COMP3702/COMP7702
Artificial Intelligence

Module 1: Search

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Thanks to Dr Alina Bialkowski and Dr Hanna Kurniawati

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Review

Recall our goal: To build a useful, intelligent agent

To start with:

- Computers perceive the world using sensors.
- Agents maintain models/representations of the world and use them for reasoning.
- Computers can learn from data.

So, to achieve our goal, we need to define our "agent" in a way that we can program it:

- The problem of constructing an agent is usually called the **agent design problem**
- Simply, it's about defining the **components** of the agent, so that when the agent acts rationally, it will accomplish the task it is supposed to perform, and do it well.

Agent design components

The following **components** are required to solve an agent design problem:

- **Action Space** (A): The set of all possible actions the agent can perform.
- **Percept Space** (P): The set of all possible things the agent can perceive.
- **State Space** (S): The set of all possible configurations of the world the agent is operating in.
- **World Dynamics/Transition Function** ($T : S \times A \rightarrow S'$): A function that specifies how the configuration of the world changes when the agent performs actions in it.
- **Perception Function** ($Z : S \rightarrow P$): A function that maps a state to a perception.
- **Utility Function** ($U : S \rightarrow \mathbb{R}$): A function that maps a state (or a sequence of states) to a real number, indicating how desirable it is for the agent to occupy that state/sequence of states.

Graph searching

When to apply search methods? When we have:

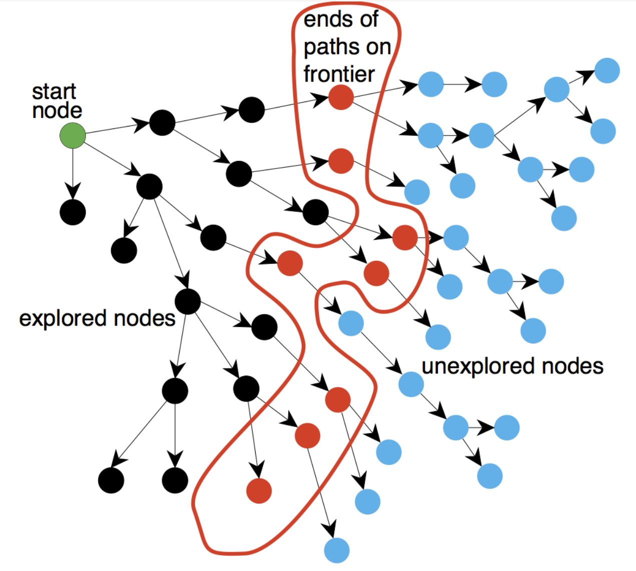
- Sensing uncertainty: **fully observable**
- Effect uncertainty: **deterministic**
- Number of agents: **single agent**

The we can drop the *percept space* and *perception function* from our design considerations.

Generic search algorithm

- Given a graph, start and goal nodes, incrementally explore paths from the start nodes.
- Maintain a **frontier** (fringe) of paths from the start node that have been explored.
- As search proceeds, the frontier expands into the unexplored nodes until a goal node is encountered.
- The way in which the frontier is expanded defines the **search algorithm** or search strategy.

Generic search algorithm (P&M Figure 3.3)



Types of search methods

Two types:

- **Blind search**: Do not use any additional information to “guess” cost of moving from current node to goal
- DFS, BFS, iterative-deepening DFS, and uniform cost search
- **Informed search**: Use additional information to “guess” the cost of moving from current node to goal and decide where to explore next using this information
- Greedy best-first search and A* search

Performance measures for search algorithms

Completeness

- **Complete**: The algorithm will find the solution whenever one exists.

Optimality

- **Optimal**: Return a minimum cost path whenever one exists.

Complexity

- **Time** (#steps) and **space** (memory) complexity
- Complexity analysis informs us of how the required time and memory needed to solve the problem increase as the input size increases
- **Input size**: Size of the state and action spaces of the search problem
- In state graph representation: Size of the graph
- Use computational complexity notation (e.g. Big-O)

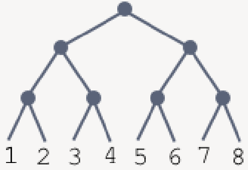
Performance measures for search algorithms

