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# Pattern Analysis

## Groups

“So our problem is to explain where symmetry comes from. Why is nature so nearly symmetrical? No one has any idea why.”

Richard Feynman (link)

(1918-1988)

Dr Shekhar “Shakes” Chandra

V1.0

# Groups

**Definition [Galois, 1830]**

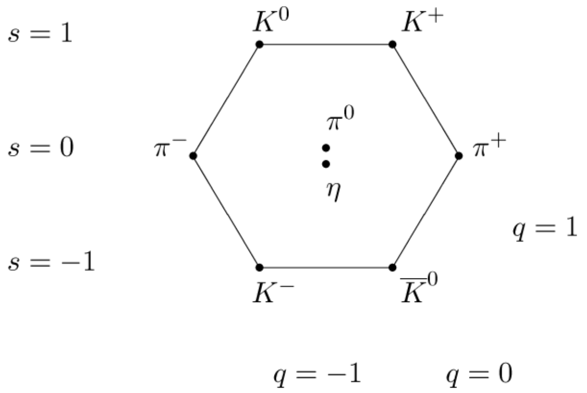
***A Group G is composed of a set of elements and a composition or product operation ·, which abide by the following properties:***

***1. The product result C = A · B is also part of the same set.***

***2. There is an identity element I in the set, so that A · I = A.***

***3. There exists an inverse operation, so that A · A$^{-1}$ = I.***

***4. The multiplication is associative, so that A · (B · C) = (A · B) · C.***




# Rubik’s Cube

- Invented by the Hungarian professor of architecture Ernő Rubik
- It has its own group, called the Rubik’s Cube group.
- All moves can be reduced to the symmetries of the 6 faces of the 8 (non-centre) facets.
- Labelling the facets 1-48, the cube follows Symmetric Group $S_{48}$
- The group is defined by the 6 face rotations – {F, B, U, D, L, R}
- Not all moves commute with each other, i.e. FR != RF

