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The METAS joule-watt balance

A combined approach

PhD project presentation

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Content

Introduction

Background

- The kilogram prototype in Paris is not stable with respect to time.
- The watt balance invented by [Kibble, 1976] provides a constant representation.
- The watt balance is nowadays named after its inventor as Kibble balance.
- The Kibble balance method is based on a comparison of electrical to mechanical power.

- From electrodynamics, the electromagnetic force

$$\boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} \quad (1)$$

  can be derived.
- The current I_s is varied such that $\boldsymbol{F}_{el} = \boldsymbol{F}_g$.
- Current I_s and voltage U_s are measured.

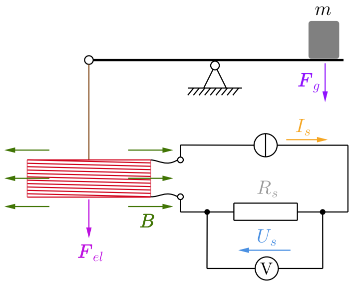

Figure 1: Static mode of the Kibble balance.

- From electrodynamics, the electromagnetic induction

$$U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \quad (2)$$

can be derived.
- The coil is moved at velocity $\boldsymbol{v}$.
- Induced voltage U_d is measured.

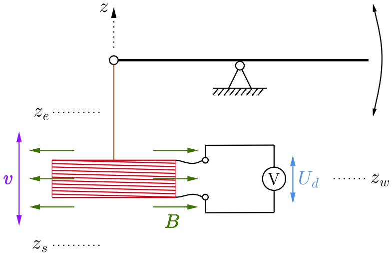

Figure 2: Dynamic mode of the Kibble balance.

- Bringing eq. (1) and eq. (2) together yields

$$\left.\begin{array}{c} \boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} = m\boldsymbol{g} \\ U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \end{array}\right\} \quad m\boldsymbol{g} \cdot \boldsymbol{v} = U_d I_s. \qquad (3)$$

- The voltage U_d and the current I_s are related to the Planck constant h by means of the quantum Hall and Josephson effects, namely

$$U_d = C_d n_{J,d} f_{J,d} \frac{h}{2e}, \qquad I_s = C_s n_{J,s} f_{J,s} \frac{n_H e}{2}. \qquad (4)$$

- Combining eq. (3) with eq. (4), one arrives at

$$m = C\frac{f_{J,d}f_{J,s}}{gv}h. \tag{5}$$

- The variables have the following meanings:

m	Mass
C	Calibration constant
$f_{J,d}$	Josephson frequency for the dynamic voltage measurement
$f_{J,s}$	Josephson frequency for the static voltage measurement
g	Gravitational acceleration at exact position of the mass m
v	Vertical velocity of movement in the dynamic phase
h	Planck constant

- The idea of the proposed project is to reduce measurement uncertainty, part of which is to operate the METAS Kibble balance as a joule balance.
- Instead of comparing mechanical to electrical power, the joule balance compares mechanical to electrical work.
- The watt balance equation $mg(z)v(t) = U(t)I(z)$ has to be integrated with respect to time with $z = z(t)$.
- One arrives at

$$m \int_{z_s}^{z_e} \frac{g(z)}{I(z)} \, \mathrm{d}z = \int_{t(z_s)}^{t(z_e)} U(t) \, \mathrm{d}t \tag{6}$$

as the joule balance equation.

Status quo

Current research status

- Several metrology institutes around the world are operating watt or joule balances.
- As [Stock et al., 2023] remark, the obtained kilogram representations however do not yet agree to a satisfactory degree.
- The measurement uncertainties are not low enough to allow for independent kilogram realizations yet.
- As of today, the kilogram is defined as a weighted consensus value of all participating watt or joule balance and Avogadro experiments worldwide.

Hypotheses

Overarching hypothesis and goal

- The overarching goal of the proposed project is therefore to reduce the measurement uncertainty of the existing METAS Kibble balance.
- This reduction is aimed to be achieved by means of four connected approaches.
- The goal is to reduce the uncertainty from currently $4.3 \cdot 10^{-8}$ to $3.0 \cdot 10^{-8}$ in relative terms.

