Building a homemade barometer

Theory and test report on building a homemade barometer

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Abstract

Barometers are devices, that measure air pressure. Usually, this is done in the field of meteorology to explore future weather predictions. Observing the atmospheric pressure evolution at some fixed point in space allows for a rough estimate of possible future weather, since a pressure drop usually is a precursor for bad, and a pressure rise an indication for good weather. This paper is concerned with development of theory coupled with a simple realization and test of a homemade barometer.

1 Introduction

Barometers are used to measure air pressure. Observing the atmospheric pressure at a fixed point in space over a given time interval allows for coarse predictions of the future weather. This is due to the observation that pressure drops are usually linked to bad weather, while pressure rises often indicate development of good weather.

High pressure air is heavier than low pressure air and thus has a tendency to sink due to buoyancy. Sinking air rises in temperature due to adiabatic and diabatic warming. Warm air can take up more water vapour and hence the relative humidity sinks. As a result, clouds tend to vanish and thus render precipitation unlikely. Low pressure air however tends to rise and thus to cool off with increasing height. Now, cold air is sooner saturated with water vapour than warm air. If air rises and cools off therefore, the air gets oversaturated with water vapour and hence clouds begin to form and precipitation becomes possible as well as likely.

As a rough estimate of the future weather helps with planning outdoor activities, it is nice to have a barometer at home to observe the pressure evolution. This paper hence aims to provide the necessary theory to build a homemade barometer using only equipment available in any hardware store. The general design of such a barometer will be provided here, whereas the details of an actual realization are left to the reader to figure out given the available tools and hardware.

2 Theoretical foundation

2.1 Background

The theoretical foundation of the proposed homemade barometer is comprised of two main equations and their theoretical background.

The first equation is the ideal gas law

$$PV = Nk_BT, \tag{1}$$

where P is the pressure of a gas, N the number of particles in the volume V, k_B the Boltzmann constant and T the temperature of the gas. There are number of assumptions processed in the derivation of this ideal gas law; but for the purpose at hand it suffices to state that air pressure in non-extreme environments on earth follows the ideal gas law very well.

The second main equation is the equation of hydrostatic pressure

$$P = \rho g h, \tag{2}$$

where P is the hydrostatic pressure of an incompressible fluid, ρ is the density of the fluid, g is the gravitational acceleration and h is the depth in the fluid, where the hydrostatic pressure P is measured.

2.2 Working principle

Consider an experimental setup as seen in fig. 1. The experimental setup consists of a U-shaped tube with the opening facing towards the sky. At the bottom of the U, a valve is placed which separates both the left part and right part of the U. The right part of the U is furthermore closable, whereas the left part remains open throughout the experiment and is therefore subject to the atmospheric pressure P_0, which is to be measured.

Initially, the red valve is closed and water is filled into the right and left part of the U. In the left end, the water is filled up to height $z_{l,0}$, whereas the water in the left tube is filled up to height $z_{r,0}$, fulfilling the condition $z_{r,0} \gg z_{l,0}$. After filling both ends of the U with water of density ρ, the right part of the U is closed, which means that at closed valve the pressure and volume of the residual air in the tube remains constant at P_0 and V_0. This situation is depicted in the left panel of fig. 1

and describes the experiment preparation required to carry out measurements.

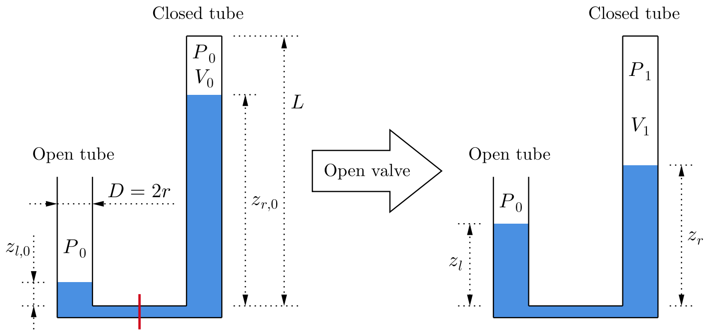


Figure 1: General working principle of the proposed homemade barometer.

Now, a measurement is carried out by first tediously preparing the experiment as described above. Then, the red valve is opened, which marks the starting point of the measurement process. Now, the water in the right part of the U is drawn to the left part of the U due to the higher hydrostatic pressure in the right part of the U. Water keeps flowing from the left part of the U to the right part until the sum of hydrostatic and atmospheric pressure in the right part of the U is exactly balanced by the sum of hydrostatic pressure and air pressure in the right (closed) part of the U. The water level in the left part of the U is now at z_l and z_r denotes the water level in the right part at equilibrium.

