UNIVERSITÄT
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METAS

The METAS joule-watt balance

A combined approach

PhD project presentation

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Content

- The kilogram prototype in Paris is not stable with respect to time.
- The watt balance invented by [Kibble, 1976] provides a constant representation.
- The watt balance is nowadays named after its inventor as Kibble balance.
- The Kibble balance method is based on a comparison of electrical to mechanical power.

- From electrodynamics, the electromagnetic force

$$\boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} \qquad (1)$$

can be derived.
- The current I_s is varied such that $\boldsymbol{F}_{el} = \boldsymbol{F}_g$.
- Current I_s and voltage U_s are measured.

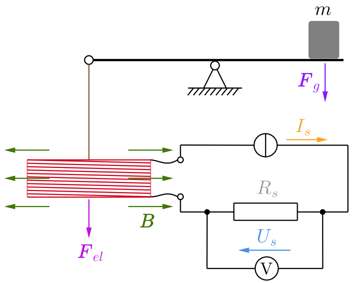

Figure 1: Static mode of the Kibble balance.

- From electrodynamics, the electromagnetic induction

$$U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \quad (2)$$

  can be derived.
- The coil is moved at velocity $\boldsymbol{v}$.
- Induced voltage U_d is measured.

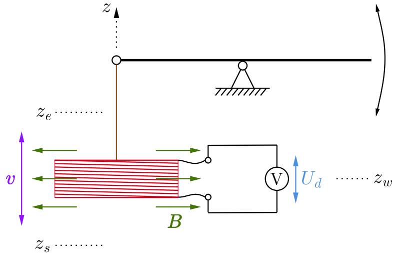

Figure 2: Dynamic mode of the Kibble balance.

- Bringing eq. (1) and eq. (2) together yields

$$\left.\begin{array}{c} \boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} = m\boldsymbol{g} \\ U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \end{array}\right\} \quad m\boldsymbol{g} \cdot \boldsymbol{v} = U_d I_s. \qquad (3)$$

- The voltage U_d and the resistance I_s are related to the Planck constant h by means of the quantum Hall and Josephson effects, namely

$$U_d = C_d n_{J,d} f_{J,d} \frac{h}{2e}, \qquad I_s = C_s n_{J,s} f_{J,s} \frac{n_H e}{2}. \qquad (4)$$

- Combining eq. (3) with eq. (4), one arrives at

$$m = C\frac{f_{J,d} f_{J,s}}{gv} h. \tag{5}$$

- The variables have the following meanings:

m	Mass
C	Calibration constant
$f_{J,d}$	Josephson frequency for the dynamic voltage measurement
$f_{J,s}$	Josephson frequency for the static voltage measurement
g	Gravitational acceleration at <u>exact</u> position of the mass m
v	Vertical velocity of movement in the dynamic phase
h	Planck constant

- The idea of the proposed project is to reduce measurement uncertainty, part of which is to operate the METAS Kibble balance as a joule balance.
- Instead of comparing mechanical to electrical power, the joule balance compares mechanical to electrical work.
- The watt balance equation $mg(z)v(t) = U(t)I(z)$ has to be integrated with respect to time with $z = z(t)$.
- One arrives at

$$m \int_{z_s}^{z_e} \frac{g(z)}{I(z)} \, \mathrm{d}z = \int_{t(z_s)}^{t(z_e)} U(t) \, \mathrm{d}t \qquad (6)$$

as the joule balance equation.

Status quo

Current research status

- Several metrology institutes around the world are operating watt or Kibble balances.
- As [Stock et al., 2023] remark, the obtained kilogram representations however do not yet agree to a satisfactory degree.
- The measurement uncertainties are not low enough to allow for independent kilogram realizations yet.
- As of today, the kilogram is defined as a weighted consensus value of all participating watt or joule balance and Avogadro experiments worldwide.

- The overarching goal of the proposed project is therefore to reduce the measurement uncertainty of the existing METAS Kibble balance.
- This reduction is aimed to be achieved by means of four connected approaches.
- The goal is to reduce the uncertainty from currently $4.3 \cdot 10^{-8}$ to $3.0 \cdot 10^{-8}$ in relative terms.

- Such a relative accuracy of order 10^{-8} is equivalent to picking a single paper from a $10\,\mathrm{km}$ stack of paper.
- A relative measurement uncertainty of $3.0 \cdot 10^{-8}$ also means that a kilogram can then be measured with an accuracy of $30\,\mathrm{\mu g}$.

