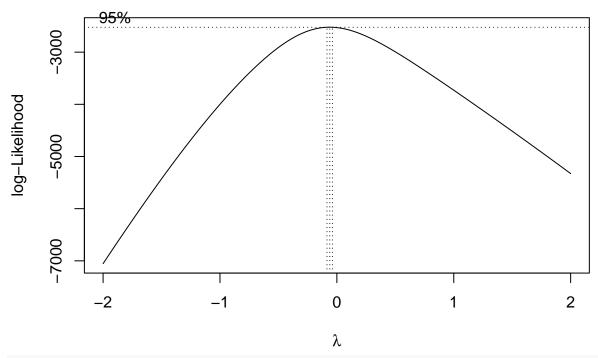
```
library(car)
## Loading required package: carData
library(reshape2)
library(ggplot2)
library(MASS)
library(interactions)
source("clean_data.R")
df <- remove_cols(df, c("Color", "Model"))</pre>
# Remove columns with only one observation and affected rows
res <- convert_categorical(df, categorical)</pre>
design <- as.data.frame(res$dummy)</pre>
singles <- c()
bad_idx <- c()</pre>
for (col in colnames(design)) {
 if (sum(design[, col] != 0) <= 1) {</pre>
    singles <- c(singles, col)</pre>
    bad_idx <- c(bad_idx, which(design[, col] != 0))</pre>
  }
}
singles
## [1] "MakeFiat"
                                                           "Fuel.TypePetrol + CNG"
                                  "MakeLexus"
## [4] "LocationDak. Kannada" "LocationFaizabad"
                                                           "LocationGorakhpur"
## [7] "LocationPurnea"
                                                           "LocationRudrapur"
                                 "LocationRohtak"
## [10] "LocationSamastipur" "LocationValsad"
bad_idx
## [1] 662 1009 1169 1077 779 569 728 245 510 264 162
df <- df[-bad_idx, ]</pre>
```

Box Cox

```
bc <- boxcox(Price ~., data = df)</pre>
```



bc\$x[which.max(bc\$y)]

[1] -0.06060606

For the sake of interpretability, we will use a log transformation, since $\lambda \approx 0$.

Iteratively Remove Multicollinear Regressors

```
x \leftarrow df
model \leftarrow lm(log(Price) \sim ., data = x)
removed <- c()
finished <- F
while(!finished) {
  temp <- car::vif(model)[, "GVIF^(1/(2*Df))"]</pre>
  worst <- names(which.max(temp))</pre>
  if (length(temp) > 0 && temp[worst] > sqrt(5)) {
    x <- remove_cols(x, c(worst))</pre>
    model <- lm(log(Price) ~., data = x)</pre>
    removed <- c(removed, worst)</pre>
  } else {
    finished \leftarrow T
}
removed
                                                        "Max.Power.Value"
## [1] "Max.Torque.Value"
                                "Max.Power.RPM"
## [4] "Engine"
                                "Fuel.Tank.Capacity" "Height"
which(abs(rstudent(model)) > 4)
```

```
## 39 268 345 1071 1171
```

After inspecting these data points, we do not find a good reason to remove them (i.e., they are not clerical errors).

Add Interactions

We consider only numerical interactions due to data sparsity; including categorical values produces NA's for most interactions. We do not expect kilometers driven to interact with torque or width, so those interactions are not considered.

names (model \$coefficients) [names (model \$coefficients) %in% numerical] ## [1] "Year" "Kilometer" "Max.Torque.RPM" "Length" ## [5] "Width" model <- lm(log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM + Year:Length + Year:Width + Max.Torque.RPM:Length + Max.Torque.RPM:Width + Length: Width, data = x) anova(model) ## Analysis of Variance Table ## ## Response: log(Price) ## Df Sum Sq Mean Sq F value Pr(>F)## Make 25 354.44 14.178 482.1594 < 2.2e-16 *** ## Year 1 178.52 178.525 6071.3552 < 2.2e-16 *** ## Kilometer 1 0.98 0.982 33.3800 9.430e-09 *** ## Fuel.Type 2 19.23 9.615 326.9979 < 2.2e-16 *** ## Transmission 1 20.31 20.313 690.7982 < 2.2e-16 *** 63 16.76 ## Location 0.266 9.0458 < 2.2e-16 *** ## Owner 3 0.46 0.153 5.2143 0.001397 ** ## Seller.Type 2 0.33 0.165 5.6238 0.003697 ** ## Max.Torque.RPM 1 1.67 1.669 56.7628 9.050e-14 *** ## Drivetrain 2 5.81 98.7875 < 2.2e-16 *** 2.905 1 51.72 51.724 1759.0668 < 2.2e-16 *** ## Length ## Width 5.06 5.063 172.1998 < 2.2e-16 *** ## Seating.Capacity 0.02 0.6507 0.419991 1 0.019 ## Year:Kilometer 1 1.10 1.100 37.4187 1.251e-09 *** ## Year:Max.Torque.RPM 0.99 0.993 33.7868 7.690e-09 *** 1 ## Year:Length 1 2.04 2.038 69.2926 < 2.2e-16 *** ## Year:Width 0.08 2.5959 0.107377 1 0.076 ## Max.Torque.RPM:Length 1 0.28 0.276 9.3872 0.002229 ** ## Max.Torque.RPM:Width 1 0.01 0.012 0.3960 0.529287 ## Length:Width 1 0.01 0.014 0.4748 0.490893 ## Residuals 1329 39.08 0.029 ## ---

Interactions with width don't seem to be significant; remove them.

