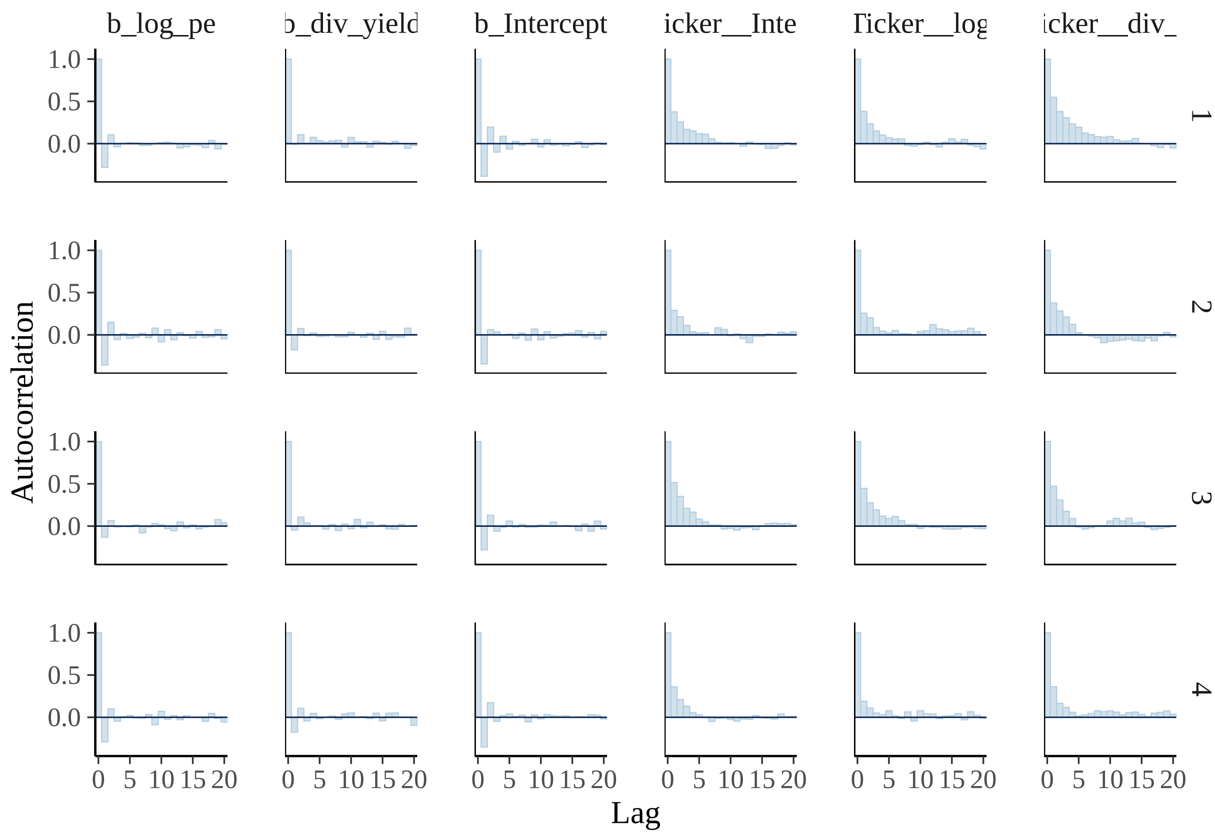
```r
draws_bayes_mixed = bayes_model3 |>
  as_draws_array()

mcmc_trace(draws_bayes_mixed,
           pars = c("b_log_pe", "b_div_yield", "b_Intercept","sd_Ticker__Intercept", "sd_Ticker__log_pe
```



No discernable pattern from trace plots

```
mcmc_acf_bar(
  draws_bayes_mixed,
  pars = c("b_log_pe", "b_div_yield", "b_Intercept","sd_Ticker__Intercept", "sd_Ticker__log_pe", "sd_Ti
```



Model 4 (Pooled + sector FE)

```
bayes_fe = brm(
  beat_spy ~ log_pe + div_yield + factor(gics_sector_name),
  data = m3_df,
  family = bernoulli(link = "logit"),
  seed = 123
)
```

```
## Compiling Stan program...
```

```
## Trying to compile a simple C file
```

```
## Start sampling
```

```
summary(bayes_fe)
```