Mathematically speaking, the total pressure P_l at the bottom of the U in the left part is given by

$$P_l = P_0 + \rho g z_l, \tag{3}$$

where analogously the total pressure P_r at the bottom of the U in the right part is accounted for by

$$P_r = P_1 + \rho g z_r. \tag{4}$$

Now, at equilibrium, these two pressures must be equal to one another. Now, the pressure P_1 can further be expressed by means of the ideal gas law. Consider for this purpose the evolution of the residual air parcel located in the right part of the U. Initially, this air parcel exhibits pressure P_0 and a volume V_0. At equilibrium then, the pressure has dropped to P_1 due to an expanded volume $V_1 > V_0$. Considering the ideal gas law, the number of particles in the air parcel does not change from the initial condition to equilibrium, hence $N_0 = N_1 \doteq N$. Furthermore, the assumption $T_0 = T_1 \doteq T$ shall be made which means, that the temperature of the air parcel is assumed to stay constant throughout the measurement procedure. Writing out the ideal gas law for both initial and equilibrium conditions, one obtains

$$P_0 V_0 = N k_B T = P_1 V_1 \quad \Leftrightarrow \quad P_1 = P_0 \frac{V_0}{V_1}. \tag{5}$$

The volume V_1 can be expressed by means of the equation

$$\begin{aligned} V_1 &= V_0 + r^2\pi(z_{r,0} - z_r) \\ &= r^2\pi(L - z_{r,0}) + r^2\pi(z_{r,0} - z_r) \\ &= r^2\pi(L - z_r). \end{aligned} \tag{6}$$

The pressure P_1 can thus be expressed as

$$P_1 = P_0 \frac{L - z_{r,0}}{L - z_r}, \tag{7}$$

which renders the pressure P_r at the bottom of the right U-part at equilibrum as

$$P_r = P_0 \frac{L - z_{r,0}}{L - z_r} + \rho g z_r. \tag{8}$$

From the equilibrium condition $P_l \overset{!}{=} P_r$, the reasoning

$$\begin{gathered} \overbrace{P_0 + \rho g z_l}^{P_l} = \overbrace{P_0 \frac{L - z_{r,0}}{L - z_r} + \rho g z_r}^{P_r} \\ \Leftrightarrow \\ P_0 \left(1 - \frac{L - z_{r,0}}{L - z_r}\right) = \rho g (z_r - z_l) \end{gathered} \tag{9}$$

Thus, the final barometer expression $P_0(z_{r,0}, z_r, z_l, L)$ results as a function of initial water height $z_{r,0}$, equilibrium water heights z_r and z_l in both parts of the U and the total length L of the right U-part. This final equation is given by

$$P_0(z_{r,0}, z_r, z_l, L) = \rho g (z_r - z_l) \left(1 - \frac{L - z_{r,0}}{L - z_r}\right)^{-1}. \tag{10}$$

In fig. 2, a visualization of the barometer equation eq. (10) for the parameters $L = 1.99\,\mathrm{m}$, $z_{r,0} = 1.16\,\mathrm{m}$, $g = 9.81\,\mathrm{m\,s^{-2}}$ and $\rho = 1000\,\mathrm{kg\,m^{-3}}$ can be seen. As one

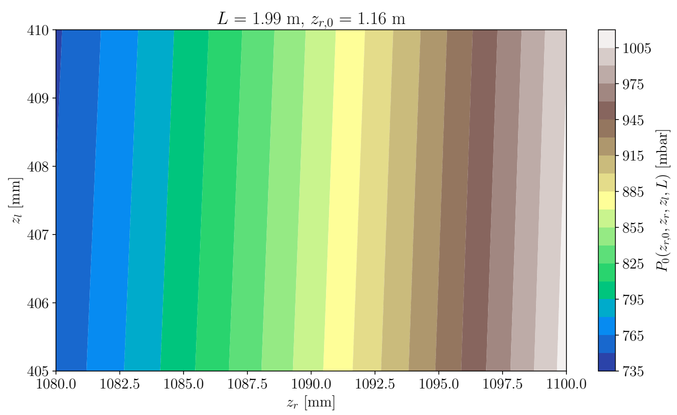


Figure 2: Visualization of the barometer equation eq. (10).

can see from this figure, the difference $\Delta z \doteq z_{r,0} - z_r$ in water level height between initial and equilibrium conditions in the right part of the U increases with dropping ambient pressure P_0. This is not surprising insofar as