Hypotheses

Overarching hypothesis and goal

- Such a relative accuracy of order 10^{-8} is equivalent to picking a single paper from a $10\,\text{km}$ stack of paper.
- A relative measurement uncertainty of $3.0 \cdot 10^{-8}$ also means that a kilogram can then be measured with an accuracy of $30\,\mu\text{g}$.

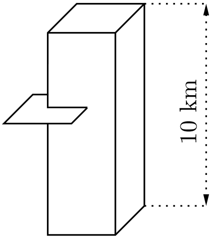

Figure 3: Visualization of a relative accuracy of order 10^{-8}.

Background: Vertical velocity v of the center of mass of the coil needs to be measured.

Problem: Captured velocities v due to tilt of the coil not captured by induced voltages U_d.

Hypothesis: Elimination of Abbe error using a weighting technique using 3 interferometers leads to reduction of noise in the $G_e = U_d/v$ profile.

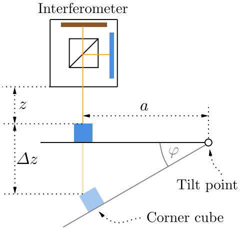

Figure 4: Visualization of a so-called Abbe offset error $\Delta z \approx a\varphi$.

Background: Voltage to velocity ratio U_d/v needs to be fitted with a polynomial to determine value at weighting position.

Problem: Choice of polynomial order for fit of G_e profile is not substantiated.

Hypothesis: Obtaining a force profile $G_m = g/I_s$ for the dynamic range substantiates choice of polynomial order and hence reduces uncertainty associated to choice of order.

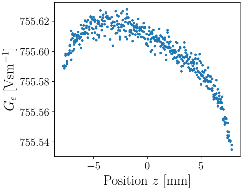

Figure 5: Example datapoints obtained for the $G_e = U_d/v$ profile.

Background: Given two methods m_1 and m_2 with associated uncertainties σ_1 and σ_2 to determine some quantity m, $\sigma_m \leq \min(\sigma_1, \sigma_2)$ holds.

Problem: Currently, there is only one method (Kibble balance) available to trace the kilogram back to the Planck constant h.

Hypothesis: By implementation of a joule balance mode for the existing Kibble balance, the associated uncertainty on kilogram measurements might be reducible.

Hypotheses

Hypothesis and aim 4

Background: Tedious alignment of the watt balance experiment is needed to perform accurate measurements.

Problem: Non-ideal alignment of the Kibble balance has effects contributing to uncertainties of G_e, G_m and thus $m = G_e G_m^{-1}$.

Hypothesis: Systematically varying the alignment parameters $p_1, \ldots, p_k$ and studying the associated outcome for G_e and G_m, an artificial neural network can be trained to correct for alingment errors.

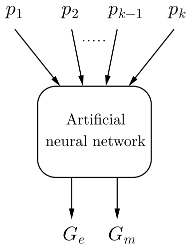

Figure 6: Proposed architecture of the neural network; $\boldsymbol{p}$ are the alignment parameters and $\boldsymbol{q}$ are the geometrical factors associated to the alignment parameters.

Methods and research plan

Plan

	FS24	HS24	FS25	HS25	FS26	HS26	FS27
Achieve familiarity with existing experimental setup							
Implement 3-interferometer technique, possibly write publication							
Attend lectures and conduct gravimetric measurements, possibly write publication							
Determine suitable polynomial order for G_e profile fit by FEM and measurements							
Gather data for AI model, build, train and test it							
Develop the METAS joule balance mode and test it, possibly write publication							
Compile findings into PhD thesis							

Significance

Significance of the proposed project

- As of today, no independent realization of the kilogram using a Kibble or joule balance experiment is possible.
- The proposed project aims for a reduction of the associated uncertainty and hence contributes towards independent realization of the kilogram.
- Industries around the world rely on traceability of their measurement equipment to the SI units.
- The METAS joule-watt balance contributes to this traceability.

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Questions

Thank you for your time and attention!

