The METAS joule-watt balance

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Content

1. Introduction
2. Status quo
3. Hypotheses and aims
4. Methods and research plan
5. Significance

Introduction

- The kilogram prototype in Paris might not be stable with respect to time.
- The watt balance invented by [Kibble, 1976] provides a method to trace the kilogram to a constant of nature, namely h.
- The watt balance is nowadays named after its inventor as Kibble balance.
- The Kibble balance method is based on a comparison of electrical to mechanical power.

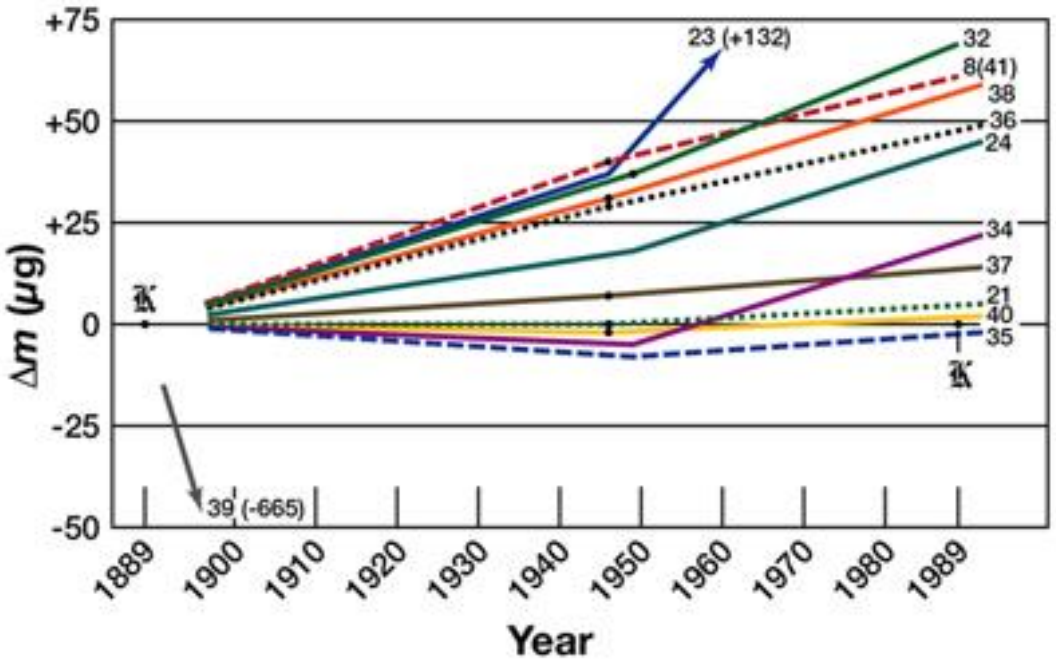

Figure 1: Mass fluctuations of the BIPM mass representations with respect to the official prototype of the mass. Image credit: NIST, https://www.nist.gov/si-redefinition/kilogram-present, last accessed 08 July, 2024.

