

# 2005-2016 MCM赛题中英文

## MCM 2005 A: Flood Planning

Lake Murray in central South Carolina is formed by a large earthen dam, which was completed in 1930 for power production. Model the flooding downstream in the event there is a catastrophic earthquake that breaches the dam.

Two particular questions:

1. Rawls Creek is a year-round stream that flows into the Saluda River a short distance downriver from the dam. How much flooding will occur in Rawls Creek from a dam failure, and how far back will it extend?
2. Could the flood be so massive downstream that water would reach up to the S.C. State Capitol Building, which is on a hill overlooking the Congaree River?

## MCM 2005 B: Tollbooths

Heavily-traveled toll roads such as the Garden State Parkway in New Jersey, Interstate 95, and so forth, are multilane divided highways that are interrupted at intervals by toll plazas. Because collecting tolls is usually unpopular, it is desirable to minimize motorist annoyance by limiting the amount of traffic disruption caused by the toll plazas.

Commonly, a much larger number of tollbooths is provided than the number of travel lanes entering the toll plaza. Upon entering the toll plaza, the flow of vehicles fans out to the larger number of tollbooths, and when leaving the toll plaza, the flow of vehicles is required to squeeze back down to a number of travel lanes equal to the number of travel lanes before the toll plaza. Consequently, when traffic is heavy, congestion increases upon departure from the toll plaza. When traffic is very heavy, congestion also builds at the entry to the toll plaza because of the time required for each vehicle to pay the toll.

Make a model to help you determine the optimal number of tollbooths to deploy in a barrier-toll plaza. Explicitly consider the scenario where there is exactly one tollbooth per incoming travel lane. Under what conditions is this more or less effective than the current practice? Note that the definition of "optimal" is up to you to determine.

## ICM 2005 C: The Exhaustible Resource Problem

Select a vital nonrenewable or exhaustible resource (water, mineral, energy, food, etc.) for which your team can find appropriate world-wide historic data on its endowment, discovery, annual consumption, and price. The modeling tasks are:

- (1) Using the endowment, discoveries, and consumption data, model the depletion or degradation of the commodity over a long horizon using resource modeling principles.
- (2) Adjust the model to account for future economic, demographic, political and environmental factors. Be sure to reveal the details of your model, provide visualizations of the model's output, and explain limitations of the model.
- (3) Create a fair, practical "harvesting/management" policy that may include economic incentives or disincentives, which sustain the usage over a long period of time while avoiding severe disruption of consumption, degradation or rapid exhaustion of the resource.
- (4) Develop a "security" policy that protects the resource against theft, misuse, disruption, and unnecessary degradation or destruction of the resource. Other issues that may need to be addressed are political and security management alternatives associated with these policies.
- (5) Develop policies to control any short- or long-term "environmental effects" of the harvesting. Be sure to include issues such as pollutants, increased susceptibility to natural disasters, waste handling and storage, and other factors you deem appropriate.
- (6) Compare this resource with any other alternatives for its purpose. What new science or technologies could be developed to mitigate the use and potential exhaustion of this resource? Develop a research policy to advance these new areas.

## MCM 2005 A题：水灾计划

南卡罗来纳州中部的磨累河是由北部的一个巨大水坝形成的，这是在 1930 年为了发电而修建的，模拟一起洪水淹没下游的事件，这起事件是由于一次灾难性的地震摧毁了水坝造成的。要求解决两个问题：

- (1) Rawls Creek 是水坝下游流入 Saluda 河的一条终年流动的河流，则当水坝损毁后在 Rawls Creek 将会出现多大的洪流，洪水的波及面将有多大？
- (2) S.C. 国会大厦大楼在一座小山上，在 S.C. 国会大厦大楼能俯视 Congaree 河。洪水能如此巨大顺流以致于水将扩展到 S.C. 国会大厦大楼吗？

## MCM 2005 B题：收费站

收费站交通流量大的收费道路，比如：the Garden State Parkway, Interstate 95 等等，都是多车道的高速公路，司机需要在收费处停车交费。因收取过路费不是件得人心的事，所以有关部门期望通过减少由于收费站造成的交通混乱来使司机的不满情绪降到最小值。

通常情况下，收费站收费窗口的数量会远大于高速公路上的车道数。当进入收费站时，车辆分散开，进入各个收费窗口；而出站时，这些车辆有不得不挤回车道上。于是，当交通流量大时，出收费站时往往出现交通拥挤。当交通非常繁忙时，收费站入口处也会出现交通拥挤，因为每部车交费都需要一定的时间。

请你帮助建立一个模型，来确定对于一个交通繁忙的收费处多少个收费窗口才是最佳数目。考虑每一条路只有一个收费站的情况，在什么情况下会比现行的有效，在什么情况下会比现行的效率低？至于“最佳”这个定义由参赛队员自己判别。

### ICM 2005 C题：不可再生资源管理问题

选择一种对人类生存和发展至关重要的不可再生，或者即将耗尽的资源（例如水资源、矿产资源、能源、食物等），需要找到有关这种资源在全世界范围内的预计储量、探明储量、年度消耗量和价格等数据。要完成的任务如下：

- (1) 使用预计储量、探明储量和年度消耗量的数据，运用资源建模方法，对世界范围的资源的耗尽问题或者资源储量减少问题进行建模分析。
- (2) 考虑到未来的经济、人口、政策和环境因素的影响，调整你的数学模型，要求模型要有足够的细节，提供可视化的结论，并且讨论模型的局限性。
- (3) 提出一个公平、可操作的“开采/管理”策略，例如经济激励和惩罚措施，从而确保资源的长期可用性，而且能避免这种资源的极度混乱的消耗、退化或者迅速枯竭。
- (4) 提出一种“安全开采”策略以保护资源，防止被偷盗、滥用、破坏、不必要的退化或耗尽。此外，讨论与这些策略相应的政策和安全管理方面的可行方案。
- (5) 资源开采过程可能带来短期或者长期的环境危害，请提出一种控制这种危害的有效策略。制定策略时需要考虑：环境污染问题、对自然灾害敏感性逐步加剧、资源废料的处理和存放，以及你们认为重要的其他因素。
- (6) 将这种资源和相近用途的其他资源进行比较，为减少你们选择资源的消耗或潜在的耗尽危险，应该发展什么样的新科学或技术？提出一种能推动这些新领域进一步的研发策略。

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### MCM 2006 A: Positioning and Moving Sprinkler Systems for Irrigation

There are a wide variety of techniques available for irrigating a field. The technologies range from advanced drip systems to periodic flooding. One of the systems that is used on smaller ranches is the use of “hand move” irrigation systems. Lightweight aluminum pipes with sprinkler heads are put in place across fields, and they are moved by hand at periodic intervals to insure that the whole field receives an adequate amount of water. This type of irrigation system is cheaper and easier to maintain than other systems. It is also flexible, allowing for use on a wide variety of fields and crops. The disadvantage is that it requires a great deal of time and effort to move and set up the equipment at regular intervals.

Given that this type of irrigation system is to be used, how can it be configured to minimize the amount of time required to irrigate a field that is 80 meters by 30 meters? For this task you are asked to find an algorithm to determine how to irrigate the rectangular field that minimizes the amount of time required by a rancher to maintain the irrigation system. One pipe set is used in the field. You should determine the number of sprinklers and the spacing between sprinklers, and you should find a schedule to move the pipes, including where to move them.

A pipe set consists of a number of pipes that can be connected together in a straight line. Each pipe has a 10 cm inner diameter with rotating spray nozzles that have a 0.6 cm inner diameter. When put together the resulting pipe is 20 meters long. At the water source, the pressure is 420 Kilo- Pascal’s and has a flow rate of 150 liters per minute. No part of the field should receive more than 0.75 cm per hour of water, and each part of the field should receive at least 2 centimeters of water every 4 days. The total amount of water should be applied as uniformly as possible.

### MCM 2006 B: Wheelchair Access at Airports

One of the frustrations with air travel is the need to fly through multiple airports, and each stop generally requires each traveler to change to a different airplane. This can be especially difficult for people who are not able to easily walk to a different flight’s waiting area. One of the ways that an airline can make the transition easier is to provide a wheelchair and an escort to those people who ask for help. It is generally known well in advance which passengers require help, but it is not uncommon to receive notice when a passenger first registers at the airport. In rare instances an airline may not receive notice from a passenger until just prior to landing.

Airlines are under constant pressure to keep their costs down. Wheelchairs wear out and are expensive and require maintenance. There is also a cost for making the escorts available. Moreover, wheelchairs and their escorts must be constantly moved around the airport so that they are available to people when their flight lands. In some large airports, the time required to move across the airport is nontrivial. The wheelchairs must be stored somewhere, but space is expensive and severely limited in an airport terminal. Also, wheelchairs left in high traffic areas represent a liability risk as people try to move around them.

Finally, one of the biggest costs is the cost of holding a plane if someone must wait for an escort and becomes late for their flight. The latter cost is especially troubling because it can affect the airline’s average flight delay which can lead to fewer ticket sales as potential customers may choose to avoid an airline.

Epsilon Airlines has decided to ask a third party to help them obtain a detailed analysis of the issues and costs of keeping and maintaining wheelchairs and escorts available for passengers. The airline needs to find a way to schedule the movement of wheelchairs throughout each day in a cost effective way. They also need to find and define the costs for budget planning in both the short term and in the long term.

Epsilon Airlines has asked your consultant group to put together a bid to help them solve their problem. Your bid should include an overview and analysis of the situation to help them decide if you fully understand their problem. They require a detailed description of an algorithm that you would like to implement which can determine where the escorts and wheelchairs should be and how they should move throughout each day. The goal is to keep the total costs as low as possible. Your bid is one of many that the airline will consider. You must make a strong case as to why your solution is the best and show that it will be able to handle a wide range of airports under a variety of circumstances.

Your bid should also include examples of how the algorithm would work for a large (at least four concourses), a medium (at least two concourses), and a small airport (one concourse) under high and low traffic loads. You should determine all potential costs and balance their respective weights. Finally, as populations begin to include a higher percentage of older people who have more time to travel but may require more aid, your report should include projections of potential costs and needs in the future with recommendations to meet future needs.

### ICM 2006 C: The HIV/AIDS Problem

As the HIV/AIDS pandemic enters its 25th year, both the number of infections and number of deaths due to the disease continue to rise. Despite an enormous amount of effort, our global society remains uncertain on how to most effectively allocate resources to fight this epidemic.

You are a team of analysts advising the United Nations (UN) on how to manage the available resources for addressing HIV/AIDS. Your job is to model several scenarios of interest and to use your models to recommend the allocation of financial resources. The narrative below provides some background information, and outlines specific tasks.

**Task 1:** For each of the continents (Africa, Asia, Europe, North America, Australia, and South America), choose the country you believe to be most critical in terms of HIV/AIDS. Build a model to approximate the expected rate of change in the number of HIV/AIDS infections for these countries from 2006 to 2050, in the absence of any additional interventions. Fully explain your model and the assumptions that underlie your model. In addition, explain how you selected the countries to model.

Use as a list of countries for inclusion in your analysis the countries included in the attached spreadsheet, which include all member states of the World Health Organization (WHO) as of 2003.

Data: "list\_WHO\_member\_states.xls".

Reliable data on HIV prevalence rates by county are generally difficult to obtain. The attached spreadsheet includes several worksheets of data which you may use in your analysis. Data: "hiv\_aids\_data.xls".

a. "Global HIV- AIDS cases, 1999": These data come from UNAIDS (the Joint United Nations Program on HIV/AIDS) and report the estimated number of HIV positive 0 to 49 year olds by country at the end of 1999.

b. "HIV- AIDS in Africa over time": These data come from the US government and give some piecemeal time series data on measured HIV prevalence rates among women of childbearing age, in urban areas, over time for some African countries.

c. "HIV- AIDS subtypes": These data come from UNAIDS and give the geographic distribution of HIV-1 subtypes by country. Also attached, for your use, are some basic population and demographic data.

Data:

(1) "fertility\_data.xls": These data come from the UN and give age-specific fertility rates by major area, region, and country, 1995-2050 (births per thousand women)

a. Estimates for 1995-2005.

b. Projections (under the assumption of medium fertility levels) for 2005-2050.

(2) "population\_data.xls": These data come from the UN and give total population (both sexes combined) by major area, region, and country, annually for 1950-2050 (thousands).

a. Estimates for 1950-2005.

b. Projections (under the assumption of medium fertility levels) for 2006-2050.

(3) "age\_data.xls": These data come from the UN and give population (for both sexes, and by gender) by five-year age groups, major area, region, and country, 1950-2050 (thousands).

a. Estimates, 1950-2005.

b. Projections (under the assumption of medium fertility levels) for 2010-2050.

(4) "birth\_rate\_data.xls": These data come from the UN and give crude birth rates by major area, region, and country, 1950-2050 (births per thousand population)

a. Estimates, 1950-2005.

b. Projections (under the assumption of medium fertility levels) for 2005-2050.

(5) "life\_expectancy\_0\_data.xls": These data come from the UN and give life expectancy at birth (by sex and both sexes combined) by major area, region, and country, 1950-2050 (years).

a. Estimates, 1950-2005.

b. Projections (under the assumption of medium fertility levels) for 2005-2050.

There are a number of interventions that HIV/AIDS funding could be directed towards -- including prevention interventions (voluntary counseling and testing, condom social marketing, school-based AIDS education, medicines to prevent mother-to-child transmission, etc.) and care interventions (treating other untreated sexually transmitted diseases, treating opportunistic infections, etc.). You should focus on only two potential interventions: provision of antiretroviral (ARV) drug therapies, and provision of a hypothetical HIV/AIDS preventative vaccine.

**Task 2:** First, estimate the level of financial resources from foreign aid donors that you realistically expect to be available to address HIV/AIDS, by year, from 2006 to 2050, for the countries you selected in Task 1. Then use the

model you developed in Task #1 and these estimates of financial resources to estimate the expected rate of change in the number of HIV/AIDS infections for your selected countries from 2006 to 2050 under realistic assumptions for the following three scenarios:

- (1) Antiretroviral (ARV) drug therapy.
- (2) A preventative HIV/AIDS vaccine.
- (3) Both ARV provision and a preventative HIV/AIDS vaccine.

Assume in these scenarios that there is no risk of emergence of drug-resistant strains of HIV (you will examine this issue in Task 3).

Be sure to carefully describe the assumptions that underlie your model.

You can choose whether these scenarios should be implemented for all of the countries you selected in Task 1, or for certain subsets of countries based on income cut-offs, disease burden, etc. Available for use if you wish is a spreadsheet of country-level income data.

Data: "income\_data.xls" .

These data are from the World Bank (2002) and give per-capita gross national product (GNP) data as well as broad income classifications that you are free to use in your analysis if you wish.

ARV drug therapies can have tremendous benefits in terms of prolonging the lives of individuals infected with HIV/AIDS. ARVs are keeping a high proportion of HIV/AIDS-infected individuals in rich countries alive, and policy makers and international institutions are facing tremendous political pressure to increase access to ARVs for individuals in poor countries. Health budgets in low-income countries are very limited, and it seems unlikely that poor countries will be able to successfully expand these programs to the majority of their populations using their own resources. Appendix 1 presents country-specific data from UNAIDS on current access to ARVs for a number of countries.