Big-O notation

- Suppose $f(n)$ is the required time/space required to solve the problem if the input size is n
- Then, we say $f(n)$ is of complexity $O(g(n))$ if there is a constant k and n_0 such that:

$$0 \geq f(n) \leq k\, g(n) \quad \text{for all } n_0$$

Branching factor: used to characterise graph topologies



$b = 2 \qquad n = 8$

$b = 8 \qquad n = 8$

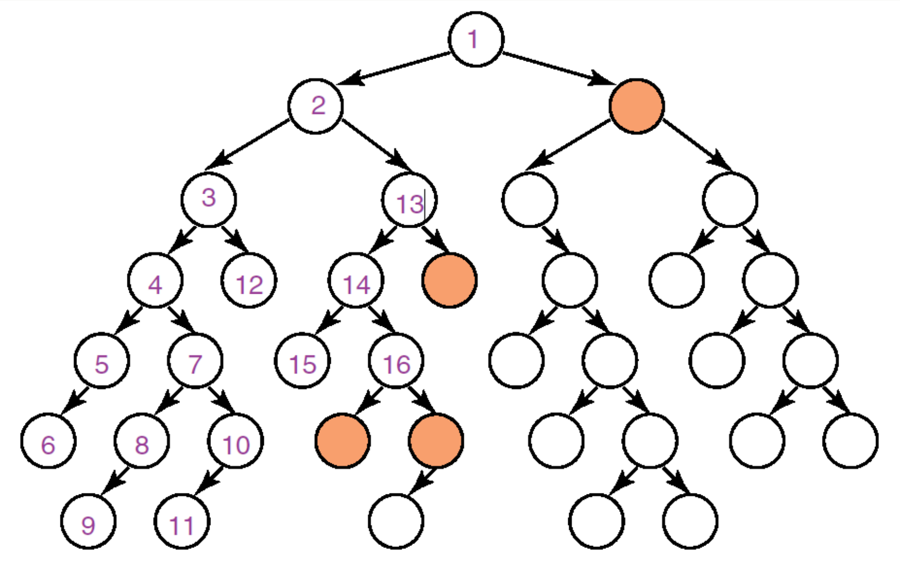
Blind search: DFS and BFS

Depth-first search

Depth-first search

- **Depth-first search** treats the frontier as a stack, or a last-in first-out queue.
- It always selects one of the last elements added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots]$
  - p_1 is selected. Paths that extend p_1 are added to the front of the stack (in front of p_2).
  - p_2 is only selected when all paths from p_1 have been explored.

Illustrative Graph — Depth-first Search



Depth-first search: Properties and analysis

Parameters: b, branching factor; m, maximum depth; d, depth of shallowest goal node

Complete? Will DFS find a solution?

- Complete, if m and b are finite and nodes are not revisited

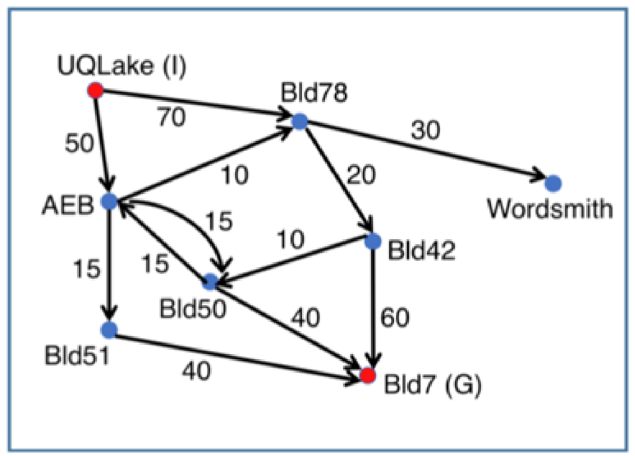
Generate optimal solution? Does DFS guarantee to find the path with fewest edges?

- No

Complexity:

- **Time**: $O(b^m) \quad \Leftarrow \quad 1 + b + b^2 + \ldots + b^m = \frac{b^{m+1-1}}{b-1}$
- **Space**: Can be implemented using $O(bm)$, or $O(m)$ using *backtracking* DFS. But be careful of revisiting vertices (states)!
- **Efficient in use of space**