Introduction: Static mode

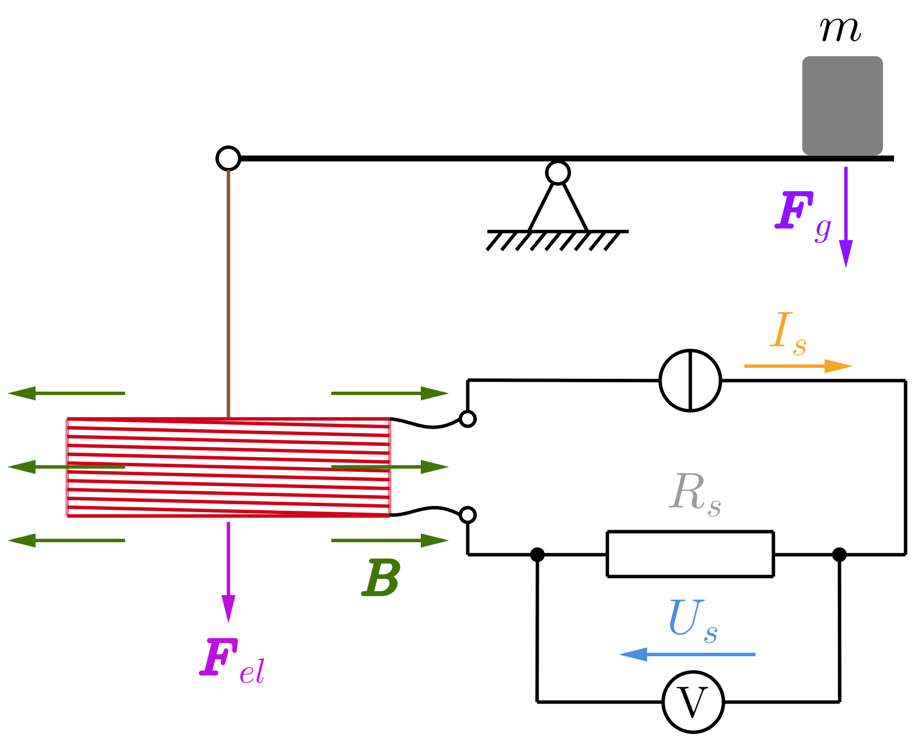

Figure 2: Static mode of the Kibble balance.

- From electrodynamics, the electromagnetic force

$$\boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l}$$

  can be derived.
- The current I_s is varied such that

$$\boldsymbol{F}_{el} = \boldsymbol{F}_g$$

  holds.
- Voltage U_s and current I_s are measured.

Introduction: Dynamic mode

METAS

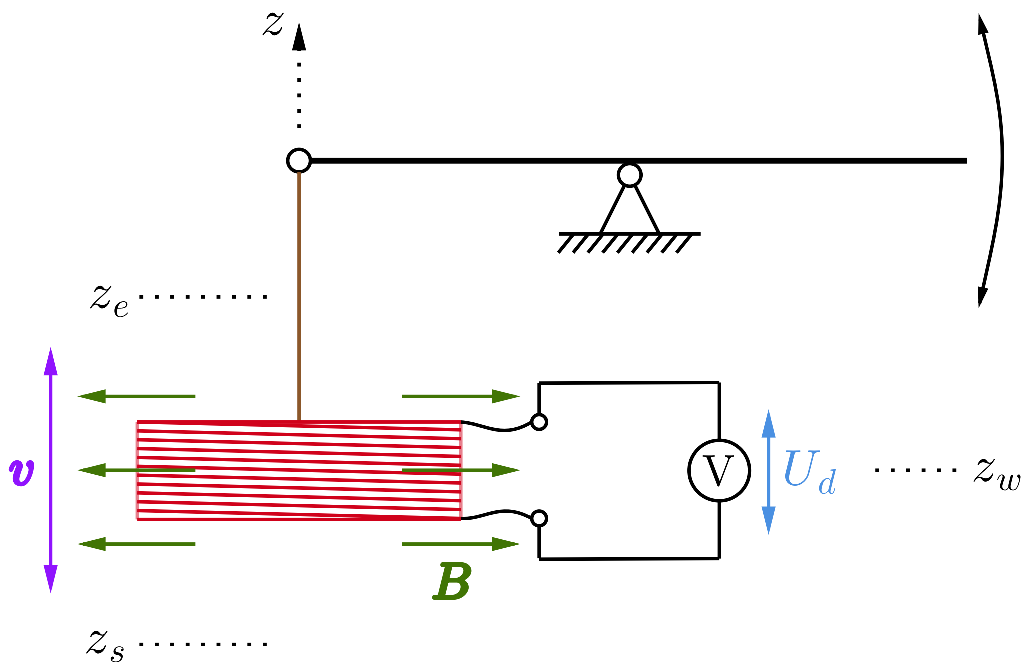

Figure 3: Dynamic mode of the Kibble balance.

- From electrodynamics, the electromagnetic induction

$$U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v}$$

  can be derived.
- The coil is moved at velocity $\boldsymbol{v}$.
- Induced voltage U_d and velocity $\boldsymbol{v}$ are measured.