The efficacy of ARVs depends in large part on adherence to the treatment regimen and to proper monitoring. The most favorable conditions for ARVs are structured programs with extensive counseling and physician care, as well as regular testing to monitor for disease progression and the onset of opportunistic infections. Non-adherence or inadequate treatment carries with it two very serious consequences. First, the treatment may not be effective for the individual undergoing treatment. Second, partial or inadequate treatments are thought to directly lead to the emergence of drug-resistant strains of HIV.

The price of the drugs initially used to treat patients has come down to several hundred dollars a year per patient, but delivering them and providing the necessary accompanying medical care and further treatment is the key administrative and financial challenge. It is estimated that purchasing and delivering antiretroviral using the clinically-recommended approach (DOTS, or directly observed short course treatments) which is intended to minimize the emergence of drug-resistant strains would cost less than \$1,100 per person per year. (Adams, Gregor et al. [2001]. "Consensus Statement on Antiretroviral Treatment for AIDS in Poor Countries," [http://www.hsph.harvard.edu/bioethics/pdf/consensus\\_aids\\_therapy.pdf](http://www.hsph.harvard.edu/bioethics/pdf/consensus_aids_therapy.pdf) )

For a preventative HIV vaccine, make assumptions you feel are reasonable about the following (in addition to other factors you may choose to include in your model):

- (1) The year in which an HIV/AIDS preventative vaccine might be available.
- (2) How quickly vaccination rates might reach the following steady-state levels of vaccination:
  - a. If you wish to immunize new cohorts (infants), assume the steady-state level for new cohorts of the country-by-country immunization rates for the third dose of the diphtheria-pertussis-tetanus vaccine (DTP3), as reported by the WHO (2002).

Data: "vaccination\_rate\_data.xls".

- b. If you wish to immunize adults (any group over age 5), assume the steady-state level for older cohorts is the second dose of the tetanus toxoid (TT2) rate, as reported by the WHO (2002).

Data: "vaccination\_rate\_data.xls".

- (3) The efficacy and duration of protection of the vaccine.
- (4) Whether there would be epidemiological externalities from vaccination.
- (5) Assume the vaccine is a three-dose vaccine, and can be added to the standard package of vaccines delivered under the WHO's Expanded Programme on Immunization (EPI) at an incremental cost of addition of \$0.75.

Task 3: Re-formulate the three models developed in Task 2, taking into consideration the following assumptions about the development of ARV-resistant disease strains.

Current estimates suggest that patients falling below 90-95 percent adherence to ARV treatment are at a "substantial risk" of producing drug resistant strains. Use as an assumption for your analysis that a person receiving ARV treatment with adherence below 90 percent has a 5 percent chance of producing a strain of HIV/AIDS which is resistant to standard first-line drug treatments.

Second- and third-line ARV drug therapies are available, but assume for your analysis that these drugs are prohibitively expensive to implement in countries outside of Europe, Japan, and the United States.

Task 4: Write a white paper to the United Nations providing your team's recommendations on the following:



(1) Your recommendations for allocation of the resources available for HIV/AIDS among ARV provision and a preventative HIV vaccine.

(2) Your argument for how to weigh the importance of HIV/AIDS as an international concern relative to other foreign policy priorities.

(3) Your recommendations for how to coordinate donor involvement for HIV/AIDS.

For (1): assume that between now and 2010 the available financial resources could be allocated so as to speed the development of a preventative HIV vaccine – through directly-financing vaccine research and development (R&D), or through other mechanisms. Any gains from such spending would move the date of development you assumed in Task 2 to some earlier date.

### **MCM 2006 A题：灌溉喷洒系统的布置与移动**

目前有很多种田间灌溉的技术，从先进的滴灌系统到周期性的漫灌等各种技术。用于较小农场的技术之一就是使用“手动”灌溉系统，带有喷头的轻质铝管放置在田间，定时用手移动它们以确保所有农田都能够得到充足的水。这种灌溉系统比其他系统更加便宜，更加容易管理、维护，它们的使用非常灵活，可用于各种农田和农作物的灌溉。其缺点是，每过一段时间，就要花费很多时间和精力来移动和安装设备。

考虑到要使用这种灌溉系统，怎样安装才能用最少的时间去灌溉一片 80m×30m 的农田？为完成这项任务，请求你们去寻求一种确定怎样灌溉这块矩形农田使得农场主管理、维护该灌溉系统所需要的时间最少。这块农田上将使用一套管组，你们需要确定喷头的数量以及喷头之间的距离，同时还要给出一个移动管道，包括需要把管道移动到什么位置的工作进度表。

一套管组由若干互相连接成直线形的管子组成，每根管子的内壁直径为 10cm，并带有一个内壁直径 0.6cm 的可旋转喷嘴。把管子连接在一起，其总长为 20 米长。水源处的压力为 420 千帕 (Kilo-Pascal)，流率为每分钟 150 升。农田任何部分接受的水量不得超过每小时 0.75cm，同时农田的每个部分每 4 天至少要接受 2cm 的水量，尽可能均匀地使用洒水的总量。

### **MCM 2006 B题：在机场使用轮椅的问题**

乘飞机旅行令人头疼的事情之一就是需要在多个机场转机，而且每到一个机场通常都要求旅客去转乘另外一架飞机。对那些行动有困难的旅客而言，从一个候机区走到另外一个候机区就特别困难了。解决办法之一是航空公司为请求帮助的这些旅客提供轮椅和陪同人员，使得中转更加方便。通常都能预先知道那些乘客要求帮助，但也常有旅客宁愿在到机场登记时才请求帮助。在很罕见的情形，知道飞机就要降落前，航空公司可能还没有接到需要帮助的旅客的请求。

航空公司面临着降低成本的持续的压力。轮椅会磨损、昂贵而且还需要管理和维护，提供陪同人员也需要费用。另外，为给需要帮助的旅客在他们的航班到达机场时能及时提供帮助，轮椅和陪同人员还要不断在机场移动。在一些大机场，人员和设备在机场内部移动所花费的时间也是不容忽视的。轮椅还需要有存放的地方，但是机场候机大厅场地的租费昂贵而且极其有限。还有，把轮椅留在客流繁忙的通道，当过往旅客试图绕过轮椅时也会妨碍行人通过。最后，最大的代价之一就是，如果某位旅客必须等候陪同人员的到来而导致飞机等他而延误航班的代价。这种代价特别令人烦恼，因为它有可能影响到航空公司的平均航班延误时间，其后果是有些潜在乘客会因此而避开这个公司的航班，造成该航空公司机票销售的减少。

Epsilon 航空公司决定请求第三方帮助他们就为旅客提供轮椅和陪同人员服务的管理和维护中的各种问题和成本进行详细的分析。这家公司希望得到一个讲求成本效益的每天的轮椅调度方法，并找出和定义短期和长期的预算规划所需的各种成本。

Epsilon 航空公司要求你们的咨询小组汇集你们的分析形成一个投标以帮助该航空公司解决他们的问题。你们的投标书应该包括对实际情况的概述和分析，以便这家航空公司能够确定你们是否已经完全了解他们的问题。他们需要你们提供将要执行的算法的详细叙述，该算法能确定轮椅和陪同人员应该安置在那里，以及每天应该怎样移动。目标是使总的成本尽可能低。你们的投标书是 Epsilon 航空公司将会考虑的许多投标书之一。你们必须提供一个强有力的案例，以说明为什么你们的解决方案是最佳的，而且能够处理各种环境下的各种机场的问题。

你们的投标书还应该包括该算法如何处理大型（至少 4 个候机大厅）、中型（至少 2 个候机大厅）和小型（1 个候机大厅）机场在客流高峰和低谷时段的各种例子。你们应该确定所有潜在的成本并权衡它们各自的权重。最后，因为老年旅客在旅客总数中开始占有更大的比重，因为他们有较多的时间外出旅行，但也可能提出更多的帮助要求，所以你的报告还应该包括对未来潜在成本和乘客需求的规划，以及怎样满足未来需求的建议。

### **ICM 2006 C题：HIV/ADIS 问题**

从 HIV/AIDS 开始大范围流行至今已经 25 年了，这种疾病导致的感染人数和死亡人数不断攀升。人们在抗击 HIV/ADIS 方面付出了巨大的努力，也取得了一定的成果，但目前国际社会对于如何有效地配置各种资源以实现更有效防控 HIV/ADIS 相关方面的研究仍然欠缺。

如果你们是联合国的一个专家组，请就怎样管理可利用的资源来防控 HIV/AIDS 问题，向联合国提出合理的建议。你们需要对目前引起大家关注的几种 HIV/ADIS 防控方案进行建模，并用你们的模型就资金分配问题给出合理建议。

问题 (1)：从每个大洲（非洲、亚洲、欧洲、北美洲、澳洲和南美洲）中，各选择一个你们认为 HIV/AIDS 流行程度最严重的国家。建立数学模型粗略估计这些国家在没有任何有效干预措施时，2006~2050 年 HIV/AIDS 感染人数的变化规律。在模型中，要详细地叙述你们的模型并准确给出你们模型的基础假设。另外，解释一下你们选择这些国家的原因。

问题附件中的电子表格提供了 WHO（世界卫生组织）成员国家在相关领域直到 2003 年的数据，你们的模型中可能要用到这些数据。

数据文件：list\_WHO\_member\_states.xls，给出了 WHO 成员国家列表。

数据文件：hiv\_aids\_data.xls，给出了 HIV/ADIS 相关数据表。

(1) Global HIV-AIDS cases, 1999, 全球 HIV/ADIS 病例汇总 (1999 年)。该数据来自 UNAIDS (联合国 HIV/AIDS 项目研究组)，给出了到 1999 年底，由各个国家提供的 0 到 49 岁 HIV 检验为阳性的大致人数。

(2) HIV-AIDS in Africa over time, 非洲 HIV/ADIS 流行情况。该数据来自美国政府，该表格给出了非洲一些城市育龄妇女零碎的艾滋病发病率时间序列统计数据。(3) HIV-AIDS subtypes, HIV/ADIS 亚型表格。该数据来自 UNAIDS，表中按国家划分，给出了 HIV-1 子类型的地理分布。

附件还提供了这些国家的人口统计学数据。这些数据都来自于联合国，包括了世界主要地区和国家在 1995~2050 年的人口统计数据，

其中 1995~2005 年的数据为统计估计数据, 2006~2050 年的数据为按照中等生育率假设下的预测数据。

相关的数据文件如下:

(1) fertility\_data.xls, 给出了生育率数据, 指特定年龄的妇女的生育率(每千名妇女生育子女的数目)。

(2) population\_data.xls, 给出了人口统计表(分性别统计, 单位: 千人)。(3) age\_data.xls, 给出了年龄分布统计数据, 每隔5年划分一个年龄组, 统计各个年龄组的人数。

(4) birth\_rate\_data.xls, 给出了出生率统计数据, 单位: 出生人数/千人\*年。

(5) life\_expectancy\_0\_data.xls, 给出了人均寿命统计数据, 单位: 年。 HIV/AIDS 防控专款主要以两种方式介入 HIV/AIDS 防控——预防介入和治疗介入。预防介入主要包括: 健康咨询、HIV 检测服务、避孕套推广、以学校为基础的艾滋病教育、防止母-婴传染的药物等。治疗介入包括: 治疗其它性传染病、控制高致病感染等。你们工作要集中于两种最主要的介入方式: 提供抗逆转录酶病毒 (Antiretroviral: ARV) 药物治疗和提供 HIV/AIDS 预防疫苗。

问题(2): 落后国家难以承担本国的 ADIS 防控资金, 在 UNADIS 的倡导下, 发达国家建立了多项 ADIS 国际援助项目, 用于帮助和支持落后国家开展 ADIS 防控工作。

对你们在问题(1) 中选择的国家, 估计这些国家在 2006~2050 年期间期望得到多少国际援助资金用于 ADIS 防控。

用问题(1)中所建立的模型和这些资金数据, 在以下三种控制方案下, 分别估计所选定国家在 2006~2050 年 HIV/AIDS 感染人数预期的变化率。

(1) ARV 药物治疗;

(2) 预防性 HIV/AIDS 疫苗;

(3) 同时采用 ARV 治疗和预防性 HIV/AIDS 疫苗。

假设上述三种方案都不会有产生 HIV 抗药菌株的风险(问题 3 中需要考察该问题).务必详细、准确地叙述你们模型的基础假设。

你们可以考虑是否这些方案适用于所有的国家, 或者综合考虑国家的国民收入水平和医疗负担判断这些方案是否适用。附件中提供了有关各国收入水平的表格, 或许可以为你的模型所用。

数据文件: income\_data.xls, 给出了世界国民收入表。

该数据来自世界银行 (2002), 表中给出各地区国民人均生产总值 (GNP) 以及全球的收入分布。你们可以自由地使用这些数据到你们的模型中。

ARV 药物治疗可以延长 HIV/AIDS 感染者的生命, 给 HIV/AIDS 感染者带来极大的福音。在发达国家, ARV 药物使用率很高, 延长了大量的 HIV/AIDS 感染者的生命。有关部门和国际组织正面临着巨大的压力, 他们一直致力于如何拓展落后国家的 HIV/AIDS 感染者获得 ARV 药物的渠道。经济落后国家的卫生保健预算非常有限, 这些国家不大可能仅仅依靠自己对大多数 HIV/AIDS 感染者开展这些防治计划。附件 1 的数据来自 UNAIDS, 这些数据给出了当前正在使用 ARV 药物治疗的国家的有关数据。

ARV 的疗效, 在很大程度上依赖于感染者能否严格坚持治疗, 并且定期观察疗效。一种公认有效的 ARV 治疗条件是: 一份能够提供全面咨询和医生护理的治疗计划、定期的检查以观察病情的发展和条件感染人群的发病情况。间断的治疗或者不恰当的治疗方法会导致两方面的严重后果: 一是可能没有治疗效果; 二是可能会直接导致产生 HIV 抗药性菌株。

ARV 药物的制药厂商一直在努力降低制药成本从而降低药物价格, 虽然用于治疗 HIV/AIDS 早期感染的药物价格已经降到每个病人每年几百美元, 但如何分发这些药物, 并提供必要的辅助医疗保健措施以及进一步的治疗, 是考验政府的行政和财政能力的关键。经过比较, 直接观察短疗程治疗方案产生抗药菌株的可能性最小, 采用该方法的 ARV 药物成本可以控制在每人每年不超过 1100 美元。

针对预防性 HIV 疫苗问题, 请对以下参数给出你们认为合理的估计, 除了这些情况以外, 还可以考虑其他的情况。

(1) 预防性 HIV/AIDS 疫苗可以投入实用的时间;

(2) 疫苗接种普及速度;

如果你们希望使新生儿得到免疫, 那么假设国家的新生儿 HIV/AIDS 疫苗的稳定接种水平和 WHO(2002) 报告的 DTP3 (白喉-百日咳-破伤风疫苗第三剂)的接种率相同。数据文件: vaccination\_rate\_data.xls, 给出了新生儿疫苗接种率数据。如果你们希望使成人 (5 岁以上的人群)得到免疫, 那么假设这部分人群 HIV/AIDS 疫苗的稳定接种水平与 WHO(2002) 报告的 TT2(破伤风类毒素第二剂)的免疫率相同。数据文件: vaccination\_rate\_data.xls, 给出了成人疫苗接种率数据。

(3) 疫苗的功效及其有效期;

(4) 是否存在来自疫苗接种的流行病学的外部偶然因素;

(5) 假设疫苗是三剂的 (three-dose), 而且可以放进疫苗的标准封装, 随 WHO 的免疫扩展计划 (EPI) 一起, 以额外 0.75 美元的增加成本发放。

问题(3): 在如下关于 ARV 抗药性菌株的假设前提下, 重新阐明你们在问题(2) 中建立的三个模型。

当前的研究表明, 采用 ARV 治疗的病人的坚持程度低于 90%-95% 的话, 有产生抗药性菌株的“巨大风险”。在你们的分析中可以采用假设: 接受 ARV 治疗的病人的坚持程度低于 90%, 就有 5% 的可能性产生抗药性。

人们也可以使用二线和三线药物治疗。但在你们的模型中应假设, 除欧洲、日本和美国以外的国家要使用这些药物将会因为过于昂贵而无法负担。

问题(4): 向联合国相关部门写一份建议书, 就如下几个方面给出你们的建议:

(1) 就在抗击 HIV/AIDS 中 ARV 药物的供应和预防 HIV 疫苗的资源分配问题提出合理建议;

(2) 相对于其他外交政策而言, 应该赋予 HIV/AIDS 防控政策哪些优先权?