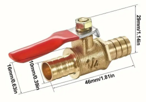
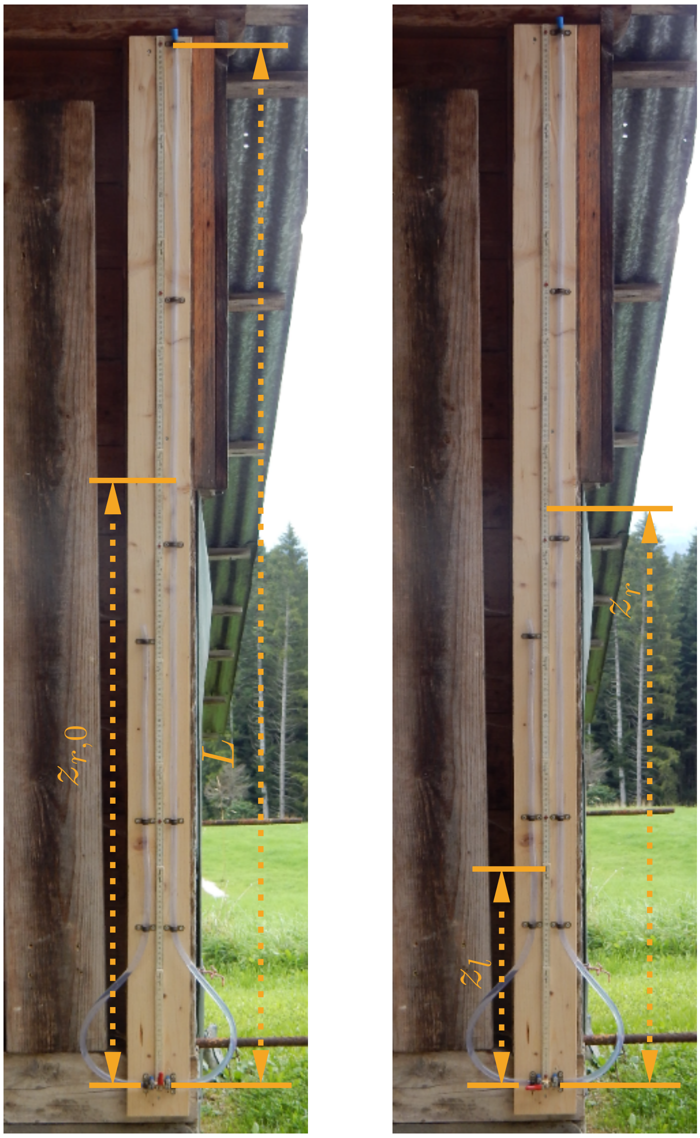
Description	Image	Link
Tube		tube-link
Valve		valve-link
Plug		plug-link

Table 1: Used materials for the homemade barometer reported in the paper at hand.



(a) Barometer during stage 1 of a measurement.

(b) Barometer during stage 2 of a measurement.

Figure 3: Pictures of the homemade barometer during both stages of a measurement procedure. At the top of the images, the blue plug used to seal the right part of the barometric U airtight, whereas on the bottom of the pictures, the red valve with the red lever can be perceived. In the left picture, the valve is closed, whereas in the right picture it is visualized in its open position.

with less ambient pressure P_0, the pressure exerted on the left water level is lower and hence the water level in the left part of the U will rise stronger as with less pressure. As a result, the water level in the right part of the U will drop lower at lower pressure, which makes the difference Δz large.

The pressure regime in a practical experiment will range somewhere between 750 mbar and 1000 mbar. Hence, following fig. 1 and fig. 2, the barometer design will feature a tube length of $L \approx 2.0$ m, an initial right water level of $z_{r,0} \approx 1.1$ m and an initial water height in the left part of the U of $z_{l,0} = 33$ mm. This setup should result in a difference Δz in the range of 60 mm to 90 mm between equilibrium and initial water level height in the right part of the barometric U.

3 Methods

3.1 Experimental apparatus

A homemade barometer was built according to the general sketch given in fig. 1. The barometric U was made from transparent plastic tubing of inner diameter $D = 10$ mm, which was attached to a wooden board of around 2 meters length using commercially available standard pipe clamps. At the bottom of the U, the valve was placed. Furthermore, a conic silicon plug of appropriate diameter was used to seal the right part of the barometric U airtight prior to measurements. The used materials are summarized in table 1 and were bought at the online shop temu.com[1].