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

```
model <- lm(
 log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM
                  + Year:Length + Max.Torque.RPM:Length,
 data = x
)
anova(model)
## Analysis of Variance Table
## Response: log(Price)
##
                          Df Sum Sq Mean Sq
                                              F value
                                                         Pr(>F)
## Make
                          25 354.44 14.178 481.9160 < 2.2e-16 ***
## Year
                           1 178.52 178.525 6068.2914 < 2.2e-16 ***
## Kilometer
                           1 0.98
                                      0.982
                                              33.3632 9.505e-09 ***
## Fuel.Type
                           2 19.23
                                      9.615 326.8329 < 2.2e-16 ***
## Transmission
                           1 20.31 20.313 690.4496 < 2.2e-16 ***
## Location
                          63 16.76
                                     0.266
                                               9.0412 < 2.2e-16 ***
## Owner
                           3
                              0.46
                                      0.153
                                               5.2117 0.001402 **
## Seller.Type
                           2 0.33
                                               5.6210 0.003708 **
                                      0.165
                           1 1.67
                                              56.7341 9.165e-14 ***
## Max.Torque.RPM
                                      1.669
                           2 5.81
                                              98.7377 < 2.2e-16 ***
## Drivetrain
                                      2.905
## Length
                           1 51.72 51.724 1758.1791 < 2.2e-16 ***
## Width
                           1 5.06
                                      5.063 172.1129 < 2.2e-16 ***
## Seating.Capacity
                           1 0.02
                                      0.019
                                              0.6504 0.420108
## Year:Kilometer
                           1 1.10
                                              37.3998 1.262e-09 ***
                                      1.100
## Year:Max.Torque.RPM
                           1 0.99
                                      0.993
                                              33.7697 7.752e-09 ***
## Year:Length
                           1
                               2.04
                                      2.038
                                              69.2576 < 2.2e-16 ***
## Max.Torque.RPM:Length
                           1
                               0.27
                                      0.270
                                               9.1768 0.002498 **
## Residuals
                        1332 39.19
                                      0.029
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Seems reasonable now. We will double check our assumptions
vif(model, type = "predictor")
## GVIFs computed for predictors
##
                         GVIF Df GVIF^(1/(2*Df))
## Make
                   119.109068 25
                                        1.100320
## Year
                    75.935730 8
                                        1.310774
## Kilometer
                   330.635985 3
                                        2.629612
## Fuel.Type
                     5.334325 2
                                        1.519742
## Transmission
                     1.851632 1
                                        1.360747
## Location
                    16.448508 63
                                        1.022473
## Owner
                    1.673586 3
                                        1.089619
## Seller.Type
                     1.341372 2
                                        1.076186
## Max.Torque.RPM
                  207.518246 6
                                        1.559869
## Drivetrain
                     4.003689 2
                                        1.414539
## Length
                   207.518246 6
                                        1.559869
## Width
                                        2.150498
                     4.624641 1
## Seating.Capacity 2.882982 1
                                        1.697934
##
                                      Interacts With
```

Make

```
## Kilometer
                                                   Year
## Fuel.Type
## Transmission
## Location
## Owner
## Seller.Type
                                          Year, Length
## Max.Torque.RPM
## Drivetrain
## Length
                                  Year, Max.Torque.RPM
## Width
## Seating.Capacity
## Make
                    Year, Kilometer, Fuel. Type, Transmission, Location, Owner, Seller. Type, Max. Torque.
## Year
                                                         Make, Fuel. Type, Transmission, Location, Owner,
## Kilometer
                                Make, Fuel. Type, Transmission, Location, Owner, Seller. Type, Max. Torque.
## Fuel.Type
                         Make, Year, Kilometer, Transmission, Location, Owner, Seller. Type, Max. Torque.
                             Make, Year, Kilometer, Fuel. Type, Location, Owner, Seller. Type, Max. Torque.
## Transmission
                        Make, Year, Kilometer, Fuel. Type, Transmission, Owner, Seller. Type, Max. Torque.
## Location
## Owner
                     Make, Year, Kilometer, Fuel. Type, Transmission, Location, Seller. Type, Max. Torque.
## Seller.Type
                           Make, Year, Kilometer, Fuel. Type, Transmission, Location, Owner, Max. Torque.
## Max.Torque.RPM
                                             Make, Kilometer, Fuel. Type, Transmission, Location, Owner,
## Drivetrain
                          Make, Year, Kilometer, Fuel. Type, Transmission, Location, Owner, Seller. Type,
                                             Make, Kilometer, Fuel. Type, Transmission, Location, Owner,
## Length
## Width
                     Make, Year, Kilometer, Fuel. Type, Transmission, Location, Owner, Seller. Type, Max.
## Seating.Capacity
                                 Make, Year, Kilometer, Fuel. Type, Transmission, Location, Owner, Seller
```

We get that $GVIF^{(1/(2*Df))} > 2.236068 \approx \sqrt{5}$, for "Kilometer," but this is known to be important is does not exceed the threshold by a lot, so we keep it.