(3) 针对如何合理配置 HIV/AIDS 的国际援助资金问题给出建议。

对于第 (1) 个方面, 假设从现在起到 2010 年期间, 可利用的财政资源将专门划拨相关经费, 用以加速预防性 HIV 疫苗的研制工作—或直接资助疫苗的研发, 或通过其他的机制来实现这个目的。这种投入的回报都将促进问题 (2) 中疫苗研制成功日期的提前。

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### **MCM 2007 A: Gerrymandering**

The United States Constitution provides that the House of Representatives shall be composed of some number (currently 435) of individuals who are elected from each state in proportion to the state's population relative to that of the country as a whole. While this provides a way of determining how many representatives each state will have, it says nothing about how the district represented by a particular representative shall be determined geographically. This oversight has led to egregious (at least some people think so, usually not the incumbent) district shapes that look "unnatural" by some standards.

Hence the following question: Suppose you were given the opportunity to draw congressional districts for a state. How would you do so as a purely "baseline" exercise to create the "simplest" shapes for all the districts in a state? The rules include only that each district in the state must contain the same population. The definition of "simple" is up to you; need to make a convincing argument to voters in the state that your solution is fair. As an application of your method, draw geographically simple congressional districts for the state of New York.

### **MCM 2007 B: The Airplane Seating Problem**

Airlines are free to seat passengers waiting to board an aircraft in any order whatsoever. It has become customary to seat passengers with special needs first, followed by first-class passengers (who sit at the front of the plane). Then coach and business-class passengers are seated by groups of rows, beginning with the row at the back of the plane and proceeding forward.

Apart from consideration of the passengers' wait time, from the airline's point of view, time is money, and boarding time is best minimized. The plane makes money for the airline only when it is in motion, and long boarding times limit the number of trips that a plane can make in a day.

The development of larger planes, such as the Airbus A380 (800 passengers), accentuate the problem of minimizing boarding (and deboarding) time.

Devise and compare procedures for boarding and deboarding planes with varying numbers of passengers: small (85–210), midsize (210–330), and large (450–800).

Prepare an executive summary, not to exceed two single-spaced pages, in which you set out your conclusions to an audience of airline executives, gate agents, and flight crews.

Note: The 2 page executive summary is to be included IN ADDITION to the reports required by the contest guidelines.

An article appeared in the NY Times Nov 14, 2006 addressing procedures currently being followed and the importance to the airline of finding better solutions. The article can be seen at:

<http://travel2.nytimes.com/2006/11/14/business/14boarding.html>

### **ICM 2007 C: The Kidney Exchange Problem**

Transplant Network: Despite the continuing and dramatic advances in medicine and health technology, the demand for transplantation drastically exceeds the number of donors. To help this situation, the U.S. Congress passed the National Organ Transplant Act in 1984, establishing the Organ Procurement and Transplantation Network (OPTN) to match organ donors to patients with organ needs. Even with all this organizational technology and service in place, there are nearly 94,000 transplant candidates in the U.S. waiting for an organ transplant and this number is predicted to exceed 100,000 very soon. The average wait time exceeds three years—double that in some areas, such as large cities. Organs for transplant are obtained either from a cadaver queue or from living donors. The keys for the effective use of the cadaver queue are cooperation and good communication throughout the network. The good news is that the system is functioning and more and more donors (alive and deceased) are identified and used each year with record numbers of transplants taking place every month. The bad news is that the candidate list grows longer and longer. Some people think that the current system with both regional and national aspects is headed for collapse with consequential failures for some of the neediest patients. Moreover, fundamental questions remain: Can this network be improved and how do we improve the of a complex network like OPTN? Different countries processes and policies, which of these work best? What is the future status of the current system?

Task 1: For a beginning reference, read the OPTN Website (<http://www.optn.org>) with its policy description and data banks

(<http://www.optn.org/data> and <http://www.optn.org/latestData/viewDatareports.asp>).

Build a mathematical model for the generic U.S. transplant network(s). This model must be able to give insight into the following: Where are the potential bottlenecks for efficient organ matching? If more resources were available for improving the efficiency of the network function better if it was divided into smaller networks (for instance at the state level)? And finally, can you make the system more effective by saving and prolonging more lives? If so, suggest policy changes and modify your model to reflect these improvements.

Task 2: Investigate the transplantation policies used in a country other than the U.S. By modifying your model from Task 1, determine if the U.S. policy would be improved by implementing the procedures used in the other country. As an members of an expert analysis team (knowledge of public health issues and network science) hired by Congress to perform a study of these questions, write a one-page report to Congress addressing the questions and issues of



Task1 and the information and possible improvements you have discovered from your research of the different country's policies. Be sure to reference how you used your models from Task1 to help address the issues.

Focusing on Kidney Exchange: Kidneys filter blood, remove waste, make hormones, and produce urine. Kidney failure can be caused by many different diseases and conditions. People with end-stage kidney disease face death, dialysis (at over \$60,000/yr), or the hope for a kidney transplant. A transplant can come from the cadavers of an individual who agreed to donate organs after death, or from a live donor. In the U.S. about 68,000 patients are waiting for a kidney from a deceased donor, while each year only 10,000 are transplanted from cadavers and 6,000 from living individuals (usually relatives of the patients). Hence the median wait for a matching kidney is three years-unfortunately, some needy patients do not survive long enough to receive a kidney.

There are many issues involved in kidney transplantation-the overall physical and mental health of the recipient, the financial situation of the recipient (insurance for transplant and post-operation medication), and donor availability (is there a living donor willing to provide a kidney?). The transplanted kidney must be of a compatible ABO blood type. The 5-year survival of the transplant is enhanced by minimizing the number of mismatches on six HLA markers in the blood. At least 2,000 would-be-donor/recipient pairs are thwarted each year because of blood-type incompatibility or poor HLA match. Other sources indicate that over 6,000 people on the current waiting list have a willing but incompatible donor. This is a significant loss to the donor population and worthy of consideration when making new policies and procedures.

An idea that originated in Korea is that of a kidney exchange system, which can take place either with a living donor or with the cadaver queue. One exchange is paired-kidney donation, where each of two patients has a willing donor who is incompatible, but each donor is compatible with the other patient ; each donor donates to the other patient, usually in the same hospital on the same day. Another idea is list paired donation, in which a willing donor, on behalf of a particular patient, donates to another person waiting for a cadaver kidney; in return, the patient of the donor-patient pair receives higher priority for a compatible kidney donation to 3-way, 4-way, or a circle (n-paired) in which each donor gives to the next patient around the circle. On November 20, 2006, 12 surgeons performed the first-ever 5-way kidney swap at Johns Hopkins Medical Facility. None of the intended donor-recipient transplants were possible because of incompatibilities between the donor and the originally intended recipient. At any given time, there are many patient-donor pairs(perhaps as many as 6,000)with varying blood types and HLA markers. Meanwhile,the cadaver queue receives kidneys daily and is emptied daily as the assignments are made and the transplants performed.

Task 3: Devise a procedure to maximize the number and quality of exchanges, taking into account the medical and psychological dynamics of the situation. Justify in what way your procedure achieves a maximum. Estimate how many more annual transplants your procedure will generate, and the resulting effect on the waiting list. Strategies: Patient can face agonizing choices. For example, suppose a barely compatible-interms of HLA mismatches-kidney becomes available from the cadaver queue. Should they take it or wait for a better match from the cadaver queue or from an exchange? In particular, a cadaver kidney has a shorter half-life than a live donor kidney.

Task 4: Devise a strategy for a patient to decide whether to take an offered kidney or to even participate in a kidney exchange. Consider the risks, alternatives, and probabilities in your analysis.

Ethical Concerns: Transplantation is a controversial issue with both technical and political issues that involve balancing what is best for society with what is best for the individual. Criteria have been developed very carefully to try to ensure that people on the ethical concerns of who should go on to the policies try to address the ethical concerns of who should go on to the list or who should come off. Criteria involved for getting on or coming off the list can include diagnosis of a malignant disease, HIV infection or AIDS, severe cardiovascular disease, a history of non-compliance with prior treatment, or poorly controlled psychosis. Criteria used in determining placement priority include: time on the waiting list, the quality of the match between donor and recipient, and the physical distance between donor and recipient. As a result of recent changes in policy, children under 18 years of age receive priority on the waiting list and often receive a transplant within weeks or months of being placed on the list. The United Network for Organ Sharing Website recently(Oct. 27,2006)showed the age of waiting patients as:

Under 18: 748

18 to 34: 8,033

35 to 49: 20,553

50 to 64: 28,530

65 and over: 10,628

One ethical issue of continual concern is the amount of emphasis and priority on age to increase overall living time saved through donations. From a statistical standpoint, since age appears to be the most important factor in predicating length of survival, some believe kidneys are being squandered on older recipients.

Political Issues: Regionalization of the transplant system has produced political ramifications(e.g., someone may desperately need a kidney and is quite high on the queue, but his or her deceased neighbor's kidney still can go to an alcoholic drug dealer 500 miles away in a big city). Doctors living in small communities, who want to do a good job in transplants, need continuing experience by doing a minimum number of transplants per year. However, the kidneys from these small communities frequently go to the hospitals in the big city and, therefore, the local doctors cannot maintain their proficiency. This raises the question, perhaps transplants should be performed only in a few large centers, by a few expert and experienced surgeons? But would that be a fair system and would it add or detract from system efficiency?

Many other ethical and political issues are being debated. Some of the current policies can be found at<http://www.unos.org/policiesandbylaws/policies.asp?reaources=true>. For example, recent laws have been passed in the U.S. that forbid the selling or mandating the donation of organs,yet there are many agencies advocating for donors to receive financial compensation for their organ. The state of Illinois has a new policy that assumes everyone desires to be an organ donor (presumed consent)and people must opt out if they do not. The Department of Health and Human Services Advisory Committee on Organ Transplantation is expected to recommend that all states adopt



policies of presumed consent for organ donation. The final decision on new national policies rests with the Health Resource and Services Administration within the U.S. Department of Health and Human Services.

Task 5: Based on your analysis, do you recommend any changes to these criteria and policies? Discuss the ethical dimensions of your recommended exchange procedure and your recommended patient strategy(Task3 and Task4).Rank order the criteria you would use for priority and placement, as above, with rationale as to why you placed each where you did. Would you consider allowing people to sell organs for transplantation? Write a one-page paper to the Director of the U.S. Health Resources and Services Administration with your recommendations.

Task 6: From the potential donor's perspective, the risks in volunteering involve assessing the probability of success for the recipient, the probability of survival of the donor, the probability of future health problems for the donor, the probability of future health risks(such as failure of the one remaining kidney), and the post-operative pain and recovery. How do these risks and others affect the decision of the donor? How do perceived risks and personal issues (phobias, irrational fears, misinformation, previous experiences with surgery, level of altruism, and level of trust) influence the decision to donate? If entering a list paired network rather than a direct transplant to relative or friend, does the size n of the n-paired network have any effect on the decision of the potential donor? Can your models be modified to reflect and analyze any of these issues? Finally, suggest ways to develop and recruit more altruistic donors.

### MCM 2007 A题：不公平的选区划分

美国宪法规定众议院由一定数目的众议员（目前是 435 人）组成，他们是由各州按照该州人口占全国总人口的百分比选出来的。尽管这种规定提供了确定每个州有多少众议员的方法，但是一点也没有说及有关一个特定的众议员所代表的选区应该怎样按地区决定的问题。这种疏忽已经导致了按某种标准看来是违反常情的很不好的（至少某些人认为通常是不必这样做的）选区安排。因此就向你们提出了以下的问题：假设你们有机会去制定一个州的众议院的选区。你们会怎样把它作为一种纯“基础性”的练习来创建一个州的所有选区的“最简单”的划分。这些划分规则中至少要包含一条：该州的每个选区必须有同样的人口。“简单”的定义要由你们来下；但是你们必须就你们的解决方法是公正的做出一个能够使该州选民信服的论证。作为你们的方法的应用，试创建纽约州的按地域来说是简单的选区划分。

### MCM 2007 B题：飞机就座问题

航空公司允许引领候机乘客以任何次序就座。已经成为惯例的是首先引领有特殊需要的乘客就座，然后是头等舱的乘客就座（他们坐在飞机的前部）。然后引领持经济舱和商务舱机票的乘客从飞机后排开始向前按照排结组就座。

从航空公司的角度来看，除了考虑乘客的等候时间外，时间就是金钱，所以登机时间最好要减到最少。飞机只有在飞行的时候才能为航空公司赚钱，而长的登机时间限制了一架飞机一天中可以飞行的次数。

诸如Airbus A380（空中客车 A380，可以容纳 800 名乘客）的大型机的发展就更要强调缩短登机（以及下机）时间的问题了。

就乘客人数不同的飞机：小型机（85-210），中型机（210-330）和大型机（450-800），设计登机和下机时间的步骤并进行比较。

准备一份不超过两页纸（不空行打印）的实施概要，以便向航空公司业务主管、登机口执法人员以及空（地）勤人员阐明你们的结论。

在 2006 年 11 月 14 日的《纽约时报》上刊登的一篇文章报告了当前遵循的步骤以及航空公司寻求更好的解决方案的重要性。该文可以在如下网址找到：

<http://travel2.nytimes.com/2006/11/14/business/14boarding.html>

### ICM 2007 C题：肾移植问题

移植网络：在医学和医疗技术方面，尽管有着持续且引人注目的进步，但移植的需求量大大超过了捐助者的数量。为了解缓这种情况，美国国会通过了《1984 年全国器官移植法》，并且建立了器官供应移植网络（OPTN），以便匹配器官捐赠者和器官需要患者。即使所有这些技术和服务到位，在美国仍有近 94000 需要移植的患者等待器官移植，并且这个数字预计很快将超过 100000。在一些大城市，患者无论是从死捐赠者或从活捐赠者获得移植器官，平均等待时间均超过三年。有效使用死捐赠者的器官的关键是各机构间的合作和整个网络的良好沟通。好消息是，该体系正在发挥作用并且每年越来越多的捐助者（活着的和死去的）被认定和使用，每个月都会出现创记录的移植数量。坏消息是，等待移植的患者名单越来越长。随着一些最需要移植的患者在等待中死去，一些人认为，当前地方和国家的移植体系正在走向灭亡。此外，基本的问题仍然存在：这个网络是否可以得到改善？我们如何改善一个像 OPTN 这样复杂的网络？针对不同国家的处理方法和政策，哪些是更值得被实施的？未来的主流移植体系是什么？