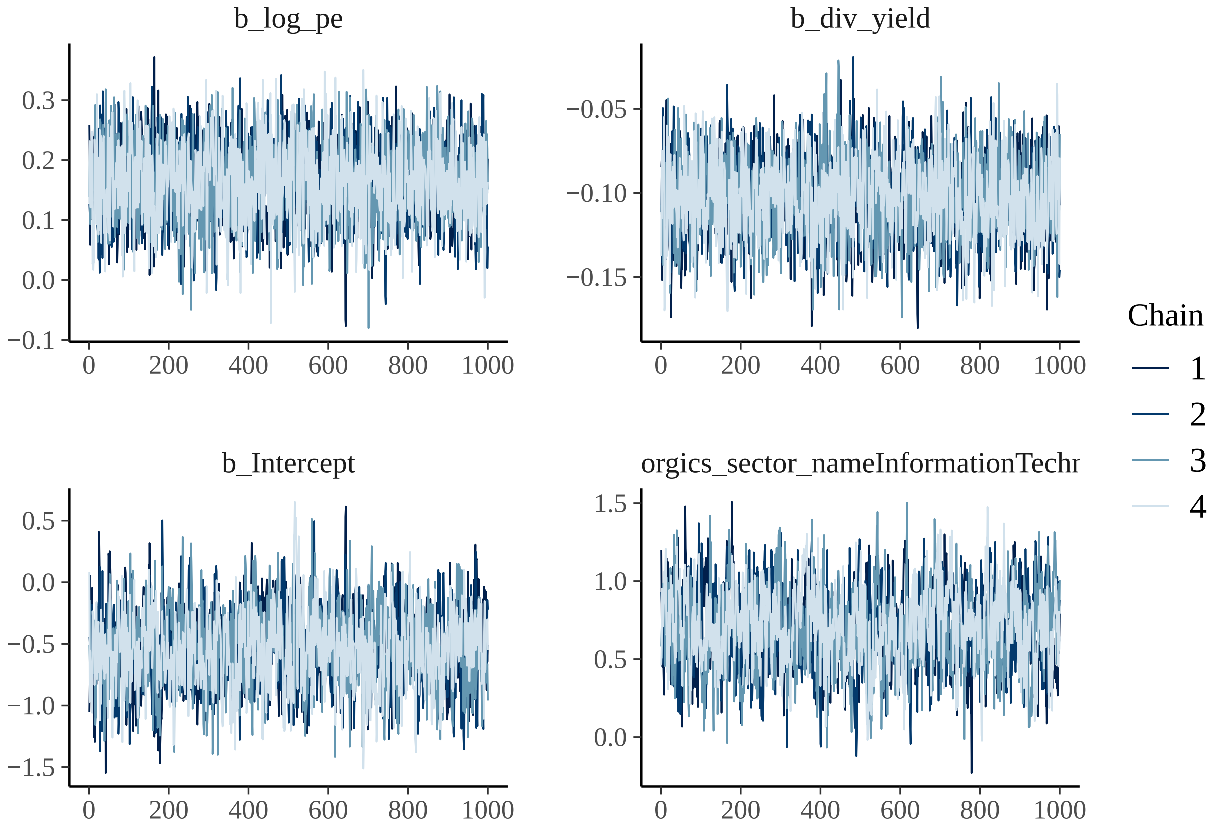
```
##  Family: bernoulli
##   Links: mu = logit
## Formula: beat_spy ~ log_pe + div_yield + factor(gics_sector_name)
##    Data: m3_df (Number of observations: 3400)
##   Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
```

```
##          total post-warmup draws = 4000
##
## Regression Coefficients:
##                                              Estimate Est.Error l-95% CI
## Intercept                                       -0.53      0.31    -1.15
## log_pe                                           0.16      0.07     0.04
## div_yield                                       -0.10      0.02    -0.15
## factorgics_sector_nameConsumerDiscretionary      0.28      0.25    -0.23
## factorgics_sector_nameConsumerStaples           -0.25      0.25    -0.75
## factorgics_sector_nameEnergy                    -0.01      0.27    -0.53
## factorgics_sector_nameFinancials                 0.45      0.23    -0.00
## factorgics_sector_nameHealthCare                -0.11      0.25    -0.61
## factorgics_sector_nameIndustrials                0.33      0.23    -0.13
## factorgics_sector_nameInformationTechnology      0.69      0.25     0.21
## factorgics_sector_nameMaterials                  0.01      0.26    -0.52
## factorgics_sector_nameRealEstate                -0.38      0.27    -0.91
## factorgics_sector_nameUtilities                  0.15      0.25    -0.35
##                                              u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept                                        0.08 1.00     1206     1782
## log_pe                                           0.29 1.00     2550     2373
## div_yield                                       -0.06 1.00     2367     2615
## factorgics_sector_nameConsumerDiscretionary      0.78 1.00      889     1279
## factorgics_sector_nameConsumerStaples            