Symmetry and self-similarity.

Symmetry Group

Definition: the symmetry group of a geometric object is the group of all transformations under which the object is invariant.

Example: equilateral triangle, starfish shape.

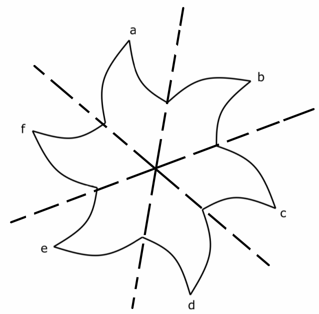
1 Groups

For the given starfish shape below:

1. determine the symmetries of this shape
2. construct the Cayley table for this shape

What is the main difference in symmetries of this starfish when compared to the symmetries of an equilateral triangle?

1.



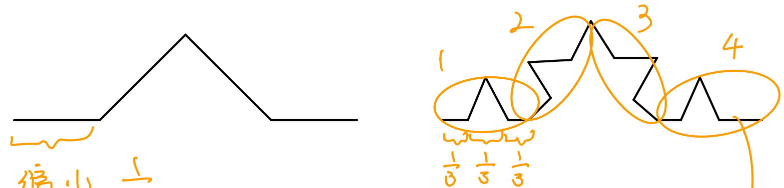
1. 60°, 120° 180° 240° 300° 360°

2.

	a	b	c	d	e	f
60°	b	c	d	e	f	a
120°	c	d	e	f	a	b
180°	d	e	f	a	b	c
240°	e	f	a	b	c	d
300°	f	a	b	c	d	e
360°	a	b	c	d	e	f

The main difference is the equilateral is invariant after flipping.

Fractal Dimension



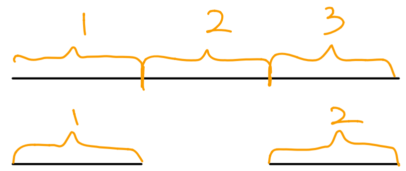
边缩小 $\frac{1}{3}$

scale by $\frac{1}{3}$ creating 4 pieces.

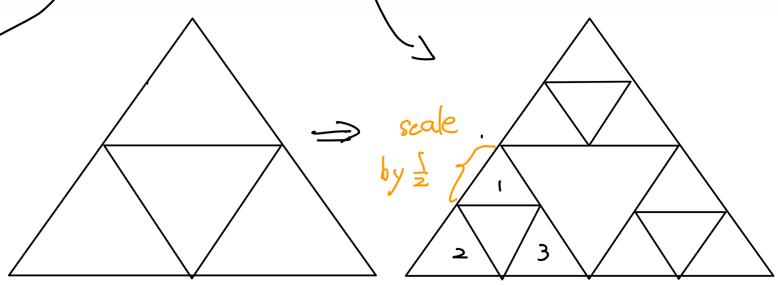
Show that the fractal dimension of the Koch curve is

$$D = \frac{\log 4}{\log 3} = 1.2619 \tag{1}$$

Likewise, compute the fractal dimensions of the Cantor set and the Sierpinski gasket. [Hint: See chapter 4.3 of PJS]



$\frac{\log 2}{\log 3}$.



scale by $\frac{1}{2}$

— — — —

$$\frac{\log 3}{\log 2}$$

We can use box counting algorithms to determine fractal dimensions.

1 Fourier Transform

Describe the Fourier transform (FT) and the physical principles used in constructing this transform, making sure to also include:

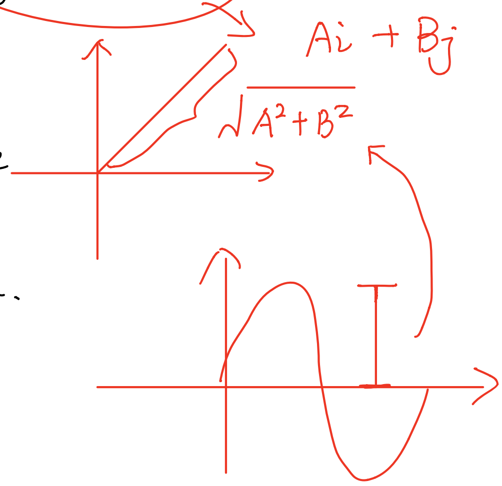
1. key equations of the transform
2. how the Fourier domain can be used to process the input signal
3. what type of signals would the FT be useful for
4. an example of its use

1. $$X[k] = \sum_{n=0}^{N-1} x[n] e^{-2\pi i n k / N}$$ → length of signal.

sin, cos

Discrete

Fourier transform projects data onto the unit circle. It maps harmonics to the data.