任务 1：参考 OPTN 网站 (<http://www.optn.org>) 的政策和数据库 (<http://www.optn.org/data>

和<http://www.optn.org/latestData/viewDatareports.asp>)，建立关于美国移植网络的数学模型，模拟美国肾交换的过程，解决以下问题：

找出影响器官移植的主要问题；

如果可以提供更多的投资提高匹配效率，该如何使用它们？

如果此网络被分成一些小网络，例如州级规模的，该系统是否可以更有效的运作？

是否可以建立更有效的系统以延长与挽救更多的生命？如果可以，提出建议并且改动所建的模型以反映这些改进。

任务 2：基于其它国家的肾移植政策，修改上面模型，判断美国政策是否需要借鉴其它国家的政策来完善。通过以上的研究，以一名特聘专家的身份写一页报告给议会，阐述所存在的问题，以及可以进行的改进。

关注肾交换：肾脏过滤血液，排除垃圾，制造激素，产生尿液。肾衰竭可由许多不同的疾病和条件引起，患有晚期肾脏疾病的患者面临死亡、透析（超过 60000 美元/年）或者肾移植。一个移植器官可以来自同意死后捐献器官的捐赠者，或一个活体捐赠者。在美国大约有 68000 病人在等待死捐赠者的肾，而每年来自死捐赠者的肾移植只有 10000 个，来自活体（通常是患者的亲戚）的肾移植有 6000 个。因此等待合适肾脏的时间平均是三年。不幸的是，一些病人不能活足够长的时间来接收一个肾。

进行肾移植存在许多问题，如接受者的身体和心理健康，接受者的财务状况（移植的保险及术后的药物），捐赠者的可得到性（是否有一个活体愿意提供一个肾）。移植肾必须满足兼容的 ABO 血型。为了确保移植后有 5 年的存活期，要求 6 个 HLA 的血液标记的不匹配数量最小化。每年，由于血型不相容或可怜的 HLA 匹配，至少 2000 对供体-受体移植失败。其他资料指出，当前等候名单中，有超过 6000 愿意捐赠的人由于不匹配，而无法捐赠。这是捐赠人口上的一个重大损失，在制定新的政策和规程时值得考虑。

在任何给定的时间，有许多有着不同的血型 and HLA 标记的病人供体对（可能多达 6000）。因为捐献者与原来接收者之间的不兼容性，打算捐赠给受体的移植是不可能的。与此同时，每天接收的死者肾脏，又随着分配与移植的进行而被用尽。为了解决如此的状况，起源于韩国的一个想法是一个肾交换系统，它可以通过活体和等待死者肾的队列进行。一种交换是成对肾脏捐赠，每个病人有一个愿意捐赠但不相容的捐赠者，但该捐赠者与另一病人兼容；每个捐赠者捐赠给另一病人，通常是在同一个医院同一天进行手术。另一个想法是列表成对捐赠，捐赠者代表一个特定的病人，捐赠给一个等待肾源的患者，作为回报，这个供体病人，因为捐赠了一个兼容的肾脏，获得更高的优先级；3 对，4 对一个圈 (n 配对), 每个捐赠者捐赠肾给在圈上的下一位病人。2006 年 11 月 20 日，在约翰霍普金斯医疗机构，12 个外科医生首次进行了 5 对肾交换。

任务 3：从医学与心理学的角度考虑，设计一种处理方法，使肾交换的数量最大且质量最好。判断采用什么措施使处理方法收益最大；用你的处理方案估计，每年将增加多少肾交换，从而将缩短多少等待队列的长度。

策略：病人可能面临痛苦的选择。例如，假设可以获得一个来自死者的几乎不兼容的 HLA 不匹配肾脏。他们应该用它还是等待另一个死者或一个交换系统中的匹配更好的肾？特别是，一个死者肾比一个活体肾有更短的半衰期。

任务 4：从风险、变化与概率方面考虑，为病人设计一种方案，选择接受提供的肾还是参与肾交换。

道德忧虑：从技术和道德两个不同的角度看，移植是一个有争议的问题，其涉及一个平衡：如何对个体、社会都最有利。标准已被仔细地制定，试图确保人们间的道德关怀：谁应该继续排队或谁应该脱离。关于继续或脱离列表的标准可以包含一个恶性疾病的诊断：艾滋病、严重的心血管疾病、有不符合要求的病史、或控制不佳的精神病。用于确定优先位置的标准包括：在等候名单上的时间、供体和受体之间的匹配质量、供体和受体之间的地理距离。由于近期政策变化，18 岁以下的儿童在候补名单上获得优先，经常在被放置名单上数周或数月内，就可接受移植。器官分享联合网站最近 (2006 年 10 月 27 日) 的研究表明，等待病人的年龄为：

18 岁以下：748

18 到 34 岁：8033

35 到 49 岁：20553

50 到 64 岁：28530

65 与超过 65 岁：10628

持续担忧的一个道德问题是如何确定特权，以及如何确定年轻人的优先权等级。从一个统计角度来看，基于生存时间的考虑，年龄是最重要的因素，有些人认为年长的患者接受肾移植是挥霍资源。

政治因素：区域化的移植系统已经产生政治影响（如：有人可能急需一个肾脏并且在队列中相当靠前，但他的或她的已故邻居的肾脏还可能去 500 公里外的一个大的城市中的一个酒精药物的经销商那里）。住在小社区医生，在移植方面想有所作为，需要通过每年的最低数量的移植手术维持他们的技术水平。然而，这些小社区的肾脏通常被送到大城市的医院。因此，当地医生不能维持他们的进行移植手术的能力。这就提出了一个问题，难道移植只应在几个大中心，由几个专家和有经验的外科医生执行吗？但这一个公平的体系吗？这样会增加体系的效率还是减少体系的效率？

其他许多伦理和政治问题正在讨论。现有的一些政策可以在<http://www.unos.org/policiesandbylaws/policies.asp?resources=true>找到。例如，最近在美国已经通过了禁止销售和强制捐献器官的法律，然而有许多机构倡导捐赠者为他们的器官索取经济赔偿。伊利诺斯州有一个新政策，假设每个人都愿意成为器官捐献者（假定同意），如果某些人不同意，他们必须选择退出。在器官移植上，美国卫生与人类服务咨询委员会有可能建议所有州采取假定同意器官捐赠的政策。国家新政策的最后制定取决于美国健康与人类服务部门的卫生资源和服务管理局。

任务 5：基于对各民族传统与宗教信仰的分析，讨论肾交换方案与个人意识之间的矛盾（任务 3 与任务 4）：为保证交换的公正与合理性，制定优先级的策略。由于肾源的匮乏，是否可以进行器官交易；就此敏感问题的研究，给美国健康资源与服务协会的主任写一页短文。

任务 6：潜在的捐赠者就捐赠问题有许多担心，例如接收者能成功移植的概率、捐赠者生存的概率、捐赠者将来能否健康生活的概率、将来健康风险（所留下的肾失去功能）的概率以及手术后的痛苦与恢复过程。

这些担心与他们的个人问题（恐怖症、无理的害怕、错误的信息、以前手术的经历、无私度、信任度）将如何影响他们的决定？

如果是进入配对网络表，而不是交换给亲属或朋友，那么 n- 配对网络的大小对潜在的捐赠者有任何影响吗？

模型可以被改进并且分析这些问题中的一些吗？

最后，为了发展与招募更多的无私的捐赠者，提出建议。

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## MCM 2008 A: Take A Bath

Consider the effects on land from the melting of the north polar ice cap due to the predicted increase in global temperatures. Specifically, model the effects on the coast of Florida every ten years for the next 50 years due to the melting, with particular attention given to large metropolitan areas. Propose appropriate responses to deal with this. A careful discussion of the data used is an important part of the answer.

## MCM 2008 B: Creating Sudoku Puzzles

Develop an algorithm to construct Sudoku puzzles of varying difficulty. Develop metrics to define a difficulty level. The algorithm and metrics should be extensible to a varying number of difficulty levels. You should illustrate the algorithm with at least 4 difficulty levels. Your algorithm should guarantee a unique solution. Analyze the complexity of your algorithm. Your objective should be to minimize the complexity of the algorithm and meet the above requirements.

## ICM 2008 C: Finding the Good in Healthcare Systems

Nations have systems for providing healthcare for their residents. Issues that are often of concern to people and are often in the news include which system is better and whether current systems can be improved. Aspects of these

systems vary widely between nations: how they are funded; whether services are delivered through public, private, or non-profit organizations; whether public insurance is universal for all residents; who is eligible for assistance; what care is covered; whether the latest medical procedures are available; and how much is required as user fees. Other factors that are often debated in determining the quality of care include: coverage for complementary care (glasses, dental, prostheses, prescription drugs, etc.); which diseases are the most critical in improving overall health; percentage of GDP spent on healthcare; percentage of healthcare costs that goes toward labor/administration/malpractice insurance; ratio of public to private spending on healthcare; per capita spending on healthcare; growth of per capita spending on healthcare; number of participating physicians; per capita sick days; fairness of care in terms of age, race, gender, socio-economic class; and many more. Adding to the complications are health related factors such as personal exercise, food availability, climate, occupations of citizens, and smoking habits. The World Health Organization (WHO), an agency of the United Nations, is a source of data on health factors. The annual World Health Report

<http://www.who.int/whr/en/index.html>

assesses global health factors, and World Health Statistics

[http://en.wikipedia.org/wiki/World\\_Health\\_Organisation](http://en.wikipedia.org/wiki/World_Health_Organisation)

provides health statistics for the countries in the U.N. The production and dissemination of health statistics is a major function of WHO. To many people, these data and the associated analyses are considered unbiased and very valuable to the world community. There are many other sources of reliable health data available.

Part I: Describe several different outcomes (metrics) that could be used to evaluate the effectiveness of a country's healthcare system, such as average life expectancy of its residents. What metric would you use to make comparisons between existing and potential systems? Can you combine your metrics to make them even more useful in measuring quality?

Part II: Identify current sources of data that provide the raw data needed to compute the metrics you have identified above. You may need to modify your list of metrics based on the availability of data. Explain why you have selected those data and demonstrate how they can be used to assess and compare the relative effectiveness of healthcare systems as they exist in different countries.

Part III: Choose at least three of the most important and viable metrics for comparing healthcare systems. Justify why these are the most useful for this purpose. Can any of these help measure the historical change in an existing healthcare system? Are they measurable and can the data be easily collected?

Part IV: Use your three (or more) metrics to compare the United States healthcare system with one other country which is considered to have good healthcare using the most recent year for which you have data. Which country has the better healthcare system? Is your answer definitive?

Part V: Using your metrics compare the United States and one other country which is considered to have poor healthcare using the most recent year for which you have data. Which country has the better healthcare system?

Part VI: Pick a country's (U.S. or other) healthcare system and restructure it to improve the system based on your metrics. Build predictive models to test various changes to determine if the changes will improve the overall quality of the system. Suggest major change(s) that can improve the system.

### **MCM 2008 A题：冰盖融化问题**

请考虑由于全球气温升高所造成的北极冰帽融化对大陆的影响。特别地，请建立模型讨论北在未来 50 年内每 10 年极冰帽融化对佛罗里达海岸的影响，尤其要关注对大型都市区的影响。就此问题给出适宜的回答，对所用数据作详细讨论应是重要的组成部分。

### **MCM 2008 B题：数独谜题生成问题**

开发一个产生数独题目的算法，该算法能产生不同难度级别的数独题目。要求首先定义数独难度级别，并将其作为产生不同难度级别数独题目的标准。算法和度量标准应该具有可扩展性，能够很容易扩展到更多难度级别。并且至少用 4 个难度级别来进行例证说明。另外一定要保证数独题目具有唯一解。特别注意要分析算法的复杂性，并在满足前面要求的条件下使算法复杂性达到最小。

### **ICM 2008 C题：卫生保健系统评估**

每个国家都有各自的一些卫生保健系统，为它们的居民提供卫生保健服务。人们通常关注的、也经常出现在新闻报道中的问题是：什么样的系统更好，以及现有的系统是否可以得到进一步改善？在不同国家，卫生保健系统在很多方面是不同的，例如：它们是如何提供资助；是否通过公共、私人或非营利组织来提供服务；是否所有居民都享有公共保险；谁有资格寻求帮助；都有哪些保健项目；最新的医疗措施是否可供使用，以及有多少需要交纳费用？还有一些其他因素，在讨论和判定医疗服务质量时往往也需要考虑，包括：补充护理（配镜，牙科，假肢，处方药等）的覆盖率；哪些疾病是影响公共健康的最关键因素；国民生产总值中用于医疗保健部分所占的百分比；医疗保健费用中用于劳动/行政/医疗事故保险部分所占的百分比；公共与私人医疗服务支出的比例；人均卫生保健支出及其增长率；参与医师人数；人均病假天数；针对不同年龄、种族、性别、社会经济阶层的保健状况等。除了上述因素决定医疗服务质量外，还应考虑与健康相关的其它因素的影响，例如个人锻炼、粮食供应、气候、公民就业和吸烟习惯等。

联合国辖下的世界卫生组织（WHO），具有建立和公布卫生统计报告是 WHO 的重要功能，是世界卫生因素统计资料的重要来源。世界卫生报告每年针对全球健康因素进行评估（<http://www.who.int/whr/en/index.html>），世界卫生统计报告提供了联合国各成员国的卫生统计数据（[http://en.wikipedia.org/wiki/world\\_health\\_organisation](http://en.wikipedia.org/wiki/world_health_organisation)）。这些数据以及相关的分析通常被认为是针对整个世界卫生保健状况的公正且十分有价值的信息。除此之外，还有许多其他来源可靠的卫生统计数据可用。请完成下述几个部分的要求：

1、描述几个不同的指标，用来有效评估一个国家的卫生保健系统，如居民的平均预期寿命。考虑使用什么指标来比较现有的和潜在的系统？试着将多种衡量标准有机结合，使它们在能更好地衡量卫生保健系统的服务质量？

2、筛选当前的数据来源提供的原始数据，以计算第一部分提到的度量标准，基于这些可用的数据可能需要修改上述指标列表。解释为什么选择这些数据，并说明它们如何能够被用来评估和比较存在于不同国家的卫生保健系统。

3、选取至少三个最重要的、可行的指标来比较不同的卫生保健系统，并说明这些指标的有效性。考虑这些指标能否反映出卫生保健系统的历史变化以及搜集这些指标的难易程度。



- 4、使用选取的三个 (或更多) 的指标, 用能找到的最新的数据资料, 来对美国与另外一个被认为具有较好卫生保健系统的国家进行比较, 哪个国家有更好的卫生保健系统? 并评价结果的合理性。
- 5、使用选取的三个 (或更多) 的指标, 用能找到的最新的数据资料, 来对美国与另外一个被认为具有较差卫生保健系统的国家进行比较。
- 6、根据选择的指标的计算结果, 针对一个国家 (美国或其他) 的卫生保健系统提出改进意见, 建立预测模型来确定这些改进意见对总体卫生保健系统的提高作用。

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### **MCM 2009 A: Designing a Traffic Circle**

Many cities and communities have traffic circles—from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road that gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a Traffic Engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

### **MCM 2009 B: Energy and the Cell Phone**

This question involves the “energy” consequences of the cell phone revolution. Cell phone usage is mushrooming, and many people are using cell phones and giving up their landline telephones. What is the consequence of this in terms of electricity use? Every cell phone comes with a battery and a recharger.