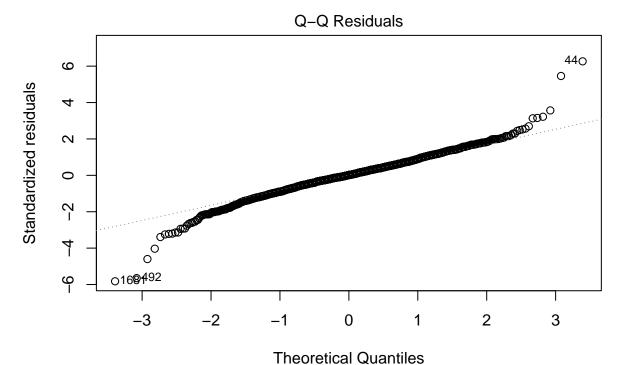
Kilometer, Max. Torque. RPM, Length

plot(model)

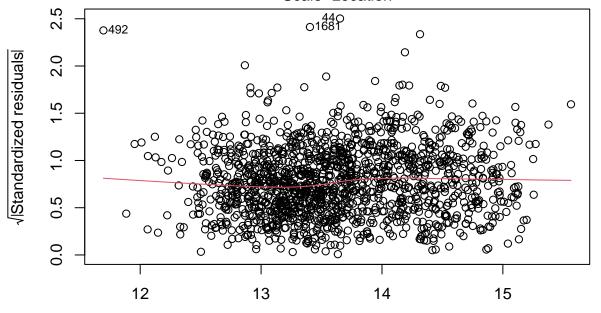
Year

Residuals vs Fitted 0 440 0 2 o. Residuals 0.0 S ġ. 0 0492 01681 12 13 14 15

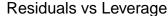
Fitted values
Im(log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM + Year:Length + Ma ...

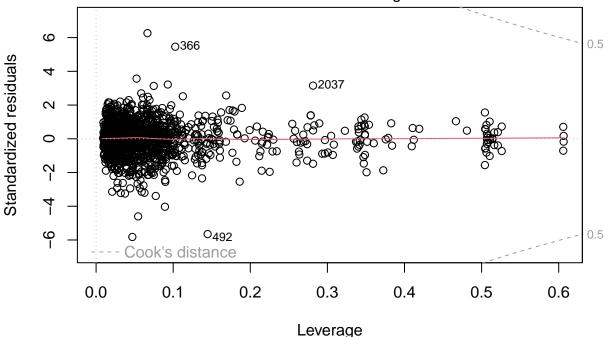


Im(log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM + Year:Length + Ma ... Scale-Location



Fitted values Im(log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM + Year:Length + Ma ...





Im(log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM + Year:Length + Ma ...

Backwards Stepwise Search

We do a backwards search since our model already conforms to the linear assumptions and is performing well. We simply wish to reduce the model size now.

```
reduced_model <- step(model, direction = "backward", data = x, trace = 0)
old <- names(model$coefficients)
new <- names(reduced_model$coefficients)
old[!(old %in% new)]</pre>
```

```
## [1] "Seller.TypeCorporate" "Seller.TypeIndividual" "Seating.Capacity"
```

Inspect Model Coefficients

```
summary(reduced_model)
##
## Call:
  lm(formula = log(Price) ~ Make + Year + Kilometer + Fuel.Type +
##
##
       Transmission + Location + Owner + Max. Torque. RPM + Drivetrain +
##
       Length + Width + Year:Kilometer + Year:Max.Torque.RPM + Year:Length +
##
       Max.Torque.RPM:Length, data = x)
##
## Residuals:
        Min
##
                  1Q
                       Median
                                     3Q
                                             Max
   -0.96723 -0.09029 0.00315 0.09727
##
## Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
##
```