0.24 1.00      837     1187
## factorgics_sector_nameEnergy                     0.52 1.00      945     1424
## factorgics_sector_nameFinancials                 0.92 1.00      784     1064
## factorgics_sector_nameHealthCare                 0.39 1.00      877     1298
## factorgics_sector_nameIndustrials                0.79 1.00      764     1071
## factorgics_sector_nameInformationTechnology      1.18 1.00      855     1431
## factorgics_sector_nameMaterials                  0.52 1.00      915     1599
## factorgics_sector_nameRealEstate                 0.16 1.00      913     1482
## factorgics_sector_nameUtilities                  0.65 1.00      871     1256
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

Rhats are ~1 and effective sample sizes » 100

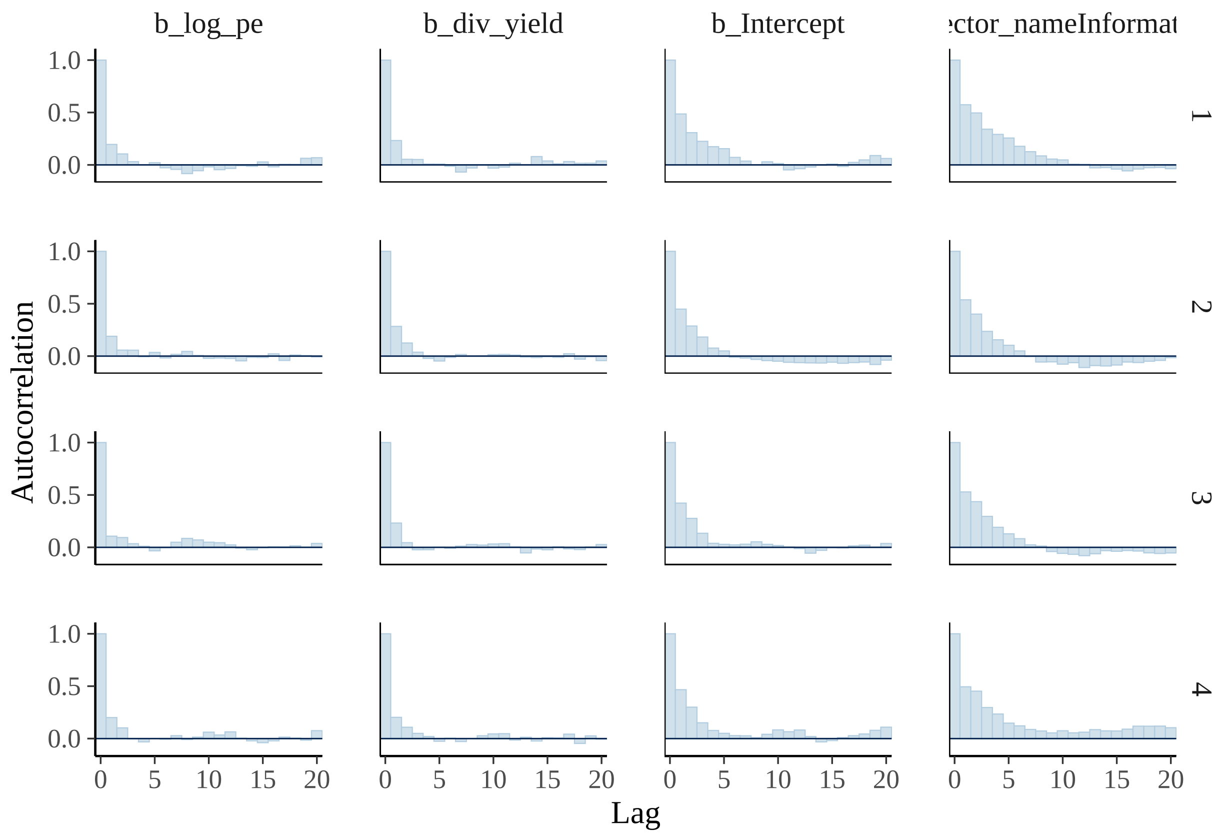
```r
draws_bayes_fe = bayes_fe |>
  as_draws_array()

mcmc_trace(draws_bayes_fe,
           pars = c("b_log_pe", "b_div_yield", "b_Intercept","b_factorgics_sector_nameInformationTechnol
```



No discernable pattern from trace plots

```
mcmc_acf_bar(
  draws_bayes_fe,
  pars = c("b_log_pe", "b_div_yield", "b_Intercept","b_factorgics_sector_nameInformationTechnology")
)
```



Model Comparison

```
loo_compare(loo(bayes_model1),
            loo(bayes_model2),
            loo(bayes_model3),
            loo(bayes_fe))
```

```
##                elpd_diff se_diff
## bayes_fe         0.0       0.0
## bayes_model2    -0.3       1.2
## bayes_model1   -22.5       8.1
## bayes_model3   -24.9       7.9
```

LOOCV favors the `pooled model with sector fixed effects` and the `model with nested random intercepts` over the `baseline pooled model` and `mixed effects model w/ firm-level random intercepts and slopes`.

The sector FE and nested random intercepts models are generally comparable (firm-level variation may be small)