F turns the front clockwise

B turns the back clockwise

U turns the top clockwise

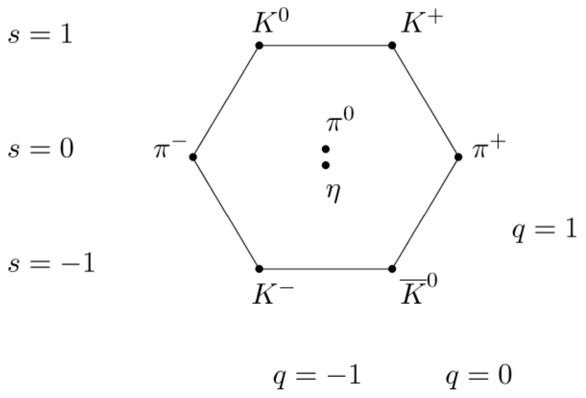
D turns the bottom clockwise

L turns the left face clockwise

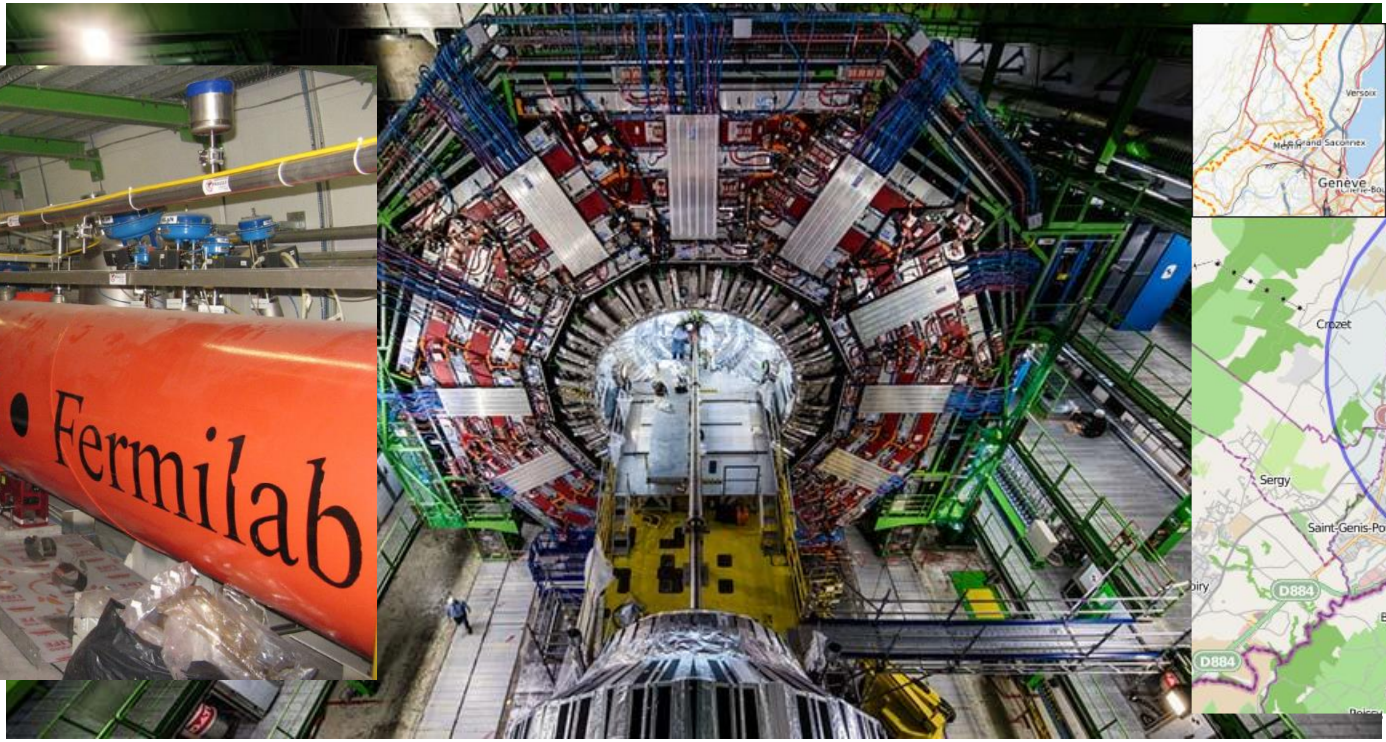
R turns the right face clockwise

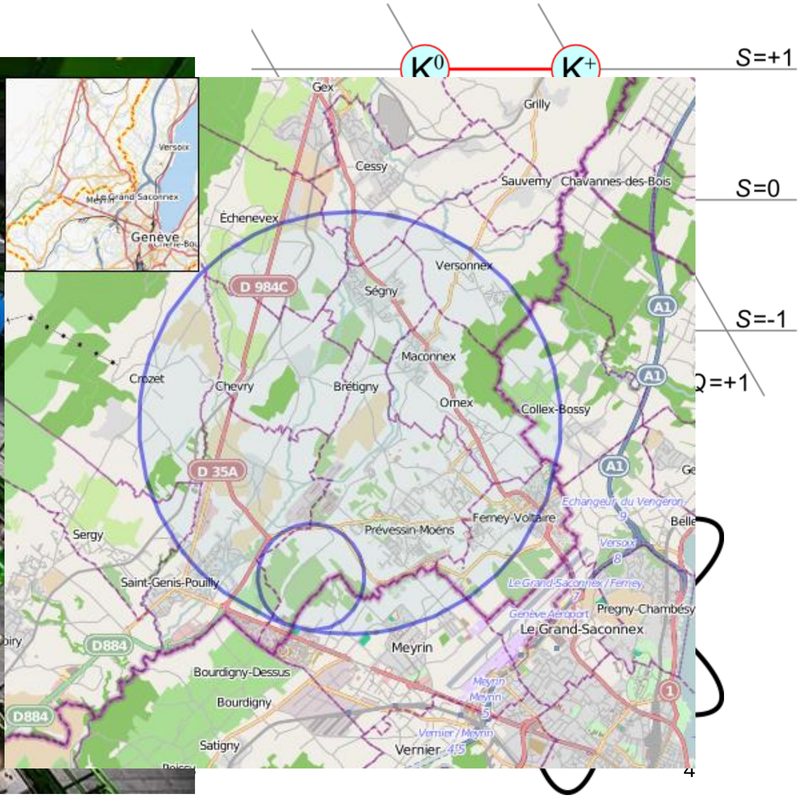
43,252,003,274,489,856,000 possibilities but can always be solved in 20 or fewer moves!