#### **Requirement 1**

Consider the current US, a country of about 300 million people. Estimate from available data the number  $H$  of households, with  $m$  members each, that in the past were serviced by landlines. Now, suppose that all the landlines are replaced by cell phones; that is, each of the  $m$  members of the household has a cell phone. Model the consequences of this change for electricity utilization in the current US, both during the transition and during the steady state. The analysis should take into account the need for charging the batteries of the cell phones, as well as the fact that cell phones do not last as long as landline phones (for example, the cell phones get lost and break).

#### **Requirement 2**

Consider a second “Pseudo US”—a country of about 300 million people with about the same economic status as the current US. However, this emerging country has neither landlines nor cell phones. What is the optimal way of providing phone service to this country from an energy perspective? Of course, cell phones have many social consequences and uses that landline phones do not allow. A discussion of the broad and hidden consequences of having only landlines, only cell phones, or a mixture of the two is welcomed.

#### **Requirement 3**

Cell phones periodically need to be recharged. However, many people always keep their recharger plugged in. Additionally, many people charge their phones every night, whether they need to be recharged or not. Model the energy costs of this wasteful practice for a Pseudo US based upon your answer to Requirement 2. Assume that the Pseudo US supplies electricity from oil. Interpret your results in terms of barrels of oil.

#### **Requirement 4**

Estimates vary on the amount of energy that is used by various recharger types (TV, DVR, computer peripherals, and so forth) when left plugged in but not charging the device. Use accurate data to model the energy wasted by the current US in terms of barrels of oil per day.

#### **Requirement 5**

Now consider population and economic growth over the next 50 years. How might a typical Pseudo US grow? For each 10 years for the next 50 years, predict the energy needs for providing phone service based upon your analysis in the first three requirements. Again, assume electricity is provided from oil. Interpret your predictions in term of barrels of oil.

### **ICM 2009 C: Creating Food Systems: Re-Balancing Human-Influenced Ecosystems**

Click the Title Below To View a PDF of Problem C

[Creating Food Systems: Re-Balancing Human-Influenced Ecosystems](#)

 ICM\_2009.pdf



### MCM 2009 A题：设计一个交通环路

许多城市和社区都存在交通圈，从圈中（如在巴黎凯旋门，胜利纪念碑在曼谷）到许多大的小巷中圈与一个或两个小的通道。这些交通圈立场的一些停止标志或道路上的每个传入让步标志优先的交通圈已经有的位置上的圆圈在每个传入的道路交通优先传入让步标志，有的位置A红绿灯对每个传入路（右转上没有红灯允许）。其他设计也是可能的。

这个问题的目的是用一个模型，以确定如何最好地控制交通，绕流，并走出了一条循环。国家明确的目标，你在你的模型的使用做出的最佳选择时，应考虑影响这种选择的因素。包括一个不超过两个双页纸，应说明了交通工程师如何使用你的模型，以帮助选择适当的流量控制任何特定交通圈的方法更多的技术总结。也就是说，总结一下，每种类型交通控制方法的应该在什么条件下使用。当推荐使用交通灯时，说明确定每个指示灯应多少秒为绿色（可根据不同的日期和时间等因素）的方法。用具体例子说明您的模型是如何工作的。

### MCM 2009 B题：能源与手机

这个问题涉及到手机革命的“能源”后果。使用手机是雨后春笋，许多人使用手机，放弃了他们的固定电话。从用电方面考虑这种行为的结果是什么？每部手机配备了一个电池和一个充电器。

#### 要求 1

考虑到目前美国，约300万人口的国家。从现有户数小时的数据估计，每个成员有手机，这在过去是由固定电话服务。现在，假设所有的座机被手机取代，那就是因为家庭的每个成员都有一部手机。用模型估计用电在美国目前的变化后果，无论是在过渡期间还是稳定期间。分析家应该考虑到为手机电池充电的需要，以及一个事实，即手机不会像固定电话长时间被使用（例如，手机丢失或者损坏）。

#### 要求 2

考虑第二个“伪美国”他是一个与当前美国具有相同的经济地位约300万人口的国家。然而，这个新兴国家既没有固定电话也没有手机。从能源的角度考虑，什么是提供电话服务的最佳方式？当然，手机有很多社会后果，并且使用该固定电话不允许的。关于只有一个有固定电话，只使用一个手机，或者是两者的混合物所引起的广泛和隐藏的后果的讨论。

#### 要求 3

手机需要定期充电。然而，许多人始终保持自己的充电器插上此外，很多人每天晚上自己的手机充电，无论是否需要充电。根据你对要求2的答案，为这种伪美国的浪费做法建立模型。假设伪美国用供应的石油发电。从石油桶的方面，解释你的结果。

#### 要求 4

估计由使用不同的充电器类型（电视，录像机，计算机外围设备，等等）但离开时不充电的设备插入使用的能源量。使用准确的数据为由当前美国浪费了每天的石油桶上的能量建立模型。

#### 要求 5

现在考虑在未来50年的人口和经济增长。一个典型的伪美国怎么可能发展壮大？在未来50年中每10年间，根据你在要求1和要求3的分析，预测提供电话服务的能源需要。同样，假设提供电力来自石油。从石油桶的方面，解释你的预测。

### ICM 2009 C题：建立食品系统：重新平衡人类影响的生态系统

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### MCM 2010 A: A Primer on the Physics of Baseball

Explain the “sweet spot” on a baseball bat. Every hitter knows that there is a spot on the fat part of a baseball bat where maximum power is transferred to the ball when hit. Why isn’t this spot at the end of the bat? A simple explanation based on torque might seem to identify the end of the bat as the sweet spot, but this is known to be empirically incorrect.

Develop a model that helps explain this empirical finding. Some players believe that “corking” a bat (hollowing out a cylinder in the head of the bat and filling it with cork or rubber, then replacing a wood cap) enhances the “sweet spot” effect. Augment your model to confirm or deny this effect.

Does this explain why Major League Baseball prohibits “corking”? Does the material out of which the bat is constructed matter? That is, does this model predict different behavior for wood (usually ash) or metal (usually aluminum) bats? Is this why Major League Baseball prohibits metal bats?

### MCM 2010 B: Criminology

In 1981 Peter Sutcliffe was convicted of thirteen murders and subjecting a number of other people to vicious attacks. One of the methods used to narrow the search for Mr. Sutcliffe was to find a “center of mass” of the locations of the attacks. In the end, the suspect happened to live in the same town predicted by this technique. Since that time, a number of more sophisticated techniques have been developed to determine the “geographical profile” of a suspected serial criminal based on the locations of the crimes.

Your team has been asked by a local police agency to develop a method to aid in their investigations of serial criminals. The approach that you develop should make use of at least two different schemes to generate a geographical profile. You should develop a technique to combine the results of the different schemes and generate a useful prediction for law enforcement officers. The prediction should provide some kind of estimate or guidance about possible locations of the next crime based on the time and locations of the past crime scenes. If you make use of any other evidence in your estimate, you must provide specific details about how you incorporate the extra information. Your method should also provide some kind of estimate about how reliable the estimate will be in a given situation, including appropriate warnings.

In addition to the required one-page summary, your report should include an additional two-page executive summary. The executive summary should provide a broad overview of the potential issues. It should provide an overview of your approach and describe situations when it is an appropriate tool and situations in which it is not an appropriate tool. The executive summary will be read by a chief of police and should include technical details appropriate to the intended audience.

### **MCM 2010 A题：解释棒球棒上的“最佳击球点”**

每一个棒球手都知道在棒球棒比较粗的部分有一个击球点，这里可以把打击球的力量最大程度地转移到球上。为什么这个点不在棒球棒的最末端？基于力矩的解释或许可以解释确定棒球棒的最末端就是最佳的击球点，但是实际当中并不是这样的。

构建一个模型帮助解释实际当中的这个发现。有一些棒球手相信在最佳击球点添充上软木塞可以提高打击效果（在球棒头部挖一个圆柱状槽，填充上软木塞或者橡皮。进一步扩展模型确认或者否定该结论。这个解释是否可以解释为什么棒球联盟否定这种做法。

球棒是否和材质有关系，模型是否可以预测木头和金属球棒的不同打击效果？这是否是联盟禁止金属球棒的原因？

### **MCM 2010 B题：犯罪学问题**

1981年Peter Sutcliffe(萨克利夫)被判刑因为他参与了十三起谋杀和对其他人的恶毒攻击。缩小搜索Sutcliffe的方法之一是发现一个攻击位置的“质心”。最终犯罪嫌疑人恰好生活在该方法预测的同一个小镇。从那时起，已经发展出一系列更加复杂的技术用来预测基于犯罪地点的具有地理效应（地理轮廓）的系列犯罪行为。

你的团队被一个当地警察局要求发展出一种方法来帮助他们的系列犯罪调查。你们的方法应该至少需要利用两种不同的情景以生成地理效应（地理轮廓），进而根据不同情况下的分析结果对执法人员提供有效的预测。基于以往犯罪的时间和位置，预测信息应该提供一些估计或指导下次可能的犯罪地点。如果在预测中用到了其它的信息，必须提供特别的细节说明告诉我们这些信息是如何被整合的。你们的方法中也应该包括在给定条件下（包括适当警告信息）下预测的可靠性估计。

除了必要的一页小结，你们的报告应该包括两页额外的总结。这两页总结应该提供潜在问题的概述，它应该提供什么情况下，你们提出的方法是一个恰当的工具，在哪些情况下它不是。执行摘要将宣读了警察局长，并应包括适当的目标受众的技术细节。

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### **MCM 2011 A: Snowboard Course**

Determine the shape of a snowboard course (currently known as a “halfpipe”) to maximize the production of “vertical air” by a skilled snowboarder.

“Vertical air” is the maximum vertical distance above the edge of the halfpipe.

Tailor the shape to optimize other possible requirements, such as maximum twist in the air.

What tradeoffs may be required to develop a “practical” course?

### **MCM 2011 B: Repeater Coordination**

The VHF radio spectrum involves line-of-sight transmission and reception. This limitation can be overcome by “repeaters,” which pick up weak signals, amplify them, and retransmit them on a different frequency. Thus, using a repeater, low-power users (such as mobile stations) can communicate with one another in situations where direct user-to-user contact would not be possible. However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies.

In addition to geographical separation, the “continuous tone-coded squelch system” (CTCSS), sometimes nicknamed “private line” (PL), technology can be used to mitigate interference problems. This system associates to each repeater a separate subaudible tone that is transmitted by all users who wish to communicate through that repeater. The repeater responds only to received signals with its specific PL tone. With this system, two nearby repeaters can share the same frequency pair (for receive and transmit); so more repeaters (and hence more users) can be accommodated in a particular area.

For a circular flat area of radius 40 miles radius, determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users. Assume that the spectrum available is 145 to 148 MHz, the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, and there are 54 different PL tones available.

How does your solution change if there are 10,000 users?

Discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

### **MCM 2011 A题：滑雪场**

请设计一个单板滑雪场（现为“半管”或“U型池”）的形状，以便能使熟练的单板滑雪选手最大限度地产生垂直腾空。

“垂直腾空”是超出“半管”边缘以上的最大的垂直距离。

定制形状时要优化其他可能的要求，如：在空中产生最大的身体扭曲。

在制定一个“实用”的场地时哪些权衡因素可能需要？

### MCM 2011 B题：中继站的协调

特高频无线电频谱包含信号的发送和接受。这种限制可以被中继站所克服。中继站可以捕捉到微弱的信号，然后把它放大，再用不同的频率重新发送。这样，低功耗的用户，例如移动电话用户，在不能直接与其他用户联系的地方可以通过中继站来保持联系。然而，中继站之间会互相影响，除非彼此之间有足够远的距离或通过充分分离的频率来传送。

除了地理的分离、“连续编码音调控制系统”(CTCSS),有时被称为“私人专线”(PL)、通过这项技术可以减轻干扰问题。该系统连接每个中继站，靠的是所有通过同一个中继站连接的用户发送的独立的亚音频音调来连接。中继站只回应接收到的具有特殊PL的语调的信号。通过这个系统，两个附近的中继站可以共享相同的频率对（包括接收和发送）；对于更多的中继站（并且更多的用户）可以安置在一个特定的区域。

在一个半径 40 英里的圆形区域，请你设计一个方案，用最少量的中继站来容纳 1000 同时在线用户。假设频谱范围是 145 到 148 兆赫，在中继站中的发射机的频率要么是 600 千赫以上，要么低于接收机频率 600 千赫、并且这里有 54 个不同的 PL 可用。

如果这里有 10,000 个用户，如何改变你的解决方案。

在由于山区引起信号传播的阻碍的地区，讨论这样的情形。

## 2012.note

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### MCM 2012 A: The Leaves of a Tree

"How much do the leaves on a tree weigh?" How might one estimate the actual weight of the leaves (or for that matter any other parts of the tree)? How might one classify leaves? Build a mathematical model to describe and classify leaves. Consider and answer the following:

Why do leaves have the various shapes that they have?

Do the shapes "minimize" overlapping individual shadows that are cast, so as to maximize exposure? Does the distribution of leaves within the "volume" of the tree and its branches effect the shape?

Speaking of profiles, is leaf shape (general characteristics) related to tree profile/branching structure?

How would you estimate the leaf mass of a tree? Is there a correlation between the leaf mass and the size characteristics of the tree (height, mass, volume defined by the profile)?

In addition to your one page summary sheet prepare a one page letter to an editor of a scientific journal outlining your key findings.

### MCM 2012 B: Camping along the Big Long River

Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar-powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish.. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river. Currently, X trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park managers have been asked to allow more trips to travel down the river. They want to determine how they might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other words, how many more boat trips could be added to the Big Long River's rafting season? The river managers have hired you to advise them on ways in which to develop the best schedule and on ways in which to determine the carrying capacity of the river, remembering that no two sets of campers can occupy the same site at the same time. In addition to your one page summary sheet, prepare a one page memo to the managers of the river describing your key findings.