3.2 Measurement procedure

The measurement procedure consists of two stages.

In the first stage visualized in fig. 3a, the valve is closed and water is filled into both arms of the barometric U, ensuring that $z_{l,0} \ll z_{r,0}$, such that the difference in hydrostatic pressure between both barometer arms is sufficient to be balanced by the difference in air pressure in stage 2 of the measurement. After the water has been filled into the barometric U, the top of the right part of the U is sealed by the blue plug. At this point, three measurements each for both L and $z_{r,0}$ are taken.

In the second stage of a measurement, the valve is opened, which leads to an equilibration of hydrostatic and gas pressure in both barometric arms. A sufficient equilibrium is achieved as soon as the water levels are no more observed to oscillate in height. As soon as this condition has been reached, three measurements each for both z_r and z_l are acquired.

The successful completion of both stages will lead to 12 measurement values in total, three measurements per measured parameter L, $z_{r,0}$, z_r and z_l. These values

[1]See `https://www.temu.com/`.

are then propagated through an uncertainty evaluation leading to a final value $P_0 \pm \sigma_{P_0}$ associated with an uncertainty σ_{P_0} of measurement. The uncertainty σ_{P_0} is given for a 95 percent confidence interval.

3.3 Uncertainty evaluation

The measurement uncertainty for the experiment at hand is evaluated according to the standard procedure explained in [JCGM, 2023].

4 Results

The photos depicted in fig. 3 correspond to a measurement on August 21, 2024 at 16:22 in the afternoon; fig. 4 shows the results obtained by a Monte Carlo simulation of uncertainty propagation.

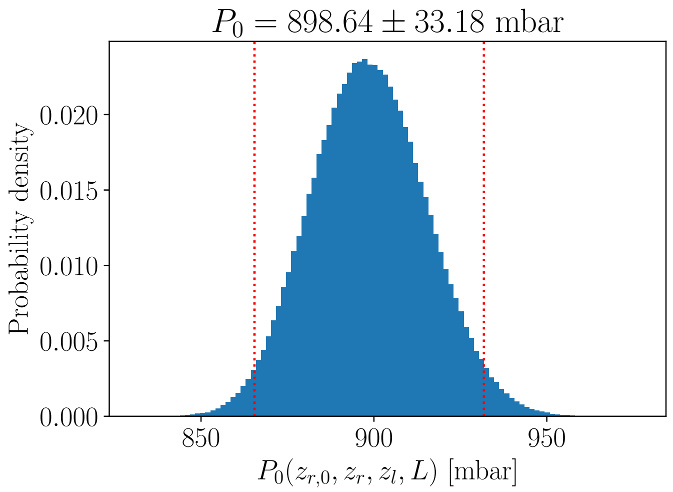


Figure 4: Measurement results associated to the measurement depicted in fig. 3 with measurement uncertainty obtained by a Monte Carlo simulation.

4.1 Uncertainty evaluation

There are four measured parameters, namely the total length L of the right part of the barometric U, the initial right water height $z_{r,0}$, the final right water height z_r and the final left water height z_l. Each of those measurements is in the units of length and measured by the same yard stick and thus subject to the same uncertainty contributions. However, some of the uncertainty contributions scale with measured length and are therefore different for each of the four parameters.

In the following material, each uncertainty contribution to each one of the four measured parameters is briefly described and explained. Details can be seen in the excel file used for the uncertainty evaluation available in the online repository[2]. The total uncertainty reported is calculated by the formula

$$\sigma_{P_0} = 2\sqrt{U_{z_{r,0}}^2 + U_{z_r}^2 + U_{z_l}^2 + U_L^2}, \tag{11}$$

with the involved quantities being explained below.

Initial right water height $z_{r,0}$ The initial right water height is subject to six considered uncertainty contributions. First of all, there is the statistical uncertainty contribution $\delta z_{r,0}$ given by the standard deviation of the three taken measurements. Second, there is an uncertainty contribution $\delta z_{r,0,rd}$ due to possibly incorrect reading of the yard stick. Third, there is an uncertainty associated to the surface tension of the water yielding a non-flat surface, given by $\delta z_{r,0,st}$. Fourth, a scaled uncertainty $\delta z_{r,0,mu}$ due to the accuracy class of the used yard stick contributes to the budged. Fifth, there is a contribution $\delta z_{r,0,or}$ due to a possible misidentification of the measurement origin down at the bottom of the barometric U. Sixth, there is a contribution $\delta z_{r,0,tilt}$ due to a possible tilt of the experimental apparatus, thus also scaling with measured length.