Consider an electric field $\boldsymbol{E}(\boldsymbol{r},t)$, a magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$, a charge density $\rho(\boldsymbol{r},t)$ and a current density $\boldsymbol{j}(\boldsymbol{r},t)$. The four so-called Maxwell equations

$$\boldsymbol{\nabla}\cdot\boldsymbol{E}=\frac{\rho}{\epsilon_0} \quad\Leftrightarrow\quad \oint_{\partial V}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{S}=\frac{1}{\epsilon_0}\int_V \rho\,\mathrm{d}V \tag{7}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0 \quad\Leftrightarrow\quad \oint_{\partial V}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}=0 \tag{8}$$

$$\boldsymbol{\nabla}\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t} \quad\Leftrightarrow\quad \oint_{\partial S}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}=-\int_S\frac{\partial\boldsymbol{B}}{\partial t}\cdot\mathrm{d}\boldsymbol{S} \tag{9}$$

$$\boldsymbol{\nabla}\times\boldsymbol{B}=\mu_0\boldsymbol{j}+\mu_0\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t} \quad\Leftrightarrow\quad \oint_{\partial S}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{l}=\mu_0\int_S\boldsymbol{j}\cdot\mathrm{d}\boldsymbol{S}+\mu_0\epsilon_0\int_S\frac{\partial\boldsymbol{E}}{\partial t}\cdot\mathrm{d}\boldsymbol{S} \tag{10}$$

govern these physical quantities.

In order to derive an expression for electromagnetic induction, it is necessary to invoke Maxwell's equations; in particular the Lorentz force equation

$$\boldsymbol{F}(\boldsymbol{r},t) = q\left[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v}(t)\times\boldsymbol{B}(\boldsymbol{r},t)\right], \tag{11}$$

where $\boldsymbol{r}$ and t denote the position and time of evaluation; and where q is a charge probe, $\boldsymbol{v}$ is the velocity of q and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field respectively. Furthermore, the third Maxwell equation is of interest for the watt and joule balance techniques, namely

$$\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}. \tag{12}$$

In addition to these equations, also the integral theorem of Stokes is relevant; let Σ denote an arbitrary surfae in $\mathbb{R}^3$ and let $\partial\Sigma$ denote its boundary, which in this case is a line in $\mathbb{R}^3$. If then $\boldsymbol{V}$ is an arbitrary vector field in $\mathbb{R}^3$, the integral theorem of Stokes states that

$$\int_{\Sigma} (\boldsymbol{\nabla} \times \boldsymbol{V}) \cdot \mathrm{d}\boldsymbol{S} = \oint_{\partial\Sigma} \boldsymbol{V} \cdot \mathrm{d}\boldsymbol{l}, \tag{13}$$

where $\mathrm{d}\boldsymbol{S}$ denotes the oriented surface element of Σ and $\mathrm{d}\boldsymbol{l}$ is a line element of $\partial\Sigma$. With this theorem, Maxwell's third equation eq. (12) can be transformed to the integral form

$$\oint_{\partial\Sigma} \boldsymbol{E}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{l} = -\int_{\Sigma} \frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} \cdot \mathrm{d}\boldsymbol{S}. \tag{14}$$

If the surface Σ becomes time-dependent $\Sigma \to \Sigma(t)$, the magnetic flux Φ_B through the surface $\Sigma(t)$ can be written as

$$\Phi_B = \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S}. \tag{15}$$

The total time derivative of the magnetic flux in turn gives the negative induced voltage $U(t)$, namely

$$U(t) = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S}. \tag{16}$$

In order to evaluate this total time derivative, both a change in the magnetic field aswell as in the surface element have to be considered. The change in the surface element $\mathrm{d}\boldsymbol{S}$ with time t can be written as $\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{v} \times \mathrm{d}\boldsymbol{l}$, as fig. 7 indicates.

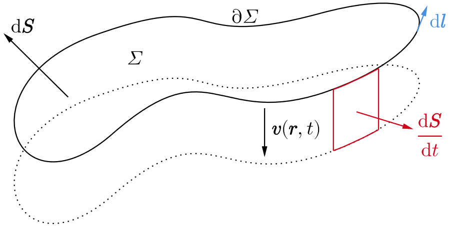

Figure 7: Illustration for the derivation of the expression for the change in the surface element $\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{v} \times \mathrm{d}\boldsymbol{l}$.