# Gell-Manns 8 fold way




- Atoms are made of neutrons, protons and electrons
- Neutrons and Protons are a set of particles called Hadrons
- The Large Hadron Collider (LHC) found many other particles

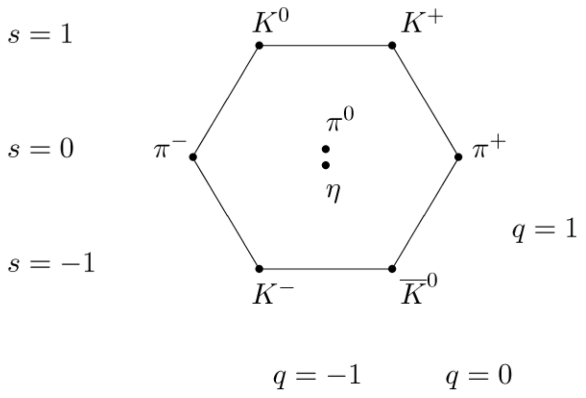



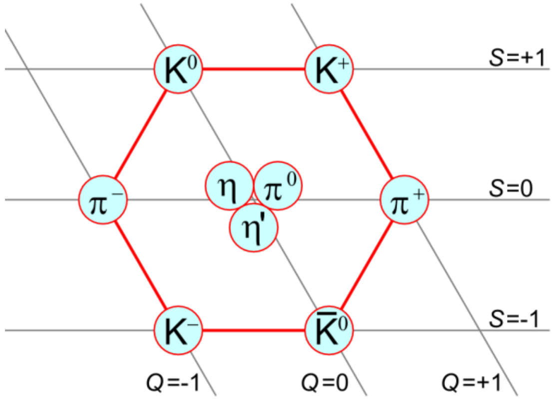


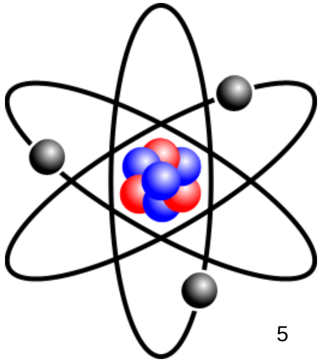

 

# Gell-Manns 8 fold way

- No one could explain why there were so many smaller different particles produced at the LHC
- Murray Gell-Mann, inspired by the 8-fold way from Buddhism, proposed a group structure that predicted particles called Quarks
- His model also predicted more particles which he called ‘strange’ quarks and these were eventually found!
- No one had found them before because they are very short lived (radioactively decay rapidly)