Example — Navigating UQ



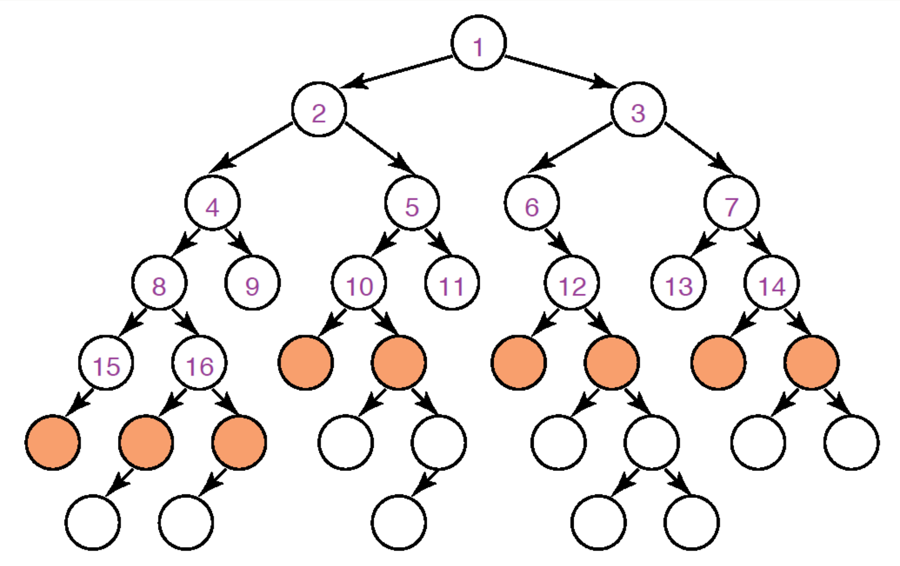
Ignore costs on these edges

Breadth-first search

Breadth-first search

- **Breadth-first search** treats the frontier as a first-in first-out queue.
- It always selects one of the earliest elements added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots, p_n]$:
    - p_1 is selected. Its neighbours are added to the end of the queue, after p_n.
    - p_2 is selected next.

Illustrative Graph — Breadth-first Search



Breadth-first search: Properties and analysis

Parameters: b, branching factor; d, depth of shallowest goal node

Complete? Will BFS find a solution?

- Complete if b is finite

Generate optimal solution? Does BFS guarantee to find the path with fewest edges?

- **Yes** in #steps

Complexity:

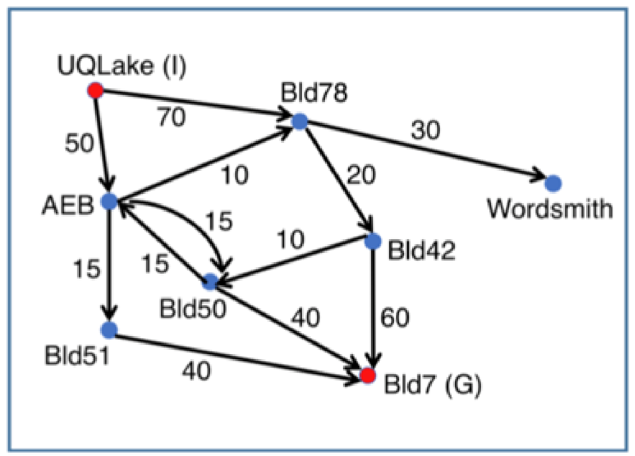
- **Time**: $O(b^d) \quad \Leftarrow \quad 1 + b + b^2 + \ldots + b^d = \frac{b^{d+1-1}}{b-1}$
- **Space**: $O(b^d)$ nodes to remember: $O(b^{d-1})$ explored nodes $+$ $O(b^d)$ unexplored nodes
- **Finds minimum step path, but requires exponential space!**

Let's get some intuition: Practical numbers for BFS

d	# Nodes	Time	Memory
2	110	.11 msec	107 Kbytes
4	11,110	11 msec	10.6 Mbyte
6	$\sim 10^6$	1 sec	1 Gbytes
8	$\sim 10^8$	~2min	103 Gbytes
10	$\sim 10^{10}$	~2.8 hours	10 Tbyte
12	$\sim 10^{12}$	~11.6 days	1 Pbytes
14	$\sim 10^{14}$	~3.2 years	99 Pbytes

Assumptions: b = 10; 1 Kbytes/node; 1million nodes/sec

Example — Navigating UQ



Ignore costs on these edges

Iterative deepening depth-first search

Iterative deepening DFS (IDDFS)

DFS:Efficient in space, but no path length guarantee.

BFS: Finds minimum step path, but requires exponential space.