$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-2\pi i f t}\, dt$$ continuous.

2. Fourier domain has many advantage, including filtering and convolution. we can filter high frequency part of images or filter low frequency part of images, which can get the edge of the image.
The resulting Fourier space is a one stop shop for filtering and preprocessing operations, such as smoothing, downsampling and edge detection.

3. Any images or voice or some data can be tranfromed by "wave".
Fourier features are ideal for signals and data from physical systems.

4. example : MRI.

2 Principal Component Analysis

Describe Principal Component Analysis (PCA) and the mathematical principles used in constructing this transform, making sure to also include:

1. key equations of the transform
2. how the learned eigen-space can be used
3. what type of data would the PCA be useful for
4. an example of its use

1. $\tilde{x}_i = \sum_{i=0}^{K-1} b_i u_i$

2. An eigenspace is the collection of eigenvectors associated with each eigenvalue for linear transformation applied to the eigenvector.

3. high dimension data
correlation data

4. example : eigen-face.

Eigen-decomposition

compute the eigen-decomposition.

$$Au = \lambda u$$

A is a transformation matrix.

u is a vector called an eigenvetor

λ is a scalar value called eigenvalue.

If we slove the eigenvector u, we get the expression $(A - \lambda I)u = 0$

I is the identity matrix.

to compute eigen-decomposition

use singular value decomposition (SVD) algorithm that decomposes the matrix A as

$$A = UWV^T$$

W is a diagnoal matrix whose entries are singular value in descending order that correspond to the eigenvalues of the decomposition.

matrix U corresponds to the eigenvectors of the decomposition.

The PCA finds the most compact representation of our data.

1 Convolution

Describe the concept of convolution, making sure to include:

1. key equations and the fundamental concept behind it
2. differences between discrete and circular convolution
3. an example where it might be useful

1. Convolution is defined as the integral of the product of the two function after one is reversed and shifted.

   equation : $(f * g)(t) \triangleq \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$

   1. Express each function in term of a dummy variable τ
   2. Reflect one of the function $g(\tau) = g(-\tau)$
   3. add a time-offset, t, which allow $g(t-\tau)$ to slide along the τ-axis.
   4. start t at $-\infty$ and slide it all the way to $+\infty$. Wherever the two function intersect. find the integral of their product.

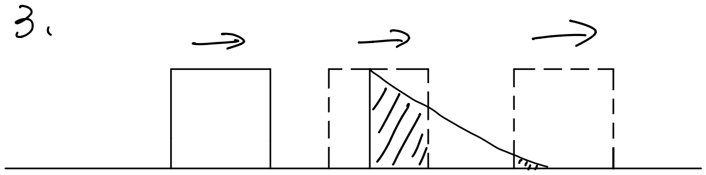
2. ① circular convolution $y[n]$ contain the same number of samples as that of $x[n]$ and $h[n]$ (zero padding)

   But in discrete convolution, the number of samples in $x[n]$ and the number of samples in $h(n)$ need not be the same.

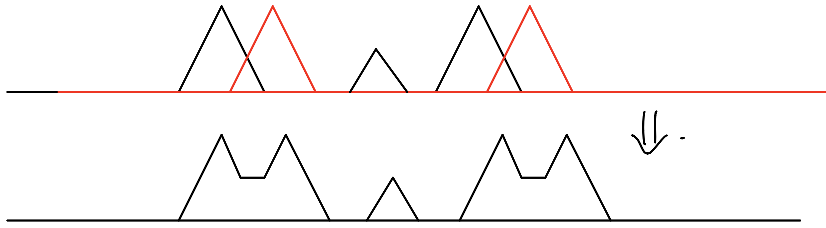
   ② Discrete Convolution : linear shift, result length: $N_1 + N_2 - 1$
   the functions f and g are not periodic.