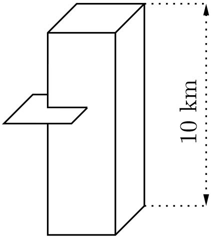

Figure 3: Visualization of a relative accuracy of order 10^{-8}.

Background: Text.

Problem: Captured velocities v due to tilt of the coil not captured by induced voltages U_d.

Hypothesis: Elimination of Abbe error using a weighting technique using 3 interferometers leads to reduction of noise in the $G_e = U_d/v$ profile.

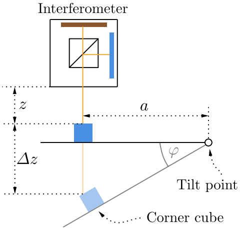

Figure 4: Visualization of a so-called Abbe offset error $\Delta z \approx a\varphi$.

Background: Text.

Problem: Choice of polynomial order for fit of G_e profile is not substantiated.

Hypothesis: Obtaining a force profile $G_m = g/I_s$ for the dynamic range substantiates choice of polynomial order and hence reduces uncertainty associated to choice of order.

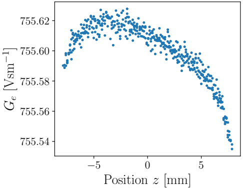

Figure 5: Example datapoints obtained for the $G_e = U_d/v$ profile.

Background: Text.

Problem: Currently, there is only one method (Kibble balance) available to trace the kilogram back to the Planck constant h.

Hypothesis: By implementation of a joule balance mode for the existing Kibble balance, the associated uncertainty on kilogram measurements might be reducible.

Background: Text.

Problem: Non-ideal alignment of the Kibble balance has effects contributing to uncertainties of G_e, G_m and thus $m = G_e G_m^{-1}$.

Hypothesis: Systematically variying the alignment parameters $p_1, \ldots, p_k$ and studying the associated outcome for G_e and G_m, an artificial neural network can be trained to correct for alingment errors.

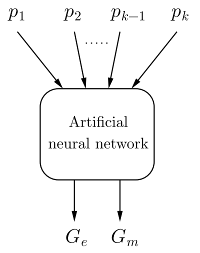

Figure 6: Proposed architecture of the neural network; $\boldsymbol{p}$ are the alignment parameters and $\boldsymbol{q}$ are the geometrical factors associated to the alignment parameters.

Methods and research plan

Milestones

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Text.

Text.

Literature

Eichenberger, A., Baumann, H., Mortara, A., Tommasini, D., Reber, D., Klingelé, E., Jeanneret, B., and Jeckelmann, B. (2022).
First realisation of the kilogram with the metas kibble balance.
Metrologia, 59(2):025008.

Fang, H., Bielsa, F., Li, S., Kiss, A., and Stock, M. (2020).
The bipm kibble balance for realizing the kilogram definition.
Metrologia, 57(4):045009.

Haddad, D., Seifert, F., Chao, L. S., Possolo, A., Newell, D. B., Pratt, J. R., Williams, C. J., and Schlamminger, S. (2017).
Measurement of the planck constant at the national institute of standards and technology from 2015 to 2017.
Metrologia, 54(5):633.

Jeckelmann, B. and Jeanneret, B. (2001).
The quantum hall effect as an electrical resistance standard.
Reports on Progress in Physics, 64(12):1603.

Kajastie, H., Riski, K., and Satrapinski, A. (2009).
Mass determination with the magnetic levitation method—proposal for a new design of electromechanical system.
Metrologia, 46(3):298.

Literature

Kibble, B. P. (1976).
A Measurement of the Gyromagnetic Ratio of the Proton by the Strong Field Method, pages 545–551.
Springer US, Boston, MA.

Stock, M., Conceição, P., Fang, H., Bielsa, F., Kiss, A., Nielsen, L., Beaudoux, F., Espel, P., Thomas, M., Ziane, D., Baumann, H., Eichenberger, A., Marti, K., Bai, Y., Hu, M., Li, Z., Lu, Y., Peng, C., Wang, J., Wang, Y., Wu, D., Abbott, P., Haddad, D., Kubarych, Z., Mulhern, E., Schlamminger, S., Newell, D., Fujita, K., Inaba, H., Kano, K., Kuramoto, N., Mizushima, S., Okubo, S., Ota, Y., Zhang, L., Davidson, S., Green, R. G., Liard, J. O., Murnaghan, N. F., Wood, B. M., Borys, M., Eppers, D., Knopf, D., Kuhn, E., Hämpke, M., Müller, M., Nicolaus, A., Scholz, F., Spoors, M., and Ahmedov, H. (2023).
Final report on the ccm key comparison of kilogram realizations ccm.m-k8.2021.
Metrologia, 60(1A):07003.