### MCM 2012 A题：树的叶子

"一棵树的叶子有多重？"怎么能估计树的叶子（或者树的任何其它部分）的实际重量？怎样对叶子进行分类？建立一个数学模型来对叶子进行描述和分类。模型要考虑和回答下面的问题：

为什么叶子具有各种形状？

叶子之间是要将相互重叠的部分最小化，以便可以最大限度的接触到阳光吗？树叶的分布以及树干和枝杈的体积影响叶子的形状吗？

就轮廓来讲，叶形（一般特征）是和树的轮廓以及分枝结构有关吗？

你将如何估计一棵树的叶子质量？叶子的质量和树的尺寸特征（包括和外形轮廓有关的高度、质量、体积）有联系吗？

除了你的一页摘要以外，给科学杂志的编辑写一封信，阐述你的主要发现。

### MCM 2012 B题：大长河沿岸露营

大长河（225 英里）可以欣赏风景的意见和令人兴奋的白色水激流。这条河无法通过远足活动访问，所以享受它的唯一方法就是去河旅行，需要几天的露营。河 trips 一切开始在第一次启动和退出在最终退出，河下游 225 英里。乘客拿桨-动力橡胶木筏，平均旅行 4 英里每小时，或机动的船，平均旅行 8 英里每小时。Trips 的范围是从 6 到 18 夜的露营在河上的开始完成...负责管理这条河的政府机构希望每次去享受荒野的经验，与其他团体的船在河上最少接触。目前，X trips 旅行下大长条河每年在六个月期间（今年太冷河旅行的其余部分）。大长江，相当均匀分布于整个河流廊道上有 Y 营地。鉴于河漂流的兴起，公园经理被要求允许更多的旅行，沿着这条河。他们想要确定如何，他们可能会安排班次，不同的持续时间（以夜河上衡量）的最优组合和推进（电机或桨）将利用露营可能的最佳方式。换句话说，多少更多船旅行能被添加到大长河漂流季节？河经理雇你，向他们提供关于各种方法，制定最佳计划和各种方法，确定承载力的河，记住没有两个集的露营者可以占用相同的站点，在同一时间。除了您一页摘要表外，还准备管理者的描述您的主要调查结果的河的一页备忘录。

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### MCM 2013 A: The Ultimate Brownie Pan

When baking in a rectangular pan heat is concentrated in the 4 corners and the product gets overcooked at the corners (and to a lesser extent at the edges). In a round pan the heat is distributed evenly over the entire outer edge and the product is not overcooked at the edges. However, since most ovens are rectangular in shape using round pans is not efficient with respect to using the space in an oven.

Develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes - rectangular to circular and other shapes in between.

Assume

1. A width to length ratio of  $W/L$  for the oven which is rectangular in shape.
2. Each pan must have an area of  $A$ .
3. Initially two racks in the oven, evenly spaced.

Develop a model that can be used to select the best type of pan (shape) under the following conditions: 1. Maximize number of pans that can fit in the oven ( $N$ ) 2. Maximize even distribution of heat ( $H$ ) for the pan 3. Optimize a combination of conditions (1) and (2) where weights  $p$  and  $(1-p)$  are assigned to illustrate how the results vary with different values of  $W/L$  and  $p$ .

In addition to your MCM formatted solution, prepare a one to two page advertising sheet for the new Brownie Gourmet Magazine highlighting your design and results.

### MCM 2013 B: Camping along the Big Long River

Fresh water is the limiting constraint for development in much of the world. Build a mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the projected water needs of [pick one country from the list below] in 2025, and identify the best water strategy. In particular, your mathematical model must address storage and movement; de-salinization; and conservation. If possible, use your model to discuss the economic, physical, and environmental implications of your strategy. Provide a non-technical position paper to governmental leadership outlining your approach, its feasibility and costs, and why it is the "best water strategy choice."

Countries: United States, China, Russia, Egypt, or Saudi Arabia

### ICM 2013 C: Network Modeling of Earth's Health

**Background:** Society is interested in developing and using models to forecast the biological and environmental health conditions of our planet. Many scientific studies have concluded that there is growing stress on Earth's environmental and biological systems, but there are very few global models to test those claims. The UN-backed *Millennium Ecosystem Assessment Synthesis Report* found that nearly two-thirds of Earth's life-supporting ecosystems--including clean water, pure air, and stable climate--are being degraded by unsustainable use. Humans are blamed for much of this damage. Soaring demands for food, fresh water, fuel, and timber have contributed to dramatic environmental changes; from deforestation to air, land, and water pollution. Despite the considerable research being conducted on local habitats and regional factors, current models do not adequately inform decision makers how their provincial policies may impact the overall health of the planet. Many models ignore complex global factors and are unable to determine the long-range impacts of potential policies. While scientists realize that the complex relationships and cross-effects in myriad environmental and biological systems impact Earth's biosphere, current models often ignore these relationships or limit the systems' connections. The system complexities manifest in multiple interactions, feedback loops, emergent behaviors, and impending state changes or tipping points. The recent Nature article written by 22 internationally known scientists entitled "Approaching a state shift in Earth's biosphere" outlines many of the issues associated with the need for scientific models and the importance of predicting potential state changes of the planetary health systems. The article provides two specific quantitative modeling challenges in their call for better predictive models:

- 1) To improve bio-forecasting through global models that embrace the complexity of Earth's interrelated systems and include the effects of local conditions on the global system and vice versa.
- 2) To identify factors that could produce unhealthy global state-shifts and to show how to use effective ecosystem management to prevent or limit these impending state changes.

The resulting research question is whether we can build global models using local or regional components of the Earth's health that predict potential state changes and help decision makers design effective policies based on their potential



impact on Earth's health. Although many warning signs are appearing, no one knows if Planet Earth is truly nearing a global tipping point or if such an extreme state is inevitable.

The *Nature* article and many others point out that there are several important elements at work in the Earth's ecosystem (e.g., local factors, global impacts, multi-dimensional factors and relationships, varying time and spatial scales). There are also many other factors that can be included in a predictive model — human population, resource and habitat stress, habitat transformation, energy consumption, climate change, land use patterns, pollution, atmospheric chemistry, ocean chemistry, bio diversity, and political patterns such as social unrest and economic instability. Paleontologists have studied and modeled ecosystem behavior and response during previous cataclysmic state shifts and thus historic-based qualitative and quantitative information can provide background for future predictive models. However, it should be noted that human effects have increased significantly in our current biosphere situation.

### Requirements:

You are members of the International Coalition of Modelers (ICM) which will soon be hosting a workshop entitled "Networks and Health of Planet Earth" and your research leader has asked you to perform modeling and analysis in advance of the workshop. He requires your team to do the following:

**Requirement 1:** Build a dynamic global network model of some aspect of Earth's health (you develop the measure) by identifying local elements of this condition (network nodes) and appropriately connecting them (network links) to track relationship and attribute effects. Since the dynamic nature of these effects is important, this network model must include a dynamic time element that allows the model to predict future states of this health measure. For example, your nodes could be nations, continents, oceans, habitats, or any combination of these or other elements which together constitute a global model. Your links could represent nodal or environmental influences, or the flow or propagation of physical elements (such as pollution) over time. Your health measure could be any element of Earth's condition to include demographic, biological, environmental, social, political, physical, and/or chemical conditions. Be sure to define all the elements of your model and explain the scientific bases for your modeling decisions about network measures, nodal entities, and link properties. Determine a methodology to set any parameters and explain how you could test your model if sufficient data were available. What kinds of data could be used to validate or verify the efficacy of your model? (Note: If you do not have the necessary data to determine parameters or perform verification, do not throw out the model. Your supervisor realizes that, at this stage, good creative ideas and theories are as important as verified data-based models.) Make sure you include the human element in your model and explain where human behavior and government policies could affect the results of your model.

**Requirement 2:** Run your model to see how it predicts future Earth health. You may need to estimate parameters that you would normally determine from data. (Remember, this is just to test and understand the elements of your model, not to use it for prediction or decision making.) What kinds of factors will your model produce? Could it predict state change or tipping points in Earth's condition? Could it provide warning about global consequences of changing local conditions? Could it inform decision makers on important policies? Do you take into account the human elements in your measures and network properties?

**Requirement 3:** One of the powerful elements of using network modeling is the ability to analyze the network structure. Can network properties help identify critical nodes or relationships in your model? If so, perform such analysis. How sensitive is your model to missing links or changing relationships? Does your model use feedback loops or take into account uncertainties? What are the data collection issues? Does your model react to various government policies and could it thus help inform planning?

**Requirement 4:** Write a 20-page report (summary sheet does not count in the 20 pages) that explains your model and its potential. Be sure to detail the strengths and weaknesses of the model. Your supervisor will use your report as a major theme in the upcoming workshop and, if it is appropriate and insightful to planetary health modeling, will ask you to present at the upcoming workshop. Good luck in your network modeling work!

### Potentially helpful references include:

Anthony D. Barnosky, Elizabeth A. Hadly, Jordi Bascompte, Eric L. Berlow, James H. Brown, Mikael Fortelius, Wayne M. Getz, John Harte, Alan Hastings, Pablo A. Marquet, Neo D. Martinez, Arne Mooers, Peter Roopnarine, Geerat Vermeij, John W. Williams, Rosemary Gillespie, Justin Kitzes, Charles Marshall, Nicholas Matzke, David P. Mindell, Eloy Revilla, Adam B. Smith. "Approaching a state shift in Earth's biosphere," *Nature*, 2012; 486 (7401): 52 DOI: 10.1038/nature11018

Donella Meadows, Jorgen Randers, and Dennis Meadows. *Limits to Growth: The 30-year update*, 2004.

Robert Watson and A.Hamid Zakri. *UN Millennium Ecosystem Assessment Synthesis Report*, United Nations Report, 2005.

University of California - Berkeley. "Evidence of impending tipping point for Earth." *ScienceDaily*, 6 Jun. 2012. Web. 22 Oct. 2012.

### MCM 2013 A题: 最佳巧克力蛋糕烤盘

当你使用一个矩形的烤盘烘烤食物时, 热量会集中在烤盘的四个角落, 于是角落处的食物就会被烤糊(烤盘边缘处也有类似情形, 但程度轻一些)。当使用一个圆形烤盘时, 热量会均匀地分布在整个边缘上, 就不会再有边缘上烤糊的现象发生。然而, 由于大多数烤箱内部是矩形的, 如果使用圆形烤盘, 就不能充分利用烤箱的内部空间了。

建立一个模型, 来描述热量在不同形状的烤盘表面的分布。这些形状包括矩形、圆形以及两者之间的过渡形状。

假设, 1、矩形烤箱的宽长比为  $W/L$ 。2、每个烤盘的面积为  $A$ 。3、先考虑烤箱内有两个搁架且间隔均匀的情形。

建立一个模型用以选择满足下列条件的最佳烤盘的形状: (1)、使得烤箱中可以容纳的烤盘数量( $N$ )最大。(2)、使得烤盘上的热量分布( $H$ )最均匀。3、综合(1)、(2)两个条件, 并且为(1)、(2)分别设置权重  $p$  和  $(1-p)$ , 寻求最优。然后描述结果随着  $W/L$  和  $p$  的值的变化是如

何变化的。

除了撰写 MCM 论文之外,你还要为新一期巧克力蛋糕美食杂志准备一个一至两页的广告,阐述你的设计和结果的亮点所在。

### **MCM 2013 B题: 水, 水, 无处不在**

淡水资源是世界上许多地方持续发展的限制因素。建立数学模型来确定一个有效的,可行的,低成本的2013年用水计划,来满足某国(从下方的列表中选择一个国家)未来(2025年)的用水需求,并确定最优的淡水分配计划。特别的,你的数学模型必须包括储存、运输、淡化和节水等环节。如果可能的话,用你的模型来讨论你的计划对经济,自然和环境的影响。提供一个非技术性的意见书给政府领导概述你的方法,以及方法的可行性和成本,以及它为什么是“最好的用水计划的选择”。

国家: 美国、中国、俄罗斯、埃及或者沙特阿拉伯。

### **ICM 2013 C题: 地球健康的网络建模**

**背景:** 全社会都在关注如何研究与应用模型来预测我们地球的生物和环境的健康状况。许多科学研究表明地球的环境和生物系统所面对的压力正在增加,但是能够验证这一观点的全局性模型却很少。联合国支持的千年生态系统评估综合报告指出:由于无节制的开采利用,地球上近三分之二的生命支持生态系统正在逐渐退化,这一系统包含干净的水,纯净的空气,和稳定的气候等。人类在很大程度上要对这种环境破坏负责。对粮食,淡水,燃料,以及木材的海量消耗将导致地球环境的剧烈变化,如森林遭砍伐,土地,空气和水遭污染。尽管已有大量关于局部栖息地和地域因素的环境健康研究工作,但是目前所有的模型还是不能明确告知决策者各地方的局部政策是如何影响地球的整体健康状况的。

许多模型忽视了复杂的整体因素,不能用于判定既定政策的长期影响。而科学家们意识到大量环境和生态系统中的复杂关系和交叉作用影响着地球的生物圈,现有的模型常常忽略了这些关系或者限制了系统的各要素之间的联系。系统的复杂性体现在多元互动,反馈回路,应急行为以及临界状态变化或者临界点的存在。最近由22个国际知名科学家在《自然》杂志上发表了题为《地球的生物圈正在接近状态的变化》的论文,该论文概述了许多需要有关科学模型解决的问题,并且阐述了预测地球健康系统的潜在状态变化的重要性。这篇文章在呼吁建立更好的预测模型时提出了两个具体的量化建模问题:

(1) 改进生态预测全局模型,应考虑地球上相互关联的系统间的复杂性,包括局部条件如何影响全局模型,以及全局系统对局部条件的影响。

(2) 确定那些可能会产生不良的整体状态变化的因素,并且展示如何利用有效的生态管理来阻止或者限制这些临界状态发生。

所导致的研究问题是我们是否可以利用局部的或者是区域性的地球健康状态以建立整体模型来预测潜在的状态改变,并且帮助决策者基于这些潜在的影响地球健康的因素制定一些有效的政策。虽然许多警告性的状况正在出现,但是没有人知道是否地球健康正在真正接近整体临界点,也没有人知道这种极端的状态是否是可避免的。

《自然》杂志的这篇文章和许多其他的文章都指出地球的生态系统中有几个重要的因素正在起作用(例如:局部因素、整体影响、多维因素和相互关系、时空变化)。还有许多其他因素,诸如人口、资源和栖息地紧缩、栖息地变迁、能源消耗、气候变化、土地开采模式、污染、空气化学物、海洋化学物、生物多样性,以及诸如社会动荡和经济不稳定的政治模式,这些都可以包括在一个预测模型中。古生物学家已经研究并且模拟了过去几个地球环境大幅度状态改变时期的生态系统的行为和反应。因此,基于历史的定性和定量的信息可以为未来的预测模型提供背景。然而,应该指出的是,在当前的生物圈,人类的影响已经比过去有显著的增加。

**要求:** 假设你们是ICM的成员,将要主办一个主题为“网络与地球的健康”的研讨会,你们的研究组长要求你们在研讨会以前预先进行建模和分析。他要求你们小组做如下事情:

**要求1:** 对于地球的健康状况的某一影响方面(你们自己选定量化指标),建立一个动态的全球网络模型,并通过选定这种情况下的局部要素(即网络节点)以及要素间的连接(即网络链路)来跟踪它们间的关系和属性效应。由于这些效应的动态本质是很重要的,这个网络模型必须包括一个动态的时间元素来使模型可以预测这个健康状况的影响因素的未来状态。例如,你们定义的节点可以是国家,洲,海洋,栖息地或这些元素的任意组合,或者是能够共同构成一个整体模型的其他元素。你们定义的链路可以表示随着时间变化的节点或环境的影响,或各种物理因素(如污染)的流动与演化。你们所选的健康影响量化指标可以是地球状况的任意一方面,包括人口统计学方面的,生物学方面的,环境的,社会的,政治的,物理的或化学方面的状况。请确保对你们模型中的所有元素都进行了定义,并解释你们关于网络方式,节点实体和链路属性的模型决策的科学依据。在设置所有参数后,如果有充足的样本数据,请用一种方法来解释如何检验你们的模型。哪些数据能够用来证实或验证你们的模型?(注意:如果你们没有必要的数据来确定参数或进行验证,也不要抛弃你们的模型。在这种情形下,你们的组长认为创造性的想法和理论模型与经过数据验证的模型是同等重要的。)(在你们的模型中一定要考虑人类活动因素,并解释人类行为和政府政策在哪些地方影响你们模型的结果。

**要求2:** 基于你们的模型,给出预测未来的地球健康状况的方法。你们首先要估计参数,通常情况下,这些参数值是通过实际数据计算而得。(请记住:这只是测试和理解你们模型的元素,不是用它来预测和决策。)(你们的模型将会受哪些因素影响?你们的模型是否能预测地球健康情况的状态变化或临界点?是否能通过改变局部条件获得对全局结果带来的影响进行预警?是否能在重要政策上对决策者提供决策参考资料?在你们所选的地球健康影响指标和网络属性中考虑了人类因素吗?