With this knowledge, one can differentiate both $\boldsymbol{B}(\boldsymbol{r}, t)$ and $\mathrm{d}\boldsymbol{S}$ in the integrand to arrive at

$$\begin{aligned}
\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t} \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S} \\
&= \int_{\Sigma(t)} \frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} \cdot \mathrm{d}\boldsymbol{S} + \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} \\
&= -\oint_{\partial\Sigma(t)} \boldsymbol{E}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{l} + \oint_{\partial\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot (\boldsymbol{v}(t) \times \mathrm{d}\boldsymbol{l}) \\
&= -\oint_{\partial\Sigma(t)} [\boldsymbol{E}(\boldsymbol{r}, t) + \boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r}, t)] \cdot \mathrm{d}\boldsymbol{l} = -U(t).
\end{aligned} \tag{17}$$

In the case where the external magnetic field is zero, i.e. $\boldsymbol{E}(\boldsymbol{r}, t) = 0$, this equation reduces to

$$U(t) = \oint_{\partial\Sigma(t)} [\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r}, t)] \cdot \mathrm{d}\boldsymbol{l} \tag{18}$$

and is herewith the first of the fundamental equations used for the Kibble balance experiment.

A second important equation is derived from the Lorentz force equation. Consider for this purpose fig. 8.

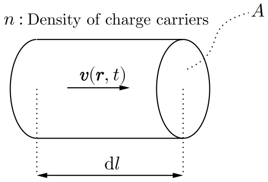

Figure 8: Illustration to derive the second important equation needed for the Kibble balance principle.

In this figure, a short element of length $\mathrm{d}l$ of a wire with charge carrier density n is shown. In principle, this wire represents any object of choice. If one assumes, that this wire or alternatively an object of choice is immersed in a magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$, a magnetic force is exerted on charge carriers moving at speed $\boldsymbol{v}(t)$ in the object. Assuming that all charge carriers move at speed $\boldsymbol{v}(t)$, the total current $I(t)$ is given by $I(t) = nAe|\boldsymbol{v}(t)$.

The electromagnetic force exerted on one electron would be given by the expression $e\boldsymbol{E}(\boldsymbol{r},t) + e\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)$; and on $nA\,\mathrm{d}l$ carriers with no external electric field ($\boldsymbol{E}(\boldsymbol{r},t) = 0$), the force differential $\mathrm{d}\boldsymbol{F}$ is given by

$$
\begin{aligned}
\mathrm{d}\boldsymbol{F} = neA\,\mathrm{d}l\,[\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)] &= I(t)\,\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}(\boldsymbol{r},t) \\
&= -I(t)\boldsymbol{B}(\boldsymbol{r},t) \times \mathrm{d}\boldsymbol{l},
\end{aligned} \tag{19}
$$

where in the last steps the definition $\mathrm{d}\boldsymbol{l} \doteq \mathrm{d}l\,\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ was used. Integrated over a closed path $\partial\Sigma(t)$ therefore one obtains the magnetic force on the object given by

$$
\boldsymbol{F}(\boldsymbol{r},t) = -I(t) \oint_{\partial\Sigma(t)} \boldsymbol{B}(\boldsymbol{r},t) \times \mathrm{d}\boldsymbol{l}. \tag{20}
$$

The quantum Hall effect can be used for resistance measurement; according to [Jeckelmann and Jeanneret, 2001], the quantum Hall resistance R_H is given by the expression

$$R_H = \frac{h}{n_H e^2}, \quad n_H \in \mathbb{N}, \tag{21}$$

where $h = 6.626\,070\,15 \times 10^{-35}$ J s is the Planck constant and e is the elementary charge.

The Josephson voltage U_J across a Josephson junction can be used for voltage measurements; according to [Kajastie et al., 2009] it is given by the equation

$$U_J = \frac{2e}{h} n_J f_J, \quad n_J \in \mathbb{N}, \tag{22}$$

where h is the Planck constant, e is the elementary charge and f_J is the frequency of the microwave radiation used to irradiate the Josephson junction with.