Iterative deepening DFS

- Multiple DFS with increasing depth-cutoff until the goal is found.
- For $k = 1, 2, \ldots$: Perform DFS with depth cutoff k.
- Only generates nodes with depth $\leq k$.

IDDFS: Properties and analysis

Parameters: b, branching factor; m, maximum depth; d, depth of shallowest goal node

Complete? Will IDDFS find a solution?

- Complete if b is finite

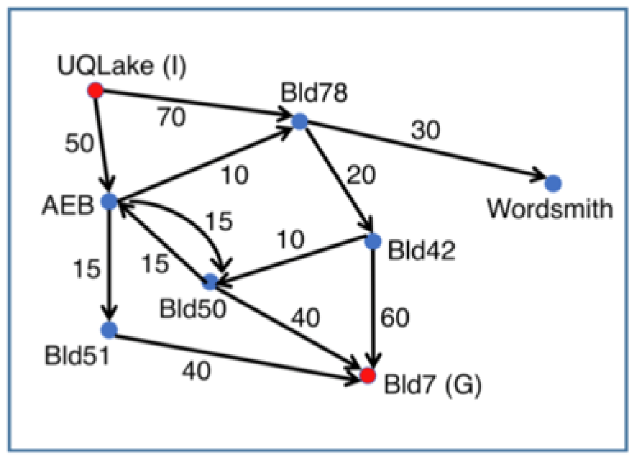
Generate optimal solution? Does IDDFS guarantee to find the path with fewest edges?

- **Yes** in #steps

Complexity:

- **Time**: $O(b^d) \quad \Leftarrow \quad db + (d-1)b^2 + \ldots + (1)b^d$
- **Space**: $O(bd)$
- **Finds minimum step path, and doesn't require exponential space!**

Example — Navigating UQ



Ignore costs on these edges

Search with edge costs: Uniform cost search

Search with edge costs: Uniform cost search

- Sometimes there are costs associated with edges.
- The **cost** of a path is the sum of the costs of its edges:

$$cost(n_0, \ldots, n_k) = \sum_{i=1}^{k} cost(n_{i-1}, n_i)$$

- An optimal solution is one with minimum cost.

Uniform cost search

- At each stage, uniform-cost search selects a path on the frontier with lowest cost.
- The first path to a goal is a least-cost path to a goal node.
- When edge costs are equal $\Rightarrow$ breadth-first search.

Uniform cost Search

- **Uniform cost search** treats the frontier as a **priority queue** ordered by path cost.
- It always selects one of the highest-priority vertices added to the frontier.
- If the list of paths on the frontier is $[p_1, p_2, \ldots]$:
  - p_1 is selected to be expanded.
  - Its successors are inserted into the priority queue.
  - The highest-priority vertex is selected next (and it might be a newly expanded vertex).

UCS: Properties and analysis

Parameters: b, branching factor; m, maximum depth; d, depth of shallowest goal node

$C*$: Cost of optimal solution, ϵ: minimum cost of a step

Complete? Will UCS find a solution?

- Complete if b is finite and all edges have a cost $\geq \epsilon > 0$

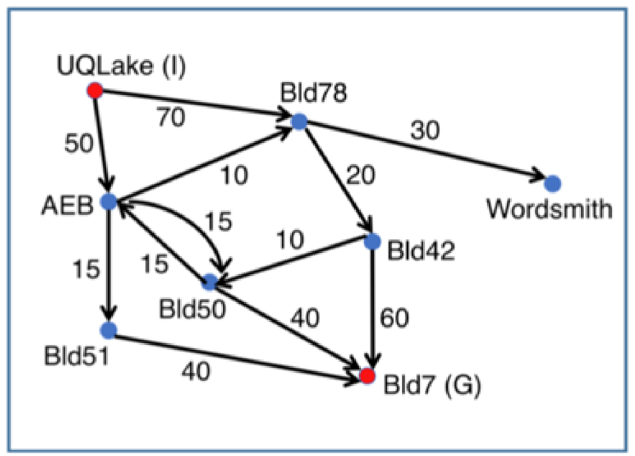
Generate optimal solution? Does UCS guarantee to find the path with **the lowest cost**?

