The METAS joule-watt balance

A combined approach

PhD project presentation

Daniel Zahnd

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Content

- The kilogram prototype in Paris is not stable with respect to time.
- The watt balance invented by [Kibble, 1976] provides a constant representation.
- The watt balance is nowadays named after its inventor as Kibble balance.
- The Kibble balance method is based on a comparison of electrical to mechanical power.

- From electrodynamics, the electromagnetic force

$$\boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} \quad (1)$$

can be derived.

- The current I_s is varied such that $\boldsymbol{F}_{el} = \boldsymbol{F}_g$.
- Current I_s and voltage U_s are measured.

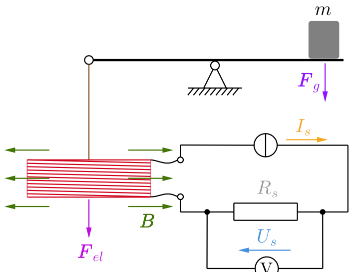

Figure 1: Static mode of the Kibble balance.

- From electrodynamics, the electromagnetic induction

$$U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \quad (2)$$

  can be derived.
- The coil is moved at velocity $\boldsymbol{v}$.
- Induced voltage U_d is measured.

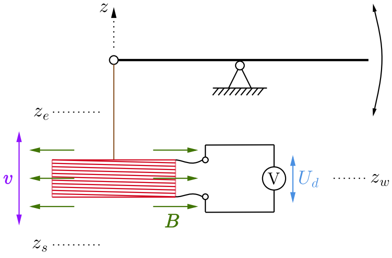

Figure 2: Dynamic mode of the Kibble balance.

- Bringing eq. (1) and eq. (2) together yields

$$\left.\begin{aligned} \boldsymbol{F}_{el} = I_s \oint \boldsymbol{B} \times \mathrm{d}\boldsymbol{l} = m\boldsymbol{g} \\ U_d = \oint (\boldsymbol{B} \times \mathrm{d}\boldsymbol{l}) \cdot \boldsymbol{v} \end{aligned}\right\} \quad m\boldsymbol{g} \cdot \boldsymbol{v} = U_d I_s. \qquad (3)$$

- The voltage U_d and the resistance I_s are related to the Planck constant h by means of the quantum Hall and Josephson effects, namely

$$U_d = C_d n_{J,d} f_{J,d} \frac{h}{2e}, \qquad I_s = C_s n_{J,s} f_{J,s} \frac{n_H e}{2}. \qquad (4)$$

- Combining eq. (3) with eq. (4), one arrives at

$$m = C\frac{f_{J,d} f_{J,s}}{gv} h. \tag{5}$$

- The variables have the following meanings:

m	Mass
C	Calibration constant
$f_{J,d}$	Josephson frequency for the dynamic voltage measurement
$f_{J,s}$	Josephson frequency for the static voltage measurement
g	Gravitational acceleration at <u>exact</u> position of the mass m
v	Vertical velocity of movement in the dynamic phase
h	Planck constant

- The idea of the proposed project is to reduce measurement uncertainty, part of which is to operate the METAS Kibble balance as a joule balance.
- Instead of comparing mechanical to electrical power, the joule balance compares mechanical to electrical work.
- The watt balance equation $mg(z)v(t) = U(t)I(z)$ has to be integrated with respect to time with $z = z(t)$.
- One arrives at

$$m \int_{z_s}^{z_e} \frac{g(z)}{I(z)} \, \mathrm{d}z = \int_{t(z_s)}^{t(z_e)} U(t) \, \mathrm{d}t \tag{6}$$

as the joule balance equation.

- Several metrology institutes around the world are operating watt or Kibble balances.
- As [Stock et al., 2023] remark, the obtained kilogram representations however do not yet agree to a satisfactory degree.
- The measurement uncertainties are not low enough to allow for independent kilogram realizations yet.
- As of today, the kilogram is defined as a weighted consensus value of all participating watt or joule balance and Avogadro experiments worldwide.

Hypotheses

Overarching hypothesis and goal

- The overarching goal of the proposed project is therefore to reduce the measurement uncertainty of the existing METAS Kibble balance.
- This reduction is aimed to be achieved by means of four connected approaches.
- The goal is to reduce the uncertainty from currently $4.3 \cdot 10^{-8}$ to $3.0 \cdot 10^{-8}$ in relative terms.
- This means that a kilogram can then be measured with an accuracy of $30\,\mu g$.

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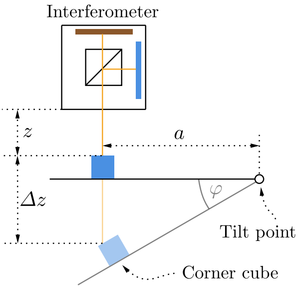

Figure 3: Visualization of a so-called Abbe offset error Δz.