Introduction: Kibble balance equation 1/2

- Bringing the equations from both the static and dynamic modes together yields

$$\left.\begin{aligned} \boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} = m\boldsymbol{g} \\ U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \end{aligned}\right\} \quad m\boldsymbol{g} \cdot \boldsymbol{v} = U_d I_s .$$

- The voltage U_d and the current I_s are related to the Planck constant h by means of the quantum Hall and Josephson effects, namely

$$U_d = C_d n_{J,d} f_{J,d} \frac{h}{2e}, \qquad I_s = C_s n_{J,s} f_{J,s} \frac{n_H e}{2}$$

Introduction: Kibble balance equation 2/2

- Finally combining equations, one arrives at

$$m = C\frac{f_{J,d} f_{J,s}}{gv}h.$$

- The variables have the following meanings:

m	Mass
C	Calibration constant
$f_{J,d}$	Josephson frequency for the dynamic voltage measurement
$f_{J,s}$	Josephson frequency for the static voltage measurement
g	Gravitational acceleration at exact position of the mass m
v	Vertical velocity of movement in the dynamic phase
h	Planck constant

Basic idea of the project

- The idea of the proposed project is to reduce measurement uncertainty, part of which is to operate the METAS Kibble balance as a joule balance.
- Instead of comparing mechanical to electrical power, the joule balance compares mechanical to electrical work.
- The watt balance equation $mg(z)v(t) = U(t)I(z)$ has to be integrated with respect to time with $z = z(t)$.
- One arrives at

$$m \int_{z_s}^{z_e} \frac{g(z)}{I(z)} \, \mathrm{d}z = \int_{t(z_s)}^{t(z_e)} U(t) \, \mathrm{d}t$$

as the joule balance equation.

Status quo

- Several metrology institutes around the world operate watt or joule balances.
- As [Stock et al., 2023] remark, the obtained kilogram representations however do not yet agree to satisfactory degree.
- The measurement uncertainties are not low enough to allow for independent kilogram realizations yet.
- As of today, the kilogram is defined as a weighted consensus value of all participating watt or joule balance and Avogadro experiments worldwide.

Hypotheses: Overarching goal

- The overarching goal of the proposed project is therefore to reduce the measurement uncertainty of the existing METAS Kibble balance.
- This reduction is aimed to be achieved by means of four connected approaches.
- The goal is to reduce the uncertainty from currently $4.3 \cdot 10^{-8}$ to $3.0 \cdot 10^{-8}$ in relative terms.

Hypotheses: Overarching goal

- Such a relative accuracy of order 10^{-8} is equivalent to picking a single paper from a $10\ \mathrm{km}$ stack of paper.
- A relative measurement uncertainty of $3.0 \cdot 10^{-8}$ also means that a kilogram can then be measured with an accuracy of $30\ \mu\mathrm{g}$.

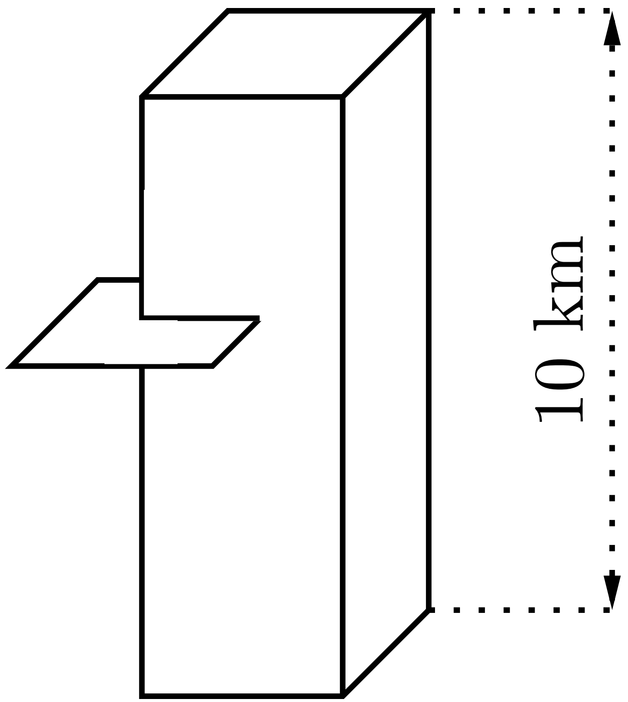

Figure 4: Visualization of a relative accuracy of order 10^{-8}.