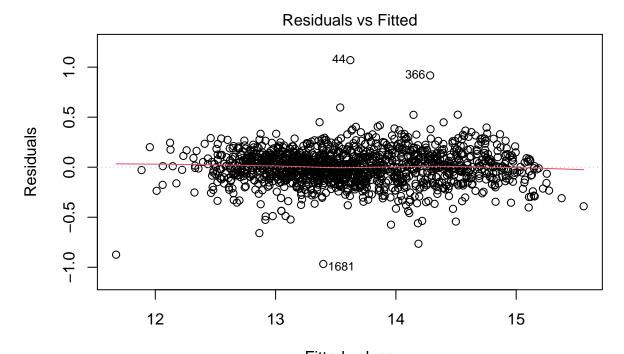
```
## (Intercept)
                           1.158e+02
                                      4.324e+01
                                                  2.678 0.007494 **
## MakeBMW
                           1.267e-02
                                      4.002e-02
                                                  0.317 0.751541
## MakeChevrolet
                                      9.078e-02
                                                         < 2e-16 ***
                          -7.982e-01
                                                 -8.793
## MakeDatsun
                          -8.571e-01
                                      6.913e-02 -12.399
                                                          < 2e-16 ***
## MakeFord
                          -5.727e-01
                                      3.836e-02 -14.932
                                                          < 2e-16 ***
## MakeHonda
                                      3.120e-02 -15.573
                          -4.859e-01
                                                         < 2e-16 ***
## MakeHvundai
                          -4.650e-01
                                      2.736e-02 -16.996 < 2e-16 ***
## MakeIsuzu
                          -4.377e-01
                                      1.275e-01
                                                 -3.433 0.000616 ***
  MakeJaguar
                           6.590e-02
                                      7.679e-02
                                                  0.858 0.390958
## MakeJeep
                          -3.336e-01
                                      5.056e-02
                                                 -6.598 6.01e-11 ***
## MakeKia
                          -4.155e-01
                                      4.444e-02
                                                 -9.350
                                                         < 2e-16 ***
## MakeLand Rover
                                                  1.761 0.078472 .
                           1.808e-01
                                      1.027e-01
## MakeMahindra
                          -5.963e-01
                                      3.300e-02 -18.072
                                                         < 2e-16 ***
## MakeMaruti Suzuki
                                                         < 2e-16 ***
                          -5.232e-01
                                      2.813e-02 -18.599
## MakeMercedes-Benz
                           1.454e-01
                                      3.286e-02
                                                  4.426 1.04e-05 ***
## MakeMG
                          -5.963e-01
                                      5.491e-02 -10.861
                                                         < 2e-16 ***
## MakeMINI
                          7.041e-01
                                      7.573e-02
                                                  9.297
                                                          < 2e-16 ***
## MakeMitsubishi
                          -2.496e-01
                                      1.123e-01
                                                 -2.223 0.026365 *
## MakeNissan
                          -7.334e-01
                                      5.153e-02 -14.232
                                                         < 2e-16 ***
## MakeRenault
                          -7.493e-01
                                      3.845e-02 -19.484
                                                         < 2e-16 ***
## MakeSkoda
                          -5.091e-01
                                      3.729e-02 -13.652
                                                         < 2e-16 ***
## MakeSsangyong
                          -8.725e-01
                                      1.130e-01
                                                 -7.719 2.29e-14 ***
## MakeTata
                                      3.690e-02 -19.485
                          -7.190e-01
                                                         < 2e-16 ***
## MakeToyota
                                      2.997e-02
                                                 -8.304 2.44e-16 ***
                          -2.488e-01
                          -5.183e-01
## MakeVolkswagen
                                      3.755e-02 -13.803 < 2e-16 ***
## MakeVolvo
                          3.094e-02
                                      5.612e-02
                                                  0.551 0.581532
## Year
                          -5.295e-02
                                      2.144e-02
                                                 -2.470 0.013648 *
## Kilometer
                          -4.525e-04
                                      1.205e-04
                                                 -3.755 0.000181 ***
                                      3.761e-02
## Fuel.TypeDiesel
                          -1.647e-01
                                                 -4.378 1.29e-05 ***
## Fuel.TypePetrol
                          -1.895e-01
                                      3.407e-02 -5.561 3.24e-08 ***
## TransmissionManual
                          -1.439e-01
                                      1.264e-02 -11.384
                                                         < 2e-16 ***
## LocationAhmedabad
                           6.225e-02
                                      4.846e-02
                                                  1.284 0.199240
## LocationAllahabad
                          -9.060e-02
                                      1.081e-01
                                                 -0.838 0.401922
## LocationAmbala Cantt
                         -9.213e-02
                                      6.973e-02
                                                 -1.321 0.186672
## LocationAmritsar
                           1.218e-01
                                      9.740e-02
                                                   1.251 0.211326
                                      1.082e-01
                                                  2.710 0.006818 **
## LocationAurangabad
                           2.931e-01
## LocationBangalore
                           2.465e-01
                                      4.583e-02
                                                  5.379 8.84e-08 ***
## LocationBhopal
                           2.905e-02
                                      1.085e-01
                                                  0.268 0.788928
## LocationBhubaneswar
                           8.012e-02
                                      1.126e-01
                                                   0.711 0.476905
## LocationBulandshahar
                                                 -2.392 0.016889 *
                          -3.137e-01
                                      1.311e-01
## LocationChandigarh
                           3.183e-02
                                      6.192e-02
                                                   0.514 0.607355
## LocationChennai
                           2.161e-01
                                      5.086e-02
                                                  4.248 2.30e-05 ***
## LocationCoimbatore
                           2.694e-01
                                      5.931e-02
                                                  4.542 6.07e-06 ***
## LocationDehradun
                           5.931e-02
                                      5.817e-02
                                                  1.019 0.308151
## LocationDelhi
                           7.161e-02
                                      4.396e-02
                                                  1.629 0.103563
## LocationDharwad
                                      1.289e-01
                                                   1.609 0.107889
                           2.073e-01
## LocationErnakulam
                           2.090e-01
                                      1.296e-01
                                                   1.613 0.106954
## LocationFaridabad
                          -6.976e-02
                                      5.634e-02
                                                 -1.238 0.215806
## LocationGhaziabad
                          -2.308e-02
                                      1.298e-01
                                                 -0.178 0.858964
## LocationGoa
                           2.481e-01
                                      9.789e-02
                                                  2.534 0.011380
                                      5.621e-02
## LocationGurgaon
                          -1.028e-02
                                                 -0.183 0.854956
## LocationGuwahati
                           1.672e-01
                                      8.915e-02
                                                  1.875 0.060970
## LocationHaldwani
                          -1.095e-02 1.292e-01
                                                 -0.085 0.932493
## LocationHyderabad
                           1.855e-01 4.650e-02
                                                  3.990 6.97e-05 ***
```

```
## LocationIndore
                           2.097e-01
                                      9.657e-02
                                                   2.172 0.030046 *
                                                   2.247 0.024785 *
## LocationJaipur
                           1.230e-01
                                      5.472e-02
                                                   3.143 0.001708 **
## LocationJalandhar
                           1.812e-01
                                      5.766e-02
## LocationJamshedpur
                          -1.725e-02
                                                  -0.192 0.847533
                                      8.972e-02
## LocationKanpur
                          -8.606e-02
                                      5.155e-02
                                                  -1.669 0.095280
## LocationKarnal
                           9.667e-02
                                      7.439e-02
                                                   1.299 0.194010
