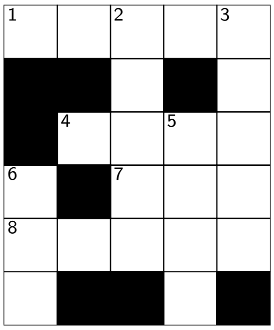
COMP3702/COMP7702 Artificial Intelligence
Semester 2, 2020
Tutorial 6

Before you begin, please note:

- Tutorial exercises are provided to help you understand the materials discussed in class, and to improve your skills in solving AI problems.
- Tutorial exercises will not be graded. However, you are highly encouraged to do them for your own learning. Moreover, we hope you get the satisfaction from solving these problems.
- The skills you acquire in completing the tutorial exercises will help you complete the assignments.
- You'll get the best learning outcome when you try to solve these exercises on your own first (before your tutorial session), and use your tutorial session to ask about the difficulties you face when trying to solve this set of exercises.

Exercises

Exercise 6.1. Recall the crossword puzzle from last tutorial:



Words: AFT, ALE, EEL, HEEL, HIKE, HOSES, KEEL, KNOT, LASER, LEE, LINE, SAILS, SHEET, STEER, TIE.

This can be modeled as a CSP, with:

Variables: 1-across, 2-down, 3-down, 4-across, 5-down, 6-down, 7-across and 8-across

Domains: The list of candidate words given above.

Constraints: Letters used in two words (nb. all constraints are binary).

Last week we generated a constraint graph for this CSP, and applied node consistency (1-consistency) to the domains of each variables to reduce the search space. The resulting node-consistent domains are:

1-across: {HOSES, LASER, SAILS, SHEET, STEER}

2-down: {HOSES, LASER, SAILS, SHEET, STEER}

3-down: {HOSES, LASER, SAILS, SHEET, STEER}

4-across: {HEEL, HIKE, KEEL, KNOT, LINE}

5-down: {HEEL, HIKE, KEEL, KNOT, LINE}

6-down: {AFT, ALE, EEL, LEE, TIE}

7-across: {AFT, ALE, EEL, LEE, TIE}

8-across: {HOSES, LASER, SAILS, SHEET, STEER}

a) Apply backtracking search to the node-consistent constraint graph (pseudocode give below). Record the number of nodes expanded in the search procedure. (You can trace the algorithm manually or develop code to answer this question).

b) Apply arc-consistency to this CSP (manually or in code; pseudocode give below). Record the number of arc-consistency check operations that are performed. What is the outcome of applying arc consistency?

c) Compare the number of search expansion or consistency check operations of backtracking search and arc-consistency.

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
       if value is consistent with assignment given CONSTRAINTS[csp] then
           add {var = value} to assignment
           result ← RECURSIVE-BACKTRACKING(assignment, csp)
           if result ≠ failure then return result
           remove {var = value} from assignment
   return failure
```

One *efficient* algorithm for arc-consistency is commonly called AC-3:

```
function AC-3(csp) returns false if an inconsistency is found and true otherwise
   inputs: csp, a binary CSP with components (X, D, C)
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (X_i, X_j) ← REMOVE-FIRST(queue)
      if REVISE(csp, X_i, X_j) then
         if size of D_i = 0 then return false
         for each X_k in X_i.NEIGHBORS - {X_j} do
            add (X_k, X_i) to queue
   return true

function REVISE(csp, X_i, X_j) returns true iff we revise the domain of X_i
   revised ← false
   for each x in D_i do
      if no value y in D_j allows (x,y) to satisfy the constraint between X_i and X_j then
         delete x from D_i
         revised ← true
   return revised
```

Figure 6.3 The arc-consistency algorithm AC-3. After applying AC-3, either every arc is arc-consistent, or some variable has an empty domain, indicating that the CSP cannot be solved. The name "AC-3" was used by the algorithm's inventor (Mackworth, 1977) because it's the third version developed in the paper.

Exercise 6.2. Mr Jones finds three trunks A, B, and C in a cave. Based on studying the history of where these trunks came about, he knows that one trunk contains gold, while two are empty. On the wall of the cave, he found three clues: "A is empty", "B is empty", and "gold is in B". From studying the historical social norm of the villages around the cave, Mr Jones knows that only one of the clues is true, while the other two are false". Which trunk has the gold? (source: `http://disi.unitn.it/~ldkr/ml2014/ExercisesBooklet.pdf`)

Exercise 6.3. Are the following entailments correct? Please provide the proof.

a) $(A \wedge B) \models (A \Leftrightarrow B)$

b) $(A \Leftrightarrow B) \models (A \wedge B)$

Hint: When we are asked to determine if an entailment is correct (or holds, or is true) we can convert the entailment into an implication, and check if the implication is valid. Checking whether or not the implication is valid means solving a validity problem. Remember that a sentence is valid when every combination of variable assignments in the sentence causes it to be true.

Exercise 6.4. UQPark is a theme park with 5 rides: Bumper cars, carousel, haunted class, roller coaster, and ferris wheel, where each ride can be turned on and off independently of the other rides. After performing a cost-benefit analysis, UQPark Management decided that only 3 rides should be open at any given day, and the set of rides that are open/closed must satisfy the following constraints:

1. Either bumper cars or carousel must be open.
2. If bumper cars is closed, then roller coaster must open.
3. If carousel is open, then either bumper cars or haunted class must be open too.
4. If haunted class is open, then ferris wheel must be open too.
5. Bumper cars and ferris wheel cannot both be open at the same day.
6. If roller coaster is open, then ferris wheel must be open too.
7. If roller coaster is closed, then either haunted class or ferris wheel must be open.

UQPark Facilities thinks there is no combination of the rides that can satisfy all of Management's constraints. Question: Please frame this problem as a satisfiability problem with propositional logic representation.

This week we concentrate on representation, while next week (Tutorial 7) we will continue with this example to answer the question of which of UQPark's Facilities or Management teams is correct.