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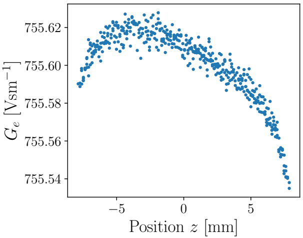

Figure 4: Example datapoints obtained for the $G_e = U_d/v$ profile.

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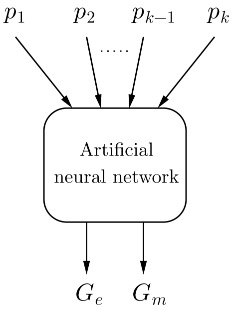

Figure 5: Proposed architecture of the neural network; $\boldsymbol{p}$ are the alignment parameters and $\boldsymbol{q}$ are the geometrical factors associated to the alignment parameters.

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Questions

Thank you for your time and attention!

Consider an electric field $\boldsymbol{E}(\boldsymbol{r},t)$, a magnetic field $\boldsymbol{B}(\boldsymbol{r},t)$, a charge density $\rho(\boldsymbol{r},t)$ and a current density $\boldsymbol{j}(\boldsymbol{r},t)$. The four so-called Maxwell equations

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \frac{\rho}{\epsilon_0} \quad\Leftrightarrow\quad \oint_{\partial V}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{S} = \frac{1}{\epsilon_0}\int_V \rho\,\mathrm{d}V \tag{7}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0 \quad\Leftrightarrow\quad \oint_{\partial V}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S} = 0 \tag{8}$$

$$\boldsymbol{\nabla}\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \quad\Leftrightarrow\quad \oint_{\partial S}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l} = -\int_S \frac{\partial\boldsymbol{B}}{\partial t}\cdot\mathrm{d}\boldsymbol{S} \tag{9}$$

$$\boldsymbol{\nabla}\times\boldsymbol{B} = \mu_0\boldsymbol{j} + \mu_0\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t} \quad\Leftrightarrow\quad \oint_{\partial S}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{l} = \mu_0\int_S \boldsymbol{j}\cdot\mathrm{d}\boldsymbol{S} + \mu_0\epsilon_0\int_S\frac{\partial\boldsymbol{E}}{\partial t}\cdot\mathrm{d}\boldsymbol{S} \tag{10}$$

govern these physical quantities.

In order to derive an expression for electromagnetic induction, it is necessary to invoke Maxwell's equations; in particular the Lorentz force equation

$$\boldsymbol{F}(\boldsymbol{r},t) = q\left[\boldsymbol{E}(\boldsymbol{r},t) + \boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)\right], \tag{11}$$

where $\boldsymbol{r}$ and t denote the position and time of evaluation; and where q is a charge probe, $\boldsymbol{v}$ is the velocity of q and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field respectively. Furthermore, the third Maxwell equation is of interest for the watt and joule balance techniques, namely

$$\boldsymbol{\nabla} \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}. \tag{12}$$

In addition to these equations, also the integral theorem of Stokes is relevant; let Σ denote an arbitrary surfae in $\mathbb{R}^3$ and let $\partial\Sigma$ denote its boundary, which in this case is a line in $\mathbb{R}^3$. If then $\boldsymbol{V}$ is an arbitrary vector field in $\mathbb{R}^3$, the integral theorem of Stokes states that

$$\int_{\Sigma} (\boldsymbol{\nabla} \times \boldsymbol{V}) \cdot \mathrm{d}\boldsymbol{S} = \oint_{\partial\Sigma} \boldsymbol{V} \cdot \mathrm{d}\boldsymbol{l}, \tag{13}$$

where $\mathrm{d}\boldsymbol{S}$ denotes the oriented surface element of Σ and $\mathrm{d}\boldsymbol{l}$ is a line element of $\partial\Sigma$. With this theorem, Maxwell's third equation eq. (12) can be transformed to the integral form

$$\oint_{\partial\Sigma} \boldsymbol{E}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{l} = -\int_{\Sigma} \frac{\partial \boldsymbol{B}(\boldsymbol{r}, t)}{\partial t} \cdot \mathrm{d}\boldsymbol{S}. \tag{14}$$

If the surface Σ becomes time-dependent $\Sigma \to \Sigma(t)$, the magnetic flux Φ_B through the surface $\Sigma(t)$ can be written as

$$\Phi_B = \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S}. \tag{15}$$

The total time derivative of the magnetic flux in turn gives the negative induced voltage $U(t)$, namely

$$U(t) = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r}, t) \cdot \mathrm{d}\boldsymbol{S}. \tag{16}$$

In order to evaluate this total time derivative, both a change in the magnetic field aswell as in the surface element have to be considered. The change in the surface element $\mathrm{d}\boldsymbol{S}$ with time t can be written as $\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{v} \times \mathrm{d}\boldsymbol{l}$, as fig. 6 indicates.