## LocationKharar
                           5.630e-02
                                      1.293e-01
                                                   0.435 0.663336
## LocationKheda
                          -8.822e-02
                                      1.289e-01
                                                  -0.684 0.493901
## LocationKolkata
                          -7.600e-02
                                      4.854e-02
                                                  -1.566 0.117611
## LocationKollam
                           2.268e-01
                                      1.292e-01
                                                   1.755 0.079410
## LocationKota
                           1.171e-01
                                      1.409e-01
                                                   0.831 0.405932
## LocationLucknow
                           2.163e-02
                                      4.870e-02
                                                   0.444 0.656920
## LocationLudhiana
                           4.327e-02
                                      5.132e-02
                                                   0.843 0.399288
## LocationMangalore
                           5.662e-02
                                      9.648e-02
                                                   0.587 0.557384
## LocationMeerut
                           3.854e-02
                                      7.468e-02
                                                   0.516 0.605902
## LocationMirzapur
                           9.829e-02
                                      1.402e-01
                                                   0.701 0.483506
## LocationMohali
                                      5.559e-02
                                                   2.086 0.037195 *
                           1.159e-01
## LocationMumbai
                           1.007e-01
                                      4.387e-02
                                                   2.295 0.021871 *
## LocationMuzaffurpur
                          -6.818e-02
                                      1.290e-01
                                                  -0.528 0.597330
## LocationMysore
                           3.209e-01
                                      7.812e-02
                                                   4.107 4.25e-05 ***
## LocationNagpur
                           1.676e-01
                                      1.081e-01
                                                   1.550 0.121461
## LocationNashik
                                      7.787e-02
                           5.820e-02
                                                   0.747 0.454960
## LocationNavi Mumbai
                                                   1.464 0.143490
                           9.202e-02
                                      6.287e-02
## LocationNoida
                           9.597e-02
                                      6.413e-02
                                                   1.497 0.134746
## LocationPanchkula
                          -2.740e-02
                                      1.079e-01
                                                  -0.254 0.799604
## LocationPatna
                           6.216e-02
                                      5.200e-02
                                                   1.195 0.232170
## LocationPune
                           1.310e-01
                                      4.601e-02
                                                   2.848 0.004468 **
## LocationRaipur
                           9.927e-02
                                      6.040e-02
                                                   1.644 0.100513
## LocationRanchi
                          -4.610e-02
                                      5.965e-02
                                                  -0.773 0.439798
## LocationRanga Reddy
                           6.674e-02
                                      1.300e-01
                                                   0.514 0.607655
## LocationRoorkee
                           4.763e-02
                                      1.098e-01
                                                   0.434 0.664466
## LocationSalem
                           4.209e-01
                                      1.290e-01
                                                   3.264 0.001128 **
## LocationSurat
                           4.111e-02
                                      8.797e-02
                                                   0.467 0.640325
## LocationThane
                           1.021e-01
                                      6.216e-02
                                                   1.642 0.100858
## LocationUdupi
                           1.480e-01
                                      7.408e-02
                                                   1.998 0.045881
## LocationVadodara
                          -9.552e-03
                                      1.081e-01
                                                  -0.088 0.929598
## LocationVaranasi
                          -4.888e-02
                                      6.047e-02
                                                 -0.808 0.419060
## LocationWarangal
                                      1.290e-01
                                                  -0.490 0.624449
                          -6.318e-02
## LocationYamunanagar
                                      1.088e-01
                                                   0.212 0.832520
                           2.302e-02
## LocationZirakpur
                          -2.603e-02
                                      6.193e-02
                                                  -0.420 0.674326
## OwnerSecond
                          -2.011e-02
                                      1.340e-02
                                                  -1.501 0.133607
## OwnerThird
                                                  -2.488 0.012972 *
                          -1.212e-01
                                      4.872e-02
## OwnerUnRegistered Car
                          8.222e-02
                                      9.193e-02
                                                   0.894 0.371321
## Max.Torque.RPM
                                                   3.661 0.000261 ***
                           1.067e-02
                                      2.914e-03
## DrivetrainFWD
                          -2.637e-01
                                      2.231e-02 -11.820
                                                         < 2e-16 ***
## DrivetrainRWD
                          -2.210e-01
                                      2.694e-02
                                                  -8.204 5.44e-16 ***
## Length
                          -7.860e-02
                                      9.661e-03
                                                  -8.136 9.26e-16 ***
## Width
                           1.503e-03
                                      1.077e-04
                                                  13.951
                                                         < 2e-16 ***
## Year:Kilometer
                           2.237e-07
                                      5.974e-08
                                                   3.745 0.000188 ***
## Year:Max.Torque.RPM
                          -5.232e-06
                                      1.441e-06
                                                  -3.631 0.000293 ***
                                                   8.212 5.07e-16 ***
## Year:Length
                           3.933e-05
                                      4.789e-06
## Max.Torque.RPM:Length -3.811e-08
                                      1.344e-08
                                                 -2.834 0.004661 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

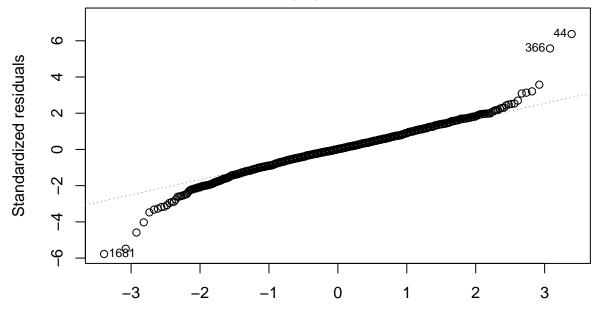
```
##
## Residual standard error: 0.1715 on 1335 degrees of freedom
## Multiple R-squared: 0.9439, Adjusted R-squared: 0.9394
## F-statistic: 213.7 on 105 and 1335 DF, p-value: < 2.2e-16
anova(reduced_model)
## Analysis of Variance Table
##
## Response: log(Price)
##
                         Df Sum Sq Mean Sq
                                          F value
                                                     Pr(>F)
## Make
                         25 354.44 14.178 482.3002 < 2.2e-16 ***
## Year
                         1 178.52 178.525 6073.1284 < 2.2e-16 ***
## Kilometer
                         1 0.98 0.982
                                          33.3898 9.375e-09 ***
                         2 19.23
                                    9.615 327.0934 < 2.2e-16 ***
## Fuel.Type
## Transmission
                        1 20.31 20.313 691.0000 < 2.2e-16 ***
## Location
                         63 16.76 0.266 9.0484 < 2.2e-16 ***
## Owner
                        3 0.46
                                    0.153
                                          5.2159 0.001394 **
## Max.Torque.RPM
                         1
                            1.73
                                    1.728
                                           58.7925 3.357e-14 ***
## Drivetrain
                        2 5.72
                                   2.859
                                          97.2752 < 2.2e-16 ***
## Length
                        1 52.04 52.042 1770.3899 < 2.2e-16 ***
## Width
                        1 5.07
                                   5.072 172.5249 < 2.2e-16 ***
                         1 1.12
## Year:Kilometer
                                    1.119
                                           38.0572 9.091e-10 ***
## Year:Max.Torque.RPM
                       1 0.95
                                    0.953
                                           32.4200 1.525e-08 ***
## Year:Length
                        1 2.09
                                    2.092 71.1609 < 2.2e-16 ***
## Max.Torque.RPM:Length
                       1 0.24
                                    0.236
                                           8.0336 0.004661 **
## Residuals
                      1335 39.24
                                   0.029
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Final Model Verification