The total uncertainty contribution $U_{z_{r,0}}$ of the initial right water height $z_{r,0}$ is thus given by

$$U_{z_{r,0}}^2 = \left(\frac{\partial P_0(z_{r,0}, z_r, z_l, L)}{\partial z_{r,0}}\right)^2 \sum_i \delta z_{r,0,i}^2. \tag{12}$$

Final right water height z_r The final right water height z_r is subject to the same, but appropriately scaled uncertainty contributions as $z_{r,0}$. Thus, there is the statistical contribution δz_r, the contribution $\delta z_{r,rd}$ due to reading, the quantity $\delta z_{r,st}$ due to surface tension, the contribution $\delta z_{r,mu}$ given by the accuracy class of the yard stick, the uncertainty $\delta z_{r,or}$ due to possible origin misplacement and finally the contribution $\delta z_{r,tilt}$ caused by possible tilt.

The total uncertainty contribution U_{z_r} of the final right water height z_r is thus given by

$$U_{z_r}^2 = \left(\frac{\partial P_0(z_{r,0}, z_r, z_l, L)}{\partial z_r}\right)^2 \sum_i \delta z_{r,i}^2. \tag{13}$$

Final left water height z_l The final left water height z_l is subject to the same, but appropriately scaled uncertainty contributions as z_r. Therefore, there is the statistical contribution δz_l, the contribution $\delta z_{l,rd}$ due to reading, the quantity $\delta z_{l,st}$ due to surface tension, the contribution $\delta z_{l,mu}$ given by the accuracy class of the yard stick, the uncertainty $\delta z_{l,or}$ due to possible origin misplacement and finally the contribution $\delta z_{l,tilt}$ caused by possible tilt.

The total uncertainty contribution U_{z_l} of the final left water height z_l is thus given by

$$U_{z_l}^2 = \left(\frac{\partial P_0(z_{r,0}, z_r, z_l, L)}{\partial z_l}\right)^2 \sum_i \delta z_{l,i}^2. \tag{14}$$

[2]See `https://github.com/danielzahnd/various-documents/tree/main/homemade-barometer/code`.

Tube length L The tube length L is subject to the same, but appropriately scaled uncertainty contributions as z_l, except for the surface tension contribution, since with L no water level measurements are involved. Hence, there is the statistical contribution δL, the contribution δL_{rd} due to reading, the uncertainty δL_{mu} given by the accuracy class of the yard stick, the uncertainty δL_{or} due to possible origin misplacement and finally the contribution δL_{tilt} caused by possible tilt.

The total uncertainty contribution U_L of the tube length L is thus given by

$$U_L^2 = \left(\frac{\partial P_0(z_{r,0}, z_r, z_l, L)}{\partial L}\right)^2 \sum_i \delta L_i^2. \tag{15}$$

5 Discussion

Text.

6 Conclusions

Text.

A Derivation of the ideal gas law

The ideal gas law can be derived from the so-called kinetic gas theory. The kinetic gas theory is based on three postulates, which can be found in any physics textbook such as [Tipler and Mosca, 2007]:

(1) The volume of the gas under consideration contains a large number of molecules.

(2) The adjacent molecules are separated at distances much larger than their sizes.

(3) The only interactions between molecules are elastical collisions.

Given these three postulates, the so-called Maxwell-Boltzmann distribution for the absolute value v of molecules in a gas can be derived; it is given by

$$p(v) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{mv^2}{2k_B T}}, \tag{16}$$

where $p(v)$ is the probability, that the velocity v of a molecule in the gas is found to be in the interval $v \in [v, v + \mathrm{d}v]$. Furthermore, m denotes the mass of a gas molecule. Since $p(v)$ is a probaility density function, the mean squared velocity $\overline{v^2}$ of a gas molecule can be calculated by means of performing the integral

$$\overline{v^2} = \int_0^\infty v^2 p(v)\,\mathrm{d}v = \frac{3k_B T}{m}, \tag{17}$$

where k_B is known as the Boltzmann constant.

Now, in order to derive an expression for the pressure P, one can consider a cubic box of length l filled with

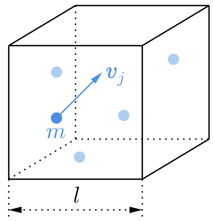


Figure 5: Illustration of a gas container.