Hypothesis and aim 1

- **Background:** Vertical velocity v of the center of mass of the coil needs to be measured.

- **Problem:** Captured velocities v due to tilt of the coil not captured by induced voltages U_d.

- **Hypothesis:** Elimination of Abbe error using a signal weighting technique using 3 interferometers leads to reduction of noise in the $G_e = U_d/v$ profile.

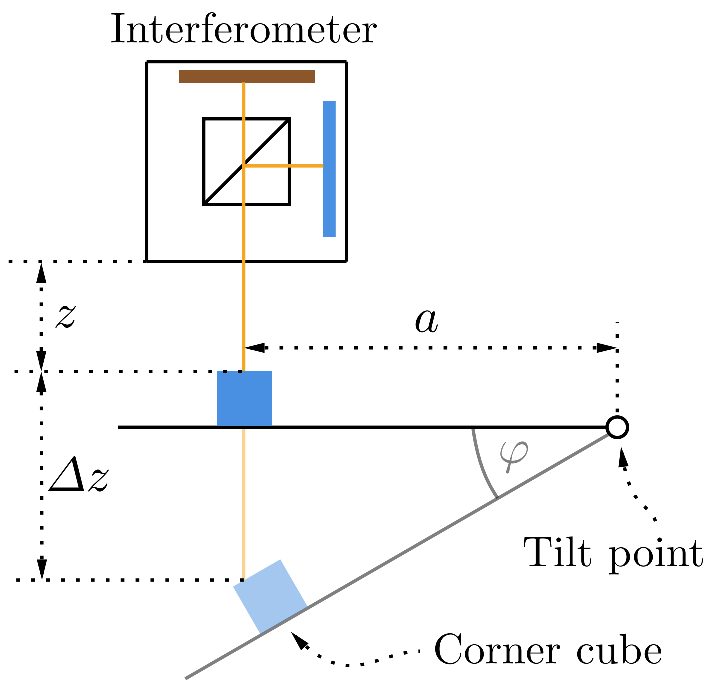

Figure 5: Visualization of a so-called Abbe offset error.

Hypothesis and aim 2

- **Background:** Voltage to velocity ratio $G_e = U_d/v$ needs to be fitted with a polynomial to determine value at weighting position.

- **Problem:** Choice of polynomial order for fit of G_e is not substantiated.

- **Hypothesis:** Obtaining a force profile $G_m = g/I_s$ for the dynamic range substantiates choice of polynomial order and hence reduces uncertainty associated to choice of order.

