Simple Linear Regression

This example shows how to perform simple linear regression using the `accidents` dataset. The example also shows you how to calculate the coefficient of determination R^2 to evaluate the regressions. The `accidents` dataset contains data for fatal traffic accidents in U.S. states.

Linear regression models the relation between a dependent, or response, variable y and one or more independent, or predictor, variables $x_1, ..., x_n$. Simple linear regression considers only one independent variable using the relation

$$y = \beta_0 + \beta_1 x + \epsilon,$$

where β_0 is the y-intercept, β_1 is the slope (or regression coefficient), and ϵ is the error term.

Start with a set of n observed values of x and y given by (x_1, y_1), (x_2, y_2), ..., (x_n, y_n). Using the simple linear regression relation, these values form a system of linear equations. Represent these equations in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}.$$

Let

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}, B = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}.$$

The relation is now $Y = XB$.

In MATLAB, you can find B using the docid:matlab_ref.btg5qam operator as `B = X\Y`.

From the dataset `accidents`, load accident data in `y` and state population data in `x`. Find the linear regression relation $y = \beta_1 x$ between the accidents in a state and the population of a state using the `\` operator. The `\` operator performs a least-squares regression.

```
load accidents
x = hwydata(:,14); %Population of states
y = hwydata(:,4); %Accidents per state
format long
b1 = x\y
```

```
b1 =
    1.372716735564871e-04
```

b1 is the slope or regression coefficient. The linear relation is $y = \beta_1 x = 0.0001372x$.

Calculate the accidents per state yCalc from x using the relation. Visualize the regression by plotting the actual values y and the calculated values yCalc.

```
yCalc1 = b1*x;
scatter(x,y)
hold on
plot(x,yCalc1)
xlabel('Population of state')
ylabel('Fatal traffic accidents per state')
title('Linear Regression Relation Between Accidents & Population')
grid on
```

Improve the fit by including a y-intercept β_0 in your model as $y = \beta_0 + \beta_1 x$. Calculate β_0 by padding x with a column of ones and using the \ operator.

```
X = [ones(length(x),1) x];
b = X\y
```

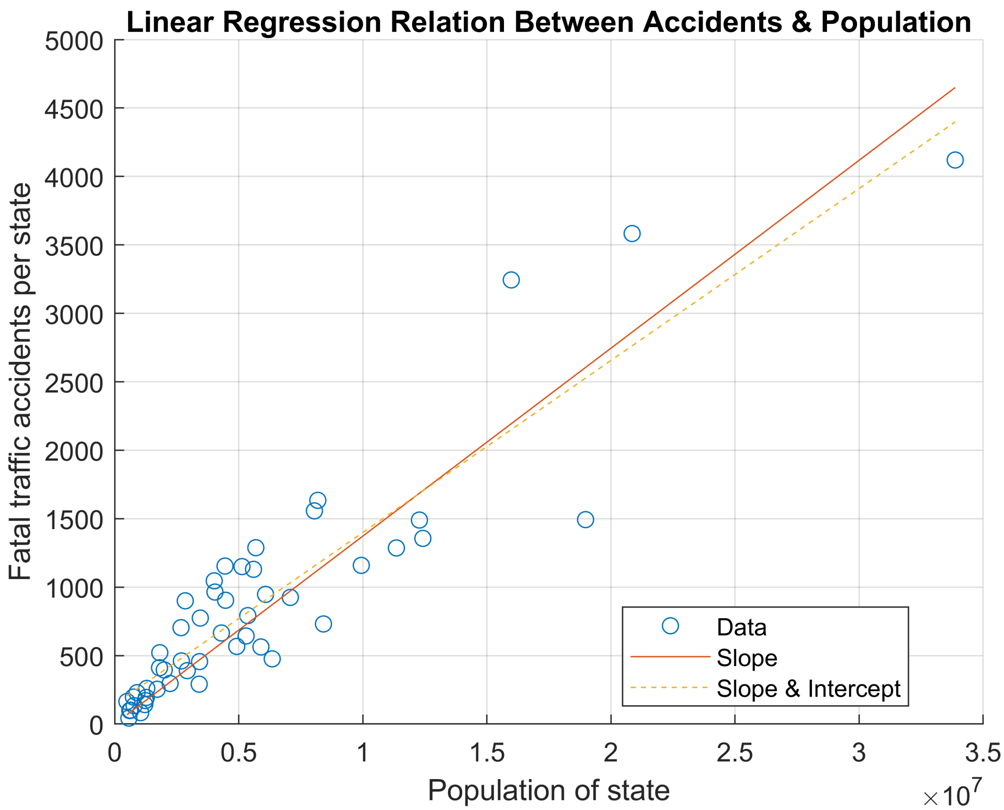
```
b =
    1.0e+02 *

    1.427120171726538
```

```
0.000001256394274
```

This result represents the relation $y = \beta_0 + \beta_1 x = 142.7120 + 0.0001256x$.

Visualize the relation by plotting it on the same figure.

```
yCalc2 = X*b;
plot(x,yCalc2,'--')
legend('Data','Slope','Slope & Intercept','Location','best');
```



From the figure, the two fits look similar. One method to find the better fit is to calculate the coefficient of determination, R^2. R^2 is one measure of how well a model can predict the data, and falls between 0 and 1. The higher the value of R^2, the better the model is at predicting the data.

Where $\hat{y}$ represents the calculated values of y and $\overline{y}$ is the mean of y, R^2 is defined as

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \overline{y})^2}.$$

Find the better fit of the two fits by comparing values of R^2. As the R^2 values show, the second fit that includes a y-intercept is better.

```
Rsq1 = 1 - sum((y - yCalc1).^2)/sum((y - mean(y)).^2)
```

```
Rsq1 =
   0.822235650485566
```

```
Rsq2 = 1 - sum((y - yCalc2).^2)/sum((y - mean(y)).^2)
```

```
Rsq2 =
   0.838210531103428
```