1. Let us consider that A is countably infinite and take A = N = {1, 2, 3, 4, ...} the set of natual numbers without loosing its generality.
2. Here consider that N is equinumerous with its power set P(N) and it looks like,

P(N) = {Ç¾, {1, 2, 3}, {4, 5}, {1, 3, 5}, {2, 4, 6}, ...}.

Here P(N) contains infinite subsets of N, like odd numbers {1, 3, 5} and empty setÂ Â {Ç¾}.

3. Now will attempt the pair of N with the set of P(N) such that no element remain unpair to each set as show in below,

N P(N)

1 <-> {1, 2, 3}

2 <-> {4, 5}

3 <-> {1, 3, 5}

4 <-> {2, 4, 6}

⋮ ⋮

4. In our example the number 1 is paired with the subset {1, 2, 3}, which contains 1 as a member, which is called as *selfish* number.

5. Similar way 2 is paired with the subset {4, 5}, which does not contain 2 as a member, which is called as *non-selfish* number.

6. Using this method, we can create a special set of natural numbers and this set will provide the contradiction here.

7. As there is no natural number that can be paired with A, we have contradicted our original consideration, that there is a bijection between N and P(N).

8. Note here, the set A may be empty. It means that every natural number n maps to a subset of natural numbers which contains n.

9. Here, every number maps to a non-empty set and no number maps to the empty set, but the empty set is a member of P(N), and mapping still does not cover the P(N).

10. Using this contradiction we prove that the cardinality of N and P(N) does not equal.

11. Also the cardinality of N does not less than the cardinality of P(N) because P(N) include all singletons, by definition.

12. Hence finally only one possibility remains, which is the cardinality of N is always smaller than the cardinality of P(N), that proving the Cantor's theorem.

PIN) 1<->{1, 2, 3} 2<-> {4,5) 3<-> {1,3,5} 4<-> {2,4,6}