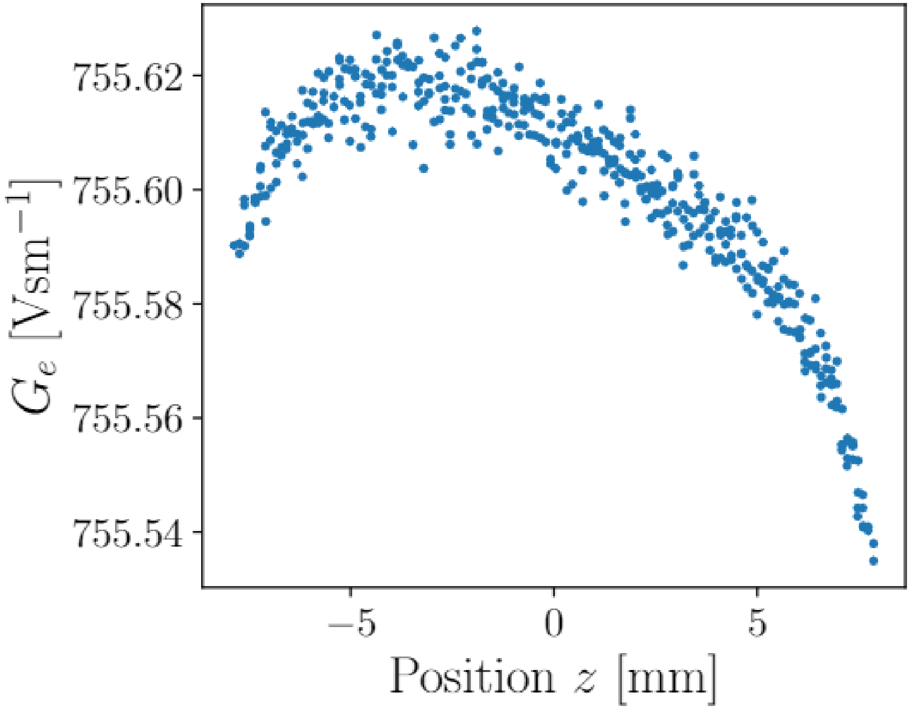

Figure 6: Example datapoints obtained for the voltage to velocity ratio.

Hypothesis and aim 3

- **Background:** Given two methods m_1 and m_2 with associated uncertainties σ_1 and σ_2 to determine some quantity m , $\sigma_m \leq \min(\sigma_1, \sigma_2)$ holds.

- **Problem:** Currently, there is only one method (Kibble balance mode) available to trace the kilogram back to the Planck constant h .

- **Hypothesis:** By implementation of a joule balance mode for the existing METAS Kibble balance, the associated uncertainty on kilogram measurements might be reducible.

Hypothesis and aim 4

- **Background:** Tedious alignment of the watt balance experiment is needed to perform accurate measurements.

- **Problem:** Non-ideal alignment of the Kibble balance has effects contributing to uncertainties of G_e, G_m and thus $m = G_e G_m^{-1}$.

- **Hypothesis:** Systematically varying the alignment parameters $p_1, \ldots, p_k$ and studying the associated outcome for G_e and G_m, an artificial neural network can be trained to correct for alignment errors.

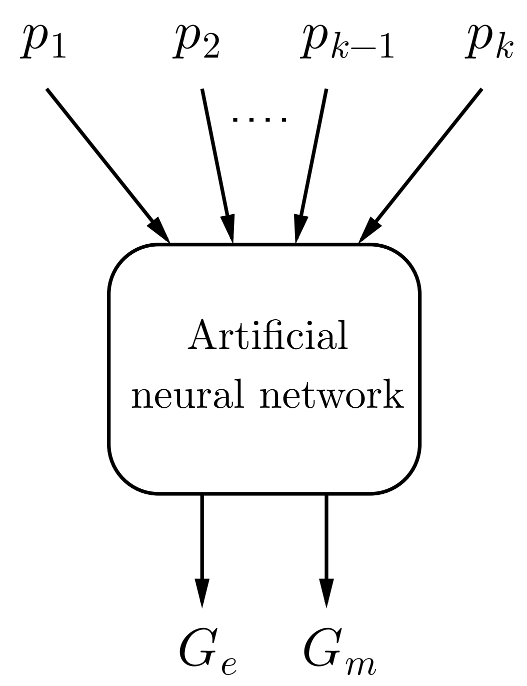

Figure 7: Proposed architecture of the neural network. Input are the alignment parameters and output are the geometrical factors.

Research plan

METAS

	FS24	HS24	FS25	HS25	FS26	HS26	FS27
Achieve familiarity with existing experimental setup							
Implement 3-interferometer technique, possibly write publication							
Attend lectures and conduct gravimetric measurements, possibly write publication							
Determine suitable polynomial order for G_e profile fit by FEM and measurements							
Gather data for AI model, build, train and test it							
Develop the METAS joule balance mode and test it, possibly write publication							
Compile findings into PhD thesis							

Significance

- As of today, no independent realization of the kilogram using a Kibble or joule balance experiment is possible.
- The proposed project aims for a reduction of the associated uncertainty and hence contributes towards independent realization of the kilogram.
- Industries around the world rely on traceability of their measurement equipment to the SI units.
- The METAS joule-watt balance contributes to this traceability.

Literature

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Thanks for your attention!

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