- **Yes** if all edges have positive cost

Complexity:

- **Time** and **Space**: $O(b^{1+\lfloor \frac{C*}{\epsilon} \rfloor})$

Example — Navigating UQ



Blind search: Summary

- Depth-first search
- Breadth-first search
- Iterative-deepening depth-first search
- Uniform cost search

Informed search

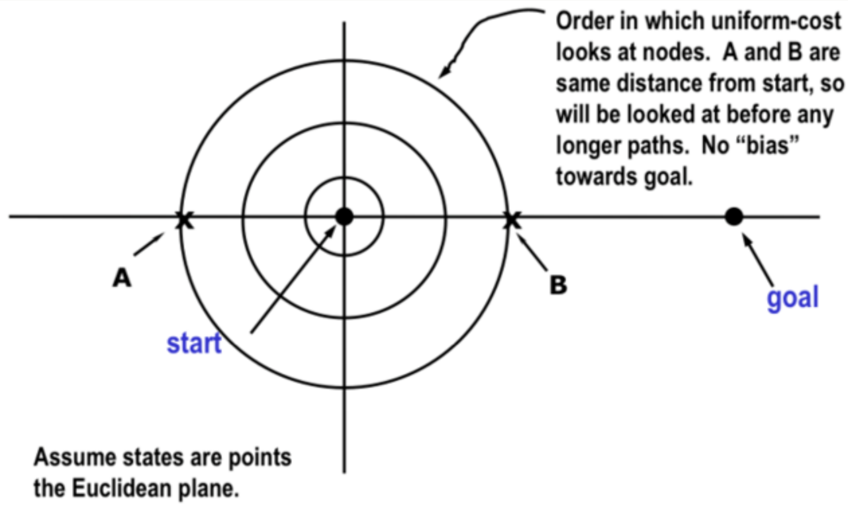
Blind vs informed search

- Blind search algorithms (e.g. UCS) use the cost from the root to the current node, $g(n)$ to prioritise which nodes to search.
- Informed search algorithms rely on **heuristics**, $h(n)$ that give an estimated cost from the current node to the **goal** to prioritise search.
- In general, informed search is faster than blind search
- However, it is more difficult to prove properties of informed search algorithms, as their performance highly depends on the heuristics used

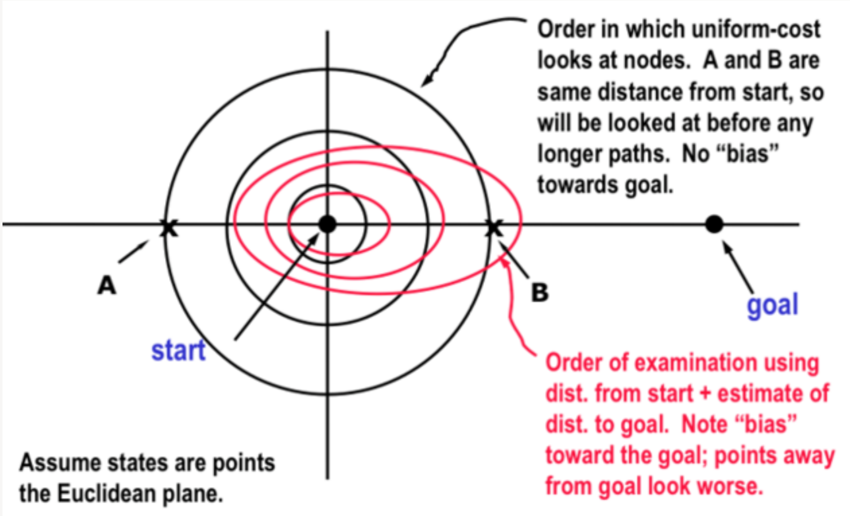
Informed search algorithms

- Greedy best-first search
- A* search

Blind vs informed search



Blind vs informed search



Informed search using heuristics

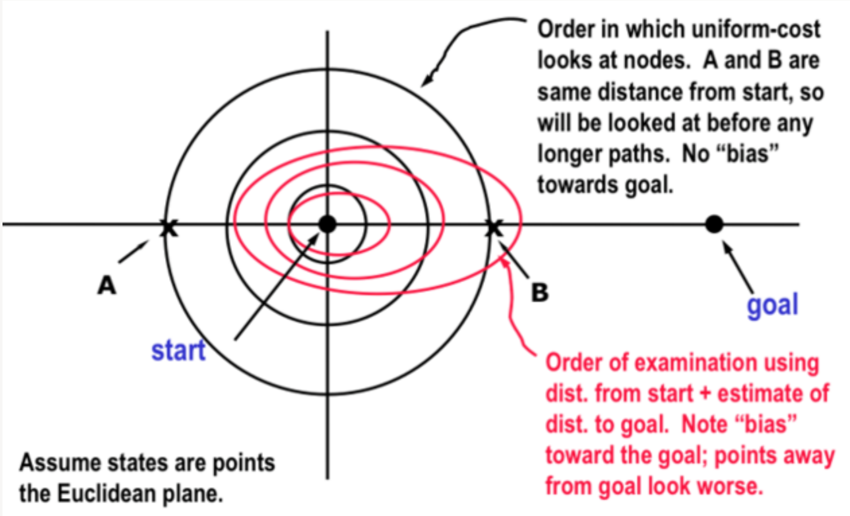
Idea: Don't ignore the goal when selecting paths.