   Circular Convolution : Circular shift, result length: $\mathrm{Max}(N_1, N_2)$ zero padding
   one of the function f or g is periodic.

3.



linear convolution



circular convolution.

2 Discrete Convolution

If $x[n] = [1,1,1]$ and $h[n] = [1,1,1]$ and $y[n] = x[n] * h[n]$, where $*$ is the discrete convolution

1. what is the length of $y[n]$?
2. Calculate $y[n]$

1. length of $y[n]$ is $3 + 3 - 1 = 5$

2.

```
        1 1 1
    1 1 1
      1 1 1
        1 1 1
          1 1 1
            1 1 1
```

$y[n] = [1, 2, 3, 2, 1]$

$y[n] = [1, 2, 3, 2, 1]$

若 circular :

$[1, 2, 3, 2, 1]$

$+ \quad +$

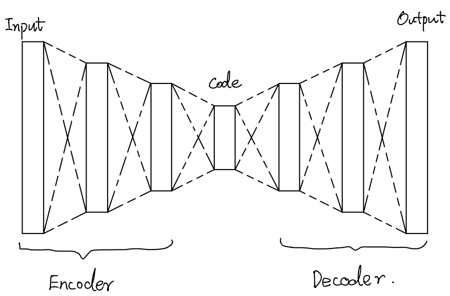
$[3, 3, 3]$.

1 Autoencoders

Describe the concept of the autoencoder neural network, making sure to include descriptions of

1. its network architecture as a schematic
2. the goal(s) of such a network
3. its main components
4. how these components can be used separately
5. an example where it might be useful

1.



2. An autoencoder is a type of artifical neural network used to learn efficient data coding in an unsupervised manner
The aim of an autoencoder is to learn a representation (encoding) for a set of data, typically for dimensionality reduction, by training the network to ignore signal "noise".

3. There are three components in Autoencoder. They are Encoder, Decoder, and code.

4. Encoder: In which the model learns how to reduce the input dimensions and compress the input data into an encoded representation

Code: which is the layer that contains the compressed representation of the input data. This is the lowest possible dimension of the input data.

Decoder: In which the model learns how to reconstruct the data from the encoded representation to be as close to the original input as possible.

Reconstruction Loss: This is the method that measures measure how well the decoder is performing and how close the output is to the original input.

5. Dimensionality Reduction. Denoisy

Relationship with PCA. Anomaly Detection

2 Convolutional Neural Networks

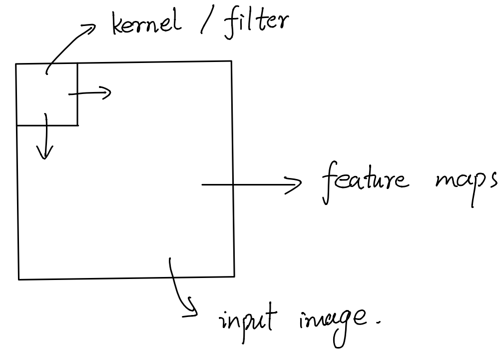
A convolutional neural network (CNN) is a very powerful type of network for pattern recognition problems.

(a) List and describe the main components of a convolutional layer within a CNN.

(b) When is a CNN likely to be useful in pattern recognition problems and give an example.

(c) Draw a flow chart/graph (making sure to include each layer) of a typical CNN for solving such a problem in your example above.

(a)



Filters:

The filters are the "neurons" of the layer. They have input weights and output a value. The input size is a fixed square called a patch or a receptive field.

Feature Maps:

The Feature map is the output of one filter applied to the previous layer.

(b) Use CNNs For:

- Image data
- classification prediction problems.
- Regression prediction problems.

(c),