Xu, J., Zhang, Z., Li, Z., Bai, Y., Wang, G., Li, S., Zeng, T., Li, C., Lu, Y., Han, B., Wang, N., and Zhou, K. (2016).
A determination of the planck constant by the generalized joule balance method with a permanent-magnet system at nim.
Metrologia, 53(1):86.

Questions

Thank you for your time and attention!

Consider an electric field $\boldsymbol{E}(\boldsymbol{r},t)$, a magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$, a charge density $\rho(\boldsymbol{r},t)$ and a current density $\boldsymbol{j}(\boldsymbol{r},t)$. The four so-called Maxwell equations

$$\boldsymbol{\nabla}\cdot\boldsymbol{E}=\frac{\rho}{\epsilon_0} \quad\Leftrightarrow\quad \oint_{\partial V}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{S}=\frac{1}{\epsilon_0}\int_V \rho\,\mathrm{d}V \tag{7}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0 \quad\Leftrightarrow\quad \oint_{\partial V}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}=0 \tag{8}$$

$$\boldsymbol{\nabla}\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t} \quad\Leftrightarrow\quad \oint_{\partial S}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}=-\int_S \frac{\partial\boldsymbol{B}}{\partial t}\cdot\mathrm{d}\boldsymbol{S} \tag{9}$$

$$\boldsymbol{\nabla}\times\boldsymbol{B}=\mu_0\boldsymbol{j}+\mu_0\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t} \quad\Leftrightarrow\quad \oint_{\partial S}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{l}=\mu_0\int_S \boldsymbol{j}\cdot\mathrm{d}\boldsymbol{S}+\mu_0\epsilon_0\int_S \frac{\partial\boldsymbol{E}}{\partial t}\cdot\mathrm{d}\boldsymbol{S} \tag{10}$$

govern these physical quantities.

In order to derive an expression for electromagnetic induction, it is necessary to invoke Maxwell's equations; in particular the Lorentz force equation

$$\boldsymbol{F}(\boldsymbol{r},t) = q\left[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)\right], \tag{11}$$

where $\boldsymbol{r}$ and t denote the position and time of evaluation; and where q is a charge probe, $\boldsymbol{v}$ is the velocity of q and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field respectively. Furthermore, the third Maxwell equation is of interest for the watt and joule balance techniques, namely

$$\boldsymbol{\nabla} \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}. \tag{12}$$

In addition to these equations, also the integral theorem of Stokes is relevant; let Σ denote an arbitrary surfae in $\mathbb{R}^3$ and let $\partial\Sigma$ denote its boundary, which in this case is a line in $\mathbb{R}^3$. If then $\boldsymbol{V}$ is an arbitrary vector field in $\mathbb{R}^3$, the integral theorem of Stokes states that

$$\int_\Sigma (\boldsymbol{\nabla} \times \boldsymbol{V}) \cdot \mathrm{d}\boldsymbol{S} = \oint_{\partial\Sigma} \boldsymbol{V} \cdot \mathrm{d}\boldsymbol{l}, \tag{13}$$

where $\mathrm{d}\boldsymbol{S}$ denotes the oriented surface element of Σ and $\mathrm{d}\boldsymbol{l}$ is a line element of $\partial\Sigma$. With this theorem, Maxwell's third equation eq. (12) can be transformed to the integral form

$$\oint_{\partial\Sigma} \boldsymbol{E}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{l} = -\int_\Sigma \frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} \cdot \mathrm{d}\boldsymbol{S}. \tag{14}$$

If the surface Σ becomes time-dependent $\Sigma \to \Sigma(t)$, the magnetic flux Φ_B through the surface $\Sigma(t)$ can be written as

$$\Phi_B = \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S}. \tag{15}$$

The total time derivative of the magnetic flux in turn gives the negative induced voltage $U(t)$, namely

$$U(t) = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S}. \tag{16}$$

In order to evaluate this total time derivative, both a change in the magnetic field aswell as in the surface element have to be considered. The change in the surface element $\mathrm{d}\boldsymbol{S}$ with time t can be written as $\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{v} \times \mathrm{d}\boldsymbol{l}$, as fig. 7 indicates.