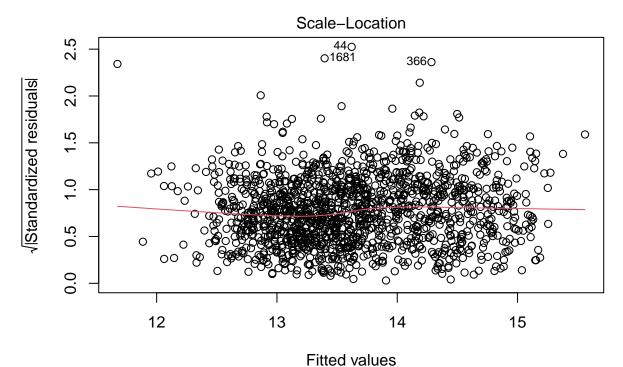
```
plot(reduced_model)
```



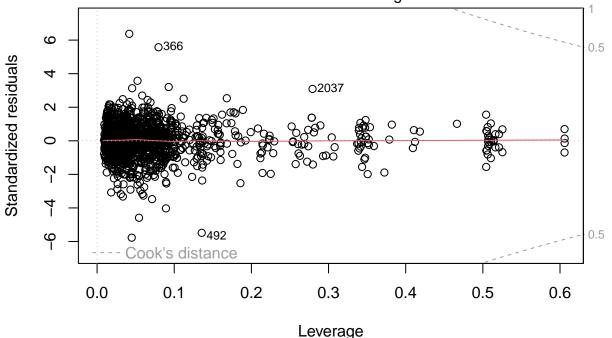
Fitted values
Im(log(Price) ~ Make + Year + Kilometer + Fuel.Type + Transmission + Locati ...
Q-Q Residuals



Theoretical Quantiles
Im(log(Price) ~ Make + Year + Kilometer + Fuel.Type + Transmission + Locati ...



Im(log(Price) ~ Make + Year + Kilometer + Fuel.Type + Transmission + Locati ...
Residuals vs Leverage



Im(log(Price) ~ Make + Year + Kilometer + Fuel.Type + Transmission + Locati ...

Normalized Coefficients

```
normalized <- remove_cols(x, old[!(old %in% new)]) # removed from multicollinearity test
normalized <- remove_cols(x, c("Seating.Capacity", "Seller.Type"))</pre>
```

```
for (col in colnames(normalized)) {
  if (col != "Price" && !(col %in% categorical)) {
    normalized[, col] <- (normalized[, col] - mean(normalized[, col])) / sd(normalized[, col])</pre>
  }
}
normalized_model <- lm(log(Price) ~ . + Year:Kilometer + Year:Max.Torque.RPM</pre>
                    + Year:Length + Max.Torque.RPM:Length, data = normalized)
sort(abs(normalized_model$coefficients), decreasing = T)
##
              (Intercept)
                                   MakeSsangyong
                                                             MakeDatsun
##
            14.479239336
                                     0.872512749
                                                            0.857115032
##
           MakeChevrolet
                                     MakeRenault
                                                             MakeNissan
##
             0.798238688
                                     0.749256440
                                                            0.733447399
##
                 MakeTata
                                        MakeMINI
                                                                  MakeMG
##
             0.718998608
                                     0.704054157
                                                            0.596345412
##
            MakeMahindra
                                        MakeFord
                                                      MakeMaruti Suzuki
##
             0.596297888
                                     0.572746578
                                                            0.523222676
##
          MakeVolkswagen
                                       MakeSkoda
                                                              MakeHonda
##
             0.518324656
                                     0.509078901
                                                            0.485869081
             MakeHyundai
                                                          LocationSalem
##
                                       MakeIsuzu
             0.465042221
##
                                     0.437700433
                                                            0.420870279
##
                  MakeKia
                                        MakeJeep
                                                         LocationMysore
##
             0.415501427
                                     0.333574675
                                                            0.320886296
##
                     Year
                           LocationBulandshahar
                                                     LocationAurangabad
##
             0.313783661
                                     0.313704055
                                                            0.293113281
##
      LocationCoimbatore
                                   DrivetrainFWD
                                                         MakeMitsubishi
##
             0.269393557
                                     0.263692508
                                                            0.249563035
##
              MakeToyota
                                     LocationGoa
                                                      LocationBangalore
##
              0.248826742
                                     0.248093841
                                                            0.246496783
##
                                                          DrivetrainRWD
          LocationKollam
                                          Length
##
             0.226788237
                                     0.226439416
                                                            0.221018253
##
         LocationChennai
                                  LocationIndore
                                                      LocationErnakulam
##
             0.216077914
                                     0.209724881
                                                            0.208988890
##
         LocationDharwad
                                 Fuel.TypePetrol
                                                      LocationHyderabad
##
             0.207341744
                                     0.189472148
                                                            0.185525949
##
       LocationJalandhar
                                  MakeLand Rover
                                                         LocationNagpur
##
             0.181224043
                                     0.180846132
                                                            0.167588274
##
        LocationGuwahati
                                 Fuel.TypeDiesel
                                                          LocationUdupi
##
             0.167182283
                                     0.164673775
                                                            0.148042941
##
       MakeMercedes-Benz
                             TransmissionManual
                                                                   Width
##
                                                            0.135371333
             0.145432782
                                     0.143902253
##
            LocationPune
                                  LocationJaipur
                                                       LocationAmritsar
##
             0.131030713
                                     0.122967074
                                                            0.121805119
##
              OwnerThird
                                    LocationKota
                                                         LocationMohali
##
             0.121210169
                                     0.117146416
                                                            0.115934495
##
           LocationThane
                                  LocationMumbai
                                                         LocationRaipur
##
             0.102055354
                                     0.100701797
                                                            0.099272043
        LocationMirzapur
##
                                  LocationKarnal
                                                          LocationNoida
##
             0.098291704
                                     0.096670247
                                                            0.095972455
##
    LocationAmbala Cantt
                            LocationNavi Mumbai
                                                      LocationAllahabad
##
             0.092130013
                                     0.092020794
                                                            0.090599537
```