- Often there is extra knowledge that can be used to guide the search: **heuristics.**
- $h(n)$ is an estimate of the cost of the shortest path from node n to a goal node.
- $h(n)$ needs to be efficient to compute.
- h can be extended to paths: $h(n_0, \ldots, n_k) = h(n_k)$.
- $h(n)$ is an **underestimate** if there is no path from n to a goal with cost less than $h(n)$.
- An **admissible heuristic** is a nonnegative (≥ 0) heuristic function that is an underestimate of the actual cost of a path to a goal.

Example heuristic functions

- If the nodes are points on a Euclidean plane and the cost is the distance, $h(n)$ can be the straight-line distance from n to the closest goal (i.e. ignoring obstacles).
- If the nodes are locations and cost is *time*, we can use the distance to a goal divided by the maximum speed (underestimate).
- If the goal is complicated, simple decision rules that return an approximate solution and that are easy to compute can make for good heuristics
- A heuristic function can be found by solving a simpler (less constrained) version of the problem.

Blind vs informed search



Greedy best-first search

Greedy best-first search

Greedy Best-First search is *almost* the same as UCS, with some key differences:

- Uses a **priority queue** to order expansion of fringe nodes
- The highest priority in priority queue for greedy best-first search is the node with the smallest **estimated cost** from the current node to the goal
- The estimated cost-to-goal is given by the heuristic function, $h(n)$
- If the list of paths on the frontier is $[p_1, p_2, \ldots]$:
  - p_1 is selected to be expanded.
  - Its successors are inserted into the priority queue.
  - The highest-priority vertex is selected next (and it might be a newly expanded vertex).

Greedy best-first search: Properties and analysis

Complete? Will greedy best-first search find a solution?

- **No** (it depends on the heurisitc)

Generate optimal solution? Is greedy best-first search guaranteed to find the lowest cost path to the goal?

- **No**

Complexity:

- Depends highly on the heuristic
- Worst case if the tree depth is finite: $O(b^m)$ where b is branching factor and m is maximum depth of the tree (i.e. it can be exponentially bad).

Example — Navigating UQ

Heuristic values (to Bld7)

h(UQLake) = 100

h(Bld78) = 50

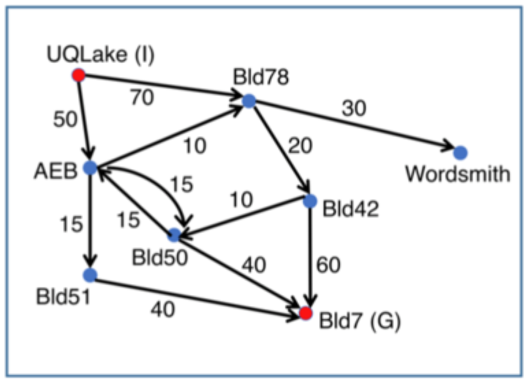
h(AEB) = 53

h(Wordsmith) = 1000 h(Bld42) = 50

h(Bld50) = 38

h(Bld51) = 30

h(Bld7) = 0



A* Search

A* Search

A* search uses both path cost and heuristic values

- It is a mix of uniform-cost and best-first search
- $g(p)$ is the cost of path p from initial state to a node (UCS)
- $h(p)$ estimates the cost from the end of p to a goal (GBFS)
- A* uses: $f(p) = g(p) + h(p)$

$f(p)$ estimates the **total** path cost of going from a start node to a goal via p

$$\underbrace{\underbrace{\mathit{start} \xrightarrow{\text{path } p} n}_{g(p)} \; \underbrace{\xrightarrow{\text{estimate}} \mathit{goal}}_{h(p)}}_{f(p)}$$