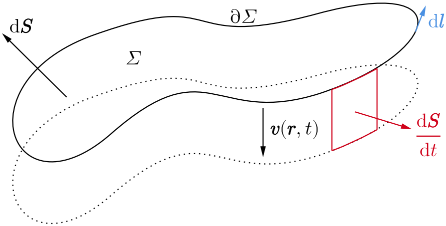

Figure 7: Illustration for the derivation of the expression for the change in the surface element $\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{v} \times \mathrm{d}\boldsymbol{l}$.

With this knowledge, one can differentiate both $\boldsymbol{B}(\boldsymbol{r}, t)$ and $\mathrm{d}\boldsymbol{S}$ in the integrand to arrive at

$$\begin{aligned}
\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t} \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S} \\
&= \int_{\Sigma(t)} \frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} \cdot \mathrm{d}\boldsymbol{S} + \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} \\
&= -\oint_{\partial\Sigma(t)} \boldsymbol{E}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{l} + \oint_{\partial\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot (\boldsymbol{v}(t) \times \mathrm{d}\boldsymbol{l}) \\
&= -\oint_{\partial\Sigma(t)} [\boldsymbol{E}(\boldsymbol{r}, t) + \boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r}, t)] \cdot \mathrm{d}\boldsymbol{l} = -U(t).
\end{aligned} \tag{17}$$

In the case where the external magnetic field is zero, i.e. $\boldsymbol{E}(\boldsymbol{r},t) = 0$, this equation reduces to

$$U(t) = \oint_{\partial\Sigma(t)} [\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)] \cdot \mathrm{d}\boldsymbol{l} \tag{18}$$

and is herewith the first of the fundamental equations used for the Kibble balance experiment.

A second important equation is derived from the Lorentz force equation. Consider for this purpose fig. 8.

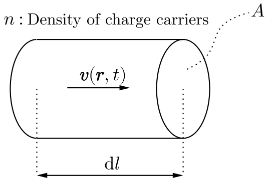

Figure 8: Illustration to derive the second important equation needed for the Kibble balance principle.

In this figure, a short element of length $\mathrm{d}l$ of a wire with charge carrier density n is shown. In principle, this wire represents any object of choice. If one assumes, that this wire or alternatively an object of choice is immersed in a magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$, a magnetic force is exerted on charge carriers moving at speed $\boldsymbol{v}(t)$ in the object. Assuming that all charge carriers move at speed $\boldsymbol{v}(t)$, the total current $I(t)$ is given by $I(t) = nAe|\boldsymbol{v}(t)$.

The electromagnetic force exerted on one electron would be given by the expression $e\boldsymbol{E}(\boldsymbol{r},t) + e\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)$; and on $nA\,\mathrm{d}l$ carriers with no external electric field ($\boldsymbol{E}(\boldsymbol{r},t) = 0$), the force differential $\mathrm{d}\boldsymbol{F}$ is given by

$$\begin{aligned} \mathrm{d}\boldsymbol{F} = neA\,\mathrm{d}l\,[\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)] &= I(t)\,\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}(\boldsymbol{r},t) \\ &= -I(t)\boldsymbol{B}(\boldsymbol{r},t) \times \mathrm{d}\boldsymbol{l}, \end{aligned} \tag{19}$$

where in the last steps the definition $\mathrm{d}\boldsymbol{l} \doteq \mathrm{d}l\,\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ was used. Integrated over a closed path $\partial\Sigma(t)$ therefore one obtains the magnetic force on the object given by

$$\boldsymbol{F}(\boldsymbol{r},t) = -I(t) \oint_{\partial\Sigma(t)} \boldsymbol{B}(\boldsymbol{r},t) \times \mathrm{d}\boldsymbol{l}. \tag{20}$$

The quantum Hall effect can be used for resistance measurement; according to [Jeckelmann and Jeanneret, 2001], the quantum Hall resistance R_H is given by the expression

$$R_H = \frac{h}{n_H e^2}, \quad n_H \in \mathbb{N}, \tag{21}$$

where $h = 6.626\,070\,15 \times 10^{-35}$ J s is the Planck constant and e is the elementary charge.

Add explanatory picture and think about explanation.

The Josephson voltage U_J across a Josephson junction can be used for voltage measurements; according to [Kajastie et al., 2009] it is given by the equation

$$U_J = \frac{2e}{h} n_J f_J, \quad n_J \in \mathbb{N}, \tag{22}$$

where h is the Planck constant, e is the elementary charge and f_J is the frequency of the microwave radiation used to irradiate the Josephson junction with.

Add explanatory picture and think about explanation.