##	LocationKheda	LocationKanpur	${\tt OwnerUnRegistered\ Car}$
##	0.088221751	0.086058622	0.082215522
##	LocationBhubaneswar	LocationKolkata	LocationDelhi
##	0.080119830	0.076002757	0.071607021
##	${ t Location} { t Faridabad}$	LocationMuzaffurpur	LocationRanga Reddy
##	0.069764717	0.068179314	0.066742019
##	MakeJaguar	LocationWarangal	LocationAhmedabad
##	0.065901543	0.063180846	0.062246218
##	LocationPatna	LocationDehradun	LocationNashik
##	0.062160128	0.059305473	0.058196769
##	Max.Torque.RPM	LocationMangalore	LocationKharar
##	0.056697498	0.056620215	0.056298195
##	LocationVaranasi	LocationRoorkee	LocationRanchi
##	0.048878372	0.047629067	0.046096495
##	${ t Location Ludhiana}$	Year:Length	LocationSurat
##	0.043273053	0.041510876	0.041112278
##	LocationMeerut	Kilometer	LocationChandigarh
##	0.038541407	0.034232361	0.031825300
##	MakeVolvo	LocationBhopal	LocationPanchkula
##	0.030938615	0.029053023	0.027400801
##	LocationZirakpur	LocationGhaziabad	LocationYamunanagar
##	0.026027821	0.023076512	0.023015393
##	LocationLucknow	OwnerSecond	Year:Max.Torque.RPM
##	0.021634758	0.020113875	0.018733584
##	Max.Torque.RPM:Length	Year:Kilometer	LocationJamshedpur
##	0.017715033	0.017494262	0.017254036
##	${ t MakeBMW}$	LocationHaldwani	${\tt LocationGurgaon}$
##	0.012673995	0.010949026	0.010276737
##	${ t Location Vadodara}$		

Confidence Intervals

##

confint(reduced_model, level = 0.95)