A* Search Algorithm

- A* is a mix of uniform-cost and best-first search, algorithmically similar to both
- It treats the frontier as a **priority queue** ordered by $f(p)$
- Highest priority is the node with the lowest f value The function $f(p)$ is the the shortest path length from root to the node ($g(p)$) plus the estimated future reward from node p to the goal ($h(p)$)
- It always selects the node on the frontier with the lowest estimated distance from the start to a goal node constrained to go via that node

Example — Navigating UQ

Heuristic values (to Bld7)

h(UQLake) = 100

h(Bld78) = 50

h(AEB) = 53

h(Wordsmith) = 1000 h(Bld42) = 50

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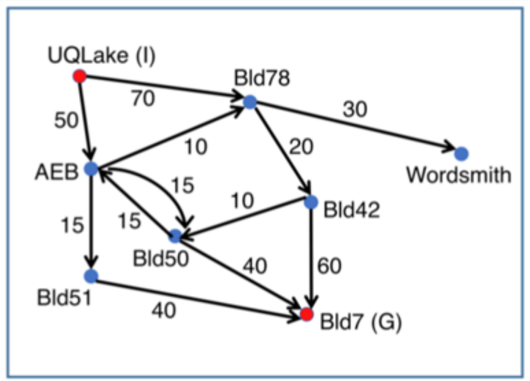


A* Search: Properties and analysis

- Will A* search find a solution?
- Is A* search guaranteed to find the shortest path or the path with fewest arcs?
- What is the time complexity as a function of length of the path selected?
- What is the space complexity as a function of length of the path selected?
- How does the goal affect the search?

Admissibility of A*

If there is a solution, A* always finds an optimal solution —as the first path to a goal selected— if the following conditions are met:

1. the search graph branching factor b is finite
2. edge costs are bounded above zero (there is some $\epsilon > 0$ such that all of the edge costs are greater than ϵ), and
3. $h(n)$ is > 0 and an underestimate of the cost of the shortest path from n to a goal node.

. . . *we have seen 2 and 3 before:*

A heuristic is admissible if it never overestimates the cost-to-goal

Why is A* admissible?

If a path p to a goal is selected from a frontier, can there be a shorter path to a goal?

- Suppose path p' is on the frontier.
- Because p was chosen before p', and $h(p) = 0$:

$$g(p) \le g(p') + h(p').$$

- Because h is an underestimate:

$$g(p') + h(p') \le g(p'')$$

  for any path p'' to a goal that extends p'.

So $g(p) \le g(p'')$ for any other path p'' to a goal.

How do good heuristics help?

Suppose c is the cost of an optimal solution. What happens to a path p where

- $g(p) + h(p) < c$
- $g(p) + h(p) = c$
- $g(p) + h(p) > c$

How can a better heuristic function help?

Example — Navigating UQ

Heuristic values (to Bld7)

$h(\text{UQLake}) = 100$

$h(\text{Bld78}) = 50$

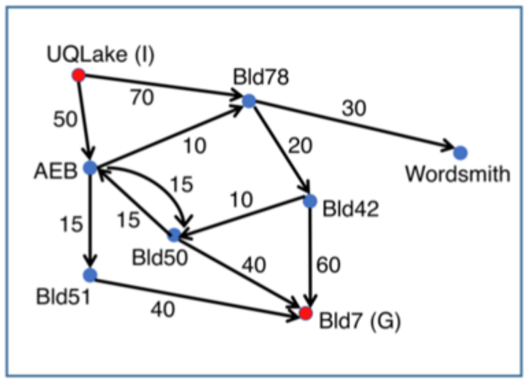
$h(\text{AEB}) = 53$

$h(\text{Wordsmith}) = 1000$ $h(\text{Bld42}) = 50$

$h(\text{Bld50}) = 38$

$h(\text{Bld51}) = 30$

$h(\text{Bld7}) = 0$



Attributions and References

Thanks to Dr Alina Bialkowski and Dr Hanna Kurniawati for their materials.

Many of the frames in Module 1 are adapted from David Poole and Alan Mackworth, *Artificial Intelligence: foundations of computational agents*, 2E, CUP, 2017 `http://https://artint.info/`.

Other materials derived from Stuart Russell and Peter Norvig, *Artificial Intelligence: A Modern Approach*, 3E, Prentice Hall, 2009.

All remaining errors are Archie's — please email if you find any: archie.chapman@uq.edu.au