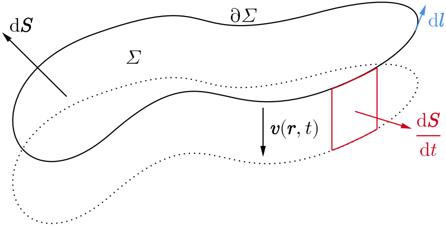

Figure 6: Illustration for the derivation of the expression for the change in the surface element $\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{v} \times \mathrm{d}\boldsymbol{l}$.

With this knowledge, one can differentiate both $\boldsymbol{B}(\boldsymbol{r},t)$ and $\mathrm{d}\boldsymbol{S}$ in the integrand to arrive at

$$\begin{aligned}\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} &= \frac{\mathrm{d}}{\mathrm{d}t}\int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r},t)\cdot\mathrm{d}\boldsymbol{S} \\ &= \int_{\Sigma(t)} \frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}\cdot\mathrm{d}\boldsymbol{S} + \int_{\Sigma(t)} \boldsymbol{B}(\boldsymbol{r},t)\cdot\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} \\ &= -\oint_{\partial\Sigma(t)} \boldsymbol{E}(\boldsymbol{r},t)\cdot\mathrm{d}\boldsymbol{l} + \oint_{\partial\Sigma(t)} \boldsymbol{B}(\boldsymbol{r},t)\cdot(\boldsymbol{v}(t)\times\mathrm{d}\boldsymbol{l}) \\ &= -\oint_{\partial\Sigma(t)} \left[\boldsymbol{E}(\boldsymbol{r},t)+\boldsymbol{v}(t)\times\boldsymbol{B}(\boldsymbol{r},t)\right]\cdot\mathrm{d}\boldsymbol{l} = -U(t).\end{aligned} \tag{17}$$

In the case where the external magnetic field is zero, i.e. $\boldsymbol{E}(\boldsymbol{r}, t) = 0$, this equation reduces to

$$U(t) = \oint_{\partial\Sigma(t)} [\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r}, t)] \cdot \mathrm{d}\boldsymbol{l} \tag{18}$$

and is herewith the first of the fundamental equations used for the Kibble balance experiment.

A second important equation is derived from the Lorentz force equation. Consider for this purpose fig. 7.

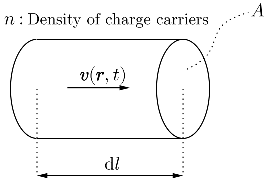

Figure 7: Illustration to derive the second important equation needed for the Kibble balance principle.

In this figure, a short element of length $\mathrm{d}l$ of a wire with charge carrier density n is shown. In principle, this wire represents any object of choice. If one assumes, that this wire or alternatively an object of choice is immersed in a magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$, a magnetic force is exerted on charge carriers moving at speed $\boldsymbol{v}(t)$ in the object. Assuming that all charge carriers move at speed $\boldsymbol{v}(t)$, the total current $I(t)$ is given by $I(t) = nAe|\boldsymbol{v}(t)$.

The electromagnetic force exerted on one electron would be given by the expression $e\boldsymbol{E}(\boldsymbol{r},t) + e\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)$; and on $nA\,\mathrm{d}l$ carriers with no external electric field ($\boldsymbol{E}(\boldsymbol{r},t) = 0$), the force differential $\mathrm{d}\boldsymbol{F}$ is given by

$$\begin{aligned} \mathrm{d}\boldsymbol{F} = neA\,\mathrm{d}l\,[\boldsymbol{v}(t) \times \boldsymbol{B}(\boldsymbol{r},t)] &= I(t)\,\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}(\boldsymbol{r},t) \\ &= -I(t)\boldsymbol{B}(\boldsymbol{r},t) \times \mathrm{d}\boldsymbol{l}, \end{aligned} \tag{19}$$

where in the last steps the definition $\mathrm{d}\boldsymbol{l} \doteq \mathrm{d}l\,\frac{\boldsymbol{v}}{|\boldsymbol{v}|}$ was used. Integrated over a closed path $\partial\Sigma(t)$ therefore one obtains the magnetic force on the object given by

$$\boldsymbol{F}(\boldsymbol{r},t) = -I(t) \oint_{\partial\Sigma(t)} \boldsymbol{B}(\boldsymbol{r},t) \times \mathrm{d}\boldsymbol{l}. \tag{20}$$

The quantum Hall effect can be used for resistance measurement; according to [Jeckelmann and Jeanneret, 2001], the quantum Hall resistance R_H is given by the expression

$$R_H = \frac{h}{n_H e^2}, \quad n_H \in \mathbb{N}, \tag{21}$$

where $h = 6.626\,070\,15 \times 10^{-35}$ J s is the Planck constant and e is the elementary charge.

The Josephson voltage U_J across a Josephson junction can be used for voltage measurements; according to [Kajastie et al., 2009] it is given by the equation

$$U_J = \frac{2e}{h} n_J f_J, \quad n_J \in \mathbb{N}, \tag{22}$$

where h is the Planck constant, e is the elementary charge and f_J is the frequency of the microwave radiation used to irradiate the Josephson junction with.