$N \in \mathbb{N}$ gas molecules following the kinetic gas theory. Such a situation is visualized in fig. 5. Each molecule $j \in \{1, \ldots, N\}$ shall have a velocity $\boldsymbol{v}_j = (v_{j,x}, v_{j,y}, v_{j,z})^\top$ and a mass m. The pressure exerted on a container wall is hence given by the total force F generated by molecules hitting the wall divided by the surface $A = l^2$ of the wall. The force F exerted by molecules hitting a container wall is given by the change in momentum of the hitting particles divided by the interval of time, during which the change in momentum occurs; therefore, one can write down

$$F = \sum_{j=1}^N \frac{mv_{j,i}^2}{l} = \frac{1}{3}\sum_{j=1}^N \frac{mv_j^2}{l}, \tag{18}$$

where in the last step

$$\sum_{j=1}^N v_{j,x} = \sum_{j=1}^N v_{j,y} = \sum_{j=1}^N v_{j,z} = \frac{1}{3}\sum_{j=1}^N v_j^2 \tag{19}$$

with $v_j = |\boldsymbol{v}_j|$ was used, since no particular direction of movement is preferred by the gas molecules. Now, the pressure P can be calculated by the force F divided by the surface area $A = l^2$ as

$$P = \frac{F}{A} = \frac{m}{3V}\sum_{j=1}^N v_j^2. \tag{20}$$

The average kinetic energy $\frac{1}{2}m\overline{v^2}$ can be calculated by means of

$$\frac{1}{2}m\overline{v^2} = \frac{1}{2N}\sum_{j=1}^N mv_j^2, \tag{21}$$

which allows for a reformulation of the former equation as

$$PV = \frac{N}{3}m\overline{v^2} = \frac{2N}{3f}\bar{E}_f, \tag{22}$$

where $\bar{E}_f \doteq \frac{f}{2}m\overline{v^2} = f\bar{E}$ with $\bar{E} = \frac{1}{2}k_B T$ the average energy per degree of freedom f of movement was defined. Thus, $\bar{E}_f$ is the average kinetic energy of a molecule with f degrees of freedom of movement. In the case of single-atom molecules, the result eq. (17) holds and one has

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T = 3\bar{E}. \tag{23}$$

From this, $f = 3$ can be concluded for single-atom molecules, which corresponds to the expected three degrees of translational freedom. Inserting the above expressions for $\bar{E}$ and $\bar{E}_f$ into eq. (22) yields the ideal gas law

$$PV = Nk_BT. \tag{24}$$

B Derivation of hydrostatic pressure

Hydrostatic pressure is the pressure an object experiences on its surface due to a medium of different density ρ surrounding it, the whole system being subject to a gravitational field. Consider therefore for a derivation of the mathematical expression for the hydrostatic pressure fig. 6.

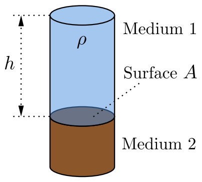


Figure 6: Illustration of an object (medium 2) experiencing hydrostatic pressure exerted on it by an other medium (medium 1) of constant density ρ.

The hydrostatic pressure P exerted on medium 2 by means of medium 1 is given by the gravitational force generated by the mass of medium 1 divided by the surface area of medium 2 facing medium 1. Hence, one can write

$$P = \frac{\rho h A g}{A} = \rho g h. \tag{25}$$

C Propagation of uncertainties

The following formulae are derived in detail in standard references such as [JCGM, 2023].

Consider $n \in \mathbb{N}$ random variables $x_1, \ldots, x_n$ and a function $f(x_1, \ldots, x_n)$ relating those variables. Let furthermore σ_{x_i} be the uncertainties (standard deviations) of the random variables x_i, which are assumed to be uncorrelated. In this case, the general propagation of uncertainties σ_{x_i} on the outcome $f(x_1, \ldots, x_n)$ is quantified by the uncertainty σ_f as

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2} = \sqrt{\sum_{i=1}^{n} c_i^2 \sigma_i^2}, \tag{26}$$

where the short-hand notation $\sigma_i \doteq \sigma_{x_i}$ and $c_i \doteq \partial f / \partial x_i$ was introduced. Note, that the c_i factors are commonly called sensitivity coefficients.

References

[JCGM, 2023] JCGM (2023). *Guide to the Expression of Uncertainty in Measurement*. Bureau International des Poids et Mesures (BIPM). JCGM 100:2008 with 2023 editorial corrections.

[Tipler and Mosca, 2007] Tipler, P. A. and Mosca, G. (2007). *Physics for Scientists and Engineers*. W. H. Freeman and Company, New York, 6th edition.