**要求3:** 网络模型的一个强有力优势在于它具有分析网络结构的能力。在你们的模型中,网络属性能帮助识别关键节点或节点间的关系吗?如果能,对此进行分析。你们的模型对于链路的缺失与节点间关系的改变的敏感性如何?你们的模型使用反馈回路或考虑了不确定因素吗?数据收集方面有哪些问题?你们的模型能对各种政府政策作出反应并因此有助于政府制定计划吗?

**要求4:** 写一个20页(不包括摘要)的报告来解释你们的模型和它的潜力。一定要详细说明你们模型的优缺点。在将要到来的研讨会上,你们的组长将会以你们的报告作为一个主要议题,如果你们的模型是合理的并且对地球的健康状况建模方面有深刻的见解,组长会请你们在研讨会上作报告。

祝你们的网络建模工作顺利!

可能会用到的参考文献如下:

- Anthony D. Barnosky, Elizabeth A. Hadly, Jordi Bascompte, Eric L. Berlow, James H. Brown, Mikael Fortelius, Wayne M. Getz, John Harte, Alan Hastings, Pablo A. Marquet, Neo D. Martinez, Arne Mooers, Peter Roopnarine, Geerat Vermeij, John W. Williams, Rosemary Gillespie, Justin Kitzes, Charles Marshall, Nicholas Matzke, David P. Mindell, Eloy Revilla, Adam B. Smith. "Approaching a state shift in Earth's biosphere, ". Nature, 2012; 486 (7401): 52 DOI: 10.1038/nature11018
- Donella Meadows, Jorgen Randers, and Dennis Meadows. Limits to Growth: The 30-year update, 2004
- Robert Watson and A. Hamid Zakri. UN Millennium Ecosystem Assessment Synthesis Report, United Nations Report, 2005.
- University of California at Berkeley. "Evidence of impending tipping point for Earth. " Science Daily, 6 Jun. 2012. Web. 22 Oct. 2012.

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## MCM 2014 A: The Keep-Right-Except-To-Pass Rule

In countries where driving automobiles on the right is the rule (that is, USA, China and most other countries except for Great Britain, Australia, and some former British colonies), multi-lane freeways often employ **a rule that requires drivers to drive in the right-most lane unless they are passing another vehicle, in which case they move one lane to the left, pass, and return to their former travel lane.**

Build and analyze a mathematical model to analyze the performance of this rule in light and heavy traffic. You may wish to examine tradeoffs between traffic flow and safety, the role of under- or over-posted speed limits (that is, speed limits that are too low or too high), and/or other factors that may not be explicitly called out in this problem statement. Is this rule effective in promoting better traffic flow? If not, suggest and analyze alternatives (to include possibly no rule of this kind at all) that might promote greater traffic flow, safety, and/or other factors that you deem important.

In countries where driving automobiles on the left is the norm, argue whether or not your solution can be carried over with a simple change of orientation, or would additional requirements be needed.

Lastly, the rule as stated above relies upon human judgment for compliance. If vehicle transportation on the same roadway was fully under the control of an intelligent system – either part of the road network or imbedded in the design of all vehicles using the roadway – to what extent would this change the results of your earlier analysis?

## MCM 2014 B: College Coaching Legends

*Sports Illustrated*, a magazine for sports enthusiasts, is looking for the “best all time college coach” male or female for the previous century. Build a mathematical model to choose the best college coach or coaches (past or present) from among either male or female coaches in such sports as college hockey or field hockey, football, baseball or softball, basketball, or soccer. Does it make a difference which time line horizon that you use in your analysis, i.e., does coaching in 1913 differ from coaching in 2013? Clearly articulate your metrics for assessment. Discuss how your model can be applied in general across both genders and all possible sports. Present your model’s top 5 coaches in each of 3 different sports.

In addition to the MCM format and requirements, prepare a 1-2 page article for *Sports Illustrated* that explains your results and includes a non-technical explanation of your mathematical model that **sports fans** will understand.

### 问题A：车辆右行

在一些规定汽车靠右行驶的国家（即美国，中国和其他大多数国家，除了英国，澳大利亚和一些前英国殖民地），多车道的高速公路经常使用这样一条规则：要求司机开车时在最右侧车道行驶，除了在超车的情况下，他们应移动到左侧相邻的车道，超车，然后恢复到原来的行驶车道（即最右车道）。

建立和分析一个数学模型，来分析这一规则在轻型和重型交通中的性能（即车辆较少和交通较拥堵时）。你可以研究交通流量和安全二者间的平衡，最高或最低车速限制的作用，和其它在这个问题陈述中没有明确说明的影响因素。这条规则能否有效地提升交通流量？如果不能，请分析并建议一个替代方案，这个方案可以提升交通流量、安全性，或您认为重要的其他因素。

在规定汽车靠左行驶的国家，证明您的解决方案能否简单地改变方向就可应用在这些国家，或是否要考虑额外的要求。

最后，如上所述的规则依赖于人的行为标准（即人们是否遵守这样的交通规则）。如果相同的交通情况完全在一个智能系统的控制之下——无论是道路网的部分或是行驶在道路上的车辆都嵌入了这个系统——在何种程度上，这会改变你刚才分析的结果？

### 问题B：大学教练的故事

体育画报，为运动爱好者杂志，正在寻找上个世纪堪称“史上最优秀大学教练”的男性或女性。建立数学模型，选出在大学曲棍球，足球，棒球或垒球，篮球，橄榄球领域（过去或现在）最好的一个或多个、男性或女性大学教练。你在你的分析中使用的时间范围对结果有影响吗？比如说，在1913年执教的情况不同于2013年？清楚地说明您的评估指标。讨论你的模型怎样在男女性别和所有可能的运动中应用。展示由你的模型得到的3个不同的运动各自排名前5的教练。

除了MCM的格式和要求，准备体育画报一份1-2页的文章，用非技术性的解释向您的体育迷阐述你的结果。

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## MCM 2015 A: Eradicating Ebola

The world medical association has announced that their new medication could stop Ebola and cure patients whose disease is not advanced. Build a realistic, sensible, and useful model that considers not only the spread of the disease, the quantity of the medicine needed, possible feasible delivery systems, locations of delivery, speed of manufacturing of the vaccine or drug, but also any other critical factors your team considers necessary as part of the model to optimize the eradication of Ebola, or at least its current strain. In addition to your modeling approach for the contest, prepare a 1-2 page non-technical letter for the world medical association to use in their announcement.

## MCM 2015 B: Searching for a lost plane

Recall the lost Malaysian flight MH370. Build a generic mathematical model that could assist “searchers” in planning a useful search for a lost plane feared to have crashed in open water such as the Atlantic, Pacific, Indian, Southern, or Arctic Ocean while flying from Point A to Point B. Assume that there are no signals from the downed plane. Your model should recognize that there are many different types of planes for which we might be searching and that there are many different types of search planes, often using different electronics or sensors. Additionally, prepare a 1-2 page non-technical paper for the airlines to use in their press conferences concerning their plan for future searches.



### 问题A：根除埃博拉病毒

世界医学协会已经宣布他们的新药物能阻止埃博拉病毒并可治愈那些得非晚期疾病的患者。建立一个可行的，明智的，有用的模型，模型不仅要考虑疾病的蔓延、药物的需求量、可能可行的输送系统、输送的位置、疫苗或药物的生产速度，也要考虑你的团队认为有必要作为模型的一部分来优化根除埃博拉病毒，或者至少解决目前压力的其他重要因素。除了你比赛论文中的建模方法，你还要为世界医学协会在他们的公告中使用准备一封1-2页的科学技术性的信。

### 问题B：寻找失踪的飞机

回忆失踪的马航MH370，建立一个通用的数学模型，可以帮助“搜索者”规划一个有用方案，寻找从A点飞到B点可能坠毁在开放水域如大西洋、太平洋、印度洋，南大洋，或北冰洋的失联飞机。假设坠落的飞机没有信号。你的模型应该认识到因为有许多不同类型的飞机所以我们可能会有许多不同类型的搜索和通常使用不同电子或传感器的搜索飞机。另外，准备1-2页的科学技术性的发言稿，为航空公司在他们的新闻发布会上关于他们的未来搜索计划进行演讲。

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### MCM 2016 A: A Hot Bath

A person fills a bathtub with hot water from a single faucet and settles into the bathtub to cleanse and relax. Unfortunately, the bathtub is not a spa-style tub with a secondary heating system and circulating jets, but rather a simple water containment vessel. After a while, the bath gets noticeably cooler, so the person adds a constant trickle of hot water from the faucet to reheat the bathing water. The bathtub is designed in such a way that when the tub reaches its capacity, excess water escapes through an overflow drain.

Develop a model of the temperature of the bathtub water in space and time to determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water.

Use your model to determine the extent to which your strategy depends upon the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, and the motions made by the person in the bathtub. If the person used a bubble bath additive while initially filling the bathtub to assist in cleansing, how would this affect your model's results?

In addition to the required one-page summary for your MCM submission, your report must include a one-page non-technical explanation for users of the bathtub that describes your strategy while explaining why it is so difficult to get an evenly maintained temperature throughout the bath water.

### MCM 2016 B: Space Junk

The amount of small debris in orbit around earth has been a growing concern. It is estimated that more than 500,000 pieces of space debris, also called orbital debris, are currently being tracked as potential hazards to space craft. The issue itself became more widely discussed in the news media when the Russian satellite Kosmos-2251 and the USA satellite Iridium-33 collided on 10 February, 2009.

A number of methods to remove the debris have been proposed. These methods include small, space-based water jets and high energy lasers used to target specific pieces of debris and large satellites designed to sweep up the debris, among others. The debris ranges in size and mass from paint flakes to abandoned satellites. The debris' high velocity orbits make capture difficult.

Develop a time-dependent model to determine the best alternative or combination of alternatives that a private firm could adopt as a commercial opportunity to address the space debris problem. Your model should include quantitative and/or qualitative estimates of costs, risks, benefits, as well as other important factors. Your model should be able to assess independent alternatives as well as combinations of alternatives and be able to explore a variety of important "What if?" scenarios.

Using your model, determine whether an economically attractive opportunity exists or no such opportunity is possible. If a viable commercial opportunity exists as an alternative solution, provide a comparison of the different options for removing debris, and include a specific recommendation as to how the debris should be removed. If no such opportunity is possible, then provide innovative alternatives for avoiding collisions.

In addition to the required one-page summary for your MCM submission, your report must include a two-page Executive Summary that describes the options considered and major modeling results, and provides a recommendation for a particular action, combination of actions, or no action, as appropriate from your work. The Executive Summary should be written for high level policy makers and news media analysts who do not have a technical background.

### MCM 2016 C: The Goodgrant Challenge

The Goodgrant Foundation is a charitable organization that wants to help improve educational performance of undergraduates attending colleges and universities in the United States. To do this, the foundation intends to donate a total of \$100,000,000 (US\$100 million) to an appropriate group of schools per year, for five years, starting July 2016. In doing so, they do not want to duplicate the investments and focus of other large grant organizations such as the Gates Foundation and Lumina Foundation.

Your team has been asked by the Goodgrant Foundation to develop a model to determine an optimal investment strategy that identifies the schools, the investment amount per school, the return on that investment, and the time duration that the organization's money should be provided to have the highest likelihood of producing a strong positive effect on student performance. This strategy should contain a 1 to N optimized and prioritized candidate list of schools you are recommending for investment based on each candidate school's demonstrated potential for effective use of private funding, and an estimated return on investment (ROI) defined in a manner appropriate for a charitable organization such as the Goodgrant Foundation.



To assist your effort, the attached data file (ProblemCDATA.zip) contains information extracted from the U.S. National Center on Education Statistics ([www.nces.ed.gov/ipeds](http://www.nces.ed.gov/ipeds)), which maintains an extensive database of survey information on nearly all post-secondary colleges and universities in the United States, and the College Scorecard data set (<https://collegescorecard.ed.gov>) which contains various institutional performance data. Your model and subsequent strategy must be based on some meaningful and defensible subset of these two data sets.

In addition to the required one-page summary for your MCM submission, your report must include a letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation, Mr. Alpha Chiang, that describes the optimal investment strategy, your modeling approach and major results, and a brief discussion of your proposed concept of a return-on-investment (ROI) that the Goodgrant Foundation should adopt for assessing the 2016 donation(s) and future philanthropic educational investments within the United States. This letter should be no more than two pages in length.

Note: When submitting your final electronic solution DO NOT include any database files. The only thing that should be submitted is your electronic (Word or PDF) solution.