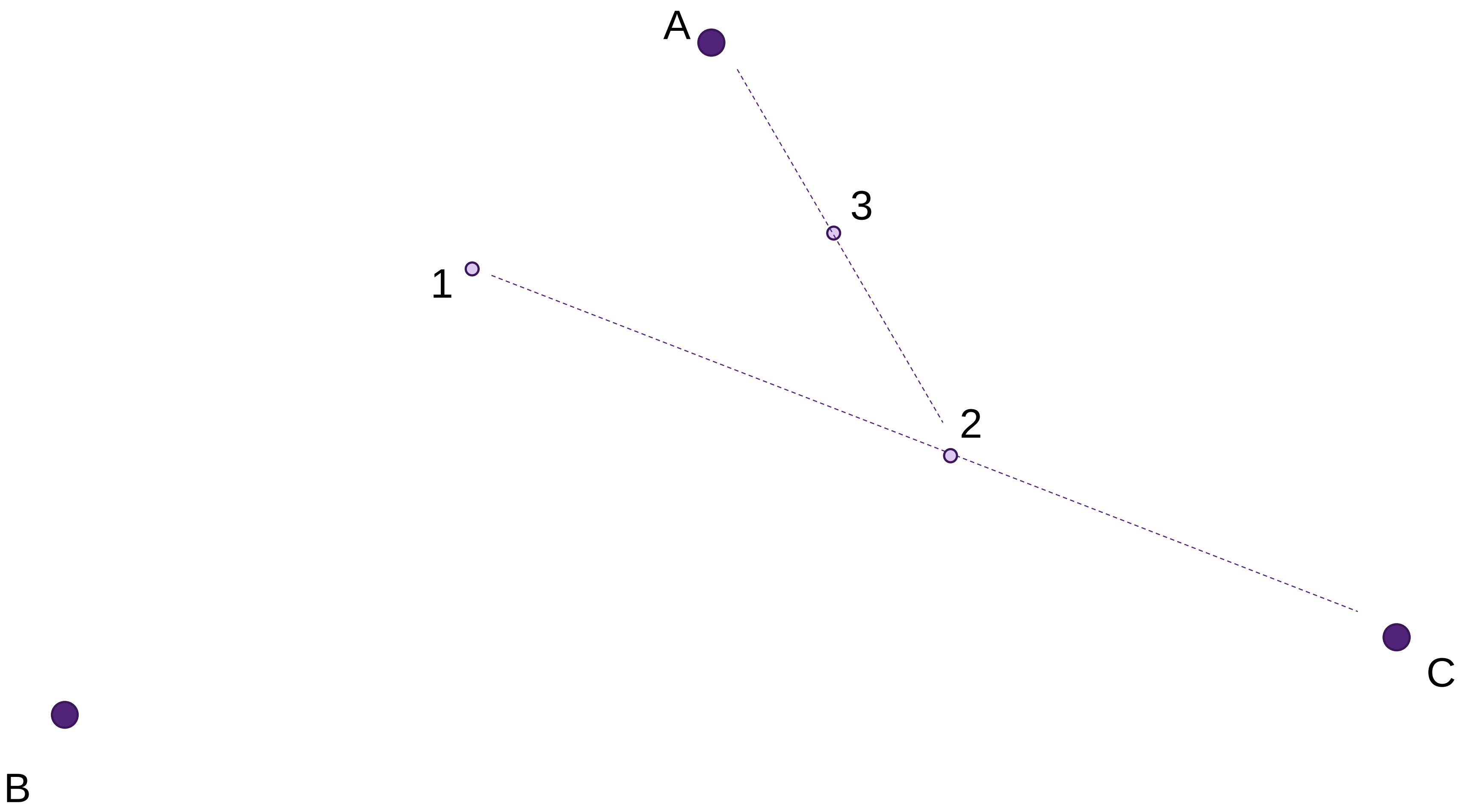



# Emily Noether’s Theorem

- Groups give rise to conservation laws!
- All continuous symmetries will have an associated conservation law.
- When there is (local) phase invariance, we get the conservation of charge. [Video]
- When there is invariance to spatial translation in a set of equations of motion, we get a conservation of momentum.
- Emily Noether was a genius and arguably one of the greatest mathematicians of her time, but was largely ignored because she was a woman. ☹

# Fractals




CHAOS

# Conclusion

- Nature has a remarkable ability to utilize symmetry in nearly all creations
- Groups can be applied to a number of areas – Physics, maths, computer science etc.
- Noether's Theorem points to the fact that (continuous) invariance gives rise to conservations, which in turn govern the way nature works!
- There is another type of symmetry that nature uses that is not directly quantified by groups, self-similarity.

# What’s Next?

How Nature creates information and an infinitude of chaotic structures from just a set of simple rules – self-similarity.

# Thank you

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