0.009552204

```
##
                                 2.5 %
                                              97.5 %
## (Intercept)
                          3.097671e+01 2.006291e+02
## MakeBMW
                         -6.583948e-02 9.118747e-02
## MakeChevrolet
                         -9.763334e-01 -6.201439e-01
## MakeDatsun
                         -9.927263e-01 -7.215037e-01
## MakeFord
                         -6.479947e-01 -4.974984e-01
## MakeHonda
                         -5.470757e-01 -4.246625e-01
## MakeHyundai
                         -5.187203e-01 -4.113642e-01
## MakeIsuzu
                         -6.878548e-01 -1.875461e-01
                         -8.474833e-02 2.165514e-01
## MakeJaguar
## MakeJeep
                         -4.327616e-01 -2.343877e-01
                         -5.026782e-01 -3.283247e-01
## MakeKia
## MakeLand Rover
                         -2.061831e-02 3.823106e-01
## MakeMahindra
                         -6.610257e-01 -5.315700e-01
## MakeMaruti Suzuki
                         -5.784090e-01 -4.680363e-01
## MakeMercedes-Benz
                          8.097184e-02 2.098937e-01
## MakeMG
                         -7.040558e-01 -4.886350e-01
                          5.554995e-01 8.526088e-01
## MakeMINI
## MakeMitsubishi
                         -4.697699e-01 -2.935620e-02
```

```
## MakeNissan
                         -8.345425e-01 -6.323523e-01
## MakeRenault
                         -8.246941e-01 -6.738188e-01
## MakeSkoda
                         -5.822294e-01 -4.359284e-01
## MakeSsangyong
                         -1.094264e+00 -6.507613e-01
## MakeTata
                         -7.913884e-01 -6.466088e-01
## MakeToyota
                         -3.076110e-01 -1.900425e-01
## MakeVolkswagen
                         -5.919894e-01 -4.446599e-01
## MakeVolvo
                         -7.915660e-02 1.410338e-01
## Year
                         -9.500900e-02 -1.088987e-02
## Kilometer
                         -6.888294e-04 -2.160802e-04
## Fuel.TypeDiesel
                         -2.384552e-01 -9.089237e-02
## Fuel.TypePetrol
                         -2.563113e-01 -1.226330e-01
## TransmissionManual
                         -1.687006e-01 -1.191039e-01
## LocationAhmedabad
                         -3.282940e-02 1.573218e-01
## LocationAllahabad
                                         1.213755e-01
                         -3.025746e-01
## LocationAmbala Cantt
                         -2.289298e-01
                                         4.466978e-02
## LocationAmritsar
                         -6.927460e-02
                                         3.128848e-01
## LocationAurangabad
                           8.091882e-02
                                         5.053077e-01
## LocationBangalore
                           1.565970e-01 3.363965e-01
## LocationBhopal
                          -1.838064e-01
                                         2.419125e-01
## LocationBhubaneswar
                         -1.407892e-01
                                        3.010288e-01
## LocationBulandshahar
                         -5.709691e-01 -5.643906e-02
## LocationChandigarh
                          -8.964572e-02
                                        1.532963e-01
## LocationChennai
                           1.162988e-01
                                         3.158570e-01
## LocationCoimbatore
                           1.530404e-01
                                         3.857467e-01
## LocationDehradun
                         -5.481136e-02
                                        1.734223e-01
## LocationDelhi
                         -1.463001e-02
                                         1.578440e-01
## LocationDharwad
                         -4.548198e-02
                                         4.601655e-01
## LocationErnakulam
                         -4.516543e-02
                                         4.631432e-01
## LocationFaridabad
                         -1.802835e-01
                                         4.075409e-02
## LocationGhaziabad
                         -2.777928e-01
                                         2.316398e-01
## LocationGoa
                           5.605099e-02
                                         4.401367e-01
## LocationGurgaon
                         -1.205426e-01
                                         9.998910e-02
## LocationGuwahati
                         -7.706673e-03
                                         3.420712e-01
## LocationHaldwani
                          -2.644659e-01
                                         2.425678e-01
## LocationHyderabad
                                         2.767447e-01
                           9.430717e-02
## LocationIndore
                           2.028461e-02
                                         3.991652e-01
## LocationJaipur
                           1.562376e-02
                                         2.303104e-01
## LocationJalandhar
                           6.811753e-02
                                         2.943306e-01
## LocationJamshedpur
                         -1.932662e-01
                                         1.587581e-01
## LocationKanpur
                         -1.871899e-01
                                         1.507262e-02
## LocationKarnal
                          -4.926889e-02
                                         2.426094e-01
## LocationKharar
                         -1.973544e-01
                                         3.099507e-01
## LocationKheda
                         -3.411327e-01
                                         1.646892e-01
## LocationKolkata
                         -1.712186e-01
                                         1.921307e-02
## LocationKollam
                         -2.665001e-02
                                         4.802265e-01
## LocationKota
                         -1.592889e-01
                                         3.935818e-01
## LocationLucknow
                         -7.389690e-02
                                         1.171664e-01
## LocationLudhiana
                         -5.740803e-02
                                         1.439541e-01
## LocationMangalore
                         -1.326422e-01
                                         2.458826e-01
## LocationMeerut
                         -1.079708e-01
                                         1.850536e-01
## LocationMirzapur
                         -1.768274e-01
                                         3.734108e-01
## LocationMohali
                           6.890658e-03
                                         2.249783e-01
## LocationMumbai
                           1.463348e-02 1.867701e-01
```

```
## LocationMuzaffurpur
                         -3.213162e-01 1.849576e-01
                          1.676259e-01 4.741467e-01
## LocationMysore
                         -4.456527e-02 3.797418e-01
## LocationNagpur
## LocationNashik
                         -9.455767e-02 2.109512e-01
## LocationNavi Mumbai
                         -3.130472e-02
                                       2.153463e-01
## LocationNoida
                         -2.983226e-02 2.217772e-01
## LocationPanchkula
                         -2.391027e-01 1.843011e-01
## LocationPatna
                         -3.985521e-02 1.641755e-01
## LocationPune
                         4.077207e-02
                                       2.212894e-01
## LocationRaipur
                         -1.922212e-02 2.177662e-01
## LocationRanchi
                         -1.631170e-01 7.092398e-02
## LocationRanga Reddy
                                       3.216963e-01
                         -1.882123e-01
## LocationRoorkee
                         -1.677348e-01
                                       2.629929e-01
## LocationSalem
                          1.678870e-01
                                       6.738535e-01
## LocationSurat
                                       2.136842e-01
                         -1.314596e-01
## LocationThane
                         -1.988465e-02
                                        2.239954e-01
## LocationUdupi
                         2.711925e-03
                                       2.933740e-01
## LocationVadodara
                         -2.216110e-01 2.025066e-01
                         -1.675057e-01 6.974894e-02
## LocationVaranasi
## LocationWarangal
                         -3.163003e-01
                                       1.899386e-01
## LocationYamunanagar
                         -1.904491e-01 2.364799e-01
## LocationZirakpur
                         -1.475090e-01 9.545337e-02
## OwnerSecond
                         -4.640287e-02 6.175123e-03
## OwnerThird
                         -2.167867e-01 -2.563362e-02
## OwnerUnRegistered Car -9.813258e-02 2.625636e-01
## Max.Torque.RPM
                         4.950203e-03 1.638198e-02
## DrivetrainFWD
                         -3.074587e-01 -2.199264e-01
## DrivetrainRWD
                         -2.738714e-01 -1.681651e-01
## Length
                         -9.755450e-02 -5.964957e-02
## Width
                          1.291687e-03 1.714377e-03
## Year:Kilometer
                          1.065315e-07 3.409337e-07
## Year:Max.Torque.RPM
                         -8.057828e-06 -2.405186e-06
## Year:Length
                          2.993538e-05 4.872614e-05
## Max.Torque.RPM:Length -6.448120e-08 -1.173199e-08
```