The ProblemCDATA.zip data file contains:

- Problem C - IPEDS UID for Potential Candidate Schools.xlsx
- Problem C - Most Recent Cohorts Data (Scorecard Elements).xlsx
- Problem C - CollegeScorecardDataDictionary-09-08-2015.xlsx
- IPEDS Variables for Data Selection.pdf

You can download the data (ProblemCDATA.zip) on the following websites:

- <http://www.comap-math.com/mcm/ProblemCDATA.zip>
- <http://www.mathismore.net/mcm/ProblemCDATA.zip>
- <http://www.mathportals.com/mcm/ProblemCDATA.zip>
- <http://www.immchallenge.org/mcm/ProblemCDATA.zip>

### **ICM 2016 D: Measuring the Evolution and Influence in Society's Information Network**

Information is spread quickly in today's tech-connected communications network; sometimes it is due to the inherent value of the information itself, and other times it is due to the information finding its way to influential or central network nodes that accelerate its spread through social media. While content has varied -- in the 1800s, news was more about local events (e.g., weddings, storms, deaths) rather than viral videos of cats or social lives of entertainers -- the prevailing premise is that this cultural characteristic to share information (both serious and trivial) has always been there. However, the flow of information has never been as easy or wide-ranging as it is today, allowing news of various levels of importance to spread quickly across the globe in our tech connected world. By taking a historical perspective of flow of information relative to inherent value of information, the Institute of Communication Media (ICM) seeks to understand the evolution of the methodology, purpose, and functionality of society's networks. Specifically, your team, as part of ICM's Information Analytics Division, has been assigned to analyze the relationship between speed/flow of information vs inherent value of information based on consideration of 5 periods: in the 1870s, when newspapers were delivered by trains and stories were passed by telegraph; in the 1920s, when radios became a more common household item; in the 1970s, when televisions were in most homes; in the 1990s, when households began connecting to the early internet; in the 2010s, when we can carry a connection to the world on our phones. Your supervisor reminds you to be sure to report the assumptions you make and the data you use to build your models.

Your specific tasks are:

- (a) Develop one or more model(s) that allow(s) you to explore the flow of information and filter or find what qualifies as news.
- (b) Validate your model's reliability by using data from the past and the prediction capability of your model to predict the information communication situation for today and compare that with today's reality.
- (c) Use your model to predict the communication networks' relationships and capacities around the year 2050.
- (d) Use the theories and concepts of information influence on networks to model how public interest and opinion can be changed through information networks in today's connected world.
- (e) Determine how information value, people's initial opinion and bias, form of the message or its source, and the topology or strength of the information network in a region, country, or worldwide could be used to spread information and influence public opinion.

Possible Data Sources:

As you develop your model and prepare to test it, you will need to assemble a collection of data. Below are just some examples of the types of data you may find useful in this project. Depending on your exact model, some types of data may be very important and others may be entirely irrelevant. In addition to the sample sources provided below, you might want to consider a few important world events throughout history -- if some recent big news events, such as the rumors of country-turned-pop singer Taylor Swift's possible engagement had instead happened in 1860, what percentage of the population would know about it and how quickly; likewise, if an important person was assassinated today, how would that news spread? How might that compare to the news of US President Abraham Lincoln's assassination?

Sample Circulation Data and Media Availability:

- [http://media-cmi.com/downloads/Sixty\\_Years\\_Daily\\_Newspaper\\_Circulation\\_Trends\\_050611.pdf](http://media-cmi.com/downloads/Sixty_Years_Daily_Newspaper_Circulation_Trends_050611.pdf)
- <http://news.bbc.co.uk/2/hi/technology/8552410.stm>
- <http://www.gov.scot/Publications/2006/01/12104731/6>
- <http://www.technologyreview.com/news/427787/are-smart-phones-spreading-faster-than-any-technology-in-human-history/>
- <http://newsroom.fb.com/content/default.aspx?NewsAreaId=22>
- <http://www.poynter.org/news/mediawire/189819/pew-tv-viewing-habit-grays-as-digital-news-consumption-tops-print-radio/>
- <http://www.people-press.org/2012/09/27/section-1-watching-reading-and-listening-to-the-news-3/>

- <http://theconversation.com/hard-evidence-how-does-false-information-spread-online-25567>

#### Historical Perspectives of News and Media:

- <https://www.quora.com/How-did-news-get-around-the-world-before-the-invention-of-newspapers-and-other-media>
- <http://2012books.lardbucket.org/books/a-primer-on-communication-studies/s15-media-technology-and-communication.html>
- <http://firstmonday.org/article/view/885/794>
- Richard Campbell, Christopher R. Martin, and Bettina Fabos, *Media & Culture: An Introduction to Mass Communication*, 5th ed. (Boston, MA: Bedford St. Martin's, 2007)
- Marshall T. Poe, *A History of Communications: Media and Society from the Evolution of Speech to the Internet* (New York: Cambridge, 2011)
- Shirley Biagi, *Media/Impact: An Introduction to Mass Media* (Boston, MA: Wadsworth, 2007)

**Your ICM submission should consist of a 1 page Summary Sheet and your solution cannot exceed 20 pages for a maximum of 21 pages. Note: The appendix and references do not count toward the 20 page limit.**

#### ICM 2016 E: Are we heading towards a thirsty planet?

Will the world run out of clean water? According to the United Nations, 1.6 billion people (one quarter of the world's population) experience water scarcity. Water use has been growing at twice the rate of population over the last century. Humans require water resources for industrial, agricultural, and residential purposes. There are two primary causes for water scarcity: physical scarcity and economic scarcity. Physical scarcity is where there is inadequate water in a region to meet demand. Economic scarcity is where water exists but poor management and lack of infrastructure limits the availability of clean water. Many scientists see this water scarcity problem becoming exacerbated with climate change and population increase. The fact that water use is increasing at twice the rate of population suggests that there is another cause of scarcity – is it increasing rates of personal consumption, or increasing rates of industrial consumption, or increasing pollution which depletes the supply of fresh water, or what? \*\*

Is it possible to provide clean fresh water to all? The supply of water must take into account the physical availability of water (e.g., natural water source, technological advances such as desalination plants or rainwater harvesting techniques). Understanding water availability is an inherently interdisciplinary problem. One must not only understand the environmental constraints on water supply, but also how social factors influence availability and distribution of clean water. For example, lack of adequate sanitation can cause a decrease in water quality. Human population increase also places increased burden on the water supply within a region. When analyzing issues of water scarcity, the following types of questions must be considered. How have humans historically exacerbated or alleviated water scarcity? What are the geological, topographical, and ecological reasons for water scarcity, and how can we accurately predict future water availability? What is the potential for new or alternate sources of water (for example, desalinization plants, water harvesting techniques or undiscovered aquifers)? What are the demographic and health related problems tied to water scarcity?

#### Problem Statement

The International Clean water Movement (ICM) wants your team to help them solve the world's water problems. Can you help improve access to clean, fresh water?

Task 1: Develop a model that provides a measure of the ability of a region to provide clean water to meet the needs of its population. You may need to consider the dynamic nature of the factors that affect both supply and demand in your modeling process.

Task 2: Using the UN water scarcity map (<http://www.unep.org/dewa/vitalwater/jpg/0222-waterstress-overuse-EN.jpg>) pick one country or region where water is either heavily or moderately overloaded. Explain why and how water is scarce in that region. Make sure to explain both the social and environmental drivers by addressing physical and/or economic scarcity.

Task 3: In your chosen region from Task 2, use your model from Task 1 to show what the water situation will be in 15 years. How does this situation impact the lives of citizens of this region? Be sure to incorporate the environmental drivers' effects on the model components.

Task 4: For your chosen region, design an intervention plan taking all the drivers of water scarcity into account. Any intervention plan will inevitably impact the surrounding areas, as well as the entire water ecosystem. Discuss this impact and the overall strengths and weaknesses of the plan in this larger context. How does your plan mitigate water scarcity?

Task 5: Use the intervention you designed in Task 4 and your model to project water availability into the future. Can your chosen region become less susceptible to water scarcity? Will water become a critical issue in the future? If so, when will this scarcity occur?

Task 6: Write a 20-page report (the one-page summary sheet does not count in the 20 pages) that explains your model, water scarcity in your region with no intervention, your intervention, and the effect of your intervention on your region's and the surrounding area's water availability. Be sure to detail the strengths and weaknesses of your model. The ICM will use your report to help with its mission to produce plans to provide access to clean water for all citizens of the world. Good luck in your modeling work!

#### Possible Resources

An Overview of the State of the World's Fresh and Marine Waters. 2nd Edition, 2008. (<http://www.unep.org/dewa/vitalwater/index.html>).

The World's Water: Information on the World's Freshwater Resources. (<http://worldwater.org>).

AQUASTAT. Food and Agriculture Organization of the United Nations. FAO Water Resources.

([http://www.fao.org/nr/water/aquastat/water\\_res/index.stm](http://www.fao.org/nr/water/aquastat/water_res/index.stm)).

The State of the World's Land and Water Resources for food and agriculture. 2011. (<http://www.fao.org/docrep/017/i1688e/i1688e00.htm>).

GrowingBlue: Water. Economics. Life. (<http://growingblue.com>).

World Resources Institute. [www.wri.org](http://www.wri.org).

**\*\*Note that the 2013 Mathematical Competition in Modeling (Problem B) and the 2009 High School Modeling Competition in Modeling (Problem A) were related to modeling different aspects of water scarcity.**

**Your ICM submission should consist of a 1 page Summary Sheet and your solution cannot exceed 20 pages for a maximum of 21 pages. Note: The appendix and references do not count toward the 20 page limit.**

### **ICM 2016 F: Modeling Refugee Immigration Policies**

With hundreds of thousands of refugees moving across Europe and more arriving each day, considerable attention has been given to refugee integration policies and practices in many countries and regions. History has shown us that mass fleeing of populations occur as a result of major political and social unrest and warfare. These crises bring a set of unique challenges that must be managed carefully through effective policies. Events in the Middle East have caused a massive surge of refugees emigrating from the Middle East into safe haven countries in Europe and parts of Asia, often moving through the Mediterranean and into countries such as Turkey, Hungary, Germany, France, and UK. By the end of October 2015, European countries had received over 715,000 asylum applications from refugees. Hungary topped the charts with nearly 1,450 applications per 100,000 inhabitants, but with only a small percentage of those requests granted (32% in 2014), leaving close to a thousand refugees homeless per every 100K residents of the country. Europe has established a quota system where each country has agreed to take in a particular number of refugees, with the majority of the resettlement burden lying with France and Germany.

The refugees travel multiple routes – from the Middle East through (1) West Mediterranean, (2) Central Mediterranean, (3) Eastern Mediterranean, (4) West Balkans, (5) Eastern Borders, and (6) Albania to Greece (See these routes mapped out in <http://www.bbc.com/news/world-europe-34131911>). Each route has different levels of safety and accessibility, with the most popular route being Eastern Mediterranean and the most dangerous, Central Mediterranean. Countries that have been burdened the most are concerned about their capacity to provide resources for the refugees such as food, water, shelter, and healthcare. There are numerous factors that determine how the refugees decide to move through the region. Transportation availability, safety of routes and access to basic needs at destination are considered by each individual or family in this enormous migration.

The UN has asked your team, the ICM-RUN (RefUgee aNalytics) to help develop a better understanding of the factors involved with facilitating the movement of refugees from their countries of origin into safe- haven countries.

#### **Your Specific Tasks:**

1. Metrics of refugee crises. Determine the specific factors which can either enable or inhibit the safe and efficient movement of refugees. There are attributes of the individuals themselves, the routes they must take, the types of transportation, the countries' capacity, including number of entry points and resources available to refugee population. This first task requires ICM-RUN to develop a set of measures and parameters and justify why they should be included in the analysis of this crisis.
2. Flow of refugees. Create a model of optimal refugee movement that would incorporate projected flows of refugees across the six travel routes mentioned in the problem, with consideration of transportation routes/accessibility, safety of route and countries' resource capacities. You can include different routes, different entry points, single or multiple entry points, and even different countries. Use the metrics that you established in Task 1 to determine the number of refugees, as well as the rate and point of entry necessary to accommodate their movement. Be sure to justify any new elements you have added to the migration and explain the sensitivities of your model to these dynamics.
3. Dynamics of the crisis. Refugee conditions can change rapidly. Refugees seek basic necessities for themselves and their families in the midst of continuously changing political and cultural landscapes. In addition, the capacity to house, protect, and feed this moving population is dynamic in that the most desired destinations will reach maximum capacity the quickest, creating a cascade effect altering the parameters for the patterns of movement. Identify the environmental factors that change over time; and show how capacity can be incorporated into the model to account for these dynamic elements. What resources can be prepositioned and how should they be allocated in light of these dynamics? What resources need priority and how do you incorporate resource availability and flow in your model? Consider the role and resources of both government and non-government agencies (NGOs). How does the inclusion of NGO's change your model and strategy? Also consider the inclusion of other refugee destinations such as Canada, China, and the United States. Does your model work for these regions as well?
4. Policy to support refugee model. Now that you have a working model, ICM-RUN has been asked to attend a policy strategy meeting where your team is asked to write a report on your model and propose a set of policies that will support the optimal set of conditions ensuring the optimal migration pattern. Your UN commission has asked you to consider and prioritize the health and safety of refugees and of the local populations. You can include as many parameters and considerations as you see fit to help to inform the strategic policy plan, keeping in mind the laws and cultural constraints of the effected countries. Consider also the role and actions of non- governmental organizations (NGOs).
5. Exogenous events. In addition to endogenous systemic dynamics, exogenous events are also highly likely to occur and alter the situation parameters in these volatile environments, For example, a major terrorist attack in Paris, France has been linked to the Syrian refugee crisis, and has resulted in substantial shifts in the attitudes and policies of many European countries with respect to refugees. The event has also raised concerns among local populations. For example, Brussels, Belgium was placed in a lockdown after the Paris raids in attempts to capture possible terrorists.
  1. What parameters of the model would likely shift or change completely in a major exogenous event?
  2. What would be the cascading effects on the movement of refugees in neighboring countries?
  3. How will the immigration policies that you recommend be designed to be resilient to these types of events?

6. Scalability. Using your model, expand the crisis to a larger scale – by a factor of 10. Are there features of your model that are not scalable to larger populations? What parameters in your model change or become irrelevant when the scope of the crisis increases dramatically? Do new parameters need to be added? How does this increase the time required to resolve refugee placement? If resolution of the refugee integration is significantly prolonged, what new issues might arise in maintaining the health and safety of the refugee and local populations? What is the threshold of time where these new considerations are in play? For example, what policies need to be in place to manage issues such as disease control, childbirth, and education?

The Report: The UN Commission on Refugees has asked your ICM-RUN team to provide them a 20- page report that considers the factors given in your tasks. Each team should also write a 1 page policy recommendation letter which will be read by the UN Secretary General and the Chief of Migration.

**Your ICM submission should consist of a 1 page Summary Sheet, a 1 page letter to the UN, and your solution (not to exceed 20 pages) for a maximum of 22 pages. Note: The appendix and references do not count toward the 22 page limit.**

The Commission has also provided you with some on-line references that may be helpful:

- <http://www.bbc.com/news/world-europe-34131911>
- <http://www.iom.int/>
- <http://iusp2009.princeton.edu/papers/90854>
- <http://www.unhcr.org/pages/49c3646c4d6.html>
- [http://www.nytimes.com/2015/08/28/world/migrants-refugees-europe-syria.html?\\_r=0](http://www.nytimes.com/2015/08/28/world/migrants-refugees-europe-syria.html?_r=0)
- <http://www.who.int/features/qa/88/en/>
- <http://www.euro.who.int/en/health-topics/health-determinants/migration-and-health/migrant-health-in-the-european-region/migration-and-health-key-issues>
- <https://www.icrc.org/en/war-and-law/protected-persons/refugees-displaced-persons>